A METHOD FOR RAPID MEASUREMENT OF THE RATE OF DECAY OF FREE VIBRATIONS

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A METHOD FOR RAPID MEASUREMENT OF THE RATE OF DECAY OF FREE VIBRATIONS

By

WILLIAM L. JAMES, Physicist

Forest Products Laboratory,[1] Forest Service
U. S. Department of Agriculture

Summary

A method for rapid measurement of the rate of decay of free vibration is described, and use of the method for determining internal friction in wood is outlined. Special problems connected with exciting and measuring longitudinal vibration in wood specimens are outlined and solutions suggested. The most consistent measurements of vibration damping were obtained by using vibrations excited by a single sharp blow and detected by a microphone. A special apparatus for indicating logarithmic decrement instantaneously is described.

Introduction

Internal friction is the term applied to mechanisms whereby energy of deformation in a material is converted to heat. The fundamental mechanism of internal friction may differ with different types of material, and in a given material different mechanisms may predominate under different conditions. The total measurable internal friction in a material, regardless of the mechanisms to which it is due, in general depends markedly on such variables as temperature, stress amplitude, and history of the material.

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[1] Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

Among the recognized mechanisms that contribute to internal friction are magnetoelastic coupling in ferromagnetic materials, thermoelastic coupling (most commonly illustrated by thermal expansion), migration of crystal dislocations under stress, and resisted relative motion of structural elements of the material. In all cases, internal friction results in a net increase in the entropy of the deformed system, either through generation of irreversible eddy currents in the case of ferromagnetics, or irreversible heat flow between regions of high and low stress or high and low rate of slipping in nonmagnetic materials.

The predominant mechanism of internal friction in wood is probably the resisted relative movement of structural elements; as such, the amount of internal friction should indicate something of the tightness of the bonds between structural elements. It follows that there should be some correlation between internal friction and other physical properties that depend upon the nature of the interparticle bonds. For this reason, the measurement of internal friction in wood is of interest, not only from a theoretical standpoint, but also as a possible means of predicting certain mechanical properties of wood from knowledge of the internal friction.

Internal friction manifests itself in a number of ways, notably by limiting the amplitude of forced resonant vibration and causing a gradual decay of free vibration. Any such measurable manifestation of internal friction offers a potential quantitative measure of the internal friction itself. This report describes apparatus and techniques that permit rapid measurement of internal friction by measuring the rate of decay of free vibration.

Techniques

The general method described here is to support the specimen, drive it into vibration at its natural frequency, detect the vibration, and measure the frequency and rate of decay of the free vibration with the driving force zero.

The most important technical consideration is that the rate of decay of free vibrations is a reliable measure of internal friction only when this friction is the only significant cause for the vibration decrement. This means that loss of vibrational energy through the specimen support and through the driving and detecting mechanisms must be small. Ordinarily, this would require that the specimen be supported at a vibration node, and that the driving and detecting devices not touch the specimen during the time of free vibrations. Energy loss through radiation of sound is considered negligible for materials, such as wood, with fairly high internal friction.

Loss of energy through the driver, detector, and support devices can be appreciable, however, and can apparently be quite erratic and elusive. In the development of the techniques and equipment described in this report, the extreme sensitivity of the measured rate of decay of vibration to seemingly minor changes in the experimental configuration became apparent, and made a number of precautions necessary.

In particular, when the driving method used involved a force transducer that remained in contact with the specimen, the measured logarithmic decrement of vibration varied widely and erratically whenever the driver was moved, or removed and replaced. Further, such measured values of logarithmic decrement were significantly larger than values obtained with a driver that did not remain in contact with the specimen. These results were especially surprising in view of the small mass and low stiffness of the driving device.

Similar results were obtained when using high-quality phonograph pickups of small moving mass and high mechanical compliance for detecting devices; the measured value of logarithmic decrement varied widely as a result of relocating the pickup at slightly different positions on the specimen, or even from removing the pickup and replacing it as nearly as possible in the same location.

In general, values of logarithmic decrement that were repeatable and consistent could be obtained only by using driving and detecting methods that did not require physical contact with the specimen during the time of free vibration.

Support of Specimen

The technique used to support the vibrating specimen depends on whether transverse or longitudinal vibration is being used. Since the method described here was developed primarily for studying longitudinal vibration, these techniques are emphasized.

The specimens commonly used are bars with uniform, nearly square cross section and lengths of no less than 10 times the maximum transverse dimension. In the fundamental mode, the only vibration node is at the middle of the bar, and thus the obvious means of support is to balance the beam on a suitable surface.

The surface on which the vibrating specimen is supported can be either rigid or flexible. A rigid support, such as a knife edge, theoretically will absorb no energy because it does not move, and should be preferable. Experience has shown, however, that a rigid support often causes the envelope of the decaying vibration to be distorted from the natural exponential form. This is probably because the point of support is not exactly the nodal point, and the relatively large alternating force between the specimen and the support causes reflections of sound waves from the support that conflict with those associated with the fundamental vibration of the specimen. Since measurement of the rate of decay of free vibration requires an undistorted decay envelope, the rigid support is unsatisfactory for wood specimens.

The required flexible support ideally should be made of material low in damping characteristics, but again experience indicates that the rate of decay of free vibration in wood is not highly sensitive to internal friction in the support itself, provided that the support stiffness is low. The support adopted here as most convenient is made of three foam rubber feet, each a nominal 1/4-inch cube. These are cemented to a flat wood base and positioned at the vertices of an equilateral triangle with sides of about 3/4 inch. This arrangement provides enough support area so that the specimen can easily be balanced.

Results obtained by using this form of support were compared to those obtained with the specimen supported on a pair of threads about 3/4 inch apart and those with the specimen supported on a pair of flexible knife edges made of shim stock. There was no consistent difference between results obtained with the three different support methods.

Driving Techniques

For maximum reliability, the driving technique should not require physical contact with the specimen during the time of decaying free vibration. With this restriction imposed, three driving techniques appear feasible that do not require modification of the specimen. These are (1) driving with sound from a small, concentrated source, (2) driving with a force transducer that contacts the specimen during the forced vibration but is removed during free vibration, and (3) exciting vibration with a single sharp blow. Other techniques requiring extraneous attachments to the specimen have been entirely satisfactory, but somewhat inconvenient.

Driving with high-intensity sound at the natural frequency of the specimen easily produces the desired amplitude of vibration, but requires either that the vibration detector be insensitive to sound or that the experiments be done in an anechoic chamber. Otherwise, the measurements would be of room reverberation time rather than rate of decay of vibration of the specimen. Because of the desirability of a microphone for vibration detecting, this limitation is objectionable.

A force transducer that delivers a periodic force while in contact with the specimen, and can be separated rapidly from the specimen, provides a satisfactory means of driving. With the transducer contacting the specimen, the driving frequency is adjusted for maximum response of the specimen. The driving force is then switched off and the transducer separated from the specimen, which then vibrates freely. This procedure permits direct measurement of the specimen's natural frequency, but requires a certain amount of time to search for the correct driving frequency.

Exciting vibration in the specimen by a single sharp blow is probably the simplest technique and has given the most consistent measurements of the rate of decay of free vibration. The natural frequency of the specimen is not easily measured because the vibration decays quickly, but there is no time lost in seeking the correct driving frequency.

Detecting Techniques

Ideally, the amplitude of vibration of the specimen should be measured without physically contacting it. If, in addition, it is desirable to avoid extraneous attachments to the specimen, there remain two obvious techniques, optical and acoustical, for measuring the amplitude of vibration.

Optical techniques have the advantage of permitting absolute calibration by elementary means, but their application requires rather stringent alinement and location of the specimen within the optical system.

Acoustical techniques, using an ordinary microphone, are easy to apply; although calibration is difficult, accurate measurements of the rate of decay of free vibration can be made if the acoustical system is linear.

The acoustical technique has been used predominantly in the equipment described, although attempts were made to measure amplitude of vibration of the specimen with high-quality phonograph pickups. The difficulty of obtaining consistent measurements with phonograph pickups in contact with the specimen was described earlier.

Data Reduction

The quantity commonly used for expressing the rate of decay of vibration is the logarithmic decrement, $\underline{\delta}$, defined as the logarithm of the ratio of displacements at times differing by one period of vibration. The quantity so defined is chosen because the decay of free vibration in nearly all naturally occurring cases is approximately an exponential function of the number of cycles of vibration. This is equivalent to saying that the energy dissipated per unit time is a constant fraction of the instantaneous total energy. Therefore the value of δ is independent of the amplitude of vibration, as long as the internal friction itself is independent of amplitude.

The most fundamental and probably most accurate way of determining δ after suitable driving and detecting techniques have been assembled is to display the amplitude of vibration as a function of time with an oscilloscope, and photograph the oscilloscope display. The photograph will be of the form shown in figure 1. Successive maximums can be measured and plotted on semilogarithmic coordinates, the slope of the resulting straight line being equal to $-\delta$. Or, with reduced reliability, two amplitude measurements can be made a known number of cycles apart, and δ computed from these values. Details of the mathematical basis for these and other calculations are in Appendix I.

Data reduction from photographs permits high accuracy but is tedious and costly in time. When a great many specimens are involved, the cost of data reduction from oscillograms may be prohibitive. There are potentially many ways to obtain a numberical measurement of δ directly, and a satisfactory one is described in this report.

In principle, this data-reducing system determines the number of cycles of vibration between two preselected amplitude maximums. As shown in Appendix I, if the ratio of these two maximums is set equal to e, or about 2.72, the number of cycles between the two will equal numerically the reciprocal of δ. With such a system, values of δ can be determined almost instantly and replicated in rapid succession for improved reliability.

Apparatus

A block diagram of the apparatus is shown in figure 2, and figure 3 is a photograph of the apparatus. A variable audiofrequency generator drives a power amplifier through an electronic switch. The electronic switch is used to start and stop the audio power sharply, and also is used to provide the pulses for shock-exciting vibration. The power amplifier operates the driver that supplies the force to excite vibration of the specimen. The output from the microphone that detects the vibration of the specimen is amplified and fed to an amplitude discriminator. The amplified microphone output is also available for observation and recording with an oscilloscope and camera. An electronic counter records from the amplitude discriminator the number of cycles of vibration elapsing as the amplitude of vibration decays by a factor of 2.72. This number is the reciprocal of the log decrement, δ. The electronic counter also has a standard time gate that permits measuring accurately the number of cycles of vibration per second.

The specimen is supported on three 1/4-inch cubes of foam rubber, spaced at the vertices of a triangle 3/4 inch on a side, as previously described.

Driver

The driver (fig. 4) consists of a large permanent magnet, soft iron pole pieces, and a cylindrical coil on a flexible mount. The pole pieces form a radial magnetic field in which the coil is suspended, similar to the usual loud speaker. The coil is constrained by its suspension to move axially. Fixed to the coil is a small wood tup that transmits the force generated by the coil to the specimen.

The driver can be so located that the quiescent position of the tup is a few millimeters from the specimen, or with the tup pressing against the specimen. With the tup away from the specimen, amplified pulses from the electronic switch through the coil cause the tup to strike the specimen and excite transient vibration in the specimen. With the tup touching, alternating current of the correct frequency through the coil will excite continuous resonant vibration in the specimen, permitting the natural frequency of the specimen to be determined.

Details of construction of this and other parts of the apparatus are presented in Appendix II.

Detector

The vibration detector (fig. 5) is the element of an ordinary dynamic microphone with a diaphragm about 2 inches in diameter. An integral transformer matches the microphone coil impedance to the high impedance of the preamplifier. In practice, the microphone is mounted about 1/16 inch from the end of the specimen, so the sound generated by the vibrating specimen predominates over extraneous room noise.

Amplifier and Amplitude Discriminator

The output from the vibration-detecting microphone is amplified by a simple two-stage resistance-coupled preamplifier with a voltage gain variable from 0 to about 200. A circuit diagram of the amplifier and discriminator are given in Appendix II.

The amplitude discriminator utilizes the sharp cut-off characteristics of gated beam tetrodes. With these tubes, the output amplitude changes rapidly from maximum to zero as the input amplitude decreases by a small amount below a certain critical value. This critical value of input amplitude can be adjusted by varying the fixed bias on the control grid of the tube. In the circuit described here, four gated beam tetrodes, type 6BN6, are used. The output of the microphone preamplifier is transformer coupled to the control grids of one pair of gated beam tetrodes. The grids of these tubes are returned to ground through the low-resistance transformer to minimize the DC grid voltage resulting from rather large control grid current with large input signals.

These 6BN6's are cathode biased from a regulated supply through low-resistance variable voltage dividers. The resistance and bias currents are so selected that the cathode current of the 6BN6 has a negligible effect on the bias voltage. The cathode bias values of these 6BN6's are so adjusted that their cutoff amplitudes differ by a factor of 2.72; it is these tubes that provide the amplitude discrimination. The second pair of 6BN6's operate in the same manner, except that both have the same bias and therefore the same cutoff amplitude. These tubes, as described in Appendix II, serve to increase the sharpness of the amplitude discrimination.

In operation, when the signal from the preamplifier exceeds the cutoff amplitude of both of the first pair of 6BN6's, a signal appears at the plate of each tube. The form of this signal is a sequence of clipped half-sine pulses, one pulse for each cycle of vibration of the specimen. When the amplitude of the preamplifier signal decreases (as by decreasing amplitude of vibration of the specimen) to the cutoff amplitude of the 6BN6 with the higher bias, the signal at the plate of this tube rapidly decays. The amplitude of vibration of the specimen at which this occurs is called the first critical amplitude.

Similarly, as the amplitude of vibration decreases further, specifically by a factor of 2.72, the signal at the plate of the 6BN6 with the lower bias decays rapidly. The amplitude of vibration at which this occurs is the second critical amplitude. Thus, when the amplitude of vibration exceeds the first critical amplitude, two pulses are generated per cycle of vibration, one from each of the first pair of 6BN6's. When the amplitude is between the first and second critical values, one pulse is generated per cycle of vibration. Below the second critical amplitude, no pulses are generated. The pulses generated by the first pair of 6BN6's are applied through amplifiers to the second pair of 6BN6's. Through amplitude discriminating action this produces a more abrupt cutoff of the pulses at the critical amplitudes (Appendix II).

The output of the amplitude discriminator is applied to an electronic counter and timer. This device is essentially a high-speed pulse counter with an electronic switch or gate to control the signal to the counters. The gate can be opened and closed either by external signals or by internally generated precision timing signals. The signal applied to the counter through the gate also can be either external or the internally generated signals of precisely controlled frequency.

Each time the gate opens, the counter is automatically reset to zero, the input signal from the gate to the counters being delayed to prevent loss of the earliest information. When the gate closes, it is automatically disabled for a length of time that is variable from a finite to an indefinite period. This is termed the display time, its purpose being to permit the counters to be read before the count is erased. For special applications, it is often desirable to dispense with the automatic zero reset, the automatic display time, or both; for this purpose, the counter was modified by adding switches that activate or deactivate these automatic circuits.

In this apparatus, the gate is opened and closed by the pulses from the amplitude discriminator, the display time is made zero by deactivating the display time circuit, and the counters are set to count external signals. The zero reset circuit remains in operation.

The pulses from the amplitude discriminator that stop at the first critical amplitude are connected to close the gate, and the pulses that stop at the second critical amplitude are connected to open the gate. The pulses that open the gate are also connected to the external signal input where they are applied through the gate to the counters. Therefore, when the amplitude of vibration exceeds the first critical amplitude, the gate receives open and close signals in rapid succession. The counters record each open signal, but this one count is erased each time the gate opens, so at most one count will be recorded under these conditions. If the display time circuit were not deactivated, it would prevent the rapid open-and-close sequence of the gate.

However, as the amplitude of vibration decays past the first critical amplitude, the supply of close signals fails; the gate then opens and remains open, and the counters continue to accumulate the open signals. At the second critical amplitude, the open signals also stop and, although the gate remains open, there is no signal to count. The counters then display the total number of open pulses that occurred after the failure of the close pulses. This is equal to the number of cycles of vibration between the first and second critical amplitude. Since the first and second critical amplitude differs by a factor of 2.72, this number is the reciprocal of $\underline{\delta}$. As the counters are automatically reset only by the opening of the gate, the recorded number, $1/\underline{\delta}$, remains displayed until the amplitude of vibration is again driven to a value exceeding the first amplitude. The process can be repeated indefinitely with no intermediate operations at a rate of about one reading per second, permitting a number of measurements to be made for increased reliability.

An apparatus similar to this has been developed independently by Pattison,[2] who used a method of rectifying the decaying AC signal and counting the number of cycles occurring between two values of the rectified signal.

The system for direct reading of $1/\underline{\delta}$ is calibrated by adjusting the bias values of the first pair of 6BN6 tubes. This is most easily done by using a variable amplitude audio generator and an audio frequency voltmeter. At some convenient amplitude of input signal, the bias on one of the first pair of 6BN6's is set just high enough to cut off the signal. The input amplitude is then reduced by a factor of 2.72, as indicated by the voltmeter, and the bias of the other 6BN6 is set in the same way.

To check the calibration and to provide a rapid means of maintaining calibration, a bar of glass-reinforced phenolic resin is used as a standard specimen. This bar is 1-1/2 inches square in cross section and 26-1/2 inches long. Its value of $\underline{\delta}$ at 70° F. was found to be 0.031 by plotting the amplitude of vibration as a function of number of cycles on semilog coordinates.

Discussion of Method

The method of measuring the rate of decay of free vibrations described herein provides quick and reasonably simple measurements of internal friction. It is faster than measuring the width of the frequency response curve, but requires more elaborate instrumentation. On the other hand, the techniques and instrumentation are less complicated than those associated with directed measurement

[2] Pattison, John. An Apparatus for Accurate Measurement of Internal Friction. Review of Scientific Instruments 25(5): 490-494, 1954.

of energy input to maintain vibration, or with measurement of the resistive component of an electromagnetic circuit magnetomechanically coupled to the vibrating specimen.

The instrumentation described here is usable with vibrations over a range of frequencies limited by the microphone to between 50 and 10,000 cycles per second; with a suitable vibration detector it would be usable from a few cycles per second to about 100 kilocycles.

In applying the method to specimens of wood it is found that, when using shock excitation, the frequency range is limited by a characteristic of the wood that is not readily explained. As the length of a given wood specimen is reduced in order to increase the resonant frequency, the form of the envelope of the decaying vibration becomes less distinct. When the specimen's length is reduced to about 20 inches or less, the decay envelope is practically without regular form. This corresponds to frequencies in the order of 4,000 cycles per second. The form of the decay envelope is somewhat better when using interrupted continuous excitation of the specimen (instead of the shock excitation usually employed) but the improvement is not highly significant. Good results can be obtained at frequencies of 4,000 cycles per second and higher by using longer specimens driven at harmonic frequencies with continuous excitation. This suggests that the difficulty in using shorter specimens is directly related to their size, not to the higher frequency of vibration. This size limitation is associated with the length or size in the direction of compression waves, and not with lateral dimensions. Reducing the cross sectional area of a specimen does not adversely affect the form of the decay envelope as long as the specimen is mechanically suited for longitudinal vibration.

Accuracy

The accuracy of the system for indicating the logarithmic decrement rapidly is limited primarily by the stability of electronic components, particularly the resistors that determine the bias on the 6BN6's and the 6BN6 tubes themselves. In addition there is a small but finite uncertainty in the indications because the amplitude cutoff of the 6BN6 tubes is not absolutely sharp, even with the two sharp cutoff stages in series. Thus, there may be an uncertainty of one or more cycles in the indicated number of cycles between the two critical amplitudes, the uncertainty depending on the rate of decay of the vibrations being observed. The error introduced by this uncertainty is minimized by the fact that for rapidly decaying vibration the uncertainty is small; for slowly decaying vibration, where the uncertainty is larger, the counted number of cycles is also proportionately larger, so the relative error does not increase greatly. Figure 6 shows the discriminator output associated with vibrations with high and low rate of decay.

One further source of error arises from the characteristics of the specimens themselves. In many cases, particularly with wood specimens, the envelope of the decaying vibrations is not perfectly smooth and regular, with the result that there is random variation in the indicated number of cycles between the two critical amplitudes. Repeating the measurement several times, as is easily done, minimizes this error.

The total error resulting from these considerations is estimated to be no larger than ±5 percent, provided the system is correctly calibrated and a sufficient number of readings are made on specimens with noisy decay patterns.

APPENDIX I

Mathematical Basis for Calculations

In considering damped vibration, it is usually assumed and in most cases verified experimentally that the rate of dissipation of vibrational energy is proportional to the vibrational energy, $\underline{E}$. That is

$$\frac{dE}{dt} = -k_1 E \qquad (1)$$

where the minus sign shows the energy to be decreasing. This can be rewritten

$$\frac{dE}{E} = -k_1 \, dt \qquad (2)$$

and integrated to give

$$\log E = -k_1 t + c \qquad (3)$$

Since at time $\underline{t} = o$, $\underline{E}$ has an initial value of $\underline{E_o}$, it follows that $\underline{c} = \log \underline{E_o}$, and

$$\log \frac{E}{E_o} = -k_1 t$$

or

$$\frac{E}{E_o} = e^{-k_1 t} \qquad (4)$$

In practical applications, when the vibrational energy is measured in terms of the amplitude it is convenient to determine the rate of decay of vibrational energy by measuring changes over 1-cycle intervals. Here, for simplicity, calculations are based on the assumption that energy loss per cycle is a small fraction of the total energy, and that the ratio of amplitudes separated in time by one period of vibration is near to unity.

The vibrational energy is continually alternating between kinetic and elastic and, if the vibrating system is linear (force proportional to displacement), the elastic energy is proportional to the square of the displacement. At maximum displacement the energy is all elastic, so the total energy then is proportional to the square of the maximum displacement or amplitude. That is,

$$E = k_2 y^2 \qquad (5)$$

where E is total energy and y is the amplitude.

Differentiating:

$$dE = 2k_2 y \, dy$$

Dividing by equation (5):

$$\frac{dE}{E} = \frac{2\,dy}{y} \qquad (6)$$

But if ΔE and Δy (over one period) are small, then

$$\frac{\Delta E}{E} \cong \frac{2\Delta y}{y} \qquad (7)$$

This is the fraction of the total energy dissipated per cycle.

Now, if $\Delta y = y_n - y_{n+1}$, where y_n and y_{n+1} are the amplitudes of the nth and (n+1)th cycle, and $\frac{y_n}{y_{n+1}}$ is nearly unity,

$$\frac{\Delta y}{y} \cong \frac{y_n - y_{n+1}}{y_{n+1}} = \frac{y_n}{y_{n+1}} - 1$$

But $\log \frac{y_n}{y_{n+1}} = \frac{y_n}{y_{n+1}} - 1 - 1/2\left(\frac{y_n}{y_{n+1}} - 1\right)^2 + 1/3\left(\frac{y_n}{y_{n+1}} - 1\right)^3 - - - - -$

And if $\frac{y_n}{y_{n+1}}$ is nearly unity, powers of $\left(\frac{y_n}{y_{n+1}} - 1\right)$ beyond the first may be ignored.

In this case, $\log \frac{y_n}{y_{n+1}} \cong \delta \cong \frac{y_n}{y_{n+1}} -1 \cong \frac{\Delta y}{y}$. (8)

Therefore from (7) and (8),

$$\frac{\Delta E}{E} \cong 2\delta \tag{9}$$

Equation (9) relates energy dissipation to logarithmic decrement <u>δ</u>. From equation (8),

$$\frac{y_n}{y_{n+1}} = e^{\delta} \tag{10}$$

but as equation (10) holds for all values in <u>n</u>, or for all adjacent amplitude maximums, it follows that

$$\frac{y_n}{y_{n+k}} = e^{k\delta} \tag{11}$$

or

$$\log \frac{y_n}{y_{n+k}} = k\delta, \text{ or } \delta = \frac{1}{k} \log \frac{y_n}{y_{n+k}} \tag{12}$$

thus, if the amplitude can be measured at two points <u>k</u> cycles apart <u>δ</u> can be computed from equation (12).

Further, if

$$\frac{y_n}{y_{n+k}} = e,$$

then

$$\log \frac{y_n}{y_{n+k}} = 1,$$

and

$$\delta = 1/k \tag{13}$$

Therefore, if k is the number of cycles of vibration occurring while the amplitude falls by a factor of e, then 1/k is equal to the logarithmic decrement δ. This is the basis for the adjustment of the amplitude discriminators described in the section on apparatus.

From equation (12), we have

$$\log y_{n+k} = \log y_n - \delta k \tag{14}$$

This is of the form

$$f(x) = f(x_o) - mx$$

which is the equation of a straight line with a slope of -m. Therefore (14) is the equation of a straight line with slope of -δ.

Thus, if $\log y_{n+k}$ is plotted as a function of k on cartesian coordinates, or y_{n+k} versus k on semilog coordinates, the resulting plot has a slope of -δ.

APPENDIX II

Details of the Apparatus

Driver

The magnetic field for the driver is established by a large horseshoe permanent magnet. Soft iron pole pieces, which are held against the magnet simply by magnetic attraction, form the required radial magnetic field. As shown in figure 4, the larger pole piece is roughly keyhole shaped. This pole piece is 1-1/4 inches square in cross section at the end in contact with the magnet, and tapers to 1/2 inch thick at the other (circular) end. The circular end is 2 inches in diameter and has a 1-inch hole. A brass disk, 3/4 inch thick and 2 inches in diameter with a 5/8-inch axial hole, is secured with screws to the circular end of the heyhole-shaped pole piece. This brass disk serves as a spacer between the pole piece and the pole of the magnet, and the 5/8-inch hole locates the other pole piece. The other pole piece is a 5/8-inch rod of soft iron, 1-1/4 inches long, inserted in the hole in the brass spacer, and in contact with the magnet pole. The surfaces of both pole pieces where contact is made with the magnet are ground to a smooth finish. The magnetic field is established between the inner surface of the 1-inch hole in the keyhole-shaped pole piece and the 5/8-inch-diameter iron rod. The gap in which the field is concentrated is thus 3/16 inch wide, permitting comparatively bulky coils to be suspended in the field.

The coils used in the driver have been wound with 26-gage wire, about 50 turns on a cylindrical plastic form 3/4 inch in diameter. The coil is suspended in the magnetic field with two flexible spiders made of heavy paper or thin sheet plastic. Connection to the coil is through braided voice coil lead wire.

Preamplifier and Discriminator

A circuit diagram of the preamplifier and discriminator is shown in figure 7, and the power supply in figure 8.

The preamplifier is a conventional two-stage resistance-coupled circuit. The preamplifier output is amplified by a single-stage amplifier that is transformer coupled to the grids of the first, or amplitude discriminating, pair of 6BN6 tubes.

The amplitude discriminator stages use the sharp cutoff 6BN6 in a resistance-coupled circuit that is conventional except that adjustable fixed cathode bias is provided. The bias is adjusted to a value beyond cutoff so that plate current flows in the 6BN6 only when the positive grid signal exceeds the difference between the bias and cutoff voltages. The special design of the 6BN6 results in a comparatively narrow range of grid voltage to change the plate current from zero to maximum.

Transformer coupling to the 6BN6 grids is used to provide low-resistance grid return to ground, as the grid current of the 6BN6 is relatively large for signals large enough to cause the tube to conduct. Unless the grid return was low resistance, the DC grid voltage would vary, causing erratic cutoff values. Similarly, the cathode bias voltage is derived across low resistances so that the cathode current of the 6BN6 does not materially change the bias value.

The cathode bias values of the first pair of 6BN6's are so adjusted that the input signals are just sufficient to cause the tubes to differ in amplitude by a ratio of e. Usually when the 6BN6 conducts, the plate current switches rapidly from zero to maximum, so the output of the 6BN6 is a negative pulse in the form of a clipped half-sine wave. For a narrow range of input amplitude, however, the plate current of the 6BN6 reaches values intermediate between zero and maximum, and over this range the output is approximately proportional in amplitude to the input. It follows that the amplitude discriminator would give information that was uncertain by an amount governed by the number of cycles of input signal that fell within this quasi-linear range. The magnitude of this uncertainty is greatly reduced by the simple means of using a second pair of 6BN6's to discriminate the output of the first pair.

Since the output from the 6BN6 is in the form of negative pulses, and positive pulses are required to drive the 6BN6, a single stage of amplification is used between the two 6BN6 stages in order to invert the phase of the pulses. This stage is direct coupled to the output of the first 6BN6's to prevent the shift of the average plate current in the amplifier stage that would result if the highly unsymmetrical output of the 6BN6 were applied to this stage through a capacitor. This DC shift under some conditions would cause faulty behavior of the following 6BN6 stages. The phase-inverting amplifier drives the final 6BN6's through blocking capacitors in order to remove instability from drift in the inverted stage. The DC shift resulting from the capacitor coupling is not serious here, because it is not amplified before reaching the final 6BN6 grids.

The final 6BN6 tubes operate as do the first pair, except that both tubes have the same fixed cathode bias.

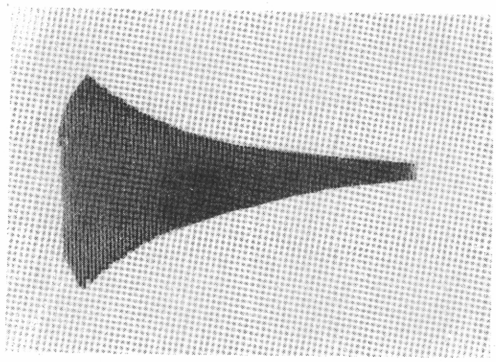

Figure 1.--Oscilloscope screen, showing the plot of a decaying vibration against time.

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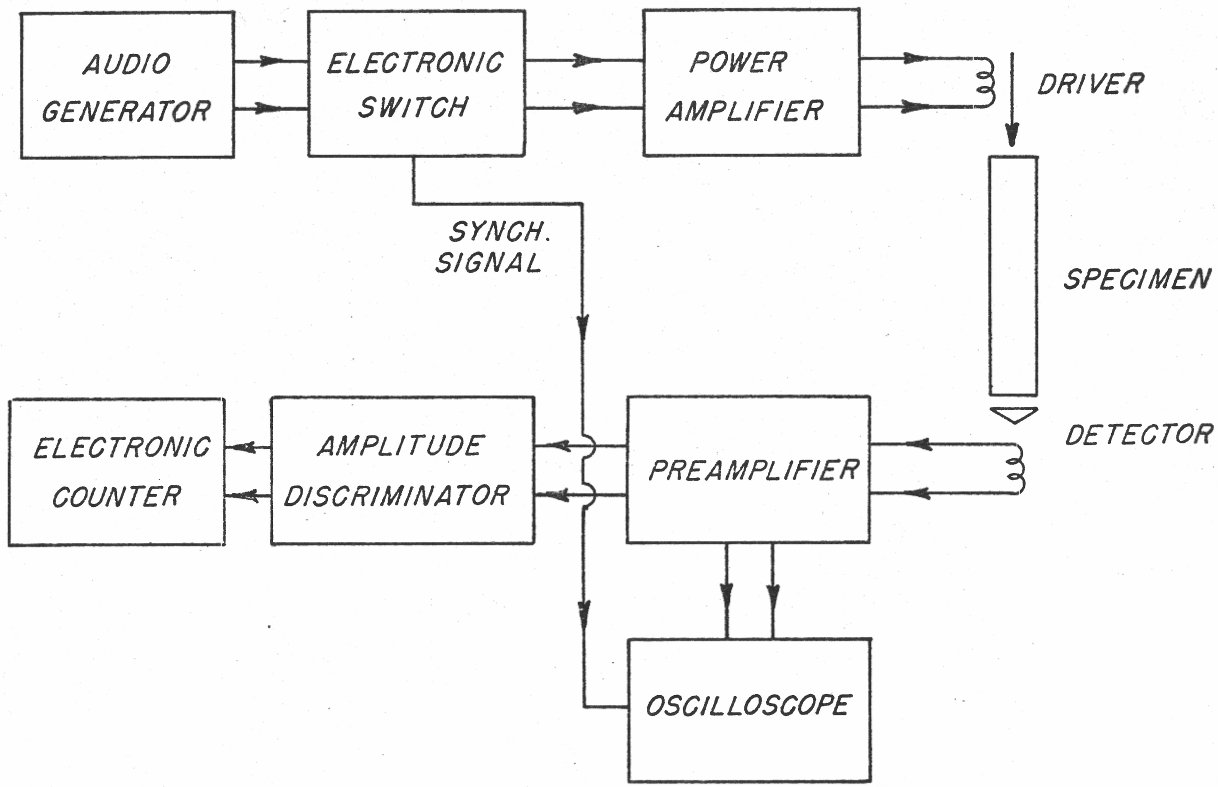

Figure 2. --Block diagram of apparatus for rapid measurement of log decrement($\underline{\delta}$).

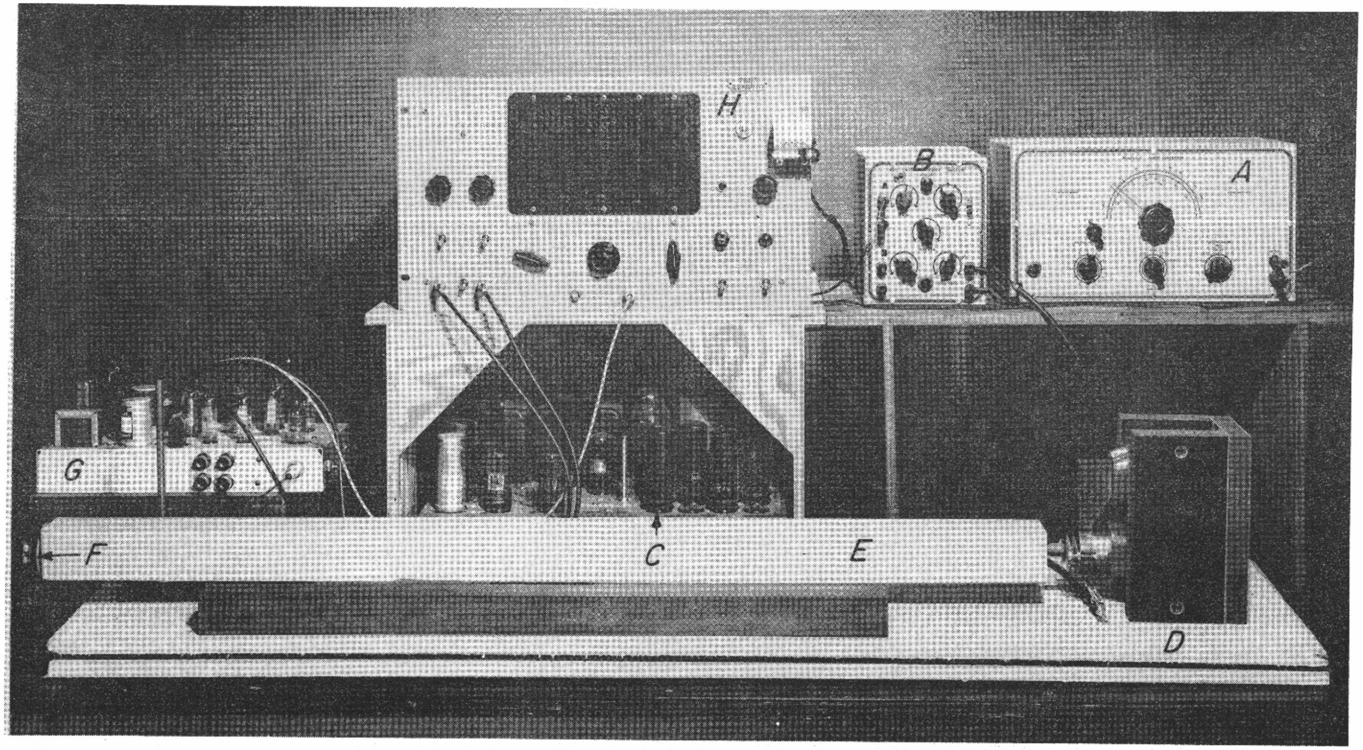

Figure 3. --General view of apparatus for rapid measurement of log decrement (δ). A -- Audio generator; B -- electronic switch; C -- power amplifier; D -- driver; E -- specimen; F -- detector; G -- pre-amplifier and amplitude discriminator; and H -- counter.

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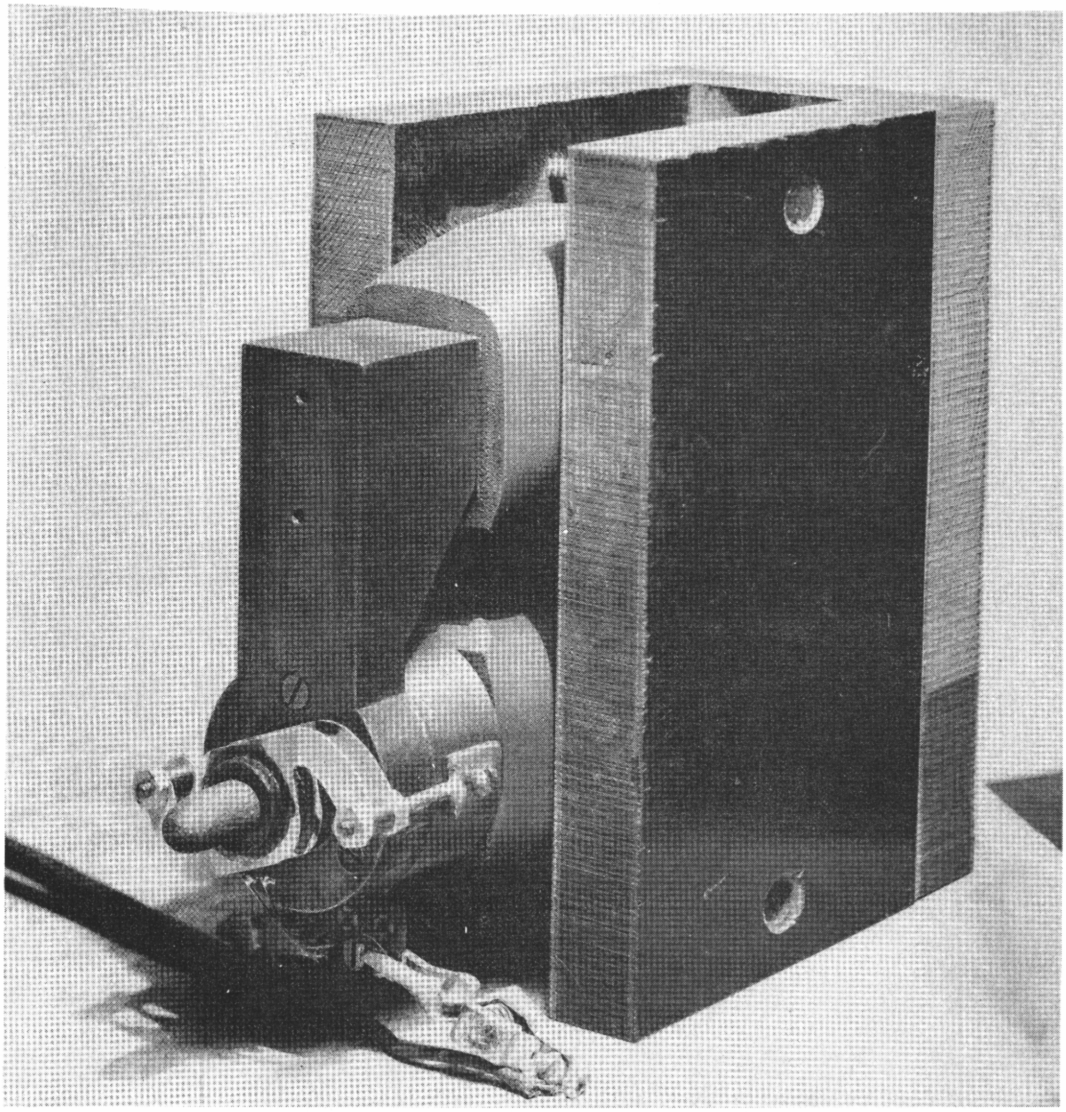

Figure 4. --Close-up view of driver. The large horseshoe permanent magnet is fastened in the frame, and the keyhole-shaped pole piece helps to form the magnetic field. The circular wood tup (lower left) transmits to the specimen the force generated by a coil.

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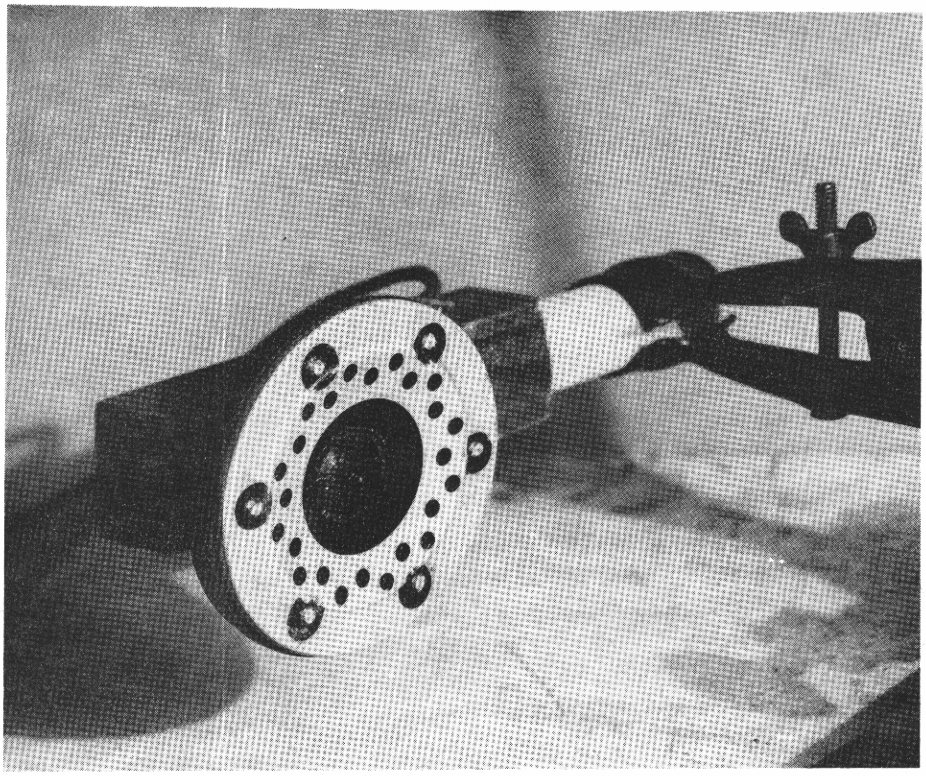


Figure 5. --Close-up view of detector.

ZM 115 777

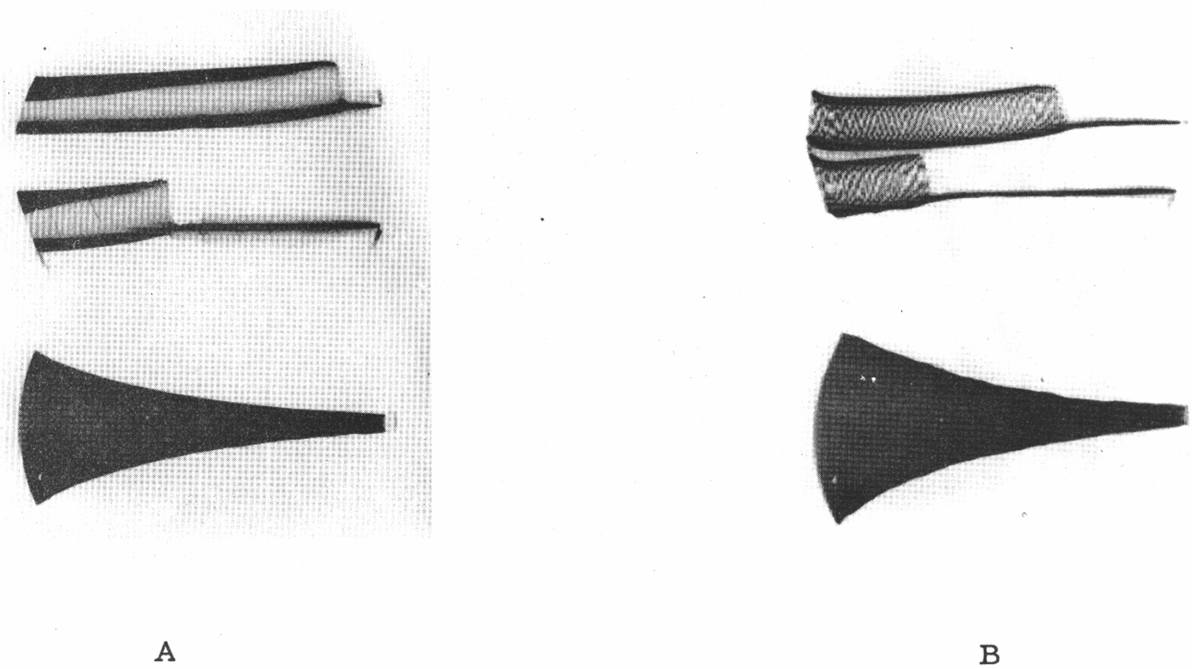
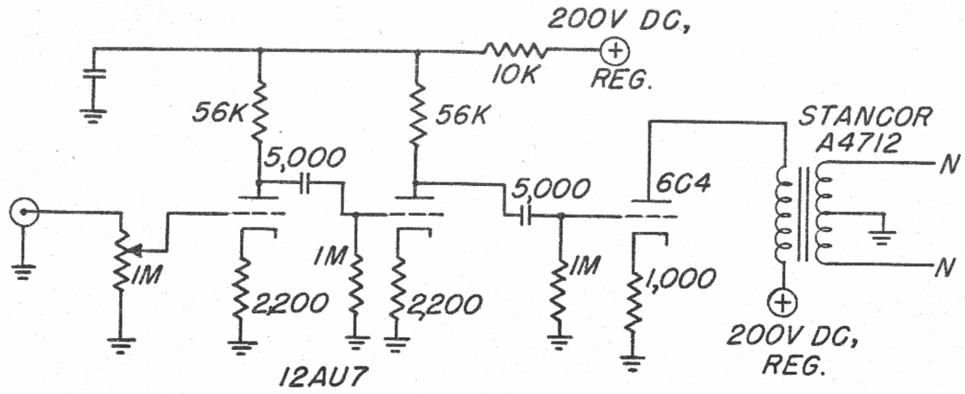
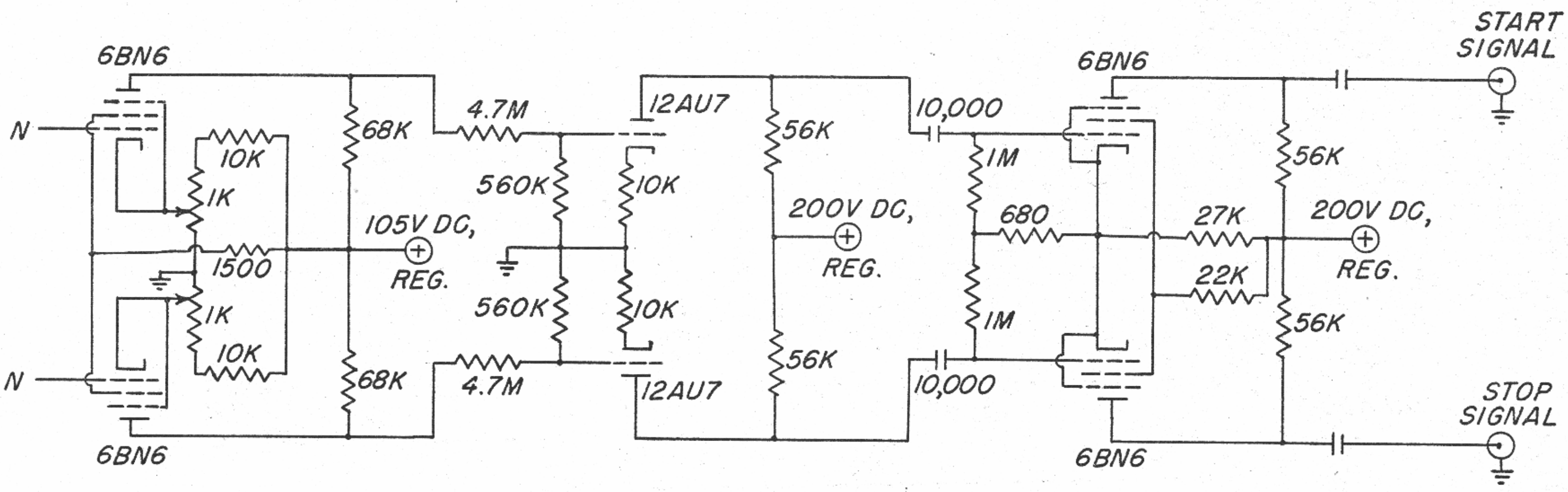


Figure 6. --Plot of amplitude versus time for a slowly decaying vibration A (from a steel bar) and a rapidly decaying vibration B (Douglas-fir beam). The sharpness of cutoff of the corresponding trains of discriminator pulses above each plot illustrates the precision that can be expected by this method under widely different conditions. Value of δ in A is 0. 00048 and in B 0. 018.

Z M 116 193



Preamplifier



Amplitude discriminator

Figure 7. --Circuit diagram of preamplifier and amplitude discriminator.

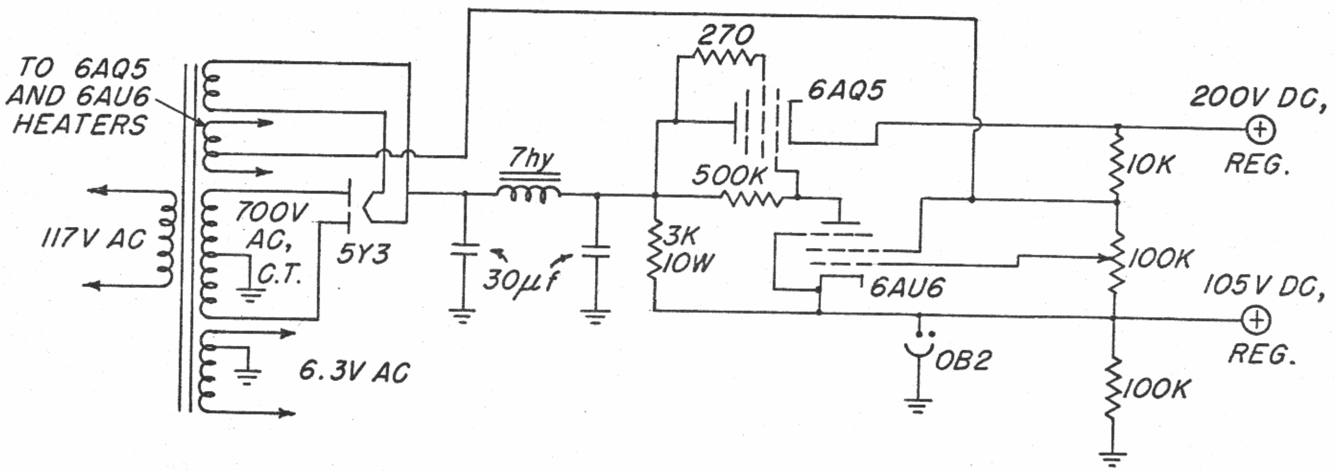


Figure 8. --Circuit diagram of power supply.

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