DIGITAL INVESTIGATION OF SWITCHING TRANSIENTS FROM AN IMPEDANCE SHUNTED CIRCUIT-BREAKER

by

PETER HARVEY

A THESIS

submitted to

OREGON STATE UNIVERSITY

in partial fulfillment of
the requirements for the
degree of

MASTER OF SCIENCE

June 1962

APPROVED:

Redacted for privacy

Professor of Electrical Engineering Department

In Charge of Major

Redacted for privacy

Head of Electrical Engineering Department

Redacted for privacy

Chairman of School Graduate Committee

Redacted for privacy

Dean of Graduate School

Date thesis is presented November 14, 1961

Typed by Sheila Harvey

ACKNOWLEDGEMENT

The author wishes to express his gratitude to Professor J. F. Engle for his inspiration, suggestions and guidance.

TABLE OF CONTENTS

FIGURES

DIGITAL INVESTIGATION OF SWITCHING TRANSIENTS FROM AN IMPEDANCE SHUNTED CIRCUIT-BREAKER

INTRODUCTION

The theoretical operation of an alternating current circuit-breaker is basically quite simple. At a random time in the cycle the contact points separate. The inductive elements in the circuit will sustain an arc between the contacts until the current reaches a zero point in the cycle. The arc is then extinguished and the gap between the contact points is insulated against the resultant rise in voltage by the forced entry of a dielectric medium - usually oil or air.

The chief complication is that the arc creates a conducting ionized path through the dielectric medium and no matter how ingenious the design of the breaker, it is difficult to remove all the arc products at each interruption. If the separation of the contact points occurs just before current zero, there will be few arc products but the points will still be relatively close together at the natural current zero. Conversely, if there is large separation, there will be a large accumulation of arc products.

Howatson (1, p.12) has shown that there is an optimum time in the current cycle to initiate the break, in order to obtain a balance between the separation of the points and the arc products left in the gap. He also

found the rate of contact separation to be subordinate to the rate of rise of recovery voltage, which is perhaps the most critical factor in circuit-breaker operation.

If the insulating properties of the gap do not build up at a faster rate than the recovery voltage across the gap, the gap will break down. This will cause a resumption or re-strike of the arc with consequent high voltage and current transients throughout the circuit. These may prove injurious to the breaker and associated equipment, and in an extreme case of repeated re-strikes, the circuit-breaker could be destroyed.

In applications where it is feared that the external circuits will cause a high rate of rise of recovery voltage, resistance (3, p.705) may be switched across the gap to decrease this rate of rise of recovery voltage. The operation is done either mechanically or electrically by diverting the arc through a resistor. The resistor is removed from the circuit a few cycles later by a further switching operation.

In direct current relay circuits, it is an established practice to put a resistor in series with a capacitor across the gap. These will absorb and dissipate the energy from the inductive elements of the circuit and reduce the magnitude of the over-voltage and the duration of the arc.

The purpose of this thesis is to study the effect of a resistor and capacitor in series across the gap of an alternating current circuit-breaker. The recovery voltage curves are calculated and compared for various resistance-capacitance combinations.

METHOD OF INVESTIGATION

Rather than set up and test an actual circuit, a mathematical analysis was chosen for several reasons:

(1) The lack of facilities for accurate synchronization of the start of the break with the phase angle of the current.

(2) On severe recovery voltage rises, an actual breaker would re-strike; probably before its essential rate of rise curve could be obtained.

(3) The capricious behaviour of circuit-breakers and dielectric breakdown characteristics lend themselves only to statistical averages of performance. This would lead to an excessive number of tests, if many circuit configurations were to be investigated.

Because of the complexity of the circuit and number of cases to be investigated, the problem was written up as a computer program for the Alwac III-E computer at Oregon State University.

Figure 1 is the circuit that was investigated. Its configuration is a simple approximation to actual circuits encountered in the field.

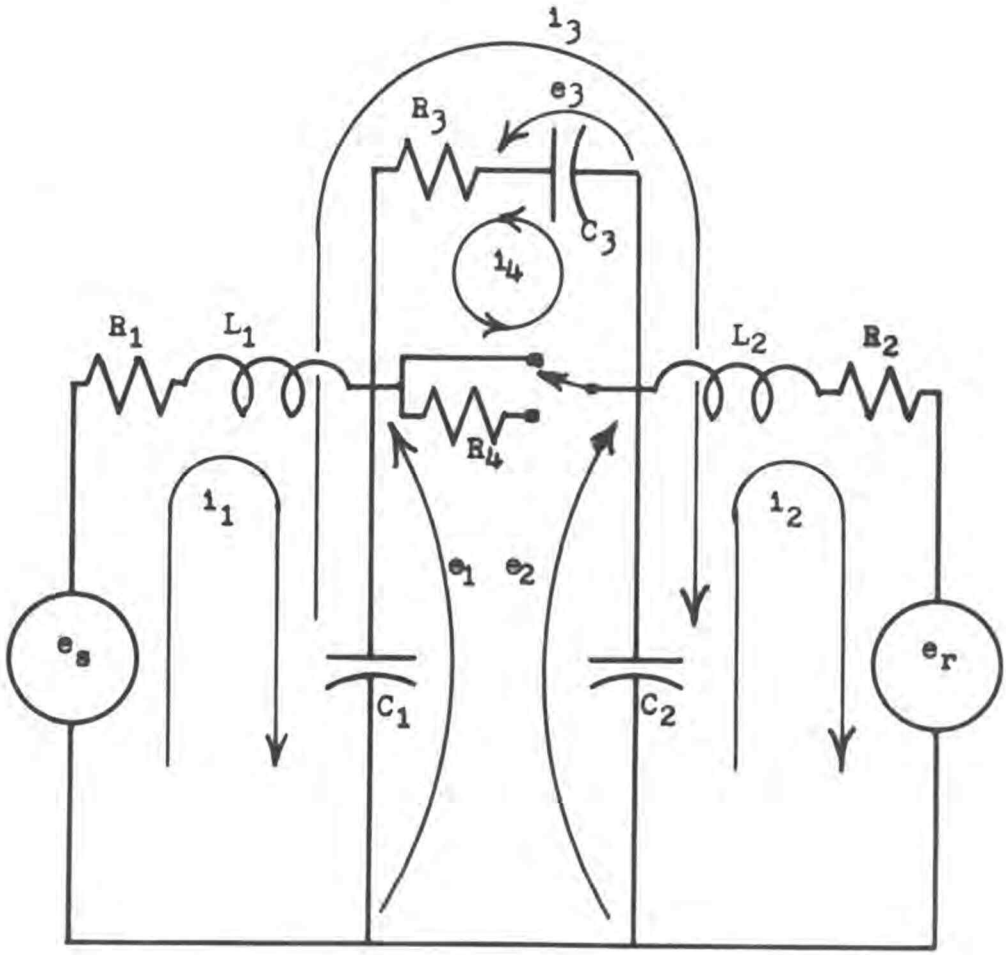


Figure 1. Circuit with simulated circuit-breaker.

The simulated breaker gap is represented by R_4, which is varied in discrete steps, $R_4 = 0$ for the breaker closed and a very high value ($R_4 = 10^6$ ohms) for the breaker open. Ideal operation is assumed by allowing the break to be initiated at current zero and by having an infinitely fast separation of the contact points. All arc-drop phenomena and re-strike complications are thereby eliminated. It was also assumed that steady state conditions prevailed before the breaker opened.

The program can accommodate restrikes by substituting a third value of R_4 which would approximately represent the voltage drop across the arc. Its insertion would depend upon the contact separation and the recovery voltage. In order to keep the investigation general and to conserve computer time, it was decided not to include restrikes, since each type of circuit-breaker would have its own breakdown characteristics.

The sending end and receiving end voltages are of the form

$$e_s = E_{s\ max} \cos \omega T$$
$$e_r = E_{r\ max} \cos(\omega T + \theta) \qquad (1)$$

where $\omega = 120\pi$ radians/sec (synchronous frequency) and θ is the displacement angle of the receiving end voltage in radians.

The sending end parameters are R_1, L_1 and C_1 and the receiving end parameters are R_2, L_2 and C_2.

The resistor and capacitor shunting of the breaker contact points is done by R_3 and C_3.

The loop currents are i_1, i_2, i_3 and i_4, with e_1, e_2 and e_3 being the voltages across the capacitors.

The voltage across the breaker contacts is e_1-e_2 and the plot of its magnitude versus time is the recovery voltage curve of the breaker.

The loop equations from Figure 1 are:

$$\begin{aligned} e_s &= L_1 i_1' + R_1 i_1 + e_1 \\ e_r &= -L_2 i_2' - R_2 i_2 + e_2 \\ 0 &= -e_1 + e_2 + e_3 + R_3 i_3 - R_3 i_4 \\ 0 &= -e_3 - R_3 i_3 + (R_3 + R_4) i_4 \end{aligned} \tag{2}$$

The prime symbol represents the order of the derivative and

$$\begin{aligned} e_1 &= \frac{1}{C_1} \int_0^t (i_1 - i_3) + e_1(0) \\ e_2 &= \frac{1}{C_2} \int_0^t (i_3 - i_2) + e_2(0) \\ e_3 &= \frac{1}{C_3} \int_0^t (i_3 - i_4) + e_3(0) \end{aligned} \tag{3}$$

where e(0) is the voltage across the capacitor at time 0.

The method of solution of the differential equations (sets 2 and 3) follows the "classical" (4, p.179) rather than the "operational" approach. This approach was chosen because of the ability of a computer to handle systems of linear equations and the fact that there were many intermediate steps which could be checked. This facilitated program "debugging" and was also a check upon the proper operation of the computer. The final print-out would give all values of voltage and current if desired, at arbitrary intervals of time, Δt. A detailed explanation of the mathematical method used for the solution of the equations is given in the Appendix.

Figure 2. is the flow chart of the computer program. After accepting the circuit parameters and other input data, the constants of the characteristic polynomial are calculated. A sub-routine using Newton's Method is called in to find the roots of this polynomial. When found, they are tested to see if there are one real and two pairs of complex roots of the form

$$a_1, \quad a_2 \pm jb_2, \quad a_3 \pm jb_3,$$

as this is the only combination of roots that the program was designed to handle. Otherwise an alarm is given and the computer stops. It was assumed from the beginning that the circuit parameters chosen would have roots of this form.

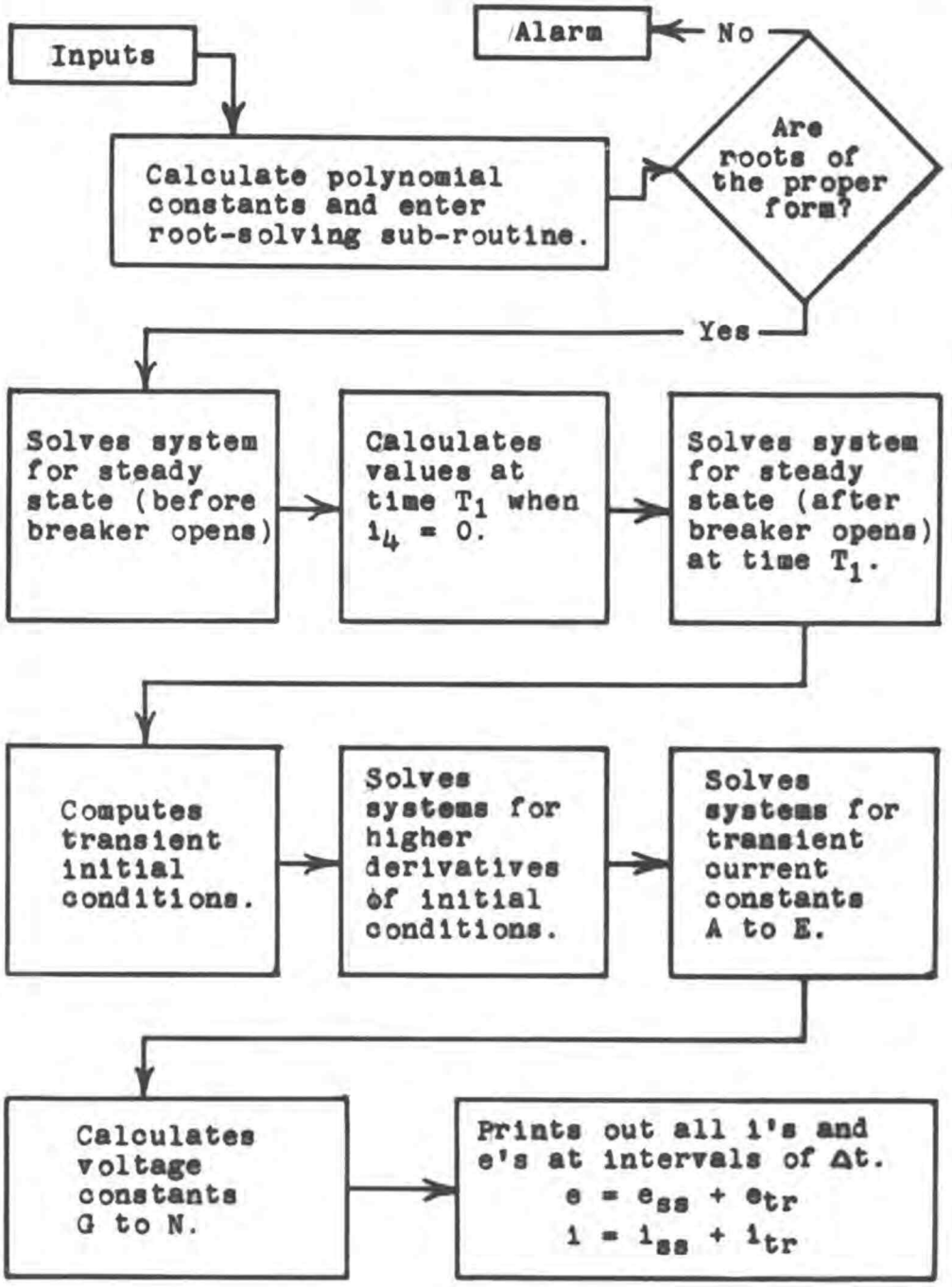


Figure 2. Flow diagram of computer program.

The magnitude of the currents and voltages before the breaker opens is calculated by finding the general steady state solution in the form

$$\begin{aligned} i_{ss} &= I_{ss\ max} \cos(\omega T + \phi_i) \\ e_{ss} &= E_{ss\ max} \cos(\omega T + \phi_e) \end{aligned} \qquad (4)$$

The time (T_1) is found when i_4, which is the current through the breaker, equals zero. The particular values of all voltages and currents at this instant of time are computed and stored.

The steady state solution after R_4 is increased, simulating the breaker opening, is also calculated at time T_1. The difference between the two steady state solutions of the values e_1, e_2, e_3, i_1 and i_2, represents the initial conditions of the resulting transient. The transient is of the form

$$\begin{aligned} i_{tr} &= A\epsilon^{-a_1 t} + \epsilon^{-a_2 t}(B \sin b_2 t + C \cos b_2 t) \\ &\quad + \epsilon^{-a_3 t}(D \sin b_3 t + E \cos b_3 t) \\ e_{tr} &= G\epsilon^{-a_1 t} + \epsilon^{-a_2 t}(H \sin b_2 t + K \cos b_2 t) \\ &\quad + \epsilon^{-a_3 t}(M \sin b_3 t + N \cos b_3 t) \end{aligned} \qquad (5)$$

The constants A to E and G to N are calculated from the initial conditions and the roots of the characteristic polynomial.

The steady state and transient values are calculated at intervals of Δt, then added together and printed out.

The interval Δt may be shortened or lengthened as desired during the print-out, and the whole print-out can be stopped and started at a new time.

Each curve takes about one half hour of computer time. Approximately half this time is spent on the initial calculations and the remaining time is spent on the print-out with its associated computations.

RESULTS

The values chosen for the test circuit of Figure 1 were:

R_1 = 4.6 ohms	R_2 = 3.06 ohms
L_1 = 0.0307 henrys	L_2 = 0.0205 ohms
C_1 = 0.107 microfarads	C_2 = 0.071 microfarads

These are the equivalent lumped constants for an average 69-Kv line (5, p.280), with a length of 15 miles on the sending end side of the breaker and a length of 10 miles on the receiving end side. One half the equivalent lumped capacitance of each line section was put adjacent to the circuit-breaker. The sending and receiving ends of the line were assumed to terminate on infinite buses. Lumped capacitors at these ends would have no effect and were, therefore, eliminated. A sending end voltage (e_s) of 55 Kv peak-to-ground was used.

The above line constants were held for all the calculations except two. The quantities varied were the breaker shunting resistance (R_3) and capacitance (C_3).

A short circuit on the receiving end of the line ($e_r = 0$) was found to give by far the most severe transient. A preliminary calculation showed the maximum gap voltage to reach only 5 Kv when load current was interrupted, compared to almost 100 Kv for interrupting fault current. This is because, on short circuit, the voltage

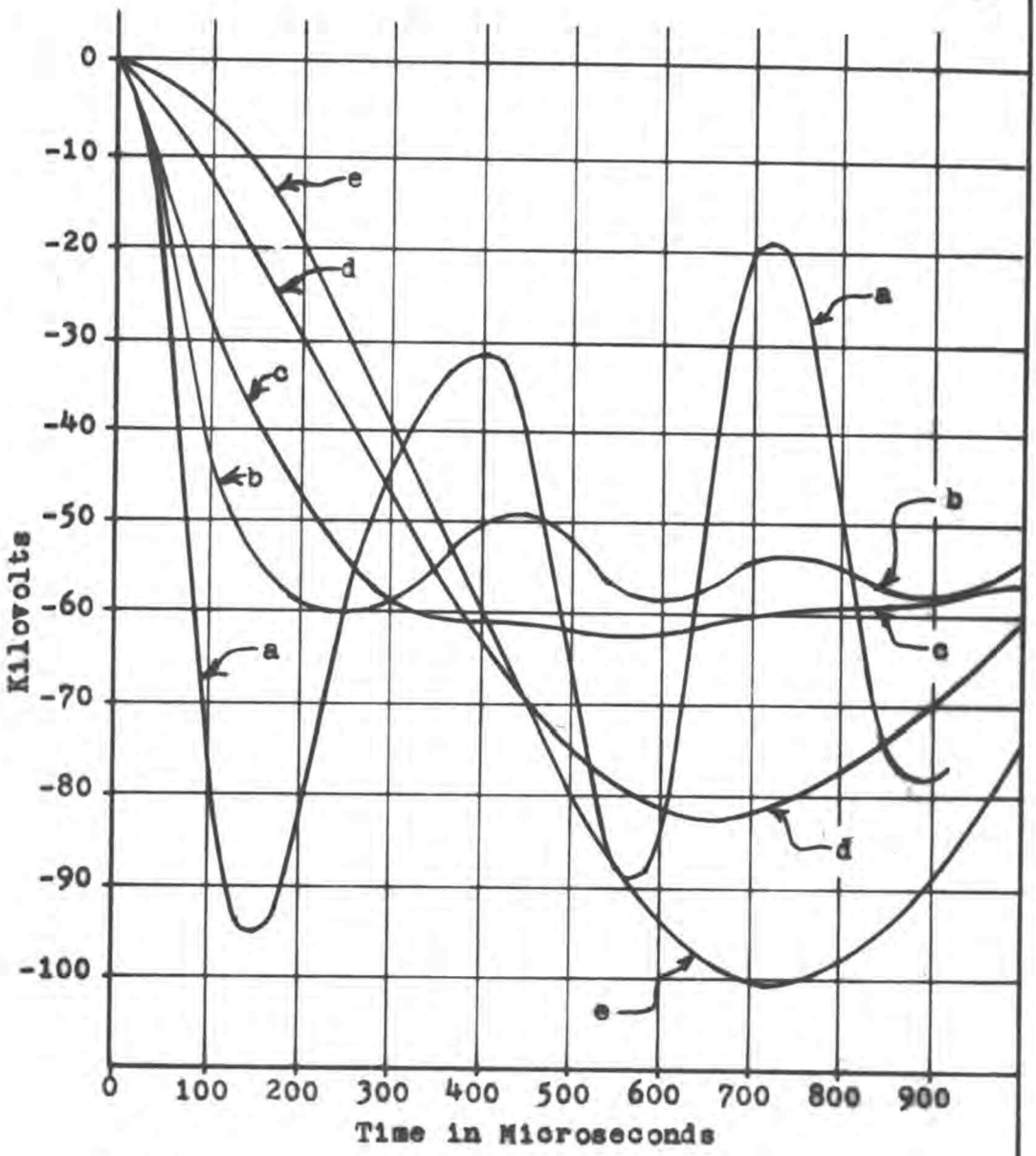
phasor leads the current phasor by approximately 90°. When the interruption occurs at current zero, the voltage will be close to maximum, leaving a maximum charge on the line-to-ground capacitors (C_1 and C_2) to initiate the resulting transient.

Figure 3. (Cases "a" to "e") shows the results of varying R_3 from 10,000 ohms to 10 ohms, while holding C_3 at a large value of 1 microfarad.

The curve of Case "a" shows the effect of a 10,000 ohm resistor. The resistance is so large that its effect is similar to that of an open circuit in the circuit-breaker shunt path. A calculation with a small value of C_3 (0.01 microfarad) and a small value of R_3 (10 ohms) was made and essentially the same curve as "a" was produced. The size of C_3 was too small to have any effect on the circuit recovery voltage transient. Curve "a" can, therefore, be taken as a close approximation to the response of the system with no shunt connection around the breaker.

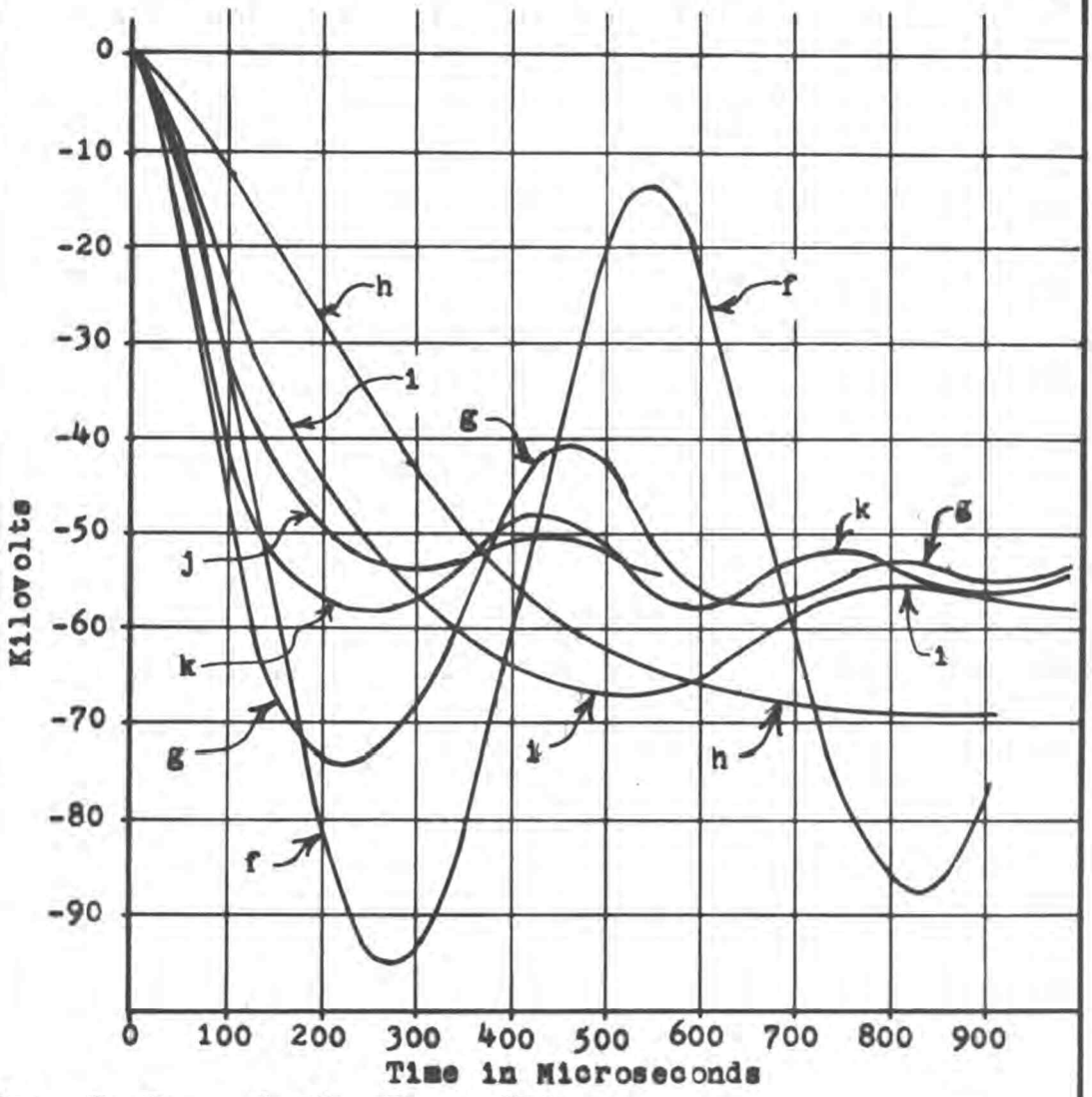
The voltage (e_3), across the shunting capacitor (C_3), will follow the gap voltage with a time lag dictated by the R-C time constant of the shunting elements. It is impossible for it ever to exceed the maximum gap voltage.

The capacitor voltage (e_3), as well as the recovery voltage, was printed out at each interval of



Case a	C_3 = 1 μf	R_3 = 10,000 Ω
Case b	C_3 = 1 μf	R_3 = 1,000 Ω
Case c	C_3 = 1 μf	R_3 = 400 Ω
Case d	C_3 = 1 μf	R_3 = 100 Ω
Case e	C_3 = 1 μf	R_3 = 10 Ω

Figure 3. Calculated recovery voltage curves Cases a to e.



Case f	$C_3 = .1\ \mu f$	$R_3 = 100\ \Omega$	
Case g	$C_3 = .1\ \mu f$	$R_3 = 1000\ \Omega$	
Case h	$C_3 = 1\ \mu f$	$R_3 = 400\ \Omega$	L_1 and L_2 doubled
Case i	$C_3 = 1\ \mu f$	$R_3 = 400\ \Omega$	$.37\ \mu f$ added to C_2
Case j	$C_3 = 10^6\ \mu f$	$R_3 = 600\ \Omega$	
Case k	$C_3 = 10^6\ \mu f$	$R_3 = 1000\ \Omega$	

Figure 4. Calculated recovery voltage curves Cases f to k.

time (Δt) in the calculation. In Case "e" (R_3 = 10 ohms), the curve of capacitor voltage (e_3) conformed almost exactly to the recovery voltage curve. As R_3 was increased, the curve of capacitor voltage followed the curve of recovery voltage with an increased time lag. In Case "c" (R_3 = 400 ohms), the capacitor voltage rose to the magnitude of the gap voltage curve only after an interval of 1,000 microseconds.

In Figure 4., Cases "f" and "g" show the results of a capacitor of 0.1 microfarad in series with a resistor of 100 ohms and 1,000 ohms respectively. By comparison with the curves of Cases "d" and "b", where the shunting capacitor (C_3) is 1 microfarad, the curves "f" and "g" show the effect of a reduction in the size of the shunting capacitor, by the increased frequency of the recovery voltage transient. The curves ("f" and "g") are closer in shape and values to the limiting curve of Case "a", where the breaker shunting impedance has no noticeable effect.

Case "h" represents a calculation with L_1 and L_2 doubled. The resistor (R_3) of 400 ohms and the capacitor (C_3) of 1 microfarad are the same values as in Case "c". The effect of the increased inductance in the circuit is to reduce the rate of rise of recovery voltage and to increase the peak value slightly.

The same result is observed in Case "i". The line-to-ground capacitor (C_2) on the bus at the receiving side of the breaker is increased by 0.37 microfarad, the equivalent of a 225 KVA capacitor. The breaker shunting values (R_3 and C_3) are the same as in Cases "c" and "h".

The effect of increasing the breaker shunting capacitance (C_3) is approximately the same as increasing the inductance or capacitance in the connected circuits. The rate of rise of recovery voltage is reduced and the peak recovery voltage is increased.

Curves "j" and "k" show the results equivalent to short-circuiting the shunt capacitor (C_3). This is achieved in the computer program by setting C_3 to an extremely high value of 10^6 microfarads, thereby almost eliminating its effect from the circuit.

With the effect of the capacitor thus eliminated, the computer program could not handle a value of R_3 lower than about 500 ohms, because the characteristic polynomial had three real roots. This would indicate that the coupling between the two circuits on each side of the breaker had become so great that there was one transient oscillating term instead of two.

A comparison of Cases "j" and "k", with Cases "c" and "b", shows that the effect of the one microfarad shunting capacitor is very small when it is in series with a pure resistor of about 500 ohms or greater.

There is, however, considerable difference in the final steady state current through the shunting branch, with and without the capacitor. There would be 13.8 amperes rms in Case "b" where a 1 microfarad capacitor is in series with the 1,000 ohm resistor and 39 amperes rms in Case "k" where the capacitor is essentially short-circuited.

Two calculations were made to emphasize the difference between interrupting direct and alternating current. The current (i_4) through the breaker was interrupted by inserting R_4 into the circuit at the maximum current point in the cycle, thereby approximating a direct-current interruption. The interrupted current was a load current of approximately 100 amperes. With 0.1 microfarad and 10 ohms shunt impedance, the peak voltage was 22 Kv. When the shunt resistance was increased to 1,000 ohms, the peak recovery voltage went up to 250 Kv.

Table I. is a summary of the results. The value given for the rate of rise of recovery voltage is the slope of a straight line, passing through the zero point and lying tangent to the recovery voltage curve.

TABLE I

RESULTS OF RECOVERY-VOLTAGE STUDY

Case	C_3 μf	R_3 ohms	Peak rec. v. rise Kv	Rate rec. v. rise $\frac{\text{volts}}{\mu\text{sec}}$	Peak i_3 amp	Peak e_3 Kv	Comments
a	1	10000	95	710	-	-	i_3 and e_3 beyond limits of computer.
b	1	1000	60	416	53	-	e_3 peak not reached by end of run.
c	1	400	62	278	97	55	
d	1	100	83	156	167	79	
e	1	10	101	158	215	101	
f	.1	100	95	400	52	95	
g	.1	1000	75	478	35	64	
h	1	400	69	145	86	-	L_1 and L_2 doubled in size.
i	1	400	67	244	95	-	.37 μf added to C_2.
j	10^6	600	54	322	90	0	
k	10^6	1000	58	416	59	0	
-	.1	10	100	416	42	100	Curve similar to "f" with higher peaks.
-	.01	10	93	670	-	-	Curve similar to "a".

CONCLUSIONS

A resistor-capacitor shunt, across the contact points of a circuit-breaker, performs a different function in interrupting direct current than it does in interrupting alternating current.

In the direct current case, an attempt is made to break the current flowing through an inductance. A capacitor, shunting the contact points, is needed to prevent the theoretically infinite overvoltages that could occur.

In the alternating current case, the current through the inductance has come to its natural zero when the interruption is made. The shunt capacitor merely modifies the nature of the resulting transient, caused by the stored energy in the equivalent line-to-ground capacitors.

In both cases, a resistor in series with the shunt capacitor will tend to introduce damping into the circuit, and cut down the magnitude of the transient oscillations of voltage and current.

The results of this investigation will only be qualitatively applicable to an actual three-phase circuit. Two modifying factors that would have to be considered are the effect of the interaction between the phases and the effect of the distributed line constants.

Several general conclusions may, however, be drawn about the action of a resistance-capacitance shunt connected across an alternating current circuit-breaker:

(1) A capacitor shunt will reduce the rate of rise of recovery voltage. To produce a noticeable effect, however, the capacitor must be relatively large compared with the equivalent lumped capacitance of the circuit. The peak recovery voltage, with a pure capacitor shunt, will be as high as, or higher than, that produced with no shunt whatsoever.

(2) Adding resistance in series with the shunt capacitance decreases the peak value of recovery voltage but increases its rate of rise. There appears to be an optimum value of resistance (about 100 ohms in this circuit), where, due to the shape of the recovery voltage curve, the effect of reducing the peak value of recovery voltage also causes a slight reduction in its rate of rise.

(3) Increasing the resistance of the circuit-breaker shunt tends to decrease the effect of the capacitance. A point is soon reached where the capacitance, in series with the resistance, has no more effect on the transient performance than the resistance alone.

With a shunting resistance only, there is a value of resistance (about 400 ohms in this circuit), below

which the peak recovery voltage is no greater than the steady state gap voltage. This value of resistance also causes a reduction in the rate of rise of recovery voltage, compared with the case where there is no shunt connection. If the shunting resistor is made very small, the current passing through it becomes a significant fraction of the total current interrupted. The breaking of this current then becomes as major an operation as making the original break. This, however, is done in many cases (3, p.705); the resistance being inserted and removed all in one operation.

(4) In this experimental circuit, a large enough shunt capacitor to modify sufficiently the rate of rise of recovery voltage, would have to have a value of approximately 0.5 microfarad and be able to withstand a temporary voltage peak of twice normal line-to-ground voltage. If this condition could be met by an ordinary 40-Kv rms capacitor, it would have to have a rating of 300 KVA, or 900 KVA for a three-phase bank, which, at an approximate value of $7.00 per KVA (2, p.235), would be $6,300.00.

The physical size of this unit would be too large to include within the circuit-breaker case, as is done with a resistor shunt.

The practical use of a resistor-capacitor shunt would depend upon the characteristics and cost of the

circuit-breaker. The advantages of the decrease in the rate of rise of recovery voltage and reduced final steady state current through the shunt, would have to outweigh the disadvantages of the high peak recovery voltage, external circuitry, and cost. The alternatives would be pure resistor shunting or a higher voltage classification breaker.

BIBLIOGRAPHY

1. Howatson, A. M. The effect of contact synchronization. American Institute of Electrical Engineers Transactions 80:10-13. 1961.

2. Johnson, A. A. Application of capacitors to power systems. In: Westinghouse Electric Corporation's Electrical transmission and distribution reference book. 4th ed. East Pittsburgh, 1950. p. 233-264.

3. Kane, R. E. and J. K. Walker. A 69-Kv compressed air circuit breaker for 5,000,000 KVA. American Institute of Electrical Engineers Transactions 74:705-710. 1955.

4. Lago, Gladwyn V. and Donald L. Waidelich. Transients in linear circuits. New York, Ronald Press, 1958. 393 p.

5. McCann, G. D. Regulation and losses of transmission lines. In: Westinghouse Electric Corporation's Electrical transmission and distribution reference book. 4th ed. East Pittsburgh, 1950. p. 265-289. (Rev. by R. F. Lawrence)

APPENDIX

APPENDIX

MATHEMATICS OF THE COMPUTER PROGRAM

The computer program is written to calculate the transient oscillations of all currents and voltages that occur in the alternating-current circuit of Figure 1., when the value of resistance (R_4) is suddenly changed.

The program reduces a system of differential equations into systems of linear equations, which can be solved by calling in a system-solving sub-routine.

In the following derivations, when the symbol of the variable of current or voltage or associated constant has a subscript, the symbol refers to one particular current or voltage in the circuit. If it has no subscript, the explanation applies equally well to every appropriate current or voltage variable in the circuit, and the procedure will usually have to be applied to each current or voltage variable in turn.

Figure 2. gives the order in which the computer program goes through the sets of calculations. This is not necessarily the order used in the following explanations.

The method of solution for the transient response is the usual one applied to linear differential equations with constant coefficients. A solution for the currents

of the form

$$i = A\epsilon^{st} \tag{6}$$

is assumed, in which case the higher derivatives are:

$$i' = As\epsilon^{st}, \qquad i'' = As^2\epsilon^{st}$$

As the transient response is the free response of the system in the absence of any forcing function, the sending and receiving end voltages (e_s and e_r) of Figure 1, are set to zero. By differentiating the original loop equations (sets 2 and 3), the constant terms e(0) are eliminated and the resulting system of differential equations, when the current substitution ($i = A\epsilon^{st}$) is made, becomes, in matrix form

$$\epsilon^{st} \quad (M) \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = 0 \tag{7}$$

where

$$(M) = \begin{pmatrix} \left(s^2L_1+sR_1+\frac{1}{C_1}\right) & 0 & \left(-\frac{1}{C_1}\right) & 0 \\ 0 & \left(-s^2L_2-sR_2-\frac{1}{C_2}\right) & \left(\frac{1}{C_2}\right) & 0 \\ \left(-\frac{1}{C_1}\right) & \left(-\frac{1}{C_2}\right) & \left(sR_3+\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right) & \left(-sR_3-\frac{1}{C_3}\right) \\ 0 & 0 & \left(-sR_3-\frac{1}{C_3}\right) & \left(s\left(R_3+R_4\right)+\frac{1}{C_3}\right) \end{pmatrix}$$

For a non-zero solution, the determinant of the matrix (M) must equal zero.

$$\left[M \right] = 0$$

The evaluation of this determinant gives the characteristic polynomial for the circuit in the form:

$$a_6s^6 + a_5s^5 + a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0 = 0$$

This characteristic polynomial will have six separate values of "s", or roots, in its solution. This implies that the transient current (i_{tr}) will contain six terms,

$$i_{tr} = U\epsilon^{s_1t} + V\epsilon^{s_2t} + W\epsilon^{s_3t} + X\epsilon^{s_4t} + Y\epsilon^{s_5t} + Z\epsilon^{s_6t}$$

For the particular circuit configuration of Figure 1. a_0 will always equal 0. One root will then be $s_6 = 0$, and as the nature of the transient response is such that it dies out as the time (t) becomes very large, "Z" must be zero.

The rest of this discussion will consider only the five remaining terms in the transient. The characteristic polynomial will now have the form

$$a_6s^5 + a_5s^4 + a_4s^3 + a_3s^2 + a_2s + a_1 = 0 \qquad (8)$$

It was assumed, from the nature of the circuit parameters, that the solution of the characteristic

polynomial would have one real and four complex roots

$$s_1 = -a_1$$

$$s_2 = -a_2 + jb_2 \qquad s_4 = -a_3 + jb_3$$

$$s_3 = -a_2 - jb_2 \qquad s_5 = -a_3 - jb_3$$

The computer program tests the roots to verify that they are of this form before allowing the computations to continue.

The transient component of current can be written,

$$i_{tr} = U\epsilon^{-a_1 t} + V\epsilon^{(-a_2 + jb_2)t} + W\epsilon^{(-a_2 - jb_2)t} + X\epsilon^{(-a_3 + jb_3)t} + Y\epsilon^{(-a_3 - jb_3)t} \tag{9}$$

which can be modified to the form

$$i_{tr} = A\epsilon^{-a_1 t} + \epsilon^{-a_2 t}(B \sin b_2 t + C \cos b_2 t) + \epsilon^{-a_3 t}(D \sin b_3 t + E \cos b_3 t) \tag{10}$$

The constants A to E must be determined from the initial conditions existing in the circuit at the instant the discontinuity is introduced (R_4 changed).

When the value of R_4 is suddenly changed at time T_1, the voltages e_1, e_2 and e_3 across the capacitor and the currents i_1 and i_2 through the inductor cannot change instantaneously, because of the nature of the circuit elements. The transient initial conditions are these values of current and voltage, minus the new steady

state values calculated for the changed circuit at time T_1.

$$i_{tr}(0) = i\left(T_1^-\right) - i_{ss}\left(T_1^+\right)$$
$$e_{tr}(0) = e\left(T_1^-\right) - e_{ss}\left(T_1^+\right) \tag{11}$$

To calculate the twenty constants A through E for the four transient currents, higher derivatives of the initial conditions must be found. The equation (10) of the transient current is evaluated at time t = 0, as well as its four higher derivatives.

$$i_{tr} = A + C + E$$

$$i'_{tr} = -a_1A + b_2B - a_2C + b_3D - a_3E$$

$$i''_{tr} = a_1^2A + (-2a_2b_2)B + (a_2^2 - b_2^2)C + (-2a_3b_3)D + (a_3^2 - b_3^2)E$$

and so on.

This operation is performed in the computer by building up a matrix of the form

$$(N) = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ c_1 & d_1 & e_1 & f_1 & g_1 \\ c_2 & d_2 & e_2 & f_2 & g_2 \\ c_3 & d_3 & e_3 & f_3 & g_3 \\ c_4 & d_4 & e_4 & f_4 & g_4 \end{pmatrix}$$

where

$c_1 = -a_1$ $\quad d_1 = b_2$ $\quad e_1 = -a_2$

$c_2 = c_1^2$ $\quad d_2 = 2d_1e_1$ $\quad e_2 = (e_1^2 - d_1^2)$

$c_3 = c_1^3$ $\quad d_3 = (d_1e_2 + e_1d_2)$ $\quad e_3 = (e_1e_2 - d_1d_2)$

$c_4 = c_1^4$ $\quad d_4 = 2d_2e_2$ $\quad e_4 = (e_2^2 - d_2^2)$

The "f" and "g" columns are compiled identically to the "d" and "e" columns, with b_3 substituted for b_2 and a_3 for a_2.

This results in the system of equations,

$$(N) \qquad \begin{pmatrix} A \\ B \\ C \\ D \\ E \end{pmatrix} = \begin{pmatrix} i_{tr} \\ i'_{tr} \\ i''_{tr} \\ i'''_{tr} \\ i^{(v}_{tr} \end{pmatrix} \qquad (12)$$

This system of equations can be solved for the current constants A to E, after first finding the initial values of the four transient currents and their higher derivatives.

To calculate the initial values of the transient currents at time t = 0, and their higher derivatives, the original loop equations (2) are re-written with the forcing voltages (e_s and e_r) set at zero.

$$
\begin{aligned}
L_1 i'_{1tr} &= -R_1 i_{1tr} - e_{1tr} \\
-L_2 i'_{2tr} &= R_2 i_{2tr} - e_{2tr} \\
R_3 i_{3tr} - R_3 i_{4tr} &= e_{1tr} - e_{2tr} - e_{3tr} \\
-R_3 i_{3tr} + (R_3 + R_4) i_{4tr} &= e_{3tr}
\end{aligned}
\tag{13}
$$

After substituting the transient initial conditions (11) of e_{1tr}, e_{2tr}, e_{3tr}, i_{1tr} and i_{2tr} in the above set of equations (13), they can be solved for the values of i'_{1tr}, i'_{2tr}, i_{3tr} and i_{4tr}.

The higher derivatives of the transient currents are calculated by first forming a matrix

$$
(P) = \begin{pmatrix} L_1 & 0 & 0 & 0 \\ 0 & -L_2 & 0 & 0 \\ 0 & 0 & R_3 & -R_3 \\ 0 & 0 & -R_3 & (R_3 + R_4) \end{pmatrix}
$$

Then, by differentiating the above set of equations (13) and making use of the current-voltage relationships (3), a system of equations is constructed and solved.

$$
(P) \begin{pmatrix} i''_{1tr} \\ i''_{2tr} \\ i'_{3tr} \\ i'_{4tr} \end{pmatrix} = \begin{pmatrix} -R_1 i'_{1tr} - \frac{1}{C_1}\left(i_{1tr} - i_{3tr}\right) \\ R_2 i'_{2tr} - \frac{1}{C_2}\left(i_{3tr} - i_{2tr}\right) \\ \frac{1}{C_1}\left(i_{1tr} - i_{3tr} - \frac{1}{C_2}\left(i_{3tr} - i_{2tr}\right) - \frac{1}{C_3}\left(i_{3tr} - i_{4tr}\right)\right) \\ \frac{1}{C_3}\left(i_{3tr} - i_{4tr}\right) \end{pmatrix}
\tag{14}
$$

Successively higher derivatives of current are calculated in this way, placing the results of one calculation into the constants of the next.

The relation (3) of the loop equations shows the voltages to be an integrated function of the currents. Because of the nature of sine and exponential functions, the transient voltages will have the same form as the transient currents, ie:

$$e_{tr} = G\epsilon^{-a_1 t} + \epsilon^{-a_2 t}(H \sin b_2 t + K \cos b_2 t) + \epsilon^{-a_3 t}(M \sin b_3 t + N \cos b_3 t) \tag{15}$$

The transient voltages are derived from the transient currents by a further manipulation of the loop equations (13),

$$\begin{aligned} e_{1tr} &= -L_1 i'_{1tr} - R_1 i_{1tr} \\ e_{2tr} &= L_2 i'_{2tr} + R_2 i_{2tr} \\ e_{3tr} &= e_{1tr} - e_{2tr} - R_3\left(i_{3tr} - i_{4tr}\right) \end{aligned} \tag{16}$$

By suitably combining the A, B, C, D and E constants of the four transient currents with the circuit parameters and roots of the characteristic polynomial, the G, H, K, M and N constants of the three voltages may be directly determined.

Conventional alternating-current phasor technique is applied to calculate the steady state values of current and voltage. The loop equations from Figure 1.

when written in terms of complex quantities are:

$$\left(Q\right) \begin{pmatrix} \bar{i}_{1ss} \\ \bar{i}_{2ss} \\ \bar{i}_{3ss} \\ \bar{i}_{4ss} \end{pmatrix} = \begin{pmatrix} \bar{E}_s \\ \bar{E}_r \\ 0 \\ 0 \end{pmatrix} \tag{17}$$

The bar ($^{-}$) indicates a complex quantity and

$$\left(Q\right) = \begin{pmatrix} (a + jb) & 0 & (0 + jc) & 0 \\ 0 & (d + je) & (0 + jf) & 0 \\ (0 + jc) & (0 + jg) & (h + jk) & (m + jn) \\ 0 & 0 & (m + jn) & (p + jq) \end{pmatrix}$$

where

$a = R_1$ $\quad b = \left(\omega L_1 - \frac{1}{\omega C_1}\right)$ $\quad c = \frac{1}{\omega C_1}$

$d = -R_2$ $\quad e = \left(-\omega L_2 + \frac{1}{\omega C_2}\right)$ $\quad f = -\frac{1}{\omega C_2}$

$g = \frac{1}{\omega C_2}$ $\quad h = R_3$ $\quad k = -\frac{1}{\omega}\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$

$m = -R_3$ $\quad n = \frac{1}{\omega C_3}$ $\quad p = \left(R_3 + R_4\right)$ $\quad q = -\frac{1}{\omega C_3}$

This complex-number system of equations (17) is expanded into a real-number system by the following manipulation:

$$\begin{pmatrix} a & 0 & 0 & 0 & -b & 0 & -c & 0 \\ 0 & d & 0 & 0 & 0 & -e & -f & 0 \\ 0 & 0 & h & m & -c & -g & -k & -n \\ 0 & 0 & m & p & 0 & 0 & -n & -q \\ b & 0 & c & 0 & a & 0 & 0 & 0 \\ 0 & e & f & 0 & 0 & d & 0 & 0 \\ c & g & k & n & 0 & 0 & h & m \\ 0 & 0 & n & q & 0 & 0 & m & p \end{pmatrix} \begin{pmatrix} (Re)i_{1ss} \\ (Re)i_{2ss} \\ (Re)i_{3ss} \\ (Re)i_{4ss} \\ (Im)i_{1ss} \\ (Im)i_{2ss} \\ (Im)i_{3ss} \\ (Im)i_{4ss} \end{pmatrix} = \begin{pmatrix} (Re)E_s \\ (Re)E_r \\ 0 \\ 0 \\ (Im)E_s \\ (Im)I_r \\ 0 \\ 0 \end{pmatrix} \tag{18}$$

After solving the above system (18), the computer program calls in a sub-routine handling complex numbers, to put the currents in polar form. The voltages are then calculated from the circuit constants and associated currents.

At each interval of time $(T_1 + n\Delta t)$ the steady state values of current and voltage are calculated in the form

$$\begin{aligned} i_{ss} &= I_{ss\ max} \cos\left(\omega(T_1 + n\Delta t) + \phi_i\right) \\ e_{ss} &= E_{ss\ max} \cos\left(\omega(T_1 + n\Delta t) + \phi_e\right) \end{aligned} \tag{19}$$

The absolute value of the current phasor is I_{max} and ϕ_i is its phase angle.

The transient values of current and voltage are similarly calculated at each interval of time ($n\Delta t$).

$$i_{tr} = A\epsilon^{-a_1(n\Delta t)} + \epsilon^{-a_2(n\Delta t)}\left(B \sin b_2(n\Delta t) + C \cos b_2(n\Delta t)\right) + \epsilon^{-a_3(n\Delta t)}\left(D \sin b_3(n\Delta t) + E \cos b_3(n\Delta t)\right)$$

$$e_{tr} = G\epsilon^{-a_1(n\Delta t)} + \epsilon^{-a_2(n\Delta t)}\left(H \sin b_2(n\Delta t) + K \cos b_2(n\Delta t)\right) + \epsilon^{-a_3(n\Delta t)}\left(M \sin b_3(n\Delta t) + N \cos b_3(n\Delta t)\right)$$

The steady state and transient values of current and voltage are added together at each interval of time (Δt) to form the total values. This process is carried on, increasing "n" by 1 each step, until enough points are obtained to plot the desired curves. The computer is then manually stopped.

The program can be modified while it is running through the computer by using the three jump switches provided on the control panel.

If the second switch is on during the calculation part of the program, the constants of the characteristic polynomial, the roots of the characteristic polynomial, and the results of all the system-solving operations will be printed out. All the intermediate constants may then be checked. If the first switch is on, the matrices of the systems to be solved are printed out, allowing further checks to be made.