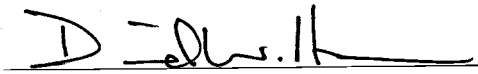
AN ABSTRACT OF THE DISSERTATION OF

Sean J. Canavan for the degree of Doctor of Philosophy in Forest Resources presented on December 20, 2001. Title: The Presence and Characterization of Measurement Error in Forestry.

Abstract approved: 

David W. Hann

Measurement error (ME) is a component of any study involving the use of actual measurements, but is often not recognized or is ignored. The consequences of MEs on estimates of tree and stand attributes and the parameters and predictions of forest models can be varied and severe, including the presence of bias and increased variance. While correction methods do exist for countering the effects of MEs, these methods require knowledge of the distribution of the errors. A new method for directly modeling error distributions called the two-stage error distribution (TSED) method is presented. This method is compared with traditional methods for error modeling through examples using diameter (D) and height (H) MEs. Comparisons are done based on a measure of dissimilarity between their fitted error distribution surfaces and the empirical error surface. Results indicate that the TSED method produces a much more accurate characterization of the ME distributions than traditional methods when a high percentage of errors are zero. When few measurements are exactly correct, the TSED method works as well as the most accurate form of the traditional method. The TSED method also performs better at characterizing distributions with asymmetric tails. It is therefore more adaptable than traditional methods and should be used for future error modeling.

Variables included in forest models are often not simple variables such as D and H, but rather transformations of these variables. Applying correction techniques to account for MEs in these transformed variables requires distributions for the MEs of these variables. This information is often not available and may be time-consuming and expensive to collect and model. Indirect derivation methods are examined for obtaining the error distributions of transformed variables when the error distributions for their component variables are known. Statistical error distributions for D^2 and D^2H are first indirectly

derived using known error distributions for D and H, and then directly estimated using the TSED method and under the assumption of normally distributed errors. Results indicate that indirect derivation can lead to error characterizations of the transformed variables as accurate or more accurate than direct estimation of the error distributions.

The Presence and Characterization of Measurement Error in Forestry

by
Sean J. Canavan

A DISSERTATION

Submitted to

Oregon State University

in partial fulfillment of
the requirements of the
degree of

Doctor of Philosophy

Presented December 20, 2001
Commencement June 2002

ACKNOWLEDGEMENTS

I've been here at OSU for a little over four years and believe it or not, school is finally going to end. After 10 years of higher education, it's time to move out into some version of the 'real world' and earn a living wage. This road has been longer than I would have imagined at any point prior to being on the different parts of it. There are many people who deserve thanks for helping me stay on it and for helping me not go crazy during the journey. Okay, so maybe I haven't been entirely successful on that last point. Life in the dungeon, er, basement, will do that to anyone!

My thanks, first and foremost, goes to my major advisor, Dr. David Hann. His ideas and guidance made it possible to get through this dissertation process. My thanks to him go beyond this, however, to include appreciation for many great meals and conversations which broadened my education well beyond the area of forest modeling. The wallpaper of information he has provided in my cubicle has served as a series of lessons in applied math, life in the workplace, and male-female relationship dynamics. It would be grossly incorrect for one to assume that the scope of the conversations in Peavy 053A is limited to topics in the area of quantitative forestry.

I would like to express my gratitude also to my committee members, Doug Maguire, Paul Murtaugh, Dan Schafer, and Eldon Olsen, for their comments which have led to an improved version of this dissertation. Thanks also to Doug for lots of advice, future employment, a heck of a lot of teaching experience, and for reminding me that data are plural and the word 'use' should only be used so often. Scott Urquhart, a former committee member, also deserves thanks here for a large amount of help with the simulation aspect of this project.

The square root of 5248 is 72.4430811050993

Completing this degree would not have been possible without the funding I received during the process. In this respect, thanks are also due to Jack Walstad and the Forest Resources department here at Oregon State University for my graduate student appointment and the freedom I was afforded in choosing

my research direction. I am also very grateful to the families of Dorothy D. Hoener, Lu Alexander, and Alfred W. Moltke for the fellowships I received during my time here.

Thanks are also owed to Susan Theirl for her careful reading of the different chapters of this dissertation and her comments with how to improve the grammatical aspects of it. I'm often guilty of writing like I speak when not careful. So, I'd like to thank her for her many suggestions which helped tighten up the readability of the papers contained here, and also for always encouraging me to take time off and spend the holidays with family despite other pressures to not do so <g>.

There are many friends without whom this whole ordeal would have been much more difficult. Thanks to Partha, Maya, Karthik, and Julian for coming to visit, having me at your places and continuing to be great friends. Thanks also to Cat, Josh, Ryan, Marie, Matt, Christina, John, Jeff, Mark, Shay, Brian, various Harris Lab folks, and many others for basketball, climbing, biking, hiking, crabbing, fishing, movies, parties, barbecues and other meals, games, music, conversations, and loads of hanging out. Spoon left.

A very large debt of gratitude is also owed to my family. Thanks to my parents for your support through the thick parts and the thin parts of this experience and to my brother and sisters who are some of my best friends. Knowing that you're always there if I need you keeps me going.

And last, but not least, thanks to Timber, my 110lb. yellow lab, for keeping me humble by acting as a constant reminder that, no matter how many degrees I accumulate, my only real purpose in life is to play with him, make sure he's fed on a regular basis, and provide him suitable accommodations to sleep for at least 15 hours a day. Which makes me wonder, are humans really the superior species?

Thanks again everyone.

Sean

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

LIST OF FIGURES

LIST OF TABLES

THE PRESENCE AND CHARACTERIZATION OF MEASUREMENT ERROR IN FORESTRY

INTRODUCTION

This dissertation is presented in the form of three distinct manuscripts. While each manuscript may stand alone, together they take the reader through the history of measurement error research in forestry up through the latest in measurement error distribution modeling. The goal of this combined work is to convince forest managers and researchers that measurement errors should not be taken lightly, that correction methods do exist, and that traditional methods for producing error distribution models (i.e., the normal distribution) to be used in the correction techniques may not be satisfactory in comparison to the new method described here.

The practice of forestry requires accurate and precise information concerning the current or future state of many tree and stand attributes. Some of these attributes are routinely measured directly (e.g., a tree's current diameter at breast height). Other attributes are either too costly or impossible to measure directly. As a result, either sampling is done to estimate the attribute (e.g., a stand's current basal area per acre) or a statistical model is used to predict the attribute (e.g., a tree's current total stem cubic foot volume) from measured attributes. All attributes, whether measured directly, sampled, or predicted, are subject to error. All direct measurements are subject to the misuse or poor choice of measurement tools (this type of error will be termed "mensuration error") or to rounding the measurement (to be termed "grouping error"). By definition, sampling will result in an estimate with "sampling "error. Predicted values are subject to modeling error.

The error resulting from the sampling process or dwelling in the response variable of a regression model has seen considerable recognition and discussion in the forestry literature (e.g., Schreuder et al. 1993, Philips 1994, Shiver and Borders 1996, Avery and Burkhart 2001). The description, consequences, and control of error in the predictor, or independent, variables in regression modeling has received less attention in the forestry literature than it deserves. Work from econometrics (e.g., Kmenta 1997) and statistics (e.g., Fuller 1987 and Carroll et al. 1995) have shown that error in the predictor variables can

bias the estimates of the parameters and the resulting model output, and such error can increase the standard errors for the parameter estimates.

The objective of Chapter 1 is to examine the consequences and possible correction techniques for stochasticity and/or measurement error in predictor variables when developing regression models. The presence of, consequences of, and correction methods for stochastic predictor variables are examined first. This is followed by an examination of the presence of, consequences of, and correction methods for measurement errors in fixed predictor variables. Published information on the sizes and variability of mensuration errors in common forest measurements is also presented.

Stochasticity in a variable can arise as a result of the process by which the values of the variable were selected from the population, and is typically ignored. Predictor variables are assumed to be fixed (nonstochastic) at known levels prior to selection. The response variable(s) are then randomly selected from the population defined by these fixed levels of the predictor variable(s). If the values of the predictor variable(s) are randomly selected along with the response variables, then the predictor variables are considered to be stochastic. Consequences of stochastic model predictor variables are discussed in detail as well as methods for correction to account for stochasticity.

A measurement error arises when there is a difference between an observed or estimated value for an attribute and the actual, or population, value for the attribute. Measurement errors are unavoidable due to the level of precision to which measurements are made. In situations where the measurement errors do not result from the sampling procedure, increasing the sample size is not a viable method for reducing their effects; rather than canceling out, the measurement error effects may be cumulative. Consequences of measurement errors in model predictor variables are discussed in detail as well as methods for correction to account for their presence.

Procedures exist to correct for the effects on models of measurement errors in the predictor variables. Implementing these methods often requires knowledge about the distributions of the errors. This information is typically not readily available and may be time-consuming and expensive to collect and characterize. The objective of Chapter 2 is to compare a new method for modeling error distributions to traditional methods. A description of the traditional methods and their positive and negative aspects is followed by a description of a new method called the two-stage error distribution (TSED) method. The

two methods are then applied to actual measurement error data for two variables and compared based on their abilities to model the empirical error distributions. These methods are particularly germane to the characterization of mensuration error, though they may have utility for characterizing the distributions of grouping or sampling errors.

A complicating factor in dealing with measurement errors is that predictor variables in forest models are often not simple variables such as diameter (D), total height (H), or height to crown base (HCB). Instead, transformed variables that are functions of these and other simple variables are typically used. These functions may be univariate, such as D^2 or ln(D), or they may be multivariate, such as D^2H or crown ratio (CR), where CR =1.0-HCB/H. Although there are many transformations of potential interest, information for the measurement error distributions of these transformed variables is extremely rare. This problem is further exaggerated by the number of alternative procedures for measuring each simple variable, with each variable and measurement method combination having its own measurement error distribution. A potential solution to this problem is to indirectly derive the measurement error distributions for transformed variables from the measurement error distributions of known simple variables.

The objective of Chapter 3 is to determine whether the error distributions for transformed variables could be obtained through indirect derivation via error propagation formulas or simulation when the error distributions for the component simple variables are known. The intent is to provide a method that avoids the time-consuming processes of having to collect error data for transformed variables and then perform the complex modeling needed to directly estimate their distributions. To explore the potential of the indirect derivation method, the measurement error distributions for D^2 and D^2H, two common predictor variables in individual tree volume equations, are indirectly derived through error propagation formulas or, if necessary, simulation. These indirect derivations are then compared to directly estimated measurement error distributions for D^2 and D^2H. These directly estimated distributions are developed under the assumption of normally distributed errors and using the (TSED) method.

Chapter 1

AN OVERVIEW OF STOCHASTICITY AND MEASUREMENT ERROR IN FOREST MODELS

Sean J. Canavan and David W. Hann

Abstract

Forest management decisions require accurate and precise information concerning the current or future state of many tree and stand attributes. Whether they are measured directly, sampled, or predicted from an equation, all attributes are subject to error. This error can manifest itself as a systematic (i.e., biased) error, a random error, or a combination of both. Stochasticity and/or measurement error in model predictor variables can have severe effects on estimates of model parameters, yet they are often ignored. Stochasticity arises as a result of the process used to select the values of a variable from the population. Consequences of its presence can include biased and inconsistent parameter estimates, making traditional model fitting procedures inappropriate. Measurement error arises through the misuse or poor choice of measurement tools, rounding of measurements, or through sampling. Its effects can include biased and imprecise model parameter estimates, with the resulting models indicating invalid relationships between variables. The objective of this paper is to examine the consequences and possible correction techniques for stochasticity and measurement error in predictor variables when developing regression models in forestry. The presence of, consequences of, and correction methods for stochastic predictor variables are presented first, followed by the presence of, consequences of, and correction methods for measurement errors in model predictor variables. Published information on the sizes and variability of mensuration errors in common forest measurements is also presented.

Introduction

The practice of forestry requires accurate and precise information concerning the current or future state of many tree and stand attributes. Some of these attributes are routinely measured directly (e.g., a tree's current diameter at breast height, current total height, current height to crown base, or current breast height age). Other attributes are either too costly or impossible to measure directly. As a result, either a sample is taken to estimate the attribute (e.g., a stand's current number of trees per acre or current basal area per acre) or a statistical model is used to predict the attribute (e.g., a tree's current total stem cubic foot volume, future diameter growth rate, or future height growth rate, and a stand's future number of trees per acre, future basal area per acre, future volume per acre, or future top height).

All attributes, whether measured directly, sampled, or predicted, are subject to error. All direct measurements are subject to the misuse or poor choice of measurement tools (this type of error will be termed "mensuration error") or to rounding the measurement (to be termed "grouping error"). By definition, sampling will result in an estimate with "sampling "error. Statistical regression models recognize and assume that error is present in the response, or dependent, variable but not the predictor, or independent, variable. Regardless of the source, the resulting error can manifest itself as a systematic (i.e., biased) error, a random error, or a combination of both.

The error resulting from the sampling process or dwelling in the response variable of a regression model has seen considerable recognition and discussion in the forestry literature (e.g., Schreuder et al. 1993, Philips 1994, Shiver and Borders 1996, Avery and Burkhart 2001). The description, consequences, and control of error in the predictor variables in regression modeling has received less attention in the forestry literature than it deserves. Work from econometrics (e.g., Kmenta 1997) and statistics (e.g., Fuller 1987 and Carroll et al. 1995) has shown that error in the predictor variables can bias the estimates of the parameters and the resulting model output, and such error can increase the standard errors for the parameter estimates.

Much of this past work has concentrated on testing mensuration, sampling, or estimation procedures for possible bias (e.g., see the Appendix and Schreuder et al. 1993, Philips 1994, Shiver and Borders 1996, Avery and Burkhart 2001). However, the presence of bias in predictor variables is not of large concern because it can be removed by correction or by the regression parameters. The latter solution assumes that the equation will always be applied with the biased form of the predictor variable. It is when the errors are random that their effects on attribute estimates or forest models become more problematic.

Stochasticity in a variable, in the econometric sense, can arise as a result of the process by which the values of the variable in question are selected from the population. The response variable in a model is a stochastic variable and the predictor variables are assumed to be fixed (nonstochastic) at known levels prior to selection. Values of the response variable(s) are then randomly selected from the population based on the fixed levels of the predictor variable(s). Such an equation in which the values of the predictor variables are fixed is said to be a functional form model (Kmenta 1997). If the values of the predictor variable(s) are randomly selected along with the response variables, then the predictor variables are considered to be stochastic and the resulting equation is said to be a structural form model (Kmenta 1997). In this paper, stochastic variables will be denoted by y ($y_1, y_2, \ldots, y_m$) and fixed variables will be denoted by x ($x_1, x_2, \ldots, x_n$).

Mensuration error, grouping error and sampling error are sources of measurement error. Measurement error can occur in the stochastic response variable and in stochastic and fixed predictor(s). The random component of measurement error in the response variable will increase the amount of unexplained (and unexplainable) variation about the regression equation. If the measurement errors in the response variable are unbiased, then the resulting regression parameters are also unbiased. Therefore, this paper will only examine measurement error in the predictor variable(s).

The objective of this paper is to examine the consequences and possible correction techniques for stochasticity and/or measurement error in predictor variables when developing regression models in forestry. The paper does not deal with the consequences of introducing measurement error during the

application of existing regression models (e.g., Kangas 1996), but rather on the consequences during model development. The presence of, consequences of, and correction methods for stochastic predictor variables are examined first. This is followed by an examination of the presence of, consequences of, and correction methods for measurement errors in fixed predictor variables. Published information on the sizes and variability of mensuration errors in common forest measurements is presented in the Appendix.

Stochasticity

The Presence Of Stochastic Predictor Variables In Forestry Models

Models may vary in the number of stochastic variables they contain. In the majority of cases, one stochastic variable is taken to be the response variable and any others present in the equation are predictor variables. These equations can be called 'unidirectional,' 'bi-directional,' or 'multidirectional' depending upon their intended application.

Unidirectional Equations

A structural equation involving two or more stochastic variables is unidirectional if the equation will be applied in a manner such that only one of the stochastic variables will be predicted by the equation. In other words, the relationship between the variables is of interest in only one direction. An example is:

$$y_1 = f(y_2, \ldots, y_m, x_1, \ldots, x_n) + \varepsilon_1 \qquad (1)$$

The function f can be linear or nonlinear in form, have parameters to be estimated, and may contain other predictor variables. The y_i's on the right hand side of Equation (1) may be predicted with separate equations, but these equations may not contain y_1 as a predictor variable. They must also be unidirectional functions of fixed variables and/or other stochastic variables. The resulting structural coefficients of the stochastic variables can take on a triangular form (Kmenta 1997) in that the matrix of stochastic variable coefficients, for which each row represents, may be represented by a triangular matrix. The majority of forestry models are unidirectional.

Bi-directional Equations

Two structural equations are bi-directional if the equations will be used to predict two of the stochastic variables with the response variable of each equation acting as a predictor variable in the other equation. For example, along with interest in predicting y_1 from Equation (1), it may also be desired to predict y_2:

$$y_2 = g(y_1, y_3, y_4, \ldots, y_m, x_1, \ldots, x_n) + \varepsilon_2 \quad (2)$$

The relationships in f and g can be linear or nonlinear in form, have parameters to be estimated, and may contain other predictor variables. The function g can be the result of mathematically (or numerically) solving function f for y_2 (i.e., the two are inverses) in which case the two functions share a common set of parameter estimates, or they can be two separate functions, each with its own set of parameter estimates. Examples of bi-directional equations in forestry include:

1. The "bark thickness" equation in which diameter inside bark at breast height (DBH_{ib}) can be predicted from diameter outside bark at breast height (DBH_{ob}) or DBH_{ob} can be predicted from DBH_{ib} (e.g., Ritchie and Hann 1984, Larsen and Hann 1985).

2. The "maximum size-density" equation in which maximum size can be predicted from maximum density or maximum density can be predicted from maximum size (e.g., Weller 1987).

3. The "site index conversion" equation in which the site index of species "A" can be predicted from the site index of species "B" or the site index of species "B" can be predicted from the site index of species "A" (e.g., Hann and Scrivani 1987, Nigh 1995).

4. The "stump diameter" equation in which stump diameter inside bark can be predicted as a function of DBH_{ob} or DBH_{ob} can be predicted as a function of stump diameter inside bark (e.g., Walters and Hann 1986a). This example can include a third predictor variable, stump height, in the relationship.

5. The "dominant height/site index" equation in which dominant height can be predicted from site index and age, or site index can be predicted from dominant height and age (e.g., Curtis et al. 1974, Payendeh 1978, Smith 1984, Hann and Scrivani 1987, Goelz and Burk 1996). This example of a bi-directional bivariate equation also includes a third predictor variable, age, in the relationship.

Multi-directional Equations

Three or more structural equations are multi-directional if the equations will be used to predict three or more of the stochastic variables with the response variable of each equation acting as a predictor variable in the other equations. For example, along with interest in predicting y_1 and y_2 from Equations (1) and (2), it may also be desired to predict y_3, y_4, etc:

$$y_3 = h(y_1, y_2, y_4, y_5, \ldots, y_m, x_1, \ldots, x_n) + \varepsilon_3$$

$$y_4 = k(y_1, \ldots, y_3, y_5, \ldots, y_m, x_1, \ldots, x_n) + \varepsilon_4$$

$$\vdots$$

$$y_j = l(y_1, \ldots, y_{j-1}, y_{j+1}, \ldots, y_m, x_1, \ldots, x_n) + \varepsilon_j$$

A need to fit these or any other set of three or more equations of similar relation would produce a multi-directional situation. The functional relationships in $f, g, h, k, \ldots, l$ can be linear or nonlinear in form, have parameters to be estimated, and may contain other fixed predictor variables. The functions $g, h, k, \ldots, l$ can be the result of mathematically (or numerically) solving function f for $y_2, y_3, \ldots, y_i$ in which case

all of the functions share a common set of parameter estimates, or they can be separate functions, each with its own set of parameter estimates The number of stochastic variables and the number of ways to express their interdependence is limited only by the number of predictor and response variables in the relationship.

An example of multi-directional equations in forestry is the "dominant height/site index" equations which can also be solved to predict age as a function of site index and dominant height (e.g. Monserud 1975, Martin and Ek 1984, Ritchie and Hann 1986, Wensel et al. 1987, Hann and Ritchie 1988, Ritchie and Hann 1990). This example would be deemed a tri-directional equation involving three stochastic variables in which any of the three can serve as the response variable and the other two as the predictor variables.

Consequences Of Stochastic Predictor Variables

The vast majority of the variables in forest models are stochastic variables. When a predictor variable in a model is stochastic, the resulting effects on the properties of the model parameter estimates depend upon the relationship between the stochastic variable, y_j, and the error term of the model, ε:

1. If y_j and ε are independent [so that $E(y_j\varepsilon) = E(y_j)E(\varepsilon)$ and $Cov(y_j,\varepsilon) = 0$] then the inclusion of the stochastic predictor variable will not affect the desired properties of the least squares estimates of the parameters (Kmenta 1997).

2. If, for each observation i $y_{i,j}$ is uncorrelated with ε_i (i.e., y_j and ε are contemporaneously uncorrelated: $Cov(y_1, \varepsilon_1) = Cov(y_2, \varepsilon_2) = \cdots = Cov(y_n, \varepsilon_n) = 0$), then the inclusion of the stochastic predictor variable will provide biased but consistent (asymptotically unbiased) estimates (Kmenta 1997).

3. If neither of the above holds, then the least squares estimates will be biased and inconsistent (Kmenta 1997).

Unidirectional Equations

Draper and Smith (1998) note that, if y_1 and y_2 are both stochastic variables and the modeler is only interested in y_1 as a function of y_2, then the maximum likelihood estimates of the parameters for this fit are still the least squares estimates if (y_1, y_2) has a bivariate normal distribution.

Bi-directional Equations

In the bi-directional situation, the maximum likelihood estimates of the parameters are no longer the least squares estimates. Curtis *et al.* (1974) state that the traditional site index curves do not provide optimum estimates of site index because of the minimization of different sums of squares: fitting height as a function of site index minimizes one sum of squares, while fitting site index as a function of height minimizes a different sum of squares (these will only be the same if perfect correlation exists between the two variables). They found that estimates of site index made using a site index estimation equation fitted to individual tree values were superior with regards to both accuracy and precision to those based on an inverted height growth equation and a site index equation forced through the origin. Estimates of site index based on the inverted height growth equation were found to be highly variable for young ages, while those for the site index fit were considerably better for ages less than one-half of index age.

Smith (1984) illustrates the bias which is incorporated into site index curves when a correlation exists between age class and site index. He states that an earlier method presented by Clutter et al. (1983) will suffer from the same bias unless site, age, and height relationships can be exactly specified.

With respect to this issue, a need is mentioned for taking into account whether trees that are dominant during sampling were dominant at earlier ages. Examples are given for which the traditional calculation method is inappropriate for calculating site index and dominant height growth equations due to the presence of errors in both variables. Smith concludes that the asymptotic property of height equations may be over-emphasized as a result of ignoring the correlation between age and site index. He suggests that using an anamorphic proportional relationship between height and age might result in more stable estimates of site index in some cases.

Leduc (1987) notes that least-squares regression is not appropriate when the main interest is in the description of the functional form of the relationship between two variables. In other words, if the intent is to test predictions about the theoretically expected values of equation coefficients, then OLS is not appropriate if both variables are stochastic. A numerical example is presented in which some of the techniques from the section 'Correction Methods for Stochastic Variables' are employed. Comparison with actual values of predicted slopes and intercepts from seven different prediction methods indicated that predicted slopes varied by over 500% for the different techniques. OLS regression of each variable on the other resulted in 95% confidence intervals for the slope parameter which did not contain the true value because of errors in both variables, furthering the argument that OLS is inappropriate for cases in which both variables contain error. Other techniques, which will be discussed in section 'Correction Methods for Stochastic Variables', were found to be satisfactory for this bi-directional bivariate situation.

Weller (1987) encountered bi-directional bivariate measurement error in his assessment of the $-3/2$ power rule for plant self-thinning (i.e., the maximum size-density equation). He acknowledges the inappropriateness of linear regression for fitting thinning lines since both variables in this context are subject to errors and neither is experimentally controlled. He states that since most thinning lines in the past have been fit by regression, they may contain a bias toward underestimating the steepness of the true thinning slope (attenuation). In his analysis, Weller looks at the maximum size-density relationship for 63 thinning lines reported in the literature as supporting the $-3/2$ power rule. Of these, 62% were found to be either useless for testing the rule or in quantitative disagreement with it, and the other 38% were

found to potentially support it when reanalyzed with a method that accounts for the measurement error in both variables. In addition to this, Weller argues that the results do not support the theory of a single thinning rule for all plants, and that slopes and intercepts are not constant within a particular species. Weller's technique for fitting the thinning line in the presence of measurement error, principal components analysis, will be discussed in the section 'Correction Methods for Stochastic Variables'.

Nigh (1995) states that linear regression is an unsuitable technique for performing site index conversions when a simple compatible system is needed. A comparison is made between fitting conversion lines with linear regression and geometric mean regression (GMR). GMR is a technique that accounts for error in both variables and will be discussed below. Nigh's analysis shows that the least squares lines are different from the GMR line for the spruce/pine data analyzed, and that differences increased as the correlation between variables (site indices for the two species) decreased. He concludes that using least squares regression can lead to incorrect and incompatible results, and that a technique such as GMR, which allows for variability in both variables and produces a compatible relationship, is preferred.

Multi-directional Equations

No studies in forestry could be found which examined the consequences of having stochastic variables in multi-directional equations. The fact that they are an extension of the bi-directional case suggests that the consequences presented for bi-directional equations with stochastic variables will carry over into the multi-directional case.

Correction Methods For Stochastic Predictor Variables

Most studies involving a stochastic predictor variable have assumed that the stochastic variable is independent of the residual error of the model, thereby allowing for the use of least squares modeling techniques. If this is not true, the negative impact of a stochastic variable can be reduced by selection procedures that cover as wide a range as possible in the values of the predictor variable (Draper and Smith 1998). The objective is to maximize the variance in the predictor variable y_i in relation to the size of the stochastic error in the response variable y_j.

Unidirectional Equations

If the stochastic predictor variables are independent of the model error term, ε_i, then no correction method is necessary, and the model can be fit without disturbing the least squares properties of the fitted parameters. If a stochastic predictor variable is not independent of the model error term, then a possible solution is to replace the stochastic variable with an instrumental variable. An effective instrumental variable should be closely related to the predictor variable but independent of ε_i. In forestry, predicted values are common instrumental variables (e.g., Monserud 1976, Walters and Hann 1986b) in those situations where observed values of the stochastic predictor variable are available during equation development but not its application.

Using predicted values as instruments for stochastic predictor variables creates a system of equations. Forestry problems are frequently represented by systems of equations rather than individual equations (e.g., Furnival and Wilson 1971, Borders 1989). If the predicted values for the predictor variables are created based on pre-existing equations, the parameters of the regression equation under development may be solved using two-stage least squares (2SLS) techniques (Kmenta 1997). If the equations for the predicted values of the predictor variables are to be estimated simultaneously along with the regression for the response variable, then three stage least squares (3SLS) may be used (Kmenta

1997). LeMay (1990) presents an alternative method called multistage least squares (MSLS) for fitting the equations in the presence of non-*iid* error terms. She compares fits to a volume equation with MSLS and 3SLS. The results indicate that, as expected, the parameter estimates are similar for the two techniques, but 3SLS overestimated the variances of the parameters. In addition, parameter estimation is more efficient under MSLS, although the degree of improvement with regards to variance estimation and efficiency is dependent upon the degree to which the error terms of the individual equations are non-*iid*.

As another example, Hasenauer *et al.* (1998) compared simultaneous regression techniques (2SLS, 3SLS) to OLS for fitting three individual tree models. Models were fit for individual tree basal area increment, height increment, and crown ratio. Results indicated that strong cross-equation correlations were present, and consequently the OLS estimates were biased and less efficient. In addition, OLS identified two parameters as being significant which were not significant when 3SLS was run. Estimated coefficient values differed by sizeable amounts between the OLS and 3SLS fits for several of the predictor variables. A slight improvement in the precision of the estimates was found when 3SLS was used.

Hasenauer and Monserud (1997) looked at the effect of replacing observed values with predicted values for both the response variable and one of the predictor variables during model estimation. Both measured height growth rate and predicted height growth rate from a height/diameter equation were used as response variables. In addition, both measured crown ratio and predicted crown ratio were included as one of the predictor variables. The authors found that the fit statistics measuring prediction error about smoothed (predicted) height increment data, specifically R^2, are incorrect and strongly biased upward. This appears to result from the removal of much of the natural variation in the height increment data. In addition to this problem, model predictions for fits with a predicted response variable were biased, leading to an overestimation of height increment. This was especially true for intermediate and suppressed trees. A comparison of fits using observed and predicted CR showed that the effect of CR as a predictor was considerably reduced when its predicted form was included in the model. The authors concluded that smoothed or transformed input data should be avoided for modeling purposes.

Process error (Gertner 1986, 1991) can occur if an equation is calibrated with a measured predictor variable and then applied with predicted values of the same predictor variable, or vice versa. This problem results from the parameter value for a predictor variable being dependent upon the form of the predictor variable used in model calibration. The population parameters and their estimates differ based on whether the predictor variable was measured or predicted. As an example, assume that the following set of relationships is true:

$$y = \hat{y} + \varepsilon \tag{3}$$

$$\hat{y} = \alpha_0 + \alpha_1 x_1 + \cdots + \alpha_k x_k \tag{4}$$

$$z = \beta_0 + \beta_1 \hat{y} + \gamma \tag{5}$$

It is also assumed that experimental data exist for y, the x's and z. Equation (5) contains the predictor variable $\hat{y}$ instead of y because $\hat{y}$ will be the value available in application for making predictions. Replacing $\hat{y}$ with y in Equation (5) to estimate β_0 and β_1 would result in the following:

$$\hat{y} = y - \varepsilon$$

$$z = \beta_0 + \beta_1(y - \varepsilon) + \gamma$$

$$z = \beta_0 + \beta_1 y + (\gamma - \beta_1 \varepsilon)$$

As a result, the least squares estimates of β_0 and β_1 obtained using y in place of $\hat{y}$ are biased and inconsistent (Kmenta 1997).

Process error arises in Forestry in various ways. Along with filling in missing values using predicted values from a separate equation, a common situation is that predictor variables in an equation are quantities that have been measured on the modeling data set but which will not or cannot be directly measured in the application of the equation. An example of the former includes filling in missing values for measurements of height and height to crown base (e.g., Walters and Hann 1986b, Hasenauer and Monserud 1997). Examples of the latter include tree measurements of future growth rates (e.g., Monserud 1976). DBH_{ib} (Walters and Hann 1986b), stem volume, leaf area, foliage weight, and upper stem diameters. Monserud (1976) presented a nonlinear example in which individual tree mortality was estimated with predicted rather than actual future diameter growth. Walters and Hann (1986b) developed taper equations with predicted DBH_{ib} instead of measured DBH_{ib}. They also presented two sets of regression parameters for their equations, one set for measured crown ratio and the other for predicted crown ratio.

Bi-directional Equations

A common approach to dealing with a bi-directional relationship is to fit separate equations to each of the two stochastic variables of interest as a function of the other variables in the relationship (e.g., Curtis et al. 1974, Payendeh 1978, Ritchie and Hann 1984, Larsen and Hann 1985, Walters and Hann 1986a, Hann and Scrivani 1987). This also represents a system of equations that can be solved with methods such as 2SLS and 3SLS described above. The common criticism of these methods in a situation such as this is that the equations are not consistent through inversion. Solving Equation (1) for y_2, for example, would not give Equation (2). The reason for this is the different sums of squares minimized in fitting the two equations, as mentioned previously.

A second approach to fitting a bi-directional relationship is to estimate the parameters of an invertible single equation that can be solved to predict either of the two stochastic variables. Methods that have been developed to estimate the parameters of this type of equation are described below:

Geometric Mean Regression (GMR)

The GMR line should be used when the predictor variable is uncontrolled (stochastic) and when both the dependent and predictor variables are subject to natural variability (Nigh 1995). The line produced with this method has a slope equal to the geometric mean of the slope generated by regressing Y on X and the negative inverse of the slope generate by regressing X on Y, where Y and X are variables related by a simple linear model. The objective of this method is to minimize the sum of the absolute values of the products of the horizontal and vertical deviations from the line. It is equivalent to minimizing the sum of the areas of the triangles with legs equal to these deviations. The method has two assumptions:

1. The errors in X and Y are independent.

2. The ratio of the actual error variances of Y and X is equal to S_{yy}/S_{xx}.

The estimate of the slope produced using GMR may be calculated as follows:

$\beta = \text{sign}(r)(S_y/S_x)$, where: r = the correlation coefficient for Y and X

$$\text{sign}(r) = \begin{cases} -1 & \text{if } r < 0 \\ 0 & \text{if } r = 0 \\ 1 & \text{if } r > 0 \end{cases}$$

$$S_i = \sqrt{S_{ii}}$$

The properties of the GMR line are such that the equation produced allows for two-way predictions (compatible), and is therefore well suited for the site index conversion equations to which Nigh (1995) applies it. The line produced by GMR is also known as the standard trend line, the standard major axis regression line, the reduced major axis regression line, and the functional mean regression line (Ricker 1984).

Principal Components Analysis (PCA)

Weller (1987) used PCA to fit a line to biomass-density data. The essence of PCA is that it redefines the axes of the data space along the directions of maximum variation, thereby collapsing the space into one of lower dimension while still explaining a high amount of variation. In the case of bivariate data, the data space is reduced from two down to one dimension. PCA allows for error in both variables, but makes the assumption that the ratio of the measurement error variances for the two variables is 1. This assumption limits the number of situations in which the technique may be applied. The principal components are calculated as follows:

$$\mathbf{P}_j = (\mathbf{X} - \mathbf{1}\,\overline{\mathbf{X}})\boldsymbol{\Gamma}_j, \quad j = 1,2 \text{ (there are only two variable here)}$$

where: $\mathbf{P}_j$ = the j^{th} principal component

$\mathbf{X}$ = $n \times 2$ matrix of observed values (columns are data vectors)

$$= \begin{bmatrix} y_1 & x_1 \\ y_2 & x_2 \\ \vdots & \vdots \\ y_n & x_n \end{bmatrix}$$

$\mathbf{1}$ = $n \times 1$ vector of 1's

$\overline{\mathbf{X}}$ = 1×2 vector of column means of $\mathbf{X}$

Γ_j = the j^{th} eigen vector of Σ

Σ = the 2×2 covariance matrix of $\mathbf{X}$

The sign and magnitude of the elements of $\mathbf{P}_j$ tell how to interpret the j^{th} principal component with respect to the two original variables. The resulting principal components are linear combinations of the original variables. The first principal component can therefore be solved to determine the best line that allows for variation in both variables. After some algebraic manipulation, it can be shown that:

$$y_i = (P_{1i} + \bar{y}\,\gamma_1 + \bar{x}\,\gamma_2) - \gamma_2 x_j$$

where γ_i = the i^{th} element of Γ_1

This is the line defined by the first principal component, and takes into account the variation present in both variables. This method is equivalent to major axis regression.

Major Axis Regression (MAR) *(from Leduc 1987)*

This method, also called Orthogonal Regression, is equivalent to the bivariate case of principal components analysis (PCA) (Mohler et al. 1978). Geometrically, this method minimizes the squared perpendicular distances from the data points to the line. The slope of this line can be estimated as follows:

1. Calculate: $T = \arctan\left(\frac{2s_{xy}}{s_x^2 - s_y^2}\right)/2$

where: s_x^2 = the sample variance of the X's

s_y^2 = the sample variance of the Y's

s_{xy} = the sample covariance of X and Y

2. The slope is then: $\beta = \tan(T)$

This method is often not recommended because it produces slopes that are symmetric (compatible) but not scale invariant. MAR is often applied when the data have been logarithmically transformed because of the incorrect assumption that logarithmic data are dimensionless (Leduc 1987). MAR is appropriate when testing for the equality of two independent measures of the same quantity, in which it is expected that Y = X (Rayner 1985), such as site index and dominant height.

Carroll and Ruppert (1996) argue that this method should be applied with caution as it may overcorrect for measurement error if the error resulting from the inexactness of the fitted equation is not taken into account. Their estimate of the slope parameter in the equation $Y = \beta_0 + \beta_1 X + \varepsilon + q$, where ε is the error due to measurement error and q is the error present after measurement error is removed resulting from model inexactness, is:

$$\hat{\beta}_1 = \frac{s_y^2 - \eta s_x^2 + \{(s_y^2 - \eta s_x^2)^2 + 4\eta s_{xy}\}^{1/2}}{2s_{xy}}$$

where: $\eta = \dfrac{\sigma_q^2 + \sigma_\varepsilon^2}{\sigma_u^2}$

σ_q^2 = the variance of the equation errors

σ_ε^2 = the variance of Y

σ_u^2 = the variance of the measurement errors

σ_ε^2 can be estimated ($\hat{\sigma}_\varepsilon^2$) from the data. σ_u^2 can be estimated ($\hat{\sigma}_u^2$) if there are replicates in the data set, or it can be taken from existing knowledge. σ_q^2 can be estimated as follows:

$$\hat{\sigma}_q^2 = s_v^2 - \hat{\sigma}_\varepsilon^2 - \hat{\beta}_1^* \times \hat{\sigma}_u^2$$

where: $$s_v^2 = (n-2)^{-1} \sum_{i=1}^{n} \{Y_i - \overline{Y} - \hat{\beta}_1^* \times (X_i - \overline{X})\}^2$$

$$\hat{\beta}_1^* = \left(\frac{s_X^2}{s_X^2 - \hat{\sigma}_u^2} \right) \hat{\beta}_1^{OLS}$$

$\hat{\beta}_1^{OLS}$ = the ordinary least squares estimate of β from the regression of Y on X

Goelz and Burk's Ad Hoc Method *(Goelz and Burk 1996)*

In a prior study, Goelz and Burk (1992) found that height at base age is underestimated for high site indices and overestimated for low site indices. Their ad hoc method was found to recover the shape of the data more realistically than other difference-based and integral-based methods. Their method employs a difference equation format. All possible differences are used as a data structure, the error structure is then modeled, and standard errors of the parameter estimates are then inflated to account for the exaggerated sample size. Fitting all possible differences in a linear equation, they state, is equivalent

to fitting simultaneous Y on X and X on Y regressions (MAR). Two aspects of the error structure are modeled in this method, autocorrelation and heteroskedasticity. The model includes first-order autoregressive terms, and a weighting function is included to provide homoskedasticity. The standard errors are inflated to account for artificially inflating the number of observations through the use of all possible differences. The method itself is algebraically intensive. As a result, the algorithm is not presented here. The authors present an example that compares fits for a height at base age model using OLS to fits from the authors' ad hoc method with a five-step minimization method presented by Fuller (1987). Results indicated that the standard error estimate for one of the two parameters was underestimated by a considerable amount under OLS, but not when the authors' method was used. In addition, bias in the OLS estimates appeared to increase as age increased and/or site index decreased.

Multi-directional Equations

As with the bi-directional situation, a multi-directional relationship can be characterized by fitting separate equations to each of the stochastic variables of interest as a function of the other variables in the relationship, producing a system of equations which can be solved with methods such as 2SLS and 3SLS described above. The criticism of inconsistency in the equations still exists. No methods have been developed for fitting the parameters of a single invertible equation that can be solved to predict any of the stochastic variables in the equation.

Measurement Error

In addition to the assumption of nonstochasticity, regression analysis assumes that the predictor variables are measured without error. A measurement error arises when there is a difference between an observed or estimated value for an attribute and the actual, or population, value for the attribute. Measurement errors can occur in both fixed and stochastic predictor variables and in the response variable. There are many ways in which measurement errors can arise in forestry applications. These ways can be grouped into three categories. These three categories, along with the process error mentioned in the section on stochasticity, comprise the four types of errors discussed by Gertner (1986, 1991). A description of these categories is followed by a discussion of the consequences that result from not correcting for the presence of measurement errors. Current methods for measurement error correction are then presented.

The Presence Of Measurement Error In Forestry

Forestry measurement errors can be the result of mensuration error, grouping error, and sampling error. Subgroups of errors exist within these categories. In the following sections, each category is defined and examples of each are presented from the forestry literature.

Mensuration Error

This is the error that arises when a recorded value is not exactly the same as the true value due to a flaw in the measurement process. Errors of this type may arise from misuse or poor choice of a measurement tool, what McRoberts et al. (1994) call 'variation in the use of instruments', or when it is

not possible to measure an attribute exactly. These situations may be the result of a lack of proper measurement tools, a lack of adequate training, carelessness, or other factors. Mensuration error also may occur when tree or stand characteristics are visually estimated. In addition, time or financial constraints may also prevent accurate and precise measurements.

Numerous studies have examined the accuracy and precision of tree height, diameter (or basal area), and crown attribute measurements. A summary of these studies is found in the Appendix. This information is provided to act as a partial library of mensuration error distribution attributes that may be useful in applying correction techniques. In addition, the information can be useful for testing model sensitivity and for error budget analyses. It may also affect decisions regarding which measurement tools to use. Equipment costs and time involved can be balanced against attainable levels of accuracy and precision.

Grouping Error

Under certain conditions, the process of grouping measurements of a predictor variable into classes can introduce error into the predictor variable (Swindel and Bower 1972). Rounding of continuous variables, a form of grouping, is a common procedure in forestry. Dbh's are often recorded as tallies by one- or two-inch diameter class, measured to the nearest tenth of an inch, or measured to the last whole tenth of an inch. Heights are often recorded to the nearest foot, and in some cases to the nearest five feet. Crown ratios are sometimes recorded to the nearest five or ten percent.

Grouping error can occur if an equation is calibrated with a predictor variable rounded to one level of precision and then applied with the same predictor variable rounded to a different level of precision. The error arises from the fact that the level of precision to which the variable was rounded prior to model calibration affects the parameter value for that predictor variable. Population parameters and their estimates will differ as a result of rounding in the predictor variable.

Ritchie (1997) presents three methods for estimating stems per acre when trees are point sampled and their diameters are rounded and tallied by diameter class. He found that errors can be minimized if diameter-class widths are made narrower for small diameter trees and made wider for large diameter trees.

Sampling Error

When sampling methods are used to estimate a predictor variable, the variable will contain sampling error (Jaakkola (1967), Smith and Burkhart (1984), Stage and Wykoff 1997). The size of this error depends upon the sampling design, including sample size, plot size, plot shape, sampling distribution, definition of the population, etc. (Gertner 1991). Basal area per acre, the number of trees per acre, stand volume, and site index are examples of variables which are subject to sampling error.

A number of studies have examined the effect of sample design on sampling error. Kulow (1966) compared the accuracy and precision of basal area per acre estimates from 144 different simulated sampling designs using three mapped 10.4-acre timber tracts. The designs included six different plot sizes and six different plot shapes for the fixed area plots along with six different basal area factors and two different edge bias correction methods for the variable radius plots. Each combination was simulated under random, systematic and multiple random start sampling distributions.

Jaakkola (1967) looked at the effect of small basal area factors in variable radius plot sampling on estimates of stand density. His goal was to determine whether density measured on a small variable radius plot remained constant in the area immediately adjacent to the plot, and to decide whether the plot size employed was large enough to include any variation.

Smith (1975) carried out a study to determine the effects of plot size on minimum and maximum amounts of basal area sampled per unit area. Based on previous works originating with Smith (1938), Reich and Arvanitis (1992) conducted a study to look at the relationship between plot size and sampling

variance as affected by spatial patterns in forests in an effort to determine the best combination of sample size, plot area, or BAF in forest sampling. Spetich and Parker (1998) examined the relationship between plot size and the number of plots needed to accurately estimate total biomass for old-growth forests within specified error levels. This was done for three different disturbance regimes (grazing, low mortality, high mortality) at five different time periods between 1926 and 1992.

In addition to these, several studies have looked at the effect of failure to account for edge bias in a sampling design on the accuracy and precision of stand attribute estimates, and at the numerous methods that exist for altering the sampling design to remove this bias (Finney and Palca 1948, Spurr 1952, Grosenbaugh 1958, Haga and Maezawa 1959, Barrett 1964, O'Regan and Palley 1965, Barrett and Allen 1966, Beers 1966 & 1977, Arvanitis and O'Regan 1967 & 1972, Wensel and John 1969, Schmid 1969 & 1982, Monserud and Ek 1974, Martin *et al.* 1977, Fowler and Arvanitis 1979 & 1981, Hahn et al. 1995, Scott and Bechtold 1995, and Moisen et al. 1995).

Sampling members from an incorrect population can introduce error into predictor variables. For example, Nigh and Love (1999) looked at the effect on site index estimation resulting from incorrectly including trees that had previously experienced height damage (i.e., applying the equation to a population different from the one sampled for model development).

Consequences Of Measurement Error In Predictor Variables

The consequences of having measurement error in variables are varied and include producing biased and inefficient parameter estimates as well as resulting in incorrect interpretations such as invalid statistical tests of model coefficients. As Carroll et al. (1995) state, depending on the relationships between the measured and true values of a variable and the other variables in a model, it may be the case that (1) the real effects of measurement errors are hidden, (2) observed data display relationships that are not present in the true data, or (3) the signs of the estimated coefficients are changed. In simple linear

regression, the effect of measurement error in the predictor variable is to attenuate the value of the slope parameter (Fuller 1987):

Suppose: σ^2_u = the variance of the measurement error of x in the population

σ^2_x = the variance of x in the population

The OLS regression estimate of β_1, the true slope parameter for the model, is instead an estimate of β_1^*, where:

$$\beta_1^* = \lambda\beta_1, \quad \text{with } \lambda = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2} \leq 1 \tag{6}$$

From this, it can be seen that the estimated slope will always be less than the actual slope unless $\sigma^2_u = 0$. In other words, the relationship between the predictor and response variables will appear weaker than it actually is unless there is no measurement error in the predictor variable. A similar result occurs in both the multiple linear regression and nonlinear regression cases. Estimates of the true relationship indicate a weaker relationship than actually exists. In addition to an attenuated slope estimate, variance about the fitted line also increases. It can be shown that, for σ_ε^2 = the variance of Y and X containing measurement error, andβ_x = the coefficient of Y regressed on X, then:

$$\text{var}(\mathbf{Y} \mid \mathbf{X}) = \sigma_\varepsilon^2 + \frac{\beta_x^2 \sigma_u^2 \sigma_x^2}{\sigma_u^2 + \sigma_x^2} \geq \sigma_\varepsilon^2$$

Below are the same three types of measurement error seen previously. Described beneath each type are studies that have demonstrated consequences of that particular type of measurement error in a forestry context.

Mensuration Error

Garcia (1984) included random errors in predictor variables of a stand-level growth model. He looked at the effect of the errors on basal area, number of trees, and top height predictions. Large deviations in basal area predictions for old stands are attributed to large relative measurement errors in height increment. Errors in predictions of numbers of trees are less severe, but do appear to increase for the same reason.

McRoberts et al. (1994) found that, of errors in dbh, crown ratio, and site index, it was site index errors that contributed most to the variation in the output of a projection model. They emphasize the importance of recognizing that measurement errors propagate through models over time, and that confidence in model predictions decreases over time as a result.

Mowrer and Frayer (1986) developed a general rule stating that the combined coefficient of variation for sampling and mensuration error for input variables should not exceed 20%. Gertner *et al*'s. (1995) guideline required that 95% confidence intervals for predicted values be within 20% of the mean. Williams and Schreuder (2000) concluded "...that measurement errors up to ±40% of the true height can be tolerated before inclusion of estimated height in volume prediction is no longer warranted."

Kangas (1998) studied the biases in diameter growth model coefficients under different measurement error assumptions for the predictor variables. Results indicated that measurement error in one predictor variable affected the regression coefficients of all predictor variables with which that predictor variable was correlated. Significance of model coefficients also depended on the degree of

measurement error, with the significance level decreasing as measurement error variance increased. Bias was found to increase with the proportion of the stand thinned.

Gertner and Dzialowy (1984) performed a similar study in which random measurement errors were placed on the predictor variables of STEMS, an individual-tree distance-independent projection model. The effects of these errors on individual equation predictions, as well as on the growth projection system as a whole, were examined. Their overall conclusion was that both accuracy and precision can be seriously affected by measurement errors in the predictor variables. Basal area projections were found to be very sensitive to slight biases in DBH and site index. Only errors in site index were influential when predicting numbers of trees per acre.

Gertner (1984) found that using traditional methods for estimating sample size will not result in an optimal sampling design if measurement error is present. The aim of his study was to determine how the amount of time allocated to measure a variable radius plot influenced the amount of measurement error in an estimate of mean stand basal area. His results indicated that, while no significant bias was found, the variance of the estimates was found to dramatically increase as a result of hasty measurements. Gertner concludes that sampling designs that are optimal for research situations in which there is little measurement error may be sub optimal for non-research situations in which a great deal of measurement error may be present.

Smith (1986) looked at the effect on stand volume predictions of measurement errors in aerial photo estimates of height and crown closure. Three error levels were used for each variable. Height errors were either 0%, 5%, or 15% of the average height for all plots. Crown closure errors were either 0%, 5%, or 15%. Using a linear equation with an intercept and height and crown closure terms to predict stand volume, Smith found that the general tendency was for the values of the coefficients of the fitted model to move toward zero as the amount of measurement error increased.

Snowdon (1987) looked at the effect of errors in diameter measurements on estimates of basal area increment in a study predicting foliar biomass from basal area increment. His results indicated that basal area increment is a better predictor of foliar biomass than the traditional sectional area at breast height

when the increment period is four years or more. In periods less than four years, the effect of measurement error introduced by using a diameter tape clouds the relationship to the point that sectional area at breast height is a better predictor.

Biging (1988) describes a method for partitioning the error in log volume estimation into that resulting from measurement error and the portion due to model misspecification that arises when the actual log form does not match the geometric assumptions made by the model. Along with describing how this partitioning may be done, he demonstrates the effect of measurement error on log volume estimation. This is done by assuming a log length measurement error of 0.1' and a log diameter measurement error of 0.1" and showing that percent errors for a 16ft. log vary from approximately 4.5% for a small diameter log (6") to 1.3% for a large diameter log (30").

Päivinen and Yli-Kojola (1989) looked at the effect of diameter and height measurement errors on the standard deviation of stem volume estimates. Based on information from prior studies (Nousiainen 1986, and Päivinen 1987) on the sizes of measurement error standard deviations, they showed that the standard deviation in stem volume measurements could range from 10.2% for small trees (Dbh = 3.9", Ht = 27.9') to 4.3% for larger trees (Dbh = 11.8", Ht = 83.7'). They state that the effect of measurement error can be reduced when estimating volume increment if permanent plots are used over temporary plots on which trees are bored, and if the period of time involved is long.

Two common areas of study regarding the consequences of measurement error are sensitivity and uncertainty analysis. Sensitivity analysis seeks to determine standards for maximum or allowable random error through examination of the amount of random error a model can tolerate before its predictions degenerate to an unacceptable level. Uncertainty analysis is a method for quantifying the effects of uncertainty in input values on the quality of model predictions (Wallach & Genard 1998). It examines the quality of model predictions to determine whether additional or more precise measurements are needed. Both analysis types are designed to evaluate the overall predictive ability of a model in the presence of measurement error. The error in either analysis may result from any of the three types of measurement error described above.

Procedures have also been developed to judge the performance of models formulated in different ways for the same process (Gertner 1986, Wallach and Genard 1998). Multiple studies of this type have been done in forestry. Gertner (1986) presents a method for approximating the precision of predictions when random errors are present in the data. This method partitions the total variance in the predicted values into the variance from estimating the model's parameters and the variance derived from errors in the input variables. It is then possible to assess the reduction in precision resulting from this second source of variation. Using individual tree volume equations, Gertner shows that when predictor variables are not free from error, an "optimal" model may produce worse predictions than a "suboptimal" model.

In a separate study, Gertner (1990) describes a method for calculating the approximate bias in the response resulting from systematic bias in each of the predictor variables. He also provides a method similar to that in his 1986 publication for looking at the effects of sampling error, measurement error, and regression error on response precision via the mean squared error.

Gertner (1991) presents a method for analyzing the sensitivity of functions to random errors in the regressor variables. He shows that when the input variables for a function contain measurement error (biased or unbiased), outputs will be biased if the response surface has curvature. Bias will increase as the degree of curvature increases. An approximation for the bias is given based on first and second order Taylor series.

Wallach and Genard (1998) performed an analysis to look at the effect of uncertainty on the mean squared error of prediction (MSEP). They also examined how the effects on some variables due to uncertainty were changed as other parameters were fitted to the data. Their results show that uncertainty affects the model variance part of the MSEP directly, and affects the model bias part of the MSEP when there are non-linearities in the model. Their conclusion was that the effect of variability in published parameter values on the MSEP will usually be decreased when other parameters are adjusted to the data due to compensation between different components in the variance term.

Kozak (1998) looked at the effect of measurement errors in both upper stem diameter measurements (at 50% of tree height) and in the measurement of the height above breast height for those diameters on

the precision and accuracy of taper predictions. Four levels of measurement error in diameter, 5%, 10%, 15%, and 20% of the mean, were simulated. Actual height measurements were used, and compared with additional heights for interpolated diameters. He found that both types of measurement errors greatly affected the precision of model predictions. The bias of model predictions was also affected, although less significantly. While the general conclusion from the study was that including an upper stem diameter measurement did improve the effectiveness of the equation, Kozak notes that measurement errors appear to increase as height from the ground increases. The height at which to take the additional measurement should therefore be considered carefully.

Kangas and Kangas (1999) looked at the effect of measurement errors in stand mean Dbh and basal area on bias in the criterion variables and objective function in a forest yield optimization simulation study. They found that the measurement errors had a notable effect on the bias of the criterion variables on which the optimization was based, with overestimation of basal area leading to an underestimate of stand growth. The effect of the measurement errors on the observed utility function was, however, small.

Grouping Error

Swindel and Bower (1972) showed that rounding can lead to biased parameter estimates in linear models. They provided a theoretical derivation of the form of the bias and of bounds on the bias and the efficiency of parameter estimates for variables measured with error.

Kmenta (1997) describes the effect of rounding into groups on linear regression parameter estimates. He considers two types of grouping: groups summarized by their means, and groups in the form of intervals within which the values fall. In the case of using group means, OLS estimators of the parameters are unbiased. If the number of observations in each group is not equal though, the errors are heteroskedastic. In either situation, the estimators are not efficient because of loss of the within-group variation. In the case of categorized data (data grouped into intervals), the OLS estimators are biased

unless the midpoints of the intervals are uncorrelated with the deviations of the values of the categorized variable from their respective midpoints.

Ritchie (1997) compared methods for estimating trees per acre from diameter class data measured on point samples. He found that tallying trees by diameter classes does lead to a bias in trees per acre estimates that is most severe for small trees or when diameter classes are wide. For narrower diameter classes and larger trees, the bias decreased. In addition, results indicated that the traditional method, in which expansion is done with the midpoint of the diameter class, could lead to considerable underestimates when trees are distributed roughly uniformly within intervals. Using the harmonic mean of the interval endpoints led to overestimates of a similar magnitude. Expansion based on the geometric mean of the endpoints was the most accurate under the uniformity assumption.

Sampling Error

Kulow (1966) found that sampling precision and accuracy were proportional to the size of the sampling unit, with 0.2ac, 0.1ac plots, and 5 BAF and 10 BAF point samples producing significantly better results than smaller sampling units. No significant differences were found between the best fixed-area plot design and the best point-sampling design for two of the three forests sampled. Point samples were superior in the third forest. Sampling distribution (random, systematic, multiple random start) was also found to affect precision and accuracy, "but not decisively so". Using Grosenbaugh's (1958) peripheral zone technique to account for edge bias led to predictions that were neither accurate nor precise. Plot shape was found to have no effect on accuracy or prediction. Kulow recommended that a random distribution of BAF=10 point samples with no adjustment for edge bias be used to achieve maximum precision and accuracy.

In looking at sampling for growth increment, Jaakkola (1967) concluded that plot size should be sufficiently "large" so that it includes within its limits much of the "close-range density variation of the

stand". He used both OLS and major axis regression to relate stand basal area (BA) at a point to the mean of four BA estimates from surrounding points. In the event of little within-stand variation in density, this line would have an intercept of 0.0 and a slope of 1.0. The error present in both estimates of BA was accounted for with major axis regression. By also fitting an OLS line, he demonstrates the bias which results in the slope and intercept parameters for this method. For the major axis regression fit, the intercept was significantly different from 0.0 and the slope parameter was significantly different from 1.0, indicating that. as expected, deviations in density do occur in the stand over small areas. In fitting lines relating growth to stand density, Jaakkola found that the coefficient for basal area (competition) varied with the size of the plot from which it was estimated. In addition, he found that 2 m^2/ha BAF plots ($\approx 9ft^2$/ac) were not adequate for accurately estimating the true density because of the patchy nature of stand density. His supposition is that this can be nullified by increasing the size of the plots.

Smith (1975) looked at the effect of plot size on the minimum and maximum amounts of basal area sampled per unit area. Plot sizes ranged from 0.001 to 1.0 acres. With regards to setting upper limits on potential yields, Smith recommended setting a goal for management based on the basal area per acre representing the 95% point in a frequency distribution for medium-sized sample plots. Observing that the maximum amount of basal area observed increased as the number of plots increased, he noted that small plot sizes result in very skewed distributions of basal area per acre estimates with maximum values about twenty times larger than those of the largest plots. While no plot size was clearly optimal for establishing upper limits for basal area, Smith suggested that the optimum size was probably around 0.01 acres.

In their examination of the effect of sampling for predictor values on the accuracy and precision of yield predictions, Smith and Burkhart (1984) applied simulation analysis to compare the average of plot yields to the yield of the average plot for estimating the true yield. They found that large differences in bias resulted between the two methods. The average of the plot yields was found to be more accurate, although slightly less precise, than the alternative. Total sample size had little effect on yield estimates. Precision was increased only as a result of a reduction in standard deviation by a factor of approximately

$1/\sqrt{n}$, which is expected from an increase in sample size. Stratifying samples by site index reduced the average bias by approximately 50%, while stratifying by the number of trees only reduced the bias by about 10%. Stratifying on both variables led to a bias reduction of about two-thirds. Estimates obtained with stratified sampling were nearly equal for the two mean estimator methods. Using the average of plot yields was superior in combination with random sampling.

Päivinen and Yli-Kojola (1989) concluded that sampling error in mortality measurements was the single most significant factor preventing accurate estimation of changes in growing stock. In addition, they state that when sample size is small, the size of the sampling error is large enough to dwarf the effect of mensurational error, making its role in the total error insignificant.

Hann and Zumrawi (1991) found that sampling unit designs different from the modeling dataset design can lead to large differences in predictions for components of growth models. They termed this error "differential design error". The authors warn that this could have the effect of distorting decisions regarding stand treatments. They found that problems resulting from errors incurred by alternate sampling designs were most severe when the designs were based on small sampling units. Results also indicated that estimating mean growth rate by predicting sampling units separately and then averaging predictions could help minimize differential design error. This supports the use of averaged predictions found by Smith and Burkhart (1984). In addition, the authors mention the potential for problems in model validation due to an inability to separate the error due to the model form from the differential design error. The authors state that errors derived from the application of alternative sampling unit designs will most likely be smaller for stands with more uniform spacing, and larger for stands with high degrees of clumping.

Reich and Arvanitis (1992) examined the relationship between plot size and sampling variance as affected by spatial patterns in forests and showed that the relationship between sample variance and plot size/BAF is strongly influenced by the spatial pattern of the forest attribute of interest. They found that the degree of aggregation is important for determining the coefficient of variation (CV) of a given plot size. In addition, the CV for the particular forest attribute was found to decrease with increasing plot

size in most cases. A process is presented for determining the optimal plot size or BAF for a particular spatial pattern, stand size, and CV. Sampling efficiency was also considered. The results indicate that, as the degree of spatial variation (aggregation) increases, plot size should decrease and sample size should increase.

Stage and Wykoff (1993, 1997) looked at a method (Structural Based Prediction, SBP) for continually correcting model coefficients for error over time in a growth model. Their correction method accounted for both measurement error in the predictor variables and sampling error resulting from the sampling design employed. A comparison analysis was done to look at a model created in this way and an OLS model. The models were run with data from a uniform even-aged regeneration stand and a stand with a patchy, uneven-aged distribution. Long-term comparisons of basal area showed the two models to produce similar results. Large differences are present, however, at young ages and at extremes of point density. The SBP model was found to be fairly well behaved for different plot sizes in the uniform stand, although plot size had much more of an effect in projections for the patchy stand. These results were found in runs for both thinned and unthinned stands.

Spetich and Parker (1998) looked at the relationship between plot size and the number of samples needed to achieve a certain level of accuracy in estimating total stand biomass across disturbance levels in old-growth stands. They found that the most efficient plot sizes requiring measurement of the least total area ranged from 0.02 to 1.6 acres. Small plot sizes are recommended after grazing disturbances, and large plots after high mortality. They note that recommended plot sizes are typically larger than 0.1 ac., making them larger than most current second-growth forest inventory plots.

Nigh and Love (1999), in a study on selecting undamaged site trees for estimating site index, found that over 50% of the lodgepole pine trees and over 75% of the white spruce trees in their sample had experienced damage. Causes of damage were unclear, but insects and frost were suspected. The results of the study showed that damage did not significantly reduce height growth in the spruce trees, but that lodgepole pine tree heights may have been reduced as a result of damage, thereby biasing site index estimates downward. The authors concluded that the amount of damage in site trees was considerably

higher than anticipated, but that this damage most likely had little impact on site tree height and, consequently, site index.

Correction Methods For Measurement Error

A number of methods have been presented in recent years dealing with measurement error correction. Fuller's (1987) book Measurement Error Models describes the effects of measurement errors on linear models. In their book, Measurement Error in Nonlinear Models, Carroll et al. (1995) extend this body of literature into the area of nonlinear models. The techniques presented deal primarily with correcting the bias in regression coefficients that results from having measurement error in the predictor variables. Methods for calculating the standard errors of these new parameter estimates are also generally provided.

One of the dominant factors in determining the effect of measurement error on parameters is the size of the measurement error variance for a variable. In many situations, it is the size of this variance relative to the natural variance of the predictor that is measured with error, x_i, which is important. Thus, a common recommendation is to sample as wide a range of the predictor as possible, so as to maximize the variance of x_i, and therefore minimize the ratio of the measurement error variance to the variance of x_i (Draper and Smith 1998). This is evident in Equation (6). As σ^2_u decreases in relation to σ^2_x, λ becomes closer to 1, and β_1^* becomes closer to β_1.

Methods for eliminating the bias due to different types of measurement error are presented below. It is important to note that, as Stefanski (1985) points out, although measurement error models perform better than methods which do not take measurement error into account, the degree of improvement depends on the given model, the distribution of the predictor variables, and the sample size. As a result, gains resulting from measurement error correction may range from substantial to negligible. Methods for reducing but not eliminating the bias due to measurement errors have also been examined (Kangas

1996). These techniques are not discussed here, however, as the focus here is on methods for eliminating bias. For the correction methods presented below, the following notation will be used:

$\mathbf{Y}$ = response variable, a stochastic variable which may also contain measurement error

$\mathbf{1}$ = vector of 1's having the same dimension as $\mathbf{Y}$

$\mathbf{Z}$ = predictor variable(s) assumed to be measured without error

$\mathbf{X}$ = predictor variable(s) that cannot be observed exactly and contain error

$\mathbf{W}$ = an additional predictor variable related to $\mathbf{X}$. In the case of unbiased additive measurement error: $\mathbf{W} = \mathbf{X} + \mathbf{U}$, where $E(\mathbf{U}) = 0$, and $Var(\mathbf{U}) = \sigma_u^2$.

$\mathbf{T}$ = an additional independent and observed variable called an instrumental variable, which must satisfy three requirements:

(*i*) It must be correlated with $\mathbf{X}$.

(*ii*) It must be independent of $\mathbf{W} - \mathbf{X}$.

(*iii*) It must be independent of $(\mathbf{Y}|\mathbf{Z},\mathbf{X})$.

$\tilde{\mathbf{X}} = (\mathbf{1},\mathbf{Z}^t,\mathbf{X}^t)^t$, $\tilde{\mathbf{W}} = (\mathbf{1},\mathbf{Z}^t,\mathbf{W}^t)^t$, $\tilde{\mathbf{T}} = (\mathbf{1},\mathbf{Z}^t,\mathbf{T}^t)^t$

$\beta^t_{Y|1\underline{Z}X}$ = the regression coefficients of the $\mathbf{Z}_i$'s in the regression of $\mathbf{Y}$ on $(\mathbf{1},\mathbf{Z},\mathbf{X})$

$$\beta^{t}_{Y|\tilde{X}} = (\beta_{Y|1ZX}, \beta^{t}_{Y|1ZX}, \beta^{t}_{Y|1ZX})^{t}$$

Regression Calibration *(from Carroll et al. 1995)*

The main purpose of this method is to derive an approximate model for the observed data (**Y**, **Z**, **W**) in terms of the fundamental model parameters (from the model for the actual data (**Y**, **Z**, **X**)). The basis for this method is the replacement of **X** by the regression of **X** on (**Z**, **W**). It is well suited to problems with additive measurement error, and is potentially applicable to any regression model if the approximation of **X** is accurate.

(*i*) Using replication, validation, or instrumental data, estimate the regression of **X** on (**Z**, **W**), $m(\mathbf{Z},\mathbf{W},\gamma_{cm})$, depending on parameters γ_{cm} which are estimated by $\hat{\gamma}_{cm}$ ("cm" stands for "calibration model").

(*ii*) Replace the unobserved **X** by its estimate $m(\mathbf{Z},\mathbf{W},\hat{\gamma}_{cm})$ in the model for **Y** and then run a standard regression analysis to obtain parameter estimates.

(*iii*) Adjust the resulting standard errors to account for the estimation of $\hat{\gamma}_{cm}$ with either the bootstrap or sandwich method. 'Resampling pairs' is one bootstrap method, which can be appliec. This method calls for forming bootstrap data sets by sampling at random with replacement from $\{(Y_1, Z_1), \ldots, (Y_n, Z_n)\}$. Standard errors produced from this method are still asymptotically valid in the presence of heteroskedasticity. The sandwich method is algebraically involved and not presented here. For more information on this method, see Carroll et al. (1995), pp259-265.

Adjusting the standard errors requires knowing the measurement error variance. If the value is not known, it must be estimated. One way this may be done is with replication data. For the case of replication data and a single variable measured with error, σ_u^2 can be calculated in the following way. It is assumed there are k replicate measurements, $\mathbf{W}$, of $\mathbf{X}$, and $\overline{\mathbf{W}}$ is their mean.

$$\sigma_u^2 = \frac{\sum_{j=1}^{k}(\mathbf{W}_j - \overline{\mathbf{W}})^2}{k}$$

For the case when there are n predictor variables measured with error, this can be modified as follows to estimate the measurement error variance-covariance matrix. It is assumed there are $j = 1,\ldots,k_i$ replicate measurements, $\mathbf{W}_{ij}$, of $\mathbf{X}_i$, and $\overline{\mathbf{W}}_{i\cdot}$ is their mean.

$$\hat{\Sigma}_{uu} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{k_i}(\mathbf{W}_{ij} - \overline{\mathbf{W}}_{i\cdot})(\mathbf{W}_{ij} - \overline{\mathbf{W}}_{i\cdot})^t}{\sum_{i=1}^{n}(k_i - 1)}$$

In the event that data are not available to estimate the measurement error variance(s), outside information from other studies may be required.

SIMEX Algorithm *(from Carroll et al. 1995)*

This is a simulation-based method of estimating and reducing bias from measurement error. Estimates are obtained by adding additional measurement error to the data, establishing a trend for induced bias versus the variance of the added error, and extrapolating back to the case of no measurement error.

(*i*) The first step is to generate new predictors. Additional datasets of increasingly larger measurement error $(1+\lambda_i)\sigma_u^2$ are created through simulation. For $0 = \lambda_1 < \lambda_2 < \cdots < \lambda_n$:

$$W_{b,i}(\lambda) = W_i + \sqrt{\lambda}U_{b,i}, \qquad i = 1,\ldots,n, \quad b = 1,\ldots B$$

B is the number of random errors drawn for a particular W_i with a particular λ value. The generated pseudo errors, $\{U_{b,i}\}_{i=1}^{n}$, are mutually independent, independent of the observed data, and *iid* normal random variables with mean 0 and variance σ_u^2.

(*ii*) Naïve parameter estimates are then computed. Define $\hat{\Theta}_b(\lambda)$ to be the estimator of the parameters when the $\{W_{b,i}(\lambda)\}_{i=1}^{n}$ are used, and define the average of these estimators as $\hat{\Theta}(\lambda) = \frac{1}{B}\sum_{b=1}^{B}\hat{\Theta}_b(\lambda)$. This value is then the average of a large number of estimates with the same amount of measurement error.

(*iii*) Each component (parameter) of $\hat{\Theta}(\lambda)$ is then modeled as a function of λ for $\lambda \geq 0$. Extrapolation is then done back to the case of $\lambda = -1$. A common and stable extrapolation function is to fit $\gamma_1 + \gamma_2\lambda + \gamma_3\lambda^2$. The vector of extrapolated values is the SIMEX estimator. The resulting standard errors should then be adjusted to account for the estimation of $\hat{\gamma}_{cm}$ with either the bootstrap or sandwich method.

This process assumes that σ_u^2 is known. When it is not, it must be estimated from auxiliary data as described above for replication data. A forestry example employing the SIMEX procedure is given by Kangas (1998). Solow (1998) provides an additional natural resources example.

Instrumental Variables *(from Carroll et al. 1995)*

In the regression calibration and SIMEX procedures it is assumed possible to estimate the measurement error variance σ_u^2. It is important to note that this is not always possible. When information for estimating σ_u^2 is not present, it is still possible to estimate the regression model parameters provided that the data contain an instrumental variable, **T**, in addition to the unbiased measurement **W**. A possible source of **T**, for example, is a second measurement of **X** obtained by an independent method. This second measurement does not need to be unbiased for **X**.

Carroll *et al.* restrict their discussion to the case where there is a generalized linear model (GLM) relating **Y** to (**Z**,**X**). In other words, functions for the mean and variance of (**Y**|**Z**,**X**) depend on a linear function of the covariates and predictors. There is also assumed to be a linear regression of **X** on (**Z**,**T**,**W**). Examples of these GLM's relating **Y** to (**Z**,**X**) include linear, logistic, and Poisson regression. These models depend on a linear combination of the predictors along with an additional parameter (θ) describing the variability in the response.

Carroll *et al.* provide a two-step algorithm to estimate the model parameters for the regression of **Y** on (**Z**,**X**) using **W** and **T**:

(i) Perform multivariate regression of $\widetilde{\mathbf{W}}$ on $\widetilde{\mathbf{T}}$ to obtain $\hat{\beta}_{\widetilde{W}|\widetilde{T}}$

(ii) Fit the generalized linear regression of **Y** on the predicted values $\hat{\beta}_{\widetilde{W}|\widetilde{T}}\widetilde{\mathbf{T}}$ to obtain an estimator of $\hat{\beta}_{Y|\widetilde{X}}$

***Distribution Correction Method** (from Lappi 1991)*

This method is based on the fact that the distribution of a variable, **W**, containing measurement error is wider than the true distribution of the corresponding variable without error, **X**. The wider shape results from the increased variance of **W**. This variance is equal to the variance of **X** plus the measurement error variance. The method moves the measured values of **W** closer to their mean so that the resulting distribution is more similar to the true distribution. An estimate of the measurement error variance is required in order to estimate the true population variance. The method works as follows:

(i) Estimate the mean of the true values of **X**, μ, using the mean of the observed values **W**, which are assumed to be unbiased.

(*ii*) Estimate the measurement error variance σ_u^2. It is recommended that this be done with additional information, possibly from a separate study to evaluate the magnitude of measurement errors. If no additional information is available, then σ_u^2 can be estimated by dividing the sample variance by the number of sample trees.

(*iii*) Calculate $v = \sqrt{\frac{\sigma_w^2 - \sigma_u^2}{\sigma_w^2}}$

(*iv*) Form predicted values for the sampled values of **W** that are "corrected" for measurement error:

$$\hat{\mathbf{X}}_i = v\mathbf{W}_i + (1 - v)\,\mu$$

How close the distribution of $\hat{\mathbf{X}}_i$ is to the distribution of **X** depends on the distributions of **W** and **U**. If the predicted values are found to be satisfactory, they can then be treated as independent values measured

without error in modeling. This process is very similar to what is occurring in regression calibration if the errors are assumed to be normally distributed.

Dynamic Error Correction Over Time *(Stage and Wykoff 1993, 1997)*

This method corrects for sampling error. It updates the estimates of the sampling errors at each time interval and employs measurement error correction techniques to correct for these errors at each interval before estimating model parameters. The method is presented in the context of sampling for competition measures (basal area, and basal area in larger trees) since, as the authors state, ignoring sampling properties of competition biases estimates of stand development and management responses. The process has three steps:

(*i*) The first step is to model the sampling error of the variables representing competition. The approach used is similar to that of Lappi (1991) presented above. In their example, Stage and Wykoff assumed a poisson-distributed forest, with the sampling errors for the competition variables based on this distribution.

(*ii*) Next the parameters of the structural model relating growth to competition are estimated using measurement error correction to account for the sampling errors determined in the first step. The specific correction method is not given. By assuming a structural model, the observed values are treated as random samples from an underlying distribution, with the observed values differing from their true values by an unknown stochastic error.

(*iii*) The third step occurs at each time interval of the projection model. New estimates of the sampling errors are first calculated based on current tree attributes and the original sampling

design. Revised regression coefficients are then computed by incorporating current sampling error variances. New estimates of growth conditional on updated competition variable values are made with the revised coefficients.

This leads to a reduction in the biases that result from a failure to account for changing error structures over time because of forest growth.

Literature Cited

Abetz, P. 1960. The accuracy of measurements of radial increments and annual ring width on stem discs and increment cores by means of the Eklund machine. Allg. Forst. Jagdztg. 74-80.

Arvanitis, L.G., and W.G. O'Regan. 1967. Computer simulation and economic efficiency in forest sampling. Hilgardia, 38(2):133-164.

Arvanitis, L.G., and W.G. O'Regan. 1972. Cluster or satellite sampling in forestry: a Monte Carlo computer simulation study. Proc. Of Int. Union For. Res. Organ. 3rd Conference, Advisory Group of Forest Statisticians. Sept. 1970. Joux-en-Josas, France. pp. 191-205.

Assmann, E. 1970. The principles of forest yield study. Pergamon Press. New York. 503pp.

Avery, T.E. 1958. Composite aerial photo volume tables for souther pines and hardwoods. J. For. 56(10):741-745.

Avery, T.E., and H.E. Burkhart. 2001. Forest Measurements. McGraw Hill. Boston. 456pp.

Barrett, J.P. 1964. Correction for Edge Effect Bias in Point-Sampling. For. Sci. 10:52-55.

Barrett, J.P., and P.H. Allen. 1966. Angle-gauge Sampling a Small Hardwood Tract. For. Sci. 12:83-89.

Beers, T.W. 1966. The direct correction for boundary-line slop-over in horizontal point sampling. Purdue Univ. Agric. Exp. Stn. Res. Progress Rep. 224. Lafayette, IN.

Beers, T.W. 1977. Practical Correction of Boundary Overlap. SJAF. 23:16-18.

Behre, C.E. 1926. Comparison of diameter tape and caliper measurements in second-growth spruce. J. of For. 24:178-182.

Bell, J.F. and R. Gourley. 1980. Assessing the accuracy of a sectional pole, Haga altimeter, and alti-level for determining total height of young coniferous stands. Southern Journal of Applied Forestry 4:136-138.

Biggs, P.H. 1991. Aerial tree volume functions for eucalypts in Western Australia. Can. J. For. Res. 21:1823-1828.

Biging, G.S. 1988. Estimating the accuracy of volume equations using taper equations of stem profile. Can. J. For. Res. 18:1002-1007.

Biging, G.S. and L.C. Wensel. 1988. The effect of eccentricity on the estimation of basal area and basal area growth. Forest Science 34:338-342.

Borders, B.E. 1989. Systems of equations in forest stand modeling. For. Sci. 35:548-556.

Brown, D.G., J.D. Duh, and S.A. Drzyzga. 2000. Estimating error in an analysis of forest fragmentation change using North American landscape characterization (NALC) data. Remote Sens. Environ. 71(1):106-117.

Carroll, R.J., and D. Ruppert. 1996. The use and misuse of orthogonal regression in linear errors-in-variables models. Am. Stat. 50(1)1-6.

Carroll, R.J., D. Ruppert and L.A. Stefanski. 1995. Measurement error in nonlinear models. Monographs on Statistics and Applied Probability 63. Chapman & Hall. London. 305pp.

Chacko, V.J. 1961. A study of the shape of cross section of stems and the accuracy of caliper measurement. Indian Forester 87:758-762.

Clutter, J.L., J.C. Fortson, L.V. Pienaar, G.H. Brister, and R.L. Bailey. 1983. Timber management: a quantitative approach. John Wiley & Sons. New York. 333p.

Crockford, R.H., and M.E. Johnson. 1983. Some errors in the measurement of precipitation, throughfall and stemflow and the implications for estimation of interception. Hydrology and Water Resources Symposium 1983. 236-242.

Curtis, R.O. and D. Bruce. 1968. Tree heights without a tape. Journal of Forestry 66:60-61.

Curtis, R.O., D.J. DeMars and F.R. Herman. 1974. Which dependent variable in site index-height-age regressions? Forest Science 20:74-87.

Draper, N.R., and H. Smith. 1998. Applied regression analysis. J. Wiley & Sons. New York. 709pp.

Finlayson, W., and Z. Tchanou. 1975. Sources of error in Biltmore stick type instruments. Commonw. For. Rev. 54(3/4):290-295.

Finney, D.J., and H. Palca. 1948. The elimination of bias due to edge-effects in forest sampling. Forestry 23:31-47.

Fowler, G.W., and L.G. Arvanitis. 1979, Aspects of statistical bias due to the forest edge: fixed-area circular plots. Can. J. For. Res. 9:383-389.

Fowler, G.W., and L.G. Arvanitis. 1981, Aspects of statistical bias due to the forest edge: horizontal point sampling. Can. J. For. Res. 11:334-341.

Fuller, W. 1987. Measurement error models. John Wiley & Sons, New York. 440pp.

Furnival, G.M.. and R.W. Wilson. 1971. Systems of equations for predicting forest growth and yield. IN Statistical Ecology 3:43-55. G.P. Patil, E.C. Pielou, W.E. Waters, eds., Penn State Univ. Press, University Park.

Garcia, O. 1984. New class of growth models for even-aged stands: *Pinus radiata* in Golden Downs Forest. New Zealand Journal of Forestry Science 14:65-88.

Gertner, G.Z. 1984. Control of sampling error and measurement error in a horizontal point cruise. Can. J. For. Res. 14:40-43.

Gertner, G.Z. 1986. Postcalibration sensitivity procedure for regressor variable errors. Canadian Journal of Forest Research 16:1120-1123.

Gertner, G.Z. 1990. The sensitivity of measurement error in stand volume estimation. Canadian Journal of Forest Research 20:800-804.

Gertner, G.Z. 1991. Prediction bias and response surface curvature. Forest Science 37:755-765.

Gertner, G.Z. and P.J. Dzialowy. 1984. Effects of measurement errors on an individual tree-based growth projection system. Canadian Journal of Forest Research 14:311-316.

Gertner, G.Z., X. Cao, H. Zhu. 1995. A quality assessment of a Weibull based growth projection system. For. Ecol. Manag. 71:235-250.

Ghosh, S., J.L. Innes, and C. Hoffmann. 1995. Observer Variation as a source of error in assessments of crown condition through time. For. Sci. 41(2):235-254.

Goelz, J.C.G., and T.E. Burk. 1992. Development of a well-behaved site index equation: jack pine in north central Ontario. Can. J. For. Res. 22:776-784.

Goelz, J.C.G., and T.E. Burk. 1996. Measurement error causes bias in site index equations. Canadian Journal of Forest Research 26:1585-1593.

Gregoire, T.G., S.M. Zedaker, and N.S. Nicholas. 1990. Modeling relative error in stem basal area estimates. Can. J. For. Res. 20:496-502.

Grosenbaugh, L.R. 1958. Point-sampling and line sampling: probability theory, geometric implications, synthesis. USDA Forest Serv. South Forest Exp. Stn. Occas Pap. 160, 34p.

Haga, T., and K. Maezawa. 1959. Bias due to edge effect in using the Bitterlich method. For. Sci. 5:370-376.

Hahn, J.T., C.D. MacLean, S.L. Arner, and W.A. Bechtold. 1995. Procedures to handle FIA cluster plots that straddle two or more conditions. For. Sci. Monogr. 31:12-25.

Hann, D.W., and J.A. Scrivani. 1987. Dominant height growth and site index equations for Douglas-fir and ponderosa pine in southwest Oregon. For. Res. Lab., Oreg. State Univ., Corvallis. Res. Bull. 59. 13p.

Hann, D.W., and M.W. Ritchie. 1988. Height growth rate of Douglas-fir: A comparison of model forms. For. Sci. 34(1):165-175.

Hann, D.W. and A.A. Zumrawi. 1991. Growth model predictions as affected by alternative sampling-unit designs. Forest Science 37:1641-1655.

Hasenauer, H. and R.A. Monserud. 1997. Biased predictions for tree height increment models developed from smoothed 'data'. Ecological Modeling 98:13-22.

Hasenauer, H., R.A. Monserud, and T.G. Gregoire. 1998. Using simultaneous regression techniques with individual-tree growth models. For. Sci. 44(1)87-95.

Howe, G.T. and W.T. Adams. 1988. Clinometer versus pole measurement of tree heights in young Douglas-fir progeny tests. Western Journal of Applied Forestry 3(3):86-88.

Hyppönen, M., and P. Roiko-Jokela. 1978. On the accuracy and effectivity of measuring sample trees. Folia Forestalia 356:1-25.

Jaakkola, S. 1967. On the use of variable size plots for increment research. P. 371-378 *in* Proc. 14th World Congress of the International Union of Forestry Research Organizations. Div. 6, Sect. 25

Kangas, A.S. 1996. On the bias and variance in tree volume predictions due to model and measurement errors. Scand. J. For. Res. 11:281-290.

Kangas, A.S. 1998. Effect of errors-in-variables on coefficients of a growth model and on predictions of growth. Forest Ecology and Management 102:203-212.

Kangas, A.S., and J. Kangas. 1999. Optimization bias in forest management planning solutions due to errors in forest variables. Silva Fennica 33(4):303-315.

Kärkkäinen, M. 1975. Measurement of the cross-sectional area of birch and aspen logs. Silva Fenn. 9:212-232.

Kärkkäinen, M. 1976. Auxiliary observations on the measurement of the cross-sectional area of aspen logs. Silva Fenn. 10:257-265.

Keeland, B.D. and R.R. Sharitz. 1993. Accuracy of tree growth measurements using dendrometer bands. Can. J. For. Res. 23:2454-2457.

Kmenta, J. 1997. Elements of econometrics. Macmillan Publishing Co. New York. 786pp.

Kozak, A. 1998. Effects of upper stem measurements on predictive ability of a variable-exponent taper equation. Canadian Journal of Forest Research 28:1078-1083.

Krauch, H. 1924. Comparison of tape and caliper measurements. J. of For. F22:537-538.

Kulow, D.L. 1966. Comparison of Forest Sampling Designs. J. For. 64:469-474.

Lappi, J. 1991. Estimating the distribution of a variable measured with error: stand densities in a forest inventory. Can. J. For. Res. 21:469-473.

Larsen, D.R., and D.W. Hann. 1985. Equations for predicting diameter and squared diameter inside bark at breast height for six major conifers of Southwest Oregon. Forest Research Laboratory, Oregon State University, Corvallis. Research Note 77. 4p.

Larsen, D.R., D.W. Hann and S.C. Stearns-Smith. 1987. Accuracy and precision of the tangent method of measuring tree height. Western Journal of Applied Forestry 2:26-28.

Leduc, D.J. 1987. A comparative analysis of the reduced major axis technique of fitting lines to bivariate data. Canadian Journal of Forest Research 17:654-659.

LeMay, V.M. 1990. MSLS: a linear least squares technique for fitting a simultaneous system of equations with a generalized error structure. Can. J. For. Res. 20:1830-1839.

Martin, G.L., and A.R. Ek. 1984. A comparison of competition measures and growth models for predicting plantation red pine diameter and height growth. For. Sci. 30(3):731-743

Martin, G.L., A.R. Ek and R.A. Monserud. 1977. Control of plot edge bias in forest stand growth simulation models. Can. J. For. Res. 7(1):100-105.

Matern, B. 1956. MeddfStSk 46(11). (cited in Assman 1970)

McArdle, R.E. 1928. Relative accuracy of calipers and diameter tape in measuring Douglas-fir trees. Journal of Forestry 26:338-342.

McRoberts, R.E., J.T. Hahn, G.J. Hefty, and J.R. Van Cleve. 1994. Variation in forest inventory field measurements. Can. J. For. Res. 24:1766-1770.

Mohler, C.L., P.L. Marks, and D.G. Sprugel. 1978. Stand structure and allometry of trees during self-thinning of pure stands. J. Ecol. 66:599-614.

Moisen, G.G., A.R. Stage, and J.D. Born. 1995. Point Sampling with Disjunct Support Near Population Boundaries. For. Sci. Monogr. 31:62-82.

Monserud, R.A. 1975. Methodology for simulating Wisconsin northern hardwood stand dynamics. PhD dissertation. The University of Wisconsin-Madison. 156p.

Monserud, R.A. 1976. Simulation of Forest Tree Mortality. For. Sci. 22(3):438-444.

Monserud, R.A., and A.R. Ek. 1974. Plot edge bias in forest stand growth simulation models. Can. J. For. Res. 4(4):419-423.

Mowrer, H.T., and W.E. Frayer. 1986. Variance propagation in growth and yield projections. Can. J. For. Res. 16:1196-1200.

Muller, D. 1957. FwCbl 76(34). (cited in Assman 1970)

Nester, M.R. 1981. Assessment and measurement errors in slash pine research plots. Department of Forestry, Queensland, Technical Report No. 26, 10p.

Nicholas, N.S., T.G. Gregoire, and S.M. Zedaker. 1991. The reliability of tree crown position classification. Can. J. For. Res. 21:698-701.

Nigh, G.D. 1995. The geometric mean regression line: A method for developing site index conversion equations for species in mixed stands. Forest Science 41:84-98.

Nigh, G.D., B.A. Love. 1999. How well can we select undamaged site trees for estimating site index? Can. J. For. Res. 29:1989-1992.

Nousiainen, M. 1986. Metsän koealainventoinnissa tarvittavien koepuunittausten luotettavuus. Pro gradu-työ. Helsingin yliopiston metsänarvioimistieteen laitos. 60p.

Omule, S.A.Y. 1980. Personal bias in forest measurements. For. Chron. 56:222-224.

O'Regan, W.G., and M.N. Palley. 1965. A computer technique for the study of forest sampling methods. For. Sci. 11(1):99-114.

Paine, D.P. 1981. Aerial photography and image interpretation for resource management. John Wiley & Sons, New York, N.Y., 571p.

Päivinen, R. 1987. Metsän inventoinnin sunnittelumalli. Summary: A planning model for forest inventory. Joensuun yliopiston luonnontieteellisiä julkaisuja – University of Joensuu, Publications in Sciences 11. 179p.

Päivinen, R., and H. Yli-Kojola. 1989. Permanent sample plots in large-area forest inventory. Silva Fennica 23(3):243-252.

Päivinen, R., M. Nousiainen, and K.T. Korhonen. 1992. Accuracy of certain tree measurements. Folia Forestalia 787:1-18.

Payandeh, B. 1978. A site-index formula for peatland black spruce in Ontario. For. Chron. 54:39-41.

Philip, M.S. 1994. Measuring trees and forests. University Press. Cambridge. 310pp.

Phillips, D.L., S.L. Brown, P.E. Schroeder, and R.A. Birdsey. 2000. Toward error analysis of large-scale forest carbon budgets. Global Ecology and Biogeography 9(4):305-313.

Plamondon, J.A. 1999. Log-length measurement errors with six single-grip harvester heads. Technical Note Forest Engineering Research Institute of Canada – FERIC, No. TN-297. 6p.

Rayner, J.M.V. 1985. Linear relations in biomechanics: the statistics of scaling functions. J. Zool. Soc. London, 206:415-439.

Reich, R.M., and L.G. Arvanitis. 1992. Sampling unit, spatial distribution of trees, and precision. North. J. Appl. For. 9(1):3-6.

Ricker, W.E. 1984. Computation and uses of central trend lines. Can. J. Zool. 62(10):1987-1995.

Ritchie, M.W. 1997. Minimizing the rounding error from point sample estimates of tree frequencies. W. J. Appl. For. 12(4):108-114.

Ritchie, M.W., and D.W. Hann. 1984. Nonlinear equations for predicting diameter and squared diameter inside bark at breast height for Douglas-fir. Forest Research Laboratory, Oregon State University, Corvallis. Research Note 47. 12p.

Ritchie, M.W., and D.W. Hann. 1986. Development of a tree height growth model for Douglas-fir. For. Ecol. Manag. 15:135-145.

Ritchie, M.W., and D.W. Hann. 1990. Equations for predicting the 5-year height growth of six conifer species in southwest Oregon. For. Res. Lab., Oreg. State Univ., Corvallis. Res. Pap. 54. 13p.

Ross, R.D. 1968. On the accuracy of penta prism calipers. J. of For. 66:932-933.

Schmid, P. 1969. Stichproben am Waldrand. Mitt. Schweiz. Anst. Forstl. Versuchswes. 45, 3:234-303.

Schmid-Haas, P. 1982. Sampling at the Forest Edge. Swedish Univ. of Agric. Sci. S-901 83 Umeå, Sweden.

Schreuder, H.T., T.G. Gregoire, and G.B. Wood. 1993. Sampling methods for multiresource forest inventory. John Wiley & Sons, Inc. New York. 464pp.

Scott, C.T., and W.A. Bechtold. 1995. Techniques and computations for mapping plot clusters that straddle stand boundaries. For. Sci. Monogr. 31:46-61.

Shiver, B.D., and B.E. Borders. 1996. Sampling techniques for forest resource inventory. John Wiley & Sons, Inc. New York. 356pp.

Smith, H.F. 1938. An empirical law describing heterogeneity in yields of agricultural crops. J. Agric. Sci. 28:1-23.

Smith, J.H.G. 1975. Use of small plots can overestimate upper limits to basal area and biomass. Canadian Journal of Forest Research 5:503-505.

Smith, J.L. 1986. Evaluation of the effects of photo measurement errors on predictions of stand volume from aerial photography. Photog. Eng. Rem. Sens. 52(3):401-410.

Smith, J.L. and H.E. Burkhart. 1984. A simulation study assessing the effect of sampling for predictor variable values on estimates of yield. Canadian Journal of Forest Research 14:326-330.

Smith, V.G. 1984. Asymptotic site-index curves, fact or artifact? For. Chron. 60:150-156.

Snowdon, P. 1987. Predicting foliar biomass of *Pinus radiata* from basal area increment. Aust. For. Res. 17(3):277-281.

Solow, A.R. 1998. On fitting a population model in the presence of observation error. Ecology 79(4):1463-1466.

Spetich, M.A. and G.R. Parker. 1998. Plot size recommendations for biomass estimation in a Midwestern old-growth forest. North. J. Appl. For. 15(4):165-168.

Spurr, S.H. 1952. Forest Inventory. Ronald Press Co., New York. 476 pp.

Spurr, S.H. 1960. Photogrammetry and photo-interpretation. The Ronald Book Company, New York, N.Y., 427p.

Spurr, S.H. and C.T. Brown. 1946. Tree height measurements from aerial photographs. J. For. 44(6)716-721.

Stage, A.R., and W.R. Wykoff. 1993. Procedure for representing spatial variability within distance-independent stand growth models: An application of measurement error models. Pages 131-144, *In*: Modeling stand response to silvicultural practices. Proceedings of the IUFRO s4.01 Conference. September 27 – October 1, 1993. Blacksburg, VA.

Stage, A.R., and W.R. Wykoff. 1997. Adapting distant-independent forest growth models to represent spatial variability: Effects of sampling design on model coefficients. Unpublished report. 50p.

Stefanski, L.A. 1985. The effects of measurement error on parameter estimation. Biometrika 72:583-592.

Swindel, B.F., and D.R. Bower. 1972. Rounding errors in the independent variables in a general linear model. Technometrics 14(1):215-218.

Wallach, D. and M. Genard. 1998. Effect of uncertainty in input and parameter values on model prediction error. Ecological Modelling 105:337-346.

Walters, D.K., and D.W. Hann. 1986 (a). Predicting merchantable volume in cubic feet to a variable top and in Scribner board feet to a 6-inch top for six major conifers of southwest Oregon. Forest Research Laboratory, Oregon State University, Corvallis. Research Bulletin 52. 107p.

Walters, D.K., and D.W. Hann. 1986 (b). Taper equations for six conifer species in southwest Oregon. Forest Research Laboratory, Oregon State University, Corvallis. Research Bulletin 56. 41p.

Weller, D.E. 1987. A reevaluation of the -3/2 power rule of plant self-thinning. Ecological Monographs 57:23-43.

Wensel, L.C., and H.H. John. 1969. A statistical procedure for combining different types of sampling units in a forest inventory. For. Sci. 15(3):307-317.

Wensel, L.C., W.J. Meerschaert, and G.S. Biging. 1987. Tree height and diameter growth models for northern California Conifers. Hilgardia. 55(8):1-20.

Williams, M.S., and H.T. Schreuder. 2000. Guidelines for choosing volume equations in the presence of measurement error in height. Can. J. For. Res. 30:306-310.

Williams, M.S., W.A. Bechtold, and V.J. LaBau. 1994. Five instruments for measuring tree height: An evaluation. South. J. Appl. For. 18(2):76-82.

Williams, M.S., K.L. Cormier, R.G. Briggs, and D.L. Martinez. 1999. Evaluation of the Barr & Stroud FP15 and Criterion 400 laser dendrometers for measuring upper stem diameters and heights. For. Sci. 45(1)53-61.

Woods, K.D., A.H. Feiveson, and D.B. Botkin. 1991. Statistical error analysis for biomass density and leaf area index estimation. Can. J. For. Res. 21:974-989.

Zarnovican, R. and J. DeGrâce. 1995. Mesure de la hauteur des arbres avec la perche Sokkisha: Test de précision. The Forestry Chronicle 71(5):616-620.

Appendix: Mensuration Error In Common Forestry Measurements

Mensuration errors can occur in predictor variables due to instrument error, observer error or errors arising from the measurement environment. Instrument errors can be caused by mis-manufacture of the instrument, past damage to the instrument, mis-calibration of the instrument, or poor design of the instrument. Observer error (i.e., human error) can be caused by poor training in the use of the instrument, poor training in the application of the measurement method, poor work habits, errors in using calculators, or errors in recording values on paper forms or in entering them into data recorders. Errors resulting from the measurement environment can occur if the tree being measured is large in size, if the tree is on a steep slope, if the tree is in a dense stand, if it is difficult to determine appropriate points of measurement on the tree, if the workday is dark, or if the workday is windy. Mensuration errors can also vary between measurement methods. An overly complex method, a method requiring more measurements, or a method requiring precise execution of measurements can lead to increased frequency and size of mensuration errors.

There have been a number of studies in forestry that have examined the accuracy and precision of different procedures and tools for measuring tree height, diameter, and crown attributes. These studies have examined errors that arise from the use of different instruments, from the use of different measurement methods, or that are caused by the observer. Data collection for these studies was done under either a controlled or operational environment. Controlled studies attempt to minimize some of the sources of error in order to concentrate on the source(s) of error that are of interest. Operational studies make no attempt to control any sources of error and are typically meant to mimic standard field situations. Both types of studies make comparisons against either a "true" measurement, an "expert's" measurement, or a measurement from a "standard" method.

Tree Height

Tree heights can be measured either directly or indirectly. Direct methods include falling the tree and measuring the length of the stem with a tape (the "felled tree" method), or standing a graduated pole next to the stem of the standing tree and determining which graduation corresponds to the height of the tree (the "direct pole" method). Indirect techniques include the "distance" method, the "indirect pole" method, the "dendrometer" method, or the "laser" method. The distance method consists of measuring the horizontal distance from the observer to the tree (often correcting for slope), determining the angles to the top and base of the tree, and calculating the height by trigonometry. The "indirect pole" method consists of standing a pole of known height next to the stem of the tree, measuring the angles to the top and base of the tree and to the top of the pole, and calculating the height by trigonometry. In the "dendrometer" method, an optical dendrometer is set up approximately one tree height from the tree, the distance to the tree is measured with the instruments optical range finder, the angles to the top and bottom of the tree are determined by the instrument's clinometer, and the height is calculated by trigonometry. Similarly, the "laser" method measures the distance to the top and bottom of the tree with a laser, the angle between the two distance measurements by an internal clinometer, and automatically calculates tree height by trigonometry. Of these methods, the felled tree method is considered the most accurate, and as a result, often takes the role of the "true" standard against which other methods are compared.

Curtis and Bruce (1968) used the "indirect pole" method and reported on the change in the standard error of height estimates with changing pole length. They also compared height estimates using the "indirect pole" method with those from the "distance" method and the Barr & Stroud dendrometer, reporting that the "indirect pole" method gave accuracy comparable to the "distance" method.

Hyppönen and Roiko-Jokela (1978) conducted an operational study that compared two measurement methods and five instruments for measuring angles against "true" values from felled trees. A total of 38 trees were included in the study, with heights ranging from 8.6 meters to 18.2 meters. The " direct pole" method, using a sectional pole, was compared to the "distance" method, using the Suuntakehä, the

Suunto altimer with tripod, the Suunto altimeter hand held, and Lönnroth hypsometer to measure angles. They found that the "direct pole" method was the most accurate and precise of the six methods studied.

Bell and Gourley (1980) conducted a controlled study that compared two measurement methods and two instruments for measuring angles against felled-tree values from 35 Douglas-fir trees. Trees ranged from 27 feet to 45 feet in height. The "direct pole" method, with a sectional height pole, was compared to the "distance" method, with both a Haga altimeter and an alti-level to measure angles. The study was controlled in that the direct pole measurements included climbing up an adjacent tree to make sure the tip of the pole was coincident with the tip of the tree. Based on the results of a chi-squared test, Bell and Gourley found that maximum anticipated errors were 0.49 feet with the "direct pole" method, 1.22 feet with the "distance" method using the alti-level, and 2.17 feet with the "distance" method using the Haga altimeter. The three tools had the same relative ranking with respect to precision.

Omule (1980) conducted an operational study that compared height estimates from students based on the "distance" method with a Suunto clinometer and a nylon tape to his own more experienced measurements with the same instruments. Approximately 500 trees were included in this study. The average tree height for the sample trees was 32.33 meters, with 15% of the measurements having a bias larger than 6 meters in magnitude. He found a significant negative bias and a sizeable variance in the students' measurements. No trend of increasing bias with increasing height was evident.

Nester (1981) conducted an operational study to look at mensurational errors of direct height measurements with a duraluminum height pole in a 12-year old slash pine stand. The measurements for the individual crews were compared to the average across crews using an ANOVA. None of the crews were found to have a statistically significant bias in their height measurements. The standard deviation of height measurement errors was found to be 1.18 feet. In addition, the tallest tree on each plot was not selected for height measurement on 3.1% of the quadrats on average.

In their publications on forest inventory in Finland, Nousiainen (1986) and Päivinen (1987) provided information about height measurement errors. Their data appear to indicate that height measurement errors are unbiased with a standard deviation of 2.3 feet. (In Finnish, cited in Päivinen and

Yli-Kojola 1989). The type of study and measurement tools used were not presented in the citing publication.

Larsen et al. (1987) conducted an operational study that compared height measurements from the "indirect pole" method with a Suunto clinometer and telescoping height pole to "true" heights from 794 felled conifers. Trees ranged from 17.9 feet to 193.8 feet in height. They found a statistically significant average underestimation bias of 0.769 feet from the "indirect pole" method, and that the magnitude of the bias increased with the square of tree height. Their results also indicated that the variation in percent error was approximately constant across height.

Howe and Adams (1988) conducted a semi-operational study that compared the "distance" method with clinometer and a 10 meter rope to "standard" measurements from the "direct pole" method for 459 Douglas-fir trees in plantations. The average height of these trees was 7.99 meters. They found that heights from the "distance" method were underestimated by 0.31 meters on average when compared to heights from the "direct pole" method.

Päivinen et al. (1992) conducted an operational study that compared height estimates using the "distance" method with a Suunto clinometer to values from the "felled tree" method for 152 predominantly conifer trees. A total of 1309 measurements ranging in height from approximately 12 meters to 32 meters were made on these trees by five observers. The authors found that the "distance" method consistently underestimated height by 32 centimeters. The standard deviation of the height mensuration errors was 67 centimeters. A positive correlation between the errors and the height of the tree was also found. Bias and precision varied between observers. Bias ranged from an overestimation of 27 centimeters to an underestimation of 46 centimeters for the five observers, with standard deviations of the differences ranging from 40 centimeters to 56 centimeters. Five-year height growth was overestimated by a mean of 14 centimeters, with a standard deviation of 27 centimeters.

Zarnovican and Grâce (1995) conducted an operational study that compared heights measured using the "direct pole" method with a Sokkisha (telescoping) pole to heights from the "felled tree" method to determine accuracy and precision for this tool in young thinned balsam fir stands. The study included measurements on 223 trees ranging in height from 3.20 meters to 8.16 meters, and in diameter from 5.5

centimeters to 14.5 centimeters. Errors in height ranged from –0.26 meters to 0.22 meters, with an average error of –0.06 meters and an error standard deviation of approximately 0.1 meters. Error variation was found to increase with height.

With improvements in technology have come new and more precise tools for height measurement that need to be compared against older tools. Williams et *al.* (1994) conducted a controlled study that compared the accuracy and precision of two different height measuring methods using five instruments (the "laser" method and the "distance" method with the Suunto clinometer on a tripod, the Enbeeco clinometer on a tripod, the Speigel-relaskop on a tripod, and the Speigel tele-relaskop on a tripod to measure angles) to standing heights measured with the "direct" pole method. A total of 100 trees were included in the study, ranging in height from approximately 11 feet to 98 feet. The study was controlled in that a tripod was used to steady the angle measurement tools. The Suunto and Enbeeco clinometers demonstrated significant biases in certain basic height classes. When the height classes were more finely broken down, the tele-relaskop demonstrated the largest bias for the majority of the classes, and was the least precise of the angle measuring devices. Results for the "laser" method were inconclusive because of the small number of trees measured with this tool. Bias for each technique was tested using linear height models to model actual height as a function of predicted height. Bias was indicated by significant differences either of an intercept parameter from zero or a slope parameter from one.

Williams et al. (1999) conducted a second controlled study that compared height measurements for 369 trees using both the "dendrometer" method (Barr & Stroud dendrometer) and the "laser" method (Criterion 400 laser devise) to previously measured marks on standing trees as a means of determining accuracy and precision. Mean errors in height prediction were 0.03 meters for the Barr & Stroud and –0.01 meters for the Criterion. The error standard deviation was 0.11 meters for both tools. Their results indicated that neither tool is capable of producing both unbiased height and diameter measurements without calibration. Analysis of accuracy at varying distances from the tree for each device showed that mensuration errors for the Barr & Stroud increased with increasing distance from the tree. Precision was not found to change for either instrument as distance from the tree or tree height changed. Errors for individual observers were accounted for with a mixed effects linear model to determine bias.

In addition to these studies regarding the sizes of errors in height measurements, there have also been studies done to look at errors in tree heights estimated from aerial photos (Spurr and Brown 1946, Paine 1981, Biggs 1991). In the study by Biggs, the presence of measurement error was recognized and a correction factor was applied to the height estimates of each observer based on ground measurements from a few stands, resulting in improved fits of stand volume.

Tree Diameter / Basal Area

Tree diameters or basal areas can also be measured directly or indirectly. Direct methods of measuring diameters include using a mechanical caliper on the bole of a standing tree (the "mechanical caliper" method), an optical caliper at a distance from the bole of a standing tree (the "optical caliper" method), or falling and sectioning the tree and measuring the diameter at the section with a ruler or tape measure (the "felled tree" method). The direct method for measuring basal area requires felling and sectioning the tree and then measuring the cross sectional area with a planimeter. Indirect methods for measuring diameter include measuring the circumference of the stem and converting the circumference measurement to a diameter by the assuming the stem is circular in cross section (the "d-tape" method), or with either an optical or a laser dendrometer (the "dendrometer" method). The indirect method for measuring basal area requires one or two measurements of diameter and calculates basal area based on an assumption of circularity (the single diameter case) or with the formula for an ellipse (the two diameter case).

A problem common to all methods is deciding what constitutes a tree's true diameter in the absence of circularity. At a given point on a tree's bole, there are an infinite number of possible diameter measurements that could be made with any one of the direct methods. All of these measurements are identical for a circular bole, while most would differ for non-circular boles. As a result, the diameter from the "d-tape" method, the largest diameter, the smallest diameter, the arithmetic mean of the largest and smallest diameters from the other methods, or the geometric mean of the largest and smallest

diameters from the other methods have all been used as indicators of the tree's "true" diameter for non-circular boles.

Krauch (1924) conducted an operational study that compared caliper measurements to those from a diameter tape for measuring diameter at breast height (Dbh). Mean differences are presented by diameter class. A total of 846 trees were measured, ranging in size from approximately 4 inches to 30 inches. The results indicate that differences between tape and caliper measurements are small, and that tape measurements are slightly larger than caliper measurements by an average of 0.05 inches. The magnitude of mean errors appeared to increase as diameter class increased, although the sign of the errors was not consistent.

Behre (1926) conducted an operational study that compared diameter tape and caliper measurements for Dbh. A total of 72 trees were examined, ranging in size from 4.7 inches to 16.8 inches. The d-tape produced measurements that were on average 0.03 inches larger than the average of two caliper measurements, with an error standard deviation of 0.075 inches. In addition to a diameter comparison, Behre also looked at the effect of different diameter measurement tools on volume predictions. A method is presented for computing percent changes in volume needed to convert from volumes derived from d-tapes to volumes derived from caliper measurements, and vice versa. He concludes, however, that the choice of tool is of no practical importance.

Finlayson and Tchanou (1975) analyzed sources of error in Biltmore stick measurements based on 1:10 scale drawings of cross-sections of tree breast height basal areas. The results of their simulation study strongly suggest the presence of a bias in estimates of diameter for non-circular stems, but that the bias is very small and its sign depends on the particular shape. Based on this, different shapes can be expected to have different biases. In addition, they note that Biltmore stick measurements are more variable than those of calipers which in turn are more variable that taped measurements.

In his study, Omule (1980) also compared the students' estimates of Dbh made with a d-tape to his own measurements and found a small negative bias and small variance. For the approximately 500 trees measured, the students underpredicted Dbh by 0.05 centimeters on average. Over 95% of the students'

measurements were within 5.0 centimeters of the "true" value, although biases as large as ±30 centimeters were present. Bias in Dbh measurements appeared to increase with Dbh.

Nester (1981) also looked at mensurational errors in taped diameter measurements in a 12 year old slash pine stand. The measurements for the individual crews were again compared to the average across crews using an ANOVA. One crew was found to have a statistically significant bias in their Dbh measurements. Observing this crew revealed that the person doing the measuring had a tendency to allow the tape to droop during measurement, leading to an average inflation of the Dbh measurements by 0.3 inches. The standard deviation of Dbh measurement errors was found to be 0.11 inches.

In addition to information on mensurational errors associated with height measurements, Nousiainen (1986) and Päivinen (1987) also provided information about diameter measurement errors. Their data appear to indicate that breast height diameter measurement errors are unbiased with a standard deviation between 0.16 inches and 0.28 inches. (In Finnish, cited in Päivinen and Yli-Kojola 1989). The type of study and measurement tools used for the diameter analysis were not presented in the citing publication.

Examination of mensuration errors in diameter estimates has not been limited to diameters at breast height. Ross (1968) conducted an operational study that compared the accuracy and precision of diameter measurements at multiple points along the stem using two methods of alignment for a penta prism caliper, with diameter tape measurements as the reference. Diameter measurements were made at multiple heights on 29 trees. Both methods of alignment led to mean underpredictions of diameter at all heights examined. Overall mean bias was -0.03 inches for inward alignment and –0.23 inches for outward alignment. Bias was also found to increase with the height of the diameter measurement. The standard deviations of the errors were 0.027 inches for inward alignment and 0.310 inches for outward alignment. Ross found that outward alignment resulted in diameter estimates at all heights that were all significantly different from those obtained with a tape. Inward alignment only produced significant errors at 17 feet.

In their study, Hyppönen and Roiko-Jokela (1978) also compared accuracy and precision of upper stem diameters at a height of 6 meters made with both curved and precision calipers and the suuntakehä.

The basis for comparison was not apparent. There were three observers, each taking measurements on the same 38 trees. Average bias for the curved caliper was –0.04 centimeters with a standard deviation of 0.80 centimeters. Both bias and standard deviation varied widely across observers. Average bias for the precision caliper was –0.40 centimeters with a standard deviation of 0.84 centimeters. Bias and precision were much more consistent across observers than for the curved caliper. Average bias for the suuntakehä was 2.06 centimeters with a standard deviation of 3.47 centimeters. Bias and precision again varied widely across observers. The bias for the curved caliper and the suuntakehä were both significant at the α=0.001 level, while the bias for the curved caliper was not significant at the α=0.05 level.

Päivinen et *al.* (1992) also examined the accuracy of stump, breast height, and six-meter height diameter measurements. The measurements were made with steel calipers for the first two and a curved caliper for the third one. Two data sets were collected. The first data set had 520 Dbh measurements on 64 trees. A "carefully measured" value for each tree was considered the true value. The second data set had two parts. In the first part, 14 observers measured two diameters at right angles on 50 trees. In the second part, breast height was marked on 33 trees prior to measurement. The average of all measurements on a tree was considered to be the correct value for that tree in both parts. Trees ranged in Dbh from approximately 17 centimeters to 34 centimeters. The mean bias for Dbh measurements for the combined data set was 0.08 centimeters with a standard deviation of 0.69 centimeters. When the diameter was the average of two measurements, the standard deviation of the errors dropped to 0.46 centimeters. A comparison of the two parts of the second data set indicated that error standard deviations were reduced, in some cases by half, when breast height was marked on the tree prior to measurement. Upper stem diameters, also measured 520 times across 64 trees, were compared with felled-tree measurements made with a steel caliper. The curved caliper was found to underestimate diameter at 6 meters by an average of 0.1 centimeters, with an error standard deviation of 0.7 centimeters. Stump diameter measurements, defined as the diameter measured in a "certain direction", were measured 1162 times across 83 trees. The mean bias was 0.15 centimeters with an error standard deviation of 1.2 centimeters.

McRoberts et al. (1994) looked at the effect of mensuration error on 20-year predictions of basal area and basal area growth. Part of their analysis included an operational study to estimate Dbh mensuration error variation. Plots were measured by multiple crews, resulting in eight Dbh measurements for trees on some plots and nine Dbh measurements for trees on the remaining plots. A total of 61 trees were measured for Dbh, ranging in size from approximately 2 centimeters to 70 centimeters. The mean value for each tree across crews was considered to be the "true" value. An examination of mensuration errors found that 60% of measurements were within 0.13 centimeters of the mean value, and 90% were within 0.38 centimeters of the mean. The bias for each crew was less than 0.25 centimeters. The standard deviation of within tree measurements was considered an estimate of the mensuration error variation, and was used to generate random errors in the simulation study. The standard deviation of within tree Dbh measurements was also found to increase nonlinearly with Dbh.

In addition to looking at heights, Williams et al. (1999) also provided accuracy statistics for upper stem diameter measurements made with the Barr & Stroud FP15 and the Criterion 400 laser dendrometers. A total of 1187 measurements from 369 trees were included in this part of the study. Callipered diameter measurements were considered the correct values. Heights were marked along the stem to control error in the location of diameter and height measurements. Mean errors in Dbh prediction were 0.34 centimeters for the Barr & Stroud and 0.12 centimeters for the Criterion. The error standard deviations were 0.88 centimeters and 1.43 centimeters respectively. Mean error for the Barr & Stroud was significant ($\alpha = 0.05$) while mean error for the Criterion was not. Errors were found to increase with distance from the tree for both instruments, with errors increasing more rapidly for the Criterion. Bias decreased and precision increased for the Criterion as experience with this tool increased. No change was apparent in the bias and precision for the Barr & Stroud as operator experience increased.

Of concern equal to that of accurate diameter measurements, is the concern for accurate measures of diameter growth. Abetz (1960) conducted an operational study that looked at the effect of mensuration error on the accuracy of radial increment measurements made with the Eklund machine. Across 5282

observations, the machine was found to underestimate diameter growth by 0.08 millimeters on average. The standard for comparison was not apparent.

Keeland and Sharitz (1993) conducted a controlled study that tested the accuracy of dendrometer bands for determining diameter growth in the first growing season after installation and found a consistent tendency to underestimate first-year growth. A total of 49 trees having a mean Dbh of 34.45 centimeters were included in their analysis. The study was controlled in that the bark of each tree was smoothed prior to installation where the bands were to be placed. The mean bias for Dbh growth measurements in the first year after attachment was found to be 0.45millimeters, with a standard deviation of 0.63millimeters. The bias was significant at the α=0.001 level.

Resulting from the dilemma over which diameter measurement to consider the true diameter, some researchers have instead looked at the accuracy of basal area measurements. McArdle (1928) conducted a controlled study that looked at the consistency in Dbh measurements, and examined the accuracy of basal area estimates from diameter tape measurements compared to those from caliper measurements. A total of 200 Douglas fir trees were included, ranging in size from 3 inches to 36 inches. The study was controlled in that all trees were tagged at breast height. He found that the tape produced basal area estimates that were 1.5% larger than those from calipers on average. The results also indicated that differences between the two tools increased in size as the trees increased in size. Basal area estimates obtained with the tape were more consistent though, and McArdle recommended using a tape when Dbh growth is the value of interest.

Chacko (1961) conducted a controlled study that compared basal areas derived from three different diameters to actual basal areas determined with a planimeter and found that calipers lead to an overestimate of true sectional area. A total of 111 sections from four species with Dbh's ranging from 10.3 inches to 24.2 inches were used. The study was controlled in that all logs were peeled prior to measurement. An analysis of the correlation between under-bark perpendicular diameters for each section to assess circularity revealed that the sections were more circular than ellipsoid, with coefficients ranging from 0.52 to 0.79 for the four species examined. Basal areas calculated from caliper measurements based on circularity were then compared with actual basal areas from planimeter

measurements. The results revealed that each of the four caliper measurements lead to an overestimation of basal area. The average of a random diameter and the diameter perpendicular to it was the most accurate method, and overestimated basal area by an average of 3.2% (standard deviations ranging from 3.0% to 7.0% for different species). The average of the largest and smallest diameters was the least accurate, overestimating basal area by an average of 5.2% (standard deviations ranging from 3.3% to 7.5%).

Kärkkäinen (1975) conducted a controlled study that looked at accuracy and precision of diameter tape and caliper derived estimates of basal area. Calculated values were compared to values based on interpolation between 16 radial measurements. A total of 660 debarked cross-sections were included in the study ranging in size from 14.8 centimeters to 34.9 centimeters. The study was controlled in that trees were sectioned allowing for major, minor and other axes to be exactly identified and measured. Their results indicated that calculation methods based on the assumption of an elliptical stem should not be used. The most accurate results were obtained by taking the average of the area of a circle calculated from the smallest diameter and the area of a circle calculated from the diameter perpendicular to it, with a mean bias of -1.51 centimeters^2 for the whole data set and 0.95 centimeters^2 for an eccentric tree subset (173 trees). This method produced the most accurate estimates by a large margin for trees with highly eccentric stem cross-sections. In addition, errors for this method were only "slightly dependent" on the size of the cross-section. The largest bias, 15.97 centimeters^2 for the whole data set and 19.94 centimeters^2 for the eccentric tree subset, was obtained using basal areas derived from d-tape measurements of circumference. Errors for this and other methods were found to increase with the trees' cross-sectional area.

In a continuation of the previous study, Kärkkäinen (1976) again examined basal area estimation accuracy based on different diameter measurements. Multiple techniques for calculating basal area are compared with exact areas for each disc measured with a planimeter. A total of 174 cross-sections were included in the study, ranging in diameter from 16.9 centimeters to 36.9 centimeters. All basal areas were calculated under the assumption of circularity. The average of the largest and smallest diameters overestimated actual basal area by an average of 1.7%. Estimates based on the diameter from a random

direction overestimated basal area by an average of 1.8%. A weighted average of the longest and shortest diameters led to the most precise estimates, which underestimated basal area by an average of 0.8%.

In order to determine the best method for obtaining unbiased estimates of tree basal area and basal area increment, Biging and Wensel (1988) conducted a controlled study to decide the number of cores that should be taken, their locations, and the appropriate geometric formulae for computing increment. Two data sets were examined. The first consisted of 45 trees that were noticeably out-of-round (mean eccentricity of 0.906, where eccentricity was defined as the length of the diameter perpendicular to the longest diameter over the length of the longest diameter). The second data set consisted of 50 randomly chosen breast height sections from a previous study (mean eccentricity of 0.946). Actual cross-sectional areas based on planimeter measurements ranged from 8.30 inches^2 to 399.90 inches^2 for the out-of-round data set, and from 20.38 inches^2 to 521.57 inches^2 for the random data set. Actual basal area increments from the difference between planimeter measurements ranged from 5.50 inches^2 to 79.30 inches^2 for the out-of-round data set, and from 11.14 inches^2 to 149.75 inches^2 for the random data set. The study was controlled in that trees were sectioned allowing for major, minor and other axes to be exactly identified and measured. They found that simple circular- or elliptical-based calculation methods produced consistently significant ($\alpha = 0.05$) overestimates of tree basal area for both data sets except for those based on the shortest diameter with the out-of-round data. The geometric mean of the shortest and longest diameters produced the most accurate estimates on average for the random data set. Basal area calculations with the longest diameter led to the largest average overestimation for both data sets: 20.42 inches^2 for the out-of-round data set (standard error of 3.52 inches^2) and 11.87 inches^2 for the random data set (standard error of 1.76 inches^2). Bias in basal area increment was less consistent. Only the longest diameter led to significant errors in estimation for both data sets: 8.65 inches^2 for the out-of-round data (standard error of 1.19 inches^2) and 5.72 inches^2 for the random data (standard error of 2.08 inches^2). Basal area increment accuracy was dependent on both the number of increment cores taken and their location around the stem.

Gregoire et al. (1990) conducted an operational study that compared basal areas from diameter taping and four different caliper measurements to actual basal areas and looked at errors relative to tree basal area. A total of 101 trees ranging in breast height basal area from 17.5 centimeters^2 to 675.4 centimeters^2 outside bark were used. Measurements were compared to areas from digitized cross sections. They found that calipers estimates of Dbh were negatively biased, contrary to the results of Chacko (1961) and Biging and Wensel (1988). Mean errors for caliper measurements ranged from –0.6% for North-South measurements (SD = 9.3%) to –3.7% for East-West measurements (SD = 14.2%). Different averaging techniques produced errors that fell between these values. They also found that a diameter tape overestimated basal area on average by 3.1% (SD = 6.6%). An important point from their results is the need for continuity in the measurement tools used for repeated measures. Using a caliper to measure Dbh on a tree previously measured with a d-tape, a 5% increase in basal area could be masked by the effect of changing instruments.

An additional reference worth mentioning is Assman (1970). He provides a discussion of multiple early German studies in which the accuracy and precision of Dbh and basal area estimates based on measurements with calipers and a steel tape were examined. In his summary he cites Matern's (1956) work showing that taped measurements are equivalent to taking the arithmetic mean of all diameters, and that elliptical trees are best measured by callipering the largest and shortest diameters. He also cites Muller's (1957) work showing the systematic positive error that results from using a tape on these elliptical trees.

Tree Crown

In addition to diameter and height, crown measurements are another area in which true mensuration error has been explored. Mensuration error in this area refers to the inability to classify or estimate crown characteristics consistently. As an example, Nicholas et al. (1991) conducted a controlled study that examined the repeatability of estimating crown class on multiple visits in uneven-aged stands. The

study was controlled in that, if two observers could not agree on a crown class for a particular tree, a third observer was brought in to cast the deciding vote. A total of 662 trees were included in the study, of which 23% were classified differently at remeasurement. Approximately 80% of codominant, intermediate, and suppressed trees were reclassified in the same categories during a second visit. Only 38% of dominant trees were reclassified as dominant at remeasurement, with the rest being classified as codominant. Attempts were made to relate successful classification percentages to stand level variables such as stand age, stand density, stand basal area, mean diameter, and slope. No "meaningful" trends were observed. Percent successful reclassification did appear to be related to stand complexity, with trees in plantations correctly reclassified 89.7% of the time. This value decreased as stand complexity increased, with trees in multi-aged stands and stands with break-up from insect damage being difficult to reclassify. The authors suggested that clarifying the accepted crown class definitions would improve consistency of remeasurement. For uneven-aged stands, they suggested that crown classification be defined solely by the amount and type of light available to a tree, with no consideration given to crown size or relative canopy position. For single-canopy stands, they suggest that classification be based solely upon the level of the canopy surrounding the tree in question.

In their study, McRoberts et al. (1994) also looked at distributions of differences between crown ratio estimates and their within-tree modes. The same 61 trees included in the Dbh analysis were included in this analysis. Approximately 87% of estimates were within 10% of the tree's mode. The coefficient of variation for differences was 73%. No relationship between within-tree standard deviation of crown ratio and mean tree Dbh or mean tree crown ratio was found. Crown ratio errors were also not significantly correlated with Dbh errors.

Ghosh et al. (1995) conducted a controlled study that looked at observer variation as a source of error in assessing three continuous and ten categorical crown characteristics through time. All characteristics but one were assessed on over 1000 trees. The characteristic 'Causes of Damage' was assessed on only 242 trees in 1991 and 201 trees in 1992 because of the limited number of trees with observable damage. The study was controlled in that crown evaluations were made from the same position each year to minimize variability due to viewing location. Measured values were checked

against values from "control" crews of similar training to look at repeatability. Between-team discrepancies varied between years and indicators. Significant differences between field team estimates and control values were found in at least one year for each of the thirteen crown indicators examined. The proportion of disagreement was over 50% in some years for certain categorical indicators. The authors concluded that, even following extensive training, experienced observers differ significantly in their estimates of common indices, and therefore the reliability of crown measurement data in some published studies may be questionable.

In addition to these studies regarding errors in crown measurements, there have also been studies done to look at errors in crown measurements estimated from aerial photos (Avery 1958, Spurr 1960). The general consensus of researches seems to be that only two out of three crown closure estimates from aerial photos will be within 10% of the true value (Spurr 1960).

Other Measurements

Studies involving measurement error in forestry have not been limited to basic tree measurements. Brown et al. (2000) looked at error in forest fragmentation metrics for overlapping scene pairs from the same time frame. Plamondon (1999) looked at errors in log-length measurements using different single-grip harvester heads. Crockford and Johnson (1983) examined errors in measurements of precipitation, throughfall, and stemflow and found that confidence in resulting estimates of interception was limited based on the size of the errors. Woods et al. (1991) performed a statistical error analysis for estimates of leaf area index and biomass density, with consideration given to mensurational errors, model selection errors, sampling error, and regression model error. Phillips et al. (2000) examined sources of error, including mensurational error, sampling error, and regression model error, for large-scale carbon budgets.

Chapter 2

TOWARDS A CHARACTERIZATION OF MEASUREMENT ERROR

Sean J. Canavan and David W. Hann

Abstract

Measurement error (ME) is a component of any study involving the use of actual measurements, but is often not recognized or is ignored. The consequences of ME's on models can be severe, affecting estimates of tree and stand attributes as well as model parameters. While correction methods do exist for countering the effects of ME's, these methods typically require knowledge of the distribution of the errors. A new method for modeling error distributions called the two-stage error distribution (TSED) method is presented here. This method is compared with traditional methods for error modeling through an application example using diameter and height ME's. Comparisons are done based on the sum of squared differences, a measure of dissimilarity between their fitted error distribution surfaces and the empirical error surface. The results indicate that the TSED method produces a much more accurate characterization of the ME distribution than traditional methods when a high percentage of errors are zero. For cases in which this does not occur, the TSED method works as well as the most accurate form of the traditional method. The TSED method is also expected to perform better at characterizing distributions with asymmetric tails. It is therefore more adaptable than traditional methods and should be used for error modeling in the future.

Introduction

A measurement error (ME) arises when there is a difference between an observed or estimated value for an attribute and the actual, or population, value for the attribute. ME's are unavoidable due to the level of precision by which measurements are made. In situations where the ME's do not result from the sampling procedure, increasing the sample size is not a viable method for reducing their effects; rather than canceling out, the ME effects may be cumulative. ME's may be sorted into three categories (Gertner 1986, 1991):

Mensuration Error This error arises when a recorded value is not exactly the same as the true value resulting from a flaw in the measurement process. Causes include error in the choice or use of a measurement tool, the inability to measure an attribute exactly as a result of a lack of proper measurement tools, a lack of adequate training, carelessness, visual estimation of tree or stand characteristics, time or financial constraints, or other factors.

Grouping Error This error occurs when a recorded value is not exactly the same as the true value due to grouping or rounding. For example, a model is calibrated with a predictor variable rounded to one level of precision and is then applied with the same variable rounded to a different level of precision.

Sampling Error This is an error in an estimate of a population parameter that results when the estimate is based on only a portion of the population (Jaakkola 1967, Smith and Burkhart 1984, Gertner 1986, Stage and Wykoff 1997). The size and distribution of the error depends upon the sampling design, including sample size, plot size, plot shape, sampling method, definition of the population, etc. (Kulow 1966, Gertner 1991), and can also be affected by grouping error.

Gertner's fourth measurement error category, process error, has not been included here based on the discussion in Chapter 1 detailing its relationship to stochastic error, which is not the focus of this analysis.

Considerable work has been done outside of forestry on the consequences of ME's (i.e. Fuller 1987, Carroll et al. 1995). As Kangas (1998) cites, depending on the relationships between the measured and true values of a variable and the other variables in a model, it may be the case that (1) the real effects of ME's are hidden, (2) observed data display relationships that are not present in the true data, or (3) the signs of the estimated coefficients are changed (Carroll et al. 1995). In simple linear regression the effect of ME's in the predictor variable is to attenuate the value of the slope parameter (Fuller 1987), causing the relationship between the predictor and response variables to appear weaker than it actually is.

The consequences of ME's in forestry data have also been examined. Biased and imprecise estimates of stand and tree attributes may result from the presence of ME's, possibly affecting management decisions as a result. In addition, forest models can be strongly affected by ME's. An assumption of regression is that the predictor variables are measured without error. Several studies have examined the effects on forest models when this assumption is violated. Bias has been demonstrated in model parameter estimates and model predictions resulting from mensuration error (Gertner and Dzialowy 1984, Gertner 1991, Wallach and Genard 1998, and Kozak 1998, Kangas & Kangas 1999), grouping error (Swindel and Bower 1972, Smith and Burkhart 1984, Kangas 1996, Ritchie 1997), and sampling error (Kulow 1966, Jaakkola 1967, Nigh and Love 1999). Precision of estimates has also been shown to change due to mensuration error (Gertner 1984, Gertner and Dzialowy 1984, Garcia 1984, Päivinen and Yli-Kojola 1989, McRoberts et al. 1994, Kangas 1998, Wallach and Genard 1998, and Kozak 1998), grouping error (Kmenta 1997), and sampling error (Kulow 1966, Smith 1975, Hann and Zumrawi 1991, and Reich and Arvanitis 1992).

Smith (1986) demonstrated the attenuation derived by Fuller with forestry data. Kangas (1998) found that mensuration error in one predictor variable of a model affected the regression coefficients of all predictor variables with which the contaminated predictor variable was correlated. Hasenauer and Monserud (1997) did a comparison of fits using observed and predicted crown ratio, and showed that the effect of crown ratio as a predictor was considerably reduced when its actual sampled value was replaced with its predicted value.

To evaluate and possibly correct for the effects of ME's, the distribution of the errors is needed. The distribution of a variable can be specified by either a probability density function (PDF) or a cumulative

distribution function (CDF). The PDF and CDF of many common distributions can be completely specified if their central tendency (i.e. the mean) and dispersion (i.e. the standard deviation) are known.

Whether a variable is continuous or discrete affects the form of the variable's PDF and CDF. Theoretically, most ME's are continuous variables. If measurements are made to a high enough level of precision, any value could be achieved. In practice all ME's are constrained to a particular set of values by the measurement process. For example, diameters at breast height (D) are typically measured in the U.S.A. to the nearest 0.1-inch causing the resulting ME's to appear to be limited to values that are multiples of 0.1-inch. In this way, they appear discrete. This operational form of the errors is the form that must be dealt with in practice. The theoretical distributions may be continuous, but their practical construction will reflect the discrete nature of the data sets on which they are modeled.

The objective of this analysis is to compare a new method for modeling error distributions to traditional methods. A description of the traditional methods and their positive and negative aspects is followed by a description of a new method called the two-stage error distribution (TSED) modeling method. The two methods are then applied to actual ME data for two variables and compared based on their abilities to model the empirical error distributions. These methods are particularly germane to the characterization of mensuration error, though they may have utility for characterizing the distributions of grouping or sampling errors.

Error notation in existing publications varies to a wide degree. The notation employed here will be as follows. The true value of a variable x will be denoted by x_T, and the measured value of x will be denoted by x_M. The error in a measurement of x will be denoted by δ_x, where $x_M - x_T = \delta_x$. The negative, zero, and positive values of δ_x will be denoted by δ_x^-, δ_x^0, and δ_x^+ respectively. The total number of observed values of δ_x will be denoted by n, and the number of errors in δ_x^-, δ_x^0, and δ_x^+ will be denoted by n^-, n^0, and n^+ respectively.

Traditional Methods

Several studies in forestry have examined the effects of randomly generated ME's on models. These studies have assumed that the errors were normal in distribution (Nester 1981, Garcia 1984, Smith 1986, Päivinen and Yli-Kojola 1989, Gertner 1991, McRoberts et al. 1994, Kozak 1998, Kangas 1996, Kangas 1998, Kangas and Kangas 1999, Phillips et al. 2000, Williams and Schreuder 2000). The PDF for the distribution of any normally distributed random variable, γ, is given by the equation:

$$f_\Gamma(\gamma) = \frac{1}{\sqrt{2\pi}\sigma(\gamma)} e^{\frac{-(\gamma-\mu(\gamma))^2}{2\sigma(\gamma)^2}}, \quad -\infty < \gamma < \infty, \quad \infty < \mu(\gamma) < \infty, \quad \sigma(\gamma) > 0$$

The parameter $\mu(\gamma)$ represents the mean of the distribution. As the distribution is symmetric, $\mu(\gamma)$ is also at the center of the distribution. The parameter $\sigma(\gamma)$ represents the standard deviation of the distribution, providing a measure of the distribution's spread. The square of the standard deviation, $\sigma^2(\gamma)$, represents the variance of the distribution. This distribution is completely specified by $\mu(\gamma)$ and $\sigma(\gamma)$, and additional information is not needed to model it. The PDF for the normal distribution looks like the typical bell curve. The CDF for the normal distribution is shown in Figure 2.1.

The ME studies cited above in which a normal distribution was assumed, for which γ in the above distribution would be δ_x, may be separated into three categories regarding $\mu(\delta_x)$: (1) those that have assumed $\mu(\delta_x)$ to be zero; (2) those that have assumed $\mu(\delta_x)$ is a constant other than zero; (3) those that have assumed $\mu(\delta_x)$ is not constant. A value of $\mu(\delta_x)$ other than zero implies that the distribution of the ME's contains a bias. The average of a random sample of values from the distribution is not expected to be zero in this case. A non-constant value of $\mu(\delta_x)$ may be expressed as a function of x_M as well as other attributes, $z_1, z_2, \ldots, z_k$, that may have an effect on the accuracy of the measurements. For example, the steepness of the ground's slope under the plot or tree, the weather conditions during measurement (sunny, cloudy, rainy, etc.), and/or variation in the proficiency of measurement personnel can all affect the degree of accuracy of forest measurements. The variable x_M is used instead of x_T because, in application, x_T is

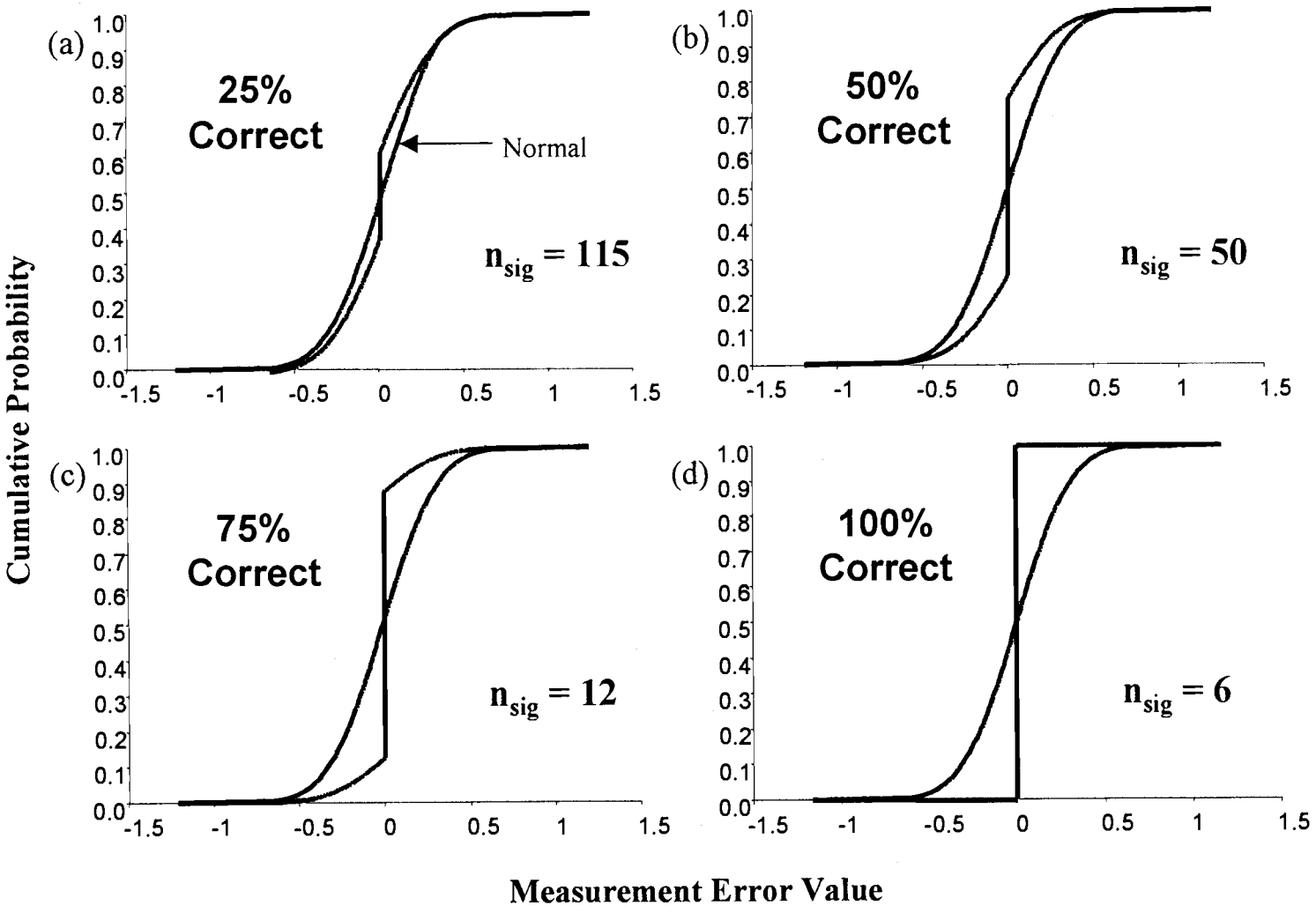


Figure 2.1. - Comparison between the normal distribution and distributions with increasing percentages of correct measurements

unknown. The majority of ME studies in forestry have assumed that $\mu(\delta_x)= 0.0$. An example of a case in which the authors allowed $\mu(\delta_x)$ to vary is Larsen et al. (1987). In this study the mean of the ME's for tree height (H) was found to change with H_M.

The three characterizations of $\mu(\delta_x)$ may each be further separated into those that assume $\sigma(\delta_x)$ is constant and those that allow $\sigma(\delta_x)$ to vary, giving six possible types of normal distributions. The assumption that $\sigma(\delta_x)$ is constant implies the errors do not become more or less variable as the value of x_M changes. Allowing $\sigma(\delta_x)$ to vary in size indicates that the kurtosis of the distribution is different for different values of x_M. Similar to $\mu(\delta_x)$, non-constant values of $\sigma(\delta_x)$ may be assumed to be a function of x as well as other variables. Nearly all studies in forestry involving ME's have assumed that $\sigma(\delta_x)$ was constant. An example of a study in which the authors allowed $\sigma(\delta_x)$ to vary is McRoberts et al. (1994).

Characterizing the normal distribution requires modeling $\mu(\delta_x)$ and $\sigma(\delta_x)$. Traditionally $\mu(\delta_x)$ is modeled first. The function for $\mu(\delta_x)$ may be a constant, a linear function, or a nonlinear function. The fitted values for $\mu(\delta_x)$ are then subtracted from the observed values of x_M. $\sigma(\delta_x)$ is then modeled with the resulting bias-corrected data. This is done through experimentation to find a transformation of the bias corrected data that produces values with homogeneous variance. Back-transforming a successful transformation will give a function for the heterogeneous variance of the bias-corrected data.

Although it may often be easy to model the normal distribution, it is not always the appropriate choice. The normal assumes the distribution is symmetric about $\mu(\delta_x)$, making it inappropriate for asymmetric distributions. It becomes a poorer choice as the degree of asymmetry increases. The normal also does not allow for relatively high concentrations of individual values. In a situation for which the distribution is highly concentrated at a single value, the PDF would show a spike and the CDF would have a vertical section at that value. The normal does not allow for this situation to occur. This behavior is a concern in ME modeling as certain situations may be expected to result in high percentages of correct measurements. A comparison between the normal distribution and a distribution with increasing amounts of correct measurements is shown in Figure 2.1. In Figure 2.1a, the case of 25% correct measurements, the differences between this and the Normal CDF are significant at the α=0.05 level according to the Kolmogorov test if the sample size is greater than 115 observations (Conover 1971). This number

decreases to 28 observations for the 50% correct case (Figure 2.1b), 12 observations for the 75% correct case (Figure 2.1c), and six observations for the "ideal" case (Figure 2.1d) in which all the measurements are correct.

For the "ideal" case, $\sigma(\delta_x)$ would be equal to zero. The PDF would be entirely concentrated at $\mu(\delta_x)$ with no other values of δ_x having positive probability. If $\mu(\delta_x)= 0.0$ then the resulting CDF would describe the ideal situation of no ME (Figure 2.1d). This situation cannot be modeled with the normal distribution, however, due to the requirement that the standard deviation be strictly greater than zero. Other common distributions suffer from the same limitation.

Two-Stage Error Distribution (TSED)

The new approach to modeling ME's presented here, which allows for the vertical behavior of the CDF at zero or any other point, uses a two-stage modeling process. Models of this type are not new in forestry. Hamilton and Brickell (1983) described a two-stage model for a two state system and applied it to estimation of cull volume in standing trees. The model described here is similar in design, however, it may be applied to problems with two or more states. In the first stage, the probabilities of the different types of errors are modeled. These probabilities provide the heights of the different sections of the CDF corresponding to the different error types. In the second stage the portions of the CDF curve corresponding to the different error types are modeled. For an error type comprised of a single value, such as δ_x^0, the shape of the CDF for this type is a vertical line as seen in Figure 2.1d.

Stage 1: Error Type Probability Modeling

In the first stage, the probabilities of the different types of errors are modeled. These error types may be any partition of the range of error values for which the partition elements are each a single value or a

single segment of the real number line. The simplest partition with meaning would include two error types, for example $\{\delta_x^-, \delta_x^{\geq 0}\}$. The partition that will be used in the Example Application section is $\{\delta_x^-, \delta_x^0, \delta_x^+\}$.

A description of the modeling procedure will begin by considering the case for which the probabilities of the error types, $Pr(\delta_x^-)$, $Pr(\delta_x^0)$, and $Pr(\delta_x^+)$ for example, are constant across x. This will be followed by a description of the modeling procedure for the case in which the error type probabilities change as x changes.

Case 1: Constant Error Type Probabilities

Constant error type probabilities across x_M, the size of the objects being measured, will arise when the size of the object does not affect the ability to measure it accurately. Modeling the probabilities [Pr( )] in this case is done using simple proportions. $Pr(\delta_x^-)$ is estimated by $(n^-)/n$, and $Pr(\delta_x^+)$ is estimated by $(n^+)/n$. By definition, $Pr(\delta_x^0) = 1.0 - Pr(\delta_x^-) - Pr(\delta_x^+)$. When no bias is present, $Pr(\delta_x^-) = Pr(\delta_x^+)$ (for example Figure 2.2a). Bias is present when $Pr(\delta_x^-) \neq Pr(\delta_x^+)$ (for example Figure 2.2b). $Pr(\delta_x^-)$ and $Pr(\delta_x^+)$ are estimated by the proportion of observed errors in each of the two error types.

Case 2: Non-Constant Error Type Probabilities

In forestry situations, it is more likely that the error type probabilities will not be constant. Measurements such as D and H are increasingly more difficult to make without error as the size of the tree being measured increases. Errors of this type may also be unbiased (for example Figure 2.2c) or they may be biased (for example Figure 2.2d). In either situation, simple proportions are no longer appropriate solutions. When only two error types, for example $\{\delta_x^-, \delta_x^{\geq 0}\}$, are involved, the probabilities may be characterized with logistic regression as a function of tree size [$g(x)$]:

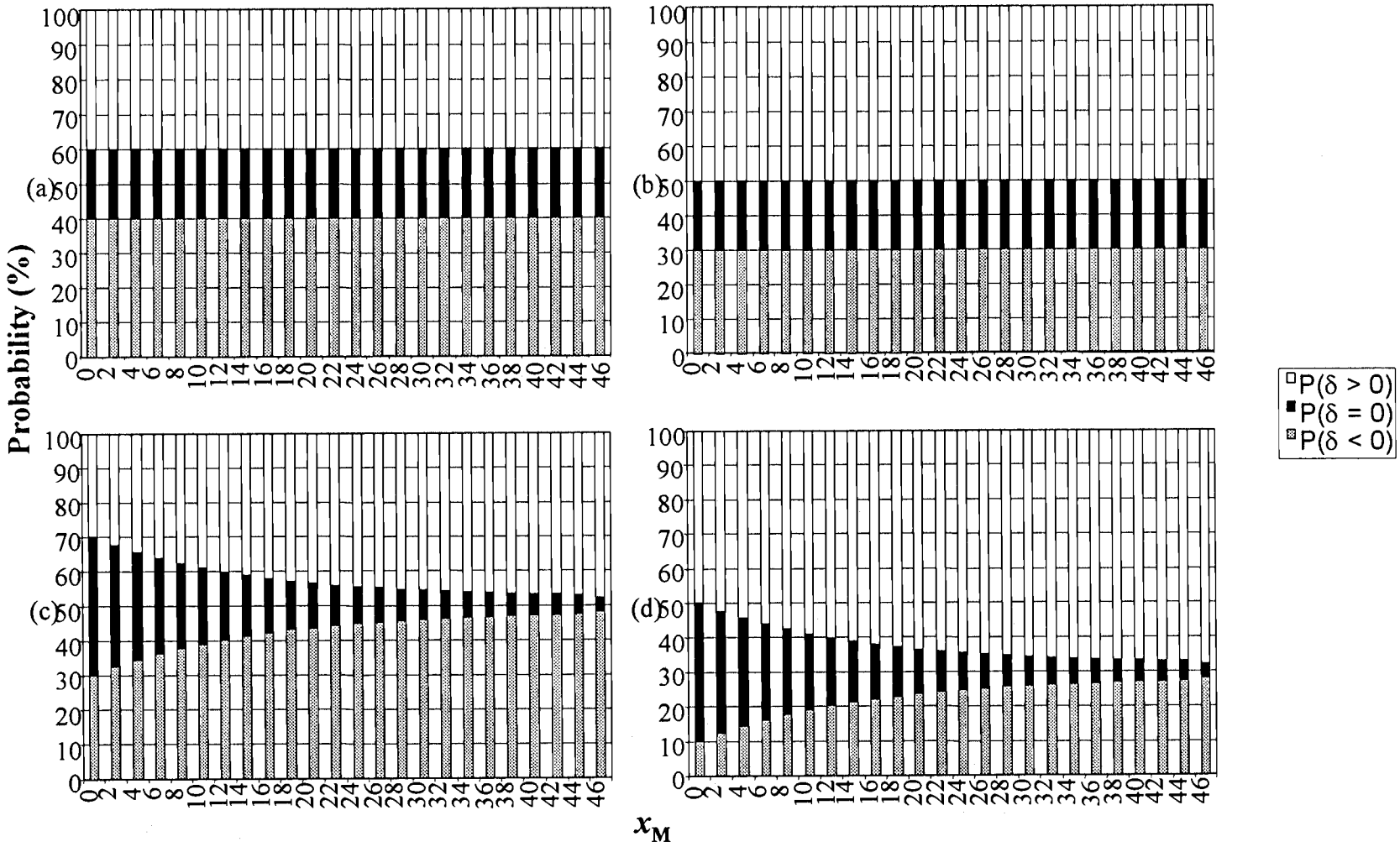


Figure 2.2 – Error type probability graphs; (a) constant and unbiased, (b) constant and biased, (c) non-constant and unbiased, (d) non-constant and biased

$$\Pr(\delta_x^{-}) = \frac{1}{1+e^{g(x_M)}} \qquad \Pr(\delta_x^{\geq 0}) = \frac{e^{g(x_M)}}{1+e^{g(x_M)}}$$

When there are more than two error types, multinomial regression as a function of tree size may be used to characterize them:

$$\Pr(\delta_x^{-}) = \frac{1}{1+e^{g_1(x_M)}+e^{g_2(x_M)}} \quad \Pr(\delta_x^{0}) = \frac{e^{g_1(x_M)}}{1+e^{g_1(x_M)}+e^{g_2(x_M)}} \quad \Pr(\delta_x^{+}) = \frac{e^{g_2(x_M)}}{1+e^{g_1(x_M)}+e^{g_2(x_M)}} \qquad (1)$$

These models are an extension of the logistic model. Logistic regression and multinomial regression meet the requirements of non-negative fitted probabilities that sum to 1.0. This modeling may also be done with other procedures meeting the same requirements.

Stage 2: Modeling the CDF Curves Within the Error Types

In the second stage, the parts of the CDF curve corresponding to the different error types are modeled and combined with the error type probabilities from the first stage to build the fitted CDF surface. Modeling is done separately for each error type as curve forms may differ across error types. For modeling purposes, the curve in each error type is considered a CDF itself. This allows each curve to be fit easily with the same equation form and then scaled to the correct height using the error type probabilities.

Modeling the CDF for a particular error type begins with grouping the errors into classes based on the size of x. The classes need to be defined in a manner such that there are enough observations in each to reasonably approximate the form of the corresponding CDF for that class. Therefore, the widths of the classes may need to change as x changes. Errors within each size class are then further separated into error classes. Widths of these error classes should be small in order to provide precise information about the form of the CDF for the size class.

Cumulative probabilities are then calculated for the errors in a size class. A CDF takes in values from $-\infty$ to $+\infty$ and the fitted models must do the same. The δ_x^- CDF function must be fit such that it is 0.0 for large δ_x^- approaching $-\infty$, increasing up to 1.0 as δ_x^- approaches zero. The δ_x^+ function must be fit such that it is zero for values near zero and approximately 1.0 for large δ_x^+ approaching $+\infty$. Cumulative probabilities are therefore calculated beginning with the largest δ_x^- class and moving toward the smallest δ_x^- class. Likewise, cumulative probabilities for the δ_x^+ are calculated beginning with the smallest class and moving toward the largest class. Each error class is assigned a value corresponding to the proportion of the total number of observations in the size class that are within that class or are less than the values in that class. The form of the CDF for the size class is approximated using these cumulative probabilities.

The next step in the curve modeling process is to choose a model form for the size class CDFs within an error type. Plotting the empirical CDFs for different size classes gives an indication of what model forms may be appropriate. Errors are unintentional and larger values are much less likely to occur than smaller values, but their likelihood often increases with x. Therefore, the model form chosen must favor small errors over large errors, but must allow this relationship to change as the size of x_M changes. The exponential CDF would be an appropriate choice for modeling CDFs of this form:

$$F_{\Delta_x^-}(\delta_x^-) = \Pr(\Delta_x^- \le \delta_x^-) = 1 - e^{\delta_x^- \beta^-}, \quad \beta^- > 0 \tag{2}$$

$$F_{\Delta_x^+}(\delta_x^+) = \Pr(\Delta_x^+ \le \delta_x^+) = 1 - e^{-\delta_x^+ \beta^+}, \quad \beta^+ > 0 \tag{3}$$

The exponential CDF is flexible in form and has a single parameter which appears only once in the function. Fitting the model is easily done as a result. The exponential PDF may appear a better choice in certain cases, but is not so easily fit. Its functional form is:

$$f_{\Delta_x^-}(\delta_x^-) = \Pr(\Delta_x^- = \delta_x^-) = \beta^- e^{\delta_x^- \beta^-}, \quad \beta^- > 0$$

$$f_{\Delta_x^+}(\delta_x^+) = \Pr(\Delta_x^+ = \delta_x^+) = \beta^+ e^{-\delta_x^+ \beta^+}, \quad \beta^+ > 0$$

The parameters β^+ and β^- appear in two places in each equation and must be the same in both places in order for the equations to integrate to 1.0 as required. Our experience indicates that this restriction makes the PDF less tenable than the CDF function and can lead to considerably poorer fits when modeling the distribution of ME's. Functions other than the exponential may be fit provided they have the desired curve form, produce cumulative probabilities beginning at zero and summing to 1.0, and are monotone increasing, right continuous, and flexible in form.

A constant β^- or β^+ value in Equation (2) or (3) results in a two-dimensional exponential CDF function. Modeling β^- or β^+ as a function of x_M and possibly other variables $z_1, \ldots, z_k$ results in a multi-dimensional exponential CDF, where the number of dimensions is k+3. To model a three-dimensional exponential CDF equation, two-dimensional exponential CDF equations are first fitted to each size class of x. The resulting parameter estimates of $\beta_1^-, \ldots, \beta_t^-$, or , $\beta_1^+, \ldots, \beta_j^+$ are then modeled as a function of the size class midpoints. β^- or β^+ in Equation (2) or (3) are then replaced by this function and the overall function can be fit across all size classes values of x_M simultaneously. As a result of this process, each unique value of x_M is characterized by a unique two-dimensional exponential CDF for both δ_x^+ and δ_x^-.

Discrete measurements affect how the CDF models should be fit in order to maximize their predictive ability. For example, measurements of both x_T and x_M are made to the nearest 0.1-unit, then δ_x is restricted to integer multiples of 0.1-units. The smallest value of δ_x^+, $\min(\delta_x^+)$, is 0.1-unit. Assigning positive probability to the range of impossible values between zero and $\min(\delta_x^+)$ will bias the other positive error probabilities downward. This is also true for the negative errors. To eliminate this bias, the ME CDF should be horizontal in the intervals $[\max(\delta_x^-), 0)$ and $(0, \min(\delta_x^+)]$. This is accomplished by fitting the model with the transformation of $x_M - \max(\delta_x^-)$ for the negative errors and $x_M - \min(\delta_x^+)$ for the positive errors. The exponential CDF model then becomes:

$$F_Y^-(y) = 1 - e^{y\beta}, \quad y \le 0, \quad \beta > 0 \qquad = 1 - e^{(x_M - \min(\delta_x^-))\beta} \quad x_M \le \max(\delta_x^-), \quad \beta > 0 \tag{4}$$

$$F_Y^+(y) = 1 - e^{-y\beta}, \quad y \ge 0, \quad \beta > 0 \qquad = 1 - e^{-(x_M - \min(\delta_x^+))\beta} \quad x_M \ge \min(\delta_x^+), \quad \beta > 0 \tag{5}$$

The result is to shift the CDF curves in Equations (2) and (3) so that positive probability is not assigned to negative errors greater than $\max(\delta_x^-)$ or positive errors less than $\min(\delta_x^+)$.

The probability of δ_x^+ values falling at $\min(\delta_x^+)$ is often large. When this occurs, a vertical section at $\min(\delta_x^+)$ must be incorporated into $F_Y^+(y)$ to accurately model the distribution of the positive errors. The size of the vertical section for an observation of size x is equal to the probability of the smallest positive error, $\Pr(\min(\delta_x^+))$. Without this section, $\Pr(\min(\delta_x^+))$ would be assigned a value of zero because $F_Y^+(y)$ begins at 0.0 before increasing to 1.0. Building the vertical section into $F_Y^+(y)$ is done by first modeling $\Pr(\min(\delta_x^+))$. Logistic regression is an appropriate choice for this, with the logistic model being a function of x_M and possibly other variables:

$$\Pr(\min(\delta_x^+)) = \frac{e^{h(x,z_1,\ldots z_k)}}{1+e^{h(x,z_1,\ldots z_k)}} \tag{6}$$

This is then included in the CDF model for positive errors as follows:

$$F_Y^+(y) = 1-[1-\Pr(\min(\delta_x^+))]e^{y\beta}, \quad y = x - \min(\delta_x^+) \geq 0, \quad \beta > 0 \tag{7}$$

This model has a value of $\Pr(\min(\delta_x^+))$ when y is zero and approaches 1.0 as y approaches $+\infty$, as desired.

Once the positive and negative error CDF models have been fit for the upper and lower portions of the overall curve, they can be combined with the error type probabilities to give the function for the entire ME CDF surface as follows:

$$F_{\Delta_x}(\delta_x) = \Pr(\Delta_x \leq \delta_x) = \begin{cases} \Pr(\delta_x^-) \times F_Y^-(\delta_x) & \delta_x < 0 \\ \Pr(\delta_x^-) + \Pr(\delta_x^0) & \delta_x = 0 \\ \Pr(\delta_x^-) + \Pr(\delta_x^0) + \Pr(\delta_x^+) \times F_Y^+(\delta_x) & \delta_x > 0 \end{cases} \tag{8}$$

An example of the CDF for a particular size class resulting from an application of Equation (8) is shown in Figure 2.3.

Traditional Method and TSED Application Comparison

To demonstrate the traditional and TSED procedures, both were applied to the problem of characterizing the distribution of mensuration errors arising from the measurement of D and H taken before and after they were felled for stem analysis. Therefore, D_M and H_M were defined as the measurements taken on the standing trees, and D_T and H_T were defined as the measurements taken after stem analysis.

The data come from two studies associated with the development of the Southwest Oregon version of the ORGANON growth and yield model. A detailed description of the geographic and age ranges of the data is provided in Hanus et al. (2000). D and H were measured on all trees to the nearest 0.1in and 0.1ft respectively. D_M measurements were made with a diameter tape. H_M measurements were made directly using a 25- to 45-ft telescoping pole for smaller trees, and indirectly with the pole-tangent method (Larsen et al. 1987) for larger trees. Felled tree D measurements were made with a caliper to measure both the longest and shortest axes after sectioning the tree at 4.5 feet. D_T was defined as the arithmetic mean of the two measurements. H_T measurements were made with a tape. Five species groups were included in the analysis: Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco), True fir (*Abies grandis* (Dougl. Ex D. Don) Lindl. and *Abies concolor* (Gord. & Glend.) Lindl. Ex Hildebr.), Ponderosa pine (*Pinus ponderosa* Dougl. Ex Laws.), Sugar pine (*Pinus lambertiana* Dougl.), and Incense cedar (*Calocedrus decurrens* Torr.). There were 2175 trees included in the D application example ranging in D_M from 0.8 inches to 72.1 inches. All D_M measurements were taken on the same day the tree was felled and measured for D_T to eliminate confounding effects due to diameter growth and drying. Standing and felled height measurements were taken on different days for some trees. To avoid errors confounded with height growth, trees for which the standing height measurement was taken in the growing season, May 1 – August 15 (Jablanczy 1971, Emmingham 1977), were only included if the felled height measurement was

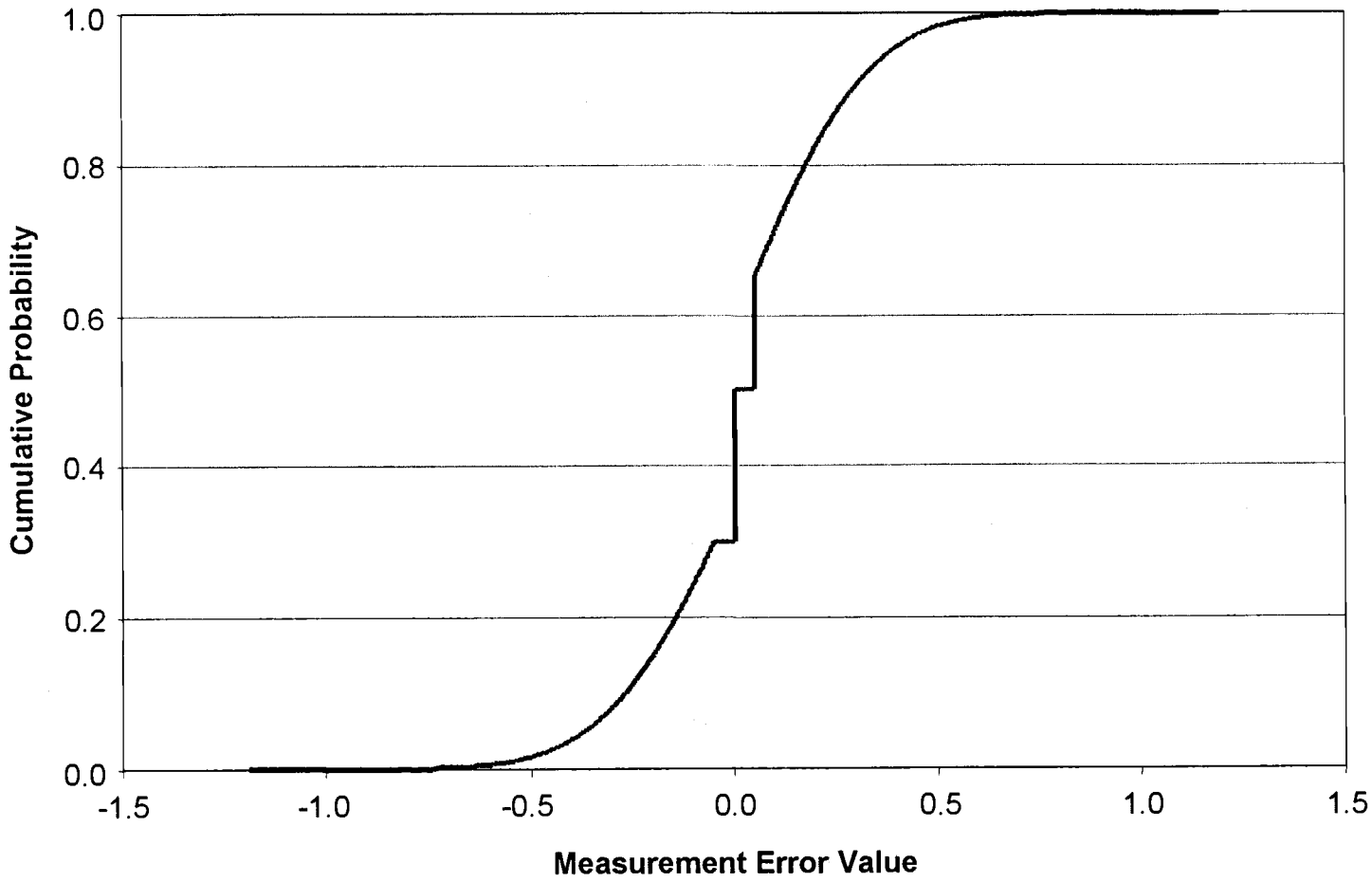


Figure 2.3 – Example of combined CDF graph for a single size class

taken within five days of the standing measurement. H_T to the tip of the tree was used in this case and for cases in which both measurements were taken in the same non-growing season. H_T to the last whorl was used if H_M was taken outside the growing season and H_T was taken within the growing season or if measurements were made in consecutive non-growing seasons. The resulting dataset contained 1238 tree records with H_M ranging from 8.4 feet to 231.7 feet.

Eight alternate ME CDF surfaces were compared to each of the actual ME CDF surfaces for the samples of observed values of δ_D and δ_H. The first alternative surface, called the "Base Fit", assumed all errors to be zero. The resulting Base Fit CDF has a vertical section at zero of length 1.0 for each value of δ_D and δ_H. This was considered the simplest fit to the data as well as the ideal fit in that it assumes no measurement error. Its purpose was to provide a measure of the improved or worsened fit of the TSED over the six traditional method fits when compared to the actual ME CDF surfaces for the samples of observed values of δ_D and δ_H.

D Analysis

By definition, $\delta_D = D_M - D_T$, which led to there being 1278 positive, 368 zero, and 529 negative δ_D values.

Traditional Distribution Modeling

A plot of the actual δ_D CDF surface (Figure 2.4) indicates that a distribution with a non-constant $\mu(\delta_D)$ and a non-constant $\sigma(\delta_D)$ is appropriate. In this example, the normal distribution will be applied with each of the six $(\mu(\delta_D), \sigma(\delta_D))$ combinations discussed previously in order to examine how increased information improves the CDF surface model. This will also provide an indication of how well previous studies have approximated this distribution.

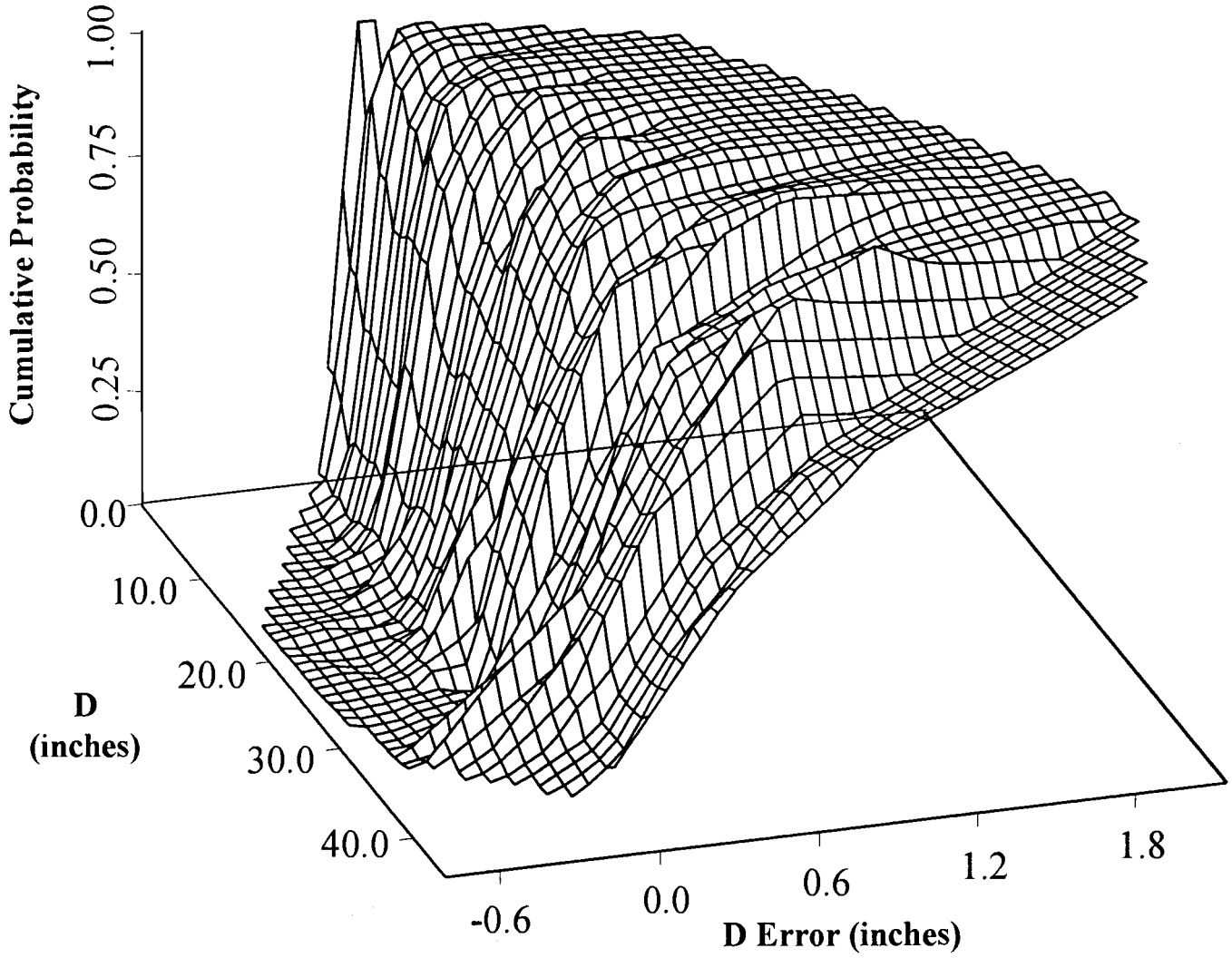


Figure 2.4 – The empirical D measurement error CDF surface

$\mu(\delta_D)$ was 0.0901 inches, with a $\sigma(\delta_D)$ of 0.2237 inches. The bias was found to increase with D according to the model:

$$\mu(\delta_D) = 0.00398331 \times D + 0.00012141 \times D^2$$

(p-values < 0.0001 for both coefficients). In addition, $\sigma^2(\delta_D)$ was found to be heterogeneous and increased with D (p-values for Levene's and Brown and Forsythe's tests were < 0.0001):

$$\sigma(\delta_D) = 0.03877408 \times e^{0.1145 \times D}$$

The resulting variance stabilizing transformation was:

$$\delta_D / e^{0.1145 \times D_M}$$

(Levene's test p-value = 0.02, Brown and Forsythe's test p-value = 0.05).

Six alternative normal distributions were created by varying their definitions of $\mu(\delta_D)$ and $\sigma(\delta_D)$ (Table 2.1). CDF surfaces were generated for each of the six distributions. Figure 2.5 illustrates the resulting CDF for Normal 6, the normal distribution which best matched the behavior observed with the actual CDF.

TSED Distribution Modeling

Stage 1

The changing location and slope of the actual CDF as D changes (Figure 2.4) indicate that the error type probabilities are not constant. As there are three error types, $\{\delta_x^-, \delta_x^0, \delta_x^+\}$, the error type

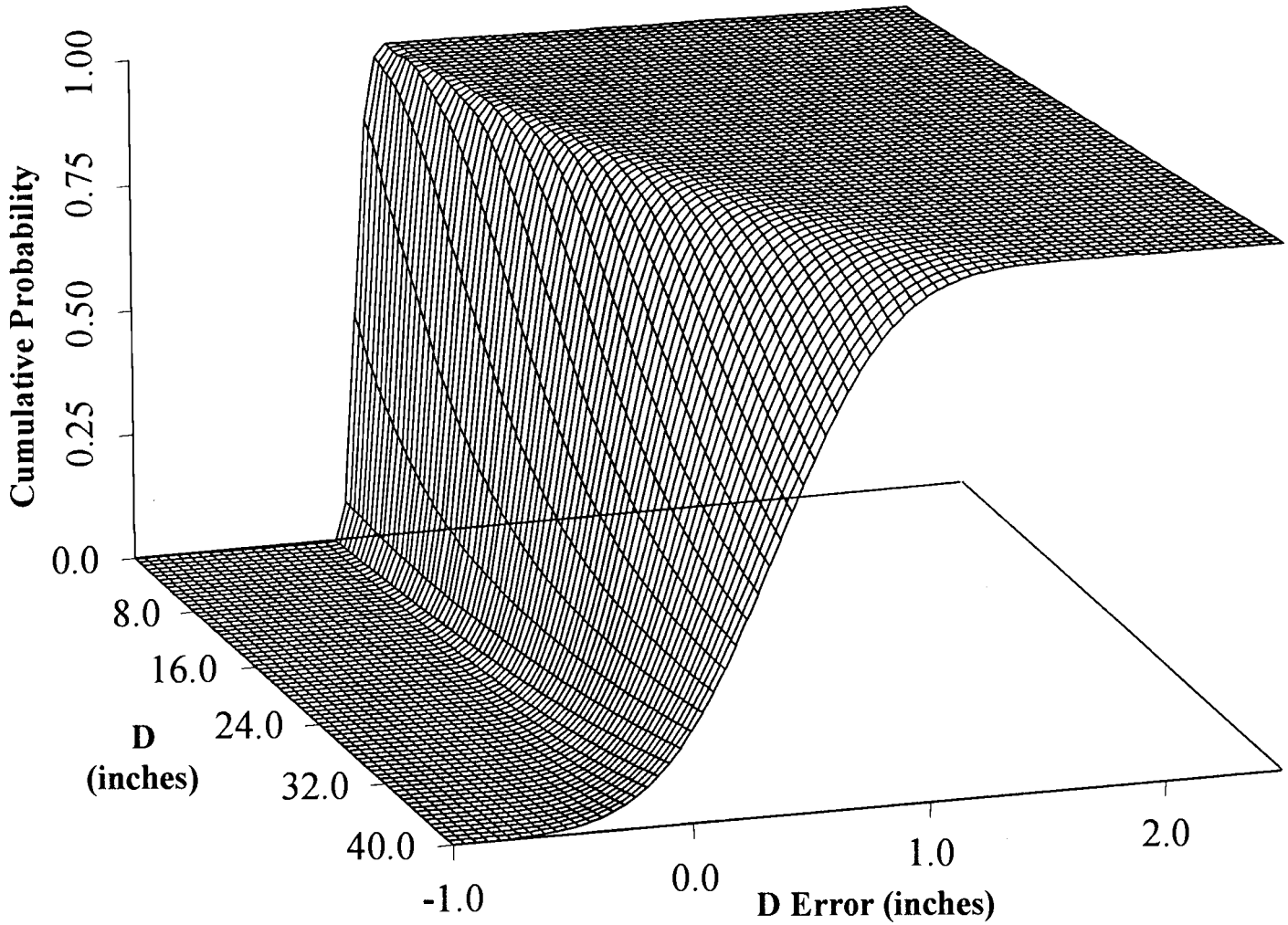


Figure 2.5 – D CDF surface plot of Normal 6 (non-constant mean, non-constant variance)

Table 2.1. Means and standard deviations for the six D measurement error normal distributions.

Alternative Distribution	D	
	μ_D (in.)	σ_D (in.)
Normal 1	0	0.2237
Normal 2	0.0901	0.2237
Normal 3	$0.00398331 \times D + 0.00012141 \times D^2$	0.2237
Normal 4	0	0.03877408×exp[0.1145×D]
Normal 5	0.0901	0.03877408×exp[0.1145×D]
Normal 6	$0.00398331 \times D + 0.00012141 \times D^2$	0.03877408×exp[0.1145×D]

probabilities were modeled with multinomial regression. This was done in S-plus using a generalized linear model with the Poisson link function (Schafer 1998). The data were grouped by error type into 1-inch D classes for trees less than 26.1 inches. Due to the smaller number of larger trees, trees greater than 26.0 inches and less than 38.1 inches were grouped into two-inch classes. The remaining 31 trees with diameters greater than or equal to 38.1 inches and having a mean of approximately 45.0 inches were grouped together into one class and assigned a value of 45.0 inches.

Equation (1) was fit with g_1 and g_2 being linear functions of D, D^2, $D^{½}$, and D^{-1}. These variables and a variable indicating error type were included as factor variables. Interactions between the D variables and the error type variable were also included. Parameter estimates for the interaction terms from this fit are equal to the multinomial maximum likelihood estimates of the model parameters based on multinomial likelihood (McCullagh and Nelder 1995). Fitted models were tested for overdispersion using quasilikelihood to determine whether extra-multinomial variation was present. The overdispersion parameter was estimated to be 1.0367, with a 95% confidence interval of (0.6645, 1.4030), indicating a lack of overdispersion. The models were then refit without quasilikelihood. The D^{-1} terms in g_1 and g_2 were insignificant and were dropped from the model. Refitting indicated that g_1 was not significantly different from zero; the probability of a negative error is not significantly different from the probability of a zero error within each size class. The appropriate reduced model forms were then:

$$\Pr(\delta_x^-) = \frac{1}{2 \times (1 + e^{g_2(x)})} \qquad \Pr(\delta_x^0) = \frac{1}{2 \times (1 + e^{g_2(x)})} \qquad \Pr(\delta_x^+) = \frac{e^{g_2(x)}}{1 + e^{g_2(x)}} \tag{9}$$

Counts for the negative and zero errors were combined and Equation (9) was fit. The resulting error type probability model coefficients are given in Table 2.2.

Table 2.2. Fitted multinomial regression coefficients for the reduced D error type probability models.

Function	Variable	Coefficient	Std. Error	p-value[1]
g_2	Intercept	-2.58503924	0.637901559	0.0004
	D	-0.32173674	0.099804471	0.0033
	$D^{½}$	1.87615818	0.489346805	0.0007
	D^2	0.00280732	0.001048412	0.0125

[1] - based on 27 degrees of freedom

Stage 2

Modeling the CDF curves within the error types began by creating separate data sets for the negative and positive errors. The Stage 1 D classes were used here. The first class (0 – 1 in.) was removed as it had no trees for the positive errors and only one tree for the negative errors. Within each D size class, error classes were created to model the error CDF. Errors ranged in size from –0.80 in. to 2.15 inches. Error classes with a width of 0.025 inches extended from –0.90 inches to -0.05 inches for the negative errors and from 0.05 inches to 2.25 inches for the positive errors.

Graphing the cumulative probabilities by error class for each of the D classes indicated that the exponential forms in Equations (2) and (3) were appropriate. The means and variances of the curve forms differ across D classes, indicating the need for the β^- and β^+ parameters to change with D. The minimum absolute error size based on the definition of a correct measurement was 0.05 inches, and the correction incorporated into Equations (4) and (5) was included with $\max(\delta_x^-) = -0.05$ inches and $\min(\delta_x^+) = 0.05$ inches.

Modeling began by fitting a function for the $Pr(\min(\delta_x^+))$. This model was fit with Proc Logistic in SAS. Predictor variables included in the model building were D, D^2, $D^{1/2}$, and D^{-1}. The final model form was:

$$Pr(\min(\delta_D^+)) = \frac{e^{a_1 \times D + a_2 \times D^{-1}}}{1 + e^{a_1 \times D + a_2 \times D^{-1}}} \tag{10}$$

All other parameters, including a constant term, were insignificant at the α=0.10 level. Equation (10) was included in Equation (7) to produce the updated CDF equations with $\max(\delta_x^-)$ and $\min(\delta_x^+)$ as defined above:

$$F_Y^-(\delta_D^-) = 1 - e^{-(\delta_D^- - \max(\delta_D^-))^* \beta^-} \qquad \delta_D < 0, \quad \beta^- > 0 \tag{11}$$

$$F_Y^+(\delta_D^+) = 1 - \left(1 - \frac{e^{a_1 D + a_2 D^{-1}}}{1 + e^{a_1 D + a_2 D^{-1}}}\right) e^{-(\delta_D^+ - \min(\delta_D^+))^* \beta^+}, \quad \delta_D > 0, \quad \beta^+ > 0 \tag{12}$$

Fitting Equations (11) and (12) and plotting the estimates for β^- and β^+ against D class showed that the parameters could be modeled with an exponential function in both cases. The best model forms were:

$$\beta^- = b_0 \times e^{b_1 \times D^2 + b_2 \times D^{-1}} \qquad \text{for } \delta_D < 0$$

$$\beta^+ = b_0 \times e^{b_1 \times D^2 + b_2 \times D^{-1}} \qquad \text{for } \delta_D > 0$$

These functions were put in place of β^- and β^+ in Equations (11) and (12) and the power on the error variable was allowed to vary, producing the final CDF equation forms:

$$F_Y^-(\delta_D^-) = 1 - e^{-(\delta_D^- - (-0.05))^{c_1} \times (b_0 \times e^{b_1 \times D^2 + b_2 \times D^{-1}})} \qquad \text{for } \delta_D < 0 \tag{13}$$

$$F_Y^+(\delta_D^+) = 1 - \left(1 - \frac{e^{a_1 \times D + a_2 \times D^{-1}}}{1 + e^{a_1 \times D + a_2 \times D^{-1}}}\right) e^{-(\delta_D^+ - (0.05))^{c_1} \times (b_0 \times e^{b_1 \times D + b_2 \times D^{-1}})} \qquad \text{for } \delta_D > 0 \qquad (14)$$

The parameters of Equations (13) and (14) were fit in SAS using Proc NLIN and the resulting estimates are given in Table 2.3. Starting values for this fit came from prior fits of β^-, β^+, and $Pr(\min(\delta_D^+))$. The starting value for the c_1 parameter was 1.0 in each equation. Combining the fitted error type probabilities from stage 1 with these models according to Equation (8) produced the fitted CDF surface in Figure 2.6.

Table 2.3. Fitted nonlinear regression coefficients for the modeled D measurement error CDF curves within the error types.

	Negative Error Fit		Positive Error Fit	
Parameter	Estimate	Std Error	Estimate	Std Error
a_1	----	----	0.10842178	0.00394588
a_2	----	----	-3.53708466	0.28698672
b_0	0.01215524	0.00115901	12.41092908	0.61720091
b_1	0.00091755	0.00004805	-0.04861734	0.00110454
b_2	-1.95002800	0.30062975	3.47824077	0.43714443
c_1	-1.27338986	0.03135226	1.07090649	0.01253076
Adjusted R^2	0.8858		0.9486	

No statistical test exists for comparing the predictive ability of CDF surfaces. As a result, the TSED fitted surface was compared with the different normal distributions by calculating the sum of squared differences between the eight fitted surfaces and the actual surface for all 2175 observations in the data set. The results of these comparisons are given in Table 2.4.

H Analysis

The height (H) error data set contained 722 negative errors, 30 zero errors, and 486 positive errors. Contrary to the D example, the H application example evaluates the capability of the TSED and the

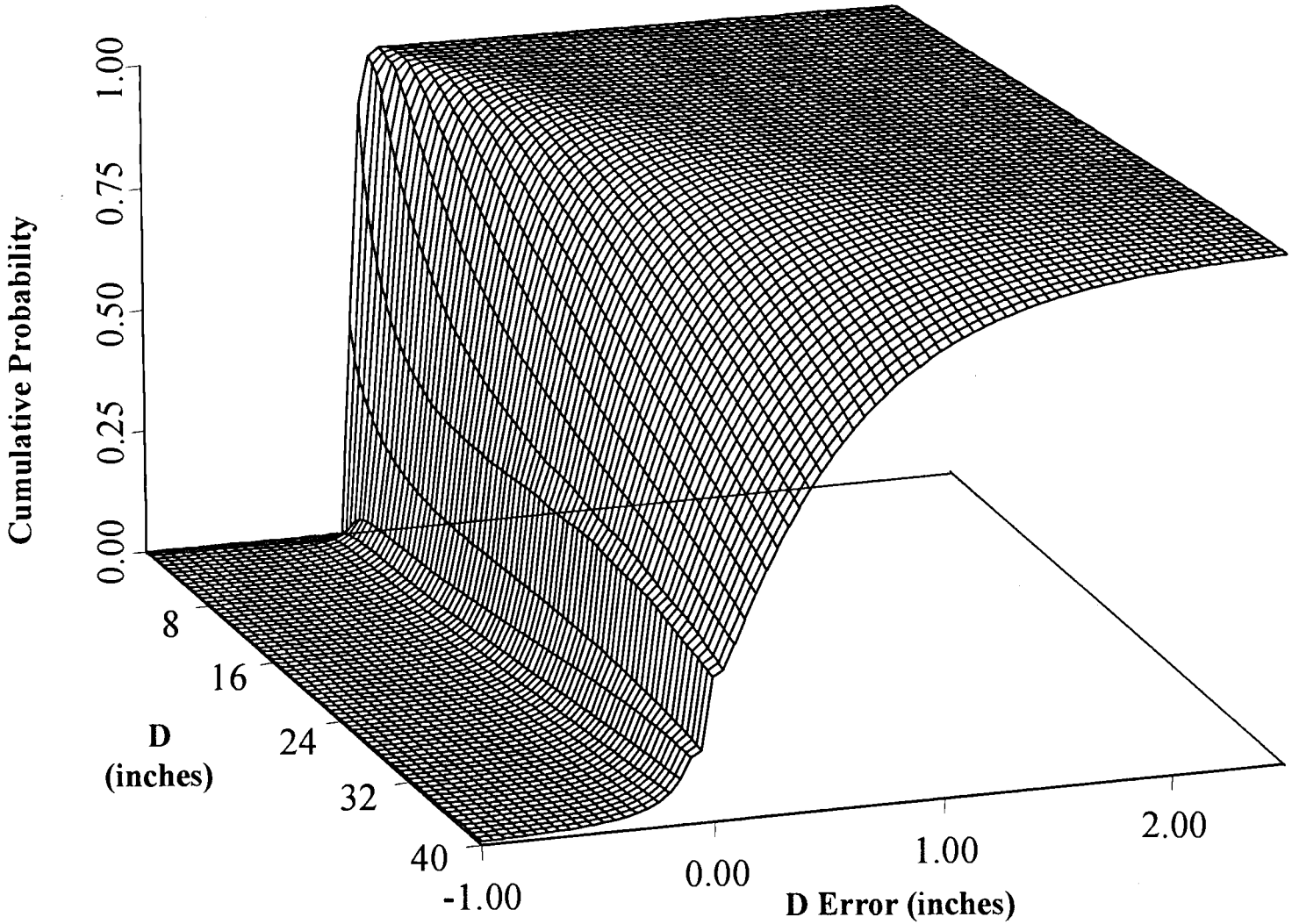


Figure 2.6 – Fitted TSED surface for the D errors

Table 2.4. Sums of squared differences between fitted D measurement error CDF surfaces and the empirical surface, with the percent reduction in the Base Fit sum of squared differences offered by each model.

Distribution	SS Diff	% Reduction
Base Fit	244.8921	0.0
TSED	4.6392	98.1
Normal 1	41.1679	83.2
Normal 2	61.5424	74.9
Normal 3	39.1445	84.0
Normal 4	40.5096	83.5
Normal 5	134.0695	45.3
Normal 6	36.1398	85.2

normal distributions at characterizing the distribution of a ME with a small percentage of zero errors. The actual δ_H CDF surface, Figure 2.7, indicates that a distribution with non-constant $\mu(\delta_H)$ and non-constant $\sigma(\delta_H)$ is appropriate for modeling δ_H.

Traditional Distribution Modeling

$\mu(\delta_H)$ was –0.5950 feet, with a $\sigma(\delta_H)$ of 2.7950 feet. The bias increased with H according to the model:

$$\mu(\delta_H) = -0.007390377 \times H \qquad \text{(p-value} < 0.0001\text{)}.$$

In addition, $\sigma^2(\delta_H)$ was found to be heterogeneous and to increase with H (p-values for Levene's and Brown and Forsythe's tests were < 0.0001):

$$\sigma(\delta_H) = 0.02015762 \times H^{1.1}$$

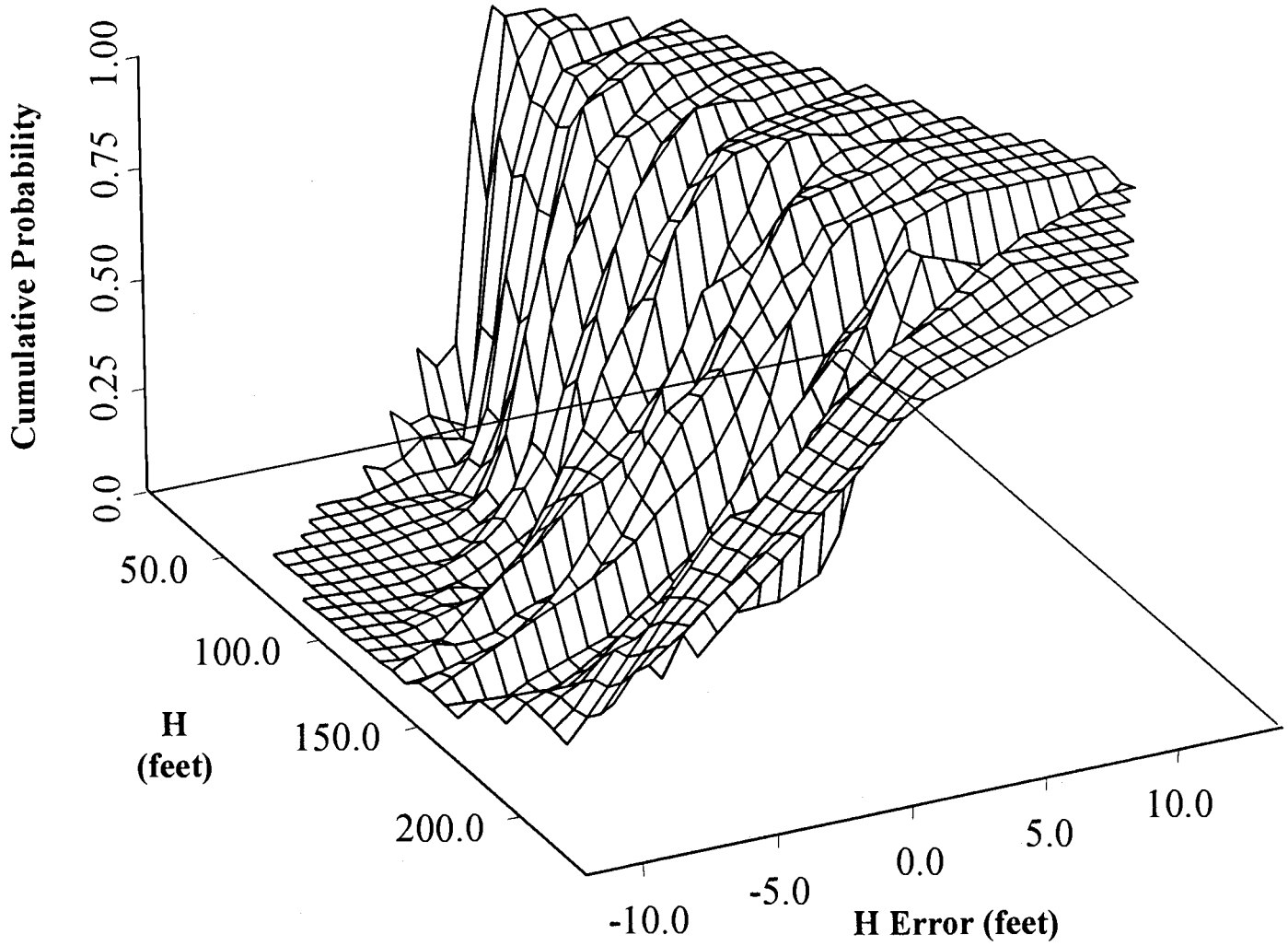


Figure 2.7 – The empirical H measurement error CDF surface

Therefore, dividing all bias-corrected errors by $H^{1.1}$ was found to successfully stabilize the variance (Levene's test p-value = 0.3199, Brown and Forsythe's test p-value = 0.4041). Analogs to the six normal distributions in the D analysis were also created in this analysis based on these estimates of $\mu(\delta_H)$ and $\sigma(\delta_H)$, and are summarized in Table 2.5.

Table 2.5. Means and standard deviations for the six H measurement error normal distributions.

Alternative Distribution	H	
	Mean (ft.)	Std Dev. (ft.)
Normal 1	0	2.7950
Normal 2	-0.5950	2.7950
Normal 3	-0.007390377×H	2.7950
Normal 4	0	$0.02015762 \times H^{1.1}$
Normal 5	-0.5950	$0.02015762 \times H^{1.1}$
Normal 6	-0.007390377×H	$0.02015762 \times H^{1.1}$

TSED Distribution Modeling

Stage 1

As with the D errors, the H error type probabilities were not constant and, therefore, they were modeled with multinomial regression in S-Plus. Trees less than 130.1ft in height were grouped into five-foot classes. Trees greater than 130.0 ft. and less than 200.1 ft. were grouped into ten-foot classes due to the smaller number of trees in this range. The remaining trees with heights greater than 200.0 ft., ranging from 204.2 ft. to 231.7 ft. and having a mean of approximately 220.0 ft., were grouped into one class and assigned a value of 220.0 ft.. No trees were present in the 2.5 ft. class (0 – 5 ft.) and only one tree was present in the 7.5 ft. class. Both classes were therefore removed.

Equation (1) was fit with g_2 and g_3 being linear functions of H, H^2, $H^{1/2}$, and H^{-1}. Fitting was done in the same manner as described for the D error analysis. The H^2, $H^{1/2}$, and H^{-1} terms were not significant and were removed. Fitted models were again tested for overdispersion. The overdispersion parameter was estimated to be 1.0142 with a 95% confidence interval of (0.6066, 1.31180), indicating a lack of extra-multinomial variation. Fitting the final model without quasilikelihood resulted in the parameter estimates in Table 2.6. Both g_2 and g_3 are significantly different from zero and the models therefore cannot be simplified as was done in the D analysis.

Table 2.6. Fitted multinomial regression coefficients for the H error type probability models.

Function	Variable	Coefficient	Std. Error	p-value[1]
g_1	Intercept	-2.20982444	0.41758552	< 0.0001
	H	-0.01375464	0.00589249	0.0231
g_2	Intercept	-0.57546102	0.13676892	< 0.0001
	H	0.00221736	0.00151917	0.1498

[1] - based on 58 degrees of freedom

Stage 2

Separate data sets were created for the positive and negative H errors. Stage 1 H classes were used in this stage. Errors ranged in size from –12.1 ft. to 14.0 feet. Error classes of width 0.2 feet were created in each H class extending from –12.2 feet to –0.1 feet for the negative errors and from 0.1 feet to 14.2 feet for the positive errors. Graphing the cumulative probabilities by error class for each of the H classes indicated that the general exponential forms in Equations (2) and (3) were appropriate for modeling $F_Y^-(\delta_H^-)$ and $F_Y^+(\delta_H^+)$ with β^- and β^+ changing over H.

H_M and H_T were measured to the nearest 0.1 feet. The values of $\max(\delta_H^-)$ and $\min(\delta_H^+)$ were therefore –0.1ft and 0.1ft, respectively. Equation (6) was fit for the $\Pr(\min(\delta_H^+))$ using Proc Logistic in

SAS. Predictor variables included in the original model were H, H^2, $H^{½}$, and H^{-1}. The final model form was:

$$\Pr(\min(\delta_H^+)) = \frac{e^{a_0+a_1\times H+a_2\times H^{0.5}}}{1+e^{a_0+a_1\times H+a_2\times H^{0.5}}}$$

Equations (5) and (7) with $\max(\delta_H^-)$, $\min(\delta_H^+)$, and $\Pr(\min(\delta_H^+))$ as defined above became:

$$F_Y^-(\delta_H^-) = 1 - e^{-(\delta_H^- - \max(\delta_H^-))^*\beta^-} \qquad \delta_H^- < 0,\ \beta^- > 0 \tag{15}$$

$$F_Y^+(\delta_H^+) = 1 - \left(1 - \frac{e^{a_0+a_1H+a_2H^{0.5}}}{1+e^{a_0+a_1H+a_2H^{0.5}}}\right) e^{-(\delta_H^+ - \min(\delta_H^+))^*\beta^+} \qquad \delta_H^+ > 0,\ \beta^+ > 0 \tag{16}$$

Fitting these two equations by H class and plotting the resulting estimates for β^- and β^+ against H class showed that both β^- and β^+ could be modeled with an exponential function in both cases:

$$\beta^- = b_0 \times e^{b_1\times H+b_2\times H^{-1}+b_3\times H^{-2}} \qquad \text{for } \delta_H < 0$$

$$\beta^+ = b_0 \times e^{b_1\times H+b_2\times H^{-1}} \qquad \text{for } \delta_H > 0$$

These functions were put in place of β in Equations (15) and (16) and the power on the error variable was allowed to vary:

$$F_Y^-(\delta_H^-) = 1 - e^{-(\delta_H^- - (-0.1))^{c1}\times(b_0\times e^{b_1\times H+b_2\times H^{-1}+b_3\times H^{-2}})} \qquad \text{for } \delta_H < 0$$

$$F_Y^+(\delta_H^+) = 1 - \left(1 - \frac{e^{a_0+a_1\times H+a_2\times H^{0.5}}}{1+e^{a_0+a_1\times H+a_2\times H^{0.5}}}\right) e^{-(\delta_H^+ - 0.1)^{c1}\times(b_0\times e^{b_1\times H+b_2\times H^{-1}})} \qquad \text{for } \delta_H > 0$$

Fitting these equations was done using Proc NLIN in SAS. The resulting parameter estimates are given in Table 2.7. Starting values for this fit came from prior fits of β^-, β^+, and $Pr(\min(\delta_H^+)$. The starting value for the c_1 parameter was 1.0 in both equations. Combining the fitted error type probabilities from stage 1 with these models according to Equation (8) produced the fitted surface in Figure 2.8.

Table 2.7. Fitted nonlinear regression coefficients for the modeled H measurement error CDF curves within the error types.

	Negative Error Fit		Positive Error Fit	
Parameter	Estimate	Std Error	Estimate	Std Error
a_0	----	----	3.31790195	0.85276164
a_1	----	----	0.07696922	0.01740963
a_2	----	----	-0.85457141	0.24453370
b_0	1.43353558	0.18535801	0.49336219	0.04100471
b_1	0.01284910	0.00061222	-0.00575389	0.00036801
b_2	-88.93230138	6.46004660	44.99513555	3.01427504
b_3	503.24182680	46.17903859	----	----
c_1	-1.65017572	0.03216920	1.14224207	0.02350058
Adjusted R^2	0.8735		0.8771	

The TSED fitted surface was compared with the different normal distributions in the same manner as in the D example. Sums of squared differences for the comparisons are given in Table 2.8.

Table 2.8. Sums of squared differences between fitted H measurement error CDF surfaces and the empirical surface, with the percent reduction in the Base Fit sum of squared differences offered by each model.

Distribution	SS Diff	% Reduction
Base Fit	127.1764	0.0
TSED	7.3767	94.2
Normal 1	26.6320	79.1
Normal 2	19.4876	84.7
Normal 3	19.2318	84.9
Normal 4	17.7715	86.0
Normal 5	12.1585	90.4
Normal 6	7.2584	94.3

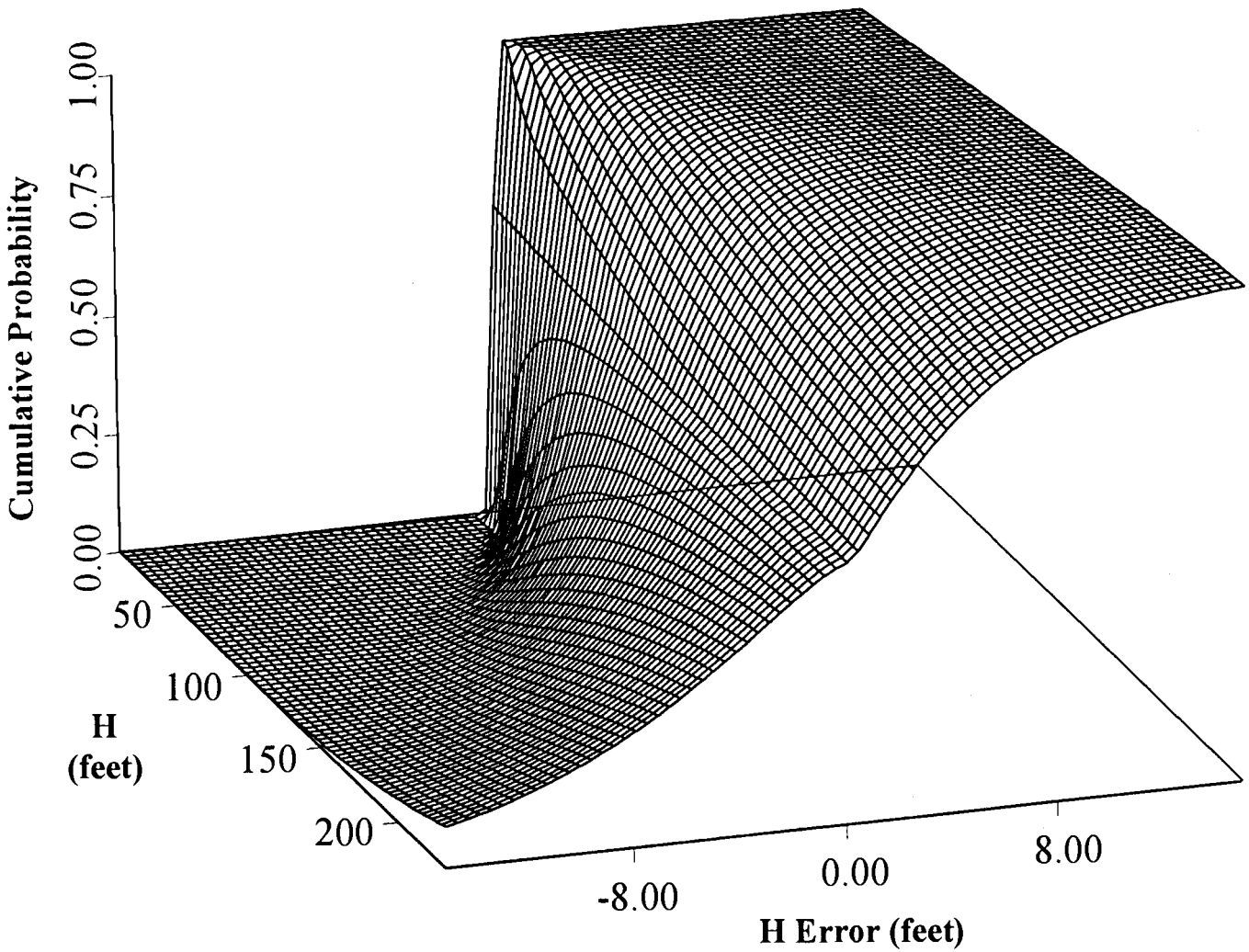


Figure 2.8 – Fitted TSED surface for the H errors

Discussion

Comparing the performance of the TSED method to the normal fits for the D example (Table 2.4) indicates that the TSED method provides a much better approximation to the actual error distribution than any of the six normal distributions examined, explaining 13% more of the unexplained variation in the Base Fit than the best normal distribution. This improved performance results from the TSED characterization of the vertical section of the distribution at zero. Approximately 17% of the D errors were zero, with the percentage being very high for small trees. Examining the differences between the normal surfaces and the empirical surface indicates that the region around zero is the primary reason why the sum of squared differences is higher and the percent reduction in the Base Fit sum of squared differences is lower for these fits compared to the TSED fit. In the H example, the fit statistics for the normal distributions are vastly improved. Fewer than 2.5% of the H errors were zero, making the H error CDF surface nearly continuous. Consequently, the normal surfaces are able to approximate the empirical surface much more closely. Normal 6, the best fit of the six normal distributions, characterized the H error CDF surface as well as The TSED method, leading to a 94.2% reduction in the Base Fit sum of squared differences compared to a 94.1% reduction for the TSED fit.

Comparing the sums of squared differences for the different normal fits to each other for the application examples reveals several interesting results. In the D example, Normal 2 and Normal 5 produced much poorer error distribution characterizations than the other normal distributions, explaining 74.9% and 45.3% of the Base Fit sum of squared differences respectively. These two fits assumed $\mu(\delta_D)$ to be a constant other than zero. For small D classes, mean errors were very close to zero and the CDF was nearly vertical resulting from a small standard deviation. The steepness of the fitted and empirical surfaces, combined with the differences in their means, led to large vertical differences between them for these size classes causing the sum of squared differences to be large. The other four normal surfaces did not have this problem. In the case of Normal 1 and Normal 4, the distributions were centered at zero thereby minimizing surface differences in the nearly vertical sections of the distributions. In the case of Normal 3 and Normal 6, the distributions had variable means allowing them to also be centered near zero for small trees. This problem for Normal 2 and Normal 5 was not seen in the H example for which their

percent reductions in the Base Fit sum of squared differences were 84.7% and 90.4% respectively. The smaller percentage of zero errors led to a smaller vertical section in the CDF surface. Vertical differences between the normal surfaces and the empirical surface were reduced as a result.

The benefits resulting from including more detailed information about the error distributions were not the same for the D and H examples. In the D example, allowing $\mu(\delta_D)$ to vary led to a 0.8% improvement in explaining the Base Fit sum of squared differences over the $\mu(\delta_D) = 0$ fit when $\sigma(\delta_D)$ was held constant (Normal 3 vs. Normal 1) and an 1.7% improvement when $\sigma(\delta_D)$ was allowed to vary (Normal 6 vs. Normal 4). The largest improvement over the simplest case (Normal 1) came from allowing both μ and σ to vary (Normal 6). This improvement in the explained Base Fit variation was only 2.0% however. The small sizes of the improvements are due to the inability of any of the six normal distributions to characterize the vertical section of the empirical CDF surface. Allowing $\mu(\delta_H)$ and $\sigma(\delta_H)$ to vary in the H example each result in improvements of 5.6% and 8.3% respectively. The improvement from allowing both to vary simultaneously, 15.2%, is larger than the sum of their individual improvements indicating a synergistic effect from applying both corrections. The lower sums of squared differences and larger improvements from including additional information can be attributed to the more continuous nature of the empirical H error CDF surface.

Differences between the six normal CDFs and the empirical error CDF throw doubt on the validity of results presented in published studies for variables with a relatively large percentage of correct measurements. McRoberts et al.'s (1994) study is the most similar to the TSED method presented here. Their method involved drawing a number from a Uniform(0,1) distribution to determine whether the corresponding observation contained error. If an error was present, a value for the error was drawn from a heterogeneous Normal distribution. This method has potential problems given the fact that the probability of a zero error does not change as tree size changes and with the assumption of no bias in the errors. Based on the comparison above, allowing for changing error type probabilities and including a bias correction within the normal distribution may have a significant impact on the ability to characterize the error distribution accurately.

Several of the studies mentioned previously used errors drawn randomly from an assumed distribution in order to better understand their effect on a model or system of models. This type of analysis is also possible with the TSED distribution model. Errors may easily be drawn randomly from the appropriate CDF curve in the fitted surface corresponding to the desired D or H. This may be done by drawing a random number, p, from a Uniform(0,1) distribution, treating p as a cumulative probability from the CDF, and using the inverted form of the distribution to determine the error size which corresponds to a cumulative probability of p. A tree size is required first to specify the CDF curve in the surface from which the error is to be drawn. Once this is determined and a random number is drawn, the next step in this process is to calculate the error type probabilities. If the random number is less than the probability of a negative error, then the inverted form of the negative error CDF portion of the curve is used. If the random number is greater than the probability of a negative error and less than the probability of a negative error plus the probability of a zero error, the error is then zero. Otherwise, the inverted form of the positive error portion of the curve is used. For example, the inverted forms for δ_D are as follows:

For a random number $p < \Pr(\delta_D^-)$:

$$\text{error} = \left(\frac{\ln(p/\Pr(\delta_D^-))}{b_0 \times \exp[b_1 \times D + b_2 \times D^{-1}]} \right)^{\frac{1}{c_1}}$$

For a random number p for which $\Pr(\delta_D^-) < p < \Pr(\delta_D^-) + \Pr(\delta_D^0)$:

$$\text{error} = 0$$

For a random number $p > \Pr(\delta_D^-) + \Pr(\delta_D^0)$:

$$\text{error} = \sqrt[c_1]{\frac{\ln\left(\dfrac{p-\Pr(\delta_D^-)-\Pr(\delta_D^0)}{\Pr(\delta_D^+)} \Big/ 1-\dfrac{\exp[-(a_1\times D+a_2\times D^{-1})]}{1+\exp[-(a_1\times D+a_2\times D^{-1})]}\right)}{b_0\times\exp[b_1\times D+b_2\times D^{-1}]}} \tag{17}$$

The coefficients for these equations are the same as those produced in the model fitting described above in the Application section. Values of δ_H are drawn following the same procedure, with the component δ_H equations substituted in the appropriate places.

The level of precision to which measurements are made has an effect on the ability of different CDF models to characterize the ME's. Had the D measurements been taken to the nearest inch, the number of correct measurements would have increased to 1087 out of 2175. If the H measurements had been made to the nearest foot, the number of correct measurements would have increased from 30 to 274 out of 1238. In both cases, the need for a non-standard distribution that can characterize the vertical section of the CDF surface at zero would have been greater. It must be remembered that the objective of measurement technology and quality control techniques is to increase the $\Pr(\delta_x^0)$. For example, had the currently available laser technology been used to measure H_M instead of the techniques used in this study, the $\Pr(\delta_H^0)$ would probably have increased and, therefore, the length of the vertical segment of the CDF surface of δ_H.

Even with a small vertical section at zero, the TSED method can provide a better fit for many ME distributions due to its ability to characterize an asymmetric distribution. For example, the distribution of δ_D was very asymmetric, with errors ranging from –0.80 inches to 2.15 inches. As a result, TSED is also recommended for modeling error distributions that are suspected of either being asymmetric, having large proportions of zero errors, or both.

As evidenced by it's ability to characterize the vertical section of the ME CDF at zero and allow for asymmetric tails in the distribution, the TSED method provides a richer description of the errors than traditional methods. This improvement is further seen in examining the results of the first stage of the fitting process. The first stage provides a description of the way in which the error type probabilities change with x_M If a library of information from this stage were put together for different measurement

techniques of the same variable, decisions regarding which technique to use in certain situations could be made. For example, when faced with choosing between two measurement techniques with error type probabilities as shown in Figures 2.2a and 2.2c, the decision of which technique to use could be made based on the size of the trees to be measured. If the trees are generally small in size, then the technique represented by Figure 2.2c would lead to a higher proportion of correct measurements. If the trees are large, however, this technique results in fewer correct measurements than that represented by Figure 2.2a. Since both techniques are unbiased, assuming the standard deviations of the errors in each case behave the same, the normal distribution would not differentiate between them with regards to accuracy, and therefore would not be helpful in determining which technique would lead to the most accurate measurements.

It is important to also note that species differ in bole shape at breast height and in crown shape and density. As a result, species may differ in the forms of their error distributions for D and/or H. What is presented here is intended as a demonstration of the proposed error evaluation technique. The errors were combined across six conifer species in order to get a more complete description of the error distribution within each D or H class and to better illustrate the method proposed here. Further application of the TSED method would be most appropriate with models calibrated by species.

TSED's more detailed characterization of the distribution of ME's requires the estimation of substantially more parameters than any of the normal distribution formulations. For example, TSED required 14 and 16 parameters to estimate the CDFs of D and H respectively, while Normal 6 required only 4 and 2 respectively. This requirement combined with the proposed method for identifying and characterizing each component of TSED dictates the need for a large data set of ME's in order to fit a TSED model. These ME's must also cover as full a range of x and δ_x as possible.

This study would be improved by a rigorous statistical test comparing CDF surfaces for significant differences. The Kolmogorov test exists for comparing CDF curves, but no analog for comparing surfaces exists. Application of such an analog or an entirely new test would indicate the significance of the improvement offered by the TSED method over different assumed Normal distributions, thereby providing a measure of the importance of characterizing the vertical section which appears when a considerable portion of the measurements are done correctly.

Literature Cited

Carroll, R.J., D. Ruppert and L.A. Stefanski. 1995. Measurement error in nonlinear models. Monographs on Statistics and Applied Probability 63. Chapman & Hall. London. 305pp.

Conover, W.J. 1971. Practical Nonparametric Statistics. John Wiley & Sons. New York. 493pp.

Emmingham, W.H. 1977. Comparison of selected Douglas-fir seed sources for cambial and leader growth patterns in four western Oregon environments. Can. J. For. Res. 7:154-164.

Fuller, W. 1987. Measurement error models. John Wiley & Sons, New York. 440pp.

Garcia, O. 1984. New class of growth models for even-aged stands: *Pinus radiata* in Golden Downs Forest. New Zealand Journal of Forestry Science 14:65-88.

Gertner, G.Z. 1984. Control of sampling error and measurement error in a horizontal point cruise. Can. J. For. Res. 14:40-43.

Gertner, G.Z. 1986. Postcalibration sensitivity procedure for regressor variable errors. Canadian Journal of Forest Research 16:1120-1123.

Gertner, G.Z. 1991. Prediction bias and response surface curvature. Forest Science 37:755-765.

Gertner, G.Z. and P.J. Dzialowy. 1984. Effects of measurement errors on an individual tree-based growth projection system. Canadian Journal of Forest Research 14:311-316.

Hamilton, D.A., and J.E. Brickell. 1983. Modeling methods for a two-state system with continuous responses. Can. J. For. Res. 13:1117-1121.

Hann, D.W. and A.A. Zumrawi. 1991. Growth model predictions as affected by alternative sampling-unit designs. Forest Science 37:1641-1655.

Hanus, M.L., D.W. Hann, and D.D. Marshall. 2000. Predicting height to crown base for undamaged and damaged trees in southwest Oregon. Research Contribution 29, Forest Research Laboratory, Oregon State University, Corvallis.

Hasenauer, H. and R.A. Monserud. 1997. Biased predictions for tree height increment models developed from smoothed 'data'. Ecological Modeling 98:13-22.

Jaakkola, S. 1967. On the use of variable size plots for increment research. P. 371-378 *in* Proc. 14th World Congress of the International Union of Forestry Research Organizations. Div. 6, Sect. 25

Jablanczy, A. 1971. Changes due to age in apical development in spruce and fir. Department of Fisheries and Forestry, Canadian Forestry Service Research Note. 27(2):10.

Kangas, A.S. 1996. On the bias and variance in tree volume predictions due to model and measurement errors. Scand. J. For. Res. 11:281-290.

Kangas, A.S. 1998. Effect of errors-in-variables on coefficients of a growth model and on predictions of growth. Forest Ecology and Management 102:203-212.

Kangas, A.S., and J. Kangas. 1999. Optimization bias in forest management planning solutions due to errors in forest variables. Silva Fennica 33(4):303-315.

Kmenta, J. 1997. Elements of econometrics. Macmillan Publishing Co. New York. 786pp.

Kozak, A. 1998. Effects of upper stem measurements on predictive ability of a variable-exponent taper equation. Canadian Journal of Forest Research 28:1078-1083.

Kulow, D.L. 1966. Comparison of Forest Sampling Designs. J. For. 64:469-474.

Larsen, D.R., D.W. Hann and S.C. Stearns-Smith. 1987. Accuracy and precision of the tangent method of measuring tree height. West. J. Appl. For. 2:26-28.

McCullagh, P., and J.A. Nelder 1995. Generalized Linear Models. Monographs on Statistics and Applied Probability 37. Chapman & Hall. London. 511pp.

McRoberts, R.E., J.T. Hahn, G.J. Hefty, and J.R. Van Cleve. 1994. Variation in forest inventory field measurements. Can. J. For. Res. 24:1766-1770.

Nester, M.R. 1981. Assessment and measurement errors in slash pine research plots. Department of Forestry, Queensland, Technical Report No. 26, 10p.

Nigh, G.D., B.A. Love. 1999. How well can we select undamaged site trees for estimating site index? Can. J. For. Res. 29:1989-1992.

Päivinen, R., and H. Yli-Kojola. 1989. Permanent sample plots in large-area forest inventory. Silva Fennica 23(3):243-252.

Phillips, D.L., S.L. Brown, P.E. Schroeder, and R.A. Birdsey. 2000. Toward error analysis of large-scale forest carbon budgets. Global Ecology and Biogeography 9(4):305-313.

Reich, R.M., and L.G. Arvanitis. 1992. Sampling unit, spatial distribution of trees, and precision. North. J. Appl. For. 9(1):3-6.

Ritchie, M.W. 1997. Minimizing the rounding error from point sample estimates of tree frequencies. W. J. Appl. For. 12(4):108-114.

Schafer, D.W. 1998. Unpublished course notes. Oregon State University, Corvallis.

Smith, J.H.G. 1975. Use of small plots can overestimate upper limits to basal area and biomass. Canadian Journal of Forest Research 5:503-505.

Smith, J.L. 1986. Evaluation of the effects of photo measurement errors on predictions of stand volume from aerial photography. Photog. Eng. Rem. Sens. 52(3):401-410.

Smith, J.L. and H.E. Burkhart. 1984. A simulation study assessing the effect of sampling for predictor variable values on estimates of yield. Canadian Journal of Forest Research 14:326-330.

Stage, A.R., and W.R. Wykoff. 1997. Adapting distant-independent forest growth models to represent spatial variability: Effects of sampling design on model coefficients. Unpublished report. 50p.

Swindel, B.F., and D.R. Bower. 1972. Rounding errors in the predictor variables in a general linear model. Technometrics 14(1):215-218.

Wallach, D. and M. Genard. 1998. Effect of uncertainty in input and parameter values on model prediction error. Ecological Modelling 105:337-346.

Williams, M.S., and H.T. Schreuder. 2000. Guidelines for choosing volume equations in the presence of measurement error in height. Can. J. For. Res. 30:306-310.

Chapter 3

CHARACTERIZING MEASUREMENT ERROR DISTRIBUTIONS OF TRANSFORMED PREDICTOR VARIABLES

Sean J. Canavan and David W. Hann

Abstract

Measurement error (ME) is a component of any study involving the use of actual measurements, but it is often not recognized or even ignored. The consequences of MEs on models can be severe, affecting estimates of tree and stand attributes and well as model parameters. While correction methods do exist for countering the effects of MEs, these methods require knowledge of the distribution of the errors. Predictor variables included in forest models are often not simple variables such as diameter at breast height (D) and total height (H), but rather combinations of these variables. Applying correction techniques to account for MEs in these transformed predictor variables requires distributions for their MEs. This information is often not available for the numerous transformed variables that may act as predictor variables, and may be time-consuming and expensive to collect and model. Indirect methodologies are examined for deriving the error distributions of transformed variables when the error distributions for their simple, component variables are known. Error distributions for D^2 and D^2H are first directly estimated both under the assumption of normality and with the TSED method. Indirect derivation methods based on error distributions for D and H are then used to derive the D^2 and D^2H error distributions. Results indicate that indirect derivation can lead to error characterizations as accurate or more accurate than direct estimation of the error distributions of the transformed variables. This was found to be true for predictor variables with high percentages of observations without ME (e.g., D^2) and predictor variables with very low percentages of observations without ME (e.g., D^2H).

Introduction

Measurement errors (ME) are unavoidable and can have serious effects on estimates of tree and stand attributes as well as on forest model parameters and output (see Chapter 2). Sources of ME include flaws in the measurement process, grouping, sampling, and the forms of variables included in models (Gertner 1986, 1991). Consequences of ME may include biased and imprecise estimates of tree and stand attributes, model parameters, and model predictions. Procedures exist for correcting the effects of ME in the predictor variables on models, but they require knowledge of the distributions of the errors. This information is often not readily available and may be time-consuming and expensive to collect and characterize.

A complicating factor in dealing with ME is that predictor variables in forest models are often not simple variables such as diameter (D), total height (H), or height to crown base (HCB). Instead, transformed variables that are functions of these and other simple variables are typically included. These functions may be univariate, such as D^2 or ln(D), or they may be multivariate, such as D^2H or crown ratio (CR), where CR =1.0-HCB/H. Although there are many transformations of potential interest, information for the ME distributions of these transformed variables is extremely rare. This problem is further complicated by the number of alternative procedures for measuring each simple variable, with each variable and measurement method combination having its own ME distribution. A potential solution to this problem is to indirectly derive the ME error distributions for transformed variables from the ME distributions of known simple variables. Two possible methods for doing this are to through error propagation formulas (e.g., Taylor 1997) or simulation. Error propagation formulas (EPFs) will give an exact formula for the errors of the transformed variable in terms of the errors of the component variables. In some cases, however, it may not be possible to derive the exact ME distribution of a transformed variable using an EPF. The form of the distribution may then be approximated through simulation, although the process will not result in an explicit equation for the ME distribution. Nevertheless, the ME approximation from simulation can serve many of the uses of an exact equation, including some of the ME correction procedures.

The objective of this analysis was to determine whether error distributions for transformed variables could be obtained through indirect derivation with EPFs or simulation when the error distributions for their component simple variables are known. The intent is to provide a method that avoids the time-consuming processes of having to collect error data for transformed variables and then perform the complex modeling needed to directly estimate their ME distributions. To explore the potential of the indirect derivation method, the ME distributions for D^2 and D^2H, two predictor variables commonly found in individual tree volume equations, will be indirectly derived through EPFs or, if necessary, simulation. These indirect derivations will be compared to directly estimated ME distributions for D^2 and D^2H. These directly estimated distributions will be developed both under the assumption of normally distributed errors and with the Two-Stage-Error Distribution (TSED) method previously described in Canavan and Hann (2002). The TSED method has been shown to produce a much more accurate characterization of the ME distribution than traditional methods when a high percentage of errors are zero or when the ME distribution is asymmetric. For cases in which this does not occur, the TSED method works as well as the most accurate form of the traditional method. (Canavan and Hann 2002).

Direct Estimation Of Measurement Error Distributions

The first step of the analysis was to directly estimate the ME distributions for the transformed variables. These then acted as a basis of comparison for the indirectly derived ME distributions. The ME distributions for the transformed variables were estimated with data from two studies associated with the development of the Southwest Oregon version of the ORGANON growth and yield model. A detailed description of the geographic and age ranges of the data is provided in Hanus et al. (2000). D_M and H_M were measured on all standing trees to the nearest 0.1 inch and 0.1 feet respectively. D_M measurements at 4.5 feet above the ground were made with a D tape which is calibrated to automatically convert a measurement of circumference into an estimate of D through the formula for a circle. H_M measurements were made directly using a 25- to 45-ft telescoping pole for smaller trees, and indirectly with the pole-tangent method (Larsen et al. 1987) for larger trees. The trees were then felled, sectioned and D

measurements along both the longest and shortest axes at 4.5 feet above ground were made with a regular tape. D_T was defined as the arithmetic mean of these two measurements. H_T measurements were made with a tape after felling. D_T and H_T were defined as the true measurements and, therefore, D_M and H_M potentially contained ME. Five species groups were included in the analysis: Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco), True fir (*Abies grandis* (Dougl. Ex D. Don) Lindl. and *Abies concolor* (Gord. & Glend.) Lindl. Ex Hildebr.), Ponderosa pine (*Pinus ponderosa* Dougl. Ex Laws.), Sugar pine (*Pinus lambertiana* Dougl.), and Incense cedar (*Calocedrus decurrens* Torr.).

There were 2175 trees included in the D^2 measurement error (δ_{D^2}) analysis ranging in D_M^2 from 0.64 inches^2 to 5198.41 inches^2. δ_{D^2} was defined to be $D_M^2 - D_T^2$. All D_M and D_T measurements were completed on the same day the tree was felled to eliminate confounding effects due to diameter growth or drying. H_M and H_T were measured on different days for some trees. To avoid errors confounded with height growth, trees for which the H_M measurement was made in the growing season May 1 – August 15 (Jablanczy 1971, Emmingham 1977) were only included if H_T was measured within five days of H_M. H_T was defined by the tip of the tree in this case, as well as for cases in which both measurements were made in the same non-growing season. H_T was defined by the last whorl if H_M was measured outside the growing season and H_T was measured within the growing season, or if measurements were made in consecutive non-growing seasons. These restrictions reduced the data set for the D^2H error (δ_{D^2H}) analysis to 1238 tree records with H_M ranging from 8.4 feet to 231.7 feet.

Univariate Transformation: D^2

The actual ME CDF surface for the sample of observed values of δ_{D^2} is shown in Figure 3.1. Population CDF surfaces were directly estimated with both an unbiased normal distribution with heterogeneous variance and the TSED method.

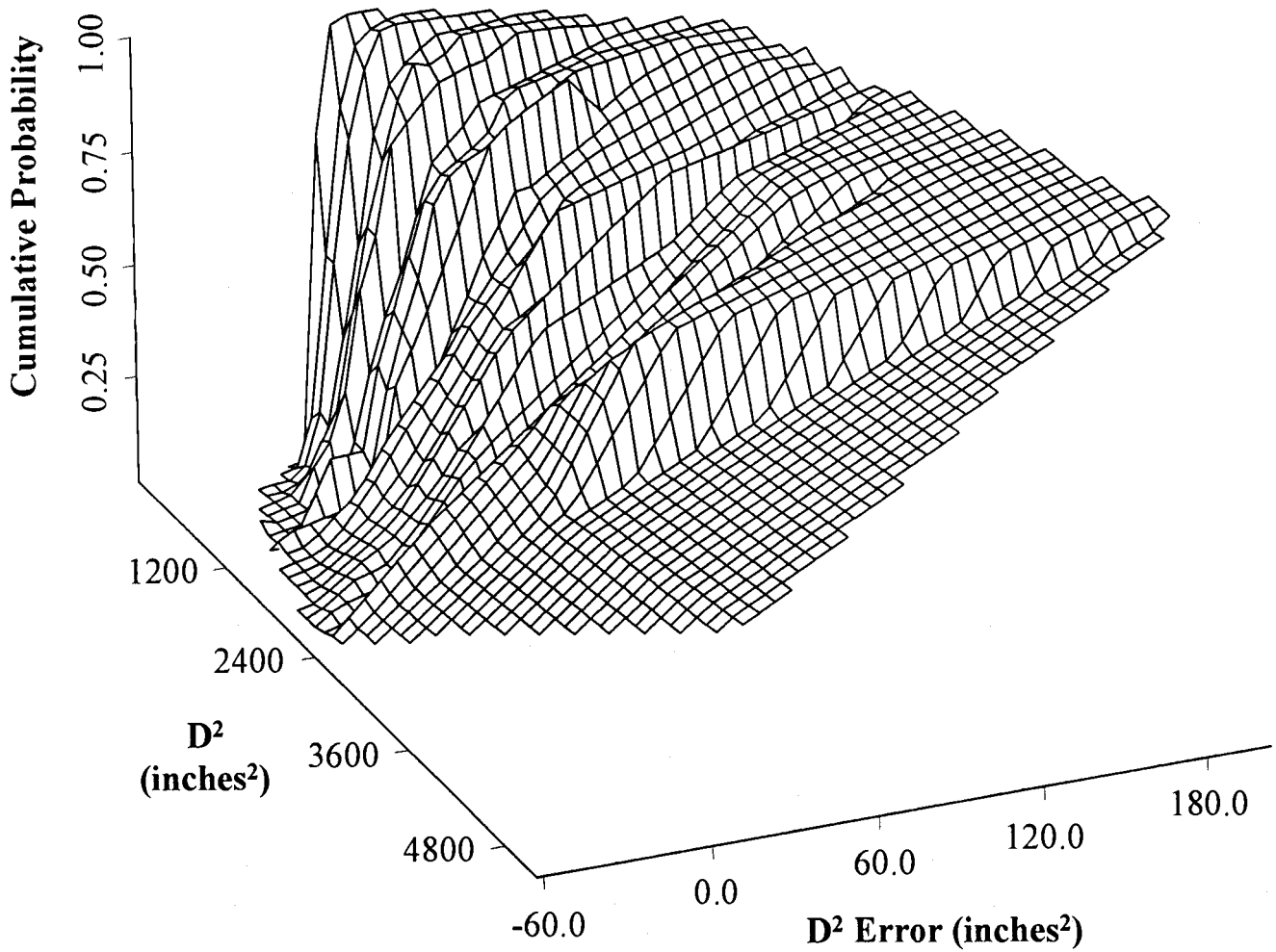


Figure 3.1 - The empirical D^2 measurement error CDF surface

Normal Modeling

The distribution of δ_{D^2} was first modeled with a normal distribution. The mean of the distribution was modeled using different functions of D^2. The best fit was obtained with an unbiased normal distribution:

$$\mu(\delta_{D^2}) = 0.0 \tag{1}$$

The standard deviation of the mean-corrected distribution was allowed to change with D^2 according to the equation:

$$\sigma(\delta_{D^2}) = 0.02602541*(20+D^2) \tag{2}$$

Dividing the mean-corrected errors by the right side of Equation (2) led to the highest degree of homogeneity of the transformations examined (p-values for Levene's and Brown and Forsythe's tests were 0.0254 and 0.0215 respectively). The resulting equation for the normal PDF surface was:

$$f_{\Delta_{D^2}}(\delta_{D^2}) = \frac{1}{\sqrt{2\pi} \times \sigma(\delta_{D^2})} e^{\frac{-(\delta_{D^2} - \mu(\delta_{D^2}))^2}{2\times(\sigma(\delta_{D^2}))^2}}$$

This normal PDF function has no closed form integral solution. The CDF corresponding to this distribution was therefore approximated with the 'NORMDIST' function in Microsoft Excel.

TSED Modeling

The distribution of δ_{D^2} was next modeled using the TSED method (Canavan and Hann 2002). In the first stage of the TSED method, the probabilities of the different types of errors, negative ($\delta^{-}_{D^2}$), zero ($\delta^{0}_{D^2}$), and positive ($\delta^{+}_{D^2}$), are modeled. We have found that multinomial regression works well for this, although other methods could be used. These error type probabilities provide the heights of the different sections of the CDF corresponding to the different error types. In the second stage the portions of the overall CDF curve corresponding to the different error types are modeled. For error types not composed of a single value, these curves are modeled with exponential CDF equations with parameter values that change as a function of tree size, in this case as a function of D^2. For an error type comprised of a single value, such as δ_x^0, the shape of the CDF for this type is a vertical line. Once the curves for the different error types are modeled, they are combined with the error type probabilities from the first stage to build the fitted CDF surface.

In Stage 1, the error type probabilities were fitted as functions of D^2 with multinomial regression. The resulting model forms were:

$$\Pr(\delta^{-}_{D^2}) = \frac{1}{(1+e^{g_0+g_1D}+e^{g_2+g_3D})} \qquad \Pr(\delta^{0}_{D^2}) = \frac{e^{g_0+g_1D}}{(1+e^{g_0+g_1D}+e^{g_2+g_3D})}$$

$$\Pr(\delta^{+}_{D^2}) = \frac{e^{g_2+g_3D}}{(1+e^{g_0+g_1D}+e^{g_2+g_3D})}$$

In Stage 2, the portions of the overall CDF curve for the negative and positive errors were fitted as functions of D^2 using nonlinear regression in SAS. The resulting model forms were:

$$\delta^{-}_{D^2}: \ \beta^{-} = b_0^{-} \times e^{b_1^{-}D^2+b_2^{-}D^{-1}+b_3^{-}\sqrt{D}} \qquad \delta^{+}_{D^2}: \ \beta^{+} = b_0^{+} \times e^{b_1^{+}D^2+b_2^{+}D^{-1}}$$

Combining the results from Stage 1 and Stage 2 to build the overall CDF for δ_{D^2} and allowing the power on the error term to vary produced the following:

$$F_{\Delta_{D^2}}(x) = \begin{cases} \left[1 - e^{-\beta^- \times (-x+\alpha_2)^{c_1}}\right] \times \Pr(\delta_{D^2} < 0) & \delta_{D^2} < 0 \\ \Pr(\delta_{D^2} < 0) + \Pr(\delta_{D^2} = 0) & \delta_{D^2} = 0 \\ \left[1 - (1-\alpha_1) \times e^{-\beta^+ \times (x-\alpha_3)^{d_1}}\right] \times \Pr(\delta_{D^2} > 0) + \Pr(\delta_{D^2} = 0) + \Pr(\delta_{D^2} < 0) & \delta_{D^2} > 0 \end{cases}$$

The fitted coefficients for these functions are given in Table 3.1. Adjusted R^2 values for the negative and positive portions of this fit were 0.9293 and 0.9719 respectively.

Table 3.1 – Parameter values for the directly estimated TSED fit of δ_{D^2}.

Parameter	Value	Parameter	Value	Parameter	Value
g_0	0.01273928	c_1	-1.88093307	α_1	0.0
g_1	-0.02937247	d_1	1.27381597	α_2	0.0
g_2	0.74470195			α_3	0.0
g_3	0.00969213	b_0^-	0.29374559		
		b_1^-	0.00091067	b_0^+	0.02624078
		b_2^-	-5.06482388	b_1^+	-0.00125278
		b_3^-	0.74314399	b_2^+	24.05503535

Multivariate Transformation: D^2H

The actual ME CDF surface for the sample of observed values of δ_{D^2H} is shown in Figure 3.2. Population CDF surfaces were again estimated using both an unbiased normal distribution with heterogeneous variance and the TSED method.

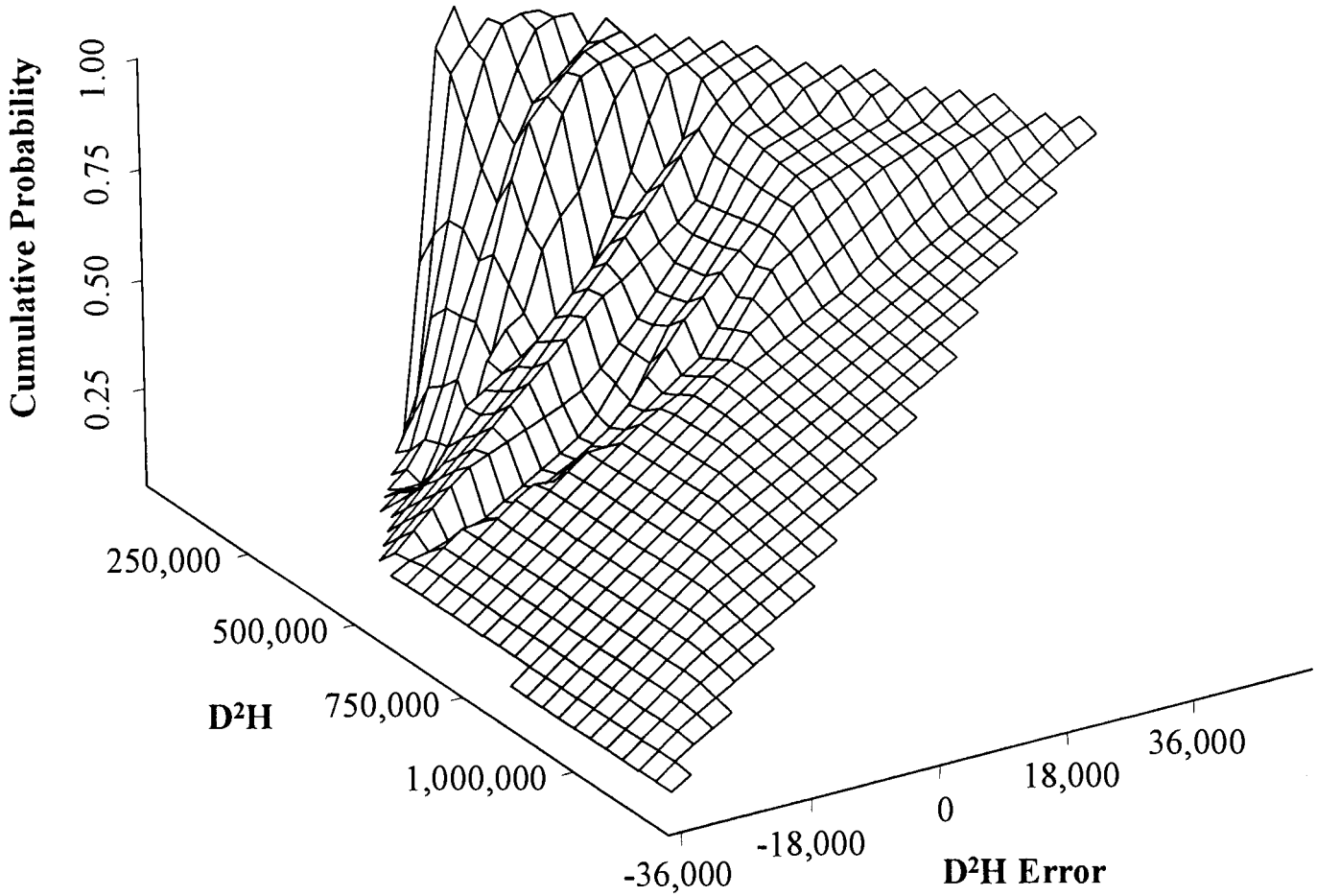


Figure 3.2 - The empirical D^2H measurement error CDF surface

Normal Modeling

Modeling the distribution of δ_{D^2H} with a normal distribution began by modeling the mean of the distribution as a function of D^2H. The best fit was again obtained with an unbiased normal distribution:

$$\mu(\delta_{D^2H}) = 0.0 \tag{3}$$

The standard deviation of the mean-corrected distribution was allowed to change with D^2H according to the equation

$$\sigma(\delta_{D^2H}) = 0.09880220*(100 + D^2H) \tag{4}$$

Dividing the mean-corrected errors by the right side of Equation (4) led to the highest degree of homogeneity of the transformations examined (p-values for Levene's and Brown and Forsythe's tests were 0.4552 and 0.5110 respectively). The resulting equation for the normal PDF surface was:

$$f_{\Delta_{D^2H}}(\delta_{D^2H}) = \frac{1}{\sqrt{2\pi} \times \sigma(\delta_{D^2H})} e^{\frac{-(\delta_{D^2H} - \mu(\delta_{D^2H}))^2}{2\times(\sigma(\delta_{D^2H}))^2}}$$

The normal PDF has no closed form integral solution and an explicit equation for the CDF of δ_{D^2H} may not be obtained directly. Several software programs such as S-Plus, Microsoft Excel, Matlab, and many others offer close numerical approximations for the normal CDF based on the normal PDF however. Comparisons between this approximated CDF and the empirical CDF are therefore still possible. The CDF corresponding to this PDF was approximated with the 'NORMDIST' function in Microsoft Excel.

TSED Modeling

In Stage 1, the following error type probabilities were fitted as functions of D^2H using multinomial regression:

$$\Pr(\delta_{D^2H}<0)=\frac{1}{(1+e^{g_0}+e^{g_1})} \qquad \Pr(\delta_{D^2H}=0)=\frac{e^{g_0}}{(1+e^{g_0}+e^{g_1})} \qquad \Pr(\delta_{D^2H}>0)=\frac{e^{g_1}}{(1+e^{g_0}+e^{g_1})}$$

In Stage 2, the portions of the overall CDF curve for the negative and positive errors were fitted as functions of D^2H with nonlinear regression in SAS:

$$\delta^{-}_{D^2H}:\ \beta^{-}=b_0^{-}\times e^{b_1^{-}(D^2H)^{-1}+b_2^{-}\sqrt{D^2H}} \qquad \delta^{+}_{D^2H}:\ \beta^{+}=b_0^{+}\times e^{b_1^{+}(D^2H)^{-1}+b_2^{+}\sqrt{D^2H}}$$

Combining the results from Stage 1 and Stage 2 to build the overall CDF for δ_{D^2H} and allowing the power on the error term to vary produced the following:

$$F_{\Delta_{D^2H}}(x)=\begin{cases}\left[1-e^{\beta^{-}\times(-x+\alpha_2)^{c_1}}\right]\times\Pr(\delta_{D^2H}<0) & \delta_{D^2H}<0\\ \Pr(\delta_{D^2H}<0)+\Pr(\delta_{D^2H}=0) & \delta_{D^2H}=0\\ \left[1-(1-\alpha_1)\times e^{\beta^{+}\times(x-\alpha_3)^{d_1}}\right]\times\Pr(\delta_{D^2H}>0)+\Pr(\delta_{D^2H}=0)+\Pr(\delta_{D^2H}<0) & \delta_{D^2H}>0\end{cases}$$

The fitted coefficients for these functions are given in Table 3.2. Adjusted R^2 values for the negative and positive portions of this fit were 0.9436 and 0.9613 respectively.

Table 3.2 – Parameter values for the directly estimated TSED fit of δ_{D^2H}.

Parameter	Value	Parameter	Value	Parameter	Value
g_0	-4.55560700	c_1	-1.07728592	b_0^+	0.00241447
g_1	0.14636460	d_1	1.17045965	b_1^+	1879.79906087
				b_2^+	-0.00905964
α_1	0.0	b_0^-	91.28929570		
α_2	0.0	b_1^-	-1609.5073313		
α_3	0.0	b_2^-	0.00862553		

Indirect Derivation Of Measurement Error Distributions

Error Propagation Formulas

In his book An Introduction to Error Analysis, Taylor (1997) presents formulas for the propagation of errors in measurements. According to Equation (3.23) in Taylor (1997), if q is a univariate transformation of x, then the uncertainty in q (δ_q) is related to the uncertainty in x (δ_x) through the equation:

$$\delta_q = \left|\frac{dq}{dx}\right|\delta_x \tag{5}$$

According to Equation (3.47) in Taylor (1997), if q is a multivariate transformation of $a, b, c, \ldots, g$ then the uncertainty in q (δ_q) is related to the uncertainty in $a, b, c, \ldots, g$ ($\delta_a, \delta_b, \delta_c, \ldots, \delta_g$) through the equation:

$$\delta_q = \sqrt{\left(\frac{\partial q}{\partial a}\times\delta_a\right)^2 + \left(\frac{\partial q}{\partial b}\times\delta_b\right)^2 + \left(\frac{\partial q}{\partial c}\times\delta_c\right)^2 + \cdots + \left(\frac{\partial q}{\partial g}\times\delta_g\right)^2} \tag{6}$$

These formulas will be used to produce distributions for the errors in D^2 and D^2H as functions of the errors in D and H.

Univariate Transformation: D^2

Applying Equation (3.23) in Taylor (1997) in the D^2 case, $q = D^2$, $x = D$, and the derivative of q with respect to x is 2D, the errors in D^2 may therefore be related to the errors in D by:

$$\delta_{D^2} = |2D|(\delta_D)$$

D is always greater than zero so the absolute value may be removed. The equation can be rearranged to show:

$$\delta_D = \frac{\delta_{D^2}}{2D}$$

This indicates that the probability density function (PDF) for δ_{D^2} may be arrived at through a univariate transformation of the PDF of δ_D. Theorem 2.1.3 in Casella & Berger (1990) describes how to derive the distribution of a variable y, which is a univariate function of x, from the distribution of x. Applying this theorem to the case of D^2 as a function of D indicates that the distributions are related as follows:

$$f_{\Delta_{D^2}}(\delta_{D^2}) = f_{\Delta_D}\left(\frac{\delta_{D^2}}{2D}\right) \times \frac{1}{2D} \tag{7}$$

The distribution for δ_{D^2} is therefore equivalent to the distribution for δ_D with δ_D replaced by $\delta_{D^2} \times (2D)^{-1}$ and the resulting equation multiplied by $(2D)^{-1}$.

There are two options available for the PDF of δ_D. The first option is a normal distribution. In their publication, Canavan and Hann (2002) provide fitted normal distributions for δ_D. The second option is to differentiate the TSED cumulative distribution function (CDF) for δ_D as reported by Canavan and Hann (2002). These options will be compared.

Normal PDF for δ_D

The form of the best normal PDF fit for δ_D in Canavan and Hann (2002) is given by:

$$f_{\Delta_D}(x) = \frac{1}{\sqrt{2\pi}(\sigma(\delta_D) \times e^{0.1145 \times D})} e^{\frac{-\left(x-(0.00398331 \times D+0.00012141 \times D^2)\right)^2}{2(\sigma(\delta_D) \times e^{0.1145 \times D})^2}}$$

In this equation, $\sigma(\delta_D)$ is the standard deviation of the errors after dividing them by $e^{0.1145 \times D}$ to stabilize their variance, and equals 0.03877408. According to the univariate transformation in Equation (7), the PDF for δ_{D^2} is:

$$f_{\Delta_{D^2}}(\delta_{D^2}) = \frac{1}{\sqrt{2\pi}(\sigma(\delta_D) \times e^{0.1145 \times D})} e^{\frac{-\left(\frac{\delta_{D^2}}{2D}-(0.00398331 \times D+0.00012141 \times D^2)\right)^2}{2(\sigma(\delta_D) \times e^{0.1145 \times D})^2}} \times \left(\frac{1}{2D}\right)$$

This is equivalent to:

$$f_{\Delta_{D^2}}(\delta_{D^2}) = \frac{1}{\sqrt{2\pi}(2D \times \sigma(\delta_D) \times e^{0.1145 \times D})} e^{\frac{-\left(\delta_{D^2}-2D \times (0.00398331 \times D+0.00012141 \times D^2)\right)^2}{2(2D \times \sigma(\delta_D) \times e^{0.1145 \times D})^2}} \quad (8)$$

This is the PDF for a normal with mean $2D \times [0.0039833 \times D + 0.00012141 \times D^2]$, and standard deviation $2D \times \sigma(\delta_D) \times e^{0.1145 \times D}$. The CDF corresponding to the CDF in Equation (8) was approximated with the 'NORMDIST' function in Microsoft Excel.

TSED PDF for δ_D

The form of the TSED CDF for δ_D in Canavan and Hann (2002) is given by:

$$F_{\Delta_D}(x) = \begin{cases} \left[1 - e^{-k_{\bar{\beta}}(D)\times[-1(x+\alpha_2)]^{c_1}}\right] \times \Pr(\delta_D < 0) & \delta_D < 0 \\ \Pr(\delta_D < 0) + \Pr(\delta_D = 0) & \delta_D = 0 \\ \left[1 - (1-\alpha_1) \times e^{-k^{+}_{\beta}(D)\times[x-\alpha_3]^{d_1}}\right] \times \Pr(\delta_D > 0) + \Pr(\delta_D = 0) + \Pr(\delta_D < 0) & \delta_D > 0 \end{cases} \quad (9)$$

Where:

$$\Pr(\delta_D<0) = \frac{1}{2 \times (1 + e^{g_0+g_1D+g_2\sqrt{D}+g_3D^{-1}})} \qquad \Pr(\delta_D=0) = \frac{1}{2 \times (1 + e^{g_0+g_1D+g_2\sqrt{D}+g_3D^{-1}})}$$

$$\Pr(\delta_D>0) = \frac{e^{g_0+g_1D+g_2\sqrt{D}+g_3D^{-1}}}{(1 + e^{g_0+g_1D+g_2\sqrt{D}+g_3D^{-1}})}$$

$$\alpha_1 = \left(1 - \frac{e^{a_1D+a_2D^{-1}}}{1 + e^{a_1D+a_2D^{-1}}}\right), \quad k^{-}_{\beta}(D) = b_0^{-} \times e^{b_1^{-}D+b_2^{-}D^{-1}}, \quad k^{+}_{\beta}(D) = b_0^{+} \times e^{b_1^{+}D+b_2^{+}D^{-1}}$$

The fitted values for these parameters are given in Table 3.3. The PDF for δ_D may be obtained by taking the derivative of Equation (9) with respect to x:

$$f_{\Delta_D}(x) = \frac{d}{dx} F_{\Delta_D}(x)$$

Table 3.3 – Parameter values from Canavan and Hann (2002) for the directly estimated TSED fit of δ_D.

Parameter	Value	Parameter	Value	Parameter	Value
g_0	-2.58503924	c_1	-1.27338986	α_2	0.05
g_1	-0.32173674	d_1	1.07090649	α_3	0.05
g_2	1.87615818				
g_3	0.00280732	b_0^-	0.01215524	b_0^+	12.41092908
		b_1^-	0.00091755	b_1^+	-0.04861734
a_1	0.10842178	b_2^-	-1.95002800	b_2^+	3.47824077
a_2	-3.53708466				

Taking the derivative of Equation (9) leads to the following equation for the PDF of δ_D:

$$f_{\Delta_D}(x) = \begin{cases} -\Pr(\delta_D < 0) \times (c_1 k_\beta^-(D) \times [-1(x+\alpha_2)]^{c_1-1}) e^{-k_\beta^-(D) \times [-1(x+\alpha_2)]^{c_1}} & \delta_D < 0 \\ 0 & \delta_D = 0 \\ \Pr(\delta_D > 0) \times (1-\alpha_1) \times (c_1 k_\beta^+(D) \times [x-\alpha_3]^{c_1-1}) e^{-k_\beta^+(D) \times [x-\alpha_3]^{c_1}} & \delta_D > 0 \end{cases} \quad (10)$$

Combining Equations (7) and (10) gives the PDF for δ_{D^2} :

$$f_{\Delta_{D^2}}(x) = \begin{cases} -\Pr(\delta_D < 0) \times \left(c_1 k_\beta^-(D) \times \left[-1(\frac{x}{2D} + \alpha_2) \right]^{c_1-1} \right) e^{-k_\beta^-(D) \times \left[-1(\frac{x}{2D}+\alpha_2) \right]^{c_1}} \times \frac{1}{2D} & \delta_D < 0 \\ 0 & \delta_D = 0 \\ \Pr(\delta_D > 0) \times (1-\alpha_1) \times \left(c_1 k_\beta^+(D) \times \left[\frac{x}{2D} - \alpha_3 \right]^{c_1-1} \right) e^{-k_\beta^+(D) \times \left[\frac{x}{2D} - \alpha_3 \right]^{c_1}} \times \frac{1}{2D} & \delta_D > 0 \end{cases}$$

Integrating this with respect to x gives the following form of the CDF for δ_{D^2} :

$$F_{\Delta_{D^2}}(x) = \begin{cases} \left[1 - e^{-k_{\beta}^{-}(D)\times\left[-1(\frac{x}{2D}+\alpha_2)\right]^{c1}}\right] \times \Pr(\delta_D < 0) & \delta_D < 0 \\ \Pr(\delta_D < 0) + \Pr(\delta_D = 0) & \delta_D = 0 \\ \left[1 - (1-\alpha_1)\times e^{-k_{\beta}^{+}(D)\times\left[\frac{x}{2D}-\alpha_3\right]^{c1}}\right] \times \Pr(\delta_D > 0) + \Pr(\delta_D = 0) + \Pr(\delta_D < 0) & \delta_D > 0 \end{cases} \quad (11)$$

Multivariate Transformation: D^2H

Applying equation (3.47) in Taylor (1997), in this case $q = D^2H$, $a = D^2$, $b = H$, the derivative of q with respect to a is H, and the derivative of q with respect to b is D^2. The uncertainty in D^2H may therefore be related to the uncertainty in D and H by:

$$\delta_{D^2H} = \sqrt{(H \times \delta_D)^2 + (D^2 \times \delta_H)^2} \quad (12)$$

A bivariate transformation is necessary to produce the PDF for δ_{D^2H} from the PDFs for δ_{D^2} and δ_H. Normal and TSED forms for the CDFs of δ_{D^2} and δ_H may act as sources for the PDFs in the bivariate transformation. Both types of distributions will be used here for comparison.

Normal PDFs for δ_{D^2} and δ_H

The distributions for δ_{D^2} and δ_H were first assumed to follow normal distributions. The normal distribution for δ_{D^2} is given in Equation (8). Canavan and Hann (2002) provide a good normal

approximation of the distribution of δ_H (Normal 6). Performing the transformation of these distributions required integrating the following function with respect to *y*:

$$f_{X,Y}(x,y) = C_2 \times u(D) \times w(H) \times e^{-\frac{(y-H\times\mu(\delta_{D^2}))^2}{2H^2\sigma^2(\delta_{D^2})} - \frac{(\sqrt{x^2-y^2}-D^2\times\mu(\delta_H))^2}{2D^4\sigma^2(\delta_H)}} \times \frac{x}{\sqrt{x^2-y^2}}$$

C_2 is a constant, u(D) is a function of D, w(H) is a function of H, $\mu(\delta_{D^2})$ is the non-constant mean of the D^2 errors described in Equation (1), $\mu(\delta_H)$ is the non-constant mean of the height errors (Canavan and Hann 2002), $\sigma(\delta_{D^2})$ is the standard deviation of the D^2 measurement errors described in Equation (2), and $\sigma(\delta_H)$ is the standard deviation of the height measurement errors (Canavan and Hann 2002). No closed form solution could be found for this integral. As a result, the CDF for the right side of Equation (11) could not be found using an EPF with normal distributions for δ_{D^2} and δ_H, and the distribution of δ_{D^2H} was approximated with statistical simulation instead.

TSED PDFs for δ_{D^2} and δ_H

The distributions for δ_{D^2} and δ_H were next assumed to follow the TSED forms for these variables. The TSED form of the distribution for δ_{D^2} is given in Equation (11). The TSED form of the distribution for δ_H is provided in Canavan and Hann (2002). Finding the CDF of the term on the right hand side in Equation (12) requires a bivariate transformation of these distributions. Performing the bivariate transformation necessary to find the distribution of δ_{D^2H} requires integrating the following function with respect to *v*:

$$f_{U,V}(u,v) = C_1 \times \left(\frac{\sqrt{v}}{2DH}\right)^{\gamma-1} \times \left(\frac{\sqrt{u-v}}{D^2}\right)^{\lambda-1} \times e^{s(D)\times\left(\frac{\sqrt{v}}{2DH}\right)^{\gamma}+t(H)\times\left(\frac{\sqrt{u-v}}{D^2}\right)^{\lambda}} \times \frac{1}{\sqrt{uv-v^2}}$$

C_1 is a constant, s(D) is a function of D, and t(H) is a function of H. No closed form solution could be found for this integral. The CDF for the right side of Equation (12) therefore could not be found with an EPF and TSED-based PDFs for δ_{D^2} and δ_H. Statistical simulation was used instead.

An additional point to note is that, even if a solution for the integral existed in either the TSED or normal case, the right side of Equation (6) does not indicate the sign of the uncertainty in q. As a result, it is unclear when the errors in q may be positive or negative. The equation is therefore useful only in producing the magnitudes of the errors in q. Additional information regarding how the probabilities of negative, zero, and positive errors change with D^2H would be necessary in order to continue with this method if the integrals had closed form solutions. With regards to univariate transformations, Equation (5) indicates that the sign of the uncertainty in q depends on the sign of the uncertainty in x. The result is that, while possible for univariate transformations, distributions of errors based on multivariate transformations may not be modeled with Taylor's method of error propagation. The forms of these distributions may still be estimated, however, with simulation.

Statistical Simulation

The distribution of δ_{D^2H} may be estimated through simulation, thereby avoiding problems due to integration. Distributions for δ_H and δ_{D^2} must first be assumed. The directly estimated and indirectly derived TSED and heterogeneous normal distributions for δ_{D^2} presented here were each assumed separately along with the directly estimated TSED and normal distribution for δ_H in Canavan and Hann (2002). Observations of δ_H and δ_{D^2} for a single tree were assumed to be independent. This assumption

was tested using the Pearson-product moment correlation of δ_H and δ_{D^2}. The resulting p-value for the test was 0.2346, indicating that δ_H and δ_{D^2} are independent. Random uniform(0,1) values for obtaining δ_H and δ_{D^2} were drawn independently as a result. Two separate samples of 1000 random observations were drawn from a uniform(0,1) distribution. These random observations were assumed to be cumulative probabilities from the CDFs for δ_H and δ_{D^2} and were mapped to random errors from these distributions. When the TSED forms of the CDFs were assumed for δ_H and δ_{D^2}, errors were drawn as described in Canavan and Hann (2002). When δ_H and δ_{D^2} were assumed to be normally distributed, errors were drawn by means of functions such as 'qnorm' in S-Plus and 'NORMINV' in Microsoft Excel. The resulting 1000 random values drawn for δ_H and δ_{D^2} for each pair of values measured with error (D_M^2, H_M) were used to calculate 1000 "true" values D_T^2 and H_T through the relationships: $D_M^2 - D_T^2 = \delta_{D^2}$, and $H_M - H_T = \delta_H$. A total of 1000 values of δ_{D^2H} for each observed pair of values (D_M^2, H_M) was then calculated as: $\delta_{D^2H} = D_M^2 H_M - D_T^2 H_T$. Assigning cumulative probabilities to the ranked 1000 values of δ_{D^2H} for each observed pair (D_M^2, H_M) defined the δ_{D^2H} CDF for that observed $D_M^2 H_M$.

Comparison Of Methods

Seven alternate ME CDF surfaces were compared to each of the actual ME CDF surfaces for the samples of observed values of δ_{D^2} and δ_{D^2H}. The first alternative surface, called the "Base Fit", assumed all errors to be zero. The resulting Base Fit CDF has a vertical section at zero of length 1.0 for each value of δ_{D^2} or δ_{D^2H}. This was considered the simplest fit to the data as well as the ideal fit in that it assumes no measurement error. Its purpose was to provide a measure of the improved or worsened fit of the two directly estimated and the four indirectly derived δ_{D^2} or δ_{D^2H} CDF surfaces when compared to each of the actual ME CDF surfaces for the samples of observed values of δ_{D^2} and δ_{D^2H}.

Univariate Transformation: D^2

Comparisons of the directly estimated and the indirectly derived ME CDF surfaces to the actual δ_{D^2} CDF surface were done through the sum of squared differences between CDF surfaces as described in Canavan and Hann (2002). These results are summarized in Table 3.4. The indirect derivation based on an EPF and the TSED form of the δ_D CDF gave the best approximation, explaining 94.4% of the Base Fit sum of squared differences. Figure 3.3 shows the surface described by this model. Direct estimation of the ME CDF with the TSED gave the next best fit, producing a 92.5% reduction in the sum of squared differences over the Base Fit. Characterizing the δ_{D^2} CDF using the indirect method based on an EPF and the normal distribution form for δ_D produced the third best fit, followed by direct estimation with a heterogeneous normal distribution. These last two methods lead to reductions in the Base Fit sum of squared differences of 88.4%, and 84.5% respectively.

Twenty seven percent of the total sum of squared differences for the indirect derivation based on the TSED for δ_D came from trees with a D of 5.5 inches or less, and 26% came from the same size trees for the indirect derivation based on a normal distribution for δ_D. These amounts are 47% and 15% for the TSED and normal respectively when the CDF of δ_{D^2} was directly estimated. The differences in these percentages account for the disparity between the total sums of squared differences for the indirectly

Table 3.4 – Results of the sum of squared difference surface comparisons for the δ_{D^2} fits, with the percent reduction in the Base Fit sum of squared differences offered by each model.

Distribution:	SS Diff:	% Reduction
Base Fit	276.3931	0.0
Indirect Derivation (EPF) using TSED for δ_D	15.5582	94.4
Direct Estimation using TSED	20.8628	92.5
Indirect Derivation (EPF) using normal distribution for δ_D	32.0632	88.4
Direct Estimation using normal distribution	34.1456	87.6

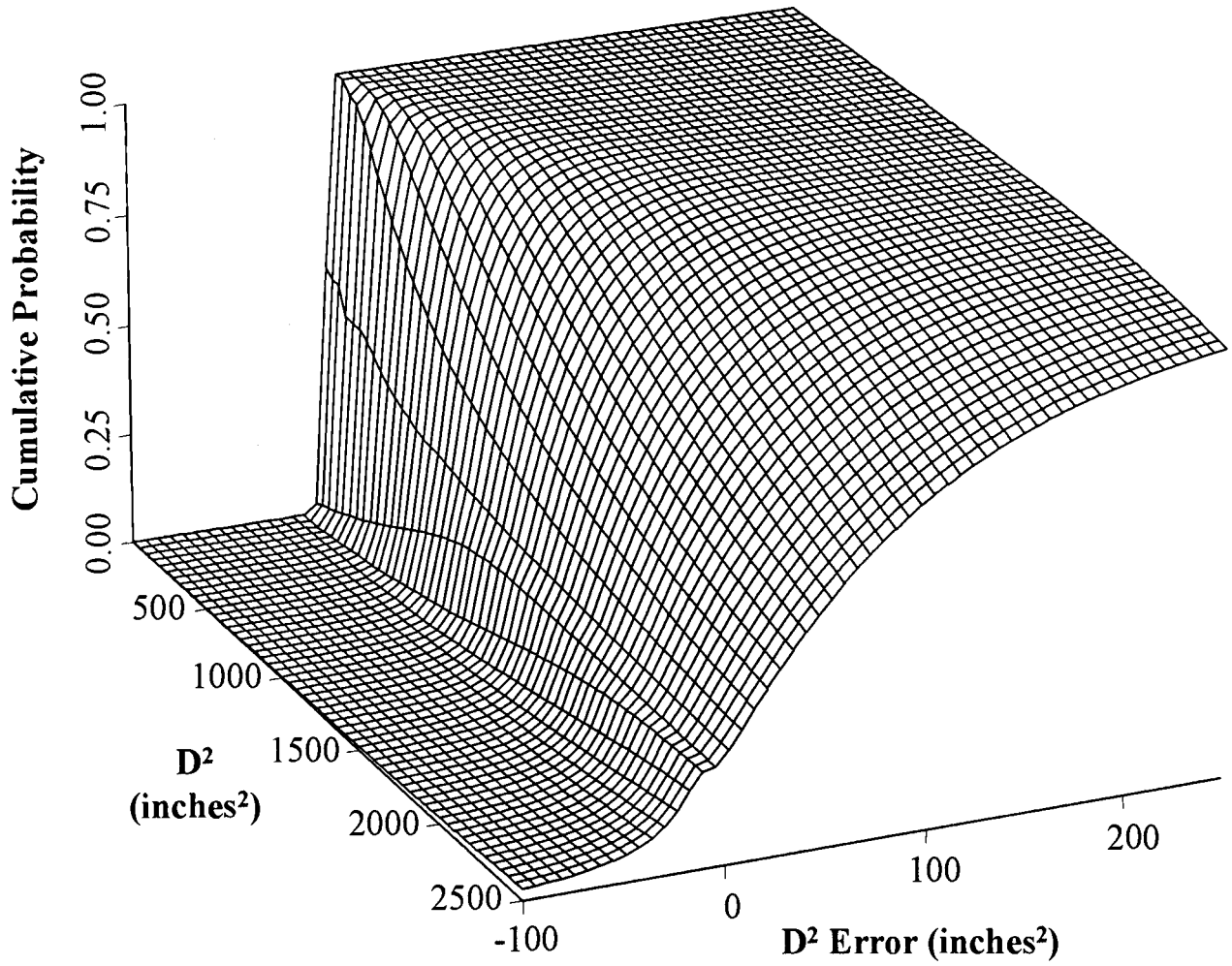


Figure 3.3. - The indirectly derived surface for the D^2 measurement error CDF based on the TSED form of the D error CDF

derived and the directly estimated CDFs using the TSED, indicating that the improvement in the indirect derivation comes from an improved characterization of the error distributions for small trees. The improvement in the indirectly derived normal distribution fit over the directly estimated normal distribution is not limited to small trees however, and is due to an improvement across all size classes.

Multivariate Transformation: D^2H

Comparisons of the directly estimated and indirectly derived ME CDF surfaces to the actual δ_{D^2H} CDF surface were again done based on the sum of squared differences between fitted CDF surfaces. Differences between the CDF surfaces built through indirect derivation with simulation and the actual CDF surface were found by subtracting the actual cumulative probability for each observed value of δ_{D^2H} from the cumulative probability the observed value would have had in the simulated CDF for the corresponding $D_M^2 H_M$ value. These differences were then squared and summed. Results are given in Table 3.5. The indirectly derived CDF based on the indirectly derived TSED CDF for δ_{D^2} and directly estimated TSED for δ_H gave the best approximation of the actual CDF surface, leading to a 90.6% reduction in the Base Fit sum of squared differences. Figure 3.4 shows the CDF surface described by this model. The indirectly derived CDF based on the directly estimated TSED CDF for δ_{D^2} gave the next best fit, leading to an 88.9% reduction in the Base Fit sum of squared differences. Directly estimating the ME CDF with the TSED gave the third best fit to the actual distribution, explaining 88.6% of the sum of squared differences of the Base Fit. Characterizing the δ_{D^2H} CDF using the indirect derivation based on the indirectly derived normal distribution for δ_{D^2} produced the fourth best fit, leading to an 87.7% reduction in the Base Fit sum of squared differences. This was followed by the indirect derivation based on the directly estimated normal CDF and then by the directly estimated heterogeneous normal distribution which led to reductions in the Base Fit sum of squared differences of 84.2% and 28.2% respectively.

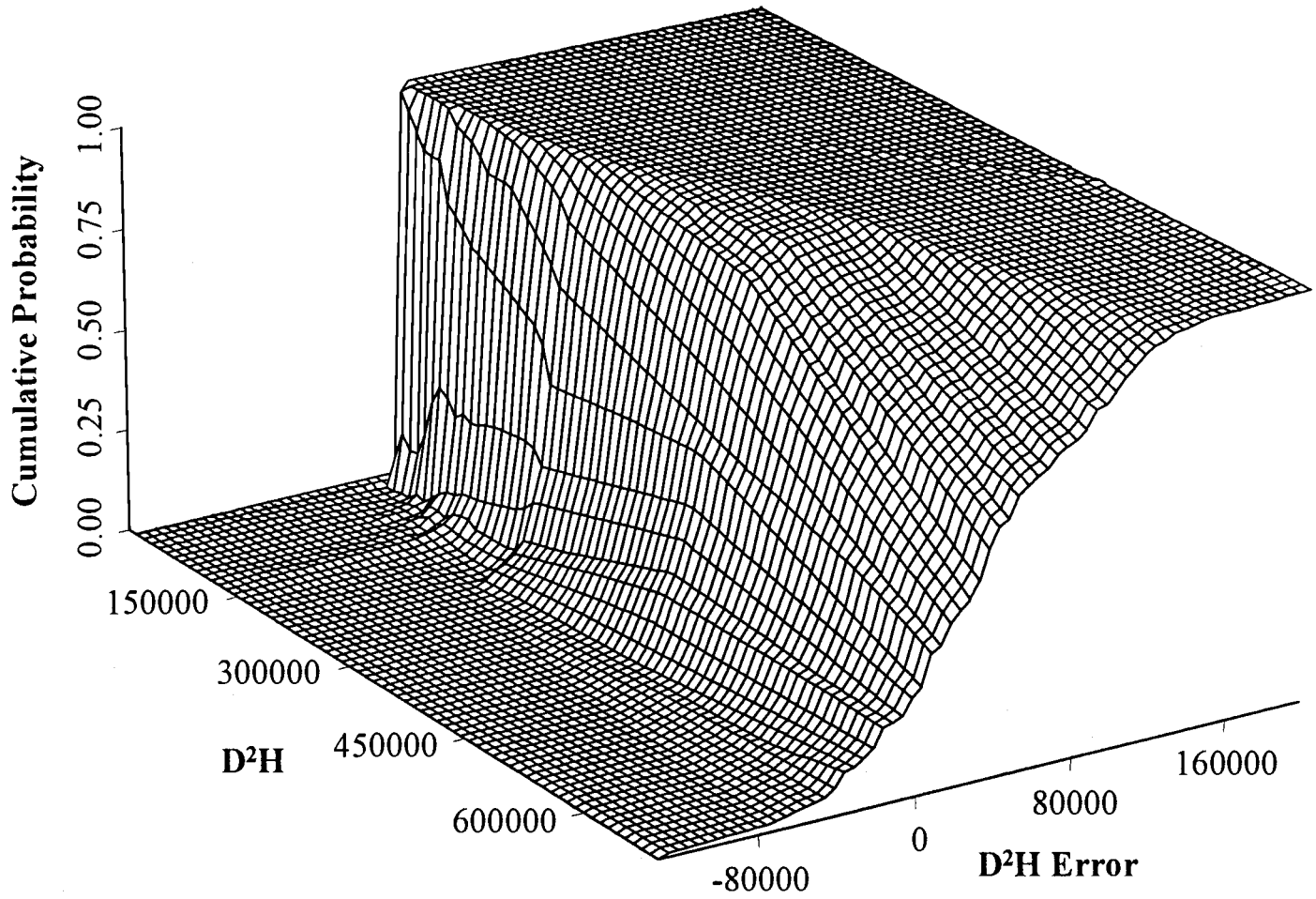


Figure 3.4 - The indirectly derived surface for the D^2H measurement error CDF based on the indirectly derived TSED form of the D^2 error CDF and the directly estimated H CDF

Table 3.5 – Results of the sum of squared difference surface comparison for the δ_{D^2H} fits, with the percent reduction in the Base Fit sum of squared differences offered by each model.

Distribution:	SS Diff:	% Reduction
Base Fit	112.2943	0.0
Indirect Derivation using the indirectly derived TSED for δ_{D^2}	10.5453	90.6
Indirect Derivation using the directly estimated TSED for δ_{D^2}	12.5166	88.9
Direct Estimation using TSED	12.8361	88.6
Indirect Derivation using the indirectly derived normal distribution for δ_{D^2}	13.8154	87.7
Indirect Derivation using the directly estimated normal distribution for δ_{D^2}	17.7154	84.2
Direct Estimation using the normal distribution	20.5005	81.7

Twenty nine percent of the total sum of squared differences for the indirect derivation based on the indirectly derived TSED for δ_{D^2} came from trees with a D of 5.5 inches or less. This amount was 22% for the indirect derivation based on the indirectly derived normal distributions for δ_{D^2}. The corresponding amounts were 38% and 51% for the indirectly derived distributions based on the directly estimated TSEDs and normal distributions for δ_{D^2} respectively. These amounts were 38% and 19% for the TSED and normal respectively when the CDF of δ_{D^2H} was directly estimated. After removing the sum of squared differences contribution of trees less than 5.5 inches in diameter, the remaining amounts are approximately equal for the three TSED fits. This indicates that it is an improved ability to predict errors for small trees that leads to the indirect derivation based on the indirectly derived δ_{D^2} TSED having the best fit. The same cannot be said for the two indirect derivations and the direct estimation based on the normal for which no pattern was visible in the sums of squared differences after accounting for small trees.

The stability of the indirectly derived CDFs using simulation were evaluated by running the simulations ten times each and examining the variability in the sums of squared differences. The standard deviation of the sum of squared differences for the simulations based on the indirectly estimated TSED CDF for δ_{D^2} and the directly estimated CDF for δ_H was 0.0763 with all values being within 1.5% of the mean value. Simulations based on the indirect derivation of the δ_{D^2} CDF with a normal distribution and

the directly estimated CDF for δ_H had a standard deviation of 0.2466 with all values being within 3.1% of the mean value. The standard deviation of the sums of squared differences for the simulations using the directly estimated TSED CDFs for δ_{D^2} and δ_H was 0.1139, with all ten values being within 1.5% of the mean value. Simulations based on the directly estimated normal CDFs for δ_{D^2} and δ_H had a standard deviation of 0.1127, with all ten values being with 1.3% of the mean value. These low degrees of variability indicated highly stable estimates of the CDF surfaces.

Discussion

Comparing the performances of the different CDF forms indicates that normal and TSED distributions based on indirect derivation outperformed corresponding normal and TSED distributions directly estimated with actual data. In the case of the TSED, this result was seen more strongly for D^2 than for D^2H. In the case of the normal distribution, the reverse was true with the improvement from using indirect derivation methods instead of the direct estimation methods being much larger in the D^2H case than in the D^2 case.

The superiority of indirect derivation over direct estimation may be explained by the fact that the distributions of δ_{D^2} and δ_{D^2H} are combinations of ME distributions for simple variables that are complex in their characterizations. In the case of δ_D, the distribution is asymmetric and has a large vertical section. Both aspects carry over to the δ_{D^2} case, with the asymmetry being amplified by the square transformation. For δ_{D^2H}, the component parts are complex in different ways. The values of δ_{D^2} are asymmetric toward positive errors while the values of δ_H are somewhat asymmetric toward negative errors. In addition, while the CDF for δ_{D^2} has a large vertical section, the CDF for δ_H does not (Canavan and Hann 2002). The complexity of the component parts is made more complex in combination, leading to error structures that are not easily modeled. The indirect derivation relationship provided by Taylor for δ_{D^2} in terms of δ_D directly incorporates the added complexity into the model

through the derived relationship between the two errors. This avoids the difficulty of having to model the effect of the transformation. In the case of the δ_{D^2H} simulation, the complexity of the δ_{D^2H} surface is also avoided by drawing errors in D^2 and H using the distributions of δ_{D^2} and δ_H in order to calculate the values of δ_{D^2H}. The best models for δ_{D^2} and δ_{D^2H}, therefore, are not forced to fit the complex forms of δ_{D^2} or δ_{D^2H}. Rather, they accomplish this through models developed at a more basic level. In doing so, they allow the complexity of the transformed error distribution to be modeled through the rules that govern how the component parts of the transformation combine to produce the transformed variable. In the δ_{D^2} case, the result is an explicit form for the CDF of the errors. In the δ_{D^2H} case, the result is an approximation of the CDF surface without producing an explicit model for it.

The results from the comparisons of the sums of squared differences also indicate that distributions based on TSED CDFs led to better characterizations of the actual CDFs than distributions based on normal CDFs. The performance of the TSED over the normal in this study is not surprising in light of the results of Canavan and Hann (2002). The latter results indicated that the surface for δ_D was well modeled with a TSED, but not with a normal distribution due to a large proportion of correct measurements and asymmetry in the distribution of the errors. The surface for δ_H, for which the CDF was more symmetric than for δ_D and had virtually no vertical section, was modeled equally well by the normal and the TSED methods. The improved fit of δ_D by the TSED fit carried over into both the δ_{D^2} and the δ_{D^2H} portions of this analysis. This is evidenced by the largest discrepancies between the normal fits and the empirical surface being clustered around the portion of the surface representing zero or very small values of δ_{D^2} and δ_{D^2H}. This portion of the surface corresponds to zero or small values of δ_D. The size of trees to which these error distributions are applied should therefore be carefully considered.

The Base Fits in the analysis were provided in order to gauge the amount by which the indirectly derived models improve or worsen the degree of fit over the directly estimated models. They were defined to be the situation in which all errors were 0.0. The probability of a zero error is 1.0 and the CDF is entirely vertical at 0.0. This represents the ideal situation of no measurement error. Other situations, such as equal probability for all errors, may also be considered for this role, and will result in different

percent improvements. Absolute measurements of improvement, namely differences in the actual sums of squared differences, may be used as fixed measures of predictive ability. In the case of δ_{D^2}, the asymmetry in the distribution was the cause of the large sum of squared differences value for the Base Fit.

Taylor's EPFs were found to be useful for modeling the error distribution of a univariate transformation but not a multivariate transformation. The univariate transformation produced an explicit form of the error distribution and saved considerable amounts of money and time by avoiding the need to collect and model error data for the variable of interest. Taylor's formula for the multivariate error transformation could not be applied because of an inability to solve the resulting integral when TSED or normal forms of the distributions of the simple variables were assumed and because it does not indicate the sign of the errors. Simulation provided a means of approximating the error distribution for a multivariate transformation. It does not provide an explicit form for the error distribution, however.

That the errors in D^2 and H were found to be independent is not unexpected. The measurement processes for each were unrelated and a significant correlation between them would have been unexpected. Their independence justifies drawing them separately in the simulation analysis for δ_{D^2H}. It is important to note that there are cases in which this will not be true. For example, CR is a widely used transformed variable of two height measurements, H and HCB. Both measurements are typically made with the same tool and from the same location. As a result, it is very possible that errors in one measurement resulting from the measurement technique or the setting will be duplicated in the second measurement. A significant correlation will then likely exist between errors in both measurements. If this is the case and the intent is to model the measurement error in the CR variable through simulation, errors from the distribution of H and HCB should not be drawn independently but rather from a joint distribution. Independent draws of these two errors may bias the resulting surface by failing to account for the dependent relationship between them. The user must determine when two measurements may be correlated and when they may be considered to be independent based on the measurement processes.

A change in the level of precision to which D or H measurements are made can affect the relative performance of the normal distribution versus TSED (Canavan and Hann 2002). Had the D measurements been made to the nearest inch, as is sometimes done, the number of correct measurements would have

increased from 368 to 1087 out of 2175. If the H measurements had been made to the nearest foot, as is usually done, the number of correct measurements would have increased from 30 to 274 out of 1238. The resulting effect in both cases would have been to increase the length of the vertical sections in the distributions of δ_{D^2} and δ_{D^2H}. An increase in the length of the vertical section would have led to a larger disparity between the fits for the TSEDs and normal distributions for both the δ_{D^2} and the δ_{D^2H} cases. The precision level used during data collection should therefore be kept in mind when deciding how best to approximate the form of an error distribution.

Literature Cited

Canavan, S.J., 2002. The presence and characterization of measurement error in forestry. Dissertation. Oregon State Univerity. Corvallis, Oregon.

Canavan, S.J., and D.W. Hann. 2002. Towards a characterization of measurement error. For submission to Forest Science.

Casella, G. and R.L. Berger. 1990. Statistical inference. Duxbury Press. California. 650pp.

Emmingham, W.H. 1977. Comparison of selected Douglas-fir seed sources for cambial and leader growth patterns in four western Oregon environments. Can. J. For. Res. 7:154-164.

Gertner, G.Z. 1986. Postcalibration sensitivity procedure for regressor variable errors. Canadian Journal of Forest Research 16:1120-1123.

Gertner, G.Z. 1991. Prediction bias and response surface curvature. Forest Science 37:755-765.

Hanus, M.L., D.W. Hann, and D.D. Marshall. 2000. Predicting height to crown base for undamaged and damaged trees in southwest Oregon. Research Contribution 29, Forest Research Laboratory, Oregon State University, Corvallis.

Jablanczy, A. 1971. Changes due to age in apical development in spruce and fir. Department of Fisheries and Forestry, Canadian Forestry Service Research Note. 27(2):10.

Larsen, D.R., D.W. Hann and S.C. Stearns-Smith. 1987. Accuracy and precision of the tangent method of measuring tree height. West. J. Appl. For. 2:26-28.

Taylor, J.R. 1997. An introduction to error analysis: The study of uncertainties in physical measurements. University Science Books. California. 327pp.

CONCLUSION

The work presented in this dissertation concerns the area of measurement error. It is presented in a forestry context, but the information contained within is applicable to non-forestry situations as well. Consequences of errors on estimates of subject attributes and statistical models are no different in forestry than in other disciplines. As such, the information contained here should convince researches in any area that measurement errors should not be taken lightly or ignored. The new method presented here, the TSED method, is not limited to applications in forestry, and should be especially useful for modeling any error distribution that is asymmetric or contains a large concentration of observations at individual values. Traditional error distribution assumptions allow for neither of these characteristics. The indirect derivation methods presented for producing error distributions of transformed variables from the distributions of simple variables are also applicable to non-forestry contexts. These methods will be beneficial in any situation for which the error distribution of a transformed variable is required and not readily available but the error distributions of the component variables are available.

The information presented in these three manuscripts is beneficial from both a practical and a theoretical perspective. From a practical perspective, the different types of measurement errors that arise in forestry are presented along with the consequences of their presence and a description of correction methods for countering the effects they have. In addition, information from the published literature on the bias and variation in forest measurements with standard tools is presented. The new method presented for characterizing measurement error distributions improves the usefulness of measurement error correction techniques in situations for which the errors do not follow traditional distributional forms. Decisions based on information from forest measurements can be made more reliably as a result. The analysis examining characterization of error distributions for transformed variables offers practical benefits with regards to saving both time and money. Deriving the error distributions of transformed variables from the error distributions of the component variables eliminates the need to collect error data for the transformed variables and perform the task of modeling their distributions.

From a theoretical perspective, the new method for modeling error distributions presented here, TSED, is more flexible than traditional methods. This new method broadens the applicability of

measurement error analysis to situations in which the errors do not follow traditional forms. TSED modeling provides an example of a model form that is adaptable to improvements in measurement technology, for example, which result in increasing percentages of correct measurements. It also can accommodate asymmetric error distributions. These situations were found to exist naturally in the error distributions of common forest measurements. The analysis examining characterization of error distributions for transformed variables offers theoretical benefits in that it demonstrates the applicability of error propagation formulas and statistical methods for deriving the forms of these distributions.

The information presented in Chapter 1 examined the presence of, consequences of, and correction methods for stochasticity and measurement error in forest models. Stochasticity in unidirectional, bi-directional, and multi-directional models was examined and three types of measurement errors in forestry were discussed. An examination of the consequences of stochasticity and measurement error indicated that biased, imprecise, inconsistent, and inefficient estimates of tree and stand attributes and model parameters can result from their presence. Correction methods were given for dealing with stochasticity in the three types of equations and the three types of measurement errors which commonly arise in forestry.

Chapter 2 focused on a new method, TSED, for modeling measurement errors. TSED was found to produce a more accurate characterization of the measurement error distribution for cases in which either the error distribution was asymmetric or a high percentage of the measurements were correct. In addition, it performed as well as the traditional modeling approach when the errors were more symmetric and had a very low percentage of correct measurements. Random observations may be drawn from error distributions produced with the TSED method, allowing for sensitivity and uncertainty analyses. The large amount of data required to fit TSED models was noted as a potential drawback to its use. Its performance relative to traditional methods justifies any additional expenditure, however, if accurate error information is required.

The statistical approaches examined in Chapter 3 for modeling error distributions of transformed variables were found to lead to better characterizations than modeling the error distributions directly. This was true for a variable with a large percentage of correct measurements and an asymmetric distribution, as well as for a nearly symmetric variable with a very small percentage of correct measurements. Exact

forms of the error distributions were found with transformed variables based on univariate transformations. When the transformed variables were based on multivariate transformations, however, this was not possible, and simulation was used.

Based on the information presented in Chapter 1, stochasticity and measurement errors in predictor variables should not be ignored. The consequences of doing so may be severe. The results of Chapters 2 and 3 indicate that the TSED method performed as well as or better than the traditional method for modeling the distribution of simple and transformed variables. In addition, statistical methods for deriving error distributions based on the TSED forms of error distributions for other variables performed better than those based on traditional forms of the same distributions. The TSED method should therefore be used for modeling the error distributions of simple variables and in conjunction with statistical methods to produce error distributions for transformed variables.

BIBLIOGRAPHY

Abetz, P. 1960. The accuracy of measurements of radial increments and annual ring width on stem discs and increment cores by means of the Eklund machine. Allg. Forst. Jagdztg. 74-80.

Arvanitis, L.G., and W.G. O'Regan. 1967. Computer simulation and economic efficiency in forest sampling. Hilgardia, 38(2):133-164.

Arvanitis, L.G., and W.G. O'Regan. 1972. Cluster or satellite sampling in forestry: a Monte Carlo computer simulation study. Proc. Of Int. Union For. Res. Organ. 3rd Conference, Advisory Group of Forest Statisticians. Sept. 1970. Joux-en-Josas, France. pp. 191-205.

Assmann, E. 1970. The principles of forest yield study. Pergamon Press. New York. 503pp.

Avery, T.E. 1958. Composite aerial photo volume tables for souther pines and hardwoods. J. For. 56(10):741-745.

Avery, T.E., and H.E. Burkhart. 2001. Forest Measurements. McGraw Hill. Boston. 456pp.

Barrett, J.P. 1964. Correction for Edge Effect Bias in Point-Sampling. For. Sci. 10:52-55.

Barrett, J.P., and P.H. Allen. 1966. Angle-gauge Sampling a Small Hardwood Tract. For. Sci. 12:83-89.

Beers, T.W. 1966. The direct correction for boundary-line slop-over in horizontal point sampling. Purdue Univ. Agric. Exp. Stn. Res. Progress Rep. 224. Lafayette, IN.

Beers, T.W. 1977. Practical Correction of Boundary Overlap. SJAF. 23:16-18.

Behre, C.E. 1926. Comparison of diameter tape and caliper measurements in second-growth spruce. J. of For. 24:178-182.

Bell, J.F. and R. Gourley. 1980. Assessing the accuracy of a sectional pole, Haga altimeter, and alti-level for determining total height of young coniferous stands. Southern Journal of Applied Forestry 4:136-138.

Biggs, P.H. 1991. Aerial tree volume functions for eucalypts in Western Australia. Can. J. For. Res. 21:1823-1828.

Biging, G.S. 1988. Estimating the accuracy of volume equations using taper equations of stem profile. Can. J. For. Res. 18:1002-1007.

Biging, G.S. and L.C. Wensel. 1988. The effect of eccentricity on the estimation of basal area and basal area growth. Forest Science 34:338-342.

Borders, B.E. 1989. Systems of equations in forest stand modeling. For. Sci. 35:548-556.

Brown, D.G., J.D. Duh, and S.A. Drzyzga. 2000. Estimating error in an analysis of forest fragmentation change using North American landscape characterization (NALC) data. Remote Sens. Environ. 71(1):106-117.

Canavan, S.J., 200X. The presence and characterization of measurement error in forestry. Dissertation. Oregon State Univerity. Corvallis, Oregon.

Canavan, S.J., and D.W. Hann. 200X. Towards a characterization of measurement error. Chapter 2 paper

Carroll, R.J., and D. Ruppert. 1996. The use and misuse of orthogonal regression in linear errors-in-variables models. Am. Stat. 50(1)1-6.

Carroll, R.J., D. Ruppert and L.A. Stefanski. 1995. Measurement error in nonlinear models. Monographs on Statistics and Applied Probability 63. Chapman & Hall. London. 305pp.

Casella, G. and R.L. Berger. 1990. Statistical inference. Duxbury Press. California. 650pp.

Chacko, V.J. 1961. A study of the shape of cross section of stems and the accuracy of caliper measurement. Indian Forester 87:758-762.

Clutter, J.L., J.C. Fortson, L.V. Pienaar, G.H. Brister, and R.L. Bailey. 1983. Timber management: a quantitative approach. John Wiley & Sons. New York. 333p.

Conover, W.J. 1971. Practical Nonparametric Statistics. John Wiley & Sons. New York. 493pp.

Crockford, R.H., and M.E. Johnson. 1983. Some errors in the measurement of precipitation, throughfall and stemflow and the implications for estimation of interception. Hydrology and Water Resources Symposium 1983. 236-242.

Curtis, R.O. and D. Bruce. 1968. Tree heights without a tape. Journal of Forestry 66:60-61.

Curtis, R.O., D.J. DeMars and F.R. Herman. 1974. Which dependent variable in site index-height-age regressions? Forest Science 20:74-87.

Draper, N.R., and H. Smith. 1998. Applied regression analysis. J. Wiley & Sons. New York. 709pp.

Emmingham, W.H. 1977. Comparison of selected Douglas-fir seed sources for cambial and leader growth patterns in four western Oregon environments. Can. J. For. Res. 7:154-164.

Finlayson, W., and Z. Tchanou. 1975. Sources of error in Biltmore stick type instruments. Commonw. For. Rev. 54(3/4):290-295.

Finney, D.J., and H. Palca. 1948. The elimination of bias due to edge-effects in forest sampling. Forestry 23:31-47.

Fowler, G.W., and L.G. Arvanitis. 1979, Aspects of statistical bias due to the forest edge: fixed-area circular plots. Can. J. For. Res. 9:383-389.

Fowler, G.W., and L.G. Arvanitis. 1981, Aspects of statistical bias due to the forest edge: horizontal point sampling. Can. J. For. Res. 11:334-341.

Fuller, W. 1987. Measurement error models. John Wiley & Sons, New York. 440pp.

Furnival, G.M., and R.W. Wilson. 1971. Systems of equations for predicting forest growth and yield. IN Statistical Ecology 3:43-55, G.P. Patil, E.C. Pielou, W.E. Waters, eds., Penn State Univ. Press, University Park.

Garcia, O. 1984. New class of growth models for even-aged stands: *Pinus radiata* in Golden Downs Forest. New Zealand Journal of Forestry Science 14:65-88.

Gertner, G.Z. 1984. Control of sampling error and measurement error in a horizontal point cruise. Can. J. For. Res. 14:40-43.

Gertner, G.Z. 1986. Postcalibration sensitivity procedure for regressor variable errors. Canadian Journal of Forest Research 16:1120-1123.

Gertner, G.Z. 1990. The sensitivity of measurement error in stand volume estimation. Canadian Journal of Forest Research 20:800-804.

Gertner, G.Z. 1991. Prediction bias and response surface curvature. Forest Science 37:755-765.

Gertner, G.Z. and P.J. Dzialowy. 1984. Effects of measurement errors on an individual tree-based growth projection system. Canadian Journal of Forest Research 14:311-316.

Gertner, G.Z., X. Cao, H. Zhu. 1995. A quality assessment of a Weibull based growth projection system. For. Ecol. Manag. 71:235-250.

Ghosh, S., J.L. Innes, and C. Hoffmann. 1995. Observer Variation as a source of error in assessments of crown condition through time. For. Sci. 41(2):235-254.

Goelz, J.C.G., and T.E. Burk. 1992. Development of a well-behaved site index equation: jack pine in north central Ontario. Can. J. For. Res. 22:776-784.

Goelz, J.C.G., and T.E. Burk. 1996. Measurement error causes bias in site index equations. Canadian Journal of Forest Research 26:1585-1593.

Gregoire, T.G., S.M. Zedaker, and N.S. Nicholas. 1990. Modeling relative error in stem basal area estimates. Can. J. For. Res. 20:496-502.

Grosenbaugh, L.R. 1958. Point-sampling and line sampling: probability theory, geometric implications, synthesis. USDA Forest Serv. South Forest Exp. Stn. Occas Pap. 160, 34p.

Haga, T., and K. Maezawa. 1959. Bias due to edge effect in using the Bitterlich method. For. Sci. 5:370-376.

Hahn, J.T., C.D. MacLean, S.L. Arner, and W.A. Bechtold. 1995. Procedures to handle FIA cluster plots that straddle two or more conditions. For. Sci. Monogr. 31:12-25.

Hamilton, D.A., and J.E. Brickell. 1983. Modeling methods for a two-state system with continuous responses. Can. J. For. Res. 13:1117-1121.

Hann, D.W., and J.A. Scrivani. 1987. Dominant height growth and site index equations for Douglas-fir and ponderosa pine in southwest Oregon. For. Res. Lab., Oreg. State Univ., Corvallis. Res. Bull. 59. 13p.

Hann, D.W., and M.W. Ritchie. 1988. Height growth rate of Douglas-fir: A comparison of model forms. For. Sci. 34(1):165-175.

Hann, D.W. and A.A. Zumrawi. 1991. Growth model predictions as affected by alternative sampling-unit designs. Forest Science 37:1641-1655.

Hanus, M.L., D.W. Hann, and D.D. Marshall. 2000. Predicting height to crown base for undamaged and damaged trees in southwest Oregon. Research Contribution 29, Forest Research Laboratory, Oregon State University, Corvallis.

Hasenauer, H. and R.A. Monserud. 1997. Biased predictions for tree height increment models developed from smoothed 'data'. Ecological Modeling 98:13-22.

Hasenauer, H., R.A. Monserud, and T.G. Gregoire. 1998. Using simultaneous regression techniques with individual-tree growth models. For. Sci. 44(1)87-95.

Howe, G.T. and W.T. Adams. 1988. Clinometer versus pole measurement of tree heights in young Douglas-fir progeny tests. Western Journal of Applied Forestry 3(3):86-88.

Hyppönen, M., and P. Roiko-Jokela. 1978. On the accuracy and effectivity of measuring sample trees. Folia Forestalia 356:1-25.

Jaakkola, S. 1967. On the use of variable size plots for increment research. P. 371-378 *in* Proc. 14th World Congress of the International Union of Forestry Research Organizations. Div. 6, Sect. 25

Jablanczy, A. 1971. Changes due to age in apical development in spruce and fir. Department of Fisheries and Forestry, Canadian Forestry Service Research Note. 27(2):10.

Kangas, A.S. 1996. On the bias and variance in tree volume predictions due to model and measurement errors. Scand. J. For. Res. 11:281-290.

Kangas, A.S. 1998. Effect of errors-in-variables on coefficients of a growth model and on predictions of growth. Forest Ecology and Management 102:203-212.

Kangas, A.S., and J. Kangas. 1999. Optimization bias in forest management planning solutions due to errors in forest variables. Silva Fennica 33(4):303-315.

Kärkkäinen, M. 1975. Measurement of the cross-sectional area of birch and aspen logs. Silva Fenn. 9:212-232.

Kärkkäinen, M. 1976. Auxiliary observations on the measurement of the cross-sectional area of aspen logs. Silva Fenn. 10:257-265.

Keeland, B.D. and R.R. Sharitz. 1993. Accuracy of tree growth measurements using dendrometer bands. Can. J. For. Res. 23:2454-2457.

Kmenta, J. 1997. Elements of econometrics. Macmillan Publishing Co. New York. 786pp.

Kozak, A. 1998. Effects of upper stem measurements on predictive ability of a variable-exponent taper equation. Canadian Journal of Forest Research 28:1078-1083.

Krauch, H. 1924. Comparison of tape and caliper measurements. J. of For. F22:537-538.

Kulow, D.L. 1966. Comparison of Forest Sampling Designs. J. For. 64:469-474.

Lappi. J. 1991. Estimating the distribution of a variable measured with error: stand densities in a forest inventory. Can. J. For. Res. 21:469-473.

Larsen, D.R., and D.W. Hann. 1985. Equations for predicting diameter and squared diameter inside bark at breast height for six major conifers of Southwest Oregon. Forest Research Laboratory, Oregon State University, Corvallis. Research Note 77. 4p.

Larsen, D.R., D.W. Hann and S.C. Stearns-Smith. 1987. Accuracy and precision of the tangent method of measuring tree height. Western Journal of Applied Forestry 2:26-28.

Leduc, D.J. 1987. A comparative analysis of the reduced major axis technique of fitting lines to bivariate data. Canadian Journal of Forest Research 17:654-659.

LeMay, V.M. 1990. MSLS: a linear least squares technique for fitting a simultaneous system of equations with a generalized error structure. Can. J. For. Res. 20:1830-1839.

Martin. G.L.. and A.R. Ek. 1984. A comparison of competition measures and growth models for predicting plantation red pine diameter and height growth. For. Sci. 30(3):731-743

Martin. G.L.. A.R. Ek and R.A. Monserud. 1977. Control of plot edge bias in forest stand growth simulation models. Can. J. For. Res. 7(1):100-105.

Matern, B. 1956. MeddfStSk 46(11). (cited in Assman 1970)

McArdle, R.E. 1928. Relative accuracy of calipers and diameter tape in measuring Douglas-fir trees. Journal of Forestry 26:338-342.

McCullagh, P., and J.A. Nelder 1995. Generalized Linear Models. Monographs on Statistics and Applied Probability 37. Chapman & Hall. London. 511pp.

McRoberts. R.E.. J.T. Hahn. G.J. Hefty, and J.R. Van Cleve. 1994. Variation in forest inventory field measurements. Can. J. For. Res. 24:1766-1770.

Mohler. C.L.. P.L. Marks. and D.G. Sprugel. 1978. Stand structure and allometry of trees during self-thinning of pure stands. J. Ecol. 66:599-614.

Moisen, G.G., A.R. Stage, and J.D. Born. 1995. Point Sampling with Disjunct Support Near Population Boundaries. For. Sci. Monogr. 31:62-82.

Monserud, R.A. 1975. Methodology for simulating Wisconsin northern hardwood stand dynamics. PhD dissertation. The University of Wisconsin-Madison. 156p.

Monserud, R.A. 1976. Simulation of Forest Tree Mortality. For. Sci. 22(3):438-444.

Monserud, R.A.. and A.R. Ek. 1974. Plot edge bias in forest stand growth simulation models. Can. J. For. Res. 4(4):419-423.

Mowrer, H.T., and W.E. Frayer. 1986. Variance propagation in growth and yield projections. Can. J. For. Res. 16:1196-1200.

Muller, D. 1957. FwCbl 76(34). (cited in Assman 1970)

Nester, M.R. 1981. Assessment and measurement errors in slash pine research plots. Department of Forestry, Queensland, Technical Report No. 26, 10p.

Nicholas, N.S., T.G. Gregoire, and S.M. Zedaker. 1991. The reliability of tree crown position classification. Can. J. For. Res. 21:698-701.

Nigh, G.D. 1995. The geometric mean regression line: A method for developing site index conversion equations for species in mixed stands. Forest Science 41:84-98.

Nigh, G.D., B.A. Love. 1999. How well can we select undamaged site trees for estimating site index? Can. J. For. Res. 29:1989-1992.

Nousiainen, M. 1986. Metsän koealainventoinnissa tarvittavien koepuunittausten luotettavuus. Pro gradu-työ. Helsingin yliopiston metsänarvioimistieteen laitos. 60p.

Omule, S.A.Y. 1980. Personal bias in forest measurements. For. Chron. 56:222-224.

O'Regan, W.G., and M.N. Palley. 1965. A computer technique for the study of forest sampling methods. For. Sci. 11(1):99-114.

Paine, D.P. 1981. Aerial photography and image interpretation for resource management. John Wiley & Sons, New York, N.Y., 571p.

Päivinen, R. 1987. Metsän inventoinnin sunnittelumalli. Summary: A planning model for forest inventory. Joensuun yliopiston luonnontieteellisiä julkaisuja – University of Joensuu, Publications in Sciences 11. 179p.

Päivinen, R., and H. Yli-Kojola. 1989. Permanent sample plots in large-area forest inventory. Silva Fennica 23(3):243-252.

Päivinen, R., M. Nousiainen, and K.T. Korhonen. 1992. Accuracy of certain tree measurements. Folia Forestalia 787:1-18.

Payandeh, B. 1978. A site-index formula for peatland black spruce in Ontario. For. Chron. 54:39-41.

Philip, M.S. 1994. Measuring trees and forests. University Press. Cambridge. 310pp.

Phillips, D.L., S.L. Brown, P.E. Schroeder, and R.A. Birdsey. 2000. Toward error analysis of large-scale forest carbon budgets. Global Ecology and Biogeography 9(4):305-313.

Plamondon, J.A. 1999. Log-length measurement errors with six single-grip harvester heads. Technical Note Forest Engineering Research Institute of Canada – FERIC, No. TN-297. 6p.

Rayner, J.M.V. 1985. Linear relations in biomechanics: the statistics of scaling functions. J. Zool. Soc. London, 206:415-439.

Reich, R.M., and L.G. Arvanitis. 1992. Sampling unit, spatial distribution of trees, and precision. North. J. Appl. For. 9(1):3-6.

Ricker, W.E. 1984. Computation and uses of central trend lines. Can. J. Zool. 62(10):1987-1995.

Ritchie, M.W. 1997. Minimizing the rounding error from point sample estimates of tree frequencies. W. J. Appl. For. 12(4):108-114.

Ritchie, M.W., and D.W. Hann. 1984. Nonlinear equations for predicting diameter and squared diameter inside bark at breast height for Douglas-fir. Forest Research Laboratory, Oregon State University, Corvallis. Research Note 47. 12p.

Ritchie, M.W., and D.W. Hann. 1986. Development of a tree height growth model for Douglas-fir. For. Ecol. Manag. 15:135-145.

Ritchie, M.W., and D.W. Hann. 1990. Equations for predicting the 5-year height growth of six conifer species in southwest Oregon. For. Res. Lab., Oreg. State Univ., Corvallis. Res. Pap. 54. 13p.

Ross, R.D. 1968. On the accuracy of penta prism calipers. J. of For. 66:932-933.

Schafer, D.W. 1998. Unpublished course notes. Oregon State University, Corvallis.

Schmid, P. 1969. Stichproben am Waldrand. Mitt. Schweiz. Anst. Forstl. Versuchswes. 45, 3:234-303.

Schmid-Haas, P. 1982. Sampling at the Forest Edge. Swedish Univ. of Agric. Sci. S-901 83 Umeå, Sweden.

Schreuder, H.T., T.G. Gregoire, and G.B. Wood. 1993. Sampling methods for multiresource forest inventory. John Wiley & Sons, Inc. New York. 464pp.

Scott, C.T., and W.A. Bechtold. 1995. Techniques and computations for mapping plot clusters that straddle stand boundaries. For. Sci. Monogr. 31:46-61.

Shiver, B.D., and B.E. Borders. 1996. Sampling techniques for forest resource inventory. John Wiley & Sons, Inc. New York. 356pp.

Smith, H.F. 1938. An empirical law describing heterogeneity in yields of agricultural crops. J. Agric. Sci. 28:1-23.

Smith. J.H.G. 1975. Use of small plots can overestimate upper limits to basal area and biomass. Canadian Journal of Forest Research 5:503-505.

Smith, J.L. 1986. Evaluation of the effects of photo measurement errors on predictions of stand volume from aerial photography. Photog. Eng. Rem. Sens. 52(3):401-410.

Smith, J.L. and H.E. Burkhart. 1984. A simulation study assessing the effect of sampling for predictor variable values on estimates of yield. Canadian Journal of Forest Research 14:326-330.

Smith, V.G. 1984. Asymptotic site-index curves, fact or artifact? For. Chron. 60:150-156.

Snowdon, P. 1987. Predicting foliar biomass of *Pinus radiata* from basal area increment. Aust. For. Res. 17(3):277-281.

Solow, A.R. 1998. On fitting a population model in the presence of observation error. Ecology 79(4):1463-1466.

Spetich, M.A. and G.R. Parker. 1998. Plot size recommendations for biomass estimation in a Midwestern old-growth forest. North. J. Appl. For. 15(4):165-168.

Spurr, S.H. 1952. Forest Inventory. Ronald Press Co., New York. 476 pp.

Spurr, S.H. 1960. Photogrammetry and photo-interpretation. The Ronald Book Company, New York, N.Y., 427p.

Spurr, S.H. and C.T. Brown. 1946. Tree height measurements from aerial photographs. J. For. 44(6)716-721.

Stage, A.R., and W.R. Wykoff. 1993. Procedure for representing spatial variability within distance-independent stand growth models: An application of measurement error models. Pages 131-144, *In*: Modeling stand response to silvicultural practices. Proceedings of the IUFRO s4.01 Conference, September 27 – October 1, 1993. Blacksburg, VA.

Stage, A.R., and W.R. Wykoff. 1997. Adapting distant-independent forest growth models to represent spatial variability: Effects of sampling design on model coefficients. Unpublished report. 50p.

Stefanski, L.A. 1985. The effects of measurement error on parameter estimation. Biometrika 72:583-592.

Swindel, B.F., and D.R. Bower. 1972. Rounding errors in the independent variables in a general linear model. Technometrics 14(1):215-218.

Taylor, J.R. 1997. An introduction to error analysis: The study of uncertainties in physical measurements. University Science Books. California. 327pp.

Wallach, D. and M. Genard. 1998. Effect of uncertainty in input and parameter values on model prediction error. Ecological Modelling 105:337-346.

Walters, D.K., and D.W. Hann. 1986 (a). Predicting merchantable volume in cubic feet to a variable top and in Scribner board feet to a 6-inch top for six major conifers of southwest Oregon. Forest Research Laboratory, Oregon State University, Corvallis. Research Bulletin 52. 107p.

Walters, D.K., and D.W. Hann. 1986 (b). Taper equations for six conifer species in southwest Oregon. Forest Research Laboratory, Oregon State University, Corvallis. Research Bulletin 56. 41p.

Weller, D.E. 1987. A reevaluation of the -3/2 power rule of plant self-thinning. Ecological Monographs 57:23-43.

Wensel, L.C., and H.H. John. 1969. A statistical procedure for combining different types of sampling units in a forest inventory. For. Sci. 15(3):307-317.

Wensel, L.C., W.J. Meerschaert, and G.S. Biging. 1987. Tree height and diameter growth models for northern California Conifers. Hilgardia. 55(8):1-20.

Williams, M.S., and H.T. Schreuder. 2000. Guidelines for choosing volume equations in the presence of measurement error in height. Can. J. For. Res. 30:306-310.

Williams, M.S., W.A. Bechtold, and V.J. LaBau. 1994. Five instruments for measuring tree height: An evaluation. South. J. Appl. For. 18(2):76-82.

Williams, M.S., K.L. Cormier, R.G. Briggs, and D.L. Martinez. 1999. Evaluation of the Barr & Stroud FP15 and Criterion 400 laser dendrometers for measuring upper stem diameters and heights. For. Sci. 45(1)53-61.

Woods, K.D., A.H. Feiveson, and D.B. Botkin. 1991. Statistical error analysis for biomass density and leaf area index estimation. Can. J. For. Res. 21:974-989.

Zarnovican, R. and J. DeGrâce. 1995. Mesure de la hauteur des arbres avec la perche Sokkisha: Test de précision. The Forestry Chronicle 71(5):616-620.