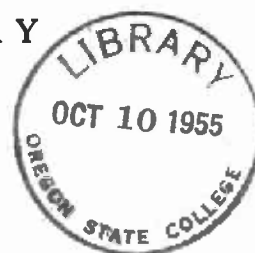# SIMPLIFIED METHOD OF SELECTING AND DESIGNING PACKAGE CUSHIONING MATERIALS

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Madison 5, Wisconsin
In Cooperation with the University of Wisconsin

SIMPLIFIED METHODS OF SELECTING AND

DESIGNING PACKAGE CUSHIONING MATERIALS

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Summary

The following methods of selecting and designing package cushioning
materials are presented: (1) an exact method of determining the
cushion thickness required to protect a given article from damage
caused by a drop of a specific height, (2) a simplified method of deter-
mining requirements when an uncertainty of ± 10 percent can be toler-
ated in the level of protection provided the article, and (3) equations
by which quick estimations of cushion-thickness requirements can be
made.

When cushion factor-stress curves are available for most cushioning
materials, the information contained herein could be used as the basis
of a cushion calculator.

Introduction

The Forest Products Laboratory, in cooperation with the Engineer
Research and Development Laboratories, Fort Belvoir, Va., has
developed three simplified methods of selecting and designing the

[1] Maintained at Madison, Wis., in cooperation with the University of
Wisconsin.

cushioning materials used to protect packaged articles from shock. The principles on which these methods are based were derived by the Laboratory in cooperation with the Packaging Section (WCRTH-5), Wright Air Development Center, Wright Patterson Air Force Base, Ohio (4). [2]

Previous reports written by the Laboratory and others contain information pertinent to this project, in which case specific reference is made to those reports. Other reports (2, 5, 6, 7) are referenced merely for background information.

A method of designing a specific cushion for a given article was previously reported (4), but the problem of selecting a cushion with the proper physical characteristics was not treated in detail. In addition to providing simplified methods of calculating cushion thickness, this report provides criteria for selecting a cushion of proper stiffness.

<div align="center">Design Considerations</div>

In order to design a package rationally to protect a given article from shock during handling and shipping operations, a level of performance in rough handling for the container, criteria of fragility for the article, and the energy absorbing capacity of cushions must be established.

Level of Performance

The determination of the hazards encountered by a package during handling operations is, in the final analysis, a statistical problem. If the statistical distribution of forces encountered in handling operations is known, the package can be designed for the successful transport of any desired percentage of the total shipment. When packaging for commercial shipment, it might be economically feasible to permit damage to a limited percentage of the goods shipped; whereas, for strategic military materials, it would be necessary to design for nearly 100 percent successful shipment.

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[2] Underlined numbers in parentheses refer to the list of numbered references, page 17.

In an analysis of performance, such factors as height of drop, type of surface, probability of a flat drop, and orientation of container at impact (edge, corner or flat drop) might be considered. Since information is not available on all these factors, height of drop is used herein as a basis for specifying the level of performance in handling operations. A 30-inch drop is usually used because of its relation to the height of a man's hands from the floor when the man is standing, but the methods used herein may be applied to any height of drop.

Article Fragility

As is the case with criteria for rough handling, numerous factors could be used to establish criteria of fragility. Variation in materials and the fatiguing effect of a series of equal or unequal shocks could again lead to a statistical approach to the problem. The effect of a certain type of shock on an article could be determined by analyzing the complete acceleration-time curve of the shock. Such an analysis, however, would be prohibitively expensive and time consuming for many applications.

To simplify this problem, the article is assumed to be rigid, and only the peak acceleration is used in this report as a criterion of fragility. It is obvious that this quantity alone is not completely sufficient. For example, an article given a constant acceleration in a centrifuge could not be damaged by collision, such as could occur in electronic equipment where a shock-mounted tube tipped far enough during shock to strike an object, such as a transformer. Thus, the fragility must vary with the pulse shape; that is, with the rate at which the acceleration is applied. This would lead to the problem of selecting the pulse shapes for which an article must be tested. A practical solution to this problem suggested by Masel (8) consists essentially of determining fragility by dropping an article on various cushions until damage occurs. The advantage here is that the article is tested under a situation similar to actual packaging conditions, and the pulse shape need not be considered unless the fragility expressed by the maximum acceleration varies sharply as a function of pulse shape.

While the use of maximum acceleration as a criterion for fragility is an oversimplification, it is thought that its use does lead to improved design. It also provides a simple basis for comparison of cushioning materials as to their effectiveness in limiting peak acceleration.

Another reason for selecting maximum acceleration is that it can be related to the maximum force for a rigid body through Newton's second law of motion, as shown in appendix I. Thus, it is convenient to use the quantity, maximum g-value, so that

$$G_m = \frac{a_m}{g} \tag{1}$$

where a_m is the maximum acceleration an article can withstand without damage in feet per second per second, g is acceleration due to gravity, 32.2 feet per second per second, and G_m is the maximum g-value, which is dimensionless. When using G_m, Newton's law becomes

$$F_m = W\, G_m \tag{2}$$

where W is the weight in pounds and F_m is the maximum force in pounds. Therefore, a rigid article having a maximum g-value of 50 and weighing 2 pounds could sustain without damage forces up to 100 pounds.

If the maximum force an article can sustain dynamically without damage is known, that quantity can be used just as effectively as maximum g-value or acceleration. Acceleration is used here since the quantities displacement, velocity, and acceleration have been generally used in shock and vibration studies. The use of g-value and acceleration also emphasizes the dynamic characteristics of the problem.

Cushions as Energy Absorbers

A cushioned pack, as shown in figure 1, consists essentially of (a) an article, often placed in a chipboard or fiberboard container; (b) a cushioning material; and, (c) an outer container. In order to analyze a cushioned pack with simple physical laws, it is necessary to use an idealized mechanical system representing a package during a drop, as shown in figure 2. In this system, the following assumptions have been made: The article, assumed to be rigid and of uniform density, is represented by the weight, W; the cushion, assumed to be massless, is represented by a spring; the outer container and floor are assumed to be rigid; the outer container is assumed to strike the floor flat so that no rotation of the article or container occurs.

When a cushioned pack is dropped from a height, h, (fig. 2, A) it receives a constant acceleration of 1g until the instant the container strikes the floor (fig. 2, B). At this instant, the container stops and the article continues to fall a distance, S, as shown in figure 2, C. The acceleration (negative) or force experienced by the article will depend on how quickly it is stopped. In general, the acceleration will be much greater than 1g. At the maximum displacement, the velocity of the article will be zero. If the force-displacement curve is monotonic, as it often is, the maximum acceleration or force will also be encountered at the maximum displacement.

The energy absorbed or stored by the cushion is equal to the area under the force-displacement curve for that cushion. Force-displacement curves are obtained by recording force and displacement on a universal testing machine as the thickness of the cushion is decreased. The change of thickness of the cushion is the displacement. Thus, when the outer container stops, the force on the packaged article increases until there is sufficient area under the force-displacement curve to equal the potential energy possessed by the article prior to drop.

The force-displacement curves of three types of cushions are shown in figure 3. Few cushions are known to possess the characteristics of an ideal cushion (fig. 3, A), and when they do they often have undesirable recovery characteristics. An example of this type is plastic foam cushioning. Although no real cushion shows a discontinuity at zero displacement, cushions that approach discontinuity, as shown by the dashed line, can be considered ideal. A steel spring is an example of a linear cushion (fig. 3, B). Anomalous cushions (fig. 3, C) are designed with the aid of graphical analysis because their force-displacement curve cannot be reasonably fit by mathematical equations. Many anomalous cushions are called tangent because their force-displacement curve resembles the trigonometric tangent function. In fact, an extensive cushioning analysis of tangent functions has been made by Mindlin (9). Except for certain approximations, however, the graphical analysis associated with anomalous elasticity will be used as a basis for all cushion design in this report.

For some purposes the data are left as force-displacement curves, but it is often more convenient to divide the force by the area of the cushion to obtain stress, thus:

$$f = \frac{F}{A} \tag{3}$$

where $\underline{F}$ is the force in pounds, $\underline{A}$ is the area in square inches, and $\underline{f}$ is the stress in pounds per square inch; and to divide the displacement by the cushion thickness to obtain strain, thus:

$$s = \frac{S}{T} \tag{4}$$

where $\underline{S}$ is the displacement in inches, $\underline{T}$ is the measured thickness of cushion in inches, and $\underline{s}$ is the strain in inch per inch, which is dimensionless.

The use of stress-strain curves has several distinct advantages over stress-displacement or force-displacement curves when they are applied to cushions whose area and thickness can be changed. For example, by changing the bearing area of a cushion that is too stiff or too soft for a given problem, the cushion can be used more efficiently. Another advantage is that the stress-strain curve is independent of the actual area and thickness of cushion tested to obtain the force-displacement data. Thus, only one curve is required to describe the energy absorbing properties of the cushion, which results in greatly simplified design curves.

The stress-strain curves of the materials used to illustrate the proposed methods of design in this report are shown in figure 4, and the materials are described in table 1. The strain axis is shown up to values of 1, since that is the maximum value of strain that could result from displacement of a cushion.

The procedure for obtaining the force-displacement data from which the stress-strain curves are obtained is important because most cushions are not perfectly resilient and do not fully recover their initial height after displacement. The procedure used to obtain the data from which the curves in figure 4 were obtained is given in reference (4). The curves all represent the initial displacement of the respective cushions. Although a procedure for repeated displacement is also given in reference (4), very little data have been obtained with it. A different procedure for obtaining static force-displacement data now being developed by ASTM Committee D-10, Subcommittee VI appears to have a good possibility of wide acceptance.

It should also be emphasized that the data on which the curves in figure 4 were based were obtained from static force-displacement tests. It appears quite possible that data resulting from dynamic tests, for at

least some materials, may be different than data resulting from static
tests. It is expected, however, that the methods presented in this re-
port can be used for dynamic data when they are available, and it also
appears that design based on static rather than dynamic data will be
conservative. Other factors that affect the force-displacement data of
various cushions, such as temperature and humidity, have not been
carefully examined.

In discussing the choice of proper cushion, the relative ease with which
cushions can be displaced is often referred to qualitatively as softness
while the difficulty of displacement is referred to as stiffness. These
quantities are defined more exactly as the slope of the force-displacement
curve (often called spring rate) or as the slope of the stress-strain curve
(often called modulus of elasticity). For a series of stress-strain curves
of similar shape, the cushion with the higher slope at a given strain is
called the stiffer cushion.

<center>
Three Methods of Selecting and
Designing Package Cushioning
</center>

The simplicity of cushion calculations depends in part on the extent to
which the stress-strain curves of the cushions are similar or are
members of a family of curves and in part on the uncertainty that can
be tolerated in the cushion calculations. This will be illustrated by
means of sample calculations involving the following:

1. An exact method of selecting and designing package cushioning.

2. A simplified method of selecting and designing package cushioning
when an uncertainty of ± 10 percent can be tolerated in the level of pro-
tection provided for the packaged article.

3. Several equations for quick estimations of cushion thickness.

1. Exact Cushion Design

The thickness of cushion required to protect a given article from damage
caused by a drop of specific height is directly proportional to a ratio of
$\underline{h}$ to G_m, as shown in appendix II. Thus,

$$T = \frac{C\,h}{G_m} \tag{5}$$

where $\underline{C}$ is the cushion factor, which is dimensionless.[3] As described on page 5 and in appendix II, it is assumed that the equation applies only to flat drops, and it is valid only for cushions whose thickness is approximately one-fifth the height of drop or less. For example, if $\underline{h}$ is 30 inches, then equation (5) holds for thicknesses up to 6 inches.

Since the cushion factor varies as a function of stress for a given cushion, the maximum stress must be known. This is obtained by combining equations (2) and (3) to obtain,

$$f_m = \frac{W \, G_m}{A} \qquad (6)$$

where f_m is the maximum stress in pounds per square inch.

Equations (5) and (6) are related to each other by cushion factor-stress curves. These curves are obtained from stress-strain curves as follows:

The area under the stress-strain curve up to several values of strain is obtained with a planimeter or by counting squares. From the area under the curve, the energy per unit volume is obtained by a suitable conversion factor. Then, knowing the stress corresponding to the energy per unit volume at each value of strain, the ratio of stress to energy per unit volume can be obtained. The cushion factor-stress curves for nine materials are shown in figure 5.

Note that a wide range of cushion stiffness is presented. The difference in stress for a soft cushion, such as material No. 4, and a relatively stiff cushion, such as material No. 6, may vary by a factor of at least 1,000. With equations (5) and (6) and with cushion factor-stress curves for a series of materials, it is possible to select and design package cushioning. Cushion design by this method is as precise as graphical analysis will permit, assuming that the cushion factor-stress curves and the assumptions made in deriving the formulae are valid.

To illustrate the use of equations (5) and (6) in connection with cushion factor-stress curves, assume that weight $\underline{W}$ is 3 pounds, area $\underline{A}$ is 20

[3] The use of the phrase "cushion factor" has been suggested by Slaughter (13). However, the quantity defined as "cushion factor" by Slaughter is the reciprocal of the cushion factor used in this report. The cushion factor used in this report is the same as the "J_s" used by Janssen (3) and the same as the "ratio of force to energy" used by Orensteen (10, 11, 12).

inches^2, maximum g-value G_m is 40, and height of drop $\underline{h}$ is 30 inches, then from equation (6)

$$f_m = \frac{W\,G_m}{A} = \frac{6\ \text{lb.}}{\text{in.}^2}$$

From figure 5, it is evident that at a stress of 6 pounds per square inch, material No. 5 has the smallest cushion factor; therefore, the least thickness will be required with this material. The required thickness of material No. 5 may be calculated with equation (5) as follows:

$$T = \frac{C\,h}{G_m} = \frac{4.7 \times 30\ \text{in.}}{40} = 3.5\ \text{in.}$$

If material No. 8 were substituted for No. 5, the resulting thickness would be

$$T = \frac{C\,h}{G_m} = \frac{5.5 \times 30\ \text{in.}}{40} = 4.1\ \text{in.}$$

A thickness of 4.5 inches would be needed with material No. 3, which is too soft, and 4.3 inches would be needed with material No. 6, which is too firm. These thicknesses are based on a maximum stress of 6 pounds per square inch. However, if the bearing area were changed so that maximum stress corresponded to minimum cushion factor (3), the thickness requirements of materials No. 3 and No. 6 could be reduced. The minimum cushion factor for material No. 3 occurs at a stress of 2.5 pounds per square inch. With equation (6), the bearing area would have to be changed from 20 to 48 square inches, as follows:

$$A = \frac{W\,G_m}{f_m} = \frac{3\ \text{lb.} \times 40}{\dfrac{2.5\ \text{lb.}}{\text{in.}^2}} = 48\ \text{in.}^2$$

With the increased bearing area, the thickness required of material No. 3 would be

$$T = \frac{C\,h}{G_m} = \frac{4.6 \times 30\ \text{in.}}{40} = 3.5\ \text{in.}$$

The minimum cushion factor for material No. 6 occurs at a stress of
10.2 pounds per square inch; therefore, the bearing area must be
reduced from 20 to 11.8 square inches. With the decreased area, a
3.8-inch thickness of material No. 6 would be required, which is
slightly greater than requirements for material No. 3. Material No. 6
might still be a better cushion, however, because its bearing area was
reduced. The increased area of No. 3 would require extra blocking,
and it might increase the cube of the pack.

2. Approximate Cushion Design

If some uncertainty can be tolerated in the protection given an article,
it is possible to simplify cushion selection and design. Let $\bar{C}$ be an
approximate cushion factor such that $\bar{C}$ - 0.1 $\bar{C}$ equals the minimum
cushion factor for a given cushion. Then it is possible by the method
shown in appendix IV to establish a range of stress over which $\bar{C}$ may
be used with a maximum error of ± 10 percent. Figure 5 shows that
the cushion factor of tangent (Nos. 1 through 8) cushions varies slowly
around its minimum as a function of stress. This effect has also been
observed by Best (1). Table 2 shows average cushion factors and stress
ranges for materials Nos. 1 through 8. This tabulation, along with equa-
tions (5) and (6), gives sufficient information to design cushioned packs
with a maximum uncertainty in the level of protection (maximum stress)
of ± 10 percent.

To illustrate the use of this method the problem given on page 8 is again
solved. The maximum stress of 6 pounds per square inch falls within
the range listed for materials Nos. 5, 6, and 8. Of these materials,
No. 5 has the lowest approximate cushion factor so that it will require
the least thickness. The thickness for material No. 5 will be, using
equation (5),

$$T = \frac{C\,h}{G_m} = \frac{5 \times 30 \text{ in.}}{40} = 3.75 \text{ in.}$$

Since the correct answer, based on the calculations shown for method
1, for material No. 5 is 3.5 inches, the error is 7.2 percent. It is
possible, of course, to change the bearing area as described on page
9, so that the maximum stress falls within the stress range of a given
cushion.

With an uncertainty of ± 10 percent, a cushion might be underdesigned, but this would probably be compensated for by other factors. For example, it seems unlikely that the error in g-values, which may vary from 50 to 100 percent, and the uncertainty of the height of drop, which is an assumed value, will ever be less than ± 10 percent. Furthermore, several factors, such as the cushioning effect of the container and dropping surface and the friction between the cushion and the sides of the article, tend to make design conservative.

A disadvantage in the approximate method of design, however, is that it is difficult to establish stress limits and approximate cushion factors for materials with stress-strain curves that are not all members of a single family of curves. The curves for material No. 9 (figs. 4 and 5), for example, do not belong to the same family as the other eight materials. Although material No. 9 has a low minimum cushion factor, the ± 10 percent limitation on stress would give it a narrow stress range, which would put it in unfavorable competition with other materials. With a wide range of stress, however, material No. 9 could compete favorably.

Since the stress-strain curve of material No. 9 is unusual, this difficulty is not often encountered with commonly used cushions. This demonstrates, however, the advantage in using the exact method when cushion design is not confined to cushions with one type of stress-strain curve.

3. Quick Estimation of Cushion Thickness

The coefficient, $\underline{C}$, in equation (5) depends on the shape of the force-displacement or stress-strain curve, whichever is used to describe the energy absorbing capacity of a cushion. If the general shape of these curves for available cushions is known, then a specific value of this coefficient can be used to estimate cushion thickness.

Ideal and Linear Cushions

Because the force-displacement curves for ideal and linear cushions can be represented mathematically, the cushion thickness can be calculated exactly if $\underline{h}$ and G_m are known. As shown in appendix III, the minimum displacement of an ideal cushion (fig. 3) is

$$S_{ideal} = \frac{h}{G_m} \qquad (7)^{\underline{4}}$$

where $S_{\underline{ideal}}$ is minimum displacement of an ideal cushion.

Since equation (7) allows no room for the cushion at maximum displacement, it is convenient to combine equations (4) and (7) to obtain the minimum thickness of an ideal cushion as

$$T_{ideal} = \frac{1}{s_m} \; \frac{h}{G_m} \qquad (8)$$

where $s_{\underline{m}}$ is the strain at maximum displacement in inch per inch.

For a linear cushion (fig. 3), it is shown in appendix III that the minimum cushion displacement is

$$S_{linear} = \frac{2\,h}{G_m} \qquad (9)^{\underline{5}}$$

where $S_{\underline{linear}}$ is the minimum displacement of a linear cushion. By combining equations (4) and (9), we obtain the minimum thickness of a linear cushion as,

$$T_{linear} = \frac{2}{s_m} \; \frac{h}{G_m} \qquad (10)$$

The application of equations (8) and (10) will be illustrated in a paragraph that follows.

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[4] This equation holds only if a cushion of proper stiffness is used. The force constant must equal the maximum force exerted on the article, that is, if a spring of improper stiffness was used, equation (7) would not hold.

[5] This equation holds only if a cushion of proper stiffness is used; the spring rate must equal the maximum force exerted on the article divided by the maximum displacement.

Anomalous Cushions

Of the 9 cushions analyzed for this report, 8 of them are described as having tangent-type elasticity even though they are not analyzed by exact tangent functions. Cushion No. 9 is the exception. Over the wide range of stiffnesses included by these eight materials, it is always possible to obtain a cushion factor of approximately 5 by choosing a cushion with the proper stiffness. Therefore, it is assumed that, for approximately tangent-type materials, the thickness of a cushion with proper stiffness can be estimated in general by

$$T = \frac{5\,h}{G_m} \tag{11}$$

The selection of a cushion of proper stiffness for equation (11) to be valid has already been discussed in the section on exact cushion design. For the tangent-type materials analyzed in this report, the stiffness will generally be proper if the strain at maximum displacement is between 0.4 and 0.65.

Illustration

As an example of estimating cushion thickness with equations (8), (10), and (11), let the height of drop equal 30 inches and the maximum g-value equal 50. Since equations (8) and (10) include strain, it is necessary to assume a strain in order to compare them with equation (11). If we assume s_m to be 0.5, then equation (8) would require a 1.2-inch thickness for an ideal cushion, equation (10) would require 2.4 inches for a linear cushion, and equation (11) would require 3 inches for a tangent cushion. Although 3 inches may be the best thickness for many cushioning materials, it can be assumed that the thickness could be cut in half, if a cushion of proper stiffness with a force-displacement curve that is partly concave downward, such as the curve for material No. 9, were used. Over a limited range, this type of cushion somewhat resembles the ideal cushion.

This method of estimating cushion thickness would be useful to engineers who set up packaging requirements. For example, once they knew that a 48-inch drop and a g-value of 10 would call for 24 inches of tangent cushioning, according to equation 11, they would realize such requirements were impractical. (Actually, 24 inches would be less than required, because the cushion thickness would be greater than one-fifth the height of drop.) Even if this figure were cut in two through the

use of special cushions, it would still not generally yield a practical answer. It may seem naive to juggle height of drop and maximum g-value to fit a preconceived value of proper cushion thickness rather than just guess the thickness and stiffness of cushion required for a given article. However, by determining $\underline{h}$ and G_m, each type of cushion is given an equal chance to meet the requirements by the methods given in this report.

Thickness Efficiency

The cushion design resulting from different types of cushion can be compared by comparing the thickness efficiency of the cushions. Thickness efficiency has been defined in (4) as

$$\text{Efficiency} = \frac{T_{ideal}}{T} \times 100 \text{ percent} \qquad (12)$$

where $\underline{T}$ is the measured thickness of cushion in inches, and T_{ideal} is the thickness of cushion with ideal elasiticity (maximum strain = 1) for the same cushion design problem in inches. Thus, the efficiency of a linear spring can be obtained from equations (8) and (10) as 50 percent when s_m is 1. For a linear spring that bottoms at a strain of 0.5, the efficiency would be 25 percent. From equations (8) and (11) the efficiency of cushions is

$$\text{Efficiency} = \frac{1}{C} \times 100 \text{ percent} \qquad (13)$$

For the commonly available tangent cushions, such as Nos. 1 through 8, the maximum efficiency will be approximately 20 percent (1/5 x 100 percent).

Conclusions

The three methods of cushion design presented are in many respects complementary. The formulae for "estimating cushion thickness" will help the package engineer determine reasonable design criteria. They will be particularly useful if reliable data on article fragility and rough handling are not available. The "exact cushion design" method will be most useful when it is desirable to obtain maximum thickness efficiency. The "approximate cushion design" method will be the simplest to use because it is completely tabular, and no curves are required by the

user. The most convenient way to use this method would be to list the average cushion factor and stress range for each material, along with other pertinent information, such as cost, density, fire resistance, effect of moisture, compression-set characteristics, and availability data.

Although a calculator could at any time be set up for computing $\frac{W\,G_m}{A}$ and $\frac{C\,h}{G_m}$, the operations are purely arithmetic and can be performed by longhand, slide rule, or calculating machine. When cushion factor-stress curves based on a generally accepted test procedure are available for most commonly used cushioning materials, it may be practical to construct various tables and calculators for use by cushion designers.

NOTATION

A = area in square inches

a = acceleration in feet per second per second

C = cushion factor, which is dimensionless

E = energy in inch-pounds

e = energy per unit volume in inch-pounds per cubic inch

F = force in pounds

f = stress in pounds per square inch

G = g-value, which is dimensionless

g = acceleration due to gravity, which is 32. 2 feet per second per second

h = height of drop in inches

m = mass in slugs

S = displacement in inches

s = strain in inch per inch, which is dimensionless

T = measured cushion thickness in inches

W = weight in pounds

Subscripts -

m = maximum value for a given problem. Other subscripts are defined in the particular section in which they are used.

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APPENDIX I

From Newton's second law of motion we obtain,

$$F_m = ma_m \tag{1}$$

where F_m is the maximum force in pounds, m is mass in slugs, and a_m is maximum acceleration in feet per second per second.

Since,
$$W = mg \tag{2}$$

where W is the weight in pounds and g is acceleration due to gravity or 32.2 feet per second per second, by combining equations (1) and (2), we obtain

$$F_m = \frac{Wa_m}{g} \tag{3}$$

To facilitate the use of acceleration in this formula, the term g-value is defined as the ratio of an acceleration to the acceleration due to gravity, thus

$$G = \frac{a}{g} \tag{4}$$

where G is g-value, a dimensionless ratio, a is acceleration in feet per second per second, and g is acceleration due to gravity or 32.2 feet per second per second. The maximum g-value and acceleration a given article can sustain without being damaged are given by

$$G_m = \frac{a_m}{g} \tag{5}$$

Thus, if the maximum acceleration a given article can withstand is 3,220 feet per second per second, or 100 g, the maximum g-value would be 100.

Substituting equation (5) into equation (3), we obtain

$$F_m = WG_m \tag{6}[a]$$

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[a] Equation (6) could be modified to take into account the weight of the article. For a falling body, a vertical force acting upward on the article would equal the sum of the inertial force and weight of the article, thus $F_m = WG_m + W$ or $F_m = W (G_m + 1)$. Since the maximum g-values for articles are generally assumed to be greater than 10, equation (6) can be used to a good approximation.

Dividing both sides of equation (6) by the area, $\underline{A}$, gives the maximum stress,

$$f_m = \frac{W\, G_m}{A}$$

where $\underline{f_m}$ is maximum stress in pounds per square inch.

APPENDIX II

The energy absorbed or stored per unit area per unit thickness of cushion can be equated to the integral of the stress-strain curve. Thus,

$$\frac{E}{AT} = \int_0^{s_m} f \, ds \tag{1}$$

where E is the energy absorbed by cushion in inch-pounds, A is the bearing area of cushion in square inches, T is the measured thickness of cushion in inches, and s_m is maximum strain in inch per inch. Because stress is an unknown function of strain, the $\int f \, ds$ is also unknown. However, the $\int f \, ds$ for each value of strain can be obtained graphically from the area under the stress-strain curve. For simplicity in notation, let

$$e_m = \int_0^{s_m} f \, ds \tag{2}$$

Since

$$E = W(h + s_m T) \tag{3}$$

where W is the weight of the article in pounds and h is height of drop in inches, we obtain, by combining equations (1), (2), and (3),

$$T = \frac{Wh}{Ae_m - Ws_m} \tag{4}$$

But, if $h > 10 \, s_m T$, then to a good approximation

$$E = Wh \tag{5}$$

and we obtain by combining equations (1), (2), and (5)

$$T = \frac{Wh}{Ae_m} \tag{6}$$

The maximum stress exerted on the article, as shown in appendix I, is given by

$$f_m = \frac{W \, G_m}{A} \tag{7}$$

where f_m is maximum stress in pounds per square inch, and $\underline{G_m}$ is maximum g-value, the dimensionless ratio of the maximum acceleration of the article to the acceleration due to gravity. Acceleration due to gravity is 32.2 square feet per second.

Combining equations (6) and (7) yields

$$T = \frac{f_m}{e_m} \ \frac{h}{G_m} \tag{8}$$

Again for convenience in notation, let

$$C = \frac{f_m}{e_m} \tag{9}$$

where $\underline{C}$ is the cushion factor, which is dimensionless.

Equations (8) and (9) yield

$$T = \frac{C\ h}{G_m} \tag{10}$$

APPENDIX III

Ideal Cushions

When an article is packed with a properly designed ideal cushion, it receives a constant acceleration equal to the maximum permissible acceleration for the article from the time the container strikes the floor until the article is stopped. Thus, the article is stopped in the minimum distance without exceeding its maximum g-value. The equation for an ideal cushion (fig. 3, A) is

$$F = K_{ideal} \tag{1}$$

where K_{ideal} is force constant in pounds for an ideal cushion, and F is force exerted by the cushion on the article in pounds.

From the law of conservation of energy, the potential energy of the article at height, h, must equal the energy stored or absorbed by the cushion when the article comes to rest after impact, thus

$$Wh = \int_{0}^{S_{ideal}} F \, dS \tag{2}$$

where S is displacement of the cushion, S_{ideal} is maximum displacement of an ideal cushion, and F is force which, for a cushion with ideal elasticity, is also the maximum force, F_m, in pounds.

Integrating equation (2) and solving for S_{ideal} we obtain

$$S_{ideal} = \frac{Wh}{F_m} \tag{3}$$

Substituting WG_m for F_m in equation (3) gives

$$S_{ideal} = \frac{h}{G_m} \tag{4}$$

Thus, equation (4) gives the minimum stopping distance or cushion thickness required for a given cushion design problem. Note that for $G_m = 1$, the article must fall a distance equal to the height of drop

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after the container stops and that the displacement, S_{ideal}, does not include any space for the compressed cushion at the maximum displacement.

Linear Cushions

When an article is cushioned with a linear cushion, it is subject to an increasing acceleration from the time the container strikes the floor until the article is stopped. If the cushion is properly designed, and the height of drop is the same as that for which the cushion is designed, the maximum acceleration that will occur just as the article stops will equal the maximum permissible acceleration for the article. The equation for a linear cushion (fig. 3, B) is

$$F = K_{linear}\, S \qquad\qquad (5)$$

where K_{linear} is the spring rate in pounds per inch of a linear cushion. The energy equation (2) becomes

$$Wh = \int_0^{S_{linear}} F\, dS \qquad\qquad (6)$$

where S_{linear} is the maximum displacement of a linear cushion. Substituting equation (5) for F in equation (6) and integrating, we obtain

$$Wh = \frac{K_{linear}\, S^2_{linear}}{2} \qquad\qquad (7)$$

Using equation (5), we can write $F_m = K_{linear}\, S_{linear}$ $\qquad$ (8)

From equations (7) and (8) and letting $F_m = WG_m$ we obtain

$$S_{linear} = \frac{2h}{G_m} \qquad\qquad (9)$$

APPENDIX IV

Let $\overline{C}$ be a number such that $\overline{C} - 0.1 \overline{C}$ equals the minimum cushion factor for a given material. Once $\overline{C} + 0.1 \overline{C}$ is known, the upper and lower stress limits can be obtained from a cushion factor-stress curve so that the maximum error in cushion thickness would be ± 10 percent. It is not the error in cushion thickness, however, but the uncertainty in the level of protection (stress) that is the true criterion of design error. When the approximate cushion factor + 10 percent was used to obtain stress limits from cushion factor-stress curves for materials Nos. 1 through 8, the maximum error was only 20 percent, and it was generally less than 15 percent. Therefore, the stress limits obtained from the thickness error did not differ greatly from those obtained from the stress uncertainty.

If $\overline{C}$ is the approximate cushion factor, T_1 the actual calculated thickness based on an approximate cushion factor, e_1 the actual energy per unit volume based on T_1, f_1 the actual stress corresponding to e_1, f the correct stress based on article g-value, h the height of drop, and G_m the maximum g-value

then
$$T_1 = \frac{\overline{C} h}{G_m} \tag{1}$$

also
$$e_1 = \frac{Wh}{A T_1} \tag{2}$$

but
$$\frac{W}{A} = \frac{f}{G_m} \tag{3}$$

so
$$e_1 = \frac{f h}{G_m T_1} \tag{4}$$

From equations (1) and (4) we obtain

$$e_1 = \frac{f}{\overline{C}} \tag{5}$$

With a curve for energy per unit volume of strain and a stress-strain curve, the actual stress, f_1, is found at the strain corresponding to e_1. The uncertainty in stress will be

$$\frac{f - f_1}{f} \times 100 \text{ percent} \tag{6}$$

A plot of $\frac{f - f_1}{f}$ versus f for materials Nos. 1, 2, and 3 is shown in figure 6. From these curves, the stress limits of ± 10 percent can be obtained.

Table 1. --Cushioning materials

Mate-rial No.	Description	Density based on dry weight	Thickness of samples tested	Moisture content at 75° F., 64 percent relative humidity
		Lb. per cu. ft.	Inches	Percent
1	Curled cattle hair bonded with natural latex or neoprene rubber	1.17	1.53	10.1
2	............do..........	1.56	1.43	8.7
3	............do..........	2.62	1.35	7.3
4	Glass fibers bonded together with a resin	.80	1.30	1.4
5	............do..........	3.20	1.00	3.6
6	............do..........	5.00	1.00	2.6
7	Reclaimed sponge rubber; ground and bonded together	7.70	1.06	Less than 1
8	............do..........	12.00	1.11	Less than 1
9	Festooned sheet of curled cattle hair bonded with latex into a 1-inch thick pad (one ply)	1.94	.91	10.9

Table 2. --Summary of stress ranges and
approximate cushion factors
for materials 1 through 8

Material	Approximate cushion factor $\overline{C}$	Stress range $f = \dfrac{W\,G_m}{A}$
	Dimensionless	P. s. i.
1	5. 1	0. 28 - 1. 4
2	4. 3	. 6 - 2. 0
3	5. 1	1. 0 - 4. 7
4	5. 8	. 05 - . 4
5	5. 0	2. 3 - 10. 0
6	5. 6	5. 0 - 30. 0
7	6. 2	. 95 - 5. 8
8	6. 1	2. 0 - 13. 0
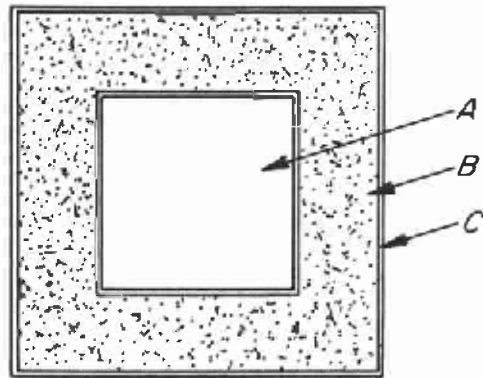
Figure 1. --A cushioned pack consists essentially of <u>A</u>, an article; <u>B</u>, a cushion; and <u>C</u>, an outer container.
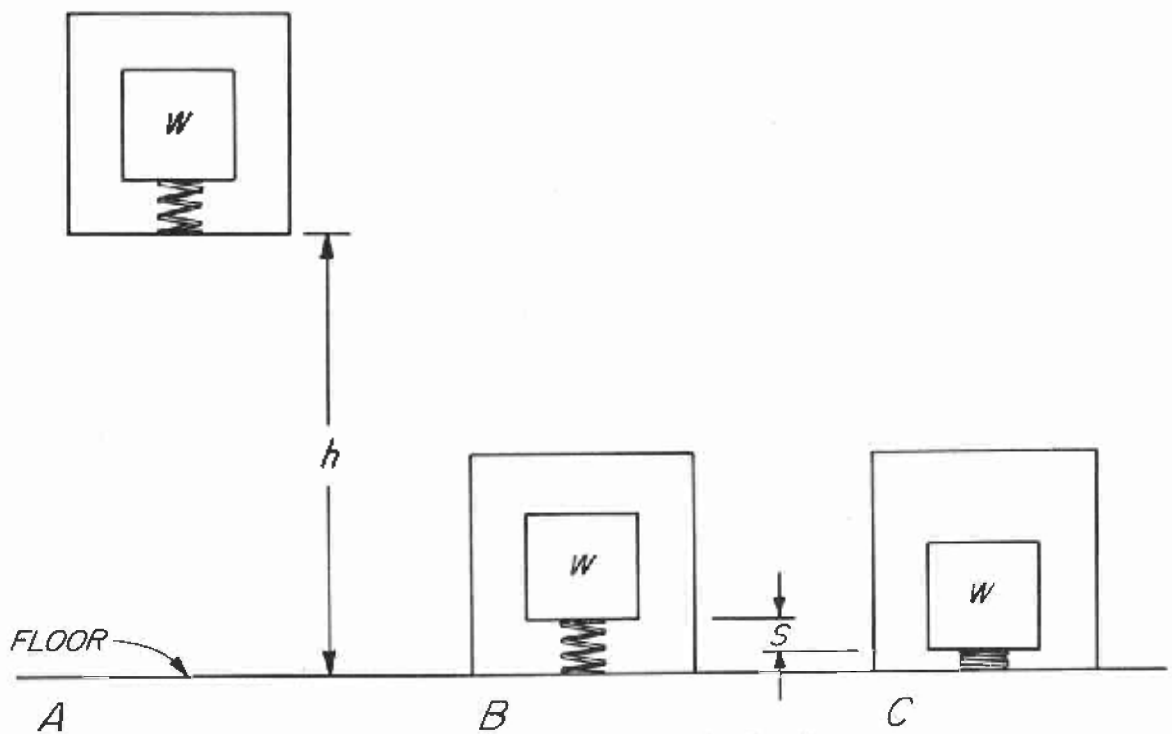
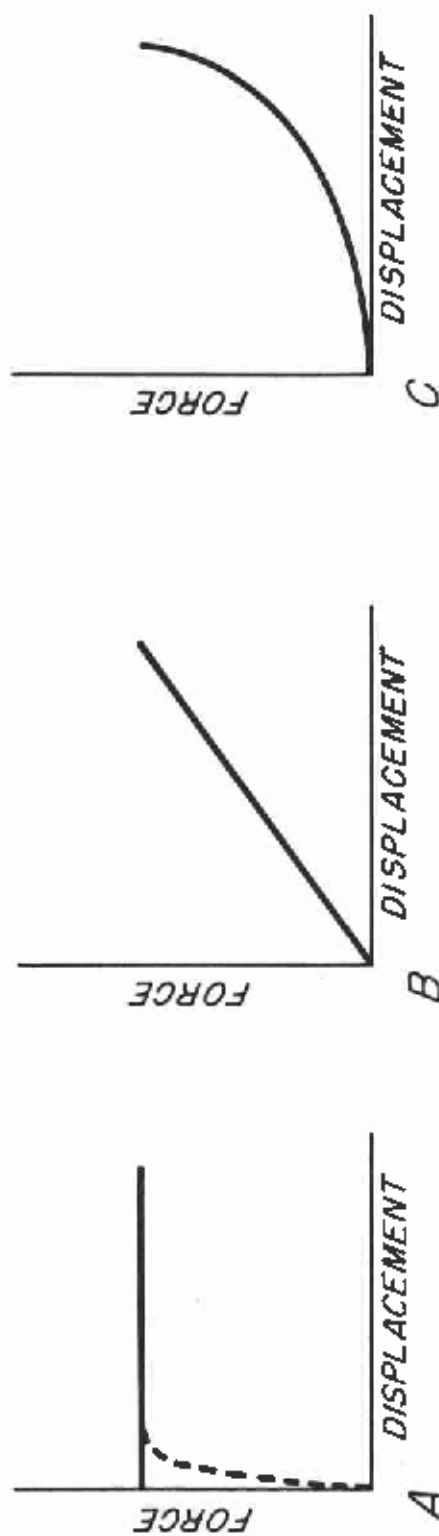Figure 2. --An idealized representation of a package during a drop.

Figure 3. --Force-displacement curves of three types of cushions. A, ideal cushion;
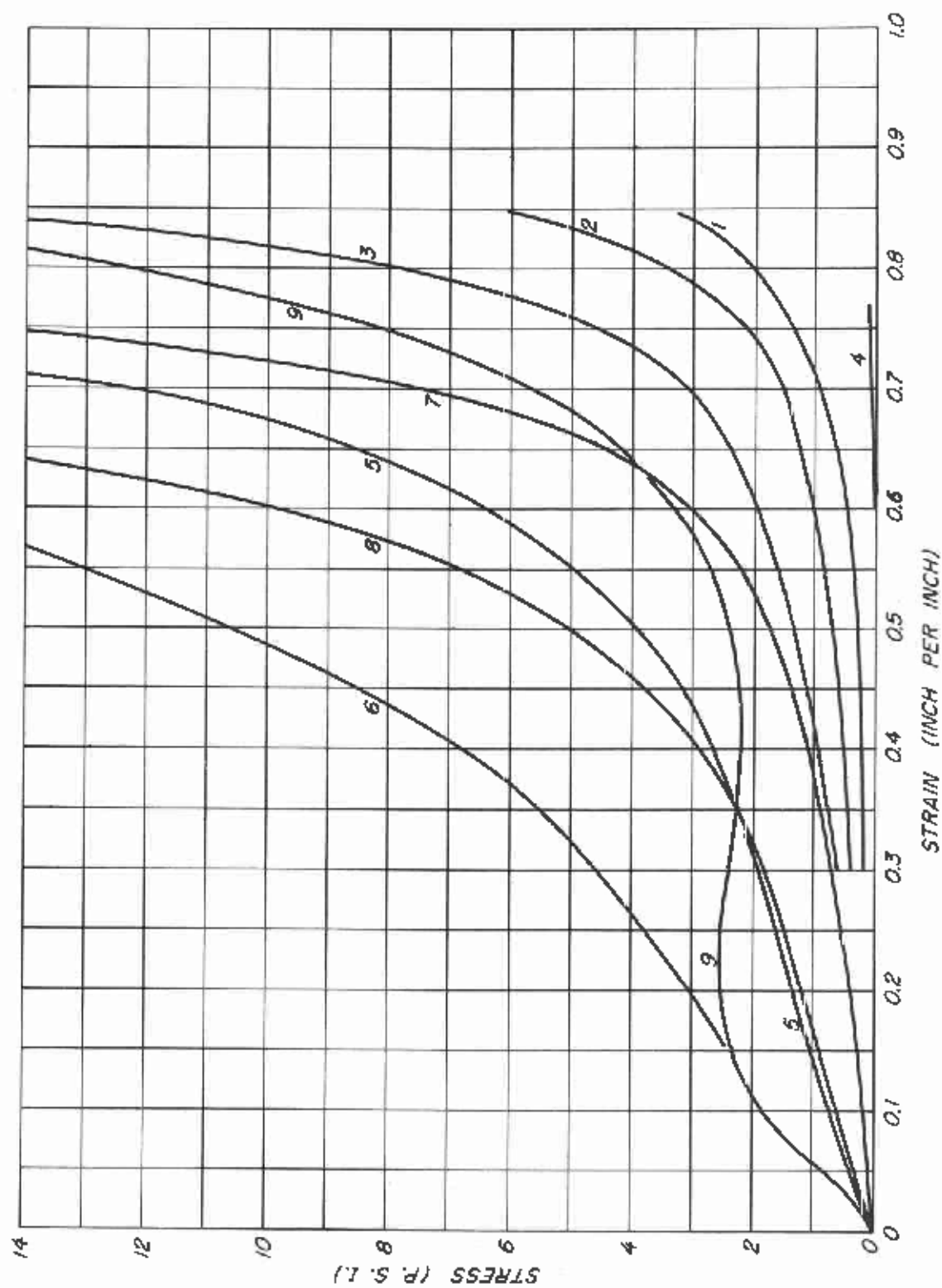B, linear cushion; C, anomalous.

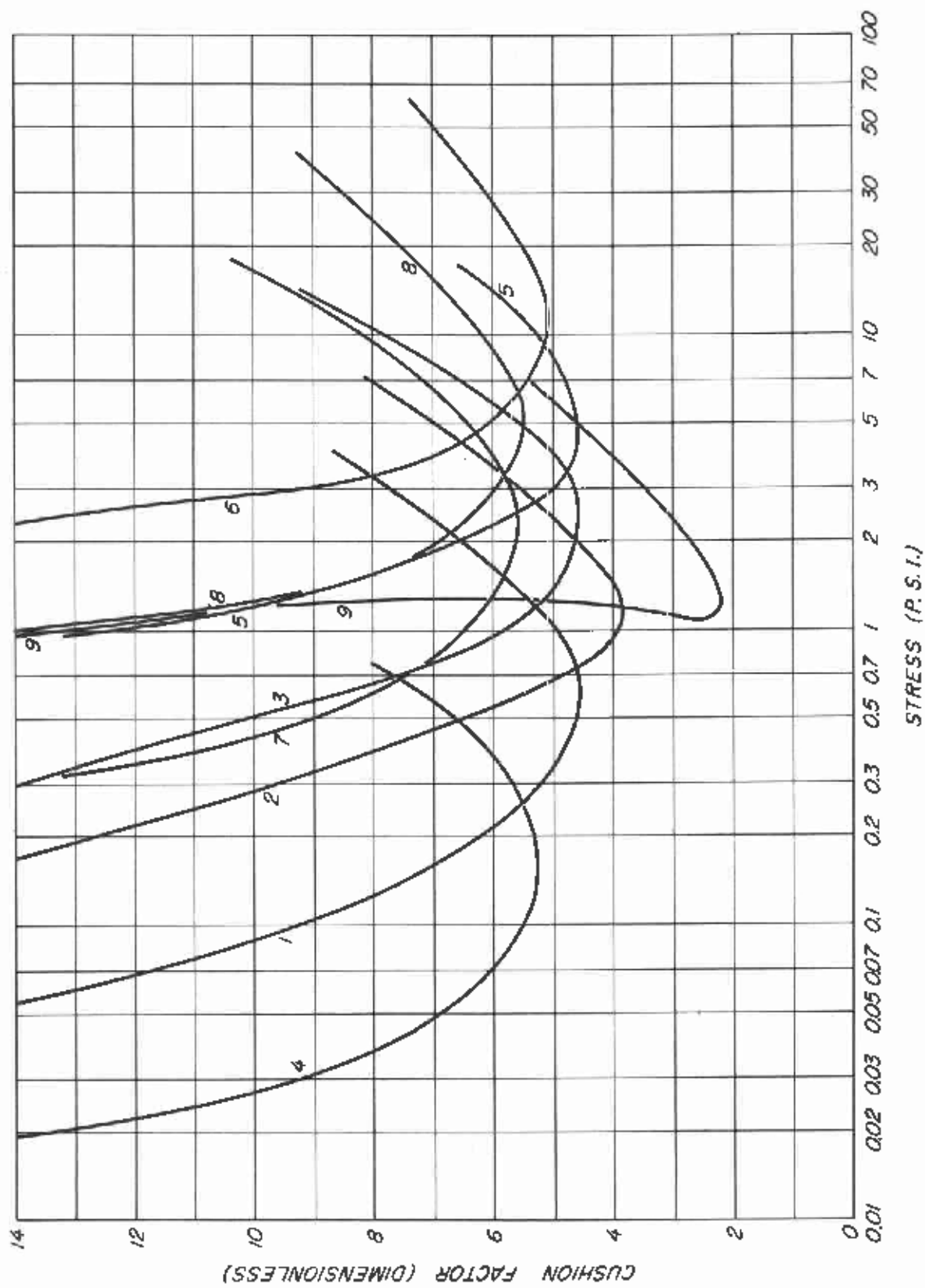Figure 4. --Stress-strain curves for the nine materials analyzed for this report.

Figure 5. -- Cushion factor-stress curves for the nine materials analyzed for this report.

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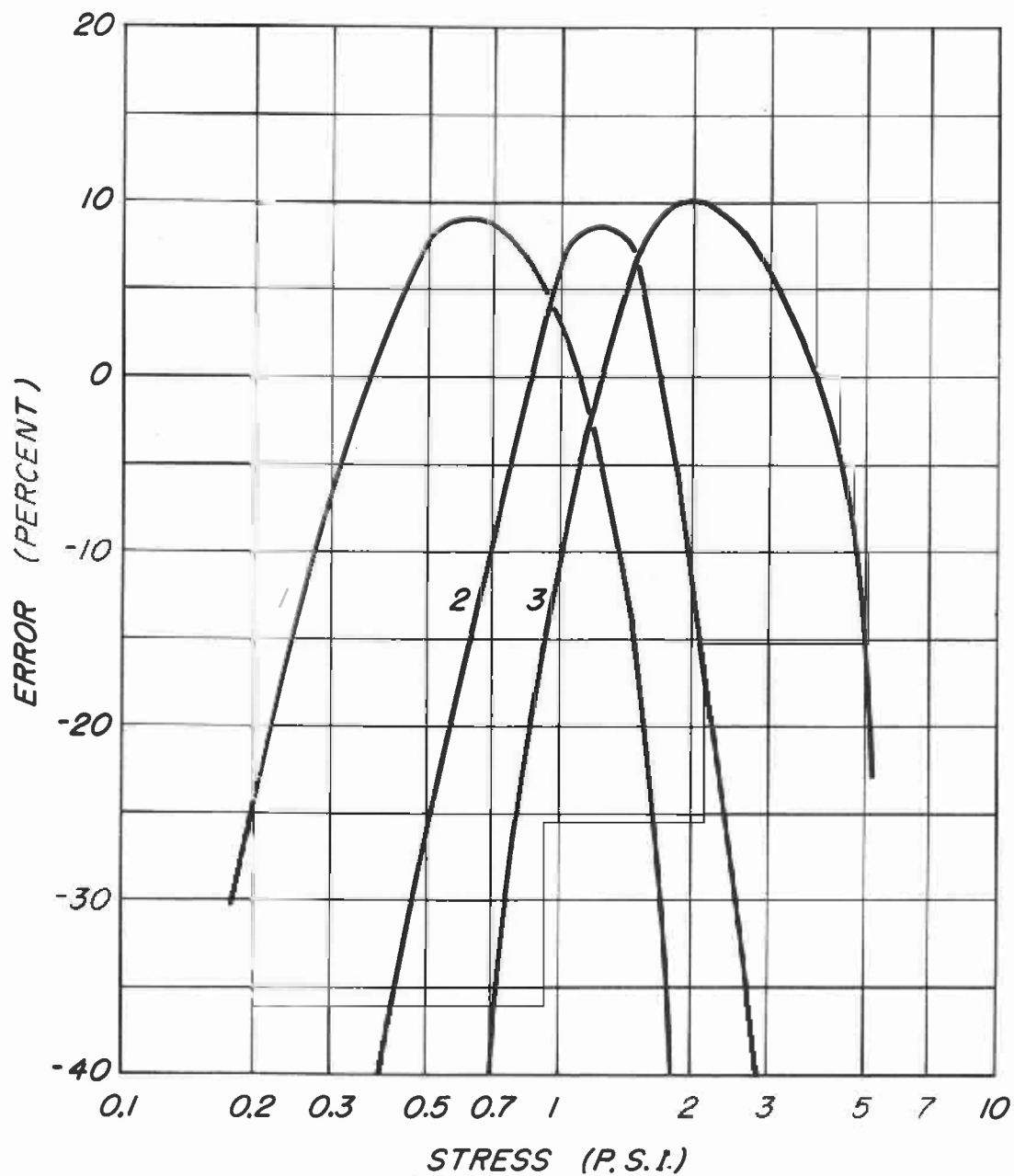*STRESS (P. S. I.)*

CUSHION FACTOR (DIMENSIONLESS)

Figure 6. --Error-stress curves for materials Nos. 1, 2, and 3.
Average cushion factors used to obtain the error were 5.1 for
material No. 1, 4.3 for material No. 2, and 5.1 for material
No. 3.

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