

AN ABSTRACT OF THE THESIS OF

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Title: Construction of Regression Models for Predicting Crown Development in Southwestern Oregon Douglas-fir

Abstract approved: _____
David W. Hann

A branch mortality dating technique and whorl sampling strategy were implemented to model five-year crown recession from data collected on temporary plots. Twenty-eight Douglas-fir from two levels-of-growing-stock studies in Oregon and Washington were first dissected to validate the proposed dating technique and assess alternative sampling strategies. Branch mortalities in 10-15 whorls below crown base were dated by locating discontinuities between branch and bole growth rings in stem cross-sections. Along with height measurements to the sample whorls, this technique allowed reconstruction of past crown base positions. Backdated heights to crown base

corresponded closely with 15-year repeat crown measurements taken on the same trees.

Seven sampling strategies (sampling scheme and estimator) were assessed for their ability to estimate past five-year crown recession by sampling only two to four whorls per tree. Simple linear regressions of estimated on actual recession for various five-year intervals suggested that a two-whorl sampling scheme with an appropriate estimator would perform adequately on temporary growth plots.

This sampling strategy was applied to 357 Douglas-fir from temporary growth plots in southwestern Oregon. Numerous nonlinear and logarithmic models were developed to predict five-year crown recession from other tree, stand, and site variables. Residual analyses and indices of fit demonstrated that a multiplicative model with lognormal errors was the most appropriate model form.

Sapwood taper above breast height was modeled with a quadratic-quadratic segmented polynomial. This taper function allowed extrapolation or interpolation of sapwood area measurements near crown base to sapwood area at crown base. Transformation of gross crown dimensions into expressions of conic surface area

yielded accurate predictions of sapwood area at crown base. These expressions were therefore inferred to reflect equally well the total leaf area of individual Douglas-fir trees in southwestern Oregon. Modeling at the resolution of gross crown dimensions therefore possesses both the physiological appeal of providing an accurate index of the tree's relative photosynthetic capacity and the conceptual appeal of portraying competition for light and aerial growing space.

Construction of Regression Models for Predicting
Crown Development
in Southwestern Oregon Douglas-fir

by

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A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Doctor of Philosophy

Completed April 9, 1986

Commencement June 1986

APPROVED:

Associate Professor of Forest Management in charge of
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Date thesis is presented _____

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To my parents,
David W. Maguire and Muriel T. Maguire,
who opened the door to opportunity
by instilling in me the value of an education

Acknowledgements

Funding and logistical support for this research were provided by the Southwest Oregon Forestry Intensified Research Cooperative. I especially thank Jim Boyle and David Hann for providing the flexibility needed to fund my research through its completion. I also thank Jamie Schaup, Susie Lewis, and Janet Brown for maintaining the vital communication link between the Department of Forest Management and somewhere in Maine.

Numerous individuals were subjected to cruel and unusual field work during the summer of 1983. To them I direct a sincere note of appreciation: Alan Matricardi, Kevin McNamara, John Scrivani, Harvey Huber, Paul Gremand, Rick Knight, Brian Ferguson, and Merlise Clyde. In addition, since I concede that there is a place for nonviolent resistance in all social interactions, I also thank Red Wheelis and Rick Perkins for the samples that they did carry out. None of the field work, however, could have been completed so successfully without the able supervision of Steve Stearns-Smith and Dave Larsen.

The Oregon State Department of Forestry provided critical assistance in the form of work space, shop

time, and basic tolerance of the "Lord of the Rings."
I extend a note of special thanks to Fred Robinson and Lee Ohlman. Rand Sether provided additional shop space and access to the band saw in the Forest Research Lab at Oregon State University. His tolerance of my sloppy wood and his sense of humor made the tedium a bit more bearable.

Bob Curtis and Don Reukema of the USFS Pacific Northwest Research Station provided many helpful comments during the planning stage of my research. In addition, I gratefully acknowledge the Pacific Northwest Research Station, in particular Bob Curtis and Jim Wilcox, for generously providing data from the Stampede Creek and Iron Creek Levels-Of-Growing Stock Studies. I similarly thank Gaston Porterie and Ernie Rotter of the USFS Pacific Northwest Region for their logistical support during sampling on the LOGS plots.

Past and present inmates of cell 053 provided generous assistance and helpful advice throughout the data analysis and model building phases of my research. I therefore acknowledge with gratitude my interactions with Martin Ritchie, John Scrivani, Dave Walters, Merlise Clyde, Arlene Hester, and Dave Larsen. As was true of the initial field work, the results presented here are truly a team effort.

No note of appreciation would be complete without mention of The Great Cell Master, David Hann. It is true that whenever things were not going well, he was always there to remind me of this. Whenever I was limping, he could slide in from nowhere, administer the venerable scissor kick, and slam my face to the floor. Yet in the end, his true intentions were more accurately reflected in his unfaltering support and generous contributions. To him, therefore, I extend a sincere and warm thanks.

Many helpful suggestions were also contributed by other faculty members and fellow graduate students, in particular, John Tappeiner, Phil Sollins, Dan Schafer, David Marshall, and Fiona Hamilton. During my investigation of sapwood taper, I was continually inspired by the contagious enthusiasm of Dick Waring, Ram Oren, and John Marshall. To them and all others at Oregon State University who helped me to maintain a positive outlook, I extend a warm thanks.

Finally, the love and devotion of my little family proved my greatest asset during the years at Oregon State. Despite numerous occupational hazards, Possum remained dedicated to a lifetime of unrelenting field work. When a porcupine used his face for a pin cushion

on Grayback Mountain, and when he approached hypothermia at Stampede Creek, he stuck with me. Less glorious, but nevertheless amusing, Yamhill's pathological need for a pillow rendered my computer printouts always warm and well-protected. Most important of all, however, was the love and moral support of my wife, Chris, who always seemed to transform the difficult and restless days into moments of hope and peace. Her patience and affection made it all possible.

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Construction of Regression Models
for Predicting Crown Development
in Southwestern Oregon Douglas-fir

Chapter I

Introduction

Introduction

Accurate prediction of stand growth and yield is an important aspect of sound forest management. Projection of future inventories, the scheduling of forest outputs, and prediction of silvicultural responses rely heavily on the quality of growth and yield information available. Increasing demands placed on the forest resource and the advancing technology of electronic data processing have provided the impetus for rapid expansion of growth and yield projection techniques. In particular, computer simulation of individual tree and stand growth is quickly replacing the relatively coarse summarizations of stand growth and yield tables. The effects of a wide array of controlling variables, such as stand density and site quality, can now be summarized in a set of prediction equations comprising simulation models. With the aid of high speed computers, growth and yield can be predicted for a specific set of stand conditions and management regimes. Regression equations from which these simulators are constructed can incorporate a variety of relevant predictor variables. Thus, the behavior of stands grown under varying densities and thinning regimes, for example, can be predicted more accurately by introducing stand density as a variable in the regression equations.

As models operating at the resolution of individual trees are further refined, crown size invariably emerges as a key predictor of individual tree growth. In addition, crown size and crown geometry facilitate computation of various competition indices and have been demonstrated as major determinants of stem form. More recently, this latter connection between tree form and crown size has been applied to improve volume estimates by incorporation of crown ratio into stem volume equations. The effect of crown size on tree form and volume becomes particularly pronounced as stand densities vary more widely under intensive management regimes, in turn producing a greater range in crown size.

Unfortunately, advances in the modeling of crown dynamics have not kept pace with the recognition and application of crown size as a key element in growth and yield models. Long term crown development studies are extremely rare, and the few repeat crown measurement data sets available cover a very restricted range in species, stand age, and geographic location. Likewise, techniques for estimating past changes in height to crown base on temporary plots have received little attention. These problems are further aggravated by the inconsistency among definitions of crown base, and the subjectivity involved in their

application, rendering many repeat or single measurement data bases less than ideal for modeling purposes.

Starting in Chapter II, minimization of the subjectivity problem is attempted by introducing objective definitions of two crown points, crown base (CB) and lowest contiguous live whorl (LCLW). Chapter II then proceeds to describe a stem and branch dissection technique by which dates of branch mortality can be estimated. This technique is applied to 10-15 whorls below CB on 28 Douglas-fir to reconstruct past positions of CB and LCLW. Comparison of these estimates to repeat crown base measurements on the same trees validate the efficacy of both the dating technique and the definitions of CB and LCLW.

As growth and yield modeling efforts move into new geographic locations and forest types, the necessary data are usually collected on temporary growth plots. These data typically include past five-year diameter growth and, sometimes, past five-year height growth. Although reconstruction by the dating technique provides a detailed description of past CB and LCLW behavior, including their positions five years previous, dissection of 10-15 whorls below CB to estimate past five-year crown recession would be prohibitive. Hence the potential value of the dating

technique to growth and yield modeling efforts relies on effective modification of the technique into a more operationally feasible procedure for estimating past five-year crown recession on temporary plots. Chapter III therefore explores seven sampling strategies which entail dissection of only two to four whorls per tree. Several of these strategies appear potentially useful for constructing a crown change data base.

Chapter IV applies one of the two-whorl sampling schemes and its corresponding estimator of past five-year crown recession to 357 Douglas-fir in southwestern Oregon. Crown change was then modeled directly as a function of other tree, stand, and site variables, offering a severely needed alternative to the traditional static modeling approach in which height to crown base is simply predicted at the beginning and end of the growth period. The various nonlinear and logarithmic models developed from this data base are then compared and discussed.

Application of crown size as a predictor variable in growth equations presumes its correlation with relative photosynthetic capacity of the tree, usually conceived of as total leaf area. This relationship, however, has not been well documented in the literature. Sapwood cross-sectional area at breast height has been suggested as a more accurate predictor

of total tree leaf area in Douglas-fir, as well as in other coniferous species. More recently, however, this relationship has been claimed to hold only for sapwood area at crown base, due to considerable taper occurring from breast height up the branch-free portion of the bole. Chapter V therefore develops a quadratic-quadratic segmented polynomial taper function which describes sapwood area taper above breast height. In Chapter VI, this equation is applied to estimate crown base sapwood area for 189 trees on which sapwood area was measured two whorls below CB and/or LCLW. Crown base sapwood area regressed on various transformations of crown length, crown radius, and crown base stem diameter (outside bark) establish the strong relationship between sapwood area and crown size expressions representing conic surface area. Since total leaf area has been shown to be a closely linear function of crown base sapwood area, gross crown dimensions under appropriate transformation are inferred to offer an equally effective estimator of total leaf area for southwestern Oregon Douglas-fir. In addition, modeling gross crown dimensions, rather than sapwood area or leaf area directly, provides the conceptual advantage of portraying the three dimensional competition for light and aerial growing space.

Chapter II

A Stem Dissection Technique for Dating Branch Mortality
and Reconstructing Past Heights to Crown Base in
Southwestern Oregon Douglas-fir

Abstract

Twenty-eight Douglas-fir trees from two levels-of-growing-stock studies in Oregon and Washington were dissected to validate a technique for dating branch mortality and estimating past crown recession on temporary plots. Years of branch mortality were estimated by locating discontinuities between branch and bole growth rings in stem cross-sections. Along with height measurements to the 10-15 whorls sampled per tree, this technique allowed reconstruction of past crown base positions. Backdated heights to crown base corresponded closely with 15-year repeat crown measurements taken on the same trees.

Introduction

Crown size and position influence growth layer patterns and the resulting bole form of forest trees (Duff and Nolan 1953, Larson 1963, Fayle 1985). Under various stand conditions and site histories, therefore, knowledge of crown development facilitates interpretation of the growth dynamics of individual trees. In this regard many silviculturists measure crown size both as a response to silvicultural treatment (Briegleb 1952, Curtis and Reukema 1970, Stiel 1966) and as an index of future growth potential (Hamilton 1969, van Laar 1969, Weaver and Pool 1979). Consistent with this latter relationship, individual tree growth models routinely employ crown size as an explanatory variable in various prediction equations (Botkin et al. 1972a, Daniels and Burkhart 1975, Krumland and Wensel 1981, Belcher et al. 1982, Wycoff et al 1982, Wensel and Koehler 1985). In addition, due to the intimate relationship between the crown and stem growth patterns, crown size has been found to improve stem taper and volume prediction in numerous species (Naslund 1947, Farrar 1984, Burkhart and Walton 1985, Walters et al. 1985).

In spite of the widely recognized value of crown

dimensions in modeling tree growth and interpreting silvicultural responses, long term crown data are conspicuously lacking. Crowns have typically been remeasured for only parts of the duration of experimental trials or throughout only a limited period of particular stands' development. More often, crown measurements have not been taken at all. Quality of crown remeasurements can also be a problem due to the subjectivity involved in visually estimating a biologically meaningful crown base. In all these situations the benefits to be derived from concurrent monitoring of stem growth dynamics and crown development have apparently been forfeited. However, where considerable investment has been made on research plots and the gains from detailed crown development analyses are potentially large, a branch mortality dating technique may prove a viable alternative for reconstructing crown size.

In the late 1930's there appeared in the American forestry literature a stem dissection technique by which the year of branch mortality could be estimated from sectioned knots or branches (Koehler 1936, Andrews and Gill 1939, Rapraeger 1939). Longitudinal or cross-sectional cuts were first made through the bole so as

to include a longitudinal branch section. The growth ring marking the transition from a tight (red) knot to loose (black) knot was then interpreted as the year of branch mortality. Application of this technique to dead branches below the crown base would provide a rather detailed history of branch mortality and crown recession on individual trees. Furthermore, an abbreviation of this technique could potentially provide estimates of past periodic crown recession on temporary plots.

The objectives of the present study, therefore, were: 1) to describe and apply a proposed technique for estimating past positions of crown base on intensively dissected sample trees; and 2) to validate the technique by comparing the reconstructed heights to crown base to actual repeat crown measurements taken on the same trees.

Study Sites

Stampede Creek

The first set of permanent plots from which trees were sampled occurred on the Tiller Ranger District of the Umpqua National Forest, approximately 11 km (7 mi) east of Tiller, Oregon. The 27 .08-ha (.2 ac) plots were established in 1968 as the Stampede Creek installation of the regional Douglas-fir Levels-of-Growing-Stock Study (Williamson and Staebler 1971).

The stand originated 10 years after a wildfire in 1929. At the time of study initiation in 1968 the stand was 16.8 m (55 ft) high (17.2 m (56.5 ft) for crop trees). Precipitation averages 700-800 mm (27.5-31.5 in) annually, and the plots are situated at an elevation of approximately 915 m (3000 ft). Temperature ranges from a January mean minimum of -2° C to a July mean maximum of 27° C. Although the stand falls within the region described by Franklin and Dyrness (1973) as mixed conifer, overstory composition is 100 percent Douglas-fir (Pseudotsuga menziesii (Mirbel) Franco), and the understory contains a strong component of Gaultheria shallon Pursh.. Slopes are gentle, averaging about 25 percent with a general northeast aspect. The soils are of a heavy loam

texture overlying heavy clay loam (Williamson and Staebler 1971). Site quality (King 1966) has been estimated at low site class III which is equivalent to a site index (base age 50 yrs) of 30.5 m (100 ft) (Williamson and Curtis 1984). Plot descriptions by treatment, up to 1973, are given by Williamson (1976). More recent volume, diameter, height, and density data are presented by Williamson and Curtis (1984).

Iron Creek

Trees were also sampled from the Iron Creek installation of the regional Douglas-fir Levels-of-Growing-Stock Study (Williamson and Staebler 1971). These 27 .08 ha plots (.2 ac) were established on the Randle Ranger District of the Gifford Pinchot National Forest, approximately 14 km (9 mi) south of Randle, Washington.

The stand was planted in 1949, and at the start of the calibration period in 1966 the crop trees averaged 11.1 m (36.4 ft) in height. Precipitation averages about 1900 mm (65 in) annually and temperature ranges from a January mean minimum of -4°C to a July mean maximum of 23.5°C . The stand occupies a midslope position at approximately 760 m (2500 ft) in elevation.

Slopes average 25 percent with a general east aspect. The deep, well-drained volcanic soils range from sandy loam to loam interbedded with pumice (Williamson and Staebler 1971). Site quality (King 1966) has been estimated at a high site class II, which is equivalent to a site index (base age 50 yrs) of approximately 38.7 m (127 ft) (Williamson and Curtis 1984). Plot descriptions by treatment, up to 1973, are given by Williamson (1976). More recent volume, diameter, height, and density data are presented by Williamson and Curtis (1984).

Methods

Data Collection

The calibration thinning for Stampede Creek was done in 1968, with the first, second, and third treatment thinnings following in 1973, 1978, and 1983, respectively. At Iron Creek, the calibration and five treatment thinnings were implemented in 1966, 1970, 1973, 1977, 1980, and 1984. Starting in 1973, height to the base of live crown was measured on a subset of the trees in each plot just prior to treatment thinnings. Crown base was approximated by visual reconstruction of the crown, whereby any gaps in the crown were filled in with branches from below so as to produce a crown with an even base (Robert O. Curtis, personal communication). Twenty-four trees felled in the 1983 thinning at Stampede Creek received all three repeat crown measurements and were therefore chosen for further crown analysis. Four additional trees with four repeat measurements each were felled for analysis just prior to the 1984 thinning at Iron Creek.

On each of the 28 sample trees, two crown points were marked: 1) crown base (CB), defined as the lowest whorl which had live branches at least three quarters of the way around the circumference of the stem, and

above which all whorls had the same (cf. Curtis and Reukema 1970, Curtis 1983); and 2) lowest contiguous live whorl (LCLW), defined as the lowest live whorl above which all whorls had at least one live branch. Starting with the first whorl below crown base, 10-15 successive whorls were marked for removal. Heights to all whorls were recorded (nearest .03 m), including those whorls specified as CB and LCLW. The whorls were then sawed out, leaving at least 5 cm of any protruding branches.

After removing the sample whorls and transporting to the lab, individual branch stubs and knots were split out of the bole section. Oblique cross-sectional cuts were then made through each wedge on a band saw, longitudinally through the branch and knot (Fig. 1). The area of discontinuity between the bole growth rings and dead branch growth rings was carved to a smooth surface. Finally, the year of branch mortality was estimated with the aid of a 13X power hand lens, assuming that the year before the initial growth ring discontinuity was the year in which the branch died (Koehler 1936, Andrews and Gill 1939, Rapraeger 1939). Andrews and Gill (1939) present data indicating that this estimated year of mortality, which actually

represents the year before the branch cambium dies all the way back to the bole cambium, corresponds closely to year of actual branch mortality. Further experiments are currently under way to test the validity of this assumption.

Identification of the growth ring discontinuity was facilitated by three other phenomena associated with branch mortality. First, the trees typically respond to branch suppression mortality by forming a "barrier zone" of resinous deposits in the first growth ring after branch mortality, consistent with the concept of decay compartmentalization (Shigo and Marx 1977, Shigo 1979, 1985). This appears as localized darkening of the bole growth ring (Fig. 2). In red pine, Fayle (1981) describes a similar presence of resin ducts in the growth ring corresponding to the year after estimated branch mortality.

The second indicator involves discoloration of the branchwood. The zone of discoloration in the longitudinal branch section includes the entire branch profile until, moving from the outside of the bole inward, the width of the darkened zone rapidly tapers toward the middle of the branch once the first growth ring subsequent to mortality is reached (Fig. 2). This discoloration zone appears to represent the protection

zone containing resin-based substances previously described by Shigo (1985) in other conifers.

Lastly, the cross-sectional shape of the bole growth rings in the vicinity of the branch alter dramatically after branch mortality. Typically, the resinous annual ring discussed above is followed by an annual ring of the same shape, but conspicuously narrower than subsequent, and often previous, annual rings in the vicinity of branch insertion. Then, rather than tapering into the branch, the next bole growth ring begins to bulge around and encase the base of the dead branch (Fig. 2). The local reduction in annual ring width around the year of branch mortality is also consistent with observations in red pine (Fayle 1981).

On an occasional branch, live cambium persists as a short collar (up to two cm) around the base of the branch, underscoring the advantage of including at least three to four cm of the branch base in the cross-sectional cuts. In addition, dating of branch mortality in branches which have died recently was facilitated by inspection of the cambium edge on the outside of the dead branch base. Rings of resin or thin layers of previous years' growth often record the

continued slight recession of live cambium for one to several years after branch mortality. Interpretation of these patterns provided a more expedient way to date very recent mortality (usually 1/2 to 3 yrs), and was found consistent with growth ring analyses.

Once the estimated year of branch mortality was established, the number of growth layers which accrued subsequent to mortality was recorded. Since a given branch could have died any time during the estimated year of mortality, half a year was added to each record.

Comparison of Crown Reconstruction

and Repeat Measurements

Branch mortalities were backdated and crown positions reconstructed annually. This process began with the last year that the whorl below present CB had been CB and continued back until the postdated LCLW moved below the lowest whorl sampled. For each year of backdating, the CB and LCLW were identified. Field observations indicated that a given whorl generally lost status as a potential CB when one branch died out of a total of four or less, when two branches died out of a total of five to seven, or when three branches died out of a total of eight or more.

CB, LCLW, and repeat measurements of crown base were plotted over time, and for each year of repeat crown measurement the deviation of CB and LCLW from this measurement was computed. The relationships between repeat crown measures and reconstructed CB and LCLW were also analyzed by expressing the repeat measures as the following proportions after determining the appropriate weighting factor by likelihood criteria (Furnival 1961): 1) proportion of height to CB; 2) proportion of height to LCLW; 3) proportion of height midway between LCLW and CB; and 4) proportion of the distance between LCLW and CB added onto height to LCLW.

Results

Reconstructed behavior of CB and LCLW for nine representative trees are shown in Fig. 3, along with the three or four field estimates of height to crown base on each tree. These nine trees represent the three poorest, three average, and the three closest degrees of correspondence between the repeat measures and the crown reconstruction estimates. Repeat measurements of height to crown base were consistent with past positions of CB and LCLW over time as estimated through branch mortality dating.

For all observations, repeat measurements averaged 0.53 m (1.7 ft) below CB and 1.70 m (5.8 ft) above LCLW (Table 1). As a result, repeat measures of crown base were significantly higher ($p < .001$) than midway between CB and LCLW (Table 1). This was true even for the 1983/84 observations in which LCLW and CB were identified exactly. In expressing repeat measures as the height to LCLW plus a proportion of the distance between CB and LCLW, Furnival's (1961) index suggested that residual variance was most nearly proportional to the squared distance between CB and LCLW. Hence the appropriate mean of ratios estimator indicated that repeat crown measurements occurred on average at 79.7

percent of the height between LCLW and CB (s.e = 5.9 percent). In addition, repeat crown measures occurred at 92 percent of height to CB, 123 percent of height to LCLW, and 106 percent of the mean between height to LCLW and CB. However, Furnival's index indicated inferiority of these last three expressions for the relationship between field estimated crown base and the reconstructed positions of CB and LCLW.

A total of 79 repeat crown observations had corresponding estimates of both CB and LCLW positions. (The 1973 LCLW in five trees dropped below the last sample whorl.) Fifty, or 63 percent, of the repeat measurements fell between LCLW and CB. Almost all repeat crown base measurements (96 percent) resulted in heights above the corresponding LCLW estimate, but 37 percent were also above the CB estimates. For these 37 percent, or 31 observations, the mean height above CB (after eliminating one obvious recording error) was 0.61 m (2.0 ft). Similarly, the mean distance below LCLW for the four lower repeat crown measurements was only 0.16 m (0.5 ft).

Discussion

Comparison of Crown Reconstruction and Repeat Measures

Comparison of the repeat crown measurements with the behavior of CB and LCLW as reconstructed by the branch mortality dating technique supports several conclusions. First, the past behavior of CB and LCLW is accurately described by application of branch mortality dating in conjunction with the definitions of CB and LCLW. Thus, mortality of individual branches is estimated quite effectively by the dating technique. In addition, close correspondence of the visually estimated repeat measures with each of CB and LCLW alone, as well as with the height midway between these two points (Table 1, Fig. 3), illustrate that the two approaches to defining crown size and position, though not equivalent, are quite compatible in recognition of the different criteria for defining crown base. Rates of change in CB and LCLW can therefore be translated into rate of change in repeat measures.

Given the definitions of CB and LCLW, one might expect visually estimated crown bases to average halfway between these two points. However, systematic departures of repeat crown measurements from the midpoint between CB and LCLW, as well as the frequent

occurrence of repeat measures of crown base above reconstructed CB, can be attributed to four possible factors: 1) inherent differences between the definition of visually estimated crown base applied in repeat measurements and the definitions of CB and LCLW applied in the branch dating technique; 2) bias in estimated dates of branch mortality from the dating technique; 3) failure of an actual whorl to conform to the CB recession criteria which were built into the backdating and crown reconstruction procedure; and 4) bias due to consideration of only branch mortality in the dating technique vs. a minimal vigor in the visual reconstruction approach.

The first source of inconsistency would apply, for example, if there often existed a number of whorls above CB with a three-quarter complement of branches but also with numerous gaps containing dead branches. These gaps could then absorb a number of branches from below CB in the visual estimation process. If few live whorls with only one or very few small branches per whorl occurred from CB to LCLW, the height of visually estimated crown base may exceed the actual position of CB. In a less drastic scenario, a large number of whorls below CB with only one small live branch would

result in a visually estimated crown base well above the midpoint between LCLW and CB. Conversely, however, whorls below CB with nearly a full three-quarters complement of branches would produce a visually estimated crown base below this same midpoint.

The relative contribution of this first source of bias towards explaining occurrences of CB below repeat crown base measurements appears minimal. The branch mortality data reveal that branches seldom began to die more than one or two whorls above CB before CB recedes through that level. Therefore, very few branches will be absorbed into these gaps during visual reconstruction of crown base. Similarly, since whorls between LCLW and CB should average a half complement of live branches, this source of bias appears to contribute minimally to the mean occurrence of repeat measurements at 80 percent of the distance between LCLW and CB.

Given a systematically low or high bias in the estimated number of years since mortality, the second source of bias would translate into a downward or upward shift, respectively, in CB and LCLW. One factor that could contribute to the observed downward shift, therefore, would be a routine delay of greater than one year between the loss of all remaining green needles on

a branch and the dieback of branch cambium to within 5 cm of the bole. However, this source of error likewise appears capable of explaining only some occurrences of CB below visually estimated crown base. If one year is added onto the estimated times since mortality, only nine of the 31 CB's which were estimated to occur below the corresponding repeat measurement move above the repeat measurement. If two years are added, an additional four CB's move above the visually estimated crown base.

Next, the criteria in the backdating procedure which dictate when CB movement occurs represent a generalization of field observations to avoid the tedious or prohibitive tracking of relative branch positions. Therefore, estimated movement of CB may suffer from lack of knowledge about the exact spatial arrangement of branches. This source of error would be restricted primarily to cases in which the first rather than second branch from a total of greater than four actually signifies loss of CB status. This also would produce a low bias in position of CB for short time intervals. However, only eight of the CB observations move above the visually estimated crown base if, indeed, the first branch mortality in whorls with

greater than four branches shifts CB to the next whorl. These eight observations are included in the set which additionally or alternatively could be explained by bias in branch mortality dating.

The last source of discrepancy would prevail if there commonly existed a large number of sparsely foliated, lower branches, or branches otherwise subjectively deemed below minimum vigor by the observer. The resulting shift would again increase the level of visually reconstructed crown base relative to CB and LCLW. This seems the strongest explanation for the remaining unexplained CB observations which fall below the corresponding repeat measurements, as well as for the average occurrence of repeat measures above the point midway between LCLW and CB. That is, branches of poor vigor and sparse foliage are intentionally, or perhaps inadvertently, disregarded. This factor is especially likely given the numerous field observations that live branch cambium invariably accompanies even a few green needles at the tip of a branch. These branches would almost certainly be disregarded due to low vigor or possibly be undetectable from ground level. Thus, visual estimation during repeat crown measurement gives a higher estimate of height to crown base, contrasted with the strict live/dead criterion of

mortality dating. This is even more clearly indicated by the fact that the 1983 estimates, which were direct observations in both procedures, exhibited the same relationships (Fig. 3).

In summary, the dissection technique for dating branch mortality apparently provides an accurate picture of the historic sequence of branch mortality. By reinterpreting this information on the level of CB and LCLW criteria, past movements of the crown base are also accurately portrayed. However, the objective of estimating a more biologically meaningful or functional crown base in repeat crown measures leads to a systematic upward shift in crown base, relative to the stem dissection estimates, due to elimination of live branches of unsuitable vigor. Assuming that CB and LCLW, as here defined, are equally or more meaningful in regard to deriving a functional crown size and growth potential, measurement to these points may be advantageous since they would be more consistent among different observers. In addition, inconsistencies common to repeat crown measurement data, such as periodic reductions in height to crown base, would be eliminated or at least rendered less frequent.

Applications

Others have applied stem dissection techniques in a manner supportive of the proposed crown reconstruction method. Schopf (1954) and Dietrich (1973) presented stem and crown profile diagrams to illustrate the process of natural pruning and knot formation. Through inclusion of a black knot zone in stem profiles, past positions of live crown base are clearly implied. In addition, Dietrich (1973) demonstrated his technique by presenting a branch/bole longitudinal section on which he labeled bole growth rings corresponding to initial branch suppression and final branch mortality. Extensions of the dissection technique for crown reconstruction similarly may include estimation of suppression onset and the implied contribution of various branches to stem growth. Forward and Nolan (1961a, b) provided a detailed framework for studying branch increment patterns. Ultimately such analyses can provide branch ring-width criteria for concurrently defining functional crown base along with live crown base, assuming branch radial growth is correlated with its contribution to stem growth. Alternatively, the relative contribution of various branches or parts of the crown can be modeled

as a continuous function of branch growth, as dictated by branch position and size (cf. Labyak and Schumacher 1954).

Intensive stem analysis has also been applied to reconstruction of past heights and diameters in stand development studies (Oliver and Stephens 1977, Wierman and Oliver 1979). Branch mortality dating can supplement such analyses by providing details on the historic recession of both CB and LCLW. Crown width can also be easily estimated by recording branch diameters and applying branch diameter-branch length equations (e.g., Smith et al. 1965). In this manner, detailed reconstruction of interactions among adjacent crowns can provide further insight into the mechanisms of stand development.

Finally, growth modelers have emphasized the need for more comprehensive, long term studies of crown development (Hatch 1971, Krumland and Wensel 1981, Krumland 1982). The described stem and branch dissection technique offers a promising way to reconstruct the long term crown dynamics of individual trees; however, this technique also suggests an approach for creating a crown change data base from trees on temporary plots by subsampling one to several branch whorls below present crown base. Thus, in

addition to traditional measurement of past diameter increment on cross-sectioned trees and height increment on felled trees, estimation of past periodic crown recession may also be feasible on temporary growth plots by abbreviating the described branch dating/crown reconstruction technique.

Fig. II.1. Schematic diagram of tree bole section illustrating dissection technique.

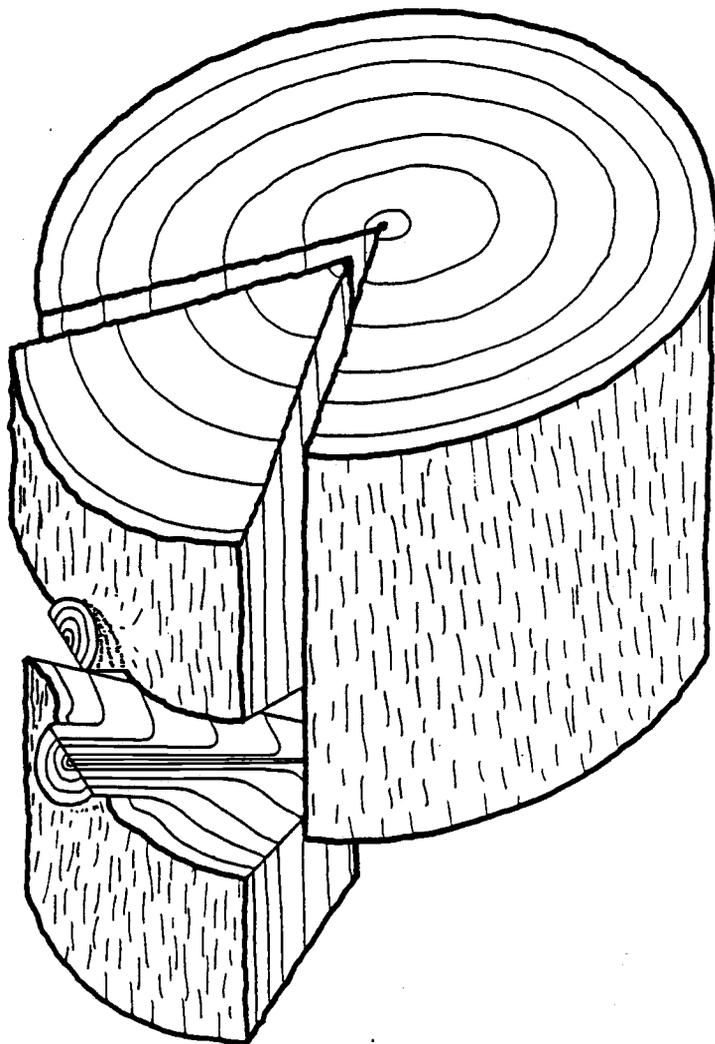


Fig. II.1.

Fig. II.2. Photograph of oblique cross-section through a Douglas-fir bole, exposing the longitudinal section of an included branch.

Fig. II.2.

Fig. II.3. Reconstructed recession of CB and LCLW over time and their positions relative to repeat measures of height to crown base for nine trees. The first three represent the three poorest degrees of correspondence (a), the second three represent three average degrees of correspondence (b), and the last three represent the closest degrees of correspondence (c). (Heavy solid line = CB recession by dating technique, light solid line = LCLW recession by dating technique, and dashed line = repeat crown measurements.)

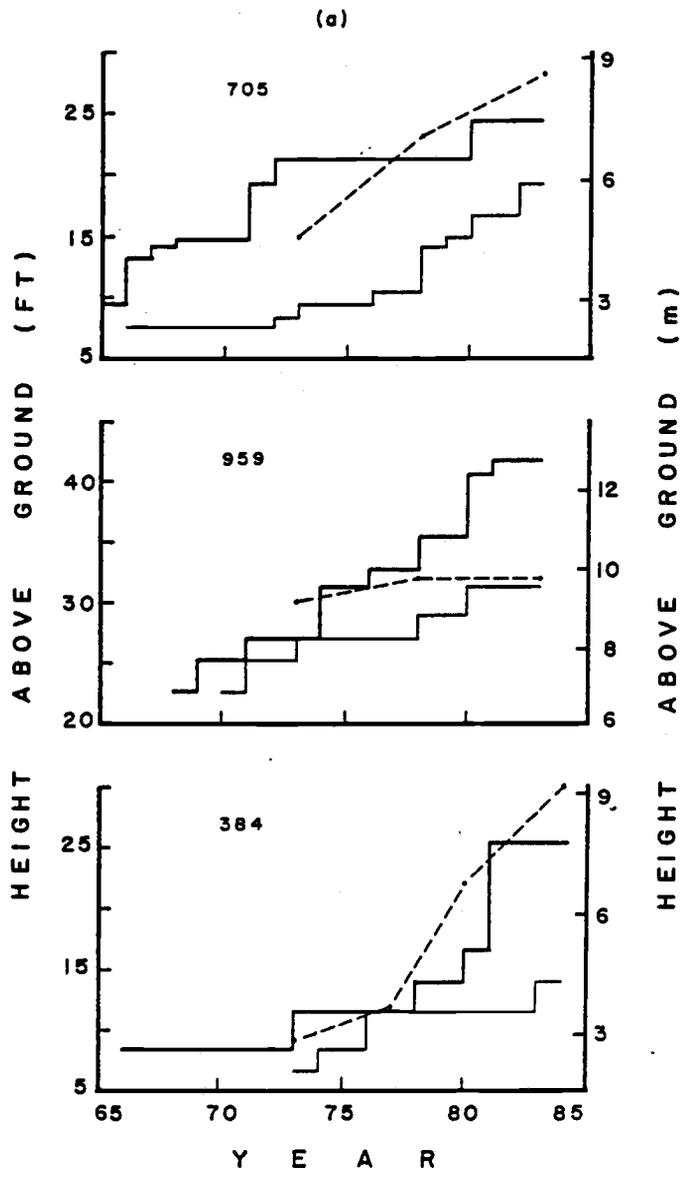


Fig. II.3

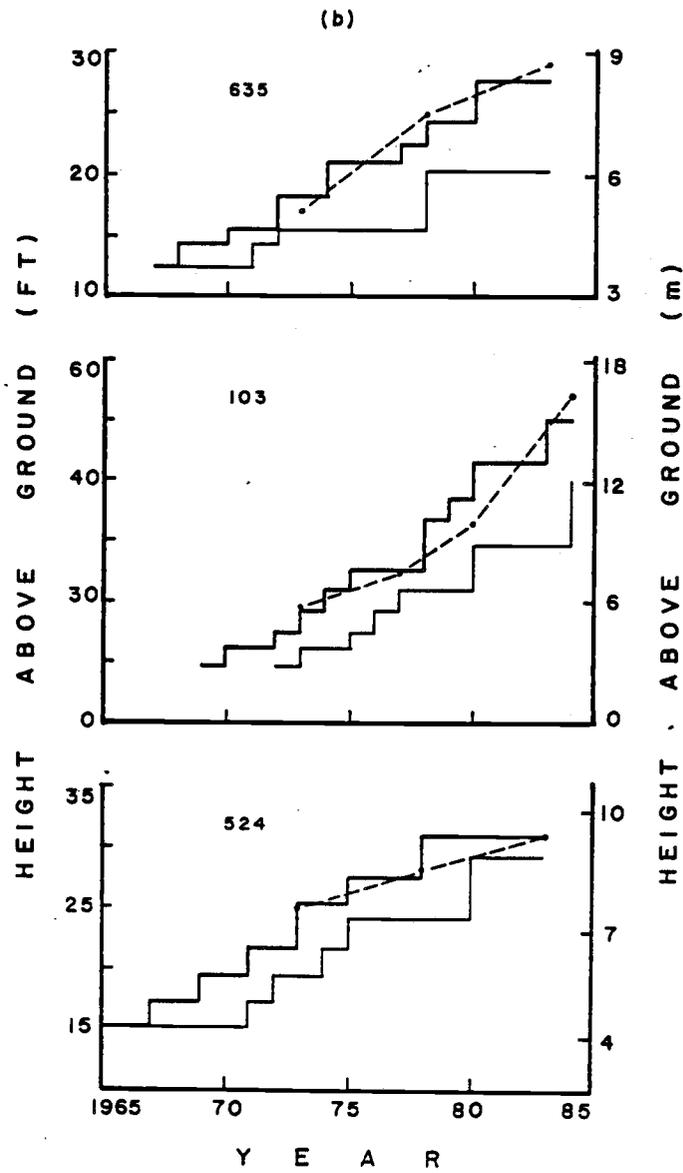


Fig. II.3. Continued.

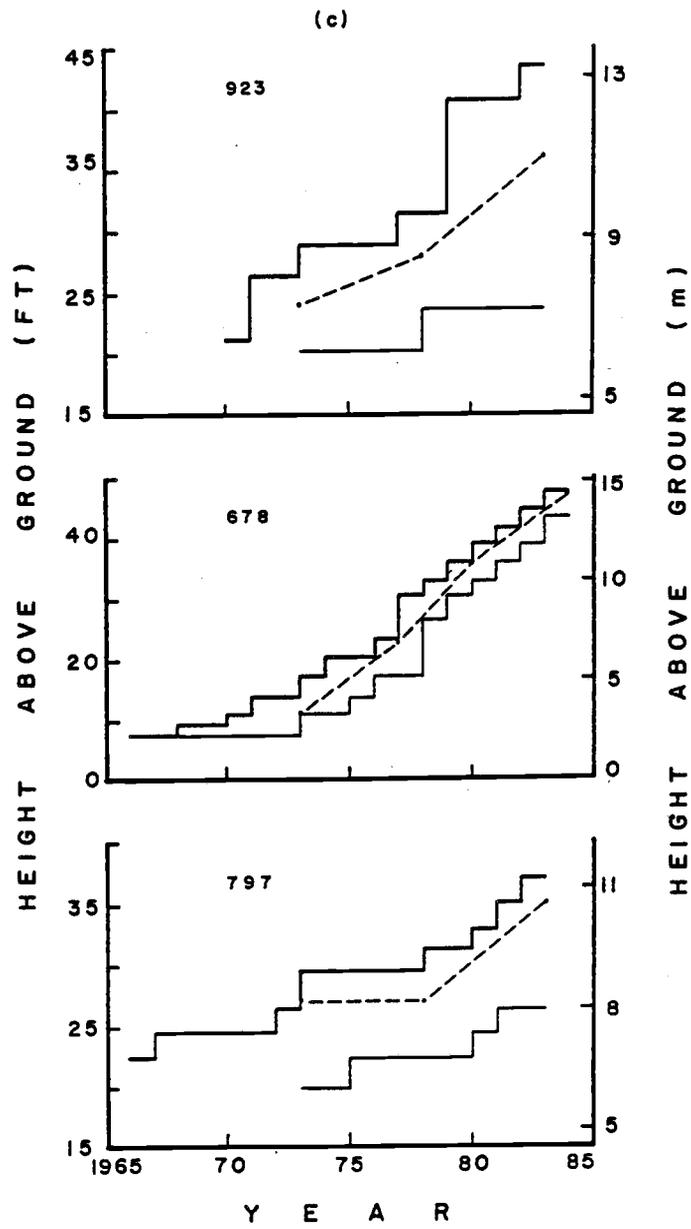


Fig. II.3. Continued.

Table II.1. Summary of relative positions of repeat measures of height to crown base (HT(RM)) and heights to CB and LCLW (HT(w)) as reconstructed through the branch mortality dating technique.

Crown point	Relative position	Number of Observations	Percent of Observations	Mean difference HT(RM)-HT(w) in m (s.e.)
CB	Above or equal to repeat measure	53	63.1	-1.20 (.153)
	Below repeat measure	31	36.9	0.61 (.107)
	Total	84	100.0	-0.53 (.141)
LCLW	Above repeat measure	3	3.8	-0.16 (.062)
	Below or equal to repeat measure	76	96.2	1.78 (.155)
	Total	79	100.0	1.70 (.155)
Midway between CB and LCLW	Above repeat measure	22	27.8	-0.65 (.148)
	Below repeat measure	57	72.2	1.04 (.126)
	Total	79	100.0	0.57 (.131)

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Chapter III

A Sampling Strategy for Estimating Past
Five-year Crown Recession on Temporary Growth Plots

Abstract

The potential efficacy of seven whorl sampling strategies and a branch mortality dating technique for estimating past five-year crown recession on temporary plots was explored. Past movements of crown base were first reconstructed on 28 Douglas-fir trees by application of the dating technique to all branches in 10-15 whorls below present crown base. Seven schemes were then proposed which entailed sampling only two or four whorls per tree. The corresponding estimators for each scheme allowed computation of past crown recession for each tree over various five-year intervals. Simple linear regressions of estimated on actual recession suggest that several of the sampling strategies (sampling scheme and estimator) were appropriate for estimating past five-year crown recession on temporary growth plots.

Introduction

Individual tree growth and yield models typically contain four major subcomponents that drive the dynamics of tree dimensions and stand structure: 1) height growth; 2) diameter growth; 3) crown change; and 4) tree mortality. The crown change submodel (often in conjunction with the height growth submodel) provides current estimates of crown length, crown ratio or some other expression of crown size. These crown dimensions alone are often of significance to silviculturists and other model users, but they also serve as predictor variables in other model subcomponents. Thus crown dimensions improve predictions of height growth (Arney 1972, Daniels and Burkhart 1975, Mitchell 1975, Wensel and Koehler 1985), diameter growth (Botkin et al. 1972, Daniels and Burkhart 1975, Krumland and Wensel 1981, Belcher et al. 1982, Wensel and Koehler 1985), mortality (Arney 1972, Daniels and Burkhart 1975), volume growth (Mitchell 1975), and even the vertical distribution of bole increment on the stem (Arney 1972, Mitchell 1975). Consistent with the latter relationship, crown size may improve individual tree taper and volume equations (Naslund 1947, Farrar 1984, Burkhart and Walton 1985, Walters et al. 1985).

Finally, individual tree crown size has also been found an indispensable parameter for computing collective stand competition measures (Wycoff et al. 1982, Krumland and Wensel 1981, Wensel and Koehler 1985) as well as individual subject tree competition indices (Hatch 1971, Arney 1972, Botkin et al. 1972, Mitchell 1975, Krumland and Wensel 1981, Wensel and Koehler 1985).

Since growth models commonly project tree and stand growth over numerous growth periods, a submodel of crown development must continuously update crown size to maintain dimensions commensurate with the dynamics of other tree and stand variables. Thus, crown size remains available for predictions in any subsequent growth periods. However, despite the widespread conclusion that crown dimensions are critical to accurate prediction of stand and tree growth in individual tree models, data from which crown development submodels can be constructed are extremely scarce. Most distance independent models have relied on static predictions of height to crown base (Daniels and Burkhart 1975) or crown ratio (Belcher et al. 1982, Wycoff et al. 1982). In such models, updated crown dimensions are predicted from other tree, stand, and site variables at the end of the growth period. Thus

temporary plot data which include the desired crown dimension and relevant predictor variables have served as the data base for model construction. Under a variety of intensive management regimes, however, these models are plagued by inherent inconsistencies such as lowering of height to crown base after thinning. Although clauses can be included within the model to ameliorate these problems, direct prediction of crown change seems more appealing, especially when modeling or extrapolating to repeated intermediate stand entries.

Krumland and Wensel (1981) and Krumland (1982) have presented apparently the only models capable of predicting crown change directly. The data base for both models derived from repeat crown measures, although the crown measures were admittedly of a coarse resolution and were recorded over a relatively narrow range of stand ages (Krumland and Wensel 1981). In contrast to the common practice of periodically remeasuring diameters and heights on permanent plots, periodic changes in crown size have rarely been monitored in most forest types. Crown reconstruction techniques analogous to stem analysis for height and diameter have received little attention, although

growth layer patterns have been shown to relate in a general manner to the size and position of crown (Duff and Nolan 1953, Larson 1963, Fayle 1985). More recently, Maguire and Hann (1986b) expanded a branch mortality dating technique (Andrews and Gill 1939) into a procedure for reconstructing past heights to crown base. Although the position of crown base at a given year in the past can be reconstructed by full stem dissection, this procedure may be prohibitive for most regional growth and yield studies in which a wide range of site and stand conditions needs to be sampled.

As growth and yield modeling efforts move into new forest types or geographic locations, crown data which cover the target population will usually be collected from temporary growth plots. Such temporary plot data have restricted crown modeling to static approaches in distance-independent models: direct measures of past crown recession on temporary plots, from which crown change can be modeled directly, remain elusive. The objective of the present paper, therefore, is to explore various operational sampling strategies by which branch mortality dating and crown reconstruction techniques (Maguire and Hann 1986b) can be abbreviated for estimating past five-year crown recession on temporary growth and yield plots. More specifically,

actual crown recession on a set of trees will be established by dissection of the full complement of whorls below present crown base. The efficacy of numerous whorl sampling schemes and corresponding estimators for estimating actual periodic change in height to crown base will then be analyzed.

Study Sites

Stampede Creek

The first set of permanent plots from which trees were sampled occurred on the Tiller Ranger District of the Umpqua National Forest, approximately 11 km (7 mi) east of Tiller, Oregon. The 27 .08-ha (.2 ac) plots were established in 1968 as the Stampede Creek installation of the regional Douglas-fir Levels-of-Growing-Stock Study (Williamson and Staebler 1971).

The stand originated 10 years after a wildfire in 1929. At the time of study initiation in 1968 the stand was 16.8 m (55 ft) high (17.2 m (56.5 ft) for crop trees). Precipitation averages 700-800 mm (27.5-31.5 in) annually, and the plots are situated at an elevation of approximately 915 m (3000 ft). Temperature ranges from a January mean minimum of -2° C to a July mean maximum of 27° C. Although the stand falls within the region described by Franklin and Dyrness (1973) as mixed conifer, overstory composition is 100 percent Douglas-fir (Pseudotsuga menziesii (Mirbel) Franco), and the understory contains a strong component of Gaultheria shallon Pursh.. Slopes are gentle, averaging about 25 percent with a general

northeast aspect. The soils are of a heavy loam texture overlying heavy clay loam (Williamson and Staebler 1971). Site quality (King 1966) has been estimated at low site class III which is equivalent to a site index (base age 50 yrs) of 30.5 m (100 ft) (Williamson and Curtis 1984). Plot descriptions by treatment, up to 1973, are given by Williamson (1976). More recent volume, diameter, height, and density data are presented by Williamson and Curtis (1984).

The calibration thinning for Stampede Creek was implemented in 1968, with the first, second, and third treatment thinnings following in 1973, 1978, and 1983, respectively. Starting in 1973, height to the base of live crown was measured on a subset of the trees in each plot, just prior to treatment thinnings. Crown base was approximated by visual reconstruction of the crown, whereby any gaps in the crown were filled in with branches from below so as to produce a crown with an even base (Robert O. Curtis, pers. com.).

Iron Creek

Trees were also sampled from the Iron Creek installation of the regional Douglas-fir Levels-of-Growing-Stock Study (Williamson and Staebler 1971). These 27 .08 ha plots (.2 ac) were established on the

Randle Ranger District of the Gifford Pinchot National Forest, approximately 14 km (9 mi) south of Randle, Washington.

The stand was planted in 1949, and at the start of the calibration period in 1966 the crop trees averaged 11.1 m (36.4 ft) in height. Precipitation averages about 1900 mm (65 in) annually and temperature ranges from a January mean minimum of -4°C to a July mean maximum of 23.5°C . The stand occupies a midslope position at approximately 760 m (2500 ft) in elevation. Slopes average 25 percent with a general east aspect. The deep, well-drained volcanic soils range from sandy loam to loam interbedded with pumice (Williamson and Staebler 1971). Site quality (King 1966) has been estimated at a high site class II, which is equivalent to a site index (base age 50 yrs) of approximately 38.7 m (127 ft) (Williamson and Curtis 1984). Plot descriptions by treatment, up to 1973, are given by Williamson (1976). More recent volume, diameter, height, and density data are presented by Williamson and Curtis (1984).

Methods

Data Collection

The 28 sample trees in the present study were those previously described and analyzed by Maguire and Hann (1986b). In brief, twenty-four trees felled in the 1983 thinning at Stampede Creek, and four additional trees felled just prior to the 1984 thinning at Iron Creek, were selected for detailed stem and branch dissection.

On each of the 28 sample trees, two crown points were marked: 1) crown base (CB), defined as the lowest whorl which had live branches at least three quarters of the way around the circumference of the stem, and above which all whorls had the same (cf. Curtis and Reukema 1970, Curtis 1983); and 2) lowest contiguous live whorl (LCLW), defined as the lowest live whorl above which all whorls had at least one live branch. Starting with the first whorl below crown base, 10-15 successive whorls were marked for removal. Heights to all whorls were recorded (nearest .03 m), including those whorls specified as CB and LCLW. The whorls were then sawed out, leaving at least 5 cm of any protruding branches.

After removing the sample whorls and transporting

to the lab, individual branch stubs and knots were split out of the bole section. Oblique cross-sectional cuts were then made through each wedge on a band saw, longitudinally through the branch and knot. The area of discontinuity between the bole growth rings and dead branch growth rings was carved to a smooth surface. Finally, the year of branch mortality was estimated with the aid of a 13X power hand lens, assuming that the year before the initial growth ring discontinuity was the year in which the branch died (Koehler 1936, Andrews and Gill 1939, Rapraeger 1939). Maguire and Hann (1986b) provide further details and validation of the branch mortality dating technique.

Once the estimated year of branch mortality was established, the number of growth rings which accrued subsequent to mortality was recorded. Since a given branch could have died any time during the estimated year of mortality, half a year was added to each record.

Estimation of Five-Year Crown Recession

Branch mortalities were backdated and crown positions reconstructed annually. This process began with the last year that the whorl below present CB had

been CB and continued back until the postdated CB moved below the lowest whorl sampled. For each year of backdating, the CB and LCLW were identified. Field observations indicated that a given whorl generally lost status as a potential CB when one branch died out of a total of four or less, when two branches died out of a total of five to seven, or when three branches died out of a total of eight or more.

Modification of the branch mortality dating technique into an operationally feasible procedure for estimating past five-year crown recession requires the development of a whorl sampling scheme and appropriate estimators of crown recession.

Sampling Schemes

One to four whorls per tree were assumed to represent the range of feasible whorl sampling intensities which would not seriously reduce the tree sampling intensity in a regional growth and yield study. Therefore, two sets of sampling schemes were applied to reconstructed crown and whorl positions, the first set requiring analysis of four sample whorls per tree and the second set requiring only two. The whorls sampled and the specific data required for rate

estimation are as follows:

Four sample whorls -

1. Sample: First two whorls below CB and first two all-dead whorls below LCLW.

Data: Estimated years of mortality for all branches in four sample whorls; heights to CB, whorl below CB, LCLW, and whorl above second all-dead whorl below LCLW.

2. Sample: First and third whorls below CB and first and third all-dead whorls below LCLW.

Data: Estimated years of mortality for all branches in four sample whorls; heights to CB, second whorl below CB, LCLW, and whorl above third all-dead whorl below LCLW.

3. Sample: First and fourth whorls below CB and first and fourth all-dead whorls below LCLW.

Data: Estimated years of mortality for all

branches in four sample whorls; heights to CB, third whorl below CB, LCLW, and whorl above fourth all-dead whorl below LCLW.

Two sample whorls -

4. Sample: Second whorl below CB and second all-dead whorl below LCLW.

Data: Estimated years of mortality for all branches in two sample whorls; heights to CB, first whorl below CB, LCLW, and whorl above second all-dead whorl below LCLW.

5. Sample: Same as (5b) except that if two or less whorls occurred between CB and LCLW, only the second all-dead whorl below LCLW was analyzed.

Data: Estimated years of mortality for all branches in two sample whorls; heights to CB, first whorl below CB, LCLW, and whorl above second all-dead whorl below LCLW.

6. Sample: Third whorl below CB and third all-dead whorl below LCLW.

Data: Estimated years of mortality for all branches in two sample whorls; heights to CB, second whorl below CB, LCLW, and whorl above third all-dead whorl below LCLW.

7. Sample: Fourth whorl below CB and fourth all-dead whorl below LCLW.

Data: Estimated years of mortality for all branches in two sample whorls; heights to CB, third whorl below CB, LCLW, and whorl above fourth all-dead whorl below LCLW.

Crown Recession Estimators

Several characteristics of the progression of CB and LCLW are of significance in the development of appropriate estimators. First, both CB and LCLW recede up the bole in a stair-step fashion: a given whorl retains status as CB (or LCLW) until approximately one quarter (or all) of the branches die (Fig. 1).

Second, it is biologically appealing to approximate this discrete process with the continuous dotted line shown in Fig. 1. Before a given whorl or

branch dies, total foliage biomass continually declines and the photosynthetic efficiency of the remaining foliage begins to drop. Therefore, although CB as defined in the present study does not arrive at the next whorl above technically until that first or second branch in the whorl dies, the effective CB lifts more continuously, commensurate with the gradual progression of individual branch suppression mortality. Note also that repeat observations on either the CB or LCLW would occur as points along the horizontal sections of the steps; hence, the least squares regression line through repeat observations of either the CB or LCLW would fall well below the continuous approximation in Fig. 1.

Lastly, lines A and B in Fig. 1 illustrate two possible trajectories of actual past periodic crown recession, assuming validity of the continuous approximation and the concept of effective CB or LCLW. Accurate estimation of any periodic five-year rate is seen to rely on knowledge of: 1) height of effective CB five years previous; and 2) height of effective CB at present. In actual application of the various sampling schemes, the latter will never be known since the longevity of present CB is unknown. The former will only occasionally be known if any

sampled whorl in schemes [1] to [7] died exactly five years previous, or if the two sampled whorls in scheme (1) died earlier and later than five years ago.

Given the biological appeal of tracking the effective CB (or LCLW) represented by the continuous approximation in Fig. 1, the following estimators were developed for corresponding sampling schemes, compatible with this interpretation (Fig. 2):

$$[1] \quad \Delta CB = 5 \frac{H(CB) - H(CB-1)}{Y(CB-2) - Y(CB-1)}$$

$$\Delta LC = 5 \frac{H(LC) - H((LC-2)+1)}{Y(LC-2) - Y(LC-1)}$$

$$[2] \quad \Delta CB = 5 \frac{H(CB) - H(CB-2)}{Y(CB-3) - Y(CB-1)}$$

$$\Delta LC = 5 \frac{H(LC) - H((LC-3)+1)}{Y(LC-3) - Y(LC-1)}$$

$$[3] \quad \Delta CB = 5 \frac{H(CB) - H(CB-3)}{Y(CB-4) - Y(CB-1)}$$

$$\Delta LC = 5 \frac{H(LC) - H((LC-4)+1)}{Y(LC-4) - Y(LC-1)}$$

$$[4] \quad \Delta CB = 5 \frac{H(CB) - H(CB-1)}{Y(CB-2)}$$

$$\Delta LC = 5 \frac{H(LC) - H((LC-2)+1)}{Y(LC-2)}$$

[5] if greater than two whorls between CB and LCLW, same as [4]

if two or less whorls between CB and LCLW,

$$\Delta CB = 5 \frac{H(CB) - H((LC-2)+1)}{YCB(LC-2)}$$

$$\Delta LC = 5 \frac{H(LC) - H((LC-2)+1)}{Y(LC-2)}$$

$$[6] \quad \Delta CB = 5 \frac{H(CB) - H(CB-2)}{Y(CB-3)}$$

$$\Delta LC = 5 \frac{H(LC) - H((LC-3)+1)}{Y(LC-3)}$$

$$[7] \quad \Delta CB = 5 \frac{H(CB) - H(CB-3)}{Y(CB-4)}$$

$$\Delta LC = 5 \frac{H(LC) - H((LC-4)+1)}{Y(LC-4)}$$

where ΔCB = estimated past five-year CB
recession

- ΔLC = estimated past five-year LCLW recession
- $H(CB)$ = height to CB
- $H(LC)$ = height to LCLW
- $H(CB-w)$ = height to wth whorl below CB
- $H((LC-w)+1)$ = height to whorl above wth all-dead whorl below LCLW
- $Y(CB-w)$ = number of years since wth whorl below CB ceased to be a potential CB
- $Y(LC-w)$ = number of years since wth all-dead whorl below LCLW died (ceased to be a potential LCLW)
- $YCB(LC-w)$ = number of years since wth all-dead whorl below LCLW ceased to be a potential CB

Two other sets of estimators corresponding to the seven sampling schemes were also computed initially. The first set was intuitively based on distances between the actual sampled whorls or between the sampled whorls and CB (for example, [1b] and [4b] in Fig. 2); note, however, that these estimators are not compatible with the above interpretation of effective

CB and LCLW.

The second set applied only to estimators [4] to [7], but replaced height to CB with the height midway between CB and the whorl above CB (for example, [7c] in Fig. 2b.). The probability of observing CB (or LCLW) in any one year, conditional on the present location of CB (or LCLW), is uniformly distributed on the interval over which that whorl remains CB. Therefore, since the expected value of the year of observation is the midpoint of this interval (Johnson and Kotz 1970), the latter estimate is statistically consistent (Fisher 1956).

On average estimators [1] - [7] performed equally well or better than these latter two sets of estimators. Hence only estimators [1] - [7] were further pursued.

For each year of backdating on the 28 dissected trees, past five-year recession of CB and LCLW were estimated by application of each sampling strategy. Mean estimated crown recession was computed as:

$$[8] \quad \Delta HCB = p (\Delta CB) + (1-p) (\Delta LCLW)$$

where HCB = estimated past five-year mean
crown recession

Δ CB = estimated past five-year CB
recession

Δ LCLW = estimated past five-year LCLW
recession

p = estimated or hypothesized
ratio, $[HCB-HT(LC)]/[HT(CB2)-$
 $HT(LC)]$, where HCB = field
estimated crown base

If the desired mean crown base occurs either theoretically or empirically at a proportion, p, of the distance between LCLW and CB, this estimator yields the slope of the correct trajectory of past five-year crown recession. Therefore, two mean recession estimates were computed for each set of nonzero CB and LCLW estimates: 1) the arithmetic mean (p=.5), hypothesizing that the visually estimated crown base should occur midway between CB and LCLW; and 2) a weighted mean in which the proportion, p, was determined empirically from the relationship of repeat measures to CB and LCLW.

Actual Crown Recession

Actual past five-year recession corresponding to each tree and year was also computed, as far back as the effective crown base five years ago was still

within the height range of sampled whorls. Two types of CB (and LCLW) trajectories were derived (Fig. 3):

(A1a-d) the slope of the trajectory with an origin at the height of the whorl above the lowest sampled whorl on the year in which the lowest sampled whorl ceased to be CB (or LCLW), and endpoint at the height to effective CB (or LCLW) between CB (or LCLW) and the next CB (or LCLW) (not necessarily the first whorl above CB (or LCLW)). This first version of actual rate varies by sampling scheme, so the rates are designated (A1a) for schemes (1) and (4), (A1b) for schemes (2) and (6), (A1c) for schemes (3) and (7), and (A1d) for scheme (5) (Fig. 3; (A1d) not shown since the number of whorls between CB and the second all-dead whorl below LCLW will be variable);

(A2) the slope of the trajectory with an origin at the height of the effective CB (or LCLW) exactly five years previous

and end point at the height of present effective CB (or LCLW). The past and present effective CB's (or LCLW's) were located between CB (or LCLW) and the next actual CB (or LCLW), which were not necessarily the first whorl above each CB (or LCLW).

Versions (A1a-d) give recession rates that may represent more or less than five years, depending on when the lowest sample whorl lost CB or LCLW status. However, the trajectory corresponding to (A2) gives the exact distance which CB or LCLW moved in the past five years, as revealed by the dating technique. These two variations on actual rate were calculated for each CB and LCLW recession estimate. Arithmetic mean and weighted mean actual recession were then calculated from equation [8] as described for mean recession estimates.

Assessment of Estimator Accuracy

Simple linear regressions of estimated on actual rates were computed for each of the seven rate estimators on both variations of actual rate ((A1a-d)

and (A2)). This resulted in 14 regressions for each of CB recession, LCLW recession, arithmetic mean recession, and weighted mean recession. All observations predicted to have a negative estimated recession rate were eliminated from the analysis. Quality of fit was judged by both R^2 (coefficient of multiple determination) and RMS (residual mean square). In addition, it was desirable for the expected value of the estimated rate not to deviate significantly from the actual rate (that is, for the estimates to be unbiased); hence, the parameter estimate vector, (intercept, slope), was tested for significant departure from the vector (0, 1) by an appropriate F-statistic (Draper and Smith 1981).

Only 10 repeat crown measurements could be found with corresponding reconstructed crown change over the same time period. The estimated crown changes from these repeat measures were regressed on reconstructed arithmetic mean recession to provide a rough assessment of the variability of repeat measure estimates about actual recession.

Finally the mean, minimum, and maximum of the following variables were computed to gain insight into the relative performance of the seven sampling

strategies: 1) number of internodes between present CB's (and LCLW's) and previous CB's (and LCLW's); 2) years since each of the four sample whorls below CB (and below LCLW) lost status as CB (or LCLW); and 3) whorl longevity as CB (and LCLW).

Results

Estimators [1] - [3] performed no better or more poorly than estimators [4] - [7] in regard to R^2 , RMS, and significant deviation of the parameter estimate vector from (0, 1) (Tables 1 - 4). Furthermore, since corresponding sampling schemes (1) - (3) necessitated removal and dissection of four whorls, vs. two whorls in schemes (4) - (7), sampling strategies 1 - 3 were dropped from further analysis.

Of the remaining four sampling strategies, one notable pattern involving actual rates (A1a-d) was the progressive increase in R^2 from estimators [4] to [6] to [7] (Tables 1-4; one exception was the slightly greater R^2 for LCLW estimator [6] than for estimator [7]). This pattern of improving fits from [4] to [6] to [7] was reflected to a lesser extent in RMS trends as well. In each of CB, LCLW, arithmetic mean, and weighted mean recession, over 96 percent of the variation in estimated rate [7] were explained by actual rate (A1c).

The relationships between the various sets of estimates and actual rate (A2) were more variable. The greatest R^2 and lowest RMS were .736 and 8.462, respectively, for LCLW estimator [7]. By both R^2 and

RMS criteria, the best fits on actual rate (A2) were provided by estimator [5] using either the arithmetic mean (Table 3) or the weighted mean (Table 4) of CB and LCLW recessions.

The intercept-slope vector was not significantly different from (0, 1) for arithmetic mean estimates from [4] - [7] regressed on actual rate (A2) (Table 3). Similar results were obtained for weighted mean estimates (Table 4), except for those from estimator [6].

Although only ten observations were available, the regression of repeat measure estimates on actual 5-yr crown change indicated a very poor correspondence between the two ($R^2 < .001$).

As shown in Table 5, for the time span over which the trees were backdated, CB actually moved an average of three whorls (that is, CB five years previous to each "present" CB averaged three whorls below "present" CB). The minimum for this five year change was zero (no change) and the maximum was nine. Similarly, previous LCLW's averaged two whorls below present LCLW's, with a minimum of zero and maximum of seven.

The mean number of years since each of the four sample whorls below CB and LCLW lost status as CB and LCLW increases moving downward, as would be expected

(Table 5). On average, the five-year-previous CB and LCLW were within the range of sampled whorls since the means extended back 5.5 and 6.5 years, respectively. However, the fact that all minima are 0.5 years concurs with the above observation that the previous CB or LCLW occurred as low as nine and seven whorls, respectively, below present positions. Finally, whorls remained CB and LCLW for an average of 2.3 and 2.8 years, respectively, with minima of one for both, and maxima of 8 and 15, respectively (Table 5).

Discussion

The relative degrees of fit among the various estimators on actual rates (A1a-d) provide insight into the general behavior of the estimators. The variability in estimators [4] - [7] around actual rates [A1a-c] derives from the inability to know effective CB or LCLW, as depicted in Fig. 4. Thus, the progressively better fits from estimator [4] to estimator [7] (Tables 1-4) derive from the fact that the estimators cover progressively longer time intervals (Table 5, Fig. 3); hence, the effect of uncertainty in the exact endpoint of the trajectory diminishes from estimator [4] to [6] to [7].

The drop in R^2 and increase in RMS from regressing the same estimated rates on (A2) portrays the additional variability introduced by the variable time intervals of the estimates. Estimates from [4] - [7] and corresponding actual rates [A1a-c] are the slopes, converted to a five-year basis, of trajectories covering anywhere between 0.5 and 18.5 years (Table 5). The true five-year recession varies somewhat from the average rate of change over these intervals of variable length. Thus, an estimator will perform most satisfactorily if the whorl sampled lost CB or LCLW.

status to the whorl above exactly five years previous (in other words, when the position of effective CB or LCLW five years ago is established exactly). Any deviation from this situation will allow the estimator to vary further from the true five-year change in height to CB. For estimators [4] - [7], therefore, uncertainty in the actual present location of effective CB introduces a relatively small amount of variation, and uncertainty in the location of effective CB five years ago introduces a relatively large amount of variation into the estimator.

Given this pattern in R^2 and RMS from the simple linear regressions, a reasonable strategy to minimize the variation and bias in the rate estimator would be to sample the whorls directly below the average position of CB and LCLW five years previous. For the range in site, age, and stand conditions covered in the backdating process at Stampede Creek and Iron Creek, the third or fourth whorl below CB and second or third all dead whorl below LCLW appear closest to optimal (Table 5).

For estimation of actual rate (A2) in the original population at the two LOGS studies, it may be appropriate to elaborate on the best regressions (e.g., by transformation) to obtain an optimal fit of

estimates on actual rate. The final equation can then be inverted to provide conversion from estimated to actual five-year rate. However, this calibration equation will vary to some degree among populations. Thus in applying a scheme and estimator to a new population for which a performance analysis is not available, it would be best to implement a sampling strategy which yields an apparently unbiased estimator. In a modeling context, additional "errors in measurement" of the estimated response variable will be absorbed into the error term of the subsequent crown change prediction model. As long as the estimated rate is unbiased, unbiased and consistent parameter estimates should result. With respect to both bias and RMS in the present analysis, estimator [5] appears optimal, as both the arithmetic mean and weighted mean estimators on actual rate (A2) yield parameter estimate vectors not significantly different than (0, 1), and, of all estimators satisfying these parameter estimate conditions for the arithmetic and weighted mean estimators, this estimator produced the lowest RMS and highest R^2 . Estimator [4] similarly met the parameter estimate conditions for the two mean estimators on actual recession (A2) and had a relatively high R^2 and

low RMS.

Estimators [4] and [5] and their corresponding sampling schemes thus appear satisfactory for constructing a data base for modeling crown recession in the Stampede Creek/Iron Creek population. As stated above, where estimation of past five-year recession is the objective, those sampling schemes which, on average, sample whorls at or just below CB and LCLW five years previous will perform best. The fact that for this time interval, previous CB and LCLW averaged three and two whorls, respectively, below present positions suggests that estimator [6] or [7] should have been optimal for CB and estimator [4], [5], or [6] should have been optimal for LCLW. With the intention of designing an easily implemented operational sampling procedure, the present analysis maintained a uniform sampling interval below both CB and LCLW. One may have expected design (6) to be optimal, therefore. However, since design (5) combines the optimal interval of two whorls below LCLW and greater than two (but less than four) whorls below CB, it is not surprising that mean estimator (5) proves optimal by yielding the lowest RMS.

For the diversity of recession patterns in the Stampede Creek plots, estimators [4] -[7] produce

apparently unbiased estimates of crown change, with the possible exception of arithmetic mean estimator [7]. For populations of similar structure, therefore, these estimators can be expected to similarly provide unbiased 5-yr crown change estimates. In drastically different populations, the type of analysis presented here can be performed to either construct a calibration equation or establish the unbiasedness of a given estimator.

Other possible schemes, for example, those in which the sampled whorls vary according to stand age and density to capture the previous crown position more closely, may improve estimator variance, but would be logistically more complex, and perhaps prohibitive. More intensive whorl sampling schemes may also prove advantageous, especially where tree, stand, and site variability is relatively small. In these situations, sampling frequency across the range in potential predictor variables can be sacrificed to obtain more accurate estimates of past crown recession. However, where a wide variety of conditions prevail, substantial benefits may accrue from intensifying the sampling of trees rather than whorls.

In the context of alternative data bases and past

crown modeling approaches, the branch mortality dating technique compares quite favorably. Due to the usually slow rates of change and subjectivity involved, repeat crown measurements often indicate negative changes in height to crown base. One major aspect of this subjectivity includes variation introduced during visual reconstruction of the crown by filling in crown gaps with branches from below. The poor correlation between actual periodic crown recession and repeat crown estimates underscores the inherently coarser resolution of repeat crown measurements. Where accurate crown measurements are a concern, perhaps repeat measures to both CB and LCLW will adequately improve the resolution of and consistency among serial observations. Inconsistencies among observations may alternatively be ameliorated, but also possibly biased, by carrying past heights to crown base into the field during remeasurement.

Finally, static approaches to crown change modeling inevitably must break down under management regimes that include repeated intermediate treatments of varying intensity. Crown lengths will reflect stand structure, in some cases, of several growth periods past rather than the current structure. Furthermore, it is precisely under these conditions of widely

varying stand structure that crown size as a predictor variable in growth, taper, and volume equations becomes most critical. Until long term crown studies yield improved data bases, the described whorl sampling approach can extract the necessary data from temporary plots.

Fig. III.1. Progressions of CB and LCLW through time and their relationship to periodic trajectories (A and B).

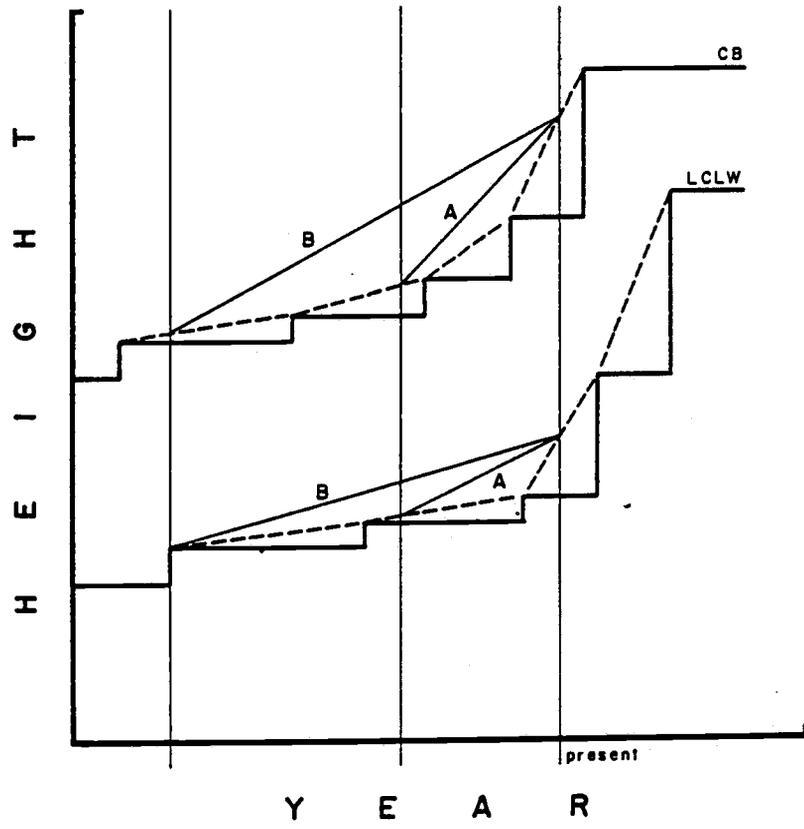


Fig. III.1.

Fig. III.2. Two possible progressions of CB (or LCLW) through time and their relationship to estimators [1], [2], [3], [4], [6], and [7]. The actual position of estimator [5] for CB recession will vary depending on the number of whorls between CB and the second all-dead whorl below LCLW. LCLW recession estimator [5] is identical to [4].

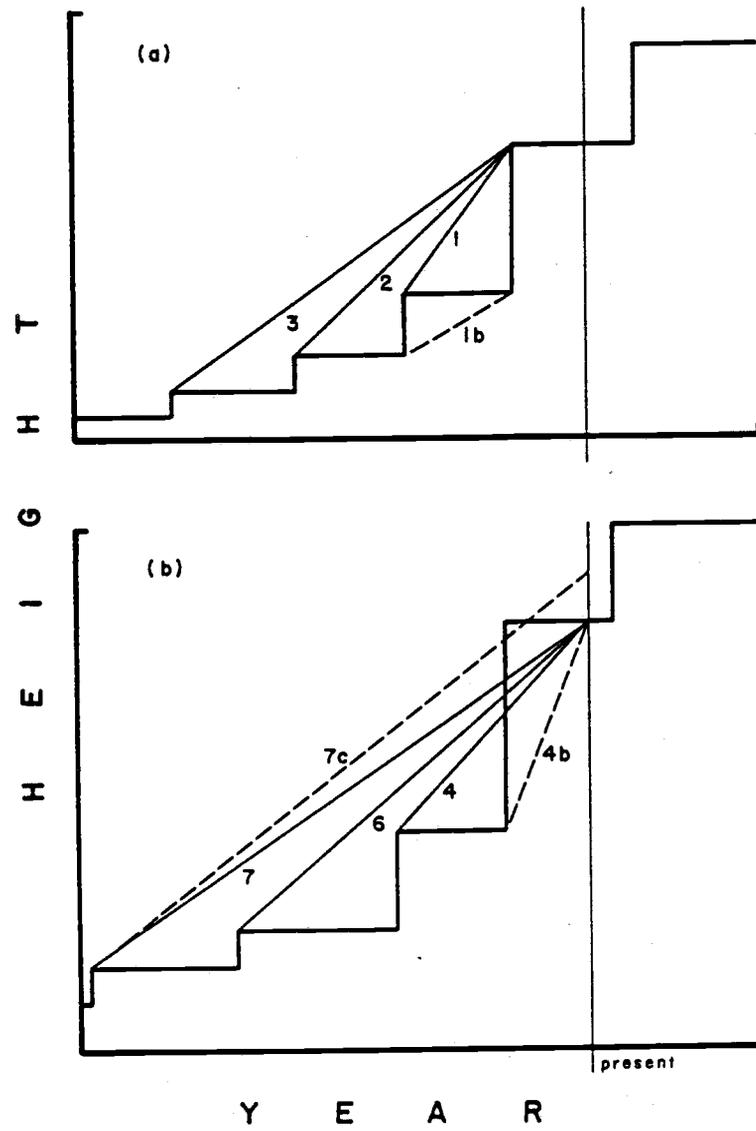


Fig. III.2.

Fig. III.3. The two basic versions of actual CB (or LCLW) recession, (A1a-c) and (A2). Actual rate (A1d) is not shown since it will vary depending on the position of the second all-dead whorl below LCLW (see text).

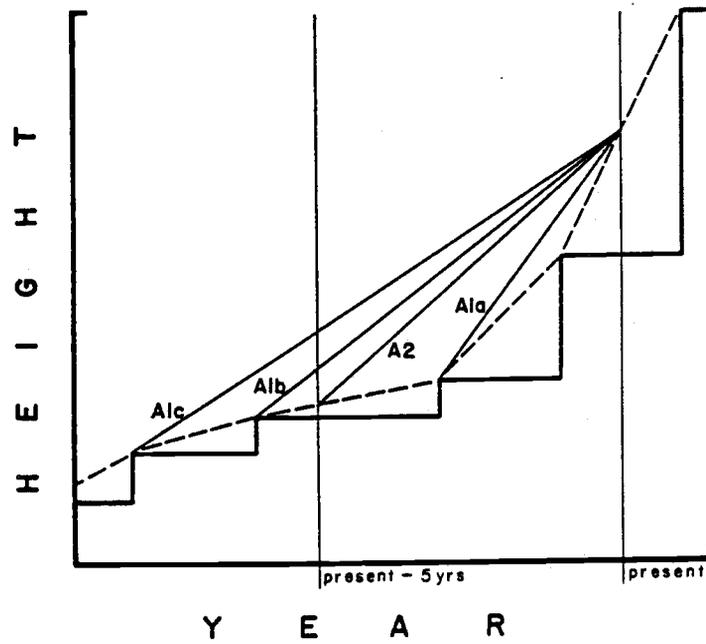


Fig. III.3.

Fig. III.4. Four possible years for observation of present CB, corresponding trajectories from estimator [7] (light solid lines), and actual rate (A1b) for the fourth year (dark solid line).

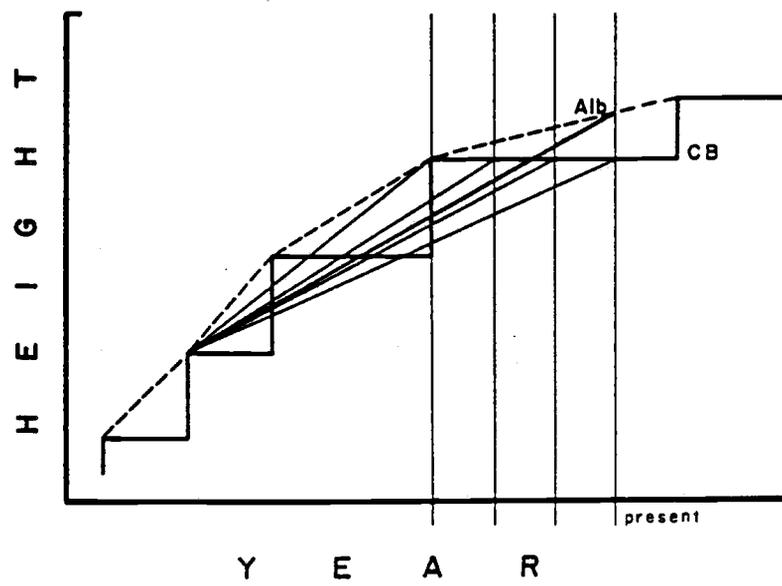


Fig. III.4.

Table III.1. Results of simple linear regressions of CB recession estimates on two types of actual CB rates.

Estimator	Actual rate	RSQ	RMS	N	Coefficients		F
					a (s.e.)	b (s.e.)	
4	A1a	.875	5.421	237	-.2695 (.2161)	.6372 (.0157)	589.3*
5	A1d	.882	3.371	199	.3724 (.1913)	.6552 (.0171)	381.3*
6	A1b	.932	6.051	226	-.3931 (.2312)	.7811 (.0141)	281.8*
7	A1c	.978	3.013	213	-.3807 (.1554)	.8546 (.0089)	278.4*
4	A2	.220	35.750	205	1.2819 (.7614)	.8085 (.1070)	1.7
5	A2	.391	16.387	207	.6293 (.5076)	.8218 (.0716)	4.2*
6	A2	.145	75.360	205	2.7311 (1.1476)	.9188 (.1565)	6.9*
7	A2	.097	115.724	213	3.3767 (1.3573)	.8970 (.1881)	7.1*

* Intercept-slope vector significantly different than (0,1), $p < .05$.

Table III.2. Results of simple linear regressions of LCLW recession estimates on two types of actual LCLW rates.

Estimator	Actual rate	RSQ	RMS	N	Coefficients		F
					a (s.e.)	b (s.e.)	
4	A1a	.888	5.374	166	-.1833 (.2410)	.6521 (.0181)	349.9*
5	A1d	.888	5.374	166	-.1833 (.2410)	.6521 (.0181)	349.9*
6	A1b	.979	1.435	108	-.2198 (.1486)	.7750 (.0109)	385.7*
7	A1c	.969	1.799	66	.0235 (.2383)	.7836 (.0177)	154.5*
4	A2	.354	25.257	107	.4951 (.7440)	.8677 (.1145)	0.7
5	A2	.354	25.257	107	.4951 (.7440)	.8677 (.1145)	0.7
6	A2	.420	33.474	82	-1.0442 (1.0385)	1.0826 (.1422)	0.6
7	A2	.736	8.462	55	-.4153 (.6734)	.9644 (.0793)	1.5

* Intercept-slope vector significantly different than (0,1), $p < .05$.

Table III.3. Results of simple linear regressions of mean recession estimates on two types of actual mean rates.

Estimator	Actual rate	RSQ	RMS	N	Coefficients		F
					a (s.e.)	b (s.e.)	
4	A1a	.897	2.9089	148	-.2235 (.2212)	.6349 (.0178)	567.0*
5	A1d	.896	2.675	151	.1632 (.2068)	.6489 (.0181)	427.3*
6	A1b	.953	2.045	100	-.4346 (.2431)	.7879 (.0176)	264.0*
7	A1c	.977	1.227	66	-.0529 (.1554)	.8031 (.0089)	214.0*
4	A2	.486	16.513	91	.2835 (.7253)	.8982 (.0980)	0.8
5	A2	.537	14.011	93	.1291 (.6534)	.9169 (.0892)	0.9
6	A2	.283	36.911	75	2.2211 (1.3206)	.8686 (.1618)	2.1
7	A2	.279	40.700	55	2.3580 (1.6103)	.8126 (.1796)	1.1

* Intercept-slope vector significantly different than (0,1), $p < .05$.

Table III.4. Results of simple linear regressions of weighted mean recession estimates on two types of actual weighted mean rates (mean rate= $(.797)(\text{CB rate})+(1-.797)(\text{LCLW rate})$).

Estimator	Actual rate	RSQ	RMS	N	Coefficients		F
					a (s.e.)	b (s.e.)	
4	A1a	.888	3.702	148	-.2711 (.2419)	.6267 (.0185)	519.9*
5	A1d	.879	2.871	151	.4306 (.2161)	.6371 (.0194)	364.6*
6	A1b	.936	4.204	100	-.8004 (.3379)	.8130 (.0216)	171.8*
7	A1c	.980	2.005	66	-.0723 (.2453)	.8105 (.0146)	171.8*
4	A2	.423	22.593	91	.3183 (.8640)	.8957 (.1110)	0.7
5	A2	.560	13.490	93	.1847 (.6493)	.9067 (.0843)	1.2
6	A2	.157	70.141	75	3.8001 (1.9153)	.8162 (.2213)	3.5*
7	A2	.171	95.812	55	2.8015 (2.5102)	.8857 (.2675)	1.1

* Intercept-slope vector significantly different than (0,1), $p < .05$.

Table III.5. Means, minima, and maxima for number of internodes between present and previous CB and LCLW, and years since loss of CB or LCLW status for the four whorls below CB and LCLW.

Variable	Mean	Minimum	Maximum
Number of internodes between present CB and previous CB	3.0	0.0	9.0
Number of internodes between present LCLW and previous LCLW	2.0	0.0	7.0
Years since whorl lost CB status			
CB - 1	2.2	0.5	13.5
CB - 2	3.4	0.5	17.5
CB - 3	3.9	0.5	18.5
CB - 4	5.5	0.5	18.5
LC - 2	6.5	0.5	13.5
Years since whorl lost LCLW status			
LC - 1	2.1	0.5	9.5
LC - 2	3.8	0.5	11.5
LC - 3	5.7	0.5	14.5
LC - 4	6.5	0.5	13.5

Table III.5. Continued.

Variable	Mean	Minimum	Maximum
Whorl longevity as CB	2.3	1	8
Whorl longevity as LCLW	2.8	1	15

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Chapter IV

Models for Predicting Five-year Change
in Height to Crown Base
in Southwestern Oregon Douglas-fir

Abstract

Numerous nonlinear and logarithmic models were developed for predicting five-year crown recession rates in Douglas-fir. The data base was constructed through application of a branch and stem dissection technique previously explored by Maguire and Hann (1986a, b). Residual analysis and indices of fit demonstrated that a multiplicative model with lognormal errors was the most appropriate model form. A full log model was presented which predicted five-year change in height to crown base from diameter at breast height (dbh), total height, crown ratio, breast height age, basal area of trees with a larger dbh, crown competition factor, height growth, and elevation. A reduced log model is also discussed which predicts crown recession from only crown ratio, total height, breast height age, height growth, and crown competition factor.

Introduction

Crown dimensions reflect the total leaf area, and hence relative photosynthetic capacity, of individual trees within a forest stand. In addition, crown size records the relative degree of past competition experienced by the tree. Not surprisingly, therefore, crown size is an effective predictor of individual tree growth within a stand (Hamilton 1969, van Laar 1969, Weaver and Pool 1979), and has been applied widely in individual tree growth models (Hatch 1971, Arney 1972, Botkin et al. 1972a, b, Daniels and Burkhart 1975, Mitchell 1969, 1975, Krumland and Wensel 1981, Belcher et al. 1982, Wycoff et al. 1982, Wensel and Koehler 1985). In addition, crown size exerts considerable control over stem form (Larson 1963) and hence may improve volume estimates within individual tree growth and yield models (Burkhart and Walton 1985, Farrar 1985, Walters et al. 1985). However, since growth models commonly project tree and stand growth over more than one growth period, a submodel of crown development is required to update crown size commensurate with changes in other tree and stand variables. In this way current crown size is continuously available for predictions in subsequent

growth periods.

Data from which crown development models can be constructed are relatively scarce and, if they do represent a time series, usually cover only a very short period of the stand's total life or rotation. In contrast to the common practice of periodically remeasuring diameters and heights on permanent plots, long-term changes in crown size have rarely been monitored. Although the general relationship between stem growth layer patterns and the size and position of crown has been documented (Duff and Nolan 1953, Larson 1963, Fayle 1985), crown reconstruction techniques analogous to stem analysis for height and diameter have received little attention. One exception has been the branch/stem dissection technique described by Maguire and Hann (1986b). This technique is an expansion of previously published sectioning methods for studying knot formation and dating branch mortality (Koehler 1936, Andrews and Gill 1939, Rapraeger 1939).

In the past, crown development has been modeled most commonly from strictly cross-sectional data; that is, the data were collected on temporary plots distributed across a range of stand ages. The crown dimensions of interest were regressed on stand, site, and tree variables which were all predicted in

other components of the models. The resulting static crown model then predicted crown size at the end of each growth period from the updated predictor variables. Change in crown size was simply the difference between the crown size at the end and beginning of a given growth period. Alternatively, this prediction equation can be differentiated to yield change as a direct function of change in the predictor variables. The temporary plot/static model approach has been applied to change in crown ratio (Belcher et al. 1982, Wycoff et al. 1982), change in crown length (Stage 1974), and change in height to crown base (Daniels and Burkhardt 1975). The simplest version of this approach predicted total leaf weight and leaf area for a given species from squared diameter (Botkin et al. 1972a,b).

The only known repeat measurement data sets from which crown models have been developed included a relatively short range in stand ages (Hatch 1971, Krumland and Wensel 1981). Hatch (1971) fit available data to a probability distribution from which the proportion of the maximum possible crown length (defined as the sum of initial crown length and height growth) actually realized was assigned to trees

stochastically. Although their data were admittedly very coarse in resolution, Krumland and Wensel (1981) and Krumland (1982) constructed a nonlinear regression model to predict directly the five-year change in height to crown base from present crown length, five-year height growth and canopy closure at present crown base level. Each of these authors have emphasized the need for longer term crown development studies to provide data bases for modeling (Hatch 1971, Krumland and Wensel 1981, Krumland 1982).

Data collected on a finer resolution than gross crown dimensions have also provided a basis for modeling crown development, but only in distance-dependent models. Arney (1972) measured "crown competition quotient" at various levels within stands to determine the level of competition above which individual whorls could no longer survive. Mortality of each whorl was thereby predicted as trees grew in height, as branch whorls expanded in width, and as crown competition intensified at the subject whorl. At a similar resolution, Mitchell (1969, 1975) modeled crown expansion and recession from branch length and height growth data, although crown recession rate was based strictly on assumptions concerning competition effects on branch longevity.

As growth and yield modeling efforts move into new forest types or geographic locations, crown data which cover the target population will usually be collected from temporary growth plots. Such temporary plot data have restricted crown modeling either to static approaches in distance-independent models or to more elaborate branch/whorl growth and survival approaches in distance-dependent models. For distance independent models, the latter approach usually is either more elaborate than necessary or requires other stand features tracked only in distance dependent models. On the other hand, static models become logically inconsistent under intensive management regimes such as those including repeated thinnings. Under these conditions, models which predict crown change directly are clearly preferable. The objectives of the present study, therefore, were: 1) to apply the technique and sampling strategy explored by Maguire and Hann (1986a,b) to generate a crown change data base for a growth and yield study in southwestern Oregon Douglas-fir; and 2) to construct from these data a crown development submodel which predicts directly the five year change in height to crown base from other tree, stand and site variables.

Methods

Study Site and Data Collection

Trees were sampled from cluster plots established by the Southwest Oregon Forestry Intensified Research (FIR) Growth and Yield Project during the summer of 1983. Most plots consisted of Douglas-fir (Pseudotsuga menziesii (Mirbel) Franco) with varying mixtures of ponderosa pine (Pinus ponderosa Dougl.), grand fir (Abies grandis (Dougl.) Forbes), white fir (Abies concolor (Gord. & Glend.) Lindl.), sugar pine (Pinus lambertiana Dougl.), and incense cedar (Calocedrus decurrens (Torr.) Florin.). The study area covered an elevational range from 275 to 1550 m (900 to 5100 ft), and extended from near the California border ($42^{\circ} 10'$ N) north to Cow Creek ($43^{\circ} 00'$ N), and from the Cascade crest ($122^{\circ} 15'$ W) to approximately 15 miles west of Glendale ($123^{\circ} 50'$ W). This region is characterized by January mean minimum temperatures of -5° to 0° C, and July mean maximum temperatures of 26° to 32° C. Annual precipitation varies from 76 to 210 cm, with less than 10 percent of the total falling during June, July, and August.

Each cluster plot consisted of four to ten sampling points located at the apices of equilateral

triangles with 150-ft (45.72 m) sides. At each point, all trees greater than 8.0 in (20.3 cm) were measured on a variable radius plot of BAF 20. In addition, all trees greater than 4.0 in (10.2 cm) but less than or equal to 8.0 in were measured on a nested fixed area plot with a radius of 15.56 ft (4.74 m). Finally, all trees less than or equal to 4.0 in were measured on a nested fixed area plot with a radius of 7.78 ft (2.37 m). Diameter at breast height (dbh, nearest 0.1 in) was recorded for all trees, and both total height and height to crown base were measured directly with a height pole (nearest 0.1 ft) if either was 25 ft (7.62 m) or less. On trees with total height or crown base height greater than 25 ft, these heights were determined by the pole-tangent method (Curtis and Bruce 1962).

A subsample of trees on each plot was felled for height growth measurement, sectioning, and crown sampling. Past five-year height growth, ending with the previous growing season, was measured directly on the felled tree, and breast height age was determined from a cross-sectional cut through the bole. The following points were then identified on the felled tree to facilitate implementation of the crown sampling

scheme (Fig. 1):

Crown Base (CB) - lowest whorl in the crown which had live branches at least three quarters of the way around the circumference of the stem, and above which all whorls had the same;

Lowest Contiguous Live Whorl (LCLW) - lowest live whorl above which all whorls had at least one live branch;

Intermediate Live Whorl (ILW) - second whorl down from crown base (CB); and

Second All Dead Whorl (SADW) - second all dead whorl below the lowest contiguous live whorl (LCLW).

Based on expected mean stand conditions and average tree growth, and later as reinforced by the analysis of Maguire and Hann (1986a), a sampling scheme was chosen which called for removal of the SADW and conditional removal of the ILW. Therefore, on 357 felled Douglas-fir, total height and the heights to CB, LCLW, and SADW were recorded. The SADW was then cut out of the bole, leaving at least 2 in (5 cm) of any protruding branches. If greater than two whorls occurred between

CB and LCLW, height to ILW was similarly recorded and this whorl was removed from the bole.

After removing the sample whorls and transporting to the lab, individual branch stubs and knots were split out of the bole section. The technique described by Maguire and Hann (1986b) was then applied to date the mortality of each dead branch. Briefly, longitudinal cuts were made through each branch, cross-sectionally through the encasing bolewood. This allowed identification of the first year of discontinuity between growth rings in the branch and those in the bole. Once this estimated year of mortality was established, the number of growth rings which accrued subsequent to mortality was recorded. Since a given branch could have died any time during the estimated year of mortality, half a year was added to each record.

Total basal area per acre and crown competition factor (Krajicek et al. 1967) for all conifers and hardwoods taller than 4.5 ft (1.3 m) were computed for each sample point. Crown competition factor (CCF) was computed from maximum crown width equations presented by Paine and Hann (1982). In addition, point basal area and CCF in all trees with a larger dbh than each of the 357 sample trees were computed to serve as

indices of relative stand position. Finally, estimated canopy closures at increments of 10 percent of the height of the tallest tree were computed for each point with equations presented by Ritchie and Hann (1985). Extensive data on soils, climate, and other site factors were collected and initially screened in the modeling process described below. These included, but were not limited to, slope, aspect, elevation, soil water holding capacity, and Scrivani's (1986) site index. Means, minima, and maxima for the major variables are presented in Table 1.

Estimation of Crown Recession

Maguire and Hann (1986a) present an analysis of the relative performance of several possible estimators of past five-year crown recession for the described sampling scheme. Based on this analysis, the following estimator was chosen for five-year change in height to crown base:

$$\Delta HCB = \frac{\Delta CB + \Delta LCLW}{2}$$

where ΔHCB = estimated five-year change in
height to point midway between
H(CB) and H(LCLW)

if ILW was sampled

$$\begin{aligned} \Delta CB &= \text{estimated 5-yr CB recession} \\ &= 5 \frac{[H(CB) - H(ILW)]}{YCB(ILW)} \end{aligned}$$

and if ILW was not sampled

$$\Delta CB = 5 \frac{[H(CB) - H(SADW)]}{YCB(SADW)} \frac{W_{cb}}{W_{cb} + 1}$$

$$\begin{aligned} \Delta LCLW &= \text{estimated 5-yr LCLW recession} \\ &= 5 \frac{[H(LCLW) - H(SADW)]}{YLC(SADW)} \frac{W_{lc}}{W_{lc} + 1} \end{aligned}$$

$H(x)$ = height to whorl x

W_{cb} = number of whorls between SADW and
CB

W_{lc} = number of whorls between SADW and
LCLW

$YCB(ILW)$ = years since ILW ceased to be a
potential CB

$YCB(SADW)$ = years since SADW ceased to be a
potential CB

$YLC(SADW)$ = years since SADW died (ceased to
be a potential LCLW)

A given whorl is assumed to lose status as a potential

CB when one branch died out of a total of four or less, when two branches died out of a total of five to seven, or when three branches died out of a total of eight or more (see Maguire and Hann 1986b). By definition, SADW dies when its last live branch succumbs to mortality.

Models

Three models for predicting crown change directly were extracted from the literature (Hatch 1971, Krumland and Wensel 1981, Krumland 1982). Two of these models represent different versions estimated from the same repeat crown measurement data (Krumland and Wensel 1981, Krumland 1982).

Hatch (1971) presented a model which was apparently fit to repeat crown length measurements and is similar in concept to the models of Krumland and Wensel (1981) and Krumland (1982). Although Hatch (1971) introduced stochastic features into his predictions of crown length change, his basic model can be rearranged and modified to produce a deterministic nonlinear model consistent with his original logic. He first summed crown length and height growth, then defined crown length ratio as the proportion of this sum which the subject tree realized:

$$\text{CLR} = (\text{CL} + \Delta\text{H} - \Delta\text{HCB}) / (\text{CL} + \Delta\text{H})$$

where CLR = crown length ratio

CL = crown length at beginning of
growth period

ΔH = height growth

ΔHCB = periodic change in height to
crown base

This ratio is restricted to the interval (0,1) by modeling with a beta probability distribution (Johnson and Kotz 1970):

$$f(x) = \frac{x^{a-1}(1-x)^{b-1}}{B(a,b)}$$

where $B(a,b)$ = beta function

The b parameter is constant across stand conditions, but the a parameter is modeled as a nonlinear function of competition index (Hatch et al. 1975):

$$a = k_0(1 - k_1 \exp(-k_2 \text{CI}^2))$$

where CI = competition index

This index of competition is more refined than, but analogous to, crown ratio. Therefore, for a tree of

given competition index or crown ratio, the mean of the beta distributed CLR is

$$u = a / (a + b)$$

or

$$u = \frac{k_0(1 - k_1 \exp(-k_2 CR^2))}{k_0(1 - k_1 \exp(-k_2 CR^2)) + k_3}$$

where CR = ratio of crown length to total tree height

But since

$$CL = CLR(CL + \Delta H) - \Delta CL$$

where ΔCL = periodic change in crown length

CLR, CL, ΔH as above

after some algebraic manipulation, the following nonlinear model is implied:

$$\frac{\Delta CL + CL}{CL + \Delta H} = \frac{1}{1 + k_0[k_1 - k_2 \exp(-k_3 CR^2)]^{-1}}$$

That is, change in crown length can be expressed as a nonlinear function of initial crown length, height

growth, and crown ratio.

Krumland and Wensel (1981) took a similar approach to modeling five year change in height to crown base. Recession was modeled as a nonlinear function of present crown length, five-year height growth and estimated canopy closure percent at present crown base (CC), the latter being a measure of competition experienced by the subject tree:

$$\Delta HCB = \frac{m_1 CL + m_2 H}{1 + \exp(m_3 + m_4 CC)}$$

This model can be interpreted as potential crown recession (numerator) modified by an expression of surrounding crown competition (denominator). The model was later revised to evoke more reasonable behavior in regard to long term predictions and the direction of predictor variable effects (Krumland 1982):

$$\Delta HCB = \frac{n_1 [1 - \exp(-n_2 CL)] + n_3 p\Delta H^4}{1 + \exp[n_5 + n_6 CC]}$$

where $p\Delta H$ = predicted five-year height
growth

These three models were fit by nonlinear least squares.

Crown closure at subject tree crown base was determined by linear interpolation between canopy closures at the height intervals described above.

These models were elaborated on by expanding the array of potential predictor variables in the modifier portion of the model, while maintaining the restriction of crown change to a proportion of initial crown length plus height growth. The basic model, therefore, was

$$\Delta\text{HCB} = \frac{\text{CL} + \Delta\text{H}}{1 + \exp(\underline{\text{X}}\underline{\text{B}})}$$

where $\underline{\text{X}}$ = row vector of predictor variables

$\underline{\text{B}}$ = column vector of regression coefficients

The most promising predictor variables were first identified by transforming the above equation into a log model and applying ordinary least squares. Various combinations and transformations of the following best variables were then explored by nonlinear least squares:

CCF = crown competition factor

CR = live crown ratio

BA = basal area

H = total height

D = diameter at breast height

AGE = breast height age

Next, the estimated crown recession rates were transformed by natural logarithm and regressed in an all subsets routine on variables representing both the original and log transformed scales. The tree and stand variables screened included dbh, height, crown ratio, breast height age, height growth, basal area, CCF, basal area in trees with larger diameter than subject tree, and CCF in trees with larger diameter than the subject tree. In addition, numerous site variables were available, including, but not limited to, elevation, Stage's (1976) slope/aspect transformations, Scrivani's (1986) site index and numerous soil and climatic variables. This procedure identified the model with lowest Mallows' C_p (Draper and Smith 1981) of those models with parameter estimates all significantly different from zero. In addition, a reduced model was selected which had the lowest C_p and was expected to exhibit a lower degree of multicollinearity by prohibiting inclusion of both height and diameter. Finally, the largest log model

was also selected which had the lowest C_p of those models which yielded parameter estimate convergence in the nonlinear least squares procedure.

Various nonlinear models representing variations on , or reductions of, the latter log model were explored. The best nonlinear models were then further refined with techniques described by Jensen and Homeyer (1970).

Residual plots for the various nonlinear models indicated both positive skewness and constant coefficient of variation, rather than the constant variance assumptions of unweighted least squares. These attributes suggest that logarithmic transformation of a multiplicative regression model with lognormal errors would be most appropriate. In addition, two other approaches were explored for ameliorating skewness and/or heteroscedasticity: 1) variously weighted nonlinear least squares models; and 2) a generalized linear model assuming gamma-distributed observations (McCullagh and Nelder 1983).

The model building procedure thus led to seven basic models: 1) modification of maximum potential recession ($CL + \Delta H$); 2) a largest log model with lowest C_p for which all variables were significant; 3) a largest log model with lowest C_p , but having no height-

diameter combinations; 4) a largest log model with lowest C_p for which all parameters could be estimated by nonlinear least squares; 5) the nonlinear model corresponding to (4); 6) a bell-shaped nonlinear model developed through methods of Jensen and Homeyer (1970); and 7) a generalized linear model assuming gamma-distributed observations, also corresponding to models (4) and (5).

Payandeh (1981) discusses application of the following "generalized coefficient of determination" for comparing nonlinear and logarithmic models on the original scale:

$$S = 1 - \frac{\sum(y - \hat{y})^2}{\sum(y - \bar{y})^2}$$

This measure was computed for each of the above models on the original scale to indicate the relative variance about each regression surface. Three sets of predictions were analyzed for each log model: 1) predictions without correction for log bias; 2) predictions corrected for log bias through the "naive" estimator discussed by Flewelling and Pienaar (1981); and 3) predictions corrected for log bias with the minimum MSE estimators suggested by Teekens and Koerts

(1972) and Evans and Shaban (1976) (see Flewelling and Pienaar 1981).

Results

The best versions of each basic model were chosen on the basis of mean squared error (MSE) and coefficient of multiple determination (RSQ). A relatively low proportion of the variation in recession rate is explained by the models due to the error associated with measuring the response variable (Maguire and Hann 1986a). Parameter estimates, MSE, RSQ, Furnival's (1961) index, and residual skewness and kurtosis coefficients are shown for the best versions of each nonlinear model in Table 2, and for the best logarithmic models in Table 3. Table 4 presents parameter estimates, reduction in deviance, and residual skewness and kurtosis coefficients for the generalized linear model with a log link function (McCullagh and Nelder 1983).

The original forms of the Krumland and Wensel (1981), Krumland (1982), and modified Hatch (1971) models fit the data quite poorly. The generalization of these models, however, exhibited substantial improvement. The best model of this form was as follows:

$$[N1] \quad \Delta HCB = \frac{CL + \Delta \bar{h}}{1 + \exp(\underline{XB})}$$

where ΔHCB = estimated 5-yr change
in height to crown base

$$\underline{\text{XB}} = a_0 + a_1 \log(\text{CR}) + a_2 \text{CR} + a_3 \text{AGE} + a_4 \log(\text{CCF})$$

CL = crown length (ft)

H = height growth (ft)

CR = crown ratio (CL/H),

expressed as a percent

AGE = breast height age of tree in
years

CCF = crown competition factor

$\log()$ = natural logarithm

Providing for coefficients and powers on CL and ΔH separately or together did not improve the fit. Likewise, weighting by the sum of ΔH and CL raised to different powers always produced a poorer Furnival's index.

Despite considerable exploration of alternative variables, no five variable nonlinear models yielded parameter estimate convergence. As indicated in Table 2, after considerable exploration of alternatives, the best four variable nonlinear model, excluding those

created by the methods of Jensen and Homeyer (1970), was:

$$[N2] \quad \Delta HCB = c_0 H^{c_1} CCF^{c_2} \exp[c_3 CR + c_4 AGE]$$

Again, no weighting schemes provided a lower index of fit. The corresponding log model was

$$[L1] \quad \log(\Delta HCB) = b_0 + b_1 \log(H) + b_2 \log(CCF) + b_3 CR + b_4 AGE$$

This model also proved the best four variable log model.

Graphs of response variable cell means, where cells were defined by different variables, revealed a strong peaking behavior of crown change over CR. This was repeatedly indicated by more complex log models as well, due to inclusion of both CR and $\log(CR)$ terms with negative and positive coefficients, respectively. Therefore, [N2] was modified into the following nonlinear model:

$$[N3] \quad \Delta HCB = d_0 H^{d_1} CCF^{d_2} \exp[d_3 (1 - CR/38)]^{d_4} + d_5 AGE$$

The initial parameter estimates for d_3 , d_4 and the denominator of the CR expression were determined as

described by Jensen and Homeyer (1970). The latter estimate defines the peak of crown base change with respect to crown ratio. The optimal peak in regard to MSE was found to occur at approximately 38 percent crown ratio by changing the estimated value in increments of one percent and iteratively refitting the model. Allowing for a two segmented bell-shaped curve did not substantially improve the fit. All weighting schemes tried were inappropriate as judged by Furnival's index.

The log model which had the lowest Mallows Cp of those models with parameter estimates significantly different from zero was

$$\begin{aligned}
 \text{[L2] } \log(\Delta\text{HCB}) = & e_0 + e_1D + e_2H + e_3CR \\
 & + e_4AGE + e_5BAL + e_6\Delta H + \\
 & e_7\log(D) + e_8\log(H) + e_9\log(CR) \\
 & + e_{10}\log(BA) + e_{11}\log(EL) + e_{12}\log(\Delta H)
 \end{aligned}$$

As shown in Table 3, the reduction of this model, which was intended to ameliorate multicollinearity, contained the following six variables and resulted in a MSE midway between [L1] and [L2]:

$$\begin{aligned}
 \text{[L3] } \log(\Delta\text{HCB}) = & f_0 + f_1CR + f_2AGE + f_3\log(H) \\
 & + f_4\log(CR) + f_5\log(\text{CCF}) + f_6\log(\Delta H)
 \end{aligned}$$

The high degree of multicollinearity in the full log model, [L2], is reflected in the large variance inflation factors (VIF = diagonal element of the inverse correlation matrix; see Neter et al. 1983). The largest VIF was 54.4 for $\log(H)$, but six others were greater than 10. All seven variables corresponding to these VIF's entered the model in both original and log form: diameter, height, crown ratio, and height growth. By expressing these 8 variables as 4 linear combinations of the log and untransformed pairs (as defined by the original parameter estimates), all VIF's were reduced below 10. This suggests that a large degree of the multicollinearity occurs between the untransformed variables and their logarithms. However, the diameter and height variables still exhibited relatively large VIF's (9.4 and 9.0, respectively), since they maintained a high correlation (-.925). In contrast, after removing the effect of the crown ratio X $\log(\text{crown ratio})$ correlation, VIF's in the reduced log model, [L3], exhibited a maximum of only 3.9.

As shown in Table 3, skewness and kurtosis coefficients indicated close conformity of residuals from each log model to the normal distribution (Bowman

and Shenton 1975). In contrast, the residuals from all nonlinear models had a positive skewness and extreme kurtosis, indicating very significant departure from normality (Table 2). The substantially lower indices of fit for the log models corroborate their apparent superiority, although these indices also take into account correctness of model form and constancy of variance. The residual plots for nonlinear models, together with the fact that numerous attempts at weighted nonlinear least squares failed to improve the fits, suggest that lognormality of residuals was the predominant problem rather than strictly nonconstant variance. Although nonlinear models [N1] and [N2] yielded the highest S (generalized coefficient of determination), that of log model [L2] with the MSE log bias adjustment was also relatively large (Table 5). For each log model, the minimum MSE adjustment gave the lowest residual variance about the regression predictions, as measured by S.

As revealed by inspection of the full log model parameter estimates, the response "surface" described by this model peaks with respect to three predictor variables: total height, crown ratio, and height growth. These peaks occur at 113.7 ft, 28.0 percent, and 7.8 ft for height, crown ratio, and height growth,

respectively. In contrast, dbh exhibits a minimum at 19.6 in. As seen by comparison to the data base ranges (Table 1), these response surface maxima and minimum occur slightly above the corresponding means.

The gamma model was fit assuming both a reciprocal link function and log link function (McCullagh and Nelder 1983); however, a lower deviance was obtained from the log link. As stated by McCullagh and Nelder (1983), a close connection exists between log-transformed ordinary least squares and the generalized linear model with gamma distributed observations and log link (gamma-theory multiplicative model). Although more complex models were found with all parameter estimates significantly different from zero, only the multiplicative gamma model corresponding to the log model [L1] is presented for comparison (Table 4). Parameter estimates from [N2], [L1], and [G1] exhibit a reasonably close correspondence.

Discussion

Despite significant departure from normality of residuals in the nonlinear models, all three nonlinear models and the three log models theoretically yield unbiased and consistent parameter estimates. However, the full and reduced log models also produce efficient estimators due to their constant variance, as well as uniformly minimum variance unbiased parameter estimates due to the close conformity of their residuals to the normal distribution. Thus the tests on parameter estimates are better and assessment of variable significance is more powerful in the log models. Size and normality of residuals, constancy of variance, and appropriateness of model form all influence the index of fit by which the log and nonlinear models are compared (Furnival 1961). One or a combination of these characteristics were therefore more favorably met by the log models, and most favorably by the full log model.

Given that the crown recession data conform most favorably to a multiplicative model with lognormal errors, trade-offs among the log models still exist in regard to interpretability of the coefficients, degree of fit, and the dangers of extrapolation. Clearly, the

full log model, [L2], explains the largest amount of variation in five-year crown recession; however, the relatively high degree of multicollinearity suggests that: 1) predictions of crown change may extrapolate poorly to sets of predictor variables outside the original data range; and 2) the actual effect of a given variable is not easily interpretable from its regression coefficient (Neter et al. 1983). In absence of a validation data set, these concerns are of particular importance. Conversely, to the extent that the intercorrelations among variables such as height and diameter will be inherent in any population to which the crown model is applied, predictions from the full model will be more refined and reflective of the complex interactions operating in the natural system.

Flewelling and Pienaar (1981) discuss numerous adjustment factors for correcting log bias in logarithmic models. The factor most commonly encountered in past research, $\exp(s^2/2)$, yields their "naive" estimator, which has been demonstrated to give biased estimates. Teekens and Koerts (1972) and Evans and Shaban (1976) provide correction factors which still produce biased estimates, but provide a lower mean squared error than the minimum variance unbiased estimator presented by Bradu and Mundlak (1970). Since

the residual mean square is relatively large for the crown change model ($>.25$), the MSE adjustor is judged most appropriate in predicting five-year change in height to crown base. In addition, further advantage of this adjustment factor seems to be suggested by the higher S given by this factor over the unadjusted and naive estimators (Table 5).

Applied to the original data, the full and reduced log models ([L2] and [L3]) with the minimum MSE correction factors predict five year crown changes of up to 11.4 and 12.4 ft, respectively (Table 5). The maximum height growth of 16.9 ft for the same data base at least suggests that the crown recession predictions will fall within reasonable limits.

Similarly, the regression coefficients for [L3] describe a behavior which supports past observations and theory on forest growth and development. Height to crown base has been shown to follow a sigmoid pattern of development which is roughly proportional to the cumulative height growth curve (Kramer 1962). Although total height has a positive effect on crown change in model [L3], breast height age has the opposite effect. Therefore, the net effect of these variables results in a peak in crown recession through

stand development. For example, assuming a site index of 100 ft (base age 50), Hann and Scirvani's (1986) height growth model can be solved to express dominant height as a function of age. Substituting this expression into the crown model produces a peak in crown recession at approximately 6 years breast height age, all other variables held constant. The actual location of the peak, however, will obviously depend on other tree, stand, and site variables.

The intensity of competition for aerial growing space, and hence the rate of crown recession, would be expected to correspond roughly to rate of height growth. The increase in predicted crown change with height growth is therefore reasonable; however, the approximate correspondence between height growth and crown recession maxima would be tempered by other variables such as stand density and relative vertical position of the crown, as well as the accumulation and vertical distribution of foliage and branch biomass within the crown itself (Kinerson et al. 1974, Schreuder and Swank 1974). Whereas the latter effect can only be implied through other variables in model [L3], stand density is directly represented by point CCF. As is consistent with spacing trials (Stiell 1966), thinning experiments (Barclay et al. 1982), and

static crown base models (Ritchie and Hann 1986), increasing stand density accelerates crown recession.

As demonstrated by Stiel (1978), heterogeneity in stand density can also produce large differences in crown recession rates and resulting crown lengths. Crown ratio can therefore be serving in this capacity as an index of both local stand density and relative crown position. The fact that a maximum in the predicted crown recession surface occurs at approximately 29 percent again supports observations on stand structure and development. At a very high local density and inferior crown position, crown ratio will be quite low, indicating a relatively suppressed position. Trees under such conditions grow very slowly in height, are well below the general stand canopy level, and, as predicted by the model, exhibit relatively gradual crown recession. However, for trees in intermediate to codominant positions, crown ratios are typically .25 to .40. These trees are competing most intensively to maintain their position in the upper stand canopy; hence their crowns would recede at rates representing relatively high proportions of height growth. At the upper extreme, trees of larger crown ratios, but equal height, exhibit

relatively low rates of recession, since they are expanding or maintaining their occupation of aerial growing space, often at the expense of smaller trees.

Finally, the interaction of breast height age, total height, and height growth probably reflects site quality to some degree as well. Both site index and habitat type have been found to significantly influence crown length (Wycoff et al. 1982, Ritchie and Hann 1986).

In summary, this sampling strategy and modeling approach offer a first approximation to an alternative to the temporary plot/static model procedure. Although static models of height to crown base have been the most common approach to incorporating crown size into growth and yield models, direct models of crown recession offer several practical and conceptual advantages. From a modeling perspective, it should be noted that while recently disturbed stands would have disastrous effects on efforts to develop static models of height to crown base, this problem is at least partially alleviated when modeling crown change directly. Nonlinear functions which consider current crown ratio relative to current stand density and other relevant variables can provide predictions ranging from the logical extreme of zero to any theoretical or

empirical maximum. Thus, as silviculture becomes more intensive and as thinning regimes become more variable, the response of crown structure to these regimes can still be modeled by replacing static crown models with direct models of crown recession. In addition, with application of the described sampling procedure and subsequent modeling approach, temporary plots can provide the data base necessary to construct a crown change data base where long term crown studies are lacking. The balance between whorl sampling intensity and tree sampling intensity in this approach can be tailored to the particular conditions of the target population, thereby achieving any desired accuracy in recession estimates.

Fig. IV.1. Schematic diagram of sample tree illustrating locations of crown base (CB), intermediate live whorl (ILW), lowest contiguous live whorl (LCLW), and second all dead whorl (SADW).

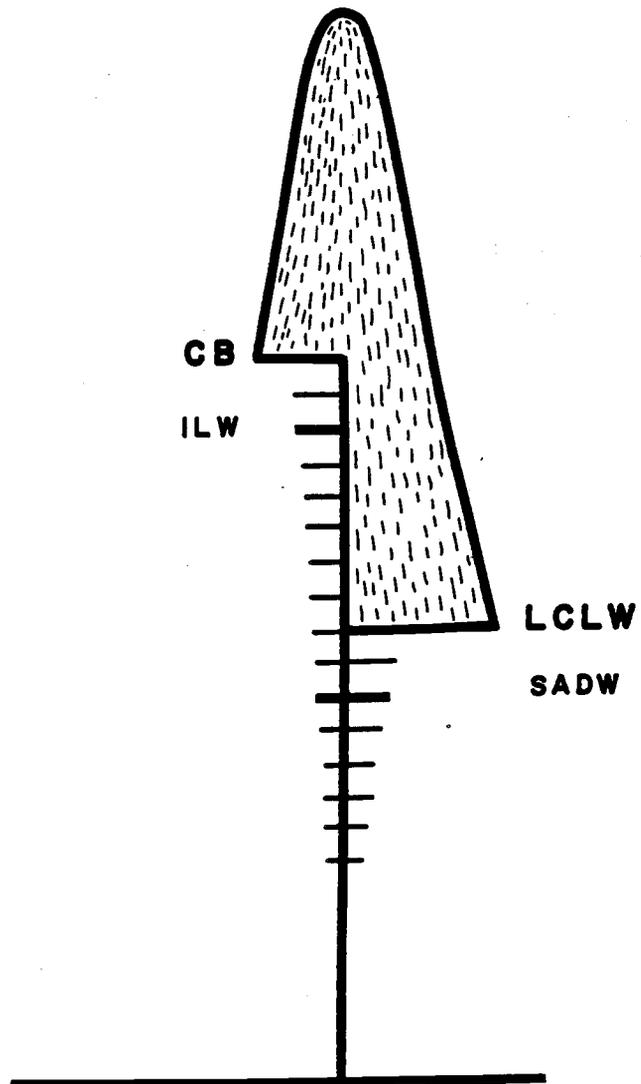


Fig. IV.1.

Table IV.1. Mean, minima, and maxima for variables in the model construction data base.

Variable	Minimum	Mean	Maximum
D	0.9	12.5	44.2
H	10.1	77.0	200.9
CR	8.0	53.5	98.0
AGE	12.0	51.1	150.0
CL	4.7	39.2	112.3
BAL	0.0	86.6	288.0
BA	4.6	181.7	359.2
CCFL	0.0	100.5	765.8
CCF	30.0	277.1	921.4
EL	960.	2,702.	4,600.
HG	1.0	7.2	16.9
SI	49.2	94.0	139.5
CC	11.5	77.7	342.8
SLOPE	5.0	41.3	74.0
Estimated crown change	0.2	3.8	20.0

Table IV.2. Parameter estimates (with approximate s.e.), mean squared error (MSE), coefficient of multiple determination (RSQ and adjusted RSQ), Furnival's index (F.I.), and residual skewness and kurtosis coefficients for the three nonlinear models.

Model	Parameter estimates (s.e.)	MSE	RSQ (adj.RSQ)	F.I.	Skew.	Kurt.
[N1]	$a_0 = 4.81413$ (2.18487)	8.5837	.2922 (.2842)	2.9298	1.731	8.285
	$a_1 = -1.59586$ (.751242)					
	$a_2 = 0.0783432$ (.0159416)					
	$a_3 = 0.0310715$ (.0029753)					
	$a_4 = -0.344278$ (.108236)					
[N2]	$c_0 = 0.525778$ (.0439229)	9.2246	.2394 (.2307)	3.0372	1.699	7.359
	$c_1 = 0.987868$ (.137429)					

Table IV.2. Continued.

Model	Parameter estimates (s.e.)	MSE	RSQ (adj.RSQ)	F.I.	Skew.	Kurt.
	$c_2 = 0.409694$ (.099319)					
	$c_3 = -0.0163938$ (.0028625)					
	$c_4 = -0.0274767$ (.0036156)					
[N3]	$d_0 = 0.107657$ (.084262)	8.53566	.2982 (.2902)	2.9216	1.716	8.187
	$d_1 = 0.885770$ (.122156)					
	$d_2 = 0.277555$ (.097098)					
	$d_3 = -1.00388$ (.144040)					
	$d_4 = -0.0290691$ (.0032980)					
	$d_5 = 2.03385$ (.474265)					

Table IV.3. Parameter estimates (with s.e.), mean squared error (MSE), coefficient of multiple determination (RSQ and adjusted RSQ), Furnival's index (F.I.), and residual skewness and kurtosis coefficients for the two log models.

Model	Parameter estimates (s.e.)	MSE	RSQ (adj.RSQ)	F.I.	Skew.	Kurt.
[L1]	$b_0 = -4.29528$ (.732339)	.60228	.3552 (.3479)	1.5049	0.158	3.126
	$b_1 = 1.04585$ (.108967)					
	$b_2 = 0.0783432$ (.092538)					
	$b_3 = -0.0160529$ (.0028954)					
	$b_4 = -0.0278226$ (.0026637)					
[L2]	$e_0 = -13.3904$ (2.066299)	.52276	.4531 (.4340)	1.3062	0.013	3.347
	$e_1 = 0.680396$ (.0260572)					
	$e_2 = -0.0255088$ (.0069922)					

Table IV.3. Continued.

Model	Parameter estimates (s.e.)	MSE	RSQ (adj.RSQ)	F.I.	Skew.	Kurt.
	$\theta_3 = -0.0430625$ (.0096561)					
	$\theta_4 = -0.0231087$ (.0033644)					
	$\theta_5 = -0.00253595$ (.0008935)					
	$\theta_6 = -0.0976659$ (.0408282)					
	$\theta_7 = -1.33351$ (.33464)					
	$\theta_8 = 2.90079$ (.53344)					
	$\theta_9 = 1.20378$ (.41009)					
	$\theta_{10} = 0.444607$ (.104488)					
	$\theta_{11} = 0.294167$ (.137809)					
	$\theta_{12} = 0.757876$ (.236329)					

Table IV.3. Continued.

Model	Parameter estimates (s.e.)	MSE	RSQ (adj.RSQ)	F.I.	Skew.	Kurt.
[L3]	$f_0 = -7.57512$ (1.10683)	.559784	.4041	1.8694	0.165	3.105
	$f_1 = -.0564085$ (.008407)					
	$f_2 = -.0240214$ (.003007)					
	$f_3 = .781921$ (.133313)					
	$f_4 = 1.64102$ (.365558)					
	$f_5 = .448734$ (.092020)					
	$f_6 = .304741$ (.126557)					

Table IV.4. Parameter estimates (with approximate s.e.), reduction in deviance, and residual skewness and kurtosis coefficients for the gamma model with log link.

Model	Parameter estimates (appr s.e.)	Reduction in deviance	Skew.	Kurt.
[G1]	$f_0 = -3.261$ (.7290)	290.5 - 210.1 = 80.4 (4 df)	1.664	7.365
	$f_1 = 0.9389$ (.1085)			
	$f_2 = 0.4755$ (.09212)			
	$f_3 = -0.01749$ (.002882)			
	$f_4 = -0.02298$ (.002652)			

Table IV.5. Mean, maxima, and minima for "actual" and predicted five year change in height to crown base. Also given in the last column is the generalized coefficient of determination (S). Predictions are for original modeling data base.

Model	Minimum	Mean	Maximum	S
[N1]	.31	3.68	9.99	.2922
[N2]	.32	3.15	8.51	.1665
[N3]	.38	3.70	9.55	.2982
[L1]	.32	2.88	7.67	.1695
naive	.44	3.90	10.36	.2291
MSE	.42	3.85	10.21	.2326
[L2]	.33	3.00	9.74	.2477
naive	.43	3.89	12.65	.2866
MSE	.38	3.80	12.38	.2900
[L3]	.36	2.95	8.73	.2248
naive	.48	3.90	11.55	.2725
MSE	.45	3.85	11.36	.2743
[G1]	.49	3.77	9.32	.2293
"Actual"	.20	3.75	20.00	

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Chapter V

Equations for Predicting Sapwood Taper and Volume in Douglas-fir

Abstract

Two basic taper models were analyzed for their ability to describe sapwood taper above breast height. Sapwood areas were estimated on stem cross-sections by measuring sapwood radii on the longest and perpendicular-to-longest axes, and by assuming conformity to an ellipse. These data were collected on 2 to 14 points along the stems of 72 Douglas-fir trees in southwestern Oregon. Across the range in dbh, total height, and height to crown base, quadratic-quadratic segmented polynomials (Max and Burkhart 1976) provided more consistent monotonic taper from breast height to tree tip than Bennett and Swindel (1972) models. A model for predicting breast height sapwood area from only dbh, total height, and height to crown base is also presented. Finally, integrals of the segmented polynomials gave reasonable estimates of above-breast-height sapwood volumes which could be applied to ecophysiological or forest product studies.

Introduction

Foresters and ecologists often require accurate estimates of leaf area or foliage biomass to study net primary production (Whittaker 1966), to inventory potentially utilizable biomass (Young et al. 1980), to assess tree growth efficiency (Waring 1983), or to predict post-harvest fuel loadings (Snell and Brown 1980). Although individual tree foliage weight has been predicted from numerous crown and tree dimensions (Storey 1955, Brown 1978), one especially promising technique derives from the sapwood area-foliage mass relationship first observed by German foresters near the turn of the century (Busgen and Munch 1929, first German edition 1897).

Busgen and Munch (1929) observed that "the formation of sapwood conforms to the quantity of foliage" in forest trees. These same authors cited several earlier European studies illustrating the correlation between sapwood area and foliage biomass. However, early regression equations, such as those developed by Kittredge (1944) for numerous American and European species, relied on dbh as the only predictor variable. Whittaker and Woodwell (1969) similarly predicted tree biomass components, including foliage,

from stem diameter only. This method was termed dimension analysis and has been used extensively by ecologists studying net primary production.

In contrast, Grier and Waring (1974) developed foliage mass prediction equations for Douglas-fir, noble fir, and ponderosa pine employing sapwood cross-sectional area as the predictor variable. This approach has since been applied to a wide variety of conifers and angiosperms (Waring et al. 1977, Snell and Brown 1978, Whitehead 1978, Rogers and Hinckley 1979, Kaufman and Troendle 1981, Waring et al. 1982, Albrektson 1984, and Whitehead et al. 1985). Waring et al. (1982) note that the efficacy of cross-sectional conducting area in predicting foliage biomass corroborates the pipe model theory of Shinozaki et al. (1964a,b). In part, the pipe model theory states that a given unit of leaf is supported by a pipe whose cross-sectional area is constant (Shinozaki et al. 1964a).

Shinozaki et al. (1964b) found that stands of differing ages and on differing site qualities exhibited different relationships between breast-height cross-sectional area and foliage weight; however, trees from all stands converged to the same regression line if cross-sectional area at crown base was substituted

for breast-height cross-sectional area. The age and site dependence of the former relationship is attributed to stem taper resulting from the accumulation of disused pipes downward from crown base as branches are shed. Although Shinozaki and coworkers consider only total cross-sectional area, Busgen and Munch (1929) observed that "in spruce and silver fir the cross-sectional area of the sapwood falls off from below upwards, at first rapidly, then more slowly up to the region of the crown, inside which it again diminishes very rapidly". Waring et al. (1982) present data which also illustrate the taper of sapwood area from breast height to crown base in seven conifer species. Vertical changes in sapwood permeability at least partially reconcile sapwood area taper with the pipe model theory (Whitehead et al. 1984).

Given the implication that, from crown base upward, sapwood area does exhibit a constant relationship with supported leaf area, the utility of sapwood area will depend on the ability to accurately predict sapwood area taper. Therefore, the first objective of the present study was to construct a regression equation capable of predicting sapwood area at any point in the stem, in particular at crown base,

for Douglas-fir in southwestern Oregon. Although the ratio of foliage biomass to crown base sapwood area may also depend on other factors such as local climate, correcting for sapwood area taper will reduce a major portion of the error associated with this method of leaf area prediction.

In addition, sapwood volume has been hypothesized to be important for calculating the maintenance respiration load of the tree bole (Jarvis and Leverenz 1981). The second objective, therefore, was to explore the integral forms of the sapwood taper models in regard to sapwood volume estimation.

Methods

Trees were sampled from plots established by the Southwest Oregon Forestry Intensified Research (FIR) Growth and Yield Project during the summer of 1983. Most plots consisted of Douglas-fir (Pseudotsuga menziesii (Mirbel) Franco) with varying mixtures of ponderosa pine (Pinus ponderosa Dougl.), grand fir (Abies grandis (Dougl.) Forbes), white fir (Abies concolor (Gord. & Glend.) Lindl.), sugar pine (Pinus lambertiana Dougl.), and incense cedar (Calocedrus decurrens (Torr.) Florin.). The study area covered an elevational range from 275 to 1550 m (900 to 5100 ft), and extended from near the California border ($42^{\circ} 10'$ N) north to Cow Creek ($43^{\circ} 00'$ N), and from the Cascade crest ($122^{\circ} 15'$ W) to approximately 15 miles west of Glendale ($123^{\circ} 50'$ W) (Fig. 1). This region is characterized by January mean minimum temperatures of -5° to 0° C, and July mean maximum temperatures of 26° to 32° C. Annual precipitation varies from 76 to 210 cm, with less than 10 percent of the total falling during June, July, and August.

A subsample of trees on each growth and yield plot were felled for height growth measurement, stem analysis, and crown sampling. The following

definitions were applied to facilitate the crown sampling scheme (Maguire and Hann 1986a):

Crown Base (CB) - lowest whorl in the crown which had live branches at least three quarters of the way around the circumference of the stem, and above which all whorls had the same;

Lowest Contiguous Live Whorl (LCLW) - lowest live whorl above which all whorls had at least one live branch;

Intermediate Live Whorl (ILW) - second whorl down from crown base (CB); and

Second All Dead Whorl (SADW) - second all dead whorl below the lowest contiguous live whorl (LCLW).

On 72 Douglas-fir from 27 plots, sapwood cross-sectional area was estimated at two points on the stem: 1) at breast height and 2) just beneath the SADW. In addition, on trees with greater than two whorls between crown base and the lowest contiguous live whorl, sapwood area was estimated just beneath the ILW. Finally, on three of the trees, sapwood area was determined every 2.56 m (8.4 ft) above breast height.

Tree dbh, total height and height to the sampled whorls were also recorded.

Sapwood areas were estimated on cross-sectional cuts made through the bole at each of the described points. The diameter inside bark and two sapwood radii (delineated by color) were recorded first along the longest axis of the cross-section, then along the axis perpendicular to the longest, all to the nearest .13 cm (.05 in). Total stem cross-sectional area and heartwood cross-sectional area were calculated at each point assuming conformity to an ellipse. Sapwood area was then taken as the difference. Breast height sapwood areas for the sample trees ranged from 29.5 sq cm to 1649.0 sq cm. The sampled trees ranged in diameter from 7.4 cm (2.9 in) to 92.0 cm (36.2 in) and in total height from 7.74 m (25.4 ft) to 52.06 m (170.8 ft). Crown length was defined as the distance between tree tip and halfway between CB and LCLW. Crown ratios therefore varied from .11 to .85.

The two basic taper models to which the data were fit express sapwood area at any point above breast height as a proportion of breast height sapwood area. Since breast height sapwood area is often not measured directly, the data were first fit to two additional models to allow breast height sapwood area

prediction from only dbh, total height, and height to crown base.

The direct relationship between sapwood area at a point within the crown and the cumulative foliage above that point has been previously documented (Waring et al. 1982). Crown surface area or volume, as approximated by various geometric solids, was assumed to adequately represent the contained foliage area. Total leaf area can therefore be approximated by the following model, since most of the solids commonly applied in crown modeling (e.g., cone, cylinder) are special cases of this general formula (Mawson et al. 1976):

$$FA = k_0 CW^{k_1} CL^{k_2}$$

where FA = predicted foliage area

CW = crown width (m)

CL = crown length (m)

Now since the ratio of foliage area to sapwood area within the crown is implied to be constant (Waring et al. 1982),

$$sa_{CB} = k_3 FA = k_4 CW^{k_1} CL^{k_2}$$

where sa_{CB} = predicted sapwood area at
crown base (sq cm)

Next, it was hypothesized that sapwood area at breast height could be expressed as:

$$PSA = sa_{CB} \exp[k_5 HCB]$$

where HCB = height from breast height (1.37 m)
to crown base (m)

PSA = predicted breast height sapwood
area (sq cm)

Finally, past research has documented the relationship between crown width and diameter at breast height for open grown trees (Krajicek et al. 1961). In addition, Maguire and Hann (1986e) found a high correlation between crown radius (or diameter) and stem diameter at crown base. Therefore, taper equations developed for the study area (Walters and Hann 1986) facilitated prediction of crown base bole diameter (inside bark) which then served as a surrogate for crown width (see Appendix A):

expected for the normal distribution (Bowman and Shenton 1975).

The first taper model was developed by Bennett and Swindel (1972) to describe dib taper of old-field slash pine. This model was modified to produce the following sapwood area taper model:

$$sa = c_1 SA X_1 + c_2 X_2 + c_3 X_3 + c_4 X_4$$

where SA = sapwood area at breast height
(sq cm)

sa = predicted sapwood area at height
h (sq cm)

$$X_1 = (H-h)/(H-1.37)$$

$$X_2 = (H-h) (h-1.37)$$

$$X_3 = H (H-h) (h-1.37)$$

$$X_4 = (H-h) (h-1.37) (H+h+1.37)$$

h = height from ground (m)

H = total height (m)

The variance in sapwood area increases with increasing sapwood area; hence, the model was transformed to predict the ratio of sapwood area at a given height to breast height sapwood area. In addition, when $h=1.37$ m (4.5 ft), predicted sapwood area should equal actual

sapwood area at breast height. The following model provided this constraint and was expected to exhibit homogeneity of variance:

$$[3a] \text{ sa/SA} - X_1 = c_2 (X_2 / \text{SA}) + c_3 (X_3 / \text{SA}) + c_4 (X_4 / \text{SA})$$

with X_1 , X_2 , X_3 , and X_4 as above

A variation on [3a] allowed actual breast high sapwood area to be replaced by breast high sapwood area predicted from [1] or [2]:

$$[3b] \text{ sa/PSA} - X_1 = d_2 (X_2 / \text{PSA}) + d_3 (X_3 / \text{PSA}) \\ + d_4 (X_4 / \text{PSA})$$

where PSA = predicted breast height
sapwood area (sq cm)

Since the relative sapwood area profile varies commensurate with relative crown size, the three parameters in [3a] were expected to vary substantially among combinations of height, diameter, and height to crown base. Therefore, an expanded model was fit, allowing the parameters to vary as functions of these three variables:

$$[4a] \quad sa/SA - X_1 = c_2^* X_2/SA + c_3^* X_3/SA + c_4^* X_4/SA$$

$$\text{where } c_2^* = e_0 + e_1 D + e_2 H + e_3 H/D + e_4 HCB + e_5 D HCB$$

$$c_3^* = f_0 + f_1 D + f_2 H + f_3 H/D + f_4 HCB + f_5 D HCB$$

$$c_4^* = g_0 + g_1 D + g_2 H + g_3 H/D + g_4 HCB + g_5 D HCB$$

and $D = \text{dbh (cm)}$

$H = \text{total height (m)}$

$HCB = \text{ht to crown base (m)}$

As in [3b], a variation on [4a] was explored replacing actual breast-height sapwood area with sapwood area predicted from [1] or [2]:

$$[4b] \quad sa/PSA - X_1 = d_2^* (X_2/PSA) + d_3^* (X_3/PSA) + d_4^* (X_4/PSA)$$

where

$$d_2^* = e_0^* + e_1^* D + e_2^* H + e_3^* H/D + e_4^* HCB + e_5^* D HCB$$

$$d_3^* = f_0^* + f_1^* D + f_2^* H + f_3^* H/D + f_4^* HCB + f_5^* D HCB$$

$$d_4^* = g_0^* + g_1^* D + g_2^* H + g_3^* H/D + g_4^* HCB + g_5^* D HCB$$

The other taper model considered was a variation on the segmented polynomial taper equation previously explored by Max and Burkhart (1976). Two segments of a quadratic-quadratic equation were joined at various points defined relative to crown base, such that the first and second derivatives were continuous at the join point. To satisfy these constraints, the model takes the following form:

$$[5a] \quad Y = 1 + Z_1 + j_1 Z_2 + j_2 Z_3$$

$$\text{where } Y = sa/SA$$

$$Z_1 = I [A (1 + B) - 1]$$

$$Z_2 = W + I [A (W + J B) - W]$$

$$Z_3 = W^2 + I [J A (2 W - J + J B) - W^2]$$

$$J = \text{join point}$$

$$= j_3 (HCB - 1.37)/(H - 1.37)$$

$$I = 1 \quad \text{if } W > J$$

$$0 \quad \text{if } W < J$$

$$A = (W - 1)/(J - 1)$$

$$B = (J - W)/(J - 1)$$

$$W = (h - 1.37)/(H - 1.37)$$

In addition, j_1 and j_2 were allowed to vary as a function of other tree dimensions:

$$j_1 = m_0 + m_1 D + m_2 H + m_3 H/D + m_4 HCB + m_5 D HCB$$

$$j_2 = n_0 + n_1 D + n_2 H + n_3 H/D + n_4 HCB + n_5 D HCB$$

Allowing for prediction of breast height sapwood area, a final variation on [5a] was:

$$[5b] \quad Y = sa/PSA = 1 + Z_1 + j_1^* Z_2 + j_2^* Z_3$$

$$j_1^* = m_0^* + m_1^* D + m_2^* H + m_3^* H/D + m_4^* HCB + m_5^* D HCB$$

$$j_2^* = n_0^* + n_1^* D + n_2^* H + n_3^* H/D + n_4^* HCB + n_5^* D HCB$$

Full and reduced versions of models [4a], [4b], [5a], and [5b] were fit in an all subsets routine. For certain combinations of total height, dbh, and crown ratio, the taper curves from models [4a], [4b], [5a], and [5b] could conceivably peak between breast height and the tree tip. This would indicate a bulge in sapwood area rather than monotonic taper to tree tip. In addition, predictions in the upper crown may become negative before climbing back to zero at tree tip. The

first derivatives of models [4a] and [4b] and the segments of models [5a] and [5b] were therefore calculated for various combinations of total tree height, dbh, and crown ratio. The full range from 1.5 to 60 m (5 to 200 ft) in height, 2.5 to 100 cm (1 to 40 in) in dbh, and 10 to 100 percent crown ratio was screened for the presence and severity of sapwood area bulges. Screenings for all models were based on predicted breast height sapwood areas. The final fits of each model were selected on the basis of having the lowest Mallows's Cp (Draper and Smith 1981) within the set of models whose variables were all statistically significant ($p < .05$), and whose profiles contained no bulges or negative sapwood areas for observable combinations of height, dbh, and crown ratio.

The Bennett-Swindel models were compared to the Max-Burkhardt models by Furnival's (1961) index of fit and residual mean squares. Sapwood volumes above breast height were predicted by integration of the best model (see Appendix B). These volumes were then expressed as a percentage of total volume above breast height (inside bark) as predicted by Walters et al. (1985).

Results

The best weight for model [1], as judged by Furnival's (1961) index of fit, was provided by dividing each observation by the square of DCB. In addition, approximate t-tests on the non-linear parameter estimates indicated that all are significantly different from zero (Table 1).

All variables in model [2] were highly significant (Table 1), and residual plots indicated no departure from the assumption of constant variance. Comparison of Furnival's index for models [1] and [2] suggests that model [2] is more appropriate in regard to both the size of residuals and possible departures from normality and constant variance. In addition, skewness and kurtosis coefficients for residuals were $-.060$ and 1.924 , respectively, for [1] and $.121$ and 1.074 for [2]. Thus both sets of residuals fall outside the 95 percent confidence contours for a sample size of 72, primarily due to the low kurtosis coefficients (Bowman and Shenton 1975). On the basis of Furnival's index, model [2] was chosen for breast height sapwood area predictions in models [3b], [4b], and [5b]. When applying predictions from this equation, one half of the residual mean square from model [2] was added to

predicted log(sapwood area) to correct for log bias (Flewelling and Pienaar 1981).

The parameter estimates associated with all three variables in Bennett-Swindel model [3a] proved significantly different from zero (Table 2). However, for model [3b], which is based on predicted breast height sapwood area rather than actual, the final model contained only two variables (Table 2). Both of these models had sapwood bulges for at least some height-diameter combinations. In addition reducing model [3a] to two or even one predictor variable did not remove predicted bulges. Similarly, no single variable model for [3b] eliminated all sapwood bulges.

The increased flexibility of expanded Bennett-Swindel models [4a] and [4b] reduced residual mean squares substantially when compared to [3a] and [3b] (from .010073 to .007282, and from .015874 to .011938, respectively); however, no versions of these models could be found which had both all statistically significant variables ($p < .05$) and lack of bulges or negative sapwood areas.

The optimal join point for the segmented polynomial model [5a] occurred at approximately $j_3 = .8$; therefore, this join point was maintained throughout the analysis for models [5a] and [5b]. The final

models retained three and four variables, respectively (Table 3). Despite the fact that more complex versions (up to six variables) contained all highly significant variables, these models predicted sapwood bulges in the lower portions of stems with small crown ratios. Furthermore, only an additional 1.0 and 1.9 percent of the variation in sapwood area were accounted for by the expanded versions of models [5a] and [5b], respectively. In the reduced final model no problems were encountered with predictions of negative sapwood areas for observable combinations of height, diameter, and crown ratio.

Since the segmented polynomial taper equation provided the only acceptable fit to these data, only the integrals of [5a] and [5b] were derived (Appendix B). The integral of equation [5b] was applied to estimate sapwood volumes for numerous diameter-height-crown ratio combinations typical of stand grown trees. Sapwood volumes ranged from 36 to 92 percent of total volume above breast height (Table 4).

Discussion

Gross crown dimensions have long been measured as indicators of a tree's vigor or photosynthetic capacity. The implication typically is that these dimensions provide a crude estimate of total leaf area. Assuming that crown base sapwood area indeed correlates well with total leaf area (Waring et al. 1982), the strong relationships between crown dimensions and sapwood area as modeled by [1] and [2] support this assumption (Table 1).

The predicted reduction in sapwood area from breast height to crown base varies considerably among trees of various diameters, heights, and crown ratios (Fig. 1). Waring et al. (1982) provide an average sapwood area:leaf area conversion factor of .47 sq m/sq cm at breast height vs. .54 sq m/sq cm at crown base. This implies an average taper of approximately 87 percent. For typical stand grown Douglas-fir, at least in southwestern Oregon, this would still apparently result in overestimation of crown base sapwood area as estimated by the sapwood area taper curves (Fig. 1). For those trees presented in Fig. 1, the segmented polynomial predicts taper to crown base of 39 to 74 percent of predicted breast height sapwood area. Since

leaf area is typically estimated as a linear function of sapwood area, the bar graphs in Fig. 1 can be interpreted as alternatively as the relative magnitudes of leaf area estimates by each technique. Thus correction for sapwood area taper will have a drastic effect on leaf area estimates.

Actual bulges in sapwood area are probably not prominent especially on the scale suggested by the Bennett-Swindel models and the more complex segmented polynomials; however, variations in xylem permeability (Whitehead et al. 1984) suggest the possibility of small scale fluctuations. In fact, in those trees sectioned every 2.56 m, departures from monotonic taper from breast height to crown base were observed. In contrast, taper in sapwood area above crown base was remarkably smooth. Sapwood areas estimated by the photocopy and area meter technique (Waring et al. 1982) on sectioned Douglas-fir from the Washington Cascades support the contention that these small scale bulges in the data are not simply an artifact of the estimation technique applied here (David Marshall, personal communication).

Sapwood volumes expressed as percentage of total volume for various heights, diameters, and crown ratios yield several noteworthy but expected patterns (Table

4). First, for a given height and dbh, the proportion of sapwood increases with increasing crown ratio. In addition, for a given crown ratio trees representing median heights for each dbh are predicted to contain a lesser proportion of sapwood volume as dbh increases. Finally, for a given dbh the the increase in estimated sapwood percentage from crown ratios .3 to .6 is less for taller trees (higher h/d).

In absolute terms, trees with either larger crown ratios or greater heights, other dimensions remaining equal, will have larger sapwood volumes. To the extent that the mean metabolic activities of sapwood in various trees within a stand are approximately equivalent, this implies higher respiration costs to the trees with greater heights or crown ratios. More interesting, however, is the greater predicted sapwood volume, and presumably respiration, of trees with larger diameters vs. trees of smaller diameter but equal height and crown ratio. For example, a tree of 30 cm diameter, 30 m height, and .4 crown ratio is predicted to have .504 cu m (17.82 cu ft) of sapwood vs. 1.045 cu m (36.91 cu ft) in a 50 cm tree with the same height and crown ratio (Table 4).

In conclusion, both segmented polynomial

models prove effective for estimating sapwood area taper from breast height to crown base. These stem taper models provide the desired logical constraints at tree tip and at breast height. Where the primary objective is to portray the full sapwood area profile, however, data which are more systematically distributed over the length of the stem can undoubtedly improve the parameter estimates presented here. In conjunction with better representation of extreme height-dbh-crown ratio combinations, such data sets will reduce the problem of bulges in both the segmented polynomial and the Bennett-Swindel models. Finally, the sapwood area taper models can easily be integrated to yield sapwood volume estimates applicable to ecophysiological or forest product studies.

Fig V.1. Sapwood areas predicted at breast height by model [2] (bh), at crown base by model [5b] (cb), and at crown base by coefficients from Waring et al. (1982) (co). Predictions from model [2] are consistently the highest and predictions from [5b] are consistently the lowest.

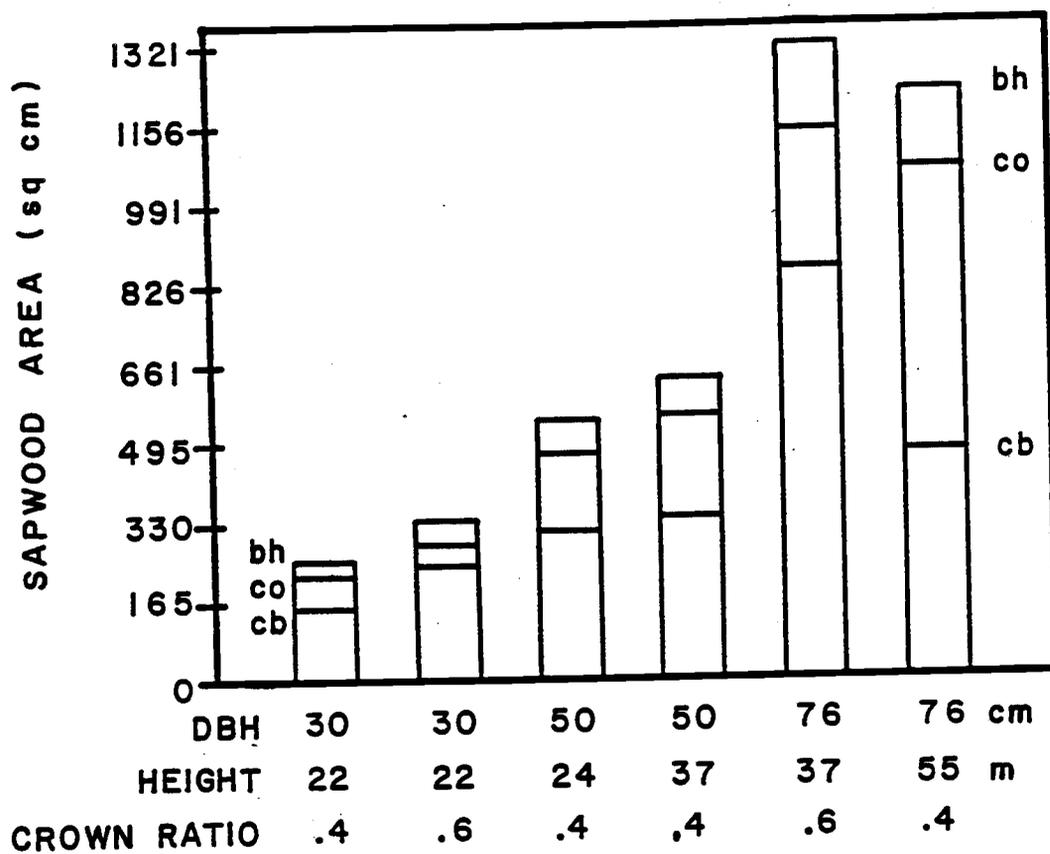


Fig.V.1.

Table V.1. Parameter estimates (and standard errors)
for models [1] and [2].

Model	Parameter	Parameter estimate	Standard error	Adj RSQ	MSE	FURN I
[1]	a ₀	2.085528	.312640			
	a ₁	1.988579	.126011			
	a ₂	-.389997	.140392			
	a ₃	.010577	.002767			
				.9435	8,981.84	54.5788
[2]	b ₀	.837717	.105664			
	b ₁	1.96673	.13000			
	b ₂	-.419168	.148744			
	b ₃	.0123346	.0036488			
				.9731	.032807	51.3422

Table V.2. Parameter estimates (and standard errors)
for basic Bennett-Swindel taper models [3a]
and [3b].

Model	Parameter	Parameter estimate	Standard error	MSE	Adj. RSQ
[3a]	c ₂	.362901	.047368		
	c ₃	.0146073	.0059868		
	c ₄	-.0110286	.0036950		
				.010073	.6712
[3b]	d ₂	.402309	.058943		
	d ₃	-			
	d ₄	-.0033126	.00132284		
				.015874	.5428

Table V.3. Parameter estimates (and standard errors) for segmented polynomial models.

Model [5a]			Model [5b]		
Parameter	Parameter estimate	Standard error	Parameter	Parameter estimate	Standard error
m_0	-.632598	.058512	m_0^*	-.597595	.094024
m_1	-		m_1^*	-	
m_2	-		m_2^*	-	
m_3	-		m_3^*	-	
m_4	.0182344	.0060860	m_4^*	-	
m_5	-		m_5^*	-	
n_0	-		n_0^*	-	
n_1	-.0170059	.0036933	n_1^*	-	
n_2	-		n_2^*	-.0447115	.0144645
n_3	-		n_3^*	-	

Table V.3. Continued.

Model [5a]			Model [5b]		
Parameter	Parameter estimate	Standard error	Parameter	Parameter estimate	Standard error
n_4	-		n_4^*	.0842778	.0159344
n_5	-		n_5^*	-.000485598	.0001653
MSE		.011081			.017245
Adj R^2		.8599			.8214

Table V.4. Sapwood volume as a percentage of total volume inside bark for various heights, diameters, and crown ratios.

	Diameter at breast height								
	30 cm (11.81 in)			50 cm (19.69 in)			75 cm (29.53 in)		
	Height (m)			Height (m)			Height (m)		
Crown ratio	18 (59')	24 (79')	30 (98')	30 (98')	36 (118')	42 (138')	36 (118')	42 (138')	48 (157')
.30	57.8	59.4	59.2	46.5	48.3	49.6	35.9	37.1	38.2
.40	70.9	69.1	66.5	55.2	55.2	55.0	43.9	43.8	43.8
.50	81.8	75.8	70.6	62.0	59.9	57.9	50.8	49.2	47.8
.60	91.9	81.4	73.5	68.1	63.8	60.2	57.5	54.3	51.6

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Chapter VI

Regression Analysis of the Relationship between
Gross Crown Dimensions and Sapwood Area at Crown Base

Abstract

Past research has established the close correlation between sapwood area at crown base and total leaf area of the tree. Therefore, various transformations of crown length, crown radius, and crown base stem diameter (outside bark) were assessed for their ability to predict crown base sapwood area, and, presumably, total leaf area. Regression analyses indicated that each variable without transformation was a rather poor predictor of sapwood area at crown base, but that various transformations representative of conic surface area performed quite well. Allowing exponents on the variables to vary through nonlinear regression improved the fits only slightly. Gross crown dimensions under the appropriate transformations are inferred to reflect quite accurately the total leaf area of the tree.

Introduction

Various expressions of crown size often exhibit significant correlation with volume, height, or diameter growth of trees within a given stand. Although the predictive power of these expressions are typically enhanced by correcting for relative position among competitors, gross crown dimensions presumably reflect the relative photosynthetic capacity of the tree. This photosynthetic capacity is most commonly conceived of as the total leaf area of the tree.

The particular expressions of gross crown size screened during growth model development have varied widely. These variables have included: 1) crown length (Holsoe 1948, van Laar 1969, Hamilton 1969, Arney 1972, Daniels and Burkhart 1975, Weaver and Pool 1979, Krumland and Wensel 1981); 2) crown ratio (Daniels and Burkhart 1975, Weaver and Pool 1979, Belcher et al. 1982, Wycoff et al. 1982, Wensel and Koehler 1985); 3) crown surface area (Hamilton 1969, Holsoe 1948, van Laar 1969); 4) crown volume (Hamilton 1969, van Laar 1969, Weaver and Pool 1979, Wensel and Koehler 1985); 5) crown diameter (Holsoe 1948, van Laar 1969, Weaver and Pool 1979); 6) foliage area (Botkin et al. 1972, Dissescu 1973); 7) foliage weight (Botkin et

al. 1972, Dissescu 1973); 8) crown projection area (Weaver and Pool 1979, Hamilton 1969); and 9) the ratio of crown width to tree diameter at breast height (Weaver and Pool 1979). Definition of "effective" crown base as the widest part of the crown has also allowed subsequent exploration of "effective" crown length, "effective" crown surface area, and "effective" crown volume as more biologically meaningful predictors of periodic growth (van Laar 1969). Finally, Mitchell (1975) computed the volume of hypothetical shells of annual foliage production, then expresses this foliage volume as a percent of the maximum possible volume (in absence of competition) to predict bole volume increment.

Despite the recognized utility of various crown dimensions, the relationship between crown size expressions and total tree leaf area has seldom been investigated. In individual tree growth models, the crown variables selected will depend strongly on other tree variables included in the model, and hence may even represent ultimate factors other than total leaf area. In addition, since the spatial arrangement and resulting density of foliage within the geometric solid described by the crown can vary (Stiell 1962, Brown 1978), the total leaf area of two trees with the same

gross crown dimensions may differ substantially. If this is true, further modification of crown size expressions to reflect any systematic variation in foliage density within the crown would emerge as a potentially more powerful predictor of periodic growth.

Direct determination of tree leaf area is quite expensive and laborious. However, recent research suggests that total tree leaf area is a linear function of sapwood area at crown base (Waring et al. 1982). In addition, the factor for converting crown base sapwood area to leaf area appears fairly constant among sites. These findings imply that transformations of crown dimensions which provide accurate predictions of crown base sapwood area will also yield strong correlations with total leaf area. Therefore, the objectives of the present study are: 1) to explore the relationship between crown base sapwood area and various transformations of crown dimensions in southwestern Oregon Douglas-fir (Pseudotsuga menziesii (Mirbel) Franco); and 2) to thereby assess the adequacy of previously applied expressions of crown size and ascertain the need for modification of these expressions to reflect more accurately the total leaf area of the tree.

Study Site and Data Collection

Trees were sampled from cluster plots established by the Southwest Oregon Forestry Intensified Research (FIR) Growth and Yield Project during the summer of 1983. Most plots consisted of Douglas-fir (Pseudotsuga menziesii (Mirbel) Franco) with varying mixtures of ponderosa pine (Pinus ponderosa Dougl.), grand fir (Abies grandis (Dougl.) Forbes), white fir (Abies concolor (Gord. & Glend.) Lindl.), sugar pine (Pinus lambertiana Dougl.), and incense cedar (Calocedrus decurrens (Torr.) Florin.). The study area covered an elevational range from 275 to 1550 m (900 to 5100 ft), and extended from near the California border ($42^{\circ} 10'$ N) north to Cow Creek ($43^{\circ} 00'$ N), and from the Cascade crest ($122^{\circ} 15'$ W) to approximately 15 miles west of Glendale ($123^{\circ} 50'$ W). This region is characterized by January mean minimum temperatures of -5° to 0° C, and July mean maximum temperatures of 26° to 32° C. Annual precipitation varies from 76 to 210 cm, with less than 10 percent of the total falling during June, July, and August.

Each cluster plot consisted of four to ten sample points located at the apices of equilateral triangles with 45.72 m (150 ft) sides. A subsample of trees on

each plot was felled for height growth measurement, sectioning, and crown sampling. Prior to felling, diameter at breast height (nearest 0.25 cm (0.1 in)) and four crown radii were recorded. The first two crown radii were measured along the line defined by the sample point and the center of the bole at breast height. The vertical projection of the crown perimeter onto this line was estimated by clinometer and recorded to the nearest 0.15 m (0.5 ft). Two other crown radii were similarly estimated along the line perpendicular to the first.

Crown sampling involved removal and dissection of whorls containing dead branches to determine time since branch mortality and rate of crown recession (Maguire and Hann 1986a, b). Therefore, after the tree was felled the following points were identified on the felled tree to facilitate implementation of the crown sampling scheme:

Crown Base (CB) - lowest whorl in the crown which had live branches at least three quarters of the way around the circumference of the stem, and above which all whorls had the same;

Lowest Contiguous Live Whorl (LCLW) - lowest live whorl above which all whorls had at least one live branch;

Intermediate Live Whorl (ILW) - second whorl down from crown base (CB); and

Second All Dead Whorl (SADW) - second all dead whorl below the lowest contiguous live whorl (LCLW).

On 189 felled Douglas-fir, total height and the heights to CB, LCLW, and SADW were recorded. In addition, diameter outside bark was measured just below CB and LCLW (nearest .25 cm). The SADW was then cut out of the bole. If greater than two whorls occurred between CB and LCLW, height to ILW was similarly recorded and this whorl was also removed from the bole.

After transporting the whorl sections to the lab, sapwood radii (delineated by color) were measured just below the sample whorl on the cross-sectional cuts. Two sapwood radii were measured along the longest axis, and two sapwood radii were measured along the axis perpendicular to the longest (nearest .13 cm). Sapwood cross-sectional area was then computed assuming an elliptical stem cross-section:

$$SA = \text{PI} (D_1 D_2 - d_1 d_2) / 4$$

where SA = cross-sectional sapwood area (sq cm)

D_1 = diameter inside bark along long axis (cm)

D_2 = diameter inside bark along short axis (cm)

$$d_1 = D_1 - r_1 - r_2$$

r_1, r_2 = sapwood radii along long axis (cm)

$$d_2 = D_2 - r_3 - r_4$$

r_3, r_4 = sapwood radii along short axis (cm)

$$\text{PI} = 3.14159265$$

In past analyses of the leaf area-crown base sapwood area relationship, crown base has been defined as the lowest live branch (Shinozaki et al. 1964). However, since crowns typically exhibit some degree of lopsidedness, crown base has usually been measured at a visually reconstructed, even crown base in more

traditional crown studies. More recently, it has been suggested that crown base be defined as the lowest point in the crown which still has live branches in three of the four quadrants around the stem (Curtis 1983). Since sapwood area consistently increases downward even through the branch-free portion of the bole (Waring et al. 1982), the increase in sapwood area from CB to LCLW (as defined above) cannot be attributed to only the increase in leaf area serviced. Hence, for practical application toward leaf area estimation, the most appropriate definition of crown base has not been well defined. Three possibilities exist in the present context: 1) CB; 2) LCLW; and 3) the level midway between CB and LCLW. Therefore, the sapwood taper equation presented by Maguire and Hann (1986c) was applied to estimate sapwood cross-sectional areas at these three levels by interpolating between, or extrapolating beyond, the sapwood areas measured at ILW and SADW.

Analysis and Results

Sapwood areas were first related to crown dimensions by assuming conformity of the crown to certain basic geometric solids. Simple linear regressions through the origin were performed for each transformation. Since variance around the regression line was clearly not constant, optimal weights were chosen by Furnival's (1961) index. The weights tested included the inverses of the predictor variables raised to integral powers from one to six. In addition, the unweighted coefficients of multiple determination (RSQ) provided an approximate assessment of the proportion of variation in sapwood area explained.

Each analysis was performed three times, once each for sapwood area at CB, LCLW, and midway between CB and LCLW. The crown length (CL) applied in each case was defined as the distance between the tip of the tree and the stem level at which sapwood was measured. Both arithmetic and geometric mean crown radius (one half the geometric mean crown diameter) were initially screened as predictor variables; however, slightly better fits were provided by the geometric mean, except for a negligible advantage of the arithmetic mean in some crown volume computations. The geometric mean is

also theoretically more flexible since it should handle both circular and elliptical crown cross-sections. Therefore, only the results from geometric mean radii are presented. The mean crown radius was assumed equal among the three levels of sapwood area determination. Stem diameters outside bark at CB and LCLW, as well as the interpolated diameter midway between, were also analyzed as possible surrogates for crown radius or diameter. As shown in Table 1, strong correlations existed among all three dimensions, suggesting that the variables may be interchangeable in many of the geometric formulae.

Lateral surface area was first computed assuming both a cone and a paraboloid. At each sapwood level, parabolic surface area proved a slightly better predictor of sapwood area (Tables 2-4). The conic surface area model ([1A-C]) was then modified by approximating the product of crown radius and slant height by the product of crown radius and crown length. This improved the degree of fit between crown dimensions and sapwood area beyond the fit obtained for either parabolic or conic surface area formulae. Substituting stem diameter for crown radius improved the fit even further (models [4A-C], Table 2-4).

Next, sapwood area was regressed on the square of

each variable. Again, due to the high correlations among the variables, these transformations could be construed in several ways, from crown projection area, as in the case of crown radius squared, to conic surface area as before. The squares of all three variables at all levels, however, predicted sapwood area more poorly than the product of crown length and crown radius or the product of crown length and stem diameter. Squared stem diameter ([10A-C]) performed quite well, but still not as well as the product of stem diameter and crown length.

As shown for purpose of comparison, crown length, crown radius, or stem diameter without transformation represent rather poorly the sapwood area at any of the three crown levels.

Inspection of the formulae for computing volumes of a cone, paraboloid, neiloid, or ellipsoid reveals that all reduce to the following (Mawson et al. 1976):

$$V = k CL R^2$$

where V = crown volume

CL = crown length

R = geometric mean crown radius

k = constant which varies by geometric solid

Therefore, sapwood area was regressed on the product of crown length and crown radius squared (models [11A-C]). As shown in Tables 2-7, likelihood criteria (Furnival 1961) indicate the superiority of surface area over crown volume.

Light intensities interior to larger crowns typically are too low to support foliage, resulting in a shell of volume which actually contains foliage (Larcher 1975). In a limited way, the general formula for crown volume can handle crowns with a hollow core as well. First, let

$$V = k_1 CL R^2 - k_2 cl r^2$$

where cl = length of hollow core

r = radius of hollow core

V, CL, R as above

Then assume

$$cl = a CL$$

$$r = b R$$

where $0 < a < 1$

$0 < b < 1$

which implies

$$V = k_1 CL R^2 - k_2 a CL b R^2$$

$$= k^* CL R^2$$

This structure assumes, however, that the radius and length of the hollow core are a constant proportion of the corresponding full crown dimensions for all crowns. In reality, this proportion increases with crown size. The following model was therefore introduced to allow greater flexibility in this regard:

$$V = k_{10} \exp(k_{11} CL) CL R^2$$

A marked improvement was effected through this modification (models [12A-C]); however, surface area remained superior (Tables 2-7).

Past research has also shown that foliage density varies in the crown profile (Stiell 1962, Jensen and Long 1983). Where inferences as to the actual density of foliage on the plane bisecting the crown can be made, integration of foliage density over the crown volume should provide an estimate of the total mass or leaf area. A first approximation for describing this planar foliage area density was derived by assuming

that the density was maximal at the outer surface of conical crown, and declined linearly to zero at the interior of the crown. Since the crown radius at any level within the crown can be described as

$$R_z = CL - z \quad R$$

$$\frac{\text{-----}}{CL}$$

where R_z = crown radius at height z

z = height above crown base

it follows that

$$\begin{aligned} d(r, \theta, z) &= m \frac{r}{R_z} \\ &= m \frac{r CL}{(CL-z) R} \end{aligned}$$

where $d(r, \theta, z)$ = foliage density at
cylindrical coordinates,
 (r, θ, z)

m = foliage density at the outer
edge of the crown

This density can then be integrated over cylindrical

coordinates:

$$M = \int_0^{CL} \int_0^{2\text{PI}} \int_0^{\frac{CL-z}{CL} R} d(r,\theta,z) r dr d\theta dz$$

$$= m \frac{2}{9} \text{PI} CL R^2$$

where PI = 3.14159265

Hence, the general volume formula again includes this particular assumption of foliage distribution. Advancing to more complex foliage distribution functions, however, yields more complex expressions. For example, allowing foliage density to peak vertically in a certain manner produces the following:

$$d(r,\theta,z) = m \frac{r CL}{CL - z} [1 - z/(p CL)]^2$$

$$= m \frac{r}{p^2 CL} \frac{(p CL - z)^2}{CL - z}$$

where p defines the location of the foliage density peak as a proportion of CL and

$$0 < p < 1$$

Hence,

$$M = \int_0^{CL} \int_0^{2\pi} \int_0^R \frac{CL-z}{CL} d(r, \theta, z) r dr d\theta dz$$

$$= \frac{2\pi CL R^3}{15}$$

As the results of the regression analysis show (models [13A-C]), however, sapwood area remained more strongly correlated with surface area (Tables 2-7).

Improvements in sapwood area prediction recovered by allowing exponents on crown dimensions to vary was assessed by nonlinear least squares. Optimal weights were again chosen on the basis of Furnival's (1961) index. As shown in Tables 2-7, very slight improvements were obtained for nonlinear models containing only crown length ([13A-C]) or crown radius ([15A-c]) over the simple square of the variables (models [6A-C] and [8A-C]). In contrast, the increased flexibility of allowing the exponent of stem diameter to vary (models [16A-C] vs. [10A-C]) improved the fits on this variable substantially (Tables 2-7). To a

lesser extent, nonlinear least squares estimation of exponents on crown length and stem diameter together tightened the fit to sapwood area as well (models [18A-C] vs. [4A-C]). However, as suggested by unweighted RSQ's, only about one or two percent of the variation in sapwood area is gained. The increased flexibility of the nonlinear fits was only slightly beneficial for the crown length with crown radius (models [17A-C] vs. [3A-c]).

Finally, general patterns in the indices of fit and unweighted RSQ's suggest that the power of crown dimensions for predicting sapwood area does not differ appreciably among the different crown base levels.

Discussion

The superior performance of crown surface area in predicting sapwood area (relative to crown width, crown length, crown projection area, or crown volume) corroborates past research establishing its superiority for predicting periodic growth (Holsoe 1948, Hamilton 1969). In contrast, Weaver and Pool (1979) found that the best dimension for predicting diameter increment of tropical trees depended on the particular species. However, their analysis was complicated by the fact that logarithmic equations were mixed in with both simple and multiple linear equations. In four of the five species, either a logarithmic equation with crown diameter as the predictor, or a multiple linear equation with both projection area and crown length, explained the largest amount of variation in diameter growth.

The efficacy of crown surface area in portraying total leaf area may depend, at least in part, on species characteristics such as shade tolerance and arrangement or display of leaves and leaf layers (Monsi et al. 1973, Horn 1974, Honda and Fisher 1978). Thus to a greater or lesser degree, the surface area of a cone or paraboloid represents the area of leaf surface

that can be cut up and rearranged within the crown without shading underlying foliage below the light compensation point. Since the foliage surface area is in fact finely divided and distributed over a volume, functions which accurately describe this distribution of leaf area over the crown space should facilitate computation of total area by triple integration. In particular, if the crown can be assumed symmetrical, a function which describes foliage area density on a half plane vertically bisecting the crown can be integrated in cylindrical coordinates relatively easily.

Schreuder and Swank (1974) and Kellomaki et al. (1980) model vertical distribution of crown biomass with a Weibull and a beta distribution, respectively, thus representing integration of a density function over both the tangential and horizontal dimensions. In contrast, Stiell (1962) and Jensen and Long (1983) present two dimensional foliage mass diagrams; that is, their profiles represent integration of a foliage biomass density function over only one dimension, rotation around the axis. As can be intuited from Jensen's and Long's (1983) density diagram, integration of their foliage density along the horizontal dimension roughly corresponds to the functions described by Schreuder and Swank (1974) and Kellomaki et al. (1980).

A major difference, however, is that the latter vertical distribution functions describe the relative distribution of foliage, and hence require independent estimates of total foliage biomass.

An obvious alternative to the triple integration approach described above would be double integration of planar functions capable of defining profiles such as those diagrammed by Stiell (1962) and Jensen and Long (1983). However, the spatial arrangement of the foliage for a given species, especially in regard to a maximum leaf area density, is to a large extent dictated by the physics of light penetration and the physiology of the species (Monsi et al. 1973). Therefore the theoretical basis for defining a density function over aerial growing space may be stronger than that for starting with a function defined on the crown profile. In the former case a true maximum should exist, whereas in the latter, the foliage area at any point is determined both by the theoretical spatial maximum and by the circumference over which that maximum is integrated. Either approach, however, promises to be quite complex: Stiell's (1962) research illustrates the wide variation in two dimensional foliage distribution among trees of different ages and initial spacings. Similarly, Brown (1978) found

foliage bulk density (foliage biomass per unit crown volume) to vary among levels within a single tree, but also among trees of different crown classes. Therefore, in view of the remarkably tight fits obtained with the relatively simple approaches above, little appears to be gained through the increased complexity of these latter approaches, especially in regard to the intended application. In other applications, more elaborate foliage distribution functions and concurrent light regime predictions facilitate the modeling of tree growth at the physiological level (Kellomaki 1980, Kellomaki and Oker-Blom 1983).

These more detailed analyses of foliage distribution do render less surprising the fact that crown volume is not particularly powerful for predicting sapwood area. If foliage biomass were uniformly distributed over the crown volume, integration around the stem axis would yield density profiles that exhibit increasing density from stem to crown perimeter, and from tree tip to crown base: this is clearly not consistent with the profile diagrams mentioned above (Stiell 1962, Jensen and Long 1983). Of the various crown dimensions and transforms applied in past growth models, conical surface area, as

approximately by the product of crown length and crown radius, correlates most closely with sapwood area. Crown width or radius raised to various powers is a particularly poor predictor, and although crown length without transformation is similarly poor, crown length squared or raised to some other empirically determined power performs satisfactorily. Finally, stem diameter squared or raised to a power as determined by nonlinear least squares is also quite useful. However, the best predictor of sapwood area is clearly the product of stem diameter and crown length. The strength of stem diameter squared for predicting sapwood area might be expected due to its obvious relationship to total cross-sectional area; however, a vast majority of the trees studied contained substantial and varying proportions of heartwood at all three crown base levels. In addition, noticeable improvement was provided by crown length. The high correlation between crown radius and stem diameter, along with the fact that greater error almost certainly was associated with crown radius measurements, suggests that stem diameter may be interpreted as a more accurately measured surrogate for crown diameter.

Although leaf area commands greater biological appeal and is often implied more relevant to tree

growth and physiology than gross crown dimensions, the present results emphasize that the two are virtually interchangeable for Douglas-fir in southwestern Oregon, given appropriate transformation. Problems would obviously arise where disease or insect defoliation occur. However, the practicality of leaf area estimation relies almost exclusively on its relationship to sapwood cross-sectional area. Although equations for predicting sapwood area at crown base can now correct for errors previously derived from sapwood taper (Maguire and Hann 1986c), the dynamics between leaf area and sapwood area at crown base have not been well defined. Thus under extreme or selective disturbance, much more intricate approaches to estimation of residual photosynthetic surface area must still be applied. From a modeling perspective, the virtual equivalence of leaf area and certain crown dimension transforms is tipped in favor of gross crown dimensions since it is conceptually more appealing to model crown recession and height growth than simply change in leaf area. Modeling crown recession relates more immediately to the ultimate determinant of crown size and leaf area, that is, competition for light and aerial growing space.

Table VI.1. Correlations among crown length, geometric mean crown radius, and diameter outside bark at CB, LCLW, and midway between CB and LCLW.

Variables	Correlation at:		
	CB	Midpoint between CB and LCL	LCLW
Crown length X crown radius	.7301	.7695	.7760
Crown length X stem diameter	.9279	.9269	.9219
Crown radius X stem diamete	.8628	.8922	.9036

Table VI.2. Results of zero intercept linear regressions of sapwood area at CB on crown dimensions. Parameter estimates (with s.e.'s), final weight, and Furnival's index are presented for each weighted regression, as well as the coefficient of multiple determination (RSQ) for the corresponding unweighted regression.

Model	Parameter estimates	Optimal weight	Index of fit	RSQ
[1A] $a_1 R \sqrt{R^2 + CL^2}$	$a_1 = 5.16443$ (.13918)	X^{-2}	32.32	.921
[2A] $a_2 R [(R^2 + 4 CL^2)^{1.5} - R^3] / CL^2$	$a_1 = .656303$ (.017387)	X^{-2}	31.64	.925
[3A] $a_3 CL R$	$a_3 = 5.46240$ (.13855)	X^{-2}	29.93	.932
[4A] $a_4 CL DCB$	$a_4 = .845515$ (.012979)	X^{-1}	22.06	.954
[5A] $a_5 CL$	$a_5 = 8.15631$ (.32961)	X^{-4}	50.08	.660
[6A] $a_6 CL^2$	$a_6 = 1.62570$ (.04978)	X^{-3}	37.39	.870

Table VI.2. Continued.

Model	Parameter estimates	Optimal weight	Index of fit	RSQ
[7A] a_7R	$a_7 = 33.6216$ (2.0511)	X^{-4}	72.83	.525
[8A] a_8R^2	$a_8 = 19.7867$ (.8365)	X^{-4}	56.93	.693
[9A] a_9DCB	$a_9 = 5.45415$ (.21834)	X^{-3}	41.57	.805
[10A] $a_{10}DCB^2$	$a_{10} = .435077$ (.008036)	X^{-2}	25.20	.913

Table VI.3. Results of zero intercept linear regressions of sapwood area at point midway between CB and LCLW on crown dimensions. Parameter estimates (with s.e.'s), final weight, and Furnival's index are presented for each weighted regression, as well as the coefficient of multiple determination (RSQ) for the corresponding unweighted regression.

Model	Parameter estimates	Optimal weight	Index of fit	RSQ
[1B] $b_1 R \text{ SQRT}(R^2 + CL^2)$	$b_1 = 5.13713$ (.13522)	X^{-2}	35.92	.920
[2B] $b_2 R [(R^2 + 4 CL^2)^{1.5} - R^3] / CL^2$	$b_1 = .649668$ (.016914)	X^{-2}	35.44	.923
[3B] $b_3 CL R$	$b_3 = 5.34260$ (.13568)	X^{-2}	34.40	.927
[4B] $b_4 CL \text{ DMID}$	$b_4 = .778523$ (.010701)	X^{-1}	22.71	.950
[5B] $b_5 CL$	$b_5 = 7.82204$ (.35888)	X^{-4}	57.70	.649

Table VI.3. Continued.

Model	Parameter estimates	Optimal weight	Index of fit	RSQ
[6B] $b_6 CL^2$	$b_6 = 1.40153$ (.03696)	X^{-3}	37.53	.860
[7B] $b_7 R$	$b_7 = 37.6065$ (2.1532)	X^{-4}	76.46	.552
[8B] $b_8 R^2$	$b_8 = 21.9958$ (.8583)	X^{-4}	58.41	.713
[9B] $b_9 DMID$	$b_9 = 5.72828$ (.22601)	X^{-3}	47.23	.803
[10B] $b_{10} DMID^2$	$b_{10} = .417224$ (.007288)	X^{-2}	27.22	.917

Table VI.4. Results of zero intercept linear regressions of sapwood area at LCLW on crown dimensions. Parameter estimates (with s.e.'s), final weight, and Furnival's index are presented for each weighted regression, as well as the coefficient of multiple determination (RSQ) for the corresponding unweighted regression.

Model	Parameter estimates	Optimal weight	Index of fit	RSQ
[1C] $c_1 R \text{ SQRT}(R^2 + CL^2)$	$c_1 = 5.05925$ (.13217)	X^{-2}	39.31	.917
[2C] $c_2 R [(R^2 + 4 CL^2)^{1.5} - R^3] / CL^2$	$c_1 = .638235$ (.016543)	X^{-2}	38.94	.918
[3C] $c_3 CL R$	$c_3 = 5.21872$ (.13297)	X^{-2}	38.15	.921
[4C] $c_4 CL \text{ DLC}$	$c_4 = .708177$ (.010490)	X^{-1}	27.00	.931
[5C] $c_5 CL$	$c_5 = 7.64459$ (.36455)	X^{-4}	65.80	.639
[6C] $c_6 CL^2$	$c_6 = 1.22394$ (.03339)	X^{-3}	43.21	.828

Table VI.4. Continued.

Model	Parameter estimates	Optimal weight	Index of fit	RSQ
[7C] c_7R	$c_7 = 41.2717$ (2.2777)	X^{-4}	80.88	.565
[8C] c_8R^2	$c_8 = 24.0286$ (.8988)	X^{-4}	61.17	.719
[9C] c_9DLC	$c_9 = 5.86572$ (.23082)	X^{-3}	52.37	.797
[10C] $c_{10}DLC^2$	$c_{10} = .393676$ (.007059)	X^{-2}	30.77	.915

Table VI.5. Results of linear and nonlinear regressions of sapwood area at CB on crown dimensions, including transformations representative of volume and mass. Parameter estimates (with s.e.'s), final weight, and Furnival's index are presented for each weighted regression, as well as the coefficient of multiple determination (RSQ) for the corresponding unweighted regression.

Model	Parameter estimates	Optimal weight	Index of fit	RSQ
[11A] $a_{11}CL R^2$	$a_{11} = 1.75662$ (.06222)	X^{-1}	46.44	.699
[12A] $a_{12}\exp(a_{13}CL) CL R^2$	$a_{12} = 3.04759$ (.232540) $a_{13} = -.038655$ (.0056497)	X^{-1}	40.99	.810
[13A] $a_{14}CL R^3$	$a_{14} = .4242989$ (.030863)	X^{-1}	69.92	.272
[14A] $a_{15}CL^{a_{16}}$	$a_{15} = 2.82710$ (.31522) $a_{16} = 1.75398$ (.05030)	X^{-3}	35.73	.871

Table VI.5. Continued.

Model	Parameter estimates	Optimal weight	Index of fit	RSQ
[15A] $a_{17}R^{a_{18}}$	$a_{17} = 16.2589$ (1.5559)	X^{-4}	56.10	.702
	$a_{18} = 2.24019$ (.09765)			
[16A] $a_{19}DCB^{a_{20}}$	$a_{19} = 1.16712$ (.08452)	X^{-2}	18.52	.953
	$a_{20} = 1.69256$ (.02321)			
[17A] $a_{21}CL^{a_{22}}R^{a_{23}}$	$a_{21} = 3.24477$ (.29393)	X^{-2}	27.04	.939
	$a_{22} = 1.30143$ (.05960)			
	$a_{23} = .890809$ (.08293)			

Table VI.5. Continued.

Model	Parameter estimates	Optimal weight	Index of fit	RSQ
[18A] $a_{24}^{GL} a_{25}^{DCB} a_{26}^{DCB}$	$a_{24} = 1.22318$ (.08413)	x^{-1}	17.94	.964
	$a_{25} = .390721$ (.065469)			
	$a_{26} = 1.36851$ (.05690)			

Table VI.6. Results of linear and nonlinear regressions of sapwood area at midpoint between CB and LCLW on crown dimensions, including transformations representative of volume and mass. Parameter estimates (with s.e.'s), final weight, and Furnival's index are presented for each weighted regression, as well as the coefficient of multiple determination (RSQ) for the corresponding unweighted regression.

Model	Parameter estimates	Optimal weight	Index of fit	RSQ
[11B] $b_{11}CL R^2$	$b_{11} = 1.71727$ (.06106)	X^{-1}	52.39	.701
[12B] $b_{12}\exp(b_{13}CL) CL R^2$	$b_{12} = 3.04168$ (.238449) $b_{13} = -.036739$ (.005329)	X^{-1}	46.13	.803
[13B] $b_{14}CL R^3$	$b_{14} = .417447$ (.030352)	X^{-1}	78.78	.264
[14B] $b_{15}CL^{b_{16}}$	$b_{15} = 1.69098$ (.21228) $b_{16} = 1.92121$ (.05214)	X^{-3}	37.43	.860

Table VI.6. Continued.

Model	Parameter estimates	Optimal weight	Index of fit	RSQ
[15B] $b_{17}R^{b_{18}}$	$b_{17} = 18.5007$ (1.6286)	X^{-4}	57.66	.726
	$b_{18} = 2.21216$ (.09072)			
[16B] $b_{19}DMID^{b_{20}}$	$b_{19} = 1.06922$ (.07935)	X^{-2}	20.48	.960
	$b_{20} = 1.71150$ (.02332)			
[17B] $b_{21}GL^{b_{22}}R^{b_{23}}$	$b_{21} = 2.63632$ (.30130)	X^{-2}	30.67	.932
	$a_{22} = 1.39678$ (.07187)			
	$a_{23} = .815801$ (.090120)			

Table VI.6. Continued.

Model	Parameter estimates	Optimal weight	Index of fit	RSQ
[18B] $b_{24}CL^{b_{25}}DMID^{b_{26}}$	$b_{24} = 1.04889$ (.07319)	X^{-1}	19.29	.964
	$a_{25} = .456597$ (.065081)			
	$a_{26} = 1.34872$ (.05468)			

Table VI.7. Results of linear and nonlinear regressions of sapwood area at LCLW on crown dimensions, including transformations representative of volume and mass. Parameter estimates (with s.e.'s), final weight, and Furnival's index are presented for each weighted regression, as well as the coefficient of multiple determination (RSQ) for the corresponding unweighted regression.

Model	Parameter estimates	Optimal weight	Index of fit	RSQ
[11C] $c_{11} CL R^2$	$c_{11} = 1.67478$ (.05973)	X^{-1}	57.49	.706
[12C] $c_{12} \exp(c_{13} CL) CL R^2$	$c_{12} = 3.01809$ (.238754) $c_{13} = -.034450$ (.004914)	X^{-1}	50.38	.794
[13C] $c_{14} CL R^3$	$c_{14} = .4092479$ (.029721)	X^{-1}	86.33	.269
[14C] $c_{15} CL^{c_{16}}$	$c_{15} = 1.37942$ (.19611) $c_{16} = 1.95202$ (.05635)	X^{-3}	43.25	.832

Table VI.7. Continued.

Model	Parameter estimates	Optimal weight	Index of fit	RSQ
[15C] $c_{17}R^c c_{18}$	$c_{17} = 20.5663$ (1.7322)	X^{-4}	60.51	.736
	$c_{18} = 2.19110$ (.08746)			
[16C] $c_{19}DLC^c c_{20}$	$c_{19} = 1.01204$ (.08387)	X^{-2}	24.17	.951
	$c_{20} = 1.71568$ (.02553)			
[17C] $c_{21}CL^c c_{22} R^c c_{23}$	$c_{21} = 2.47119$ (.32111)	X^{-2}	34.46	.923
	$c_{22} = 1.38492$ (.07661)			
	$c_{23} = .841082$ (.09263)			

Table VI.7. Continued.

Model	Parameter estimates	Optimal weight	Index of fit	RSQ
[18C] $c_{24}^{CL} c_{25}^{DLC} c_{26}^{DLC}$	$c_{24} = .953189$ (.076329)	X^{-1}	25.27	.956
	$c_{25} = .442596$ (.069456)			
	$c_{26} = 1.37021$ (.05833)			

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APPENDICES

DBH = diameter at breast height (in)

H = total height (ft)

HCB = height to crown base (ft)

Appendix B

The integral of models [5a] and [5b] can be expressed as follows:

$$\begin{aligned} SV &= \text{sapwood volume above breast height (cu m)} \\ &= SV_1 + SV_2 \end{aligned}$$

where

$$\begin{aligned} SV_1 &= \text{sapwood volume, breast height to join} \\ &\quad \text{point R (cu m)} \\ &= (.0001) \text{ PSA } [h_R + q_2(h_R^2/2H) \\ &\quad \quad \quad + q_3(h_R^3/3H^2)] \end{aligned}$$

$$\begin{aligned} SV_2 &= \text{sapwood volume, join point to tip (cu m)} \\ &= (.0001) \text{ PSA } [1/(R - 1)^2] [E(H^3 - h_R^3) \\ &\quad \quad \quad + F(H^2 - h_R^2) + G(H - h_R)] \end{aligned}$$

PSA = breast height sapwood area predicted
from [2] (sq cm)

$$R = .8 (HCB - 1.37) / (H - 1.37)$$

h_R = height to join point R (m)

$$E = (k_3 R^2 - 2k_3 R - k_2 - 1) / 3H^2$$

$$F = [k_2(R^2 + 1) + k_3 2R + 2R]/H$$

$$G = (-k_2 R^2 - k_3 R^2 - 2R - 1)$$

$$q_1 = j_1 \text{ for model [5a]}$$

$$= j_1^* \text{ for model [5b]}$$

$$q_2 = j_2 \text{ for model [5b]}$$

$$= j_2^* \text{ for model [5b]}$$