AN ABSTRACT OF THE THESIS OF

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Title: Simulation of National Cow Inventories and Calf Crop, 1965 to 1981: Projections to 1987

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Ray Brokken

This study simplified, refined and updated the cattle cycle simulation model developed by Thomas L. Nordblom (1981). This refined model was used to forecast the numbers of beef cows, calves born, heifers recruited and cows culled through 1987.

The hypothesis that the historical cattle cycle has been related to investment incentive differences across cow ages through time, resulting each year in changes in herd age structure, performance and potentials for adjustment in subsequent years, was maintained in the current model.

To reduce the deviations of the simulated annual inventories of beef cows and heifers, calves born and cows culled from the historical data series a review of the general model structure, the biological constraints and the intermediate functions was performed. Many model modifications resulted, including a reduction in the simulated time series from 28 years to 17 years, changes in the definition of the simulated cow inventory and simplification of many of the intermediate functions.

The model is built on the economically important biological attri-

butes of conception rates, health rates, cow survival rates, cull cow body weights, calf survival rates and weaning weights. Based on these biological functions cow culling rates and expected calf sales are defined as management expectation and producer profit expectation functions, respectively.

Present salvage value (PSV_j) estimates for pregnant and nonpregnant cows of each age are defined as the product of their respective body weights and prices. Present value of breeding (PVB_j) estimates, based on a two year maximum planning horizon are calculated as the sum of the expected net culling revenues and the present value of expected calf sales minus maintenance costs for each age of cow. The ratio of the PVB_j estimates to PSV_j estimates is calculated for each of the 26 discrete age and pregnancy classes of heifers and cows, in each year from 1965 through 1982. These V-ratios, in turn, are decision variables for determining the proportions of animals in each class to be retained in the herd, simulated by the national age distribution inventory model.

The age distribution inventory model produces annual summations of the four simulated cattle inventories for comparison with the objective historical series of January 1 inventories of beef cows and replacement heifers, and annual numbers of cull cows slaughtered and beef calves born. Given its few exogenous price and cost variables, simple biological relationships and management expectations, the model is able to track the historical numbers of beef cows and calves born quite well.

Mean proportional absolute deviations (MPAD) of the simulated series from the objective historical series were computed in addition to simple correlation coefficients and Theil's coefficients of inequality. The tracking behavior of the model with respect to the historical series of beef cows, heifers, culls and calves born, improved considerably over Nordblom's model. The MPAD for beef cows declined from the previous model's low of .026 to .009. The MPAD for heifers recruited, cows culled and calves born declined from .172, .261, and .036 to .075, .227, and .023, respectively. Theil's coefficients of inequality for beef cows, calves born, heifers recruited, and cows culled were .300, .767, .568, and .823, respectively.

To test the age structure hypothesis, a simulation run was made with parameters set to reflect the assumption that cows of all ages perform the same. The model's tracking ability was not improved by the homogeneity assumption. This could be due, in part, to a lack of the resources (time and money) needed for proper fine tuning of the model. The homogeneous cow run performed better than a naive forecast, with Theil's U statistic all below 1.0. Thus, it should not be easily discarded.

The final simulation run known as STRINGHAM was used as a base for forecasting the four cattle inventory series through 1987. In order to forecast, the exogenous price and cost series were extrapolated in real 1983 dollars. Prices of several of the inputs, identified in the cost of production budgets, followed the CPI quite closely from 1950 to 1983. Thus, the current 1983 price for these items (salt and minerals, fuel and lubrication and building and machinery) was extrapolated through 1987. The U.S.D.A. corn and choice slaughter steer price forecasts, deflated to 1983 dollars, were used for projecting the cost series for inputs whose prices were highly correlated, directly or indirectly, with these forecasts. This group included utility cow prices, feeder calf prices, bull charges, pasture rent and hay prices.

Using the projected exogenous cost series two alternative forecasts of cattle inventories numbers were made. The scenarios varied only in the projected cost of loans. The first forecast was made holding the cost of short-term loans constant at its 1981 level. The second forecast was made assuming a five percent per year decline in the interest rate. Forecast one showed a continual decline in cow numbers from 1981 through 1987. Forecast two showed a decline until 1986 with cow numbers increasing in 1987.

The simulated numbers of beef cows, calves born, heifers recruited and cows culled from 1965 through 1981 and the forecasts through 1987 are shown graphically. Simulated national beef cow herd age structure changes through cattle cycles are also shown from 1965 through 1981.

Simulation of National Cow Inventories and Calf Crop, 1965 to 1981: Projections to 1987

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> "Whoever speaks, let him speak, as it were,
> the utterances of God; whoever serves, let
> him do so as by the strength which God supplies;
> so that in all things God may be glorified through
> Jesus Christ, to whom belongs the glory and
> dominion forever and ever. Amen.
>
> 1Peter4:11

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CHAPTER 1

INTRODUCTION

Since shortly after 1880, thirteen years after annual data on cattle numbers became available, the number of cattle on farms and ranches has fluctuated up and down in a remarkably regular cyclical pattern. The first peak in cattle numbers occurred in 1890, the last in 1975 with intervening peaks in 1904, 1918, 1934, 1945, 1955 and 1965 (Figure 1).

Since the late 1940s, the cyclical nature of cattle numbers has been confined almost completely to the beef cattle sector. Triggered by falling per capita consumption of dairy products and increasing production per cow, dairy cattle numbers, in the mid-1940s, began a pattern of almost continuous decline [Petritz et al. 1981]. Thus, the cattle cycle can be more precisely thought of as a beef cattle cycle.

Figure 2 not only illustrates the remarkable regularlity of beef cow inventory cycles in the U.S., it also portrays the pronounced upward secular trend in beef cow numbers. The simulation and projection of these beef cow inventory cycles are the primary concern of this study. Following is a brief review of the hypotheses which have been purported to explain the cyclical phenomena in the number of beef cattle.

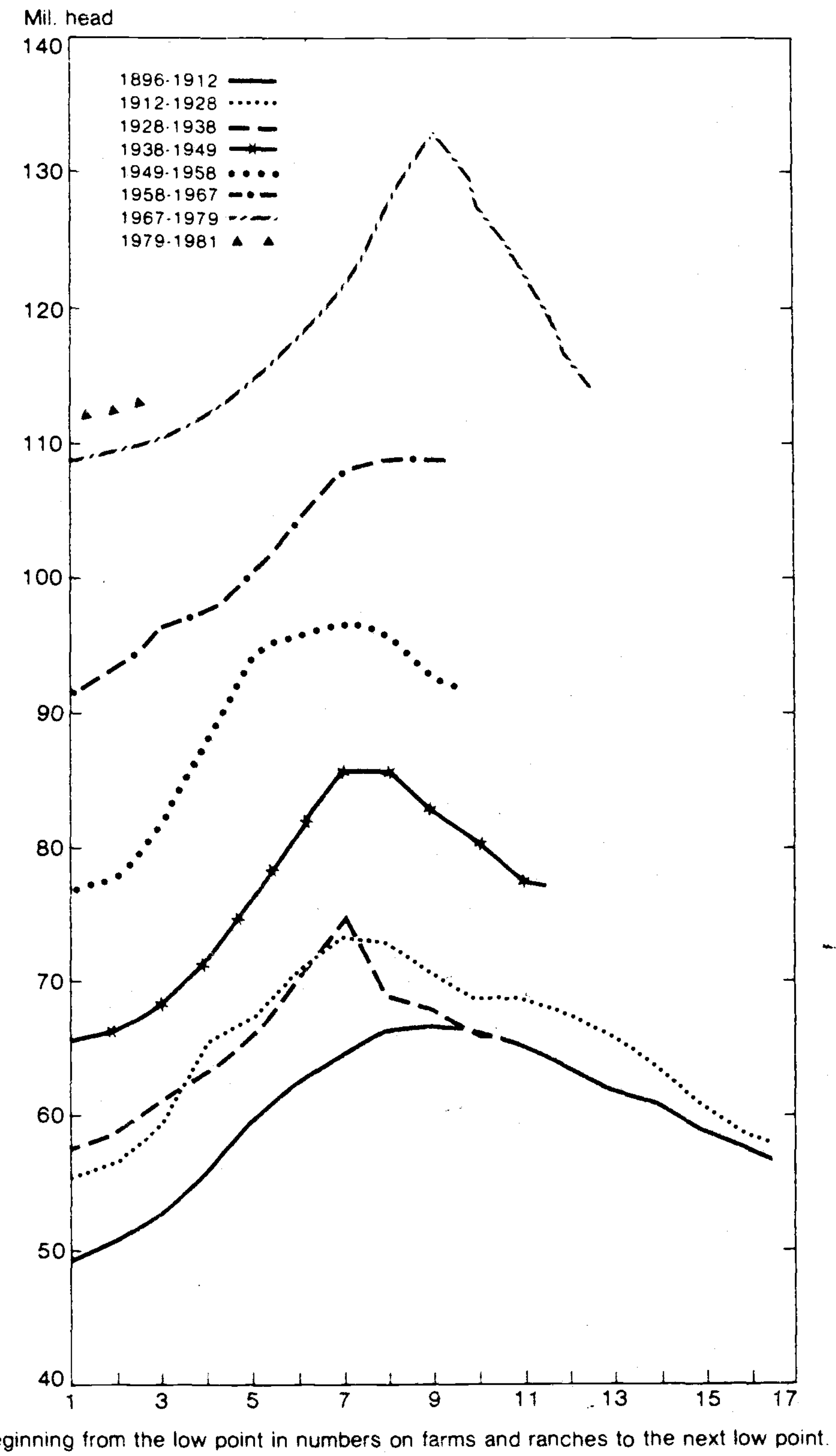

Figure 1. Cattle Inventory Cycles, 1896-1981.

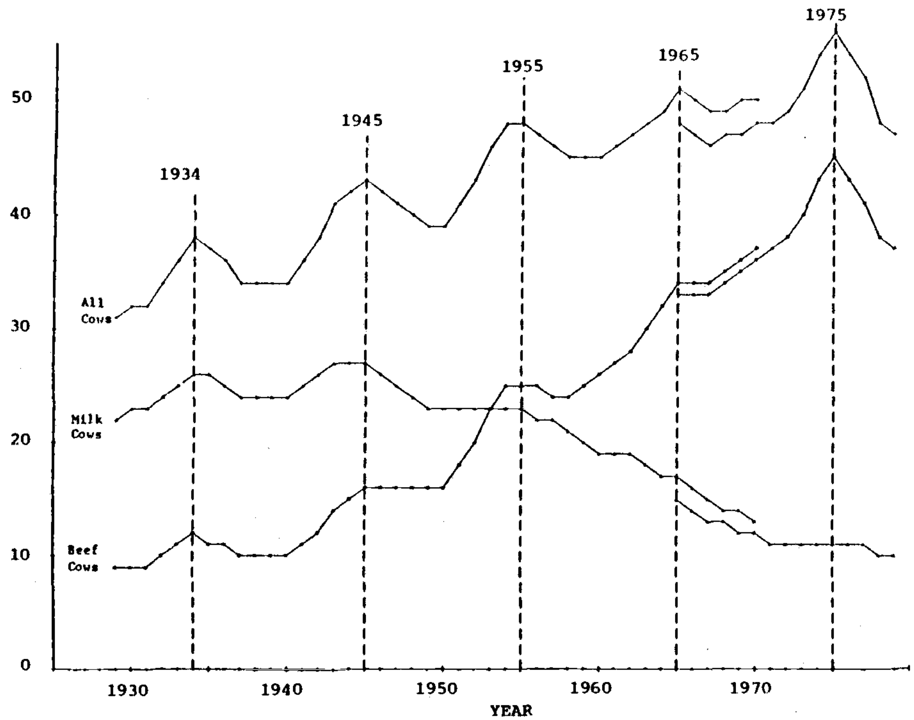

Figure 2. Cow Inventories in the United States, 1929-1979.[a/]

a/ Reported until 1970 as cows and heifers 2 years old and over; but, since 1965 reported as cows and heifers that have calved. Source: U.S.D.A..

National Cattle Cycle Literature

The periodic or wavelike patterns in cattle numbers and prices are a well-known phenomenon of the livestock industry. Various explanations concerning the nature and the causes of these cycles have been presented over the last 80 years.

For convenience, Kim's (1970) classification of the cattle cycle literature into three separate categories, has been used here. First, there are those studies that hypothesize that the cattle cycle is fundamentally the result of causal forces that are primarily exogenous to the cattle industry, e.g. shortage of grazing land, changes in animal husbandry practices, wars, etc. The second category is founded on the belief that the repetitive nature of the cattle cycle is self-generating. It focuses on such endogenous factors as producers response to price and production lags. The third category is a combination of the first two. It purports to explain the cattle cycle in terms of both internal and external factors. These various explanations of the cattle cycle will be considered under Kim's (1970) headings of exogenous hypotheses, endogenous hypotheses, and exogenous-endogenous hypotheses.

Exogenous Hypotheses

Hopkins (1926), Burmeister (1949), and Pearson (1953) are among those advancing the hypotheses that external phenomena are the major factors contributing to the cyclical fluctuations of cattle numbers.

Hopkins examined the alternating periods of increase and decrease in cattle numbers and prices from the late 1800s through the mid 1920s. He attempted to explain these cyclical occurrences in terms of such exogenous variables as the amount and availability of grazing land, the inelasticity of the industry, the changes in conventional methods of raising and fattening cattle, wars, and other factors (business cycle).

Burmeister attempted to examine each of the five cycles that had occurred in the United States between 1890 and 1949. He concluded that the cattle cycle is primarily a function of physical constraints and the economic climate of the times.

Pearson wrote a series of publications addressing the recurring phenomena of the cattle cycle. He began by submitting the hypothesis that the cycle is caused by changes in the aggregate demand for beef. Pearson [1953, p. 4948] states "... When building is active, business is good and labor is fully employed. It is therefore concluded that consumers, with plenty of purchasing power, are willing to spend larger amounts of money on beef." However, after testing the above hypothesis with respect to empirical evidence Pearson [1953, pp. 4948-50] concluded, "... There were seven periods when building activity and demand for cattle moved in opposite directions and seven when they moved in the same direction . . . The farmer demand for cattle was not consistently related to urban demand as measured by building which exhibited cyclical fluctuations. Nor was it related to urban demand as measured by the price level of business activity, neither of which exhibited regular cyclical fluctuations ..."

As an alternative to his original hypothesis, Pearson [1953, p. 4951] submitted a revised hypothesis that states that the "fluctuations in the farmer demand for cattle are due to forces found on the farms and ranges rather than urban homes . . . All cattlemen want to increase their herds as much as possible because they are optimists . . . One of the dampers on this enthusiasm for expansion is the supply of feed." Pearson supports this hypothesis by applying conventional linear regression to the supply of roughages and the change in demand for cattle. He reported that the supply of roughages as measured by the Jennings series was positvely related to changes in demand for cattle.

Endogenous Hypotheses

Included in this category are such major works as Ezekiel (1938), Lorie (1947) and Ehrich (1966). Ezekiel based his endogenous hypothesis on the self-generating mechanism known as the Cobweb Theorem. He identified [pp. 437-8] the following three conditions under which the Cobweb Theorem applies to the cattle cycle; "(1) where production is completely determined by the producers' response to price, under conditions of pure competition ... (2) where the time needed for production requires at least one full period before production can be changed, once the plans are made; and (3) where the price is set by available supply."

Lorie (1947), in his landmark study on the causes of the cattle cycle, attacks Ezekiel's (1938) assertion that the Cobweb Theorem provides an adequate explanation of the cattle cycle. He discounts Ezekiel's paper on two interrelated points, first, Ezekiel considered only two variables, prices and production. This allowed him to reason [p. 266], with respect to lags in supply, that "cases ... with a one-year lag in response all produce two-year cycles ... Case Ia, with a two-year lag in production, has a four-year period from peak to peak ..." Secondly, the word "lag" is loosely used and not clearly defined. Lorie [1947, p. 52] points out that careful reading of Ezekiel text indicates that these "lags" are "equal to the period of gestation plus the marketing age." If so, Lorie [1947, p. 52] further asserts that Ezekiel "would have had to maintain that the 'lag' in the case of cattle is seven to eight years." This certainly is not true.

Lorie [1947, p. 53] concludes that the failure of the Cobweb Theorem to satisfactorily explain the observed cycles in cattle numbers is due to Ezekiel's "failure to distinguish between production and marketings and the different effects of these factors on prices and the responses of producers."

Lorie's theory of the cattle cycle, on the other hand, attempted to make clear the nature of the interrelationships among value (price), marketings, and production. First, he assumed that there existed complete stability in the cattle industry. Into this stable equilibrium he introduced a disturbance that caused producers to accumulate animals causing a decline in marketings. This decrease in marketings led to an increase in the value of marketed animals.

Lorie theorized that a "normal" price or trend line existed, above which producers tended to accumulate cattle and below which they began liquidating. Nordblom [1981, p. 10] suggested that this "normal" price "might be considered as the price level at which all out-of-pocket costs would be covered by the sale of steer calves, non-pregnant or unsound cows, and about 75 percent of the heifer calves."

Thus, the "normal" price type of reaction by producers to the initial rise in value generates further accumulations, further declines in marketings, and further rises in values. After three to four years the increased production on ranchers resulting from the larger breeding capacity of the herds can be expected to reverse the downward trend in marketings. As slaughter prices begin to fall herd growth slows. Marketings continue to increase, however, due to the still increasing productive capacity of the breeding herd. These continued increases in marketings are accompanied by further declines in prices and slower herd growth. As slaughter prices fall below the "normal" line, herd growth stops and liquidation of breeding animals begins.

As the number of cattle on farms reach the equilibrium level the productive capacity of the herd levels out, marketings peak and values hit bottom. With reduced herd size and reduced slaughter, values begin to rise. As values rise, the rate of liquidation lessens. Marketings will continue to decline and slaughter prices to rise until both simultaneously reach equilibrium. At this point, cattle numbers are at their lowest. Herd expansion will start again as prices rise above the "normal" level. This brings Lorie's (1947) model of the cattle cycle process back to its beginning.

Nordblom [1981, p. 11] commented that "the decision process behind the 'reaction' of farmers to increase or decrease their breeding herd inventories was not defined by Lorie other than in terms of general tendencies." However, Nordblom concludes that Lorie's study is the landmark work on the cattle cycle. He states, "Lorie's (1947) study has endured as the foundation of our understanding of the cattle cycle process . . . our received knowledge of the cattle cycle has expanded since 1947 little more than in terms of our observations on its vigorous continuation" [Nordblom 1981, p. 11].

The last major study of endogenous causation, presented by Ehrich (1966), framed the cattle cycle in terms of another cycle-generating mechanism, i.e., Servomechanism Control System. Kim [1970, p. 12] describes the operation of the Servomechanism Control System as follows:

> "The harmonic oscillations generated by Servomechanism Control Systems are but one example of a widely occurring phenomenon called 'feedback.' It occurs whenever a signal produces a response that acts after a delay to alter that signal... Particularly in relation to inventory control, it is a major topic in operations rsearch, and ... it has been advanced as the mechanism of the inventory cycles of the general economy ... The essential requirement for 'feedback' is an unvarying response to a signal, which acts through a fixed delay to alter the signal in a predetermined manner."

In this study, Ehrich [1966, p. 25] concluded that producers tend to change their rate of planned production in response to the deviation of price from equilibrium (stimulus). The change in output is realized after a delay (physical growth limitations), and the price stimulus is altered by the new level of production.

In all essence, Ehrich's (1966) study is just a more empirical version of Lorie's [1947, p. 56] model of the interrelations of beef market prices, quantities marketed, and beef cow numbers. However, Ehrich [1966, p. 25] went a step further than Lorie by concluding that the producers response to deviations of price from equilibrium served to "deny the existence of a conventional supply function for beef cattle." Furthermore, since the Cobweb Theorem depends on the existence of a conventional supply curve, Ehrich's conclusion once again proves it to be an inadequate model of either the hog or cattle industry.

Ehrich's statistical analysis allowed him to determine that the "internal behavioral structure — not exogenous force — is the primary mechanism that generates cyclical fluctuations in the beef economy" [Ehrich 1966, p. 17].

Exogenous and Endogenous Hypotheses

Breimeyer (1954) and Nordblom (1981) are two of the authors which consider both endogenous and exogenous variables as determinants of the cattle cycle. Breimeyer [1954, p. 16] expressed this general theme as follows:

> "Quite naturally, theories with respect to cycles in cattle, are divided into those emphasizing outside factors and those favoring automatic self-generating properties ... Objections to the automatic interpretation are that it disregards outside factors such as demand, feed supply, and competitive position. It would be unfair and uncomplimentary to cattlemen to suggest that they are insensitive to such factors."

However, in regards to the positive points of the endogenous hypotheses, Breimeyer [1954, p. 16] suggests that cattle producers are "motivated by price . . . mostly they respond to the expectations of future prices." Thus, Breimeyer [1954, p. 16] summarizes his hypotheses as follows:

> "... cattle producers respond to all factors affecting them including current prices and

> expectations of future prices. They act within limitations imposed by the characteristics of the industry — a long life cycle, high investment, and few alternative enterprises to most producers. Because of these characteristics, responses are not quick, simple, or direct but take on the slow evoluations known as the cattle cycle."

In 1956, the average price of cattle dropped an enormous 45 percent from its high in 1951. In response to this tremendous drop in price, the American National Cattlemens Association organized a major study of the marketing questions associated with the cattle cycle. This comprehensive study [DeGraff, 1960], titled Beef Production and Distribution, belongs also the the school of exogenous-endogenous causation.

DeGraff [1960, pp. 41-42] suggests two circumstances that might trigger the swings of a cattle cycle. The first is a change in the demand for beef which manifests itself in the softening or strengthening of cattle prices. DeGraff hypothesizes that this change in cattle prices may start the chain reaction of a cycle [p. 42]. The second impetus to the cattle cycle that DeGraff points out is a change in the supply of feed — especially feed on pastures and ranges. DeGraff [1960, p. 42] further states that "while such influences ... may initiate a cycle, they do not explain ... why a cycle follows its standardized pattern [this] is found not in economics, but in biology."

Nordblom (1981) developed a simulation model of the cattle cycle from 1950 to 1978. He hypothesized that the historical patterns of the cattle cycle "have been related to investment incentive differences across cow ages through time, resulting each year in changes in herd age structure, performance and potential for adjustment in subsequent years" [Nordblom 1981, abstract].

The model developed by Nordblom (1981) is a synthesis of the biological attributes of cows, across cow ages, and the economic value of cattle across sex and age. The biological attributes are conception rates, health rates, cow survival rates and weaning weights. Using these biological parameters, Nordblom (1981) defines his management expectation parameters, retainment and culling rates. These biological and expectation parameters are the foundation of Nordblom's model. The biological parameters vary across cow ages but are constant through time, thus, they are exogenous to Nordblom's cattle cycle simulation model.

Furthermore, Nordblom [1981, p. 115] develops a budget generator that produces estimates of expected net annual revenues for each of his age and pregnancy classes of heifers and cows. This budget generator is based on an exogenous price and cost series. These estimates are used to project the present values of expected future net revenues for each class of breeding animals [Nordblom 1981, abstract]. The ratio of future breeding value to present slaughter value is calculated for each class of breeding animals, for each year of the simulation. These value ratios or "V"-ratios are the decision variables in Nordblom's (1981) model for determining the number of animals in each class to be retained.

Nordblom (1981) bases his V-ratios on the concept of Lorie's (1947) "normal" line. V-ratio values less than 1.0 suggest incentives for heavy culling, while a V-ratio above 1.5 suggest a high incentive for retainment [Nordblom 1981, p. 132]. These exogenous value ratios are used to control Nordblom's model of the internal age structure dynamics of the aggregate U.S. beef cow herd through cattle cycles. It is this internal age structure and subsequent inventory levels which are generated endogenously.

Nordblom's (1981) model appears to track the historical numbers of beef cows and calves born quite well, producing mean proportional absolute deviations (MPAD) of .029 and .036 respectively. However, "the tracking performance in heifer and cull numbers were much less accurate." The MPAD with respect to culls was 26.1 percent.

Nordblom's (1981) model, however, has shown the likely aggregate consequences of producers investment response toward beef cows. Given its few exogenous price and cost variables, and simple management expectation relationships and biological parameters, Nordblom's model behaves relatively well.

The current study is a further examination of the national cattle cycle, using Nordblom's (1981) model as a base. It focuses on a thorough critique of Nordblom's assumptions, model structure and functional forms, with the intention of improving on its tracking ability.

Thesis Objectives

Three objectives are defined for the present study:

(1) To simplify, refine and update the cattle cycle simulation model developed by Thomas L. Nordblom (1981);

(2) To project, under alternative short-term scenarios, the exogenous prices and costs that drive the model; and

(3) To use the projected exogenous price and cost series to forecast cattle inventory numbers through 1987.

Methodology

In order to fulfill the above objectives a considerable amount of time and energy was spent on an iterative process that consisted of (1) the analysis of the deviations of the simulated annual inventories of beef cows and heifers, the production of calves and the marketings of cull cows from the historical data series and (2) the systematic modification of the model's functional forms and parameters in an effort to reduce the deviation. This iterative process was repeated numerous times beginning with Thomas L. Nordblom's final simulation run known as DISPLAY and ending with the current model structure and simulation run known as STRINGHAM.

The analysis of the deviations began with a review of the general model structure including such items as the definitions of the simulated inventories and the historical data series, proceeded through the biological constraints such as the conception rate function and the calf survival rate function, and ended with a review of the intermediate functions.

This process brought to light immediately a major problem with the definition of Nordblom's simulated cow inventory. His model included pregnant yearling heifers in the cow inventory. Hence, while the simulated cow inventory was quite close to the historical series, there was an important difference in the definition of the simulated cows on inventory and the data series of cows on inventory. The definition used in Livestock and Meat Statistics publications is

"Cows and Heifers that have Calved." Clearly, pregnant yearlings should not be included in the simulated cow inventory. This modification of Nordblom's model created errors in excess of 25 percent in the simulated cow inventory and increased the complexity of the problem at hand.

Furthermore, review of the historical series showed a major change in the definition of cows on inventory in 1970, overlapping back to 1965. Prior to 1970, the definition of cows on inventory used in Livestock and Meat Statistics publications was "Cows and Heifers Two Years Old and Over," beginning in 1970 the definition changed to "Cows and Heifers that have Calved." This change in definition created severe problems in the simulation of the cow inventory and the heifer recruit inventory. Thus, the earlier period of 1950 to 1964 was dropped from the model.

After these two major corrections were completed the iterative process of review began again. The analysis of the biological constrains was accomplished through a review of the cited literature and validation of the functional forms and parameters.

The intermediate functions presented unique problems because there exists no empirical research in these areas thus, Nordblom based the development of the equations on logical, theoretical ideas. However, according to Nordblom [1981, p. 23] "There is a considerable element of art, and a strong role for intuition, in the choice of model structure." Therefore a good deal of effort was spent on the analysis of the intermediate functions. More specifically, the investment decision variables ($g_{28,j}$ and $g_{30,j}$) which link the value model to the age distribution inventory model were scrutinized thoroughly.

In summary, a systematic analysis of the model was conducted, beginning with the most general points and continuing down to the fine workings of the system. This analysis resulted in the redefinition of many of the functional forms and the correction of definitional problems and the misspecifications of parameters.

FLEX/REFLEX

The effectiveness with which simulation techniques may be applied to systems is highly dependent on the way a system is structured. The FLEX/REFLEX simulation modeling paradigm developed by Dr. Scott W. Overton, Curtis White and others at Oregon State University, lends itself well to modeling the dynamic nature of the national cattle cycle. Appendix A provides a more thorough discussion of the FLEX/REFLEX paradigm.

The synchronization of the separate tasks of modeling and programming is accomplished through FLEXFORM model documentation. The FLEXFORM document of the present model is given in Appendix B. This document provides a concise description, display and cross-reference of every variable, parameter and equation contained in the model. The FLEX/REFLEX documentation scheme (FLEXFORM) was designed solely for the purpose of creating and preserving useable documentation.

The FLEX/REFLEX notational convention is introduced here and used throughout the remainder of the text.

z_i = input variables

$x_{i,j}$ = state variables

$g_{i,j}$ = internal or intermediate functions

$f_{i,j}$ = flux functions to update state variables

$Y_{i,j}$ = output functions

b_i = parameters.

Plan of the Thesis

Chapter 1 describes the general cattle cycle phenomenon, the objectives of the thesis and its methodology.

Chapter 2 discusses the biological attributes of beef cows and describes the functions ($g_{1,j}$ through $g_{8,j}$) used to model these characteristics. Management expectation and producer profit expectation function are also defined here.

Chapter 3 defines the exogenous input variables (z_i). The expected feeder steer price and utility cow price functions are described. Annual cost budgets, based on the year 1978, are developed for five classes of breeding animals: weaned heifers kept for breeding, pregnant yearling heifers, non-pregnant yearling heifers, pregnant and non-pregnant mature cows. Cost indices (1978 = 1.0) for each of the ten costs are developed and specified as annual input variables (z_i).

Chapter 4 provides an equation-by-equation description of the cow value and age distribution inventory model. The model operates with a time resolution of one year, receiving annual input variables each year of the 17 year run.

Chapter 5 presents the simulation results and model validation. Statistical and graphical comparisons of simulated versus historical data series are also given here.

Chapter 6 presents the simulated forecasts through 1987, of cow inventories, calf crops, beef heifer recruits and slaughter cow numbers. Forecasting techniques are discussed and alternative future scenarios of the prices of cattle, corn, and other production inputs are described.

Chapter 7 contains conclusions and indications for further research.

CHAPTER 2

BIOLOGICAL AND MANAGEMENT EXPECTATION PARAMETERS PRODUCER PROFIT EXPECTATION FUNCTIONS

Many changes have taken place in the cattle industry over the past century, but the cattle cycle has continued to persist. The question arises as to why the cattle industry cannot attain a sustainable growth pattern that would smooth out these cyclical swings in cattle numbers. It is a major theme of this thesis that the cattle cycle has persisted because it is based on two unchanging factors: (1) the profit motive which prompts producers to make production decisions on the basis of expected prices; and (2) the biological characteristics of bovine reproduction and growth necessitates a lag of three to five years for the results of production decisions to cause changes in the number of cattle slaughtered. In the investment phase production exceeds sales, causing actual slaughter to fall below potential and prices to rise. This rise in price further aggravates the profit motive causing successive overadjustments. The same reasoning applies to the disinvestment phase where sales exceed production, actual slaughter increases above normal, prices fall further and successive overadjustment of liquidation occurs.

The biological characteristics of beef cows across age classes, management expectations and the producer profit expectation functions lay the foundation for the development of the simulation model. It should be noted that the biological characteristics and their mathematical forms have been adopted from Nordblom's (1981) model. The purpose of this chapter is to discuss these functions.

Following is a list of the biological, management expectation, and the producer profit expectation functions.

Biological Functions

Conception rates by cow age	$g_{1,j}$
Unimpaired health rates by cow age	$g_{2,j}$
Cow survival rates by cow age	$g_{3,j}$
Cow culling weights by cow age	$g_{4,j}$
Maximum aggregate average cow weight by cow age	g_5
Calf weaning weights by cow age	$g_{6,j}$
Weight of weaned heifers kept for breeding	g_7
Calf survival rates by cow age	$g_{8,j}$

Management Expectation Functions

Cull sales in the coming year by cow age	$g_{9,j}$
Cull sales in second year by cow age	$g_{10,j}$

Producer Profit Expectation Functions

Calf sales in the coming year by cow age	$g_{11,j}$
Calf sales in second year by cow age	$g_{24,j}$

The numerical values of these functions vary by age but are constant through the length of the simulation.

Table 1 gives a concise listing of the mathematical expressions for the biological, management expectation and producer profit expectation functions. Reported in Table 2 are the functional values from the current simulation run.

Table 1. Biological, Management Expectation and Producer Profit Expectation Functions.

Biological Functions	
Conception rate (C_j) as a function of age (j) at breeding	j = 1,14 = age at breeding $g_{1,j} = b_1 + b_2(j-b_3) + b_4(j-b_3)^2$
Unimpaired health rate (H_j) in the year prior to age j.	j = 1,15 = age at breeding $g_{2,j} = 1.0 - (b_5 + \frac{b_6}{j} + b_7 \cdot j^2)$
Cow survival rate (S_j) in the year prior to age j.	j = 2,15 = age becoming $g_{3,j} = b_8 + b_9 \cdot j$
Cow culling weight (CW_j) at culling time prior to age j.	j = 2,15 = age becoming $g_{4,j} = b_{10} \cdot b_{11} \cdot (b_{12} + (b_{13} \cdot j) + \frac{b_{14}}{j}) + (1.0 - b_{10})$ $\cdot\ b_{15} \cdot (b_{16} + (b_{17} \cdot j) + (b_{18} \cdot j^2) + (b_{19} \cdot j^3)$
Maximum aggregate cow body weight (MA)	$g_5 = b_{10} \cdot b_{11} + (1.0 - b_{10}) \cdot b_{15}$
Calf weaning weights (WW_j) expected for cows aged (j) years at calving.	j = 2,14 = age at calving time $g_{6,j} = g_5 \cdot b_{20} \cdot (b_{21} + (b_{22} \cdot j) + (b_{23} \cdot j^2) + (b_{24} \cdot j^3)$
Estimated weight of weaned heifers kept for breeding	$g_7 = g_5 \cdot b_{25}$
Calf survival rate (CS_j) by cow age j.	j = 2,14 = age at calving $g_{8,j} = b_{26} + (b_{27} \cdot j) + (\frac{b_{28}}{j})$
Management Expectation Functions	
Expected cull sales in coming year by cow age j.	j = 1,15 = age becoming $g_{9,j} = 2 - g_{1,j} - g_{2,j}$
Expected cull sales in second year by cow age j.	j = 1,13 = age becoming $g_{10,j} = [g_{1(j+1)} + g_{2(j+1)} + g_{3(j+1)} - 2]\ g_{3(j+2)}$
Producer Profit Expectation Functions	
Expected calf sales in coming year by cow age j.	j = 2,15 = age becoming $g_{11,j} = g_{8,j} \cdot g_{6,j} \cdot g_{12,1} \cdot b_{38}$
Expected calf sales in second year by cow age j.	j = 1,14 = age becoming $g_{24,j} = g_{10,j} \cdot g_{8(j+1)} \cdot g_{6(j+1)} \cdot g_{12,1} \cdot b_{38}$

Table 2. Biological and Management Expectation Function Values.

	j = age	=	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Conception rates	$g_{1,j}$	=	.814	.864	.902	.928	.942	.943	.933	.911	.877	.831	.773	.703	.621	.527	0.000
Unimpaired health rates	$g_{2,j}$	=	.794	.916	.952	.966	.969	.966	.958	.947	.933	.916	.896	.874	.849	.823	.794
Cow survival rates	$g_{3,j}$	=	0.000	.988	.987	.986	.985	.984	.983	.982	.981	.980	.979	.978	.977	.976	.975
Cow culling weights	$g_{4,j}$	=	0.000	7.053	8.448	9.194	9.644	9.919	10.074	10.141	10.141	10.087	9.993	9.868	9.721	9.562	9.397
Maximum cow weight	g_5	=	10.225	-----	-----	-----	-----	-----	------	------	------	------	-----	-----	-----	-----	-----
Calf weaning weights	$g_{6,j}$	=	0.000	3.877	4.050	4.181	4.257	4.338	4.374	4.388	4.385	4.371	4.349	4.325	4.304	4.291	0.000
Weaned HKB weight	g_7	=	4.295	-----	-----	-----	-----	-----	------	------	------	------	-----	-----	-----	-----	-----
Calf survival rates	$g_{8,j}$	=	0.000	.879	.908	.922	.929	.934	.936	.938	.938	.939	.938	.938	.937	.936	0.000
Cull sales now	$g_{9,j}$	=	.392	.221	.146	.107	.090	.091	.108	.142	.190	.253	.331	.423	.529	.650	0.000
Cull sales in 2 years	$g_{10,j}$	=	.757	.829	.866	.881	.878	.859	.824	.775	.712	.634	.542	.437	.318	0.000	0.000

Conception Rates by Cow Age

Nordblom (1981) reviewed several studies on the relationship of cow fertility and age. These include, Lasely and Bogart (1943), Burke (1954), Stonaker (1958), Crockett (1967) Long et al. (1975), Rogers (1971), and Bentley (1976). These studies considered conception rates and calving rates or cow fertility rates for individual herds.

The data on cow fertility presented in these studies indicated that fertility rates rise to a peak at some age between four and ten years of age and decline continuously thereafter. The empirical data was described by a quadratic function of cow age in all cases. Nordblom (1981) used the following quadratic form:

$$Y = \beta_1 + \beta_2(j-\beta_3) + \beta_4(j-\beta_3)^2$$

For the convenience of utilizing the existing program, the same form is used in the current analysis. However, the interpretation of parameters offered by Nordblom is in error.[1/]

Data from Rogers (1971) used in estimating the conception rate functions was re-estimated for the current simulation. The conception rate function ($g_{1,j}$) and its current parameter values are given below.

$$g_{1,j} = b_1 + b_2(j-b_3) + b_4(j-b_3)^2$$

where

$b_1 = 0.94$

1/ For example, Nordblom stated that the parameter β_3 represents the age at which the function becomes a maximum. However, the correct age is $j = \beta_3 - \beta_2/2\beta_4$.

$b_2 = 0.01$

$b_3 = 4.833$

$b_4 = -0.006$

Unimpaired Health Rates by Cow Age

Unimpaired health rates are defined by Nordblom [1981, p. 43] "as the maximum proportion of surviving cows in an age class that would be retained in a herd under the most favorable economic conditions." His definition takes into account the importance that producers place on the current economic outlook when making retainment decisions based on animal health. In times when high profits seem in store a marginal animal may be retained in the breeding herd if there is some prospect of her weaning a calf one year hence. However, in times when losses seem inevitable, such a cow would almost certainly be culled. Furthermore, a cow which is judged to have very poor prospects of weaning a calf in the next year will most likely be culled regardless of the economic outlook [Nordblom 1981, p. 43].

Nordblom reviewed several studies on the reasons for culling including a report by Greer et al. (1980) which summarized data on the proportions of cows culled because of physical impairment by cow age. Again the data were described by a quadratic function of age, after deleting the abnormally high observation for nine year olds.[2/]

$$IH_j = 0.00539 + 0.0010437j^2 \qquad R^2 = 0.9867$$

where: IH_j = Proportion of cows j years old culled for impaired health

2/ Greer acknowledged an error for this observation.

Nordblom modified this fitted equation by adding a hyperbolic term to express the assumption that some minimum proportion of weaned heifers will be sold, even in the face of favorable economic conditions. Thus, the function is the minimum number of surviving animals culled in each age class. The constant term was modified to give cows becoming five years of age the lowest rates of impaired health. The modified equation is:

$$IH_j = -0.045 + \frac{.25}{j} + 0.0010437j^2$$

The complement of the above equation is used in the simulation model and is referred to as the unimpaired health rate function, $g_{2,j}$.

$$g_{2,j} = 1.0 - (b_5 + \frac{b_6}{j} + b_7 j^2)$$

where

$b_5 = -0.045$

$b_6 = 0.25$

$b_7 = 0.0010437$

This function is used only in the retainment decision functions of Chapter 4.

Survival Rates by Cow Age

Because of economic considerations most cows are culled before natural death can claim them. Drawing on this conclusion, Nordblom (1981) estimated that "most cow deaths on farms (other than intentional slaughter) are accidental and unusual; on the order of one to two percent in the aggregate" [p. 48].

Nordblom canvassed the results of several studies on cow mortality, including Greer et al. (1980), Preston and Willis (1970), and Ensminger (1976). Greer et al. [1980, p. 18] summarized the individual records for 4,660 heifers and cows during the period 1943 through 1976. He reported death losses ranging from 0.95 percent (for five year olds) to 1.65 percent (for two year olds) for cows two to ten years of age.

The cow survival rate function adopted here, expresses cow survival as a linearly decreasing function of age. The rate of decrease is estimated at 0.1 percent per year of age. This small decrease in cow survival is justified on the grounds that infirm cows are sold before natural death occurs.

$$g_{3,j} = b_8 + b_9 \cdot j$$

where

$g_{3,j}$ = cow survival rate from natural and accidental death in the year prior to age (j). j=2 to 15.

b_8 = 0.99

b_9 = -0.001

j = age in years.

Cow Weight by Cow Age

Livestock marketing research has focused almost solely on the primary product of the beef cow-calf industry, stocker and feeder cattle, while marketing the important joint product cull cows, has received minor attention. The sale of cull cows, however, represents an important source of revenue. According to Yager et al. [1980, p. 456], cull cows constitute 15 percent to 25 percent of the annual gross revenue of a given firm. The purpose of this section is to define the cow weight functions used in the current study for simulating culling weights by cow age.

Nordblom reviewed a considerable amount of data on the important factors (age and breed of the cow) affecting cow weight. He accepted the growth pattern for the 5H3B cows, reported by Brown et al. (1971), as representative of the extreme for early maturing breeds. He also assumed a gradual decline in the early maturing cow body weight, to about 90 percent of maximum by the age of 14½ years, in contrast to some of the functions which reported abrupt declines in weight after ten years of age. A hyperbolic function of age (EW_j) was fitted to the modified growth pattern for the 5H3B cows to define cow body weight as a proportion of the maximum for early maturing breeds [Nordblom 1981, p. 54]:

$$EW_j = b_{12} + b_{13}j + \frac{b_{14}}{j}$$

where

EW_j = early maturing cow body weight as a proportion of maximum (ME)

b_{12} = 1.33015

b_{13} = -0.0239

b_{14} = -1.1399.

For the late maturing extreme, Nordblom fitted a cubic function of age (LW_j) to the cow weight data reported by Clay Center, the U.S. Meat Animal Research Center (1974 to 1979). He assumed a gradual decline in late maturing cow weights to approximately 95 percent of maximum.

$$LW_j = b_{16} + b_{17}j + b_{18}j^2 + b_{19}j^3$$

where

LW_j = late maturing cow body weight as a proportion of maximum (ML)

b_{16} = 0.4107

b_{17} = 0.1446

b_{18} = -0.01124

b_{19} = 0.0002673.

To derive the specific culling weight estimates for each age of cow, Nordblom specified a linear combination of the two extreme patterns times their respective maximum weights.

$$CW_j = g_{4,j} = b_{10}\ b_{11}\ (b_{12} + b_{13}j + b_{14}/j)$$
$$+ (1.0 - b_{10})\ b_{15}\ (b_{16} + b_{17}j + b_{18}j^2 + b_{19}j^3)$$

where

CWj = culling weight of a cow becoming j years of age = $g_{4,j}$

b_{10} = 0.62 = proportion of the cow herd comprised of early maturing breeds

b_{11} = = maximum mature cow weight of early maturing breeds

b_{15} = = maximum mature cow weight of late maturing breeds.

The value Nordblom assigned to the b_{10} parameter was adopted from Ensminger's estimate that as much as 62 percent of the gene pool of the U.S. commercial beef cattle industry consists of early-maturing Hereford and Angus breeds [Ensminger et al., 1955, p. 46]. The 5H3B cow data provided the maximum early maturing cow body weight estimate ($ME = b_{11}$) of 975 pounds [Brown et al., 1980, p. 44]. The highest Clay Center cow weights provided Nordblom with the maximum of 1,100 pounds for the late maturing breeds ($ML = b_{15}$). The simulated expected cow culling weights are used in the calculation of expected cull cow sales values, by the $g_{13,j}$ and $g_{14,j}$ functions described in Chapter 4.

The aggregate maximum mature cow weight ($MA = g_5$) is calculated with the terms defined above.

$$MA = g_5 = b_{10} \cdot b_{11} + (1.0 - b_{10})\, b_{15}.$$

Given the parameter values assumed by Nordblom, the aggregate maximum mature cow weight is 1,023 pounds. It is an important factor in the determination of calf weaning weights discussed next.

Calf Weaning Weights by Cow Age

Nordblom reviewed a considerable amount of literature on the relationship of calf weaning weight and cow age. This relationship is important because calf sales are the primary source of revenue for a commercial cow-calf operation.

Using data from seven of 15 studies compiled by Preston and Willis (1970) on calf weaning weights by cow age, calf weight indices were calculated. These indices were based on the observation that the heaviest calves were weaned by eight year old cows. A cubic function was fitted to these calf weight indices.

$$WI_j = b_{21} + b_{22}j + b_{23}j^2 + b_{24}j^3 \qquad R^2 = .971$$

where

WI_j = estimated weaning weight of a calf from a cow j years old, as a proportion of the calf weaning weight from an eight year old cow

b_{21} = 0.770156

b_{22} = 0.0678788

b_{23} = -0.00642507

b_{24} = 0.000187646

To estimate the link between cow weight and maximum calf weight, Nordblom compiled a set of calf weaning weight to mature cow weight ratios reported by various authors. The lowest value listed in this set of ratios is 0.364 and the highest is 0.502 with a mean value of 0.437. A ratio of 0.43 was used as the value for the parameter b_{20} which links maximum calf weight to maximum aggregate mature cow weight in the simulation model. The ratio of the heifers kept for breeding (HKB) weight to maximum aggregate mature cow weight, b_{25}, was 0.42. Thus, HKBs are among the heaviest calves weaned, but are still slightly lighter than the heaviest of their male siblings.

The weight of heifers kept for breeding and calf weaning weights are linked to MA (g_5), the maximum aggregate mature cow weight as follows:

$$WW_j = (MA \cdot MC \cdot WI_j) = g_{6,j} = g_5 \cdot b_{20} \cdot (b_{21} + b_{22}j + b_{23}j^2 + g_{24,}j^3)$$

and

$$HW = (MA \cdot HC) = g_7 = g_5 \cdot b_{25}$$

where

WW_j = Calf weaning weight for cow aged $(j + \frac{1}{2})$ years = $g_{6,j}$

HW = Estimated weight of a weaned heifer kept for breeding (HKB) = g_7

MA = Maximum aggregate mature cow weight = g_5

MC = Maximum calf weight as proportion of MA = b_{20}

WI_j = Calf weaning weight for a cow aged $(j + \frac{1}{2})$ years as a proportion of maximum calf weight

MC = HKB weight as proportion of MA = b_{25}.

The calf weaning weights, as calculated by equation $g_{6,j}$, are assumed to be the average weaning weight of steer and heifer calves for each age of cow. Given Nordblom's [1981, p. 65] assumed maximum cow weight (MG = g_5) of 1,023 pounds, and maximum calf weight as a proportion of MA at .43, the weaning weight of a calf from an eight year old cow would be 440 pounds. The weight expected for weaned heifers kept for breeding is constant throughout the simulation run. It is computed by the g_7 function to be 430 pounds ($g_7 = g_5 \cdot b_{25} = 1{,}023 \cdot 0.42$).

Cow Age and Calf Survival From Conception to Weaning

This section defines the relationship between cow age and calf survival from conception to weaning. The literature reviewed by Nordblom indicated percent birth-to-weaning calf death losses ranging from 5.6 to 21.3 percent. Nordblom computed average calf survival rates by cow age from the data reported in the U.S. Meat Animal Research Center Progress Reports Numbers 2-7, 1976.

A hyperbolic function of cow age described the data:

$$CS_j = g_{8,j} = b_{26} + b_{27}j + b_{28}/j \qquad R^2 = .929$$

where

CS_j = calf survival rate by cow age = $g_{8,j}$

b_{26} = .975463

b_{27} = -.00184144

b_{28} = -.184779

The values generated by this function for cows two through 14 years of age are shown in Table 2. The reader will note that two year old heifers have the lowest percent calf survival and ten year olds the highest, with rates declining only slightly for cows aged past ten years old. Nordblom [1981, pp. 67-68] indicates two good reasons for the only slight decline.

First, in commercial cow herds the harsh annual culling process has the effect of eliminating all but the most exceptional cows in the older age groups [Preston and Willis 1970, p. 235]. Secondly, Nordblom states, "there seems to be no basis for assuming a discontinuous pattern of calf survival rates with cow age" [p. 71].

Management Expectation Functions

The calculation of present value for breeding purposes requires an estimate of the likelihood of a cow's continued retainment in the herd through future years. In the current model, after experimentation with alternative planning horizons, a horizon limited to two years in the future was adopted. The purpose of this section is to define these estimates known as management expectation functions.

Expected Cull Cow Sales

The estimations of expected cull cow sales are derived from recruitment and culling rates in a steady state herd, i.e., all replacements are grown within the herd, the age distribution is constant through time and herd is not in a phase of expansion or reduction. The culling rules associated with a steady state herd are completely rigid. All animals found open or suffering from physical impairments are culled.

The expected cull cow sales in the coming year are very simply functions of the rates of conception ($g_{1,j}$) and unimpaired health ($g_{2,j}$).

$$g_{9,j} = 2 - g_{1,j} - g_{2,j}$$

where

$g_{9,j}$ = expected cull sales in coming year by cow age j

$g_{1,j}$ = conception rates as a function of age

$g_{2,j}$ = unimpaired health rate as a function of age.

The expected cull cow sales two years from the present is calculated by multiplying the survival rate ($g_{3(j+2)}$) for cows j+2 years of age with the fraction of animals that are pregnant, healthy and alive from the previous year.

$$g_{10,j} = [g_{1(j+1)} + g_{2(j+1)} + g_{3(j+1)} - 2]\ g_{3(j+2)}$$

where

$g_{10,j}$ = expected cull sales in second year by cow age j

$g_{1(j+1)}$ = conception rates for cows two years old and older

$g_{2(j+1)}$ = unimpaired health rates for cows two years old and older

$g_{3(j+1)}$ = survival rates for cows two years and older

$g_{3(j+2)}$ = survival rates for cows three years and older.

It should be emphasized here that these expected cull cow sales functions have been developed by the author but use the conception, unimpaired health and survival rate functions estimated by Nordblom (1981). Both $g_{9,j}$ and $g_{10,j}$ are used in the present value of breeding (PVB) calculations defined in Chapter 4. Furthermore, $g_{10,j}$ is used in the second year calculation of producer profit expectations.

Producer Profit Expectation Functions

Revenues from calf sales are the major source of income to the cow-calf operator and thus plays an important role in determining the present value of a recruit or brood cow. The expected calf sales revenue functions developed here are primary determinants of the present value of breeding for pregnant and non-pregnant animals defined in Chapter 4.

Calf sales revenues in the coming year are defined as the product of expected calf weaning weights ($g_{6,j}$), calf survival rates ($g_{8,j}$) and expected feeder steer prices ($g_{12,1}$). The use of these functions causes the expected revenue flows of a recruit or brood cow to be adjusted in each future year for the expected change in weaning weights of calves produced as the cow ages and by the probability that the calf will survive until weaning. Trapp and King (1979) developed a herd simulation model based on Perrin's (1972) replacement rule but adjusted for the physical parameters also included here. However, no mention was made as to the accuracy of the simulated results.

$$g_{11,j} = g_{6,j} \cdot g_{8,j} \cdot g_{12,1} \cdot b_{38}$$

where

$g_{11,j}$ = calf sales revenues in the coming year

$g_{6,j}$ = calf weaning weights

$g_{8,j}$ = calf survival rates

$g_{12,1}$ = expected feeder steer price

b_{38} = ratio of heifer and steer average price to choice feeder steers.

$$g_{24,j} = g_{10,j} \cdot g_{8(j+1)} \cdot g_{6(j+1)} \cdot g_{12,1} \cdot b_{38}$$

where

$g_{24,j}$ = calf sales two years from the present

$g_{10,j}$, $g_{8(j+1)}$, $g_{6(j+1)}$, $g_{12,1}$, b_{38} } defined above

Each of the age-related biological, management expectation and the producer profit expectation functions are listed and cross referenced, function-by-function, in the FLEXFORM document contained in Appendix B.

CHAPTER 3

DRIVING VARIABLES: EXOGENOUS PRICE AND COST SERIES

Commercial cow-calf enterprises derive their income from two principle sources; calf sales and cull cow sales. Animals of different age, sex and breeding ability have different economic functions within the herd which directly effects their productive values [Jarvis 1974, p. 516]. An objective of this chapter is to describe the ways in which cow age affects the economic values of cull cows and weaned calves.

The production of cows and calves not only generates income it also generates costs. The Great Plains herd budget estimated by the Economics, Statistics, and Cooperative Service (E.S.C.S.) of the U.S.D.A. [1979, p. 44] provided the basis for determination of the variable costs of production. Variable costs of maintaining pregnant and non-pregnant cows and heifers of different age classes for the year 1978 are budgeted separately. A constant state of technology with respect to production methods and productivity in the cow-calf sector, is assumed over the entire simulation period. Therefore, annual variations in production costs are estimated by indexing prices of the various inputs in the 1978 budgets (1978 = 1.0).

Cull Cow Price Function

Cull cow values are calculated as price times weight. Rogers [1971, p. 2] stated that "there is general agreement that the market value of cows decreases with advancing age." According to Ensminger [1976, p. 182] "Old cows, irregular breeders and poor milkers sell to best advantage before they become thin and 'shelly'."

Bentley, Waters and Shumway [1976, p. 17] "in an attempt to account for deterioration in carcass quality with age" assumed two alternative patterns of linearly declining cull prices as cow age advanced. Trapp and King (1979) and Rogers (1971) both assumed cull cow price patterns that were assumed to decrease at a decreasing rate with age until they leveled out at the age of ten years [Nordblom 1981, p. 85].

As in the studies cited above, Nordblom explicitly separates the weight and price components of cull cow value. The monotonically decreasing cull cow price pattern developed by Nordblom is adopted here. "The cull price of older cows is assumed to fall at first rapidly then progressively slower with advancing age . . . the most elderly cows, by this process will have the lowest price per unit of weight ..." [p. 86].

The cull values of cows becoming two to 15 years of age are defined as the product of their respective body weights and prices. Their respective price estimates are a function of current feeder steer price and utility cow price.

$$g_{14,j} = g_{4,j} \cdot \underbrace{\underbrace{\left[Z_1 - b_{35}(Z_1 - Z_2) + \frac{b_{35}(Z_1 - Z_2)}{j \cdot b_{41}}\right]}_{\text{current estimate of price per cwt. for a non-pregnant cow culled prior to becoming j years of age, j=2 to 15}}}_{\text{Present salvage value } (PSV_j) \text{ estimates for non-pregnant cows of each age.}}$$

current estimate of price per cwt. for a non-pregnant cow culled prior to becoming j years of age, j=2 to 15

Present salvage value (PSV_j) estimates for non-pregnant cows of each age.

$$g_{25,j} = g_{4,j} \cdot [\underbrace{Z_1 - b_{33}(Z_1-Z_2) + \frac{b_{33}(Z_1-Z_2)}{j \cdot b_{41}}}_{\text{current estimate of price per cwt. for pregnant cow culled prior to becoming j years of age, j = 2 to 15}}]$$

current estimate of price per cwt. for pregnant cow culled prior to becoming j years of age, j = 2 to 15

present salvage value (PSV_j) estimates for pregnant cows of each age.

where

Z_1 = annual average per cwt. prices of choice feeder steers (600 to 700 pounds at Kansas City)

Z_2 = annual average utility cow prices per cwt. at Omaha

b_{33} and b_{35} = price spread factor

b_{41} = hyperbolic age coefficient

The specification of the price spread parameters (b_{33} and b_{35}) allow for either identical or different PSV_j estimates for pregnant and non-pregnant animals. These functions are used in the calculation of value ratios (discussed in Chapter 4) for pregnant and non-pregnant cows, which are used in making culling and recruitment decisions.

The use of the feeder steer prices (Z_1) is justified on the basis of the high correlation found between feeder steer prices and utility cow prices.

$$Z_2 = 1.222 + 0.598Z_1 \qquad R^2 = .9956 \quad n = 28$$

The present salvage value (PSV_1) of a weaned heifer kept for breeding is calculated as the product of her estimated weight and an adjusted feeder steer price. Nordblom assumed the prices per cwt.

of weaned heifers kept for breeding are 86 percent of those for feeder steers (Z_1). He based this assumption on a similar weighting presented by Rogers [1972, p. 922].

It was decided by the author that a more accurate estimate of the relationship between feeder steer prices (Z_1) and heifer calf prices could be determined through linear regression analysis. Annual average per cwt. prices of choice weaner heifers (300-500 pounds at Kansas City) were regressed against the Z_1 index of feeder steer prices for the period of 1950 to 1981.

$$\text{Heifer price} = -1.5218 + .976475 \text{ (feeder steer price)} \quad R^2 = .9865$$

The adoption of this relationship resulted in a slight modification of Nordbloms PSV equation for weaned heifers kept for breeding. The modified function is presented here.

$$g_{14,1} = [(Z_1 \cdot b_{39}) + b_{31}]\ g_7$$

where

Z_1 = current feeder steer price

b_{39} = .976475

b_{31} = -1.5218

g_7 = estimated weaning weight of a HKB

Beef Cows Annual Maintenance Cost Budgets

As stated at the beginning of this chapter, the Great Plains herd budget estimated by the E.S.C.S. provided the basis for the generation of beef cow annual maintenance cost budgets. The Great Plains cost data is used because that region has long maintained the largest number of beef cows of any region in the U.S. Thus, it is assumed that changes in the culling and recruitment of breeding animals in the aggregate can be related to changes in profitability of the beef cow herds in the Great Plains region.

The annual maintenance cost budget indices developed in this section for (1) weaned heifers kept for breeding, (2) pregnant and (3) non-pregnant yearling heifers, and (4) pregnant and non-pregnant mature cows assumes constant physical proportions of inputs for each class of animals. Budgets for each year of the simulation run are created by multiplying the cost indices for each of ten cost categories with the respective base year budgets (1978 = 1.0).

Table 3 presents a summary of the production costs included in the cost budgets and their respective base year parameter values. The following sections describe the various cost items and give their respective data sources.

Feed Costs

The E.S.C.S. Great Plains per-cow budget assumes 83 cows and 17 bred yearling heifers per 100 brood animals in the herd. In order to maintain this composition it was further assumed that 20 weaned heifers would have to be kept each year.

To estimate the feed costs associated with maintaining a herd of 83 cows, 17 bred yearling heifers and 20 weaned heifers kept for breeding, per 100 cows and heifers, animal unit measurements were used. Ensminger [1976, p. 1502] defines an animal unit as "a common animal denominator, based on feed consumption. It is assumed that one mature cow represents an animal unit. Then, the comparative (to a mature

Table 3. Base Year Budget Parameters, Great Plains, 1978.[a/]

	Cows (3 years and older)	2-Year Old Heifers	Weaned Heifers Kept for Breeding
Feed Category			
Rented Pasture	b_{64} = $ 8.94	b_{55} = $ 8.50	b_{48} = $ 6.71
Hay	b_{65} = 32.25	b_{56} = 30.65	b_{49} = 24.19
Grain, Concentrate & Silage	b_{66} = 6.24	b_{57} = 5.93	b_{50} = 4.68
Protein Supplement	b_{67} = .42	b_{58} = .40	b_{51} = .32
Salt & Minerals	b_{68} = 2.14	b_{59} = 2.03	b_{52} = 1.60
Labor & Health Costs			
Labor	b_{69} = 27.54	b_{60} = 39.54	b_{53} = 13.45
Veterinary & Medicine	b_{70} = 3.35	b_{61} = 4.80	b_{54} = 1.63
Common Cost Category		Cost Per Head (1978)	
Bull Depreciation		b_{47} = $10.00	
Marketing and Hauling		b_{44} = 2.83	
Fuel, Lubrication & Electricity		b_{45} = 6.76	
Machinery & Building Repair		b_{46} = 9.22	

a/ Except for bull depreciation, these costs are based on E.S.C.S. Costs of Producing Feeder Cattle in the U.S., U.S.D.A., 1979, p. 44. Parameter names (b_i) used in the simulation model are shown with their respective values.

cow) feed consumption of other age groups . . . determines the proportion of an animal unit which they represent." According to Ensminger [1976, p. 1502], a cow, with or without an unweaned calf at her side, or a heifer two years old or over is one animal unit. Young cattle, one to two years old and weaned calves to yearlings are 0.8 and 0.6 animal units respectively. The annual feed requirements for a growing heifer is assumed by Nordblom [1981, p. 92] to be 0.75 animal units ((1/4)(.6) + (3/4)(.8)). Similarly, the annual feed requirements for a yearling heifer is assumed to be 0.95 animal units ((1/4)(.8) + (3/4)(1)).

Feed cost allocation factors used to estimate the 1978 feed costs attributed to each of the three maturity classes in a herd of 100 cows and heifers are derived below.

Feed Cost Allocation Factors

83.00/114.15=0.7271 for the 83 mature cows

16.15/114.15=0.1415 for the 17 bred heifers

15.00/114.15=0.1314 for the 20 weaned HKB's

where 114.15 is the sum of the products of the annual animal units per class.

The decomposition of the E.S.C.S. herd feed costs is shown in Table 4. The per-head feed costs were calculated by dividing the class totals by the number of animals in each class. These base year budget parameters are given in Table 3.

Labor, Veterinary and Medicine Costs

The E.S.C.S. budget categories of labor, medicine and veterinary

Table 4. Decomposition of Herd Feed Costs, Great Plains, 1978.[a/]

Feed Category	1978 Herd Total[b/]	100 Cows and Heifers: 83 Mature Cows Allocation Factor (0.7271)	17 2-Year Old Heifers Allocation Factor (0.1415)	20 Weaned Heifers Allocation Factor (0.1314)
Rented Pasture	$1,021.00	$ 742.37	$144.47	$134.16
Hay	3,682.00	2,677.00	521.00	483.81
Grain, Concentrate & Silage	712.00	517.69	100.75	93.56
Protein Supplement	48.00	34.90	6.79	6.30
Salt & Minerals	244.00	177.41	34.52	32.06
TOTALS (1978)	$5,707.00	$4,149.37	$807.53	$749.89

a/ Developed by Nordblom (1981), Table 3.2, p. 94.

b/ Based on E.S.C.S. Costs of Producing Feeder Cattle in the U.S., U.S.D.A., 1979, p. 44.

care are distributed over the three age classes according to their numerical compositions of 83/120 for mature cows, 17/120 for bred yearlings and 20/120 for weaned heifers kept for breeding. It is assumed by Nordblom [1981, p. 97] that labor and veterinary care for pregnant cows is 85/120 of the herd's requirements. Pregnant heifers, however, are expected to require additional labor and veterinary care to the tune of 25/120 of the herd's requirements. The weaned heifers kept for breeding require the least amount of husbandry inputs, therefore, their share of the herds labor and veterinary care is assumed to be equivalent to one-half their numerical standing in the herd (10/120).

The decomposition of the herd's labor and health care costs is shown in Table 5. The per-head costs are presented as parameters in Table 3.

Common Costs

The common costs of (1) marketing and hauling; (2) fuel, lubrication and electricity; (3) machinery and building repair; and (4) bull depreciation are assumed to accrue equally to all brood animals on a per-head basis.

A bull depreciation charge of $10.00 per head in 1978 was assumed for all five brood animal classes. This charge was based on the following assumptions: $1,000.00 bull purchase price; $400.00 bull slaughter value; no bull death losses; three year herd life for bulls; and a ratio of 20 cows per bull per year [Nordblom 1981, pp. 99-101].

The remaining three common cost items are assigned in 1978 dollar values according to the composition of E.S.C.S. herd budget. The para-

Table 5. Decomposition of Herd Labor and Health Costs, Great Plains, 1978.[a/]

Labor and Health Cost Category	1978 Herd Total[b/]	100 Cows and Heifers		
		83 Mature Cows Allocation Fator (85/120)	17 2-Year Old Heifers Allocation Factor (25/120)	20 Weaned Heifers Allocation Factor (10/120)
Labor	$3,227	$2,285.79	$672.29	$268.91
Veterinary and Medicine	392	277.67	81.67	32.66
TOTALS (1978)	$3,619	$2,563.46	$753.96	$301.57

a/ Developed by Nordblom (1981), Table 3.4, p. 98.

b/ Based on E.S.C.S. Costs of Producing Feeder Cattle in the U.S., U.S.D.A., 1979, p. 44.

meter values for these four common cost categories are listed in Table 3.

Historical Input Cost Series

The purpose of this section is to describe the historical input price series (1965-1981). These input cost series are indexed to the base year 1978, and when combined with the vectors of feeder steer prices (Z_1) and utility cow prices (Z_2) they form the data set of driving variables for the simulation model. Table 6, found at the end of this chapter, gives a complete listing of the exogenous driving variables vectors.

Fuel, Lubrication and Electricity

The consumer price index (1967 = 1.0) for fuel and utilities was divided by its 1978 value of 2.16 to create the Z_3 index (1978 = 1.0) series for fuel, lubrication and electricity. The C.P.I. for fuel and utilities was provided by the Economic Research Service of the U.S.D.A..

Machinery and Building Repairs

A farm machinery price index (1910-1914 = 100) found in various issues of the U.S.D.A. Agricultural Statistics was used for the period 1965 to 1972. After 1972 a weighted average of two like indices (same source), autos and trucks and other machinery, was used to extend the series to 1981. The weights were .30 and .70, respectively. The Z_4 index (1978 = 1.0) for machinery and building repairs was created by dividing each year of the original series by its 1978 value of 1,213.0.

Bull Depreciation Charges

Annual average slaughter steer prices (1965-1981), for all weights and grades at Omaha, were divided by their 1978 price of $52.34 to create the Z_5 index series (1978 = 1.0) for bull depreciation charges. The source of this data was various issues of the U.S.D.A. Agricultural Statistics.

Pasture Rental

Cash rents per acre for pasture land in Kansas, as reported in various issues of the Economic Research Service (U.S.D.A.) publication, Farm Real Estate Market Developments, was used for indexing pasture rental costs.

The Z_6 index (1978 = 1.0) series for pasture rental was calculated by dividing the Kansas pasture rental rates (1965 to 1981) by the 1978 rate of $9.60 per acre.

Hay

Annual average prices paid by farmers for "all hay" were used for indexing hay costs. This data was found in the U.S. Department of Commerce, Bureau of the Census publication: Historical Statistics of the United States.

The "all hay" prices for each year of the 1965-1981 series was divided by the 1978 price of $49.80 per ton to create the Z_7 index (1978 = 1.0).

Grain, Concentrate and Silage

Seasonal average corn prices received by farmers in the U.S., as reported in varous issues of the U.S.D.A. Agricultural Prices, was used for indexing the cost of grain, concentrate and silage. The Z_8 index series (1978 = 1.0) was created by dividing the corn prices for each year of the 1965-1981 series by the price of $2.11 per bushel.

Protein Supplement

The average annual price of soybean meal, as reported in various issues of the U.S.D.A. Agricultural Prices, was used for indexing the cost of protein supplement. The soybean meal prices for each year of the 1965-1981 series were divided by the 1978 price of $11.70 per pound to create the Z_9 index series (1978 = 1.0) for protein supplement.

Salt and Minerals

Stock salt prices paid by farmers were used for indexing salt and mineral costs. The Z_{10} index series (1978 = 1.0) for salt and minerals was created by dividing the salt price for each year of the 1965-1981 series by the 1978 price of $3.89 per cwt.

The data source, again, was the U.S.D.A.'s Agricultural Prices.

Labor

The U.S. Composite Farm Wage Index (1967 = 1.0), found in various issues of the U.S.D.A.'s Agricultural Statistics, was used for indexing farm labor costs. The index for each year of the 1965-1981 series was divided by its 1978 value (236) to create the Z_{11} index series (1978 = 1.0) for farm labor.

It should be noted that the U.S. Composite Farm Wage Index series was discontinued in 1980. The value for 1981 is an estimate provided by the Economic Research Service of the U.S.D.A..

Medicine and Veterinary Care

The consumer price index (1967 = 1.0) for human medical care was assumed to be a close substitute for veterinary care. The Z_{12} index series (1978 = 1.0) for medicine and veterinary care was created by dividing the C.P.I. for human medical care (1965-1981) by its 1978 value of 219.4. The source of this data was various issues of the U.S.D.A. Agricultural Statistics.

Interest Rates

The Production Credit Association (P.C.A.) annual average cost of loans (in percent/100) is the last driving variable (Z_{13}) in the simulation model. The source of this interest rate data were, again, various issues of the U.S.D.A. Agricultural Statistics.

Table 6. Simulation Model Driving Variables.

Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	Z_{10}	Z_{11}	Z_{12}	Z_{13}	Z_{14}
24.1200	14.4400	.4550	.3510	.4650	.4640	.6030	.5500	.4240	.4240	.3570	.4080	.0658	1965
27.4300	17.8300	.4570	.3640	.4830	.5100	.6110	.5880	.4740	.4320	.3900	.4260	.0687	1966
26.6800	17.2200	.4630	.3810	.4750	.5000	.6130	.4880	.4880	.4370	.4150	.4560	.0729	1967
27.9200	17.9400	.4690	.3990	.5050	.5360	.5920	.5120	.4560	.4470	.4560	.4840	.0734	1968
31.7800	20.2900	.4800	.4200	.5590	.5780	.6020	.5500	.4500	.4580	.5020	.5170	.0779	1969
33.7000	21.3200	.4980	.4430	.5540	.6090	.6260	.6300	.5120	.4830	.5350	.5500	.0898	1970
34.8700	21.6200	.5320	.4720	.6120	.5940	.6640	.5120	.4850	.5140	.5560	.5850	.0728	1971
41.4000	25.2100	.5560	.5060	.6780	.6250	.6960	.7440	.5660	.5370	.5930	.6040	.0702	1972
53.1700	32.8200	.5880	.5410	.8510	.7290	.8370	1.2090	1.1970	.5760	.6510	.6240	.0809	1973
37.8800	25.5600	.6950	.6130	.8000	.8960	.9890	1.4310	.8800	.6480	.7180	.6790	.0943	1974
33.9100	21.0900	.7770	.7490	.8520	.9690	1.0650	1.2040	.7580	.7330	.7880	.7610	.0891	1975
39.4000	25.3100	.8460	.8490	.7470	.8960	1.1610	1.0190	.9150	.7970	.8630	.8420	.0824	1976
40.1800	25.3200	.9340	.9310	.7710	.9480	1.1440	.9570	1.1030	.9020	.9340	.9230	.0788	1977
58.7800	36.7800	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.0883	1978
83.0800	50.1000	1.1090	1.1000	1.2940	1.2080	1.1950	1.1200	1.1110	1.1080	1.0970	1.0930	.1071	1979
75.2300	45.7200	1.2900	1.2060	1.2790	1.2920	1.4240	1.3820	1.1710	1.3140	1.1860	1.2120	.1286	1980
66.2400	41.9300	1.4780	1.3530	1.2200	1.3130	1.3680	1.0890	1.2650	1.4780	1.2250	1.3400	.1447	1981

Content List:

Z_1 = Feeder steer prices
Z_2 = Utility cow prices
Z_3 = Fuel, lubrication & electricity index
Z_4 = Machinery & building index
Z_5 = Bull charges index
Z_6 = Pasture rental index
Z_7 = Hay index
Z_8 = Grain index
Z_9 = Protein supplement index
Z_{10} = Salt and mineral index
Z_{11} = Labor index
Z_{12} = Veterinary and medicine index
Z_{13} = P.C.A. interest rate
Z_{14} = year

CHAPTER 4

THE BEEF COW VALUE AND AGE DISTRIBUTION INVENTORY MODEL

The biological, management expectation and producer profit expectation functions, developed in Chapter 2, lay the foundation for the cow value and age distribution inventory model presented here. The cow value model makes value comparisons between present value for breeding (PVB_j) and present slaughter value (PSV_j) for each age and pregnancy class. The age distribution inventory model simulates changes in beef cow numbers and the age structure of the national cow herd through time (1965-1981).

The cow value model involves price estimation and expectation, budget generation, estimation of present values of expected future net revenues, and development of investment decision variables. These decision variables are used for determining the proportions of animals in each class to be retained in the herd, forming the major link between the cow value model and the age distribution inventory model.

The age distribution inventory model involves the calculation of heifer recruitment and cow retainment decisions, according to biological constraints and economic incentives, and the subsequent changes in the aggregate herd's beef cow numbers and age structure through time (1965-1981). The age distribution inventory model presents annual summations of the simulated numbers of (1) beef cows, (2) replacement heifers, (3) cull cows, and (4) calves born to beef cows. The comparisons of these annual summations to the objective historical series are described in Chapter 5.

The Beef Cow Value Model

Table 7 gives a list and a short description of the value model functions as presented in this section. An asterisk next to a function indicates the function is different from Nordblom's (1981) original model.

Expected Feeder and Utility Cow Prices

Nordblom's early runs with the previous model indicated the need for a two-year distributed lag in price expectations in order to accurately simulate a change in inventory trend. The functions $g_{12,1}$ and $g_{12,2}$ calculate the expected feeder steer prices and utility cow prices, respectively.

Expected feeder steer prices ($/cwt.):

$$g_{12,1} = b_{73}m_{4,1} + b_{74}z_1$$

where

$m_{4,1}$ = previous year's feeder steer price ($/cwt.)

z_1 = current year's feeder steer price ($/cwt.)

b_{73} and b_{74} are distribution parameters (b_{74}=1-b_{73})

Expected utility cow prices ($/cwt.):

$$g_{12,2} = b_{75}m_{4,2} + b_{76}z_2$$

where

$m_{4,2}$ = previous year's utility cow price ($/cwt.)

z_2 = current year's utility cow price ($/cwt.)

and b_{75} and b_{76} are distribution parameters (b_{76}=1-b_{75})

Table 7. Value Model Function List.

	$g_{12,1}$	= Expected future feeder steer price
	$g_{12,2}$	= Expected future utility cow price
*	$g_{13,j}$	= Expected future cull salvage value (FSV_j)
*	$g_{14,j}$	= Present cull salvage values (PSV_j); non-pregnant cows
*	$g_{25,j}$	= Present cull salvage values (PSV_j); pregnant cows
	g_{15}	= Interest charge factor
	g_{16}	= Costs common to all budgets
	g_{17}	= Cost budget for heifers kept for breeding (HKBs)
	g_{18}	= Costs common to yearling heifers
	g_{19}	= Cost budget for pregnant yearling heifers
	g_{20}	= Cost budget for non-pregnant yearling heifers
	g_{21}	= Costs common to cows, aged 3 years and over
	g_{22}	= Cost budget for pregnant cows
	g_{23}	= Cost budget for non-pregnant cows
	g_{26}	= Discount factor for present value calculations
*	$g_{28,1}$	= Present value of a HKB (PVB_1^N)
*	$g_{28,j}$	= Present value for breeding; pregnant cows (PVB_j^P)
*	$g_{30,j}$	= $V_j^P = PVB_j^P/PSV_j$
*	$g_{31,j}$	= Present value for breeding; non-pregnant animals (PVB_j^N)
*	$g_{32,j}$	= $V_j^N = PVB_j^N/PSV_j$ calculations

These expected prices are used in the projection of expected future salvage values.

Future and Present Cull Salvage Values

Cull cow slaughter values are functions of cow body weight, price and cow age. Cow body weight and cull cow prices per cwt, as a function of age, were both discussed in detail in Chapter 3. For convenience, the present salvage value functions for pregnant and non-pregnant heifers and cows are presented again here. The future salvage value (FSV_j) estimates, for each age and pregnancy class, are based on expected future prices. This relationship is described by function $g_{13,j}$ below.

For j = 1 to 15 = age becoming,

$$g_{13,j} = FSV_j = \begin{cases} [(g_{12,1} \cdot b_{39}) + b_{31}]g_7 & \text{,if } j = 1 \\ g_{4,j}[g_{12,1} - b_{40}(g_{12,1} - g_{12,2}) + \dfrac{b_{40}(g_{12,1} - g_{12,2})}{j \cdot b_{41}}] & \text{if } j > 1 \end{cases}$$

$$g_{14,j} = PSV_j^N = \begin{cases} [(g_{12,1} \cdot b_{39}) + b_{31}]g_7 & \text{,if } j = 1 \\ g_{4,j}\ [z_1 - b_{35}(z_1 - z_2) + \dfrac{b_{35}(z_1 - z_2)}{j \cdot b_{41}}] & \text{,if } j > 1 \end{cases}$$

$$g_{25,j} = PSV_j^p = \begin{cases} [(g_{12,1} \cdot b_{39}) + b_{31}]g_7 & \text{,if } j = 1 \\ g_{4,j}[z_1 - b_{33}(z_1 - z_2) + \dfrac{b_{33}(z_1 - z_2)}{j \cdot b_{41}}] & \text{,if } j > 1 \end{cases}$$

where

FSV_j = expected future salvage value for animals becoming j years of age at time of cull sales ($/hd.)

PSV_j^N and PSV_j^p = present salvage value for non-pregnant and pregnant animals becoming j years of age at time of cull sale ($/hd.), respectively.

b_{39} = .976475

b_{31} = -1.5218

g_7 = estimated weaning weight of a heifer kept for breeding (cwt.)

$g_{4,j}$ = estimated body weight of heifers and cows becoming j years of age (cwt.)

$g_{12,1}$ and $g_{12,2}$ = expected future feeder steer and utility cow prices, respectively, ($/cwt.)

z_1 and z_2 = current feeder steer and utility cow prices, respectively, ($/cwt.)

b_{33} = 1.0

b_{35} = 1.0

b_{40} = 1.2

b_{41} = 1.15

The future and present salvage value estimates are used in the calculation of present values of future net revenues and in the investment decision functions.

Annual Cost Budget Generator

The assumptions and data necessary for the generation of annual cost budgets was presented in Chapter 3. The purpose of this section is to describe the functional forms used in calculating the annual cost budgets for (1) weaned heifers kept for breeding, (2) pregnant yearling heifers, (3) non-pregnant yearling heifers, (4) pregnant mature cows, and (5) non-pregnant mature cows.

$b_{54} \cdot z_{12}$ = veterinary and medicine costs

g_{15} = short-term interest rate.

Next, the costs common to non-pregnant and pregnant yearling heifers are calculated.

$$g_{18} = [b_{55} \cdot z_6 + b_{56} \cdot z_7 + b_{57} \cdot z_8 + b_{58} \cdot z_9 + b_{59} \cdot z_{10}]$$

where

g_{18} = costs common to all yearling heifers ($/hd.)

$b_{55} \cdot z_6$ = pasture rental costs

$b_{56} \cdot z_7$ = purchased hay costs

$b_{57} \cdot z_8$ = grain and concentrate costs

$b_{58} \cdot z_9$ = protein supplement costs

$b_{59} \cdot z_{10}$ = salt and mineral costs.

The next two functions, using g_{18}, compute the annual cost budgets specific to pregnant and non-pregnant yearling heifers.

$$g_{19} = [g_{16} + g_{18} + b_{60} \cdot z_{11} + b_{61} \cdot z_{12}]g_{15}$$

where

g_{19} = annual cost budget for pregnant yearling heifers ($/hd.)

g_{16} and g_{18} = common costs

$b_{60} \cdot z_{11}$ = labor costs

$b_{61} \cdot z_{12}$ = veterinary and medicine costs

g_{15} = short-term interest rate.

Non-pregnant yearling heifers require less labor, veterinary care and medicine than their pregnant cohorts. The cost parameters

First, an interest rate for short-term operating costs is calculated for use with the annual cost budgets for each of the five classes of breeding animals. The Production Credit Association annual average cost of loans (z_{13}) is used to allow for changes in interest charges through time. The option of using a constant rate through time is available through the specification of the parameter b_{36}.

$$g_{15} = (1.0 + (b_{42} \cdot z_{13}) + b_{36})^{b_{43}}$$

where

g_{15} = interest rate for short-term operating costs

z_{13} = P.C.A. average cost of loans (%/100)

b_{42} = constant multiplier of the P.C.A. rate = 1.0

b_{36} = optional constant interest rate = 0

b_{43} = exponent representing the fraction of a year for which interest charges are assumed to accrue = 0.5.

The four common cost items; (1) marketing and hauling costs; (2) fuel, lubrication, and electricity; (3) machinery and building repair costs; and (4) bull charges are represented in the annually calculated function, g_{16}.

$$g_{16} = (b_{44} \cdot z_{11} + b_{45} \cdot z_3 + b_{46} \cdot z_4 + b_{47} \cdot z_5)$$

where

g_{16} = costs common to all classes

$b_{44} \cdot z_{11}$ = marketing and hauling costs

$b_{45} \cdot z_3$ = fuel, lube, and electricity costs

$b_{46} \cdot z_4$ = machinery and building repair costs

$b_{47} \cdot z_5$ = bull charges

The "b" parameters represent the per head, 1978 common costs, derived from the Great Plains herd budget. The exogenous inputs represented by the annual cost indices (1978=1.0) are the "z" terms in the g_{16} function.

Throughout the following discussion of the cost budget functions (g_{17} through g_{23}) the "b" parameters represent the respective 1978 budget levels for the five classes of breeding animals. Furthermore, the "z" terms always represent the annual exogenous input variables (cost indices, 1978=1.0).

The annual per head cost budget for a weaned heifer kept for breeding is calculated by the g_{17} function.

$$g_{17} = [g_{16} + b_{48} \cdot z_6 + b_{49} \cdot z_7 + b_{50} \cdot z_8 + b_{51} \cdot z_9 + b_{52} \cdot z_{10} + b_{53} \cdot z_{11} + b_{54} \cdot z_{12}]g_{15}$$

where

g_{17} = annual cost budget for weaned heifers kept for breeding ($/hd.)

b_{16} = costs common to all classes

$b_{48} \cdot z_6$ = pasture rental costs

$b_{49} \cdot z_7$ = purchased hay costs

$b_{50} \cdot z_8$ = grain and concentrate costs

$b_{51} \cdot z_9$ = protein supplement costs

$b_{52} \cdot z_{10}$ = salt and mineral costs

$b_{53} \cdot z_{11}$ = labor costs

contained in the next function are set to reflect this difference.

$$g_{20} = [g_{16} + g_{18} + b_{62} \cdot z_{11} + b_{63} \cdot z_{12}]g_{15}$$

where

g_{20} = annual cost budget for non-pregnant yearling heifers ($/hd.)

g_{16} and g_{18} = common costs

$b_{62} \cdot z_{11}$ = labor costs

$b_{63} \cdot z_{12}$ = veterinary and medicine costs

g_{15} = short-term interest rate.

Next, the costs common to pregnant and non-pregnant cows, becoming 3 years old and over, are calculated.

$$g_{21} = [b_{64} \cdot z_6 + b_{65} \cdot z_7 + b_{66} \cdot z_8 + b_{67} \cdot z_9 + b_{68} \cdot z_{10}]$$

where

g_{21} = common costs for pregnant and non-pregnant cows becoming 3 years old and over.

$b_{64} \cdot z_6$ = pasture rental costs

$b_{65} \cdot z_7$ = purchased hay costs

$b_{66} \cdot z_8$ = grain and concentrate costs

$b_{67} \cdot z_9$ = protein supplement costs

$b_{68} \cdot z_{10}$ = salt and mineral costs

The dollar per head amount computed by g_{21} is used in the final two functions. These calculate annual cost budgets specific to pregnant and non-pregnant mature cows.

$$g_{22} = [g_{16} + g_{21} + b_{69} \cdot z_{11} + b_{70} \cdot z_{12}]g_{15}$$

where

g_{22} = annual cost budget for pregnant mature cows, becoming 3 years old and over

g_{16} and g_{21} = common costs

$b_{69} \cdot z_{11}$ = labor costs

$b_{70} \cdot z_{12}$ = veterinary and medicine costs

g_{15} = short-term interest rate.

As with non-pregnant yearling heifers, the non-pregnant mature cows require less labor, veterinary care and medicine than their pregnant cohorts. This cost difference is reflected in the respective cost parameters.

$$g_{23} = [g_{16} + g_{21} + b_{71} \cdot z_{11} + b_{72} \cdot z_{12}]g_{15}$$

where

g_{23} = annual cost budget for non-pregnant mature cows, becoming 3 years old and over.

These annual cost budgets for the five respective breeding classes are used next in the calculation of the present value of breeding for pregnant and non-pregnant heifers and cows.

Present Value of Expected Net Future Incomes

The proportions of animals in each age and pregnancy class to be recruited or retained in the herd is determined on the basis of each respective class's V-ratio. The V-ratio for any particular class is defined as the ratio of discounted future net revenue expectations for a cow, if retained in the herd, to her present cull salvage value.

The purpose of this section is to describe how the estimates of the present value for breeding (PVB_j^P and PVB_j^N) for pregnant and non-pregnant cows in each age class, are computed. The reader is referred to the beginning of this chapter for a description of the present cull salvage value functions.

First, to express future years' costs and revenues in current dollars, a discount factor is needed. The function g_{26} is specified to represent this discount factor.

$$g_{26} = \frac{1.0}{1.0 + (b_{80} \cdot z_{13}) + b_{37}}$$

where

z_{13} = P.C.A. annual average cost of loans (%/100)

b_{80} = interest rate in base year = 1.0

b_{37} = an optional constant discount rate = Ø.

In Nordblom's (1981) model the calculation of PVB_j is based on expectations of future costs, prices and performance, as well as three limiting rules. The first rule is that all cows are culled before the age of 15 years. This limits the calculations for a cow presently becoming 14 to a one-year horizon, similarly a cow becoming 13 years of age is limited to a two-year horizon, and so on.

The second rule allows for the imposition of an arbitrary maximum limit on the planning horizon beyond the first year of the future. Assigning an integer value of zero to the control parameter b_{81} limits the maximum planning horizon, for any age group, to one year in the future. Similarly, a value of 1 limits the maximum horizon to two years in the future, and so on. Of course, the first rule (on maximum age) is still determining.

The third rule limits the planning horizon in the PVB_j calculations for younger cows through the FV_j ratio (PVB_j/FSV_j) for older cows. The FV_j ratios are calculated solely for the purpose of defining expected planning horizon limits in the oldest-to-youngest iterative calculation of PVB_j^p in each year of a simulation run. A future value (FV_j) ratio less than b_{82} (a parameter set at 1.0 in Nordblom's model) would cause the planning horizon for the PVB_{j-1} calculation to be limited to a single year. Conversely, an FV_j ratio greater than b_{82} would allow the PVB_{j-1} calculations to assume the animal would be retained in the herd as a j year old, if not limited to a shorter planning horizon by one of the other two rules [Nordblom 1981, p. 126].

In essence, the third rule causes Nordblom's model to seek to maximize the PVB_j. Starting with a cow becoming 14 years of age the model works backwards through the age classes until it locates an age group with an FV_j ratio larger than b_{82}. This age group then represents the planning horizon for that year of the simulation run. For example, assume the first age group with a FV_j greater than b_{82} is the six year old class. Given this rule, the planning horizon would be six years in the future.

The likelihood of a cattleman using a time horizon beyond two years in the future for calculating the expected worth of a cow today is indeed very slim. Nordblom realized this and thus, utilized the first two rules to limit the time horizon to two years. However, in order to incorporate these three "horizon limiting" rules into the model he had to develop <u>three</u> iterative functions ($g_{27,j}$, $g_{28,j}$, and

$g_{29,j}$). First, the sum of the discounted expected future net annual incomes and final cull sale revenue ($g_{28,j}$, PVB_j) is calculated. Next, $g_{28,j}$ is used in $g_{29,j}$ which calculates the FV_j ratios (PVB_j^p/FSV_j) for use in the expected final culling age decisions (g_{27}). This process was found to be unnecessarily complex. Thus, the current model form utilizes PVB_j functions for pregnant and non-pregnant cows ($g_{28,j}$ and $g_{31,j}$) which incorporates the concept of a maximum planning horizon of two years without the use of $g_{29,j}$ or g_{27}. This serves to simplify and yet maintain the accuracy and validity of the model.

$$g_{28,1} = \underbrace{\left[(g_{9,1}\, g_{13,2}) - g_{17}\right]}_{\substack{\text{expected net culling revenue}\\ \text{(one year in the future) for a}\\ \text{heifer kept for breeding today}}} \underbrace{g_{26}}_{\text{discount factor}} + \underbrace{\left[(g_{10,1}\, g_{13,3}) - g_{19}\right]}_{\substack{\text{expected net culling revenue (two years}\\ \text{in the future) for a heifer kept for}\\ \text{breeding today}}} \underbrace{g_{26}^{2}}_{\text{discount factor}} + \underbrace{g_{24,1}}_{\substack{\text{expected calf}\\ \text{sales for a HKB}}} \underbrace{g_{26}^{2}}_{\text{discount factor}} = PVB_1^N$$

where: PVB_1^N = The discounted present value for a heifer kept for breeding (HKB) if retained in the herd for two years.

$$g_{28,2} = \underbrace{\left[(g_{9,2} \cdot g_{13,3}) - g_{19}\right]}_{\substack{\text{expected net culling revenue}\\ \text{(one year in the future) for a}\\ \text{pregnant yearling heifer kept today}}} \cdot \underbrace{g_{26}}_{\text{discount factor}} + \underbrace{\left[(g_{10,2} \cdot g_{13,4}) - g_{22}\right]}_{\substack{\text{expected net culling revenue (two years}\\ \text{in the future) for a pregnant yearling}\\ \text{heifer kept today}}} \cdot \underbrace{g_{26}^{2}}_{\text{discount factor}} + \underbrace{g_{11,2} \cdot g_{26} + g_{24,2} \cdot g_{26}^{2}}_{\substack{\text{present value of calf sales revenues for a}\\ \text{pregnant yearling heifer if retained in}\\ \text{the herd for two years}}} = PVB_2^P \quad j=2=\text{age becoming}$$

PVB_2^P = The discounted present value expected for a pregnant yearling heifer is retained in the herd for two years.

$$g_{28,j} = \left[(g_{9,j}\ g_{13(j+1)}) - g_{22}\right] \cdot g_{26} + \left[(g_{10,j}\ g_{13(j+2)}) - g_{22}\right] \cdot g_{26}^{2} + g_{11,j} \cdot g_{26} + g_{24,j} \cdot g_{26}^{2}$$

- $\left[(g_{9,j}\ g_{13(j+1)}) - g_{22}\right]$: expected net culling revenue (one year in the future) for a pregnant cow kept today
- g_{26}: discount factor
- $\left[(g_{10,j}\ g_{13(j+2)}) - g_{22}\right]$: expected net culling revenue (two years in the future) for a pregnant cow kept today
- g_{26}^{2}: discount factor
- $g_{11,j}$: expected calf sales (one year in the future) for a pregnant cow kept today
- g_{26}: discount factor
- $g_{24,j}$: expected calf sales (two years in the future) for a pregnant cow kept today
- g_{26}^{2}: discount factor

$PVB^{P}_{(2+j)}$ = Present value of final cull sale revenue (two year horizon) + Present value of calf sales revenues (two year horizon)

j = 2 to 13

$$g_{31,j} = g_{28,j} - [g_{11,j} \cdot g_{26} + (g_{22} - g_{23}) \cdot g_{26}] = PVB^{N}_{j}$$

- $g_{28,j}$: (PVB^{P}_{j})
- $g_{11,j} \cdot g_{26}$: present value of calf sales in the coming year
- $(g_{22} - g_{23})$: constant adjustment for non-pregnant cows
- g_{26}: Discount factor

This function calculates the discounted present value of future net income (two year time horizon) expected for non-pregnant cows becoming j years of age, if kept for breeding. This is calculated by adjusting the PVB^{P} for pregnant cows of the same age by the loss of the first years' expected net calf sales revenue.

Decision Variables

The key links between the value model and the age distribution inventory model are the V-ratios (PVB_j/PSV_j) for both pregnant and non-pregnant classes. The ratios of future worth to present worth for pregnant cows, by age class, are calculated by the function $g_{30,j}$.

For j = 2 to 14 = age becoming,

$$g_{30,j} = \frac{g_{28,j}}{g_{25,j}} = V_j^p = \frac{PVB^p}{PSV_j}$$

These ratios provide an investment criteria which compares the immediate slaughter value of a cow (PSV_j) with her estimated present value of retainment (PVB_j). Any cow, pregnant or not, may be sold immediately at the cull salvage value of her age class, i.e., her present value for slaughter (PSV_j). Given a two year planning horizon, a pregnant cow obviously has the potential of weaning at least one calf and very possibly two. Assuming the cow survives and is retained in the herd, subtracting the estimated maintenance costs for each year and discounting the expected future net revenues from calf sales back to the present, yields the estimated present value of her retainment (PVB_j).

These V_j ratios (PVB_j/PSV_j) are used in the retainment rate functions of the age distribution inventory model. A V-ratio of less than 1.0 signals the model to cull heavily, while a V-ratio of 1.5 signals the model to increase retainment.

The V-ratios defined here also provide a common basis for comparison across age classes. "They answer the question: for each dollar of present liquid inventory value, how much (in present dollars) will one age class yield in the future versus all other age classes" [Nordblom 1981, p. 132].

The calculation of the present value of future net incomes for the non-pregnant classes (PVB_j^N) vary from those of the pregnant classes by the difference in maintenance costs between pregnant and non-pregnant classes and the discounted net annual incomes for the first year. The function $g_{32,j}$ computes the V-ratio for the non-pregnant classes.

$$g_{32,j} = \begin{cases} \dfrac{g_{28,1}}{g_{14,1}} & \text{,if } j = 1 \\ \dfrac{g_{31,1}}{g_{14,1}} & \text{,if } j > 1 \end{cases} \qquad V_j^N = \frac{PVB_j^N}{PSV_j}$$

The calculation of V_j^N ratios for non-pregnant heifers and cows completes the cow value model. The V-ratios for all classes of breeding animals are used in the retainment decision functions $g_{37,j}$ and $g_{38,j}$. This provides the key link between the cow value model and the beef cow age distribution inventory model described below.

Age Distribution Inventory Model

The age distribution inventory model simulates annually the numbers of beef heifers and cows in each age and pregnancy class. Table 8 lists the functions specific to the age distribution model. An asterisk next to a function indicates a change from Nordblom's (1981) model. Figure 3 presents a flowchart representation of the model. After the initial year, each year of a simulation run begins with the simulated post-culling numbers of heifers and cows in each age and pregnancy class. The state variables, $x_{1,j}$ and $x_{2,j}$, described below, carry the beginning inventory numbers of heifers and cows becoming j years old.

Table 8. Age Distribution Inventory Model Function List.

State Variables	
$x_{1,1}$	= Weaned heifers not kept for breeding
$x_{1,j}$	= Post-culling inventories of pregnant cows (j = 2 to 14)
$x_{2,j}$	= Post-culling inventories, non-pregnant heifers and cows (j = 1 to 13)
Intermediate Functions	
$*g_{33}$	= The percent of yearlings bred to calve as 2 year olds
$*g_{34,j}$	= Pre-culling inventories of pregnant animals
$*g_{35,j}$	= Pre-culling inventories of non-pregnant animals
$*g_{36}$	= The percent of yearlings not exposed for breeding
$g_{37,j}$	= Proportions of pregnant animals to be retained
$g_{38,j}$	= Proportions of non-pregnant animals to be retained
$g_{39,j}$	= Numbers of pregnant animals to be retained
$*g_{40,2}$	= Number of non-pregnant yearlings to be retained
$g_{40,j}$	= Numbers of non-pregnant animals to be retained (j = 1, 3-13)
$g_{41,j}$	= Numbers of pregnant animals to cull
$g_{42,j}$	= Numbers of non-pregnant animals to cull
$*g_{43,j}$	= Summations for output reports
FLUX Functions (Updating State Variables)	
$f_{1,j} = \Delta x_{1,j}$ and $f_{2,j} = \Delta x_{2,j}$	

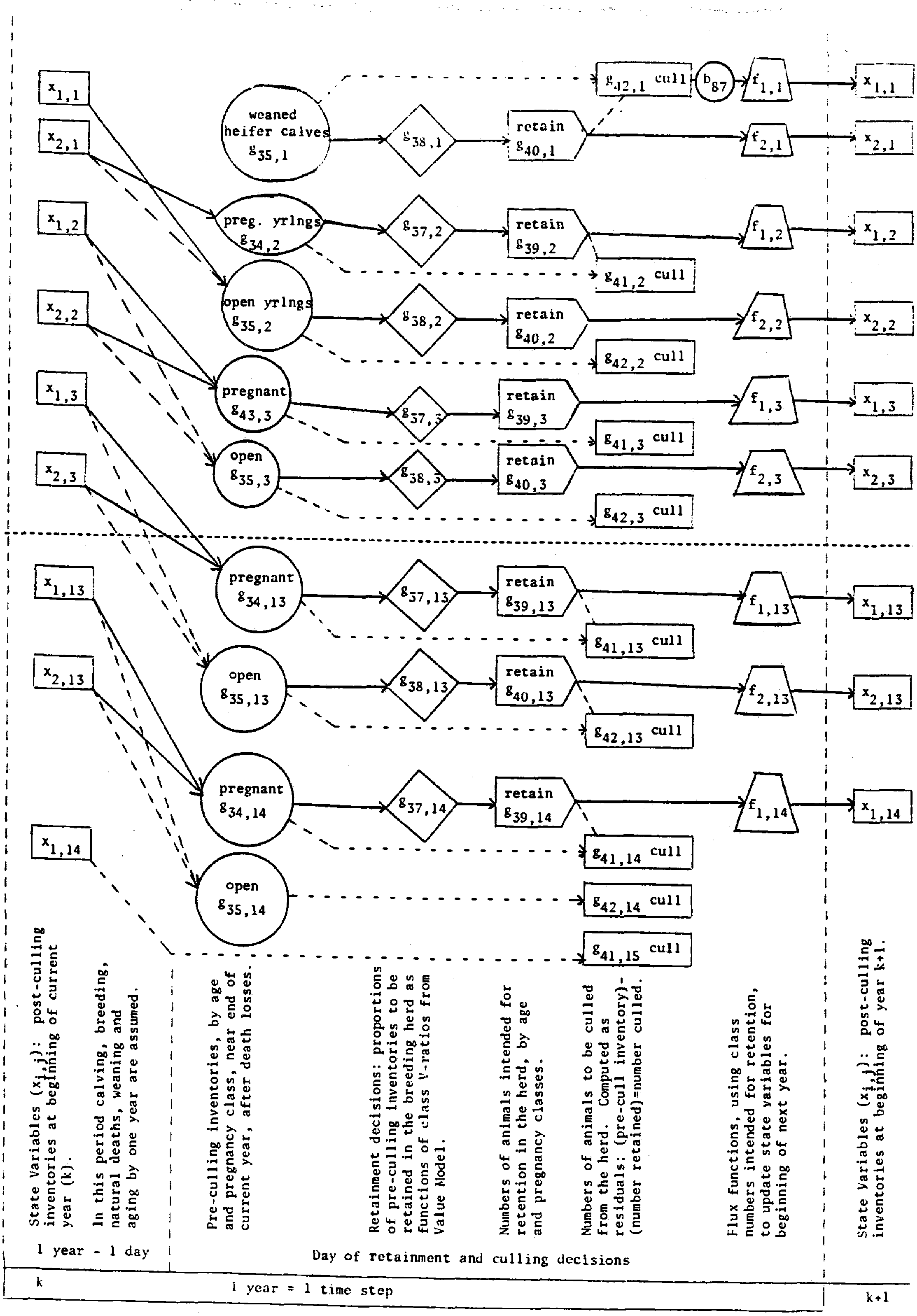

Figure 3. Beef Cow Age Distribution Inventory Model Flowchart.

State Variables

The $x_{1,j}$ state variables refer to the number of pregnant animals in each age class with the exception of $x_{1,1}$. The $x_{1,1}$ variable represents the number of weaned heifers not kept for breeding, but which are available for recruitment as non-pregnant yearlings next year.

The number of non-pregnant animals in the post-culling inventory are carried by the state variables $x_{2,j}$. It is assumed that all non-pregnant 14 year old cows are culled, thus for $x_{2,j}$, j = 1 to 13.

In the first year of a simulation run, the initial numbers of heifers and cows in each age and pregnancy class must be specified. These initial state variable values for the current simulation are listed in appendix B. All the $x_{i,j}$ variables are in units of 100,000 head.

Intermediate Functions

The function $g_{34,j}$ calculates the number of pregnant animals in the pre-culling inventory (100,000 head units). It is assumed that lactating and dry cows have identical survival rates ($g_{3,j}$) and conception rates ($g_{1,j}$), at the same ages.

For j = 2 to 14 = age becoming,

$$g_{34,j} = \begin{cases} x_{2,1} \cdot g_{3,2} \cdot g_{1,1} \cdot g_{33} & ,\text{if } j = 2 \\ [x_{1,(j-1)} + x_{2,(j-1)}]g_{3,j} \cdot g_{1,(j-1)} & ,\text{if } j > 2 \end{cases}$$

where

$x_{2,1}$ = number of heifers kept for breeding

$x_{1,1}$ = number of non-pregnant yearlings

$x_{1,j}$ = number of pregnant cows becoming 2 years old and over

$g_{1,j}$ = conception rate by age

$g_{3,j}$ = survival rate by age

g_{33} = percent of yearlings bred to calve as two year olds.

The function $g_{34,2}$ calculates the number of pregnant yearling heifers in the pre-culling inventory. The current model assumes that not all HKBs are bred to calve as 2-year-olds, therefore, the equation g_{33} has been specified to account for this assumption.

$$g_{33} = [b_{29} + b_{30}(t-t_0)]$$

where

g_{33} = the percent of yearlings bred to calve as 2 year olds as a linearly increasing function of time

b_{29} = 0.65

b_{30} = 0.009.

The pre-culling inventories of the non-pregnant classes are computed by the function $g_{35,j}$.

For j = 1 to 14 = age becoming,

$$g_{35,j} = \begin{cases} (1/2)\ [\sum_{i=2}^{14} x_{1,i} \cdot g_{8,i}] & ,\text{if } j = 1 \\ [x_{2,1} \cdot g_{3,2}(1-g_{1,1})] + x_{1,1} & ,\text{if } j = 2 \\ [x_{1,(j-1)} + x_{2,(j-1)}] \cdot g_{3,j} \cdot (1-g_{1,(j-1)}) & ,\text{if } j > 2 \end{cases}$$

where

$g_{35,j}$ = pre-culling inventory (in 100,000 head units) of non-pregnant animals becoming j years of age.

The equation $g_{35,1}$ calculates the number of weaned heifers available for recruitment into the breeding herd as the product of the number of pregnant cows in each age class $(x_{1,j})$ times their respective calf survival rates $(g_{8,j})$. Furthermore, it is assumed that heifers comprise exactly 50 percent of the calves weaned.

The number of non-pregnant yearlings in the pre-culling inventory is computed as the sum of the HKBs $(x_{2,1})$ which survived but did not conceive and the special class of heifers $(x_{1,1})$ defined above. The numbers of non-pregnant mature cows in the pre-culling inventory are computed in the same way as their pregnant age cohorts, with the exception of the allowance for non-conception $(1-g_{1,1})$.

The proportions of pregnant and non-pregnant heifers and cows which are to be retained in the herd depend on their respective retainment functions $g_{37,j}$ and $g_{38,j}$. The numbers of these pregnant and non-pregnant animals retained are subsequently calculated by the functions $g_{39,j}$ and $g_{40,j}$, respectively.

First, the number of yearlings not exposed for breeding must be accounted for. The function g_{36} calculates this percent as the product of the class of weaned heifers kept for breeding $(x_{2,1})$ times the residual of the yearlings bred to calve as 2 year olds $(1-g_{33})$.

$$g_{36} = [x_{2,1} \cdot g_{3,2} \cdot (1-g_{33})]$$

These yearling heifers which have been retained but not exposed for breeding are added into the function $g_{40,2}$; the number of non-pregnant cows kept for breeding in the coming year.

Retainment Decisions

The V-ratio for each of the five breeding classes determines the proportion of healthy animals retained out of the pre-culling inventory of a given breeding category. In general, the model increases culling pressure when V-ratios fall below 1.0 and decreases culling pressure when V-ratios are well above 1.0.

For the purpose of specifying retainment decision functions, three general categories of breeding animals have been identified. The first category is comprised of the successful breeders, i.e., pregnant heifers and cows of all ages. The second category of untried animals includes only weaned heifers and non-pregnant yearlings. All the non-pregnant cows, becoming 3 years old or over, comprise the third category of unsuccessful breeders. The first, second, and third categories are thus referred to as successful, untried and unsuccessful animals, respectively.

When compared on a V-ratio-by-V-ratio basis these three categories of brood animals face different intensities of culling pressure. The separate retainment decision functions, specified for each of these three categories, express the relative differences in culling pressure. The retainment decision functions given below are described as logistic functions of the V-ratios.

For j = 2 to 14 = age becoming

$$g_{37,j} = b_{88} + \left[\frac{g_{2,j} - b_{88}}{1.0 + e^{b_{83}(g_{30,j} - b_{84})}}\right]$$

where

$g_{37,j}$ = the proportion of the pre-culling inventory of pregnant animals becoming j years old which are to be retained in the herd

$g_{2,j}$ = the maximum proportion to be retained = the rate of unimpaired health for pregnant animals

$g_{30,j}$ = V_j^p-ratios for pregnant animals becoming j years old

b_{88} = the minimum proportion to be retained = 0

b_{84} = inflection point = .53

b_{83} = a parameter establishing the slope of the decision curve for pregnant animals = -5.5.

The retainment decisions for untried and unsuccessful animals are expressed by the $g_{38,j}$ functions.

For j = 1 to 13 = age becoming,

$$g_{38,j} = \begin{cases} b_{89} + \left[\dfrac{(b_{94} g_{2,j}) - b_{89}}{1.0 + e^{b_{92}(g_{32,j} - b_{93})}} \right] & \text{,if } j \leq 2 \text{ (untried breeders)} \\ \\ b_{90} + \left[\dfrac{(b_{91} g_{2,j}) - b_{90}}{1.0 + e^{b_{85}(g_{32,j} - b_{86})}} \right] & \text{,if } j > 2 \text{ (unsuccessful breeders)} \end{cases}$$

where

$g_{38,j}$ = proportion of the pre-culling inventory of non-pregnant animals becoming j years old which are to be retained in the herd

$b_{94} \cdot g_{2,j}$ and $b_{91} \cdot g_{2,j}$ = maximum proportions to be retained for non-pregnant classes (b_{94} = .80, b_{91} = 1.0)

b_{89} and b_{90} = minimum proportions to be retained (b_{89} = .20, b_{90} = 0).

$g_{32,j}$ = V_j^N-ratios for non-pregnant animals becoming j years old

b_{93} and b_{86} = inflection points (b_{93} = 1.1, b_{86} = .535)

b_{92} and b_{85} = parameters establishing the slope of the decision curves (b_{92} = -5.5, b_{85} = -5.5).

The class V-ratios represent the class means, allowing some exceptional cows in a low V-ratio class to be retained while culling poorer quality cows in a higher V-ratio class.

Numbers Retained and Culled

The pre-culling inventories of each age and pregnancy class times their respective retainment rates equals the post-culling inventories of all pregnant classes.

For j = 2 to 14 = age becoming for pregnant animals,

$$g_{39,j} = g_{34,j} \cdot g_{37,j}$$

where

$g_{39,j}$ = the number of pregnant animals becoming j years old to be retained in the herd (100,000 head units)

$g_{34,j}$ = pre-culling inventory of pregnant animals becoming j years old

$g_{37,j}$ = the proportion of pregnant animals becoming j years old to be retained in the herd.

The post-culling inventory of non-pregnant yearlings kept for breeding is computed by $g_{40,2}$, while the post-culling inventories for all other non-pregnant age groups (j = 1, 3-13) is calculated by function $g_{40,j}$.

$$g_{40,2} = (g_{35,2} \cdot g_{38,2}) + g_{36}$$

where

$g_{40,2}$ = number of non-pregnant yearlings kept for breeding in the coming year as j year olds

$g_{35,2}$ = pre-culling inventory of non-pregnant yearlings becoming 2 years old

g_{36} = the number of yearlings retained but not exposed for breeding.

For j = 1, 3-13 = age becoming

$$g_{40,j} = g_{35,j} \cdot g_{38,j}$$

where

$g_{40,j}$ = the number of non-pregnant animals becoming j years old to be retained in the herd (100,000 head units)

$g_{35,j}$ = pre-culling inventory of non-pregnant animals becoming j years old

$g_{38,j}$ = the proportion of non-pregnant animals, becoming j years old, to be retained in the herd.

The numbers culled from each age and pregnancy class is calculated as the difference between the pre-culling inventories and the numbers to be retained. First, the function $g_{41,j}$ computes the numbers culled from the pregnant classes.

For j = 2 to 15 = age becoming

$$g_{41,j} = \begin{cases} g_{34,j} - g_{39,j} & ,\text{if } j < 15 \\ x_{1,14} \cdot g_{3,15} & ,\text{if } j = 15 \end{cases}$$

where

$g_{41,j}$ = the number of pregnant animals culled prior to becoming j years old (in 100,000 head units)

$g_{34,j}$ = pre-culling inventory of pregnant animals becoming j years old

$g_{39,j}$ = number of pregnant animals becoming j years old which are to be retained in the herd

$x_{1,14}$ = beginning inventory of pregnant cows becoming 12 years old

$g_{3,15}$ = survival rate of cows in the year prior to becoming 15 years old.

Likewise, the numbers of non-pregnant animals culled are calculated by $g_{42,j}$.

For j = 1 to 14 = age becoming

$$g_{42,j} = \begin{cases} g_{35,j} - g_{40,j} & ,\text{if } j < 14 \\ g_{35,14} & ,\text{if } j = 1 \end{cases}$$

where

$g_{35,j}$ = pre-culling inventory of non-pregnant animals becoming j years old (all non-pregnant 14 year olds are culled)

$g_{40,j}$ = number of non-pregnant animals becoming j years old to be retained in the herd.

The complete post-culling beef breeding herd inventory is summarized by the function $g_{43,1}$. This total brood cow inventory is carried in units of a million head and is reported as one of the model's annual output functions, $Y_{1,2}$.

$$g_{43,1} = [(b_{98} \cdot g_{39,2}) + (\sum_{i=3}^{14} g_{39,i}) + (\sum_{i=3}^{13} g_{40,i}) + (b_{77} \cdot g_{43,3})](0.1)$$

where

$g_{43,1}$ = total brood cow inventory, comparable to the U.S.D.A. January 1 inventory of beef cows that have calved (million head units)

$g_{39,i}$ = pregnant cows, becoming 3 years old and over, to be retained in the herd

$g_{40,i}$ = non-pregnant cows, becoming 3 years old and over, to be retained in the herd

b_{98} = 0

$b_{77} \cdot g_{43,3}$ = simulated number of culled pregnant and non-pregnant beef cows still on inventory after January 1 (b_{77} = .50).

The next function ($g_{43,2}$) sums up the simulated numbers of weaned heifers, pregnant yearlings and non-pregnant yearlings for comparison with the objective historical series of "heifers for replacement."

$$g_{43,2} = [(b_{95} \cdot g_{39,2}) + (b_{96} \cdot g_{40,1}) + (b_{97} \cdot g_{40,2})](0.1)$$

where

$g_{43,2}$ = the number of "heifers for replacements" in million head units) for comparison with the objective historical series from the U.S.D.A. reported as one of the model's output functions, $Y_{1,3}$.

$g_{39,2}$ = number of pregnant yearling heifers in the post-culling inventory

$g_{40,1}$ = number of weaned HKB in the post-culling inventory

$g_{40,2}$ = number of non-pregnant yearlings in the post-culling inventory

$b_{95} = 1.0$

$b_{96} = .40$

$b_{97} = 0.$

For comparison with the objective historical series on annual beef cow slaughter numbers reported by the U.S.D.A., the function $g_{43,3}$ calculates the sum of the simulated cow numbers.

$$g_{43,3} = [(\sum_{i=3}^{15} g_{41,i}) + (\sum_{i=3}^{14} g_{42,i})](0.1)$$

where

$g_{43,3}$ = simulated number of culled pregnant and non-pregnant beef cows, becoming 3 years old and over (million head units). Reported as one of the model's annual output functions, $Y_{1,4}$.

$g_{41,i}$ = culled pregnant cows

$g_{42,i}$ = culled non-pregnant cows.

The number of calves weaned in the current year of the simulation is calculated by $g_{43,4}$.

$$g_{43,4} = \sum_{i=2}^{14} (x_{1,i} \cdot g_{8,i})$$

where

$x_{1,i}$ = the beginning inventory numbers of pregnant heifers and cows summed across each age group

$g_{8,i}$ = respective calf survival rates summed across each age group.

Following is a brief list and description of some model output functions which express the number of calves weaned in a given year, relative to different measures of herd size.

<u>Herd Productivity Indicators</u>

$Y_{1,6}$ = number of calves weaned in the current year per cow and heifer exposed for breeding the previous year.

$Y_{1,7}$ = number of calves weaned per cow and heifer becoming 2 years old and over in the beginning inventory.

$Y_{1,8}$ = number of calves weaned per pregnant cow and heifer in the beginning inventory.

$Y_{1,14}$ = number of calves weaned per calf born in the current year.

For specific functional forms the reader is referred to the FLEXFORM document of Appendix B.

Next, the function $g_{43,5}$ computes the total number of cows and heifers, becoming 2 years old and over, on inventory at the beginning of the current year (100,000 head units).

$$g_{43,5} = \left(\sum_{i=2}^{14} x_{1,i}\right) + \left(\sum_{i=2}^{13} x_{2,i}\right)$$

Finally, the total number of calves born to beef cows in the current year is summarized by $g_{43,6}$ (million head units).

$$g_{43,6} = \left[\sum_{i=2}^{14} (x_{1,i} \cdot g_{3,(i+1)})\right](0.1)$$

FLUX Functions

As specified by the FLEX/REFLEX modeling paradigm used here, the state variables, $x_{i,j}$ are updated by FLUX functions, $f_{i,j}$. For each respective state variable there is a corresponding FLUX function. The state variable updating process is as follows:

$$x_{i,j(K+1)} = x_{i,j(K)} + f_{i,j(K)} = x_{i,j(K)} + \Delta x_{i,j(K)}$$

where K = the current time step.

In the current model, the FLUX functions represent the number of animals, in each age and pregnancy class, to be added to or subtracted from the old value of the state variables.

For j = 1 to 14 = age becoming,

$$f_{1,j} = \begin{cases} (g_{42,1} \cdot b_{87}) - x_{1,1} & \text{,if } j = 1 \\ g_{39,j} - x_{1,j} & \text{,if } j > 1 \end{cases}$$

where

$f_{1,1}$ = the FLUX function corresponding to the state variable ($x_{1,1}$) for the class of weaned heifers not kept for breeding but available for recruitment in the future

$g_{42,1}$ = total number of heifers not to be retained

$g_{39,j}$ = number of pregnant animals becoming j years old in the post-culling inventory

$x_{1,j}$ and $x_{1,1}$ = previous years state variable values

b_{87} = .50.

Similarly for the non-pregnant classes:

j = 1 to 13 = age becoming

$$f_{2,j} = g_{40,j} - x_{2,j}$$

These FLUX functions, simply, calculate the changes in the numbers of pregnant and non-pregnant animals on inventory from the beginning of the current year to the beginning of the next.

The description of the age distribution inventory model is now complete. The reader is referred to Figure 3 for a summary picture of the simulated flow of cattle inventory numbers through time. Appendix B contains FLEXFORM documentation of all the preceeding functions.

The next chapter will discuss the objective historical data series and the statistical functions used for comparing the simulated inventories of cows, heifers, culls and calves born.

CHAPTER 5

VALIDATION AND RESULTS

The object of model construction is to create model behavior that is satisfactorily close to real world behavior. Primary validation of the current model is measured by its ability to simulate as closely as possible the objective historical data series described below. The graphic presentation of the model's ability to track the historical data series facilitates acceptance of the model after the statistical criteria have been met.

Historical Data

The historical data of January 1 inventories of beef cows and replacement heifers and annual numbers of cull cows slaughtered and beef calves born are the series against which the simulated numbers are compared. The historical data were taken from a U.S.D.A. computer file [U.S.D.A., ESS, T-DAM, 1979], and various issues of the U.S.D.A. Livestock and Meat Statistics.

The U.S.D.A., January 1 inventory, estimates of beef cows and heifers that have calved, from 1966 through 1982, are recorded in the FLEXFORM as the parameter values of b_{101} through b_{132}.

U.S.D.A., January 1 inventory, estimates of the numbers of heifers kept for breeding, from 1966 through 1982, are represented by the parameter values b_{140} through b_{171}.

The 1965 through 1981 annual estimates of beef cows slaughtered are recorded in the FLEXFORM by the parameters b_{178} through b_{209}.

The number of beef calves born was derived from U.S.D.A. esti-

Table 9. Summary Statistics.

For Beef Cows

$Y_{8,2}$ = Mean proportional absolute deviation of simulated cow numbers from historical cow numbers.

$Y_{8,3}$ = Correlation coefficient (r) between simulated and historical series of beef cow number changes.

$Y_{8,4}$ = Inequality coefficient for comparing simulated changes with historical changes in beef cow numbers = Theil's U.

$Y_{8,5}$ = Proportion of inequality due to mean bias = Theil's U^m.

$Y_{8,6}$ = Proportion of inequality due to unequal variance = Theil's U^s.

$Y_{8,7}$ = Proportion of inequality due to imperfect covariation = Theil's U^c.

NOTE: The above summary statistics are computed for heifers recruited by functions $Y_{9,2}$ through $Y_{9,7}$, for cows culled by $Y_{10,2}$ through $Y_{10,8}$ and for calves born by the output functions $Y_{11,2}$ through $Y_{11,8}$.

mates of total calves born and dairy cow numbers. The number of dairy calves born were derived by multiplying the number of dairy cows by .88. Total calves born minus the estimated number of dairy calves born yielded the number of beef calves born. This derived beef calf series is represented in the FLEXFORM by the parameter values b_{217} through b_{249}.

Statistical Comparison of Simulated and Historical Series

Table 9 gives a brief description of the six summary statistics computed by the model for comparing the series over the 17 year run.

The mean proportional absolute deviations (MPAD) represent the "average error," of each of the four simulated categories from their respective historical series, for the entire simulation run.

$$Y_{(7+i),2} = \frac{g_{44,i}}{b_{99}} = \Sigma \frac{\left|\frac{S_i - H_i}{H_i}\right|}{n}$$

where

S_i = simulated, H_i = historical

i = 1 for cow inventories

i = 2 for heifer inventories

i = 3 for cull cow numbers

i = 4 for number of calves born

b_{99} = n = 17 years.

Because they are in proportional terms, a ten percent deviation early in the run counts as heavily as a ten percent deviation near the end of the run.

Next, the model computes Theil's coefficient of inequality (U), and its decomposition statistics (U^m, U^s, and U^c). The simulated and historical series are transformed into terms of annual proportional changes to facilitate the calculation of the Theil statistics.

$$P = (S_k - S_{k-1})/S_{k-1}$$

$$A = (H_k - H_{k-1})/H_{k-1}$$

where

P = predicted changes

A = actual changes

S_k = simulated numbers in current year

S_{k-1} = simulated numbers in previous years

H_k = historical numbers in current year

H_{k-1} = historical numbers in previous years.

The intermediate functions $g_{4i,1}$ through $g_{4i,8}$ perform the transformations and summations each year of the simulation run. The FLEX-FORM document of Appendix B provides a detailed description of these calculations.

In the last year of a simulation run the standard deviations of the year-to-year proportional changes in the simulated (P) and historical (A) series are calculated by functions $g_{4i,9}$ and $g_{4i,10}$, respectively. These standard deviations (S_P and S_A) are used in calculating the correlation coefficients reported by the model output functions $Y_{8,3}$, $Y_{9,3}$, $Y_{10,3}$ and $Y_{11,3}$. A positive correlation close to 1.0 is desired.

A useful measure of the accuracy of historical simulations or ex-post forecasts is Theil's inequality coefficient, U. This statistic is calculated by the model in the last year of a simulation run.

$$U = \sqrt{(1/n)\Sigma(P-A)^2/(1/n)\Sigma A^2} = \sqrt{g_{4i,8}/g_{4i,6}}$$

The values that the inequality coefficient assumes lie between 0 and ∞. The smaller the value of the inequality coefficient the better is the predictive performance of the model. If the predicted changes equal the actual changes, then U = 0, and there is a perfect fit. If, on the other hand, P = 0 then U = 1 and the model's forecast is no better than a naive zero-change prediction. If U is greater than 1, the predictive power of the model is worse than the zero-change prediction.

The Theil inequality coefficient numerator may be decomposed into three terms each indicating a different source of simulation error. These proportions of inequality are defined as:

$$U^m = \frac{(\overline{P}-\overline{A})^2}{(1/n)\Sigma(P-A)^2}$$ = proportion of inequality due to mean bias

where $\overline{P} = \Sigma P/n$, $\overline{A} = \Sigma A/n$

$$U^s = \frac{(S_A-S_P)^2}{(1/n)\Sigma(P-A)^2}$$ = proportion of inequality due to unequal variance, and

$$U^c = \frac{2(1-r)(S_P S_A)}{(1/n)\Sigma(P-A)^2}$$ = proportion of inequality due to imperfect covariation

such that: $U^m + U^s + U^c = 1.0$.

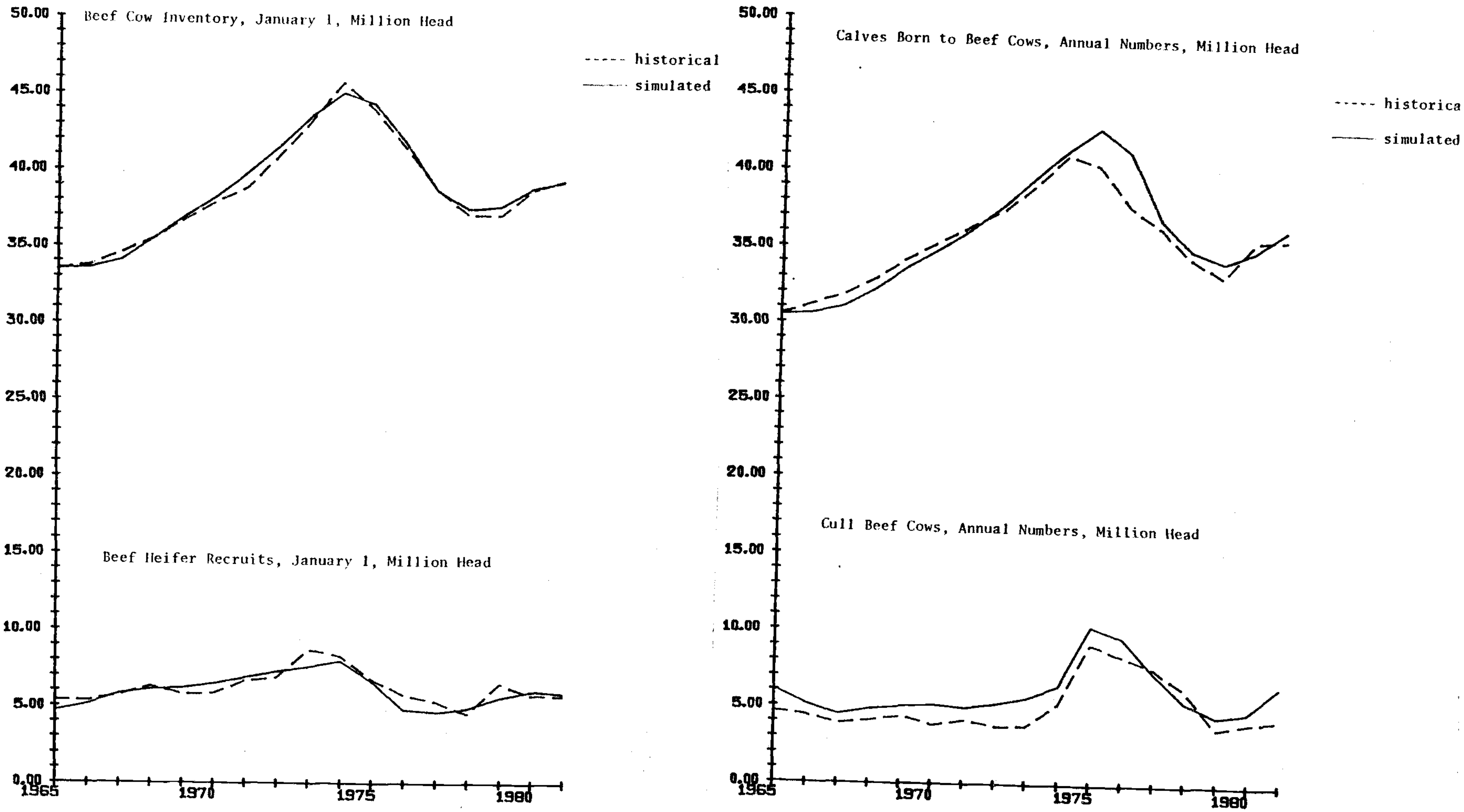

Figure 4. Simulated and Historical Inventories: 1965 to 1981.

The bias proportion U^m is an indication of systematic error, since it measures the extent of the difference between the simulated mean and the actual mean. The closer U^m is to zero the smaller the systematic bias.

The variance proportion U^s indicates that another cause of the discrepancy between the predicted and actual series is the difference between their variances. If U^s is large, it would indicate that one of the series has fluctuated considerably while the other has not.

The third source of error, the unsystematic part, is measured by the covariance proportion U^c. It is unrealistic to expect a simulation model to produce predictions that are perfectly correlated with the historical series, however, the closer U^c is to 1, the more acceptable the model.

These three inequality statistics are computed in the last year of a simulation run and reported by the output functions $Y_{7+i,4}$ through $Y_{7+i,7}$, where i is as defined above.

Results

The results of the present models' best simulation effort are presented here. The statistics used for evaluating the ability of the model to simulate the numbers of cows, heifers, culls and calves born are presented in Table 10. Figure 4 provides a graphical view of the model's tracking ability.

In the current study, major emphasis was placed on improving the tracking ability of the model's simulated numbers of cows and calves born. The mean proportional absolute deviation (MPAD) of .009 for cow inventories were by far the best. The MPAD for calves

Table 10. Comparison Statistics for Simulated and Historical Numbers.*

Comparison Class	Output Functions		j=2 MPAD	j=3 r	j=4 U	j=5 U^m	j=6 U^s	j=7 U^c
COWS	$Y_{8,j}$	=	.009 (.019)	.951 (.884)	.300 (.489)	.000 (.008)	.058 (.367)	.942 (.625)
HEIFERS	$Y_{9,j}$	=	.075 (.082)	.568 (.533)	.825 (.854)	.002 (.000)	.167 (.145)	.831 (.854)
CULLS	$Y_{10,j}$	=	.227 (.251)	.823 (.759)	.567 (.738)	.000 (.001)	.052 (.643)	.947 (.356)
CALVES BORN	$Y_{11,j}$	=	.023 (.035)	.767 (.633)	.696 (.774)	.002 (.001)	.018 (.038)	.980 (.961)

* The statistics within the parenthesis are the statistics for the homogeneous cow run.

born came in second with a value of .023.

The correlation coefficient of single period changes (r), and the coefficient of inequality (U) were also best for cows, with values of .951 and .300, respectively. The decomposition statistics U^m, U^s, and U^c indicate that the predictive power of the current model with respect to cow inventories is substantial (see Table 10).

The correlation coefficient and U statistic for calves born were, surprizingly, only third best with values of .767 and .696, respectively. However, the decomposition statistics for calves born indicate little room for improvement.

The predictive power of the model with respect to heifers is fair at best. A correlation coefficient of only .568 and a U statistic of .825 indicates there is need for improvement in this area. The proportion of inequality due to unequal variance (U^s = .167) is higher than any other class.

The MPAD for culls (.227) is the worst in all respects. However, the correlation coefficient (r = .823) and the coefficient of inequality (U = .567) were second best to that for cow inventories. This may be partly explained through visual inspection. The simulated numbers of culls track the historical numbers rather well, except for the fact that the simulated numbers are almost always too high.

Overall, the U statistics indicate that model simulates better than a zero-change prediction. All the coefficients of inequality were well below 1.0.

Fine Tuning the Model

Model behavior is sensitive, in different degrees, to changes in the various parameters contained in the model's functions. An iterative trial and error method of fine tuning, inspection of the comparison statistics, and respecification of parameters was used to achieve the current simulation results.

The model proved to be insensitive to changes made in the distributed lag parameters of the expected feeder steer and utility cow price functions ($g_{12,1}$ and $g_{12,2}$). Thus, the original values were assumed for the present model. Many other parameter changes were tried and either rejected or accepted on the basis of whether tracking performance was improved or not. However, the fine tuning of various parameters still left a MPAD of .025 for the cow inventories with most of the error occurring in the last five years of the simulation (see Figure 5).

Aggregate retainment rates in relation to the V-ratios changed in 1976. From 1974 to 1976 the V-ratios and herd retainment rates fell gradually while simulated inventories also declined. The simulated cow inventories tracked the historical inventories, up to 1976, quite well. After 1976, the V-ratios continued to fall which led to a further decline in retainment rates and a progressive departure between the simulated and historical cow inventories. In 1978, there was a reversal in the trend in V-ratios, but a large error in simulated cow inventories had already accumulated.

Analysis of the historical series showed that while the numbers of HKB remained relatively constant, there was a moderate drop in the cow slaughter rate, commencing about 1975, and a dramatic drop

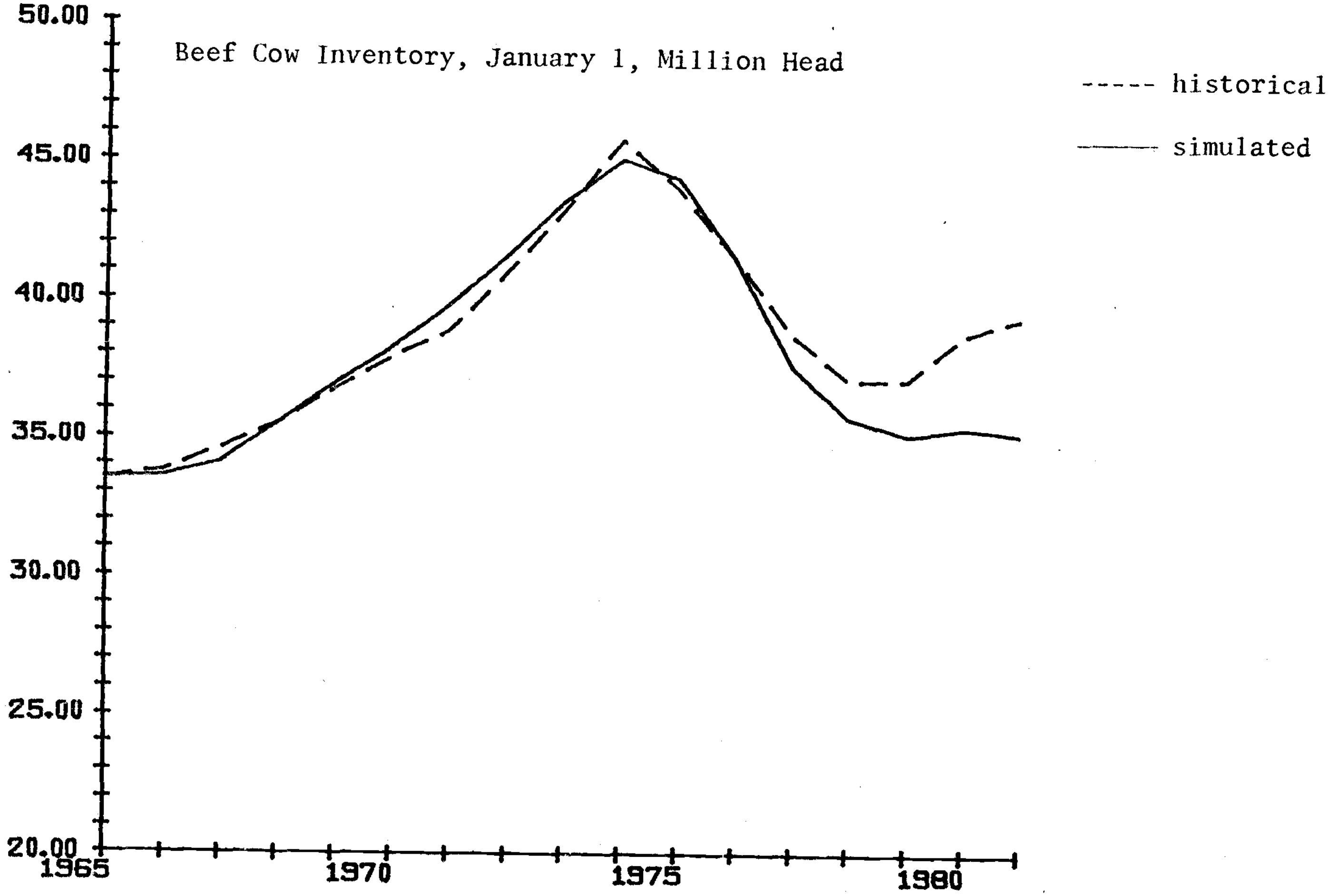

Figure 5. Simulated and Historical Cow Numbers, Before Retainment Rate Shift.

in 1978. This change in cow slaughter indicated a change in the retainment rates in relation to the V-ratios, particularly among non-pregnant mature cows.

In order to correct for this problem, the retainment rate function was shifted to the right, indirectly, through the present salvage value functions ($g_{14,j}$ and $g_{25,j}$). These changes reduced the denominator of the V-ratios. By increasing the values of the b_{33} and b_{35} parameters in the year 1976, the denominator of the value ratio functions decreased thereby increasing the V-ratio values for mature cows. This caused the model to retain more cows, producing the simulation results presented in Table 10 and Figure 4.

Figure 6 shows the alternative slaughter values for pregnant and non-pregnant mature cows after 1976. As can be seen the change in PSV of pregnant animals is slight while that of non-pregnants is more pronounced.

The ability of the model to use different parameters or time lags is convenient when attempting to fine tune. However, a large number of parameter sets yielded tracking performance quite close to the values shown here. The parameter set indicated in the FLEXFORM produced simulation results which most adequately fulfilled the validation criteria set forth by the summary statistics and the graphical analysis.

Age Structure

The simulation model presented in this text is based on the hypothesis that the national cattle cycle has been related to investment incentive differences across cow ages through time, resulting each

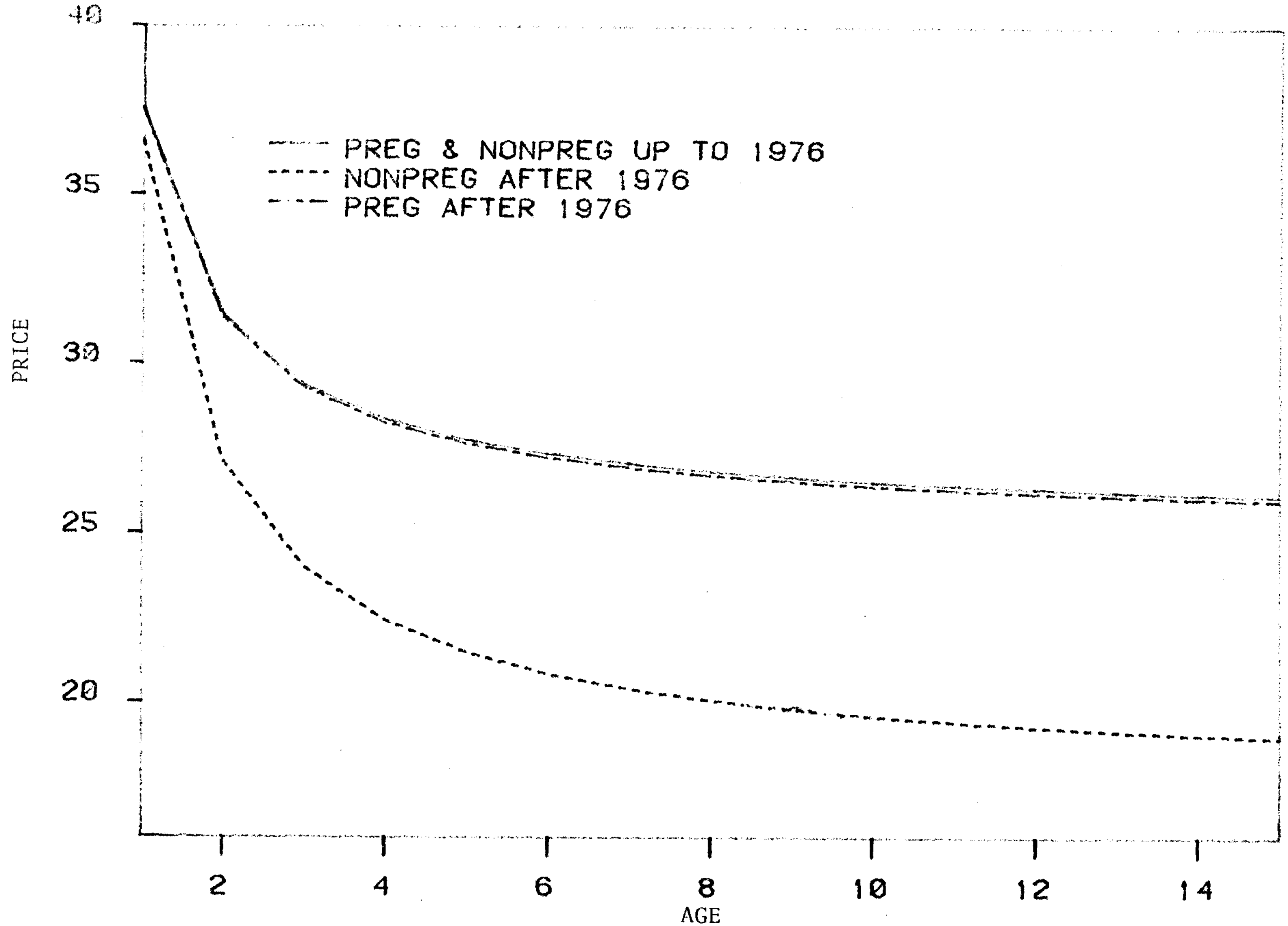

Figure 6. Alternative Slaughter Values Beginning in 1976.

year in changes in herd age structure, performance and potential adjustments in subsequent years.

Figure 7 shows the simulated age structure changes, for cows and heifers, from 1965 to 1981. The wave like pattern of the age structure changes is more pronounced for the older age classes. This is due to the variable culling pressures which increase with cow age.

As would be expected, the younger animals comprise the largest age group and the oldest animals the smallest. Attrition from intentional culling and natural death, is apparent as successively older age classes shrink. Figure 8 represents the age structure data in percentage terms.

To test the age structure hypothesis, a simulation run was made with parameters set to reflect the assumption that cows of all ages perform the same. Table 10 lists the comparison statistics for the HOMOGENEOUS run and Figure 9 provides a graphical view of the results. As can be seen, the HOMOGENEOUS simulation run produced less satisfactory results than the heterogeneous run known as STRINGHAM. However, the MPAD for the beef cow inventory of the HOMOGENEOUS run was only 1.9 percent and the U-statistics were all below 1.0. Thus, the HOMOGENEOUS cow run not only performed better than Nordbloms final simulation run DISPLAY (MPAD = 2.6 percent for cows) but also performed better than a zero change prediction. If the additional resources (time and money) needed for further fine tuning of the HOMOGENEOUS cow run were available, the possibility exists that the model could be tuned to a point where it would perform as well or better than the heterogeneous model. This would lead to a significant reduction in the complexity of the model. Thus, the HOMOGENEOUS cow run should not be easily discarded.

Given the simple biological functions, management and producer profit expectations and the few exogenous price and cost variables, the model tracks the historical inventories quite well. The simplicity of the current model, which represents the historical period, allows for simulation for as many years into the future as desired, provided independent projections of exogenous variables are available.

Chapter 6 presents the current model's simulated forecasts through 1987. Forecasting techniques are discussed and alternative future scenarios of the prices of cattle, corn and other production inputs are described.

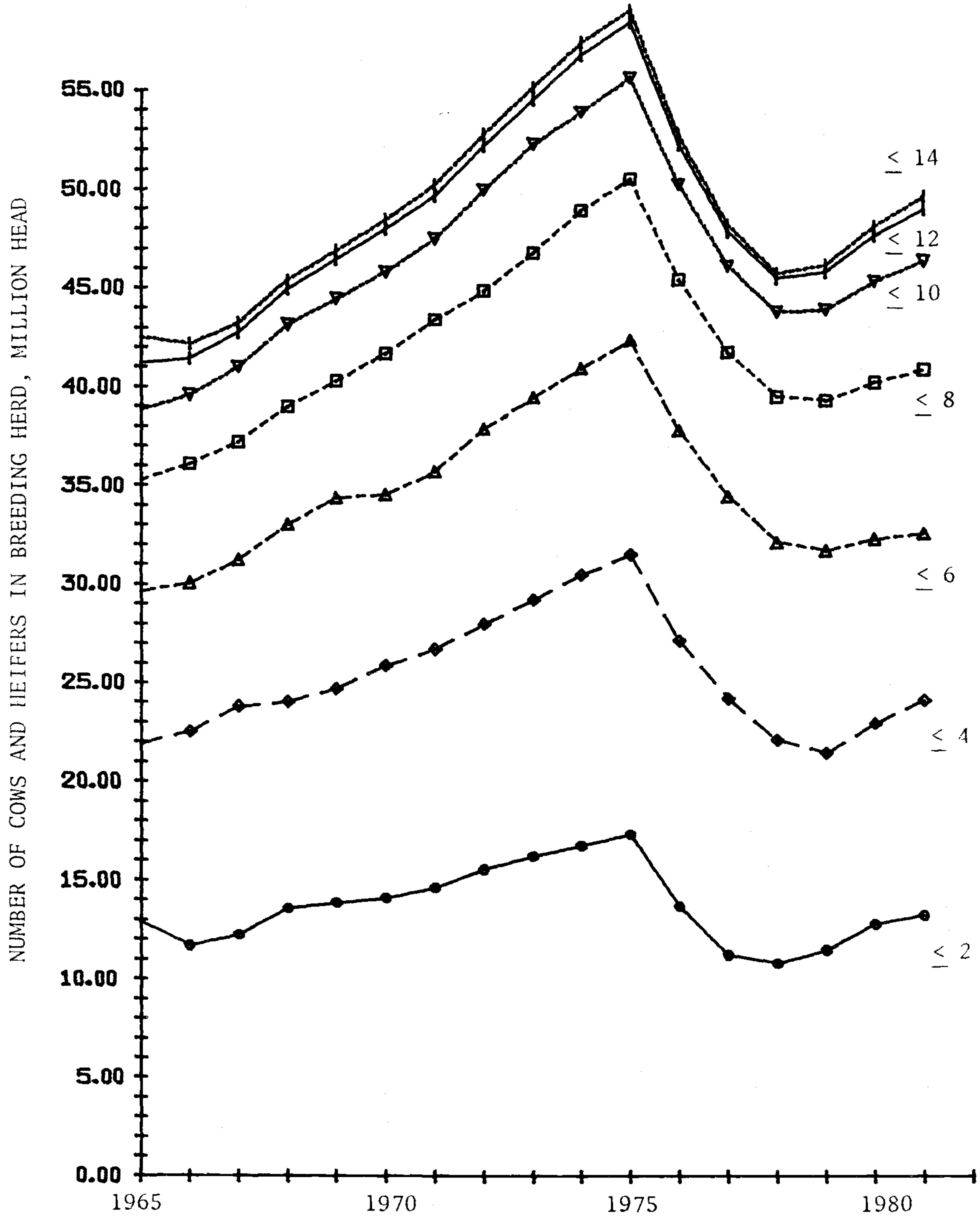

Figure 7. Simulated Cumulative Age Structure of the U.S. Beef Cow Herd, 1965-1981.

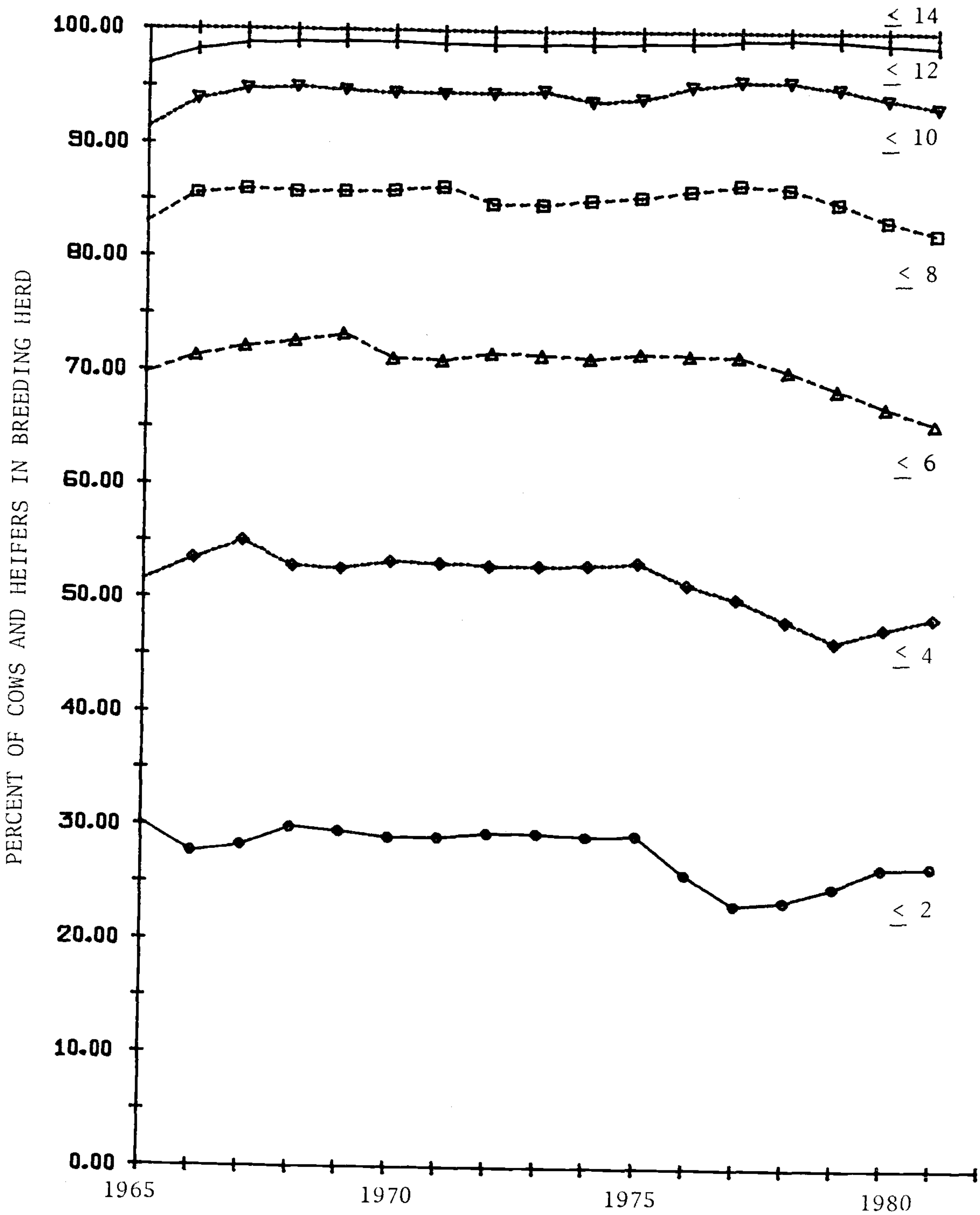

Figure 8. Simulated Age Composition of the U.S. Beef Cow Herd: 1965-1981.

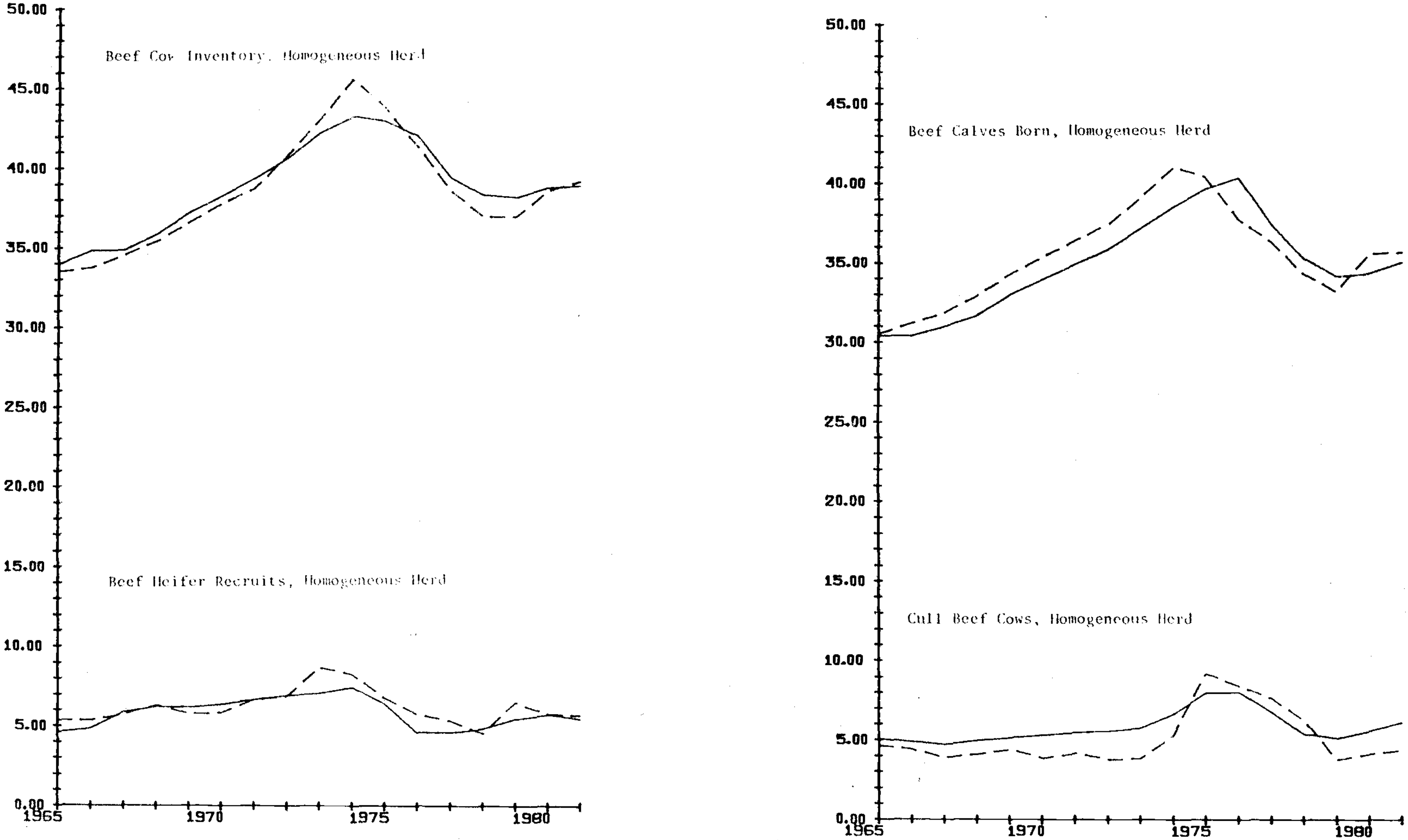

Figure 9. Simulated and Historical Inventories: HOMOGENEOUS RUN, 1965 to 1981.

CHAPTER 6

FORECAST METHODS AND RESULTS

In order to forecast, the exogenous price and cost inputs must be extrapolated. The model will function under either real or nominal price projections. The decision was made to extrapolate the various exogenous prices in real 1983 dollars.

The exogenous prices necessary for forecasting were broken down into three categories. These are:

1. The cost inputs whose prices followed the historical consumer price index closely. These include salt and minerals, labor, fuel and lubrication and building and machinery.

2. Corn and choice slaughter steer prices. The U.S.D.A., Economic Research Service, corn and choice slaughter steer price forecasts used here, were deflated to 1983 dollars by the GNP deflator.

3. The inputs whose prices are highly correlated, directly or indirectly, with prices of choice fed cattle and corn. This group includes utility cow prices, feeder calf prices, bull charges, pasture rent and hay prices.

Prices of several of the inputs, identified in the cost of production budgets, followed the CPI quite closely from 1950 to 1983. These are listed above in category one. Hence, in real terms, one must merely extrapolate the current 1983 price for these items. The price of protein was also extrapolated in this manner, even though it had not followed the CPI closely. However, it amounted to only .400 percent of total cost and the value of additional accuracy in its extrapolated price is virtually nil.

It was found that feeder steer prices are a function of choice slaughter steer prices, the price of corn and expected profit. The following equation was used to project the Z_1 vector of feeder steer prices through 1987.

$$Z_1 = b_0 + b_1 Z_5 - b_2 Z_8 - b_3[(Z_{5,t-1} - Z_{1,t-1})$$

$$+ .75(Z_{5,t-1} \cdot 11.79 \cdot Z_{8,t-1}/.93)] \qquad R^2 = .9677$$

where

Z_1 = feeder steer price

Z_5 = price of slaughter steers

Z_8 = price of corn.

b_0 = - 8.781 (- 2.530)
b_1 = 1.649 (12.886)
b_2 = - 2.541 (- 1.104)
b_3 = .0138 (- 3.938)

This equation was developed from the concept of break-even analysis. That is, the per head value of a feeder steer is imputed net of feed costs and a target return to management.

Calculated profit in the previous period is used as an approximation for expected profit $(E(\pi)]$.

$$E(\pi) = [(P - P_p) + \frac{W_g}{W_p}(P_p \cdot c \cdot P_c/A)]$$

where

P is the price of slaughter steers lagged one year

P_p is the feeder price lagged one year.

W_p is the purchase weight per head

W_g is the weight gain per head

$$\frac{W_g}{W_p} = \frac{450}{600} = .75$$

c is the number of bushels of corn required per hundredweight gain for a 600 pound steer fed to 1,050 pounds (11.79)

P_c is the national average corn price

A is a constant used to elevate the national average corn price up to the Chicago price for corn.

It was demonstrated in Chapter 3 that the price of utility cows is very highly correlated (on an annual basis) with the price of feeder steers. This relationship is expressed, again, below.

$$Z_2 = 1.222 + .598Z_1 \qquad R^2 = .9956$$
$$(2.568) \quad (58.522)$$

where

Z_1 = feeder steer price.

The pasture price was regressed on a five year moving average of utility cow prices.

$$Z_6 = -.0778 + .037X_2 \qquad R^2 = .9506$$
$$(-1.440) \quad (17.136)$$

where

X_2 = five year moving average utility cow price (Z_2).

The price of hay was determined to be a function of the price of corn. Regression of the hay price on the U.S.D.A. corn price forecast resulted in the following equation.

$$Z_7 = -.197 + 1.246Z_8 \qquad R^2 = .8757$$
$$(-3.072) \quad (13.789)$$

The last input price to be determined is the cost of bull services. Since, slaughter steer prices were used for determining bull costs in the original series, the same procedure is used for the extrapolated values derived here (see Chapter 3). All the projected prices were divided by their respective 1978 prices to create the indices necessary for cattle inventory projections.

Alternative Scenarios and Results

The shortage of time and financial resources limited the number of scenarios developed to two. Using the U.S.D.A. forecast of corn and slaughter steer prices (deflated to 1983 real dollars) and the relationships described above, the model's exogenous driving variables were projected through 1987. Tables 11 and 12 give the alternative price and cost index vectors used to project beef cow, replacement heifer, cull cow, and calves born inventories through 1987. The only difference between the two scenarios is that in scenario one the P.C.A. interest rate is maintained at its 1982 level while in scenario two the interest rate falls five percent annually.

Figures 9 and 10 present the graphical results of scenario one and scenario two, respectively. In Figure 9, the number of cows on inventory peak in 1981 and fall continuously thereafter. Some slowing in the rate of decline in cow numbers is present in the last two years of the forecast. Figure 10 also shows a peak in cow numbers occurring in 1981, however, the ensuing decline in cow numbers is more gradual and eventually reverses itself in 1986.

Table 11. Simulation Model Driving Variables Projected Through 1987: Scenario 1.

Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	Z_{10}	Z_{11}	Z_{12}	Z_{13}	Z_{14}
24.1200	14.4400	.4550	.3510	.4650	.4640	.6030	.5500	.4240	.4240	.3570	.4080	.0658	1965
27.4300	17.8300	.4570	.3640	.4830	.5100	.6110	.5880	.4740	.4320	.3900	.4260	.0687	1966
26.6800	17.2200	.4630	.3810	.4750	.5000	.6130	.4880	.4880	.4370	.4150	.4560	.0729	1967
27.9200	17.9400	.4690	.3990	.5050	.5360	.5920	.5120	.4560	.4470	.4560	.4840	.0734	1968
31.7800	20.2900	.4800	.4200	.5590	.5780	.6020	.5500	.4500	.4580	.5020	.5170	.0779	1969
33.7000	21.3200	.4980	.4430	.5540	.6090	.6260	.6300	.5120	.4830	.5350	.5500	.0898	1970
34.8700	21.6200	.5320	.4720	.6120	.5940	.6640	.5120	.4850	.5140	.5560	.5850	.0728	1971
41.4000	25.2100	.5560	.5060	.6780	.6250	.6960	.7440	.5660	.5370	.5930	.6040	.0702	1972
53.1700	32.8200	.5880	.5410	.8510	.7290	.8370	1.2090	1.1970	.5760	.6510	.6240	.0809	1973
37.8800	25.5600	.6950	.6130	.8000	.8960	.9890	1.4310	.8800	.6480	.7180	.6790	.0943	1974
33.9100	21.0900	.7770	.7490	.8520	.9690	1.0650	1.2040	.7580	.7330	.7880	.7610	.0891	1975
39.4000	25.3100	.8460	.8490	.7470	.8960	1.1610	1.0190	.9150	.7970	.8630	.8420	.0824	1976
40.1800	25.3200	.9340	.9310	.7710	.9480	1.1440	.9570	1.1030	.9020	.9340	.9230	.0788	1977
58.7800	36.7800	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.0883	1978
83.0800	50.1000	1.1090	1.1000	1.2940	1.2080	1.1950	1.1200	1.1110	1.1080	1.0970	1.0930	.1071	1979
75.2300	45.7200	1.2900	1.2060	1.2790	1.2920	1.4240	1.3820	1.1710	1.3140	1.1860	1.2120	.1286	1980
66.2400	41.9300	1.4780	1.3530	1.2200	1.3130	1.3680	1.0890	1.2650	1.4780	1.2250	1.3400	.1447	1981
67.9000	39.9600	1.6240	1.4860	1.2290	1.3330	1.3840	1.1020	1.1540	1.5600	1.2640	1.5000	.1287	1982
70.8700	43.6000	1.6240	1.4860	1.2420	1.5600	1.2430	1.1560	1.1540	1.5600	1.2640	1.5000	.1287	1983
69.2900	42.6600	1.6240	1.4860	1.2420	1.5050	1.3370	1.2310	1.1540	1.5600	1.2640	1.5000	.1287	1984
71.5400	43.8500	1.6240	1.4860	1.2840	1.4910	1.3090	1.2090	1.1540	1.5600	1.2640	1.5000	.1287	1985
73.3000	45.0600	1.6240	1.4860	1.3070	1.5140	1.2930	1.1960	1.1540	1.5600	1.2640	1.5000	.1287	1986
77.5700	47.6100	1.6240	1.4860	1.3570	1.5690	1.2650	1.1730	1.1540	1.5600	1.2640	1.5000	.1287	1987

Table 12. Simulation Model Driving Variables Projected Through 1987: Scenario 2.

Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	Z_{10}	Z_{11}	Z_{12}	Z_{13}	Z_{14}
24.1200	14.4400	.4550	.3510	.4650	.4640	.6030	.5500	.4240	.4240	.3570	.4080	.0658	1965
27.4300	17.8300	.4570	.3640	.4830	.5100	.6110	.5880	.4740	.4320	.3900	.4260	.0687	1966
26.6800	17.2200	.4630	.3810	.4750	.5000	.6130	.4880	.4880	.4370	.4150	.4560	.0729	1967
27.9200	17.9400	.4690	.3990	.5050	.5360	.5920	.5120	.4560	.4470	.4560	.4840	.0734	1968
31.7800	20.2900	.4800	.4200	.5590	.5780	.6020	.5500	.4500	.4580	.5020	.5170	.0779	1969
33.7000	21.3200	.4980	.4430	.5540	.6090	.6260	.6300	.5120	.4830	.5350	.5500	.0898	1970
34.8700	21.6200	.5320	.4720	.6120	.5940	.6640	.5120	.4850	.5140	.5560	.5850	.0728	1971
41.4000	25.2100	.5560	.5060	.6780	.6250	.6960	.7440	.5660	.5370	.5930	.6040	.0702	1972
53.1700	32.8200	.5880	.5410	.8510	.7290	.8370	1.2090	1.1970	.5760	.6510	.6240	.0809	1973
37.8800	25.5600	.6950	.6130	.8000	.8960	.9890	1.4310	.8800	.6480	.7180	.6790	.0943	1974
33.9100	21.0900	.7770	.7490	.8520	.9690	1.0650	1.2040	.7580	.7330	.7880	.7610	.0891	1975
39.4000	25.3100	.8460	.8490	.7470	.8960	1.1610	1.0190	.9150	.7970	.8630	.8420	.0824	1976
40.1800	25.3200	.9340	.9310	.7710	.9480	1.1440	.9570	1.1030	.9020	.9340	.9230	.0788	1977
58.7800	36.7800	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	.0883	1978
83.0800	50.1000	1.1090	1.1000	1.2940	1.2080	1.1950	1.1200	1.1110	1.1080	1.0970	1.0930	.1071	1979
75.2300	45.7200	1.2900	1.2060	1.2790	1.2920	1.4240	1.3820	1.1710	1.3140	1.1860	1.2120	.1286	1980
66.2400	41.9300	1.4780	1.3530	1.2200	1.3130	1.3680	1.0890	1.2650	1.4780	1.2250	1.3400	.1447	1981
67.9000	39.9600	1.6240	1.4860	1.2290	1.3330	1.3840	1.1020	1.1540	1.5600	1.2640	1.5000	.1287	1982
70.8700	43.6000	1.6240	1.4860	1.2420	1.5600	1.2430	1.1560	1.1540	1.5600	1.2640	1.5000	.1223	1983
69.2900	42.6600	1.6240	1.4860	1.2420	1.5050	1.3370	1.2310	1.1540	1.5600	1.2640	1.5000	.1164	1984
71.5400	43.8500	1.6240	1.4860	1.2840	1.4910	1.3090	1.2090	1.1540	1.5600	1.2640	1.5000	.1104	1985
73.3000	45.0600	1.6240	1.4860	1.3070	1.5140	1.2930	1.1960	1.1540	1.5600	1.2640	1.5000	.1049	1986
77.5700	47.6100	1.6240	1.4860	1.3570	1.5690	1.2650	1.1730	1.1540	1.5600	1.2640	1.5000	.0997	1987

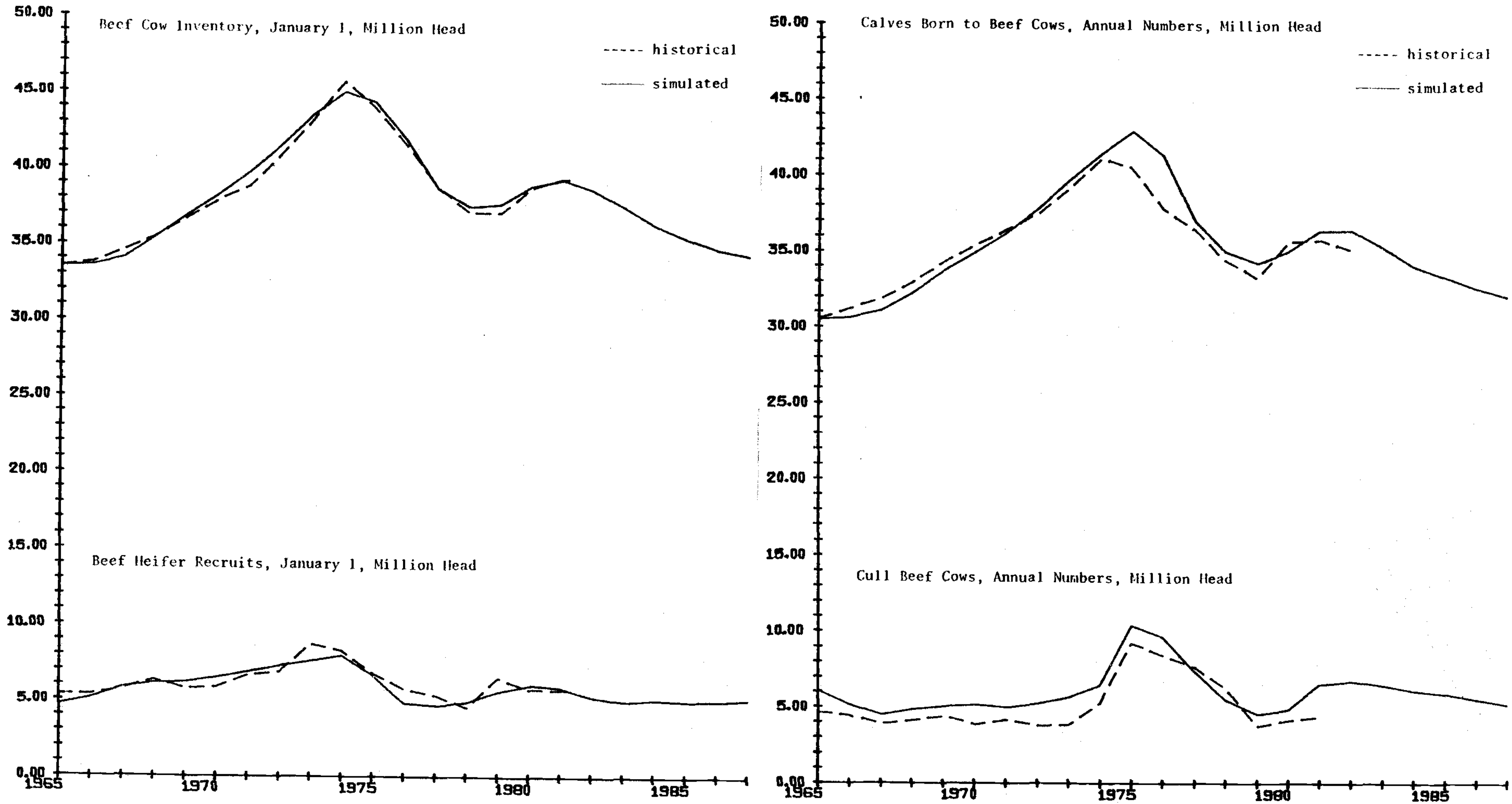

Figure 10. Simulated and Historical Inventories, 1965 to 1981: Projections to 1987: Scenario 1.

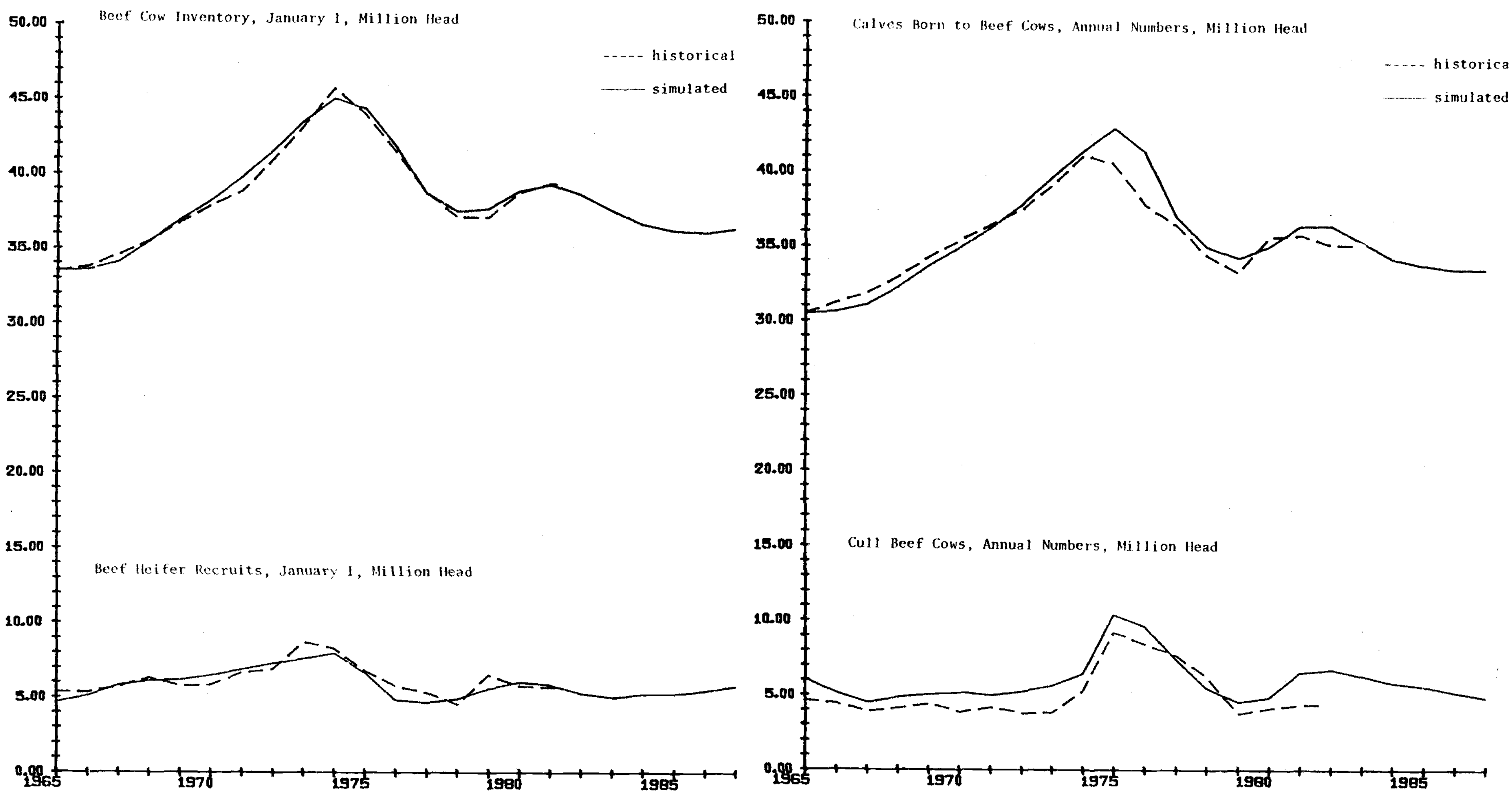

Figure 11. Simulated and Historical Inventories, 1965 to 1981: Projections to 1987: Scenario 2.

Comparison With U.S.D.A. Forecasted Cow Numbers

The U.S.D.A. forecast of beef cow numbers, through 1987, based on their slaughter steer price projections, is listed below, side by side with current model's corresponding forecast.

Cow Numbers Forecast

Year	U.S.D.A.	Scenario 1	Scenario 2	Historical
1982	38.146	38.610	38.61	38.081
1983	38.000	37.490	37.57	
1984	37.800	36.290	36.62	
1985	39.200	35.460	36.20	
1986	40.500	34.760	36.06	
1987	41.300	34.370	36.35	

As can be seen, the U.S.D.A. forecast projects a reversal in trend beginning in 1985, while the current model projects further liquidations. According to the model the U.S.D.A. slaughter steer price forecast implies continued liquidation of cow numbers. This kind of beef cow inventory adjustment would negate the U.S.D.A. slaughter steer price forecast. Thus, the model suggests that the U.S.D.A. slaughter steer price forecast is too low to generate the U.S.D.A. beef cow numbers projections. Which forecast, if either, proves to be correct remains to be seen.

CHAPTER 7

SUMMARY AND CONCLUSIONS

Simulation models may be used to organize, structure and order the knowledge already at hand. Chapter 2 developed the major economically important biological functions of conception rates, health rates, body weights, calf weaning weights and calf survival rates. These biological relationships, combined with the management expectation and producer profit expectation functions laid the foundation for the simulation model developed in this text.

The objective of simplifying the model was achieved mainly through respecification of the present value for breeding functions. However, further simplification is still possible especially concerning the retainment rate functions.

The tracking behavior of the model with respect to the historical series of beef cows, heifers, culls and calves born, improved considerably over the previous model. The mean proportional absolute deviation (MPAD) for beef cows declined from the previous model's low of .026 to .009. The MPAD for heifers recruited, cows culled and calves born declined from .172, .261 and .036 to .075, .227, .023 respectively.

The modest tracking behavior of the model, with respect to the numbers of heifers recruited, cows culled and beef calves born, may be due to several factors. The historical series against which the simulated series of calves born and heifers recruited are compared may be in error. The definition of the simulated cull cow series may not coincide well with the historical numbers of beef cows slaughtered. For example, the largest numbers of culls come from the youngest age classes, some of which go to feedlots for fattening and some to the class of non-fed slaughter.

The assumption of constant cow performance parameters throughout the study period may have introduced errors of unknown dimensions. For example, during periods when culling rates are increased one might expect calving rates and weaning weights to improve and vice versa under increasing recruitment rates.

The model is very simple in that it considers only the beef cow/calf sector and no information on dairy or other livestock sectors is used. Even given its simplicity and its few exogenous price and cost variables, the model is able to track the historical numbers of beef cows and calves born quite well.

Slaughter steer and corn price forecasts from the U.S.D.A. were used implicitly in forecasting the prices of feeder steers, utility cows, bull charges, pasture rents and hay for use in projecting cattle inventory numbers through 1987. The prices of the remaining inputs (salt and minerals, protein supplement, labor, fuel and lubrication and building and machinery), followed the consumer price index quite closely, thus, they were extrapolated at their present value through 1987. Using these projected vectors of exogenous prices two alternative forecasts of cattle inventory numbers were made through 1987.

The first forecast showed beef cow numbers peaking in 1981 and declining continuously thereafter. Such a decline in animal numbers would certainly negate the U.S.D.A. slaughter steer price forecast. The second forecast assumed a continuous five percent annual decline the interest rate on short-term operating loans from its 1981 value of .1287. Under this scenario the numbers of beef cows peak in 1981, decline until 1985 and begin to rise in 1987. Such forecasts could be useful to cow/calf producers who are attempting to act countercyclically.

Indications for Further Research

In view of the difficulties encountered in the period from 1976 to 1981, further simplification of the model may be indicated. Using V-ratios from pregnant classes to calculate retainment rates for non-pregnant classes would be a major and potentially useful simplification. A suggestion is to relate the profitability of a single five year old cow, for which the V-ratio is an indicator, to the retainment rates for non-pregnant classes. Such a simplification could possibly eliminate, or at least curtail, the heavy liquidation of non-pregnant animals that began in 1976.

The concept of investment response being determined by the present value for breeding of individual cows could be applied to the management of individual herds. An extension project could be undertaken to clarify and formalize the concept with respect to ranch management. Such information could be very useful to the cow/calf operator.

The need to further explore the ability of the model to track the objective historical series, under the assumption that cows of all ages have the same performance levels still exists. Eliminating the various age classes would be a major simplification to the model.

The forecasting ability of the model is currently determined, in part, by the accuracy of the projections of the exogenous price and cost series. There exists a substantial body of literature dealing with price formation in the beef industry. It would be interesting to construct a combined model of price formation and beef cow investment response and to compare the forecasts of both models. Of course, the accuracy of any forecast cannot be determined until the future has materialized.

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APPENDIX

APPENDIX A

The FLEX/REFLEX Paradigm

The FLEX/REFLEX Paradigm

The FLEX/REFLEX paradigm[1/] was developed by Dr. Scott W. Overton, Curtis White and others at Oregon State University. The FLEX/REFLEX modeling paradigm was created to encompass the system characteristics of ecosystems and systems in general, and to provide a framework for the construction of well defined and logically consistent models which explicate how a system works according to the applicable body of theory (Colby, 1976).

The FLEX/REFLEX paradigm is based on the general systems theory of George Klir (1969). It specifically embraces two of Klir's five compatible definitions of a system:

(1) Behavioral: the system is defined according to its behavior,

(2) Structural: the system is defined by identification of its subsystems (parts), each of which is defined according to its behavior and by identification of the links between the subsystems.

These two forms form a dual definition of the concept of a system which fundamentally supports Koestlers (1969) concept of the system as a holon.[2/]

Emphasis was placed on the holistic properties of each identified system and its hierarchical relationship to both the subsystems which compose it and the system in which it is a component (Colby, 1976).

1/ Paradigm in the sense of Kuhn (1970): "a universally recognized achievement that for a time provides model problems and solutations to a community."

2/ Holon, a derivative of the Greek root "holos" meaning whole. Webster's Dictionary (1969).

This perspective resulted in two distinct approaches to a given system. The holistic approach which accounts for the external behavioral characteristics that are functions of the interactions of the subsystems and the mechanistic approach which is concerned with the linked relations of explicitly identified subsystems. Underlying this dual approach is the assumption that all properties of a system are not recognizable as direct properties of its parts but are derived instead from the coupled interactions of the parts [Colby, 1976]. Requiring each system to be described holistically as well as mechanistically provides for logical consistency and completeness.

The FLEX/REFLEX modeling paradigm provides for model structures and techniques of system analysis that are effective when applied to the tasks of model definition (modeling), model structure (programming), model verification and validation. The synchronization of the separate tasks of modeling and programming is accomplished through FLEXFORM model documentation.

The FLEXFORM document provides a concise description, display and cross-reference of every variable, parameter and equation contained in the model. The transparency of the hierachical nature of the FLEXFORM greatly facilitates communication among those investigating the model by removing much of the extraneous material that surrounds most simulation models. Communication among investigators in turn permits constructive criticism of the characteristic and traits of the system being modeled.

The FLEXFORM documentation scheme (FLEXFORM) was designed for the sole purpose of creating and preserving useable documentation. The authors of the FLEX/REFLEX modeling paradigm insist on predocumentation of models before computer programming is commenced [Nordblom, 1981].

Following is a brief explanation of the mechanics necessary for accessing and running the cattle cycle simulation model as presented in this study. The model is programmed in Fortran 4 and is executed through the FLEX4 processor stored on the Cyber system at Oregon State University.

Once you have logged on the Cyber system the following set of commands will allow you to execute a complete run of the Cattle Cycle Simulation model.

1. Get,FLEX4/UN=AGWY5C
2. Get,Tape8=DISPL2,VCATB15,TAPE9
3. FLEX4,FI=VCATB15,P1=CATL2
4. READ=8;
5. IB(395)=1.0;
6. IB(378)=1.54
7. IB(379)=1.01
8. Run;
9. Recess;
10. Rewind,tape6,tape8,tape10,tape11,tape12

Note: It is important that you allow the computer to respond before you type in the next command.

If a hard copy of the outputs is desired type in the following commands after command number ten:

11. Title(LP)/ ____________________ / (any title desired)
12. Copy,tape6,LP
13. Copy,tape10,LP
14. Copy,tape11,LP

15. Copy,tape12,LP

16. Route,LP,DC=PR

A brief explanation of the file names and tape contents is in order here. This short list will serve as a guideline for understanding the FLEX4 processors data requirements and the output of the model.

DISPL2	=	command file
VCATB15	=	FLEX/REFLEX binary program derived from VCATS15 the source file, i.e., Fortran 4 program
Tape 9	=	input variables (Z_i)
Tape 10	=	output functions ($Y_{i,j}$)
Tape 11	=	simulated and historical numbers for each animal category for each year of the simulation run
Tape 12	=	equation values ($g_{i,j}$)
IB( )	=	a command that sets a specific value for the parameter (b_i) designated within the parentheses

Note: IB(395) must be set equal to 1.0 if Tape 12 output is desired.

APPENDIX B

FLEXFORM

FLEXFORM JUNE 1983

Tamzen K. Stringham, Dept. of Ag. and Resource Econ., Oregon State University

TITLE: Simulation of National Cow Inventories and Calf Crop, 1965 to 1981: Projections to 1987

PURPOSE: To simulate historical beef cow inventory patterns (the cattle cycle) through time and to forecast cow inventories, calf crops, beef heifer recruits and slaughter cow numbers based on alternative future scenarios of the prices of fed cattle, feeder cattle, cull cows, corn and other production inputs.

TIME RESOLUTION: one year (beginning with post-weaning/culling inventories each year)

STRUCTURE: See GROSS FLOWCHART, FUNCTION CATALOG, and BEEF COW AGE DISTRIBUTION INVENTORY MODEL FLOWCHART on following pages. FLEXFORM CONTENTS:

$x_{i,j}$ = state variables

z_i = annual variable inputs

$m_{i,j}$ = memory variables

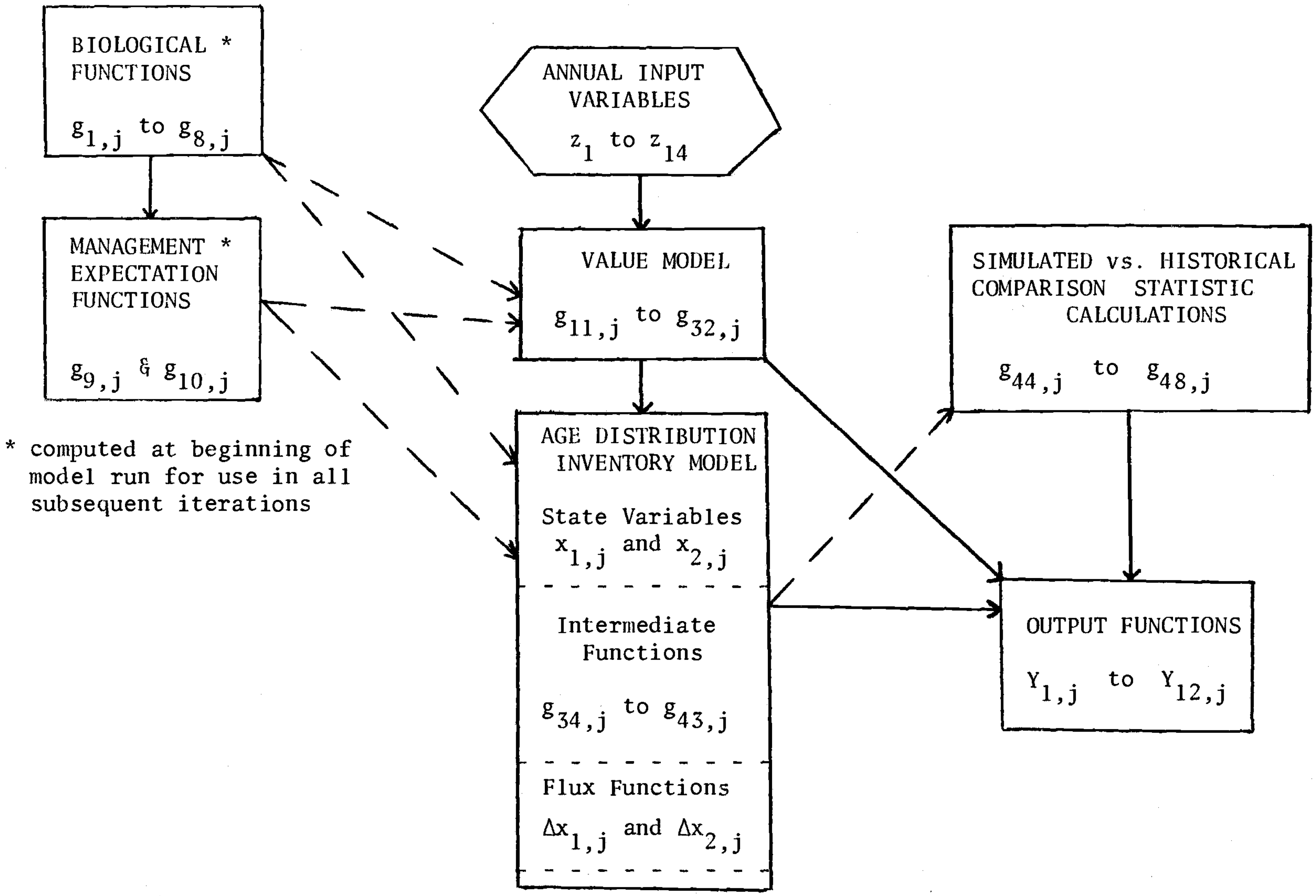
$g_{i,j}$ = intermediate functions

$f_{i,j}$ = FLUX functions (to update state variables)

$Y_{i,j}$ = output functions

b_i = parameter list

BEEF COW VALUE AND AGE DISTRIBUTION INVENTORY MODEL: GROSS FLOW CHART



BEEF COW VALUE AND AGE DISTRIBUTION INVENTORY MODEL FUNCTION CATALOG

FUNCTION LIST: BIOLOGICAL AND MANAGEMENT EXPECTATION PARAMETERS

BIOLOGICAL PARAMETERS:

- $g_{1,j}$ = conception rates, by cow age
- $g_{2,j}$ = unimpaired health rates, by cow age
- $g_{3,j}$ = cow survival rates, by cow age
- $g_{4,j}$ = cow culling weights, by cow age
- g_5 = maximum cow body weight
- $g_{6,j}$ = calf weaning weights, by cow age
- g_7 = weaning weight of a heifer kept for breeding
- $g_{8,j}$ = calf survival rates, by cow age

MANAGEMENT EXPECTATION PARAMETERS:

- $g_{9,j}$ = expected cull cow sales, by cow age, 1 year from present
- $g_{10,j}$ = expected cull cow sales, by cow age, 2 years from present

BEEF COW VALUE MODEL FUNCTION LIST

- $g_{11,j}$ = Expected calf sale revenues in the coming year
- $g_{24,j}$ = Expected calf sale revenues two years from now
- $g_{12,1}$ = Expected future feeder steer price
- $g_{12,2}$ = Expected future utility cow price

- $g_{13,j}$ = Expected future cull salvage value (PSV_j)
- $g_{14,j}$ = Present cull salvage values (PSV_j)
- $g_{25,j}$ = Present cull salvage values (PSV_j)

- g_{15} = Interest charge factor
- g_{16} = Costs common to all budgets
- g_{17} = Cost budget for heifers kept for breeding (HKB's)
- g_{18} = Costs common to yearling heifers
- g_{19} = Cost budget for pregnant yearling heifers
- g_{20} = Cost budget for non-pregnant yearling heifers
- g_{21} = Costs common to cows, aged 3 years and over
- g_{22} = Cost budget for pregnant cows
- g_{23} = Cost budget for non-pregnant cows

- g_{26} = Discount factor for present value calculations

- $g_{28,j}$ = PVB^P_j calculations
- $g_{30,j}$ = $V^P_j = PVB^P_j / PSV_j$ (→ $g_{37,j}$)
- $g_{31,j}$ = PCV^N_j calculations
- $g_{32,j}$ = $V^N_j = PBV^N_j / PSV_j$ calculations (→ $g_{38,j}$)

AGE DISTRIBUTION INVENTORY MODEL FUNCTION LIST

STATE VARIABLES

- $x_{1,1}$ = Weaned heifers not kept for breeding
- $x_{1,j}$ = Post-culling inventories of pregnant cows (j = 2 to 14)
- $x_{2,j}$ = Post-culling inventories, non-pregnant heifers and cows (j = 1 to 13)

INTERMEDIATE FUNCTIONS

- g_{33} = Percent of yearlings exposed for breeding
- g_{36} = Percent of yearlings not exposed for breeding
- $g_{34,j}$ = Pre-culling inventories of pregnant animals
- $g_{35,j}$ = Pre-culling inventories of non-pregnant animals

- $g_{37,j}$ = Proportions of pregnant animals to be retained
- $g_{38,j}$ = Proportions of non-pregnant animals to be retained

- $g_{39,j}$ = Numbers of pregnant animals to be retained
- $g_{40,j}$ = Numbers of non-pregnant animals to be retained

- $g_{41,j}$ = Numbers of pregnant animals to cull
- $g_{42,j}$ = Numbers of non-pregnant animals to cull

- $g_{43,j}$ = Summations for output reports

FLUX FUNCTIONS (Updating State Variables)

$f_{1,j} = \Delta x_{1,j}$ and $f_{2,j} = \Delta x_{2,j}$

OUTPUT FUNCTION LIST

- $Y_{1,j}$ = herd size and performance reports
- $Y_{2,j}$ = herd composition, by age class totals
- $Y_{3,j}$ = PSV_j = present cull salvage value
- $Y_{4,j}$ = $V^P_j = PCB^P_j / PSV_j$
- $Y_{5,j}$ = $V^N_j = PVB^N_j / PSV_j$
- $Y_{6,j}$ = PVB^P_j for pregnant animals
- $Y_{7,j}$ = PVB^N_n for non-pregnant animals
- $Y_{8,j}$ = sim. vs. hist. statistics for cows
- $Y_{9,j}$ = sim. vs. hist. statistics for heifers
- $Y_{10,j}$ = sim. vs. hist. statistics for culls
- $Y_{11,j}$ = sim. vs. hist. statistics for calves
- $Y_{12,j}$ = cumulative age composition of herd

STATE VARIABLE LIST FOR COW VALUE AND AGE DISTRIBUTION INVENTORY MODEL

State Variable	Initial Value	Units	Description	Used in these Functions
$x_{1,1}$	65.20	100,000 head	No. of weaned heifers not kept for breeding in the present year but available as yearlings next year	
$x_{1,2}$	37.70	"	No. of pregnant yearlings kept to calve in the present year as 2 year olds	
$x_{1,3}$	42.90	"	" " " cows " " " " " " " " 3 " "	
$x_{1,4}$	41.10	"	" " " " " " " " " " " " 4 " "	
$x_{1,5}$	37.90	"	" " " " " " " " " " " " 5 " "	
$x_{1,6}$	35.60	"	" " " " " " " " " " " " 6 " "	
$x_{1,7}$	30.40	"	" " " " " " " " " " " " 7 " "	g_{34} g_{35} g_{43} f_1
$x_{1,8}$	22.90	"	" " " " " " " " " " " " 8 " "	
$x_{1,9}$	17.80	"	" " " " " " " " " " " " 9 " "	
$x_{1,10}$	14.00	"	" " " " " " " " " " " " 10 " "	
$x_{1,11}$	10.30	"	" " " " " " " " " " " " 11 " "	
$x_{1,12}$	8.80	"	" " " " " " " " " " " " 12 " "	
$x_{1,13}$	6.40	"	" " " " " " " " " " " " 13 " "	
$x_{1,14}$	4.00	"	" " " " " " " " " " " " 14 " "	
$x_{2,1}$	53.29	"	No. of weaned heifers kept for breeding (HKB's) in the present year as 1 year olds	
$x_{2,2}$	37.50	"	No. of non-pregnant yearlings kept for breeding in the present year as 2 year olds	
$x_{2,3}$	3.96	"	" " " cows " " " " " " " " 3 " "	
$x_{2,4}$	2.68	"	" " " " " " " " " " " " 4 " "	
$x_{2,5}$	1.91	"	" " " " " " " " " " " " 5 " "	
$x_{2,6}$	1.65	"	" " " " " " " " " " " " 6 " "	
$x_{2,7}$	1.61	"	" " " " " " " " " " " " 7 " "	g_{34} g_{35} g_{43} f_2
$x_{2,8}$	1.60	"	" " " " " " " " " " " " 8 " "	
$x_{2,9}$	1.72	"	" " " " " " " " " " " " 9 " "	
$x_{2,10}$	2.01	"	" " " " " " " " " " " " 10 " "	
$x_{2,11}$	2.16	"	" " " " " " " " " " " " 11 " "	
$x_{2,12}$	2.41	"	" " " " " " " " " " " " 12 " "	
$x_{2,13}$	2.43	"	" " " " " " " " " " " " 13 " "	

INPUT LIST FOR COW VALUE AND AGE DISTRIBUTION INVENTORY

Input	Units		Used in these functions
z_1	\$/cwt.	weaned calf prices (feeder steer prices)	g_{14} g_{25} $g_{12,1}$
z_2	\$/cwt.	utility cow prices	g_{14} $g_{12,2}$
z_3	Index (78 = 1.0)	fuel, lube, and electricity C.P.I.	g_{16}
z_4	"	farm machinery C.P.I.	g_{16}
z_5	"	fed cattle price (for bull charges)	g_{16}
z_6	"	pasture rental rates	g_{17} g_{18} g_{21}
z_7	"	hay (other hay prices)	g_{17} g_{18} g_{21}
z_8	"	grain (corn price)	g_{17} g_{18} g_{21}
z_9	"	protein supplement (SBOM price)	g_{17} g_{18} g_{21}
z_{10}	"	salt and minerals (salt price)	g_{17} g_{18} g_{21}
z_{11}	"	farm labor wage rate	g_{16} g_{17} g_{19} g_{20} g_{22} g_{23}
z_{12}	"	veterinary and medicine	g_{17} g_{19} g_{20} g_{22} g_{23}
z_{13}	$\frac{\%\ \text{int.}}{100}$	P.C.A. average cost of loans (% interest / 100)	g_{15} $g_{26,1}$
z_{14}	years	year counter (beginning in 1950)	$y_{1,1}$ $g_{44,j}$ $j = 1,4$ $g_{4i,2}$ $i = 5,8$ $Y_{i,1}$ $i = 8,11$

MEMORAY VARIABLE LIST: STATE VARIABLES IN TIME (k-1)

Memory Variable	Definition	Initial Value	Units	Description	Used in these Functions
$m_{1,2}$	$= x_{1,2}(k-1)$	334.0	100,000 hd.	Post-culling inventory of pregnant yearlings kept at beginning of previous year	
$m_{1,3}$	$= x_{1,3}(k-1)$	zero	"	" " " " " cows becoming 3 years old at beginning of previous year	$Y_{1,8}$, $Y_{1,9}$, $Y_{1,10}$, $Y_{1,12}$, $Y_{2,j}$
$m_{1,4}$	$= x_{1,4}(k-1)$	"	"	" " " " " " " 4 " " " " " " "	
$m_{1,5}$	$= x_{1,5}(k-1)$	"	"	" " " " " " " 5 " " " " " " "	
$m_{1,6}$	$= x_{1,6}(k-1)$	"	"	" " " " " " " 6 " " " " " " "	
$m_{1,7}$	$= x_{1,7}(k-1)$	"	"	" " " " " " " 7 " " " " " " "	
$m_{1,8}$	$= x_{1,8}(k-1)$	"	"	" " " " " " " 8 " " " " " " "	
$m_{1,9}$	$= x_{1,9}(k-1)$	"	"	" " " " " " " 9 " " " " " " "	
$m_{1,10}$	$= x_{1,10}(k-1)$	"	"	" " " " " " " 10 " " " " " " "	
$m_{1,11}$	$= x_{1,11}(k-1)$	"	"	" " " " " " " 11 " " " " " " "	
$m_{1,12}$	$= x_{1,12}(k-1)$	"	"	" " " " " " " 12 " " " " " " "	
$m_{1,13}$	$= x_{1,13}(k-1)$	"	"	" " " " " " " 13 " " " " " " "	
$m_{1,14}$	$= x_{1,14}(k-1)$	"	"	" " " " " " " 14 " " " " " " "	
$m_{2,1}$	$= x_{2,1}(k-1)$	zero	"	Post-culling inventory of weaned heifers retained as 1 year olds in previous year	$Y_{1,9}$, $Y_{1,12}$, $Y_{2,j}$
$m_{2,2}$	$= x_{2,2}(k-1)$	"	"	Post-culling inventory of non-pregnant yearlings retained as 2 year olds in previous year	
$m_{2,3}$	$= x_{2,3}(k-1)$	"	"	" " " " " cows retained as 3 year olds in previous year	
$m_{2,4}$	$= x_{2,4}(k-1)$	"	"	" " " " " " " " 4 " " " " "	
$m_{2,5}$	$= x_{2,5}(k-1)$	"	"	" " " " " " " " 5 " " " " "	
$m_{2,6}$	$= x_{2,6}(k-1)$	"	"	" " " " " " " " 6 " " " " "	
$m_{2,7}$	$= x_{2,7}(k-1)$	"	"	" " " " " " " " 7 " " " " "	
$m_{2,8}$	$= x_{2,8}(k-1)$	"	"	" " " " " " " " 8 " " " " "	
$m_{2,9}$	$= x_{2,9}(k-1)$	"	"	" " " " " " " " 9 " " " " "	
$m_{2,10}$	$= x_{2,10}(k-1)$	"	"	" " " " " " " " 10 " " " " "	
$m_{2,11}$	$= x_{2,11}(k-1)$	"	"	" " " " " " " " 11 " " " " "	
$m_{2,12}$	$= x_{2,12}(k-1)$	"	"	" " " " " " " " 12 " " " " "	
$m_{2,13}$	$= x_{2,13}(k-1)$	"	"	" " " " " " " " 13 " " " " "	

MEMORY VARIABLE LIST: SUMMATIONS FOR MPAD TEST STATISTICS

Memory Variable		Definition	Initial Value	Units	Description	Used in these Functions
$m_{3,1}$	=	$g_{44,1}(k-1)$	Zero	dimensionless	Sum of previous years' proportional absolute deviations of model's estimate of cow numbers from the USDA estimates.	$g_{44,1}$
$m_{3,2}$	=	$g_{44,2}(k-1)$	Zero	"	Sum of previous years' proportional absolute deviations of model's estimate of heifer recruitment numbers from the USDA estimates.	$g_{44,2}$
$m_{3,3}$	=	$g_{44,3}(k-1)$	Zero	"	Sum of previous years' proportional absolute deviations of model's estimates of cull cow numbers from the USDA estimates.	$g_{44,3}$
$m_{3,4}$	=	$g_{44,4}(k-1)$	Zero	"	Sum of previous years' proportional absolute deviations of model's estimates of numbers of calves born from estimates derived from historical series.	$g_{44,4}$

NOTE: The above four memory variables are only used to carry forward "sums of proportional absolute deviations" for computation of test statistics. The USDA estimates, to which the estimates of the model are compared, have no influence on the value or age distribution inventory models.

MEMORY VARIABLE LIST: PAST CATTLE PRICES FOR EXPECTATION MODELS

Memory Variable		Definition	Initial Value	Units	Description	Used in these Functions
$m_{4,1}$	=	$z_1(k-1)$	21.92	\$/cwt.	Price of feeder steers in previous year.	$g_{12,1}$
$m_{4,2}$	=	$z_2(k-1)$	13.24	\$/cwt.	Price of utility cows in previous year.	$g_{12,2}$

NOTE: The above two memory variables are used in the cattle price expectation functions, $g_{12,1}$ and $g_{12,2}$, to represent a continuation of the most recent one year trend or a weighted average of the previous and present years' prices.

MEMORY VARIABLE LIST: SUMMATIONS FOR TEST STATISTICS ON BEEF COW NUMBERS

Memory Variable		Definition	Initial Value	Units	Description	Used in these Functions
$m_{5,1}$	=	$g_{43,1}(k-1)$	Zero	million head	Total beef cows (becoming 3 years old and over) retained at beginning of current year simulated by age distribution inventory model (S_{k-1}) for computing (P) "predicted" changes in cow numbers for comparison with (A) "actual" historical changes.	$g_{45,1}$
$m_{5,2}$	=	unassigned	----	-----	---- ---- ---- ----	-----
$m_{5,3}$	=	$g_{45,3}(k-1)$	Zero	million head	ΣP	$g_{45,3}$
$m_{5,4}$	=	$g_{45,4}(k-1)$	Zero	dimensionless	ΣP^2	$g_{45,4}$
$m_{5,5}$	=	$g_{45,5}(k-1)$	Zero	million head	ΣA	$g_{45,5}$
$m_{5,6}$	=	$g_{45,6}(k-1)$	Zero	dimensionless	ΣA^2	$g_{45,6}$
$m_{5,7}$	=	$g_{45,7}(k-1)$	Zero	dimensionless	ΣPA	$g_{45,7}$
$m_{5,8}$	=	$g_{45,8}(k-1)$	Zero	dimensionless	$\Sigma (P-A)^2$	$g_{45,8}$

NOTE:

$$P = \frac{(S_k - S_{k-1})}{S_{k-1}}$$

$$A = \frac{(H_k - H_{k-1})}{H_{k-1}}$$

MEMORY VARIABLE LIST: SUMMATIONS FOR TEST STATISTICS ON HEIFER RECRUITMENT

Memory Variable		Definition	Initial Value	Units	Description	Used in these Functions
$m_{6,1}$	=	$g_{43,2}(k-1)$	Zero	million head	Total heifers retained for breeding at beginning of current year, simulated by age distribution inventory model: (S_{k-1}) for computing (P) "predicted" changes in recruited heifer numbers for comparison with (A) "actual" historical changes.	$g_{46,1}$
$m_{6,2}$	=	Unassigned	----	---------	---- ---- ---- ----	-----
$m_{6,3}$	=	$g_{46,3}(k-1)$	Zero	million head	ΣP	$g_{46,3}$
$m_{6,4}$	=	$g_{46,4}(k-1)$	Zero	dimensionless	ΣP^2	$g_{46,4}$
$m_{6,5}$	=	$g_{46,5}(k-1)$	Zero	million head	ΣA	$g_{46,5}$
$m_{6,6}$	=	$g_{46,6}(k-1)$	Zero	dimensionless	ΣA^2	$g_{46,6}$
$m_{6,7}$	=	$g_{46,7}(k-1)$	Zero	dimensionless	ΣPA	$g_{46,7}$
$m_{6,8}$	=	$g_{46,8}(k-1)$	Zero	dimensionless	$\Sigma (P-A)^2$	$g_{46,8}$

NOTE:

$$P = \frac{(S_k - S_{k-1})}{S_{k-1}}$$

$$A = \frac{(H_k - H_{k-1})}{H_{k-1}}$$

MEMORAY VARIABLE LIST: SUMMATIONS FOR TEST STATISTICS ON BEEF COW SLAUGHTER NUMBERS

Memory Variable		Definition	Initial Value	Units	Description	Used in these Functions
$m_{7,1}$	=	$g_{43,3}(k-1)$	Zero	million head	Total beef cows culled (pregnant and non-pregnant, becoming 3 years old and over) at the end of previous year, as simulated by age distribution inventory model (S_{k-1}) for computing (P) "predicted" changes in cull beef cow numbers for comparison with (A) "actual" historical changes in numbers of beef cows slaughtered.	$g_{47,1}$
$m_{7,2}$	=	Unassigned	----	-----	---- ----- ---- ----	-----
$m_{7,3}$	=	$g_{47,3}(k-1)$	Zero	million head	P	$g_{47,3}$
$m_{7,4}$	=	$g_{47,4}(k-1)$	Zero	dimensionless	P^2	$g_{47,4}$
$m_{7,5}$	=	$g_{47,5}(k-1)$	Zero	million head	A	$g_{47,5}$
$m_{7,6}$	=	$g_{47,6}(k-1)$	Zero	dimensionless	A^2	$g_{47,6}$
$m_{7,7}$	=	$g_{47,7}(k-1)$	Zero	dimensionless	PA	$g_{47,7}$
$m_{7,8}$	=	$g_{47,8}(k-1)$	Zero	dimensionless	$(P-A)^2$	$g_{47,8}$

NOTE:

$$P = \frac{(S_k - S_{k-1})}{S_{k-1}}$$

$$A = \frac{(H_k - H_{k-1})}{H_{k-1}}$$

MEMORY VARIABLE LIST: SUMMATIONS FOR TEST STATISTICS ON NUMBER OF CALVES BORN TO BEEF COWS

Memory Variable		Definition	Initial Value	Units	Description	Used in these Functions
$m_{8,1}$	=	$g_{43,6}(k-1)$	Zero	million head	Number of calves born to beef cows and heifers in the previous year as simulated by age distribution inventory model (S_{k-1}) for computing (P) "predicted" changes in birth numbers for comparison with (A) "actual" historical changes in birth numbers.	$g_{48,1}$
$m_{8,2}$	=	Unassigned	----	-----	---- ---- ---- ----	-----
$m_{8,3}$	=	$g_{48,3}(k-1)$	Zero	million head	ΣP	$g_{48,3}$
$m_{8,4}$	=	$g_{48,4}(k-1)$	Zero	dimensionless	ΣP^2	$g_{48,4}$
$m_{8,5}$	=	$g_{48,5}(k-1)$	Zero	million head	ΣA	$g_{48,5}$
$m_{8,6}$	=	$g_{48,6}(k-1)$	Zero	dimensionless	ΣA^2	$g_{48,6}$
$m_{8,7}$	=	$g_{48,7}(k-1)$	Zero	dimensionless	ΣPA	$g_{48,7}$
$m_{8,8}$	=	$g_{48,8}(k-1)$	Zero	dimensionless	$\Sigma (P-A)^2$	$g_{48,8}$

NOTE:

$$P = \frac{(S_k - S_{k-1})}{S_{k-1}}$$

$$A = \frac{(H_k - H_{k-1})}{H_{k-1}}$$

NOTE: m_9 and m_{10} are unassigned

MEMORY VARIABLE LIST: STATE VARIABLES IN TIME (k-2)

Memory Variable		Definition	Initial Value	Units	Description	Used in these Functions
$m_{11,j}$	=	$x_{1,j}(k-2)$	Zero	100,000 head	Post-culling inventories of pregnant heifers and cows becoming j years old two years ago. j = 2,14	$Y_{1,6}$
$m_{12,1}$	=	$x_{2,1}(k-2)$	Zero	100,000 head	Post-culling inventory of weaned heifers kept for breeding (HKB's) two years ago.	$Y_{1,6}$
$m_{12,j}$	=	$x_{2,j}(k-2)$	Zero	100,000 head	Post-culling inventory of non-pregnant yearlings and cows becoming j years old two years ago. j = 2,13	$Y_{1,6}$

INTERMEDIATE FUNCTIONS: BIOLOGICAL FUNCTIONS

NOTE: An asterisk next to an equation indicates a change in the basic model developed by Tom Nordblom, the model is otherwise unchanged. Changes include: (1) the equational form and/or (2) the values used in the equation and/or (3) the interpretation of the equation.

Description	Units	Used in these functions
$j = 1,14$ = age at breeding $g_{1,j} = b_1 + b_2(j-b_3) + b_4(j-b_3)^2$ Conception Rate (C_j) as a function of age (j) at breeding	proportion $\frac{\text{cows pregnant}}{\text{cows bred}}$	g_9 g_{10} g_{34} g_{35} $Y_{1,9}$
$j = 1,15$ = age becoming $g_{2,j} = 1.0 - \left(b_5 + \frac{b_6}{j} + b_7 \cdot j^2\right)$ Unimparied health rate (H_j) (complement of the seriously impaired health rate) in the year prior to age j.	proportion $\frac{\text{healthy cows now}}{\text{live cows now}}$	g_9 g_{10} g_{37} g_{38}
$j = 2,15$ = age becoming $g_{3,j} = b_8 + b_9 \cdot j$ Cow survival rate (S_j) after natural and accidental death in the year prior to age j.	proportion $\frac{\text{live cows now}}{\text{live cows kept one year ago}}$	g_{10} g_{34} g_{35} g_{36} g_{41} $g_{34,6}$

INTERMEDIATE FUNCTIONS: BIOLOGICAL FUNCTIONS (continued)

Description	Units	Used in these Functions
$j = 2,15$ = age becoming $g_{4,j} = b_{10} \cdot b_{11} \cdot \left(b_{12} + (b_{13} \cdot j) + \frac{b_{14}}{j}\right) + (1.0 - b_{10}) \cdot b_{15} \cdot \left(b_{16} + (b_{17} \cdot j) + (b_{18} \cdot j^2) + (b_{19} \cdot j^3)\right)$ Cow culling weight (CW_j) at culling time prior to age j.	cwt.	g_{13} g_{14} g_{25} $Y_{1,11}$
$g_5 = b_{10} \cdot b_{11} + (1.0 - b_{10}) \cdot b_{15}$ (MA) Maximum aggregate cow body weight (a single value measurement depending on the proportion of early and late maturity breeds).	cwt.	g_6 g_7
$j = 2,14$ = age at calving time $g_{6,j} = g_5 \cdot b_{20} \cdot \left(b_{21} + (b_{22} \cdot j) + (b_{23} \cdot j^2) + (b_{24} \cdot j^3)\right)$ (WW_j) Calf weaning weights expected for cows aged (j) years at calving.	cwt.	g_{11} g_{24} $Y_{1,10}$

INTERMEDIATE FUNCTIONS: BIOLOGICAL FUNCTIONS (continued)

Description	Units	Used in these Functions
$g_7 = g_5 \cdot b_{25}$ Estimated weaning weight for a heifer kept for breeding (HKB) (a single value estimate linked to maximum aggregate cow body weight),	cwt.	g_{13} g_{14}
$j = 2,14$ = age at calving $g_{8,j} = b_{26} + (b_{27} \cdot j) + (\frac{b_{28}}{j})$ Calf survival rate (CS_j) (calves weaned per pregnant cow kept to calve at age j^j).	proportion $(\frac{\text{calves weaned}}{\text{pregnant cows}})$	g_{11} g_{24} g_{35} $g_{43,3}$ $Y_{1,10}$

INTERMEDIATE FUNCTIONS: MANAGEMENT EXPECTATION FUNCTIONS

Description	Units	Used in these Functions
j = 1 to 15 = age becoming * $g_{9,j} = 2 - g_{1,j} - g_{2,j}$ This function calculates cull cow sales in the coming year (cull cow sales per cow becoming age j); to be used in present value of breeding (PVB) calculations	fraction of 1 cow	g_{28}
j = 1 to 13 = age becoming * $g_{10,j} = \left[g_{1(j+1)} + g_{2(j+1)} + g_{3(j+1)} - 2 \right] g_{3(j+2)}$ This function calculates cull cow sales two years from now. Note: the function inside the parenthesis is the fraction of animals that are pregnant, healthy, and alive from the previous year. $g_{10,j}$ is used in the PVB calculations.	fraction of 1 cow	g_{24} g_{28}

INTERMEDIATE FUNCTIONS: PRODUCER PROFIT EXPECTATION FUNCTIONS

Description	Units	Used in these Functions
j = 2 to 15 = age becoming * $g_{11,j} = g_{6,j} \cdot g_{8,j} \cdot g_{12,1} \cdot b_{38}$ Calf sales in the coming year (calf sales per cow becoming age j), used in PVB calculations.	fraction of 1 cow	$g_{28(j+1)}$ g_{31}
j = 1 to 14 = age becoming * $g_{24,j} = g_{10,j} \cdot g_{8,(j+1)} \cdot g_{6,(j+1)} \cdot g_{12,1} \cdot b_{38}$ Calf sales two years from now (per cow becoming age j), used in PVB calculations.	fraction of 1 cow	g_{28}

NOTE: g_{24} follows g_{11} and preceeds g_{12}.

INTERMEDIATE FUNCTIONS: EXPECTED PRICES AND EXPECTED SALVAGE VALUES, FSV_j

Description		Units	Used in these Functions
$g_{12,1} = b_{73} \cdot m_{4,1} + (b_{74} \cdot z_1)$	Expected price of feeder steers in future years as a function of their price in the current year (z_1) and in the previous year ($m_{4,1}$).	\$/cwt.	$g_{13,j}$
$g_{12,2} = b_{75} \cdot m_{4,2} + (b_{76} \cdot z_2)$	Expected price of utility cows in future years as a function of their price in the current year (z_2) and in the previous year ($m_{4,2}$).	\$/cwt.	$g_{13,j}$

NOTE: By altering the b-parameter values in the above functions, the "expected prices" may be defined to represent a continuation of the most recent one year trend or a weighted average of last year's and this year's prices.

j = 1,15 = age becoming at time of possible salvage sale		\$/hd	g_{28}

$$
*g_{13,j} = \begin{cases} \left[(g_{12,1} \cdot b_{39}) + b_{31}\right] g_7 & ,\text{if } j = 1 \\ g_{4,j} \left[g_{12,1} - b_{40} \cdot \left(g_{12,1} - g_{12,2}\right) + \dfrac{b_{40} \cdot (g_{12,1} - g_{12,2})}{j \cdot b_{41}} \right] & ,\text{if } j > 1 \end{cases}
$$

Expected future salvage values (FSV_j), analogous to present salvage values described below, are the product of expected prices and body weights. These values are used in the net annual revenue budgets and in calculations of present values for breeding.

INTERMEDIATE FUNCTIONS: PRESENT SALVAGE VALUE (PSV_j)

Description	Units	Used in these Functions
j = 1,15 = age becoming $$*g_{14,j} = \begin{cases} \left[(z_1 \cdot b_{39}) + b_{31}\right] g_7 & ,\text{if } j = 1 \\ g_{4,j} \cdot \left[z_1 - b_{35}(z_1 - z_2) + \dfrac{b_{35}(z_1 - z_2)}{j \cdot b_{41}}\right] & ,\text{if } j > 1 \end{cases}$$ Present salvage value (PSV_j) estimates. The PSV of an HKB (a weaned heifer kept for breeding), when first retained, is her estimated weight (g_7) times an adjusted feeder steer price ($z_1 \cdot b_{39}$) + b_{31}. The cull sales values of mature non-pregnant cows (becoming j=2 to 15 years of age) are the product of their respective body weights ($g_{4,j}$) and prices. Their respective price estimates are a function of current feeder steer price (z_1) and utility cow price (z_2), declining hyperbolically with age.	\$/hd.	g_{32} Y_3
j = 2 to 15 = age becoming $$*g_{25,j} = g_{4,j} \cdot \left[z_1 - b_{33}(z_1 - z_2) + \frac{b_{33}(z_1 - z_2)}{j \cdot b_{41}}\right]$$ Present salvage value (PSV_j) estimates of pregnant cows becoming j = 2 to 15 years of age.	\$/hd.	g_{30} Y_3

NOTE: g_{25} follows g_{14} and preceeds g_{15}.

INTERMEDIATE FUNCTION: SHORT TERM INTEREST FACTOR

Description	Units	Used in these Functions
$g_{15} = (1.0 + (b_{42} \cdot z_{13}) + b_{36})^{b_{43}}$	percent	g_{17} g_{19} g_{20} g_{22} g_{23}

Factor for inflating operating costs due to short term interest charges. The current P.C.A. average cost of loans (z_{13}, a decimal fraction) is adjusted directly by b_{42}. The exponent b_{43} represents the fraction of a year for which interest is charged on short term operating costs. The option of using a constant interest rate is allowed with the b_{36} parameter.

INTERMEDIATE FUNCTIONS: COST CALCULATIONS

Description	Units	Used in these Functions
$g_{16} = (b_{44} \cdot z_{11}) + (b_{45} \cdot z_3) + (b_{46} \cdot z_4) + (b_{47} \cdot z_5)$	\$/hd./yr.	g_{17} g_{19} g_{20} g_{22} g_{23}

$(b_{44} \cdot z_{11})$: marketing & hauling costs
$(b_{45} \cdot z_3)$: Fuel, lube & electric Costs
$(b_{46} \cdot z_4)$: Maching & building repair
$(b_{47} \cdot z_5)$: charges

Costs common to all budgets.

$$g_{17} = \left[g_{16} + (b_{48} \cdot z_6) + (b_{49} \cdot z_7) + (b_{50} \cdot z_8) + (b_{51} \cdot z_9) + (b_{52} \cdot z_{10}) + (b_{53} \cdot z_{11}) + (b_{54} \cdot z_{12}) \right] g_{15}$$

g_{16}: common costs
$(b_{48} \cdot z_6)$: pasture rental cost
$(b_{49} \cdot z_7)$: Hay cost
$(b_{50} \cdot z_8)$: Grain & concentrate cost
$(b_{51} \cdot z_9)$: Protein supplement cost
$(b_{52} \cdot z_{10})$: Salt & mineral cost
$(b_{53} \cdot z_{11})$: Labor costs
$(b_{54} \cdot z_{12})$: Veterinary & medicine
g_{15}: Short term interest factor

Units: \$/hd./yr. Used in these functions: $g_{28,1}$

Costs perculiar to heifers kept for breeding.

	Units	Used in these Functions
$g_{18} = (b_{55} \cdot z_6) + (b_{56} \cdot z_7) + (b_{57} \cdot z_8) + (b_{58} \cdot z_9) + (b_{59} \cdot z_{10})$	\$/hd.	g_{19} g_{20}

$(b_{55} \cdot z_6)$: Pasture rental cost
$(b_{56} \cdot z_7)$: Hay cost
$(b_{57} \cdot z_8)$: Grain & concentrate
$(b_{58} \cdot z_9)$: Protein supplement cost
$(b_{59} \cdot z_{10})$: Salt & mineral

Costs common to yearling heifers (pregnant or not)

INTERMEDIATE FUNCTIONS: COST CALCULATIONS (continued)

Description		Units	Used in these Functions
$g_{19} = \left[g_{16} + g_{18} + (b_{60} \cdot z_{11}) + (b_{61} \cdot z_{12}) \right] \cdot g_{15}$	Cost peculiar to pregnant yearling heifers.	\$/hd./yr.	$g_{28,1}$ $g_{28,2}$
(g_{16}: Common costs; $(b \cdot z_{11})$: Labor costs; $(b \cdot z_{12})$: Veterinary & medicine; g_{15}: Short term interest factor)			
$g_{20} = \left[g_{16} + b_{18} + (b_{62} \cdot z_{11}) + (b_{63} \cdot z_{12}) \right] \cdot g_{15}$	Cost peculiar to non-pregnant yearling heifers.	\$/hd./yr.	
$g_{21} = (b_{64} \cdot z_6) + (b_{65} \cdot z_7) + (b_{66} \cdot z_8) + (b_{67} \cdot z_9) + (b_{68} \cdot z_{10})$		\$/hd./yr.	g_{22} g_{23}
($(b_{64} \cdot z_6)$: Pasture rental costs; $(b_{65} \cdot z_7)$: Hay costs; $(b_{66} \cdot z_8)$: Grain & concentrate costs; $(b_{67} \cdot z_9)$: Protein supplement costs; $(b_{68} \cdot z_{10})$: Salt & mineral costs)			

Costs common to mature cows becoming 3 years of age or older, pregnant or not.

Description		Units	Used in these Functions
$g_{22} = \left[g_{16} + g_{21} + (b_{69} \cdot z_{11}) + (b_{70} \cdot z_{12}) \right] \cdot g_{15}$	Costs for pregnant mature cows.	\$/hd./yr.	$g_{28(j+1)}$ g_{31}
(g_{16}: Common costs; $(b \cdot z_{11})$: Labor costs; $(b \cdot z_{12})$: Veterinary & medicine; g_{15}: Short term interest factor)			
$g_{23} = \left[g_{16} + g_{21} + (b_{71} \cdot z_{11}) + (b_{72} \cdot z_{12}) \right] \cdot g_{15}$	Costs for non-pregnant mature cows.	\$/hd./yr.	g_{31}

INTERMEDIATE FUNCTIONS: DISCOUNT RATE AND PVB_1^P

Description		Units	Used in these Functions
$g_{26,1} = \left(\frac{1.0}{1.0 + (b_{80} \cdot z_{13}) + b_{37}} \right)$	This discount factor is taken to the power of the i^{th} year of the future in the present value calculations which follow. Here z_{13} is the P.C.A. average cost of loans (a decimal fraction, %/100), which is taken times a constant factor b_{80}. The option of a constant discount rate is allowed with the b_{37} parameter.	percent	$g_{28,j}$ $g_{31,j}$

$$*g_{28,1} = \underbrace{\left[(g_{9,1} \cdot g_{13,2}) - g_{17} \right]}_{\text{expected net culling revenue (one year in the future) for a heifer kept for breeding today}} \underset{\uparrow \text{ discount factor}}{g_{26}} + \underbrace{\left[(g_{10,1} \cdot g_{13,3}) - g_{19} \right]}_{\text{expected net culling revenue (two years in the future) for a heifer kept for breeding today}} \underset{\uparrow \text{ discount factor}}{g_{26}^{\,2}} + \underbrace{g_{24,1}}_{\text{expected calf sales for a HKB}} \cdot \underset{\uparrow \text{ discount factor}}{g_{26}^{\,2}} = PVB_1^N$$

Units	Used in these Functions
\$/hd.	g_{30} $g_{32,1}$ Y_6

where: PVB_1^N = The discounted present value for a heifer kept for breeding (HKB) if retained in the herd for two years.

NOTE: g_{27} and g_{29} were included in Nordblom's Flexform but have been eliminated here.

INTERMEDIATE FUNCTIONS: PVB_2^P FOR PREGNANT YEARLINGS

j = 2 = age becoming   Units: $/head   Used in these Functions: g_{30} g_{31} Y_6

Description

$$*g_{28,2} = \left[(g_{9,2} \cdot g_{13,3}) - g_{19}\right] \cdot g_{26} + \left[(g_{10,2} \cdot g_{13,4}) - g_{22}\right] \cdot g_{26}^2 + g_{11,2} \cdot g_{26} + g_{24,2} \cdot g_{26}^2 = PVB_2^P$$

j=2=age becoming

- $\left[(g_{9,2} \cdot g_{13,3}) - g_{19}\right]$: expected net culling revenue (one year in the future) for a pregnant yearling heifer kept today
- g_{26}: ← discount factor
- $\left[(g_{10,2} \cdot g_{13,4}) - g_{22}\right]$: expected net culling revenue (two years in the future) for a pregnant yearling heifer kept today
- g_{26}^2: ← discount factor
- $g_{11,2} \cdot g_{26} + g_{24,2} \cdot g_{26}^2$: present value of calf sales revenues for a pregnant yearling heifer if retained in the herd for two years

PVB_2^P = The discounted present value expected for a pregnant yearling heifer if retained in the herd for two years.

INTERMEDIATE FUNCTIONS: $PVB^{P}_{(2+j)}$ FOR PREGNANT COWS

j = 3 to 14 = age becoming   Units: \$/head   Used in these Functions: g_{30} g_{31} Y_6

Description

$$*g_{28,j} = \left[(g_{9,j}\ g_{13(j+1)}) - g_{22} \right] \cdot g_{26} + \left[(g_{10,j}\ g_{13(j+2)}) - g_{22} \right] \cdot g_{26}^{2} + g_{11,j} \cdot g_{26} + g_{24,j} \cdot g_{26}^{2}$$

- $[(g_{9,j}\ g_{13(j+1)}) - g_{22}]$: expected net culling revenue (one year in the future) for a pregnant cow kept today
- g_{26}: discount factor
- $[(g_{10,j}\ g_{13(j+2)}) - g_{22}]$: expected net culling revenue (two years in the future) for a pregnant cow kept today
- g_{26}^{2}: discount factor
- $g_{11,j}$: expected calf sales (one year in the future) for a pregnant cow kept today
- g_{26}: discount factor
- $g_{24,j}$: expected calf sales (two years in the future) for a pregnant cow kept today
- g_{26}^{2}: discount factor

$PVB^{P}_{(2+j)}$ = Present value of final cull sale revenue (two year horizon) + Present value of calf sales revenues (two year horizon)

INTERMEDIATE FUNCTIONS: V_j^p

Description	Units	Used in these Functions
$j = 2,14$ = age becoming	dless	$g_{37,j}$
$*g_{30,j} = \frac{g_{28,j}}{g_{25,j}} = V_j^p = \frac{PVB_j^p}{PSV_j}$		$Y_{4,j}$

These ratios of discounted maximum net future revenue (PVB_j^p) to present salvage value (PSV_j) provide the major links between the value model and the age distribution inventory model. These V-ratios are the criteria on which the retainment rates for the pregnant cow classes are based each year in the age distribution inventory model.

NOTE: dless indicates a dimensionless constant.

INTERMEDIATE FUNCTIONS: PVB_j^N and V_j^N FOR NON-PREGNANT COWS

Description	Units	Used in these Functions
$j = 2$ to 13	$/hd.	g_{32} Y_7

$$*g_{31,j} = \underbrace{g_{28,j}}_{(PVB_j^P)} - [\underbrace{g_{11,j} \cdot g_{26}}_{\text{present value of calf sales in the coming year}} + \underbrace{(g_{22} - g_{23})}_{\text{constant adjustment for non-pregnant cows}} \cdot \underbrace{g_{26}}_{\text{Discount factor}}] = PVB_j^N$$

This function calculates the discounted present value of future net income (two year time horizon) expected for non-pregnant cows becoming j years of age, if kept for breeding. This is calculated by adjusting the PVB^P for pregnant cows of the same age by the loss of the first years' expected net calf sales revenue.

Description	Units	Used in these Functions
$j = 1,13$	dimensionless	g_{38} Y_5

$$*g_{32,j} = \begin{cases} \dfrac{g_{28,1}}{g_{14,1}} & ,\text{if } j = 1 \\[2ex] \dfrac{g_{31,j}}{g_{14,j}} & ,\text{if } j > 1 \end{cases} \qquad V_j^n = \frac{PVB_j^N}{PSV_j}$$

This function calculates the value ratios (V_j^N) for non-pregnant heifers and cows becoming (j) years of age.

INTERMEDIATE FUNCTIONS: PERCENT OF YEARLINGS EXPOSED AND NOT EXPOSED FOR BREEDING

Description	Units	Used in these Functions
$*g_{33} = [b_{29} + b_{30} (t - t_0)]$ This function calculates the percent of yearlings bred to calve as 2 year olds as a linearly increasing function of time NOTE: t_0 is a constant set equal to the beginning year of the simulation run. t is variable, equaling each year of the simulation run as that year is processed.	percent	$g_{34,2}$ g_{36}
$*g_{36} = [x_{2,1} \cdot g_{3,2} \cdot (1 - g_{33})]$ This functions calculates the number of yearlings not exposed for breeding. NOTE: g_{36} follows g_{33} and preceeds g_{34}.	percent	$g_{40,2}$

INTERMEDIATE FUNCTIONS: PRE-CULLING INVENTORY OF PREGNANT COWS

Description	Units	Used in these Functions
j = 2,14 = age becoming	100,000 head	g_{39}

$$*g_{34,j} = \begin{cases} \underset{\text{(HKB's)}}{x_{2,1}} \cdot g_{3,2} \cdot g_{1,1} \cdot g_{33} & ,\text{if } j = 2 \\ [x_{1,(j-1)} + x_{2,(j-1)}] \cdot g_{3,j} \cdot g_{1,(j-1)} & ,\text{if } j > 2 \end{cases}$$

($x_{1,(j-1)}$, $x_{2,(j-1)}$:) pregnant and non-pregnant cows (j-1) years old at breeding.

This function calculates the number of pregnant animals that would be j years old at calving if not culled now. Here it is assumed that lactating and dry cows have identical survival rates ($g_{3,j}$) and conception rates ($g_{1,j}$), at the same ages.

INTERMEDIATE FUNCTIONS: PRE-CULLING INVENTORIES OF NON-PREGNANT COWS

Description	Units	Used in these Functions
$j = 1,14$ $g_{35,j} = \begin{cases} (1/2) \overbrace{\sum_{i=2}^{14} x_{1,i} \cdot g_{8,i}}^{\text{calves weaned}} & \text{,if } j = 1 \\ [(x_{2,1} \cdot g_{3,2} \cdot (1-g_{1,1})] + x_{1,1} & \text{,if } j = 2 \\ [x_{1,(j-1)} + x_{2,(j-1)}] \cdot g_{3,j} \cdot (1-g_{1,(j-1)}) & \text{,if } j > 2 \end{cases}$	100,000 head	g_{40}

This function calculates the number of non-pregnant heifers and cows that would be (j) years old in the next breeding season if not culled now. The proportions of these non-pregnant classes which are retained for breeding in the next season depend on their respective retainment functions. (see g_{38} description below.)

INTERMEDIATE FUNCTIONS: RETAINMENT DECISION (linking the value model with the age distribution inventory model)

Description	Units	Used in these Functions
$j = 2,14$	dimensionless	$g_{39,j}$
$$g_{37,j} = b_{88} + \left[\frac{g_{2,j} - b_{88}}{1 + e^{b_{83}(g_{30,j} - b_{84})}} \right]$$	hd. to keep / hd. in pre-culling inventory	
This function determines the proportion of the pre-culling inventory of pregnant cows (becoming j years old) to be retained for calving and rebreeding: depending on V_j^p ($g_{30,j}$) the proportion with unimpaired health, ($g_{2,j}$ = asymptotic max.) and an arbitrary minimum proportion kept.		
(b_{88} = asymptotic min.) (b_{84} = V at inflection.)		

INTERMEDIATE FUNCTION: RETAINMENT DECISIONS FOR NON-PREGNANT CLASSES

Units

$j = 1,13$

$$g_{38,j} = \begin{cases} b_{89} + \left[\dfrac{(b_{94} \cdot g_{2,j}) - b_{89}}{1 + e^{b_{92}(g_{32,j} - b_{93})}} \right] & ,\text{if } j \leq 2 \\ b_{90} + \left[\dfrac{(b_{91} \cdot g_{2,j} - b_{90}}{1 + e^{b_{85}(g_{32,j} - b_{86})}} \right] & ,\text{if } j > 2 \end{cases}$$

The proportion of weaned and yearling heifers to be kept for breeding: depending on V_j^N ($g_{32,j}$), the proportion with unimpaired health ($g_{2,j}$ = asymptotic max.), and an arbitrary minimum proportion kept (b_{89} = asymptotic min.). b_{94} = max. proportion of healthy weaned heifers that may be kept for breeding. b_{93} = V at inflection.

Units: dimensionless — $\frac{\text{Heifers kept}}{\text{Heifers on hand}}$

The proportion of pre-culling inventory of open cows (becoming j years old) to be retained for breeding: depending on V_j^N ($g_{32,j}$), the proportion with unimpaired health ($g_{2,j}$) times an arbitrary factor (b_{91}) (providing an asymptotic max.), and an arbitrary minimum proportion kept (b_{90} = asymptotic min.) b_{86} = V at inflection.

Units: dimensionless — $\frac{\text{head to keep}}{\text{hd. in pre-culling inventory}}$

$g_{38,j}$ is used in function $g_{40,j}$

INTERMEDIATE FUNCTIONS: NUMBER OF ANIMALS KEPT FOR BREEDING

Description	Units	Used in these Functions
$j = 2,14$ $g_{39,j} = g_{34,j} \cdot g_{37,j}$ (pre-culling inventory of pregnant cows becoming j years old) x (proportion of pregnant cows kept to calve in the coming year as j years old) = (Number of pregnant cows kept to calve in the coming year as j years old)	100,000 head	g_{41} $g_{43,1}$ $g_{43,2}$ f_1
$j = 1,3\text{-}13$ $g_{40,j} = g_{35,j} \cdot g_{38,j}$ (pre-culling inventory of non-pregnant heifers and cows becoming j years old) x (proportion of non-pregnant cows kept for breeding in the coming year as j year olds) = (Number of non-pregnant cows kept for breeding in the coming year as j year olds.)	100,000 head	g_{42} $g_{43,1}$ $g_{43,2}$ f_2
$j = 2$ $^*g_{40,2} = (g_{35,2} \cdot g_{38,2}) + g_{36}$ (pre-culling inventory of non-pregnant yearlings becoming 2 years old) x (proportion of non-pregnant yearlings kept for breeding in the coming year as 2 year olds) + (the number of yearlings not exposed for breeding) = (Number of non-pregnant yearlings kept for breeding in the coming year as j year olds)	100,000 head	g_{42} $g_{43,1}$ $g_{43,2}$ f_2

INTERMEDIATE FUNCTIONS: NUMBERS OF ANIMALS TO BE CULLED

Description			Units	Used in these Functions
$j = 2,15$ = age becoming				
$g_{41,j} = g_{34,j} - g_{39,j}$	, if $j < 15$	number of pregnant animals in pre-culling inventory (becoming j years old) *minus* number of pregnant cows kept for calving as j year olds in the coming year gives the number of pregnant cows culled before reaching j years of age.	100,000 head	$g_{43,3}$
$x_{1,14} \cdot g_{3,15}$	, if $j = 15$	Number of cows calving as 14 year olds this year *times* the survival rate for 14 year old cows gives the number of cows which comprise the class of animals becoming 15 years old. The model culls all of these with the arbitrary final culling age rule.		
$j = 1,14$ = age becoming				
$g_{42,j} = g_{35,j} - g_{40,j}$	, if $j < 14$	Number of non-pregnant heifers and cows becoming j years old in pre-culling inventories *minus* number of non-pregnant cows to be kept for breeding as j year olds, gives the numbers culled.	100,000	$g_{43,3}$ f_1
$g_{35,14}$	, if $j = 14$	Number of non-pregnant cows becoming 14 years old in the pre-culling inventory. All are culled here by the arbitrary rule that non-pregnant 13 year olds should not be kept another year.		

INTERMEDIATE FUNCTIONS: SUBTOTALS FOR TEST STATISTICS AND OUTPUT REPORTS

Description	Units	Used in these Functions
$*g_{43,1} = \left[(b_{98} \cdot g_{39,2}) + (\sum_{i=3}^{14} g_{39,i}) + (\sum_{i=3}^{13} g_{40,i}) + (b_{77} g_{43,3}) \right] (0.1)$ Total = pregnant yearling heifers + pregnant cows + non-pregnant cows + cull cows on inventory after Jan. 1 Number retained in herd after this year's culling. This number should simulate the USDA estimates of beef cow numbers in the January 1 inventory in the year (z_{14+1}).	million head	$g_{44,1}$ $m_{5,1}$ $g_{45,1}$ $Y_{1,2}$ $Y_{1,13}$
$g_{43,2} = \left[(b_{95} \cdot g_{39,2}) + (b_{96} \cdot g_{40,1}) + (b_{97} \cdot g_{40,2}) \right] (0.1)$ Total = pregnant yearling heifers + weaned heifers kept for breeding + non-pregnant yearling heifers Reported as beef heifers recruited into the breeding herd. The respective weighting factors b_{95}, b_{96}, and b_{97} allow the inclusion of more or less of the numbers simulated in these categories in the total to be compared with the USDA estimates of "heifers for replacement" on January 1 in year (z_{14+1})	million head	$g_{44,2}$ $g_{46,1}$ $Y_{1,13}$ $m_{6,1}$ $Y_{1,14}$
$g_{43,3} = \left[(\sum_{i=3}^{15} g_{41,i}) + (\sum_{i=3}^{14} g_{42,i}) \right] (0.1)$ Total cows (pregnant + non-pregnant) culled during the current simulated year. This number should simulate the USDA estimates of beef cow slaughter numbers for the year z_{14}.	million	$g_{44,3}$ $g_{47,1}$ $Y_{1,4}$ $m_{7,1}$ $Y_{1,11}$ $Y_{1,15}$

INTERMEDIATE FUNCTIONS: SUBTOTALS FOR TEST STATISTICS AND OUTPUT REPORTS

Description	Units	Used in these Functions
$g_{43,4} = \sum_{i=2}^{14} (x_{1,i} \cdot g_{8,i})$ (pregnant cow numbers · calf survival rates) This function determines the number of calves weaned in the current year, z_{14}.	100,000 head	$Y_{1,5}$ $Y_{1,6}$ $Y_{1,7}$ $Y_{1,8}$ $Y_{1,10}$ $Y_{1,14}$
$g_{43,5} = \left(\sum_{i=2}^{14} x_{1,i} \right) + \left(\sum_{i=2}^{13} x_{2,i} \right)$ Total pregnant and non-pregnant cows and heifers (becoming 2 years old and older) on inventory at beginning of current year.	100,000 head	$Y_{1,7}$ $Y_{1,9}$ $Y_{1,12}$
$g_{43,6} = \left\{ \sum_{i=2}^{14} (x_{1,i}\ g_{3,(i+1)}) \right\} (0.1)$ numbers x cow survival rates = Estimated number of calves born to beef cows in the current year (total). This number should simulate estimates of calves born to beef cows, derived from USDA data on total calves born and dairy cow numbers. (pregnant cows & heifers retained at beginning of current year)	million head	$g_{48,1}$ $Y_{1,13}$ $Y_{1,14}$ $m_{8,1}$

INTERMEDIATE FUNCTIONS: TEST STATISTICS (Sum accumulations for MPAD'S)

Description	Units	Used in these Functions
$g_{44,1} = m_{3,1} + \left\| \frac{g_{43,1} - b_{(z_{14}-1849)}}{b_{(z_{14}-1849)}} \right\|$ Previous sum + This years proportional absolute deviation of the model's estimates of cow numbers from the USDA estimates. $(b_{101} \rightarrow b_{132})$	dimensionless	$m_{3,1}$ $Y_{8,2}$
$^*g_{44,2} = m_{3,2} + \left\| \frac{g_{43,2} - b_{(z_{14}1810)}}{b_{(z_{14}-1810)}} \right\|$ Previous sum + This years proportional absolute deviation of the model's estimates of heifer recruitment numbers from the USDA estimates. $(b_{140} \rightarrow b_{171})$	dimensionless	$m_{3,2}$ $Y_{8,2}$
$^*g_{44,3} = m_{3,3} + \left\| \frac{g_{43,2} - b_{(z_{14}-1772)}}{b_{(z_{14}-1772)}} \right\|$ Previous sum + This years proportional absolute deviation of the model's estimates of cull cow numbers from the USDA estimates $(b_{178} \rightarrow b_{209})$	dimensionless	$m_{3,3}$ $Y_{9,2}$

INTERMEDIATE FUNCTIONS: TEST STATISTICS (Sum accumulations for MPAD's) (continued)

Description	Units	Used in these Functions
$*g_{44,4} = m_{3,4} + \left| \dfrac{g_{43,6} - b_{(z_{14}-1733)}}{b_{(z_{14}-1733)}} \right|$ Previous sum + This years proportional absolute deviation of the model's estimates of number of calves born to beef cows from those derived from USDA statistics. ($b_{217} \rightarrow b_{248}$)	dimensionless	$m_{3,4}$ $Y_{11,2}$

INTERMEDIATE FUNCTIONS: TRANSFORMATIONS AND SUMMATIONS FOR TEST STATISTICS ON BEEF COW NUMBERS

Note: when z_{14} = 1965, $g_{45,1}$ through $g_{45,8}$ are not to be computed.

Description		Used in these Functions
$g_{45,1} = (g_{43,1} = m_{5,1}) / m_{5,1}$ Proportional change in simulated cow numbers	$= P = (S_k - S_{k-1}) / S_{k-1}$	$g_{45,j}$ j=3,4,7 & 8
$g_{45,2} = \left(b_{(z_{14}-1849)} - b_{(z_{14}-1850)} \right) / b_{(z_{14}-1850)}$ Proportional change in historical beef cow numbers	$= A = (H_k - H_{k-1}) / H_{k-1}$	$g_{45,j}$ j=5,6,7 & 8
$g_{45,3} = m_{5,3} + g_{45,1}$	$= \Sigma P$	$g_{45,j}$ j=9,11 & 13 $m_{5,3}$
$g_{45,4} = m_{5,4} + (g_{45,1})^2$	$= \Sigma P^2$	$g_{45,9}$ $m_{5,4}$
$g_{45,5} = m_{5,5} + g_{45,2}$	$= \Sigma A$	$g_{45,j}$ j=10,11 & 13 $m_{5,5}$
$g_{45,6} = m_{5,6} + (g_{45,2})^2$	$= \Sigma A^2$	$g_{45,j}$ j=10 & 12 $m_{5,6}$
$g_{45,7} = m_{5,7} + (g_{45,1}\, g_{45,2})$	$= \Sigma PA$	$g_{45,11}$ $m_{5,7}$
$g_{45,8} = m_{5,8} + (g_{45,1} - g_{45,2})^2$	$= \Sigma\ (P-A)^2$	$g_{45,j}$ j=12,13,14&15 $m_{5,8}$

INTERMEDIATE FUNCTIONS: STANDARD STATISTICS AND THEIL'S MEASURES OF INEQUALITY FOR COW NUMBERS

($b_{100}=16\cdot n$) Note: $g_{45,9}$ through $g_{45,10}$ are computed only when $z_{14}=1981$, otherwise set at zero.

Description		Used in these Functions
$g_{45,9} = \left(\sqrt{(b_{100}\, g_{45,4}) - (g_{45,3})^2}\right) \Big/ b_{100}$	$= S_P = \frac{1}{n}\sqrt{n\,\Sigma\, P^2 - (\Sigma P)^2}$ Standard deviation of simulated changes	$g_{45,j}$ j=11,14 & 15
$g_{45,10} = \left(\sqrt{(b_{100}\, g_{45,6}) - (g_{45,5})^2}\right) \Big/ b_{100}$	$= S_A = \frac{1}{n}\sqrt{n\,\Sigma\, A^2 - (\Sigma A)^2}$ Standard deviation of historical changes	$g_{45,j}$ j=11,14 & 15
$g_{45,11} = \frac{(b_{100}\, g_{45,7}) - (g_{45,3}\, g_{45,5})}{(b_{100})^2\, g_{45,9}\, g_{45,10}}$	$= r = (n\,\Sigma PA - (\Sigma P)(\Sigma A))/n^2(S_P\, S_A)$ Correlation coefficient	$Y_{8,3}$ $g_{45,15}$
$g_{45,12} = \sqrt{g_{45,8} / g_{45,6}}$	$=$ Theil's $U = \sqrt{\Sigma(P-A)^2 / \Sigma A^2}$	$Y_{8,4}$
$g_{45,13} = \frac{(g_{45,3} - g_{45,5})^2}{(b_{100}\, g_{45,8})}$	$=$ Theil's $U^m = \left(\frac{\Sigma P}{n} - \frac{\Sigma A}{n}\right)^2 \Big/ \frac{1}{n}\,(P-A)^2$ Proportion of inequality due to mean bias	$Y_{8,5}$

INTERMEDIATE FUNCTIONS: STANDARD STATISTICS AND THEIL'S MEASURES OF INEQUALITY FOR COW NUMBERS (continued)

($b_{100}=16 \cdot n$) Note: $g_{45,9}$ through $g_{45,10}$ are computed only when $z_{14}=1981$, otherwise set at zero.

Description	Used in these Functions
$g_{45,14} = b_{100}(g_{45,9} - g_{45,10})^2 / g_{45,8}$ = Theil's $U^S = (S_P-S_A)^2 / \frac{1}{n} \Sigma(P-A)^2$ Proportion of inequality due to unequal variance	$Y_{8,6}$
$g_{45,15} = (2\ b_{100}(1 - g_{45,11})(g_{45,9}\ g_{45,10})) / g_{45,8} = 2\ (1-r)(S_P\ S_A) / \frac{1}{n} \Sigma(P-A)^2$ Theil's U^C = Proportion of inequality due to imperfect covariation	$Y_{8,7}$

INTERMEDIATE FUNCTIONS: TRANSFORMATIONS AND SUMMATIONS FOR TEST STATISTICS ON HEIFER NUMBERS RECRUITED

Note: when z_{14} = 1965, $g_{46,1}$ through $g_{46,8}$ are not to be completed

Description			Used in these Functions
$g_{46,1} = (g_{43,2} - m_{6,1}) / m_{6,1}$	$= P = (S_k - S_{k-1}) / S_{k-1}$	Proportional changes in simulated numbers of heifers recruited	$g_{46,j}$ j=3,4,7&8
$*g_{46,2} = (b_{(z_{14}-1810)} - b_{(z_{14}-1811)}) / b_{(z_{14}-1811)}$	$= A = (H_k - H_{k-1}) / H_{k-1}$	Proportional changes in historical numbers of heifers recruited	$g_{46,j}$ j=5,6,7&8
$g_{46,3} = m_{6,3} + g_{46,1}$	$= \Sigma P$		$g_{46,j}$ j=9,11&13 $m_{6,3}$
$g_{46,4} = m_{6,4} + (g_{46,1})^2$	$= \Sigma P^2$		$g_{46,9}$ $m_{6,4}$
$g_{46,5} = m_{6,5} + g_{46,2}$	$= \Sigma A$		$g_{46,j}$ j=10,11&13 $m_{6,5}$
$g_{46,6} = m_{6,6} + (g_{46,2})^2$	$= \Sigma A^2$		$g_{46,j}$ j=10 & 12 $m_{6,6}$
$g_{46,7} = m_{6,7} + [g_{46,1}\ g_{46,2}]$	$= \Sigma PA$		$g_{46,11}$ $m_{6,7}$
$g_{46,8} = m_{6,8} + [g_{46,1} - g_{46,2}]^2$	$= \Sigma (P-A)^2$		$g_{46,j}$ j=12,13,14&15 $m_{6,8}$

INTERMEDIATE FUNCTIONS: STANDARD STATISTICS AND THEIL'S MEASURES OF INEQUALITY FOR HEIFERS RECRUITED

Note: $g_{46,9}$ through $g_{46,15}$ are to be computed only when z_{14}-1981, otherwise set at zero. (b_{100}=16=n)

Description		Used in these Functions
$g_{46,9} = \sqrt{(b_{100} g_{46,4}) - (g_{46,3})^2} \;/\; b_{100}$	$= S_P$	$g_{46,j}$ j=11,14&15
$g_{46,10} = \sqrt{(b_{100}\, g_{46,6}) - (g_{46,5})^2} \;/\; b_{100}$	$= S_A$	$g_{46,j}$ j=11,14&15
$g_{46,11} = \left((b_{100}\, g_{46,7}) - (g_{46,3}\, g_{46,5})\right) \Big/ (b_{100})^2 g_{46,9}\, g_{46,10}$	$= r$	$g_{46,15}$ $Y_{9,3}$
$g_{46,12} = \sqrt{g_{46,8}/g_{46,6}}$	= Theil's U	$Y_{9,4}$
$g_{46,13} = (g_{46,3} - g_{46,5})^2 / (b_{100}\, g_{46,8})$	= Theil's U^m	$Y_{9,5}$
$g_{46,14} = (b_{100})\,(g_{46,9} - g_{46,10})^2 / g_{46,8}$	= Theil's U^s	$Y_{9,6}$
$g_{46,15} = [2\, b_{100}(1-g_{46,11})\, g_{46,9}\, g_{46,10}] / g_{46,8}$	= Theil's U^c	$Y_{9,7}$

INTERMEDIATE FUNCTIONS: TRANSFORMATIONS AND SUMMATIONS FOR TEST STATISTICS ON CULL BEEF COW SLAUGHTER NUMBERS

NOTE: when z_{14} = 1965, $g_{47,1}$ through $g_{47,8}$ are not to be computed.

Description	Used in these Functions
$g_{47,1} = (g_{43,3} - m_{7,1}) / m_{7,1} \quad = P = (S_k - S_{k-1}) / S_{k-1}$ Proportional change is simulated numbers of beef cows culled	$g_{47,j}$ j=3,4,7&8
$^*g_{47,2} = [b_{(z_{14}-1772)} - b_{(z_{14}-1773)}] / b_{(z_{14}-1773)} = A = (H_k - H_{k-1}) / H_{k-1}$ Proportional change in historical numbers of beef cows slaughtered	$g_{47,j}$ j=5,6,7&8
$g_{47,3} = m_{7,3} + g_{47,1} \quad = \Sigma P$	$g_{47,j}$ j=9,11,13 $m_{7,3}$
$g_{47,4} = m_{7,4} + (g_{47,1})^2 \quad = \Sigma P^2$	$g_{47,9}$ $m_{7,4}$
$g_{47,5} = m_{7,5} + g_{47,2} \quad = \Sigma A$	$g_{47,j}$ j=10,11&13 $m_{7,5}$
$g_{47,6} = m_{7,6} + (g_{47,2})^2 \quad = \Sigma A^2$	$g_{47,j}$ j=10 & 12 $m_{7,6}$
$g_{47,7} = m_{7,7} + [g_{47,1}\, g_{47,2}] \quad = \Sigma PA$	$g_{47,11}$ $m_{7,7}$
$g_{47,8} = m_{7,8} + [g_{47,1} - g_{47,2}]^2 = \Sigma (P-A)^2$	$g_{47,j}$ j=12,13,14&15 $m_{7,8}$

INTERMEDIATE FUNCTIONS: STANDARD STATISTICS AND THEIL'S MEASURES OF INEQUALITY FOR CULL BEEF COW NUMBERS FOR SLAUGHTER

$(b_{100}=16=n)$ NOTE: $g_{47,9}$ through $g_{47,15}$ are computed only when $z_{14}=1981$, otherwise set at zero.

Description		Used in these Functions
$g_{47,9} = \sqrt{(b_{100}\ g_{47,4}) - (g_{47,3})^2} \Big/ b_{100}$	$= S_P$	$g_{47,j}$ j=11,14 & 15
$g_{47,10} = \sqrt{(b_{100}\ g_{47,6}) - (g_{47,5})^2} \Big/ b_{100}$	$= S_A$	$g_{47,j}$ j=11,14 & 15
$g_{47,11} = \left((b_{100}\ g_{47,7}) - (g_{47,3}\ g_{47,5})\right) \Big/ (b_{100})^2\ g_{47,9}\ g_{47,10}$	$= r$	$g_{47,15}$ $Y_{10,3}$
$g_{47,12} = \sqrt{g_{47,8} / g_{47,6}}$	= Theil's U	$Y_{10,4}$
$g_{47,13} = (g_{47,3} - g_{47,5})^2 \Big/ (b_{100}\ g_{47,8})$	= Theil's U^m	$Y_{10,5}$
$g_{47,14} = b_{100}\ (g_{47,9} - g_{47,10})^2 \Big/ g_{47,8}$	= Theil's U^s	$Y_{10,6}$

INTERMEDIATE FUNCTIONS: STANDARD STATISTICS AND THEIL'S MEASURES OF INEQUALITY FOR CULL BEEF COW NUMBERS FOR SLAUGHTER (continued)

(b_{100}=16=n) NOTE: $g_{47,9}$ through $g_{47,15}$ are computed only when z_{14}=1981, otherwise set at zero.

Description		Used in these Functions
$g_{47,15} = \left(2\, b_{100}\, (1-g_{47,11})\, g_{47,9}\, g_{47,10}\right) / g_{47,8}$	= Theil's U^C	$Y_{10,7}$

INTERMEDIATE FUNCTIONS: TRANSFORMATIONS AND SUMMATIONS FOR TEST STATISTICS ON NUMBERS OF CALVES BORN TO BEEF COWS

NOTE: when z_{14}=1965, $g_{48,1}$ through $g_{48,8}$ are not computed.

	Description	Used in these Functions
$g_{48,1} = (g_{43,6} - m_{8,1}) / m_{8,1}$	$= P = (S_k - S_{k-1}) / S_{k-1}$ Proportional change in simulated numbers of calves born to beef cows	$g_{48,j}$ j-3,4,7&8
$^*g_{48,2} = [b_{(z_{14}-1733)} - b_{(z_{14}-1734)}] / b_{(z_{14}-1734)}$	$= A = (H_k - H_{k-1}) / H_{k-1}$ Proportional change in historical numbers of calves born to beef cows	$g_{48,j}$ j=5,6,7&8
$g_{48,3} = m_{8,3} + g_{48,1}$	$= \Sigma P$	$g_{48,j}$ j=9,11&13 $m_{8,3}$
$g_{48,4} = m_{8,4} + (g_{48,1})^2$	$= \Sigma P^2$	$g_{48,9}$ $m_{8,4}$
$g_{48,5} = m_{8,5} + g_{48,2}$	$= \Sigma A$	$g_{48,j}$ j=10,11&13 $m_{8,5}$
$g_{48,6} = m_{8,6} + (g_{48,2})^2$	$= \Sigma A^2$	$g_{48,j}$ j=10 & 12 $m_{8,6}$
$g_{48,7} = m_{8,7} + [g_{48,1}\ g_{48,2}]$	$= \Sigma PA$	$g_{48,11}$ $m_{8,7}$
$g_{48,8} = m_{8,8} + [g_{48,1} - g_{48,2}]^2$	$= \Sigma (P-A)^2$	$g_{48,j}$ j=12,13,14&15 $m_{8,8}$

INTERMEDIATE FUNCTIONS: STANDARD STATISTICS AND THEIL'S MEASURES OF INEQUALITY FOR NUMBERS OF CALVES BORN TO BEEF COWS

NOTE: $g_{48,9}$ through $g_{48,15}$ are computed only when z_{14}=1981, otherwise set at zero. (b_{100}=16=n)

Description		Used in these Functions
$g_{48,9} = \sqrt{(b_{100}\, g_{48,4}) - (g_{48,3})^2} \Big/ b_{100}$	$= S_P$	$g_{48,j}$ j=11,14&15
$g_{48,10} = \sqrt{(b_{100}\, g_{48,6}) - (g_{48,5})^2} \Big/ b_{100}$	$= S_A$	$g_{48,j}$ j=11,14&15
$g_{48,11} = \left((b_{100}\, g_{48,7}) - (g_{48,3}\, g_{48,5})^2\right) \Big/ (b_{100})^2\, g_{48,9}\, g_{48,10}$	= r	$g_{48,15}$ $Y_{11,3}$
$g_{48,12} = \sqrt{g_{48,8} / g_{48,6}}$	= Theil's U	$Y_{11,4}$
$g_{48,13} = (g_{48,3} - g_{48,5})^2 / b_{100}\, g_{48,8}$	= Theil's U^m	$Y_{11,5}$
$g_{48,14} = b_{100}\, (g_{48,9} - g_{48,10})^2 / g_{48,8}$	= Theil's U^s	$Y_{11,6}$
$g_{48,15} = (2\, b_{100}\, (1 - g_{48,11})\, g_{48,9}\, g_{48,10}) / g_{48,8}$	= Theil's U^c	$Y_{11,7}$

FLUX FUNCTIONS FOR POST-CULLING INVENTORIES: UPDATING THE STATE VARIABLES $x_{1,j}$ and $x_{2,j}$

Description					Units
(Number of weaned heifers not kept for breeding in the coming year.)	$\cdot$	(Fraction of these which may be candidates next year for recruitment to the breeding herd as yearling heifers.)	=	Number of animals in the special class of non-pregnant heifers ($x_{1,1}$) which are not selected for breeding as 1 year olds, but are potentially available to join the selection pool of non-pregnant yearling heifers becoming 2 year olds, next year.	[100,000 head]

j=1,14

$$f_{1,j} = \begin{cases} (g_{42,1}\ b_{87}) - x_{1,1} & ,\text{if } j = 1 \\ g_{39,j} - x_{1,j} & ,\text{if } j > 1 \end{cases}$$

↑

Number of pregnant animals to be kept for calving in the coming year as j year olds. = Post-culling inventories of pregnant animals in breeding herd, carried into year $z_{14}+1$

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j = 1,13

Number of non-pregnant animals to be kept for breeding in the coming year as j year olds = Post-culling inventories of non-pregnant animals in the breeding herd, carried into year $z_{14}+1$ [100,000 head]

↓

$$f_{2,j} = g_{40,j} - x_{2,j}$$

OUTPUT FUNCTIONS

	Description	Units
$Y_{1,1} = z_{14}$	Current year (1965-1981)	years
$Y_{1,2} = g_{43,1}$	Number of cows (pregnant and non-pregnant, becoming 3 years old and over) retained in the herd after this years culling. These cows will comprise the January 1 inventory in year $z_{14}+1$ comparable to USDA records. (See Y_8 for test statistics.)	million head
$Y_{1,3} = g_{43,2}$	Number of weaned heifers and pregnant and non-pregant yearling heifers simulated for comparison with USDA records. These heifers comprise the January 1 inventory of "heifers for replacement" in year $z_{14}+1$, comparable to USDA records. (See Y_9 for test statistics.)	million head
$Y_{1,4} = g_{43,3}$	Number of cows (pregnant and non-pregnant, becoming 3 years old and over) culled from the herd in the current year (z_{14}). This number of culls is comparable to USDA records of beef cow slaughter numbers. (See Y_{10} for test statistics.)	million head

OUTPUT FUNCTIONS

	Description	Units
$Y_{1,5} = (g_{43,4})\ (0.1)$	Number of calves weaned in the current year	million head
$Y_{1,6} = \dfrac{g_{43,4}}{\left(\sum_{i=2}^{14} m_{11,i} + \sum_{i=1}^{13} m_{12,i}\right)}$	Number of calves weaned in current year per cow and heifer exposed for breeding in the previous year.	$\dfrac{\text{calves}}{\text{cows}}$
$Y_{1,7} = \dfrac{g_{43,4}}{g_{43,5}}$	Number of calves weaned in current year, per cow and heifer (becoming 2 years old and over, pregnant and non-pregnant) on inventory at beginning of year.	$\dfrac{\text{calves}}{\text{cows}}$
$Y_{1,8} = \dfrac{g_{43,4}}{\left(\sum_{i=2}^{14} m_{1,i}\right)}$	Number of calves weaned in current year, per pregnant cow and heifer on inventory at beginning of current year.	$\dfrac{\text{calves}}{\text{cows}}$

OUTPUT FUNCTIONS

Description		Units
$Y_{1,9} = \dfrac{\sum_{i=2}^{14} (m_{1,i} \cdot g_{1,i}) + \sum_{i=1}^{13} (m_{2,i} \cdot g_{1,i})}{g_{43,5} + m_{2,1}}$	Average conception rate of all heifers and cows exposed for breeding in the current year.	proportion
$Y_{1,10} = \left\{ \dfrac{\sum_{i=2}^{14} (m_{1,i} \cdot g_{6,i} \cdot g_{8,i})}{g_{43,4}} \right\} (100)$	Average calf weaning weight in current year.	lbs./hd.
$Y_{1,11} = \left\{ \dfrac{\sum_{i=3}^{15} (g_{41,i} \cdot g_{4,i}) + \sum_{i=3}^{14} (g_{42,i} \cdot g_{4,i})}{g_{43,3}} \right\} (100)$	Average culling weight of cows culled in current year (that would have become 3 or more years old if not culled).	lbs./hd.
$Y_{1,12} = \dfrac{\sum_{i=2}^{14} (i \cdot m_{1,i}) + \sum_{i=1}^{13} (i \cdot m_{2,i})}{(g_{43,5} + m_{2,1})}$	Average age of breeding herd at breeding time in current year. Includes weaned heifers kept for breeding at one year of age.	years of age

OUTPUT FUNCTIONS

	Description	Units
$Y_{1,13} = g_{43,6}$	Number of calves born to beef cows in the current year. This number of calves is comparable to the historical series derived from USDA data on total calf births and dairy cow numbers. This comparison is reported in the output function Y_{11} (test statistics).	million head
$Y_{1,14} = \left(\frac{g_{43,4}}{g_{43,6}}\right)(0.1)$	Number of calves weaned per calf born to beef cows in the current year.	proportion $\frac{\text{calves weaned}}{\text{calves born}}$

OUTPUT FUNCTIONS

Description		Units
j=1,14 $Y_{2,j} = \begin{cases} m_{2,1} & ,\text{if } j = 1 \\ m_{2,j} + m_{1,j} & ,\text{if } 1 < j < 14 \\ m_{1,14} & ,\text{if } j = 14 \end{cases}$	Number of animals becoming (j) years old, in post-culling inventories at beginning of current year. These are totals of pregnant and non-pregnant classes by age groups (for age distribution plots). Used in $Y_{12,j}$	100,000 head
j = 1,15 = age becoming $Y_{3,j} = g_{14,j} = PSV_j$	= Present cull salveage value for animals becoming j years old.	\$/head
y = 2,14 = age becoming $Y_{4,j} = g_{30,j} = V_j^p = \frac{PVB_j^p}{PSV_j}$	= V-ratios for pregnant classes	dimensionless
j = 1,13 = age becoming $Y_{5,j} = g_{32,j} = V_j^N = \frac{PVB_j^N}{PSV_j}$	= V-ratios for non-pregnant classes	dimensionless

OUTPUT FUNCTIONS

Description		Units
$j = 2,14$ = age becoming $*Y_{6,j} = g_{28,j} = PVB_j^P$	Present value for breeding for pregnant animals becoming (j) years old. This is the discounted max present value of future net income expected for pregnant heifers or cows becoming (j) years of age if kept for breeding.	$/head
$j = 1,13$ = age becoming $*Y_{7,j} = \left\{ \begin{array}{ll} g_{28,1} & ,\text{if } j = 1 \\ g_{31,j} & ,\text{if } j > 1 \end{array} \right\} = PVB_j^N$	Present value for breeding for non-pregnant animals becoming (j) years old.	$/head

OUTPUT FUNCTIONS; TEST STATISTICS COMPARING SIMULATED AND HISTORICAL BEEF COW NUMBERS:

January 1 inventory, year $z_{14}+1$

	Description	Units
$Y_{8,1} = \frac{g_{43,1}}{b_{(z_{14}-1849)}}$	Simulated number of beef cows as a proportion of the historical number $(\frac{S_k}{H_k})$ for each year of the run.	dimensionless

NOTE: $Y_{8,2}$ through $Y_{8,7}$ to computed only when z_{14} = 1981, otherwise set to zero.

$*Y_{8,2} = \frac{g_{44,1}}{b_{99}}$	MPAD = mean proportional absolute deviation of simulated (S) cow numbers from historical (H) cow numbers. $= \left(\frac{\sum_{i=1966}^{1982} \left\| \frac{(S_i - H_i)}{H_i} \right\|}{17 \text{ years}} \right)$	dimensionless
$Y_{8,3} = g_{45,11}$	r = correlation coefficient between simulated and historical series of beef cow number changes.	dimensionless
$Y_{8,4} = g_{45,12}$	Theil's U = Inequality coefficient for comparing simulated changes with historical changes in beef cow numbers.	dimensionless

OUTPUT FUNCTIONS: BEEF COW NUMBER STATISTICS (continued)

	Description	Units
$Y_{8,5} = g_{45,13}$	Theil's U^m = proportion of inequality due to mean bias	dimensionless
$Y_{8,6} = g_{45,14}$	Theil's U^s = proportion of inequality due to unequal variance.	dimensionless
$Y_{8,7} = g_{45,15}$	Theil's U^c = proportion of inequality due to imperfect covariation. NOTE: $U^m + U^s + U^c = 1.0$.	dimensionless
$Y_{8,8} = g_{43,1}$	Simulated January 1 inventory of beef cows for year $z_{14}+1$ (for plots)	million head
$Y_{8,9} = b_{(z_{14}-1849)}$	Historical January 1 inventory of beef cows for year $z_{14}+1$ (for plots)	million head

OUTPUT FUNCTIONS: TEST STATISTICS COMPARING SIMULATED AND HISTORICAL HEIFER NUMBERS RECRUITED

January 1 inventory, for year $z_{14}+1$

	Description	Units
$*Y_{9,1} = \frac{g_{43,2}}{b_{(z_{14}-1810)}}$	Simulated number of heifers for replacement as a proportion of historical number. $(\frac{S_k}{H_k})$ for each year of the run.	dimensionless

NOTE: $Y_{9,2}$ through $Y_{9,7}$ to be compuated only when $z_{14} = 1981$, otherwise, set to zero.

$*Y_{9,2} = \frac{g_{44,2}}{b_{99}}$	MPAD = mean proportional absolute deviation of simulated (S) heifer numbers from historical (H) numbers: $$= \left(\frac{\sum_{i=1966}^{1982} \left\| \frac{(S_i - H_i)}{H_i} \right\|}{17 \text{ years}} \right)$$	dimensionless
$Y_{9,3} = g_{46,11}$	r = correlation coefficient between the simulated and historical series of heifer recruitment numbers.	dimensionless
$Y_{9,4} = g_{46,12}$	Theil's U = inequality coefficeint for comparing simulated changes with historical changes in numbers of heifers recruited.	dimensionless

OUTPUT FUNCTIONS: HEIFER RECRUITMENT STATISTICS (continued)

	Description	Units
$Y_{9,5} = g_{46,13}$	Theil's U^m = proportion of inequality due to mean bias	dimensionless
$Y_{9,6} = g_{46,14}$	Theil's U^s = proportion of inequality due to unequal variance	dimensionless
$Y_{9,7} = g_{46,15}$	Theil's U^c = proportion of inequality due to imperfect covariation. NOTE: $U^m + U^s + U^c = 1.0$	dimensionless
$Y_{9,8} = g_{43,2}$ (for plots)	Simulated numbers of recruits for Januaary 1 of year $z_{14}+1$	million head
$*Y_{9,9} = b_{(z_{14}-1810)}$ (for plots)	Historical numbers of recruits for January 1 of year $z_{14}+1$	million head

OUTPUT FUNCTIONS: TEST STATISTICS COMPARING SIMULATED AND HISTORICAL ANNUAL CULL BEEF COW NUMBERS SLAUGHTERED IN THE YEAR z_{14}

	Description	Units
$*Y_{10,1} = \frac{g_{43,3}}{b_{(z_{14}-1772)}}$	Simulated number of cull beef cows as a proportion of the historical number of beef cows slaughtered $(\frac{S_k}{H_k})$ for each year of run.	dimensionless

NOTE: $Y_{10,2}$ through $Y_{10,7}$ to be computed only if $z_{14} = 1981$, otherwise set to zero.

	Description	Units
$*Y_{10,2} = \frac{g_{44,3}}{b_{99}}$	MPAD = Mean proportional absolute deviation of simulated (S) cull cow numbers from historical (H) beef cow slaughter numbers: $= \left(\frac{\sum_{i=1965}^{1981} \left\lvert \frac{S_i - H_i}{H_i} \right\rvert}{17 \text{ years}} \right)$	dimensionless
$Y_{10,3} = g_{47,11}$	r = correlation coefficient between changes in simulated beef cull cow numbers and changes in historical beef cow slaughter numbers	dimensionless
$Y_{10,4} = g_{47,12}$	Theil's U = Inequality coefficient for comparing simulated changes in cull beef cow numbers and historical changes in beef cow slaughter numbers	dimensionless

OUTPUT FUNCTIONS: CULL COW STATISTICS (continued)

	Description	Units
$Y_{10,5} = g_{47,13}$	Theil's U^m = proportion of inequality due to mean bias	dimensionless
$Y_{10,6} = g_{47,14}$	Theil's U^s = proportion of inequality due to unequal variance.	dimensionless
$Y_{10,7} = g_{47,15}$	Theil's U^c = proportion of inequality due to imperfect covariation. NOTE: $U^m + U^s + U^c = 1.0$	dimensionless
$Y_{10,8} = g_{43,3}$ (for plots)	Simulated number of cull beef cows, annual for year z_{14}	million head
$*Y_{10,9} = b_{(z_{14}-1772)}$ (for plots)	Historical numbers of beef cows slaughter, annual for year z_{14}	million head

OUTPUT FUNCTIONS: TEST STATISTICS COMPARING SIMULATED WITH HISTORICAL ANNUAL NUMBERS OF CALVES BORN TO BEEF COWS, YEAR z_{14}

	Description	Units
$*Y_{11,1} = \frac{g_{43,6}}{b_{(z_{14}-1733)}}$	Simulated number of calves born to beef cows as a proportion of derived historical numbers. $(\frac{S_k}{H_k})$ for each year of run.	dimensionless

NOTE: $Y_{11,2}$ through $Y_{11,7}$ are computed only when z_{14} = 1981, otherwise set to zero.

$*Y_{11,2} = \frac{g_{44,4}}{b_{99}}$	MPAD = Mean proportional absolute deviation of simulated (S) calf numbers born to beef cows from derived historical (H) numbers: $= \left(\frac{\sum_{i=1965}^{1981} \left\| \frac{(S_i - H_i)}{H_i} \right\|}{17 \text{ years}} \right)$	dimensionless
$Y_{11,3} = g_{48,11}$	r = correlation coefficient between changes in simulated numbers of calves born to beef cows and changes in derived historical numbers	dimensionless
$Y_{11,4} = g_{48,12}$	Theil's U = Inequality coefficient for comparing simulated changes in numbers of calves born to beef cows and changes in derived historical numbers.	dimensionless

OUTPUT FUNCTIONS: BEEF CALF BIRTH STATISTICS (continued)

Description		Units
$Y_{11,5} = g_{48,13}$	Theil's U^m = proportion of inequality due to mean bias	dimensionless
$Y_{11,6} = g_{48,14}$	Theil's U^s = proportion of inequality due to unequal variance	dimensionless
$Y_{11,7} = g_{48,15}$	Theil's U^c = proportion of inequality due to imperfect covariance NOTE: $U^m + U^s + U^c = 1.0$	dimensionless
$Y_{11,8} = g_{43,6}$ (for plots)	Simulated number of calves born to beef cows, annual for year z_{14}	million head
$*Y_{11,9} = b_{(z_{14}-1733)}$ (for plots)	Derived historical number of calves born to beef cows, annual for year z_{14}	million head

OUTPUT FUNCTIONS (continued)

Description	Units
$j = 1,14$	100,000 head

$$Y_{12,j} = \begin{cases} Y_{2,j} & ,\text{if } j = 1 \\ Y_{12,(j=1)} + Y_{2,j} & ,\text{if } j > 1 \end{cases}$$

Cummulative total of heifers and cows, exposed for breeding in the year z_{14}, by age. $Y_{12,4}$, for example, is the number of cows and heifers four years old and younger exposed for breeding in the year z_{14}. These numbers are used in plotting the age compositions of the simulated herd through time.

PARAMETER LIST FOR COW VALUE AND AGE DISTRIBUTION INVENTORY MODEL

Parameter	Value	Units	Description	Used in these Functions
b_1	.940	prop.	estimate of maximum conception rate	g_1
b_2	.01	dless	linear correction factor in conception rate formula	g_1
b_3	4.83	years	age of cow at which maximum conception rate is expected	g_1
b_4	-.006	dless	parabolic bend coefficient in conception rate formula	g_1
b_5	-.045	prop.	intercept term in impaired health rate formula	g_2
b_6	.25	dless	1/j coefficient in impaired health rate formula	g_2
b_7	.00104367	dless	j^2 coefficient in impaired health rate formula	g_2
b_8	.99	prop	intercept term in survival rate formula	g_3
b_9	-.001	dless	j coefficient in survival rate formula	g_3
b_{10}	.62	dless	proportion of early maturing cows in national beef herd	g_4 g_5
b_{11}	9.75	cwt.	ME: maximum body weight for early maturing cows	g_4
b_{12}	1.33015	dless	intercept term in early maturing cow body weight function	g_4

NOTE: dless indicates dimensionless constant; prop. indicates a proportion

PARAMETER LIST FOR COW VALUE AND AGE DISTRIBUTION INVENTORY MODEL (continued)

Parameter	Value	Units	Description	Used in these Functions
b_{13}	-.0239	dless	j coefficient in early maturing cow body weight function	g_4
b_{14}	-1.1399	dless	1/j coefficient in early maturing cow body weight function	g_4
b_{15}	11.0	cwt.	ML: maximum body weight for late maturing cows	g_4 g_5
b_{16}	.4107	dless	intercept term in late maturing cow body weight function	g_4
b_{17}	.1446	dless	j coefficient in late-maturing cow body weight function	g_4
b_{18}	-.01124	dless	j^2 coefficient in late-maturing cow body weight function	g_4
b_{19}	.0002673	dless	j^3 coefficient in late-maturing cow body weight function	g_4
b_{20}	.43	prop.	max. calf weight as a proportion of cow weight	g_6
b_{21}	.770156	dless	intercept term in calf weaning weight function	g_6
b_{22}	.0678788	dless	j coefficient in calf weaning weight function	g_6
b_{23}	-.00642507	dless	j^2 coefficient in calf weaning weight function	g_6
b_{24}	.000187646	dless	j^3 coefficient in calf weaning weight function	g_6
b_{25}	.42	prop.	HKB weight as a proportion of max. aggregate cow body weight	g_7

NOTE: dless indicates a dimensionless constant; prop. indicates a proportion.

PARAMETER LIST FOR COW VALUE AND AGE DISTRIBUTION INVENTORY MODEL (continued)

Parameter	Value	Units	Description	Used in these Functions
b_{26}	.975463	prop.	calf survival rate intercept	g_8
b_{27}	-.00184144	dless	j coefficient in calf survival rate function	g_8
b_{28}	-.184799	dless	j^2 coefficient in calf survival rate function	g_8
b_{29}	.65	dless	intercept term in yearlings exposed for breeding function	g_{33}
b_{30}	.009	dless	linear coefficient in yearlings exposed for breeding function	g_{33}
b_{31}	-1.5218	dless	intercept term in the relationship between heifer and steer prices	g_{13} g_{14}
b_{32}	---		unassigned	
b_{33}	1.0	dless	scaling multiplier for price difference between feeder steers and non-pregnant cull cows	g_{25}
b_{34}	1976	dless	year of the shift in cattlemens retainment decisions	---
b_{35}	1.0	dless	scaling multiplier for price difference between feeder steers and pregnant cull cows	g_{14}
b_{36}	zero	dless	optional constant "real" interest rate for inflating cost budgets	g_{15}
b_{37}	zero	dless	optional constant "real" discount rate for present value calculations	g_{26}
b_{38}	.989	dless	ratio of heifer and steer average price to choice feeder steers	g_{11} g_{24}

NOTE: dless indicates dimensionless constant; prop. indicates a proportion.

PARAMETER LIST FOR COW VALUE AND AGE DISTRIBUTION INVENTORY MODEL (continued)

Parameter	Value	Units	Description	Used in these Functions
b_{39}	.976475	dless	linear coefficient in the heifer price function	g_{14}
b_{40}	1.2	dless	scaling multiplier for price difference between feeder steers and cull cows	g_{13}
b_{41}	1.15	dless	hyperbolic age factor for price difference between feeder steers and cull cows	g_{13} g_{14} g_{25}
b_{42}	1.0	dless	interest rate multiplier for adjusting P.C.A. interest rates for short term operating loans	g_{15}
b_{43}	0.5	years	exponential term in interest factor: represents fraction of year for which interest is charged	g_{15}
b_{44}	2.83	$/hd.	Base year (1978) marketing and hauling cost/hd. for all classes	g_{16}
b_{45}	6.76	$/hd.	Base year (1978) fuel, lube, & elec. cost/hd. for all classes	g_{16}
b_{46}	9.22	$/hd.	Base year (1978) mach. and bldg. repair cost/hd. for all classes	g_{16}
b_{47}	10.00	$/hd.	Base year (1978) bull charges cost/hd. for all classes	g_{16}
b_{48}	6.71	$/hd.	Base year (1978) pasture rental cost/hd. for weaned heifers (HKB)	g_{17}
b_{49}	24.19	R/hd.	Base year (1978) hay cost/hd. for weaned heifers (HKB)	g_{17}

NOTE: dless indicates dimensionless constant.

PARAMETER LIST FOR COW VALUE AND AGE DISTRIBUTION INVENTORY MODEL (continued)

Parameter	Value	Units	Description	Used in these Functions
b_{50}	4.68	$/hd.	Base year (1978) Grain & concentrate cost/hd. for weaned heifers	g_{17}
b_{51}	.32	$/hd.	Base year (1978) Protein supplement cost/hd. for weaned heifers	g_{17}
b_{52}	1.60	$/hd.	Base year (1978) Salt and mineral cost/hd. for weaned heifers	g_{17}
b_{53}	13.45	$/hd.	Base year (1978) Labor cost/hd. for weaned heifers	g_{17}
b_{54}	1.63	$/hd.	Base year (1978) Veterinary & medicine cost/hd. for weaned heifers	g_{17}
b_{55}	8.50	$/hd.	Base year (1978) Pasture rental cost/hd. for yearling heifers (pregnant or not)	g_{18}
b_{56}	30.65	$/hd.	Base year (1978) Hay cost/hd. for yearling heifers (pregnant or not)	g_{18}
b_{57}	5.93	$/hd.	Base year (1978) Grain & concentrate cost/hd. for yearling heifers (pregnant or not)	g_{18}
b_{58}	.40	$/hd.	Base year (1978) Protein supplement cost/hd. for yearling heifers (pregnant or not)	g_{18}
b_{59}	2.03	$/hd.	Base Year (1978) Salt & minerals cost/hd. for yearling heifers (pregnant or not)	g_{18}
b_{60}	39.54	$/hd.	Base year (1978) Labor cost/hd. for pregnant yearling heifers	g_{19}
b_{61}	4.80	$/hd.	Base year (1978) Veterinary & medicine cost/hd. for pregant yearling heifers	g_{19}

PARAMETER LIST FOR COW VALUE AND AGE DISTRIBUTION INVENTORY MODEL (continued)

Parameter	Value	Units	Description	Used in these Functions
b_{62}	13.45	$/hd.	Base year (1978) Labor cost/hd. for non-pregnant yearling heifers	g_{20}
b_{63}	1.63	$/hd.	Base year (1978) Veterinary & medicine cost/hd. for non-pregant yearling heifers	g_{20}
b_{64}	8.94	$/hd.	Base year (1978) Pasture rental cost/hd. for mature cows (pregnant or not)	g_{21}
b_{65}	32.25	$/hd.	Base year (1978) Hay cost/hd. for mature cows (pregnant or not)	g_{21}
b_{66}	6.24	$/hd.	Base year (1978) Grain & concentrate cost/hd. for mature cows (pregnant or not)	g_{21}
b_{67}	.42	$/hd.	Base year (1978) Protein supplement cost/hd. for mature cows (pregnant or not)	g_{21}
b_{68}	2.14	$/hd.	Base year (1978) Salt and minerals cost/hd. for mature cows (pregnant or not)	g_{21}
b_{69}	27.54	$/hd.	Base Year (1978) Labor cost/hd. for pregnant mature cows	g_{22}
b_{70}	3.35	$/hd.	Base year (1978) Veterinary & medicine cost/hd. for pregnant mature cows	g_{22}
b_{71}	13.45	$/hd.	Base year (1978) Labor cost/hd. for non-pregnant mature cows	g_{23}
b_{72}	1.63	$/hd.	Base year (1978) Veterinary & medicine cost/hd. for non-pregnant mature cows	g_{23}

PARAMETER LIST FOR COW VALUE AND AGE DISTRIBUTION INVENTORY MODEL (continued)

Parameter	Value	Units	Description	Used in these Functions
b_{73}	.27	dless	weight of previous year's feeder steer price in expected feeder price model	
b_{74}	.73	dless	weight of current year's feeder steer price in expected feeder price model	$g_{12,1}$
b_{75}	.27	dless	weight of previous year's utility cow price in expected utility price model	
b_{76}	.73	dless	weight of current year's utility cow price in expected utility price model	$g_{12,2}$
b_{77}	.50	prop.	proportion of cull cows still on inventory after January 1	$g_{43,1}$
b_{78}	---	-----	Unassigned	
b_{79}	---	-----	Unassigned	
b_{80}	1.0	dless	multiplier for adjusting P.C.A. interest rate in the discount terms used in PVB calculations	$g_{26,1}$
b_{81}	---	-----	Unassigned	
b_{82}	---	-----	Unassigned	

NOTE: dless indicates a dimensionless constant; prop. indicates a proportion

PARAMETER LIST FOR COW VALUE AND AGE DISTRIBUTION INVENTORY MODEL (continued)

Parameter	Value	Units	Description	Used in these Functions
b_{83}	-5.5	dless	exponential v-ratio factor in retainment function for pregnant cows	g_{37}
b_{84}	.53	dless	critical v-ratio (inflection) in retainment function for pregnant cows	
b_{85}	-5.5	dless	exponential v-ratio factor in retainment function for open cows	g_{38}
b_{86}	.535	dless	critical v-ratio (inflection) in retainment function for open cows	
b_{87}	.5	dless	fraction of weaned heifers not kept for breeding which are possibly available the following year for recruitment for breeding	$f_{1,1}$
b_{88}	0	prop.	minimum proportion of pregnant cows to be retained	g_{37}
b_{89}	.20	prop.	minimum proportion of weaned and non-pregnant yearling heifers to be retained	
b_{90}	0	prop.	minimum proportion of non-pregnant cows allowed to be retained	g_{38}
b_{91}	1.0	prop.	maximum proportion of healthy non-pregnant cows to be retained	
b_{92}	-5.5	dless	exponential v-ratio factor in retainment function for weaned and non-pregnant yearling heifers	

NOTE: dless indicates a dimensionless constant; prop. indicates proportion

PARAMETER LIST FOR COW VALUE AND AGE DISTRIBUTION INVENTORY MODEL (continued)

Para-meter	Value	Units	Description	Used in these Functions
b_{93}	1.1	dless	critical v-ratio (inflection) in retainment function for weaned and non-pregant yearling heifers	g_{38}
b_{94}	.80	prop.	maximum proportion of healthy weaned heifers allowed to be kept for breeding	
b_{95}	1.0	prop.	proportion of pregnant yearling heifers counted in sum of heifers recruited	$g_{43,2}$
b_{96}	.40	prop.	proportion of weaned heifers kept for breeding counted in sum of heifers recruited	$g_{43,2}$
b_{97}	0	prop.	proportion of non-pregnant yearling heifers counted in sum of heifers recruited	$g_{43,2}$
b_{98}	0	prop.	proportion of pregnant yearling heifers included in beef cow herd inventory	$g_{43,1}$
b_{99}	17	years	number of years in a simulation run (1965-1981)	$y_{i,2}$ i=8,9,10,11
b_{100}	16	years	number of periods for which proportional changes are computed in a simulation run, for statistical comparison of simulated and historical series	$g_{4i,j}$ i=5,6,7,8 j=9,10,11, 12,13,14, 15

NOTE: dless indicates a dimensionless constant; prop. indicates proportion

PARAMETER LIST FOR COW VALUE AND AGE DISTRIBUTION INVENTORY MODEL (continued)

Para-meter	Value	Units	Description	Used in these Functions
b_{378}	1.54	dless	scaling multiplier for price difference between feeder steers and cull cows after 1976 (non-pregnant)	g_{14}
b_{379}	1.01	dless	scaling multiplier for price difference between feeder steers and cull cows after 1976 (pregnant)	g_{25}

NOTE: dless indicates a dimensionless constant

TEST PARAMETERS: HISTORICAL SERIES OF U.S. BEEF COW NUMBERS

Parameter	Value	Units	Description		Used in these Functions
b_{101}	17.545	million head	January 1, 1951	USDA estimated inventory of beef cows on farms Source: USDA data file named "COWSNBE" (USDA, ESS, T-DAM, 1979) and various issues of Livestock and Meat Statistics. Used in test statistics and for plotting against the model's post-culling inventory of cows becoming 3 years old or older in the previous year	$g_{44,1}$ $g_{45,2}$
b_{102}	19.975	"	January 1, 1952		
b_{103}	22.490	"	January 1, 1953		
b_{104}	24.285	"	January 1, 1954		
b_{105}	24.920	"	January 1, 1955		
b_{106}	24.700	"	January 1, 1956		
b_{107}	23.895	"	January 1, 1957		
b_{108}	23.530	"	January 1, 1958		
b_{109}	24.460	"	January 1, 1959		
b_{110}	25.675	"	January 1, 1960		
b_{111}	26.655	"	January 1, 1961		
b_{112}	27.996	"	January 1, 1962		
b_{113}	29.829	"	January 1, 1963		
b_{114}	31.908	"	January 1, 1964		
b_{115}	33.400	"	January 1, 1965		
b_{116}	33.500	"	January 1, 1966		
b_{117}	33.770	"	January 1, 1967		
b_{118}	34.570	"	January 1, 1968		
b_{119}	35.490	"	January 1, 1969		
b_{120}	36.689	"	January 1, 1970		
b_{121}	37.878	"	January 1, 1971		
b_{122}	38.810	"	January 1, 1972		
b_{123}	40.932	"	January 1, 1973		
b_{124}	43.182	"	January 1, 1974		
b_{125}	45.712	"	January 1, 1975		
b_{126}	43.901	"	January 1, 1976		
b_{127}	41.443	"	January 1, 1977		
b_{128}	38.738	"	January 1, 1978		
b_{129}	37.062	"	January 1, 1979		

TEST PARAMETERS: HISTORICAL SERIES OF U.S. BEEF COW NUMBERS (continued)

Parameter	Value	Units	Description		Used in these Functions
b_{130}	37.086	million head	January 1, 1980	USDA estimated inventory of beef cows on farms Source: USDA data file named "COWSNBE" (USDA, ESS, T-DAM, 1979) and various issues of Livestock and Meat Statistics. Used in test statistics and for plotting against the model's post-culling inventory of cows becoming 3 years old or older in the previous year.	$g_{44,1}$ $g_{45,2}$
b_{131}	38.726	"	January 1, 1981		
b_{132}	39.364	"	January 1, 1982		
b_{133} – b_{139}	Unassigned				

TEST PARAMETERS: HISTORICAL SERIES ON U.S. BEEF HEIFER NUMBERS FOR BREEDING

Parameter	Value	Units	Description		Used in these Functions
b_{140}	4.246	million	January 1, 1951	USDA estimated inventory of beef heifers for breeding. Source: USDA data file named "HEISBBE" (USDA ESS, T-DAM, 1979) and various issues of <u>Livestock and Meat Statistics</u>. Used in test statistics and for plotting against the model's total post-culling inventory of heifers recruited to the breeding herd in the previous year.	$g_{44,2}$, $g_{46,2}$
b_{141}	5.435	head	January 1, 1952		
b_{142}	6.780	"	January 1, 1953		
b_{143}	5.740	"	January 1, 1954		
b_{144}	5.320	"	January 1, 1955		
b_{145}	4.716	"	January 1, 1956		
b_{146}	4.587	"	January 1, 1957		
b_{147}	3.507	"	January 1, 1958		
b_{148}	3.281	"	January 1, 1959		
b_{149}	4.124	"	January 1, 1960		
b_{150}	3.838	"	January 1, 1961		
b_{151}	4.457	"	January 1, 1962		
b_{152}	4.511	"	January 1, 1963		
b_{153}	5.409	"	January 1, 1964		
b_{154}	5.397	"	January 1, 1965		
b_{155}	5.337	"	January 1, 1966		
b_{156}	5.351	"	January 1, 1967		
b_{157}	5.710	"	January 1, 1968		
b_{158}	6.320	"	January 1, 1969		
b_{159}	5.768	"	January 1, 1970		
b_{160}	5.864	"	January 1, 1971		
b_{161}	6.675	"	January 1, 1972		
b_{162}	6.901	"	January 1, 1973		
b_{163}	8.692	"	January 1, 1974		
b_{164}	8.276	"	January 1, 1975		
b_{165}	6.793	"	January 1, 1976		
b_{166}	5.774	"	January 1, 1977		
b_{167}	5.349	"	January 1, 1978		
b_{168}	4.541	"	January 1, 1979		

TEST PARAMETERS: HISTORICAL SERIES ON U.S. BEEF HEIFER NUMBERS FOR BREEDING (continued)

Parameter	Value	Units		Description	Used in these Functions
b_{169}	6.518	million head	January 1, 1980	USDA estimated inventory of beef heifers for breeding Source: USDA data file named "HEISBBE" (USDA ESS, T-DAM, 1979) and various issues of Livestock and Meat Statistics. Used in test statistics and for plotting against the model's total post-culling inventory of heifers recruited to the breeding herd in the previous year	$g_{44,2}$ $g_{46,2}$
b_{170}	5.781	"	January 1, 1981		
b_{171}	5.700	"	January 1, 1982		
b_{172}				Unassigned	
b_{173}				Unassigned	
b_{174}				Unassigned	
b_{175}				Unassigned	
b_{176}				Unassigned	
b_{177}				Unassigned	

TEST PARAMETERS: HISTORICAL SERIES ON U.S. BEEF COW SLAUGHTER

Parameter	Value	Units	Description		Used in these Functions
b_{178}	2.204	million head	1950	USDA estimate of non-fed beef cow slaughter	
b_{179}	1.465		1951		
b_{180}	2.521	"	1952	Source: USDA data file named "COWKSNF" (USDA, ESS, T-DAM, 1979) and various issues of Livestock and Meat Statistics. Used in test statistics and for plotting against the model's total number of cows culled as becoming 3 years of age and older.	
b_{181}	4.535	"	1953		
b_{182}	4.619	"	1954		
b_{183}	5.042	"	1955		
b_{184}	5.027	"	1956		
b_{185}	4.474	"	1957		
b_{186}	2.106	"	1958		
b_{187}	1.577	"	1959		
b_{188}	2.631	"	1960		
b_{189}	1.964	"	1961		$g_{44,3}$
b_{190}	2.064	"	1962		
b_{191}	1.835	"	1963		$g_{47,2}$
b_{192}	3.279	"	1964		
b_{193}	4.629	"	1965		
b_{194}	4.397	"	1966		
b_{195}	3.876	"	1967		
b_{196}	4.099	"	1968		
b_{197}	4.411	"	1969		
b_{198}	3.845	"	1970		
b_{199}	4.174	"	1971		
b_{200}	3.777	"	1972		
b_{201}	3.832	"	1973		
b_{202}	5.298	"	1974		
b_{203}	9.186	"	1975		
b_{204}	8.414	"	1976		
b_{205}	7.650	"	1977		

TEST PARAMETERS: HISTORICAL SERIES ON U.S. BEEF COW SLAUGHTER (continued)

Parameter	Value	Units		Description	Used in these Functions
b_{206}	6.254	million head	1978	USDA estimate of non-fed beef cow slaughter Source: USDA data file named "COWKSNF" (USDA, ESS, T-DAM, 1979) and various issues of Livestock and Meat Statistics. Used in test statistics and for plotting against the model's total number of cows culled as becoming 3 years of age and older.	$g_{44,3}$ $g_{47,2}$
b_{207}	3.776		1979		
b_{208}	4.136	"	1980		
b_{209}	4.368	"	1981		

b_{210}, b_{211}, b_{212}, b_{213}, b_{214}, b_{215}, b_{216} — Unassigned

TEST PARAMETERS: DERIVED HISTORICAL SERIES ON CALVES BORN TO BEEF COWS IN THE U.S.

Parameter	Value	Units		Description	Used in these Functions
b_{217}	15.40	million head	1950	Estimate of number of calves born to beef cows in U.S., derived as the residual obtained by subtracting (.88 x dairy cow numbers) from total calves born in the U.S. annually. Source: USDA data file named "COWSNMC" and "CALSC", for dairy cow numbers and total calves born, respecitvely (USDA, ESS, T-DAM, 1979) and various issues of Livestock and Meat Statistics. Used in test statistics and for plotting against the model's total number of beef calves born.	$g_{44,4}$, $g_{48,2}$
b_{218}	16.36		1951		
b_{219}	19.40	"	1952		
b_{220}	22.10	"	1953		
b_{221}	23.24	"	1954		
b_{222}	23.14	"	1955		
b_{223}	22.93	"	1956		
b_{224}	21.90	"	1957		
b_{225}	21.71	"	1958		
b_{226}	22.72	"	1959		
b_{227}	23.70	"	1960		
b_{228}	24.70	"	1961		
b_{229}	26.23	"	1962		
b_{230}	27.52	"	1963		
b_{231}	29.60	"	1964		
b_{232}	30.53	"	1965		
b_{233}	31.20	"	1966		
b_{234}	31.86	"	1967		
b_{235}	32.91	"	1968		
b_{236}	34.26	"	1969		
b_{237}	35.35	"	1970		
b_{238}	36.39	"	1971		
b_{239}	37.44	"	1972		
b_{240}	39.08	"	1973		
b_{241}	41.05	"	1974		
b_{242}	40.42	"	1975		
b_{243}	37.75	"	1976		
b_{244}	36.36	"	1977		

TEST PARAMETERS: DERIVED HISTORICAL SERIES ON CALVES BORN TO BEEF COWS IN THE U.S. (continued)

Parameter	Value	Units		Description	Used in these Functions
b_{245}	34.34	million	1978	Estimate of number of calves born to beef cows in U.S., derived as the residual obtained by subtracting (.88 x dairy cow numbers) from total calves born in the U.S. annually.	$g_{44,4}$
b_{246}	33.22	head	1979		$g_{48,2}$
b_{247}	35.62	"	1980	Source: USDA data file named "COWSNMC" and "CALSC", for dairy cow numbers and total calves born, respectively (USDA, ESS, T-DAM, 1979) and various issues of Livestock and Meat Statistics. Used in test statistics and for plotting against the model's total number of beef calves born	
b_{248}	35.72	"	1981		
b_{249}	35.05	"	1982		

b_{250}, b_{251}, b_{252}, b_{253}, b_{254}: Unassigned

APPENDIX C

Computer Program

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      SUBROUTINE ZCOMP
      COMMON/GLOBAL/IFLAG(30)
      COMMON/RNPROC/IDRUN(2),ITIME,MTV(12)
      COMMON/MODULE/JD1,JX,JY,JZ,JB,JF,JG,JH,JD2,JJ,K,KP,KPP,
     + IPCR(187),VAPD(66),X(2,15),XU(2,15),Y(12,15),Z(1,15),B(395),
     + F(2,15),G(48,15)
C*
      DIMENSION ZINIT(14,33)
C*
      IF (MTV(1).NE.IFLAG(4)) GO TO 50
      I5=IFLAG(5)
      DO 3 J=1,I5
      READ(9,900) (ZINIT(I,J),I=1,13),IZ14
      ZINIT(14,J) = FLOAT(IZ14)
3     CONTINUE
 900  FORMAT (2F8.4,11F7.4,2X,I4)
      JPJ=1
C*
      B(376) = B(10) * B(11) + (1. - B(10)) * B(15)
      B(377) = B(376) * B(25)
      DO 30 J=1,15
      AJ = FLOAT(J)
      IF (J.GT.1) GO TO 20
      B(256+J) = B(1)+B(2)*(AJ - B(3))+B(4)*(AJ - B(3))*(AJ - B(3))
      B(270+J) = 1.0 - (B(5)+B(6)/AJ+B(7) * AJ * AJ)
      GO TO 30
C*
20    CONTINUE
      IF (J.NE.15) B(256+J) = B(1) + B(2) * (AJ - B(3)) +
     1  B(4) * (AJ - B(3)) * (AJ - B(3))
      B(270+J) = 1.0 - (B(5) + B(6)/AJ + B(7) * AJ * AJ)
      B(284+J) = B(8) + B(9) * AJ
      B(300+J) = B(10) * B(11) * (B(12) + B(13)*AJ + B(14)/AJ) +
     1  (1.0 - B(10)) * B(15) * (B(16) + B(17) * AJ + B(18) * AJ * AJ +
     2  B(19) * AJ * AJ * AJ)
      IF (J.NE.15) B(315+J) = B(376) * B(20) * (B(21) + B(22) * AJ +
     1 B(23) * AJ * AJ + B(24) * AJ * AJ * AJ)
      IF (J.NE.15) B(329+J) = B(26) + B(27) * AJ + B(28)/AJ
C*
30    CONTINUE
C*
      DO 35 J=1,15
35    B(344+J) = B(79) - B(255+J) - B(270+J)
      DO 40 J=1,13
40    B(360+J)= (B(257+J) + B(271+J) + B(285+J) -B(79))*B(286+J)
C*
50    DO 60 I=1,14
      WRITE(12,801) I,ZINIT(I,JPJ)
801   FORMAT(2X,*THIS IS Z(1,*,I2,*) = *,F20.10)
60    Z(1,I)=ZINIT(I,JPJ)
      JPJ=JPJ+1
C*
      RETURN
      END
```

```
      SUBROUTINF GCOMP
      COMMON/GLOBAL/IFLAG(30)
      COMMON/RNPROC/IDRUN(2),ITIME,MTV(12)
      COMMON/MODULE/JD1,JX,JY,JZ,JB,JF,JG,JH,JD2,JJ,K,KP,KPP,
     + IPCR(187),VARD(66),X(2,15),XU(2,15),Y(12,15),Z(1,15),B(395),
     + F(2,15),G(48,15)
C*
      IZ14 = IFIX(Z(1,14))
      DO 10 J=1,JJ
      IF (J.NE.15) G(1,J) = B(256+J)
      G(2,J) = B(270+J)
      IF (J.NE.1) G(3,J) = B(284+J)
      IF (J.NE.1) G(4,J) = B(300+J)
      IF (J.NE.1.AND.J.NE.15) G(6,J) = B(315+J)
      IF (J.NE.1.AND.J.NE.15) G(8,J) = B(329+J)
      IF (J.NE.15) G(9,J) = B(344+J)
      IF (J.NE.15) G(10,J) = B(360+J)
10    CONTINUE
      G(5) = B(376)
      G(7) = B(377)
      G(12,1) = B(73)*ZM(1,1,1) + B(74)*Z(1,1)
      G(12,2) = B(75)*ZM(1,1,2) + B(76)*Z(1,2)
      DO 30 J=1,15
      IF (J.NE.1) G(11,J) = B(329+J) * B(315+J) * G(12,1)*B(38)
      IF (J.EQ.1) G(13,J) = ((G(12,1)*B(39))+B(31))*G(7)
      IF (J.NE.1) G(13,J) = G(4,J)*(G(12,1)-B(40)*(G(12,1)-G(12,2))+
     1  B(40)*(G(12,1)-G(12,2))/(FLOAT(J)*B(41)))
      IF (J.EQ.1) G(14,J) = (Z(1,1) * B(39) + B(31)) * G(7)
      IF (J.NE.1) G(14,J)=G(4,J)* (Z(1,1)-B(35)*(Z(1,1) - Z(1,2)) +
     1  B(35) * (Z(1,1) - Z(1,2))/(FLOAT(J) * B(41)))
      IF (J.NE.1) G(25,J)=G(4,J)*(Z(1,1)-B(33)*(Z(1,1)-Z(1,2)) +
     1  B(33) * (Z(1,1) - Z(1,2))/(FLOAT(J) * B(41)))
      IF(Z(14).LT.B(34)) GOTO 27
      IF (J.NE.1) G(14,J)=G(4,J)*(Z(1,1)-B(378)*(Z(1,1) - Z(1,2)) +
     1  B(378) * (Z(1,1) - Z(1,2))/(FLOAT(J) * B(41)))
      IF (J.NE.1) G(25,J)=G(4,J)*(Z(1,1)-B(379)*(Z(1,1)-Z(1,2)) +
     1  B(379) * (Z(1,1) - Z(1,2))/(FLOAT(J) * B(41)))
   27 IF (J.NE.15) G(24,J) = B(360+J) * B(330+J) * B(316+J)
     1  * G(12,1) * B(38)
30    CONTINUE
      G(15) = (1.0 + B(42) * Z(1,13) + B(36)) ** B(43)
      G(16) = B(44) * Z(1,11) + B(45) * Z(1,3) + B(46) * Z(1,4) +
     1  B(47) * Z(1,5)
      G(17) = (G(16)+B(48)*Z(1,6) + B(43) * Z(1,7) + B(50) * Z(1,8) +
     1  B(51) * Z(1,9) + B(52) * Z(1,10) + B(53) * Z(1,11) +
     2  B(54) * Z(1,12)) * G(15)
      G(18) = B(55) * Z(1,6) + B(56) * Z(1,7) + B(57) * Z(1,8) +
     1  B(58) * Z(1,9) + B(59) * Z(1,10)
      G(19) = (G(16) + G(18)+B(60) * Z(1,11)+B(61) * Z(1,12)) * G(15)
      G(20) = (G(16) + G(18)+B(62) * Z(1,11)+B(63) * Z(1,12)) * G(15)
      G(21) = B(64) * Z(1,6)+B(65) * Z(1,7)+B(66) * Z(1,8) +
     1  B(67) * Z(1,9)+B(68) * Z(1,10)
      G(22) = (G(16) + G(21)+B(69) * Z(1,11)+B(70) * Z(1,12)) * G(15)
      G(23) = (G(16) + G(21)+B(71) * Z(1,11) + B(72) * Z(1,12)) * G(15)
40    CONTINUE
      G(26,1) = 1.0/(1.0 + B(80) * Z(1,13) + B(37))
      G(26,2) = G(25,1) * G(25,1)
      G(28,1) = ((G(9,1) * G(13,2)) - G(17)) * G(26,1) +
     1  ((G(10,1) * G(13,3)) - G(19)) * G(26,2) + G(24,1) * G(26,2)
      G(28,2) = ((G(9,2)*G(13,3))-G(19))*G(26,1)+
     1  ((G(10,2) * G(13,4)) - G(22)) * G(26,2) +
     2  G(11,2) * G(26,1) + G(24,2) * G(26,2)
45    CONTINUE
      DO 50 J= 3,14
      JKK = J + 1
      G(28,J)=((G(9,J)*G(13,JKK))-G(22))*G(26,1)
     1  +((G(10,J) * G(13,JKK+1)) - G(22)) * G(26,2)
     2  + G(11,J) * G(26,1) + G(24,J) * G(26,2)
50    CONTINUE
      DO 60 J=1,13
      JKK = J + 1
      G(30,JKK) = G(28,JKK)/G(25,JKK)
      IF (J.NE.1) G(31,J) =G(28,J)-G(11,J)*G(26,1) +(G(22)-G(23))*G(26,1)
      IF (J.EQ.1) G(32,J) = G(28,1)/G(14,1)
      IF (J.GT.1) G(32,J) = G(31,J)/G(14,J)
      G(33) = B(29) + B(30) * (IZ14 - 1965)
      IF (JKK.EQ.2) G(34,JKK) = X(2,1) * G(3,2) * G(1,1) * G(33)
```

```
60    IF(JKK.GT.2)G(34,JKK)=(X(1,JKK-1)+X(2,JKK-1))*G(3,JKK)*G(1,JKK-1)
      CONTINUE
      DO 70 J=1,14
      IF (J.NE.1) GO TO 62
      G(35,J) = 0.0
      DO 61 I=2,14
      G(35,J) = G(35,J) + X(1,I) * G(8,I)
61    CONTINUE
      G(35,J) = .5 * G(35,J)
      GO TO 65
62    IF (J.NE.2) GO TO 64
      G(35,J)=X(2,1) * G(3,2) * (1.0-G(1,1)) + X(1,1)
      G(36) = X(2,1)*G(3,2)*(1-G(33))
      GO TO 65
64    G(35,J) = (X(1,J-1) + X(2,J-1)) * G(3,J) * (1.0 - G(1,J-1))
65    CONTINUE
      IF (J.NE.1) G(37,J) = B(88) + (G(2,J) - B(88))/(1.0 + EXP(B(83) *
     1 (G(30,J) - B(84))))
      IF (J.LT.3) G(38,J)=B(89)+(B(94)*G(2,J)-B(89))/(1.0+EXP(B(92)*
     1 (G(32,J) - B(93))))
      IF (J.GT.2.AND.J.NE.14) G(38,J) = B(90)+(B(91)*G(2,J)-B(90))/
     1 (1.0 + EXP(B(85) * (G(32,J) - B(86))))
      IF (J.NE.1) G(39,J) = G(34,J) * G(37,J)
      IF (J.EQ.2) G(40,J)=(G(35,2) * G(38,2)) + G(36,1)
      IF (J.NE.14.AND.J.NE.2) G(40,J) = G(35,J) * G(38,J)
70    CONTINUE
      DO 71 J=1,14
      JKK = J + 1
      IF (JKK.LT.15) G(41,JKK) = G(34,JKK) - G(39,JKK)
      IF (JKK.EQ.15) G(41,JKK) = X(1,14) * G(3,15)
      IF (J.LT.14) G(42,J) = G(35,J) - G(40,J)
      IF (J.EQ.14) G(42,J) = G(35,14)
71    CONTINUE
      DO 72 J=1,JJ
72    G(43,J) = 0.0
      G(43,1)=B(98)*G(39,2)*0.1
      DO 75 I=2,15
      IF (I.GE.3.AND.I.LE.14) G(43,3) = G(43,3) + (G(41,I)+G(42,I))*0.1
      IF (I.EQ.15) G(43,3) = G(43,3) + G(41,I) *0.1
      IF (I.GE.3.AND.I.LE.13) G(43,1) = G(43,1)+(G(39,I)+G(40,I))*0.1
      IF (I.EQ.14) G(43,1) = G(43,1) + G(39,I) *0.1
      IF(I.EQ.15) G(43,1)=G(43,1)+B(77)*G(43,3)
      IF (I.GE.2.AND.I.LE.14) G(43,4) = G(43,4) + X(1,I) * G(8,I)
      IF (I.GE.2.AND.I.LE.13) G(43,5) = G(43,5) + X(1,I) + X(2,I)
      IF (I.EQ.14) G(43,5) = G(43,5) + X(2,I)
      IF (I.GE.2.AND.I.LE.14) G(43,6) = G(43,6) + X(1,I)*G(3,I+1)*0.1
75    CONTINUE
      G(43,2) = (B(95)*G(39,2) + B(96)*G(40,1) + B(97)*G(40,2)) *0.1
      G(44,1) = GM(44,1,1) + ABS(G(43,1) - B(IZ14-1849)) /
     1 B(IZ14-1849)
      G(44,2) = GM(44,1,2) + ABS(G(43,2) - B(IZ14-1810)) /
     1 B(IZ14-1810)
      G(44,3) = GM(44,1,3) + ABS(G(43,3) - B(IZ14-1772)) /
     1 B(IZ14-1772)
      G(44,4) = GM(44,1,4) + ABS(G(43,6) - B(IZ14-1733)) /
     1 B(IZ14-1733)
      IF (MTV(1).EQ.IFLAG(4)) GO TO 100
C
      G(45,1) = (G(43,1) - GM(43,1,1))/GM(43,1,1)
      G(45,2) = (B(IZ14-1849) - B(IZ14-1850))/B(IZ14-1850)
      G(45,3) = GM(45,1,3) + G(45,1)
      G(45,4) = GM(45,1,4) + G(45,1) * G(45,1)
      G(45,5) = GM(45,1,5) + G(45,2)
      G(45,6) = GM(45,1,6) + G(45,2) * G(45,2)
      G(45,7) = GM(45,1,7) + G(45,1) * G(45,2)
      G(45,8) = GM(45,1,8) + (G(45,1) - G(45,2)) * (G(45,1) - G(45,2))
C
      G(46,1) = (G(43,2) - GM(43,1,2))/GM(43,1,2)
      G(46,2) = (B(IZ14-1810) - B(IZ14-1811))/B(IZ14-1811)
      G(46,3) = GM(46,1,3) + G(46,1)
      G(46,4) = GM(46,1,4) + G(46,1) * G(46,1)
      G(46,5) = GM(46,1,5) + G(46,2)
      G(46,6) = GM(46,1,6) + G(46,2) * G(46,2)
      G(46,7) = GM(46,1,7) + G(46,1) * G(46,2)
      G(46,8) = GM(46,1,8) + (G(46,1) - G(46,2)) * (G(46,1) - G(46,2))
C
      G(47,1) = (G(43,3) - GM(43,1,3))/GM(43,1,3)
```

```
      G(47,2)  = (B(IZ14-1772) - B(IZ14-1773))/B(IZ14-1773).
      G(47,3)  = GM(47,1,3) + G(47,1)
      G(47,4)  = GM(47,1,4) + G(47,1)  * G(47,1)
      G(47,5)  = GM(47,1,5) + G(47,2)
      G(47,6)  = GM(47,1,6) + G(47,2)  * G(47,2)
      G(47,7)  = GM(47,1,7) + G(47,1)  * G(47,2)
      G(47,8)  = GM(47,1,8) + (G(47,1) - G(47,2)) * (G(47,1) - G(47,2))
C
      G(48,1)  = (G(43,6) - GM(43,1,6))/GM(43,1,6)
      G(48,2)  = (B(IZ14-1733) - B(IZ14-1734))/B(IZ14-1734)
      G(48,3)  = GM(48,1,3) + G(48,1)
      G(48,4)  = GM(48,1,4) + G(48,1)  * G(48,1)
      G(48,5)  = GM(48,1,5) + G(48,2)
      G(48,6)  = GM(48,1,6) + G(48,2)  * G(48,2)
      G(48,7)  = GM(48,1,7) + G(48,1)  * G(48,2)
      G(48,8)  = GM(48,1,8) + (G(48,1) - G(48,2)) * (G(48,1) - G(48,2))
C
      IF ((MTV(1)+1).NE.IFLAG(5)) GO TO 100
C
      G(45,9)  = SQRT(B(100) * G(45,4) - G(45,3) * G(45,3))/B(100)
      G(45,10) = SQRT(B(100) * G(45,6) - G(45,5) * G(45,5))/B(100)
      G(45,11) = (B(100) * G(45,7) - G(45,3) * G(45,5)) /
     1  (B(100) * B(100) * G(45,9) * G(45,10))
      G(45,12) = SQRT(G(45,8)/G(45,6))
      G(45,13) = (G(45,3)-G(45,5))*(G(45,3)-G(45,5))/(B(100) * G(45,8))
      G(45,14) = B(100)*(G(45,9)-G(45,10))*(G(45,9)-G(45,10))/G(45,8)
      G(45,15) = 2.0 * B(100)*(1.0 - G(45,11))*G(45,9)*G(45,10)/G(45,8)
C
      G(46,9)  = SQRT(B(100)*G(46,4)-G(46,3)*G(46,3))/B(100)
      G(46,10) = SQRT(B(100)*G(46,6)-G(46,5)*G(46,5))/B(100)
      G(46,11) = (B(100)*G(46,7) - G(46,3)*G(46,5)) /
     1  (B(100) * B(100)*G(46,9)*G(46,10))
      G(46,12) = SQRT(G(46,8)/G(46,6))
      G(46,13) = (G(46,3)-G(46,5))*(G(46,3)-G(46,5))/(B(100)*G(46,8))
      G(46,14) = B(100)*(G(46,9)-G(46,10))*(G(46,9)-G(46,10))/G(46,8)
      G(46,15) = 2.0 * B(100)*(1.0 - G(46,11))*G(46,9)*G(46,10)/G(46,8)
C
      G(47,9)  = SQRT(B(100)*G(47,4)-G(47,3)*G(47,3))/B(100)
      G(47,10) = SQRT(B(100)*G(47,6)-G(47,5)*G(47,5))/B(100)
      G(47,11) = (B(100)*G(47,7) - G(47,3)*G(47,5)) /
     1  (B(100) * B(100)*G(47,9)*G(47,10))
      G(47,12) = SQRT(G(47,8)/G(47,6))
      G(47,13) = (G(47,3)-G(47,5))*(G(47,3)-G(47,5))/(B(100)*G(47,8))
      G(47,14) = B(100)*(G(47,9)-G(47,10))*(G(47,9)-G(47,10))/G(47,8)
      G(47,15) = 2.0 * B(100)*(1.0 - G(47,11))*G(47,9)*G(47,10)/G(47,8)
C
      G(48,9)  = SQRT(B(100)*G(48,4)-G(48,3)*G(48,3))/B(100)
      G(48,10) = SQRT(B(100)*G(48,6)-G(48,5)*G(48,5))/B(100)
      G(48,11) = (B(100)*G(48,7) - G(48,3)*G(48,5)) /
     1  (B(100) * B(100)*G(48,9)*G(48,10))
      G(48,12) = SQRT(G(48,8)/G(48,6))
      G(48,13) = (G(48,3)-G(48,5))*(G(48,3)-G(48,5))/(B(100)*G(48,8))
      G(48,14) = B(100)*(G(48,9)-G(48,10))*(G(48,9)-G(48,10))/G(48,8)
      G(48,15) = 2.0 * B(100)*(1.0 - G(48,11))*G(48,9)*G(48,10)/G(48,8)
C
100   CONTINUE
C
      RETURN
      END
```

```
      SUBROUTINE FOCOMP
      COMMON/GLOBAL/IFLAG(30),MAP(8,81)
      COMMON/RNPROC/IDRUN(2),ITIME,MTV(12),IGPTR,IMPTR,IERR
      COMMON/MODULE/JD1,JX,JY,JZ,JB,JF,JG,JH,JD2,JJ,K,KP,KPP,
     + IPCR(187),VARD(66),X(2,15),XU(2,15),Y(12,15),Z(1,15),B(395),
     + F(2,15),G(48,15)
      IF(IPCR(76).NE.0) GO TO 5
      CALL POP2
      RETURN
    5 CONTINUE
      WRITE(3,201)
  201 FORMAT(1X,*SUBROUTINES PROCESSED*)
      IM=ITIME+1
      MTV(IM)=MTV(IM)+1
      CALL POP2
      RETURN
      END
```

```
      SUBROUTINE FCOMP
      COMMON/MODULE/JD1,JX,JY,JZ,JB,JF,JG,JH,JD2,JJ,K,KP,KPP,
     + IPCR(187),VARD(66),X(2,15),XU(2,15),Y(12,15),Z(1,15),B(395),
     + F(2,15),G(48,15)
C*
      DO 10 J=1,14
      IF (J.EQ.1) F(1,J) = G(42,1) * B(87) - X(1,1)
      IF (J.NE.1) F(1,J) = G(39,J) - X(1,J)
      IF (J.NE.14) F(2,J) = G(40,J) - X(2,J)
10    CONTINUE
      DO 110 I=1,2
110   WRITE(10,903) I,(F(I,J),J=1,15)
 903  FORMAT(* F(*,I2,*, J) =*,15(F8.3))
C*
      RETURN
      END
```

```
      SUBROUTINE YCOMP
      COMMON/GLOBAL/IFLAG(30)
      COMMON/RNPROC/IDRUN(2),ITIME,MTV(12)
      COMMON/MODULE/JD1,JX,JY,JZ,JB,JF,JG,JH,JD2,JJ,K,KP,KPP,
     + IPCR(187),VARD(66),X(2,15),XU(2,15),Y(12,15),Z(1,15),B(395),
     + F(2,15),G(48,15)
C
      IZ14 = IFIX(Z(1,14))
      IF (IPCR(49).NE.0) GO TO 1
  905 FORMAT(*1*/4X,*J = *,3X,15(3X,I3,2X))
C
    1 CONTINUE
C
904   FORMAT(*-*,3X,*TIME = *,I4)
      IF (IPCR(49).EQ.0) GO TO 100
      Y(1,1) = Z(1,14)
      Y(1,2) = G(43,1)
      Y(1,3) = G(43,2)
      Y(1,4) = G(43,3)
      Y(1,5) = G(43,4) * 0.1
      SUM = 0.0
      DO 20 I=1,13
20    SUM = SUM + XM(1,2,I+1) + XM(2,2,I)
      IF (SUM.NE.0.0) Y(1,6) = G(43,4) / SUM
      IF (G(43,5).NE.0.0) Y(1,7) = G(43,4) / G(43,5)
C
      DO 25 J=8,12
25    Y(1,J) = 0.0
C
      DO 30 I=1,13
      II = I + 1
      III = I + 2
      Y(1,8) =  Y(1,8) + XM(1,1,II)
      Y(1,9) = Y(1,9) + XM(1,1,II) * G(1,II) + XM(2,1,I) * G(1,I)
      Y(1,10) = Y(1,10) + XM(1,1,II) * G(6,II) * G(8,II)
      IF(III.NE.15) Y(1,11) = Y(1,11)+G(41,III) * G(4,III)
     1 + G(42,III) * G(4,III)
      IF(III.EQ.15) Y(1,11) = Y(1,11) + G(41,III) * G(4,III)
      Y(1,12) = Y(1,12) + FLOAT(II) * XM(1,1,II) + FLOAT(I) * XM(2,1,I)
30    CONTINUE
C
      IF (Y(1,8) .NE. 0.0) Y(1,8) = G(43,4) / Y(1,8)
      IF (G(43,5)+XM(2,1,1).NE.0.0) Y(1,9)=Y(1,9) / (G(43,5)+XM(2,1,1))
      IF (G(43,4).NE.0.) Y(1,10) = Y(1,10) * 100.0 / G(43,4)
      IF (G(43,3).NE.0.0) Y(1,11) = Y(1,11) * 10.0 / G(43,3)
      IF (G(43,5)+XM(2,1,1).NE.0.0) Y(1,12)=Y(1,12)/(G(43,5)+XM(2,1,1))
      Y(1,13) = G(43,6)
      IF (G(43,6).NE.0.0) Y(1,14) = G(43,4) * 0.1 / G(43,6)
C
      DO 40 J=1,15
      IF (J.EQ.15) GO TO 35
      IF (J.EQ.1) Y(2,J) = XM(2,1,1)
      IF (J.GT.1.AND.J.LT.14) Y(2,J) = XM(2,1,J) + XM(1,1,J)
      IF (J.EQ.14) Y(2,J) = XM(1,1,14)
35    Y(3,J) = G(14,J)
      IF (J.GE.2.AND.J.LE.14) Y(4,J) = G(30,J)
      IF (J.GE.1.AND.J.LE.13) Y(5,J) = G(32,J)
      IF (J.GE.2.AND.J.LE.14) Y(6,J) = G(28,15-J)
      IF (J.EQ.1) Y(7,J) = G(28,14)
      IF (J.GE.2.AND.J.LE.13) Y(7,J) = G(31,J)
40    CONTINUE
C
      IF (B(IZ14-1849).NE.0.0) Y(8,1) = G(43,1) / B(IZ14-1849)
      IF ((MTV(1)+1).NE.IFLAG(5)) GO TO 50
      IF (B(99).NE.0.0) Y(8,2) = G(44,1) / B(99)
      Y(8,3) = G(45,11)
      Y(8,4) = G(45,12)
      Y(8,5) = G(45,13)
      Y(8,6) = G(45,14)
      Y(8,7) = G(45,15)
50    Y(8,8) = G(43,1)
      Y(8,9) = B(IZ14 - 1849)
C
      IF (B(IZ14-1810).NE.0.0) Y(9,1) = G(43,2) / B(IZ14-1810)
      IF ((MTV(1)+1).NE.IFLAG(5)) GO TO 60
      IF (B(99).NE.0.0) Y(9,2) = G(44,2) / B(99)
      Y(9,3) = G(46,11)
```

```
      Y(9,4)  = G(46,12)
      Y(9,5)  = G(46,13)
      Y(9,6)  = G(46,14)
      Y(9,7)  = G(46,15)
50    Y(9,8)  = G(43,2)
      Y(9,9)  = B(IZ14 - 1810)
C
      IF (B(IZ14-1772).NE.0.0) Y(10,1) = G(43,3) / B(IZ14-1772)
      IF ((MTV(1)+1).NE.IFLAG(5)) GO TO 70
      IF (B(99).NE.0.0) Y(10,2) = G(44,3) / B(99)
      Y(10,3) = G(47,11)
      Y(10,4) = G(47,12)
      Y(10,5) = G(47,13)
      Y(10,6) = G(47,14)
      Y(10,7) = G(47,15)
70    Y(10,8) = G(43,3)
      Y(10,9) = B(IZ14 - 1772)
C
      IF (B(IZ14-1733).NE.0.0) Y(11,1) = G(43,6) / B(IZ14-1733)
      IF ((MTV(1)+1).NE.IFLAG(5)) GO TO 80
      IF (B(99).NE.0.0) Y(11,2) = G(44,4) / B(99)
      Y(11,3) = G(48,11)
      Y(11,4) = G(48,12)
      Y(11,5) = G(48,13)
      Y(11,6) = G(48,14)
      Y(11,7) = G(48,15)
80    Y(11,8) = G(43,6)
      Y(11,9) = B(IZ14 - 1733)
C
      DO 90 J=1,14
      IF (J.EQ.1) Y(12,J) = Y(2,J)
      IF (J.GT.1) Y(12,J) = Y(12,J-1) + Y(2,J)
90    CONTINUE
C
      WRITE(12,905) (I,I=1,15)
      WRITE(12,904) IZ14
C
      IF (B(395).NE.1.0) GO TO 100
C
      DO 97 I=1,48
97    WRITE(12,901) I,(G(I,J),J=1,15)
 901  FORMAT(* G(*,I2,*, J) =*,15(F8.3))
C
100   CONTINUE
      WRITE(10,905) (I,I=1,15)
      WRITE(10,904) IZ14
      DO 105 I=1,12
105   WRITE(10,902) I,(Y(I,J),J=1,15)
 902  FORMAT(* Y(*,I2,*, J) =*,15(F8.3))
      DO 110 I=1,2
110   WRITE(10,903) I,(X(I,J),J=1,15)
 903  FORMAT(* X(*,I2,*, J) =*,15(F8.3))
C
      WRITE(11,906) Y(1,1),Y(8,8),Y(8,9),Y(9,8),Y(9,9),
     +Y(10,8),Y(10,9),Y(11,8),Y(11,9)
  905 FORMAT(1X,F5.0,2X,8F7.2)
C
      RETURN
      END
```