Calibration and Testing of a Daily Rainfall Erosivity Model

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Abstract

A general procedure is presented for calibrating a model for rainfall erosivity based on daily rainfall. The approach is based on probability distributions of wet-day precipitation amount and monthly erosivities which are inferred from published data summaries. The calibrated model was tested by comparisons with erosivities computed from hourly precipitation data. Model results were generally consistent with values based on hourly data and explained over 85% and 70%, respectively, of the variations in annual and event erosivities. Model results for extreme values (annual erosivities exceeded in 5% of the years and 1-in-20-year event erosivities) often substantially exceeded values computed for hourly data. To facilitate general use of the daily model, calibration coefficients were calculated for 33 sites in the eastern and central U.S.

Introduction

A general equation for estimating the erosivity term in the Universal Soil Loss Equation from daily rainfall data was proposed by Richardson et al. (1983). The equation provides a simple model for calculating event soil losses from daily, rather than hourly, precipitation data. Since daily weather records are more commonly available than hourly records, the equation is potentially a valuable tool for erosion, sediment yield, and nonpoint source pollution studies. It was subsequently tested by Haith and Merrill (1987) for 23 locations in the eastern and central U.S. Long-term synthetic daily rainfall records were generated at each location, and these were used in the Richardson et al. model to compute erosivities. The testing involved comparisons of these model results with rainfall erosivities reported by Wischmeier and Smith (1978).

Although the results of the comparisons were generally favorable, they were not conclusive. The use of synthetic weather introduced an additional level of uncertainty because discrepancies between the two sets of erosivities may have been associated with defects in the weather generating procedure. Difficulties were also apparent in the estimation of parameters for the Richardson et al. model. The two required coefficients were available only for the 11 original Richardson et al. sites. Coefficients for other locations were arbitrarily assigned the values of the closest of the 11 original sites.

This article describes a more complete testing of the erosivity model. The objectives of the study were to develop a general procedure for calibrating the model to any U.S. location and to test the calibrated model for selected locations in the eastern and western U.S., respectively. Although the testing procedures were similar to those used in Haith and Merrill (1987), model erosivities were computed using historic daily weather records rather than synthetic records.

Calibration Methods

The erosivity model given by Richardson et al. (1983) is the regression equation:

$$\mathrm{Log}_{10}\,EI_t = \mathrm{Log}_{10}a + 1.81\,\mathrm{Log}_{10}R_t + \epsilon \tag{1}$$

or equivalently,

$$EI_t = a\,10^{\epsilon}\,R_t^{\,1.81} \tag{2}$$

with lower and upper bounds EI_{min} and EI_{max}, respectively:

$$EI_{min} = R_t^{\,2}\left[0.00364\,\mathrm{Log}_{10}(R_t) - 0.000062\right] \tag{3a}$$

$$EI_{max} = R_t^{\,2}\left[0.291 + 0.1746\,\mathrm{Log}_{10}(R_t)\right] \quad \text{if } R_t \le 38 \tag{3b}$$

$$EI_{max} = 0.566\,R_t^{\,2} \quad \text{if } R_t > 38 \tag{3c}$$

where

EI_t = daily rainfall erosivity on day t (MJ-mm/ha-hr),
a = seasonal erosivity coefficient,
ϵ = normally distributed random variable with mean zero and standard deviation 0.34, and
R_t = rainfall amount on day t (mm).

The coefficient **a** is given by two values, a_w and a_c, where a_w is used for the warm months of April through September, and a_c is used for the cool months of October through March. The random term ϵ, which corresponds to the ϵ' variable in Richardson et al. (1983), is a residual or error term for the regression equation.

The lower and upper bounds on EI_t given by equations 3a-3c limit erosivity to physically realistic values. The lower bound EI_{min} corresponds to a minimum rainfall intensity case in which R_t is distributed over 24 hours. Conversely, EI_{max} is produced when R_t occurs in a single

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half-hour period (Richardson et al., 1983).

The erosivity model can be calibrated for U.S. locations by appropriate selection of the coefficients a_w and a_c. The calibration procedure is based on information published in Agriculture Handbook 537 (Wischmeier and Smith, 1978). The handbook provides mean annual erosivities for the U.S. over the 22-year period, 1937-1958. The expected monthly fractions of mean annual erosivity for various regions are also provided. These fractions can be multiplied by mean annual erosivity to obtain ER_m, the mean erosivity in month m (MJ-mm/ha-hr). The Wischmeier and Smith English units are converted to SI units by: 100 ft-ton-in/ac-hr = 17.0195 MJ-mm/ha-hr.

The expected monthly erosivity can also be approximately estimated using equation 2. Assuming that in any given month the daily mean precipitation is constant throughout the month, the expected erosivity is:

$$ER_m' = d_m E\left(a_m\, 10^{\epsilon}\, P_m^{\ 1.81}\right) \tag{4}$$

where

d_m = number of days in month m,
P_m = precipitation on any day in month m (mm),
a_m = monthly value of **a**, and
E = expectation operator.

Equation 4 is an approximation since the upper and lower bounds of EI are neglected and rainfall has been replaced by precipitation (which may include both rain and snow).

The random variables $\in$ and P_m are independent and equation 4 may be rewritten as:

$$ER_m' = d_m\, a_m\, E\left(10^{\epsilon}\right) E\left(P_m^{\ 1.81}\right) \tag{5}$$

The exponent $\in$ is normally distributed and hence $10^{\in}$ is lognormally distributed. The expectation of a base ten lognormal random variable is given by Hald (1952) as:

$$E\left(10^{\epsilon}\right) = 10^{\mu}\, 10^{\left(\sigma^2/2\mathrm{Log}_{10}e\right)} = 1.359 \tag{6}$$

where μ and σ^2 are the mean and variance of the normally distributed random variable (0 and 0.1156, respectively).

The erosivity parameter a_m is obtained by setting ER_m equal to ER_m' and solving:

$$a_m = \frac{ER_m}{1.359\, d_m\, E\left(P_m^{\ 1.81}\right)} \tag{7}$$

The seasonal coefficients a_w and a_c are given by weighting the monthly values by ER_m:

$$a_w = \frac{\sum_{m=\mathrm{Apr}}^{\mathrm{Sept}} a_m ER_m}{\sum_{m=\mathrm{Apr}}^{\mathrm{Sept}} ER_m} \qquad a_c = \frac{\sum_{m=\mathrm{Oct}}^{\mathrm{Mar}} a_m ER_m}{\sum_{m=\mathrm{Oct}}^{\mathrm{Mar}} ER_m} \tag{8}$$

The expected value in equation 7 can be determined from the unconditional probability distribution of daily precipitation:

$$F_m * (p) = \mathrm{Prob}\left\{P_m \le p\right\} \tag{9}$$

in which p is a particular value of precipitation (mm). Although this distribution is generally not available, it can be determined from the more commonly used conditional distribution of wet-day precipitation amount:

$$F_m(p) = \mathrm{Prob}\left\{P_m \le p \mid P_m > 0\right\} \tag{10}$$

If w_m is the probability of a wet day in month m, then:

$$\begin{aligned} F_m * (p) &= (1 - w_m) + w_m F_m(p) \\ &= w_m\left[F_m(p) - 1\right] + 1 \end{aligned} \tag{11}$$

Letting $f_m{*}(P)$ be the density function corresponding to $F_m{*}(P)$, then:

$$E\left(P_m^{\ 1.81}\right) = \int_0^{\infty} p^{1.81} f_m^{\ *}(p)\, dp \tag{12}$$

Wet-Day Precipitation Probability Distributions

The calibration procedure requires two probability measures. The wet-day probability w_m for any month is given by the average number of wet days divided by the number of days in the month. The conditional, or wet-day probability distribution of precipitation, $F_m(p)$ can be estimated from analysis of daily precipitation records. However, if a single-parameter distribution is assumed, the distribution can be obtained from μ_m the mean wet-day precipitation in month m (mm). These monthly means are computed from precipitation summaries such as *Climates of the States* (National Oceanic and Atmospheric Administration, 1985) and *Statistical Abstract of The United States* (Bureau of the Census, 1982) by dividing mean monthly precipitation by mean number of wet days for any month. Here we evaluate the integral in equation 12 and the resulting a_m for three of these single-parameter distributions.

Exponential Distribution

The exponential distribution is probably the most widely used, single-parameter distribution of daily precipitation amount (Todorovic and Woolhiser, 1974; Richardson, 1981; Pickering et al., 1988). It is given by:

$$F_m(p) = 1 - e^{-p/\mu_m} \tag{13}$$

The associated unconditional distribution is given by equation 11 as:

$$F_m^{\ *}(p) = 1 - w_m\, e^{-p/\mu_m} \tag{14}$$

and the density function is:

$$f_m^{\ *}(p) = (w_m/\mu_m)\, e^{-p/\mu_m} \tag{15}$$

Substituting into equation 12, we obtain:

$$E\left(P_m^{\ 1.81}\right) = w_m \int_0^{\infty} \left(p^{1.81}/\mu_m\right) e^{-p/\mu_m}\, dp \tag{16}$$

With the change of variables $x = p/\mu_m$:

$$E\left(P_m^{1.81}\right) =$$

$$w_m \mu_m^{1.81} \int_0^\infty x^{1.81} e^{-x}\, dx \qquad (17)$$

$$= w_m \mu_m^{1.81} \Gamma(2.81)$$

where $\Gamma(\bullet)$ is the gamma function. With evaluation of the gamma function, $\Gamma(2.81) = 1.691$, equation 7 becomes:

$$a_m = \frac{ER_m}{2.298\, d_m w_m \mu_m^{1.81}} \qquad (18)$$

Beta-P Distribution

Several authors have observed that the exponential distribution under-predicts the number of large storm events (Haith et al., 1984; Skees and Shenton, 1974; Mielke and Johnson, 1974). A single-parameter distribution which has shown significantly better extreme value performance is the calibrated beta-P distribution proposed by Pickering et al. (1988):

$$F_m(p) = 1 - \left(1 - p/\alpha_m\right)^{-10} \qquad (19)$$

where
$\alpha_m = -9\,\mu_m$.

Employing the same arguments as before, the unconditional density function for precipitation is:

$$f_m{}^*(p) = -\left(10\, w_m/\alpha_m\right)\left(1 - p/\alpha_m\right)^{-11} \qquad (20)$$

and

$$E\left(P_m^{1.81}\right) =$$

$$-w_m \int_0^\infty p^{1.81}\left(10/\alpha_m\right)\left(1 - p/\alpha_m\right)^{-11} dp \qquad (21)$$

Letting $x = -p/\alpha_m$ and rearranging terms we find:

$$E\left(P_m^{1.81}\right) =$$

$$10\, w_m(-\alpha_m)^{1.81} \int_0^\infty x^{1.81}\,(1+x)^{-11}\, dx \qquad (22)$$

The integral in equation 22 was calculated numerically to be 0.003452, and replacing -9 μ_m for α_m produces:

$$E\left(P_m^{1.81}\right) = 1.842\, w_m \left(\mu_m\right)^{1.81} \qquad (23)$$

and

$$a_m = \frac{ER_m}{2.503\, d_m W_m \mu_m^{1.81}} \qquad (24)$$

Weibull Distribution

Selker (1989) calibrated a Weibull distribution for wet-day precipitation using summary weather data for 33 U.S. sites east of the Rocky Mountains. The distribution is:

$$F_m(p) = 1 - \exp\left[-1.191\left(p/\mu_m\right)^{0.75}\right] \qquad (25)$$

As before,

$$f_m^*(p) = 0.8933\, w_m \left(p^{-0.25}/\mu_m^{0.75}\right) \exp\left[-1.191\left(p/\mu_m\right)^{0.75}\right] \qquad (26)$$

and

$$E\left(P_m^{1.81}\right) =$$

$$0.8933\, w_m \int_0^\infty p^{1.81}\, w_m \left(p^{-0.25}/\mu_m^{0.75}\right) \exp\left[-1.191\left(p/\mu_m\right)^{0.75}\right] dp \qquad (27)$$

With the change of variables $x = 1.191\,(p/\mu_m)^{0.75}$:

$$E\left(P_m^{1.81}\right) = 0.6558\, w_m \mu_m^{1.81} \int_0^\infty x^{2.413} e^{-x}\, dx$$

$$= 0.6558\, w_m \mu_m^{1.81} \Gamma(3.413)$$

$$= 1.983\, w_m \mu_m^{1.81} \qquad (28)$$

and

$$a_m = \frac{ER_m}{2.695\, d_m w_m \mu_m^{1.81}} \qquad (29)$$

Calibration Comparisons

The three probability distributions of wet-day precipitation are shown in figure 1. Although the distributions have similar shapes, they differ substantially in their skewness, or the probabilities of very large precipitation amounts. The probabilities of such events are highest for the Weibull and lowest for the exponential.

The general form of the calibration equation for all three distributions is:

$$a_m = \frac{K\, ER_m}{d_m w_m \mu_m^{1.81}} \qquad (30)$$

in which K is a constant with the values 0.435, 0.400, and 0.371 corresponding to assumptions of exponential beta-P and Weibull wet-day precipitation distributions. Although the exponential assumption produces the most conservative (largest) erosion estimate, these values are only 17% larger than those produced by the least conservative (Weibull) assumption.

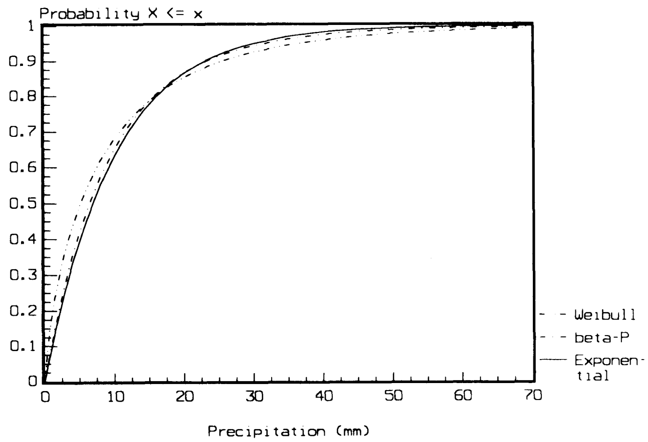

Figure 1–Comparison of Exponential, Beta-P, and Weibull wet-day precipitation probability distributions for mean precipitation = 10mm.

MODEL TESTING

Model testing involved comparisons of rainfall erosivities computed by Wischmeier and Smith (1978) from 1937-1958 hourly precipitation data with those obtained from the use of daily precipitation data in the calibrated Richardson et al. (1983) model (eq. 2 and 3).

SIMULATION PROCEDURE

Daily temperature and precipitation records for the 1937–1958 period were obtained for the 12 eastern sites and 7 western sites listed in Table 1. Complete 22-year records were available for all sites except Albany, NY, Binghamton, NY, and Portland, ME. For these three locations, records for 1939–1958, 1941–1958, and 1938–1958, respectively, were used.

Monthly erosivity coefficients a_m were calculated for each of the three precipitation distributions (exponential, beta-P, and Weibull) as given by equations 18, 24 and 29, respectively. The seasonal coefficients a_w (Apr. – Sept.) and a_c (Oct. – Mar.) were determined as in equation 8 and are listed in Table 1.

The weather records were used in a daily Monte Carlo simulation which calculated erosivity for each day in which rainfall exceeded 13 mm (0.5 in.). This threshold was also used by Wischmeier and Smith in their hourly computations. Precipitation was assumed to be rain in any day in which temperature exceeded 0° C. The random term (ϵ) in the erosivity model was sampled from the appropriate normal distribution and erosivity was calculated using equations 2 and 3. Simulations were repeated 10 times with different random sequences of ϵ for the sixteen, 22-year and two, 21-year records for a total of 220 and 210 years, respectively. The 18-year simulation for Binghamton, NY was repeated 12 times for a total of 216 years. These simulation periods assured that estimates of 20-year events would be based on at least 10 values.

TABLE 1. Erosivity coefficients for testing sites

Eastern cites	Erosivity coefficients* a_w	a_c	Western cites	Erosivity coefficients* a_w	a_c
Albany, NY	0.233	0.060	Albuquerque, NM	0.217	0.112
Binghampton, NY	0.190	0.062	Cheyenne, WY	0.265	0.050
Boston, MA	0.203	0.085	Miles City, MT	0.296	0.044
Buffalo, NY	0.230	0.050	Portland, OR	0.076	0.047
Burlington, VT	0.187	0.077	Red Bluff, CA	0.238	0.124
Concord, NH	0.183	0.106	Roswell, NM	0.321	0.151
Harrisburg, PA	0.247	0.074	Spokane, WA	0.120	0.017
Portland, ME	0.229	0.076			
Reading, PA	0.260	0.091			
Rochester, NY	0.224	0.065			
Scranton, PA	0.196	0.085			
Syracuse, NY	0.243	0.068			

* a_c: Cool season (Oct. – Mar.); a_w: warm season (Apr. – Sept.). Coefficients are based on Weibull precipitation distribution. To obtain coefficients based on exponential and beta-P, multiply by 1.173 and 1.078, respectively.

COMPARISON STATISTICS

Model testing criteria included both annual and event erosivities. Annual erosion estimates are often appropriate for soil conservation and reservoir sedimentation studies. Conversely, nonpoint source pollution is generally associated with individual storm events. Comparison statistics included mean and 95% probability levels of annual erosivities and 1-in-N-year event erosivities for N = 5 and 20. The 95% annual erosivities are values which would be expected to be exceeded in 5% of the years. A 1-in-N-year event is expected to occur on an average of once in N years.

The definition of an erosivity "event" is somewhat arbitrary. The Wischmeier and Smith (1978) results are computed from consecutive hourly storm precipitation, but the use of daily records requires "events" based on total rainfall during a single day. This is only an approximation to the Wischmeier and Smith values because the recorded daily precipitation may have been produced by more than one storm or by only a portion of a storm which lasted more than one day.

TESTING RESULTS

The simulations produced single estimates of the four erosivity values (mean and 95% quantiles of annual erosivity, 1-in-5-year and 1-in-20-year event erosivity) at each site. Errors are measured as the percentage by which model estimates differed from the corresponding Wischmeier and Smith (1978) values. Mean errors are given in Table 2. Model estimates tended to exceed the Wischmeier and Smith data for both annual and event erosivities at the 12 eastern locations. Overestimates were most severe with the 1-in-20-year events. Model coefficients calibrated from the Weibull precipitation distribution produced substantially lower mean errors. Model errors were generally lower for the western sites but no distribution produced consistently superior results.

Model results for annual erosivities with coefficients based on the Weibull distribution are compared with Wischmeier and Smith values for all 19 sites in figures 2 and 3. In these and subsequent figures a hypothetical line of perfect fit of the two erosivities is also shown. Except for a tendency to overestimate the larger 95% annual erosivities, the calibration procedure produced annual erosivities which corresponded well to the Wischmeier and Smith values. Weibull-based event erosivities are compared in figures 4 and 5. Since, in some cases, the exponential distribution produced better results, these are also shown in

TABLE 2. Mean errors in erosivity estimates with model coefficients based on exponential, beta-P, and Weibull precipitation distributions

	Mean Error (%)					
	Eastern Sites			Western Sites		
	Expo – nential	Beta-P	Weibull	Expo – nential	Beta-P	Weibull
Mean annual erosivity	16	7	0	1	-6	-12
95% quantile annual erosivity	28	18	11	14	7	1
1-in-5-yr event erosivity	18	10	3	-6	-13	-18
1-in-20-yr event erosivity	60	48	38	13	4	-3

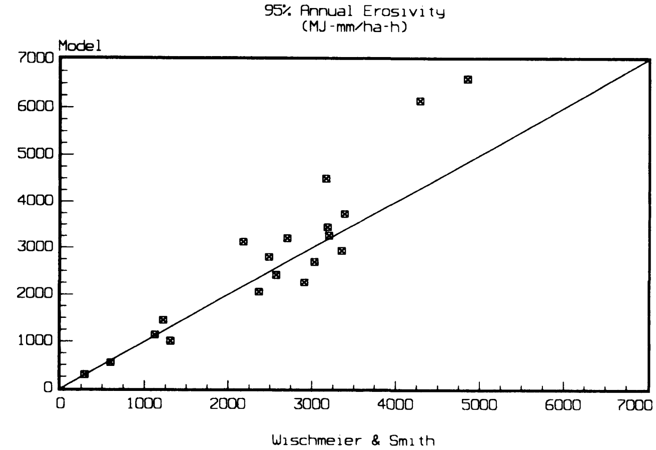

Figure 3–Comparison of Wischmeier and Smith (1978) 95% annual erosivities with model estimates for coefficients based on the Weibull distribution.

figures 6 and 7. Although the event estimates are not as accurate as the annual values, they do match the Wischmeier and Smith values fairly well for the 1-in-5-year events. Both distributions produce coefficients that substantially overpredict many of the 1-in-20-year events. However, since the Wischmeier and Smith values were based on only a single 22-year record, and "events" are measured differently in the two sets of results, these differences are not surprising.

The coefficients of determination, or R^2 values in Table 3 measure the fraction of variation in observed (Wischmeier and Smith) values explained by the model results over all 19 sites. Since the model coefficients for each distribution differ from each other by only a constant, the R^2 values are essentially identical for the three distributions. Even with the least accurate 1-in-20-year estimates, the calibrated model explains over 70% of the observed erosivity variations.

OVERVIEW

Although the testing results confirm that erosivity estimates based on daily rainfall records will not duplicate estimates based on hourly records, the calibrated Richardson et al. (1983) daily model produces estimates that are consistent with the values produced by Wischmeier and Smith's (1978) hourly computations. The model estimates capture most of the variation of the hourly results, and errors tend to be on the conservative side; i.e., model estimates are generally larger than erosivities produced from hourly records.

A better match between model estimates and Wischmeier and Smith results for extreme (1-in-20-year) events could be obtained by reducing the values of the calibration parameter a. However, since the erosivity model is linear with respect to the parameter, this would result in a poorer fit of mean erosivities.

To facilitate general use of the Richardson et al. (1983) daily erosivity model, the model coefficients a_c and a_w were calculated for 33 sites in the eastern and central U.S. using Equation 30. These sites correspond to the 33 geographic areas for which Wischmeier and Smith (1978) determined seasonal erosivities. The wet-day probabilities and means w_m and μ_m, respectively, were obtained from National Oceanic and Atmospheric Administration (1985). The coefficients listed in Table 4 are based on the Weibull precipitation distribution because that distribution

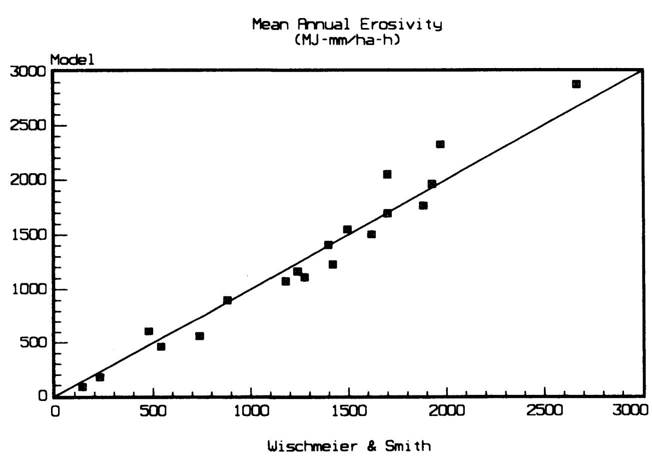

Figure 2–Comparison of Wischmeier and Smith (1978) mean annual erosivities with model estimates for coefficients based on the Weibull distribution.

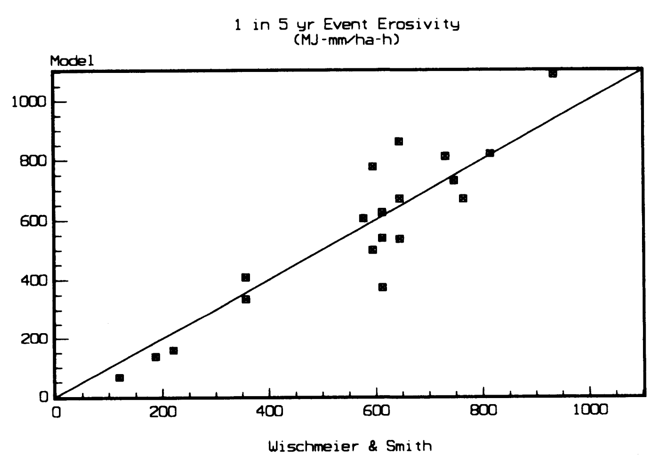

Figure 4–Comparison of Wischmeier and Smith (1978) 1-in-5-year event erosivities with model estimates for coefficients based on the Weibull distribution.

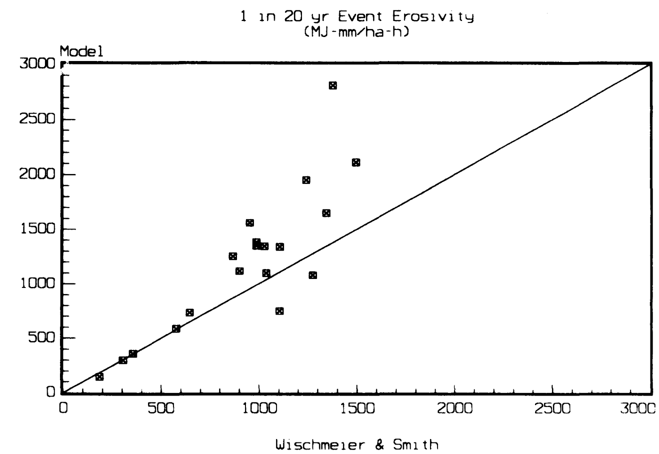

Figure 5–Comparison of Wischmeier and Smith (1978) 1-in-20-year event erosivities with model estimates for coefficients based on the Weibull distribution.

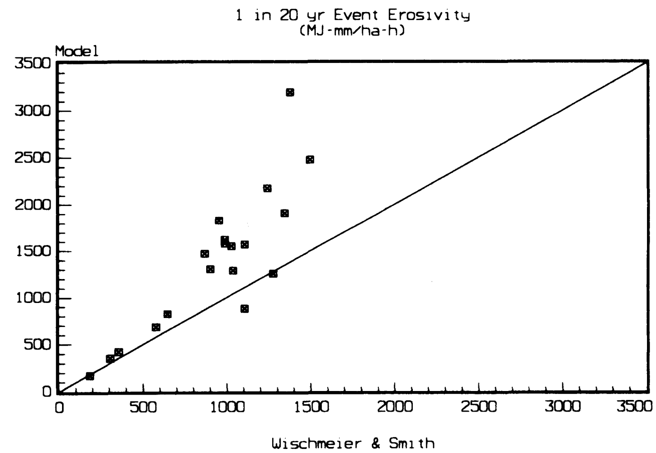

Figure 7–Comparison of Wischmeier and Smith 1(1978) 1-in-20-year event erosivities with model estimates for coefficients based on the exponential distribution.

generally produced the most accurate results for the eastern U.S.

CONCLUSIONS

This article has presented a general calibration procedure which greatly extends the general applicability of the Richardson et al. (1983) model for rainfall erosivity based on daily rainfall. The approach requires knowledge of the probability distribution of wet-day precipitation amount and monthly erosivities which can be inferred from published data summaries. Monthly erosivities are available from Wischmeier and Smith (1978) and analytical expressions for model parameters requiring only mean monthly precipitation statistics were presented in this paper for three general single-parameter precipitation distributions: exponential, beta-P and Weibull.

The calibrated Richardson et al. (1983) model was tested by comparisons of rainfall erosivities computed by Wischmeier and Smith (1978) from 1937-1958 hourly precipitation data with those obtained from running the model with daily weather data from this same period. Model results were generally consistent with Wischmeier and Smith values and explained over 85% and 70%, respectively, of annual and event erosivities. However, model results for extreme values (annual erosivities exceeded in 5% of the years and 1-in-20-year event erosivities) often substantially exceeded the comparable Wischmeier and Smith values.

Model coefficients based on the Weibull precipitation usually produced the best results. However, since coefficients based on other distributions differed from the Weibull values by less than 17%, the exact form of the precipitation distribution does not appear critical.

There are inherent limitations in the use of daily weather records for estimating the rainfall erosivity term in the Universal Soil Loss Equation. Erosivity includes kinetic energy and intensity measures which are poorly represented by daily rainfall values. Although the calibration methods could be further refined to better match the Wischmeier and Smith calculations, they would not be likely to duplicate the accuracy obtained from hourly records.

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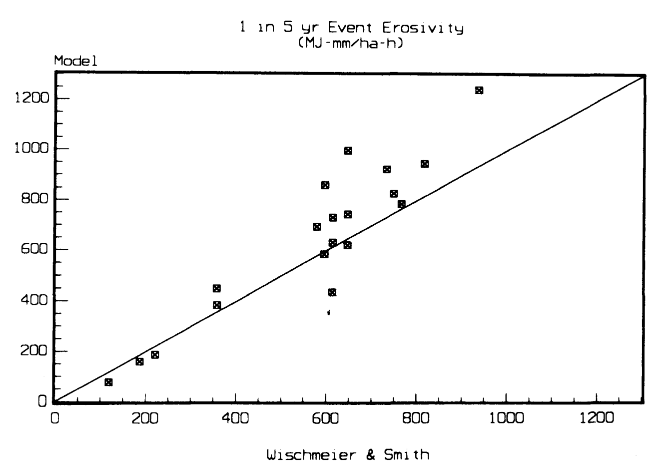

Figure 6–Comparison of Wischmeier and Smith (1978) 1-in-5-year event erosivities with model estimates for coefficients based on the exponential distribution.

TABLE 3. Coefficients of determination of erosivities with model coefficients based on exponential, beta-P, and Weibull precipitation distribution

	Coefficient of Determination (R^2)		
	Exponential	Beta–P	Weibull
Mean annual erosivity	0.961	0.960	0.960
95% quantile annual erosivity	0.866	0.864	0.865
1-in-5-yr event erosivity	0.858	0.847	0.844
1-in-20-yr event erosivity	0.735	0.727	0.726