

Supplement to
DEFLECTION AND STRESSES IN A
UNIFORMLY LOADED, SIMPLY SUPPORTED,
RECTANGULAR SANDWICH PLATE
Experimental Verification of Theory

No. 1847-A

December 1956

INFORMATION REVIEWED
AND REAFFIRMED
1962

LOAN COPY

Please return to:
Wood Engineering Research
Forest Products Laboratory
Madison, Wisconsin 53705

This Report is One of a Series
Issued in Cooperation with the
ANC-23 PANEL ON SANDWICH CONSTRUCTION
of the Departments of the
AIR FORCE, NAVY, AND COMMERCE



FOREST PRODUCTS LABORATORY
MADISON 5, WISCONSIN

UNITED STATES DEPARTMENT OF AGRICULTURE
FOREST SERVICE

In Cooperation with the University of Wisconsin

Supplement to

DEFLECTION AND STRESSES IN A UNIFORMLY LOADED,
SIMPLY SUPPORTED, RECTANGULAR SANDWICH PLATE¹

Experimental Verification of Theory

By

WAYNE C. LEWIS, Engineer

Forest Products Laboratory,² Forest Service
U. S. Department of Agriculture

Summary

Forest Products Laboratory Report No. 1847³ presented the results of the theoretical analysis for calculating deflections and strains in uniformly loaded, simply supported, sandwich panels constructed with isotropic facings and orthotropic cores. This supplementary report presents the results of tests of 24 panels that were designed so that, when tested to failure, 18 would have core shear failures and 6 would have tension and compression failures in the facings. Panels were constructed

¹-This report is one of a series (ANC-23, Item A-7) prepared and distributed by the Forest Products Laboratory under U. S. Navy Bureau of Aeronautics Order No. NAer 01684 and U. S. Air Force Contract No. DO 33(616)-56-9. Results reported here are preliminary and may be revised as additional data become available.

²-Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

³-Raville, Milton E. Deflection and Stresses in a Uniformly Loaded, Simply Supported, Rectangular Sandwich Plate, Forest Products Laboratory Report No. 1847, December 1955.

of combinations of aluminum honeycomb core and aluminum alloy facings typical of those being used for aircraft structures. The theory applies to simply supported, rectangular panels subjected to small deflections within their elastic range. The tests could not be held to these conditions, but nevertheless the results confirmed the theory within reasonable accuracy. Prediction of loads at failure were conservative.

Introduction

Formulas for calculating the deflections and stresses in uniformly loaded, simply supported sandwich panels constructed of isotropic facings bonded to orthotropic cores are developed and presented in Forest Products Laboratory Report No. 1847. Coefficients are presented graphically for computing: (1) deflection at the center of the panel, (2) strains in the facings in the x and y directions, and (3) shear strains in the cores in the xz and yz planes for rectangular, flat sandwich panels with cores in which the shear moduli in the two principle directions are related in the ratios 0.4, 1.0, and 2.5.

A series of test panels was then designed to obtain an experimental verification of the mathematical derivations presented in Report 1847. Panels 20 by 20 and 20 by 30 inches in size and of different thicknesses were tested under uniformly distributed normal load, and elastic deformation data were obtained. Tests were continued until the panels failed in order to determine if the formulas for elastic behavior could be used to estimate the ultimate strength of sandwich panels. This report describes the panels and the testing procedures used, and presents the results of the tests.

Description of Test Panels

The theoretical analysis for orthotropic cores involves the ratio of the shearing moduli associated with strains in the "TL" to the "TW" planes of the core material. Figure 17⁴ shows the orientation of the T, W, and

⁴Figures and tables in this supplementary report are numbered consecutively after those in the basic report. There were 16 figures and 1 table in the basic report.

L directions. Coefficients are presented so that deflections and strains can be computed for sandwich constructions with cores that have ratios

of $\frac{G_{TL}}{G_{TW}}$ of 2.5.

Typical aluminum honeycomb cores have ratios of moduli ($\frac{G_{TL}}{G_{TW}}$) near 2.5.

Two typical perforated aluminum foil honeycomb core materials of 3/8-inch cell size were selected for the tests. The important properties of the core materials are summarized in table 2.⁴

Panels 23 by 23 and 23 by 33 inches in size were fabricated by bonding either 2024 or 7075 clad aluminum alloy facings to cores that were 1/4, 1/2, or 1 inch thick. The blocks from which the cores were assembled were approximately 9 inches square in cross section, so the cores were assembled by bonding bandsawed sections of core material edgewise and endwise into panels approximately 1 inch wider and longer than the finished panels. Bonds between these sections were oriented carefully so that full-size blocks were located at the edges of the panel, and that bonds were as remote as possible from areas of maximum strain and where strains were to be measured during test.

The 23- by 33-inch panels were fabricated in pairs, one with the L direction of the core parallel to the long dimension of the panel and one with the L direction perpendicular to the long dimension. Panels were fabricated as indicated in columns 1 through 8 of table 3. Panels 1 through 18 were designed so that they would develop shear failures in the core before stresses in tension and compression in the facings were critical. Panels 19 through 24 were designed with cores proportionately stronger than facings so that critical stresses in tension and compression in the facings could be developed before shear failures occurred in the cores.

Method of Test

Rectangular specimens were supported in a frame 20 by 30 inches in size, and square specimens were supported in a frame 20 inches square. Uniformly distributed normal loads were applied to the bottom surface of the panels by hydraulic pressure through either a rubber bag or a thin, soft copper bag. Figure 18 shows the test setup. Specimens were loaded through the hose and needle valve shown in the lower center part of the

figure. Loads were measured by an open-tube mercury manometer. Tie rods and supporting beams held the two halves of the supporting and loading frame together during test. Nuts on the tie rods were tightened by hand only at the start of test to minimize any clamping action. The nuts became progressively tighter during test, because of the change in slope along the edges of the panel. The corners of the panels curved downwards as the centers were deflected upwards.

Deflections of the center of the panel were measured by the three-point supported bridge and micrometer dial shown in figure 19. Strains in tension and compression in the x and y directions (x is parallel to the short side of the panel, "a;" y is parallel to the long side, "b") were obtained at the center of the panel with electric resistance-wire strain gages located as shown in figure 19. The differential movement of one facing with respect to the other at the edges of the panel at midwidth and mid-length was measured by drilling a circular hole in the top facing and core. A cylindrically shaped metal plug was bonded to the inner surface of the bottom facing. The upper end of the plug was flush with the top surface of the upper facing. A Tuckerman optical strain gage was used, as shown in figure 20, to measure the differential movement of one facing with respect to the other during test. One knife edge rested on the upper facing, while the other rested on the plug bonded to the lower facing. Figure 21 shows the panel with the supporting frame in place. The load was applied upwards.

Specimens were loaded in increments, and measurements of center deflection, strains in tension and compression, and differential movement of one facing with respect to the other were determined for each increment. Values obtained were plotted as abscissas against loads in pounds per square inch as ordinates. The slopes of the straight-line portions of the curves were used for comparisons with the values of deflection and strain predicted by theory. These slopes are the "observed" values given in tables 4 and 5.

The tests were continued until the panels failed. The loads at which the panels failed are given in tables 6 and 7.

Calculations

Calculations were made by means of the curves and formulas given in figures 5 through 16. These curves and formulas apply to locations in the

panels where the deflections, strains, or stresses are maximum. Thus W_{\max} refers to the deflection at the center of the panel; $\epsilon_{x \max}$ and $\epsilon_{y \max}$ refer to the strains in the facings at the center of the panel and in the x and y directions, respectively; and $\tau_{xz \max}$ and $\tau_{yz \max}$ refer to the shear stresses in the core associated with the shear strains in the xz and yz planes located at the centers of sides b and a respectively. The associated shear strains were obtained by dividing these stresses by the appropriate moduli of rigidity. It was at these locations that the deflection, strains, and stresses were measured in the tests. The values so computed for a unit load are listed in tables 3, 4, and 5. Tables 4 and 5 also contain the experimental values.

The load carried by the facings at the center of panel was obtained by substituting computed values of $\epsilon_{x \max}$ and $\epsilon_{y \max}$ in equations (78) and (79).

The predicted ultimate strength of the panels that failed in shear (panels 1 through 18) was computed by dividing the ultimate shear strength of the core material in the TW plane by the shearing modulus associated with that plane and by the shear strain per unit load given in column 12 or 13 of table 3.

The predicted ultimate strength of the panels that failed in the facings (panels 19 through 24) was calculated on the basis of the strength of the facings in tension or compression (64,000 pounds per square inch) by means of the conventional formulas

$$X = \frac{E}{1-\nu^2} (\epsilon_x + \epsilon_y \nu)$$

$$Y = \frac{E}{1-\nu^2} (\epsilon_y + \epsilon_x \nu)$$

where ϵ_x and ϵ_y were the values presented in columns 10 and 11 in table 3, for whichever direction yielded the greater stress. In the 20- by 30-inch panels, the greater stresses were in the x direction, and in the 20-inch-square panels, the greater stresses were in the y direction.

Presentation and Discussion of Results of Tests

Table 3 summarizes the information on the construction of the panels and on the calculated values based on theory for the elastic behavior of the panels. Columns 2 through 8 present data on size of panels, and thickness and type of cores and facings. Columns 9 through 13 present the calculated values (based on theory presented in Report 1847) for center deflection, compressive and tensile strains in the facings, and shear strains in the cores.

Table 4 compares computed values for deflection and strains in the facings with those obtained from the curves derived from test. Columns 4, 7, 9, 12, and 14 present the ratios between values obtained from test and those predicted by theory. A graphical comparison of these values is given in figures 22 through 24.

Examination of these ratios and the figures shows that the agreement between calculated and observed values is reasonable when it is considered that the theory applies only to small deflections of simply supported panels, and that the test imposed large deflections on panels that were far from simply supported. In general, the calculated deflections were greater than observed deflections.

Table 5 and figure 25 compare the calculated and observed shear strains in the cores of the several panels. The observed values are consistently larger than the calculated values. Youngquist and Kuenzi reported similar difficulties in measuring shear deformations in circular sandwich panels and small beams in Forest Products Laboratory Report No. 1845-A.⁵ They found that, in small, simply supported beams, shear strains measured in a manner similar to that used in this series were larger than those indicated by theory. The difference was inversely proportional to thickness of facings. For facing thicknesses of 0.012 and 0.032 inch, the ratios between measured and actual values were as follows:

⁵Youngquist, W. G., and Kuenzi, E. W. Supplement to Stresses Induced in a Sandwich Panel by Load Applied at an Insert, Forest Products Laboratory Report No. 1845-A, September 1955.

<u>Thickness of facing</u> (In.)	<u>Ratio between measured and computed shear strains</u>	
	<u>In the TW plane</u>	<u>In the TL plane</u>
0.012	1.68	2.89
.032	1.56	1.71

It is thought that the method measures a local displacement of one facing with respect to the other, rather than the average displacement.

Facings in this study of uniformly loaded panels were 0.012, 0.020, and 0.032 inch thick. Values obtained from tests appear reasonable in view of the experience reported in 1845-A. The failures and loads on the panels were computed on the assumption that the materials remained elastic until failure occurred. The calculated and observed values are compared in tables 6 and 7 and in figure 26. The estimates made were, in general, conservative. Some of the estimates for the panels that failed in the core were very conservative.

Conclusions

The limited number of tests of simply supported, uniformly loaded sandwich panels with perforated honeycomb cores and aluminum alloy facings indicate that the following conclusions are warranted:

1. The elastic theory presented in Report 1847 is roughly confirmed for the calculation of the deflection and principal tension and compression strains in facings of panels with aluminum honeycomb cores and aluminum alloy facings within the limits of small deflections.
2. Measured shear deformations were much greater than the calculated values, but this was probably due to method of measurement rather than error in theory, because a similar experience was reported in Report 1845-A.
3. The method presented for calculating shear strengths of the panels is often very conservative.
4. The method presented for computing the bending strength of the panels yields reasonable results.

Table 2. -- Properties of core materials

Foil thickness :	Density :	Shear modulus :		Shear strength	
		TL plane :	TW plane :	TL plane :	TW plane
<u>Inch</u> :	<u>Lb. per cu. ft.</u> :	<u>P.s.i.</u> :	<u>P.s.i.</u> :	<u>P.s.i.</u> :	<u>P.s.i.</u>
0.003 :	3.96 :	42,300 :	18,600 :	238 :	118
.005 :	6.32 :	75,200 :	32,900 :	393 :	226

Table 4.--Comparison between test values and calculated deflections and strains in facings for one pound per square inch of load

Panel No.	Center deflection			Strains in faces, <u>x</u> direction ¹			Strains in faces, <u>y</u> direction ¹						
	Calculated	Observed	Ratio, <u>obs.</u> / <u>calc.</u>	Calculated	Tension	Compression	Calculated	Tension	Compression				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
	Inch	Inch		Inch per inch			Inch per inch		Inch per inch			Inch per inch	
1	0.0270	0.0255	0.94	0.000149	0.000165	1.11	0.000174	1.17	0.000063	0.000081	1.28	0.000074	1.17
2	0.0262	0.0256	0.98	0.00156	0.00172	1.10	0.00181	1.16	0.00058	0.00078	1.34	0.00066	1.14
3	0.0142	0.0137	0.96	0.00077	0.00096	1.25	0.00085	1.10	0.00081	0.00084	1.04	0.00084	1.04
4	0.0078	0.0071	0.91	0.00076	0.00078	1.03	0.00084	1.10	0.00034	0.00041	1.21	0.00038	1.12
5	0.0072	0.0068	0.94	0.00100	0.00082	0.82	0.00089	0.89	0.00029	0.00043	1.48	0.00041	1.41
6	0.0063	0.0061	0.97	0.00063	0.00062	0.98	0.00064	1.02	0.00067	0.00074	1.10	0.00078	1.16
7	0.0942	0.0940	1.00	0.00291	0.00287	0.99	0.00311	1.07	0.00117	0.00168	1.44	0.00117	1.44
8	0.0932	0.0863	0.93	0.00298	0.00370	1.11	0.00332	1.11	0.00114	0.00180	1.58	0.00114	1.58
9	0.0471	0.0435	0.92	0.00148	0.00171	1.16	0.00150	1.01	0.00151	0.00171	1.13	0.00142	0.94
10	0.0448	0.0384	0.86	0.00245	0.00240	0.98	0.00251	1.02	0.00104	0.00102	0.98	0.00110	1.06
11	0.0444	0.0380	0.86	0.00259	0.00255	0.98	0.00256	1.01	0.00095	0.00103	1.08	0.00104	1.10
12	0.0238	0.0216	0.91	0.00124	0.00121	0.98	0.00122	0.98	0.00132	0.00134	1.02	0.00139	1.05
13	0.0118	0.0118	0.92	0.00123	0.00118	0.96	0.00127	1.03	0.00056	0.00061	1.09	0.00066	1.18
14	0.0118	0.0113	0.96	0.00134	0.00130	0.97	0.00141	1.05	0.00047	0.00052	1.11	0.00048	1.02
15	0.0046	0.0056	1.22	0.00036	0.00041	1.14	0.00050	1.39	0.00044	0.00050	1.14	0.00050	1.14
16	0.1643	0.0990	0.60	0.00476	0.00476	1.00	0.00436	0.92	0.00199	0.00265	1.33	0.00199	1.33
17	0.1627	0.1260	0.78	0.00493	0.00518	1.05	0.00496	1.01	0.00188	0.00300	1.60	0.00188	1.60
18	0.0847	0.0676	0.80	0.00243	0.00258	1.06	0.00198	0.82	0.00255	0.00261	0.99	0.00198	0.78
19	0.0756	0.0500	0.66	0.00424	0.00348	0.82	0.00347	0.82	0.00171	0.00170	0.99	0.00136	0.80
20	0.0732	0.0442	0.60	0.00434	0.00353	0.81	0.00330	0.76	0.00166	0.00174	1.05	0.00150	0.90
21	0.0386	0.0254	0.66	0.00214	0.00200	0.94	0.00165	0.77	0.00219	0.00193	0.88	0.00179	0.93
22	0.0197	0.0143	0.73	0.000212	0.000156	0.74	0.000161	0.76	0.00089	0.00080	0.90	0.00059	0.66
23	0.0191	0.0141	0.74	0.000219	0.000178	0.81	0.000187	0.85	0.00084	0.00079	0.94	0.00058	0.69
24	0.0102	0.0080	0.78	0.000108	0.000094	0.87	0.000092	0.85	0.00013	0.000099	0.88	0.000094	0.83

¹With rectangular panels, x direction is parallel to the short side, y direction is parallel to long side.

²Initial slope of load-deflection curve. Because of thinness of panel, it deflected more as a diaphragm than as a panel.

³Load-strain curve was not a straight line, therefore slope could not be determined.

⁴Specimens loaded through a copper diaphragm rather than rubber bag used in initial tests of other specimens.

Table 5. -- Comparison between test values and calculated shear strains in cores for one pound per square inch of load

Panel No.	Shear strains in <u>xz</u> plane			Shear strains in <u>yz</u> plane		
	Calculated	Observed	Ratio,	Calculated	Observed	Ratio,
			<u>obs.</u>			<u>obs.</u>
			calc.			calc.
	<u>Inch per inch</u>			<u>Inch per inch</u>		
1	0.00048			0.00018		
2	.00021	0.00058	2.76	.00041	0.00090	2.19
3	.00038	.00083	2.18	.00017	.00071	4.18
4	.00025	.00049	1.96	.00010	.00017	1.70
5	.00011	.00022	2.00	.00021	.00038	1.81
6	.00020	.00043	2.15	.00009	.00039	4.33
7	.00091	.00200	2.20	.00034	.00104	3.06
8	.00040	.00158	3.95	.00078	.00145	1.86
9	.00072	.00263	3.65	.00032	.00148	4.63
10	.00087	.00148	1.70	.00033	.00099	3.00
11	.00038	.00103	2.71	.00074	.00155	2.10
12	.00069	.00120	1.74	.00031	.00090	2.90
13	.00044	.00116	2.64	.00017	.00039	2.29
14	.00020	.00062	3.10	.00038	.00049	1.29
15	.00034	.00057	1.68	.00016	(1)
16	.00168	.00360	2.14	.00064	.00196	3.06
17	.00074	.00255	3.45	.00144	.00277	1.92
18	.00133	.00446	3.35	.00059	.00304	5.15
19	.00050	.00116	2.32	.00019	.00067	3.53
20	.00022	.00120	5.46	.00043	.00140	3.26
21	.00040	(1)00018	.00016	.89
22	.00025	(1)00010	.00062	6.20
23	.00011	.00014	1.27	.00022	.00076	3.45
24	.00020	.00039	1.95	.00009	.00005	.56

¹Gage did not function properly.

Table 6. --Comparison of computed ultimate shear loads with maximum loads obtained from test for panels designed to fail in the core

Panel No.	Thickness of core	Orientation of TW plane	Ultimate shear strength of core	Ultimate load	
				Computed	Observed
	<u>Inch</u>		<u>P.s.i.</u>	<u>P.s.i.</u>	<u>P.s.i.</u>
1	0.50	xz	226	14.3	17.0
2	.50	yz	226	16.8	19.0
3	.50	xz	226	18.1	23.0
4	1.00	xz	226	27.5	33.0
5	1.00	yz	226	32.7	39.0
6	1.00	xz	226	34.4	¹ 39.0
7	.25	xz	226	7.6	16.0
8	.25	yz	226	8.8	16.5
9	.25	xz	226	9.5	23.0
10	.50	xz	118	7.3	9.0
11	.50	yz	118	8.6	11.5
12	.50	xz	118	9.2	10.0
13	1.00	xz	118	14.4	13.9
14	1.00	yz	118	16.7	18.0
15	1.00	xz	118	18.7	18.5
16	.25	xz	118	3.8	7.7
17	.25	yz	118	4.4	8.2
18	.25	xz	118	4.8	9.0

¹-Failure was incomplete, but damage was severe.

Table 7. -- Comparison of computed ultimate loads¹ with maximum loads obtained from test for panels designed to fail in the facings

Panel No.	Thickness of core	Thickness of facings	Ultimate load	
			Computed	Observed
			P.s.i.	P.s.i.
19	0.50	0.012	11.7	15.5
20	.50	.012	11.6	14.5
21	.50	.012	19.5	16.0
22	1.00	.012	23.3	² 16.0
23	1.00	.012	22.9	19.0
24	1.00	.012	37.9	(3)

¹Ultimate loads were computed on the basis of 64,000 pounds per square inch as the ultimate tensile and compressive strength of the 24S-T3 facings.

²Premature shear failure along joint in core.

³Test discontinued at 29.0 pounds per square inch because of damage to loading equipment.

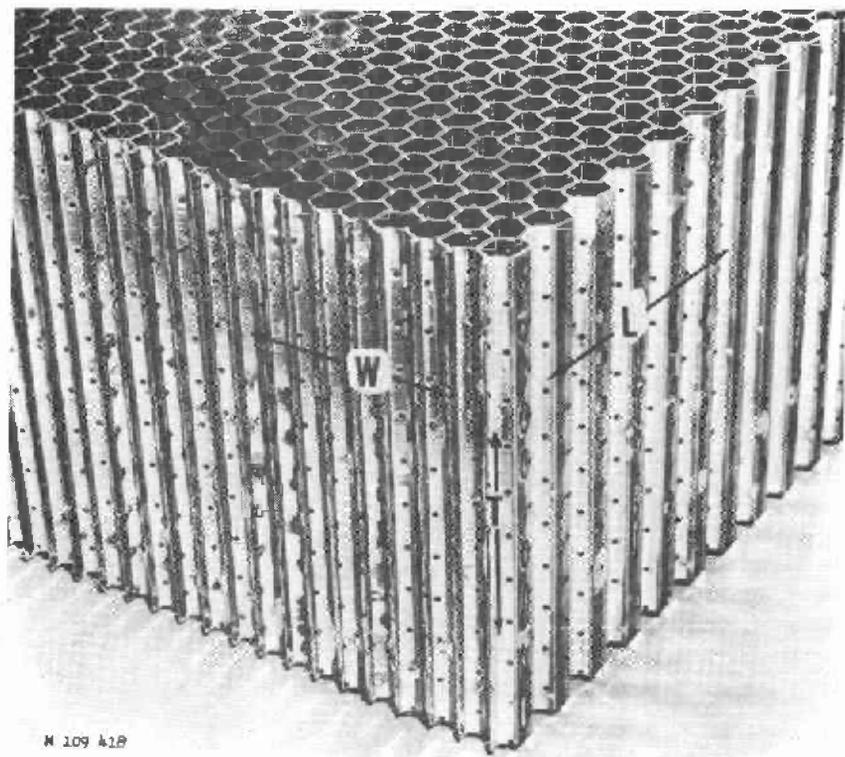


Figure 17. --Section of a block of aluminum honeycomb core material made from perforated aluminum foil. Directional orientation referred to in the text is T (thickness), W (width), and L (length).

7 M 109 418

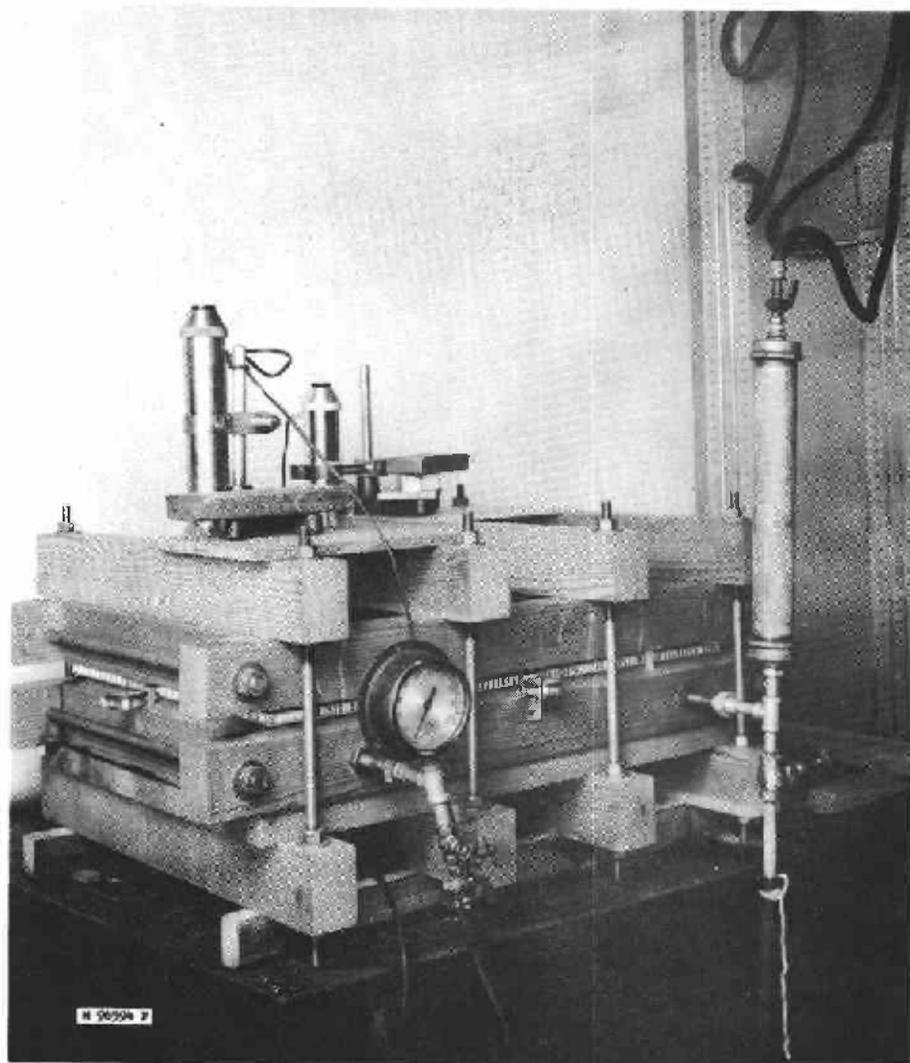


Figure 18. --General test setup for uniform loading of rectangular sandwich panel. Hydraulic pressure was applied to bottom surface of the specimen through needle valve and hose shown in lower center of figure.

Z M 96594 F

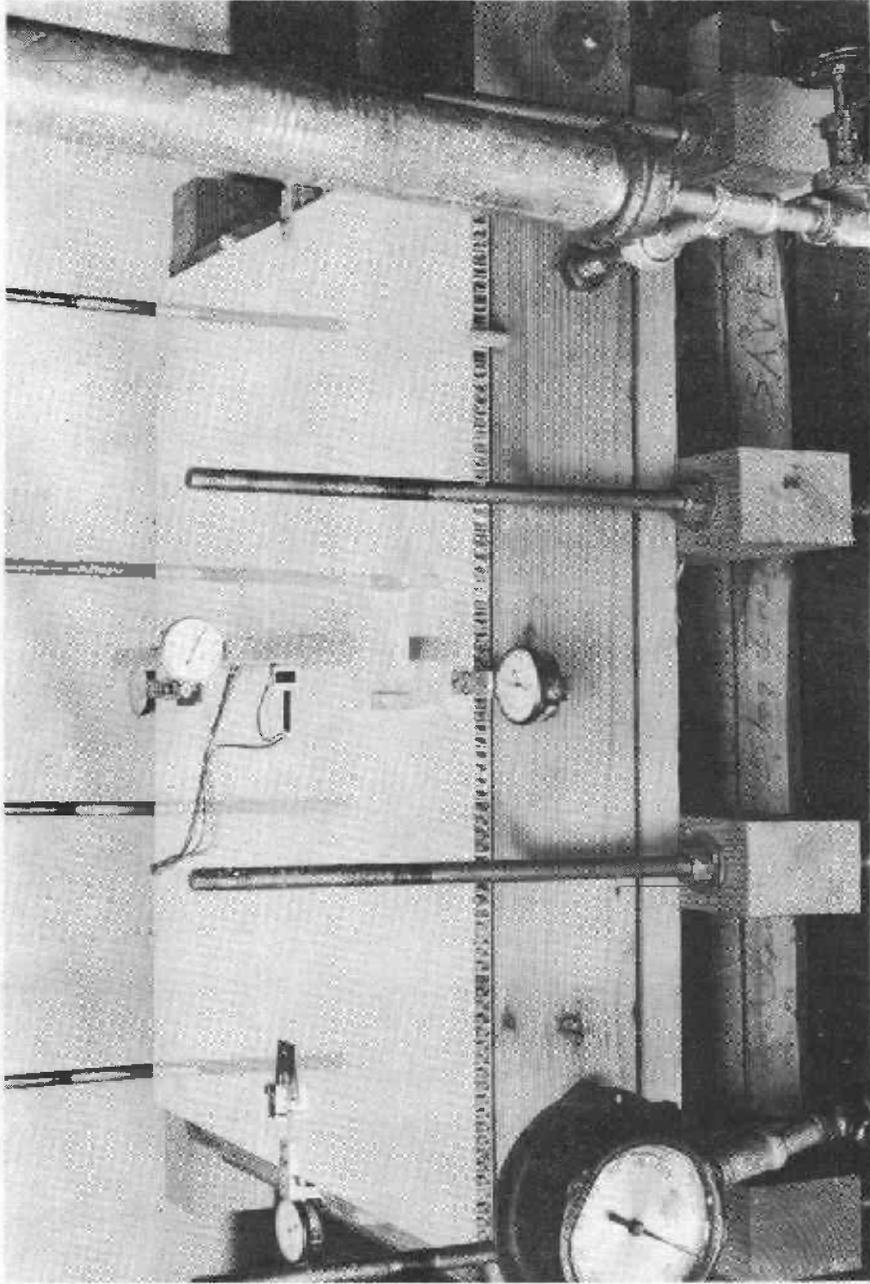


Figure 19. --Test panel in place showing location of bridge for measuring deflection of center of panel, electric resistance strain gages for measuring strains in faces, and Tuckerman optical gages for measuring movement of lower facing with respect to upper facing (shear).

Z M 96595 F

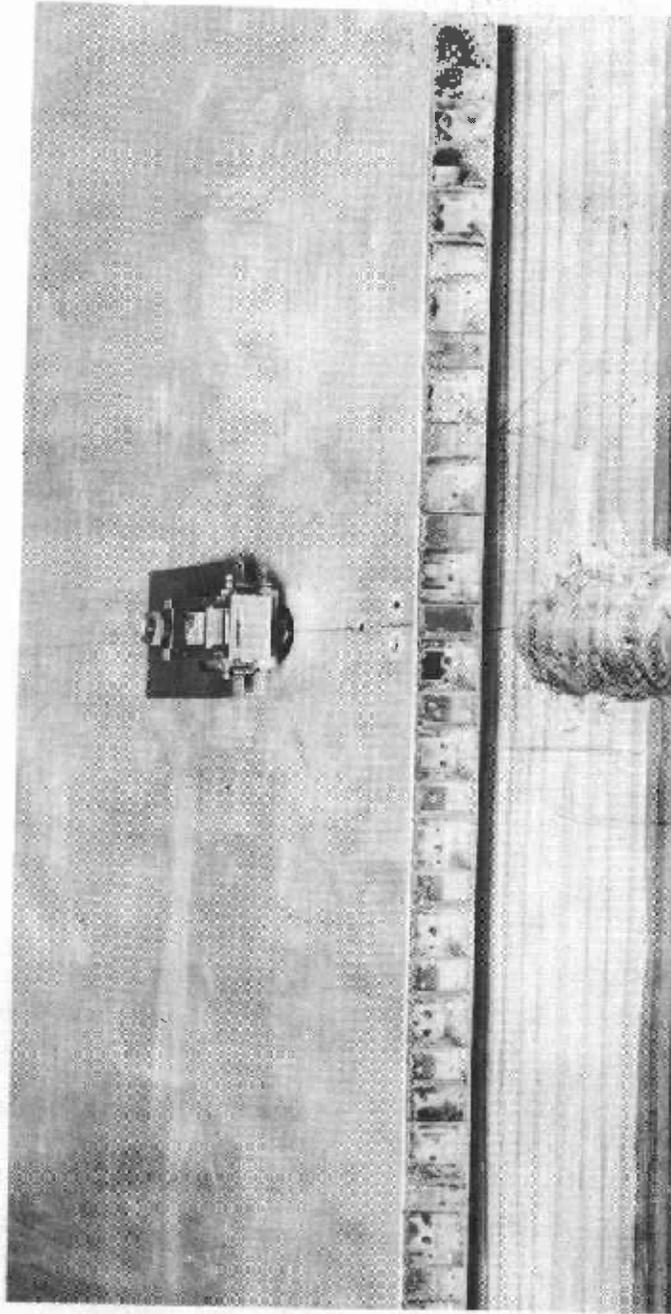


Figure 20. --Closeup view showing plug glued to lower facing of sandwich panel and method of attaching Tuckerman optical strain gage to measure differential movement of one facing with respect to the other during test.

Z M 96597 F

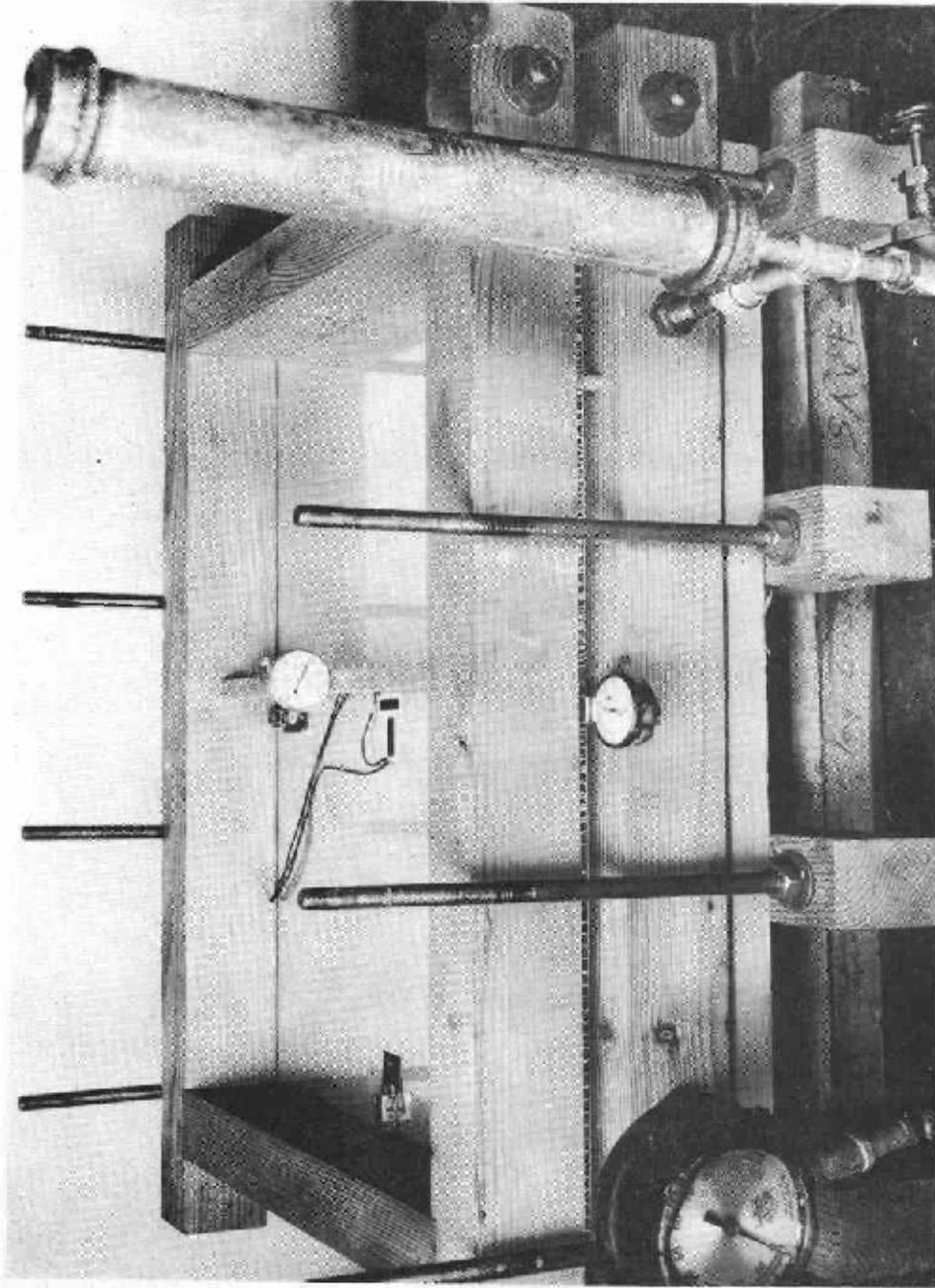


Figure 21. --- Test panel with supporting frame in place.

Z M 96596 F

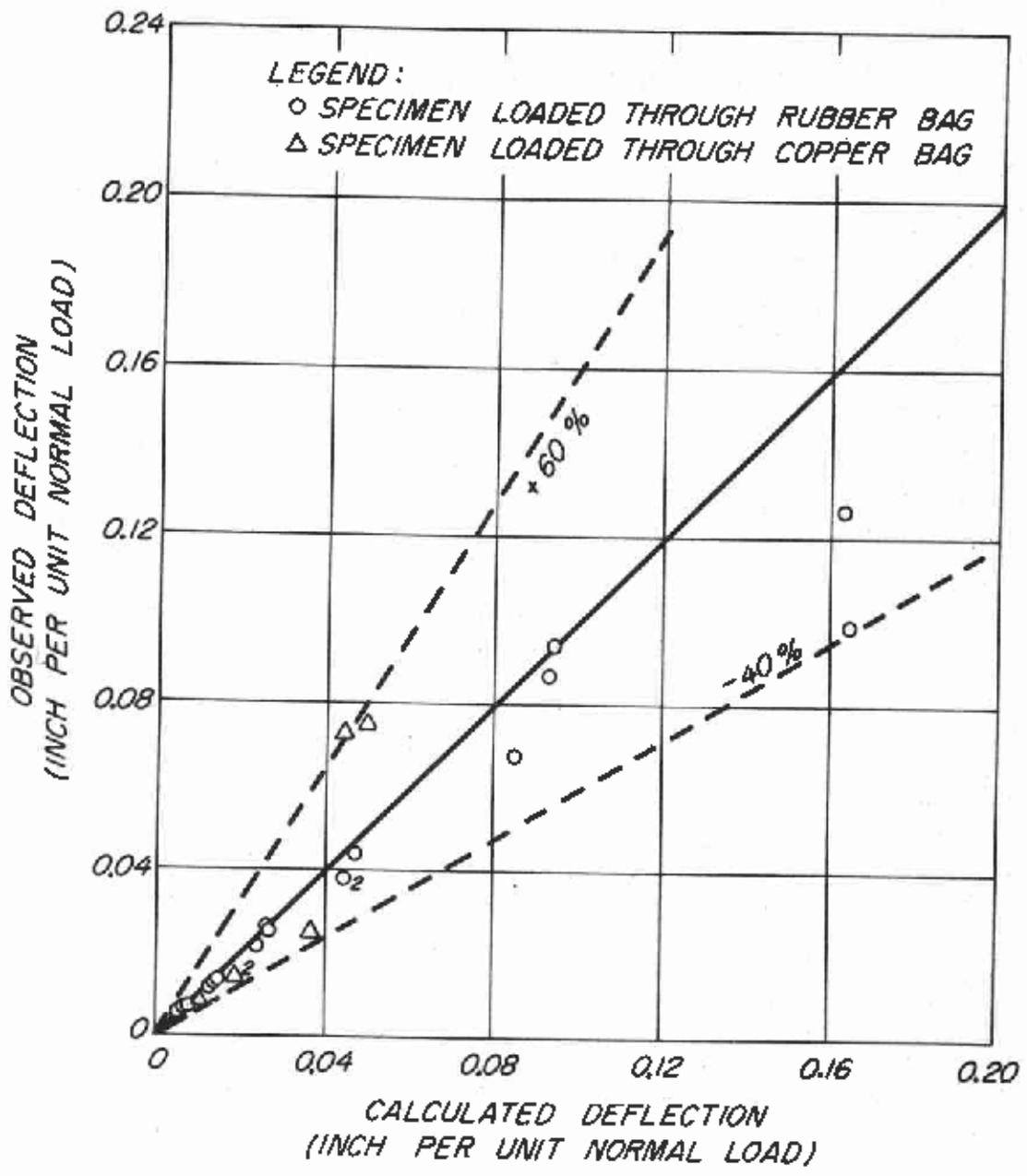


Figure 22. --Comparison between calculated and observed deflection.

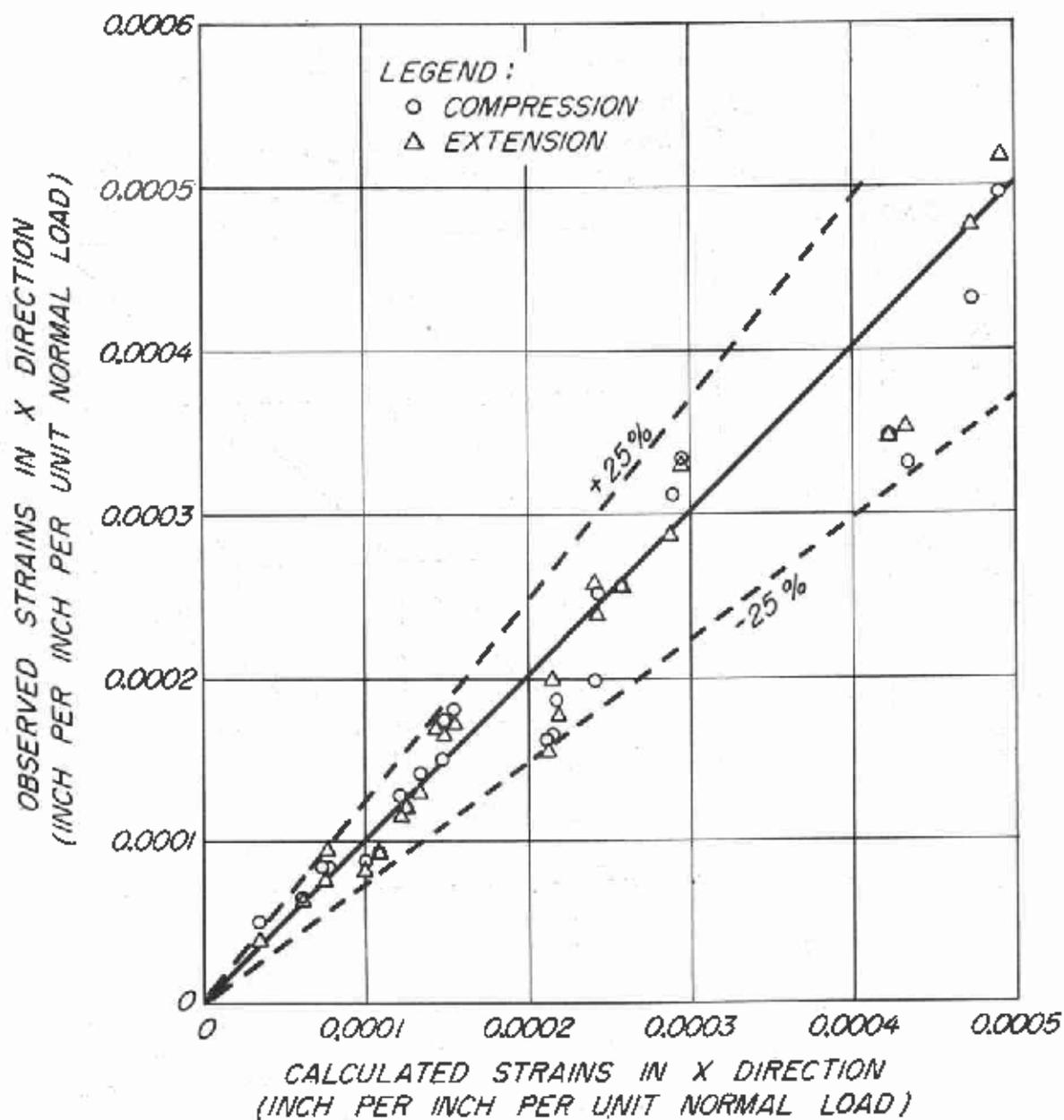


Figure 23. --Comparison between calculated and observed strains in the facings in the x direction.

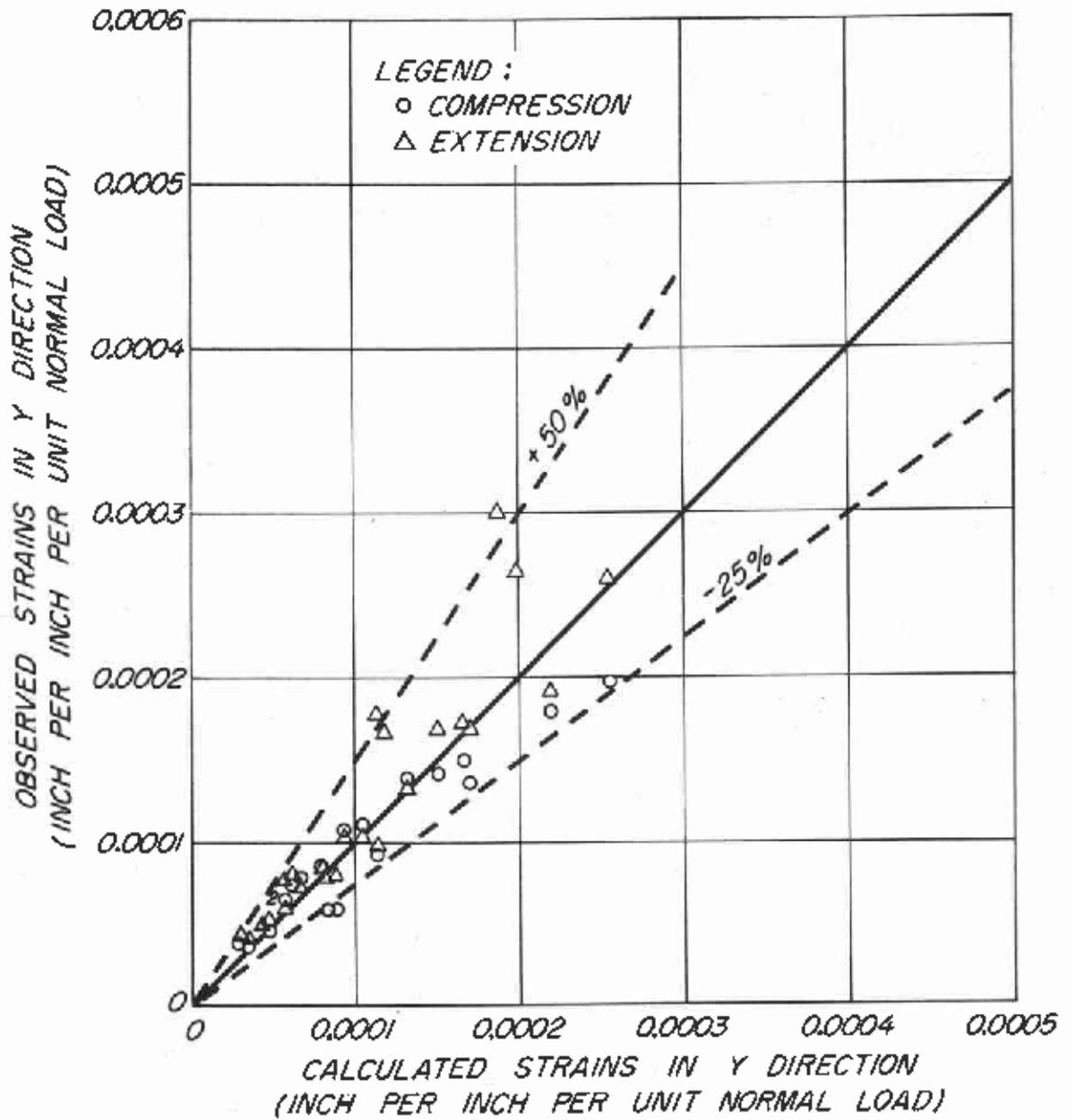


Figure 24. --Comparison between calculated and observed strains in the facings in the y direction.

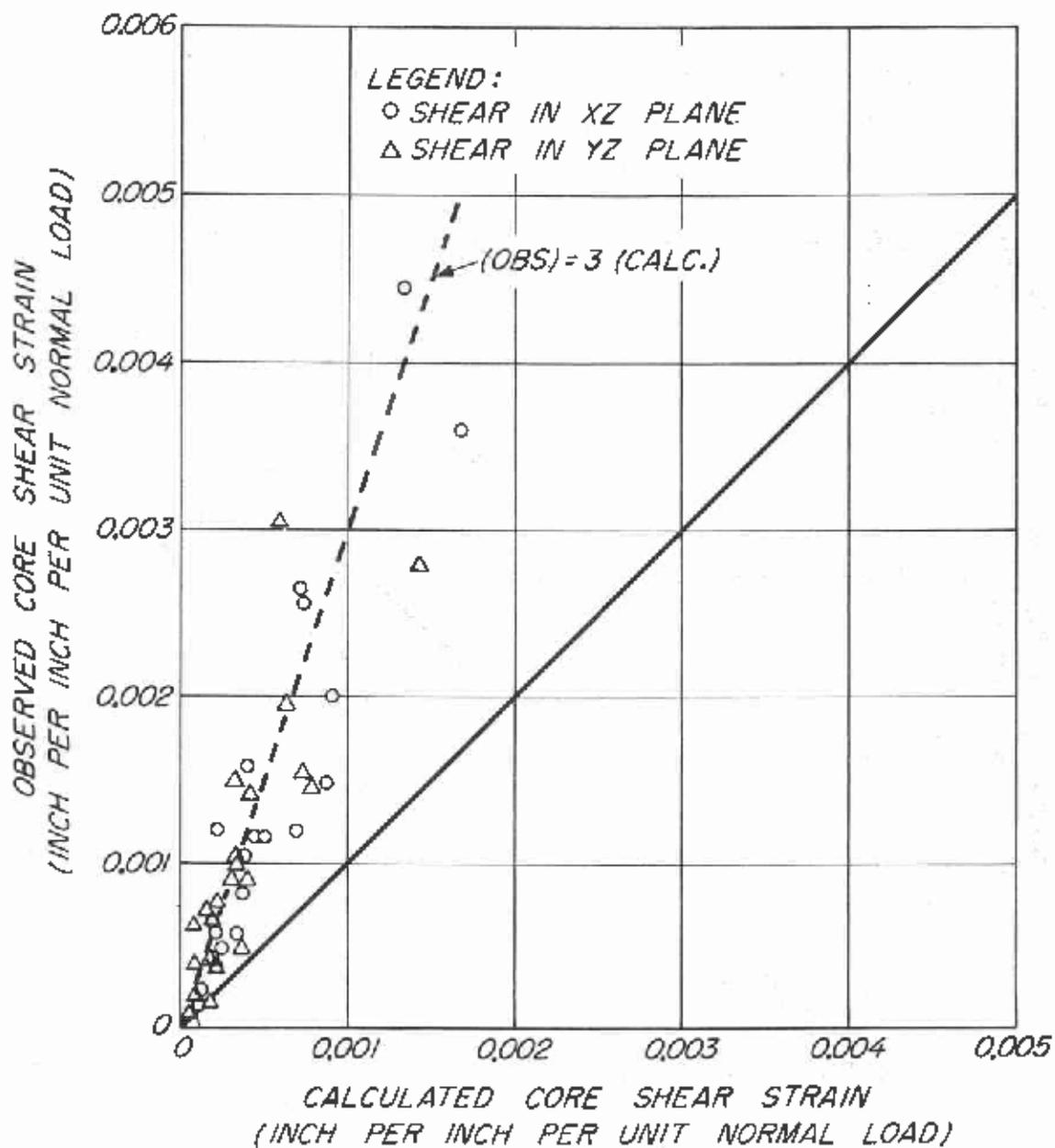


Figure 25. --Comparison between calculated and observed shear strains in the cores.

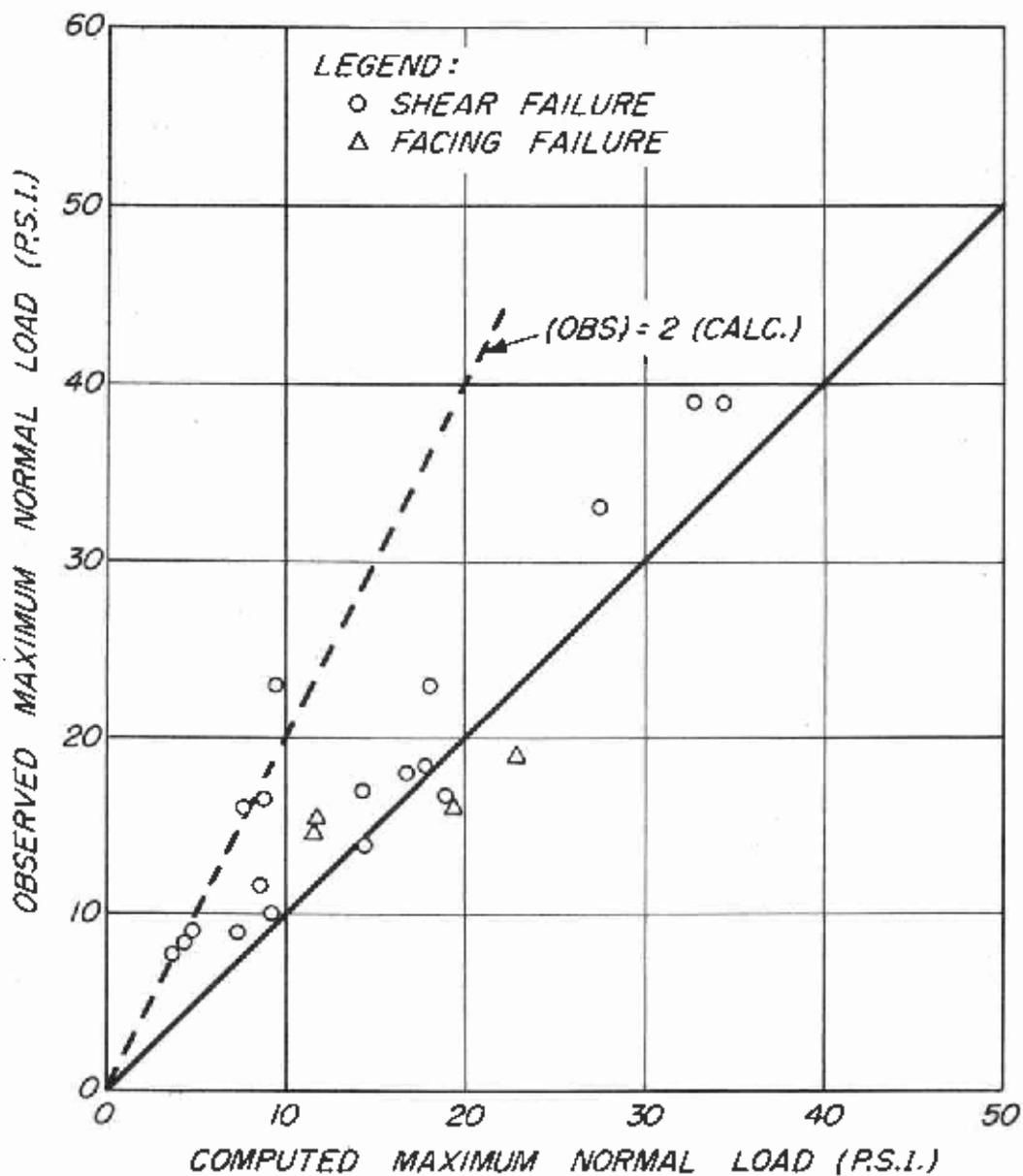


Figure 26. --Comparison between the computed and observed loads at failure.

Z M 109 423