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This thesis describes the control circuitry for a telemetry system developed to report hydrologic information from remote locations. The multiparameter, on-call system operates on a single frequency. Each remote station responds to a predetermined number of pulses from the master station. Data is transmitted by pulse-duration modulation.

In the preliminary planning, careful consideration was given to the system reliability, the energy requirements of the remote station and to system flexibility. The reliability of the system is first examined theoretically by demonstrating that only the catastrophic failure of one or more parts will result in a system failure and that the mean time between such failures is 8.9 years. The system reliability was next examined by observing its operation at reduced voltage over a wide range of temperatures. This reliability was achieved at an energy level

which allows a winter of operation from a battery of reasonable size.

The system is shown to be flexible in a variety of ways. It can be operated with various types of radio equipment and with slight changes during construction at various frequency bandwidths. The system is flexible, both as regards the number of remote stations which can be operated with a master station and the number of parameters which can be telemetered by a remote station. Furthermore, the system is compatible with a wide variety of measurement transducers.

The system presented should prove satisfactory for many applications in addition to the one for which it was designed.

**CONTROL CIRCUITRY FOR
A TELEMETRY SYSTEM**

by

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CONTROL CIRCUITRY FOR A TELEMETRY SYSTEM

Introduction

The water stored each winter in the snow packs of the Northwest is of great economic importance. The proper use of this important natural resource is guaranteed only by complete, up-to-the minute information on the condition of the snow pack. Optimum use of river storage is basically different for irrigation, hydro-electric generation, and flood control. Cooperation between those interested in each of these three areas can be improved by more complete hydrologic information. Irrigation and hydro-electric interests, for example, are reluctant to release water for flood-control purposes unless they are assured of recovering the water. Flood-control operations, on the other hand, could conceivably reduce the safety factors now used if more complete information were available.

Because of the amount of money which can be involved in a single decision concerning the usage of river storage, large expenditures can be justified for gathering hydrologic data. This is indicated by the extensive system of snow courses operated by the Soil Conservation Service and the hydro-electric utilities. Although telemetry of

hydrologic data has attracted increased interest in recent years, a negligible amount of data has been procured by this means. This is probably because the telemetry systems used to date are not flexible enough to meet the needs of the hydrologists. The majority of the telemetry systems now used in this field are capable of measuring only one parameter. There is little doubt that if a reliable multiparameter system were available, it would be used extensively. No such system has been developed and proven, although it is certainly within the state of the art.

This thesis deals with the design of the control circuitry for a hydrological telemetry system. All aspects of the telemetry system, except the measurement transducers and the radio links, are discussed in detail. Every effort has been made to achieve a telemetry system compatible with a wide variety of measurement transducers and radio systems. This will be discussed in more detail in the section on system specifications.

System Specifications

Three considerations determined the specifications of the hydrological telemetry system. These are:

1. The regulations and policies of the Federal Communications Commission
2. The needs and desires of the hydrologists who will use the system
3. The remote location of the sites to be monitored.

The primary interest of the Federal Communications Commission is that a telemetering system of this type use a minimum amount of the frequency spectrum. Telemetry systems will be restricted more stringently in this regard as their usage increases. For this reason, the wise designer will operate a maximum number of stations on one channel of minimum bandwidth.

Hydrologists agree that an "on call" telemetry system is the most desirable and that the output data should be presented in a digital form capable of being printed. At the same time, as previously indicated, the hydrologists require a multiparameter system. The number of parameters needed, however, varies from site to site, requiring the system to be flexible in this regard.

The remote location of the sites to be monitored is

the most demanding of the factors controlling the system specifications. The remote stations must operate through the winter under severe climatic conditions with no maintenance. Furthermore, this must be done at low energy levels, since it is unlikely that commercial power will be available. Although wind generators or solar cells may be used to power the remote stations, a thermoelectric generator or a battery supply will be desirable in many cases. For this reason the energy requirements at the remote site should be low enough that a battery supply of reasonable size can operate the remote station through the winter.

Pulse-duration modulation was selected as the means of transmitting data. This analog form of data transmission was selected partly because it simplified the remote-station circuitry. Another consideration was that if pulses of variable duration are used to transmit the intelligence, the minimum length of these pulses can be adjusted to control the bandwidth of the radio signal.

Transforming the decisions and requirements outlined above into system specification results in the following:

1. The system shall use only one frequency with width of 6 kilocycles or less.
2. The system shall be an "on call" system.
3. The system shall be a multiparameter system

and remote stations having different numbers of parameters should be fully compatible with the master station.

4. The data shall be presented in digital form capable of being printed.
5. The remote stations must operate unattended under severe climatic conditions for at least nine months.
6. The remote stations must operate for nine months from a battery supply of reasonable size.
7. Pulse-duration modulation will be used to transmit data.

These specifications should result in a flexible telemetry system capable of being used for many applications.

System Operation

The telemetry control circuitry consists of five parts. These are:

1. The station selection and control system
2. The measurement interrogation system
3. The data collector
4. The data presentation circuitry at the master station
5. The radio interference reduction system.

Each of these parts will be discussed in a separate section.

Station Selection and Control

The system specifications required provision for a method of calling any one of a number of remote stations at any time. This type of station selection can be accomplished if each remote station can be made to respond to a given number of pulses from the master station. A transistorized binary counter, at the remote station is used to count the pulses from the master station. Additional circuitry must be provided to accomplish the following requirements for proper operation:

1. There must be a means of recognizing when the binary counter is indicating that the remote station should be energized.
2. There must be a means of insuring that a

station does not respond to an incomplete count.

3. The binary counter must be reset any time it receives a count, whether or not that count is the correct count to energize the particular remote station.
4. There must be a means of de-energizing a remote station once it is in operation.

The simplified block diagram of Figure 1 shows a system which will meet these requirements. The circuits contained in each block are shown on Figures 2 through 8.

Pulses from the remote-station receiver have a long rise time, for reasons discussed under "Radio Interference Reduction System." The rise times of these pulses are restored by the Schmidt trigger. The reshaped pulses are applied to a fifteen-second monostable multivibrator and the binary counter. Selected outputs of the binary counter, as well as the output of the fifteen-second multivibrator, are used as inputs to an AND circuit.

The inputs from the binary counter are applied to the AND circuit through R-C filters. The filters prevent the short pulses which can occur during reset of the counter from actuating the AND circuit. At the time of reset the fifteen-second multivibrator is not blocking the AND circuit and one or more bistable multivibrators will be in a transient state. The brief undetermined

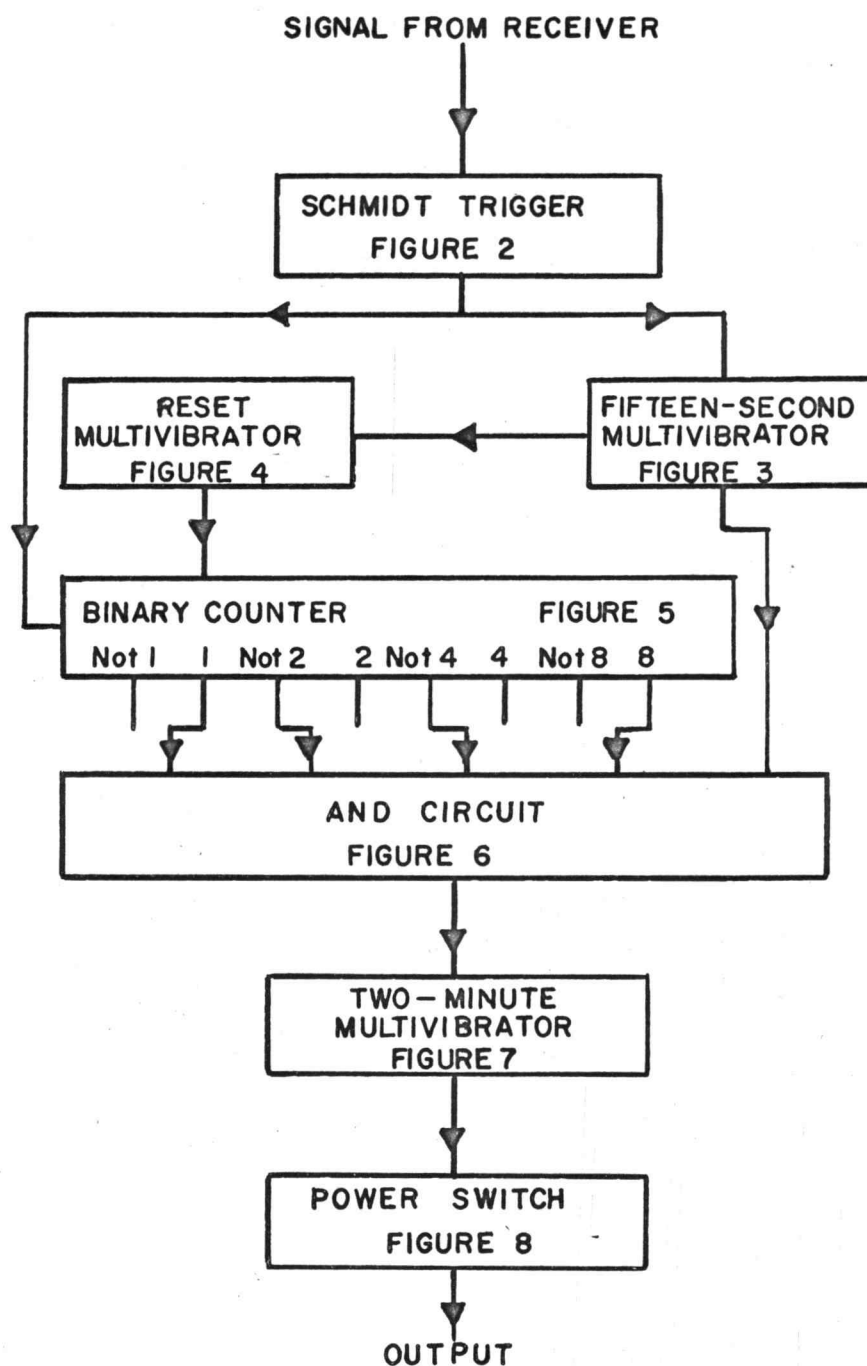


FIGURE 1. BLOCK DIAGRAM OF REMOTE-STATION SELECTION AND CONTROL SYSTEM

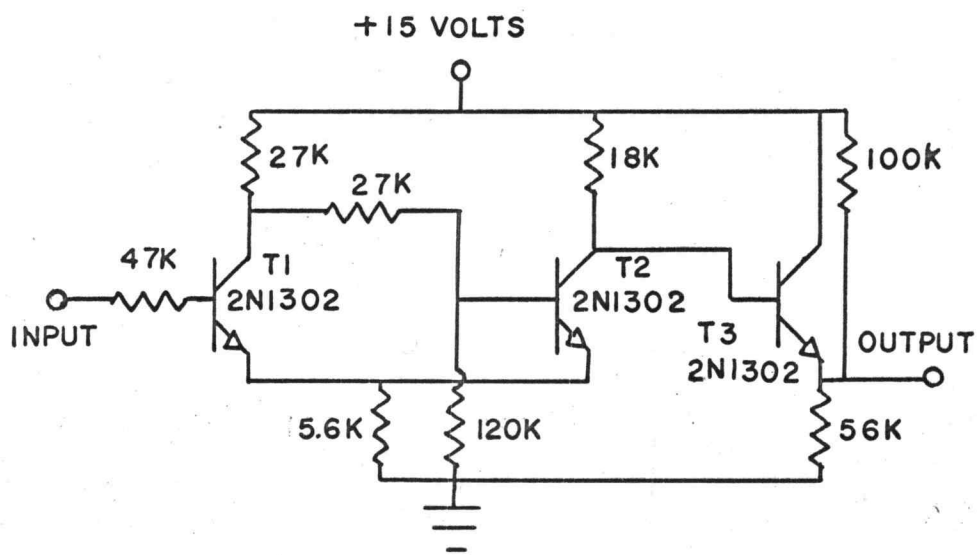


FIGURE 2. SCHMIDT TRIGGER AND EMITTER FOLLOWER

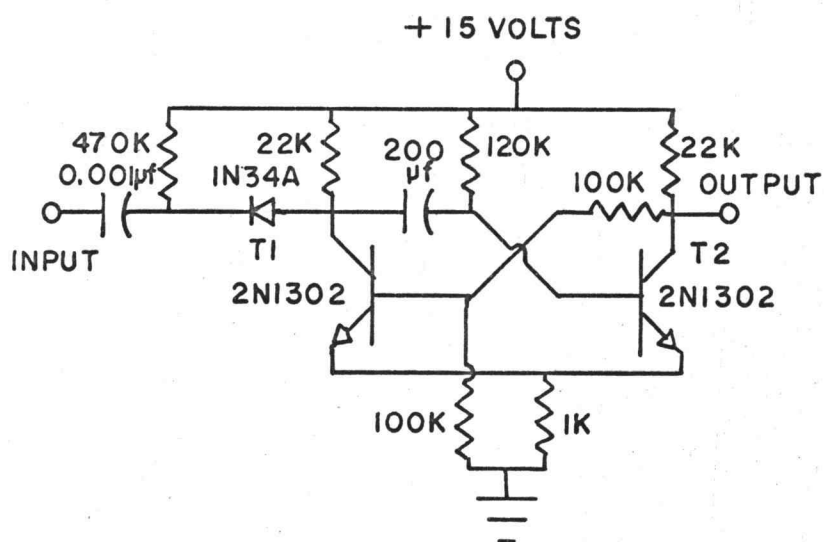


FIGURE 3. FIFTEEN-SECOND MONOSTABLE MULTIVIBRATOR

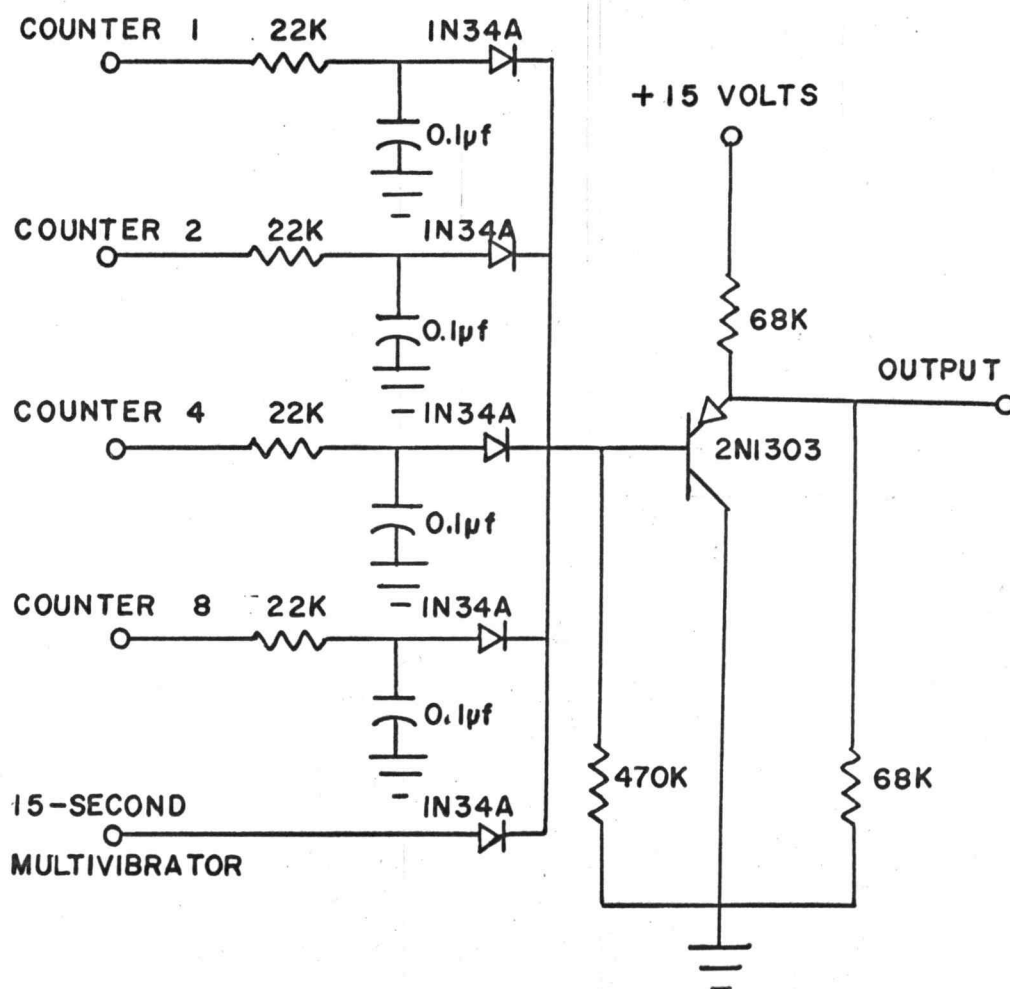


FIGURE 6. AND CIRCUIT

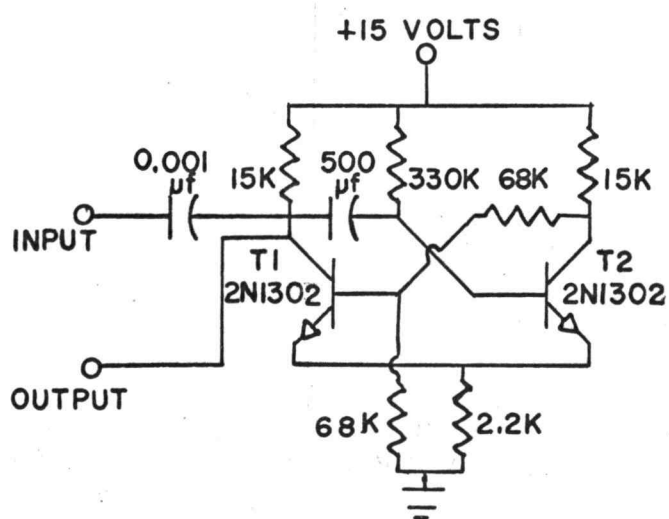


FIGURE 7. TWO-MINUTE MONOSTABLE MULTIVIBRATOR

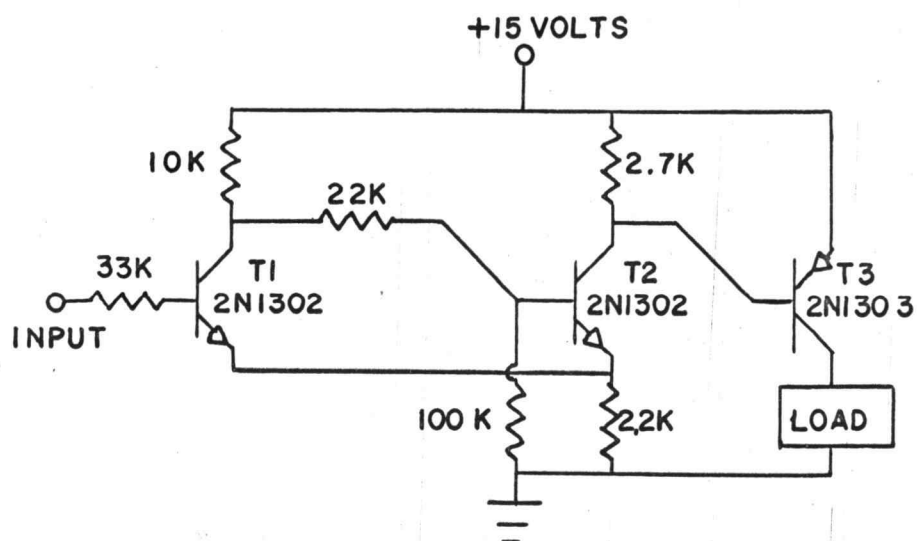


FIGURE 8. POWER SWITCH

position of the bistable multivibrators can result in wrong operation unless blocked by the filters.

The AND circuit gives an output pulse when all of its input voltages are at the lower of the two possible levels. The pulse from the AND circuit triggers a two-minute multivibrator which controls the power switch. The power switch, in turn, applies voltage to the measurement interrogation system, the measurements and the data collector. The station is automatically turned off when the two-minute multivibrator returns to its stable state.

One function of the fifteen-second multivibrator is to insure that the entire code reaches each remote station before any remote station is activated. The fifteen-second multivibrator is placed in its quasi-stable state by the first pulse received from the remote-station receiver, after which it is insensitive to additional pulses. In its quasi-stable state the fifteen-second multivibrator prevents the AND circuit from being actuated regardless of the condition of the binary counter. When the fifteen-second multivibrator returns to its stable state there is an output from the AND circuit if the binary counter has received the proper number of pulses. There is no filter between the fifteen-second multivibrator and the AND circuit. The rise time of the AND circuit output pulse

is, therefore, satisfactory for triggering the two-minute multivibrator.

In addition to blocking the AND circuit during the counting period, the fifteen-second multivibrator has the second function of initiating the counter reset sequence. When the fifteen-second monostable multivibrator returns to its stable state it triggers a reset multivibrator. This monostable multivibrator provides a pulse, after a short delay, to reset the binary counter. The length of the delay provided by the reset multivibrator is of little importance as long as there is a delay. Although the length of the reset delay controls the length of the output pulse from the AND circuit, the AND circuit must only provide a trigger pulse. The reset pulse is applied to each bistable multivibrator in the counter through a separate back-biased diode. These diodes prevent all undesired coupling through the reset circuit. The reset multivibrator resets the counter each time the fifteen-second multivibrator is triggered, whether or not the proper count is received. The counter, therefore, is always ready to receive a signal from the master station.

The code for each remote station is determined by the connections from the counter to the AND circuit. For example, a remote station will respond to nine pulses if ONE, NOT-TWO, NOT-FOUR, and EIGHT are connected to the

AND circuit. A four-stage counter of this type can control fifteen remote stations. The number of remote stations which can be controlled can be readily increased by adding stages to the counters.

Interrogation of Measurements

The measurement transducers to be used with this system, when triggered, are to produce a pulse. The duration of this pulse will be a function of the parameter to be telemetered. Although more complicated circuitry may be required for certain parameters, voltage, resistance or capacitance-controlled monostable multivibrators will be considered to be the measurement transducers for the purpose of this thesis. It then becomes the function of the measurement interrogation system to scan the measurements by triggering the monostable multivibrators in a predetermined order.

Interlacing the parameters allows complete access to each measurement whenever the remote station is transmitting. Therefore, each measurement is transmitted once in a predetermined order and the sequence is repeated continually throughout the transmit period. A relatively long transmitter off-time at the end of each measurement sequence is used to identify the start of the next sequence. The measurement interrogation system is shown on the top portion of Figure 9.

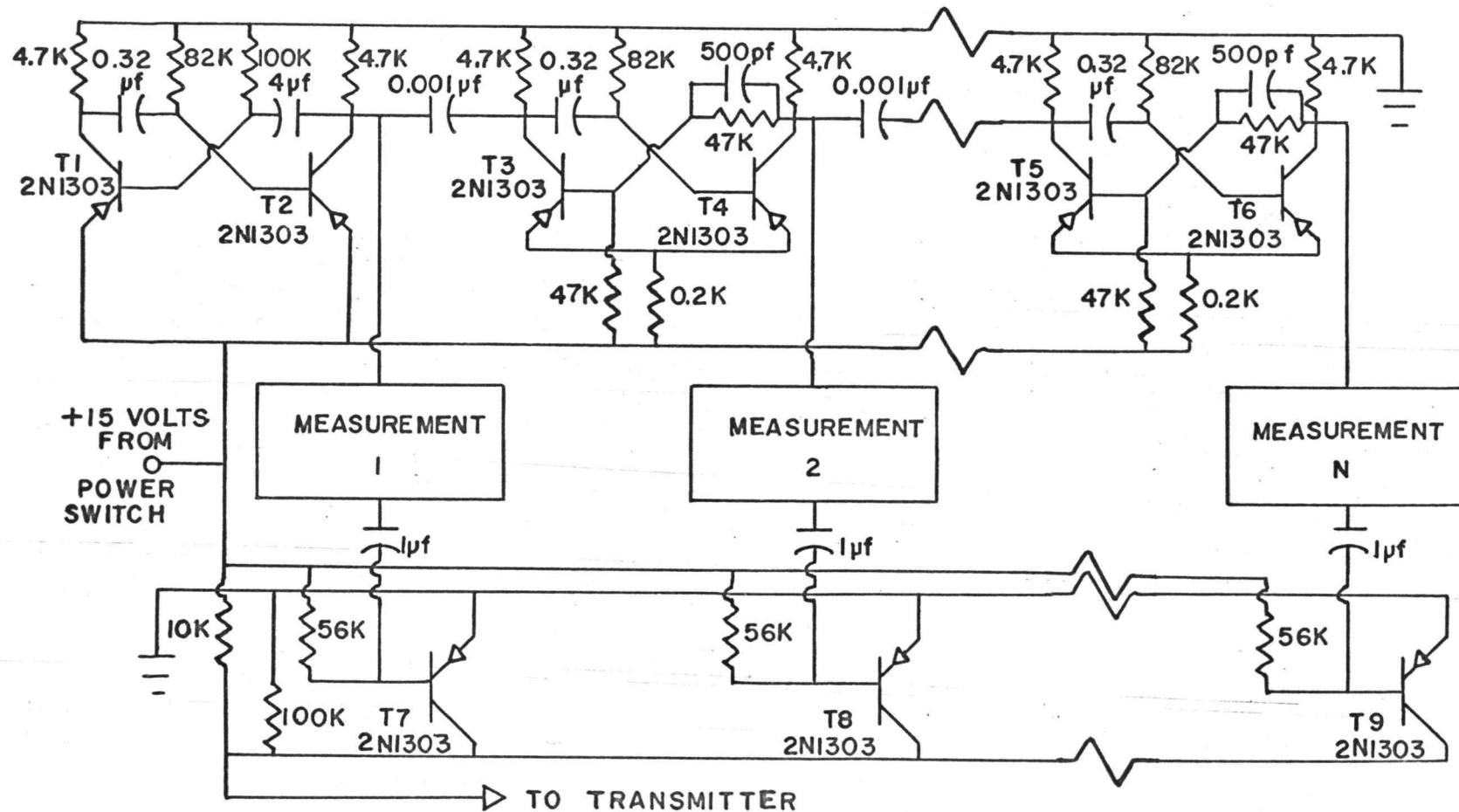


FIGURE 9. MEASUREMENT INTERROGATION SYSTEM AND DATA COLLECTOR 91

The measurement interrogation system consists of a chain of multivibrators, each of which triggers the next multivibrator and one measurement. The first multivibrator in this scanner is an astable multivibrator, while others are monostable multivibrators. The time between the start of each measurement sequence is controlled by adjusting the total period of the astable multivibrator. The time between any two measurements is controlled by adjusting the quasi-stable time of the related scanner multivibrator. The system now in use has measurement multivibrators which vary from 3 milliseconds to 13 milliseconds as a function of the parameter being measured. The scanner monostable multivibrators are set to have times of 18 milliseconds. Thus, the minimum transmitter on-time is 3 milliseconds and of equal importance, the minimum transmitter off-time is in excess of 3 milliseconds. The result is a bandwidth of 3.4 kilocycles. This can be determined by the relation $BW = \frac{10.15}{T_p}$ cycles per second (see Appendix C). BW is the frequency bandwidth containing 99 percent of the energy radiated by a single pulse of the carrier frequency T_p seconds long. This equation gives a pessimistic view of the bandwidth, first, because it assumes zero rise and fall time for the pulse and, second, because it considers a single pulse rather than a chain of pulses. Specification 1, regarding bandwidth, is thus easily met.

The astable multivibrator period is set to give a dead time in the measurement sequence slightly greater than the total time required for all of the measurements. This allows for rapid identification of the first measurement by the master station circuitry.

Data Collection

The problem of data collection in a system of this type is a simple one. The data collector must collect the output pulses from each measurement multivibrator and apply them to the remote-station transmitter in the same sequence as they are generated by the scanner measurement combination. An OR circuit will readily accomplish this.

The pulses from the OR circuit are used to control the remote-station transmitter. The transmitter is off when there is no pulse from the OR circuit and on when there is a pulse from the OR circuit. The OR circuit used is shown on the bottom portion of Figure 9. In this circuit all of the transistors are normally off and the output voltage is determined by the voltage divider. If any one of the transistors conducts, the output voltage approaches the collector supply voltage. Thus, there is an output pulse for each measurement pulse and the times of the two pulses differ by negligible amounts. Note that the number of parameters to be telemetered is easily adjustable by changing the number of stages in the scanner

and data collector.

Master-Station Data Presentation

At the master station there must be a means to present each parameter. First, the measurement pulse to be read must be separated from the other measurement pulses. This is accomplished by synchronizing a counter at the master station with the scanner at the remote station and counting to the desired pulse. Figure 10 shows a block diagram of the Master-Station Data Presentation System. The incoming pulses are passed from the master-station receiver through a radio interference reduction system to the Data Presentation Circuitry. There the pulses are applied to a binary counter, a reset multivibrator and an AND circuit. The reset monostable multivibrator has a quasi-stable state time slightly longer than the longest measurement sequence expected, but shorter than the dead time between measurement sequences. This timing arrangement allows the counters to be properly reset before the second measurement sequence is received. Thus synchronized, the counter counts to the desired pulse. The counter is connected to the AND circuit through a switching system so that it blocks the AND circuit for all but the desired count. When this count is reached, the AND circuit passes the selected pulse to the measurement device. The system shown on Figure 10 has a three-stage

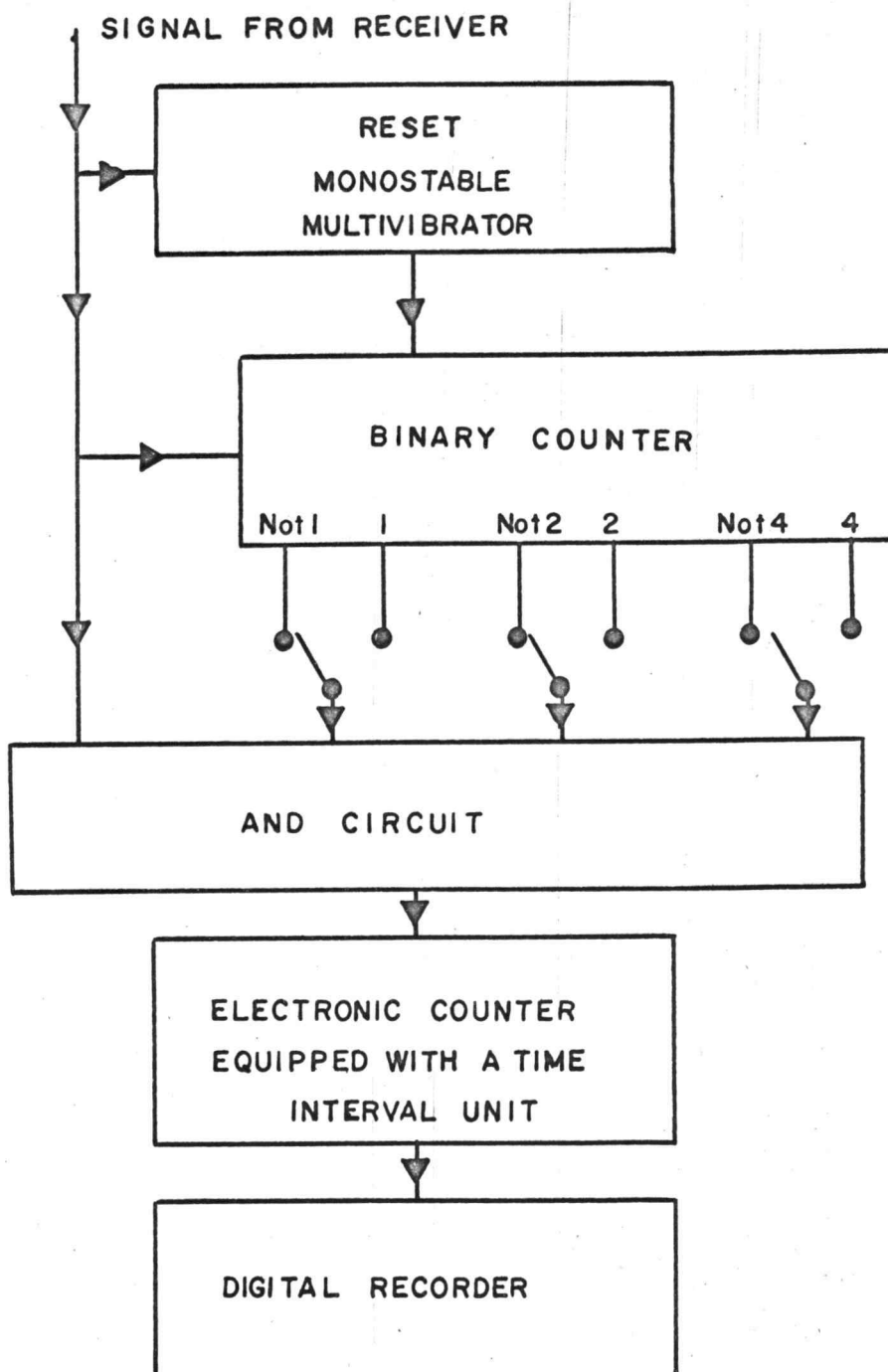


FIGURE 10. BLOCK DIAGRAM OF THE
MASTER-STATION PRESENTATION SYSTEM

counter and can be used if no remote station telemeters more than eight parameters. The number of parameters can easily be expanded to sixteen by use of a four-stage counter. An important feature of the data presentation system is that it is completely compatible with remote stations transmitting any number of parameters up to its limit.

Once the desired pulse has been separated from the measurement sequence its length can be measured by either an electronic counter equipped with a time interval unit or an oscilloscope. The electronic counter gives the desired digital presentation and can be coupled with a digital recorder which will print date and time as well as pulse data. The oscilloscope has reduced accuracy and convenience, but is much less expensive than the electronic counter.

The master station is not limited in power consumption as are the remote stations. Therefore, any of the commercially available bistable multivibrators, monostable multivibrators, and AND circuits can be used. Or, in the interest of uniformity the appropriate units of the remote-station selection circuitry can be used. In either case it is desirable to use one of the wide variety of commercially available electronic counters and its associated digital recorder.

Radio Interference Reduction System

The elimination of extraneous pulses from the master station logic circuitry is essential, since the pulse to be presented is selected by counting the preceding pulses. While it is assumed that the normal methods of combating radio interference are employed in the design of the radio equipment, the characteristics of the signal transmitted by the remote stations allow the use of an additional technique to eliminate noise. The technique is based on the elimination of short pulses.

At the remote station 3 milliseconds is added to each information pulse to limit the bandwidth of the transmitted signal. Any part of this 3 milliseconds may be subtracted at the master station with no loss of information, provided the exact time subtracted is known. A circuit is shown on Figure 11 which can be adjusted to subtract 3 milliseconds from each pulse received, and thus eliminate all pulses less than 3 milliseconds in duration. This circuit eliminates all impulse noise and all noise which results in a frequency above 170 cycles per second after demodulation.

The signal from the receiver is a series of positive pulses which have been amplified and clipped such that the information pulses have an amplitude of 15 volts from a zero-volt base. The noise between the information

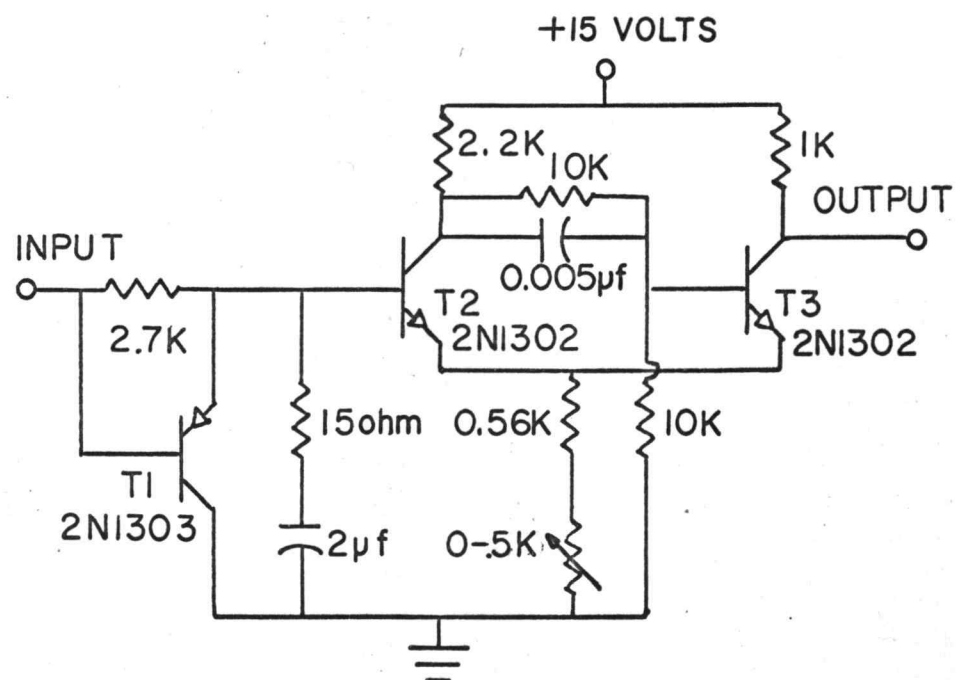


FIGURE II. SHORT PULSE ELIMINATION CIRCUIT

pulses may range in amplitude from zero to 15 volts. Although 15 volts is applied to the input of the circuit, the 2-microfarad capacitor voltage is limited to 6.3 volts, since at any higher voltage transistor T_2 is saturated and the capacitor voltage drops to 4.4 volts. A 15-volt input pulse thus causes the base of transistor T_1 to become positive with respect to the emitter, thereby cutting off the PNP transistor. Assuming that the 2-microfarad capacitor has little or no charge, transistors T_1 and T_2 are both nonconducting and the 2-microfarad capacitor is charged through the series combination of the 2,700 and 15-ohm resistors. In 3 milliseconds the voltage on the base of T_2 will become 6.3 volts and transistor T_2 will turn on. In accordance with normal Schmidt trigger action, the emitter voltage of transistor T_2 will decrease when T_2 is turned on, insuring positive switching. When the input returns to zero, the base of T_1 will be negative with respect to the emitter, the transistor will conduct, and the Schmidt trigger will be returned to the rest condition. While T_1 is conducting, the capacitor will be discharged through the 15-ohm resistor, which is included to prevent excessive collector current in transistor T_1 . The result is a circuit which subtracts 3 milliseconds from the leading edge of each pulse, while leaving the trailing edge essentially unchanged.

Note that the fast discharge system eliminates a number of difficulties which might exist if a simple R-C filter were used. For example, false triggering by a large number of closely spaced short pulses is virtually eliminated.

The technic of eliminating short pulses is used effectively at the remote stations, as well as at the master station. At the remote stations the technic is used, not only to eliminate noise, but to eliminate the measurement pulses from other remote stations. This is necessary, since the master and all remote stations transmit pulses on the same frequency. The longest pulse transmitted by the remote stations has a duration of 13 milliseconds. Maintaining the length of the call pulses from the master station to a length of at least one-fourth second provides a good margin for separation. The problem at the remote station is simpler than that at the master station, because the length of the incoming pulse need not be preserved. An R-C filter located in the audio amplifier of the receiver is, therefore, a satisfactory method of eliminating the short pulses. If more than 15 remote stations are to be controlled by a single master station, the quick-discharge method used at the master station would be advantageous because it would allow more reliable separation of pulses more nearly equal in length. This

would allow the larger number of pulses required to be sent without increasing the system "dead time".

Remote-Station Design Considerations

The function of the various parts of the remote control circuitry have been discussed in the section on system operation. The block diagrams presented meet the operational requirements of the system. It will now be shown that proper design of the circuitry will result in the high reliability specified. The word design is used in a broad sense to include packaging since this is an important part of designing reliable circuitry.

The failure rates of the components used in any circuit are a function of the stress levels at which the component is operated as well as the characteristics of the component itself. Stresses such as voltage, current, temperature, vibration and humidity must, therefore, be considered in determining the failure rate of a given component. The effects of one or more of these stress levels may be negligible due to the characteristics of the component itself or the intended useage. In other cases packaging may be used to reduce the stress levels on the components.

In general, a reduction of the electrical stress levels of voltage and current will result in a reduced failure rate. This reduction can be accomplished either by the use of components with higher ratings or by

reduction of the current and voltage. The transistor is an active element which will function properly at greatly reduced voltages and currents. This allows all of the circuit components to be derated without using components with higher ratings. Furthermore, transistors have a low failure rate even when operated under rated conditions. These factors, combined with others that will be indicated later in the discussion, prompted the use of transistors as the primary active element.

The temperature at which a component is operated has an important bearing on the failure rate of most components. The temperature of the component depends upon the ambient temperature and upon the power dissipated in the component. For most purely electrical components reliability is improved as the temperature is lowered. The improvement in reliability below temperatures of 20°C is negligible in most cases, however. The temperature variation of the remote-station control circuitry, as well as the maximum temperature, has been controlled by placing the circuitry underground. The heat sink of earth, accompanied by good insulation, results in a low circuit temperature which is fairly stable. In general, the temperature can be held below 20°C , even in mid-summer.

Vibration is not a significant stress in this application because the equipment is stationary. Although this

stress might be important during transport of the equipment prior to installation, the effects of vibration are reduced by embedding all the components in an epoxi-resin and mounting them solidly. Individual circuits, such as a monostable multivibrator, a bistable multivibrator or an AND circuit, are potted separately. The potting also greatly reduces the effects of humidity on the circuits. Humidity is further reduced by placing the circuitry in a pressurized air-tight box containing a drying agent. As long as a slight positive pressure can be maintained in the box, no moisture can enter. Any moisture trapped when the box is closed is absorbed by the drying agent.

The control of the stress levels outlined above results in all components being stressed to less than one-tenth of their rated value. The stress levels are determined by reference 5. Additional reduction of the stress levels has negligible effect on the component failure rates (5, p. 1). The mean time before failure of the remote-station control circuitry, as calculated by the method of reference 5, is 8.9 years. (Calculations are shown in Appendix A.) This figure must be qualified by the following statement.

"These failure-rate figures are the best engineering approximation of the reliability characteristics (random failures) for the parts designated when employed

repeatedly, within their specification ratings, in complex electronic equipment. Failures are considered to be opens, shorts, or radical departures from initial characteristics occurring in an unpredictable manner and in too short a period of time to permit detection through normal preventive maintenance practices" (5, p. 7).

Although the study of component reliabilities and stress levels is admittedly not exact, it does insure that the designer has considered the important factors of reliability. For example, printed circuits, pluggable units, and transistor sockets are eliminated from consideration when it is noted that a single multivibrator using these features has a failure rate of 0.085 percent failures per 1000 hours, while the same unit using soldered and wired construction has a failure rate of 0.048 percent failures per 1000 hours.

Component reliability is of little value unless the circuits used are equally reliable. The first step in obtaining reliable system has already been accomplished, in that no critical circuits are required. All of the circuits operate in an "on-off" fashion and high-speed switching is not required. Furthermore, the times of the monostable multivibrators are not critical. The scanner multivibrators are the most critical and are limited, in that the quasi-stable time should not

decrease more than 28 percent.

The fact that high-speed switching is not required allows the use of transistors which have a relatively low alpha cutoff frequency. These transistors are more rugged in their construction than high-frequency transistors and, therefore, less susceptible to damage by transients. Furthermore, since low switching speeds are acceptable, all multivibrators can be designed to operate with their transistors either solidly cut off or well into saturation. This is important for two reasons. First, the characteristics of the components, including the transistor, can vary either due to temperature or aging without appreciably changing the operation of the circuit. Second, the supply voltage can vary over an appreciable range without adversely effecting circuit operation.

Operating the transistors in saturation and cutoff reduces the effects of variations in the supply voltage because all of the voltages and currents in the circuit change in very nearly a direct proportion. The only exceptions are the emitter-base and emitter-collector voltages of the conducting transistor. These remain essentially constant, but are on the order of 0.3 volts and 0.1 volts, respectively, and have little effect on the proportionality of the larger voltages in the circuit.

In order for the study of component reliability to

be meaningful, the components must be able to vary appreciably because the failure-rate data only considers opens, shorts and radical departures from initial component characteristics. This is accomplished, as mentioned above, by operating the transistors well into saturation and cutoff. For example, transistor T_1 of the fifteen-second monostable multivibrator requires 15 microamperes base current to be saturated, but 105 microamperes are supplied. Thus, the current gain of the transistor can decrease by a factor of seven before transistor T_1 ceases to be in saturation. Or, if all the components except the transistor vary 40 percent in the worst direction for saturation of T_1 , 49 microamperes are still supplied to the base of this transistor. The current gain of the transistor can decrease 30 percent and the transistor will still be in saturation. Actually, even wider variations are possible, since the multivibrator will operate with one of its transistors in the linear region in each of its two states.

The circuits used in the remote-station control circuitry are commonly used (1, 2 and 3). For this reason, the design of the circuits has not been included in the body of the thesis. These designs may be seen in Appendix B. In addition, Appendix B includes calculations showing that each resistor and capacitor in each circuit

can vary 40 percent in the worst direction without causing a malfunction. The 40-percent variation allows for the use of 10-percent components plus an additional change of 30 percent. This additional 30-percent change allows for variations in the transistor parameters, as well as assuring that the circuits will not fail, unless one or more components vary radically from their nominal values.

In order to reduce the energy required by the station selection circuitry, the impedance levels have been increased markedly from those commonly used in such circuits. The low-level common-emitter characteristics of Figure 12 show that such reduction of the transistor currents is possible. The high impedances of the circuits may cause difficulty when one multivibrator must provide a pulse to several circuits. This is corrected by the use of emitter followers which are solidly cut off, except when they are pulsed. The emitter followers are biased off during stand-by conditions by high-impedance voltage dividers. When the base of the emitter follower is pulsed the transistor is turned on, shorting the emitter-to-collector resistor in the voltage divider and the unit operates as an emitter-follower amplifier. This technic has been used on the Schmidt trigger, the reset multivibrator and the AND circuit.

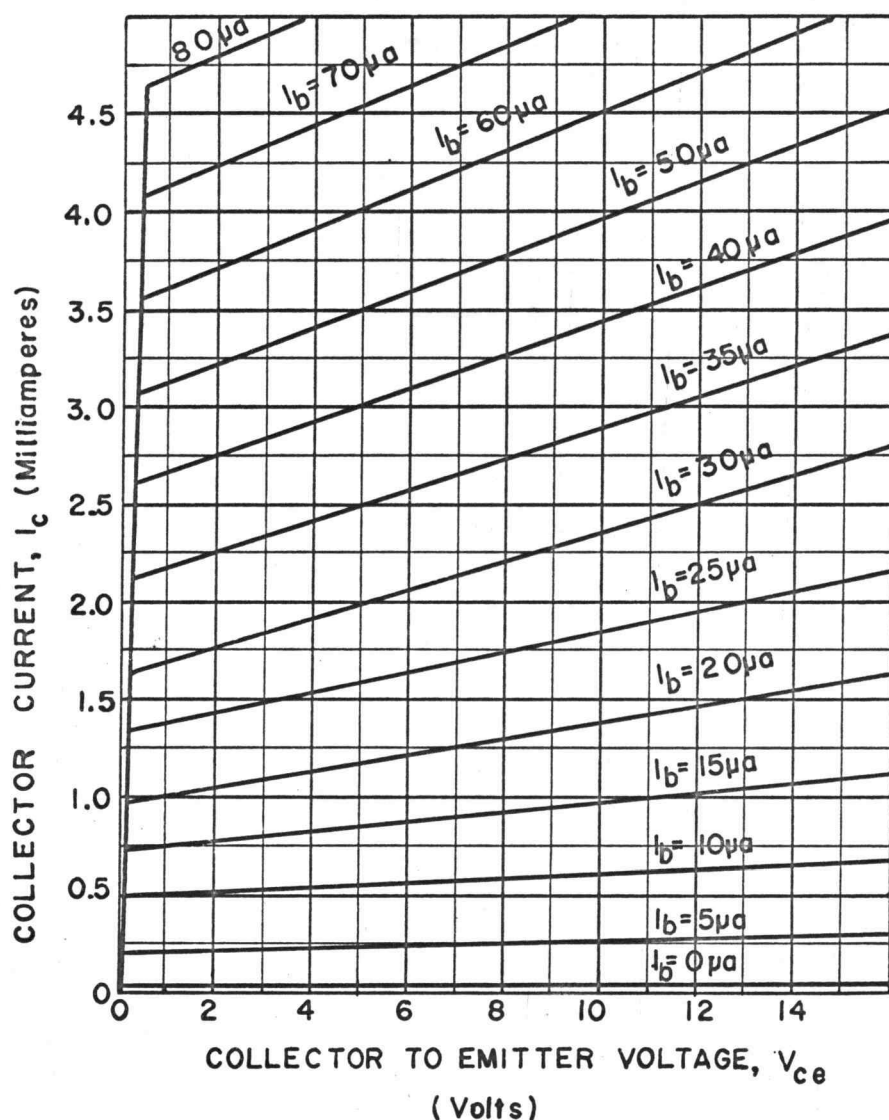


FIGURE 12. LOW-CURRENT COMMON-EMITTER CHARACTERISTICS FOR 2N1302 OR 2N1303 COMPLEMENTARY TRANSISTORS

Table 1 shows the current required for each of the circuits of station selection system. The total of 23.7 milliamperes allows the remote station to operate for nine months from a 15-volt 160-ampere-hour battery. Reduction in energy consumption beyond this point, although desirable, is not necessary because a 15-volt 160-ampere-hour battery is not unwieldy.

Table 1

**ENERGY REQUIREMENTS
FOR THE REMOTE STATION
IN THE STAND-BY CONDITION**

(Figures based on a 15-volt battery.)

Control Circuitry Unit	Current Milliamperes	Ampere Hours For 270 Days
Schmidt Trigger	0.82	
15-Second Multivibrator	0.65	
Reset Multivibrator and Amplifier	1.09	
AND Circuit	0.15	
Two-Minute Multivibrator	0.87	
Power Switch	1.23	
Four-Stage Binary Counter	4.93	
Total Control Circuitry	9.69	62.8

**Other Units at the Remote
Station**

Transmitter	3.00	19.5
Receiver	11.00	71.5
Total Remote Station	23.69	153.8

Tests of Remote-Station Control Circuitry

The nature of the circuits used in the remote-station control circuitry makes it difficult to measure the system reliability by any means other than long-term operation. The theoretical approach has indicated that the system reliability should be satisfactory. To insure that these calculations apply and that the design techniques used are sound, all circuit voltages were measured and compared with the calculated values. In addition, the quasi-stable-state times of all monostable multivibrators were measured and compared with the theoretical value of $RC \ln 2$ seconds. This time holds only when the transistors of the multivibrator are either cut off or saturated at all times (see derivation, p. 54-55).

The high-circuit stability incorporated in the design of the circuit, as well as any unexpected adverse effects of interconnection of the circuit parts, was examined by operating the circuits at reduced voltage and temperature. Figure 13 shows the results of these tests in the form of the lowest voltage at which the circuit functions properly, plotted against the temperature in degrees centigrade. The test indicates that an energy saving can be achieved by operating the circuitry at reduced voltage with little reduction of reliability.

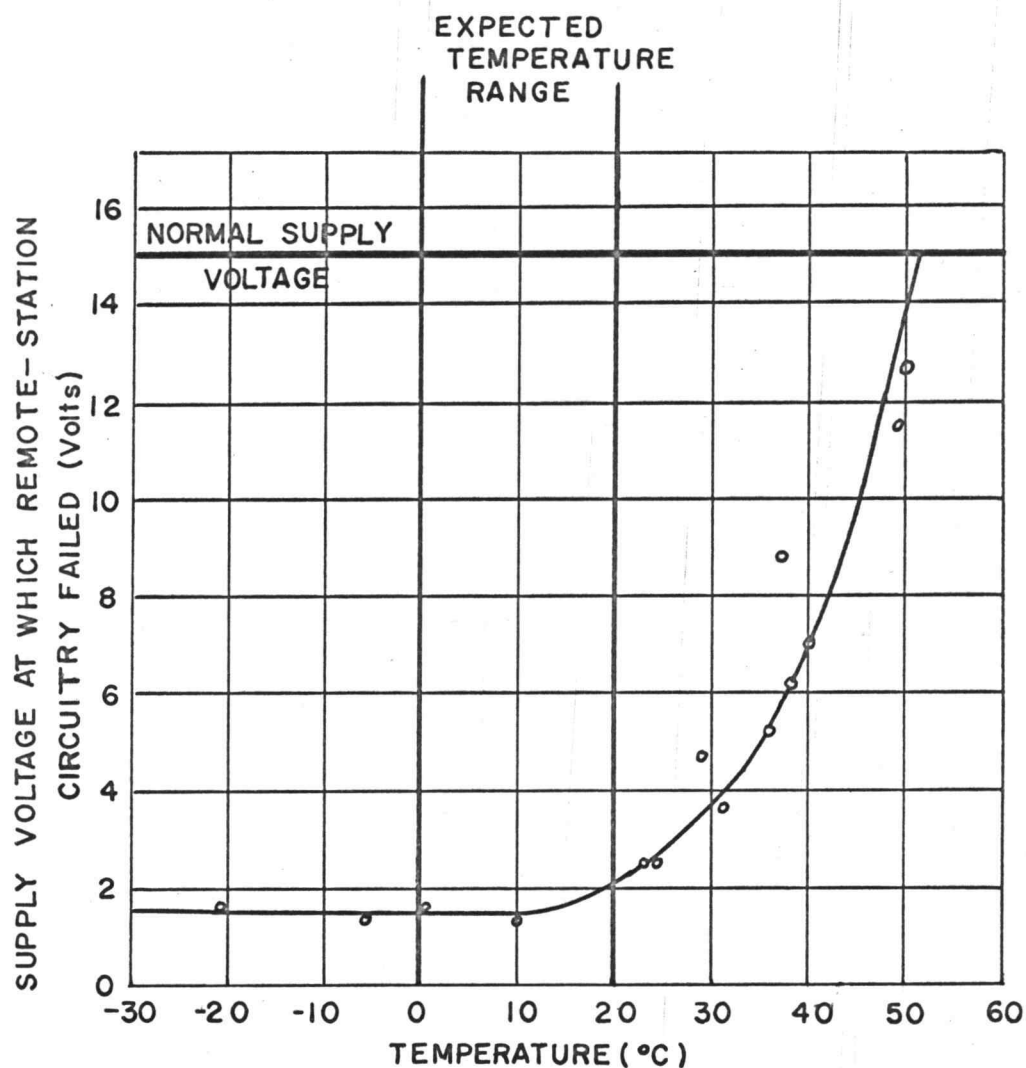


FIGURE 13. A STUDY OF RELIABILITY OVER A WIDE TEMPERATURE RANGE USING LOWEST-OPERATING VOLTAGE AS AN INDEX OF RELIABILITY

Conclusions

A telemetering system which meets the system specifications has been described. The multiparameter, on-call system operates on a single channel and is flexible, both in the number of remote stations which can be controlled and the number of parameters at each remote station.

The system is compatible with either AM or FM radio equipment. Amplitude modulation requires less bandwidth. Radio interference can be greatly reduced by short-pulse elimination.

The mean time between failures of the remote-station control unit was shown to be 8.9 years. Although this failure rate is good, it means one station will probably fail each year for each group of twelve operated (9 months of operation are considered). The energy requirements have been held to a level where 160 ampere hours from a 15-volt battery will operate the remote station for a winter season.

Equipment using the same principles of operation as described in this thesis, but with slight variations in circuitry, was operated without failure through the winter of 1962-63. During this period of operation measurements taken on site were compared with telemetered data. In spite of the use of an oscilloscope to present

the data, the system was demonstrated to be capable of reliable accuracies of ± 1 percent. The majority of this error must be charged to the transducers and the oscilloscope.

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APPENDICES

APPENDIX A

Calculations of Mean Time
Between Random Catastrophic Failures

Reliability of the equipment to be used at the remote site is of great importance. For this reason, the calculation of the mean time between catastrophic failures of any part in the remote-station control circuitry is included. These calculations are made under the assumption that the failures will be random in occurrence. This study does not include failures due to the aging of parts. Complete maintenance once a year should eliminate all failures due to aging. The failure rate of each type component is considered separately. The results are then combined to give a survival probability for the complete equipment. The problem is approached by the method outlined in reference 5.

Transistors

The transistors used are type 2N1302-03. The transistor characteristics (4, p. 1) state that the "Total Device Dissipation at (or below) 25° C Free-Air Temperature is 150 mw." The operating temperature will be below this value, so it is not necessary to derate for temperature. The transistors, which are conducting while the remote station is in stand-by, conduct a maximum of

1.3 milliamperes. The voltage drop is 0.2 volts.

The average power dissipated is:

$$P_a = (1.3 \times 10^{-3})(0.2) = 0.26 \text{ milliwatts.}$$

The stress ratio is:

$$\text{Stress ratio} = \frac{0.26}{150} = 0.0173$$

On page 1, reference 5 indicates the lowest electrical stress ratio which should be used on any component is 0.1. Thus, in all cases where the electrical stress ratio on a component is less than 0.1 it is arbitrarily increased. The transistors which are non-conducting during stand-by periods have a very low stress ratio due to the low duty cycle. The result is that all transistors used are assumed to have a stress ratio of 0.1. Reference 5, page 48-49, indicates the result is a failure rate of 0.02 percent/1000 hours.

Diodes

The diodes used are type 1N34A. These are 50-volt, general-purpose diodes, rated for an average anode current of 50 milliamperes at 25° C. Because of the high impedance levels and low operating temperature, none of the diodes have a stress ratio above 0.1. The failure rate, as taken from pages 44 and 45 of reference 5, is 0.01 percent/1000 hours.

Resistors

The resistors used are half-watt, composition resistors. The maximum current carried is 1.3 milliamperes. The stress ratio for this worst case is:

$$\text{Stress Ratio} = \frac{15 (1.3 \times 10^{-3})}{0.5}$$

$$= .039$$

Page 78 in reference 5 shows that at an ambient temperature of 20° C and a stress ratio of 0.1 the failure rate is 0.001 percent/1000 hours. This value will be used for all resistors.

Capacitors

Three types of capacitors are used. The first are 100-volt paper capacitors. The failure rate is determined by ratio of operating voltage, rather than power dissipated. The worst voltage ratio possible is:

$$\text{Voltage Ratio} = \frac{15}{100} = 0.15$$

On page 104 of reference 5, a failure rate of 0.001 is indicated for a voltage ratio of 0.15 and a temperature of 20° C, regardless of the temperature range of the capacitor.

The second types of capacitors used are 250-volt, ceramic capacitors. Here the voltage ratio is 0.06, and from reference 5, page 116, the failure rate of

0.001 is indicated for all capacitor temperature ranges.

The third types of capacitors used are 50-volt, wetslug, tantalum capacitors. Considering the voltage ratio = $\frac{15}{50} = 0.3$ and the temperature is less than 20° C, reference 5, page 123, indicates a failure rate of 0.001 percent/1000 hours.

System Reliability

Table 2 combines the failure rates for the remote-station selection unit. The failure rate for the station selection unit is 0.739 percent failures/1000 hours. This results in a mean time, before failure, of

$$\begin{aligned} \text{MTBF} &= \frac{1000}{7.39 \times 10^{-3}} = 135 \times 10^3 \text{ hours} \\ &= 16 \text{ years} \end{aligned}$$

The failure rate of the measurement interrogation and data collector unit is computed in Table 3. The mean time before failure can be calculated in the same manner as is shown above and is 20 years. The combined failure rate is the sum of the two parts or 1.333 percent failures/1000 hours, resulting in a mean time before failure of the remote-station control system of 8.87 years.

Table 2**COMBINED FAILURE RATES FOR THE
STATION SELECTION AND CONTROL UNIT**

Component	Failure Rate Percent/1000 Hours	Number of Components	Total Failure Rate Percent/1000 Hours
Transistor	0.02	22	0.44
Diode	0.01	19	0.19
Resistor	0.001	80	0.08
Capacitors	0.001	24	0.024
Connector (4 pin)	0.005	1	0.005
Unit Total			0.739

Table 3

COMBINED FAILURE RATES OF MEASUREMENT
INTERROGATION AND COLLECTION SYSTEMS
ASSUMING EIGHT MEASUREMENTS

	Failure Rate Percent/1000 Hours	Number of Components	Total Failure Rate Percent/1000 Hours
Transistors	0.02	24	.48
Resistors	0.001	80	0.08
Capacitors	0.001	24	0.024
Connector (12 pin)	0.01	1	0.01
Unit Total			0.594

APPENDIX B

Design of Remote-Station Control Circuits

The factors considered in the design of the remote-station control circuits are considered in the body of the thesis under Remote-Station Design Considerations. Numerical designs are included herein for each type of circuit. The important design values for the circuits not completely designed are tabulated. Included in the design of each circuit are worst-case variations of all resistors. The circuits are designed to operate properly if each resistor changes value 40 percent in the worst direction.

The common-emitter characteristics of the 2N1302-3 series transistors are shown on Figure 12. In saturation, the voltage from emitter to collector, V_{ec} , is nominally 0.1 volt, while 0.3 volt is an acceptable value for the emitter-to-base voltage, V_{eb} . The method of design used is a trial-and-error method. Trial circuit values are selected and the circuit analyzed to determine if the transistors are in saturation and cutoff. If the operating characteristics are not satisfactory, adjustments are made and the analysis repeated. Only the final analysis is shown for each case. The notation used follows:

V_s	supply voltage
V_e	emitter voltage for common emitter connection
V_{e1}	emitter voltage for transistor, T_1
V_{ec2}	emitter to collector voltage for transistor, T_2
I_{b1}	base current for transistor, T_1
R_c	collector resistor
R_{ble2}	resistor between the base of T_1 and the collector of T_2
R_{sc2}	resistor between the supply voltage and the collector of T_2

If the nominal value of a resistor is to be increased 40 percent, it is indicated as $(R_{sc2} \times 1.4)$, whereas $(R_{ble2} \times 0.6)$ indicates a decrease in the resistor from the base of T_1 to ground of 40 percent.

Monostable Multivibrators

The fifteen-second monostable multivibrator (Figure 3) design is shown below as an example of the design procedure for a monostable multivibrator. The calculations assume that the current through the bias resistor, R_{ble2} , is very small compared with the collector current of transistor T_2 . Stable-state calculations for nominal values are presented first. T_1 should be off and T_2 in saturation.

$$\begin{aligned}
 I_{e2} &= \frac{V_s - V_{ec2}}{R_{e2} + R_e} \\
 &= \frac{15 - 0.1}{22K + 1K} \\
 &= 0.65 \text{ milliamperes}
 \end{aligned}$$

$$\begin{aligned}
 V_e &= I_{e2} R_e \\
 &= (0.65)(1) = 0.65 \text{ volts}
 \end{aligned}$$

$$\begin{aligned}
 V_{b2} &= V_e + V_{eb2} \\
 &= 0.65 + 0.3 = 0.95 \text{ volts}
 \end{aligned}$$

$$\begin{aligned}
 I_{b2} &= \frac{V_s - V_{b2}}{R_{sb2}} \\
 &= \frac{15 - 0.95}{120K} \\
 &= 117 \text{ microamperes}
 \end{aligned}$$

Figure 12 shows $I_b = 15$ microamperes will saturate T_2 .

$$\begin{aligned}
 V_{c2} &= V_e + V_{ec2} \\
 &= 0.65 + 0.1 = 0.75 \text{ volts}
 \end{aligned}$$

$$\begin{aligned}
 V_{b1} &= \frac{V_{c2} R_{c2b1}}{R_{c2b1} + R_{b1g}} \\
 &= \frac{0.75 (100K)}{100K + 100K} = 0.38 \text{ volts}
 \end{aligned}$$

$$V_{eb1} = -0.27 \text{ volts}$$

The above calculations are repeated below, allowing each resistor to vary 40 percent in the worst direction.

Saturation of T_2 becomes more difficult as I_{c2} and V_e increase.

$$I_{c2} = \frac{V_s - V_{ec2}}{(R_{c2} \times 0.6) + (R_e \times 1.4)}$$

$$= \frac{15 - 0.1}{(22K \times 0.6) + (1K \times 1.4)}$$

$$= 1.02 \text{ milliamperes}$$

$$V_e = I_{c2} (R_e \times 1.4)$$

$$= (1.02)(1K \times 1.4)$$

$$= 1.43 \text{ volts}$$

$$V_{b2} = V_e + V_{eb}$$

$$= 1.43 + 0.3$$

$$= 1.73 \text{ volts}$$

Increasing the base resistor is the worst case.

$$I_{b2} = \frac{V_s - V_{b2}}{(R_{sb2} \times 1.4)}$$

$$I_{b2} = \frac{15 - 1.73}{(120K \times 1.4)} = 79 \text{ microamperes}$$

Figure 12 shows $I_b = 21$ microamperes will saturate T_2 .

T_1 becomes more difficult to cut off as V_e decreases.

$$V_e = \frac{(V_s - V_{ec2})(R_e \times 0.6)}{(R_e \times 1.4) + (R_e \times 0.6)}$$

$$V_e = \frac{(15 - 0.1)(1K \times 0.6)}{(22K \times 1.4) + (1K \times 0.6)}$$

$$= 0.28 \text{ volts}$$

$$\begin{aligned}
 V_{c2} &= V_e + V_{ec2} \\
 &= 0.28 + 0.1 = 0.38 \text{ volts}
 \end{aligned}$$

Maximum V_{b1} is the worst condition.

$$\begin{aligned}
 V_{b1} &= \frac{V_{c2} (R_{c2b1} \times 1.4)}{(R_{c2b1} \times 1.4) + (R_{b1g} \times 0.6)} \\
 &= \frac{0.38 (100K \times 1.4)}{(100K \times 1.4) + (100K \times 0.6)} \\
 &= 0.26 \text{ volts}
 \end{aligned}$$

$$V_{eb1} = -0.02 \text{ volts}$$

Thus, T_1 is cut off.

The fifteen-second monostable multivibrator can be placed in its quasi-stable state if a negative pulse is supplied to the input. The pulse is passed through the two capacitors and the diode to the base of T_2 , turning it off. The rise in V_{c2} caused by turning T_2 off should saturate T_1 . The following calculations examine this condition, using the nominal values of the resistors.

$$\begin{aligned}
 I_{c1} &= \frac{V_s - V_{ec1}}{R_{c1} + R_e} \\
 &= \frac{15 - 0.1}{22K + 1K} = 0.65 \text{ milliamperes}
 \end{aligned}$$

$$\begin{aligned}
 V_e &= I_{c1} R_e \\
 &= (0.65)(1) = 0.65 \text{ volts}
 \end{aligned}$$

$$V_{b1} = V_e + V_{eb1}$$

$$= 0.65 + 0.3 = 0.95$$

Since I_{c2} is negligible,

$$I_{b1} = \frac{V_s - V_{b1}}{R_{e2b1} + R_{sc2}} - \frac{V_{b1}}{R_{b1g}}$$

$$\frac{15 - 0.95}{100K + 22K} - \frac{0.95}{100K}$$

$$= 105.5 \text{ microamperes}$$

Figure 12 shows $I_{b1} = 15$ microamperes will saturate T_1 .

As T_1 saturates, V_{c1} drops from $V_s = 15$ volts to $V_e + V_{ec1} = 0.75$ volts, a drop of 14.25 volts. V_{b2} also drops 14.25 volts, because the voltage across capacitor C_{e2b1} cannot change instantaneously. V_{b2} in the stable state was determined to be 0.95 volts. Thus, V_{b2} becomes

$$V_{b2} = 0.95 - 14.25 = -13.3 \text{ volts,}$$

from which it increases exponentially toward V_s . The circuit determining the voltage V_{b2} during the quasi-stable state is shown on Figure 14.

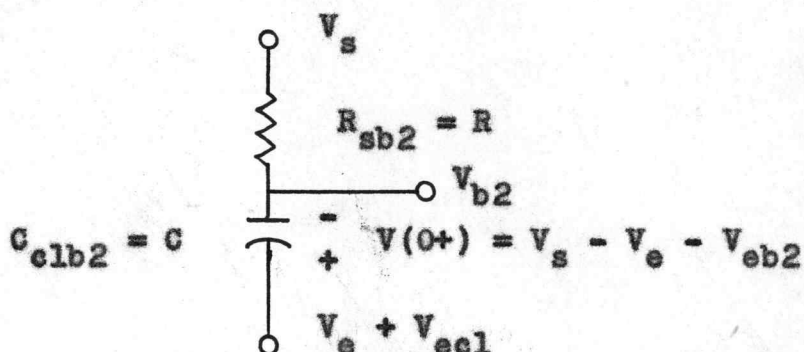


Figure 14-Monostable Timing Circuit

Using the nodal method and noting that V_s , V_e and V_{ec} are constants, results in:

$$0 = \left[V_{b2}(s) - \frac{V_e + V_{ec1}}{s} + \frac{V_s - V_e - V_{eb2}}{s} \right] Cs + \left[V_{b2}(s) - \frac{V_s}{s} \right] \frac{1}{R}$$

Solving for $V_{b2}(s)$,

$$V_{b2}(s) = \frac{-V_s + 2V_e + V_{eb2} + V_s/RCS + V_{ec1}}{s + \frac{1}{RC}}$$

$$V_{b2}(t) = \left[-2V_s + 2V_e + V_{eb2} + V_{ec1} \right] e^{-t/RC} + V_s$$

T_2 turns on when $V_{b2} = V_e$, therefore, $V_{b2}(t)$ is set equal to V_e and the equation solved for t .

$$e^{-t/RC} = \frac{V_s - V_e}{2(V_s - V_e) - V_{eb2} - V_{ec1}}$$

$$t = RC \ln \frac{2(V_s - V_e) - (V_{eb2} + V_{ec1})}{V_s - V_e}$$

Since $V_{eb2} + V_{ec1}$ is small compared with $2(V_s - V_e)$ when both transistors are saturated, the quasi-stable-state time may be approximated as

$$t \approx RC \ln 2$$

The quasi-stable-state time for the fifteen-second monostable multivibrator is

$$\begin{aligned} t &= (120 \times 10^3)(200 \times 10^6) \ln 2 \\ &= 16.6 \text{ seconds} \end{aligned}$$

The effect of component changes on the -13.3 volts, which cuts off T_2 is negligible. The quasi-stable-state time,

however, could vary from

$$t = 5.95 \text{ seconds}$$

to

$$t = 32.6 \text{ seconds}$$

for a simultaneous variation in R and C. Neither of these times would render the remote station inoperative.

Worst-case calculations for the saturation of T_1 during the quasi-stable state follow:

$$\begin{aligned} I_{c1} &= \frac{V_s - V_{ec1}}{(R_{c1} \times 0.6) + (R_e \times 1.4)} \\ &= \frac{15 - 0.1}{(22K \times 0.6) + (1K \times 1.4)} \\ &= 1.02 \text{ milliamperes} \end{aligned}$$

$$\begin{aligned} V_e &= I_c (R_e \times 1.4) \\ &= 1.02 (1K \times 1.4) \\ &= 1.43 \text{ volts} \end{aligned}$$

$$\begin{aligned} I_{b1} &= \frac{V_s - V_{b1}}{(R_{c2b1} \times 1.4) + (R_{se2} \times 1.4)} - \frac{V_{b1}}{(100 \times 0.6)} \\ &= \frac{15 - 1.73}{(100 \times 1.4) + (22 \times 1.4)} - \frac{1.73}{(100 \times 0.6)} \\ &= 49.2 \text{ microamperes} \end{aligned}$$

This base current is sufficient to insure saturation.

Bistable Multivibrators

The circuit diagram for the four bistable multivibrators, used in the counter, is shown on Figure 5. The

**Table 4 - NOMINAL DESIGN VALUES FOR MONOSTABLE MULTIVIBRATORS
USED IN THE REMOTE-STATION CONTROL CIRCUITRY**

Circuit	State	Conducting Transistor			Saturating I_b μa	V_o Volts	V_{b1} Volts	V_{b2} Volts	V_{c1} Volts	V_{c2} Volts	Quasi- Stable Time Seconds
		No.	I_c ma	I_b μa							
Fifteen- Second	Stable	T_2	0.65	117	15	0.65	0.38	0.95	15	0.75	
	Quasi- Stable	T_1	0.65	105	15	0.65	0.95		0.75	14.8	16.6
Two- Minute	Stable	T_2	0.87	398	20	1.9	7	2.2	15	2.0	
	Quasi- Stable	T_1	0.87	122	20	1.9	2.2		2.0	13.5	114
Reset	Stable	T_2	0.94	138	20	0.94	0.52	1.24	15	1.04	
	Quasi- Stable	T_1	0.94	258	20	0.94	1.24		1.04	12.2	0.14
Measure- ment Interro- gator	Stable	T_2	2.87	172	47	0.58	0.34	0.88	15	0.68	
	Quasi- Stable	T_1	2.87	253	47	0.58	0.88		0.68	13.68	0.0182

calculations are made under the assumption that the current through the bias resistors is small compared with the collector current of the conducting transistor. The bistable multivibrator is a balanced circuit, therefore, calculations need to be made for only one of the two stable states. The calculations are made for T_1 conducting and T_2 off.

$$\begin{aligned} I_{c1} &= \frac{V_s - V_{ec1}}{R_{c1} + R_e} \\ &= \frac{15 - 0.1}{12K + 1K} \\ &= 1.14 \text{ milliamperes} \end{aligned}$$

$$\begin{aligned} V_e &= I_{c1} R_e \\ &= (1.14)(1) = 1.14 \text{ volts} \end{aligned}$$

$$\begin{aligned} V_{b1} &= V_e + V_{eb1} \\ &= 1.14 + 0.3 = 1.44 \text{ volts} \end{aligned}$$

$$\begin{aligned} I_{b1} &= \frac{V_s - V_{b1}}{R_{c2b1} + R_{sc2}} - \frac{V_{b1}}{R_{b1g}} \\ &= \frac{15 - 1.44}{100K + 12K} - \frac{1.44}{100K} \\ &= 107 \text{ microamperes} \end{aligned}$$

Figure 12 indicates $I_{b1} = 22$ microamperes will saturate T_1 .

$$\begin{aligned} V_{c1} &= V_e + V_{ec1} \\ &= 1.14 + 0.1 = 1.24 \text{ volts} \end{aligned}$$

$$\begin{aligned}
 V_{b2} &= \frac{V_{e1} R_{b2g}}{R_{e1b2} + R_{b2g}} \\
 &= \frac{1.24 (100)}{100K + 100K} = 0.62 \text{ volts}
 \end{aligned}$$

$$V_{eb2} = -0.72 \text{ volts}$$

Thus, T_2 is cut off by 0.72 volts.

The above calculations are repeated below, allowing each resistor to vary 40 percent in the worst direction.

$$\begin{aligned}
 I_{e1} &= \frac{V_s - V_{b1}}{(R_{e1} \times 0.6) + (R_e \times 1.4)} \\
 &= \frac{15 - 0.1}{(12K \times 0.6) + (1K \times 1.4)} \\
 &= 1.73 \text{ milliamperes}
 \end{aligned}$$

$$\begin{aligned}
 V_e &= I_{e1} (R_e \times 1.4) \\
 &= 1.73 (1 \times 1.4) = 2.44 \text{ volts}
 \end{aligned}$$

$$\begin{aligned}
 V_{b1} &= V_e + V_{eb1} \\
 &= 2.44 + 0.3 = 2.74 \text{ volts}
 \end{aligned}$$

$$\begin{aligned}
 I_{b1} &= \frac{V_s - V_{b1}}{(R_{e2b1} \times 1.4) + (R_{se2} \times 0.6)} - \frac{V_{b1}}{(R_{b1g} \times 0.6)} \\
 &= \frac{15 - 2.74}{(100K \times 1.4) + (12K \times 0.6)} - \frac{2.74}{(100K \times 0.6)} \\
 &= 37.6 \text{ microamperes}
 \end{aligned}$$

Figure 12 shows $I_{b1} = 32$ microamperes will saturate T_1 .

In the calculations to determine cutoff of T_2 , a minimum value of V_e is the worst case. This is in opposition to the maximum value of V_e , which is the worst case in calculating the saturation of T_1 .

$$\begin{aligned} V_e &= \frac{(V_s - V_{ce1})(R_e \times 0.6)}{(R_e \times 0.6) + (R_{c1} \times 1.4)} \\ &= \frac{(15 - 0.1)(1K \times 0.6)}{(1K \times 0.6) + (12K \times 1.4)} \\ &= 0.54 \text{ volts} \end{aligned}$$

$$\begin{aligned} V_{b2} &= \frac{(V_e + V_{ce1})(R_{b2g} \times 1.4)}{(R_{c1b2} \times 0.6) + (R_{b2g} \times 1.4)} \\ &= \frac{(0.545 + 0.1)(100K \times 1.4)}{(100K \times 0.6) + (100K \times 1.4)} \\ &= 0.45 \text{ volts} \end{aligned}$$

$$V_{eb2} = -0.09 \text{ volts}$$

Thus, T_2 is cut off by 0.09 volts, even in the worst case.

AND Circuit Design

The AND circuit, shown on Figure 6, is highly dependent on the output voltages of the fifteen-second and counter multivibrators. In each case the highest voltage applied to an input diode will control the AND circuit. This voltage will be passed by the diode to which it is applied and back-bias the other diodes. If the controlling voltage is more positive than the emitter voltage of

the AND circuit, the transistor is cut off. On the other hand, if the emitter voltage is more positive than the controlling voltage the transistor will conduct and act as an emitter follower. Thus, only when all the input voltages are low will the AND-circuit output voltage drop, triggering the two-minute multivibrator.

The higher-voltage output of the fifteen-second multivibrator will be examined first using nominal values. The calculations will be made using the auxiliary circuit of Figure 15. This circuit is composed largely of fifteen-second multivibrator components, as indicated. The interrelation of the AND circuit and the fifteen-second multivibrator are thus considered. Using the nodal method to solve for V_{bA} ,

$$\begin{aligned}
 0 &= \frac{V_{bA} - V_s}{R_{sc2}} + \frac{V_{bA} - V_{b1}}{R_{c2b1}} + \frac{V_{bA}}{R_{bAg}} \\
 &= \frac{V_{bA} - 15}{22K} + \frac{V_{bA} - 0.95}{100K} + \frac{V_{bA}}{470K}
 \end{aligned}$$

$$V_{bA} = 12 \text{ volts}$$

The higher-voltage input from a counter bistable will be examined using the auxiliary circuit of Figure 16. The bistable output voltage will be determined first, assuming that T_2 is the output transistor and is not conducting. Application of the nodal method results in:

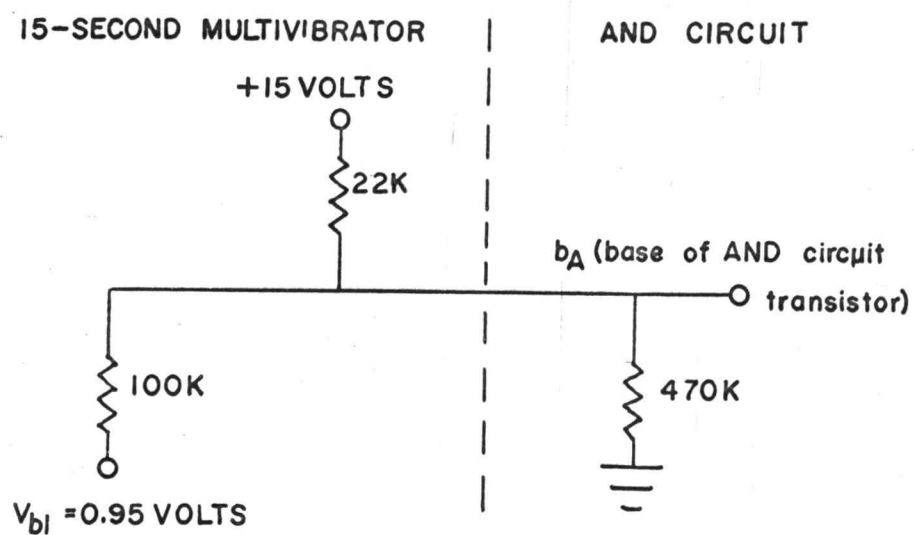


FIGURE 15. AUXILIARY CIRCUIT OF FIFTEEN-SECOND MULTIVIBRATOR INPUT TO THE AND CIRCUIT

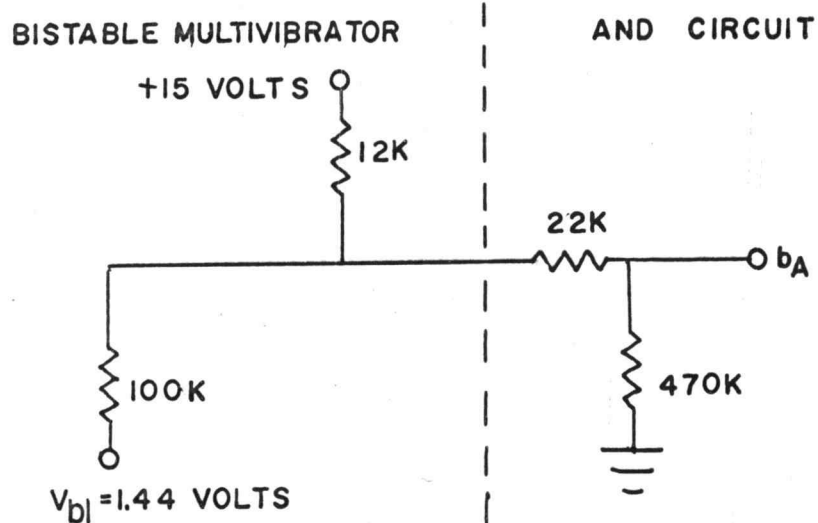


FIGURE 16. AUXILIARY CIRCUIT OF BISTABLE MULTIVIBRATOR INPUT TO THE AND CIRCUIT

$$\begin{aligned}
 0 &= \frac{V_{e2} - V_s}{R_{se2}} + \frac{V_{e2} - V_{b1}}{R_{c2b1}} + \frac{V_{e2}}{R_{c2bA} + R_{bAg}} \\
 &= \frac{V_{e2} - 15}{12K} + \frac{V_{e2} - 1.44}{100K} + \frac{V_{e2}}{470K + 22K}
 \end{aligned}$$

$$V_{e2} = 13.3 \text{ volts}$$

The base voltage of the AND circuit transistor is then

$$\begin{aligned}
 V_{bA} &= \frac{V_{e2}(R_{bAg})}{R_{c2bA} + R_{bAg}} \\
 &= \frac{13.3 (470)}{492} = 12.7 \text{ volts}
 \end{aligned}$$

The emitter voltage of the AND circuit transistor, V_{eA} , is

$$\begin{aligned}
 V_{eA} &= \frac{V_s (R_{eAs})}{R_{eAs} + R_{eAg}} \\
 &= \frac{15 (68K)}{68K + 68K} \\
 &= 7.5 \text{ volts}
 \end{aligned}$$

Thus, the AND circuit transistor can be solidly cut off by the fifteen-second multivibrator or any one of the bistable multivibrators. Cutoff of the AND circuit transistor will now be examined, allowing each resistor to vary 40 percent in the direction which results in the lowest V_{bA} . Furthermore, the lowest value of V_{b1} , obtained in the worst-case calculations for the multivibrator in question, will be used. For the case of the fifteen-second multivibrator, V_{bA} is determined as follows:

$$\begin{aligned}
 0 &= \frac{V_{bA} - V_s}{(R_{sc2} \times 1.4)} + \frac{V_{bA} - V_{b1}}{(R_{c2b1} \times 0.6)} + \frac{V_{bA}}{R_{bAg} \times 0.6} \\
 &= \frac{V_{bA} - 15}{22K \times 1.4} + \frac{V_{bA} - 0.58}{100K \times 0.6} + \frac{V_{bA}}{470K \times 0.6}
 \end{aligned}$$

$$V_{bA} = 10.35 \text{ volts}$$

The worst-case calculations for the bistable multivibrator follow:

$$\begin{aligned}
 0 &= \frac{V_{c2} - V_s}{(R_{sc2} \times 1.4)} + \frac{V_{c2} - V_{b1}}{(R_{c2b1} \times 0.6)} \\
 &\quad + \frac{V_{c2}}{(R_{c2bA} \times 1.4) + (R_{bAg} \times 0.6)} \\
 0 &= \frac{V_{c2} - 15}{12K \times 1.4} + \frac{V_{c2} - 0.84}{100K \times 0.6} \\
 &\quad + \frac{V_{c2}}{(22K \times 1.4) + (470K \times 0.6)}
 \end{aligned}$$

$$V_{c2} = 11.42 \text{ volts}$$

$$\begin{aligned}
 V_{bA} &= \frac{V_{c2} (R_{bAg} \times 0.6)}{(R_{bAg} \times 0.6) + (R_{c2bA} \times 1.4)} \\
 &= \frac{11.42 (470 \times 0.6)}{(470 \times 0.6) + (22 \times 1.4)} \\
 &= 10.35 \text{ volts}
 \end{aligned}$$

Under these conditions the highest AND circuit emitter voltage possible is the worst case. This voltage is:

$$\begin{aligned}
 V_{eA} &= \frac{V_s (R_{eAg} \times 1.4)}{(R_{eAg} \times 1.4) + (R_{eAs} \times 0.6)} \\
 &= \frac{15 (68 \times 1.4)}{(68 \times 1.4) + 68 \times 0.6} \\
 &= 10.5 \text{ volts}
 \end{aligned}$$

Thus, the emitter is 0.15 volts more positive than the base and the AND circuit transistor will conduct slightly. The output pulse, however, will have a magnitude of less than 0.15 volts, which is insufficient to trigger the two-minute multivibrator. The worst-case calculations for cutoff of the AND circuit are, therefore, satisfactory.

The "turn-on" characteristics of the AND circuit will be considered next. In this case, the output transistors of the controlling multivibrators will all be conducting. The output impedance of the multivibrators is considerably reduced in this condition and may be considered small compared with the input impedance of the AND circuit. If the highest voltage possible with the output transistor saturated is considered to be applied directly to the base of the AND circuit transistor,

$$V_{bA} = 2.42 \text{ volts.}$$

This voltage is taken from the worst-case calculations for the bistable multivibrators. The worst-case voltage from the fifteen-second multivibrator is 1.53 volts. For

$V_{bA} = 2.42$ volts and $V_{eA} = 7.5$ volts the output pulse of the AND circuit will have a magnitude of 4.78 volts. In the worst case, however, the emitter voltage will be at the lower value of:

$$\begin{aligned} V_e &= \frac{V_s (R_{eAs} \times 0.6)}{(R_{eAs} \times 0.6) + (R_{bAg} \times 1.4)} \\ &= \frac{15 (68K \times 0.6)}{(68K \times 0.6) + 68K \times 1.4} \\ &= 4.5 \text{ volts} \end{aligned}$$

The output pulse from the AND circuit will have a magnitude of 1.78 volts. This voltage is sufficient to trigger the two-minute multivibrator and the worst-case check may be considered satisfactory.

Astable Multivibrator

The astable multivibrator circuit diagram is composed of the circuitry immediately associated with transistors T_1 and T_2 on Figure 9. When T_1 is saturated,

$$\begin{aligned} I_{e1} &= \frac{V_s - V_{ecl}}{R_{e1s}} \\ &= \frac{15 - 0.1}{4.7K} = 3.17 \text{ milliamperes} \end{aligned}$$

$$V_e = 0 \text{ volts}$$

$$V_{b1} = 0.3 \text{ volts}$$

$$V_{e1} = 0.1 \text{ volts}$$

Figure 12 indicates 52 microamperes will saturate T_1 .

$$\begin{aligned}
 I_{b1} &= \frac{V_s - V_{b1}}{R_{c1g}} \\
 &= \frac{15 - 0.3}{100K} = 147 \text{ microamperes}
 \end{aligned}$$

Figure 12 indicates an I_{b1} of 55 microamperes will saturate T_1 . The calculations are repeated below, with each resistance changed 40 percent in the worst direction.

$$I_{c1} = \frac{15 - 0.1}{4.7K \times 0.6} = 5.3 \text{ milliamperes}$$

$$\begin{aligned}
 I_{b1} &= \frac{15 - 0.3}{100K \times 1.4} \\
 &= 105 \text{ microamperes}
 \end{aligned}$$

Figure 12 indicates an I_{b1} of 90 microamperes will saturate T_1 .

Considering that the opposite state is identical, except that $R_{c1g} = 100K$ is replaced by $R_{c2g} = 82K$, saturation of T_2 is assured. In each case cutoff of the transistor is assured, since the base voltage becomes approximately minus V_s during switching. This was discussed for the monostable multivibrator.

Power Switch

The circuit diagram for the power switch is shown on Figure 8. The power switch will be examined first when it is off. In this condition the station is not transmitting and T_1 is conducting while T_2 and T_3 are off.

The common-emitter voltage of T_1 and T_2 is calculated first, with T_1 conducting.

$$I_{e1} = \frac{(V_s - V_{ec1})}{R_{cls} + R_e}$$

$$= \frac{(15 - 0.1)}{12.2K} = 1.22 \text{ milliamperes}$$

$$V_e = I_{e1} (R_e) = 2.69 \text{ volts}$$

$$= (1.22 \times 10^{-3})(2.2K)$$

$$V_{b1} = V_e + V_{ec}$$

$$= 2.69 + 0.3$$

$$= 2.99 \text{ volts}$$

$$I_{b1} = \frac{V_s - V_{b1}}{R_b}$$

R_b = the 33K input resistor of the power switch plus R_{cls} of the two-minute monostable.

$$I_{b1} = \frac{15 - 2.99}{33K + 15K}$$

$$= 250 \text{ microamperes}$$

Figure 12 indicates 25 microamperes will saturate T_1 .

With T_1 saturated, T_2 will be investigated to see if it is cut off.

$$V_{c1} = V_{e1} + V_{ec1}$$

$$= 2.69 + 0.1 = 2.79 \text{ volts}$$

$$\begin{aligned}
 V_{b2} &= \frac{V_{c1} R_{b2g}}{R_{c1b2} + R_{b2g}} \\
 &= \frac{(2.79)(100K)}{100K + 22K} \\
 &= 2.29 \text{ volts}
 \end{aligned}$$

$$\begin{aligned}
 V_{be1} &= V_{b1} - V_{c1} \\
 &= 2.29 - 2.69 \\
 &= -0.4 \text{ volts}
 \end{aligned}$$

Thus, T_2 is held off by -0.4 volts. I_{b3} is then zero and T_3 will conduct 0.05 milliamperes, as indicated by Figure 12, and may be considered essentially nonconducting.

Next, the power switch will be examined when the two-minute multivibrator is in its quasi-stable state and the power switch is on. In this case, T_1 will be off while T_2 and T_3 are conducting. The calculations will begin with the assumption that T_1 is off. The large-signal transistor model for T_3 is shown in Figure 17 (3, p. 121).

In saturation, this circuit can be simplified as shown in Figure 18. Since the base is negative with respect to the collector, the collector diode conducts, shorting r_d and βi_b . In the saturation area i_b can change with no change in i_c , thus,

$$\beta = 0 \text{ and } r_{11}' = (1 + \beta) r_{11} = r_{11}$$

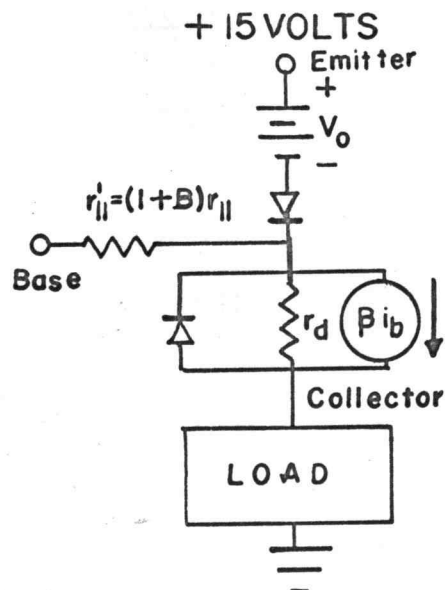


FIGURE 17. A LARGE-SIGNAL MODEL FOR TRANSISTOR T_3 OF THE POWER SWITCH

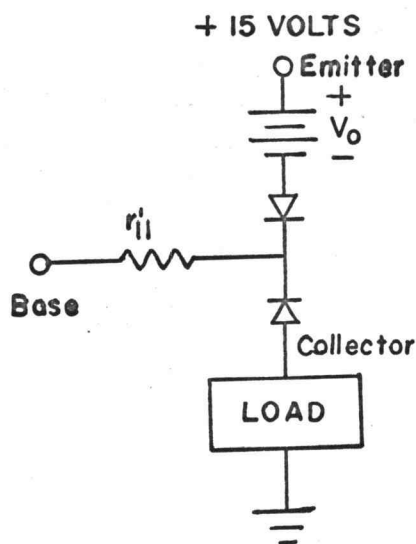


FIGURE 18. A LARGE-SIGNAL MODEL FOR TRANSISTOR T_3 OF THE POWER SWITCH WHEN SATURATED

The circuit for T_2 , including the loading effects of T_3 , is shown in Figure 19. But r_{11}' and r_{11} are small compared with the resistors external to the transistors. These may be considered negligible, allowing the circuit to be simplified as shown in Figure 20.

$$0 = \frac{V_{b2} - 0.1}{R_e} - \beta i_{b2} + \frac{V_{b2} - 15}{R_{sb2}} + \frac{V_{b2}}{R_{b2g}} + \frac{V_{b2} - 14.9}{r_d}$$

$$\text{But } i_{b2} = \frac{15 - V_{b2}}{R_{sb2}} - \frac{V_{b2}}{R_{b2g}}$$

Thus,

$$0 = \frac{V_{b2} - 0.1}{R_e} + \beta \left[\frac{V_{b2} - 15}{R_{sb2}} + \frac{V_{b2}}{R_{b2g}} \right] + \frac{V_{b2} - 15}{R_{sb2}} + \frac{V_{b2}}{R_{b2g}} + \frac{V_{b2} - 14.9}{r_d}$$

$$0 = \frac{V_{b2} - 0.1}{R_e} + (\beta + 1) \left[\frac{V_{b2} - 15}{R_{sb2}} + \frac{V_{b2}}{R_{b2g}} \right] + \frac{V_{b2} - 14.9}{r_d}$$

$$0 = (V_{b2} - 0.1)(R_{sb2} R_{b2g} r_d) + (\beta + 1) R_e r_d \left[(V_{b2} - 15) R_{b2g} + V_{b2} R_{sb2} \right] + [V_{b2} - 14.9] R_e R_{sb2} R_{b2g}$$

$$\begin{aligned} V_{b2} [R_{sb2} R_{b2g} r_d + (\beta + 1) R_e r_d (R_{b2g} + R_{sb2}) + R_e R_{sb2} R_{b2g}] \\ = 0.1 R_{sb2} R_{b2g} r_d + 15 (\beta + 1) R_e r_d (R_{b2g}) \\ + 14.9 R_e R_{sb2} R_{b2g} \end{aligned}$$

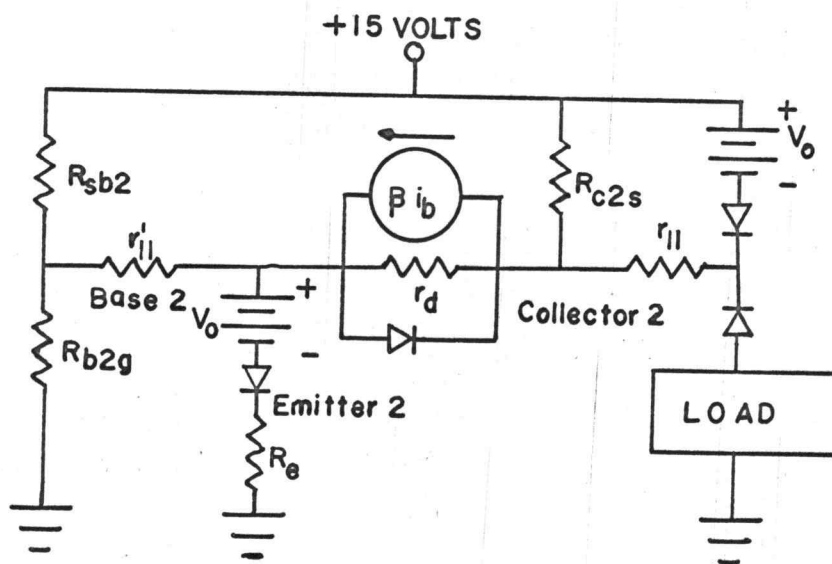


FIGURE 19. A LARGE-SIGNAL MODEL FOR TRANSISTORS T_2 AND T_3

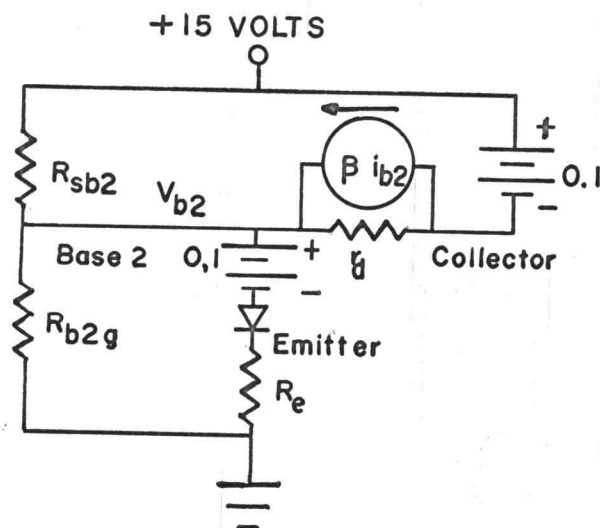


FIGURE 20. SIMPLIFIED LARGE-SIGNAL MODEL OF TRANSISTOR T_2

$$V_{b2} =$$

$$\frac{0.1 R_{sb2} R_{b2g} r_d + 15(\beta+1) R_e r_d R_{b2g} + 1.49 R_e R_{sb2} R_{b2g}}{R_{sb2} R_{b2g} r_d + (\beta+1) R_e r_d (R_{b2g} + R_{sb2}) + R_e R_{sb2} R_{b2g}}$$

Where, $\beta \approx 20$

$$R_{sb2} = 10K + 22K = 32K$$

$$R_{b2g} = 100K$$

$$R_e = 2.7K$$

$$r_d \approx 10K$$

$$V_{b2} = \frac{(0.1)(32K)(100K)(10K) + 15(21)(2.2K)(10K)(100K) + (1.49)(2.2K)(32K)(100K)}{(32K)(100K)(10K) + 21(2.2K)(10K)(132K) + (2.2K)(32K)(100K)}$$

$$V_{b2} = 8.04 \text{ volts}$$

$$I_{b2} = \frac{V_s - V_{b2}}{R_{e1} + R_{e1b2}} - \frac{V_{b2}}{R_{b2g}}$$

$$= \frac{15 - 8.04}{10K + 22K} - \frac{8.04}{100K}$$

$$I_{b2} = 138 \text{ microamperes}$$

$$I_{e2} = \beta I_{b2} + \frac{V_s - V_{b2}}{r_d}$$

$$= 20 (0.138) + \frac{15 - 8.04}{10K}$$

$$= 3.4 \text{ milliamperes}$$

$$I_{b3} = I_{e2}$$

$$I_{e3} = \beta I_{e2}$$

$$= 20 (3.4) = 68 \text{ milliamperes}$$

The original assumption that T_1 is not conducting will now be verified. The emitter of T_1 and T_2 are at the same voltage.

$$V_{e1} = R_e I_{e2}$$

$$= (2.2K)(3.64 \text{ ma})$$

$$= 8 \text{ volts}$$

The output voltage of the two-minute multivibrator is 2 volts during the quasi-stable state. T_1 is, therefore, certainly cut off. The worst-case calculations are also assured.

Schmidt Trigger

The circuit diagram for the Schmidt trigger is shown on Figure 2. The input pulse ranges from zero to twelve volts and originates in a 10-kilo-ohm source. In the stand-by condition, transistor T_1 should be off and transistor T_2 should be conducting.

When the input voltage is zero, it is obvious that T_1 will be nonconducting, regardless of any circuit parameter variations. In this event, it is necessary to show that T_2 is saturated.

$$I_{c2} = \frac{V_s - V_{ce2}}{R_{c2s} + R_{eg}} = \frac{15 - 0.1}{18K + 5.6K}$$

$$= 0.76 \text{ milliamperes}$$

$$V_e = I_{c2} R_{eg} = (0.76)(5.6)$$

$$= 4.25 \text{ volts}$$

$$V_{b2} = V_e + V_{be2}$$

$$= 4.25 + 0.3 = 4.55$$

$$I_{b2} = \frac{V_s - V_{b2}}{R_{c1s} + R_{c1b2}} - \frac{V_{b2}}{R_{b2g}}$$

$$= \frac{15 - 4.55}{27 + 27} - \frac{4.55}{120}$$

$$I_{b2} = 161 \text{ microamperes}$$

Figure 12 indicates that when I_{c2} is 0.76 milliamperes, a base current of 16 microamperes will saturate T_2 .

The above calculations are repeated below, allowing each resistor to vary 40 percent in the worst direction.

$$I_{c2} = \frac{V_s - V_{ce2}}{(R_{c2s} \times 0.6) + (R_{eg} \times 1.4)}$$

$$= \frac{15 - 0.1}{(18K \times 0.6) + (5.6K \times 1.4)}$$

$$= 0.8 \text{ milliamperes}$$

$$V_e = I_{c2} (R_{eg} \times 1.4) = (0.8)(5.6 \times 1.4)$$

$$= 6.27 \text{ volts}$$

$$V_{b2} = V_e + V_{be2}$$

$$= 6.27 + 0.3 = 6.57 \text{ volts}$$

$$I_{b2} = \frac{V_s - V_{b2}}{(R_{cls} \times 1.4) + (R_{clb2} \times 1.4)} - \frac{V_{b2}}{(R_{b2g} \times 0.6)}$$

$$= \frac{15 - 6.57}{(27K \times 1.4) + (27K \times 1.4)} - \frac{6.57}{120K \times 0.6}$$

$$= 19 \text{ microamperes}$$

Figure 12 shows that an I_{b2} of 17 microamperes will saturate T_2 . When T_1 is nonconducting, V_{b1} is equal to the input voltage. When the input voltage is 4.25 volts or more, T_1 will saturate. This is shown below.

$$\begin{aligned} V_e &= \frac{(V_s - V_{ec1})(R_e)}{R_{eg} + R_{cls}} \\ &= \frac{(15 - 0.1)(5.6K)}{5.6K + 27K} = 2.56 \text{ volts} \end{aligned}$$

V_{b1} , when T_1 is saturated, is

$$\begin{aligned} V_{b1} &= V_e + V_{eb1} \\ &= 2.56 + 0.3 \\ &= 2.86 \end{aligned}$$

$$I_{b1} = \frac{V_1 - V_{b1}}{R_{ib1} + R_1}$$

R_1 is the internal impedance of the input source and V_1 is the input voltage. An input voltage of 4.25 volts is used to demonstrate the "toggle" action of the circuit.

$$I_{b1} = \frac{4.25 - 2.86}{47K + 10K}$$

$$= 25 \text{ microamperes}$$

The collector current of T_1 is 0.46 milliamperes and Figure 12 indicates T_1 will be saturated. Once T_1 starts to conduct, it continues until the input voltage is reduced. Worst-case calculations are not included because the input voltage used above is substantially less than the expected 12 volts and saturation is assured, even under worst-case conditions. With T_1 saturated, it becomes necessary to show T_2 is nonconducting.

$$V_e = \frac{(V_s - V_{ce1}) R_{eg}}{R_{eg} + R_{cls}}$$

$$= \frac{(15 - 0.1)(5.6K)}{5.6K + 27K} = 2.56 \text{ volts}$$

$$V_{c1} = V_e + V_{ce1}$$

$$= 2.56 + 0.1 = 2.66 \text{ volts}$$

$$V_{b2} = \frac{V_{c1} R_{b2g}}{R_{b2g} + R_{c1b2}}$$

$$= \frac{2.66 (120K)}{120K + 27K} = 2.17 \text{ volts}$$

T_2 is held off by 0.41 volts. The worst-case calculations follow:

$$V_e = \frac{(V_s - V_{ce1})(R_{eg} \times 0.6)}{(R_{eg} \times 0.6) + (R_{cls} \times 1.4)}$$

$$V_e = \frac{(15 - 0.1)(5.9K \times 0.6)}{(5.6K \times 0.6) + (27K \times 1.4)}$$

$$= 1.27 \text{ volts}$$

$$V_{cl} = V_e + V_{ec1}$$

$$= 1.27 + 0.1 = 1.37 \text{ volts}$$

$$V_{b2} = \frac{V_{cl} (R_{b2g} \times 1.4)}{(R_{b2g} \times 1.4) + (R_{clb2} \times 0.6)}$$

$$= \frac{1.37 (120 \times 1.4)}{(120K \times 1.4) + (27K \times 0.6)}$$

$$= 1.25 \text{ volts}$$

T_2 is held off by 0.02 volts.

The Schmidt trigger satisfies the designs specifications in all respects.

APPENDIX C

Bandwidth Considerations

The following derivation is based on calculations by Dr. L. A. Beattie of the University of Idaho, Moscow, Idaho. The author is responsible for the arrangement and presentation.

The Federal Communications Commission specification regarding bandwidth is that 99 percent of the total energy be contained in the bandwidth. In order to consider the worst case, a single voltage pulse T_p seconds long with zero rise and fall times is considered (Figure 21).

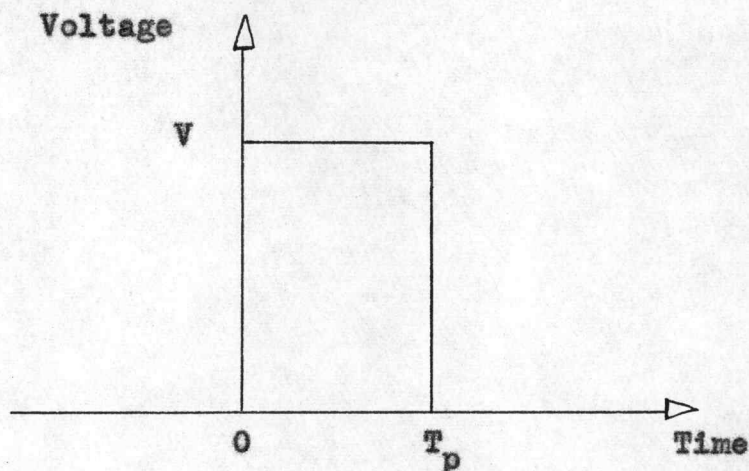


Figure 21 - A Single Voltage Pulse

Applying the Fourier integral,

$$E(\omega) = \int_0^{T_p} V e^{-j\omega t} dt$$

Solving and taking the absolute value,

$$|E(\omega)| = VT_p \left| \frac{\sin(\omega T_p/2)}{\omega T_p/2} \right|$$

Considering the radiated power to be $\frac{E^2}{R}$ and substituting X for $\frac{\omega T_p}{2}$, the

$$\text{Total Power Radiated} = \frac{V^2 T_p^2}{R} \int_{-\infty}^{\infty} \frac{\sin^2 X}{X^2} dX$$

But $\frac{\sin^2 X}{X^2}$ is an even function, thus,

$$\begin{aligned} \text{Total Power Radiated} &= \frac{2V^2 T_p^2}{R} \int_0^{\infty} \frac{\sin^2 X}{X^2} dX \\ &= \frac{V^2 T_p^2 \pi}{R} \end{aligned}$$

Next, the power in a lesser bandwidth will be calculated in terms of the bandwidth.

$$\begin{aligned} \text{Radiated Power} &= \frac{2V^2 T_p^2}{R} \int_0^a \frac{\sin^2 X}{X^2} dX \\ &= \frac{2V^2 T_p^2}{R} \int_0^a \frac{1}{2} \frac{(1 - \cos 2X)}{X^2} dX \end{aligned}$$

Let $2X = Y$

$$= \frac{V^2 T_p^2}{R} \left[\int_0^{2a} \frac{2}{Y^2} dY - \int_0^{2a} \frac{2 \cos Y dY}{Y^2} \right]$$

$$\text{Radiated Power} = \frac{2V^2 T_p^2}{R} \left| -\frac{1}{Y} + \frac{\cos Y}{Y} + \int \frac{\sin Y}{Y} dY \right|_0^{2\alpha}$$

$\int_0^{2\alpha} \frac{\sin Y}{Y} dY$ is the sine integral, $Si(2\alpha)$ and when $2\alpha \gg 1$

$$Si(2\alpha) \approx \frac{\pi}{2} - \frac{\cos 2\alpha}{2\alpha}$$

thus,

$$\begin{aligned} \text{Radiated Power} &= \frac{2V^2 T_p^2}{R} \left[-\frac{1}{Y} + \frac{\cos Y}{Y} \right]_0^{2\alpha} + \frac{\pi}{2} - \frac{\cos 2\alpha}{2\alpha} \\ &= \frac{2V^2 T_p^2}{R} \left[-\frac{1}{2\alpha} + \frac{\cos 2\alpha}{2\alpha} + \frac{1}{0} - \frac{\cos 0}{0} + \frac{\pi}{2} - \frac{\cos 2\alpha}{2\alpha} \right] \end{aligned}$$

This function will now be equated to 99 percent of the total radiated power and solved for α .

$$.99 \frac{V^2 T_p^2 \pi}{R} = \frac{2V^2 T_p^2}{R} \left[\frac{1}{2\alpha} + \frac{\pi}{2} \right]$$

$$\alpha = 31.83$$

but,

$$\alpha = \frac{\omega T_p}{2} = 31.83$$

$$\alpha = \frac{2\pi f T_p}{2} = 31.83$$

$$f = \frac{10.15}{T_p} \text{ cycles/sec}$$