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Title: The Influence of Map Complexity on Interpolation Accuracy

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In this paper "map complexity" refers to the inherent intricacy of a mapped geographic pattern. Map complexity and sample size are two variables shown to influence the accuracy of interpolated dasymetric maps.

An automated experiment was designed to investigate the precise relationship among map complexity, sample size, and the accuracy of dasymetric maps interpolated using Thiessen polygons. The results of the experiment were evaluated through regression analysis. A positive curvilinear relationship between sample size and map accuracy, and an inverse linear relationship between map complexity and map accuracy were observed. Map complexity was the more important variable influencing map accuracy and interaction between the two independent variables was indicated.

A logistic shaped curve is presented summarizing the theoretical relationship between sample size, map complexity, and dasymetric map accuracy.

The Influence of Map Complexity on Interpolation Accuracy

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INTRODUCTION

For many mapping projects it is technically impossible or prohibitively expensive to conduct an exhaustive survey of the phenomenon being mapped. One solution to this problem is to spatially sample the phenomenon and interpolate its distribution based upon the location and quality of these samples. Some amount of error is expected in maps created using the sample-interpolation procedure, and research projects designed to identify or minimize this error are needed.

Developing a technique which enables cartographers to estimate the accuracy of interpolated maps must begin by identifying potential sources of error. The number of samples collected, the sample design, the complexity of the distribution being mapped, and the specific interpolation model used are four factors influencing the accuracy of interpolated maps although others may exist. Here, the term "distribution complexity" refers to the pattern intricacy of a phenomenon's spatial distribution. For example, an area covered by ten soil types and arranged in small disjunct parcels has a more complex distribution than an area of equal size composed of only two soil types in large homogeneous parcels.

Of the four factors listed above, research directed towards how distribution complexity influences map accuracy has not been reported. This paper identifies distribution complexity as one aspect of the evolving map complexity concept which has an important influence on the accuracy of interpolated dasymetric maps. Because the influence of distribution complexity is affected by other factors, notably sample size, the influences of distribution complexity and sample size are investigated simultaneously.

Throughout this paper the complexity of the distribution being mapped is called "map complexity" in accordance with conventions already established in cartographic literature. The precise relationship between distribution complexity and map complexity is outlined below.

There are three primary research objectives in this project. First, the relationship among map complexity, sample size, and the accuracy of interpolated dasymetric maps is empirically investigated. Clearly, there should be a positive relationship between sample size and map accuracy and an inverse relationship between map complexity and map accuracy. This paper defines these relationships more precisely. Secondly, from data derived in the first research objective, the relative importance of sample size and map complexity is determined through linear multiple regression analysis. Finally, based upon these analyses and speculation, a theoretical curve relating these three variables is presented.

The Dasymetric Surface

Cartographers find it useful to conceive of a distribution that varies over space as a geographical volume described by a statistical surface (Muehrcke,1972). Many thematic maps represent a statistical surface wherein the planimetric relations of the theme are portrayed using the X and Y map dimensions, and the magnitude or Z dimension of the map theme is conveyed through the use of appropriate map symbolism which is designed with an understanding of the psychophysical and cognitive aspects of human perception.

Statistical surfaces may be divided into two types: continuous and discontinuous (Peucker,1972). Continuous surfaces are conceived as having a continuous change in slope gradient from one point to another, with very few if any vertical slopes. In contrast, discontinuous surfaces are conceived as areas of relative homogeneity separated from one another by very steep or vertical slopes. The three dimensional appearance of a discontinuous surface is step-like, whereas a continuous surface smoothly undulates.

This study was directed specifically towards a qualitative dasymetric surface. All dasymetric surfaces are discontinuous and boundaries between classes represent natural divisions in map theme (Robinson, et al.,1978). Land cover, land use, soils, and geologic formation maps are examples of this type. It is convenient to conceive of qualitative maps as statistical surfaces also. In this case, change in the Z dimension represents qualitative rather than quantitative change and the integers assigned to categories represent class type rather than numerical value.

For this project, it was necessary to distinguish between continuous and discontinuous surfaces because specific interpolation models are suitable for one type or the other and are generally not interchangeable.

LITERATURE REVIEW

For this study a survey of literature published in cartographic and geographic journals was undertaken with specific objectives in mind. If similar projects had already been completed, then this study should build upon their conclusions and avoid duplicating their effort. Also, a review of interpolation models was necessary for selecting the one most appropriate for use with a qualitative dasymetric surface. Finally, in order to quantitatively investigate the influence of "map complexity" on "map accuracy", it was necessary to know how research cartographers have precisely defined and measured these qualities.

Morrison's "Method-Produced Error"

An important study identifying which variables are most significant in influencing the accuracy of interpolated maps is Morrison's (1971) paper "Method-Produced Error in Isarithmic Mapping". In this monograph, three variables were identified as influencing isarithmic map accuracy: sample size, sample design, and interpolation model. Morrison used various combinations of sample size, sample design, and interpolation model to generate 84 different interpolated maps from four parent surfaces. The accuracy of each interpolated map was determined through comparison with the parent surface. How well the interpolated map fit the parent surface from which it was generated was measured as "the standard deviation of the residuals that occur at a 100-point square lattice of points within the study area" (Morrison,1971).

Morrison used an analysis of variance approach to show that sample design and interpolation model were the most important variables influencing isarithmic map accuracy of those tested. Although the influence of sample size was statistically significant, it was small relative to the other two variables. He concluded that a stratified random sample design was the best of the several he tested and that a double Fourier was the optimal performing interpolation model. Also, to accurately interpolate the surfaces a sample size of between 44 and 100 points was found to be adequate.

The present study departs from Morrison's project in three significant ways. Although he used four parent surfaces of different complexity, Morrison did not investigate how increased complexity influences map accuracy when the other variables are held constant. This aspect is the focus of this paper. Secondly, Morrison used continuous quantitative surfaces while in this paper discontinuous qualitative surfaces are studied exclusively. Thirdly, the present study holds the variables of sample design and interpolation model constant.

Interpolation Models

Spatial interpolation is the process of inferring a complete surface within a specified boundary from a sample of data points or subareas of known X, Y, and Z dimensions (Lam,1983). The interpolation is accomplished by using a rule or function which estimates a value Z_{est} at position X_iY_i based upon the location and quality or quantity of nearby data points or subareas. The function used to estimate the Z_{est} values is called an interpolation model.

Fundamental assumptions about the behavior of the surface being interpolated must be made prior to selecting an interpolation model. Deciding whether the surface is continuous or discontinuous is one of the most important assumptions because most interpolation models are surface specific. Figure 1 is presented as an illustration of this point.

At the top of Figure 1 the X, Y, and Z positions of ten data points on a hypothetical surface are shown. If the surface is assumed to be continuous, an appropriate interpolation model is selected, and the surface is completed. The continuous surface on the left side of Figure 1 was interpolated using the model:

$$Z_{est} = \left[\sum_{i=1}^{n} W_i Z_i\right] \Big/ \left[\sum_{i=1}^{n} W_i\right]$$

where Z_{est} is the interpolated value, Z_i is the data value at point i, and W_i is the assigned weight of data point Z_i (Monmonier,1982). In this equation, which is based on the gravity model, the weight of data point Z_i is inversely proportional to the distance-squared between $X_iY_iZ_i$ and $X_iY_iZ_{est}$. Notice that intermediate values are inferred between data points that range widely in magnitude. This is characteristic of interpolation models developed for use with continuous surfaces, and it is this quality that makes them unsuitable for discontinuous surfaces. With a discontinuous surface there is no reason to assume intermediate values exist.

One interpolation technique suitable for use with a

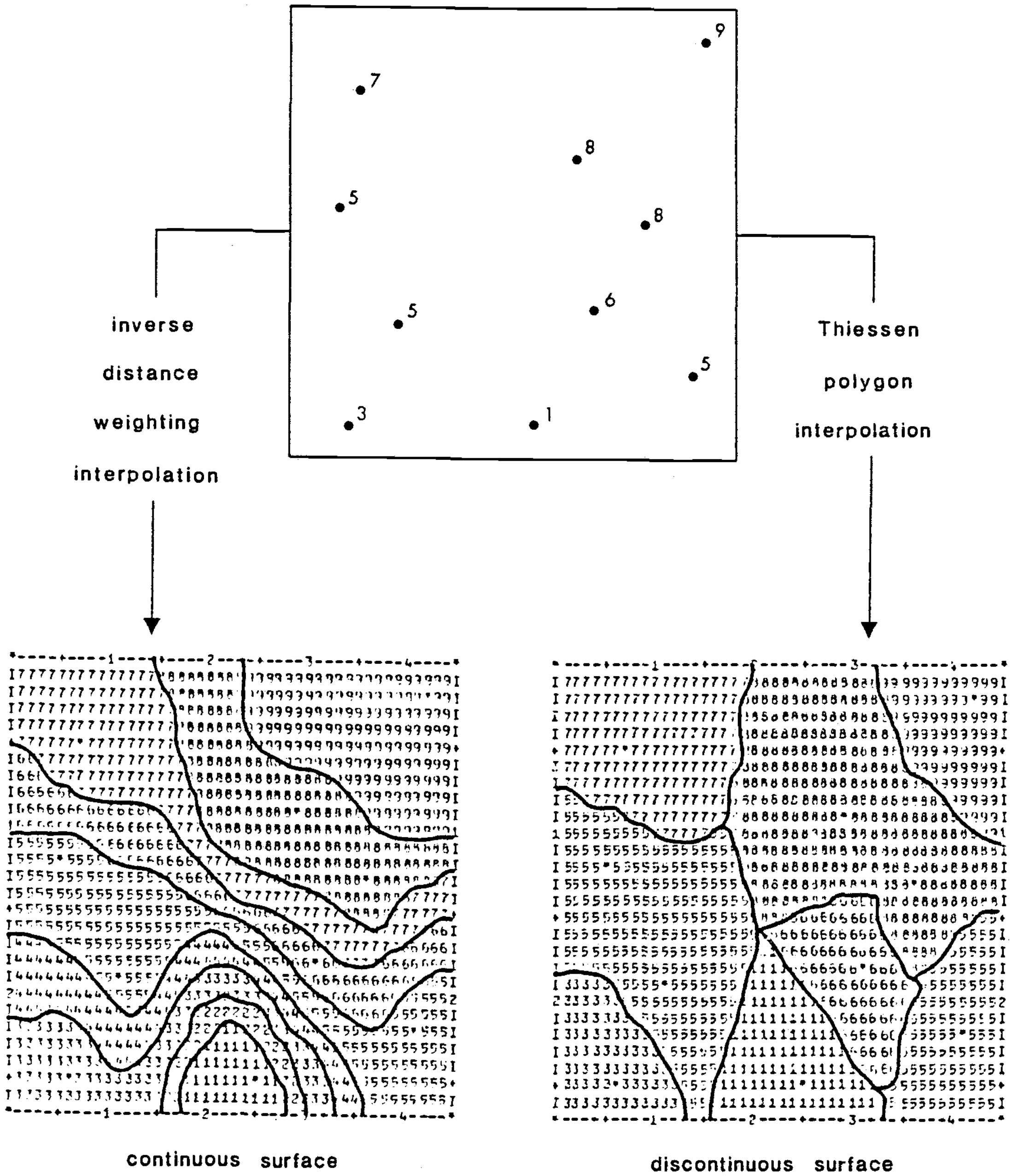


Figure 1. Two statistical surfaces interpolated from the same data set

discontinuous surface is constructing Thiessen polygons. With this method, each point with an unknown Z dimension is assigned the Z value of the nearest data point. In this manner, the study area is apportioned into many polygons using proximity to the nearest data point as the only criterion. The right side of figure 1 represents a discontinuous surface interpolated from the same ten data points. With the Thiessen polygon interpolation model, intermediate values are assumed not to exist, giving the surface a step-like appearence. Notice the discrepancy in the appearence of the surfaces depending on which interpolation model is employed.

Map Complexity Concept

The separate components of visual and intellectual map complexity are discussed in cartographic literature. Intellectual map complexity is associated with difficulty in interpreting intellectual meaning from abstract cartographic symbols whereas visual map complexity refers to the inherent intricacy of a mapped geographic pattern (MacEachern,1982).

Distribution complexity is the focus of this paper, and this quality is related directly to the visual map complexity concept. Visual map complexity is simply a cartographic representation of distribution complexity. In order to incorporate the large amount of cartographic literature devoted to measuring and defining map complexity, one must consider distribution complexity to be equivalent to visual map complexity. A discipline wide-definition of this phenomenon has not been developed and most authors offer an operational definition of visual map complexity, as will be done here.

Fundamentally, visual map complexity is a multidimensional phenomenon associated with the interconnectedness of map classes. Muehrcke (1973) describes map complexity as the "spatial variance" in a map pattern where spatial variance is a measure of the map's "internal organization". Several factors contribute to a map's spatial variance including the number of classes, the fragmentation of these classes, and variation in the proportion of map area covered by each map region. Also, these three factors seem to be interrelated. Visual map complexity is defined here as being equal to a map's spatial variance. Therefore, a measure sensitive to these three factors must be selected in order to quantify map complexity.

Numerous measures of map complexity have been proposed by academic cartographers (Olson,1972,1975), (Monmonier,1974), (Muller,1976), (Brophy,1980), (MacEachern,1982). Some of these are developed for continuous surfaces and are unsuitable for use with a discontinuous surface. For example, Monmonier (1974) used as a measure of map complexity the highest order polynomial equation necessary for a "best fit" trend surface. Other methods are designed specifically for choropleth maps and are unsuitable for dasymetric maps without modification (Muller,1976), (MacEachern,1982).

MacEachern (1982) offers the coefficient C_m as a measure of choropleth map complexity. C_m is calculated by decomposing a choropleth map into graph feature components of faces, edges, and vertices as illustrated in Figure 2. The maximum number of faces, edges, and vertices are counted using every enumeration unit on the

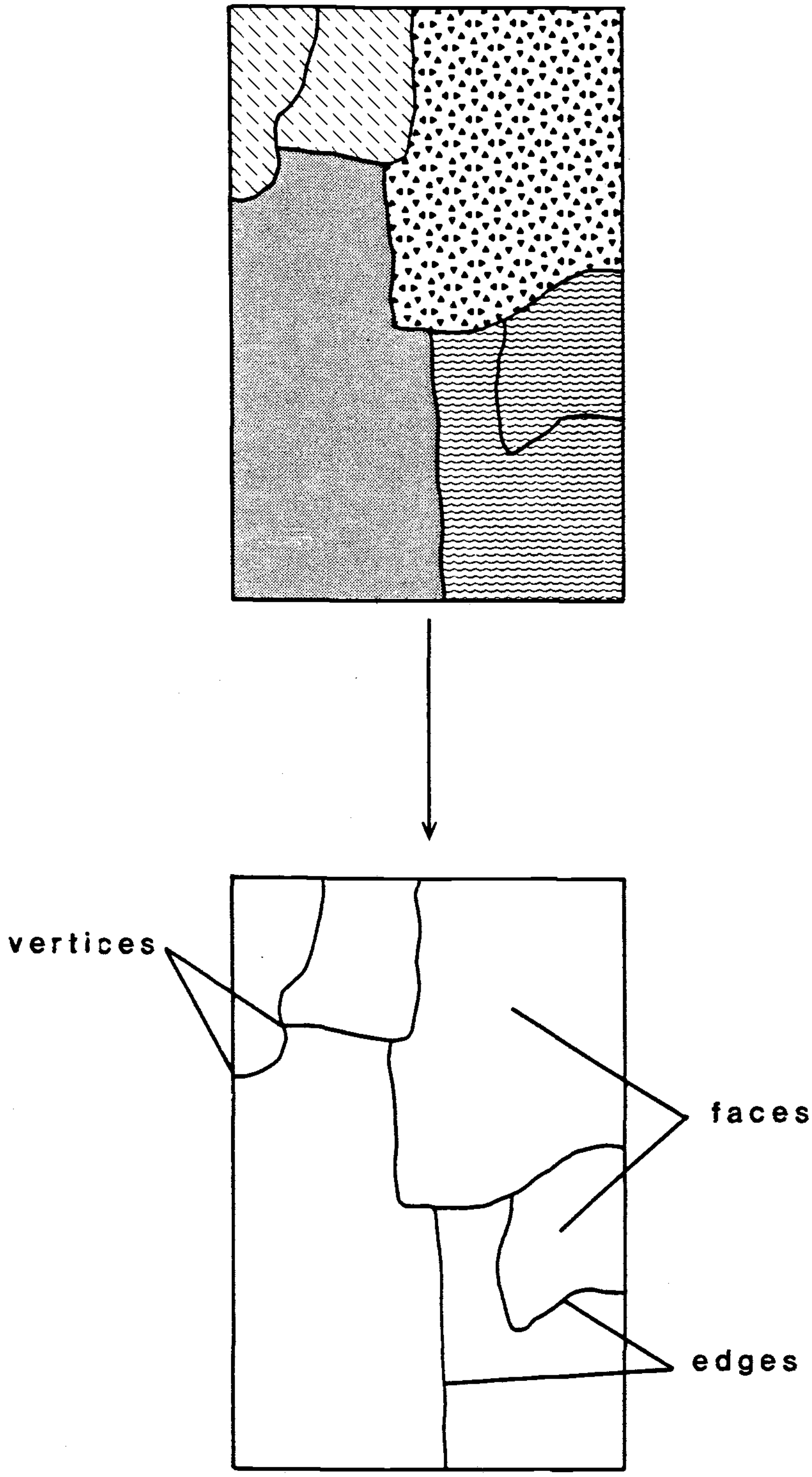


Figure 2. Decomposition of a choropleth map into faces, edges, and vertices

map. Next, all edges separating units of the same class are ignored and followed by a new count of observed faces, edges, and vertices. The complexity coefficient is as follows:

$$C_m = \frac{\text{observed number of (faces, edges, or vertices)}}{\text{maximum number of (faces, edges, or vertices)}} .$$

MacEachern (1982) reports redundancy (r = 0.94 to 0.97) among the various C_m indexes using either faces, edges, or vertices, and suggests only one count is adequate.

Another measure of choropleth map complexity is the pattern fragmentation index F (Monmonier,1974). F is calculated as :

$$F = (M-1)/(N-1)$$

where M is the number of map regions and N is the number of enumeration units. Contiguous enumeration units of the same class are considered one region. The coefficient F ranges from 0, where all enumeration units are of the same class, to 1, where no two contiguous units are of the same class. Since map enumeration units and map faces are the same feature, it should be clear that C_m using face counts and F are calculated using identical methods.

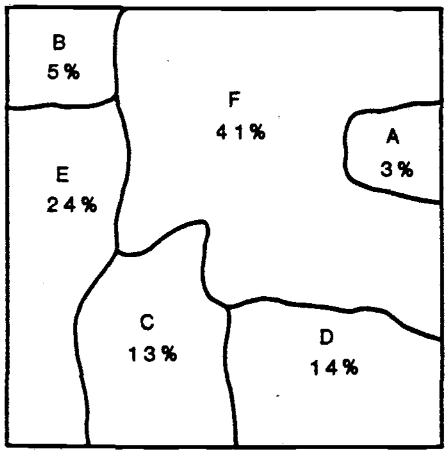
The complexity coefficients F and C_m are designed specifically for choropleth maps in which boundaries are based on political enumeration units rather than naturally occuring divisions in map theme. Without modification they are unsuitable for dasymetric maps because boundaries only separate homogeneous areas of different classes. However, C_m and F can be adapted for use with dasymetric maps by superimposing a rectangular grid over the map and treating each grid cell as a hypothetical enumeration unit. One problem introduced by this modification is the variation in F and C_m

determined by the cell size of the superimposed grid. For any dasymetric map, the numerator in both equations (number of map regions) is not affected by the superimposed grid, but the denominator (number of enumeration units) will vary depending on cell size. This fact makes method repeatability and comparison between maps difficult unless grids with identical cell size are used.

Inequality in the proportion of map area covered by each map region is a dimension of map complexity to which the indices C_m and F are not sensitive. For example, two maps with the same number of classes and regions will have identical F or C_m values even though one of the maps may be dominated by a single large map region, making this map less visually complex. Therefore, the size disparity index S_d is also necessary to reflect this second dimension of map complexity (Monmonier,1974). The size disparity index is computed by constructing a Lorenz curve (Taylor,1977) with the cumulative proportion of map area covered by each map region on the Y-axis and the proportion of map region area in rank order and equal increments on the X-axis. S_d is measured as the proportion of area between the graph diagonal and the Lorenz curve (Monmonier,1974). Figure 3 illustrates the calculation of S_d.

Unfortunately, there is no suitable method for combining the complexity indices F and C_m into a composite measure which is required for this project.

From among the various spatial autocorrelation statistics is found a measure of map complexity sensitive to the fragmentation of map classes and to the variation in the proportion of map area covered by each map region. Spatial autocorrelation is a general



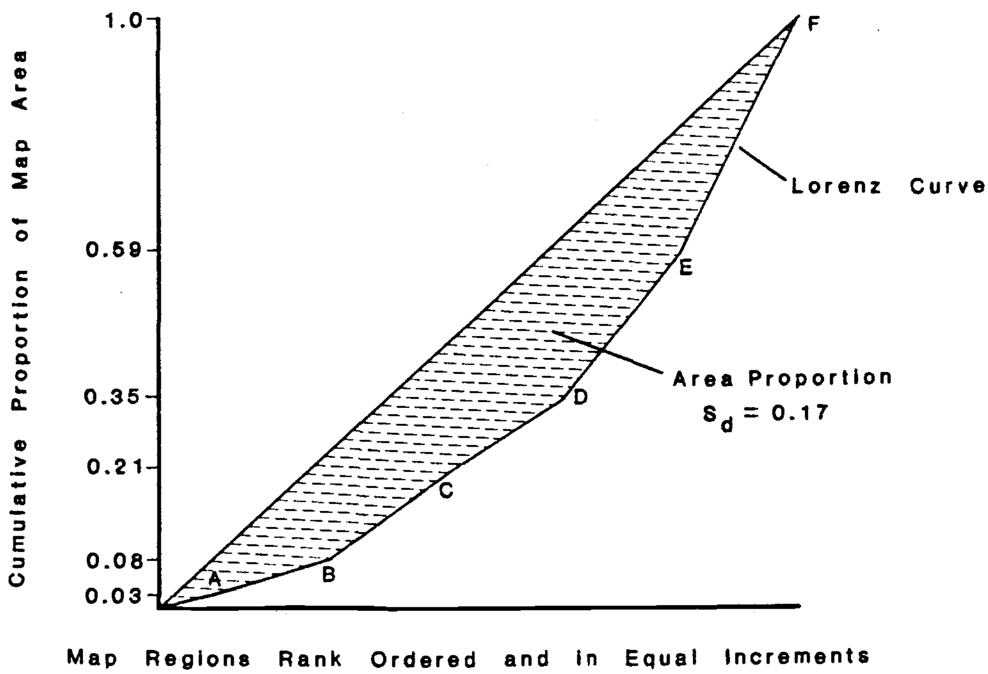


Figure 3. Calculation of the size disparity index S_d

statistical and geographic concept used to describe the degree to which a spatial pattern departs from randomness (Cliff and Ord,1973,1981), and it has been used as a measure of map complexity (Olson,1972,1975). The spatial organization of a mapped geographic pattern can be summarized by a single statistic using the correlation coefficient r, Kendall's rank correlation coefficient tau, or the proportion of map units with identical neighbors depending on whether the map theme is ratio, ordinal, or nominally measured, respectively (Olson,1975).

Additionally, spatial autocorrelation is a desirable measure of map complexity because the computation can be performed by a computer using digital grid cell maps. To illustrate this procedure, consider Figure 2 wherein neighbors are defined as cells sharing an edge. Map A is a quantitative digital map with nine cells X_{ij}, where i represents the row number and j is the column number. Cell X_{11} has two neighbors, X_{12} and X_{21}, whereas cell X_{22} has four neighbors. To calculate the coefficient r, the value of X_{11} is entered into the X column and the value of its two neighbors is entered into the Y column. Next, the value of X_{12} is entered into the X column and its neighbors' X_{11}, X_{13}, and X_{22} value is entered into the Y column. This process continues as shown in Figure 4. The coefficient r is calculated as

$$r = \frac{\sum_{i=1}^{n} (X_i - \bar{X}) \quad (Y_i - \bar{Y})}{\sqrt{\left[\sum_{i=1}^{n} (X_i - \bar{X})^2\right] \left[\sum_{i=1}^{n} (Y_i - \bar{Y})^2\right]}}$$

(Blalock, 1979).

Map A

3	3	8
3	4	8
4	9	9

X	Y
3	3
3	3
3	3
3	8
3	4
8	3
8	8
3	3
3	4
3	4
4	3
4	3
4	9
4	8
8	8
8	4
8	9
4	3
4	9
9	4
9	4
9	9
9	9
9	8

$r = 0.89$

Map B

A	A	D
A	C	E
C	E	E

X	Y
A	A
A	A
A	A
A	C
A	D
D	A
D	E
A	A
A	C
A	C
C	A
C	E
C	A
C	E
E	C
E	D
E	E
C	A
C	E
E	C
E	C
E	C
E	E
E	E

$A = 0.67$

Figure 4. Measuring map complexity with spatial autocorrelation coefficients

The value of r is interpreted in the standard fashion; large absolute values indicate map cell values are either positively or inversely correlated spatially, and values close to zero represent random spatial patterns. As an indication of map complexity, the closer r is to zero, the more complex is the map. In Figure 4, $r = 0.89$ indicating cell values with similar values are spatially correlated.

The coefficient r is not suitable for use with maps of a qualitative theme. Although qualitative map classes can be abstractly represented by a numeral, substituting these values into the preceeding equation would be nonsense. For qualitative grid cell maps, Olson (1975) proposes using the proportion of cells with identical neighbors as a measure of spatial autocorrelation. With this measure, the proportion of identical neighbors increases as map complexity decreases. Therefore, this author suggests subtracting the proportion of identical neighbors from 1 so that the magnitude of the coefficient increases as map complexity increases. This coefficient may be called "A" where

$$A = 1 - \text{proportion of identical neighbors} .$$

The right side of Figure 4 illustrates the calculation of A using qualitative Map B.

The spatial autocorrelation coefficient A appears to be the best measure of map complexity for qualitative dasymetric maps in a grid cell format. Figure 5 illustrates that A is sensitive to at least two of the three dimensions of map complexity identified. The coefficient A increases as the mapped pattern becomes more fragmented, and A decreases when one class dominates the map. Because of the interrelatedness of the three map complexity dimensions, the

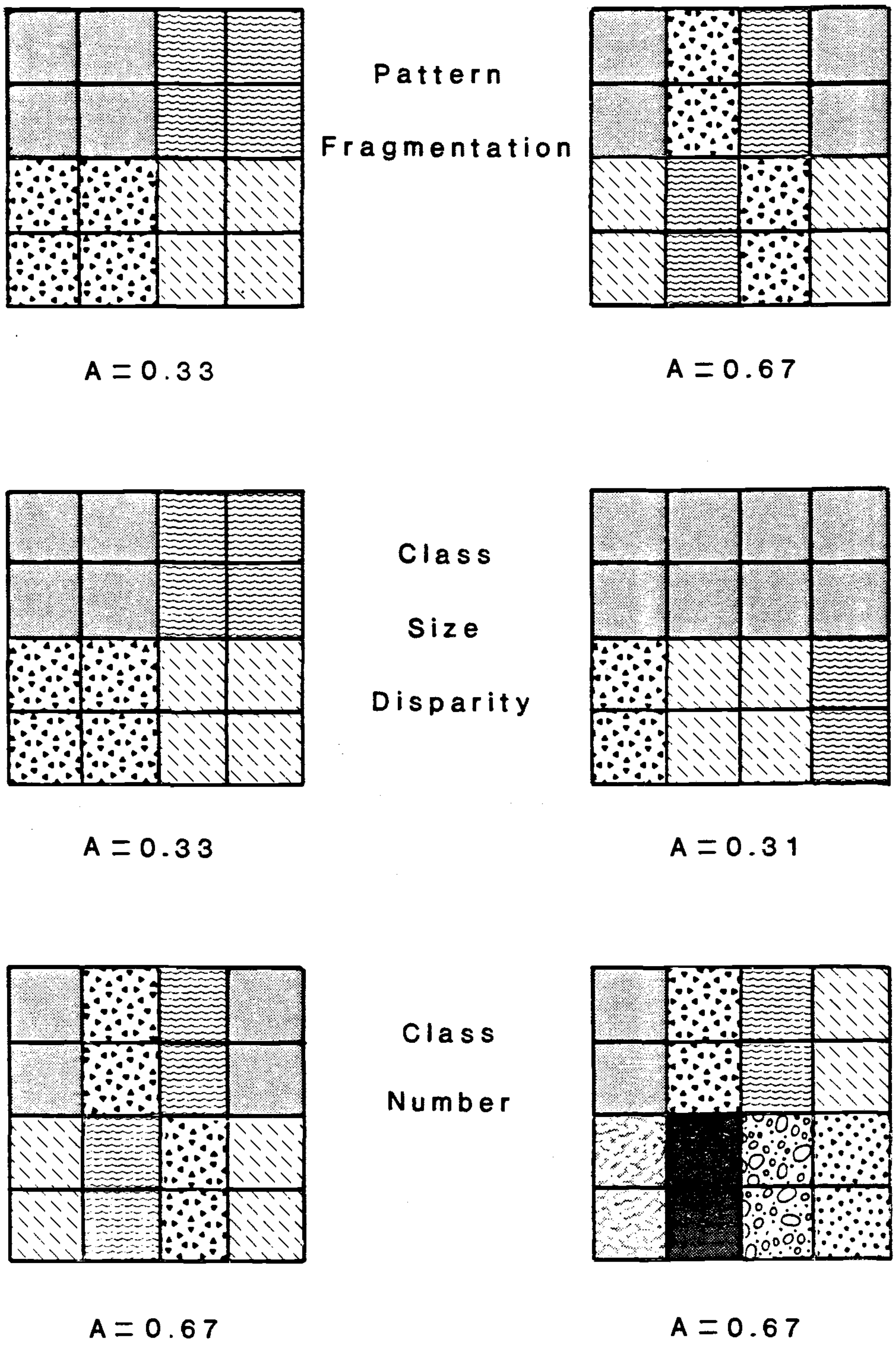


Figure 5. Three dimensions of map complexity

coefficient A may reflect the number of map classes even though it does not in this example.

Map Accuracy

This discussion of map accuracy is restricted to qualitative dasymetric maps such as those displaying land use, soils, and geologic formations. For maps of this type, Hord and Brooner (1976) have identified three types of error: polygon misclassification, boundary line misplacement, and planimetric control point errors. Not all of these are considered here because of the project's specific objectives and due to limitations imposed by using maps in a digital form. For example, in a digital grid cell map class boundaries are forced to coincide with cell outlines which often modifies their original planimetric positions slightly. For this project, map accuracy is defined as the correspondence between the parent map and the interpolated map. Therefore, map accuracy can be measured by the coefficient of areal correspondence, formally defined in set theory as

$$C_A = \frac{PM \cap IM}{PM \cup IM}$$

where C_A is a measure of map accuracy, PM is the parent map, and IM is the interpolated map. The proportion of area correctly interpolated in IM is used as the intersection of the two maps, and their union is always equal to 1 because both maps cover the same area.

RESEARCH METHODOLOGY

To investigate the influence of map complexity on map accuracy an experiment was designed in which sample size and map complexity were varied and the accuracy of interpolated maps observed. For this experiment, it was necessary to select a source map, choose a statistically unbiased method of generating point samples of various sizes, and measure the correspondence of the interpolated map and the parent map. In addition, this routine had to be performed many times.

A research methodology capable of being computerized and performed interactively was developed because of the many iterations and large amount of computation required. The experiment design developed and implemented is presented in Figure 6 as a flow diagram, and the remainder of this chapter outlines its specific details.

Map Generalization

A soils map from the South Umpqua Area, Oregon Soil Survey (U.S.D.A.,1973) was selected as the source map. The source map had to be extremely complex because it was generalized several times . Map generalization was accomplished through class reduction (combining two or more similar classes into one grouping), by smoothing class boundary outlines, and by eliminating small parcels entirely contained within a large homogeneous parcel of another class. Generalization was performed manually on the original polygonal maps which were subsequently redigitized.

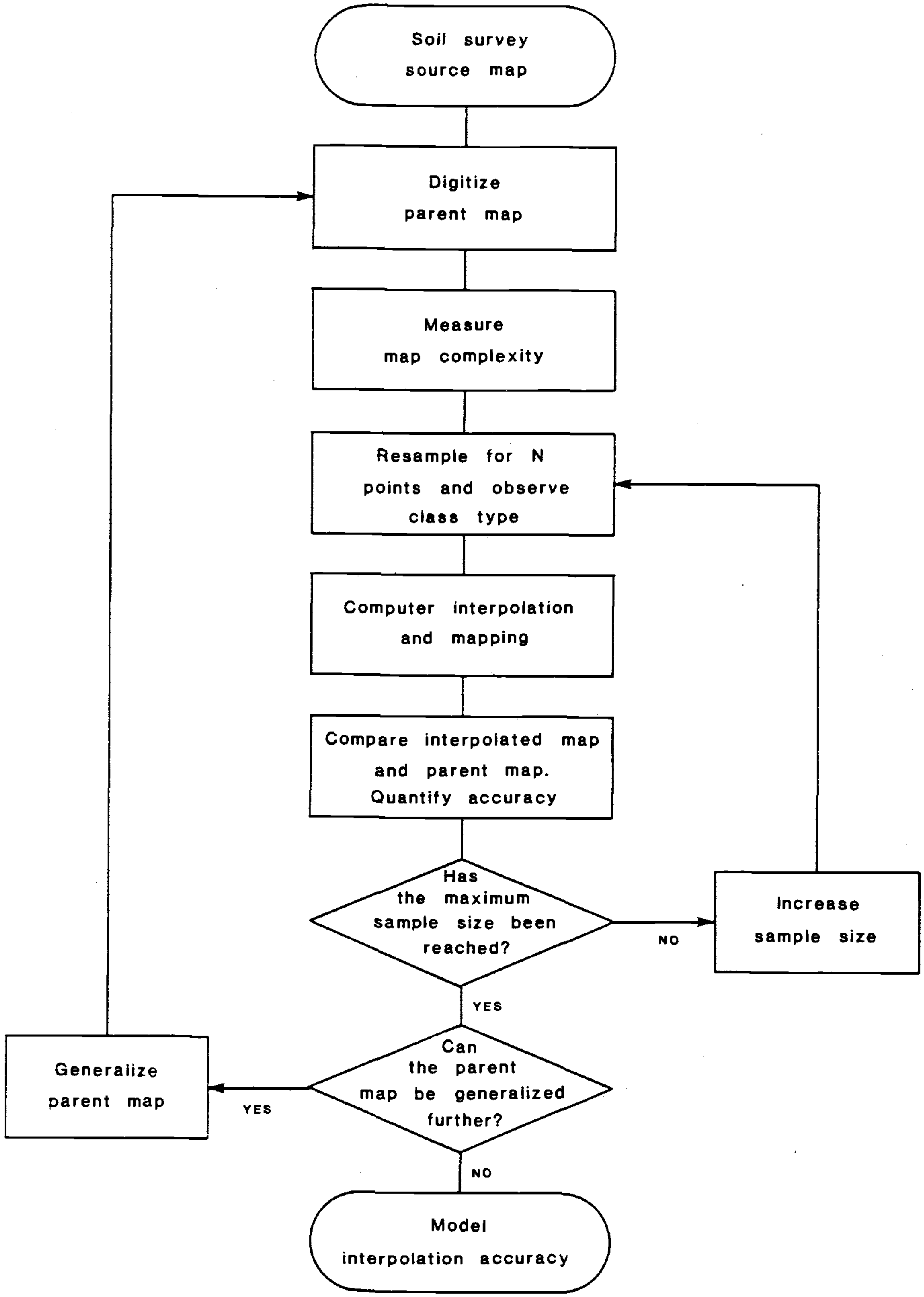


Figure 6. Experimental design

The original source map from the Oregon Soil Survey (Map 1) and its five increasingly general forms are presented in their polygonal form in Figure 7 and in their digital form in Appendix B.

Map Digitizing

Maps 1 through 6 were manually digitized by overlaying a 5x5 inch piece of mylar grid with ten cells per inch. Each soil type was assigned an integer between 0 and 9, and in each of the 2500 cells, the appropriate number was entered. Each matrix of numbers representing the digital forms of Maps 1 through 6 was entered and stored on a computer file.

Positional modification of class boundaries is unavoidable when converting a polygonal map into a digital grid cell map. Therefore, the digital representation became the parent map to which interpolated maps were compared.

Map Complexity Measurement

Once the parent maps 1 through 6 had been digitized their spatial autocorrelation coefficient A was calculated and used as a measure of map complexity. The coefficient A was computed by the program AUTO which appears in Appendix A along with the other computer programs written for this experiment.

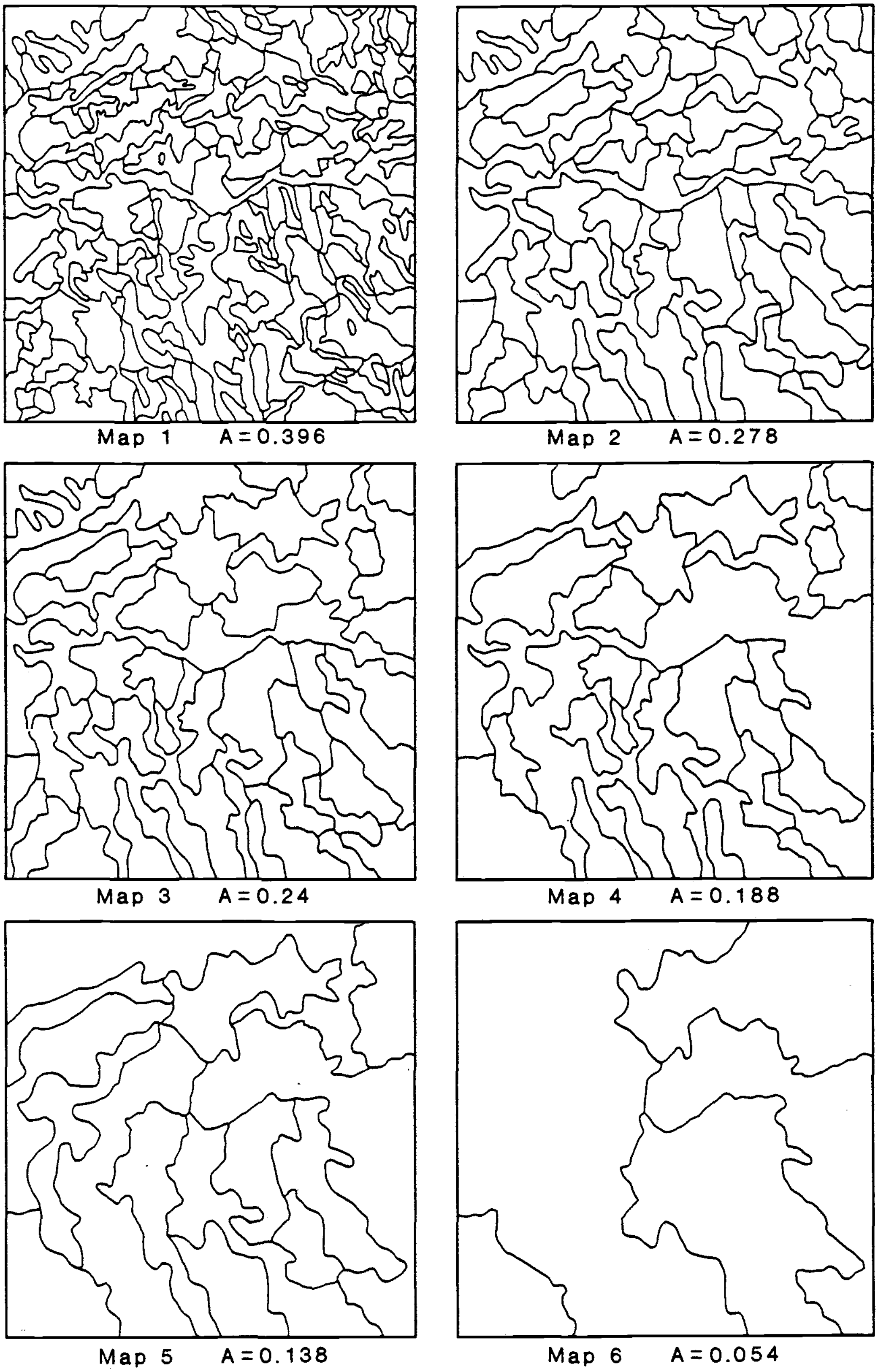


Figure 7. Source map (Map 1) and its five generalized forms with their complexity measurements

Sample Design

A stratified random sampling was selected for the experiment because Morrison (1971) found its performance to be the best for accurate interpolation from among the several he tested. Each digital map was divided into 25 square shape strata, each containing 100 cells. For a sample size of 25, one data point was selected at a random location from within each stratum as illustrated in Figure 8. Four data points were selected randomly within each stratum for a sample size of 100, and so forth. The stratified random samples were performed by program RANDOM in which random point coordinates are produced using a system supplied intrinsic function and a user supplied seed number. In addition, RANDOM also determines the soil type at the selected coordinates.

Computer Interpolation and Mapping

The data point's location and soil type were used as input for the interpolation of a new map. The interpolated map was produced using the Proximal Map options of the SYMAP computer mapping program (Dougennik and Sheehan,1977). The Proximal map package interpolates unknown values by constructing Thiessen polygons around data points supplied by the user. The Thiessen polygon interpolation model was selected because it is a well established technique for use with qualitative discontinuous surfaces. A typical deck set-up used to execute the SYMAP program appears in Appendix A.

Difficulty was encountered using SYMAP because of the

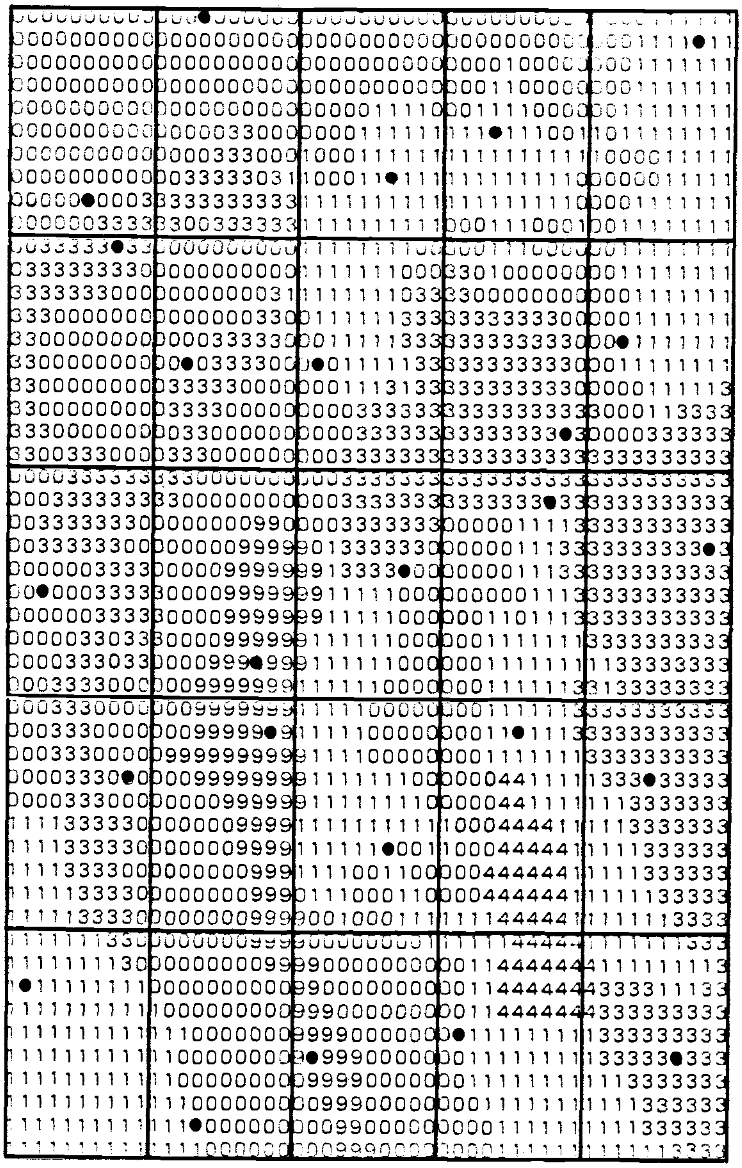


Figure 8. Stratified random sample with 25 points for Map 6

coordinate transformation of data points performed automatically to accommodate the vertical elongation typical of computer printouts. Through experimentation a 1:1 correspondence between data point coordinates and SYMAP map coordinates was achieved by misleading the program into believing the map was being printed at ten rows per inch.

Once the interpolated map was created by SYMAP, the value within each of its 2500 cells was written onto a permanent file using F-MAP elective 21. Subsequent to being reformatted by progran DAVE, the accuracy of the interpolated map was determined using this file.

Measuring Map Accuracy

To measure the accuracy of the interpolated map, the interpolated map and the parent map from which samples were taken were compared digitally. Their coefficient of areal correspondence C_A was calculated by counting the proportion of cells assigned the correct soil type, that is

$$C_A = \frac{\text{number of cells correctly interpolated}}{\text{total number of cells}} \quad .$$

The coefficient C_A was computed by program CHECK which also produces a residual map showing which cells were assigned an incorrect soil type. Figure 9 illustrates this procedure.

Range of Sample Sizes

Sample sizes of 5, 10, 25, 50, 75, 100, 250, 500, and 1000

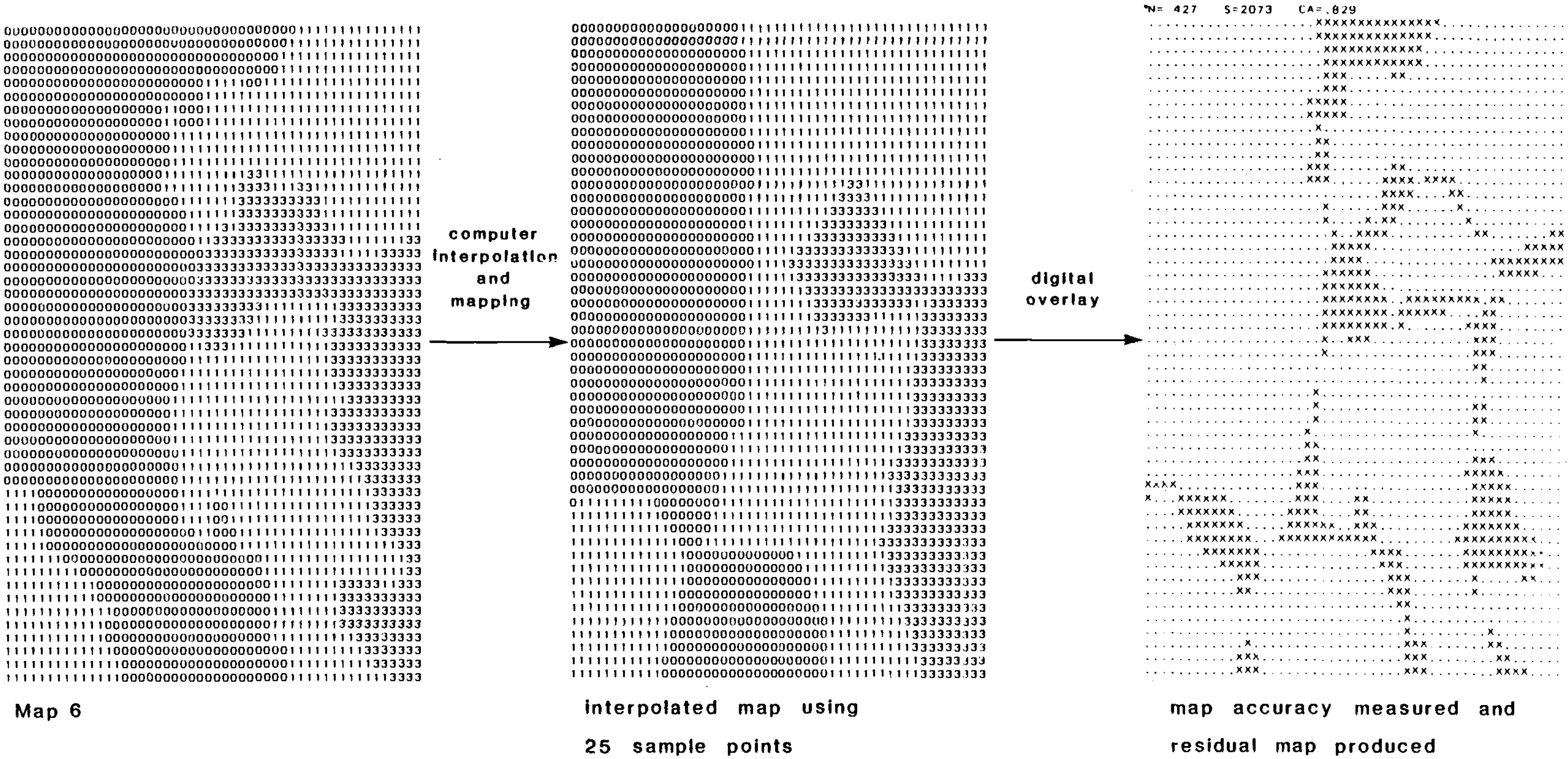


Figure 9. Comparison of parent map and interpolated map

points in various combinations were used with the maps. Five different sample sizes representing a wide range were used with each map and replicates of each sample size were made. For example, samples sizes of 25, 50, 100, 250, and 500 were used with Map 1. Independent replicates of each sample size were made in order to observe random variation in map accuracy when identical sample sizes were used.

RESULTS AND DISCUSSION

Table 1 presents the results of the experiment outlined in Figure 6 and for each map the various sample sizes and interpolated map accuracy levels are listed. In this chapter, sample size and map complexity are examined separately to see their individual effects on map accuracy. Secondly, the cumulative effect of both variables is examined to determine which has the more important influence on map accuracy.

Map Accuracy Related to Sample Size

For all six maps there was a positive curvilinear relationship between map accuracy and sample size. Figure 10 illustrates the curve for Map 6. Such curvilinear relationships may be linearized by transforming one or both variables (Neter,et al.,1983). A base ten logarithmic transformation of sample size successfully linearizes this relationship for all six maps. Figure 11 illustrates the linearizing transformation using Map 6. Linearizing the relationship is desirable because of the straightforward interpretation of the slope and intercept of linear equations.

Figure 12 presents the graphs of Maps 1 through 6 plotted on a logarithmic X-axis. Each map's least-squares fitted regression line, equation, and R^2 value are also presented. In all cases the regression line fits very well as indicated by the high R^2 values. Also, there is some indication of non-constant variance in different levels of X, that is, variation in the level of map accuracy at fixed

Table 1. Results of Experiment

Map	Sample Size	Interpolated Map Accuracy
Map 1	25	0.241
	25	0.272
	50	0.345
	50	0.318
	100	0.390
	100	0.388
	250	0.531
	250	0.500
	500	0.615
	500	0.602
Map 2	25	0.305
	25	0.305
	50	0.406
	50	0.399
	100	0.500
	100	0.499
	250	0.630
	250	0.626
	500	0.720
	500	0.721
Map 3	25	0.375
	25	0.428
	100	0.574
	100	0.554
	250	0.666
	250	0.678
	500	0.778
	500	0.754
	1000	0.835
	1000	0.827
Map 4	25	0.500
	25	0.531
	100	0.634
	100	0.659
	250	0.723
	250	0.736
	500	0.816
	500	0.786
	1000	0.867
	1000	0.869

Table 1. Results of Experiment (cont.)

Map	Sample Size	Interpolated Map Accuracy
Map 5	25	0.568
	25	0.621
	50	0.671
	50	0.600
	75	0.702
	75	0.688
	250	0.807
	250	0.818
	500	0.855
	500	0.865
Map 6	5	0.604
	5	0.668
	10	0.665
	10	0.736
	25	0.781
	25	0.821
	50	0.852
	50	0.837
	75	0.874
	75	0.883

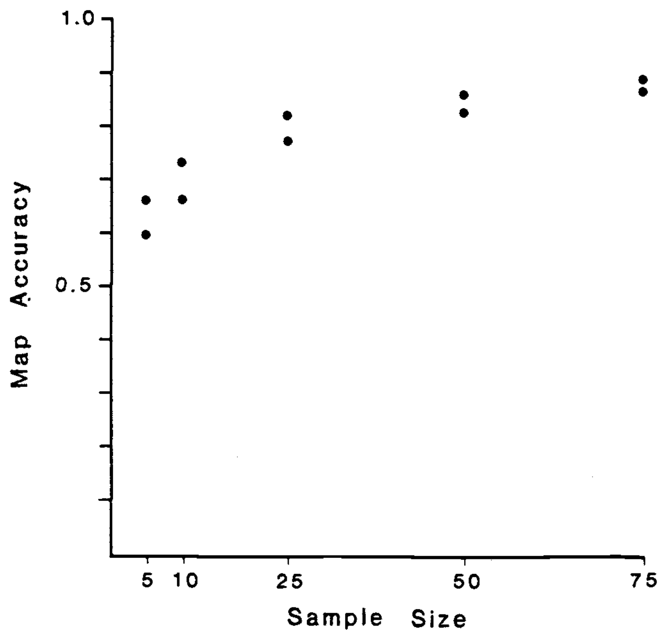


Figure 10. Scattergram showing the relationship between map accuracy and sample size for Map 6

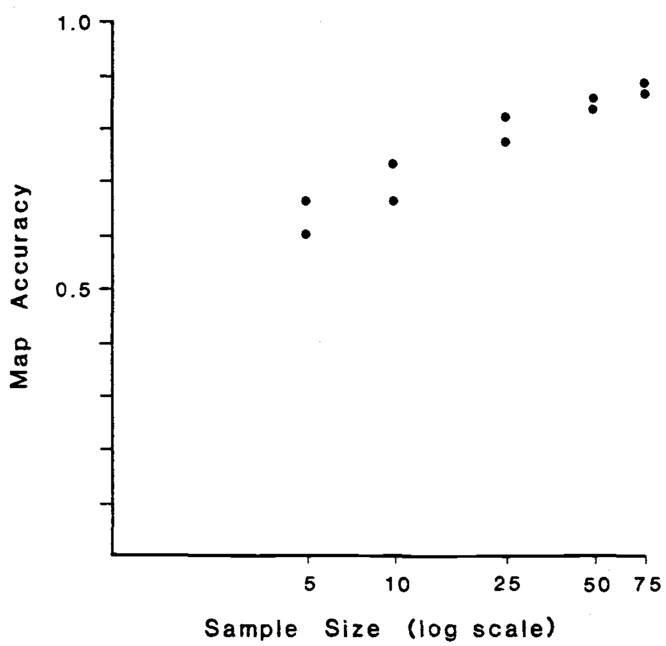
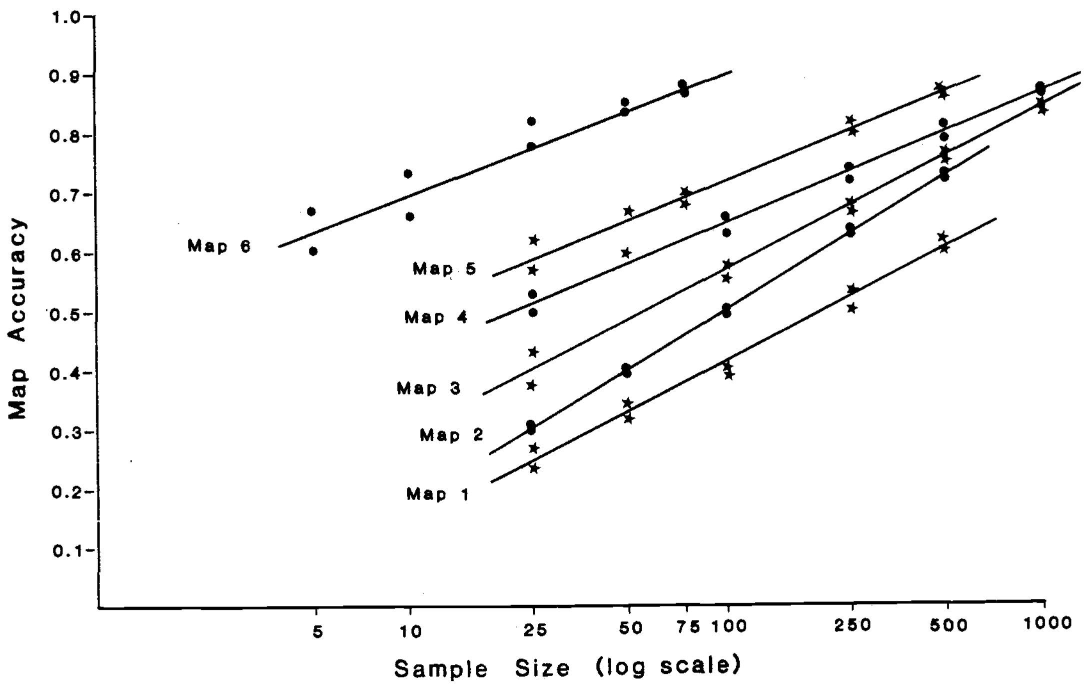


Figure 11. Scattergram showing the relationship between map accuracy and the logarithm of sample size for Map 6



Map 6	$y = 0.49 + 0.20x_{log}$	$R^2 = 0.92$
Map 5	$y = 0.28 + 0.21x_{log}$	$R^2 = 0.95$
Map 4	$y = 0.20 + 0.21x_{log}$	$R^2 = 0.99$
Map 3	$y = 0.02 + 0.27x_{log}$	$R^2 = 0.99$
Map 2	$y = -0.14 + 0.32x_{log}$	$R^2 = 0.99$
Map 1	$y = -0.12 + 0.26x_{log}$	$R^2 = 0.98$

Figure 12. Scattergram relating map accuracy to logarithm of sample size for Maps 1 through 6. Each map's least-squares regression line, equation, and R^2 value is also presented.

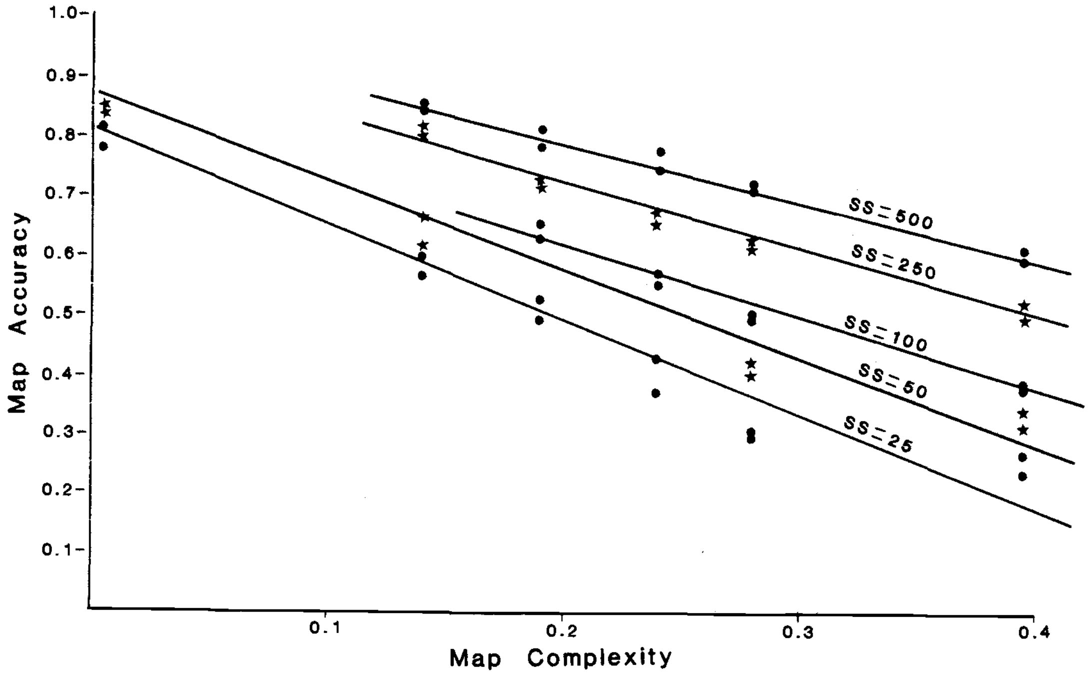
levels of sample size decreases as larger sample sizes are considered.

Notice how the slope of the regression equations is steeper for the more complex maps (Maps 1, 2, and 3). This suggests that the asymptotic leveling of map accuracy occurs at larger sample sizes as the map becomes more complex even though a clear trend is not apparent over the range of sample sizes considered.

Map Accuracy Related to Map Complexity

Map accuracy was inversely related to map complexity at all sample sizes as presented in Figure 13. No linearizing transformation was necessary . Once again the high R^2 values indicate the regression lines fit very well, although with careful examination the data seem to suggest a nonlinear trend with smaller sample sizes.

It is apparent that the influence of map complexity on map accuracy is dependent on sample size. This fact is indicated in Figure 13 by the trend of decreasing regression line slope with increased sample size. This is logical because as sample size approaches infinity the accuracy of an interpolated map should be very high irregardless of complexity. When the effect of one independent variable (map complexity) is influenced by the level of another independent variable (sample size), the two variables are said to "interact" (Neter,et al.,1983).



SS = 500	y = 0.98 - 0.96x	R^2 = 0.98
SS = 250	y = 0.95 - 1.12x	R^2 = 0.97
SS = 100	y = 0.85 - 1.21x	R^2 = 0.96
SS = 50	y = 0.87 - 1.50x	R^2 = 0.93
SS = 25	y = 0.83 - 1.60x	R^2 = 0.91

Figure 13. Scattergram relating map accuracy to map complexity for five sample sizes. Each sample size's least-squares regression line, equation, and R^2 value is also presented.

Relative Importance of the Independent Variables

Through multiple regression analysis, the individual and cumulative influence of the two independent variables on map complexity and each variable's relative importance may be determined using an analysis of variance approach (Johnson,1978), (Neter, et al.,1983). The regression analysis of variance approach is based on measuring the total amount of variation observed in map accuracy during the experiment and attributing proportions of the variation to the independent variables.

The total amount of variation (SSTO) observed in map accuracy (Y) is measured as the sum of the squared deviations from the mean value of Y, that is,

$$SSTO = \sum_{i=1}^{n} (Y_i - \bar{Y})^2 \quad .$$

By introducing an independent variable or variables X_n, and developing a regression equation relating Y to X_n, a percentage of the variation may be attributed to X_n. The amount of variation "explained" by the independent variable (SSR) is measured as the sum of the squared deviations between the least-squares fitted line and the mean value of Y, that is,

$$SSR = \sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2 \quad .$$

The relative importance of the independent variable is measured as the proportion of total variation explained by that variable. This index is called R^2 where

$$R^2 = \frac{SSR}{SSTO} \quad .$$

When regressing sample size and map complexity separately against map accuracy, their R^2 values are 0.215 and 0.466, respectively. In this experiment, map complexity is the more important variable, accounting for 46.6% of the variation in map accuracy while sample size accounts for only 21.5%. Clearly, neither variable alone can explain the majority of total variation observed in map accuracy. When the cumulative effect of sample size and map complexity is considered, 93.1% of the variation in map accuracy can be explained. Notice that their individual influence is not additive, that is, 0.215 + 0.466 = 0.671 which does not equal 0.931. This discrepancy is an expression of the interaction between the two independent variables.

SUMMARY AND CONCLUSIONS

The complexity of a geographic distribution, referred to as map complexity throughout this paper, is shown to have an important influence on the accuracy of interpolated dasymetric maps using data derived from a controlled experiment. Its effect on map accuracy is influenced by other variables in the sample-interpolation mapping procedure such as sample size. In general, there is a positive curvilinear relationship between sample size and map accuracy, and an inverse linear relationship between map complexity and map accuracy. Map complexity is shown to be the more important variable.

Based upon the results of this experiment and theoretical assumptions, Figure 14 is presented as a summary of the theoretical relationship among map complexity, sample size, and map accuracy. The data show a curvilinear relationship between sample size and map accuracy with an asymptotic leveling off at larger sample sizes. Although the experiment did not consider extremely small sample sizes, one can assume the existence of another horizontal asymptote approaching a map accuracy level of 0. Therefore, the theoretical curve proposed is logistically shaped with two horizontal asymptotes approaching map accuracy levels of 0 and 100%. Map complexity is also important in determining map accuracy. The data suggest its influence shifts the asymptotes towards larger sample sizes as map complexity increases. This has the effect of reducing the slope of the linear portion of the logistic curve.

In summary, map accuracy is proposed to be logistically related to sample size wherein the position of the asymptotes are

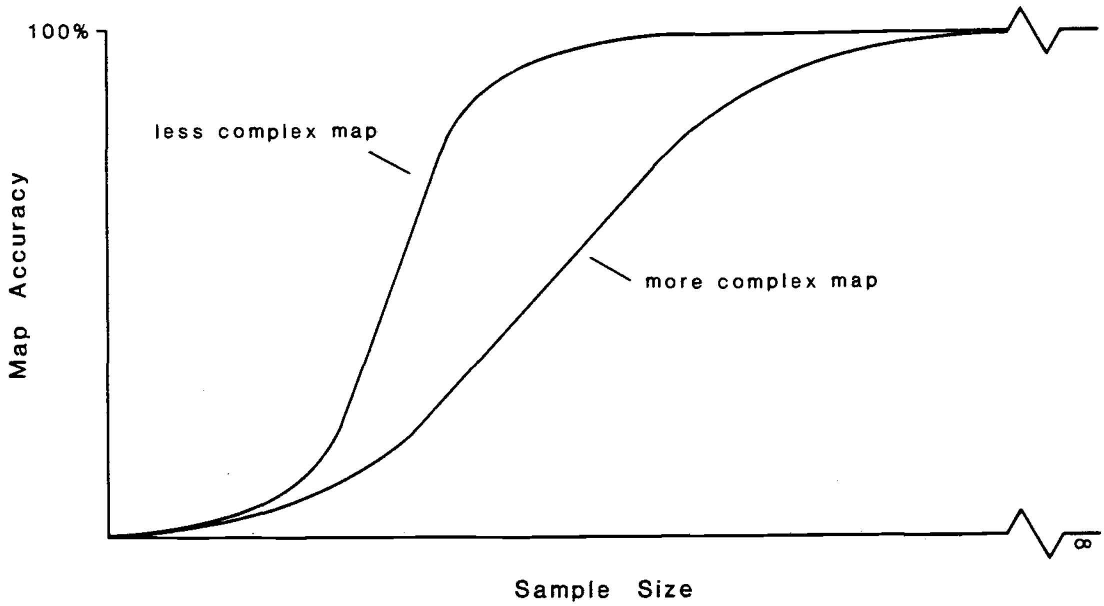


Figure 14. Theoretical relationship among map complexity, sample size, and map accuracy

determined by the level of map complexity.

The results of this experiment can be used to develop a multiple regression equation for application in the fields of computer cartography and geographic information system (GIS) design. Cartographers and GIS designers can use the regression equation as a guideline for selecting a sufficient sample size for attaining the level of accuracy required for the mapping project. The equation developed is as follows:

$$y = 0.67 - 2.39x_1 + 0.12x_{2\log} + 0.51x_1x_{2\log}$$

where y equals map accuracy, x_1 is map complexity, x_2 is sample size, and x_1x_2 is a term for the interaction between sample size and map complexity.

For most projects the complexity of a geographic distribution is not known until it has been mapped. Therefore, to use the equation presented above, it is necessary to estimate the complexity prior to mapping which may be accomplished in several ways. For example, in some mapping projects the objective is to update an existing map or to map an area at a larger scale. Under these conditions the existing map's complexity may be used as an estimate of the new map's complexity. If no previous mapping has occurred, a phenomenon's distribution complexity may be estimated during field reconaissance or through remote sensing techniques. In cases where environmental phenomena are spatially associated, such as soil type and geology or vegetation association and soil type, and one of the phenomena has previously been mapped, the distribution complexity of the mapped phenomenon may be used as an estimate of the other.

There are also applications of this research for archiving digital maps and to GIS design. A complete digital map can be stored in a skeletal form as a sample of point coordinates and their associated attribute codes. In small GIS's where computer memory must be economized, the dasymetric maps in the thematic data base may be stored in this form. Immediately prior to their use in analytical operations, the skeletal maps can be completed through interpolation. This procedure may conserve a significant amount of memory particularly when used with less complex maps. Since in this application the maps already exist, their complexity can be measured directly rather than estimated and the regression equation can be used for selecting an adaquate sample size.

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APPENDICES

```
C***************** PROGRAM AUTO **********************
C
C BRIAN K. YOUNG                      AUGUST, 1983
C
C****************************************************
C
C THIS PROGRAM READS A MATRIX OF NUMBERS AND COUNTS THE
C NUMBER OF CELLS HAVING IDENTICAL NEIGHBORS. A NEIGHBOR
C IS DEFINED AS CELLS SHARING A COMMON EDGE. THE SPATIAL
C AUTOCORRELATION COEFFICIENT "A" IS CALCULATED AS THE
C PROPORTION OF CELLS WITH IDENTICAL NEIGHBORS.
C
C
C
          REAL COEF,RS,RN
          INTEGER X(50,50),I,J,S,N,MAXI,MAXJ
          COMMON X,I,J,S,N,MAXI,MAXJ
C
          OPEN (5,FILE='MAP6')
          OPEN (10,FILE='COEF')
          I=0
          J=0
          S=0
          N=0
          COEF=0
          MAXI=50
          MAXJ=50
C
          READ (5,25) ((X(I,J), J=1,MAXJ), I=1,MAXI)
25        FORMAT(50I1)
          DO 100,I=1,MAXI
          DO 50, J=1,MAXJ
C
          IF(I .EQ. 1 .AND. J .EQ. 1) THEN
                  CALL EAST
                  CALL SOUTH
          ELSE IF (I .EQ. 1 .AND. J .NE. 1 .AND.J .NE.MAXJ) THEN
                  CALL EAST
                  CALL SOUTH
                  CALL WEST
C
          ELSE IF (J .EQ. MAXJ .AND. I .EQ. 1) THEN
                  CALL SOUTH
                  CALL WEST
C
          ELSE IF(I .NE. MAXI .AND. I .NE. 1 .AND. J .EQ. 1) THEN
                  CALL NORTH
                CALL EAST
                CALL SOUTH
C
          ELSE IF (I .EQ. MAXI .AND. J .EQ. 1) THEN
                CALL NORTH
                CALL EAST
C
          ELSE IF (I .EQ. MAXI .AND. J .NE. 1 .AND. J .NE. MAXJ) THEN
                CALL WEST
                CALL NORTH
                CALL EAST
C
          ELSE IF (I .EQ. MAXI .AND. J .EQ. MAXJ) THEN
                CALL WEST
                CALL NORTH
C
```

Figure 15. Program AUTO

```
          ELSE IF (I .NE.1 .AND. I .NE. MAXI .AND. J .EQ. MAXJ) THEN
                  CALL SOUTH
                  CALL WEST
                  CALL NORTH
C
          ELSE
                    CALL NORTH
                  CALL EAST
                  CALL SOUTH
                  CALL WEST
          END IF
C
50        CONTINUE
100       CONTINUE
C
          RS=S/2
          RN=N/2
C
          COEF=RS/(RS+RN)
          WRITE (10,200) N,S,COEF
200       FORMAT('N=',I6,3X,'S=',I6,3X,'AUTO COEF=',F7.3)
          CLOSE (10,STATUS='KEEP')
          END
C
C
C
          SUBROUTINE NORTH
          INTEGER  X(50,50),  I, J, S, N, MAXI, MAXJ
          COMMON   X, I, J, S, N, MAXI, MAXJ
          IF ( X( I, J) .EQ.  X( I-1, J)) THEN
                    S= S+1
          ELSE
                    N= N+1
        END  IF
        RETURN
        END
C
C
        SUBROUTINE EAST
        INTEGER X(50,50), I,J,S,N,MAXI,MAXJ
        COMMON X,I,J,S,N,MAXI,MAXJ
        IF(X(I,J) .EQ. X(I,J+1)) THEN
             S=S+1
        ELSE
             N=N+1
        END  IF
        RETURN
        END
C
C
        SUBROUTINE SOUTH
        INTEGER X(50,50), I,J,S,N,MAXI,MAXJ
        COMMON X,I,J,S,N,MAXI,MAXJ
        IF(X(I,J) .EQ. X(I+1,J)) THEN
               S=S+1
        ELSE
               N=N+1
        END  IF
        RETURN
        END
C
C
        SUBROUTINE WEST

        INTEGER X(50,50), I,J,S,N,MAXI,MAXJ
        COMMON X,I,J,S,N,MAXI,MAXJ
        IF(X(I,J) .EQ.X(I,J-1)) THEN
               S=S+1
        ELSE
               N=N+1
        END  IF
        RETURN
        END
```

Program AUTO

```
C************** PROGRAM RANDOM *****************
C
C BRIAN K. YOUNG                    SEPTEMBER, 1983
C
C**********************************************
C
C THIS PROGRAM PERFORMS A STRATIFIED RANDOM SAMPLE
C FROM "MAPI". THE DATA POINT COORDINATES ARE WRITTEN
C ONTO "TAPE20" AND THE DATA POINT VALUES ONTO "TAPE25",
C IN A FORMAT COMPATIBLE FOR INPUT INTO THE SYMAP PROGRAM.
C
C
C
          INTEGER MAP(50,50),I,J,N,M,X,Y,XP,YP,S,T,Q
          INTRINSIC RANF, IFIX
          OPEN (5,FILE='MAP1')
          OPEN (20,FILE='TAPE20')
          OPEN (25,FILE='TAPE25')
          CALL RANSET (145)
          READ (5,10) ((MAP(I,J),J=1,50),I=1,50)
10        FORMAT (50I1)
          DO 75, Q=1,20
          S=0
          T=0
          DO 50, N=1,5
          DO 25, M=1,5
               X= IFIX (10*RANF () )
               Y= IFIX (10*RANF () )
               IF (X .EQ. 0) THEN
                                   X=10
               END IF
               IF (Y.EQ.0)  THEN
                                   Y=10
               END IF
               XP=X
               YP=Y
               WRITE (20,15) XP+S, YP+T
15             FORMAT (10X, I2, '.', 12X, I2, '.')
               WRITE (25,20) MAP(XP+S,YP+T)
20             FORMAT (11X, I2, '.')
               T=T+10
25             CONTINUE
          S=S+10
          T=0
50        CONTINUE
75        CONTINUE
          WRITE(20,85)
85        FORMAT('99999')
          WRITE(20,95)
95        FORMAT('E-VALUES               X')
          END
```

Figure 16. Program RANDOM

```
BRIAN.
USER,DU3I8C,JANICE.
CHARGE,848040.
TITLE./BRIAN K. YOUNG
SETTL,100.
GET,SYMAPB/UN=LIBRARY.
LDSET,PRESET=ZERO.
SYMAPB.
SAVE,TAPE8.
--EOR--
B-DATA                    X
          7.                 18.
          2.                 40.
         17.                  7.
         11.                 29.
         12.                 41.
         27.                 18.
         28.                 37.
         31.                  6.
         36.                 23.
         42.                  7.
         45.                 24.
         44.                 41.
99999
E-VALUES                  X
           0.
           1.
           0.
           1.
           1.
           0.
           1.
           0.
           1.
           3.
           0.
           1.
99999
F-MAP
25 SAMPLE POINTS
MAP1.1
BRIAN K. YOUNG
    1     5.1       5.1
    2     0.0       0.0        51.0       51.0
    3     10.
    4     0.0
    5     9.0
    7
0123456789**********
                                                                C
                                                                C
                                                                 C
    8
   15     10.       10.
   21
   31
   36
   37
99999
999999
```

Figure 17. Sample SYMAP job stream

```
C****************** PROGRAM DAVE *******************
C
C THIS PROGRAM REFORMATS THE OUTPUT OF SYMAP FMAP ELECTIVE
C 21, MAKING IT COMPATIBLE WITH PROGRAM CHECK. "DAVE" WAS
C WRITTEN BY DAVE FUHRER WHOM I WISH TO THANK.
C
C
C
          INTEGER MATRIX(50,50), R,S
          REAL MAP(150,150)
          OPEN (15,FILE='INTMAP')
          READ (8) NR,NC
          DO 100 J=1,NR
          READ (8) (MAP(J,K), K=1,NC)
 100       CONTINUE
          DO 200 J=1,50
          DO 150 K=1,50
              R=J
              S=K
              MATRIX(R,S) = MAP(J,K)
150       CONTINUE
200       CONTINUE
          WRITE(15,250) ((MATRIX(R,S), S=1,50), R=1,50)
250       FORMAT(50I1)
          END
```

Figure 18. Program DAVE

```
C****************** PROGRAM CHECK ***********************
C
C BRIAN K. YOUNG                          SEPTEMBER, 1983
C
C THIS PROGRAM OVERLAYS TWO GRID CELL MAPS AND COMPARES ONE
C TO THE OTHER, CELL BY CELL, AND CALCULATES THEIR
C COEFFICIENT OF AREAL CORRESPONDENCE. FURTHER, IT PRODUCES
C A MAP OF THE RESIDUALS ON "TAPE17".
C
C
C
          CHARACTER MAP(2500)
          INTEGER X(50,50), Y(50,50), I, J, ROW, COL, N, S, Q, K,F,U
          REAL ACC, W
          OPEN (10,FILE='MAP1')
          OPEN (15,FILE='INTMAP')
          I=0
          W=0
          J=0
          ROW=0
          COL=0
          N=0
          S=0
          Q=1
          F=1
          U=50
          ACC=0
          READ (10,25) ((X(I,J), J=1,50), I=1,50)
25        FORMAT (50I1)
          READ (15,45) ((Y(ROW,COL), COL=1,50), ROW=1,50)
45        FORMAT (50I1)
          DO 155, I=1,50
          DO 150, J=1,50
          ROW=I
          COL=J
          IF(X(I,J) .EQ. Y(ROW,COL)) THEN
                          S=S+1
                         MAP(Q)='.'
               ELSE
                          N=N+1
                         MAP(Q)='X'
          END IF
          Q=Q+1
150       CONTINUE
155       CONTINUE
          Q=1
          W=S
          ACC=W/2500
          WRITE (17,160) N,S,ACC
160       FORMAT('N=',I4,3X,'S=',I4,3X,'ACC=',F4.3)
          DO 200, K=1,50
190       FORMAT(50A1)
          WRITE(17,190) (MAP(Q), Q=F,U)
          Q=Q+50
          F=F+50
          U=U+50
200       CONTINUE
          CLOSE(17, STATUS='KEEP')
          END
```

Figure 19. Program CHECK

```
00000000000111111000066070004400222220070071150777
00000000000111110000066677000942222220007771050070
00000000202111100000066660700944411200000771050000
002000002221110000000000660070000011200000711050070
0102222202222220300000000011111000111100000711101070
0100000222200003300000000441111091111100011710411470
0020200022200033300090004111111199991101111700441000
002202022000333100399004411111119999110100000011101
20222220444330211333999991111199990911100055711111
20222000443222211003333996644900009019999005577 1441
22444000430222220000999966660000099900000000537 1441
24433333370002000000333666550550090000000005555 1111
44433322272222000020066666605555000550040055570000
44400222272000000122229999999555595555504444 1570000
44000002272000111122299559995577996655500011001100
44000000000101111122022555995577616666000001011000
44000020020111111222022055669557761666110100011 1115
44000222220111322000200035655556611111111000115555
0442222299111322003020033366666611111730005555666
0542233559913300000000033366666311111733355566555
05533333539303333333003333666663333333333553555555
0553333333333000003330033331111300033333333333 5555
0355533300000000009933333333113300407700555333 35555
44333322000000007099993333333110040770355553333333
44444232200000777099993033300910040776335555555333
22222232222000070099990077700009001776655555555363
22222233332000071119990777004009081777633355557656
22222300333000001199990777004009888877633333555656
22223300302220990019900007011071881177666633565555
22233330000229979019901117011177881117633633565555
22366600000029779910111110000300001111333365556653
2336660990222777991100110000003444111533335555653
23266609222999999011000110000000449111553335355663
22226209222299079011100111990000000911115663533 5555
22222660922222077001101111199700009911116653333777
77776666000028077770011119999770009999157655577777
7778666666000080777700111099007700099445577663 77333
88886666000009000000001110099000099944457777633333
82280666000009900003011100999014499933777677733333
82200666600000909993301100222111119933777777773333
82200888600000900999300000022111111943337777777333
800008886000009900999990000928001114443337777777773
83008888850000990000290000988001104444666727377733
83388898855000999999229901088001104444466223225536
333899995555001449892299011188011000044775533 25366
333899995555001449892299011188011000044775533 25366
31144111551100014441822990008801114411777755595366
311441111111111011441188990288801144111111775555333
3999411111111110444417889902288011411111111177557739
39111111111111101114177899002288444111111111100007799
```

Map 1

```
00000000000011111100006600000442222222007777 7100077
0000000000011111000000660000044222222200077771000007
00000000202111100000066666000044441120000071 1000007
0020000022211100000066666000000001120000071 1000077
000022220222220000000000111110001111000017111100077
00000002222200033000000009111110991111011117101 11077
002020022200003330009000911111119991111110700111111
002202022000333300399000911111199991111000000011111
202222233333322333333990991111999999911100055511111
222222333333222333303399996666900999900010055551111
2243333333222222000099996666000000999000000555511111
24433333322222220000999666600550090000000005555 1111
444333222222220000299666666555555000550000055550000
444002222220000001222669999555555555555000001100000
440000022220001111222299999555555555550001 1101100
44000000000000011122222299999555566666600001 1110000
440000000001111122002209966955556666661101 000111155
44000222220111222000200036665556611111111000115555
24422222220113220000200033366666611111130005555555
22423332222133000000000333366666611111333355555555
22233333232303333333000033316666333333333355555555
22233333333300000330033333111330333333333333555555
22222233000000000993333333331133000077775553333 5555
22333332000000000999999333333300000077735555333333
22222232220000999999990333300000000777335555555333
222222332220000999999990007700000000077733555555353
22222233332000090099900777700000080077733355555555
22222300333000000999007770000088887773333555555
22223300302220999099990007770000088117777773355555
22233330002229999011001117000000881117733733555555
22336600002229999001011110000000081111133333555555
23366600002229999011001110000000081111133333555555
232666000229999990110001100000000811111333335555
2222660022299099011100111180000009111117333335555
2222266002222099001101111880000099911177333377777
8888666600002209999001111888888000999117773377777
8888666666000000099900111188008000999944177773373 33
88886666000000000000011100880000999944477777333333
8888066600000090000900110088800099994337777733333
88800666600000900999900100888111119943377777773333
8800088660000009009999000000081111111443337777777333
80000888600000990099999000000880111444433377777777 3
800088885000099000099000000800111444444333337773 3
800888885000099400099900000880011144444443333335533
0008888855555044400099990000880111111447755 5335555
00088811555500044440999900000880111111177775553355 55
044441115511000444440999900008801111111777775555555
044441111111100444444099900008000111111117755 5555
04444111111111104444440099000080000111111117755 5555
0441111111111104444440099000088000111111117777 5555
```

Map 2

Figure 20. Digital form of Maps 1 and 2

```
000000000000111110000000000000222222222200000001000000
000000000000111110000000000000222222222200000001000000
000000002022111000000000000000222221200000001100000
002000002222110000000000000000000011200000001100000
000222200222220000000000000011110001111000000011100000
000000022022200030000000009111110111111001101011 1000
00202000222000033000900099111111111111111000011111
00200202220003330009900099111111111111110000000 1111
2022222223303323333399099911111111111111000055 11111
22222223333322233333339999999911011110001105555 1111
22233333332222220000999999999900001110000000555 51111
2233333332222222000099999999900550110000000055511111
233333222222220000099999999905550000500000055511111
33333322222200000030099999999555555555506000555 1111
33300002222000033330009999995555555555550000 1111111
330000002200003333000009999955555555555500011111111
330000000003333330000009995995555555555300000111115
330002222203333000000000335555555533333330001 15555
333022222220330000000000033355555555333333000555555 5
2232233222222330000000000333355555553333333335555555
22223333222223333333300033335555333333333335555555
22223333333330000033300033333333300333333333335555
222222333000000000993333333333333300007777755333333555
22333333330000000099999333333333000000777335553333333
22233233300000009999999003330000000007773355555533 3
2222223333300000099999900011000000000777335555555333
22222233333300009900999001110000001007777333555555 5
2222223033330000000009990011100000011177773333555555
2222333033322200090099000111000000111777777335555 55
22223330000222999011901111100000011111733733555555
2222333000022299990100111100000001111113333355555 5
22236600000222999901001110000000011111113333355555
2222666000299999900110001100000000111111113333335555
22226660222229909001110011188000004411111773333355 5
22222660022222009900100111188000000441117773333 33
222226660002020099900011111888888800044411 7777333333
888886666600000099990011188008800044444177773333 33
8888866666000000009900011000800000444444777773333 3
888806666600000400000001100088800044443377777773333
8888066666000044099999001000888111144433777777777333
8880008666000044099999900000008811111144333777777733
8800008866600004400999990000000800111444443337777777773
880008888800004440009900000000800111444444333333377773
8800888888100044400099900000880011144444443333335533
00088888811110444000999990008800111111447333335555
000888811111100044400999900008001111111777555355555
0088881111111000444440999900008001111111777755555555
00888811111111000444440999000080000111111117777555555
00888811111111100444440099900008000111111117775555 5
00881111111111100444444099900008000011111117777755555
```

Map 3

```
0000000000001111100000000000000000000000000000000100000
00000000000011111000000000000000000000000000001000000
0000000000000111000000000000000000000100000000 1100000
000000000000011000000000000000000011000000001100000
000000000000000000000000000001111100011110000000 1110000 0
000000000000000000300000000911111011111100110101 11000
0000000000000003300090009911111111111111100001 1111
00000000000000333000990009911111111111111100000001111
000000000330330333339909991111111111111110000331 1111
000000033333000333333999999991101111000110333331111
0003333333300000000000999999999990000111000000033331111
00333333330000000000009999999990033011000000000333 1111
033333000000000000000999999999033300000300000033311111
33333300000000000000030099999999333333333306000333 1111
33300000000000000333300099999993333333333330000 1111111
330000000000000333300000099999333333333333000111111111
330000000000333333000000099939933333333333300000111113
33000000000033330000000000333333333333333330001 13333
3330000000000330000000000033333333333333333300033333333
0030033000000330000000000333333333333333333333333333333
000033330000033333330003333333333333333333333333333333
0000333333330000033300033333333300333333333333333333
000000333000000000993333333333333300007777733333333333
003333333300000000999993333333330000000777333333333333
000330333000000099999990033300000000000777333333333333
0000003333300000099999900011000000000777333333333333
000000333333000099009990011100000010077773333333333333
00000030333330000000099900111000000111777733333333333
0000333033300000090099000111000000111777777733333333
000033300000000099901119011111000000111117337333333333
00033333000000000999010011111000000001111113333333333
0003660000000000999901001110000000001111111333333333 3
000066600009999990011000110000000011111113333333333
0000666000000990900111001111100000441111773333333 3
0000066000000000009900100111111100000044111777333333 3
00000666000000000999000111111111100044411777733333 3
1111166666000000999900111110011000444441777773333333
1111166660000000000990001100010000044444477777733333 3
111116666000004000000001100011100004444433777777773333
1111166660000044099999001000111111114443377777777773 33
1111111666000044099999900000011111114433377777777733
111111111660000440099990000000100111444433377777777773
111111111111000044400099000000001001114444433333337773
1111111111111000444000999000000110011144444443333333333
111111111111111044400099999000110011111114473333333333
111111111111111000444009999000001001111111177733333333
1111111111111110004444409999000001001111111177773333333
11111111111111110004444409999000010000111111177733333 3
1111111111111111100444444009990000100011111111777333 33
11111111111111111004444440999000010000111111117777 3333
```


Figure 21. Digital form of Maps 3 and 4

```
000000000000000000000000000000000000000000000111111
000000000000000000000000000000000000000000001111111
000000000000000000000000000000000001000000001111111
000000000000000000000000000000000011000000001111111
000000000000000000000000001111000111100000011111111
000000000000000330000000011111111111110011011111111
000000000000003330001000111111111111111111000011111
000000000000333330311000111111111111111100000011111
000000000333333333331111111111111111111100001111111
000000333333003333331111111111000111000100111111111
003333333300000000001111111100000111000000011111111
033333333000000000001111111000330100000000011111111
333333300000000000311111111033330000000000001111111
330000000000000003300111111333333333330000011111111
330000000000003333300011111333333333333000001111111
330000000000003333000000111113333333333300011111111
330000000000333330000000011313333333333300000111113
330000000003330000000000333333333333333330001133333
333000000000330000000000333333333333333300003333333
330033300003330000000000333333333333333333333333333
000033333333300000000000333333333333333333333333333
000333333333300000000000333333333333333333333333333
003333333000000009900003333333000001111333333333333
003333330000000009999013333330000001113333333333333
000000333000000999999133330000000001113333333333333
000000333300009999999111100000000001113333333333333
000000333330000999999911110000001101113333333333333
000003303330000999999111111000000111111133333333333
000033303300000999999911111100000011111111133333333
000333300000009999999111110000000011111331333333333
000333000000009999999111110000000001111133333333333
000333000000009999999111110000000001111133333333333
000333000009999999999111100000000011111133333333333
000033300000009999999911111110000004411111333333333
000033300000000099999911111111000004411111113333333
111133333000000000999911111111110004444111113333333
111133333000000009999111111100110004444411111333333
111133330000000000999111100110000044444411113333333
111113333000000000999011100011000044444411111133333
111113333000000000999900100011111114444411111113333
111111133000000000099990000000111111444441111111333
111111113000000000999900000000000114444444111111113
111111111110000000000990000000000114444444333311133
111111111110000000009990000000000114444444333333333
111111111111100000000999900000000011111111333333333
111111111111000000000999900000000011111111133333333
111111111111100000000099990000000001111111113333333
111111111111110000000009990000000001111111111333333
111111111111111000000000990000000001111111111333333
111111111111111000000000990000000001111111111113333
```

Map 5

```
000000000000000000000000000000000000111111111111111
000000000000000000000000000000000001111111111111111
000000000000000000000000000000000011111111111111111
000000000000000000000000000000000011111111111111111
000000000000000000000000111110011111111111111111111
000000000000000000000000111111111111111111111111111
000000000000000000001100011111111111111111111111111
000000000000000000011000111111111111111111111111111
000000000000000000001111111111111111111111111111111
000000000000000000001111111111111111111111111111111
000000000000000000001111111111111111111111111111111
000000000000000000001111111113311111111111111111111
000000000000000000001111111133331113311111111111111
000000000000000000000011111113333333333111111111111
000000000000000000000001111113333333333111111111111
000000000000000000000001111313333333333311111111111
000000000000000000000000113333333333333333111111133
000000000000000000000000333333333333333331111133333
000000000000000000000000333333333333333333333333333
000000000000000000000000333333333333333333333333333
000000000000000000000003333333333333333333333333333
000000000000000000000003333333331111111113333333333
000000000000000000000003333333311111111113333333333
000000000000000000000003333333111111111333333333333
000000000000000000000001133311111111111133333333333
000000000000000000000001111111111111111133333333333
000000000000000000000011111111111111111133333333333
000000000000000000000011111111111111111113333333333
000000000000000000000111111111111111111111333333333
000000000000000000000111111111111111111133333333333
000000000000000000000111111111111111111133333333333
000000000000000000000111111111111111111133333333333
000000000000000000000011111111111111111113333333333
000000000000000000000011111111111111111111133333333
000000000000000000000011111111111111111111113333333
111100000000000000000011111111111111111111111333333
111100000000000000000011110011111111111111111333333
111100000000000000000011110011111111111111111333333
111110000000000000000001100011111111111111111133333
111110000000000000000000000011111111111111111111333
111111100000000000000000000000011111111111111111133
111111111000000000000000000000011111111111111111133
111111111110000000000000000000000111111113333311333
111111111110000000000000000000001111111133333333333
111111111111100000000000000000001111111133333333333
111111111111000000000000000000001111111133333333333
111111111111000000000000000000001111111111333333333
111111111111100000000000000000000011111111133333333
111111111111110000000000000000000011111111113333333
111111111111110000000000000000000011111111111113333
```

Map 6

Figure 22. Digital form of Maps 5 and 6