AN ABSTRACT OF THE THESIS OF

IN KYU RO for the DOCTOR OF PHILOSOPHY
(Name) (Degree)

in INDUSTRIAL ENGINEERING presented on July 24, 1978
(Major) (Date)

Title: DEVELOPMENT OF AN INFORMATION PROCESSING CAPACITY ESTIMATION METHOD FOR MANAGING A PROJECT SUBJECTED TO DISRUPTIONS

Abstract approved: Redacted for privacy
Dr. M. S. Inoue

This dissertation is an attempt at developing a method for the analysis and estimation of the effects of disruptions due to uncertainties. Such uncertainties may result from design changes and engineering drawing delays in large-scale, complex, research and development, or construction projects.

In order to provide management with a simple management tool for the analysis of disruption problems, the following works have been done in this dissertation.

Information Theory is used to provide an entropy formula and the capacity concept for a project, based on a transformation function.

A systematic prediction procedure is developed. The procedure can provide management with warning information on the likely trouble spots in the early stage of a project as well as during project execution.

A hypothesis is advanced that most of engineering drawing delays follow an exponential distribution. Using the maximum entropy principle, an exponential distribution is shown to be an adequate proba-

bility distribution for random occurring drawing delays. Furthermore, to support the hypothesis, χ^2 'goodness-of-fit' tests are conducted for actual industrial data concerning drawing delays.

An entropy formula of a triangular distribution for activity is derived. The entropy formula is used as a working vehicle for developing an information processing capacity estimation method for a project. In addition, an entropy conversion method is proposed to compare and choose project(s) among more than one project that has different measurement units for activity duration, in terms of uncertainty of the project completion date.

An information processing capacity estimation method is developed. The method can estimate the capacity of a project to handle equivocation due to design changes. It can also identify the associated activities and the necessary amount of human resources, considering the project completion date with 50% chance of success. Furthermore, the method can estimate the project slippages when the capacity is not large enough to handle the equivocation.

In an attempt to evaluate the predictive ability of the method, project slippages estimated by the information processing capacity estimation method are compared with the results obtained by a computer Monte Carlo simulation program, CRASH. A χ^2 'goodness-of-fit' test is conducted. The result of the test shows that the estimated project slippages do not significantly differ from those obtained from the CRASH computer program at the 0.005 significance level.

We conclude that the information processing capacity estimation method may be suggested as an expedient means of evaluating project status for management in the different stages of project execution.

The effect of fatigue on the capacity of a project is evaluated. In one example, it is shown that a 25% increase of total working hours over the base schedule (6-day week, 8-hour day) results in only 16% increase of the capacity of the project. The effect of fatigue is shown as a 35% time lost of the total increased working hours.

Development of An Information Processing Capacity Estimation Method for Managing A Project Subjected to Disruptions

by

In Kyu Ro

A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Doctor of Philosophy

Completed July, 1978

Commencement June, 1979

APPROVED:

Redacted for privacy

Professor of Industrial and General Engineering

in charge of major

Redacted for privacy

Head of Department of Industrial and General Engineering

Redacted for privacy

Dean of Graduate School

Date thesis is presented July 24, 1978

Typed by Sookja Ro for In Kyu Ro

ACKNOWLEDGEMENTS

My sincerest gratitude is extended to Dr. Michael S. Inoue, major professor, for his problem suggestion, valuable guidance, and encouragement, during the course of this research and years of graduate study.

Special thanks must be extended to Dr. James L. Riggs, Head of the Department of Industrial and General Engineering, for his encouragement and inspiration during my graduate studies.

Special appreciation is given to PRI, that allowed me to use CRASH program for my research.

To Mr. Steve Chou, I give my sincere thanks for his technical assistance in running computer program.

I feel a deep sense of appreciation to my wife, Sookja, who painstakingly typed, reviewed, and corrected all the material, and encouraged me throughout my study.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

DEVELOPMENT OF AN INFORMATION PROCESSING CAPACITY ESTIMATION METHOD FOR MANAGING A PROJECT SUBJECTED TO DISRUPTIONS

1. INTRODUCTION

Definition of Disruption

Disruption is defined as the "act or process of breaking apart or throwing into disorder, or interrupting to the extent of stopping or preventing normal continuance of (work)," (Webster, 1968).

Disruption in Project Management

Most project managers in industry have experienced unexpected and randomly occurring design changes that gravely affect the progress of the project. Late submissions of design specifications by the project owner, delay in reviews, approval and returns of documents to the project contractor, changes in engineering design specifications are commonplace. As a result, an experienced project manager recognizes that the design changes and engineering drawing delays do occur and take provisions to avoid serious project cost overrun and the slippage of project completion date over the original estimates. In spite of these precautions, project management is often placed in a serious condition of uncertainty in carrying out normal project operations due to the random characteristics of occurrences of design changes, the large variability of design modification activities and the uncertainty of engineering drawing delays. So the original plan must be reorganized to allow work to be done while waiting for designs to be changed and become definite, and working days must be extended.

Extra physical and human resources must be used to stay near the contracted project completion dates when scheduled activity start times are postponed. Both the design changes and the engineering drawing delays effect significantly the cost overrun and the slippage of the construction project completion time. This phenomenon has been simulated by experiments using computer simulation (Inoue, 1977). Although the cause-and-effect relationship is established between design changes and engineering drawing delays, cost overrun, and project slippage, project management still is placed in 'uncertain' situations. Because management has little information about the set of design changes to occur and the distribution of the engineering drawing delays, management can hardly prepare strategy and avoid the effects in project scheduling. Such effects are called disruption effects due to design changes that relate to project cost overrun and slippage of project completion dates. Design modification activity and engineering drawing delays are called disruption factors due to design changes in project management in this thesis.

In project resource scheduling, physical resource, human resource, and information are three important resources. It is important to consider the peculiarity of human resource in project scheduling.

CPM (Critical Path Method) and PERT (Program Evaluation and Review Technique) first appeared in the late 1950s. These two methods were readily accepted in industry. However, it has long been recognized that both CPM and PERT are unrealistic because they assume the availability of unlimited physical resources which are not valid in most real life situations. It is also true that both CPM and PERT, as well as other traditional project scheduling methods have not differentiated between human resources and physical resources. They have not taken into consideration the characteristics of human resources. Fatigue when extra shifts were added or working days extended, too much stress being given during work, boredom when waiting for equipment, orders or engineering drawing delays being too long, lack of motivation to work, frustration due to interference with normal work routines and lack of communication between workers and management are examples of factors that have usually been neglected in human resource scheduling. The results appeared as lower productivity and less efficiency in both physical resources and human resources.

Risk and Uncertainty in Project Management

Rist and uncertainty were first used in 1933 by Knight as two different concepts (Borch and Mossin, 1968). When the outcome of a decision, and the pursuant action cannot be predicted with certainty, it means that there is risk and/or uncertainty in the decision. The distinction between risk and uncertainty is not always clear and obvious. Klausner (1970) and Dean (1962) state that risk can be defined as an uncertain situation in which numerical probabilities can be assigned to possible outcomes, while it is not possible to know probability ranges for uncertain situations. By definition, while there is the possibility for good management to safeguard against risk, there is no safe way to defend against uncertainty. However, when the probability range for a possible outcome is wide enough to cover risk, it may then become so risky that the situation may be considered an uncertainty. So, the distinction between risk and uncertainty depends on the amount of information available. The same event, therefore, can be interpreted differently according to the amount of information to be supplied. Thus, the two subjects may be treated together. Woodgate (1967) claims that the profitability of a project depends on the relationship between income and expenditure; both contain risks and uncertainties. When a project is undertaken by a project contractor, there are uncertainties involved by both the project owner and the project contractor. What the project owner considers "uncertain" can be a mere risk to the project contractor by trusting the project contractor's capability to convert his uncertainty into risk. The project owner still takes a risk to trust the contractor's capability. On the other hand, what the project owner considers risk can be an uncertain event to the project contractor. If the project owner does not feel any responsibility to supply all of the necessary information to the project contractor, the uncertainty of the project contractor will be out of control. In this thesis, we define uncertainties as those things beyond the control of management in charge of the project. The uncertainty of the project contractor may become a risk depending on the amount of information supplied by the project owner.

Definitions of Design Changes and Design Modification Activity

Design change means not only revision or modification of design itself, but also some other random changes which affect the project completion date. Design modification activity includes not only action related to design revision or modification, but also some other actions which generate uncertainty in the project completion date.

Design Changes and Engineering Drawing Delays as Uncertainties

Working engineering drawings are essential to a construction project. As Feiler (1976) states the availability of working engineering drawing is one of the uncertainty examples in a large and complex project. When the project owner reviews, approves and returns the engineering drawings to the project contractor, delays may result from the design changes or long review periods. As long as the project owner does not furnish the project contractor information about designs to change and engineering drawing delays, they are completely beyond the control by the project contractor. Therefore, they are considered uncertainties by the project contractor. But still there is hope to bring them within the project contractor's control limits through establishing a good communication channel between the project owner and the project contractor. In other words, the uncertainties above may become risks according to the amount of information. However, in most cases the project owner cannot provide the project contractor with enough information to control project operations. Therefore, the project contractor needs a systematic prediction procedure which may provide the necessary information to aid project management.

The Significance of the Study

Project management may be placed in more serious position in a project execution because of disruption effects due to lack of information about the occurrences of design changes and the characteristics of engineering drawing delays. Although it is apparent from abundant practical examples in complex construction industry that design changes, engineering drawing delays, and fatigue are the major contributors to the disruptions in project operations, it is surprising that little effort has been made to study about the characteristics of disruption causes and the evaluation of disruption effects in project management. Fatigue that limits production in project operations and effects of lack of information on productivity of human resources have been mostly neglected in scheduling human resources.

The Objectives

We are therefore concerned with the investigation of the following topics:

1. Discussion of effects of fatigue on productivity of human resources involved in working schedules.

2. Development of a systematic prediction procedure of identifying the potential problem areas (possible trouble spots, e.g. designs to change) in a project.

3. Support of the hypothesis that most engineering drawing delays follow an exponential distribution.

4. Derivation of entropy formula for a triangular distribution of an activity and development of entropy conversion method.

5. Development of a capacity estimation method to estimate the capacity of a project which can handle the equivocation resulting from random design changes.

11. STATE OF ART IN PROJECT MANAGEMENT

Scheduling of Physical Resources

Gantt Bar Chart, CPM and PERT

The heart of project management methods is a graphical portrayal of the interrelationships among the elements of a project. This graphical portrayal of the interrelationships is called the project network diagram. The network diagram is essentially an outgrowth of the bar chart which was developed by Gantt in the context of a World War 1 military requirement. The Gantt bar chart is primarily designed to control the time element of a project and lists the major activities comprising a project, their scheduled start and finish times, and their current status. The Gantt bar chart was not too successful on "one-time-through projects", particularly projects with a high engineering content (Miller, 1963). The main deficiencies are: (1) the interrelationships among the activities in a project are not noted explicitly; (2) activities are not defined in detail. A detailed discussion about these two points is given in Moder and Phillips (1970). In the late 1950s two project planning and control methods first appeared. The two project management methods, CPM and PERT, are designed to correct the deficiencies of the Gantt bar chart. CPM was developed in 1957 by a joint effort of the DuPont Company and Remington Rand Univac (Moder and Phillips, 1970). The CPM research team was essentially interested in determining project duration subject to minimize total project cost. CPM was mainly applied to routine plant overhaul, maintenance, and construction work.

Characteristically, the activities comprising this type of project are subject to a small amount of variation in activity performance time. Hence, CPM treats activity performance time in a deterministic manner. PERT was developed in 1958 by a Navy research team for the development of an integrated planning and control system for the Poralis Weapon System Program. These two techniques were first introduced to the public about the same time. Malcolm, Roseboom, Clark and Fazer (1961) introduced PERT, and Kelley and Walker (1959), and Kelley (1961) introduced CPM. CPM based upon deterministic activity performance times and technological relationships between activities, is a very powerful tool to aid management in planning complex projects involving many activities. Unfortunately, it was not designed to consider the stochastic effects of risks involved in managing a project with each activity having chance variation in activity time duration. In other words, CPM lineal time estimate is a point estimate in a statistical sense of the average of the normal condition under which the project is supposed to follow. Therefore, it does not have any provision for uncertainty nor risk factor above the average condition. PERT was especially designed as a tool to evaluate the risk factors resulting from interactions of activities with probability distribution of activity durations.

Resource Constrained Project Scheduling

The previous two methods were readily applied to industry. They were quickly accepted. However, it has long been recognized that both CPM and PERT are unrealistic methods in resource scheduling because they assume unlimited availabilities of resources, which is not valid in most-life situations. When resources are scare, two problems may arise. One problem that arises when there are sufficient resources to schedule all concurrent activities competing for the same resource type; it attempts to smooth as much as possible the resource profile of resource usage over time, subject to the project completion within some specified project due date; this is the resource smoothing problem. The other problem is the resource constrained project scheduling problem. This problem occurs when the availability of resources is not sufficient to satisfy demands of concurrent activities during each time period of project duration. It aims to minimize some functions of durations. An overview of the early state of a scheduling problem was provided by Muth and Thompson (1963); particularly the project scheduling problem was addressed by Levy, Thompson and Wiest (1963a and 1963b) and Kelley (1963). A range of topics related to project scheduling (with and without resource constraints) are covered by Moder and Phillips (1970) and Ahuja (1976). Surveys of the early literature were provided by Bigelow (1962), Davis (1966) and Lerda-Oldberg (1966). Davis (1973, 1976) conducted the surveys of the recent literature. Woodgate (1973) reviewed the trends of development in the project planning systems from a practical viewpoint.

Many analytical solution procedures for the project scheduling problem have also been developed and successfully used mostly with small sized projects. The analytical methods, however, have been computationally impracticable for all but the smallest projects, either because the model is too large, or because the procedure is too lengthy or both, and because of the combinatorial nature of the project scheduling problem. Examples of analytical solution procedures are: Pritsker, Watters and Wolfe (1969). They have developed a 0-1 integer linear programming model for multi-project scheduling, but as the problem size increases the number of variables and constraints quickly increase. Schrage (1970) has applied Giffer and Thompson's (1960) branch-and-bound process to the project scheduling problem. Mason and Moodie (1971) have used an enumerative scheme for minimizing cost of changing resource usage in the project scheduling problem. Because of the relative lack of success with analytical methods, heuristic procedures are probably one of the most reasonable means for obtaining workable solutions to provide for large practical complex scheduling problems. Heuristic rules have appeared as key factors for resolving conflicts between activities that are competing for the same scarce resources in project scheduling. Since around 1960, most researchers have concentrated their massive efforts on determining which heuristic rule produces shortest project duration. Many distinct rules have been developed and used for resource constrained project scheduling, and also in related scheduling problems such as job shops and line of balancing problems. The comparisons of the effectiveness of different heuristic rules for resource constrained project scheduling have been conducted by the previous researchers: Brand, Meyer and Shaffer (1964), Mize (1964), Pascoe (1965), Crowston (1968), Fendley (1968), Gonguet (1969), Patterson (1970), Cooper (1974) and others. The eight major previous researches are briefly reviewed by discussing the type of problem examined and indicating the most effective rule found on a measure of project duration for single-project studies, or project slippage for multi-project studies.

Firstly, Brand, Meyer and Shaffer (1964) developed RSM (Resource Scheduling Method) heuristic at the University of Illinois and conducted a series of tests on construction industry projects, which were multi-resource, single project. They reported that RSM was the best effective rule. The rule was not fully discussed. Mize (1964) examined the multi-project scheduling program and he claims that the most effective rules in his research were the complex rules based on minimum activity total float. Pascoe (1965) was the first researcher to attempt to isolate the effects of project characteristics on the performance of the heuristics. He proposed first a system of project parameters ("density", "complexity", "resource utilization", etc.) for classifying project network. Pascoe (1965) conducted one of the most rigorous researches to measure the relative effectiveness of eleven heuristics rules. His general conclusions were: (1) parallel methods were better than serial methods; (2) minor sort was not important; (3) best heuristic rules were minimum LFT (Latest Finish Time) and minimum LST (Latest Start Time). However, Pascoe (1965) was not successful in magnifying the effect of project characteristics (density, complexity, resource-kind, etc.) on the capability of heuristic rules in the project scheduling. Crowston (1968) examined multi-resource, single projects. He found the best results using a rule which considered minimum latest start time. Fendley (1968) found that minimum-slack-first heuristic rule was the first one for scheduling multi-resource, multi-projects with eight different objective functions. Gonguet (1969) examined multi-resource, single projects in his experiment. He achieved the best results using minimum LFT first. Patterson (1973) tested the effectiveness of seven rules for scheduling an actual multi-project (13 different resource types) existing in a research and development organization which develops pyrotechnic items. Four different criteria (total slippage, weighted total slippage, etc.) were used in ranking each heuristic rule. The shortest job first rule scheduled activities with least total project delay; the least total float rule produced schedules with the least weighted total project slippage, the weight being determined by the resource contents required to complete the project. Cooper (1974) examined parallel methods and sampling methods. In the examination of the parallel methods he extended Pascoe's (1965) results to larger projects (60 jobs rather than 20) and a wider range of priority rules (26 instead of 11). He concluded that the sampling method was superior to the parallel methods. From the review of the previous research, it has been found that they did not consider the characteristics of human resources (fatigue, boredom, etc.) and dealt with human resources, like physical resource, in the resource constrained project scheduling. In industry, it has been the usual practice to resort to overtime in an attempt to meet project planning completion date that cannot be met within the original planned date. It is easy to understand that human beings are not machines and cannot keep on working with the same efficiency hour after hour. As a rule, fatigue is a limiting factor of a person's work output when considering a variety of working schedules. It has also been found that most of the past works have not considered information theory in project scheduling.

Scheduling of Human Resources

The objective of this section is to review previous research in fatigue and boredom and to study the importance and the effects of fatigue and to discuss the relationship between information availability and productivity of human performance.

Definition of Fatigue

As McFarland (1971) has rightfully pointed out, there appears to be as many definitions of the word "fatigue" as there are articles written about it. We will briefly review the author's opinion on the definition of fatigue. When we consider it in terms of work performance and productivity, the idea of "temporarily diminished working capacity from continued work" appears to predominate. Examples of author's opinion include: Health of Munition Workers Committee (Vernon, 1921)-- "Fatigue is the sum of results of activity which show themselves in a diminished capacity for doing work"; British Association Committee Report (Vernon, 1921)-- "A diminution of the capacity for work which follows excess of work or lack of rest,---"; Bartley (1943, 1951, 1954, and 1957), Bartley and Chute (1945, 1947) and Gross and Bartley (1951) -- They propose to reserve the term 'fatigue' for the subjective experience and suggest a definition as the sensory-cognitive syndrome, which includes tiredness, aversion to work, body discomfort, ineffectiveness in performance; Simonson and Brozek (1948a and 1948b) and Simonson and Anderson (1966) state that in many instances, decrement in performance may reveal fatigue trends better than any other measurable function; Murrell (1971) and Schmidtke (1965) say that fatigue is the detrimental effect of work upon continued work which may manifest itself as a reversible decrement in performance; Grandjean (1968) states that the condition of fatigue is accompanied by a decrease in physical and mental performance, a decrease in motivation to work, consequently fatigue is manifested by decrement in performance; Webster's Dictionary (1976)-- "the temporary loss of power to respond induced in a sensory receptor or motor end organ by continued stimulation"; Encyclopedia Britannica (1976, vol.1V)-- "a specific form of human inadequacy in which the individual experiences aversions for exertion and feels unable to carry on. Such feelings may be generated by muscular effort, exhaustion of the energy supply to the muscles of the body, however, is not an invariable precusor. Feelings of fatigue may also stem from pain, anxiety, fear, or boredom. In the later cases muscle function commonly is unimpaired." There is subtantial agreement in the definition of fatigue as temporary decrease of working capacity from continued work (Ash, 1914; Conklin and Freeman, 1939; Dill, 1942; Bartilett, 1953).

A Rationale: The Close Connection Between Fatigue and Work Output

To begin with, it is important to take into account a ratinale of a close connection between fatigue and work output. As Murrell (1971) insists, fatigue does not have the many connotations that it has been given. It refers to how human being feels in relation to some sort of aversion toward activity and the feeling of inactivity to perform. Fatigue is a self-recognized state of the individual. It is a directly experienced condition with an inferred connection between the way the individual feels and the amount to exert himself. Thus, the more work, the more tiredness. It may quickly become clear that there are close connection between fatigue and work output.

McFarland (1971) states that in most studies dealing with this topic, work output is used as one of the commonly used criteria. We will next deal with various effects of fatigue which may be important to be considered in scheduling human resources and its relationship between information and productivity.

Effects of Fatigue

1. Effects of Fatigue Resulting from Boredom

The term 'boredom', like 'fatigue', is difficult to define in precise term, because it depends on subjective feelings (Murrell, 1971). If any definition were to be given, actually it would differ very little from that already described for fatigue itself. Boredom has recieved relatively little attention because, as Murrell (1971) said, it is difficult to separate fatigue from boredom. Fatigue occurs in monotonous or bored situation (Grandjean, 1968) and similarly, too much idleness brings boredom. Boredom brigns uneasiness, a feeling of lack of purpose and, eventually, fatigue (McFarland, 1971). Wyatt and Fraser (1929) investigated monotony both in the laboratory and on the shop floor through Industrial Fatigue Research Board in England. They discussed whether it is fatigue or boredom that is the cause of variation in work output throughout the working day. They used a subjective assessment of boredom as well as measurement of output and of variability. They claim that when boredom was said to be experienced the variability of output was increased and the output was lower than that of worker fatigued. One of the serious effects of boredom is chronic fatigue which is the cumulative effects of fatigue. It decreases a person's initiative and eventually work output and is not relieved by sleep or rest (McFarland, 1971).

2. Effects of Fatigue Resulting from Stress

If a person is placed under stress, various forms of exhaustion and fatigue may result (McFarland, 1971). Heimstra (1970) conducted experiments using Crawford's (1961) concept of stress fatigue. His findings support the concept that stress brings about emotional arousal. This may cause a rather marked decrement in performance in the contingent group that got electric shock when they made a mistake. The results suggest that the fourth hour of the test may be the point at which a reduced intensity of response due to stress fatigue begins. Fatigue resulting from stress is also called chronic fatigue (McFarland, 1971). It can also be a contributor to some other ailments so characteristic of our modern life. Examples include mental illness, peptic ulcers, and higher blood pressure. Eventually, chronic fatigue due to stress from work may deteriorate mental performance and skill.

3. Effects of Fatigue Resulting from Frustration

Some situations in which performance and accomplishment are attempted may truly be frustrating. Frustration, as Bartley (1965) defined it, is a state of the individual and not a characteristic of an external situation. Certain situations are much more likely to produce frustrations. What kinds of situations could produce frustration? What effects are expected from the frustrations? Those situations which involve conflicting performance requirements could be the most likely to do this, as Bartley (1965) states. He indicates that conflicts can develop through changes in demands, such as different performances that are required at different times, no forewarning of changes, or no rationale given for the changes as design changes and engineering drawing delays in project execution processes. The effect of frustration may bring expenditure of energy, then fatigue, and finally deterioration of work accomplishment.

4. Relationship Between Information and Productivity

Under modern work conditions, decrements in productivity are more likely to be due to lack of a desire to work rather than of the ability to work (Wyatt and Fraser, 1929). Murrell (1971) states that there are many ways in which management may influence motivation. Giving knowledge of a company's goal and providing workers with expectations in terms of target are good examples. As Simmonson and Weiser (1976) indicates, the reality of a person's perception about a situation and of his expectations likewise contribute to his performance of the task and his subsequent responses to his success or failure at the task. Walster and Asronson (1967), and Snyder *et al*. (1974) showed that a person's performance and his feelings of fatigue are also related to how long a person expects to have to work. With a group of subjects given a series of tasks assumed "fatiguing", some subjects were led to believe that their work assignment was virtually done after a certain amount of work had been completed, other workers expected that they must continue for a longer period of time. The authors claimed that those subjects who believed they were nearly finished reported a greater increase in fatigue than those who expected to continue to work, when the work period was extended. Thus, when a person is palced in a situation where he must work unexpectedly longer hours or days without pre-notice or any obvious rationale for changes, he became fatigued. If he is disrupted in his normal work routine or work schedule, and furthermore, if he does not have any idea about length of working hours, he usually becomes frustrated or conflicted with his work and he may lose his motivation to work, and, finally be tired with fatigue. Obviously, his productivity will be decreased. On the contrary, when an organization establishes communication channels through which management provides workers with information and rationale behind overtime, long working hours and days, disruptions by sudden changes or repetitive changes in project schedule becuase of design changes and delays, and company's present situations and targets to be accomplished, etc., mutual understanding between management and workers will improve and the workers may be motivated and their physical and mental fatigue will probably be significantly decreased. Thus, information is also an important factor to be considered in project management.

III. INFORMATION THEORY

Information, Entropy and Uncertainty

In this section, we introduce uncertainty, information and entropy and show their relationships to each other. Consider a mutually exclusive and exhaustive set e = (E), i = 1,2,$\cdots$,n, and suppose that we assign probability $p(E_i) = p_i$ to each proposition E_i at a certain state of knowledge in such a way that

$$p(E_i) \geqq 0$$

and

$$\sum_{i=1}^{n} p(E_i) = 1$$

The probabilities p_i mentioned above are actually assigned to the E_i based on the prior knowledge about the E_i. In other words, the state of knowledge underlying p_i does not include a result of a direct experiment.

Let us assume that an experiment is actually conducted. After one of E_i's, say, E_j turns out to be true, the state of knowledge changes completely. The probabilities p_i will then be changed, based on the new state of knowledge to p_{in}, i = 1,2,$\cdots$,n,

$$p_{jn} = 1,\ p_{in} = 0 \text{ for } i \neq j$$

So we can say that the probability p_i is our degree of expectation for E_i, based on a given state of knowledge before the experiment. When we want to measure the information from a message in terms of the probability p_i that prevailed prior to the arrival of the message, the decreasing function should be selected. The function proposed by Shannon (1948) is:

$$h = \log_2(1/p_i) = -\log_2 p_i$$

The value of h decreases from ∞ (infinite information when the probability p_i prior to the message is zero) to 0 (zero information when the probability p_i is one). The function is illustrated in Figure 1. The *unit* of information is determined by the base of the logarithm. Two is used as a base in this thesis.

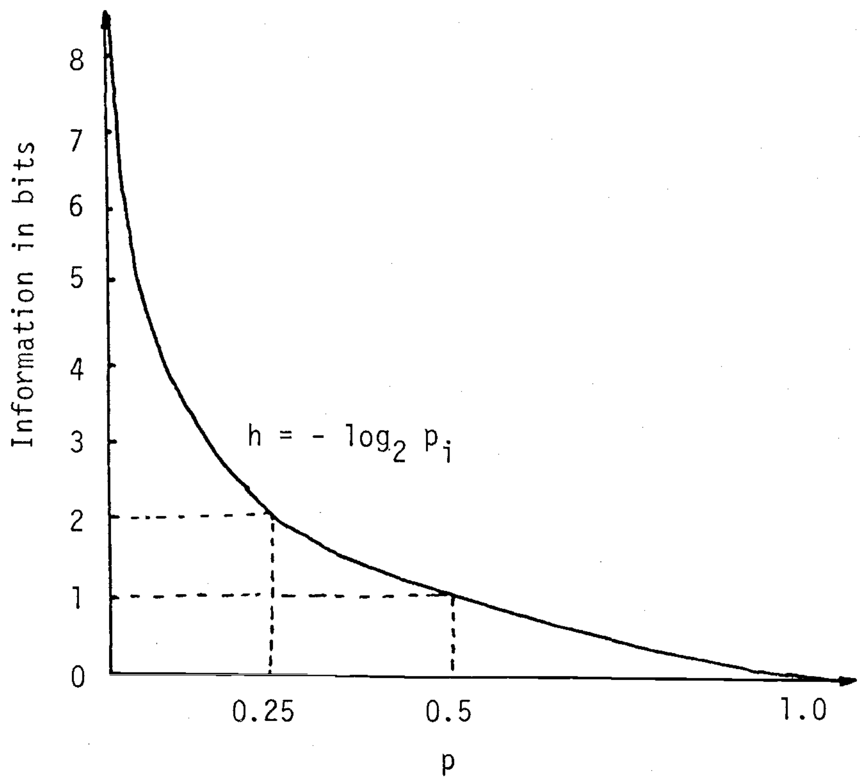


Figure 1. Information measured in bits.

If logarithms to base 2 are used, the information transmitted by two outcomes, say "head" or "tail", each with probability one-half of a fair coin tossing experiment would be:

$$h = \log_2 2 = 1$$

which does give a unit information. Information is then said to be measured in binary digits or, for short, bits. One bit is defined as a unit of information. This shows the uncertainty of the appearance of a head or tail for a coin tossing experiment before we know the result of experiment. Now, consider an experiment in which there is only one certain outcome. For example, suppose a coin has heads on both sides. There is only one outcome of this coin tossing experiments; a head appears with certainty. The information transmitted by the outcome "a head appearing", then, will be:

$$h = \log_2 1/p = \log_2 1 = 0$$

It confirms that for the certain outcome no new information is transmitted and there is no uncertainty which was already known about the outcome of the experiment. Figure 1 shows the above arguments, when $p = 0.5$, information transmitted is one bit and when $p = 1$, information transmitted is zero shown along the vertical axis.

Now, we wish to determine amount of information transmitted by a combined experiment. Consider again tossing a fair coin two times in succession. This experiment will transmit twice as much information as that of an experiment having one time tossing a fair coin which is considered as a standard of measurement. In an attempt to explain the result, we consider the sample space of outcome of the "two times successively tossing a fair coin" experiment will consist of

{HT}
{HH}
{TH}
{TT}

Since each toss is independent and each outcome is equally likely with probability one-fourth, the information transmitted by this experiment can be expressed as follows:

$$h \text{ (combined experiment)} = h \text{ (first toss)} + h \text{ (second toss)}$$

so $$h \text{ (combined experiment)} = \log_2 2 + \log_2 2$$

and $$h \text{ (combined experiment)} = 1 + 1$$

$$= 2$$

Therefore, an experiment consisting of n possible outcomes should transmit $\log_2 n$ bits of information. For the previous combined experiment, the infommation transmitted is:

$$h = \log_2 4$$

$$= 2 \text{ bits}$$

Raisbeck (1963) states that a choice of base two of logarithms as a unit of measurement gives consistent results. More rigorous justifications for the use of base two are given by Shannon and Weaver (1949), Watanabe (1969) and others.

Entropy as a Measure of Uncertainty

A measure of uncertainty (in the state of knowledge before the experiment) with regard to the outcomes of the experiment, is usually expressed in the form of negative entropy:

$$H(x) = -\sum_{i=1}^{n} p_i \log_2 p_i \; ,$$

$$\text{for } i = 1,2,3,\cdots, n$$

where H is the entropy in bits, p_i is the individual probability of each outcome of an experiment which must satisfy $\sum_{i=1}^{n} p_i = 1$ and $p_i \geq 0$.

Suppose that we have the p_i before the experiment, and the corresponding uncertainty is the entropy above mentioned. After knowing the result of the experiment, the probability $p_j = 1$, $p_i = 0$, $i \neq j$. The uncertainty, $H'(x)$, after the experiment therefore becomes:

$$H'(x) = 0$$

The reduction in uncertainty caused by the experiment can be considered as the information furnished by the experiment:

$$\begin{aligned}\text{Information} &= \text{reduction in uncertainty} \\ &= H(x) - H'(x) \\ &= -\sum_{i=1}^{n} p_i \log_2 p_i\end{aligned}$$

The equivocation of an experiment X with n possible outcomes $a_1, a_2, \cdots, a_n$ and probabilities $p_i, p_2, \cdots, p_n$ is, in general, called the entropy of the experiment and expressed as:

$$\begin{aligned}H(x) &= p_i \log_2(1/p_1) + p_2 \log_2(1/p_2) + \cdots + P_n \log_2(1/p_n) \\ &= -p_1 \log_2 p_1 - p_2 \log_2 p_2 - \cdots - p_n \log_2 p_n \\ &= -\sum_{i=1}^{n} p_i \log_2 p_i \text{ bits}\end{aligned}$$

Uncertainty and information are expressed by the same mathematical formula, although the terms "uncertainty" and "information" have opposite meanings. Uncertainty and expected information are two sides of the same coin. Uncertainty is a prevailing quantity prior to the knowledge of an event, and information is collected when the knowledge of an event is arrived. The more uncertainty prior to the message which states which of the n possible outcomes is realized, the larger is at least on the average the amount of information conveyed by it. Therefore, the entropy may be considered not only as a measure of the uncertainty associated with a distribution whose probabilities are $p_1, p_2, \cdots, p_n$, but also as the expected information of the message. Indeed, uncertainty and expected information can be regarded as complementary concepts. Figure 2 shows the change in entropy as the probability p of one outcome increases from 0 to 1.0 as the probability (1 - p) of the other decreases from 1.0 to 0.

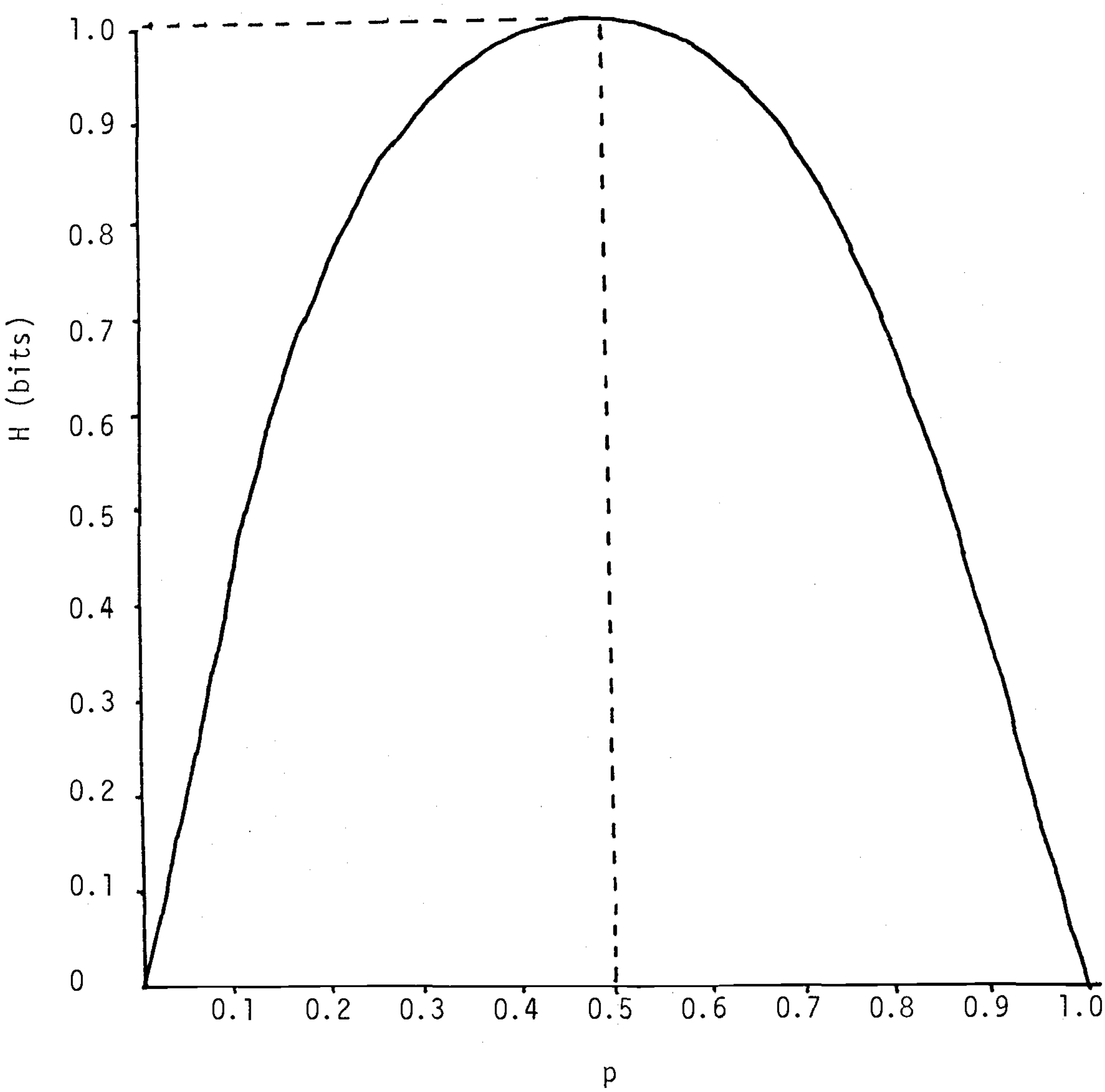


Figure 2. Entropy of an experiment having two outcomes.

It can be extended for any experiment having n outcomes and also shown that the entropy will be maximized when all probabilities of n outcomes are equal; this being the condition of greatest uncertainty or greatest equivocation. Consider an experiment with a finite number of outcomes:

$$0_1, 0_2, \cdots, 0_n \text{ having } p_1, p_2, \cdots p_n$$

The objective is to maximize the entropy function subject to the constraints

$$\sum_{i=1}^{n} p_i = 1$$

$$p_i \geqq 0 \quad \text{for } i = 1, 2, \cdots, n$$

The Largrange method of undetermined multipliers may be adopted to solve the problem.

We have:

$$\{\text{Max}\}\ H(x) = -\sum_{i=1}^{n} p_i \log_2 p_i + \lambda(1 - \sum_{i=1}^{n} p_i) \qquad (3.0)$$

$$\log_2 p_i = (\log_2 e)(\log_e p_i)$$

Therefore, defining $K = \log_2 e = 1.4427$, Equation 3.0 may be written as:

$$\{\text{Max}\}\ H(x) = -K \sum_{i=1}^{n} p_i \log p_i + \lambda(1 - \sum_{i=1}^{n} p_i)$$

Taking derivatives:

$$\frac{d\{H(x)\}}{dp_1} = -K\frac{p_1}{p_1} - K \log p_1 - \lambda = 0 \qquad (3.1)$$

$$\vdots$$

$$\frac{d\{H(x)\}}{dp_n} = -K\frac{p_n}{p_n} - K \log p_n - \quad = 0 \qquad (3.2)$$

$$\frac{d\{H(x)\}}{d\lambda} = 1 - \sum_{i=1}^{n} p_i = 0 \qquad (3.3)$$

Equation 3.1 through 3.3 may be solved to obtain:

$$p_i = e^{(-1 - \lambda/K)} \qquad (3.4)$$

for i = 1,2,···,n

Equation 3.4 is substituted into Equation 3.3, then :

$$n[e^{(-1 - \lambda/K)}] = 1$$

So $$e^{(-1 - \lambda/K)} = 1/n$$

So $$p_1 = p_2 = \cdots = p_n = 1/n$$

Therefore, entropy is maximized when all possible outcomes are equally likely. That is, uncertainty is maximized under the condition of all n probabilities are equally likely and also the largest amount of information is produced.

Information Gain

The point of maximum uncertainty is defined as zero information gain (Shirland, 1972). Therefore, we will experience an information gain when we are able to assign probabilities other than the equal probabilities to outcomes in an experiment. So information gain may be defined as the difference between the information transmitted by the equal probabilities and the information transmitted by a set of augmented probabilities based on our state of knowledge before the experiment. Information gain can be expressed in form of equation as:

$$G(x) = \log_2 n - \sum_{i=1}^{n} p_i \log_2 p_i \ , \quad \text{for } i = 1,2,\cdots,n$$

where G(x) designates information gain, n and p_i are previously defined.

The Maximum Entropy Principle

A way of Reasoning about Uncertainty

We desire logical means for reasoning about uncertainty of events. Probability theory as a general way for reasoning about uncertainty has been advocated by two approaches: One approach is a personalistic approach developed by Ramsey (1950) and Savage (1954) and the other is a logical approach supported by Keynes (1921) and Jeffreys (1939). An essential insight of both the personalistic and logical approaches is that probability must reflect the knowledge upon which they are based. Probability theory may provide a way of reasoning about one state of knowledge in the lignt of other given information. We need thus an assumption which may support the concept of translating a state of knowledge into probability assignments.

Assumption 1: A probability assignment is the reflection of our state of knowledge.

Based on the assumption 1, Bayes Theorem becomes the means by which the reasoning about uncertainty may be accomplished. Accordingly, we may use Bayes Theorem to revise a given probability if prior probability assignments and new information become available. However, there is still much controversy in Bayes Theorem over the question of how to assign prior probability by using the initial state of knowledge. For the logical derivation of probability distribution, it would be highly desirable to have a formal principle by which a state of knowledge may be translated to reach a unique choice of probability distribution. Such a principle is the maximu entropy principle originated by Jaynes (1957a, 1957b, 1962, 1963, 1965, 1967, and 1968).

The Maximum Entropy Principle

Suppose we have to assign a probability distribution which is consistent with the information that we may collect under certain circumstances. If there are several probability distribution which are consistent with the information that we have collected, we shall choose that distribution for which the entropy function $\{H(x) = - \Sigma\, p_i \log p_i\}$ is largest. This criterion is the maximum entropy principle (Jaynes, 1957; North, 1970).

The Formalism of Maximum Entropy Principle

We will confine our discussion of the maximum entropy principle formalism to the given information in the form of average values, since it most often happens that we are given the information in the form of average. Suppose that we are to choose a probability distribution that is consistent with the given information of average. To choose the best estimate of probability distribution, the problem may be stated mathematically as follows (Tribus, 1961, 1962):

Maximize $$H(x) = - \sum_i p_i \log_2 p_i \qquad (3.5)$$

where $p_i = p(x_i | A_1 A_2, \ldots, A_n\ X)$

x_i = "the value of x is x_i"

A_1 = "the mean value of $q_1(x_i)$ is A_1"

A_2 = "the mean value of $q\ (x_i)$ is A_2"

.

.

.

A_r = "the mean value of $q_r(x_i)$ is A_r"

X = "all the other background in the case"

Subject to the constraints:

$$\Sigma\ p_i = 1 \tag{3.6}$$

$$\Sigma\ p_i q_1(x_i) = A_1 \tag{3.7}$$

.

.

.

$$\Sigma\ p_i q_r(x_i) = A_r \tag{3.8}$$

This maximization of entropy H(x) is easily accomplished by the use of Lagrange method of undetermined multipliers.

To maximize the entropy in Equation 3.5 subject to the constraints in Equation 3.6, 3.7 and 3.8, Lagrange's method of undetermined multilliers is employed:

Since $\log_2 p_i = (\log_2 e)(\log_e p_i)$

we can define $K = \log_2 e = 1.4427$

Equation 3.5 can be written as:

$$H(x) = -\ K\ \Sigma\ p_i \log\ p_i \tag{3.9}$$

Differentiate Equation 3.9, 3.6 3.7 and 3.8 with respect to p_i, then we have:

$$d\{-H(x)\} = K \sum_i (\log p_i + 1)\, pd_i \tag{3.10}$$

$$\Sigma\, dp_i = 0 \tag{3.11}$$

$$\Sigma\, q_r(x_i) dp_i = 0 \tag{3.12}$$

.

.

.

$$\Sigma\, q_r(x_i) dp_i = 0 \tag{3.13}$$

Multiply Equation 3.11 by $(\lambda_o - 1)$, Equation 3.12 by λ_1, Equation 3.13 by λ_r and add them to Equation 3.10 to find:

$$K\, \Sigma\, (\log p_i + 1)\, dp_i + (\lambda_o - 1)\, \Sigma\, dp_i + \lambda_1\, \Sigma\, q_1(x_i)\, dp_i$$

$$+ \lambda_2\, \Sigma\, q_2(x_i)\, dp_i + \ldots + \lambda_r\, \Sigma\, q_r(x_i)\, dp_i = 0$$

Upon collecting terms we have:

$$K\, \Sigma\{(\log p_i + 1 + \lambda_o - 1 + \lambda_1 q_1(x_i) + \lambda_2 q_2(x_i) + \ldots$$

$$+ \lambda_r q_r(x_i)\ \}\ dp_i = 0 \tag{3.14}$$

The parenthesis must be zero to insure that Equation 3.14 is satisfied regardless of the magnitude of the individual values of dp_i. Therefore, solving for $\log p_i$ we find:

$$\log p_i = \{-\lambda_o/K - \lambda_1/K\, q_1(x_i) - \lambda_2/K\, q_2(x_i)\}\cdots \tag{3.15}$$

or

$$p_i = \exp[-\lambda_o/K - \lambda_1/K\, q_1(x_i) - \lambda_2/K\, q_2(x_i)\ \cdots \tag{3.16}$$

Tribus (1969) shows the proof that the probability distribution given in Equation 3.16 is a global maximum; the entropy, $H(x)$, is maximum.

Example 1. Suppose that we have only two constraint equations in the above constraints. Again the problem may be stated mathematically: for convenience, the base of e is taken.

Maximize $H(x) = - \Sigma p_i \log p_i$ (3.17)

Subject to $\Sigma p_i = 1$ (3.18)

$\Sigma p_i q_i(x_i) = A$ (3.19)

where $p_i = p(x_i|A,X)$: A = the mean value of $q_i(x_i)$

Differentiate Equation 3.17 with respect to the p_i:

Set equal to $dH(x) = 0$

$$d\{-H(x)\} = \sum_i (\log p_i + 1)\ dp_i = 0 \quad (3.20)$$

Differentiate Equation 3.18 and 3.19 with respect to the p_i. Multiply by Lagrangian undetermined multipliers $(\lambda_o - 1)$ and λ_1:

$$(\lambda_o - 1)\ dp_i = 0 \quad (3.21)$$

$$\lambda_1 \quad q_i(x_i)\ dp_i = 0 \quad (3.22)$$

Add Equation 3.20, 3.21 and 3.22:

$$\sum_i \{\log p_i + 1 + \lambda_o - 1 + \lambda_1 q_i(x_i)\}\ dp_i = 0$$

$$\sum_i \{\log p_i + \lambda_o + \lambda_1 q_i(x_i)\}\ dp_i = 0 \quad (3.23)$$

To guarantee that Equation 3.23 will be satisfied, regardless of the variation, dp_i, the parenthesis is equated to zero:

$$\log p_i + \lambda_o + \lambda_1 q_i(x_i) = 0$$

$$\log p_i = e^{-\lambda o - \lambda 1 q i (x i)} \quad (3.24)$$

$$p_i = e^{-\lambda_o - \lambda_1 q_i(x_i)} \quad (3.25)$$

The values of the Lagrangian undetermined multipliers are chosen to fit the given information. This procedure leads to the following equations:

$$\sum_i p_i = \sum_i e^{-\lambda_o - \lambda_1 q_i(x_i)} = 1$$

or

$$e^{-\lambda_o} \sum_i e^{-\lambda_1 q_i(x_i)} = 1$$

$$e^{\lambda_o} = \sum_i e^{-\lambda_1 q_i(x_i)} \tag{3.26}$$

Taking log on both sides of Equation 3.26:

$$\lambda_o = \log \sum_i e^{-\lambda_1 q_i(x_i)} \tag{3.27}$$

Differentiate Equation 3.27 with respect to λ_1, then:

$$\frac{d\lambda_o}{d\lambda_1} = \frac{-\sum_i q_i(x_i)\, e^{-\lambda_1 q_i(x_i)}}{\sum_i e^{-\lambda_1 q_i(x_i)}}$$

Using Equation 3.26:

$$\frac{-d\lambda_o}{d\lambda_1} = e^{-\lambda_o} \sum_i q_i(x_i)\, e^{-\lambda_1 q_i(x_i)}$$

$$= \sum_i q_i(x_i)\, e^{-\lambda_o - \lambda_1 q_i(x_i)}$$

$$= \sum_i q_i(x_i)\, p_i$$

$$= A \qquad \text{from Equation 3.25}$$

Summary of maximum entropy principle formalism:

1. Define each possible state and assign a symbol for its probability.

2. Describe the given information in the appropriate form.

For example, the average may be described:

$$\sum_i p_i q_i(x_i) = A_j$$

where A_j = the given information (average)

$q_i(x_i)$ = a function of x

x is a property that serves to identify a possible state and x_i is the value of x identified with state i.

3. Add the normalizing equation $\sum_i p_i = 1$ into constraint.
4. Maximize the entropy, $H(x) = - \sum p_i \log p_i$ agreeing with the constraint equations which represent the given information.
5. Use the probability distribution derived in Bayes formula when new information becomes available.

IV. DEVELOPMENT OF A SYSTEMATIC PREDICTION PROCEDURE OF IDENTIFYING THE POTENTIAL PROBLEM AREAS

Introduction

The purpose of this chapter is to develop a systematic prediction procedure to identify potential sets of designs to change. If management may collect and utilize information about likely trouble spots before starting a project as well as during project execution, management would be placed in a better position to plan and control the project by putting more attention on the potential trouble spots. Management may also be able to reduce effects of disruptions in the project. The procedure can systematically provide management with the necessary information to identify the potential trouble spots.

Input-output Information Channel shows inflows and outflows of information between the project owner and the project contractor. Conditional equivocation is used as a criterion to identify the likely trouble spots. A Bayesian sequential analysis is used for computing revised probability estimates. A priori probability for initial sets of designs is assigned by the concept of maximum entropy principle.

Input-Output Information Channel

Normally there exists a trust and responsibility relationship between project owner and project contractor. The project owner will supply information about potential design changes to project contractor and the project contractor tries to keep his word to complete the project within the original planned dates. In this manner they can mutually reduce risks and uncertainty. The project contractor needs an information channel to collect information about likely problem areas (possible trouble spots) in his project. We call this information channel an Input-Output Information channel. Figure 3 shows a schematic Input-Output Information channel between the project owner and project contractor. The project owner may use this information channel to send his prior knowledge or special evidence about potential design changes in the project. The project contractor can process the initial information recieved and the additional information to identify the final sets of design changes.

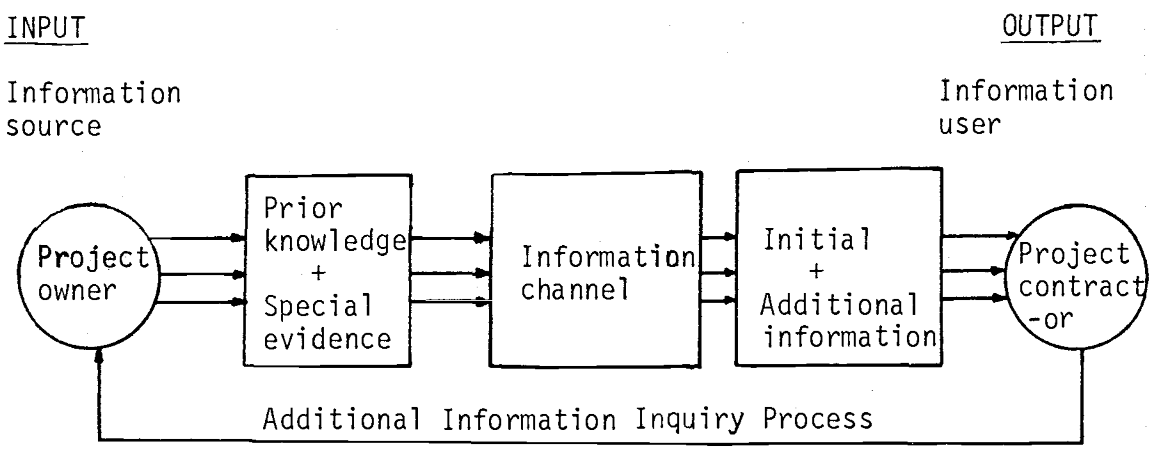


Figure 3. A schematic diagram of Input-Output Information Channel.

Design Alternatives Provided by Project Owner

Suppose that project owner reviews his project carefully before starting the project and provides the project contractor with information on problem areas (potential designs to change) in his project. At the beginning stage the project owner can only provide possible alternatives for the total number of designs to change, because he is not himself sure which designs will be changed. For example, the project owner considers T total number of designs to change. If he predicts n different kinds of design alternatives and each alternative consists of certain number of highly probable designs among T total number of designs as follows: as Figure 4 shows, E_1 design alternative consists of e_1 highly likely designs to change and E_2 design alternative e_2, E_3 design alternative e_3, and E_n design alternative e_n highly likely designs to change.

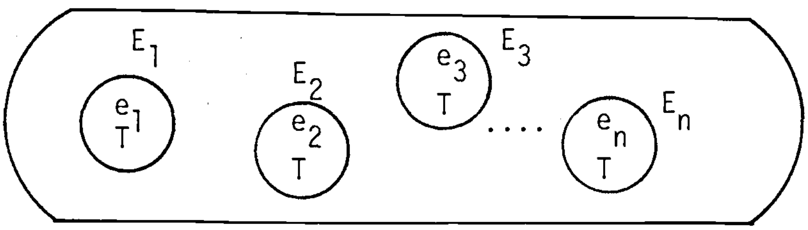


Figure 4. Design alternatives provided by project owner.

Bayesian Decision Analysis and Information Theory

Conditional Probability

Consider two experiments E and B. Experiment E consists of n outcomes and experiment B consists of m outcomes as shown in Figure 4-1.

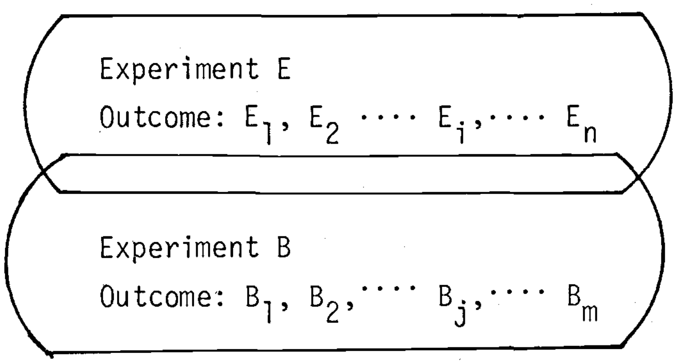


Figure 4-1. Two dependent experiments E and B.

The conditional probability distribution may be determined from the definition of conditional probability and the definition of the Bayes Theorem. Considering the above general case of two dependent experiments, the joint probability of obtaining E_i and B_j may be expressed by $p(E_iB_j)$.

$$p(E_i|B_j) = \frac{p(E_iB_j)}{p(B_j)} = \frac{p(E_i)p(B_j|E_i)}{\sum_{i=1}^{n} p(E_i)p(B_j|E_i)}$$

Bayes formula gives the posterior probabilities (revised probabilities) of outcome E_i given that outcome B_j of experiment B has already been observed. In Bayesian decision analysis the outcome B_j is considered a new additional information. A sequential sampling process is adopted. The cyclic process of obtaining additional information, revising probabilities, obtaining more information, again revising probabilities is called sequential sampling. Bayesian analysis of a sequential sampling process may continue until managemant decision-making becomes apparent within his time constraints and his capability to assume risk.

Steps for Bayesian Decision Analysis

The steps required in a Bayesian decision analysis can be listed as:

1. The possible states of nature that might exist must be described.

2. An initial probability estimates of each state of nature, the $p(E_i)$ terms must be assigned, referring to Example 2.

3. The conditional probabilities that the evidence could have

occurred, given the various states of nature, the $p(B_j|E_i)$ terms, must be computed when new information or evidence were available.

4. The revised probability estimates can be computed using Bayes Theorem and the $p(E_i)$ and $p(B_j|E_i)$.

In summary the entire Bayesian decision analysis can be performed in a standard, five column computation. Table 1 shows Column Bayesian Computation using the general nomenclature of Bayes Theorem.

Table 1. Column computation of Bayesian revised probability estimates.

(I) State of nature	(II) Original probability estimates	(III) Conditional probabilty $p(B_j|E_i)$	(IV) Joint probability (II x III)	(V) Revised probability estimates
E_1	$p(E_1)$	$p(B_j|E_1)$	$p(E_1)p(B_j|E_1)$	$p(E_1|B_j)$
E_2	$p(E_2)$	$p(B_j|E_2)$	$p(E_2)p(B_j|E_2)$	$p(E_2|B_j)$
E_3	$p(E_3)$	$p(B_j|E_3)$	$p(E_3)p(B_j|E_3)$	$p(E_3|B_j)$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
E_n	$p(E_n)$	$p(B_j|E_n)$	$p(E_n)p(B_j|E_n)$	$p(E_n|B_j)$
	1.00		$\sum_{i=1}^{n} p(B_j|E_i)p(E_i)$	1.00

In Table 1 the sum of column IV is the denominator of Bayes Theorem and all of the revised probabilities in column V are computed by dividing each term in column IV by the sum of column IV.

In each sequential Bayesian computation the revised probability estimates from the previous sample (new information) become the original probability estimates for the calculation of the next stage.

The result of Bayes Theorem was derived by Newendorp (1972, 1975), Tribus (1969) and others.

Conditional Equivocation

The conditional equivocation of outcome E_i when the additional information, B_j, becomes available is:

$$H(E_i|B_j) = p(E_i|B_j) \log p(E_i|B_j)$$

and there will be n terms of this type. In other words, we will have:

$$
\begin{aligned}
H(E_1|B_j) &= p(E_1|B_j) \log p(E_1|B_j) \\
H(E_2|B_j) &= p(E_2|B_j) \log p(E_2|B_j) \\
&\vdots \\
H(E_i|B_j) &= p(E_i|B_j) \log p(E_i|B_j) \\
&\vdots \\
H(E_n|B_j) &= p(E_n|B_j) \log p(E_n|B_j)
\end{aligned}
$$

The conditional probability, $p(E_i|B_j)$, may be determined by Bayes Theorem. In an attempt to identify outcome, E_i, given the additional information, B_j, the conditional equivocation, $H(E_i|B_j)$, is used as a criterion. The result was derived by Kunisawa (1959), Goldman (1953) and others.

Conditional Equivocation Tree

Figure 5 shows a conditional equivocation tree. The tree shows graphically the flows within the systematic prediction procedure and can be used to identify the sets of design alternatives to change in terms of the amount of conditional equivocation. The tree is explained as follows:

1. Experiment, E, consisting of n design alternatives to change, E_i, $i = 1,2,3,\cdots n$ represents Information In.

2. $p_i(E_i)$ represents prior probability for design alternative, E_i and B_j represents new information at stage i.

3. $p_i(E_i|B_j)$ represents conditional probability for design alternative, E_i at stage i, given new information B_j.

4. The conditional equivocation, $H(E_i|B_j)$, is computed for each set of design alternative.

5. At the decision box in each stage the conditional equivocation is compared and management determines two things: (a) continue or stop the decision process; (b) new information or design alternative to be chosen. If continue, new information will be asked. If stop, the design alternative that contains the least amount of conditional equivocation is identified first.

6. The design alternative to be chosen represents Information Out.

A Systematic Prediction Procedure of Identifying the Potential Sets of Design Alternatives

1. The various possible sets of design alternatives to change, E_i, $E_2, \cdots E_n$, first could be described by expertise in project owner team at the beginning of a project.

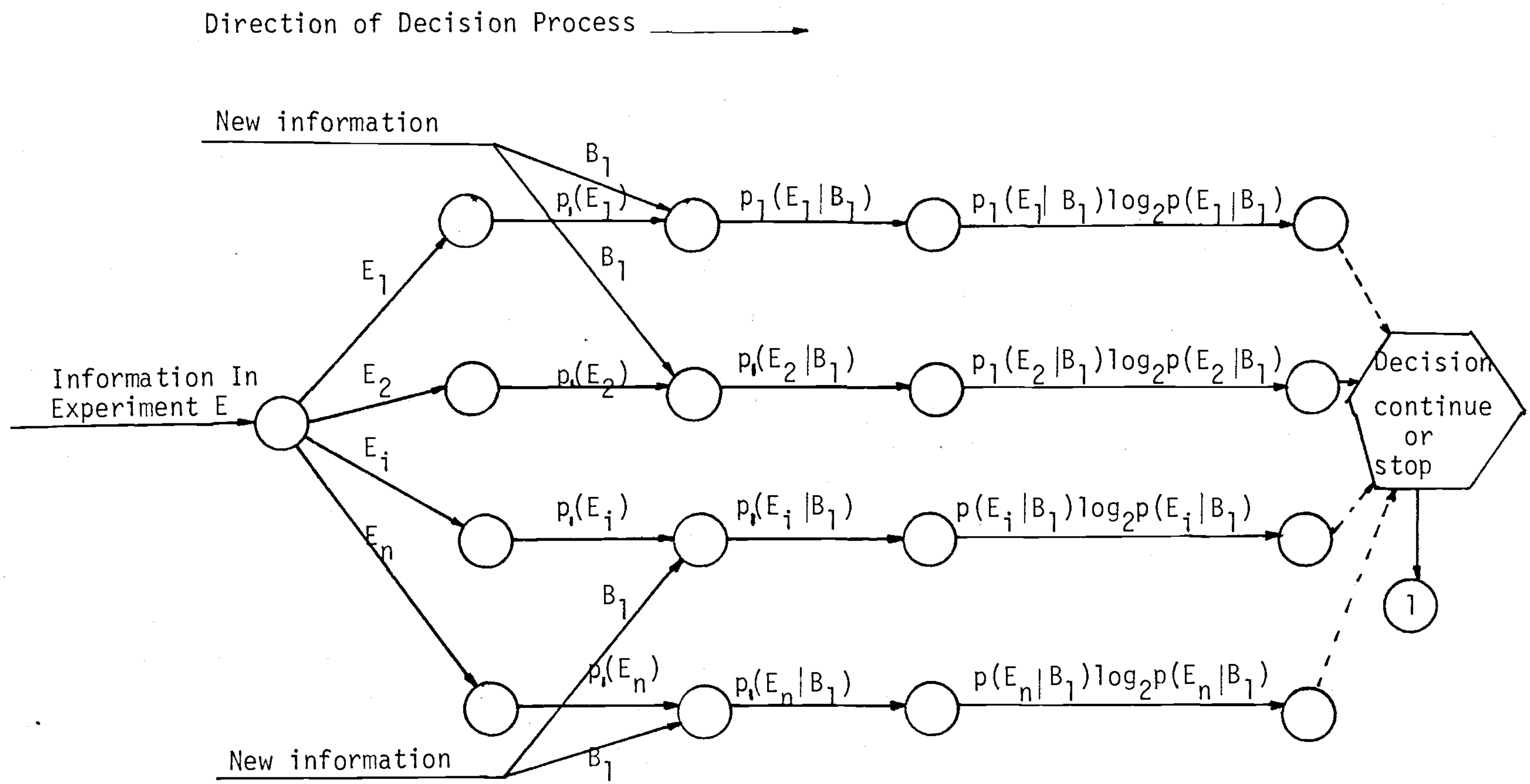
2. The original probability estimates of each set of design alternative should be estimated by project contractor. At the beginning the initial sets of design alternatives may be assigned equal probability by using the concept of the maximum entropy principle.

3. The revised probability estimates, $p(E_i|B_j)$, can be computed by Bayes Theorem and the conditional probability, $p(B_j|E_i)$ and $p(E_i)$.

4. The conditional equivocation, $H(E_i|B_j)$, for each set of design alternative should be computed and compared with each other to identify the sets of design alternatives that contain the least amount of conditional equivocation.

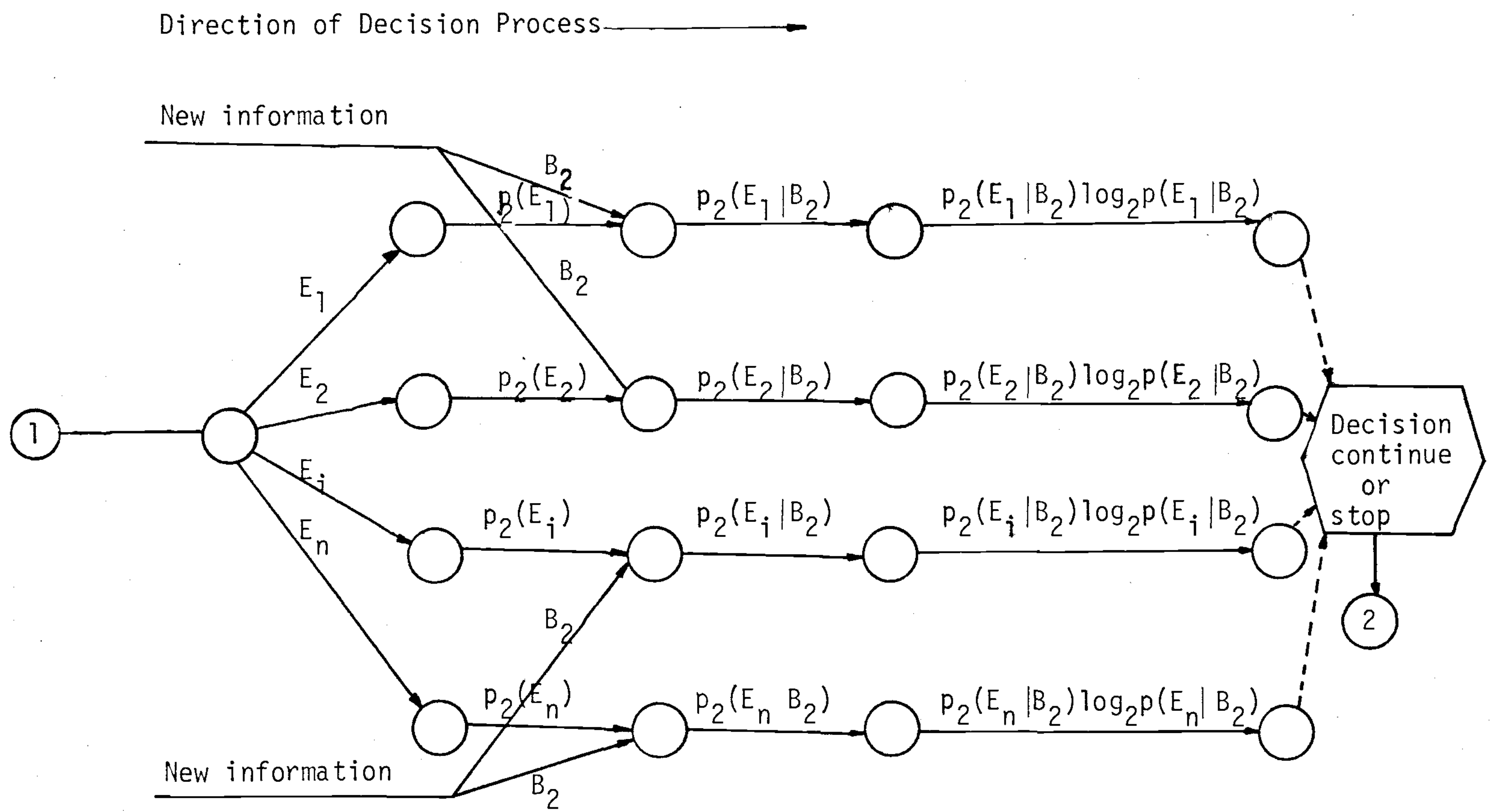
5. Repeat the process until management can identify the sets of design alternatives.

The procedure will be illustrated by a conditional equivocation tree in Figure 5 and Example 2.



STAGE 1

Figure 5. Conditional equivocation tree



STAGE 2

Figure 5.(continued) Conditional equivocation tree

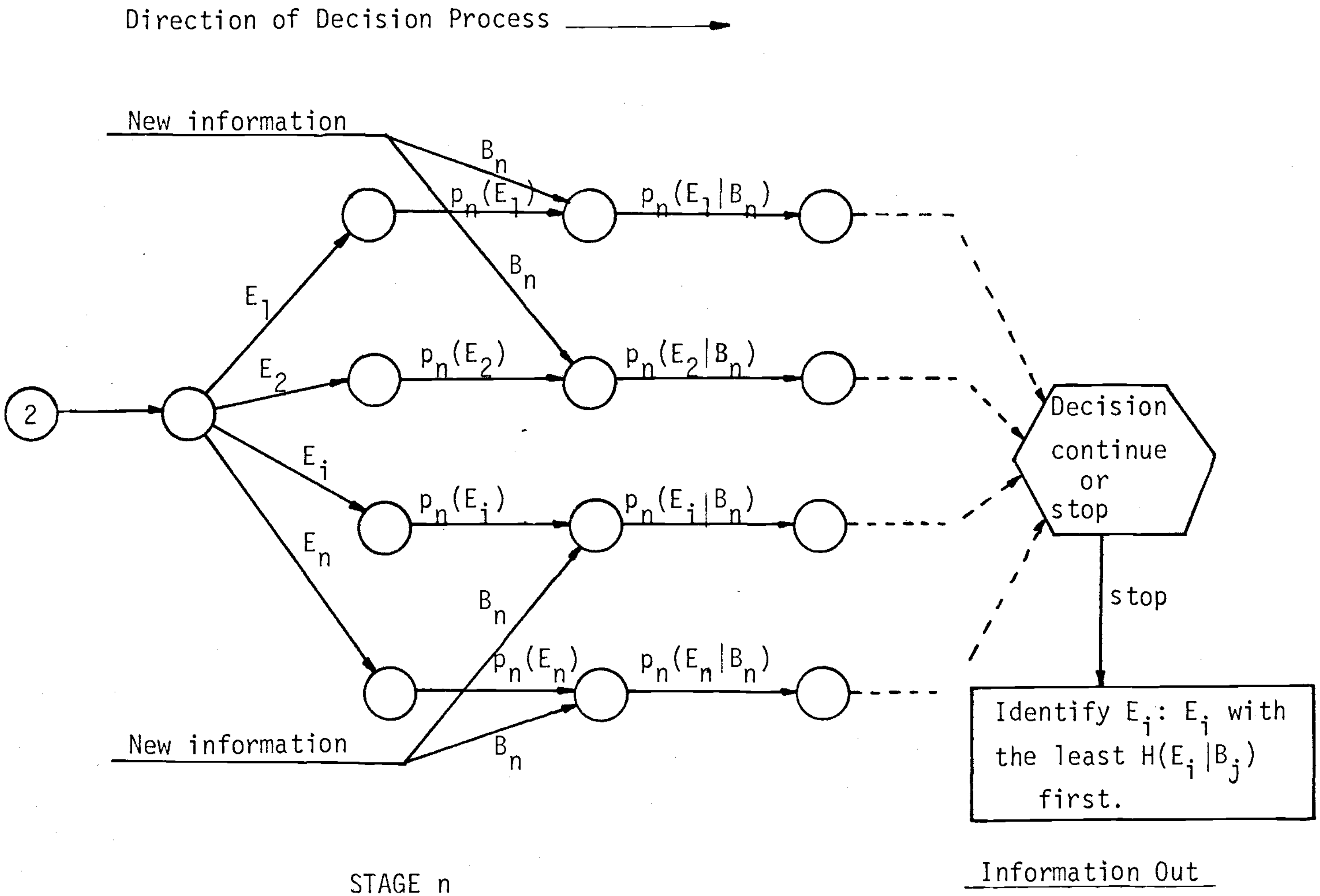


Figure 5. (continued) Conditional equivocation tree.

Example 2. Suppose that we have a contract for a construction project and established an Input-Output Information channel for the project. We expect that there may be usual design changes in the construction project. But we are uncertain about how many design changes will occur. Suppose that 20 possible changes are considered and that there are two possible alternatives as follows:

Design alternatives

E_1 : 16 designs out of 20 are to change

E_2 : 8 designs out of 20 are to change

What we really want to know is which set of design alternative above will occur by using a Bayesian sequential analysis and conditional equivocation. Since we have little information about the occurrence of the design changes and all we know is that either E_1 or E_2 will occur, for purpose of assigning prior probability estimates of the initial sets of design alternatives, $p(E_1)$ and $p(E_2)$, the concept of maximum entropy principle is used.

In an attempt to assign prior probabilities to the sets of design alternatives, E_1 and E_2, the maximum entropy principle can be used. Then, the prior probability estimates for the sets, E_1 and E_2, are:

$$p(E_1) = \frac{1}{2} \quad \text{and} \quad p(E_2) = \frac{1}{2}$$

At the first stage, the equivocation, $H_0(x)$, is:

$$H_0(x) = -\sum_{i=1}^{2} p(E_i) \log_2 p(E_i) = -(0.5 \log_2 0.5 + 0.5 \log_2 0.5) = 1 \text{ bit}$$

The stage 0 represents the situation which contains the maximum uncertainty. Therefore, there is no idea which design alternative, E_1 and E_2, occurs.

Suppose that we continue the decision process and receive new information, B_1, confirming that the first design has occurred.

We can now compute conditional probabilities $p(B_1|E_1)$ and $p(B_1|E_2)$ as:

$$p(B_1|E_1) = \frac{16}{20}$$

$$= 0.8$$

and

$$p(B_1|E_2) = \frac{8}{20}$$

$$= 0.4$$

The revised probability that E_1 is the true set of design alternative to change given the evidence of one design change, $p(E_1|B_1)$ is:

$$p(E_1|B_1) = \frac{p(B_1|E_1)\ p(E_1)}{p(E_1)p(B_1|E_1) + p(E_2)p(B_1|E_2)}$$

$$= \frac{0.8 \times 0.5}{(0.5 \times 0.8) + (0.5 \times 0.4)}$$

$$= 0.667$$

The revised probability that E_2 is the true set of design alternative to change given the evidence of one design change, $p(E_2|B_1)$ is:

$$p(E_2|B_1) = \frac{p(B_1|E_2)\ p(E_2)}{p(E_2)p(B_1|E_2) + p(E_1)p(B_1|E_1)}$$

$$= \frac{0.4 \times 0.5}{(0.4 \times 0.5) + (0.8 \times 0.5)}$$

$$= 0.333$$

At stage 1, the equivocation of the set, E_1, is:

$$H(E_1|B_1) = -\{p(E_1|B_1)\log_2 p(E_1|B_1)\}$$
$$= -(0.667 \log_2 0.667)$$
$$= 0.3896 \text{ bits}$$

The equivocation of the set, E_2, is:

$$H(E_2|B_1) = -\{p(E_2|B_1)\log_2 p(E_2|B_1)\}$$
$$= -(0.333 \log_2 0.333)$$
$$= 0.5283 \text{ bits}$$

The average equivocation for stage 1, having information about one design change, is:

$$H_1(x) = -\sum_{i=1}^{2} p(E_i|B_1)\log_2 p(E_i|B_1)$$
$$= -(0.667 \log_2 0.667 + 0.333 \log_2 0.333)$$
$$= 0.91795 \text{ bits}$$

The information gain , $G_{01}(x)$, due to the new information is:

$$G_{01}(x) = H_0(x) - H_1(x)$$
$$= 1 - 0.91795$$
$$= 0.08205 \text{ bits}$$

The equivocation, 0.08205 bits, represents the amount of reduced uncertainty resulting from the new additional information at stage 1.

Suppose that we have the second new information, B_2, the second design change to confirm the likelihood of true set of design change.

Then, new prior probabilities are:

$$p_2(E_1) = 0.667$$

and $$p_2(E_2) = 0.333$$

The conditional probabilities, $p(B_2|E_1)$ and $p(B_2|E_2)$ are:

$$p(B_2|E_1) = \frac{15}{19}$$

$$= 0.7895$$

and $$p(B_2|E_2) = \frac{7}{19}$$

$$= 0.3684$$

Thus, the revised probability that E_1 is the true set of design alternative to change given the second evidence, B_2, is:

$$p(E_1|B_2) = \frac{p(B_2|E_1)p(E_1)}{p(B_2|E_1)p(E_1) + p(B_2|E_2)p(E_2)}$$

$$= \frac{0.7895 \times 0.667}{(0.7895 \times 0.667) + (0.3684 \times 0.333)}$$

$$= 0.812$$

and $$p(E_2|B_2) = 1 - 0.812 = 0.198$$

At the second stage, the equivocation of the set, E_1, is:

$$H(E_1|B_2) = - \{p(E_1|B_2) \log_2 p(E_1|B_2)\}$$

$$= - (0.812 \log_2 0.812)$$

$$= 0.24392 \text{ bits}$$

The equivocation of the set, E_2, is:

$$H(E_2|B_2) = - \{p(E_2|B_2) \log_2 p(E_2|B_2)\}$$

$$= -\ (0.198 \log_2 0.198)$$

$$= 0.46261 \text{ bits}$$

The average equivocation for stage 2 is:

$$H_2(x) = -\sum_{i=1}^{2} p(E_i|B_2) \log_2 p(E_i|B_2)$$

$$= -\ (0.812 \log_2 0.812 + 0.198 \log_2 0.198)$$

$$= 0.70653 \text{ bits}$$

Information gain, $G_{o2}(x)$, due to the second new information, B_2, is:

$$G_{o2}(x) = H_o(x) - H_2(x) = 1 - 0.70653$$

$$= 0.29347 \text{ bits}$$

The equivocation, 0.29347 bits, represents the decrease of equivocation due to two new additional information, B_1 and B_2.

When we have third new information, the new prior probabilities are:

$$p(E_1) = 0.812$$

and $$p(E_2) = 0.198$$

The conditional probabilities are:

$$p(B_3|E_1) = \frac{14}{18}$$

$$= 0.777$$

and $$p(B_3|E_2) = \frac{6}{18}$$

$$= 0.333$$

The revised probability that the set E_1 is the true set of design alternative given the third information, B_3, is:

$$p(E_1|B_3) = \frac{p(B_3|E_1)p(E_1)}{p(B_3|E_1)p(E_1) + p(B_3|E_2)p(E_2)}$$

$$= \frac{0.777 \times 0.812}{(0.777 \times 0.812) + (0.333 \times 0.198)}$$

$$= 0.905$$

and $\quad p(E_2|B_3) = 1 - 0.905$

$$0.095$$

The equivocation for the set E_1 at stage 3 is:

$$H(E_1|B_3) = - \{p(E_1|B_3) \log_2 p(E_1|B_3)\}$$

$$= - (0.905 \log_2 0.905)$$

$$= 0.13032 \text{ bits}$$

The equivocation for the set E_2 at stage 3 is:

$$H(E_2|B_3) = - \{p(E_2|B_3) \log_2 p(E_2|B_3)\}$$

$$= - (0.095 \log_2 0.095)$$

$$= 0.32261 \text{ bits}$$

The average equivocation for stage 3 is:

$$H_3(x) = - \sum_{i=1}^{2} p(E_i|B_3) \log_2 p(E_i|B_3)$$

$$= - (0.905 \log_2 0.905 + 0.095 \log_2 0.095)$$

$$= 0.45293 \text{ bits}$$

The information gain, $G_{o3}(x)$, due to the third information, B_3, is:

$$G_{o3}(x) = H_o(x) - H_3(x) = 1 - 0.45293$$

$$= 0.54707 \text{ bits}$$

We reduce the equivocation, 0.54707 bits, by collecting three information (evidences).

Table 2 shows the summary of conditional equivocations and the

the information gain.

Table 2. Conditional equivocation, information gain and conditional probabilities in Example 1.

Stage / Class	Stage 0	1	2	3	4
$p(E_1 \mid B_j)$	0.5	0.667	0.812	0.905	0.961
$p(E_2 \mid B_j)$	0.5	0.333	0.198	0.095	0.039
$H(E_1 \mid B_j)$	0.5	0.3896	0.24392	0.13032	0.05515
$H(E_2 \mid B_j)$	0.5	0.5283	0.46261	0.32261	0.18253
$\sum_{i=1}^{2} H(E_i \mid B_j)$	1	0.9179	0.70653	0.45293	0.23768
Information gain (bit)	0	0.08205	0.29347	0.54707	0.76232

When the fourth new information is available, the prior probabilities, $p(E_1)$ and $p(E_2)$, are:

$$p(E_1) = 0.905$$

and

$$p(E_2) = 0.095$$

The conditional probabilities, $p(B_4 \mid E_1)$ and $p(B_4 \mid E_2)$ are:

$$p(B_4 \mid E_1) = \frac{13}{17}$$

$$= 0.765$$

and

$$p(B_4 \mid E_2) = \frac{5}{17}$$

$$= 0.294$$

The revised probability that the set E_1 is the true set of design alternative, given the fourth information, B_4, is:

$$p(E_1|B_4) = \frac{p(B_4|E_1)\ p(E_1)}{p(B_4|E_1)\ p(E_1) + p(B_4|E_2)\ p(E_2)}$$

$$= \frac{0.765 \times 0.905}{(0.765 \times 0.905) + (0.294 \times 0.095)}$$

$$= 0.961$$

and $p(E_2|B_4) = 1 - 0.961$

$$= 0.039$$

The equivocation for the set E_1 at stage 4 is:

$$H(E_1|B_4) = -\ \{p(E_1|B_4)\ \log_2 p(E_1|B_4)\}$$

$$= -\ \ 0.961 \log_2 0.961)$$

$$= 0.05515 \text{ bits}$$

and $H(E_2|B_4) = -\ \{p(E_2|B_4)\ \log_2 p(E_2|B_4)\}$

$$= -\ (0.039 \log_2 0.039)$$

$$= 0.18253 \text{ bits}$$

The average equivocation for stage 4 is:

$$H_4(x) = -\sum_{i=1}^{2} p(E_i|B_4)\ \log_2 p(E_i|B_4)$$

$$= -\ (0.961 \log_2 0.961 + 0.039 \log_2 0.039)$$

$$= 0.23768 \text{ bits}$$

The information gain, $G_{o4}(x)$, at stage 4 is:

$$G_{o4}(x) = H_o(x) = 1 - 0.23768$$

$$= 0.76232 \text{ bits}$$

By collecting the fourth evidence, B_4, we can reduce the equivocation by 0.76232 bits.

It can be concluded that the design alternative, E_1, is the true design set to occur. There is still the uncertainty of 0.23768 bits (about 24%).

Figure 6 shows the conditional equivocation tree for Example 2.

In summary, the systematic prediction procedure may be used to identify the potential sets of design alternatives. The procedure is conducted as follows: first, expertise in project owner team may provide the various possible sets of design alternatives; second, the prior probability for each set of design alternative should be estimated; third, Bayes Theorem may be used to compute the revised probability estimates when new information becomes available; fourth, the conditional equivocation for each set of design alternative should be computed. The conditional equivocation is used as a criterion to identify the set of design alternative that contains the least amount of conditional equivocation among the various potential sets. The conditional equivocation tree can graphically show the decision process.

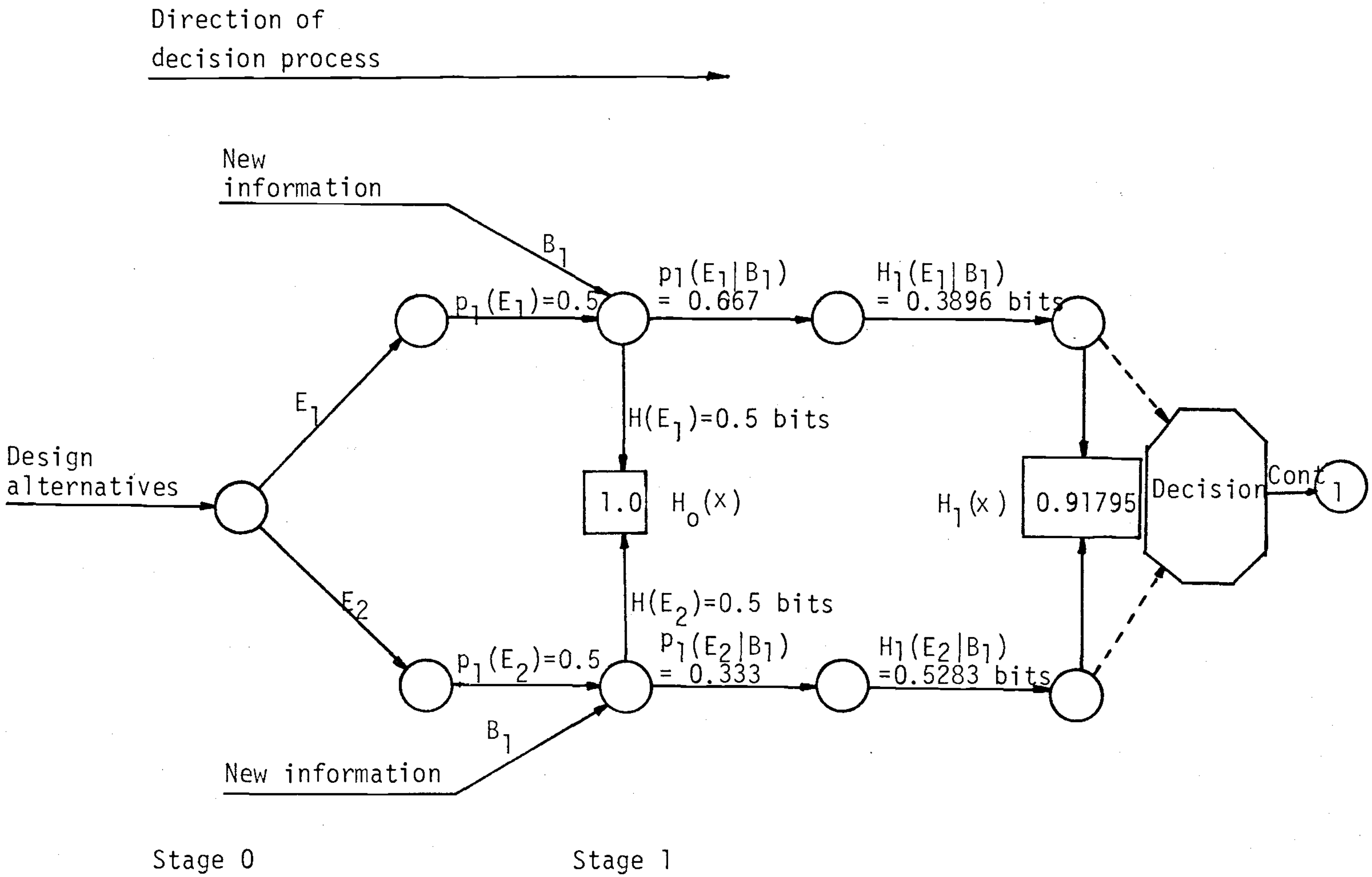
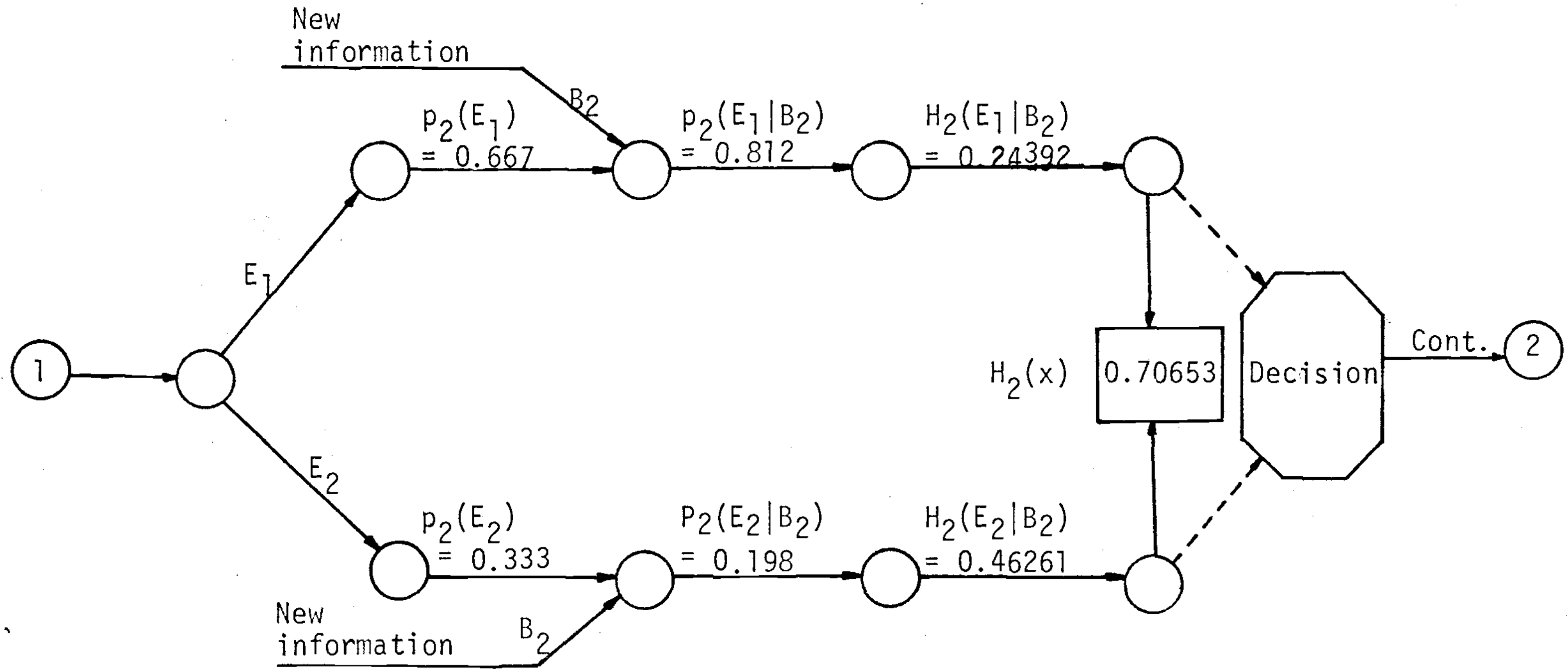
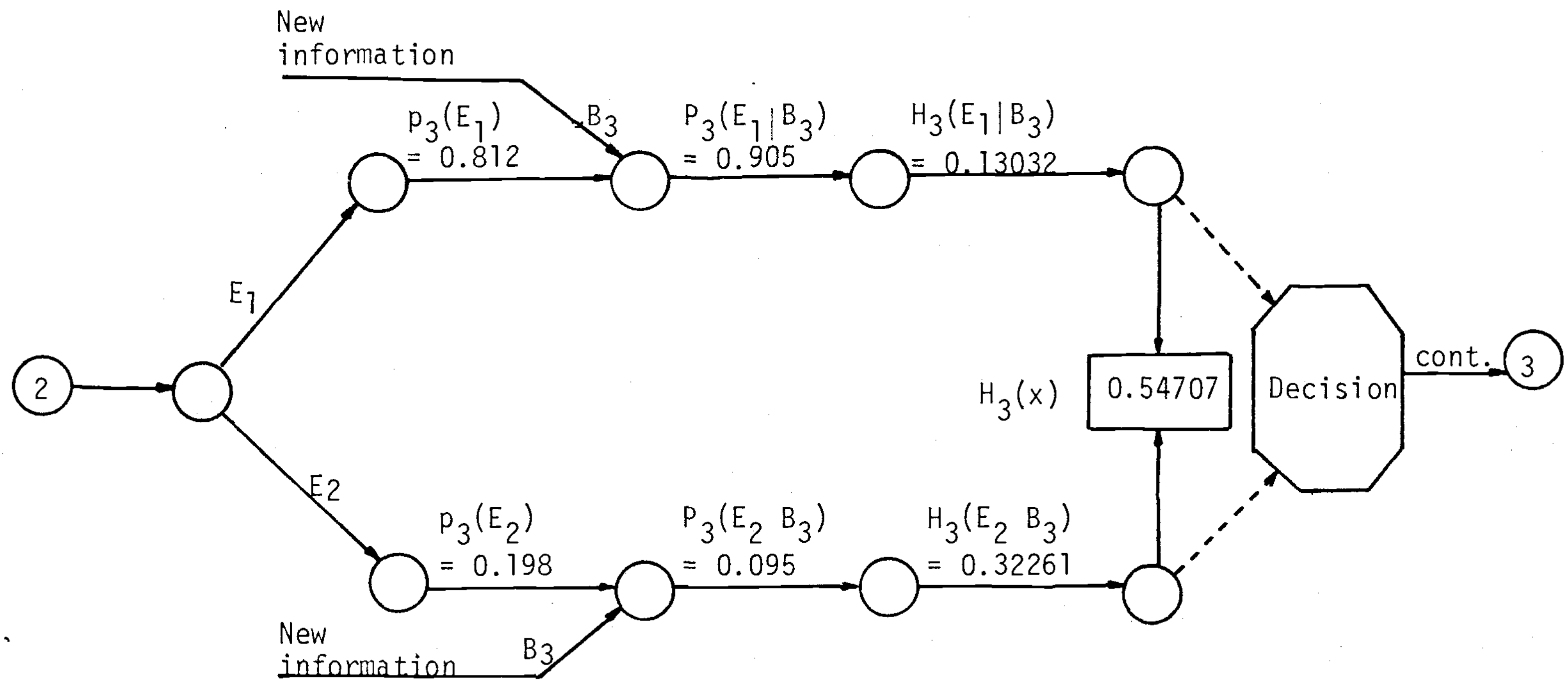


Figure 6. Conditional equivocation tree.



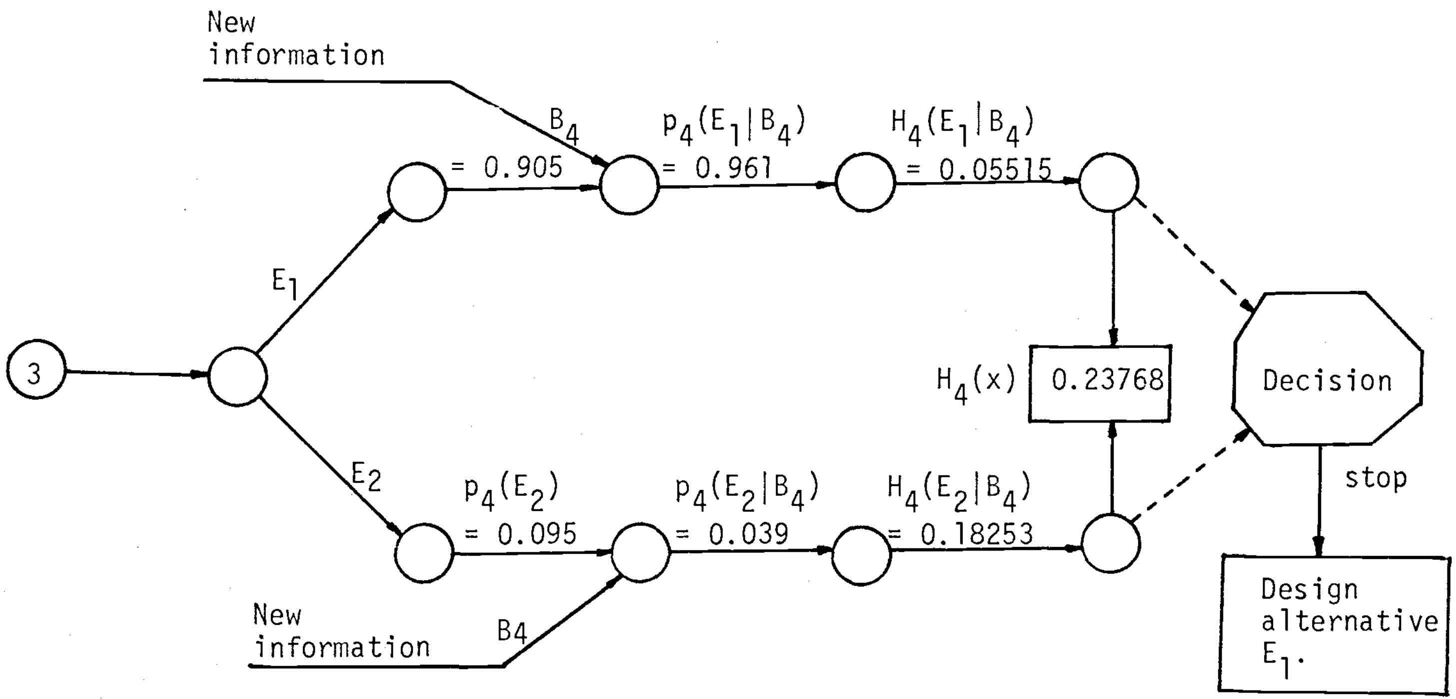
Stage 2

Figure 6. (continued) Conditional equivocation tree.



Stage 3

Figure 6. (continued) Conditional equivocation tree.



Stage 4

Figure 6. (continued) Conditional equivocation tree

V. SUPPORT OF THE HYPOTHESIS THAT MOST OF ENGINEERING DRAWING DELAYS FOLLOW AN EXPONENTIAL DISTRIBUTION

Derivation of An Exponential Distribution by Using Maximum Entropy Principle

Engineering working drawing is essential in a normal project execution. Many project managements have experienced difficulties resulting from unavailability of drawing at the right time in project operations. When engineering drawing must be revised due to design changes, delay often occurs. The delay is called engineering drawing delay. The amount of engineering drawing delay is 'uncertain' to management. In order to stay near the originally planned project completion date, management has to rearrange human resources and physical resources, and activities without any foresight about the delay. It would therefore be helpful to provide management with information about the delays. Thus, the hypothesis that most of engineering drawing delays follow an exponential distribution is advanced.

In this chapter we attempt to support the hypothesis. In order to show that an exponential distribution represents a probability distribution to reflect the maximum uncertainty of the engineering drawing delay, the maximum entropy principle is adopted. The principle of maximum entropy is able to estimate a "minimally prejudiced" probability distribution with maximum entropy. The derivation of an exponential distribution below is due to Good (1953). Actual industrial data are used (Inoue, 1977). A χ^2 'goodness-of-fit' test is conducted.

Let x be limited to positive values, and let the average value of x be a given constant, A.

The constraints are:

$$A = \int_0^\infty xPdx = G2(x) \qquad (5.1)$$

and including the normalizing condition,

$$1 = \int_0^\infty Pdx = G1(x) \qquad (5.2)$$

and the equation for entropy, H(x), is given as:

$$H(x) = - \int_0^\infty P \log Pdx$$

To maximize the entropy, H(x), subject to the constraints given in Equations 5.1 and 5.2, the Lagrange Multiplier Technique may be employed.

Now, taking derivatives:

$$\frac{\partial\{H(x)\}}{\partial P} = -(1 + \log P) \qquad (5.3)$$

$$\frac{\partial\{G1(x)\}}{\partial P} = 1 \qquad (5.4)$$

$$\frac{\partial\{G2(x)\}}{\partial P} = x \qquad (5.5)$$

Multiplying λ_1 to Equation 5.4 and λ_2 to Equation 5.5 and summing them as follows:

$$\frac{\partial\{H(x)\}}{\partial P} + \lambda_1\frac{\partial\{G1(x)\}}{\partial P} + \lambda_2\frac{\partial\{G2(x)\}}{\partial P} = 0 \qquad (5.6)$$

Substituting the values in Equations 5.3 and 5.5 into Equation 5.6, we have:

$$-(1 + \log P) + \lambda_1 + \lambda_2 x = 0 \qquad (5.7)$$

or

$$P = e^{+\lambda_2 x + \lambda_1 - 1}$$

$$= e^{\lambda_1 - 1} * e^{\lambda_2 x} \qquad (5.8)$$

To eliminate λ_1 and λ_2, we first substitute Equation 5.8 in Equation 5.2. Thus,

$$1 = \int_0^\infty e^{\lambda_1 - 1} * e^{\lambda_2 x}\, dx$$

$$= \frac{e^{\lambda_1 - 1} * e^{\lambda_2 x}}{\lambda_2}\Big|_0^\infty$$

$$= -\frac{e^{\lambda_1 - 1}}{\lambda_2} \qquad (5.9)$$

From Equation 5.9:

$$e^{\lambda_1 - 1} = -\lambda_2 \qquad (5.10)$$

Substitute Equation 5.8 and 5.10 into Equation 5.1. Then, we have:

$$A = \int_0^\infty -x\lambda_2 e^{\lambda_2 x} dx$$

$$= -\frac{e^{\lambda_2 x}}{\lambda_2}(\lambda_2 x - 1)\Big|_0^\infty$$

$$= -\frac{1}{\lambda_2}$$

Consequently,

$$\lambda_2 = -\frac{1}{A}$$

and $$e^{\lambda_1 - 1} = -\lambda_2 = \frac{1}{A}$$

Thus, $$p = \frac{1}{A} e^{-x/A}, \quad \text{for } x \geqq 0$$

It is shown that the exponential distribution with the average, A, represents a probability distribution, P, which has maximum entropy, H(x). Using the relationship $\log_2 b = \log_2 e \log_e b = 1.4427 \log_e b = k \log_e b$, the entropy, H(x) is:

$$H(x) = -\int_0^\infty p \log p dx$$

$$= -k \{ \int_0^\infty p(-\log A - \frac{x}{A}) dx \}$$

$$= k \{ \log A \int_0^\infty p dx + \frac{1}{A} \int_0^\infty x\ p dx \}$$

$$= k \{ \log A + 1 \}$$

$$= k \{ \log_e (Ae) \}$$

In summary, the result shows that the 'minimally prejudiced' distribution with the maximum entropy, H(x), is the exponential distribution with a mean of A. We may not of course always know the mean value of A. But we may at least know that such a mean value exists. This information, therefore, may be put into the formation of the equations to determine the <u>form</u> of the probability distribution, although it will not determine the numerical value of the constant appearing in the probability distribution. Thus, the condition of given value of A is not generally a strong restriction to derive the exponential distribution with the average of A for engineering drawing delays using the maximum entropy principle.

Tests for Actual Industrial Data

Test 1.

Actual industrial data for engineering drawing delays are used. A χ^2 'goodness-of-fit' test is conducted. The mean of 181.1 is known for the drawing delay data. The data represent the 'plan-issue' drawing delays. The 'plan-issue' delay is the time between the planned date to issue drawings and the actual issuing time.

Let A be the mean.

$$A = 181.1 \text{ days}$$

$$P(x) = \frac{1}{A} e^{-x/A} , \quad x \geq 0$$

Thus,

$$P(x) = \frac{1}{181.1} e^{-x/181.1}$$

$$= 0.0055 \, e^{-0.0055x} , \quad x \geq 0$$

$$F(x) = 1 - e^{-0.0055x} , \quad x \geq 0$$

Total observations of the engineering drawing delays are 177. Ten data points do not appear in the original data source. All the computations are given in Table 3 and Table 4.

Table 3. χ^2 'goodness-of-fit' test computation for Test 1.

Interval ($a_i < x \leq b_i$)	observed frequency (O_i)	exponential pdf with 1/A = 0.0055		$P(a_i<x \leq b_i)$
		$P(x < a_i)$	$P(x \leq b_i)$	
0 - 40	30	0.000	0.198	0.198
40 - 80	20	0.198	0.356	0.158
80 - 120	16	0.356	0.483	0.127
120 - 160	18	0.483	0.585	0.102
160 - 200	16	0.585	0.667	0.082
200 - 240	11	0.667	0.733	0.065
240 - 280	12	0.733	0.786	0.053
280 - 320	11	0.786	0.828	0.042
320 -	11	0.828	0.862	0.035
360 - 400	5	0.862	0.889	0.027
400 - 440	5	0.889	0.911	0.022
440 - 480	5	0.911	0.929	0.018
480 - 520	2	0.929	0.943	0.014
520 - 560	3	0.943	0.955	0.012
560 - 600	2	0.955	0.963	0.008

Data source: Inoue (1977b).

Table 4. χ^2 'goodness-of-fit' test computation for Test 1 (cont.).

Interval $(a_i < x \leq b_i)$	Expected frequency (E_i)		$\frac{(O_i - E_i)^2}{E_i}$
0 - 40	34.95		0.7011
40 - 80	28.05		2.3102
80 - 120	22.51		1.8827
120 - 160	18.06		0.0002
160 - 200	14.50		0.1552
200 - 240	11.63		0.0341
240 - 280	9.34		0.7575
280 - 320	7.49		1.6448
320 - 360	6.01		4.1431
360 - 400	4.83		
400 - 440	3.87	11.81	0.8616
440 - 480	3.11		
480 - 520	2.49		
520 - 560	2.16	6.10	0.1328
560 - 600	1.45		

$$\sum_{i=1}^{15} \frac{(O_i - E_i)^2}{E_i} = 12.6233$$

χ^2 statistic is given as:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

where O_i designates observed values and E_i designates estimated values. k indicates the number of categories in Table 4.

$$\chi^2 = \frac{(30 - 34.95)^2}{34.95} + \cdots + \frac{(7 - 6.1)^2}{6.1}$$

$$= 0.7011 + \cdots + 0.1328$$

$$= 12.6233$$

Since the critical value of $\chi^2(0.05)$ for degree of freedom, 9(11 - 1 - 1 = 9) is 16.9190, the calculated χ^2 value, 12.6233 falls in the non-critical region. Thus, we do not have enough evidence to say that the actual industrial data differ significantly from the theoretical values at the 0.05 significance level.

The entropy of the exponential distribution, H(x), with the mean of 181.1 is:

$$H(x) = 1.4427 \log_e(Ae)$$

$$= 8.943 \text{ bits}$$

Test 2.

Actual industrial data for 'issue-production go-ahead' drawing delays are used. The 'issue-production go-ahead' delay means the delaying time between the actual issuing time of engineering drawing and the time to starting production. A χ^2 'goodness-of-fit' test is conducted. All computations for χ^2 'goodness-of-fit' test are given in Table 5 and Table 6. The mean of 141.5 is known.
Let A be the mean of the data.

$$A = 141.5 \text{ days}$$

$$P(x) = \frac{1}{A} e^{-x/A}, \qquad x \geq 0$$

Thus,

$$P(x) = 0.0071\ e^{-0.0071x}, \qquad x \geq 0$$

$$F(x) = 1 - e^{-0.0071x}, \qquad x \geq 0$$

The total observations of the engineering drawing delays are 177.
In this Test 2, the five data points did not appear.

Table 5. χ^2 'goodness-of-fit' test computation for Test 2.

Interval ($a_i < x \leqq b_i$)	observed frequency (0_i)	exponential pdf with 1/A = 0.0071		$P(a_i < x \leqq b_i)$
		($P < a_i$)	($P \leqq b_i$)	
0 - 30	20	0.002	0.192	0.192
30 - 60	25	0.192	0.347	0.155
60 - 90	26	0.347	0.472	0.125
90 - 120	22	0.472	0.573	0.101
120 - 150	17	0.573	0.654	0.081
150 - 180	17	0.654	0.722	0.068
180 - 210	12	0.722	0.775	0.053
210 - 240	7	0.775	0.817	0.042
240 - 270	8	0.817	0.853	0.036
270 - 300	8	0.853	0.881	0.028
300 - 330	2	0.881	0.904	0.023
330 - 360	2	0.904	0.922	0.018
360 - 390	2	0.922	0.937	0.015
390 - 420	2	0.937	0.949	0.012
420 - 250	2	0.949	0.959	0.010

Data source: Inoue (1977b).

Table 6. χ^2 'goodness-of-fit' test computation for Test 2 (cont.)

Interval (a_i x b_i)	Expected frequency (E_i)	$\frac{(O_i - E_i)^2}{E_i}$
0 - 30	33.95	5.7320
30 - 60	27.44	0.2169
60 - 90	22.29	0.6175
90 - 120	17.92	0.9289
120 - 150	14.18	0.5608
150 - 180	12.11	1.9745
180 - 210	9.32	0.7706
210 - 240	7.55	0.0401
240 - 270	6.38	0.4113
270 - 300	5.00	1.8000
300 - 330	3.98	
330 - 360	3.23	
360 - 390	2.73	1.0321
390 - 420	2.11	
420 - 450	1.72	

$$\sum_{i=1}^{11} \frac{(O_i - E_i)^2}{E_i} = 14.0847$$

χ^2 calculated value can be computed as:

$$\chi^2 = \frac{(20 - 33.95)^2}{33.95} + \cdots + \frac{(10 - 13.77)^2}{13.77}$$

$$= (5.7320 + \cdots + 1.032)$$

$$= 14.0847$$

Since the critical value of χ^2 (0.05) for degree of freedom, 9(11 - 1 - 1 = 9) is 16.9190, the calculated χ^2 value, 14.0847 falls in the noncritical region. Thus, we do not have enough evidence to say that the actual industrial data do significantly differ from the estimated values at the 0.05 significance level.

The entropy of the exponential distribution, H(x), with the mean of 141.5 days, is:

$$H(x) = 1.4427 \log_e(Ae)$$

$$= 1.4427(\log_e 141.5 + 1)$$

$$= 8.587 \text{ bits}$$

Test 3.

Actual industrial data for 'one-time-review' engineering drawing delays are used. The 'one-time-review' delay means the time duration over the one-time-reviewing drawing (e.g. 14 days) stated in a contract. χ^2 'goodness-of-fit' test is conducted. The mean of 37 is known. All the computations for χ^2 'goodness-of-fit' test are given in Table 7 and Table 8.

Let's A be the mean of the data.

$$A = 37$$

$$P(x) = \frac{1}{A} e^{-x/A}, \qquad x \geq 0$$

Thus,

$$P(x) = 0.027e^{-0.027x}, \qquad x \geq 0$$

$$F(x) = 1 - e^{-0.027x}, \qquad x \geq 0$$

In the Test 3, the total observations for the drawing delays are 177. The 157 data points are available in the original source.

Table 7. χ^2 'goodness-of-fit' test computation for Test 3.

Interval ($a_i < x \leq b_i$)	observed frequency (O_i)	exponential pdf with 1/A = 0.027		$P(a_i < x \leq b_i)$
		$P(x < a_i)$	$P(x \leq b_i)$	
0 - 10	35	0.000	0.237	0.237
10 - 20	27	0.237	0.417	0.180
20 - 30	20	0.417	0.555	0.138
30 - 40	29	0.555	0.660	0.105
40 - 50	14	0.660	0.741	0.081
50 - 60	12	0.741	0.802	0.061
60 - 70	5	0.802	0.849	0.047
70 - 80	4	0.849	0.885	0.036
80 - 90	3	0.885	0.912	0.027
90 - 100	6	0.912	0.933	0.021
100 - 110	2	0.933	0.949	0.016
110 - 120	0	0.949	0.961	0.012

Data source: Inoue (1977b).

Table 8. χ^2 'goodness-of-fit' test computation for Test 3 (cont.).

Interval (a_i x b_i)	Expected frequency (E_i)		$\frac{(O_i - E_i)^2}{E_i}$
0 - 10	41.88		1.1302
10 - 20	31.97		0.7726
20 - 30	24.41		0.7967
30 - 40	18.63		5.7722
40 - 50	14.22		0.0034
50 - 60	10.86		0.1196
60 - 70	8.28		1.2993
70 - 80	6.32		
80 - 90	4.83		
90 - 100	3.69	19.86	1.1893
100 - 110	2.81		
110 - 120	2.21		

$$\sum_{i=1}^{7} \frac{(O_i - E_i)^2}{E_i} = 11.0833$$

χ^2 statistic can be computed as:

$$\chi^2 = \frac{(35 - 41.88)^2}{41.88} + \dots\dots\dots\dots + \frac{(15 - 19.86)^2}{19.86}$$

$$= (1.1302 + \dots\dots\dots\dots\dots\dots + 1.1893)$$

$$= 11.0833$$

Since the critical value of $\chi^2_{(0.05)}$ with degree of freedom, 7 (9-1-1) is 14.07, the claculated χ^2 value of 11.0833 falls in the noncritical region. Thus, the result of the χ^2 test shows that the actual industrial data do not significantly differ from the estimated values at the 0.05 significance level. Thus, we can conclude that the actual industrial data follow the exponential distribution with the mean of 37 days.

The entropy of the exponential distribution, H(x), with the mean of 37 days is as:

$$H(x) = 1.4427 \log_e (Ae)$$

$$= 1.4427 (\log_e 37 + 1)$$

$$= 6.652 \text{ bits}$$

VI. DERIVATION OF AN ENTROPY FORMULA FOR A TRIANGULAR DISTRIBUTION

The objective of this chapter is to provide a working vehicle for developing an information processing capacity estimation method for a project organization and to propose an entropy conversion method. The entropy conversion method can be used to compare projects, which have different measurment units for activity durations in terms of uncertainty of project completion date.

Probability Density Function of a Triangular Distribution

$$1 = \int_{-\infty}^{\infty} \text{height } dx$$

$$= \int_{a}^{m} \frac{k}{(m - a)}(x - a)dx + \int_{m}^{b} \{k - \frac{k}{(b-m)}(x - m)\} \, dx \qquad (6.1)$$

The value of k, $\frac{2}{(b-a)}$, can be easily computed. When k is replaced with $\frac{2}{(b-a)}$ into Equation 6.1, the density function of a triangular distribution, f(x), can be given as:

$$f(x) = \frac{2}{(b-a)(m-a)} (x-a) + \frac{2(b-x)}{(b-a)(b-m)} \qquad (6.2)$$

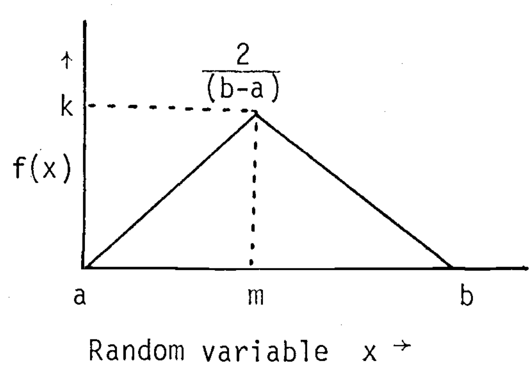


Figure 6-1. A typical triangular distribution function.

<u>Derivation of Entropy Formula for a Triangular Distribution</u>

When a distribution is discrete with probabilities, p_1, p_2, p_3,... p_n, its entropy can be defined as:

$$H(x) = \sum_{i=1}^{n} p_i \log(1/p_i)$$

$$= - \sum_{i=1}^{n} p_i \log p_i$$

For a continuous distribution with density function f(x), like a triangular distribution, the role of the probabilities is taken over by the density function evaluated at all possible values of the argument, and summation is replaced by integration. The entropy, H(x), of a continuous distribution is, therefore, defined as

$$H(x) = -\int_{-\infty}^{\infty} f(x) \log_e f(x)dx \qquad (6.3)$$

Let f(x) in Equation 6.2 be divided into two parts, $f_1(x)$ and $f_2(x)$:

$$f_1(x) = \frac{2}{(b-a)(m-a)} (x - a) \qquad (6.4)$$

$$f_2(x) = \frac{2(b-x)}{(b-a)(b-m)} \qquad (6.5)$$

Substitute Equation 6.4 and Equation 6.5 into Equation 6.3:

$$H(x) = - \int_a^m f_1(x)\log_e f_1(x)dx - \int_m^b f_2(x)\log_e f_2(x)dx$$

$$= - \int_a^m \frac{2(x-a)}{(b-a)(m-a)} \log_e \frac{2(x-a)}{(b-a)(m-a)} dx$$

$$- \int_m^b \frac{2(b-x)}{(b-a)(b-m} \log_e \frac{2(b-x)}{(b-a)(b-m)} dx \quad (6.6)$$

Let $$v = \frac{2(x-a)}{(b-a)(m-a)} \quad (6.7)$$

$$dv = \frac{2dx}{(b-a)(m-a)} \quad (6.8)$$

$$dx = \frac{(b-a)(m-a)}{2} dv \quad (6.9)$$

$$w = \frac{2(b-x)}{(b-a)(b-m)} \quad (6.10)$$

$$dw = \frac{-2dx}{(b-a)(b-m)} \quad (6.11)$$

$$dx = - \frac{(b-a)(b-m)}{2} dw \quad (6.12)$$

Substitute Equation 6.7 through 6.12 into Equation 6.6:

$$H(x) = -\frac{(b-a)(m-a)}{2} \int_a^m v \log_e v dv + \frac{(b-a)(b-m)}{2} \int_m^b w \log_e w dw$$

$$= -\frac{(b-a)(m-a)}{2} \left| \frac{v^2}{2} \log_e v - \frac{v^2}{4} \right|_a^m$$

$$+ \frac{(b-a)(b-m)}{2} \left| \frac{w^2}{2} \log_e w - \frac{w^2}{4} \right|_m^b \quad (6.13)$$

Substitute the values of v and w in Equation 6.7 and 6.10 into Equation 6.13:

$$H(x) = -\frac{(b-a)(m-a)}{2} \left| \frac{4(x-a)^2}{2(b-a)^2(m-a)^2} \log_e \frac{2(x-a)}{(b-a)(m-a)} \right.$$

$$\left. - \frac{1}{4} \frac{4(x-a)^2}{(b-a)^2(m-a)^2} \right|_a^m$$

$$+ \frac{(b-a)(b-m)}{2} \left| \frac{4(b-x)^2}{2(b-a)^2(b-m)^2} \log_e \frac{2(b-x)}{(b-a)(b-m)} - \frac{1}{4} \frac{4(b-x)^2}{(b-a)^2(b-m)^2} \right|_m^b$$

$$= - \frac{(b-a)(m-a)}{2} \left\{ \frac{2(m-a)^2}{(b-a)^2(m-a)^2} \log_e \frac{2(m-a)}{(b-a)\cdot(m-a)} - \frac{(m-a)^2}{(b-a)^2(m-a)^2} \right\}$$

$$= - \frac{(m-a)}{(b-a)} \left\{ \log_e \frac{2}{(b-a)} - \frac{1}{2} \right\} - \frac{(b-m)}{(b-a)} \left\{ \log_e \frac{2}{(b-a)} - \frac{1}{2} \right\}$$

$$= \frac{(-m + a - b + m)}{(b-a)} \left\{ \log_e \frac{2}{(b-a)} - \frac{1}{2} \right\}$$

$$= \frac{(a-b)}{(b-a)} \left\{ \log_e \frac{2}{(b-a)} - \frac{1}{2} \right\}$$

$$= \frac{1}{2} - \log_e \frac{2}{(b-a)}$$

$$= \frac{1}{2} - \log_e 2 + \log_e (b-a)$$

$$= \log_e (b-a) + 0.5 - 0.693$$

$$= \log_e (b-a) - 0.193 \text{ bits} \qquad (6.14)$$

Since $\log_a b = \log_a e \log_e b$

Then $\log_2 b = \log_2 e \log_e b$

$$= 1.4427 \log_e b$$

$$H(x) = 1.4427 \{ \log(b-a) - 0.193 \} \text{ bits} \qquad (6.15)$$

Rationale of Entropy Formula for a triangular Distribution

Uncertainty implies variability. The wider range of an activity could be representative of greater uncertainty of activity performance. Uncertainty can be expressed by a range and the distribution of possible values of a variable. Entropy can be used to measure uncertainty.

The derived entropy formula of a triangular distribution for activity is expressed as a logarithm form of a range (e.g. $b_i - a_i$) for i^{th} activity. The entropy formula derived, thus, is reasonably representative of the uncertainty of an activity distribution, which is a triangular distribution.

Entropy Conversion Method

This conversion method can be used to compute the amount of project entropy in terms of project completion date. For example, suppose there are two projects and project 1 has day measurement unit and project 2 has hour measurment unit. When project management wants to choose one of them, the management probably needs to select the project which contains the less amount of uncertainty in terms of project completion time. In such cases, the entropy conversion method will be needed. Examples will be taken for explaining the use of entropy conversion method for different measurement units in projects.

Suppose that we want to convert entropy for day measurement unit to one for hour unit. The following formula can be used.

$$H_{hr} = N x \log_2 8 + H_d$$

where H_{hr} designates entropy for hour measurement unit and H_d entropy for day measurement unit.

When the entropy for hour measurement unit, H_{hr}, is to convert to the entropy for day unit, H_d, the following conversion formula can be used.

$$H_d = H_{hr} - (N \times \log_2 8)$$

<u>Example 3.</u> Consider three activities that are involved to compute entropy in a project. The measurement unit for activities is one day. It is an attempt to compute the entropy for the three activities in terms of an hour measurement unit.

$$\begin{aligned}
H_d &= 1.4427\left[\log_e(6 - 2) - 0.193 + \log_e(8 - 3) - 0.193 \right.\\
&\qquad \left.\log_e(10 - 4) - 0.193\right]\\
&= (\log_2(6 - 2) - 0.278) + (\log_2(8 - 3) - 0.278) + (\log_2(10 - 4)\\
&\quad - 0.278)\\
&= \log_2 4 + \log_2 5 + \log_2 6 - 0.834\\
&= 2 + 2.32193 + 2.58496 - 0.834\\
&= 6.07289 \text{ bits}
\end{aligned}$$

where H_d designates the entropy for day measurement unit for activity.

$$\begin{aligned}
H_{hr} &= \log_2(8x4) + \log_2(8x5) + \log_2(8x6) - 0.834\\
&= \log_2 8 + \log_2 4 + \log_2 8 + \log_2 5 + \log_2 8 + \log_2 6 - 0.834\\
&= 3x\log_2 8 + 6.07289\\
&= (3x3) + 6.07289\\
&= 15.07289 \text{ bits}
\end{aligned}$$

In short, the entropy, H_{hr}, can also be computed.

$$H_{hr} = 3x\log_2 8 + H_d$$
$$= (3x3) + 6.07289$$
$$= 15.07289 \text{ bits}$$

And the entropy, H_d, can be computed:

$$H_d = H_{hr} - (3x\log_2 8)$$
$$= 15.07289 - 9$$
$$= 6.07289 \text{ bits}$$

Suppose that we want to convert entropy for week measurement unit to one for hour unit. The following conversion formula can be used.

$$H_{hr} = N \times \log_2 40 + H_{wk}$$
$$= N \times 5.32193 + H_{wk}$$

where H_{wk} designates the entropy for the week measurement unit and N indicates the number of activities involved to compute the entropy, H_{hr}. The 8-hour day and 5-day week are assumed.

$$H_{hr} = N \times \log_2 48 + H_{wk}$$
$$= N \times 5.58496 + H_{wk}$$

The 8-hour day and 6-day week are assumed.

$$H_{hr} = N \times \log_2 56 + H_{wk}$$
$$= N \times 5.80735 + H_{wk}$$

The 8-hour day and 7 day week are assumed.

When the entropy for hour measurement unit, H_{hr}, is conversely to convert to the entropy for week unit, H_{wk}, the following conversion formula can be used.

$H_{wk} = H_{hr} - (N x \log_2 40)$ for 5-day week (8-hour day)

$H_{wk} = H_{hr} - (N x \log_2 48)$ for 6-day week (8-hour day)

$H_{wk} = H_{hr} - (N x \log_2 56)$ for 7-day week (8-hour day)

Example 4

Consider the three activities that are used in Example 3, except week measurment unit. Suppose that the entropy, H_{hr}, is to computed. The entropy, H_{hr}, can be computed as follows:

For 40-hour week (8-hour day, 5-day week),

$$H_{hr} = 3 x \log_2 40 + H_{wk}$$

The entropy for week measurement unit, H_{wk}, is:

$$\begin{aligned} H_{wk} &= \log_2(6 - 2) - 0.278 + \log_2(8 - 3) - 0.278 + \\ &\quad \log_2(10 - 4) - 0.278 \\ &= \log_2 4 + \log_2 5 + \log_2 6 - 0.834 \\ &= 2 + 2.32193 + 2.58496 - 0.834 \\ &= 6.07289 \text{ bits} \end{aligned}$$

Thus, H_{hr} is:

$$\begin{aligned} H_{hr} &= 3 x \log_2 40 + 6.07289 \\ &= 3 x 5.32193 + 6.07289 \\ &= 22.03868 \text{ bits} \end{aligned}$$

For 48-hour a week (8-hour day 6-day week),

$$H_{hr} = 3 x \log_2 48 + H_{wk}$$

$= 3x5.58496 + 6.07289$

$= 22.82777$ bits

For 56-hour week (8-hour day, 7-day week),

$H_{hr} = 3x\log_2 56 + H_{wk}$

$= 3x5.80735 + 6.07289$

$= 23.49494$ bits

Suppose that we want to convert entropy for month measurement unit to one for hour unit. The following entropy conversion formula can be used.

For 20-day a month (160-hour a month),

$$H_{hr} = Nx\log_2 160 + H_m = 7.32193N + H_m \text{ bits}$$

where H_m designates the entropy for month measurement unit.

For 24-day a month (192-hour a month),

$$H_{hr} = Nx\log_2 192 + H_m = 7.58496N + H_m \text{ bits}$$

For 28-day a month (224-hour a month),

$$H_{hr} = Nx\log_2 224 + H_m = 7.80735N + H_m \text{ bits}$$

Conversely, the entropy, H_{hr}, is to convert to the netropy, H_m, the following formula can be used.

For 20-day month,

$$H_m = H_{hr} - 7.32193N \text{ bits}$$

For 24-day month,

$$H_m = H_{hr} - 7.58496N \text{ bits}$$

For 28-day month,

$$H_m = H_{hr} - 7.80735N \text{ bits}$$

Example 5

Consider the three activities that are used in Example 3 and 4, except month measurement unit for the activities. Suppose that the entropy, H_{hr}, is to compute. The entropy, H_{hr}, can be computed as follows:

For 20-day month (160 hours),

$$H_{hr} = 7.32193 \times 3 + H_m \text{ bits}$$

The entropy for month measurement unit, H_m, is:

$$\begin{aligned} H_m &= \log_2(6 - 2) - 0.278 + \log_2(8 - 3) - 0.278 \\ &\quad + \log_2(10 - 4) - 0.278 \\ &= \log_2 4 + \log_2 5 + \log_2 6 - 0.834 \\ &= 2 + 2.32193 + 2.58496 - 0.834 \\ &= 6.07289 \text{ bits} \end{aligned}$$

Thus, the entropy, H_m, is:

$$\begin{aligned} H_{hr} &= 7.32193 \times 3 + 6.07289 \\ &= 28.03868 \text{ bits} \end{aligned}$$

For 24-day month (192 hours),

$$\begin{aligned} H_{hr} &= 7.58496 \times 3 + H_m \\ &= 22.75488 + 6.07289 \\ &= 28.82777 \text{ bits} \end{aligned}$$

For 28-day month (224 hours),

$$H_{hr} = 7.80735x3 + H_m$$
$$= 23.42205 + 6.07289$$
$$= 29.50095 \text{ bits}$$

Suppose that entropy for week measurement unit is to convert to the entropy for day unit. The following formula can be used.

$$H_d = (N x \log_2 5) + H_{wk} = 2.32193N + H_{wk}$$

It can be applied to 5-day week period.

$$H_d = (N x \log_2 6) + H_{wk} = 2.58496N + H_{wk}$$

It can be applied to 6-day week period.

$$H_d = (N x \log_2 7) + H_{wk} = 2.80735N + H_{wk}$$

It can be applied to 7-day week period.

Inversely, when the day measurement unit is to convert to week measurement, The entropy conversion formula are as follows:

For 5-day week period, $H_{wk} = H_d - (N x \log_2 5)$
$= H_d - 2.32193N$

For 6-day week period, $H_{wk} = H_d - (N x \log_2 6)$
$= H_d - 2.58496N$

For 7-day week period, $H_{wk} = H_d - (N x \log_2 7)$
$= H_d - 2.8073N$

Example 6

Consider the three activities in a project as in Example 3, 4 and 5. The measurement unit for the activities is changed to week. Suppose that the entropy, H_d, is to compute. The entropy, H_d, can be computed as follows:

For 5-day week,

$$H_d = 2.32193N + H_{wk}$$

The entropy for week measurement, H_{wk}, is:

$$H_{wk} = \log_2(6 - 2) - 0.278 + \log_2(8 - 3) - 0.278$$
$$+ \log_2(10 - 4) - 0.278$$
$$= \log_2 4 + \log_2 5 + \log_2 6 - 0.834$$
$$= 2 + 2.32193 + 2.58496 - 0.834$$
$$= 6.07289 \text{ bits}$$

Thus, the entropy, H_d, is:

$$H_d = 2.32193x3 + 6.07289$$
$$= 6.96579 + 6.07289$$
$$= 13.03868 \text{ bits}$$

For 6-day week,

$$H_d = 2.58496x3 + 6.07289$$
$$= 7.75488 + 6.07289$$
$$= 13.82777 \text{ bits}$$

For 7-day week,

$$H_d = 2.80735x3 + 6.07289$$
$$= 8.42205 + 6.07289$$
$$= 14.49494 \text{ bits}$$

Figure 7 shows the relationships of entropy conversion for different activity measurement units in projects. Each circle designates the activity measurement unit and arrow designates the from-to relation of activity measurement unit. For instance, the tail of an arrow indicates 'From' and the head of the arrow indicates 'To'. As Figure 7 shows, the tail of an arrow indicates 'week' and the head of the arrow indicates 'day'. That is, the activity measurment unit of 'week' is changed to 'day' unit.

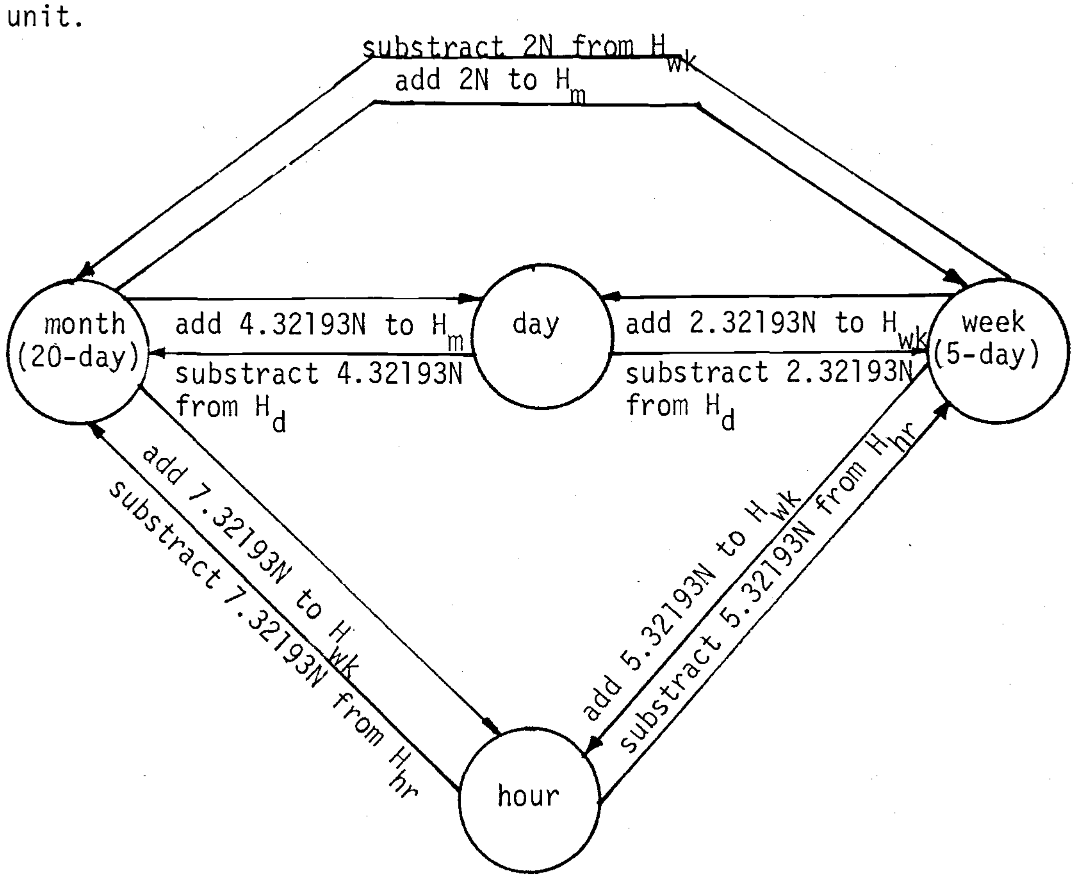


Figure 7. Entropy conversion diagarm

VII. INFORMATION PROCESSING CAPACITY ESTIMATION METHOD FOR MANAGING A PROJECT SUBJECTED TO DISRUPTIONS

Kunisawa (1959) suggests that the "transformation function" can be used to evaluate the capability of a system in an organization. He defined the transformation function as the transformation of a statistically distributed input into statistically distributed output. If the input and the output are statistically distributed, the input has statistical equivocation and the output must also have statistical equivocation. The transformation function (T.F.) was also expressed by Kunisawa (1959) in terms of equivocation as:

$$\text{T.F.} = \text{input equivocation} - \text{output equivocation}$$

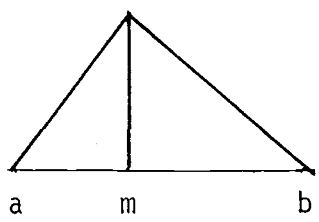
The transformation function may be applied to measure the capabilities of activities in a project and also of a project.

Capacity of Activity

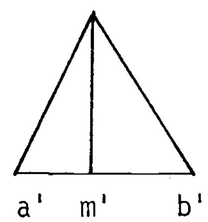
The capacity of an activity in a project can be defined as the capability of reducing equivocation by increasing the required amount of human resources for the activity. It can be expressed as the amount by which the equivocation has been decreased. The capacity of an activity can be measured by the entropy formula developed in Chapter 6.

Suppose that an activity has the distribution of original time estimates: a, minimum; m, most likely; b, maximum and the changed time estimates are, by increasing the amount of human resources for the activity, as follows:

The original time estimates:



The changed time estimates:



The original equivocation of activity (i,j), H, can be computed:

$$H(x) = 1.4427 \{ \log (b - a) - 0.193\} \quad \text{bits}$$

The changed equivocation of activity (i,j), H', can be computed:

$$H'(x) = 1.4427 \{\log (b' - a') - 0.193\} \text{ bits}$$

Thus, the capacity of the activity (i,j) is:

$$\text{Capacity of activity } (i,j) = H - H' \text{ bits}$$

<u>Capacity of a Project</u>

Suppose that a project consists of N activities. The maximum capacity of the project can be defined:

$$MC = \sum_{i=1}^{N}(H_i - H'_i) \quad \text{bits}$$

where MC designates the maximum capacity of the project.

H_i and H_i' designate the original equivocation and the changed equivocation by increasing the amount of human resorces, respectively.

Source of Equivocation

The source of equivocation is defined as a design modification activity that can generate the equivocation in a project operation.

The Statistical Characteristics

Since design changes occur randomly in time and place in a project execution, we may define the occurrence of design change as random phenomenon. A design modification activity may thus have a large variability. Thus, a triangular distribution is appropriate for the distribution of the design modification activity.

The Amount of Equivocation

Since the distribution of a design modification activity is a triangular distribution, the equivocation, H(x), can be computed:

$$H(x) = 1.4427 \quad \{\sum_{i=1}^{d} \log\ (b_i - a_i) - 0.193\}\ \text{bits}$$

where d indicates the number of design modification activities and a_i and b_i are minimum and maximum time durations in ith activity, respectively.

Example 7.

Consider a project that comprises N activities. Four design changes have occurred in the project simultaneously. The distributions of time estimates for the four design modification activities are given below:

Activity	time estimates		
	a	m	b
A	3	5	8
B	4	6	9
C	5	6	8
D	2	5	7

The amount of equivocation for design modification activities is computed:

$$H(x) = 1.4427\{\sum_{i=1}^{4} \log\,(b_i - a_i) - 0.193\}$$

$$= 1.4427\{\log(8 - 3) - 0.193 + \log(9 - 4) - 0.193$$

$$+ \log(8 - 5) - 0.193 + \log(7 - 2) - 0.193\}$$

$$= 1.4427\{1.42 + 1.42 + 0.91 + 1.42\}$$

Substantiation of Allowance Factor affecting the Compressibility of Activity Duration

The problem of reducing a project duration arises frequently in industrial projects when a project schedule is delayed. In the survey of literature, this author found that most time-cost trade off algorithms assumed that it was possible to crash the lineal time of an activity duration by increasing variable amount of resources. However, certain portions of an activity duration cannot be compressed simply by increasing resources. This uncompressible portion of the activity duration is the fixed portion; the duration of activity consists of (1) the compressible portion, and (2) the fixed portion. The fixed portion of activity duration is of interest to us in this section. In practical industrial situations, it is believed that activity durations with 30% fixed portion cannot be crashed to less than 65% of the original figure even if the human resources are doubled.

The purpose of this section is to see if the argument of "30% fixed portion of activity durations" in the project network can be supported through a literature survey. In order to develop the argument supporting industrial management's opinion, we may define an activity in a similar manner to the approach to set the standard time for a work element. An activity duration may be divided as shown in Figure 8.

The fixed portion consists of fatigue allowance, personal allowance, unavoidable delays, extra delay allowance (contingency) and machine controlled time. One industrial survey of 42 firms relative to what was orinarily included in their allowances revealed the information shown in Table 9 (Niebel, 1976).

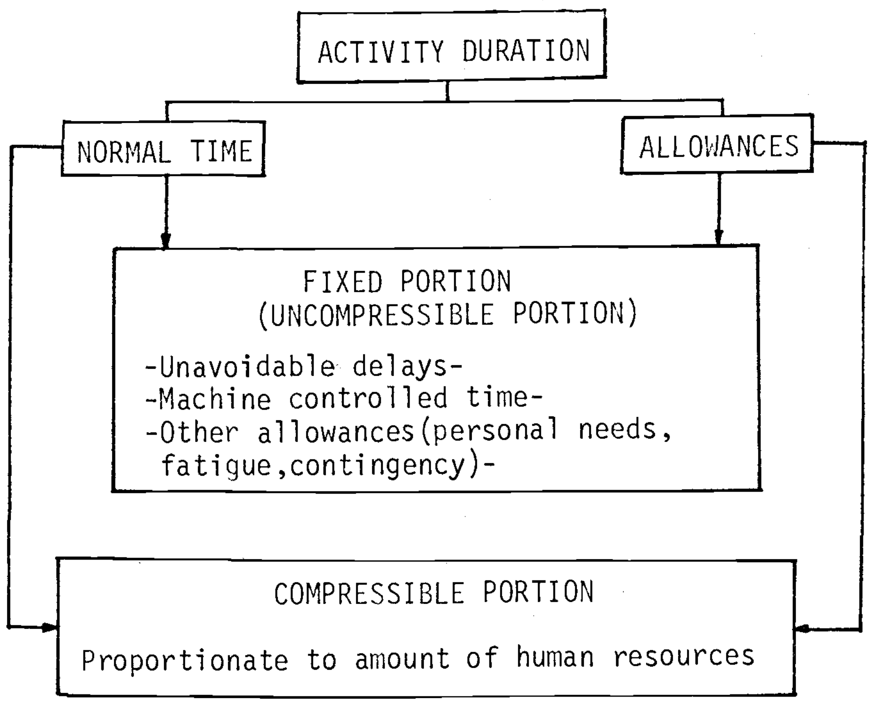


Figure 8. Relationship between fixed portion and compressible portion in activity duration.

Allowance Factors

We have surveyed literature to determine the commonly used range of major allowance factors. The survey findings are briefly presented in the following allowance factors in the order shown below:

Fatigue allowance

Pesonal allowance

Unavoidable allowance

Extra allowance

Table 9. Survey of 42 industrial firms depicting items included in their allowance factors (Cited in: Niebel, 1976).

ALLOWANCE FACTOR		NO. OF FIRMS	PERCENT
1.	Fatigue	39	93
	A. General	19	45
	B. Rest periods	13	31
	C. Did not specify A or B	7	17
2.	Time required to learn	3	7
3.	Unavoidable delay	35	83
	A. Man	1	2
	B. Machine	7	17
	C. Both man and machine	21	50
	D. Did not specify A,B,or C	6	14
4.	Personal needs	32	76
5.	Set or preparation operations	24	57
6.	Irregular or unusual operations	16	38

Fatigue Allowance

Karger (1966 gives a fatigue allowance range of 5 to 10%, although some manual operations may actually require a considerably larger allowance. Abruzzi (1956) agrees with Karger's fatigue allowance (10%). Whitmore (1970) gives a blanketed allowance factor for four categories of work as follows:

Light work ..... light assembly, office and clerical, ... 12%

Medium work ..... general assembly, electrical maintenance work, cleaning 15%

Heavy work ...... laboring, building work 20%

Very heavy work ..stocking, digging, heavy laboring 30%

Allowance for Personal Needs

Personal time is an allowance to cover the time required for such personal needs of the operator as going to the drinking fountain, the rest room, etc. The actual time required will vary with the workload, sex, age, temperature, humidity, etc. The amount of personal allowance is sometimes governed by negotiated union contracts. According to Holmes (1938) and Karger *et al*.(1966), personal allowance normally vary between 2% and 10% of the base or select time standard. Allowance for personal contingency may be increased up to 50%. Holmes (1938) mentions that work in the sun, or in an overheated room, requires a high allowance; 40% is not unusual.

Unavoidable Delay Allowance

This class of delays includes such items as (1) interruptions of worker's activities by the foreman, dispatcher, time study analyst, inspector, production personnel, and other professional staff; (2) material irregularities, difficulty in maintaining tolerances and specifications; and (3) interference delays where multiple machine assignments are made. Karger (1966) gives an example of 3 - 13% for unavoidable delay allowance. Qick et al.(1962) indicates that allowances for personal needs and unavoidable delays amount to as much as 15% of the workday.

Extra Allowance

It may be necessary to provide an extra allowance to arrive at a fair standard. A typical practice is to allow 30% extra allowance on the machine controlled portion of the cycle time (Niebel, 1976). Thus, an extra allowance must be provided to take care of the additional compound fatigue brought on by overtime or lengthening workhours.

In summary, the findings of literature survey can briefly be presented as follows:

Allowance factor	Range	Average
fatigue allowance	(5 - 10%)	7.5%
Personal allowance	(2 - 10%)	6.0%
unavoidable allowance	(3 - 13%)	8.0%
extra allowance		8.0%
Total		29.5% ≃ 30%

The average allowance for fatigue, personal needs and unavoidable delays is approximately 22%. An extra allowance must be provided to compensate for the effects of the accumulative fatigue brought on by strenuous situations. Such is often the case in delayed project schedules while trying to meet the original project completion date, work interference effects due to design changes, and engineering delays. A reasonable allowance for these effects would be 8%. Thus, the activity duration of the 30% fixed can be explained by the average allowance for the uncompressible factors of 22% and the extra allowance of 8%.

The Relationship between Fixed Portion of an Activity and its Compressibility by 100% Increase of Human Resource

Table 10 shows the relationship between fixed portion and the minimum duration of the activity when the available human resource is increased to 200% of the original requirement. When the fixed portion (100α) is 0%, the minimum activity duration is shortened to 50% of the original figure. When the fixed portion is 100%, the minimum activity duration is equal to the original activity duration. That is, the activity duration can not be compressed at all. When we have a 30% fixed activity, the most compressed activity duration (100% increase of human resources) can be calculated as follows:

$$a' = (\alpha \times a) + \frac{1}{2}(1-\alpha) \times a$$

$$m' = (\alpha \times m) + \frac{1}{2}(1-\alpha) \times m$$

$$b' = (\alpha \times b) + \frac{1}{2}(1-\alpha) \times b$$

If $\alpha = 0.30$ is subsituted into the above Equations, then we have:

$$a' = 0.65 \times a$$

$$m' = 0.65 \times m$$

$$b' = 0.65 \times b$$

where a' designates optimistic shortened activity duration, b' pessimistic shortened activity duration and m'most likely shortened activity duration; a optimistic original activity duration, b pessimistic activity duration and m most likely activity duration. α designates a compression factor.

Table 10. The relationship between fixed portion of an activity and its compressibility by 100% increase of human resources.

Fixed portion(100α)	Minimum activity duration
0%	50% (fully compressible)
10	55
20	60
30	65
40	70
50	75
60	80
70	85
80	90
90	95
100	100 (not commpressible at all)

Let us now discuss an information processing capacity estimation methodology to estimate the capacity of project organization and to predict the amount of project slippage.

Example 8

Suppose that we have a project which is comprised of nine activities. The project network is shown in Figure 9 .

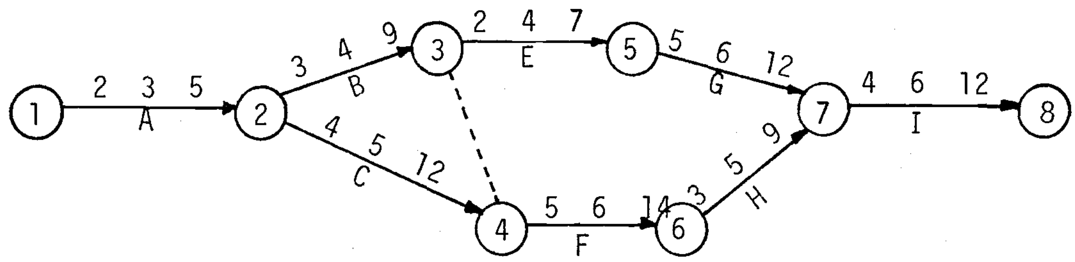


Figure 9. The project network

Table 11 shows activity durations and the requirements of human resources for each activity in the project in Example 8.

Table 11. Activity duration and the human resource requirements for the activities in the project in Example 8.

Activity					Human resource requirement
i - j	description	time estimate (week) a	m	b	
1 - 2	A	2	3	5	2
2 - 3	B	3	4	9	2
2 - 4	C	4	5	12	3
3 - 4	D	0	0	0	0
3 - 5	E	2	4	7	4
4 - 6	F	5	6	14	2
5 - 7	G	5	6	12	3
6 - 7	H	3	5	9	4
7 - 8	I	4	6	12	1

Suppose that we have just been informed by the project owner of a design change occuring on activity, B. The time estimates for the design modification activity (B*) are given as : minimum (a),6, most likely (m), 7, maximum (b), 11.

Then, $H(x) = 1.4427 \times \{\log(11-6) - 0.193\}$

$=1.4427 \times 1.416$

$= 2.04$ bits

Thus, the equivocation, H(x), due to a design change on activity,B, is

2.04 bits.

The Maximum Capacity of Project

Since any design modification activity (e.g. B* in Example 8) needs time to be performed, its successor activity, E, can not start until the design change is completed. All successors of the activity B, might thus be delayed. The project completion date, accordingly, must be prolonged. In such a case when management wants to complete the project by the originally planned completion date, the capacity of project organization should be evaluated if it has the capacity to cover the equivocation due to the design modification activity.

The capacity of a project organization was defined as the capability of reducing the equivocation by an increase of human resources and can be measured by the reduced amount of equivocation. The maximum capacity is defined as the maximum capability of reducing the equivocation by increasing human resources as much as the maximum availability of the human resources in a project organization.

In order to estimate the maximum capacity in a project, the activities that have design changes and the successors of the activities are considered to be involved. For example, in Example 8, activity B, and successors E, F, G, H and I are considered in estimating the maximum capacity with the maximum availability of human resources in the project. Then, the maximum capacity, MC, can be expressed as:

$$MC = \sum_{i=1}^{s} (H_{io} - H_{ic}) \text{ bits}, \quad \text{for } i = 1,2,\cdots, s$$

where H_{io} and H_{ic} designate the original equivocation and the changed equivocation by increasing 100% of the original requirements. The maximum capacity, MC, of the project in Example 8 can now be estimated:

$$MC = \sum_{i=1}^{6}(H_{io} - H_{ic})$$

$$= (2.31 + 2.04 + 2.89 + 2.53 + 2.31 + 2.72)$$

$$- (1.69 + 1.42 + 2.27 + 1.91 + 1.69 + 2.10)$$

$$= 14.8 - 11.08$$

$$= 3.72 \text{ bits}$$

Since the six activities B,E,F,G,H and I are considered in estimating the maximum capacity and each activity has the maximum capacity, 0.62 bits, the maximum capacity can also be computed as follows:

$$MC = 6 \times 0.62$$

$$= 3.72 \text{ bits}$$

Table 12 shows the original and the changed equivocations and the maximum capacity for each activity and the whole project in Example 8. The equivocations and the maximum capacity for each activity are computed using the entropy formula in Chapter 6. The maximum capacity for activity, A, is computed as:

$$MC = \{\text{Original Equivocation - Changed Equivocation}\}$$

$$= 1.31 \text{ bits} - 0.69 \text{ bits}$$

$$= 0.62 \text{ bits}$$

Table 12. The original and changed equivocations and the maximum capacity when doubling the availability of human resources as much as the original requirements.

Activity		Minimum activity duration(65% of the original figure)			Equivocation(in bits)		
					Original	Changed by Increase of Human resources	Maximum Capacity
i - j	desc	a'	m'	b'	H	H'	MC
1 - 2	A	1.30	1.95	3.25	1.31	0.69	0.62
2 - 3	B	1.95	2.60	5.85	2.31	1.69	0.62
2 - 4	C	2.60	3.25	7.80	2.72	2.10	0.62
3 - 4	D	0	0	0	0	0	0
3 - 5	E	1.30	2.60	4.55	2.04	1.42	0.62
4 - 6	F	3.25	3.90	9.10	2.89	2.27	0.62
5 - 7	G	3.25	3.90	7.80	2.53	1.91	0.62
6 - 7	H	1.95	3.25	5.85	2.31	1.69	0.62
7 - 8	I	2.60	3.90	7.80	2.72	2.10	0.62
Total					18.83 bits	13.87 bits	4.96 bits

Comparison of the Maximum Capacity and the Equivocation due to Design Modification Activity.

We can easily check if a project organization has enough capacity to cover the equivocation due to design modification activity.
When the equivocation is larger than the maximum capacity of the project organization the project slippage might be experienced. On the contrary, when the maximum capacity is larger than the equivocation due to design modification the project organization can handle the equivocation.
The project slippage can be predicted. The necessary amount of human resources can also be estimated in the latter case. In Example 8, since the maximum capacity of the project organization, 3.72 bits, is larger

than the equivocation due to the design modification activity, B*, we can predict that the equivocation can be handled by the project organization when doubling human resources on the related activities, B,E,F,G, H and I. In an attempt to estimate the necessary amount of human resources to resolve the contingent situation in the project, the estimation procedure qill be discussed in the next section.

<u>The Necessary Amount of Human Resources for Absorbing the Equivocation due to Design Modification Activity.</u>

Let the necessary amount of human resource by X%, then

$$X = \frac{\text{equivocation due to design modification}}{\text{Maximum capacity}} \times 100$$

$$= \frac{2.04}{3.72} \times 100$$

$$= 54.8\%$$

$$\doteq 55\%$$

Thus, we know that if the project organization can increase the original requirements of human resources by 55% on the six activities, B,E,F,G,H and I, the project may be completed by the due date with 50% chance of success.

Let the maximum capacity be Y per 10% increase of the original requirments of human resources. Then, Y can be expressed:

$$Y = \frac{\text{Maximum capacity}}{10} X = \frac{3.72}{10} X$$

$$= 0.372X \text{ bits}$$

Figure 10 shows graphically the estimated amount of human resources required for the equivocation, 2.04 bits.

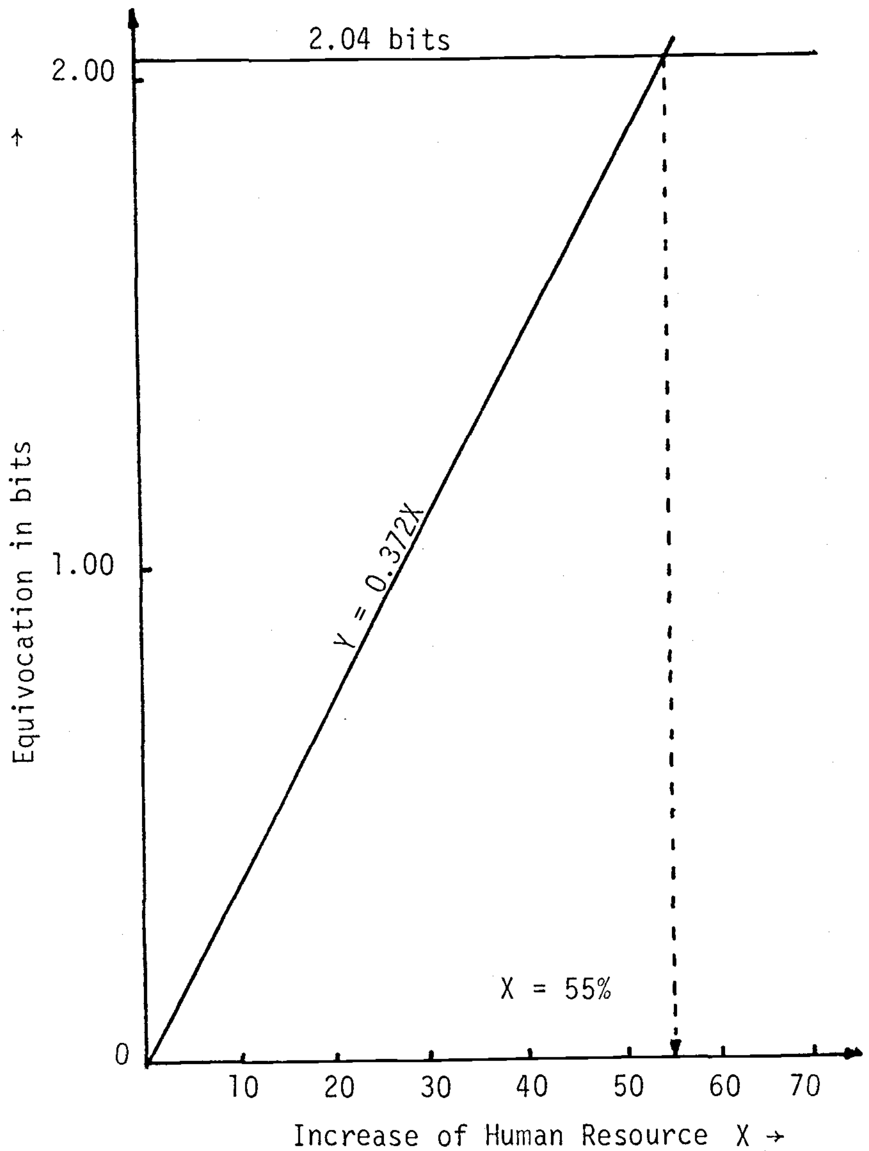


Figure 10. Estimation of Human resource requirement for the equivocation due to design modification activity.

Instead of increasing the original requirements of 55% for all six activities, we can also cover the equivocation due to design modification by imposing the human resource on the activities as they become available to the project manager. Since we know that the equivocation is 2.04 bits and the capacity of the 'first-coming' three activities, B, E, and F are 0.62, 0.62, and 0.62 bits respectively when the human resource is doubled.

Thus,

2.04 bits - (0.62 + 0.62 + 0.62) bits

= 2.04 bits - 1.86 bits

= 0.18 bits

The rest of the equivocation, 0.18 bits, can be covered by the next activity available, G, which has also the maximum capacity of 0.62 bits. So the rest of the equivocation, 0.18 bits, can be covered by increasing the human resource of 29.06% on activity, G (0.62 bits x 0.2906 =0.180 bits). In short, the equivocation, 2.04 bits, can be covered by increasing the human resource on the activities, B, E, F, and G as follows:

Activity	Percent of human resource	Equivocation
B	100%	0.62 bits
E	100%	0.62
F	100%	0.62
G	29.06%	0.18
		2.04 bits

Estimation of Disruption Effects, Project Slippage due to Design Changes

We have considered the case in which the maximum capacity of a project organization is large enough to handle the equivocation due to design modification.

Consider situations in which the equivocation is too large in comparison to its maximum capacity and may not be covered by the maximum capacity. In such a case it is inevitable to prolong the original project schedule. We might experience the effect of disruption, project slippage, in the project execution.

Suppose that three design changes are ascertained by the design expertise of project owner in the project network in Example 8. The time distributions of the three design modification activities, A*, B*, and C*, are given below:

Activity descr.	time estimates a	m	b
A*	5	6	10
B*	6	7	11
C*	4	7	12

Then, the equivocations due to the design modification activities can be computed:

$$H(x) = 1.4427 \times \sum_{i=1}^{3} \{\log(b_i - a_i) - 0.193\}$$

$$= 1.4427 \times \{ \log(10 - 5) - 0.193 + \log(11 - 6) - 0.193 + \log(12 - 4) - 0.193\}$$

$$= 1.4427 \times (1.416 + 1.416 + 1.886)$$
$$= 6.807 \text{ bits}$$

Since all the eight activities in the project are involved, the maximum capacity (MC) of the project is calculated as:

$$MC = 8 \times 0.62 \text{ bits}$$
$$= 4.960 \text{ bits}$$

Thus, we know that the maximum capacity of the project organization can not handle the equivocation, 6.807 bits, since the capacity, 4.960 bits, is smaller than the equivocation, 6.807 bits.
The difference, 1.847 bits, between the equivocation, 6.807 bits, and the maximum capacity, 4.960 bits, will appear as the effect of disruption, project slippage due to the design changes.

Conversion of the Equivocation, 1.847 bits to Man-Day

In an attempt to estimate project slippage in the project in Example 8, the average unit equivocation, AUE, is defined as:

$$AUE = \sum_{i=1}^{s} UE_i \,/\, s, \quad \text{for } i = 1,2,3,\ldots.s$$

where AUE designates the average unit equivocation and UE_i designates the unit equivocation of ith activity, which will be defined below and s is the number of activities involved.

The unit equivocation, UE_i, is defined as:

$$UE_i = H'_{ic} \,/\, M'_i$$

where H'_{ic} and m'_i designate the changed equivocation (by 100% increase of human resources) and the minimum most likely value ($m'_i = 0.65xm_i$) respectively.

The unit equivocations for activities in the project in Example 8 is shown below:

activity	H'(bits)	m'(week)	UE(bits)
A	0.69	1.95	0.35
B	1.69	2.60	0.65
C	2.10	3.25	0.65
D	0	0	0
E	1.42	2.60	0.55
F	2.27	3.90	0.58
G	1.91	3.90	0.49
H	1.69	3.25	0.52
I	2.10	3.90	0.54
Total			4.33 bits

Thus, the average unit equivocation, AUE, is computed as:

$$\text{AUE} = \frac{4.33 \text{ bits}}{9}$$

$$= 0.48 \text{ bits/week}$$

The project slippage, therefore, is calculated as:

$$\text{Project slippage} = \frac{\text{Equivocation - Maximum capacity}}{\text{AUE}}$$

$$= \frac{6.807 \text{ bits} - 4.96 \text{ bits}}{0.48 \text{ bits}}$$

$$= \frac{1.847 \text{ bits}}{0.48 \text{ bits/week}}$$

$$= 3.85 \text{ weeks}$$

$$\doteq 4 \text{ weeks}$$

Figure 11 shows the relationship between the maximum capacity of the project organization and the equivocation due to design modification, and also shows the amount of the equivocation, 1.847 bits which is equivalent to the project slippage due to design changes, 4 weeks.

The maximum capacity, Y, per 10% increase of the original human resource requirements can be expressed as:

$$Y = \frac{\text{Maximum capacity}}{10}$$
$$= \frac{4.96 \text{ bits}}{10} X$$

where X designates the percent increase of the original human resource requirement.

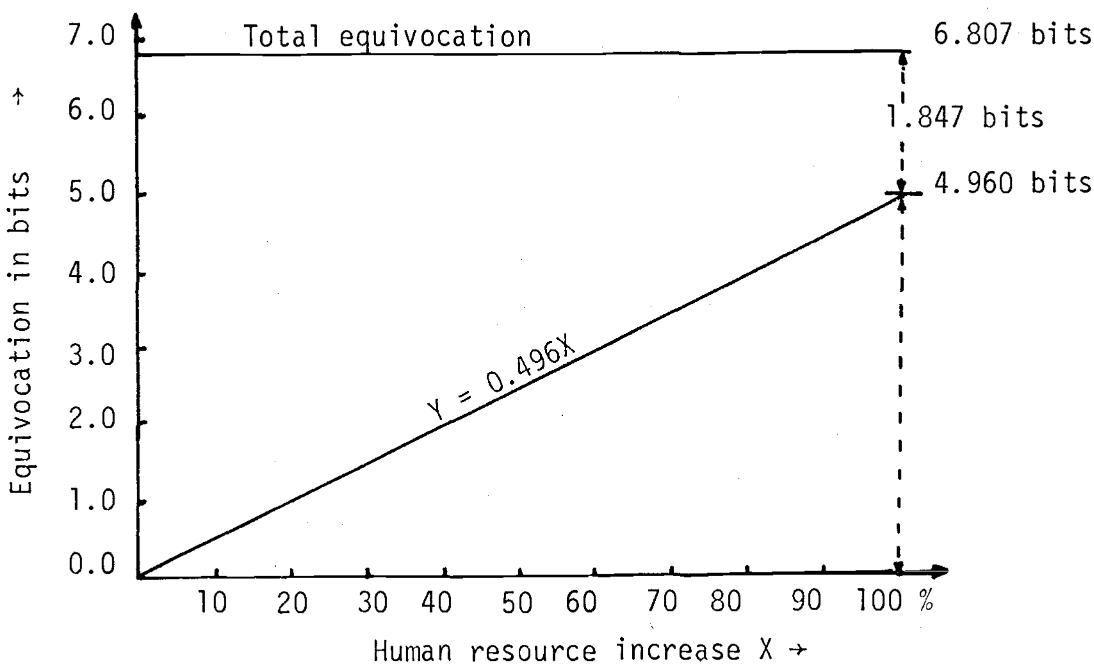


Figure 11. Relationship between the maximum capacity of the project organization, the equivocation due to design modification and the effects of disruption.

Comparison of the Results of Information Processing Capacity Estimation Method with Those of Computer Monte Carlo Simulation Program, CRASH

In an attempt to evaluate the predictive ability of the information processing capacity estimation method developed in this thesis, we compare the results produced by the method and those obtained by the computer Monte Carlo simulation program, CRASH which has been developed by Productive Resource Inc., (PRI), Corvallis, Oregon and used successfully in five super oil tankers construction projects. Consider a project which includes 17 activities. Its project network is shown in Figure 12. The maximum human resource limit is 100% of the original resource requirements. The fixed portion of activity duration, 30α, is used for all activities in the project. Suppose that design changes occurred on activity A, activity B, activity F and activity I and that the time estimates for design modification activities are given by the expertise in the project owner team in Table 13. The four design modification activities are designated by A*, B*, F* and I* respectively.

Table 13. Time estimates and equivocation for design modification activities.

Activity	Time estimates			Equivocation in bits
	a	m	b	
A*	3	7	8	2.04
B*	8	14	22	3.53
F*	7	8	15	2.72
I*	10	12	19	2.89
Total				11.18 bits

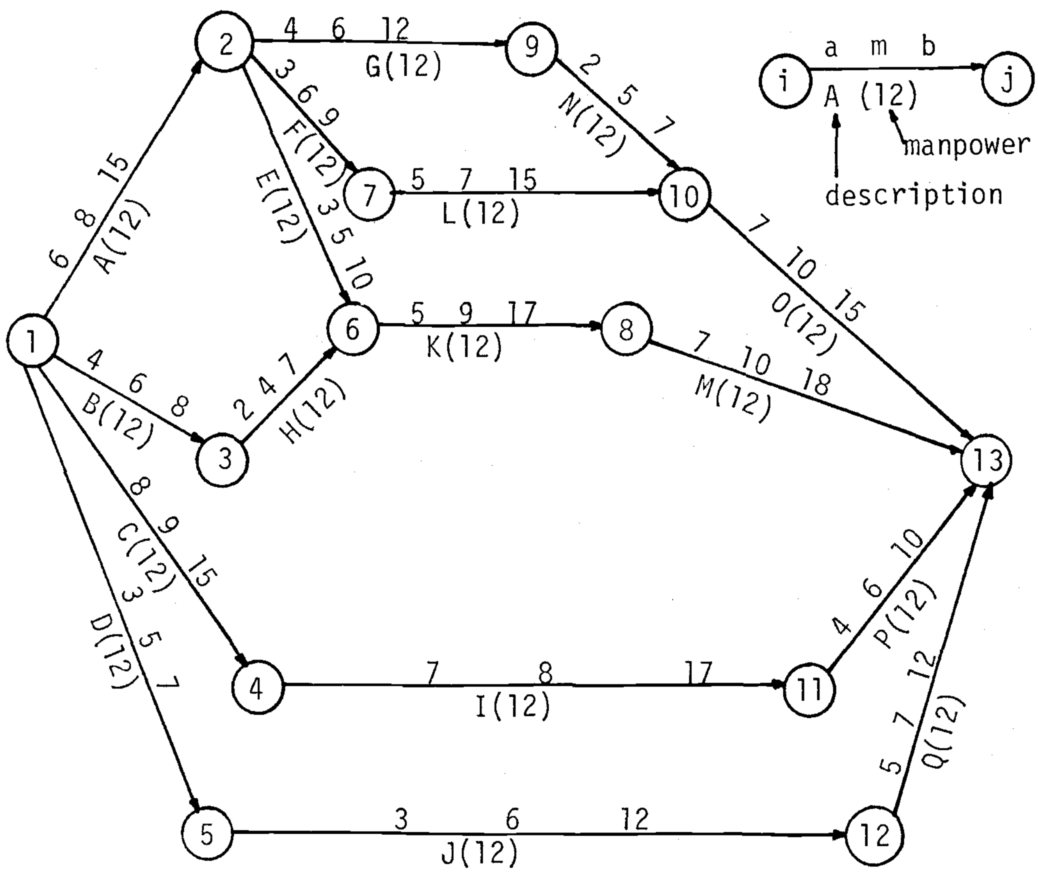


Figure 12. Project network.

Maximum Information Processing Capacity Estimation of Project

In an attempt to verify that the project organization is able to handle the contingent situation resulting from the four design changes, we first compute the amount of equivocation which is produced by the four design modification activities. This is shown in Table 13. The amount of equivocation due to the four design modification activities is 11.18 bits in Table 13. Then, the maximum information processing capacity of the project organization needs to be estimated. Since the four design changes occurred on the activities A, B, F and I, the successors of the four activities, including the activities, A, B, F and I themselves, are included to resolve the equivocation, 11.18 bits, in the project; they are 13 activities, A, B, E, F, G, H, I, K, L, M, N, O and P in the project network in Figure 12 and Figure 13. So the maximum information processing capacity, MC, can be estimated as:

$$MC = 0.62 \text{ bits} \times 13$$
$$8.06 \text{ bits}$$

The maximum capacity of each activity is 0.62 bits in Table 16 and the 13 activities as shown above are involved. Now, we can easily see that the project organization can not handle the equivocation due to design changes, since we know that the equivocation is larger than the information processing estimated capacity. In other words, although the project organization doubles the increase of human resource for the 13 activities, the maximum information processing capacity can not cover the amount of equivocation, 3.12 bits which is the difference between the equivocation due to the four design changes, 11.18 bits and the maximum capacity, 8.06 bits. The difference equivocation, 3.12 bits, will appear as the effect of the four design changes, project slippage of project completion date. Then, we need to show that the equivocation, 3.12 bits, is equivalent to how many man-days in project schedule. It will be discussed in the next section.

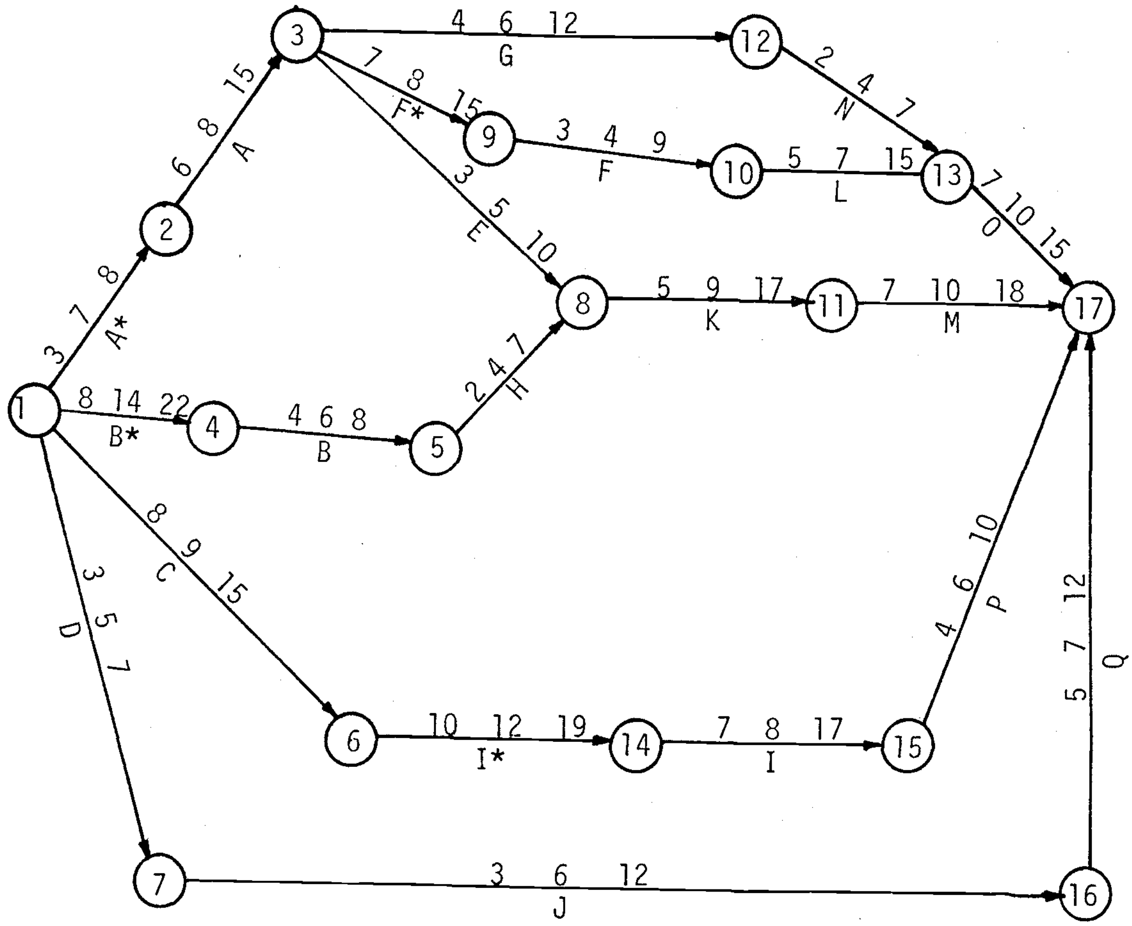


Figure 13. The Project network in Figure 12, including the four design modification activities, A*, B*, F* and I*.

Conversion of the Equivocation, 3.12 bits to Man-Day

As defined in the previous section, the average unit equivocation (AUE) can be computed as:

$$AUE = \sum_{i=1}^{13} UE_i / 13$$

$$= \frac{(0.44 + 0.28 + 0.59 + 0.43 + 0.54 + 0.44 + 0.47}{13}$$

$$\underline{+ 0.46 + 0.53 + 0.39 + 0.44 + 0.32 + 0.43)}$$

$$= \frac{5.76 \text{ bits}}{13 \text{ weeks}}$$

$$= 0.443 \text{ bits/week}$$

The unit equivocations for the associated 13 activities are shown in Table 17. The maximum information processing capacities for each activities and the project are placed in Table 16. The original equivocation in Table 14 and the changed equivocations when doubling the original requirments of human resources in Table 15 are used to compute the maximum capacity in Table 16.

Then, the project slippage can be computed as:

$$\text{Project slippage} = \frac{3.12 \text{ bits}}{0.443 \text{ bits/week}}$$

$$= 7.043 \text{ weeks}$$

$$= 35.2 \text{ man-days (for 5-day week)}$$

Table 14. The original equivocation for each activity in the project in Figure 12.

Activity i - j	descr.	time estimate a	m	b	Equivocation in bits
1 - 2	A	6	8	15	2.89
1 - 3	B	4	6	8	1.72
1 - 4	C	8	9	15	2.53
1 - 5	D	3	5	7	1.72
2 - 6	E	3	5	10	2.53
2 - 7	F	3	6	9	2.31
2 - 9	G	4	6	12	2.72
3 - 6	H	2	5	7	2.04
4 -11	I	7	8	17	3.04
5 -12	J	3	6	12	2.89
6 - 8	K	5	9	17	3.31
7 -10	L	5	7	15	3.04
8 -13	M	7	10	18	3.18
9 -10	N	2	5	7	2.04
10 -13	O	7	10	15	2.72
11 -13	P	4	6	10	2.31
12 -13	Q	5	7	12	2.53
Total					43.52 bits

Table 15. The changed equivocation and the unit equivocation for each activity in the project when doubling the original requirments of human resources in Figure 12.

Activity					Equivocation in bits, H'	Unit Equivocation (H'/m')
i - j	descr.	Time estimate a'	m'	b'		
1 - 2	A	3.9	5.2	9.75	2.27	0.44
1 - 3	B	2.6	3.9	5.2	1.10	0.28
1 - 4	C	5.2	5.85	9.75	1.91	0.33
1 - 5	D	1.95	3.25	4.55	1.10	0.34
2 - 6	E	1.95	3.25	6.5	1.91	0.59
2 - 7	F	1.95	3.9	5.85	1.69	0.43
2 - 9	G	2.6	3.9	7.8	2.10	0.54
3 - 6	H	1.3	3.25	4.55	1.42	0.44
4 -11	I	4.55	5.2	11.05	2.42	0.47
5 -12	J	1.95	3.9	7.8	2.27	0.58
6 - 8	K	3.25	5.85	2.69	2.69	0.46
7 -10	L	3.25	4.55	9.75	2.42	0.53
8 -13	M	4.55	6.5	11.70	2.56	0.39
9 -10	N	1.3	3.25	4.55	1.42	0.44
10 -13	O	4.55	6.5	9.75	2.10	0.32
11 -13	P	2.6	3.9	6.5	1.69	0.43
12 -13	Q	3.25	4.55	7.8	1.91	0.42
Total					32.98 bits	7.43 bits

Table 16. The Maximum capacity for each activity and the project in Figure 12 or 13.

i - j	Activity descr.	Original Equivocation (bits)	Changed Equivocation (bits)	Maximum Capacity (bits)
1 - 2	A	2.89	2.27	0.62
1 - 3	B	1.72	1.10	0.62
1 - 4	C	2.53	1.91	0.62
1 - 5	D	1.72	1.10	0.62
2 - 6	E	2.53	1.91	0.62
2 - 7	F	2.31	1.69	0.62
2 - 9	G	2.72	2.10	0.62
3 - 6	H	2.04	1.42	0.62
4 -11	I	3.04	2.42	0.62
5 -12	J	2.89	2.27	0.62
6 - 8	K	3.31	2.69	0.62
7 -10	L	3.04	2.42	0.62
8 -13	M	3.18	2.56	0.62
9 -10	N	2.04	1.42	0.62
10-13	O	2.72	2.10	0.62
11-13	P	2.31	1.69	0.62
12-13	Q	2.53	1.91	0.62
Total		43.52 bits	32.98 bits	10.54 bits

Table 17. The changed equivocation and the minimum most likely duration, the unit equivocation (bits) for the 13 associated activities.

Activity description	Changed Equivocation in bits, H'	Most likely duration in bits, m'	Unit Equivocation H'/m' in bits
A	2.27	5.2	0.44
B	1.10	3.9	0.28
E	1.91	3.25	0.59
F	1.69	3.9	0.43
G	2.10	3.9	0.54
H	1.42	3.25	0.44
I	2.42	5.2	0.47
K	2.69	5.85	0.46
L	2.42	4.55	0.53
M	2.56	6.5	0.39
N	1.42	3.25	0.44
O	2.10	6.5	0.32
P	1.69	3.9	0.43

Project Slippages Obtained by Computer Simulation Program

The computer Monte Carlo simulation program, CRASH, is used to evaluate the predictive ability of the information processing capacity eatimation method. The CRASH computer program was given the 30% fixed portion for activity minimum duration. Ten different runs are performed. The project slippages obtained by the CRASH computer program are shown in Table 18..

Table 18. Project slippages obtained by the computer Monte Carlo simulation progarm, CRASH.

Experiment	Project slippage(week)
Exp.1	7.2
Exp.2	7.6
Exp.3	7.4
Exp.4	7.8
Exp.5	7.6
Exp.6	7.8
Exp.7	7.4
Exp.8	6.8
Exp.9	7.8
Exp.10	7.0

χ^2 'goodness-of-fit' Test for Comparison of the Results by the Computer Simulation Program and the Information Processing Capacity Estimation Method

Table 19 shows the project slippages obtained by the computer Monte Carlo simulation program, CRASH and by the information processing capacity estimation method.

χ^2 'goodness-of-fit' test's statistic is given as:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where O_i designates the project slippages obtained by the computer simulation program and E_i the estimated project slippage by the method.

Table 19. The Project slippages obtained by the computer Monte Carlo simulation program and by the information processing capacity estimation method.

Experiment i	O_i	E_i	$(O_i - E_i)^2$
1	7.2	7.0	0.04
2	7.6	7.0	0.36
3	7.4	7.0	0.16
4	7.8	7.0	0.64
5	7.6	7.0	0.36
6	7.8	7.0	0.64
7	7.4	7.0	0.16
8	6.8	7.0	0.04
9	7.8	7.0	0.64
10	7.0	7.0	0.00
Total			3.04

χ^2 'goodness-of-fit' test's statistic is computed as:

$$\chi^2 = \frac{(7.2 - 7.0)^2}{7.0} + \frac{(7.6 - 7.0)^2}{7.0} + \ldots\ldots + \frac{(7.0 - 7.0)^2}{7.0}$$

$$= \frac{0.04 + 0.36 + \ldots\ldots\ldots\ldots\ldots\ldots + 0.00}{7.0}$$

$$= \frac{3.04}{7.0}$$

$$= 0.434$$

Result of χ^2'goodness-of-fit' Test

Since $\chi^2(0.005)$ critical value for nine degree of freedom is 1.73 and the calculated χ^2 value is 0.434 we conclude that the estimated project slippages obtained by the information processing capacity estimation method do not significantly differ from the values generated by the computer Monte Carlo simulation program at the 0.005 significance level.

The Estimated Project Slippages Stay within ± 1 Standard Deviation of the Project Slippages Obtained by the Computer Monte Carlo Simulation Program

The estimated project slippage, 7.0 weeks, is placed within the range of one standard deviation for the project slippages obtained by the computer Monte Carlo simulation program.
Table 22 shows that the estimated project slippages stay within ± one standard deviation for the project slippages generated by the computer simulation program. Figure 14 shows graphically the results of Table 22.

Table 20. The estimated project slippage stays within ±1 standard deviation (s.d.) for the project slippages obtained by the computer simulation program.

Experiment	Project slippage by computer simulation program	1 s.d.[a]	-1 s.d.	Estimated project slippage	+1 s.d.
Exp.1	7.2	1.028	6.172	7.0	8.228
Exp.2	7.6	1.045	6.555	7.0	8.645
Exp.3	7.4	1.134	6.266	7.0	8.534
Exp.4	7.8	1.335	6.465	7.0	9.135
Exp.5	7.6	1.206	6.394	7.0	8.806
Exp.6	7.8	1.315	6.485	7.0	9.115
Exp.7	7.4	1.092	6.308	7.0	8.492
Exp.8	6.8	1.068	5.732	7.0	7.868
Exp.9	7.8	1.273	6.527	7.0	9.073
Exp.10	7.0	1.270	5.730	7.0	8.270

a/ standard deviation from transient error analysis in computer outputs.

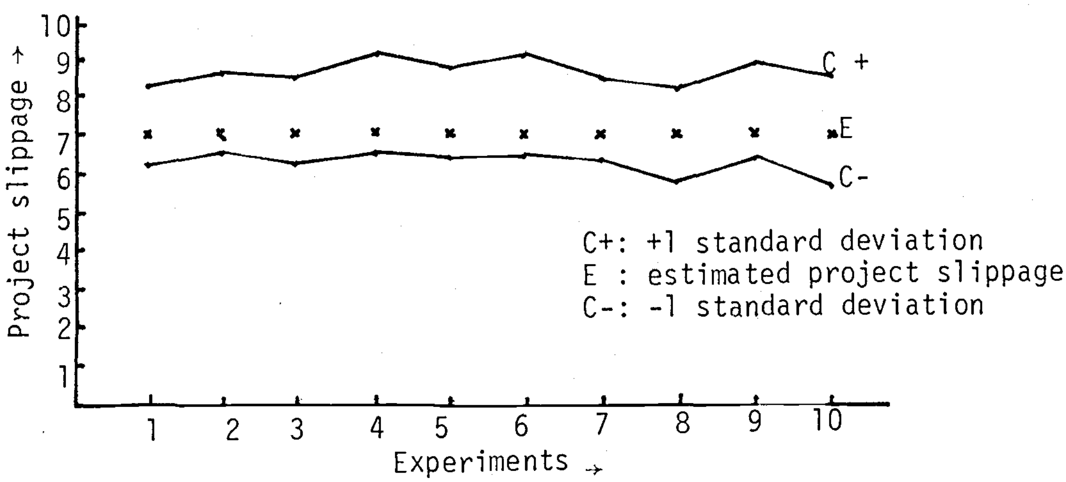


Figure 14. The graphical display for showing the estimated project slippage placed within ±1 standard deviation for the project slippages by the computer simulation program.

Summary

An information processing capacity estimation method has been developed. In an attempt to evaluate the predictive ability of the method, the numerical values from the method are compared with those from the computer Monte Carlo simulation program The result of χ^2 'goodness-of-fit' test shows that the project slippage from the information processing capacity estimation method does not significantly differ from that obtained by the computer simulation program at 99.5% significance level.

The information processing capacity estimation method can provide management with information about the necessary amount of human resources and the associated activities to cover the equivocation due to design changes. Furthermore, the method appears to be helpful in predicting the amount of project slippage, when the capacity of a project organization is not large enough to handle the equivocation due to design changes. Therefore, the method is a valuable management tool.

VIII. EFFECTS OF FATIGUE ON VARIOUS WORKING HOUR SCHEDULES

It is common practice in industry to resort to overtime in an attempt to catch on delayed project schedules. Management frequently asks the question of what schedule of working hours will bring the best result. What is presumably meant by this question is: "can we expect from overtime the same productivity that we can get regular work-time?"

As a rule, fatigue is a limiting factor in considerations of the effectiveness of varying work schedules. It is easy to understand that human being is not a machine and can not keep on working with the same efficiency hour after hour. His effectiveness will also vary with physical and mental demands which his job makes on him.

The purpose of this chapter is an attempt to collect information about how fatigue exists itself as a production limiting factor. Fatigue usually manifests itself by lowering efficiency, increasing absenteeism, increasing incidence of work injuries, and ultimately, by effecting the level of total work output.

Neither longer working hours nor overtime are as preferable as the normal 8-hour per day and 5-day per week practice (the U.S Department of Labor, 1947; Grandjean, 1968). Mcfarland (1971) also indicates that longer working hours and overtime are relatively ineffective in maintaining a worker's productivity at the regular rate.

Some necessary terminologies which were defined by the U.S. Department of Labor (1947) are given as follows:

$$\text{Efficiency} = \frac{\text{total weekley work output}}{\text{total hours actually worked}}$$

Efficiency is the average hourly work output per hour actually worked.

$$\text{Absenteeism} = \frac{\text{total employee-hours lost}}{\text{total employee-hours scheduled}}$$

The absenteeism rate is the percent of scheduled work time lost. The number of hours lost because of absenteeism is the difference between hours scheduled and hours actually worked.

Work output = scheduled hours x (100% - % of time lost through absenteeism) x efficiency.

In an attempt to show how to compare two schedules, suppose that efficiency and time lost through absenteeism, in relation to the base schedule (8-hour per day, 40-hour per week), are 95% and 8%, respectively and scheduled hours are 60 hours per week. The work output is :

$$\text{Work output} = 60 \times 0.92 \times 0.95 = 52.44$$

Suppose that during the 40-hour per week the efficiency index is 100 and absenteeism is 5%. Then, work output is :

$$\text{Work output} = 40 \times 1.00 \times 0.95 = 38$$

Comparing the two schedules, the increase in scheduled hours is $\frac{60 - 40}{40}$ or 50%. Similarly, the increase of 50% in scheduled hours results in work output increase of 38%.

Light operation: The manual handling of material up to about five pounds or the mechanical handling of somewhat heavier objects.

Heavy operation: the handling of heavy material such as operations in forge shops and foundries.

Moderately heavy operation: the operation between light and heavy operations.

Effects of Fatigue on Various Types of Workweek Schedules

5-Day Workweek Schedule

The effects of changes in daily hours from eight to nine and a half or ten hours and weekly hours from 40 to 49 or 50 on efficiency, absenteeism and work output were studied (the U.S.Department of Labor, 1947).

In the 5-day workweek schedule, the work for men operators is machine-paced and moderately heavy. The men operators are engaged in a variety of machining operations on metal working machines. As Figure 15 shows, a change from eight hours per day to ten results in a drop of 4% in hourly efficiency, even though the work is machine-paced. Scheduled worktime lost because of absenteeism rises by 7%. Women operators are engaged in light work. The work is operator-paced. The longer daily hours (nine and a half) result in decreases in efficiency, ranging from as low as 1.0% to as high as 16.2%. The changes in absenteeism are minor (average 0.56% decrease).

The increase in daily and weekly hours result in increased work output. However, the increment of work output is not in direct proportion to the additional hours worked. For the machine-paced operations, the actual machine time usually correspond very closely to the actual working time. So the increase in work output is more nearly proportional to the increase in hours. The work output-input ratio for the machine-paced operations is shown as 0.9 in Figure 15. The work output in operator-paced operations depends primarily on the speed and the endurance of operators. The work output-input ratio is shown as 0.7 in Figure 15. This is, every three additional hours of work result in an equivalent output increase of only two hours. Figure 15 further shows the effects of changes in daily and weekly hours on efficiency, absenteeism and work output during the 5-day workweek schedule.

5-Day Workweek Schedule vs. 6-Day Workweek Schedule

1. Increasing Workdays from 5 to 6: No Change in Daily Hours

Another way to increase weekly hours is by holding daily hours constant, and adding a sixth day to the workweek. Daily hours of eight and ten are considered. The work is moderately heavy and light. The operations are machine-paced and operator-controlled pace. As Figure 16 shows, the effects on efficiency and absenteeism of the lengthening in weekly hours, 40 to 48 and 50 to 58, are markedly large in both moderately heavy operations and light operations, even though the operations are operator-paced. The numerical values in Figure 15 are the averages. The source of the values is from the U.S.Department of Labor (1947). The work output during the longer workweek increases directly with the increase in weekly hours. The work output-input ratios are 0.96 and 0.93 for moderately heavy and light under operator-paced operations and 0.8 for the machine-paced, moderately operations. The results of these studies indicate that addition of a sixth day does increase total weekly output of work when daily hours are held to eight hour per day.

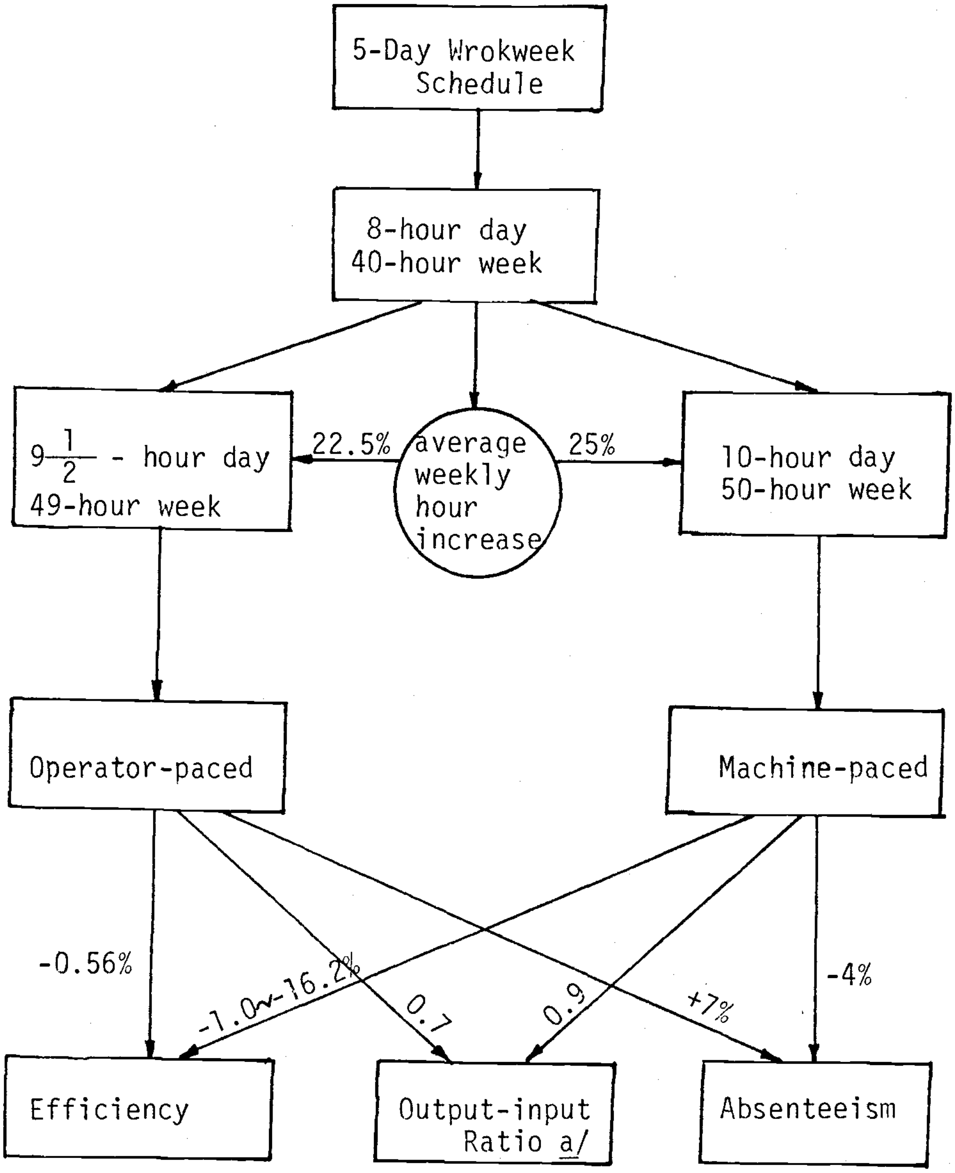


Figure 15. Effect of changes in daily and weekly hours on efficiency, absenteeism and work output-input ratio during the 5-day.
+:increase; -:decrease; a/(output/input).

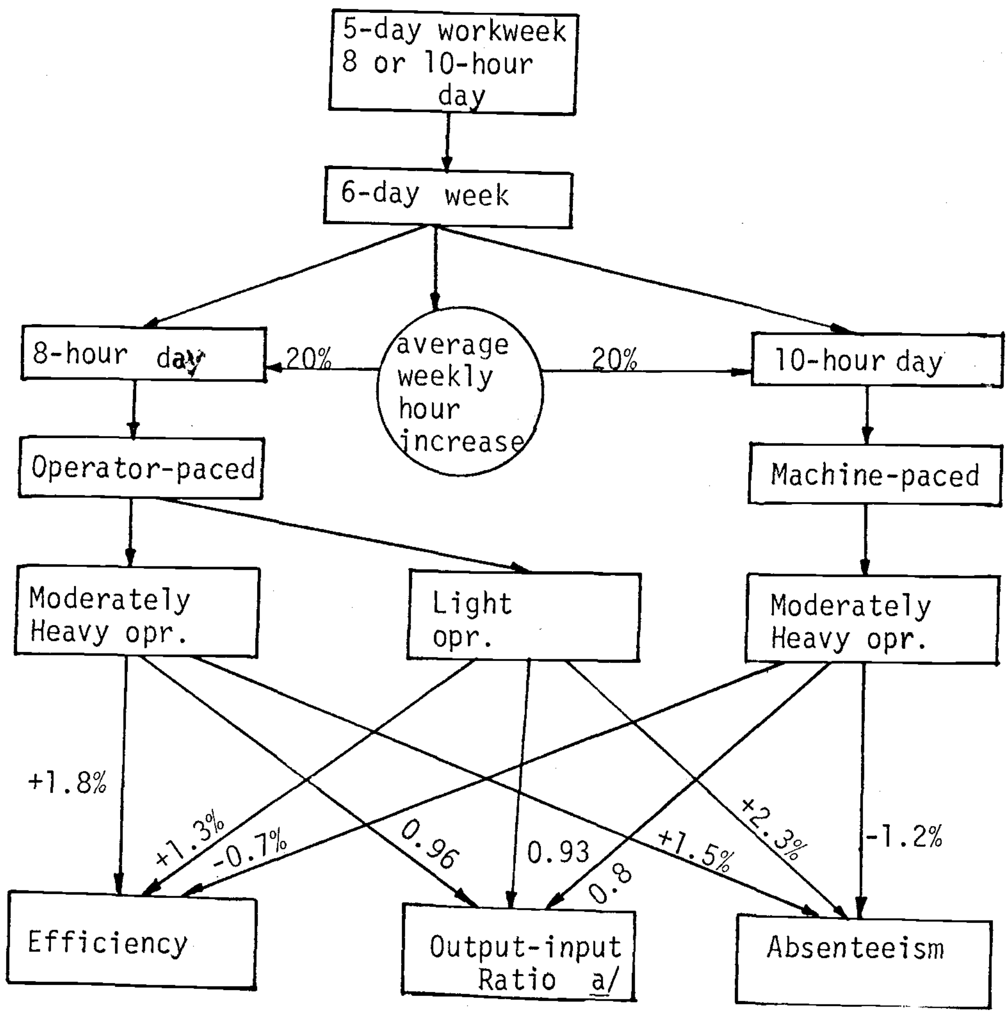


Figure 16. Effects of changes in weekly hours by increasing workdays from 5 to 6, holding daily hours constant. +: increased; -:decreased; a/ : output/hours.

2. Increasing Daily Hours and Changing Workday from 5 to 6

This alternative is to increase daily hours eight to nine or ten and weekly hours from 40 to 54 or 60. The work is moderately heavy and light. The operations are operator-paced and machine-paced.

As Figure 17 shows, that sharp increases in daily hours from eight to nine or ten result in drastic decreases in efficiencies in both operator-paced, moderately heavy and light operations and machine-paced, moderately heavy operations. Work output during the longer workweek decreases with the sharp increases in daily hours and weekly hours. As Figure 17 shows, the work output-input ratios for operator-paced, moderately heavy and light operations, and for machine-paced, moderately heavy operations are 0.5, 0.8 and 0.8 respectively. The results indicate that the sharp increase in daily hours and the addition of a sixth day has serious adverse effects on work output. The ratios of work output-input indicate that two additional hours are required to produce the work output of one hour for the moderately heavy operations (operator-paced) and three additional hours of work are required to produce the work output of two hours for the light operator-paced operations. Figure 17 shows the effects on efficiency and work output by increasing workdays from five to six, with changes in daily hours from eight to nine or ten hours. The numerical values in Figure 17 are the averages. The source of the figures is from the U.S. Department of Labor (1947).

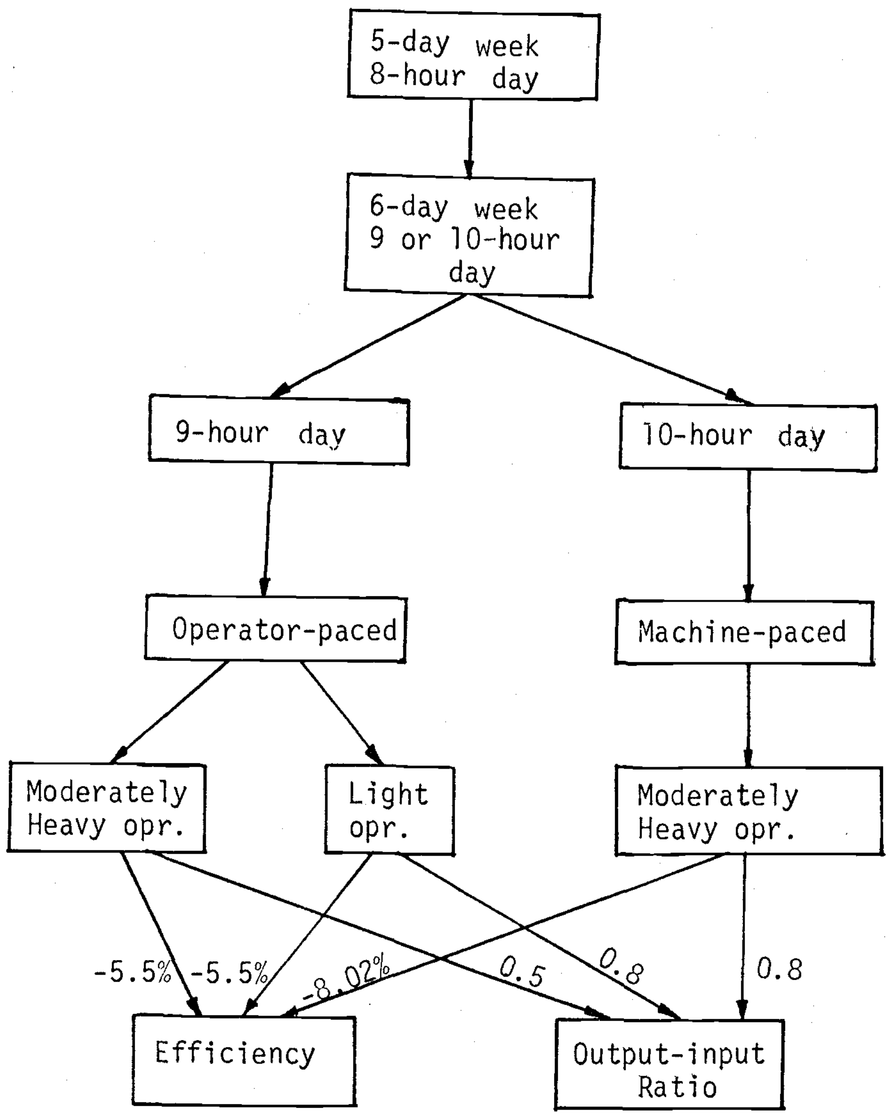


Figure 17. Effects on efficiency and work output by increasing weekly workdays. Minus signs (-) designate decrease of efficiency.

7-Day Workweek Schedule

It is generally recognized that the 7-day workweek is less productive and more wasteful. The effectiveness of Sunday work was studied by the U.S. Department of Labor (1947). The case study covered 30 men operators engaged in the production of drills and reamers. The work was light and almost entirely operator-paced. The original schedule was five days at eight hours, or 40-hour week. A 48-hour six-day week was added, followed by a 7-day week of 56 hours. It was continued for five months. The absenteeism became so bad by that time that the plant was shut down. Various 7-day work schedules were tried: first, every third Sunday, then every other Sunday, and finally every Sunday.

The findings are: as Figure 18 shows, (1) efficiency is highest during the 6-day, 48-hour week. The average efficiency level during the shorter workweek is 29% greater in comparison with that of the 7-day, 56-hour week. After a long period at seven days per week, when workers are given every third Sunday off, efficiency rises by nearly 17%. When the workers work every other Sunday, the efficiency increases 11% more. Furthermore, the efficiency during the 6-day week would be estimated higher by 6 to 8%, if there is no bonus scheme, under which production in excess of 30% above the standard is not compensated. That is, the efficiency during the shorter schedule would be one-third as high as that of the longer schedule; (2) the work output of the 30 skilled operators, under the limitations of bonus scheme, is highest during the schedule calling for work on every other Sunday. It appears that the work output level during the 6-day week and 8-hour day, would be equally as high, if the assumption of the efficiency level in the absence of the bonus limitations is sound.

Sunday work is clearly not economical. It does not increase the work output. It actually decreases the work output. Sunday work, under regular 8-hour schedule, means 8-day's pay for 6-day's work output, since work is doubly paid. Thus, it is only about 75% as efficient as the 6-day week.

In consequence, for hours above the base schedule (8-hour day, 48-hour week), the production of the two additional hours of work output takes three hours when operation is light. And when the operation is heavy, one hour of additional work output requires about two more hours of work (the U.S. Department of Labor, 1947).

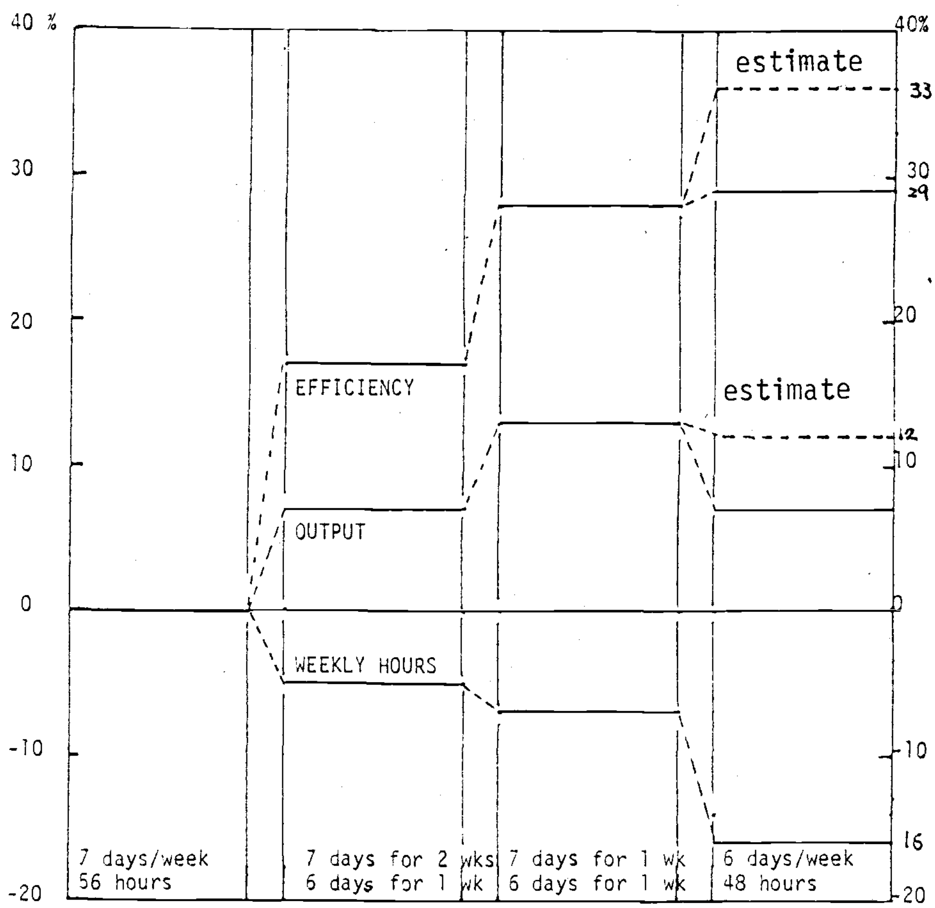


Figure 18. Effects of changes of working hours in various working schedules (cited in: The U.S.Dept. of Labor, 1947).

Effect of Mental Fatigue on Decision-Making Capacity

There are studies concerning the effects of mental fatigue on decision-making capacity. Kalsbeek (1964) investigated the effect of work duration on decision-making performance. His conclusion is that with the increase in work time and mental strain, there is a decrease in the number of information units that can be correctly processed. Grandjean (1968) experimened the effect of mental fatigue on decision-making capacity for air traffic controllers. The radar control tasks of air traffic controllers are one of mental tasks with high content of decision-making performances (Simonson and Weiser, 1976).

The results of Grandjean's experiment show that for ten hour-long work, after the sixth hour the mental performance decreases significantly. Simonson and Weiser (1976) state that choice-reaction time experiments have demonstrated particularly well the capacity of information processing on decision-making. Schmidtke(1960) studies, for example, extensively the effect of mental fatigue in terms of the reaction time and the number of choices for the subjects who arrived at a training plateau so that there is no further progress in learning for them during successive experiments. Figure 19 shows an effect of intensive mental work over a period of four hour on the reaction time. For a small (2-6) number of choice alternatives, after four hours of mental work, the reaction times show only slight differences. The effect of mental fatigue would probably not appear with a small number of choice alternatives.
On the other hand, the reaction time after mental work with a large number of choice alternatives (more than 12) increases drastically as shown in Figure 19.

In another study, Simonson and Weiser (1976) state that Babadschanjan et al.(1960) also drew a similar conclusion as Schmidtke's (1960) from choice reaction time experiments with strenuous mental load in railroad employees. The subjects had to select a critical color hue out of five different blue hues, moving behind a slit, and press a lever on its appearance before and after a work shift of 12 hours. The results of this experiment show that errors (missing reactions or reaction to a false hue) were significantly increased after the work of 12 hours, as well as reaction time. Table 21 shows the results of Babadschanjan et al. experiment. The longer choice reaction time represents a high degree of mental fatigue and massive decrease of information processing capacity (Simonson and Weiser, 1976). The average increment of the reaction time is 105.8%.

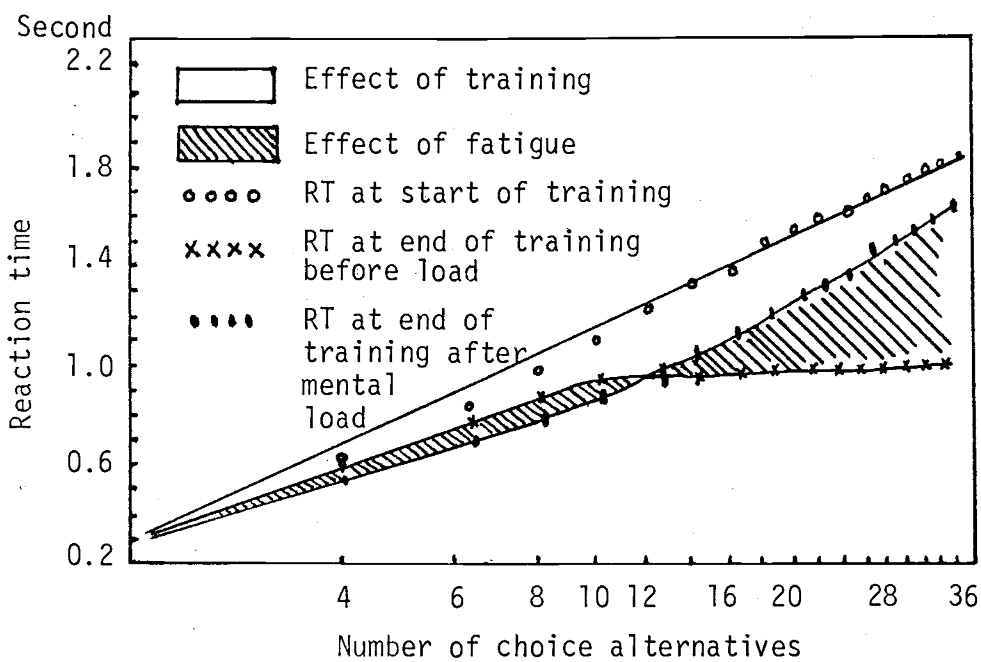


Figure 19. Effect of mental fatigue and training (learning) on choice reaction time (cited in: Simonson and Weiser,1967)

Table 21. Prolongation of reaction time after 12 hours work of train engineers (Modified; cited in Simonson and Weiser, 1976).

Subjects	Reaction time		
	Before work	After work	Ratio $\left(\frac{\text{after} - \text{before}}{\text{before}} \times 100\right)$
S-n	0.311 sec	0.682 sec	119.29%
K-ow	0.250	0.648	159.20
M-ing	0.362	0.503	38.95
Average reaction time	0.308 sec	0.611 sec	105.8 %

The effect of mental fatigue on learning (training) is also studied by Schmidtke (1960) as shown in Figure 19. Schmidtke (1960) confirms the results of the early experiments by Merkel (1885), Hick (1952) and Hyman (1953). They stated that there is a relationship between the reaction time and the logarithm of the number of possible choice alternatives which varied between 12 and 36. Schmidtke (1960) states that the relationship holds only when the subjects are not trained and it, however, does not hold after they arrived at a training plateau. Schmidtke (1960) also proposed that with the large number of choice alternatives, mental fatigue can be hypothesized to be equivalent to a loss of learning effect. The learning effect is defined as the shortening of reaction time as Schmidtke (1960) says.

Summary

We have discussed the effects of fatigue on longer working schedules to answer the question of what schedule of working hour will bring the best result. The answer is simply that there is no one best working hour schedule that applies to all cases and that it is very difficult to give clear-cut rules or principles for control or prevention of fatigue. Each situation must be carefully evaluated for its own peculiar characteristics and the effects of fatigue.

However, the U.S. Department of Labor (1947, p.1), after an intensive industrial study, takes the view that eight hours of fairly intensive work a day, five days a week, is the maximum that should be permitted in the United States for optimum productivity. Many European industrial physician believe that eight and a half hours of work a day, five days a week is more advantageous (McFarland, 1971). The longer working hours bring greater work output than that produced during the shorter work schedules, with few exceptions. However, extended longer hours of work above an optimum number, which may vary slightly with different industries, do not proportionately increase production (the U.S. Department of Labor, 1947, p.1-2).

The U.S. Department of Labor (1947, p.5) concludes that at light work for more than eight hours of work a day and 48 hours of work a week, it usually takes three hours of work to produce two additional hours of work output and for the heavy work, it takes about two more hours of work to produce one hour of additional work output.

There are studies concerning the effects of mental fatigue on decision-making capacity. After long hours of mental work, the capacity of decision-making decreases drastically.

Effects of Fatigue on Information Processing Capacity Estimation

Fatigue Factor

We have discussed that fatigue is an important production limiting factor in scheduling human resources for the longer working hours.

A base schedule is defined as 48-hour per 6-day week. A longer working hour schedule is defined as a working hour schedule which is longer than the base schedule. Fatigue factor is defined as a factor that represents the effects of fatigue on the productivity of human resources on longer working hour schedules.

We consider two categories of heavy and light operations for the fatigue factor as shown in Table 22. The definitions of the heavy operation and the light operation are given in the previous section.

Let f_i be fatigue factor for heavy operations (i=1) and light operations (i=2).

Table 22. Fatigue factors for light and heavy operations.

factor / Operations	fatigue factor
LIGHT	2/3
HEAVY	1/2

In an attempt to evaluate the effects of fatigue on the capacity estimation, an example will be used.

Example 9

Suppose the project which is discussed in Example 8 is operating under a longer working hour schedule of 60-hour per 6-day week, 10-hour a day. The operations are mostly light. We may express the effects of fatigue on the longer working schedule in terms of equivocation. Since the base schedule is 48-hour per 6-day week, 8-hour a day, the % of increased hours , INH, is shown:

$$INH(\%) = \frac{60 - 48}{48}$$

$$= 25\%.$$

We may interpret the value,25% of the increased working hours, as the human resource increment of 25% over the amount of human resource needed in the base schedule. Thus, we can now proceed with this problem in the same manner as the human resource increment problem in Chapter 7.

The fixed portion ($100\ \alpha$), 30%, is used. Then, we know that by increasing 100% human resources, the activity durations, minimum (a), most likely (m), and maximum (b) can be shortened:

$$a' = 0.65a$$

$$m' = 0.65m$$

$$b' = 0.65b$$

When human resources are doubled, the compressible portion of an activity, 70%, can be shortened to one half of 70%. That is,70% compressible portion would become 35% of the new activity duration when human resources are doubled.

In order to compute the compressed amount of an activity duration by increasing human resources of 25%, we need to know the compressed portion per unit % of human resource increment.

The compressed portion per unit % of human resource increment

= 35/100

= 0.35 % / unit % of human resource increment.

When the human resources are increased by 25%, each activity can be compressed by 8.75%. The figure,8.75%, is calculated as:

$$25\% \times 0.35 = 8.75\%$$

Then, considering 25% increment of human resources, the activity durations, without consideration of fatigue factor, are shown as:

$$0.30xa + (0.70 - 0.0875)xa = 0.9125a$$

$$0.30xm + (0.70 - 0.0875)xa = 0.9125m$$

$$0.30xb + (0.70 - 0.0875)xb = 0.9125b$$

When the fatigue factor, f_1=0.667 is considered, the compressed portion by increasing human resource of 25% would be :

$$8.75\% \times 0.667 = 5.836\%$$

Thus, the activity durations are :

$$0.30xa + (0.70 - 0.05836)xa = 0.94164a$$

$$0.30xm + (0.70 - 0.05836)xm = 0.94164m$$

$$0.30xb + (0.70 - 0.05836)xb = 0.94164b$$

Using the above formula each activity duration can be computed with and without consideration of fatigue factor, f_1=0.667. Table 26 shows the effects of fatigue on the capacity estimation of the project in Example 8. The total equivocation of the project is 18.83 bits. The equivocations of the project with and without fatigue factors are 18.83 bits and 17.77 bits respectively. When human resources are increased by 25%, the capacity of the project organization without consideration of fatigue factor is:

The capacity without fatigue factor

= 18.83 bits - 17.77 bits

= 1.06 bits

The capacity with fatigue factor

= 18.83 bits - 18.138 bits

= 0.692 bits

Now, the effect of fatigue on the capacity estimation can be evaluated:

$$\text{The effect of fatigue (\%)} = \frac{(1.06 - 0.692)\text{x}100}{1.06}$$

$$= 34.7\%$$

Thus, a 25% increase of total working hours results in only 16.325% increase (25x0.653 =16.325%) in the capacity of the project organization. The 8.675% decrease of the capacity is due to the fatigue effect (25x.347 =8.675%).

Table 23. Equivocation of fatigue effect on capacity estimation of the project in Example 8.

Activity I - J	Increment of human resource	Original time estimates(week)			Original Equivocation(bits)
1 - 2	25%	2	3	5	1.31
2 - 3	25	3	4	9	2.31
2 - 4	25	4	5	12	2.72
3 - 4	0	0	0	0	0
3 - 5	25	2	4	7	2.04
4 - 6	25	5	6	14	2.89
5 - 7	25	5	6	12	2.53
6 - 7	25	3	5	9	2.31
7 - 8	25	4	6	12	2.72
Total					18.83 bits

Activity	Changed time estimate						Equivocation		Effects of fatigue
	with f_1			without f_1					
I - J	a'	m'	b'	a'	m'	b'	with f_1	w/o f_1	
1 - 2	1.88	2.82	4.71	1.82	2.74	4.56	1.222	1.175	0.046
2 - 3	2.82	3.76	8.47	2.74	3.65	8.21	2.220	2.173	0.046
2 - 4	3.76	4.71	11.3	3.65	4.56	11.0	2.636	2.589	0.046
3 - 4	0	0	0	0	0	0	0	0	0
3 - 5	1.88	3.76	6.59	1.82	3.65	6.39	1.957	1.913	0.046
4 - 6	4.71	5.65	13.2	4.56	5.47	12.8	2.804	2.760	0.046
5 - 7	4.71	5.65	11.3	4.56	5.47	10.95	2.442	2.397	0.046
6 - 7	2.82	4.71	8.47	2.74	4.56	8.21	2.220	2.173	0.046
7 - 8	3.76	5.65	11.3	3.65	5.47	10.95	2.637	2.589	0.046
Total							18.138	17.770	0.368

Average Unit Equivocation (AUE)

Table 24 shows the computations for unit equivocation of the project in Example 8.

The Average Unit Equivocation(AUE) can be computed:

$$AUE = \sum_{i=1}^{2} UE_i/8$$

$$= 4.128/8$$

$$= 0.516 \text{ bits}$$

Table 24. Unit equivocation of the project in Example 8.

Activity I - J	Changed time estimates a'	m'	b'	Equivocation (bits)	Unit Equivocation(UE) (H'/m')
1 - 2	1.88	2.82	4.71	1.222	0.433
2 - 3	2.82	3.76	8.47	2.220	0.590
2 - 4	3.76	4.71	11.30	3.636	0.719
3 - 4	0	0	0	0	0
3 - 5	1.88	3.76	6.59	1.957	0.520
4 - 6	4.71	5.65	13.18	2.804	0.496
5 - 7	4.71	5.65	11.30	2.442	0.432
6 - 7	2.82	4.71	8.47	2.220	0.407
7 - 8	3.76	5.65	11.30	2.637	0.467
Total					4.128 bits

The effect of fatigue can be converted as follows:

$$\text{Effect of fatigue} = \frac{\text{Equivocation due to fatigue}}{\text{Average Unit Equivocation}}$$

$$= \frac{0.368}{0.516}$$

$$= 0.71317 \text{ weeks}$$

$$= 3\ 1/2 \text{ days}$$

The effect of fatigue on the capacity of a project organization is evaluated. In Example 9, it is shown that a 25% increase of total working hours over the base schedule (6-day week, 8-hour day) results in only 16% increase of the capacity of the project organization. The effects of fatigue is shown as a 35% time lost of the total increased working hours.

The Table 25 shows the usage of the information processing capacity estimation method.

Table 25. The usage of information processing capacity estimation method.

Usage	Explanation
When to use	The method can be used to estimate the capacity of a project to handle equivocation due to design changes. It can also be used to highlight the affected activities and the necessary amount of additional human resources in striving to meet the project due date. Furthermore, the method can be used to estimate the project slippages (disruption effect) when the capacity of a project is not large enough to handle the equivocation. Thus, the method can be used to evaluate the capacity of a project and the disruption effects (project slippages) due to design changes and engineering drawing delays, including fatigue, boredome and stress in scheduling human resources in project management.
When not to use	The method does not consider other disruption effects (cost effect, reverse learning effect, etc.) due to design changes and engineering drawing delays, and fatigue.

IX. IMPLICATIONS AND EXTENSIONS

Implications of Results of the Study

This dissertation has presented five specific aspects of work analysis. All five areas are related to the large-scale, complex projects disrupted by design changes and engineering drawing delays.

First, in human resource scheduling, the effects of fatigue in longer working schedules has been studied. Second, a systematic prediction procedure, which can identify potential problem areas in a project, is developed. Third, a distribution of engineering drawing delays is identified. Fourth, an entropy formula of triangular distribution for activity, as a working vehicle, is derived and an entropy conversion method is proposed. Fifth, an information processing capacity estimation method for a project is developed.

When design changes occur in a project, the project schedule is usually delayed. Then, project managers have to adopt longer working hour schedules to stay near the contracted project completion date. In scheduling human resources, traditionally, human resources have been treated indifferently from physical resources without considerations of human nature. Management should realize that fatigue and information are apparently two important factors to be considered in longer working schedules, because it is shown that effects of fatigue and lack of information are significant on the productivity of human resources. Design changes are usually random. The random occurrences of design changes bring workers boredom, frustration and stress. When the workers have to wait for regular works until the designs are firmed, they feel boredom. When normal work routines are interfered with, and information about sudden changes, such as longer working hours, or overtime, etc. is lacking, they may be frustrated and lose their desire to work. Furthermore, when they know that their working schedules are behind the original project schedule and are pushed to complete the delayed project by the contracted project completion date, they feel stress. Boredom, frustration, and stress bring fatigue to workers and eventually result in lower productivity of workers. It is also shown that fatigue effect significantly decreases the capacity of project organization. For example, the analysis of fatigue effect in Example 9 shows that a 25% increase of total working hours over the base schedule (48-hour week, 8-hour day) results in only 16% increase of the project capacity. The 9% lost from the 25% increase of the total working hours is due to fatigue effect. That is, the effect of fatigue is shown as a 35% decrement of the total increased working hours.

In consequence, fatigue, a production limiting factor, should be carefully evaluated in scheduling human resources for longer working schedules. In order to avoid the effect of lack of information in project operations, a good communication channel must be established to provide workers with information about management's sudden actions during project execution.

Since the systematic prediction procedure can provide management with warning information about potential problem areas, management would be better able to control disruption effects due to design changes in the project operations.

By the prediction procedure, the potential set of designs can be identified in a project operation. Therefore, the management may determine which designs could be changed during the project operation and place more attention to that potential set of designs. Then the occurences of design changes would not be completely uncertain to the management. If management can describe the characteristics of the occurrences of design changes and engineering drawing delays, management can get information for the strategy of the project operations. Thus, the next descussion would be implications about a distribution of engineering drawing delays.

It is shown that an exponential distribution is an adequate distribution for engineering drawing delays. This result is supported by two ways: The maximum entropy principle is used to show the adequacy of exponential distribution for engineering drawing delays. Since engineering drawing delays are completely random, a distribution for the delays must have the largest entropy. This criterion is the maximum entropy principle: The least prejudiced assignment of probability distribution based on given information is that the distribution whose entropy function is greatest. Secondly, the results of χ^2 'goodness-of-fit' tests for the actual industrial data about drawing delays show that the actual industrial data do not significantly differ from the theoretical estimated values at 0.05 significance level.

Now, the management knows the potential sets of designs of change by the systematic prediction procedure and the distribution of engineering drawing delays. As long as management can estimate average length of drawing delays through either the previous experiences or information by project owner, the management can estimate number of drawings arrived per unit time and inter-arrival time between successive drawings, provided that the management can understand Queuing Theory. Such information can be used for the management to establish strategies in an attempt to protect from cost overrun and project slippage.

In consequence, it is, therefore, very important for the management to know the potential designs to change and the distribution of engineering drawing delays in project management.

In order to provide a working vehicle for the development of the capacity estimation method, the entropy formula for a triangular distribution for activity is derived. This entropy formula is used to compute the capacity of a project. In addition, the entropy conversion method is proposed. This method would be a good means to use for comparing projects which have different measurement units in terms of uncertainty of project completion time.

When design changes occur and those designs are to be modified, management needs to have a method which can be used to evaluate effects of design modification activities on the project schedule. In an attempt to provide management with an expedient means of evaluating project status in the various stages of project execution, the capacity estimation method is developed. The method can estimate the capacity of a project organization, which can handle the equivocation resulting from design modifications in the project. The method can also estimate the amount of project slippage when the estimated maximum capacity is not large enough. In addition, the method can identify the associated activities and the required amount of human resources. The predictive ability of the method is evaluated by using a computer Monte Carlo simulation program. It is shown by a χ^2 'goodness-of-fit' test that the project slippage estimated by the capacity estimation method do not significantly differ from those obtained from the computer simulation program, CRASH, at 0.005 significance level. Thus, this capacity estimation method would be suggested as an expedient project management tool.

Extensions for Future Research

A framework for decision-making can be built by organizing the previous experiences and the newly acquired knowledge. When a decision-maker has to make a decision the framework can be used. However, when the decision-maker is mentally fatigued his framework may be considered disorganized. In such a case, the decision-maker takes longer time to make a decision. Bartlett (1943) suggested a "phantom concept" of mental fatigue as "time-correlated disorder of skill". Van Gigch (1968) proposed the integrative behavior model for evaluating information processing capacity of industrial skilled worker in modern industry. As the level of integration grows, the complexity of the information processed increases in the integrative behavior model. It implies that the amount of information processed at third level of integration is three times as great as at the first level. Van Gigch (1968) classified decision-making process as the third level of integration of his integrative behavior model and developed an entropy formula for decision-making. Therefore, it appears possible to combine Van Gigch and Bartlett concepts to further investigate effects of mental fatigue on the decision-making capacity of management from an information entropy point of view.

If design changes occur in a project, management has to make various decisions to minimize effects of disruptions on established project schedule. The decisions that are full of uncertainties would be: addition of extra shifts; extension of working hours or days; hiring extra workers; juggling the activities in the project to allow work to continue while waiting for designs; addition of extra physical resources, etc. In such situations, the management's decision-making capacity seems to affect the project completion date. If the management's decision capacity is overburdened in a unit time (day), the rest of decision-making must be postponed and eventually the project schedule would be delayed. Therefore, it is desirable to investigate delay effects on project scheduling resulting from management's decision-making capacity to handle the uncertainties. Information entropy concept can also be applied to this problem.

There is another area of interest for future research related to disruptions due to the design changes and engineering drawing delays. Effects of disruption on worker's learning can be characterized as reverse learning that was coined by Riggs (1978). The progress function was developed by the airframe industry in the 1930's as a predictive model concerning cost estimation, production scheduling, contractor's production efficiency comparisons, facilities requirement planning, personnel planning and planning guide in developing long-range projections for a wide range of programs. It is also called the "improvement curve" or "learning curve". It has been well studied that the man-hours to complete each complex assembly, such as those required in airplane construction, decrease with learning through practice of repeated tasks. Zieke (1963) reviewed the history of these curves in the airframe industry. An intensive bibliography (123 books and reports, journals and periodicals and other resources) is included. Hall (1957) studied how the cost in hours expended of major aircraft design changes can be determined. However, the effects of disruptions due to the design changes on the individual learning as reverse learning curve have never been documented. The learning curve in project operations results from familiarity and coordination developed in constructing a product: airplane, ship, etc. Since disruptions create discoordination and cause confusion in normal work routines, it is reasonable to propose to study the effects of disruptions in terms of reverse learning curve.

X. CONCLUSIONS

This dissertation presents an analysis of disruption problems due to uncertainties resulting from design changes and engineering drawing delays in large-scale, complex research and development or construction projects.

The Basic Premises

1. The distribution of activity durations, including design modification activity duration, follow a triangular distribution.

2. The maximum availability of human resources in a project can be a 100% increase of the original resource requirement for each activity.

3. Each activity duration consists of the fixed portion (uncompressible portion) and the compressible portion. An activity duration can be shortened by one half when the fixed portion is equal to zero and the human resources are doubled.

4. The fixed portion (100α) of 30% is adopted.

5. Each activity has a capability to reduce the performance equivocation, that is defined as the capacity of an activity.

6. Information Theory provides a useful concept entropy (equivocation) which can be used to measure the capacity of an activity and that of a project organization.

The Results of the Study

The following results are drawn from this study:

1. It is noticed that most literature treat human resources indifferently from physical resources without considerations of human

nature. Information, an important resource in project management, is mostly neglected in traditional studies of project management.

2. We have discussed various effects of fatigue due to boredom, stress and frustration and the relationship between lack of information and productivity of human resources. It is recognized that when workers are lacking in information about management's sudden actions (e.g. unexpected longer working hours, interruptions in normal work routines), they lose desire to work and feel fatigued and eventually, the productivity of the worker drops. Information and fatigue are apparently the important factors to be considered in scheduling human resources.

3. A systematic prediction procedure of identifying potential trouble spots in a project is developed. The prediction procedure can systematically provide management with warning information to identify the possible problem areas before starting a project as well as during the project execution. It starts to collect information about sets of potential trouble spots (designs to change) from design experts in project owner teams through an Input-Output Communication Channel. Prior probabilities for initial possible sets of designs are assigned by using the concept of maximum entropy principle. The prior probability estimates, for each likely set of designs to change, can be revised by Bayesian Sequential Decision Analysis when new information becomes available. The equivocation for each set of potential desings at each stage is computed and compared. Finally, an order list of potential design sets is identified. A design set with lower equivocation value is placed first on the order list and a design set with higher value of equivocation is placed in the last of the order list.

The management can decide how much additional information should be collected within his time and his capability to assume risk.

4. Engineering working drawing is essential in a normal project execution. The unavailability of drawings due to drawing delays at the right time creates many difficulties in normal project operations to management and eventually brings project cost overrun and project slippage. Because the characteristics of the engineering drawing delays are 'uncertain' to management, the right actions can not be taken. The hypothesis is therefore advanced that most of engineering drawing delays follow an exponential distribution. Using the maximum entropy principle, exponential distribution is shown to be an adequate probability distribution for engineering drawing delays. In order to support the hypothesis above, χ^2 'goodness-of-fit' tests for actual industrial data about the delays are conducted.

5. The entropy formula of a triangular distribution for activity is derived. This formula is used as a working vehicle for developing a capacity estimation method of a project. The entropy conversion method is proposed. When management has to choose project(s) among more than one project, the entropy conversion method is a good means to use in choosing a project in terms of uncertainty of the project completion date. A large value of entropy indicates a higher variability project.

6. The information processing capacity estimation method is developed based on the transformation function for the capacity of a project. The capacity is measured by the reduced amount of equivocation of the project completion date by increasing an amount of human

resources. The method is able to estimate the capacity of a project for handling the equivocation due to design changes in a project. When the estimated maximum capacity of a project is not large enough to handle the equivocation, the project completion time must be prolonged. In such situations, the method can estimate the amount of project slippage. Furthermore, the method can also identify the associated activities and the necessary amount of human resources.

7. In an attempt to evaluate the predictive ability of the method, the project slippages estimated by the information processing capacity estimation method are compared with the results generated by a computer Monte Carlo simulation program, CRASH. A χ^2 'goodness-of-fit' test is conducted. The results of the test shows that the estimated project slippages do not significantly differ from those obtained from the CRASH computer program at the 0.005 significance level.

We conclude that the information processing capacity estimation method may be suggested as an expedient means of evaluating project status for the management in different stages of project execution.

8. The effects of fatigue in longer working hour schedules have been discussed. It is shown, in one example, that a 25% increase of total working hours over the base schedule (48-hour week, 8-hour day) results in only 16% increase of the capacity of a project. The effect of fatigue is shown as a 35% time lost of the total increased working hours.

BIBLIOGRAPHY

Abruzzi, A. 1956. Work, workers and work measurement. New York. Columbia University Press. 318 p.

Ahuza, H.N. 1976. Construction performance control by networks. New York. Wiley-Interscience Publication, John Wiley & Sons. 636 p.

Ash, I.E. 1914. Fatigue and its effect upon control. Architectural Psychology 31:1.

Babadschanjan, M.G., W.N. Sokolowa, J.I. Kostina, M.I. Mamazaschwili and W.J. Tschirkow. 1960. Isutschenije pritschin utomaljajemosti dispetscherow. In:Psychological aspects and physiological correlates of work and fatigue,edited by Simonson, E. and P.C. Weiser. Springfield, Illinois. Charles C. Thomas. 1976.

Bartlett, F.C. 1943. Fatigue following highly skilled work. Proceedings of the Royal Society B 131:247-254. London, H.K. Lewis & Co., Ltd.

________. 1953. Psychological criteria of fatigue. In: Symposium on Fatigue edited by Floyd, W.F. and A.T. Welford. London. H.K. Lewis & Co., Ltd.

Bartley, S.H. 1943. Conflict, frustration and fatigue. Psychosom. Med. 5:160-162.

________. 1951. Fatigue and efficiency in theoritical foundations of psychology. H.Helson (ed.). New York. Van Nostrand. 787 p.

________. 1954. Understanding of Visual Fatigue. American Journal of Optometry. Monograph no. 30.

________. 1957. Fatigue and inadequacy. Psychological Review 37: 301-324.

________. 1965. Fatigue: mechanism and management. Springfield, Illinois. Charles C. Thomas. 96 p.

Bartley, S.H. and E. Chute. 1945. A preliminary classification of the concept of fatigue. Psychological Review 53:169-174.

________. 1947. Fatigue and Impairment in man. New York. McGraw-Hill Book Company, Inc. 429 p.

Bigelow, C.G. 1962. Bibliography on Project Planning and Control by Network Analysis: 1959-1961. Operations Research 10:728-731.

Borch, K. and J. Mossin (eds.) 1968. Risk and Uncertainty. Proceedings of a conference held by the International Economic Association. London, England. McMilland. 455 p.

Brand, J.D., Meyer, W.L. and L.R. Shaffer. 1964. The Resource Scheduling Method in Construction. Civil Engineering Studies, Report no.5. University of Illinois.

Conklin, E.S. and F.S. Freeman. 1939. Introductory Psychology for Students of Education. New York. Henry Holt & Co. 406 p.

Cooper, D.F. 1974. A computer analysis of some project scheduling heuristics. Proceedings of the sixth Australian Computer Conference, Sydney. May 20-24.

Crawford, A. 1961. Fatigue and Driving. Ergonomics 4:143-154.

Crowston, W.B.S. 1968. Decision Network Planning Models. Management Sciences Research Report no. 138. Pittsburgh. Carnegie-Mellon University.

Davis, E.W. 1966. Resource Allocation in Project Network Models: A survey. Journal of Industrial Engineering 17:177-188.

________. 1973. Project Scheduling under Resource Constraints: Historical Review and Categorization of Procedures. AIIE Transactions 5:297-313.

________. 1976. Project Management: Technique, Applications and Managerial Issues. American Institute of Industrial Engineers, Inc. 220 p.

Dean, J. 1962. Managerial Economics. Englewood Cliffs, New Jersey. Prentice-Hall. 621 p.

Dill, D.B. 1942. Physiology of fatigue: Factors and criteria of endurance. Journal of Laboratory Clinical Medicine 28:596.

Encyclopedia Britannica. 1976. Encyclopedia Britannica, Inc.

Feiler, A.M. 1976. Project Management Through Simulation. GCS 3-76. Covina, California. Procurement Associates Inc.

Fendley, L.G. 1966. Toward the Development of a Complete Multiproject Scheduling System. Unpublished Ph.D. thesis, Arizona State University.

Giffler, B. and G.L. Thompson. 1960. Algorithms for solving production scheduling problems. Operations Research 8:487-490.

Goldman, S. 1953. Information Theory. Prentice-Hall, Inc. 358 p.

Gonguet, L. 1969. Comparison of Three Heuristic Procedures for Allocating Resources and Producing Schedule. In:Project Planning by Network Analysis.

Good, I.J. 1950. Probability and The Weighing of Evidence. New York. Hafner Publishing Company. 119 p.

Grandjean, E. 1968. Fatigue: its physiological and psychological significance. Ergonomics 11:427-436.

Gross, I.H. and S.H. Bartley. 1951. Fatigue in house care. Journal of Applied Psychology 35:205-207.

Hall, L.H. 1957. Experience with Experience Curves for Aircraft Design Changes. N.A.A.Bulletin 39(1):59-66.

Heimstra, N.W. 1970. The Effects of 'stress fatigue' on Performance in a Simulated Driving Situation. Ergonomics 13:209-218.

Hick, E.E. 1952. On the rate of gain of information. Quarterly Journal of Experimental Psychology 4:11-19.

Holmes, W.G. 1938. Appled Time and Motion study. New York. The Ronald Press Company. 335 p.

Hyman, R. 1953. Stimulus information as a determinant of reaction time. Journal of Experimental Psychology 45:188-196.

Inoue, M.S. 1977a.Computer Simulation Study of the Effects of Design changes and Drawing Delays upon Construction Schedule of Gas Turbine Tanker. Productive Resource Inc. Corvallis, Oregon.

________. 1977b.Personal Report to a private company. Personal communication.

Jaynes, E.T. 1957a. Information Theory and Statical Mechanics. The Physical Review 106:620-630.

________. 1957b. Information theory and statistical mechanics 11. The Physical Review 108:171-190.

________. 1962. Information theory and statistical mechanics. (1962 Brandeis Lectures). Chapter 4 of Statistical Physics, 1963. K.W. Ford ed. W.A. Benjamin Inc. New York. 182-218.

________. 1963. New Engineering Applications of Information Theory. IN: Proceedings of the First Symposium on Engineering Applications of Random Function Theory and Probability. Wiley, New York, 163-203.

________. 1965. Gibbs vs. Boltzman Entropies. American Journal of Physics 33:391-398.

________. 1967. Foundations of Probability Theory and Statistical Mechanics. Chapter 6 of Delaware Seminar in the Foundations of Physics, Bunge, M. ed., Springer, Berlin.

________. 1968. Prior Probabilities. IEEE Transactions on Systems Science and Cybernetics, SSC-4:227-241.

Jeffreys, H. 1939. Theory of Probability, First ed. The Clarendon Press, Oxford. 380 p.

Kalsbeek, J.W.H. and J.H. Ettema. 1964. Physiological and Psychological Evaluation of Distribution Stress. Proceedings of Second International Congress on Ergonomics, Dortmuns, Spplement to Ergonomics. 443 p.

Karger, D.W. and F.H. Bayha. 1966. Engineered Work Measurement. Second Printing. New York. Industrial Press Inc. 722 p.

Kelley, E.E., Jr. 1961. Critical Path Planning and Scheduling Mathematical Basis. Operations Research 9:296-321.

________. 1963. The critical Path Method: Resource Planning and Scheduling. Chapter 21 of Muth, J.F. and G.L. Thompson ed. Industrial Scheduling. Englewood Cliffs, New Jersey. Prentice-Hall. 387 p.

Kelley, J.E., Jr. and M.R. Walker. 1959. Critical Path Planning and Scheduling. Proceedings of the Eastern Joint Computer Conference.

Keynes, J.M. 1962. A Treatise on Probability. New York, Harper & Row. 466 p. Reprint of 1st ed., 1921, except new introduction by Norweed Russel Hanson.

Klausner, R.F. 1970. The Evolution of Risk in Marine Capital Investiments. Marine Technology 7:449-464.

Kunisawa, K. 1959. Introduction to information theory for operations research. Japanese Union of Scientists and Engineers (JUSE). (Translated by Inoue, M.S.) 150 p.

Lerda-Oldberg, S. 1966. Bibliography on Network-Based Project Planning and Control Techniques: 1962-1965. Operations Research 14:925-931.

Levy, F.K., G.L. Thompson and J.D. Wiest. 1963a. Introduction to the Critical Path Method. Chapter 20 of Muth, J.F. and G.L. Thompson ed. Industrial Scheduling. Englewood cliffs, New Jersey. Prentice-Hall. 387 p.

________. 1963b. Mathematical basis of the Critical Path Method. Chapter 22 of Muth, J.F. and G.L. Thompson ed. Industrial Scheduling. Englewood Cliffs, New Jersey. Prentice-Hall. 387 p.

Malcolm, D.G., J.H. Roseboom, C.E. Clark and W.Fazar. 1959. Applications of a Technique for R and D Program Evaluation (PERT). Operations Research 7:646-669.

Mason, T. and C.M. Moodie. 1971. A Branch and Bound Algorithm for Minimizing Cost in Project Scheduling. Management Science 18B:158-173.

McFarland, R.A. 1971. Section 1: Fatigue in Industry-Understanding Fatigue in Modern Life. Ergonomics 14:1-10.

Merkel, J., 1885. Die Zeitlichen Verhältnisse der Willenstaetigkeit. Philos Studien 2:73.

Miller, R.W. 1963. Schedule, Cost, and Profit Control with PERT. Chapter 6. Mcgraw-Hill Book Co., Inc.

Mize, J.H. 1964. A Heuristic Scheduling Model for Multi-Project Organization. Ph.D. Thesis. Purdue University.

Moder, J.J. and C.R. Phillips. 1970. Project management with CPM and PERT. 2nd ed. New York. Van Nostrand Reinhold Co. 360 p.

Murrell, K.F.H. 1971. Ergonomics: Man in his working environment. London. Chapman and Hall. 496 p.

Muth. J.E. and G.L. Thompson. 1963. Industrial Scheduling. Englewood Cliffs, N. J., Prentice-Hall. 387 p.

Niebel, B.W. 1976. Motion and Time Study. Sixth edition. Homewood, Illinois, Richard D. Irwin, Inc. 719 p.

Newendorp, P.D. 1972. Bayesian Analysis - A Method for Updating Risk Estimates. Journal of Petroleum Technology 24:193-198.

_________. 1975. Decision Analysis for Petroleum Exploration. Petroleum Publishing Company. Tulsa, Oklahoma. 668 p.

North, D.W. 1970. The Invariance Approach to the Probabilistic Encoding of Information. Ph.D. Thesis. Stanford University.

Pascoe, Y.L. 1965. An Experimental Comparison of Heuristic Method for Allocating Resources. Ph.D. Thesis. Cambridge University, England.

Patterson, J.H. 1970. Alternative Methods of Project Scheduling with Limited Resources. Ph.D. Thesis. Indiana University.

Pritsker, A., B. Alan, L.J. Watters and P.M. Wolfe. 1969. Multi-Project Scheduling with Limited Resources: A Zero-One Programming Approach. Management Science 16:93-108.

Quick, J.H., J.H. Duncan and T.A. Malcolm, Jr. 1962. Work Factor Time Standards: Measurement of Manual and Mental Work. 458 p.

Raisbeck, G. 1963. Information theory. Cambridge, Mass., Massachusetts Institute of Technology Press. 105 p.

Ramsey, F.P. 1950. Truth and Probability. In: The Foundations of Mathematics and other Logical Essays. Braithwaite, R.B. ed. Humanities Press, New York.

Riggs, J.L. 1978. A Research Proposal to National Science Foundation (personal communication).

Rothstein, J. 1954. An Informational Approach to Organization and System Engineering Design. Transactions of the I.R.E., PGEM-1.

Savage, L. J. 1954. The Foundations of Statistics. New York, Wiley. 294 p.

Schmidtke, H. 1965. Die Ermuedung. Bern and Stuttgart, Verleg Hans Huber. 339 p.

Schrage. L. 1970. Solving Resource Constrained Network Problems by Implicit Enumeration - Non Preemptive Case. Operations Research 18:263-278.

Shannon, C.E. 1948. A Mathematical Theory of Communication. Bell System Technical Journal 27:379-423.

Shannon, C.F. and W. Weaver. 1949. The Mathematical Theory of Communication. Urbana. University of Illinois Press. 125 p.

Shirland, L.E. 1972. Multi-Factor Information Theory Models and Their Industrial Applications. Ph.D. Thesis. Pregon State University.

Simonson, E. and J. Brozek. 1948a. Effect of spectral quality of light on visual performance. Journal of Opt. Society of America 38:830-837.

_________. 1948b. Effects of illumination level on visual performance and fatigue. Journal of Opt. Society of America 38:384-392.

Simonson, E. and D.A. Anderson. 1966. Effect of age and coronary heart disease on performance and phyiological responses in mental work. 7th International Congress of Gerontology. Vienna, Austria. June 26-July 2. p. 333-336.

Simonson, E. and P.C. Weiser. 1976. Psychological Aspects and Physiological Correlates of Work and Fatigue. Springfield. Illinois. Charles C. Thomas. 445 p.

Snyder, M.R., R. Schulz and E.E. Jones. 1974. Expectancy and apparent duration as determinants of fatigue. Journal of Personal Society of Psychology 29:426.

Tribus, M. 1961. Thermostatistics and Thermodynamics. D. Van Nostrand Company, Inc. 649 p.

________. 1962. The use of the maximum entropy estimate in the estimation of reliability. In: Recent Developments in Information and Decision Processes. R.E. Machol and P. Gray ed. New York. The MacMillan Company.

________. 1969. Rational Descriptions, Decisions and Designs. Pergamon Press. 478 p.

United States Department of Labor. 1947. Hours of Work and Output. Bulletin No. 917. 160 p.

Van Gigch, J.P. 1968. The Impact of Technology on the Mental Content of Work in Industrial Operations. Ph.D. Thesis. Oregon State University.

Vernon, H.M. 1921. Industrial Fatigue and Efficiency. George Routledge & Sons.London. 264 p.

Walster, B. and E. Aaronson. 1967. Effect of Expectancy of Task Duration on the Experience of Fatigue. Journal of Exp.Society of Psychology 3:41-46.

Watanabe, S. 1969. Knowing and Guessing. New York. John Wiley & Sons. 592 p.

Webster's Third New International Dictionary. G & C Merriam Publishing Co. Springfield, Massachusetts. 1968.

Webster's Third New International Dictionary of the English Language. Supplement. Springfield, Massachusetts. G & C Merriam Co. 1976.

Whitmore, D. 1970. Work Study and Related Management Services. London. W. Heinemann Ltd. 336 p.

Woodgate, H.S. 1967. Planning by Net work: Project Planning and Control Using Network Techniques, 2nd ed. New York. Brandon/ Systems Press. 385 p.

Woodgate, S. 1973. Trends and Developments in Network Planning Systems. Management Informatics 2:105-119.

Wyatt, S. and J.A. Fraser. 1929. The effect of monotony in work. Industrial Fatigue Research Board Report 56. London. H.M.S.O.

Zieke, R.P. 1963. The Progress Function in the Aerospace Industry - A Historical Analysis. IEEE Transactions on Engineering Management EM-10:74-85.