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A Comparison of Techniques for Magnetotelluric Response Function Estimation

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Spectral analysis of the time-varying horizontal magnetic and electric field components yields the magnetotelluric (MT) impedance tensor. This frequency dependent 2x2 complex tensor can be examined for details which are diagnostic of the electrical conductivity distribution in the Earth within the relevant (frequency dependent) inductive scale length of the surface observation point. As such, precise and accurate determination of this tensor from the electromagnetic time series is fundamental to successful interpretation of the derived responses. In this paper, several analysis techniques are applied to the same data set from one of the EMSLAB Lincoln Line sites. Two subsets of the complete data set were selected, on the basis of geomagnetic activity, to test the methods in the presence of differing signal-to-noise ratios for varying signals and noises. Illustrated by this comparison are the effects of both statistical and bias errors on the estimates from the diverse methods. It is concluded that robust processing methods should become adopted for the analysis of MT data, and that whenever possible remote reference fields should be used to avoid bias due to uncorrelated noise contributions.

1. INTRODUCTION

EMSLAB has brought together in a cooperative effort many electromagnetic (EM) induction workers with diverse backgrounds and experiences. That the EMSLAB project has many facets is well illustrated by the breadth of the subject matter of the papers in this special section. One such topic has focussed interest on the problem of determining the magnetotelluric (MT) impedance tensor elements from measurements of the time-varying components of the EM field as precisely and as accurately as possible. The availability of synoptic observations of the time-varying EM field over the EMSLAB-Juan de Fuca area motivated examination of the many disparate spectral analysis methods used to analyze similar (or identical) data and also led to the development of new ways of computing MT responses (e.g., robust methods, see below). In an analogous fashion to the objectives of the mini-EMSLAB project [*Young et al.*, 1988], we wished to undertake a comparison exercise to evaluate the relative efficacies of our analysis codes given the same data.

The time-varying EM field components are, by *Maxwell's* [1892] equations, related by linear differential operators, and for certain classes of external source potentials [*Egbert and Booker*, this issue], concepts appropriate for multiple-input/multiple-output linear systems can be appealed to. The estimation of the weighting response functions, or their frequency domain equivalent the transfer functions, for a multiple-input/multiple-output linear system by analyses of the respective input and output time series is a problem that has received much attention over the past century. A tremendous boon occurred with the advent of fast Fourier transformation algorithms during the 1960s, and with some exceptions, these transfer functions are now routinely estimated in the frequency domain. While this is done mainly for computational reasons, direct estimation of the impulse response functions by cross-correlation methods is unwise because of bad statistical properties for the estimates [*Jenkins and Watts*, 1968, pp. 422-429].

For the analysis of MT data, the linear system can be thought of as having two inputs, the horizontal components of the time-varying magnetic field ($b_x(t)$ and $b_y(t)$), and two independent outputs, the horizontal components of the time-varying electric field ($e_x(t)$ and $e_y(t)$), with additive noise components on each channel ($n_{b_x}(t)$, $n_{b_y}(t)$, $n_{e_x}(t)$, and $n_{e_y}(t)$) giving our observable time-varying field components ($\tilde{b}_x(t)$, $\tilde{b}_y(t)$, $\tilde{e}_x(t)$, and $\tilde{e}_y(t)$) (see Figure 1). The true inputs and outputs are related, by a convolution operation, to the four lag-domain weighting functions $z_{xx}(\tau)$, $z_{xy}(\tau)$, $z_{yx}(\tau)$, and $z_{yy}(\tau)$. In the frequency domain, the complex frequency dependent relation between these components can be written (dependence on frequency assumed)

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} B_x \\ B_y \end{bmatrix}$$

where Z is the MT impedance tensor defined initially by *Berdichevsky* [1960, 1964] and *Tikhonov and Berdichevsky* [1966] and where $B_x(\omega)$ is the Fourier transform of $b_x(t)$ and similarly for the other components.

Generally, we have no knowledge of the true components (or latent variables) E_x, E_y, B_x, and B_y or of the noise contributions on these components (N_{E_x}, N_{E_y}, N_{B_x}, and N_{B_y}) but only of our observations of these $\tilde{E}_x$,

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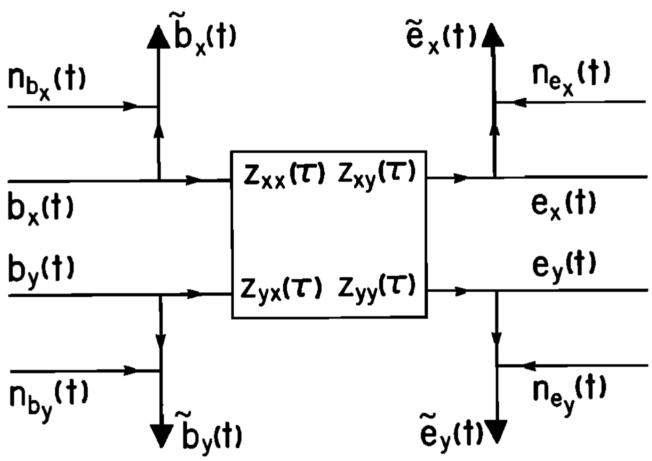

Fig. 1. Magnetotelluric linear system with the two horizontal magnetic components ($b_x(t)$, $b_y(t)$) as inputs, and the horizontal electric components as outputs ($e_x(t)$, $e_y(t)$) related by the four MT impedance weighting functions in the lag domain ($z_{xx}(\tau)$, $z_{xy}(\tau)$, $z_{yx}(\tau)$, $z_{yy}(\tau)$). Measured are $\tilde{b}_x(t)$, $\tilde{b}_y(t)$, $\tilde{e}_x(t)$, $\tilde{e}_y(t)$, i.e., the sums of these true field components perturbed by noise components ($n_{b_x}(t)$, $n_{b_y}(t)$, $n_{e_x}(t)$, $n_{e_y}(t)$).

$\tilde{E}_y$, $\tilde{B}_x$, and $\tilde{B}_y$. The problem we have then is given these observations, how do we estimate the elements of Z in some optimum manner?

Ignoring certain types of instrumentation error (nonlinear clock drift, erroneous electrode line lengths), there are two types of error inherent in the estimation of transfer functions; statistical error and bias error. Statistical error, which gives a quantitative measure of the precision of an estimate, generally will be reduced (for any reasonable processing scheme) by analyzing more data or by using robust methods (see below) which eliminate errors due to non-Gaussian residuals. Bias error, by which the accuracy of an estimate may be judged, is a more difficult quantity to estimate, but techniques exist (see below) for its elimination under certain conditions.

Standard least squares (LS) estimation of the transfer functions Z leads to estimates that are biased downward by uncorrelated noise on the inputs, i.e., noise on the magnetic field components (N_{B_x} and N_{B_y}), but that are unbiased by uncorrelated noise on the outputs, i.e., noise on the electric field components (N_{E_x} and N_{E_y}). Alternatively, the complex, frequency dependent, 2x2 MT admittance tensor A

$$\begin{bmatrix} B_x \\ B_y \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} \\ A_{yx} & A_{yy} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

can be estimated which assumes that the electric fields are inputs to the linear system with the magnetic fields as outputs. This tensor relationship was first recognized and utilized by *Neves* [1957], and alternative estimates of the elements of the MT impedance tensor are given by inverting the admittance tensor. Standard LS estimates of these elements have the property that they are upward biased by uncorrelated noise on the electric field components (N_{E_x} and N_{E_y}) but are unbiased by noise on the magnetic field components (N_{B_x} and N_{B_y}).

Obviously, the downward and upward biased estimates give an envelope within which the true transfer function should lie (to within the statistical limits of the estimates). These biases were first discussed for MT by *Sims et al.* [1971] but had been known of in other fields much earlier, particularly econometrics (*Gini* [1921]; reviewed by *Reiersøl* [1950]).

To avoid these bias errors due to noise power, remote reference processing of MT data was introduced [*Goubau et al.*, 1978*a,b*; *Gamble et al.*, 1979] in which the components of the horizontal magnetic field recorded at a remote second site are correlated with the local field components. The first use of such an estimator for the transfer function again comes from econometrics (*Reiersøl* [1941], and independently by *Geary* [1943]; see *Reiersøl* [1950] and *Akaike* [1967]), where the remote reference fields were termed the "instrumental variables." However, Gamble et al. essentially rediscovered this technique and gave it the now ubiquitous term "remote reference." It should be noted that remote reference methods are not as efficient as single-station methods in that for a given length data set, remote reference results will always have larger associated statistical errors than standard LS ones. Also, correlated noise components between the local and the remote fields (in the case of MT these could be nonuniformities in the magnetic field) are not removed by remote reference processing and can cause bias effects.

In this paper we compare various schemes (see Table 1) for estimating the elements of the MT impedance tensor. Three of these schemes are based on standard LS spectral analysis without remote reference processing; another three are robust schemes for data processing (one incorporating remote reference processing) that are less strongly affected by outliers in the output (electric field), while the other two are a nonrobust remote reference code and a cascade decimation procedure with weighted averaging. We define a robust processing procedure as one which is relatively insensitive to the presence of a moderate amount of bad data or to inadequacies in the statistical model and that reacts gradually rather than abruptly to perturbations of either. Much literature already exists on robust methods, and an exhaustive review is outside the bounds of this paper.

In order to ascertain the abilities of the different methods to extract reliable transfer function estimates in the presence of both low and high noise levels, compared to the signal levels, we identified two time segments of data of 5 days duration each; since the sampling interval was 20 s, there are 21,600 samples per segment. One of these was from a geomagnetically active period (K index typically >3), while the other was from a quiet period (K index typically 1).

Although the schemes described in this paper are probably representative of the majority of analysis techniques used by the induction community, many other schemes exist for not only estimation of the impedance

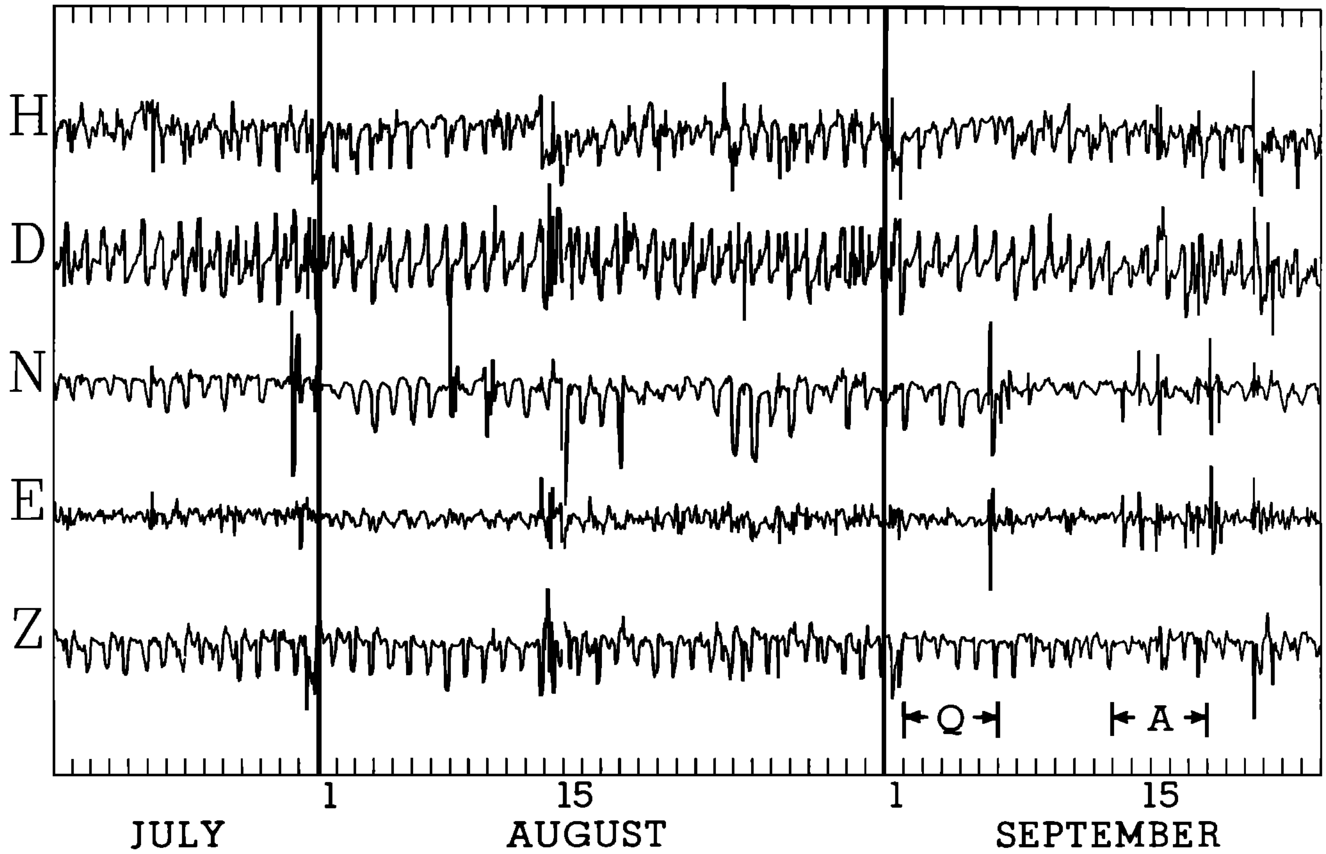

Fig. 2. Time variations of five components of the electromagnetic field observed during the interval June 18 to September 23, 1985. H, magnetic horizontal north-south; D, magnetic horizontal east-west; N, telluric horizontal north-south; E, telluric horizontal east-west; Z, magnetic vertical. The full scale deflection is 750 nT and 750 mV/km for the magnetic and electric components, respectively.

elements from time series but also for bias reduction. For example, the iterative signal-to-noise ratio enhancement schemes of *Kao and Rankin* [1977] and *Lienert et al.* [1980], the iterative bias reduction weighting scheme of *Gundel* [1977], the cross-frequency method of *Dekker and Hastie* [1981], bispectral analysis [*Haubrich*, 1965; *Hinich and Clay*, 1968], frequency-time analysis [*Welch*, 1967; *Jones and Hutton*, 1979; *Jones et al.*, 1983], complex demodulation [*Bingham et al.*, 1967; *Banks*, 1975], singular value decomposition analysis [*Park and Chave*, 1984], Cayley's factorization theorem [*Spitz*, 1984], L_1 norm analysis [*Turk et al.*, 1984], coherence sorting [*Stodt*, 1986], maximum entropy analysis [*Tzanis and Beamish*, 1986], and the most one-dimensional response [Larsen, 1989] amongst others. We suggest to all authors of processing codes that they process our data with their schemes to determine the advantages and disadvantages compared to the methods discussed herein.

2. Data

2.1. *Instrumentation*

The data analyzed and discussed in this paper were recorded at one of the EMSLAB Lincoln Line long-period land MT sites. The site was 10.6 km from the ocean and located on the Coast Range sediments (site 1 [*Wannamaker et al.*, this issue). Instrumentation consisted of an EDA flux gate magnetometer [*Trigg et al.*, 1970], Trigg telluric amplifiers [*Trigg*, 1972], active two-pole Butterworth filters (magnetic channels: low-pass -3-dB point nominally 40 s, no high-pass filters; telluric channels: low-pass -3-dB points nominally 10 and 40 s of the two cascaded filters, high pass -3-dB point nominally 30,000 s), and a 12-bit Datel cassette data logger, all housed in an insulated aluminium case for thermal protection. The electrode lines were 55 m and 65 m long for the N-S and E-W (geomagnetic) lines, respectively, and power was provided by five 1.25-V, 2000-A h air cell batteries. The five components of the time-varying electromagnetic field were sampled every 20 s with an identifying hour mark to facilitate error detection, and timing was generally accurate to better than a few seconds as noted at the weekly cassette-changing visits.

For two of the analyses, magnetic field measurements from identically instrumented locations some 30 and 136 km farther to the east (long-period sites 4 and 13 on the Lincoln Line [*Wannamaker et al.*, this issue]) were taken to facilitate remote reference processing of these data.

2.2. *Time Series*

Figure 2 illustrates the 10 weeks of data from site 1 during the EMSLAB observation period, July 18 to

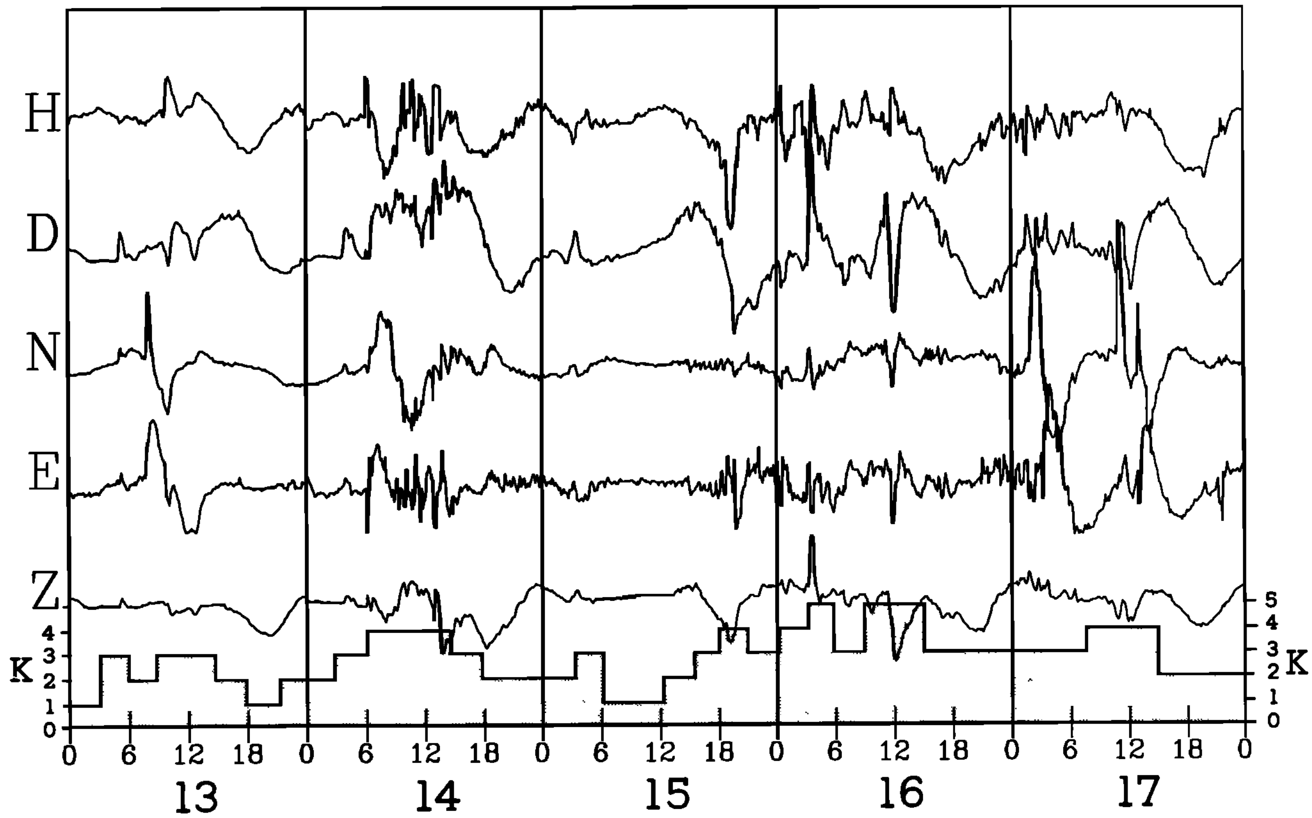

Fig. 3. Time variations of five components of the electromagnetic field observed during the quiet interval of September 2–6, 1985. The full scale deflection is 350 nT and 350 mV/km for the magnetic and electric components, respectively. The 3-hour Victoria Magnetic Observatory K indices are plotted in histogram form at the base of the figure.

September 23, 1985. These data have already been treated for gross errors using an objective scheme. Short (< 1 min, i.e., three data points) missing or erroneous data segments were interpolated based on a five-point median filter approach; that is, data gaps were infilled with the median of the two points on either side of the gap. Longer sections of missing or erroneous data were marked and left untreated. Single-point steps (boxcar shifts) were removed by despiking first differences and then reconstituting the time series using five-point median interpolation.

The quiet time diurnal variation (S_q) is apparent on all components, and storm modulations of this pattern are particularly evident at the end of July, mid-August, and mid-September. Even on the coarse scale of Figure 2 it is possible to detect several errors and problems still present in the data as released to all participants in this comparison. In particular, the spikes on the telluric components on July 30, August 8, and September 5 (see also Figure 3) obviously have no magnetic counterpart and are potentially a substantial noise source.

The quiet period chosen (Q) was the 5 days beginning 0000 UT on September 2, 1985, and the five time-varying EM components observed at the site are illustrated in Figure 3 (note that the data are plotted at more than twice the sensitivity of Figure 2). There are 21,600 samples in this window, and telluric noise spikes between approximately 1100 and 1500 UT of ≈200 mV/km amplitude completely contaminate and dominate the data. The cause of these spikes is unknown but may possibly be related to sudden discharging of the capacitors in the high-pass filter stages followed by the recharging which requires a time of order the high-pass -3-dB cutoff of 30,000 s, or 8.33 hours. Also apparent from Figure 3 is a daily event, most obvious in the B_y component (D), of two baylike features each of 1 hour duration separated by approximately 1 hour. This polar substorm event comes progressively earlier and with progressively diminishing amplitude each day and possibly resulted in contamination of the MT impedance elements in some of the analyses at these periods (see section 4.2). Shown on the base of Figure 3 in histogram form are the Victoria Observatory 3-hour K indices. K indices are a local quasi-logarithmic measure of geomagnetic activity [*Maynaud*, 1980] and have 10 classes between K=0 (magnetic quietness) and K=9 (magnetic storm) with the upper threshold for K=0 corresponding to a range of 6.5 nT and the lower threshold for K=9 to 650 nT for Victoria. These K indices confirm the visual observation that apart from the latter part of the day on September 6, there is little activity during the interval. The 24 hours beginning 0600 UT Septem-

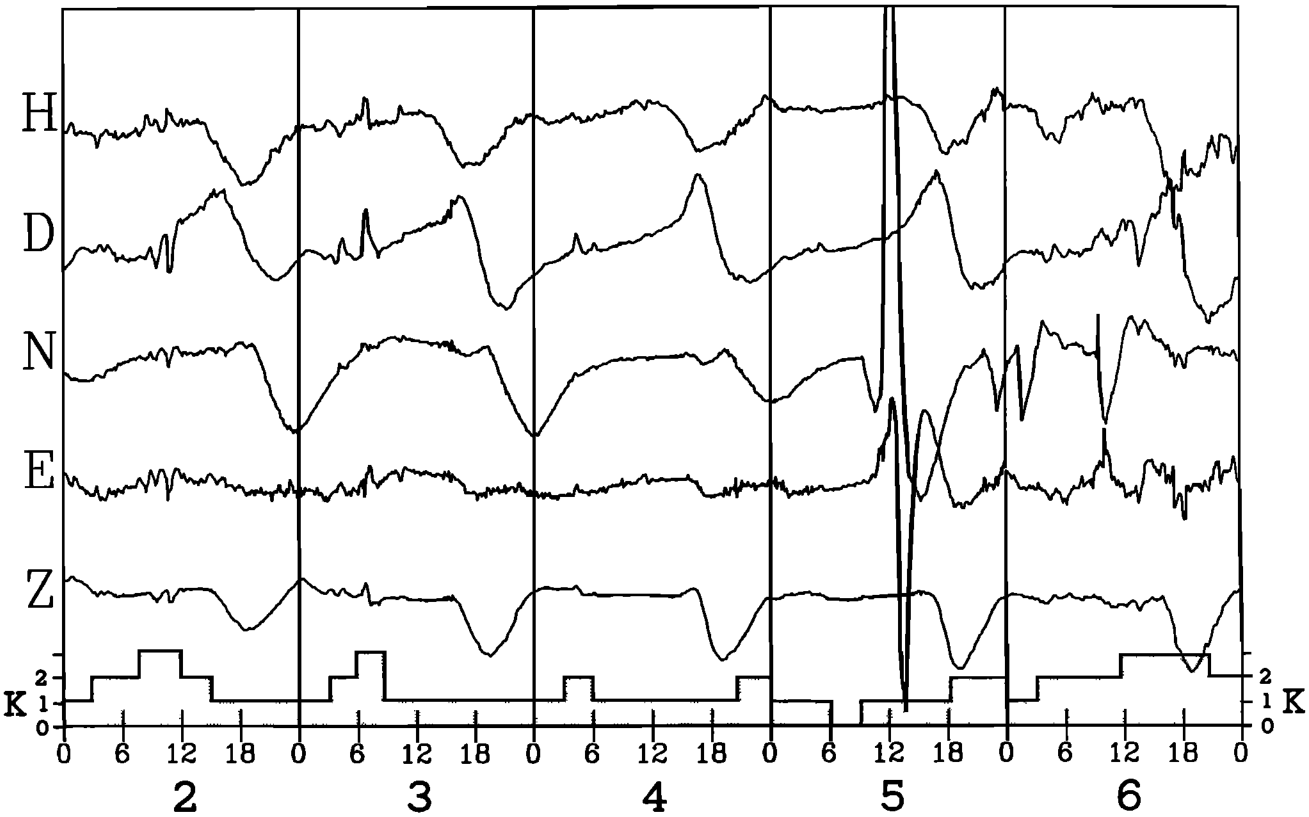

Fig. 4. Time variations of five components of the electromagnetic field observed during the active interval of September 13–17, 1985. The full scale deflection is 500 nT and 500 mV/km for the magnetic and electric components, respectively. The 3-hour Victoria Magnetic Observatory K indices are plotted in histogram form at the base of the figure.

ber 4 are defined as an Extremely Quiet Period according to the International Association of Geomagnetism and Aeronomy (IAGA) definition by having planetary 3-hour K_p indices that do not exceed 1+. (The K_p index is scaled in 28 classes from 0o, 0+, 1-, to 9o, and is a weighted average of a selection of local K indices with the weights reflecting geomagnetic latitude and local time.)

In contrast, the active period chosen (A) was the 5 days commencing 0000 UT on September 13, 1985, and the data and Victoria Observatory K indices are illustrated in Figure 4. Midday on September 16 is particularly active, with K indices of 5 and K_p indices of 7- and 6o. Obvious in the data, particularly in the telluric east-west component (E), is the ssc (sudden storm commencement) at 0601 UT on September 14, 1985. Some telluric spikes are evident in the data, e.g., ≈0730 UT on September 13, ≈0200 UT on September 17, but these are of much smaller amplitude than those during the quiet interval.

3. Analysis Techniques

Brief descriptions are given here of the processing steps taken for each of the eight analyses (see Table 1). As the -3-dB high-pass cutoff period for the telluric fields was nominally 30,000 s, the aim of the schemes were to derive estimates out to 10,000 s (in 5 days there are 43 cycles at 10,000 s).

3.1. *Method 1: Single Station Conventional Spectral Analysis*

The time series were visually inspected and nonoverlapping intervals selected with sufficiently high signal-to-noise ratio. Intentionally, outliers and spikes were not removed. For each section, (1) the first and last 10% were cosine tapered, (2) the windowed series were transformed into the frequency domain (using a standard FT algorithm), and cross-spectra computed, (3) a Parzen window in the frequency domain was applied to smooth the cross spectra such that there were typically seven estimates per decade.

TABLE 1. Short Descriptions of the Methods Used

Method	Description	Source
1	conventional spectral analysis	Bahr
2	conventional spectral analysis	Auld
3	conventional spectral analysis	Jones
4	weighted cascade decimation	Jones
5	remote reference	Chave
6	robust cascade decimation	Jones
7	robust	Egbert
8	robust remote reference	Chave

From these ensemble estimates of smoothed spectra, averaged estimates were derived by stacking (without weighting based on some quality measure) and the MT transfer functions were estimated from these averages of the spectra. The confidence intervals of the transfer functions were estimated (assuming Gaussian noise) for 5% error probability. Both upward and downward biased estimates were computed.

3.2. *Method 2: Single-Station Conventional Spectral Analysis*

The processing sequences used for method 2 are described by *Law et al.* [1980]. The data were plotted and nonoverlapping 8-hour time sections of 1440 points were selected on the basis of suitable (moderate activity) geomagnetic activity. The selected time sections were edited to correct for spurious errors. Then, for each time section, (1) the mean and trend were removed, and end effects minimized by use of a cosine taper on the first and last 10%, (2) the 1440 point data series were padded with zeroes to expand each sample to 2048 points, (3) a standard FFT algorithm was used to transform the data to the frequency domain, cross-spectra computed, and a Parzen frequency window applied to neighboring Fourier harmonics, (4) the MT transfer functions were then derived from these smoothed cross spectra.

These estimates of the transfer functions from each time section were then averaged and the mean and standard deviations calculated to give the final transfer function estimates. For the 5-day intervals, typically 5–10 time sections were analyzed, whereas 25 time sections were taken for the total data set. Additionally, for the total data set, the 15% highest and lowest estimates of the transfer functions, at each center frequency, were eliminated prior to averaging. Note that as only 1440-point length series were analyzed, estimates could only be obtained out to a maximum possible period of ≈3000 s (assuming 10 cycles are needed in the time interval).

3.3. *Method 3: Single-Station Conventional Spectral Analysis*

The processing sequences used for method 3 are described in detail by *Jones et al.* [1983]. For the 5-day intervals, the data were split into five 24-hour long time sequences of 4320 points which were then padded with zeroes to 8192 points. The smoothed cross spectra from each of the 5 days were normalized by the power in the horizontal magnetic field components, then averaged in a weighted manner using the coherence functions defined by *Jones* [1981] as weights. Note that spurious data are not rejected subjectively but are downweighted in the averaging stage.

Although methods 1, 2, and 3 represent "conventional" techniques, it should be appreciated that they differ in one important aspect; methods 1 and 3 involve averaging spectra (unweighted averaging for method 1, weighted averaging for method 3) to obtain the transfer function from the averaged spectra, whereas method 2 involves averaging the estimates of the transfer functions from each section with 15% trimming.

3.4. *Method 4: Single-Station-Weighted Cascade Decimation*

The cascade decimation scheme of *Wight et al.* [1977] (see also *Wight and Bostick* [1980]; code published by *Bostick and Smith* [1979]), was modified to incorporate ministacks of eight discrete Fourier transform harmonics. A 32-point base was used, with two estimates (sixth and eighth harmonics) from each series, with a decimation factor of 2. This yielded the first estimate at 4 times the sampling interval, or 80 s for these data, with 6-7 points/decade and seven decimates to cover the periods up to 10,000 s. The ministacks were averaged into the total stack using the inverses of the geometric means of the variances of the off-diagonal elements of Z as weights. This stacking cascade decimation scheme is thus identical to the in-field processing scheme of the MT system from Phoenix Geophysics Ltd. (Toronto). Both upward and downward biased estimates were computed.

Note that the use of inverse variances as weights incorporates not only signal criteria but also downweights events for high coherence between H and D (i.e., no independent information), γ^2_{HD}, and downweights events for low multiple coherence between the output electric component, γ^2_{NHD} and γ^2_{EHD}, and the input magnetic components (low correlation signal). The latter two are equivalent to requiring a high partial coherence between the output electric component of interest and the input magnetic component of interest, removing the effect of the other magnetic component on the electric component in an LS sense, e.g., $\gamma^2_{ND.H}$ for the Z_{xy} component. It has been the experience of one of us (A.G.J.), and of Phoenix, that inverse variances as weights gives superior estimates of Z than using multiple coherences alone as weights.

3.5. *Method 5: Remote Reference Conventional Spectral Analysis*

The processing scheme is the nonrobust method described by *Chave and Thomson* [this issue]. The data were first plotted and inspected for gross errors which were either corrected (if of short duration) or noted for exclusion in subsequent processing. A subset length of 12 hours (2160 points) was selected and a time-bandwidth 4 prolate data window [*Thomson*, 1977] was applied to each subset with 70% overlap between adjacent sections. The discrete Fourier transform was then taken for all data series, including the remote horizontal magnetic field. A set of center frequencies were selected to give eight estimates per decade, and arithmetic section and band averaging without overlap was applied in the usual way to give the remote reference impedances. The jacknife [*Chave and Thomson*, this issue], a nonparametric error estimator which is relatively insensitive to departures from the usual Gaussian assumption implicit in parametric approaches, was used to obtain error estimates.

3.6. *Method 6: Single-Station Robust Cascade Decimation*

Method 4 was modified by incorporating the transfer function improvement scheme of *Jones and Jödicke* [1984]. The eight harmonics that composed each ministack were removed and replaced, in turn, in a jacknife approach to determine which harmonic, when omitted, led to a minimum in the variances of the off-diagonal MT impedances. This was repeated iteratively for progressively fewer harmonics until the variances could not be reduced further by additional rejection. These ministacks, which contained differing numbers of harmonics, were then combined using the same jacknife scheme so as to minimize the variances of the final estimates of the off-diagonal elements of Z. Error estimates were obtained using standard statistical methods that assume the noise is Gaussian [*Goodman*, 1965; *Bendat and Piersol*, 1971, pp. 204-207] Both upward and downward biased estimates were computed.

3.7. *Method 7: Single-Station Robust Processing*

The processing scheme, an extension of the regression M-estimate, is described in detail by *Egbert and Booker* [1986]. Single-point outliers were cleaned up by a median and median absolute deviation seven-point filter scheme. Then, the data were Fourier transformed by an approach similar to cascade decimation with a 128-point length base and a decimation factor of 4. The 25% overlapping 128-point data segments were conditioned by prewhitening with a first difference filter and windowing by a time-bandwidth 1 prolate data taper [*Thomson*, 1977] prior to fast Fourier transformation. Fourier harmonics with power in the horizontal magnetic fields less than a certain minimum, chosen on instrument noise considerations, were rejected. In the estimation of the transfer functions, a combination of band and section averaging was used with a bandwidth of 25% of the center frequency using a regression M estimate implemented as described by *Egbert and Booker* [1986]. The errors were derived using the standard asymptotic approach as described by *Egbert and Booker* [1986].

3.8. *Method 8: Remote Reference Robust Processing*

This method is the robust counterpart of method 5. It differs only in the additional step of iterative reweighting of the LS response, as described in detail by *Chave et al.* [1987] and *Chave and Thomson* [this issue], and is very similar to method 7. An initial LS solution is applied to get a set of regression residuals which are compared to a Gaussian model. Residuals which are larger than expected yield weights on the corresponding data sections which reduce their influence. This continues iteratively until the residual sum of squares does not change significantly. The final residuals are Gaussian, yielding smoother impedances. The jacknife is used to get error estimates.

4. Analyses

For the purposes of comparison, we present the MT apparent resistivities and phases for the analyses of all 67 days, and the apparent resistivities only for the analyses of the two 5-day time segments. Obviously, badly scattered magnitudes with large errors will have associated badly estimated phases. It should be remembered that under the limitation that the noise components on the channels are uncorrelated, the phases are unaffected by bias errors.

4.1. *All Data*

The MT transfer functions from analyses of all 10 weeks of data available for three of the methods (2, 6, and 8), and for the first 5 weeks of data for one of the methods (7) are illustrated in Figure 5. The horizontal magnetic fields observed at site 4 were used as the remote reference fields for method 8. It is apparent that even with all 10 weeks of data, conventional spectral analysis schemes (top left), represented by method 2, may not necessarily give smooth estimates with low associated statistical error. In contrast, the three robust schemes, methods 6 (top right), 7 (bottom left), and 8 (bottom right) all give extremely smooth estimates with very low standard errors (generally <1%) in the 100–1000 s range.

Other points to note are as follows:

1. The ρ_{xy} upward and downward biased estimates for method 6 appear to be separated by a virtually frequency independent multiplicative factor. This could be explained in terms of noise sources on either the D magnetic component or the E telluric component that varies with period proportionally with the strength of the field component itself, or of a correlating noise source between the components. The downward biased ρ_{xy} estimates of method 6 are, to within its statistical estimators, virtually identical to those of method 7 which would imply that the noise source is on the E telluric component. However, when compared to method 8 the upward biased estimates are "correct" at short periods (<400 s), whereas the downward biased estimates are "correct" at the longer periods (>1000 s). This would imply that the uncorrelated noise is most significant in the magnetic fields at short periods and in the electric fields at longer periods, which is consistent with the results obtained from a multiple station analysis of a separate set of five long-period EMSLAB MT stations [*Egbert and Booker*, this issue].

2. All of the single-station B-field reference estimates (methods 2 and 7 and the downward biased estimates of method 6) appear to give significantly biased estimates of apparent resistivities at the short periods.

3. Method 7 appears to give somewhat erratic ρ_{xy} and ϕ_{xy} estimates at periods close to 1 hour. This may be due to the nonuniform and energetic source fields

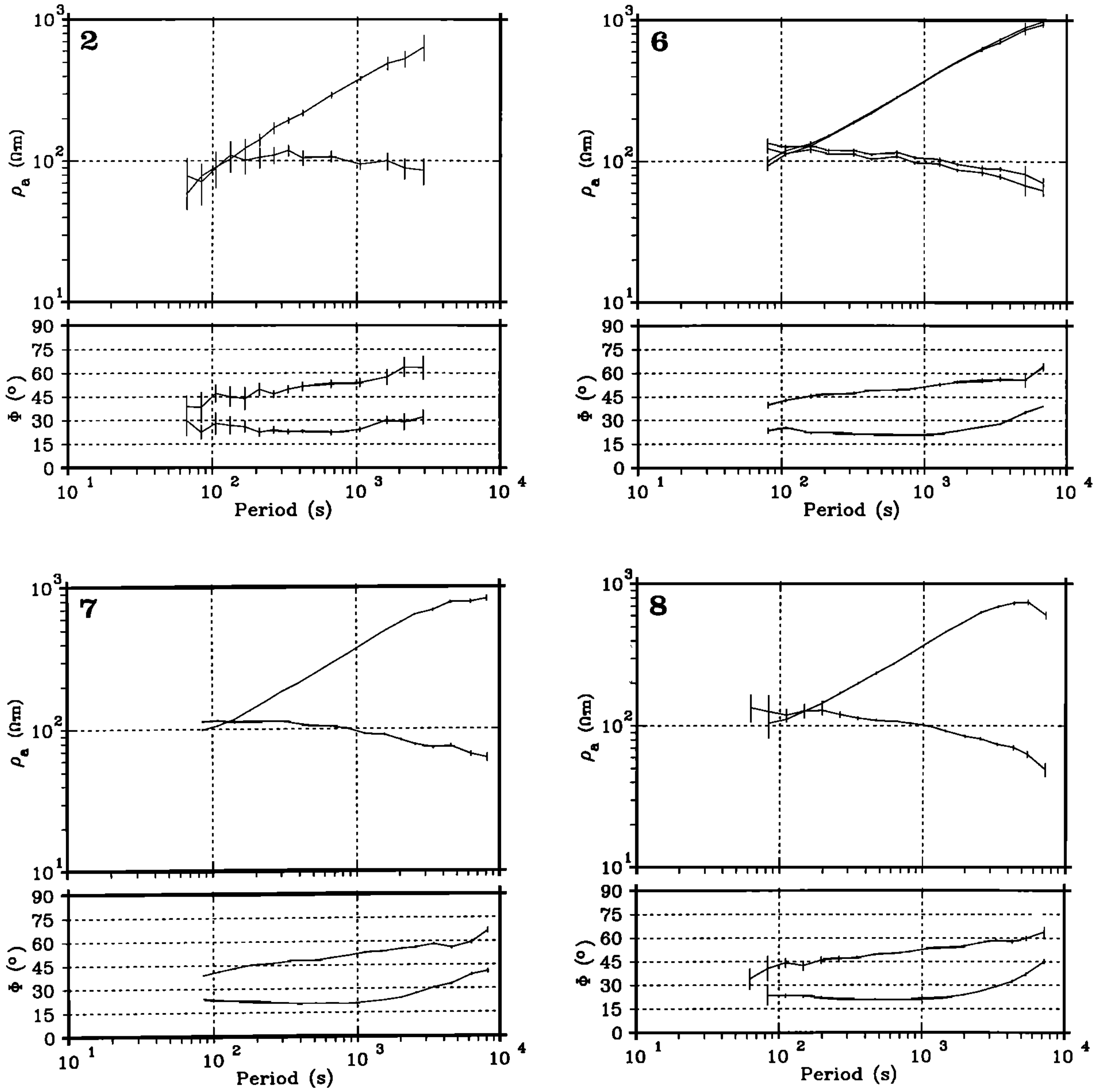

Fig. 5. MT analyses of all data using methods 2 (upper left), 6 (upper right), 7 (lower left), and 8 (lower right). Illustrated are the ρ_{xy} and ρ_{yx} apparent resistivities and their phases (note that the ϕ_{yx} phases have been rotated into the first quadrant for clarity), with associated standard errors. Note that for method 6 there are both upward and downward biased estimates.

from magnetic bay disturbances, whose characteristic periods are in this range, dominating the response over the uniform background field contributions, or it may be due to only analyzing the first 5 weeks of data instead of the whole 10 weeks as did the other three methods.

4. Method 8 gives the smoothest results at long periods, but a direct comparison between the estimates from the different techniques is difficult due to variations in bandwidth between the methods at these periods.

5. The estimates for method 8 at the longest periods (>6000 s), particularly of ρ_{yx}, also appear to "droop" and be downward biased when compared to the corresponding estimates of methods 6 and 7. The actual cause of this droop is unclear. It could imply that the inherent assumptions for remote reference processing have become invalid, i.e., that there existed a correlating "noise" source on the H component over a distance of some 30 km, although one would expect that single-station processing would also exhibit the same effects.

6. The results for method 8 at the shorter periods appear to be more scattered than those for methods 6 and 7, particularly in ϕ_{xy}. Does this just reflect the relative inefficiency of remote reference processing, or does it reflect the effects of outliers in the input (or reference?) channels. The presence of noise, and the possibility of outliers, in all channels makes this a very nonstandard robustness problem. Further work will be required to elucidate the cause of this problem.

7. The larger error bars at the shortest periods for method 8 compared to methods 6 and 7 may be due to the inherent inefficiency of remote reference method over single-station ones (see introduction) or may be due to erroneous assumptions being made about the nature of the noise contributions by methods 6 and 7, whereas jacknife errors (method 8) should be relatively more robust to violation of several assumptions. This difference is particularly noticeable in comparison with method 7 and suggests that the standard asymptotic error estimates used for method 7 are overly optimistic.

4.2. *Quiet Period*

The MT apparent resistivities from analyses of the data for the quiet interval (Figure 3) are illustrated in Figure 6. Note that the ρ_{xy} estimates have been shifted downward by one decade for clarity. Also displayed on the figure are the estimates from method 8 (excluding the first and last estimates) for all data as a reference. The horizontal magnetic fields observed at site 13 (136 km distant) were used as the remote reference fields for method 5 and 8. The following points are worthy of note:

1. Generally, where both upward and downward estimates have been computed for single-station processing (methods 1, 4, and 6), the two estimates bracket the "truth" as given by the reference lines.

2. The standard LS schemes (methods 1, 2, and 3) performed particularly badly with biased estimates that would lead to erroneous interpretations. This is particularly true for method 3 where, because of the apparent consistency, one might attempt an interpretation. The ρ_{yx} estimates are smoothly varying with period but exhibit a steeper gradient than is real, and the ρ_{xy} estimates are all some one third of a decade downward biased.

3. Remote reference processing (method 5) obviously aided correct estimation of the MT apparent resistivities, but bias effects are apparent at the shortest periods (< 100 s) and also in the ρ_{xy} estimates at around 2000 s. The scatter of the estimates is still substantial however.

4. Non robust cascade decimation processing (method 4) gives reasonable ρ_{yx} estimates, but the robust equivalent (method 6) performed far better for ρ_{xy} with the lower inherent signal-to-noise ratio.

5. Many methods appear to give obviously biased results in the period range 1000–3000 s, particularly in the ρ_{xy} estimates. This is possibly due to the nonuniform daily polar substorms discussed above (Figure 3) which appear generally to upward bias the ρ_{xy} estimates.

6. Methods 5, 6, 7, and 8 all give reasonably interpretable responses, particularly of the ρ_{yx} estimates, but the robust methods 7 and 8 are superior even though there is evident bias error in the ρ_{xy} estimates for method 7 for periods in the 100–1000 s range.

7. Remote reference robust processing gave the "best" estimates lying within a few percent of the "truth" (with the exceptions of those estimates at periods shorter than 100 s). Note that the long-period "droop" in ρ_{yx} is not evident and that these estimates of both ρ_{xy} and ρ_{yx} are more precise (smaller errors) at the shorter periods, than those for the all data analysis (Figure 5).

4.3. *Active Period*

The MT apparent resistivities from analyses of the data for the active interval (Figure 4) are illustrated in Figure 7, and once more, the ρ_{xy} estimates have been shifted downward by one decade for clarity. Again, also displayed on Figure 7 are the estimates from method 8 (excluding the first and last estimates) for all data as a reference, and the horizontal magnetic fields observed at Site 13 (136 km distant) were used as the remote reference fields for method 5 and 8. The following points are worthy of note:

1. Again, where both upward and downward biased estimates have been computed the two estimates appear to bracket the "truth."

2. The conventional schemes (methods 1, 2, and 3) do a fairly reasonable job for the ρ_{yx} estimates but exhibit large bias and random errors for the ρ_{xy} ones. The error estimates are substantially larger than for the robust methods.

3. All methods appear to have difficulty estimating the ρ_{xy} apparent resistivities between 2000 and 4000 s. That this is evident on the remote reference processed results (methods 5 and 8) as well as on the upward biased results of method 6 is indicative perhaps of nonuniform source field problems rather than noise contributions.

4. Either remote reference processing (method 5), or robust processing (methods 6 and 7), or both (method 8) can extract excellent estimates with small random errors and low bias errors from just 21,600 samples of data. However, the long-period error estimates are much smaller for the robust schemes, especially for the ρ_{xy} component.

5. Again, the long-period "droop" in ρ_{yx} for method 8 is not evident, and the estimates of both ρ_{xy} and ρ_{yx} are more precise (smaller errors) at the shorter periods, than those for the all data analysis (Figure 5). This would lead one to believe that the method 8 analyses of all the data became contaminated at the longest periods by noise sources which were correlated over 30 km, but not over 135 km (all data analysis used site 4's magnetic components for remote reference, whereas the 5-day intervals used site 13's magnetic components). Alternatively, the estimates from all of the data may be contaminated to a larger degree by source-field effects [*Egbert and Booker*, this issue].

5. CONCLUSIONS AND OTHER REMARKS

Four conclusions are obvious from this comparison.

1. *Travassos and Beamish* [1988, p. 390] state "If the (coherence-based) selection procedure can be termed adequate then simple spectral stacking of individual so-

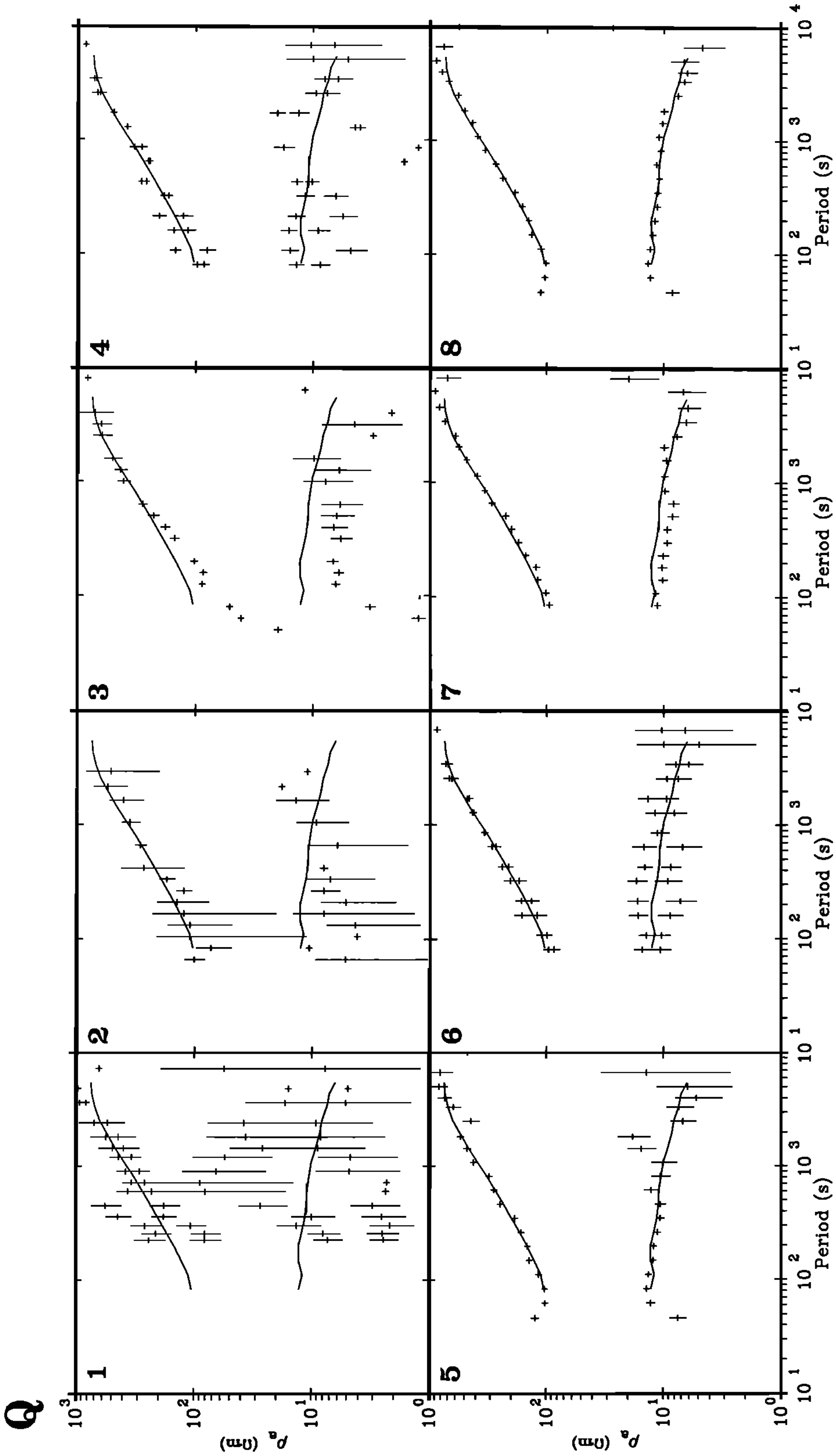

Fig. 6. Analyses of the data from the quiet interval (Figure 2). Illustrated are the apparent resistivities alone, with their associated standard errors (note for method 1 the error bars indicate 95% confidence, or approximately two standard errors), and the ρ_{xy} estimates have been shifted downward by one decade for clarity.

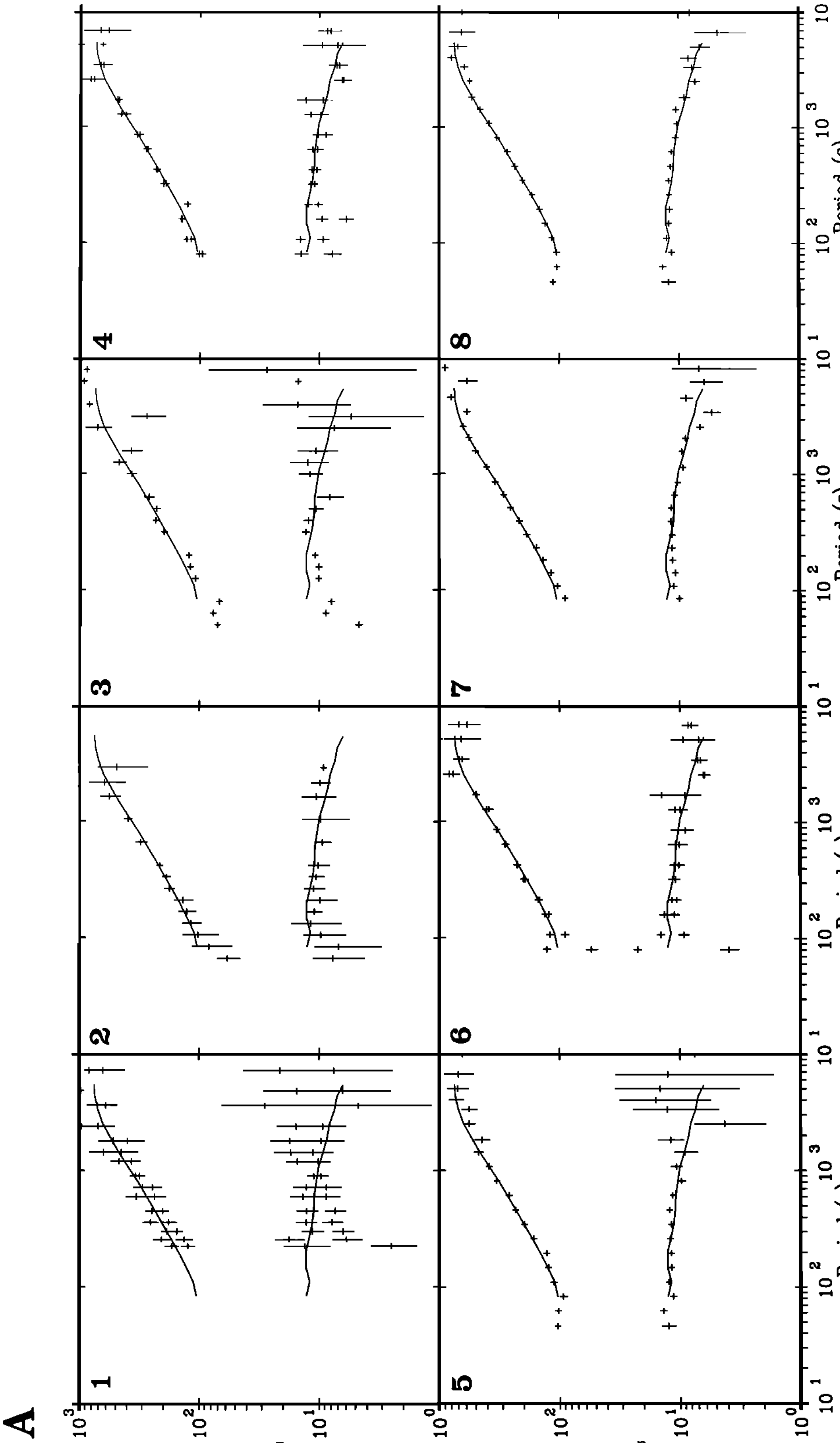

Fig. 7. Analyses of the data from the active interval (Figure 3). Illustrated are the apparent resistivities alone, with their associated standard errors (note for method 1 the error bars indicate 95% confidence, or approximately two standard errors), and the ρ_{xy} estimates have been shifted downward by one decade for clarity.

lutions works well and there is no need to resort to a statistically more robust treatment such as described by *Egbert and Booker* [1986]." In this paper we have demonstrated that spectral stacking (methods 1 and 3) can not "be termed adequate," and we have shown that robust schemes, such as methods 6, 7, or 8, give superior results. This is particularly true of short pieces of data of low signal-to-noise ratio, such as the quiet 5 days.

2. To minimize bias errors, whenever possible, remote reference processing should be undertaken. If the remote components are not available, then both the upward and downward biased estimates should be computed to give a qualitative estimate of the magnitude of the bias problem. Considering the possible sources of noise and their relative contributions in varying frequency bands, it should not be expected that either of these biased estimates are the truth, but that (hopefully) they bracket the truth (see, e.g., "all data", point 1). Robust single station estimates can still be significantly biased by noise in the input channels, particularly during periods of low activity.

3. Remote reference processing can still lead to bias errors and is not the panacea once perhaps believed. The noise correlation distances in remote reference MT have been investigated by *Goubau et al.* [1984] and *Nichols et al.* [1988], but obviously further work is necessary on the sources of noise contributions and the effects of possible nonuniform fields that are coherent between the local and remote sites. Coherent noise sources are possibly the explanation for the long-period "droop" in the ρ_{yx} estimates in the analyses of all the data for method 8 (Figure 5) which used magnetic components 30 km distant as the remote references, whereas in the analyses of the 5-day intervals (Figures 6 and 7) method 8 used magnetic components 135 km distant. The cause of the short-period "droop" in the estimates from method 8 is unknown, but we do not ascribe this to source effects.

4. Look at the data! If the data contain obvious noise, then interpolate short segments and remove large segments.

For in-field processing of data, adhoc robust schemes that require relatively few computations, such as method 6 [*Jones and Jödicke*, 1984], could be applied to the data given today's computing technology. If more advanced and more powerful computers are available in the field, then rigorous robust schemes such as methods 7 and 8 [*Egbert and Booker*, 1986; *Chave et al.*, 1987, *Chave and Thomson*, this issue], should be adopted.

Although this work has concentrated on long-period data, comparisons by one of us (A.G.J.) over the last 4 years using a Phoenix MT data acquisition system has shown that the robust scheme method 6 always gave superior results to the nonrobust scheme method 4. While this does not in itself constitute a rigorous comparative study, it does suggest that notwithstanding the very different noise sources at higher frequencies compared to those in the data studied herein, robust methods should always be used.

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