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Emmett Frank Thompson for the Ph.D. in Forest Management
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Title CONSIDERATION OF UNCERTAINTY IN FOREST MANAGEMENT DECISION-MAKING

Abstract approved Signature redacted for privacy.
(Major professor)

Management decisions are generally considered to be made under one of three categories of future knowledge: certainty, risk, or uncertainty. All three categories occur in forest management. However, forest management decisions whose outcomes are dependent upon future levels of timber yields, prices, utilization standards, or social and legal institutions are made under uncertainty. Forest managers have always recognized that uncertainty existed; however, they have not systematically included it in their decision-making process.

The objectives of the study were to: (1) establish the importance of systematically considering uncertainty in forest management decision-making and (2) illustrate and evaluate a model or procedure, for the systematic consideration of uncertainty in forest management decision-making.

A review of the present status of forest management decision-making constituted fulfillment of the first objective. Theoretical

decision-making models which are currently used in forest management, *e. g.*, present worth analysis, capital budgeting, financial maturity, and linear programming, while conceptually capable of considering uncertainty, imply certainty. That is, forestry applications of these models have employed single-valued expectations.

Fulfillment of the second objective consisted initially of a review of recent developments in the theory of decision-making under uncertainty. All decision-making problems have some common components. These components are: decision-alternatives, the actions which the decision-maker deems possible to take; states of nature, the future events which determine the outcome of the actions; and consequences, the result of taking a specific action and finding that a particular state occurs. The more popular theoretical models for decision-making under uncertainty were reviewed: minimax, minimax regret, Hurwicz index, and Laplace. While useful in some cases, each of these models has specific disadvantages. In addition, all the models have one common major disadvantage, they contain the implicit assumption that the decision-maker is completely ignorant about the states of nature which influence his problem.

In reality, forest managers and other decision-makers usually possess some information, although it may be vague, about their problems. If a decision-maker is not willing to assume complete ignorance about the occurrence of the states of nature, he cannot apply any of the above models.

There is a theoretical decision-making model which appears compatible with reality. The model, Bayesian decision theory, allows the decision-maker to arrive at a solution which is compatible with his opinions or judgements about the states of nature. Also, he can combine these opinions or judgements with experimental data to derive a solution using all available information, both subjective and objective.

Fulfillment of the second objective was completed by illustrating the application of Bayesian decision theory to a hypothetical problem. The problem, optimal degree of land ownership for an industrial forestry firm, was defined within the Bayesian model and a solution derived. Since the problem was hypothetical, the actual solution is not the primary result of the study. The resulting implications for actual situations is the primary contribution.

If forest managers are to make decisions which contain uncertainty, the uncertainty should be systematically recognized in the decision-making process. The Bayesian model is a logical procedure for such recognition. By adopting and applying such models, the efficiency of forest management decision-making will be increased.

CONSIDERATION OF UNCERTAINTY IN FOREST MANAGEMENT DECISION-MAKING

by

EMMETT FRANK THOMPSON

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APPROVED:

Signature redacted for privacy.

Associate Professor of Forest Management
In Charge of Major

Signature redacted for privacy.

Head of Department of Forest Management

Signature redacted for privacy.

Dean of Graduate School

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I. INTRODUCTION

Forest management is primarily a decision-making function. Basically, this is true of management in general. However, forest management, primarily because of the nature of the forest production process, contains unique decision characteristics. For example, forest managers must today choose the species and, to a large extent, the quality and quantity of tomorrow's timber resource. Consequently, many of today's decisions will ultimately be judged within tomorrow's environment.

Decisions, whose outcome is dependent upon some future occurrence, are generally considered to be made under one of three conditions: certainty, risk, or uncertainty. A decision is made under certainty, if each action leads to a specific, known outcome. Under risk and uncertainty an action may lead to one of several outcomes; if the objective probability of each outcome is known, the decision is made under risk, otherwise it is an uncertainty decision. Uncertainty is not synonymous with complete ignorance. The term refers to an inability, by the decision-maker, to attach objective probabilities to the various outcomes of an action.

Forest management decisions fall into all three of the above categories; however, this study will concentrate on the uncertainty category. Decision-making procedures, or models, for certainty conditions are fairly well developed in forest management; e. g., forest valuation techniques. It is the risk and uncertainty categories that have received inadequate attention and uncertainty is the more prevalent of the two.

Systematic approaches to studying and dealing with uncertainty are essentially nonexistent in forest management. It might be argued that forest managers consider uncertainty through such endeavors as research to supply basic growth and yield data and fire prevention and suppression activities. Such efforts may reduce or modify the sources of uncertainty, they will not eliminate uncertainty. Unless forest managers can objectively predict the future with respect to such elements as raw material costs, product prices, and institutional developments uncertainty will be a factor in the decision-making process and should be recognized as such.

Forest managers have always made decisions concerning the long-run future. In the absence of specified procedures or models for systematically considering the uncertainty inherent in these decisions, foresters have followed "traditional" concepts. The European idea of sustained yield, the normal forest, the regulated forest model, and the forester's inherent dislike for idle land exemplify these "traditional" concepts.

Until recently, the adherence to traditions has been understandable. That is, a comprehensive body of theory concerned with decision-making under uncertainty has not been available. However, during the past few years, a body of knowledge, generally referred to as decision theory or statistical decision theory, has been developed. This knowledge is concerned with the theory and methods for decision-making under uncertainty.

Decision theory, as a science, has not yet emerged from the formative stage. Baumol (1965, p. 552) indicated that most of the literature is still concerned with decision criteria. Nevertheless, some researchers (e.g., Grayson, 1960; Farrar, 1962) have found the theory useful in disciplines other than forestry. Forest management research should appraise and, if possible, apply current decision theory and participate in the development of improved theory.

Concurrent with the development of decision theory has been the advancement of the concept of management as a science. That is, the application of scientific methods to management problems. Spencer and Siegelman (1964, p. 16) indicated that management has two functions: coordination and supervision. They argued that coordination is the more fundamental and scientific of the two functions and that its importance derives from the presence of risk and uncertainty in management decisions. Spencer and Siegelman stated (1964, p. 17):

> . . . management in the coordination sense (as distinguished from management in the supervisory sense), exists largely as a device for adjusting to risk and uncertainty. If the future were completely known, management in the coordination sense would be needed for the most part only at the start or initial phase of an investment in order to formulate a plan for the future; thereafter, management in the supervisory sense would be all that is needed for the purpose of administering or carrying out the plan. Apparently, since risk and uncertainty constitute the environment in which businessmen operate, planning, organizing, controlling, directing, etc. are actually inseparable activities of coordination management.

In line with the above concept of management as a science directed toward decision-making under risk and uncertainty, Hertz (1965) stated:

> Using our resources most productively, then, calls for our direct involvement in the management process, and our determination to pursue our studies even in the face of powerful opposition. . . I suggest that problems to which a management scientist may address himself fall into four categories:
>
> (1) Deterministic - where predictable relations exist among the variables at hand.
>
> (2) Probabilistic - where the probabilities can be determined.
>
> (3) Stable, nondeterministic - where the probabilities can be estimated with some feeling of confidence.
>
> (4) Nonstable, nondeterministic - where estimates of the probabilities are subject to varying degrees of uncertainty.
>
> Some nontrivial problems of management fall into the third class, in which it is possible to manipulate the environment to bring about a reasonably stable system. Most nontrivial management problems fall into category 4, where there are many potential causes of changes, <u>some</u> discoverable, but in which the history of the

> system giving rise to the problem provides little or no basis for forecasting its future state.

Many of the management problems in forestry fall into Hertz' category 4. A number of recently developed theoretical models for decision-making under uncertainty in general are available. The extension or application of these models to forest management may significantly increase the level of forest management as a science.

Objectives

1. To establish the importance of systematically considering uncertainty in forest management decision-making.
2. To illustrate and evaluate a model, or procedure, for the systematic consideration of uncertainty in forest management decision-making.

Scope

The first objective may appear unnecessary from the viewpoint that uncertainty obviously exists in forest management and systematic consideration of the uncertainty is just as obviously desirable. However, the continual tendency for the previously mentioned "traditions" to dominate forest management, justifies the first objective. Fulfillment of the first objective will be accomplished by showing that the type of environment which creates uncertainty in decision-making is a primary environment surrounding forest management decision-making.

Fulfillment of the second objective will constitute the major contribution of the study. The most desirable method of illustrating decision-making under uncertainty is to take a theoretical model which is compatible with a decision-making problem, empirically implement the model, and present the solution. Unfortunately, this approach is neither practical nor possible with respect to forest management problems and theoretical models for decision-making under uncertainty.

The application of the above approach implies two basic assumptions. First, it implies that the theoretical model has been evaluated with respect to the type of decision-making problem under consideration. That is, when applied to the problem the model will produce a rational solution. Second, it implies that the data necessary to implement the model is either available or readily obtainable. Neither assumption is met in forest management.

Although a number of theoretical models, or criteria, for decision-making under uncertainty are available, none have presently been evaluated with respect to forest management problems. Since the type of data necessary to implement an uncertainty model may differ from that used in certainty or traditional approaches, data necessary to empirically implement an uncertainty model in forest management should not be expected to be readily available.

The second objective will be fulfilled by, first, reviewing the established theory for decision-making under uncertainty; second, evaluating the theoretical models for decision-making under uncertainty with respect to forest management problems; third, illustrating, through development of a hypothetical problem, how the uncertainty inherent in the forest management environment can be systematically considered in the decision-making process. A critique, evaluating the place of decision theory in forestry and the contribution of the study to forest management will complete fulfillment of the second objective.

As indicated, a hypothetical problem will be used to illustrate the consideration of uncertainty in forest management decision-making. Actually, the problem might be more accurately described as non-empirical rather than hypothetical. The problem, determination of optimum degree of forest land ownership for a pulp and paper firm, is a real enough problem, one that is faced by every industrial forestry organization. However, data to empirically implement an uncertainty model with respect to the problem is not available. Collection of such data would, itself, constitute a major research effort. Also, until the model and problem have been considered from problem formulation through solution, the relevant data to collect may not be discernible. Consequently, the problem will be solved using, whenever possible, regional or average data. When no related data are available, assumptions, which allow continuance of the solution, will be employed.

II. PRESENT STATUS OF FOREST MANAGEMENT DECISION-MAKING

The making of decisions in any one economic or business activity has much in common with other such activities. This chapter will review the generally accepted classification of decision-making situations. Forest management decisions will then be considered in relation to the classification. Also, presently used decision-making models and the need for considering new concepts in forest management will be discussed.

General Classification of Decision Situations

Decision-making is commonly separated into two areas, according to whether the decision is to be made by a group or an individual. As a result, theory distinguishes two classes of decision-making. The differentiation between individual and group, in the decision-making sense, is not numerical but rather functional. Any decision-making body, whether an individual or a collection of individuals, which desires the same end results from a decision may be considered an individual decision-maker (Luce and Raiffa, 1957, p. 13). For example, a corporation, although composed of many persons, is considered an individual decision-maker. This study will be concerned only with individual decision-making.

Since the publication of F. H. Knight's book, Risk, Uncertainty, and Profit, in 1921, the state of future knowledge has generally been considered to be in one of three categories: certainty, risk, or uncertainty. Luce and Raiffa (1957, p. 13) defined these categories within a decision-making context.

> As to the certainty-risk-uncertainty classification, let us suppose that a choice must be made between two actions. We shall say that we are in the realm of decision making under:
>
> (a) Certainty if each action is known to lead invariably to a specific outcome.
>
> (b) Risk if each action leads to one of a set of possible specific outcomes, each outcome occurring with a known probability. The probabilities are assumed to be known to the decision maker. . . Of course, certainty is a degenerate case of risk where the probabilities are 0 and 1.
>
> (c) Uncertainty, if either action or both has as its consequence a set of possible specific outcomes, but where the probabilities of these outcomes are completely unknown or are not even meaningful.

Some authors have found it convenient to modify the above classification system. For example, Johnson, et al. (1961, Ch. 3) used six knowledge situations in studying managerial processes of farmers. Shukik (1954, p. 631) divided the types of information available to economic decision-makers into three broad classes with 12 subclasses. Basically, both Johnson, et al. and Shukik were interested in further defining the decision situations lying between risk and uncertainty. In other words, there are "degrees of uncertainty" between an objective probability distribution and what has been termed "complete ignorance." For example, whether or not the future price of a commodity will be higher or lower than present price may be uncertain but exactly how much higher or lower is still more uncertain. For purposes of this study, it will be recognized that there is a range of decision-making knowledge within the uncertainty category. However, no new classification system will be developed and the standard certainty-risk-uncertainty system will be used.

The concept of certainty requires little elaboration. The distinction between risk and uncertainty, however, could stand further clarification. Worrell (1959, p. 192-195) established the following distinction between risk and uncertainty.

> A risk is an outcome whose probability of occurrence can be established in a quantitative manner. One kind of risk exists when the characteristics of the outcome are known in advance. . . two requirements must be met if a future outcome is to be classed as a risk rather than an uncertainty. The first is that one must be concerned with a large number of similar future outcomes. . . The second requirement is that the outcomes must not all be affected by the same conditions at the same time. . . A second type of risk exists when statistical probabilities of various possible outcomes can be established. . . By contrast, an uncertainty is a variable factor or outcome whose probability of occurrence cannot be established in a quantitative manner. Such things as yields, prices, technological changes, and sociological and legal changes are uncertainties.

To summarize, a decision can be said to be made under certainty, if the outcome of each specific action is known to the decision-maker. A decision is made under conditions of risk, if an objective probability distribution can be associated with the several possible outcomes of each specific action. Decisions made under any other condition fall into the uncertainty category. There are degrees of uncertainty between the two extremes of "almost risk" and complete ignorance.

General Nature of Forest Management Decisions

Forest management decisions fall into all three of the categories defined in the preceding section. Many forestry organizations treat decisions concerning continued land productivity as a series of certainty situations. That is, no matter what the future holds, the outcome from not regenerating cut-over stands is worse than the outcome from regenerating cut-over stands.

Some forest management decisions fall into the risk category. How much to pay for a tract of timber, after the timber volume has been estimated by a statistically valid cruise, may be a risk decision. Also, a company owning a large amount of forest land may, after collecting adequate data, determine the probability of a certain percent of its ownership burning within a given period. In such circumstances, decisions about fire suppression expenditures are made under risk; it should be emphasized that the risk is associated with a certain percentage of forest area burning, whether or not a specific area will burn remains uncertain.

While a number of examples can be given of forest management decisions which fall into the categories of certainty and risk, a majority of forest management decisions must be classed as uncertain. All these decisions which deal with future yields or prices or which are affected by technological, social, or legal changes are made under conditions of uncertainty. For example, a forest manager may feel certain that future stumpage prices will be higher than present stumpage prices; however, how much higher in both absolute amount and in relation to competing alternatives creates an uncertain situation. Also, until 1964, many private forest managers may have considered the continual capital gains treatment of timber as certain. Now, however, it is doubtful that this institutional factor continues to be associated with a probability of one.

Decision-Making Concepts in Forest Management

Traditionally, forest managers have not systematically considered uncertainty in their decision-making processes. There have, however, been a number of approximations which at least recognized the existence of uncertainty. This section will briefly review forest management decision-making concepts under both certainty and uncertainty.

Certainty

Most present forest management decision-making theories and techniques appear to be more closely aligned with certainty than with either risk or uncertainty. A number of these concepts which either explicitly or implicitly assume certainty will be considered.

Present Worth Analysis. The most common decision-making technique used in forest management is present worth analysis. The entire field of forest valuation or finance is based upon this concept. In its simplest form, present worth analysis is a mathematical representation of the fact that the present value of money decreases as its actual receipt is extended into the future. Foresters are familiar with the standard compound interest formula:

$$V_n = V_o(1+p)^n$$

where V_o is present value

p is the rate of interest

V_n is value after n years

Basically, the formula says that two sums of money occurring at different times are equivalent at one time, given the appropriate interest rate. The formula can be recast to bring future revenues to the present (hence the name, present worth analysis) to determine the maximum investment which can be justified to obtain a given revenue, etc.

Present worth analysis can be more sophisticated than the above example. Costs and revenues may occur singly or in series, at equal or unequal intervals, remain constant or vary with time, etc. The basic principle, however, remains the same. Costs and revenues are brought together at one point in time at some interest rate. Their algebraic sum determines the profitability of an investment.

Present worth analysis is a highly useful and very important concept in forest management. However, it is a certainty concept. Before present worth analysis may be employed, future prices, costs, and interest rates must be assumed known.

Capital Budgeting. Another decision-making concept that has been recommended for forest management is capital budgeting. Capital budgeting is essentially a system of allocating capital for efficient use. Dean (1951) has thoroughly explained the concept. In forest management, Fedkiw (1960; 1961) has been the chief proponent of capital budgeting.

Capital budgeting is very closely related to present worth analysis. In fact, it is essentially an extension of the latter concept. Basically, capital budgeting is a ranking, by means of present worth analysis, of various investment alternatives.

Since capital budgeting is based upon present worth analysis, it contains the same implied assumption of certainty regarding future knowledge. Also, since capital budgeting is, by definition, concerned with capital efficiency, it also contains the assumption that capital is the most limiting of the fixed resources.

According to economic theory, if an enterprise has a maximum profit objective, then profit should be maximized with respect to those factors of production which are fixed. There is no question that capital is an extremely important factor of production and in many cases is the limiting or fixed factor. However, other factors such as plant capacity or labor supply may also be fixed and more limiting. Profit must be maximized with respect to all fixed factors and to consider only one may sacrifice possible profit.

Financial Maturity. The capital value of a forest increases with time, assuming the forest is maintaining a positive net volume growth. This increase is due both to accretion and appreciation. (The preceding statements would not be true during a period of continuously declining stumpage prices.) When the annual increase in capital value falls below the timber capital's opportunity cost, the timber is said to be financially mature. Financial maturity is basically an extension of the economic concept of marginal analysis to the rotation decision in forest management.

The above concept of financial maturity is only one of several. Some authors extend the financial maturity label to all economic bases for the rotation decision. Duerr, Fedkiw, and Guttenberg

(1956) presented financial maturity in essentially the above form. Gaffney (1957) expanded the term to include soil-rent and other concepts. Bentley and Teeguarden (1965) have recently completed a theoretical review of the various financial maturity concepts.

Linear Programming. The previously discussed decision-making concepts are accepted and appreciated by most forest managers. However, the decision-making potentials of linear programming have not been generally recognized. Linear programming is not new; economists have been using the concept for almost two decades but the advantages of the technique have not been generally extended to forest management decisions.

Linear programming is a procedure for determining optimum resource allocation, given management objectives and availability of resources. There is no reason why certain allocation problems in forest management cannot be handled by linear programming. However, there have been few empirical examples of its use in forest management. Three examples of the application of linear programming to forest management are Coutu and Ellertsen (1960), Curtis (1962), and Kidd (1965).

The preceding list of forest management decision-making techniques under certainty is not all-inclusive; it is only illustrative. Some of the techniques are fairly elementary, others are more sophisticated. One characteristic of all the techniques, as they have been applied in forest management, is the assumption of certainty. All of the techniques could be modified to reflect consideration of risk or uncertainty, e.g., Heady and Candler (1958, Ch. 17) have indicated how risk-type decision problems may be handled by linear programming. However, in forestry all the techniques have implicitly assumed certainty, i.e. future costs, prices, yields, etc. have been single-valued. There is nothing wrong, in the appropriate circumstances, with assuming certainty and this section is not meant as an across-the-board criticism of these techniques. However, when uncertainty is prevalent in an enterprise, as it is in forest management, attempts should be made to systematically consider the uncertainty.

Uncertainty

As indicated in the preceding section, most forest management decision-making processes contain the assumption of certainty regarding future knowledge. Some techniques and procedures which consider uncertainty have, however, been advocated for forest management decision-making.

One of the factors which confuses a review of decision-making processes under uncertainty in forest management is the tendency for the forestry literature to consider the terms risk and uncertainty synonymous. For example, Chapman and Meyer (1947, p. 338) stated: "Risks caused by the operation of physical phenomena such as hurricanes, floods, or droughts are also uncertain. . . " Guttenberg (1950, p. 4), in discussing the rate of interest in forest management, stated: "There are two principal types of risk: insurable and non-insurable." A number of such examples could be cited. However, it will suffice to indicate that, when the term risk is used in most forestry literature, it is usually uncertainty that is being discussed.

To account for the presence of uncertainty, it is occasionally suggested that the interest rate used in compounding and discounting be adjusted. Duerr (1960, p. 148) has illustrated the fallacy of this approach. Compound interest rates do not maintain the same relationship through time but will diverge with time. To avoid this pitfall, Duerr recommended the adjustment of prices downward or costs upward. Vaux (1954, p. 26) used this latter technique in studying sugar pine in California. Walker (1962, p. 5) advocated using present costs and prices in discounting future values. In other words, since the future is uncertain, the present is its best estimator.

The use of present costs and prices or some modification of these factors is the most frequently recommended procedure for dealing with economic uncertainty in forest management decision-making. Under close scrutiny, however, such modifications do not actually deal with uncertainty. The general procedure is to start with the present rate of interest, cost, and price. Then, one or more of these factors is adjusted in some manner. If the adjustment changes one discrete value to another discrete value, even though in the name of considering uncertainty, the decision situation must still be classified as assuming certainty.

Recently, a few persons have recognized the need for considering uncertainty in various aspects of forest management. Dowdle (1962) has proposed the Markowitz "expected returns-variance of returns rule" in certain forest management investment situations. The Markowitz rule basically recognizes that estimates of future prices cannot be point estimates but more realistically should consist of a range of values. Dowdle's study introduced the concept and presented an illustration of its use; unfortunately, a clear distinction was never developed between risk and uncertainty.

In introducing and justifying his study, Dowdle provided a review and critique of some common forest management decision-making techniques. In discussing marginal theories of investment in forest management (i. e., soil rent, financial maturity, etc.) Dowdle pointed out their underlying assumptions (1962, p. 9):

1. Future timber-products output, by quantity and quality, and the prices of these outputs are known. Revenues accruing to each investment alternative can, therefore, be determined.

2. Costs are known at the time an investment is made. When investment expenditures are incurred over a period of time, the future costs are known.

3. Timber processing technology will not change in the future, or will change according to an assumed trend.

4. The interest or discount rate applicable to each investment alternative is known.

Dowdle, at least implicitly, recognized that the above assumptions lead to a decision-making situation under certainty. It was the unrealistic consequences of adopting such assumptions that prompted Dowdle's study.

Marty (1964) was concerned with developing procedures for analyzing timber management investments which contained uncertainty. However, Marty did not utilize the developed theory for decision-making under uncertainty. Rather he seemed to be trying to develop an independent concept for decision-making under uncertainty. Consequently, Marty violated established decision theory in a number of instances. For example, he confused the decision-maker's alternatives with the states of nature creating the decision-making environment. These components of decision theory will be considered in detail in the next chapter.

Marty's study can be summarized as an argument for discarding dominated decision alternatives. As will be indicated in the next chapter, this is where decision theory starts. Consequently, Marty's study can be viewed as bringing forest management decision-making to the point where the next logical step is to consider the application of decision theory.

Flora (1964, p. 376-380) discussed uncertainty in forest management decisions. Essentially, Flora concluded that forest managers can ignore uncertainty. He stated: "... if certain assumptions are plausible and if decisions are between forest investments whose returns are about equally distant in time, then uncertainty... can be ignored." Some of the assumptions underlying the above statement need to be scrutinized.

First, Flora considered three types of uncertainty. Type I are uncertain factors which affect both forestry and non-forestry investments. Since (according to Flora) foresters are not commonly called upon to compare forestry and non-forestry investments, Type I uncertainty does not need to be considered. An implicit assumption here is that foresters are not involved in policy considerations.

Type II uncertainties affect only forestry investments but all forestry investments to the same degree. Therefore, this type may be ignored. The assumption is that, since all investment results fluctuate the same, rate of return actually realized is unimportant. No consideration is expressed for comparing uncertain forestry returns with cut-off rates established by over-all policy or economic opportunity costs.

Type III uncertainties are those restricted to forestry but bear on some forestry investments more than others. According to Flora (p. 378), all that can be said about Type III uncertainties is, if all the uncertainties affecting various alternatives seem about equal, a "best guess" of outcomes is appropriate. Also, if two alternatives cover different time spans, there is more uncertainty associated with the longer one. Davis (1965, p. 40-41) in reviewing Flora's article, pointed out a logical inconsistency in the development of Flora's primary conclusion. Specifically, before the Type III uncertainties affecting alternative decisions can be judged to be about equal, a considerable amount of information must be available.

The authors of the above publications have made a major contribution to the forestry literature. They have brought the uncertainty in forest management decisions into the open. In addition, they have recognized some of the inadequacies inherent in traditional forest management decision processes.

Unfortunately, none of the above publications actually utilized the available theory of decision-making under uncertainty. As indicated previously, a considerable body of literature on the general theory of decision-making under uncertainty has accumulated over roughly the past decade. However, attempts to consider uncertainty in forest management decision-making have, by and large, ignored this literature; the notable exception is a recent article by Dane (1965, p. 276-279). Consequently, the forestry literature concerning decision-making under uncertainty contains inadequate formats for making such decisions. Formats or procedures have been developed for making decisions in the face of uncertain knowledge. These procedures establish consistent, logical patterns which tend to insure efficient use of available information. The procedures have proved useful in other forms of management; there is no reason to believe they would not also be helpful in forest management decision-making.

Theory in Forest Management

This study is theoretical in nature. That is, the study is concerned with the compatibility and application of decision theory to forest management problems. Before applying new theories, it is appropriate to consider why the new theories are necessary. Currently, there is a theoretical base for forest management. However, current theory is not explicitly identified and it rests largely upon the previously mentioned traditions. The main problem with present forest management theory is that it is not compatible with reality.

If forest managers have not developed a unique body of theory which realistically explains and governs the process of forest management, what has prevented this development? One of the main sources of impetus for the present study was the inability of available forest management decision-making models to provide consistent, rational solutions to many typical forest management problems. As indicated, uncertainty is an inherent factor in almost all forest management decision-making. The failure of many current forest management decision-making models is due to their implied assumption of certainty.

To illustrate the above point, a number of forest economists (e.g., Gould, 1960, 1962; Fedkiw, 1961; Dowdle, 1963) have criticized the regulated forest model as being economically inadequate. However, their criticisms have not produced any explicit substitutes for the model. On the other hand, Kidd (1965) has demonstrated that, if certainty is assumed, a linear programming model can be applied to the regulation of a forest property. That is, if considerations of uncertainty are set aside, there are no theoretical problems in forest regulation. However, the certainty assumption is obviously not compatible with reality. It would appear that efforts directed toward the systematic consideration of the uncertainty present in forest management would contribute to the underlying theory of the discipline.

This study will not provide a new and universally acceptable theory of forest management. However, such a theory is eminently called for, and, hopefully, the study will represent progress in making such a theory a reality.

Summary

Decisions are made under conditions of certainty, risk, or uncertainty. Forest management decisions fall into all three categories. However, all forest management decisions which are influenced by future yields, prices, institutional changes, etc. are in the uncertainty category. On the other hand, almost all forest management decision-making models or concepts, explicitly or implicitly, assume certainty. This discrepancy between theory and reality creates a primary stumbling block in present forest management decision-making. A few persons have recently recognized the need for considering the uncertainty in forest management. However, an evaluation of the theoretical bases for decision-making under uncertainty, with respect to forest management, has not appeared.

III. REVIEW OF CONCEPTS FOR DECISION-MAKING UNDER CERTAINTY

A general classification of the decision-making environment was presented in Chapter II. According to the classification, decisions are made under conditions of certainty, risk, or uncertainty. Forest management decisions were considered and it was concluded that uncertainty is prevalent in many such decisions. It was further argued that, in those cases where uncertainty is present, forest managers should use decision-making theories and processes which reflect consideration of the uncertainty. However, in reviewing decision-making models presently used in forest management, it was concluded that most of the models contain an implicit assumption of certainty.

The present chapter will review the general theory of decision-making under uncertainty. The theory will then be evaluated with respect to forest management decision-making.

Theoretical Concepts of Decision-making Under Uncertainty

Decision-makers in all areas of management are confronted with the problem of uncertain knowledge. The results of many decisions in these areas are dependent upon future conditions. In assessing the future, there is little, if any, certainty. Also, factors affecting business and economic decisions are commonly unique in time. That is, they are not recurring to the extent that objective probability distributions can be developed from their occurrence. Therefore, uncertainty is very prevalent in such decisions.

Recently, largely within the past ten years and almost entirely within the past 20, a considerable body of knowledge and literature has accumulated in the general area of decision-making under uncertainty. Persons trained in economics and business have made significant contributions. However, the more basic theoretical contributions have been made by mathematicians and statisticians. It seems reasonable that any study dealing with decision-making under uncertainty should begin by attempting to utilize the developed theory.

Certain properties are common to all theoretical procedures for making decisions under uncertainty. These properties are: the states of nature, the decision alternatives, and the consequences which associate each alternative to each state.

States of Nature

The states of nature, the set S, are the future conditions or occurrences which are beyond the control of the decision-maker but which influence the result of his decision. For example, whether or not it rains tomorrow will affect what is considered proper attire. There is, however, no way in which an individual can control the occurrence of rain.

Schlaifer (1959, p. 6-7) listed two requirements which the set S must fulfill. First, the set must be collectively exhaustive. By this, Schlaifer means that all the states which a decision-maker deems possible to occur must be included in the set. That is, some one element in S must occur. On the other hand, Schlaifer pointed out, if the decision-maker believes a certain state impossible, it may be excluded from consideration. Furthermore, it is not necessary to prove that the state is impossible. If a decision-maker believes a state impossible, it will not influence his decision-making and can be ignored.

The second requirement that the set S must fulfill is that it must be mutually exclusive. A mutually exclusive set of states is one with no overlapping of individual state definitions. In other words, if one element in S occurs, no other element can possibly occur.

Decision Alternatives

The second property common to theoretical decision procedures under uncertainty is the possible decision alternatives or actions, the set A. For example, in the above illustration concerning proper attire, a person might consider only two decisions alternatives: to wear a raincoat or not to wear a raincoat. Obviously, if a person knew which state of nature would obtain, he would know which decision alternative to elect. The set A consists of those alternatives which the decision-maker deems possible to elect and are not dominated by some other alternative. Dominance between acts is explained as follows. If a_1 and a_2 are two proposed elements in the set A and for every state a_1 is preferable or equivalent to a_2, then a_2 is dominated. That is, a_1 is an admissible act; a_2 is not admissible. Therefore, a_2 is not an element of the set A. Chernoff and Moses (1959), Ch. 5) have provided a graphical representation of dominance among acts.

Consequences

The third property common to theoretical procedures for decision-making under uncertainty is the set C, the consequences of choosing the various alternatives given the state of nature. The consequences may be in terms of utility, profits, losses, etc.

Whether or not the set of consequences from choosing among various alternatives, given the state of nature, should be expressed in terms of utility or some more common unit of measure, such as dollars, depends largely upon the nature of the decision. For example, if a person is to make a decision which, once made, is irrevocable, the set of consequences will most likely be expressed in terms of utility.[1] This is because many decisions have inherent consequences other than monetary ones. That is, a certain decision may, if a specific state of nature obtains, mean not only a monetary loss but also that the decision-maker loses prestige, position, or both. In such a circumstance, it is doubtful that the decision-maker could express the consequences of that act-state pair in purely monetary terms.

On the other hand, if a decision is revocable or repeatable before the true state has occurred, it may be possible to express the consequences in purely monetary terms. For example, many forest

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[1] Utility may be considered an ordinal measure of the amount of satisfaction which is associated with some thing or event. Chernoff and Moses (1959, p. 82) listed four basic assumptions behind utility.

1. With sufficient calculation an individual faced with two prospects p_1 and p_2 will be able to decide whether he prefers prospect p_1 to p_2, whether he likes each equally well, or whether he prefers p_2 to p_1.

2. If p_1 is regarded at least as well as p_2, and p_2 at least as well as p_3, then p_1 is regarded at least as well as p_3.

3. If p_1 is preferred to p_2 which is preferred to p_3, then there is a mixture of p_1 and p_3 which is preferred to p_2, and there is a mixture of p_1 and p_3 over which p_2 is preferred.

4. Suppose the individual prefers p_1 to p_2 and p_3 is another prospect. Then we assume that the individual will prefer a mixture of p_1 and p_3 to the same mixture of p_1 and p_3 to the same mixture of p_2 and p_3.

management decisions may be made in light of states whose occurrence is several years in the future. In these cases, the decision may be repeated or periodically revised as new information becomes available. As long as the series of decisions is optimal in light of the information available at the time of decision, it may be possible to express the consequences in monetary terms. As indicated, the unit of the consequences should be defined in terms of the decisions to be made. Each decision-maker must decide on the most appropriate measure of the consequences.

As to the money vs. utility measure of the consequences, Schlaifer (1959, p. 29) has devised the following test for the use of money.

> Expected monetary value should be used as the decision criterion in any real decision problem, however complex, if the person responsible for the decision would use it as his criterion in choosing between (1) an act which is certain to result in receipt or payment of a definite amount of cash and (2) an act which will result in either the <u>best</u> or the <u>worst</u> of all the possible consequences of the <u>real</u> <u>decision</u> problem.

For example, suppose a logger has the choice of working for a company at a specified salary or buying a tract of timber on his own with equal chances of a \$15,000 profit and a \$5,000 loss. Therefore, the expected value of buying the timber is \$5,000. The logger should use monetary value as his decision criterion, if: (a) he would prefer buying the timber to the salary as long as the salary is less than $5,000 and (b) he would prefer the salary to buying the timber as long as the salary exceeds $5,000.

Schlaifer (1959, p. 43) proved the above test in the sense that he showed that decision-makers who do not adhere to the test make inconsistent choices.

Pay-off or Loss Table

It is generally convenient, for decision-making purposes, to combine the sets S, A, and C in a pay-off or loss table (Table 1).

Table 1. Example of pay-off or loss table for $(A|S)$[a]

States of nature	Decision alternatives			
	a_1	a_2	...	a_n
s_1	c_{11}	c_{12}		c_{1n}
s_2	c_{21}	c_{22}		2_{2n}
⋮				
s_m	c_{m1}	c_{m2}		c_{mn}

[a] The quantity $(A|S)$ is read: A given S.

Table 1 depicts a $m \times n$ matrix with m states of nature and n decision alternatives. The mn c_{ij} reflect the consequences of choosing decision alternative a_j if s_i is the true state of nature. The pay-off or loss table can illustrate decisions under conditions of certainty, risk, or uncertainty.

Certainty, by definition, occurs in a decision-making situation when only one state of nature is considered. That is, if the definite outcome of each decision alternative is known, only one state of nature is possible. The decision rule under certainty conditions is quite obvious: choose that decision alternative with the most favorable consequence.

The table typifies decisions made under conditions of risk, if there are s_m states of nature $(m > 1)$ and each state has a probability p_i of occurring. The generally accepted decision rule under risk is to choose that alternative with the most favorable expected consequence. That is, compute the quantity $\sum_{l=1}^{m} (p_i c_{ij})$ for all a_j and choose the alternative for which it is most favorable.

Decision-making under uncertainty may be illustrated, if there are s_m states of nature $(m > 1)$ and no objective information as to the probability of the states occurring. There has never been developed a single generally acceptable procedure for making decisions under uncertainty.

Proposed Criteria for Decision-Making Under Uncertainty

As indicated, there is no one best criterion for decision-making under uncertainty. There have, nonetheless, been a number of such criteria proposed. The more popular of these proposed criteria will be examined.

Minimax. According to Luce and Raiffa (1957, p. 298), Wald was the originator of what has become known as the maximin utility or minimax loss criterion for decision-making under uncertainty. To illustrate the minimax criterion, consider the following loss table:

Table 2. Example loss table

States of nature	Decision alternatives		
	a_1	a_2	a_3
s_1	2	7	13
s_2	0	4	10
s_3	15	7	1

Using minimax, the decision-maker associates to each decision alternative its maximum possible loss. That is: $a_1 = 15$, $a_2 = 7$, $a_3 = 13$. The decision-maker then chooses that alternative for which the maximum loss is a minimum. In this case, he would choose a_2. There are various objections to minimax, a primary one being that it is very pessimistic. In other words, the decision-maker acts as though he expects the worst to happen and then chooses the "best worst." In some cases, however, minimax may be appropriate such as when no other criterion can indicate a definitely superior result

(Chernoff and Moses, 1959, p. 149).

Minimax Regret. As an alternative to minimax loss, Savage (1951, p. 55-67) proposed the minimax regret criterion. To apply this criterion, the c_{ij} in the loss table are transformed into r_{ij} where r stands for regret. The r_{ij} for a given s_i are found by subtracting from each c_{ij} the amount required to reduce the minimum c_{ij} for the specific s_i to zero. The reasoning behind this criterion is that it is the difference in losses, not the absolute amount, that is important. The regret table for the previous example would be:

Table 3. Example regret table

States of nature	Decision alternatives		
	a_1	a_2	a_3
s_1	0	5	11
s_2	0	4	10
s_3	14	6	0

The minimax procedure is then applied to the above data. As with minimax loss, minimax regret indicates that the decision-maker should choose alternative a_2. (This similarity is not a necessary condition.) Minimax regret appears to be just as much a pessimistic approach as minimax loss.

Hurwicz Index. The Hurwicz index criterion is a compromise between the pessimism of the preceding approaches and optimism (Luce and Raiffa, 1957, p. 282). The procedure for this criterion is as follows:

1. To the minimum loss, m_j, for each decision alternative, a_j, associate an index, X, such that $0 \leq X \leq 1$.
2. To the maximum loss, M_j, for each decision alternative, a_j, associate the index (1-X).
3. Determine the quantity $Xm_j + (1-X) M_j$ for each a_j.
4. Choose that alternative for which the quantity from (3) is minimum.

The Hurwicz criterion has the advantage of not being completely pessimistic as are minimax loss and minimax regret. However, it also has disadvantages. One source of difficulty is in the choice of the index. If X equals zero, the criterion is identical to minimax loss, therefore, the problem is setting a value for X which is not zero.

Luce and Raiffa (1957, p. 283) demonstrated one method of establishing an index level. They also indicated an inconsistency of the demonstrated method. Even if an index level can be specified, there are difficulties with the Hurwicz criterion. Consider the following example.

Table 4. Example pay-off table

States of nature	Decision alternatives	
	a_1	a_2
s_1	1	0
s_2	0	1
s_3	0	1
s_4	0	1
$\vdots$		
s_m	0	1

The use of any Hurwicz index in the above example will indicate that a_1 and a_2 are equivalent. It seems logical, however, to assume that most persons, operating under uncertainty, would choose a_2.

Laplace or Principle of Insufficient Reason. The Laplace criterion for decision-making under uncertainty is based on the principle that, if the decision-maker has no knowledge about the occurrence of the states, he should act as though they are equally likely. This is synonymous to choosing the decision alternative with the largest average gain or the smallest average loss.

Applying the Laplace criterion to the example used to illustrate minimax indicates that the decision-maker should choose alternative

a_1. The Laplace criterion is reasonable from the standpoint that, if there is no information about the occurrence of the states, there is no reason to believe that they will occur with different frequencies. Conversely, however, there is also no reason to believe that they will occur with equal frequency.

Essentials of Criteria for Decision-Making Under Uncertainty

The four criteria for decision-making under uncertainty reviewed in the above section are major contributions to decision theory. Most texts on the subject review the above criteria in detail. However, as indicated, each of the above criteria has certain disadvantages. The pertinent question then is whether or not current decision theory contains any criterion more acceptable than the above. Before looking for additional criteria, it is useful to consider what characteristics such criteria should possess. Luce and Raiffa (1957, p. 287-298) presented a number of axioms which they considered essential for a decision criterion under uncertainty. Their axioms, in abbreviated form, proceed as follows.

1. For any decision-making problem under uncertainty, there is some a_j in A which is optimal. That is, any problem can be resolved.
2. The optimal a_j does not depend upon the choice of origin and unit of the utility scale used to abstract the problem.

3. The optimal a_j is invariant under the labeling of acts. That is, the real a_j singled out as optimal should not depend upon the arbitrary labeling of acts used to abstract the problem.

4. If a_k belongs to a subset of optimal acts, and a_j weakly dominates or is equivalent to a_k, then a_j is also optimal.[2]

5. Given a_k, if there exists on a_j such that a_j weakly dominates a_k, then a_k cannot be optimal.

6. Adding new acts to a decision-making problem under uncertainty, each of which is weakly dominated by or is equivalent to some old act, has no effect on the optimality or non-optimality of an old act.[3]

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[2] To quote Luce and Raiffa (1957, p. 287): "Axioms 1 through 4 are quite innocuous in the sense that, if a person takes serious issue with them, then we would contend that he is not really attuned to the problem we have in mind."

[3] There is a very important assumption implicit in Axiom 6. Namely, the addition of new acts cannot alter the likelihood of occurrence of the states of nature. In other words, if a decision-making problem under uncertainty is so formulated that the choice of A influences the occurrence of the elements in S, the problem must be reformulated to remove this interaction. Of course, decision-making problems do exist in which the acts and states are not independent. However, if the definitions of the various acts influence the occurrence of the states, the problem is, by definition, no longer one of decision-making under uncertainty. This results from the fact that the decision-maker is no longer confronting "Nature" whose choice of states can be treated as a random variable. Instead, the decision-maker confronts an "intelligent adversary" who will attempt to choose that state which maximizes his position against the possible

7. The addition of new acts does not transform an old, originally non-optimal act into an optimal one, and it can change an old, originally optimal act into a non-optimal one only if at least one of the new acts is optimal.
8. In a probability mixture of two decision-making problems under uncertainty, if the second problem has payoffs which do not depend upon the act chosen, then the optimal set of the mixture problem should be the same as the optimal set of the first problem.
9. If a_j and a_j' are both optimal for a decision-making problem under uncertainty, a probability mixture of a_j and a_j' is also optimal. That is, the optimal set is convex.
10. For any decision-making problem under uncertainty, the optimal act (or acts) should not depend upon the labeling of the states of nature.
11. If a decision-making problem under uncertainty is modified by deleting a state which is a probability mixture of other states, the optimal act (or acts) is not altered.

Axiom 7 contradicts the minimax regret criterion. For

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acts. Game theory models have been developed to arrive at solutions in problems of this type. A discussion of the usefulness of game theory in forest theory in forest management is beyond the scope of this study.

example, consider the following loss table:

	a_1	a_2	a_3
s_1	5	10	15
s_2	20	12	15
s_3	19	25	16

If only alternatives a_1 and a_2 are considered, minimax regret chooses a_2. But, if a_3 is added, minimax regret chooses a_1.

Axiom 8 implies the important characteristic that adding a constant to all the consequences of any particular state does not change the choice of the optimal act. For example, the problem

	a_1	a_2	
s_1	10	5	(utility payoff)
s_2	15	2	

where the states are considered equally likely, can be changed to:

	a_1	a_2	
s_1	110	105	(utility payoff)
s_2	15	2	

without altering the fact that a_1 is the optimal act. This property of axiom 8 contradicts minimax and all Hurwicz criteria.

Luce and Raiffa (1957, p. 293) developed a basic theorem, which derives from acceptance of axioms 1, 3-5, 7-9.

> To each criterion which resolves all decision-making problems under uncertainty in such a manner as to

> satisfy axioms 1, 3, 4, 5, 7, 8, and 9, there is an appropriate *a priori* distribution over the states of nature which is independent of any new acts which might be added, such that an act is optimal (according to the criterion) only if it is best against this *a priori* distribution.

The theorem says, in effect, for each criterion which satisfies the indicated axioms, there is an *a priori* distribution[4] on the states of nature which is compatible with the criterion. This suggests that efforts to determine the distribution may be more worthwhile than efforts to choose among criteria. In other words, use of a criterion implies acceptance of the set of *a priori* probability distributions which leads to the same choice among acts. However, of the previously discussed criteria, Laplace is the only one that satisfies all the indicated axioms.

Axioms 10 and 11 characterize complete ignorance (Luce and Raiffa, 1957, p. 296. Like the first nine axioms, axiom 10 is compatible with the Laplace criterion. In fact, axiom 10 reinforces the importance of the Laplace criterion. The first nine axioms established the importance of an *a priori* probability distribution on the states of nature. Axiom 10 says that the distribution must assign each state a probabliity of 1/n, where there are n states.

Axiom 11, on the other hand, contradicts the Laplace criterion. Consider the decision-making problems (both problems expressed in

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[4]See definition of *a priori* probability distribution, page 45.

utility payoff):

I

	a_1	a_2
s_1	10	0
s_2	4	12
s_3	4	12

II

	a_1	a_2
s_1	10	0
s_2	4	12

The Laplace criterion chooses a_2 in problem I and a_1 in problem II. However, in problem I, s_2 and s_3 can be combined forming problem II. Therefore, in problem II, s_2 is twice as likely as s_1. If the decision-maker considers s_2 twice as likely as s_1, how can he maintain an illusion of complete ignorance?

As indicated, the above axiomatic treatment of a decision-making problem under uncertainty is based upon that given by Luce and Raiffa (1957, p. 287-298). The development and possible consequences of the various axioms are presented in much greater detail by Luce and Raffia.

Additional Concepts for Decision-Making Under Uncertainty

The previously discussed criteria for decision-making under uncertainty and the above essentials for criteria are in conflict. Some of the areas of discrepancy and some of the disadvantages of the criteria have been discussed. There is, however, one disadvantageous characteristic of the above criteria which has not been mentioned and which serves to further remove the criteria from possible consideration. In all the above criteria (minimax, minimax regret, Hurwicz, and Laplace) the decision-maker is forced to act as though he had no opinions regarding the states of nature.

Luce and Raiffa (1957, p. 299) stated this latter criticism as follows:

> A common criticism of such criteria as the maximum utility, minimax regret, Hurwicz index, and that based on the principle of insufficient reason is that they are rationalized on some notion of complete ignorance. In practice, however, the decision-maker usually has some vague partial information concerning the true state. No matter how vague it is, he may not wish to endorse any characterization of complete ignorance, and so the heart is cut out of criteria based on this notion.

The above quotation would seem to place the person with a decision-making problem under uncertainty in a quandary. In other words, a number of criteria have been proposed but it has also been questioned whether or not a rational decision-maker can accept any of the criteria. Fortunately, there is at least one procedure available which seems to resolve this dilemma.

Savage (1954) has proposed what he calls the theory of personalistic probability. Savage does not claim to have orginated the theory, others may call essentially the same thing subjective probability; Savage does, however, seem to be the leading proponent of the concept. According to Savage (1962, p. 163-166),

> Personal probability is a certain kind of numerical measure of the opinions of somebody about something...What are here called "probabilities" are

> not generated by the opinions of just anybody. When we theorize about probabilities, we theorize about a coherent person. By a coherent person I mean pretty much a person who does not allow book to be made against him, a person such that if we offer him various contingencies we cannot, by some sleight of hand, sell him such a bill of goods that he will be paying us money no matter what happens.

The probability distribution generated over the states of nature by the above concept is known as an _a priori_ probability distribution, _i. e._, prior to experimentation. The _a priori_ distribution may be generated in various ways. Luce and Raiffa (1957, p. 302-306) have reviewed a number of procedures for transforming opinions or vague information into a probability distribution. At the other extreme it is conceivable that the _a priori_ distribution could be based on data, such as historical data, which are consistent with the decision-maker's opinions but which do not meet the statistical requisites of an objective probability distribution.

If the decision-maker has an _a priori_ probability distribution on the states of nature, he can treat his problem as a risk situation and proceed in the usual manner. That is, the decision-maker can calculate the expected gain (or loss or utility) for each act and then choose the optimal act. However, if the decision-maker has the opportunity to gain additional information about the states of nature through experimentation, he can refine his decision-making process. For example, consider the following decision-making problem.

A forest fire suppression agency is organized in such a way that it must periodically provide standby crews of fire fighters. These crews must be paid but contribute nothing to the agency other than being available should they be needed for fire suppression.

The problem, then, is to decide upon the optimum size of standby crew. If the crew is too large, there will be excessive wage and overhead costs. If the crew is too small, there will be excessive fire losses.

Assume that the person responsible for making the decision defines the problem as follows:

The set of possible decision alternatives, in number of men per crew, is A: ($a_1 = 10$; $a_2 = 30$; $a_3 = 50$).

The set of possible states of nature is S: (s_1, s_2, s_3, s_4, s_5).[5]

The set C, the consequences of each a_j, gives s_i, is:

	a_1	a_2	a_3
s_1	5	20	45
s_2	4	18	40
s_3	10	6	14
s_4	30	12	6
s_5	50	20	4

[5] Following the U. S. Forest Service classification: s_1 refers to a fire of .25 acres or less; s_2, .26-9.99 acres; s_3, 10-99.99 acres; s_4, 100-299.99 acres; s_5, 300 or more acres.

The set of consequences is in terms of utility loss. In other words, the decision-maker will want to minimize the expected value of the above consequences. There are at least two reasons for using utility instead of monetary loss. One, the specific decision is not repeatable, i.e., mistakes cannot be rectified by subsequent decisions of the same nature. Therefore, a "bad" decision contains loss of prestige, etc., as well as a monetary loss. Second, forest fires commonly destroy values other than monetary ones. It is assumed that the above set of consequences reflects the decision-maker's appraisal of the situation.

The prior distribution on the states of nature can, in this case, be obtained from historical data, for example, monthly records on fire occurrence and size. Assume the historical data indicate the following probability distribution on the states of nature:

$$P: \; (p_1 = .85;\; p_2 = .08;\; p_3 = .04;\; p_4 = .02;\; p_5 = .01)$$

Using the prior distribution, the expected values for each act are: $a_1 = 6.07$, $a_2 = 19.12$, $a_3 = 42.17$. Obviously, with no additional information, the decision-maker should always choose a_1 and use only a small standby crew.

However, for virtually all fire suppression organizations additional information is available. The most common source of this information would be fire danger meter readings. Consider all possible fire danger readings as the set Z. Then, the elements of

Z are the categories of readings $z_1, z_2, \ldots z_k$.

If obtaining the fire danger reading is considered an experiment, the relationship between past occurrences of the states of nature and the experiment may be expressed in the following form (assuming three elements in the set Z):

	z_1	z_2	z_3
s_1	.9	.1	0
s_2	.7	.2	.1
s_3	.5	.3	.2
s_4	.3	.4	.3
s_5	.1	.5	.4

In other words, when s_1 has been the state of nature in the past, z_1 has been the observation (result of experiment) 90 percent of the time, etc.

From the above developments, it can be seen that the *a priori* probability of s_1 occurring is 85 percent and the probability of z_1 being observed, when s_1 occurs, is 90 percent. Therefore, the probability of s_1 occurring and z_1 being observed is (.85)(.90) = 76.5 percent. Similarly, the probability of any combination of s_i and z_k can be computed.

	z_1	z_2	z_3
s_1	.765	.085	0
s_2	.056	.016	.008
s_3	020	.012	.008
s_4	.006	.008	.006
s_5	.001	.005	.004
Σ	.848	.126	.026

The above probabilities add to one, in other words, one of the combinations will occur. The summation row may be considered a density function on Z, f(Z). The interpretation is that regardless which state of nature obtains z_1 will be observed 84.8 percent of the time, etc.

If z_1 is observed, then a .765/.848 percent of the time s_1 will be the state of nature. This probability is the <u>a posteriori</u> probability of s_1, given that z_1 has been observed. The set of <u>a posteriori</u> probabilities is:

	z_1	z_2	z_3
s_1	.765/.848	.085/.126	0
s_2	.056/.848	.016/.126	.008/.026
s_3	.020/.848	.012/.126	.008/.026
s_4	.006/.848	.008/.126	.008/.026
s_5	.001/.848	.005/.126	.004/.026

When combined with the set of consequences to determine the expected utility loss for each decision alternative, the a posteriori probabilities lead to a decision rule or strategy. The decision rule, for this example, is: if z_1 or z_2 is observed, choose a_1; if z_3 is observed, choose a_2. The computations are:

For $(a_1 | z_1)$: (.9021) (5) + (.0660) (4) + (.0236) (10) + (.0071) (30) + (.0012) (50) = 5.28

For $(a_2 | z_1)$: (.9021) (20) + (.0660) (18) + (.0236) (6) + (.0071) (12) + (.0012) (20) = 19.48

For $(a_3 | z_1)$: (.9021) (45) + (.0660) (40) + (.0236) (14) + (.0071) (6) + (.0012) (4) = 43.61

For $(a_1 | z_2)$: (.6746) (5) + (.1270) (4) + (.0952) (10) + (.0635) (30) + (.0397) (.50) = 8.72

For $(a_2 | z_2)$: (.6746) (20) + (.1270) (18) + (.0952) (6) + (.0635) (12) + (.0397) (20) = 17.91

For $(a_3 | z_2)$: (.6746) (45) + (.1270) (40) + (.0952) (14) + (.0635) (6) + (.0397) (4) = 37.31

For $(a_1 | z_3)$: (0) + (.3077) (4) + (.3077) (10) + (.2308) (30) + (.1538) (50) = 18.92

For $(a_2 | z_3)$: (0) + (.3077) (18) + (.3077) (6) + (.2308) (12) + (.1538) (20) = 13.23

For $(a_3 | z_3)$: (0) + (.3077) (40) + (.3077) (14) + (.0635) (6) + (.0397) (4) = 18.62

The value of the above decision rule is: (.848) (5.28) + (.126) (8.72) + (.026) (13.23) = 5.92. That is, if the decision is repeated a sufficient number of times, the utility loss over all decisions will approach 5.92. It can be proven that, given the above data, no other decision rule will produce a lower expected utility loss (Chernoff and Moses, 1959, Ch. 5-6).

The preceding method of formulating a decision rule by using the results of an experiment to modify a prior opinion has been termed the Bayesian model for decision-making under uncertainty. The name derives from the use of Bayes' theorem in converting the _a priori_ distribution into the _a posteriori_ distribution.

The theorem, which is essentially a mathematical statement of conditional probability, was introduced by Thomas Bayes in the eighteenth century (Savage, 1962, p. 162). Bayes' theorem may be stated as follows:

$$P(s_i | z_k) = \frac{P(z_k | s_i)\, P(s_i)}{P(z_k)}$$

where P = probability; s_i = the i th state of nature; z_k = the k th result of an experiment.

Further:

$P(s_i | z_k)$ is the _a posteriori_ probability of s_i occurring, given that z_k has occurred.

$P(z_k | s_i)$ is the probability with which z_k has been the result of an experiment when s_i was the state of nature

$P(s_i)$ is the _a priori_ probability of s_i occurring

$P(z_k)$ is the density function $f(Z)$ on z_k, that is, the probability of z_k being the result of an experiment.

An example should help clarify Bayes theorem. Consider one perfectly balanced die. Call the probability of an odd number occurring on one roll of the die A. Call the probability of the same roll producing a number larger than one B. Then, the probability of A, P(A) is 1/2 and the probability of B, P(B), is 5/6. The probability of both occurring, P(A, B), is 1/3. However, if it is known that B has already occurred then the probability of A occurring, $P(A|B)$, is 2/5. This is the a posteriori probability that A will occur, given that B has occurred. It may be calculated by Bayes theorem.

$$(1) \quad P(A|B) = \frac{P(B|A)\ P(A)}{P(B)} = \frac{(2/3\ (1/2)}{5/6} = 2/5$$

similarly

$$(2) \quad P(B|A) = \frac{P(A|B)\ P(B)}{P(A)} = \frac{2/5 \cdot 5/6}{1/2} = 2/3$$

Bayesian decision theory would seem to have a great number of possible uses in the broad area of decision-making under uncertainty. It provides a logical, systematic, and consistent procedure by which a decision-maker can process both experimental evidence and opinions or non-statistical data to reach a decision. In other words, it allows the decision-maker to systematically utilize all his information in a way that is consistent with the decision criterion of maximum expected utility (or minimum expected loss, etc.).

Raiffa and Schlaifer (1961, p. 3) described the type of situation to which Bayesian decision theory may be advantageously applied.

> ...we shall be concerned with the logical analysis of choice among courses of action when (a) the consequence of any course of action will depend upon the "state of the world", (b) the true state is as yet unknown, but (c) it is possible at a cost to obtain additional information about the state. We assume that the person responsible for the decision has already eliminated a great many possible courses of action as being unworthy of further consideration and thus has reduced his problem to choice within a well-defined set of contenders; and we assume further that he wishes to choose among these contenders in a way which will be logically consistent with (a) his basic preferences concerning consequences, and (b) his basic judgements concerning the true state of the world.

Raiffa and Schlaifer then enumerated the basic data necessary to make such a decision.

1. The set, A, of possible decision alternatives or acts.
2. The set, S, of the possible states of nature.
3. A set of experiments, E, any one of which may be performed to obtain additional information on S.
4. The set, Z, of the possible outcomes of the performed experiment.
5. The set, C, of the various consequences of performing a certain experiment, observing a certain outcome, taking a certain action, and finding that a certain state obtains.
6. Various probability assessments, including:
    a. P(S): the a priori probability distribution on the states of nature.
    b. $P(Z \mid E, S)$: The probability distributions of the outcomes given that a certain experiment has been performed and a certain state obtains.
    c. $P(Z \mid E)$: The probability distribution of Z given that a certain experiment has been performed.

d. $P(S|Z)$: The a posteriori probability distribution on the states of nature.

The above data can be combined as in the fire crew example to lead to a Bayesian decision strategy. The Bayesian decision strategy for the example was: if z_1 or z_2 is the outcome, choose a_1; if z_3 is the outcome, choose a_2. The value of this strategy was shown to be 5.92. In the literature, this value is referred to as the Bayes risk.

The second best strategy, in the example, was to base the decision on the a priori probabilities. The expected value from always choosing a_1 (in light of the experimental evidence) is 6.07. Consequently, performing the experiment and obtaining the set of outcomes, Z, is worth .15 units of utility loss to the decision-maker. The decision-maker can then determine whether the expected gain from the experiment is worth the cost of performing the experiment. Also, if additional experiments can be performed, the information resulting from each experiment can be used to calculate new a posteriori distributions. Thus, each bit of information is "digested" as it is obtained.

Additional support for the Bayesian approach to decision-making under uncertainty may be obtained from Baumol (1965, p. 558-567). Baumol discussed the more common decision criteria and developed a method for depicting them graphically. For example, consider the following loss table.

	a_1	a_2
s_1	3	2
s_2	1	4

Application of the minimax criterion to the above problem leads to a choice of decision alternative a_1. The minimax criterion can also be expressed graphically (Fig. 1).

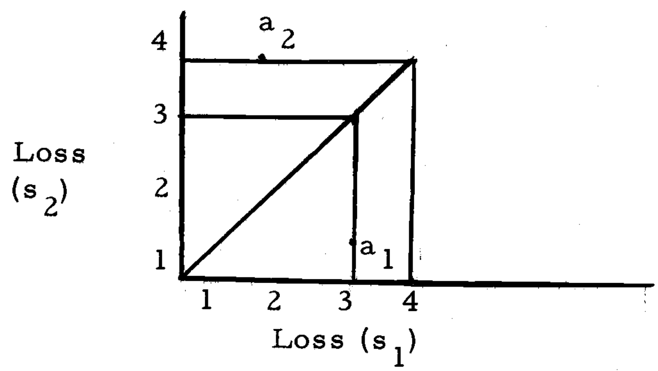

Figure 1. Graphical representation of minimax decision criterion

The reasoning behind Figure 1 is as follows. If two possible acts lie on the 45 degree line, the one nearer the origin is preferred. Since the minimax criterion considers only the maximum consequence of each act, a decision-maker is indifferent between two acts only if they have the same maximum loss. Consequently, by referencing the maximum loss for each act to the 45 degree line, perpendicular lines can be drawn to the axes, forming a square. The act (or acts) which creates the smallest square is the optimal act (or acts). The decision-maker is indifferent between those acts which lie on the lines forming the square.

The analogy of the above to the economic theory of indifference curves is obvious. The person who employs the minimax decision criterion has an implicit indifference curve like those in Figure 1; or, conversely, only a person with an indifference curve like those in Figure 1 should employ the minimax decision criterion. The curves in Figure 1, like the minimax decision criteria, reflect a pessimistic approach to decision-making under uncertainty.

Baumol concluded his discussion of decision criteria by proving that, if a person accepts the basic assumptions underlying utility,[6] he will necessarily employ a Bayesian approach to decision-making under uncertainty.

Summary

A number of theoretical concepts for decision-making under uncertainty are available in the literature. Certain properties are common to all of these theoretical concepts. The properties are: the states of nature, the decision alternatives, and the consequences which associate each alternative to each state. The properties are combined in a pay-off or loss table to identify a decision-making

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[6] See basic utility assumptions, page 30.

problem under uncertainty.

The more commonly discussed concepts for decision-making under uncertainty, i. e., minimax, minimax regret, Hurwicz, and Laplace, all require the decision-maker to assume complete ignorance about his decision-making situation. No matter how vague a decision-maker's information or opinions, it is doubtful that he can accept the complete ignorance assumption.

The Bayesian approach to decision-making under uncertainty allows the decision-maker to escape the complete ignorance assumption. The Bayesian approach allows the decision-maker to combine his opinions or subjective judgement with empirical data to arrive at a decision which is both consistent with his preferences concerning consequences and his judgements concerning the occurrence of the states of nature.

It appears intuitively logical that the Bayesian approach to decision-making under uncertainty reflects the way in which many such decisions are actually made. That is, the decision-maker identifies his problem in terms of the various alternatives he may elect. The consequences, associated with each alternative, result from possible states of nature over which the decision-maker has no control but about which he may have opinions or information relative to occurrence. The decision-maker may decide upon a strategy at this point. If so, he presumably elects a strategy which is consistent with his definition of consequences and his opinions regarding the relative occurrence of the states of nature.

On the other hand, the decision-maker may be in a position to obtain additional information about the states of nature. That is, he may be able, through experimentation, to obtain some predictive data on the states of nature. Depending upon the cost of the data, they may or may not be worthwhile obtaining. If the data are obtained, the decision-maker may use them to reinforce or revise his prior feelings regarding the states of nature. If the data becomes available in steps or if successive experiments are possible, the decision-maker can "digest" each piece of information as it is obtained.

IV. A POSSIBLE APPLICATION OF BAYESIAN DECISION THEORY TO FOREST MANAGEMENT

The existence of uncertainty in forest management decision-making has been established. That there is a theoretical framework for making decisions under uncertainty has also been established. The present chapter will illustrate how this theory can be applied in forest management decision-making.

Example Decision Problem: Optimum Degree of Land Ownership in the Southern Pulp and Paper Industry

The example to be developed is not empirical and the results cannot be interpreted as such. However, general conclusions concerning the applicability of decision theory to forest management can be drawn. When possible, published data will be used to develop the example. When such data are not available, assumptions, to permit continuation of the example, will be employed.

The southern pulp and paper industry is composed primarily of vertically integrated firms. That is, in addition to the pulp and paper manufacturing facilities, the firms also supply some of their own raw material and market the finished product. The example decision-making problem which will be considered is the decision of how much raw material to supply from within the firm.

Characteristics of the Southern Pulp and Paper Industry

Before delving into the actual decision-making problem, it is helpful to understand some of the basic characteristics of the industry. The southern pulp and paper industry is composed of those pulp and paper corporations which are located along the Atlantic and Gulf coasts from Maryland to Texas. The region includes all or part of 14 states. At present, the industry is composed of 50 individual corporations operating 81 pulp mills (Knight and Nichols, 1964, p. 25-26). In aggregate the industry owns over 23 million acres of commercial forest land, 11 percent of the commercial forest land in the region (South, Pulpwood Cons. Assn., 1964).

In a study of the pulp and paper industry in the United States, Guthrie (1950, p. 111) classified the market structure of the industry as oligopolistic. In other words, pulp and paper exhibits the distinguishing characteristic of an oligopolistic industry; there are few enough firms that the action of one can influence the others. In 1963, the firms composing the southern pulp and paper industry produced over 54 thousand tons of pulp per 24 hours, accounting for 60 percent of the nation's total (Knight and Nichols, 1964, p. 1).

Guthrie (1950, p. 1) indicated that one of the characteristics of the pulp and paper industry is its continual search for better and cheaper raw materials from which to make paper. In the region under consideration, southern yellow pine presently constitutes the largest portion of the industry's raw material. However, hardwoods are becoming increasingly important. In 1963, hardwoods accounted for 22 percent of the region's pulpwood production, an increase of five percent over the next highest year (Van Sickle and Sternitzke, 1964). The increasing use of hardwoods is illustrative of the dynamic character of the pulp and paper industry. That is, what is an acceptable management policy at one time may be unacceptable at a later time.

Before a firm can produce paper, it must acquire the raw material, in this case wood. There are, basically, only two possible sources of the raw material. The firm may obtain raw material internally or externally. The following distinction between internal and external will be used throughout the current example. Internal raw material refers to all wood produced on land owned or controlled by the firm. External raw material refers to all wood produced on land not controlled by the firm. Such wood is commonly purchased in an open market and is frequently referred to as open-market wood. Some confusion could arise from wood committed to the firm by long-term wood supply contracts. Depending upon the length and terms of the contract, such wood can be considered internal or external.

It is assumed that firms within the southern pulp and paper industry have as their over-all objective the maximization of profit subject to certain managerial and institutional constraints. Therefore, it is also assumed that the degree of internal wood supply should be justified on the same basis. However, the actual profitability of corporate forest land is difficult to determine. For example, internal wood should be priced to the mill at the cost of external wood if the internal wood were not available. But, the internal wood is available; therefore, an objective value for the substituted external wood cannot be established. It is a fact, however, that the southern pulp and paper industry owns a considerable amount of forest land. Some justifications for this ownership have appeared in the literature. An explicit reasoning for corporation ownership of forest land was given by Fedkiw (1960, p. 5).

> The main justification for investment in forest enterprises among integrated wood processing firms has been the strategic value of company control over the source of supply of its basic raw material. The term strategic is used because the major benefits are largely risk-reducing in character; real enough but difficult to quantify, accruing more or less to all other parts of the business, and extending more or less indefinitely into the future. The forest enterprise is expected to sustain a certain flow of wood to company plants, protect the firm against risks with respect to price, quantity and delivery schedule associated with outside supply sources, and supply long-term security for the firm's share of the product market and its profit position.

Segur (1960, p. 111) listed three reasons for corporate forest ownership.

1. Protection against intermittent plant shutdowns due to spot shortages caused by weather, labor, and other temporary area conditions.
2. Protection against extended plant shutdowns at some time in the future due to a basic supply deficiency.
3. Protection against runaway prices of market pulpwood.

Integration into raw material ownership, such as in the pulp and paper industry, has been termed "backward integration." Dean (1951, p. 141-143) listed four reasons for a firm practicing backward integration.

1. To produce products at a lower cost than their purchase price.
2. To reduce the risks associated with outside supply.
3. To protect against shortages of supply.
4. To protect the long-run position in supply markets.

The preceding characterization of the southern pulp and paper industry is admittedly brief. However, it introduces the industry and the industry's involvement in forest management sufficiently well to permit further development of the current example.

Defining the Decision-Making Problem

The defining of the decision-making problem is very germane to the eventual production of a desirable solution. The person who must make the decision must play a major role in the entire problem formulation process. No one else can define the problem for the actual decision-maker. Unless the alternatives, states, prior distribution, etc. accurately reflect the decision-maker's appraisal of the problem, resulting decisions will be little more than an interesting exercise.

Given that a hypothetical member firm of the southern pulp and paper industry can supply its raw material needs internally, externally, or by an internal-external combination, what degree of internal supply is optimal for the firm? Before the problem can be further refined, optimal must be identified within the framework of the particular firm. Assume that the hypothetical firm to be considered in this example has the objective of supplying that amount of wood internally which will minimize the present worth of all raw material costs over time. Actual cost, rather than utility, can be the object of minimization for two reasons. First, the decision of how much wood to supply internally is not a one-time decision. The firm can make the same decision annually or periodically as it sees fit. For this reason, past mistakes can be rectified before they create an adverse condition. Second, it is assumed that, within the relevant cost ranges, the firm's utility function for money is linear. That is, each dollar of present worth is equivalent to every other dollar of present worth.

It should be stated initially that the decision-making problem to be considered in the current example falls into the area of policy decisions rather than action decisions. That is, a member firm of the southern pulp and paper industry may establish a policy of supplying a certain portion of its raw material internally. The actual procedure for implementing the policy is independent to the extent that the established policy is capable of being implemented.

Some technical and economic assumptions are necessary in defining the example problem.[7] Following the assumptions, each set of data, *i. e.*, decision alternatives, states of nature, *a priori* probabilities, consequences, and the experiment will be developed.

Assumptions

1. The objective in the decision-making problem is to minimize the present worth of the expected cost of raw material procurement over the firm's planning period.
2. The firm's planning period is known.
3. The firm's raw material consumption over the planning period is known.
4. Current costs for internal and external raw material are known.
5. The discount rate applicable to forest management is known.
6. Volume yields for various intensities of silviculture are known.

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[7] The critique, page 117, contains an appraisal of these assumptions.

7. Logging will be done by contract loggers. Therefore, for the individual pulp and paper corporation, there is no fixed cost associated with logging which, on a cord basis, would fluctuate with the level of harvesting activity.
8. There is no interdependence among the purchasers of external wood. That is, external wood is not purchased in an oligopsonistic market.[8]
9. The set of consequences can be expressed in monetary terms.

Decision Alternatives. Since the objective of the decision is to choose that level of land ownership which will minimize the firm's expected raw material cost over its planning period, the decision

[8]This is a very critical assumption. If the market for external wood is oligopsonistic, there is interaction between the states and acts. As indicated previously, such problems require game theory rather than uncertainty models for solution. By definition, an oligopsony is a market with a small number, but more than two, buyers. However, number alone does not establish an oligopsony; there must also be an interdependence among the buyers (Henderson and Quandt, 1958, p. 164, 175). Recent attempts by the federal government to establish this interdependence among pulp and paper corporations in the Lake States have not been successful. Also, competition among sellers of external wood may tend to maintain a competitive level of external wood cost. This latter point is supported by the finding of the Battelle Memorial Institute that delivery price of pulpwood in the southeast is very close to true cost of production (Hamilton and Grimm, 1963, p. 10-11). To definitely establish the type of structure existing in the pulpwood market might require an econometric study of pulpwood supply and demand. Such an effort is outside the scope of this study.

alternatives are degrees of land ownership. The possible amount of land owned is, of course, a continuous function between no land and enough land to supply all raw material internally. For convenience, the set A will be defined as six discrete elements:

$$a_1 = 0\%;\ a_2 = 20\%;\ a_3 = 40\%;$$

$$a_4 = 60\%;\ a_5 = 80\%;\ a_6 = 100\%$$

where the individual percentages reflect enough land either owned or controlled by the firm to eventually supply that percentage of raw material internally.

Additional alternatives could have been chosen. The six chosen cover the range of relevant choices and more would not add to the illustration.

<u>States of Nature</u>. The definitions of the states of nature are not as obvious as the definitions of the acts. The hypothetical firm uses the following procedure to identify the states of nature. In order to quantify the objective of cost minimization over time, it is necessary to specify some discrete time period. The firm's planning period should be the relevant time period for this quantification.

According to Ciriacy-Wantrup (1952, p. 32) a planning period is "...any period of time over which use is planned or over which use is considered..." Ciriacy-Wantrup also indicated that the use of discounted future values and the presence of uncertainty will tend to shorten the planning period, other things equal (p. 79, 126). Of course, both of these factors operate in a forest enterprise.

Duerr (1960, p. 225) expressed some thoughts regarding planning horizons in forestry.

> . . . some firms plan further ahead than others. For some, the contours of the future dip sharply away, and the planning horizon, beyond which they peer not, is close at hand. For others, the vistas are flat and far-spreading, and the horizon is formed only by the distant, inevitable mists of uncertainty. Such differences are the product of the persons whose attitudes and vision represent the firm, and of the firm's line and facilities of production, particularly its period of production and the durability of its capital. They are also the product of the social circumstances that surround resource use, especially those determining risks.[9] . . . the rate of interest, reflecting risk may function to set a planning horizon. The professional forest manager, by virtue of such influences, is typically a long-range planner as human ranges go.

Consequently, there are a number of factors which will influence the length of planning horizon for an integrated forest products firm. Some factors are: length of rotation for timber production, depreciation period for plant and major equipment, stability of markets for final products, etc. To any individual firm, the actual planning horizon chosen will result from weighing all relevant factors. Assume, for this example, that the firm's planning horizon is 20 years; also, assume that raw material consumption over the period is known to be 500 thousand cords per year.

The latter part of the above assumption states that the firm will

[9]Duerr does not distinguish between risk and uncertainty.

continue to use a given quantity of raw material regardless of the price of that raw material. Certainly, there is some level of raw material price beyond which the firm would reduce its raw material consumption. The assumption implies, however, that within the realm of possible raw material costs, quantity of raw material used is fixed.

Given the above assumption, it follows that the absolute price of internal and external raw material is not relevant. Rather it is the relative prices of the two sources of supply that are important. For this reason, the individual states of nature are defined as follows:

> the annual percentage change in the price of external wood minus the annual percentage change in the price of internal wood over the firm's planning period.

For example, a state of nature of (-1) would be interpreted as the cost of external wood changing at a rate of one percent per year less than the cost of internal wood over the next 20 years. The absolute change in cost could be positive or negative.

The cost of external wood is easily obtained. The U. S. Forest Service has published southeastern pulpwood prices for the years 1938-1963 (Robinson, 1961; Knight, 1964). For purposes of this study, the average price of pulpwood, delivered to the pulp mills by truck, will be considered the cost of external wood (Table 5).

The cost of internal wood is not so easily obtained. Very few published reports are available on the costs of growing southern pulpwood. However, there have been at least two such studies. Worrell (1953, p. 5, 17) reported on a 1953 survey of costs of practicing forestry in the South. Somberg, Eads, and Yoho (1963, p. 6-8, 15-17), updated Worrell's study.

Using average costs of site preparation and machine planting on cutover land from the studies by Worrell and Somberg, Eads, and Yoho, it was found that these costs increased an average of eight percent each year over the decade 1953-1963. The cost of growing wood is roughly 35-40 percent of delivery cost. Therefore, the change in the cost of internal wood, due to timber growing costs, is assumed to be between 2.8 and 3.2 percent each year. Between 1953 and 1963, the U.S.D.A. index of farm wage rates increased an average of 2.8 percent per year (U.S. Bureau of the Census, 1964, p. 242). Therefore, it is assumed that the U.S.D.A. farm wage rate index reflects the annual changes in the cost of producing internal wood between 1938 and 1963 (Table 5).

A problem arises in attempting to identify discrete elements of the set S. The data on external and internal wood costs are available only for the period 1938-1963. Therefore, if the definition of the states which corresponds to the firm's planning period of 20 years is retained, only six observations are available. This

Table 5. Average external wood cost and index of internal wood production cost, 1938-1963

Year	Cost of external wood[a]	Index of internal wood production cost[b]
	Dollars	1957-59 = 100
1938	3.85	22
1939	4.40	22
1940	4.60	22
1941	5.00	27
1942	6.65	36
1943	8.00	47
1944	8.70	56
1945	9.15	63
1946	10.75	68
1947	11.70	73
1948	12.30	76
1949	11.80	74
1950	12.55	74
1951	14.70	82
1952	14.70	87
1953	14.70	88
1954	14.75	87
1955	15.05	89
1956	16.45	93
1957	16.35	96
1958	16.10	99
1959	16.60	105
1960	17.20	107
1961	16.85	110
1962	17.35	112
1963	17.40	116

a/ Robinson, 1961. Knight, 1964.

b/ U.S. Bureau of the Census, 1960, p. 280; 1964, p. 242.

situation presents three alternatives. One, use the data as originally outlined and have only six observations; two, shorten the planning period to provide more observations while maintaining agreement between the states and the planning period; or three, maintain the 20 year planning period but redefine the states to provide more observations, thus sacrificing some agreement between the planning period and states. On the bases that six observations are too few and that the 20 year planning period should be maintained, the third alternative was chosen. Consequently, the states of nature were redefined as:

> The annual percentage change in the price of external wood minus the annual percentage change in the price of internal wood over a ten year period.

Redefining the states of nature gives 16 observations. Using the cited data and the above definition, the following states occurred during the period 1938-1963.

$s_1 = -3\%$ $\qquad s_2 = -2\%$

$s_3 = -1\%$ $\qquad s_4 = 0\%$

$s_5 = 1\%$ $\qquad s_6 = 2\%$

What is known about the states of nature determines whether a decision-making problem is formulated under certainty, risk, or uncertainty. In the current example, the states of nature refer to the future levels of raw material costs. In addition, these cost levels are operative over a 20 year period. There is some historical credence to considering the six specific states. However, there is no way to associate the occurrence of the states with an objective probability distribution. There can be little doubt that the example is a decision-making problem under uncertainty.

Consequences. The consequence which associates each alternative to each state is composed of two factors: the cost of internal wood and the cost of external wood. As indicated, the cost of external wood is readily available. In 1963, the average cost of wood trucked to southeastern pulp mills was $17.40 per cord (Knight, 1964). However, as individual mills buy varying quantities of external wood, the average cost will fluctuate. How much it will fluctuate can best be answered while considering the cost of internal wood.

To establish a base from which to calculate the cost of internal wood at the mill, it is necessary to adopt additional assumptions.

1. The basic element of land ownership is a wood production unit (WPU) rather than an acre.

A WPU is equal to an acre of average productivity. Total WPU's equals total acres. This assumption neutralizes site differences. An underlying implication of the assumption is that land value reflects ability to produce net income and if one acre can produce twice as much net income as a second acre, the first acre should be worth twice as much as the second.

2. Average rotation is 30 years.

This assumption is supported by a recent study by Chapelle and Nelson (1964, p. 500-501) which indicated that optimum pulpwood rotation for loblolly pine is equal to or close to 30 years for most sites and discount rates.

3. Land value is 30 dollars per WPU.

This figure is approximately equal to the optimum soil expectation value implied by Chappelle and Nelson (1964, p. 491) for site 80 loblolly pine assuming a 28 dollar regeneration cost, five percent rate of discount, and a two dollar per acre annual expense.

4. All timber growing costs, except for regeneration, may be expensed on the corporation's federal income tax return.

5. Two levels of silvicultural intensity are possible.

   Level I produces an average of 45 cords of wood per WPU per 30 year rotation.

   Level II produces an average of 55 cords of wood per WPU per 30 year rotation.

Level I corresponds to Chappelle and Nelson's optimum stocking level for loblolly pine, site 80, with a 28 dollar per acre regeneration cost. For level II it is assumed, for this study, that ten additional cords may be produced per rotation by increasing the regeneration cost to 48 dollars per WPU.

6. Present fair market value of stumpage is six dollars per

cord.

7. The firm can purchase between 6.5 and 7.0 percent of the total land in its timbershed. Also, the firm will prefer land close to the mill.

This assumption implies that, under silvicultural level I, the firm will need to control land over a total area of approximately one million acres for each 20 percent of its raw material supply coming from internal sources. (Twenty percent of requirement equals 100 thousand cords per year.) It will also be assumed that external wood will be drawn from an area of one million acres per 20 percent of mill requirement coming from the open-market. However, if the firm elects to produce internal wood under silvicultural level II, it need only cover an area of approximately 815 thousand acres per 20 percent of raw material supplied internally.

8. The average cost of harvesting, exclusive of hauling, is presently $5.60 per cord.

In a study for Battelle Memorial Institute, Hamilton and Grimm (1963, p. 11) found that average cost to southeastern pulpwood producers, excluding stumpage, was eight dollars per cord. In a separate report, within the same study, Hamilton _et al_. (1961, p. 9) indicated that hauling accounted for approximately 30 percent of the cost of getting a cord of wood from the stump to the mill.

9. The $2.40 per cord average hauling cost implied in the

above assumption is assumed to apply to 20 percent internal supply, silvicultural level I, an average load of three cords, and an average of six loads per day.

The average haul for an area of one million acres is approximately 16 miles, resulting in a cost per mile of $.225. The cost per mile remains constant. However, as average hauling distance increases, the number of loads per day decreases; therefore, there will be an increased cost per cord for wood from farther points.

10. The firm may use any combination of silvicultural levels I and II. For convenience, the areas of land pertaining to successive increments of 20 percent internal supply are considered zones; e.g., zone 3 is that area of land which can supply 41 to 60 percent of the firm's raw material. A particular level of silvicultural intensity will be maintained throughout a zone.

Under the assumptions of this study, all possible combinations of silvicultural intensity and zones may be easily considered. In an actual situation, linear programming or some other technique capable of handling a large number of variables might be necessary. The optimal combinations of silvicultural levels, as the firm supplies more of its raw materials internally are:

If the firm supplies 20 percent of its wood internally, all production is at silvicultural level I.

If the firm supplies 40 percent of its wood internally, production is at level II in zone 1 and at level I in zone 2.

If the firm supplies 60 percent of its wood internally, production is at level II in zone 1 and at level I in zones 2 and 3.

If the firm supplies 80 percent of its wood internally, production is at level II in zones 1 and 2 and at level I in zones 3 and 4.

If the firm supplies 100 percent of its wood internally, production is at level II in zones 1 and 2 and at level I in zones, 3, 4, and 5.

The above merely quantifies what is intuitively obvious. As the firm goes farther and farther from the mill for raw material, it can afford to spend more money to produce higher volumes on land near the mill. The amount of additional money which can be spent is equal to the discounted value of the savings in transportation cost.

11. The relationship between zones of wood production and cords delivered to the mill per truck per day is assumed to be:

Zone	Cords delivered per day
1	18
2	15
3	12
4	9
5	9

Given the above assumptions, the present cost, over the planning period, of wood at the mill for the two sources of supply may be calculated (Table 6). An example of the calculations is also provided (Table 7).

The calculated costs can be expanded to the set of consequences (Table 8). For example, for state s_1 and 60 percent internal supply:

Internal wood

300M cords per year at $14.95 per cord is $4,485,000 per year. present value, at 5% interest, of $4,485,000 per year for 20 years is 55.9 million dollars.

Table 6. Cost per cord for wood delivered to mill by source of supply and state of nature

Source of supply	State of nature	Degree of land ownership in percentage of raw material supplied internally					
		0	20	40	50	80	100
		Dollars					
External	All states	18.00	17.40	16.80	16.05	15.15	0
Internal	s_1	0	13.50	14.35	14.95	15.50	15.95
	s_2	0	13.45	14.30	14.90	15.45	15.90
	s_3	0	13.35	14.20	14.80	15.35	15.80
	s_4	0	13.20	14.05	14.65	15.20	15.65
	s_5	0	13.05	13.90	14.50	15.05	15.50
	s_6	0	12.70	13.55	14.20	14.75	15.20

Table 7. Example calculation for wood cost at the mill-internal source of supply, state of nature s_4, 40 percent internal supply

Cost of growing wood (interest rate = 5%)			
Regeneration cost			$28/acre
Annual expenses (other than use of land)			$2/acre/year
After expensing for federal income tax purposes			$1.04/acre/year
Land value			$30/acre
Cost of using land			$1.50/acre/year
Total cost at 30 years			$289.77/acre
Volume yield at 30 years			45 cords/acre
Stumpage value (fair market value)			$6/cord
Value yield at 30 years			$270/acre
Less capitalized regeneration cost			$242/acre
Amount of income taxed at 25% rather than 48% due to internal timber production			$242/acre
Tax savings due to internal timber production			$55.66/acre
Total cost of growing wood			
per acre			$234.11
per cord			$5.20
Cost of harvesting (excluding hauling), per cord			$5.60
Cost of hauling			
Zone	Cords per day	Miles per day	Cost per cord
1	18	192	$2.40
2	15	270	$4.05
Weighted cost per cord for hauling			$3.25
Total cost delivered to mill, per cord			$14.05

Table 8. Present value of raw material costs by state of nature and decision alternative

State of nature	Decision alternative a_1	a_2	a_3	a_4	a_5	a_6
	Million dollars					
s_1	88.3	85.1	85.2	87.4	92.2	99.4
s_2	95.2	90.3	88.8	89.7	92.9	99.1
s_3	103.0	96.3	93.1	92.0	93.9	98.5
s_4	112.2	103.2	97.8	94.8	94.7	97.5
s_5	122.7	111.3	103.4	98.0	95.6	96.6
s_6	134.8	120.0	109.3	101.2	96.2	94.7

External wood

Average cost over first decade of planning period is

$$\left[16.05 + 16.05\,(1.03)^{-10}\right] \div 2 = \$14.00 \text{ per cord.}$$

200M cords per year at $14.00 per cord is $2,800,000 per year.

Present value, at 5% interest, of $2,800,000 per year for 10 years is 21.6 million dollars.

Average cost over second decade of planning period is

$$\left[16.05\,(1.03)^{-10} + 16.05\,(1.03^{-20}\right] \div 2 = \$10.41 \text{ per cord.}$$

200 M cords per year at $10.41 per cord is $2,082,000 per year.

Present value, at 5% interest, of $2,082,000 per year for 10 years, beginning in 10 years, is 9.9 million dollars.

Total present value of internal and external wood: 55.9 + 21.6 + 9.9 = 87.4 million dollars.

The six levels of cost for internal supply, one for each state of nature, need to be explained. The example cost calculation in Table 7 is for state s_4, no relative change in internal and external wood costs. However, if the example had been calculated for s_3, a relative decrease of one percent per year in external wood cost, there would be a change in the level of external stumpage. The level of external stumpage influences the "fair market value" of internal wood for federal income tax purposes.

Changing the state of nature from s_4 to s_3 implies that the <u>relative</u> stumpage value at the end of the firm's planning period will be: $\frac{6}{(1.01)^{20}}$ = $4.92. Therefore, the average stumpage value over the planning period will be $5.46.

Substituting $5.46 for the $6.00 stumpage value in Table 7 results in a total cost delivered to the mill of $14.20.

In Table 8 the decision alternatives are based on the decision-maker's opinion of what actions he can possibly take. The states of nature are established by checking historical data and determining those states that have occurred in the past. The consequences, associated to every act-state pair, are the costs of all the firm's raw material for the next 20 years. The cost figures are in millions of dollars present worth.

For purposes of this example, present worth was computed using a discount rate of five percent. A number of rates could have been used. Theoretically, the correct rate is the opportunity cost of the capital invested in raw material procurement. Presumably, this rate should not be less than the long-term, after-tax earning rate of the corporation. Currently, this long-term rate is around eight percent (U.S. Bureau of the Census, 1964, p. 497). However, as indicated, there is a certain strategic, but unquantifiable, aspect to a firm supplying some of its raw material. This peculiar quality of internal raw material is commonly referred to within the industry as "the insurance value of company wood." Actually, the term insurance, used in this manner, is a misnomer. To insure something infers that an expected loss can be computed, that is, risk can be insured against, uncertainty cannot. As seen, the types of phenomena which internal wood is supposed to "insure" against are primarily uncertain. The strategic aspect of internal raw material would indicate that a discount rate lower than eight percent should be used. The rate most often mentioned in this connection by southern industrial foresters is five percent. The choice of discount rate will be subsequently relaxed to ascertain its influence on the decision-making problem.

It can be observed that, regardless of the true state of nature, a_2, a_3, and a_4 are preferred to a_1 (Table 8). In decision theory terminology, a_1 is dominated. That is, a_1 will never be optimal no matter what state obtains. In other words, for this example, the firm will always supply a minimum of 20 percent of its raw material requirement internally. For this reason, a_1 will not be considered further.

A Priori Probabilities. At least one of the remaining admissible acts is optimal for some state of nature (Table 8). As pointed out in Chapter III, for each admissible act there is a set of probability distributions on the states of nature for which that set is optimal. The decision-maker's task is now to attempt to identify the probability distribution upon which he will base his decision.

As indicated, there are a number of criteria for decision-making under uncertainty which do not require the quantification of a probability distribution on the states. However, these criteria (minimax, minimax risk, Hurwicz index, and Laplace) all require an endorsement of complete ignorance which a decision-maker with a "feel" for his problem cannot accept. Chernoff and Moses (1959, p. 163) commented on this point.

> When a problem is approached, there is usually available a great deal of miscellaneous and somewhat relevant information which may lead one to think of certain states as more likely than others. With this information one may approximately measure one's "degree of belief" by some *a priori* probabilities of the states.

As brought out in Chapter III there is a range to the "degree of belief" one may hold for the *a priori* probability distribution. It may be nothing more than pure subjective judgment. On the other hand, there may be objective bases for selecting some probability distribution over others. Of course, if the basis were completely objective and the decision was to be repeated many times, the problem would be one of risk rather than uncertainty.

In the current example there is some historical data available to, at least, approximate a prior distribution on the states of nature.

The six discrete states of nature occurred with the following frequencies.

State	Frequency	Relative frequency
s_1	1	.0625
s_2	2	.1250
s_3	5	.3125
s_4	4	.2500
s_5	3	.1875
s_6	1	.0625
	16	1.0000

Sixteen observations led to the above distribution. The distribution is awkward and, with only 16 observations, there is no reason to regard it as exact. Therefore, for convenience and to better fit the decision-maker's concept of the problem, the *a priori* distribution was smoothed to:

$$P(s_1) = 0.05; \; P(s_2) = .15; \; P(s_3) = .30$$

$$P(s_4) = .25; \; P(s_5) = .20; \; P(s_6) = .05$$

Schlaifer (1961, Ch. 7) has discussed the rationale and procedures for smoothing historical frequencies to obtain prior distributions. Basically, unless the probability distribution is completely objective, the decision-maker should use that distribution in which he has the most confidence. With the above smoothed distribution, the problem of optimum level of internal raw material supply can be depicted by a cost table (Table 9).

Table 9. A priori probabilities and present value of raw material costs by state of nature and decision alternative

A priori probabilities	State of nature	Decision alternative				
		a_2	a_3	a_4	a_5	a_6
		Million dollars				
.05	s_1	85.1	85.2	87.4	92.2	99.4
.15	s_2	90.3	88.8	89.7	92.9	99.1
.30	s_3	96.3	93.1	92.0	93.9	98.5
.25	s_4	103.2	97.8	94.8	94.7	97.5
.20	s_5	111.3	103.4	98.0	95.6	96.6
.05	s_6	120.3	109.3	101.2	96.2	94.7

Experiment. At this point, the decision-maker could stop and take that decision alternative which has the lowest expected value based on the a priori probabilities. The expected value for each decision alternative is:

$$a_2 = 100.8;\ a_3 = 96.1;\ a_4 = 93.8;\ a_5 = 94.3;\ a_6 = 97.8$$

A decision criteria based only upon the a priori probabilities indicates that decision alternative a_4 is optimal. That is, the firm should adopt a policy of supplying 60 percent of its raw material from internal sources.

However, if the decision-maker can obtain additional information about the states of nature, he may be able to improve his decision-making process. That is, the decision-maker may be able to perform an experiment whereby he can predict the occurrence of the various states with some probability distribution. Then, by use of Bayes theorem, he can transform the *a priori* probability distribution on the states into an *a posteriori* distribution. Whether or not, and to what extent, to experiment is very important. Chernoff and Moses (1959, p. 182) discussed these questions.

> Suppose that there is a choice of whether or not to perform an experiment at a certain cost... In general, one should experiment if the cost of so doing is less than the consequent decrease in the Bayes risk... This notion can be extended to the case where there is available a sequence of experiments. After the i th experiment is performed leading to the *a posteriori* probability W*, one compares two quantities. The first is B(W*), the Bayes risk of selecting an action with no more experimentation; the second is C(W*), the cost of taking one more observation and proceeding thereafter in an optimal fashion. If $B(W^*) \leq C(W^*)$, *stop experimentation and take the appropriate action*. Otherwise continue experimentation. Generally, this type of comparison is mathematically unfeasible, but there are simple important examples where this idea permits us to classify optimal rules for deciding when to sample... The problem of whether to continue experimentation indicates that, in the general decision making problem, a strategy should be a rule which decides after each observation:
>
> (a) whether or not to continue experimentation
> (b) which experiment to take next if experimentation is continued, and
> (c) which action to take if experimentation is stopped.

The choice and design of an experiment to predict the occurrence of the states is very important. In fact, the degree of success in dealing with many decision problems may rest with the choice and design of experiment. The use of the word experiment may be misleading in the present context. The experiment may range from merely observing the level of some particular phenomenon (e. g., temperature) to a statistically sophisticated design. The important thing is that the results of the experiment be associated with the occurrence of the states of nature.

In order to design an experiment for the current example, it is helpful to recall the definition of the states of nature. The states are defined as relative changes in the cost of internal and external wood over the firm's planning period. Therefore, absolute changes in cost are not relevant. What is relevant is that the experiment should reflect relative cost changes.

As a general statement, most external wood is produced by small owners. The out-of-pocket costs of producing wood, i. e., taxes, site preparation, etc., may be about the same or change in the same proportions for all owners. There is, however, one cost of producing timber that is likely to fluctuate much more for a small owner than for a corporate owner. This cost is the opportunity cost--the alternative rate of return. Actually, interest on invested timber capital is the primary cost of timber production.

Since a majority of small woodland owners are farmers, fluctuations in the relative costs of internal and external wood can be hypothesized as follows. If the cost of external wood is decreasing relative to internal wood, the economic well-being of farmers is decreasing. Obviously, the reverse situation also holds. The question then is: is there any experiment which can be performed that will reflect the relative economic well-being of farmers? Also, if such an experiment is available, how well does it predict past occurrences of the states of nature?

Conceivably, one could hypothesize that observing the U. S. Department of Agriculture parity index could constitute such an experiment. Certainly the parity index should be linked to the economic well-being of farmers. Using this reasoning, the parity index for the period 1918-52 was considered (Table 10). Average annual percent changes over ten year periods were used. For example, the state of nature for the decade 1938-48 was s_3 (-1%). Using the data in Table 10, the percentage changes in parity over the ten decade periods preceding 1938 were:

Period	Percentage change in parity
1918-28	-2.7
1919-29	-1.8
1920-30	-1.8
1921-31	-1.8

Period	Percentage change in parity
1922-32	-4. 1
1923-33	-3. 3
1924-34	-1. 7
1925-35	- . 8
1926-36	. 1
1927-37	. 5

By developing information similar to the above for each occurrence of the states of nature, it was found that the percentage change in parity for ten year periods ranged from -4. 1 to 6. 2. The range of outcomes was divided into varying numbers of segments. The collection of segments is the set Z. The individual segments are the elements in Z. After considering various elements in Z, the following appear to have definite predictive value on the occurrence of the states.

z_1 = an average annual change in parity over a ten year period of less than -2 percent.

z_2 = an average annual change in parity over a ten year period between -2 percent and -1 percent.

z_3 = an average annual change in parity over a ten year period period between -1 percent and 3 percent.

z_4 = an average annual change in parity over a ten year period greater than 3 percent.

Table 10. U.S. Department of Agriculture parity index 1918-1952, 1910-1914 = 100[a]

Year	Parity index	Year	Parity index	Year	Parity index
1918	119	1930	83	1942	105
1919	110	1931	67	1943	113
1920	99	1932	58	1944	108
1921	80	1933	64	1945	109
1922	87	1934	75	1946	113
1923	89	1935	88	1947	115
1924	89	1936	92	1948	110
1925	95	1937	93	1949	100
1926	91	1938	78	1950	101
1927	88	1939	77	1951	107
1928	91	1940	81	1952	100
1929	92	1941	93		

[a] U.S. Bureau of the Census, 1960, p. 283; 1964, p. 632.

Given the above definitions, the probabilities of $(Z|S)$ follow (Table 11).

Table 11. Probability of $(Z|S)$

State of nature	Observation			
	z_1	z_2	z_3	z_4
s_1	.20	.50	.30	0
s_2	.10	.25	.40	.25
s_3	.10	.15	.50	.25
s_4	.05	.25	.40	.30
s_5	0	.25	.40	.35
s_6	0	.20	.40	.40

Table 11 may be interpreted as implying that as the parity index decreases the cost of external wood decreases relative to the cost of internal wood. That is, as the relative economic well-being of the small woodland owner decreases, the corporation can obtain the raw material produced by these owners at a relative lower cost. This relationship seems perfectly logical.

As indicated by the data, the parity index is not a perfect predictor of the states of nature. If it were, there would be six elements in the set Z and each element would associate with a unique state of nature with a probability of one. Such perfect correlation is not to be expected. What is to be expected is that the decision-maker will seek the most useful experiment which his resources will permit. In actual practice, a pulp and paper corporation might not be satisfied with the results obtained by observing the parity index. However, since the current example is only intended to be illustrative, it is not necessary to consider additional experiments.

With the above information, the probability of (s_i and z_k) may be computed along with the density function on the set Z, f(Z) (Table 12). Then, using Bayes theorem, the a posteriori probabilities on the states of nature may be calculated (Table 13).

Table 12. Probability of S and Z

State of nature	Observation			
	z_1	z_2	z_3	z_4
s_1	.0100	.0250	.0150	0
s_2	.0150	.0375	.0600	.0375
s_3	.0390	.0450	.1500	.0750
s_4	.0125	.0625	.1000	.0750
s_5	0	.0500	.0800	.0700
s_6	0	.0100	.0200	.0200
f(Z)	.0675	.2300	.4250	.2775

Table 13. A posteriori probabilities for $(S|Z)$

State of nature	Observation			
	z_1	z_2	z_3	z_4
s_1	.1481	.1087	.0353	0
s_2	.2222	.1630	.1412	.1351
s_3	.4444	.1957	.3529	.2703
s_4	.1852	.2717	.2353	.2703
s_5	0	.2174	.1882	.2523
s_6	0	.0435	.0471	.0721

Combining the cost table (Table 9) with the a posteriori probabilities produces the expected costs (Table 14).

Table 14. Expected costs based on a posteriori probabilities by decision alternative and result of experiment

Decision alternative	Observation			
	z_1	z_2	z_3	z_4
	Million dollars, present worth			
a_2	94.58	100.27	100.62	102.86
a_3	91.84	95.76	96.02	97.57
a_4	91.32	93.59	93.73	94.63
a_5	93.56	94.24	94.32	94.59
a_6	98.57	97.85	97.84	97.56

The above data lead to the following decision rule: if z_1, z_2, or z_3 is observed, choose decision alternative a_4; if z_4 is observed choose decision alternative a_5. That is, as long as the average annual change in the parity index is less than three percent, the firm should adopt a policy of supplying 60 percent of its raw material internally. This is the same strategy as indicated by the a priori probabilities. However, if the average annual change in parity over a decade is greater than three percent, the firm should adopt a policy of supplying 80 percent of its raw material internally.

The Bayes risk corresponding to the above decision rule is: $(.0675)(91.32) + (.2300)(93.59) + (42.50)(93.73) + (.2775)(94.59) = 93.77$. The Bayes risk corresponding to the strategy based only on the a priori probabilities is 93.78. Therefore, in this example, it is worth .01 million dollars, in present worth, to the decision-maker to perform the experiment (observe the parity index). With this information the decision-maker can determine whether or not he should perform the experiment. This determination is based on the cost of experimentation. In the current example, it is obviously in the decision-maker's interest to experiment.

Summary

Based upon the premise that the uncertainty present in forest management decision-making has not previously received adequate systematic consideration, a forest management decision-making problem under uncertainty was developed. The example problem was concerned with the optimum degree of land ownership for a hypothetical industrial forestry firm.

Since the example dealt with a hypothetical firm, the results are not directly applicable to actual firms. However, the example contains definite implications for forest management decision-making. The Bayesian approach to decision-making under uncertainty is a logical, consistent procedure which allows a decision-maker to systematically consider all the information relative to his problem. Applying the Bayesian procedure for decision-making under uncertainty to the hypothetical firm's problem, demonstrates the desirability of applying the procedure to the problems of actual firms.

V. MODIFICATIONS IN THE EXAMPLE

In Chapter IV a Bayesian model was developed for the decision-making problem of optimum degree of land ownership within the southern pulp and paper industry. A hypothetical example was used to illustrate the applicability of the model. In developing and solving the example, various assumptions were required. To illustrate the importance of these assumptions and to illustrate their influence on the example, some of these assumptions will be modified in this chapter. Specifically, the modifications will be concerned with the discount rate, planning period, land value, capital gains taxation, and the *a priori* probability distribution.

Discount Rate

A basic assumption of the example was that the discount rate applicable to forest management planning is known. The rate was assumed to be five percent. It was also indicated that a rate of approximately eight percent would be closer to the industry's true opportunity cost (U. S. Bureau of the Census, 1964, p. 497).

If the decision-making problem for degree of internal raw material supply is recalculated using an eight percent discount rate rather than five, the basic cost table changes considerably (Table 15).

Table 15. Present value of raw material costs by state of nature and decision alternative - eight percent discount rate

State of nature	Decision alternative					
	a_1	a_2	a_3	a_4	a_5	a_6
	Million dollars					
s_1	70.9	76.3	84.6	94.5	106.6	120.8
s_2	75.9	80.2	87.2	95.9	107.2	120.3
s_3	81.6	84.6	90.2	97.7	107.7	119.8
s_4	88.4	89.6	93.8	99.8	108.4	119.3
s_5	96.0	95.4	97.8	102.2	109.1	118.6
s_6	104.8	102.0	102.3	104.7	109.8	117.6

Expected costs for each decision alternative and outcome of the experiment may be computed by combining the costs in Table 15 with the a posteriori probabilities from Table 13 (Table 16).

Table 16. Expected costs, in present value, for $(A|Z)$ with eight percent discount rate

Decision alternative	Observation			
	z_1	z_2	z_3	z_4
	Million dollars			
a_1	80.0	80.5	85.8	88.0
a_2	83.3	97.4	97.7	89.3
a_3	89.4	92.3	92.4	93.6
a_4	97.2	98.9	99.0	100.0
a_5	107.5	108.1	108.1	108.1
a_6	120.0	119.5	119.5	119.3

Analysis of Table 16 results in the following Bayesian decision rule: regardless of the outcome of the experiment, always take decision alternative a_1. That is, the firm should supply no raw material internally.

The above analysis underscores a very important factor in forest management planning. Namely, that the choice of discount rate is very critical. It is probably a safe statement that no other single factor can influence financial decisions in forest management as much as the choice of discount rate. It is a choice that should receive considerable attention throughout a forest products firm.

Additional information is also available from analysis of Table 16. As indicated previously, one way to recognize the "insurance value" of internal supply is to use a discount rate lower than true opportunity cost. Table 16 suggests a second way of recognizing this "insurance value."

The decision-making problem can be solved using the true alternative rate of return, eight percent in this case. The corresponding Bayes risk can be determined, 86.0 in this case. Then, if the woodlands department argues that, say, 60 percent of raw material should be supplied internally for "insurance" purposes, the Bayes risk corresponding to this decision alternative can be calculated. In this case, the Bayes risk corresponding to 60 percent internal supply is 99.1. Therefore, the "premium" on the "insurance policy" would be an increase of approximately 15 percent in the present value of raw material procurement costs. Consindering internal supply in this manner may lead to possible quantification of the "insurance value of company wood."

Planning Period

Just as with the discount rate, each firm must decide upon its planning period. For the example, a planning period of 20 years was assumed. Some forest managers might consider this too short a period. This section will examine the effects of changing the planning period from 20 to 30 years.

If all other assumptions are held constant and the planning period is changed to 30 years, some changes are evident in the basic cost table (Table 17).

Table 17. Present value of raw material costs over a 30 year planning period by state of nature and decision alternative

State of nature	Decision alternative					
	a_1	a_2	a_3	a_4	a_5	a_6
	Million dollars					
s_1	100.9	99.0	100.9	105.4	112.9	123.4
s_2	111.3	106.7	106.3	108.6	113.9	122.6
s_3	123.5	116.1	112.9	112.5	115.5	121.8
s_4	138.4	127.3	120.7	116.9	116.8	120.3
s_5	156.3	140.9	130.1	122.2	118.1	118.4
s_6	178.0	157.0	141.2	128.5	120.1	116.1

The figures in Table 17 reflect ten years of additional wood procurement. Therefore, it is the resulting Bayesian decision rule rather than actual present value that is the valid means of comparison.

Expected costs for each decision alternative and outcome of experiment may be computed from the <u>a posteriori</u> probabilities and costs in Table 17 (Table 18).

Table 18. Expected costs, in present value, for $(A|Z)$ with 30 year planning period

Decision alternative	Observation			
	z_1	z_2	z_3	z_4
	Million dollars			
a_1	120.2	132.6	133.2	138.1
a_2	113.5	122.9	123.4	127.1
a_3	<u>111.1</u>	117.6	117.9	120.5
a_4	111.4	<u>115.1</u>	<u>115.3</u>	116.8
a_5	115.0	116.1	116.2	<u>116.6</u>
a_6	121.9	120.7	120.7	120.2

The Bayesian strategy resulting from analysis of Table 18 is: if z_1 is observed, choose a_3; if z_2 or z_3 is observed, choose a_4; if z_4 is observed, choose a_5. The only difference between this strategy and the one for the 20 year planning period is that a_3 is chosen instead of a_4, if z_1 is observed. However, z_1 occurs with a frequency of only .0675; therefore, the practical difference is slight.

While, in this example, the effect of changing the planning period is slight, no general statement to that effect can be made. It is reemphasized that each firm must determine its own planning period and proceed accordingly.

Land Value

The decision-making example assumed an average land value of 30 dollars per acre. This value is assumed to reflect the true opportunity cost of the land resource. There is a cost involved in using this resource which must be charged against timber production. The simplest way to determine the cost of using land is to capitalize the product of land value and discount rate to the end of the rotation. Consequently, the choice of land value is very important.

To illustrate the importance of using the appropriate land value, the decision-making problem was recomputed using a 40 dollar per acre average land value. Changes in the basic cost table are considerable (Table 19).

Table 19. Present value of raw material costs by state of nature and decision alternative--40 dollar land value

State of nature	Decision alternative					
	a_1	a_2	a_3	a_4	a_5	a_6
	Million dollars					
s_1	88.3	86.1	89.0	94.9	103.9	115.9
s_2	95.2	91.2	92.7	97.2	104.6	115.6
s_3	103.0	97.3	97.0	99.5	105.6	115.0
s_4	112.2	104.1	101.7	102.2	106.4	114.0
s_5	122.7	112.2	107.3	105.5	107.3	113.1
s_6	134.8	121.2	113.6	109.2	108.7	112.2

Expected costs are calculated in the usual manner (Table 20).

Table 20. Expected costs, in present value, for $(A|Z)$ with 40 dollar land value

Decision alternative	Observation			
	z_1	z_2	z_3	z_4
	Million dollars			
a_1	100.8	108.3	108.7	111.7
a_2	<u>95.5</u>	101.2	101.6	103.8
a_3	95.7	<u>99.7</u>	<u>99.9</u>	<u>101.5</u>
a_4	98.8	101.1	101.2	102.1
a_5	105.3	106.0	106.1	106.3
a_6	115.1	114.4	114.4	114.1

Table 20 leads to the Bayesian strategy: if z_1 is observed, choose a_2; if z_2, z_3, or z_4 is observed, choose a_3. In other words, if land value increases, other things constant, the firm cannot afford to provide as much raw material internally.

It should be pointed out that, if the true opportunity cost of land is originally included in the model, subsequent changes in land value do not affect the optimum solution. An illustration will help clarify this point.

Assume an initial land value of 30 dollars per acre and a five percent discount rate for the corporation. Assume further than an individual timber producer has a discount rate of eight percent. The cost, to the corporation, of using land is (3) (.05) = \$1.50 per acre per year. For the individual, the cost is \$2.40 per acre per year.

If land value changes to 40 dollars per acre, the costs of using land become \$2.00 and \$3.20 per acre per year for the corporation and individual, respectively. For each producer, there is an increase of one-third in the cost of using land but their relative positions remain unchanged. Therefore, the crucial point, with respect to land value, is that the appropriate value be originally included in the model.

Capital Gains Taxation

A proper treatment of timber income and timber production costs on a corporation's federal income tax form is so influential that it cannot be ignored. This influence is considered here only to emphasize the importance of this tax treatment and to demonstrate the sensitivity of the developed decision-making model to such treatment.

Income from timber qualifies for capital gains treatment under federal tax laws. For a corporation this amounts to a tax rate of 25 percent rather than 48 percent on all taxalbe income over 25 thousand dollars. In addition, most costs of timber growing, other than establishment costs, may be deducted from ordinary income in the year they occur. Various references are available which discuss the influence of federal income taxation on forest management (e.g., Williams, 1964).

To emphasize the influence of federal tax regulations, the basic cost table was recomputed with federal tax considerations omitted (Table 21).

Table 21. Present value of raw material costs by state of nature and decision alternative--no capital gains taxation

State of nature	Decision alternative					
	a_1	a_2	a_3	a_4	a_5	a_6
	Million dollars					
s_1	88.3	87.9	92.6	100.3	111.1	124.9
s_2	95.2	93.1	96.4	102.8	112.1	124.9
s_3	103.0	99.3	100.9	105.5	113.6	124.9
s_4	112.2	106.3	106.0	108.8	115.1	124.9
s_5	122.7	114.6	112.0	112.6	116.8	124.9
s_6	134.8	123.8	118.7	116.9	118.9	124.9

Expected costs are calculated from the costs in Table 21 and the a posteriori probabilities (Table 22).

Table 22. Expected costs, in present value, for $(A|Z)$--no capital gains taxation

Decision alternative	Observation			
	z_1	z_2	z_3	z_4
	Million dollars			
a_1	100.8	108.3	108.7	111.7
a_2	97.5	103.8	103.7	106.0
a_3	99.6	103.8	104.1	105.8
a_4	104.7	107.4	107.6	108.7
a_5	113.2	114.4	114.5	115.0
a_6	124.9	124.9	124.9	124.9

The Bayesian decision rule resulting from analysis of Table 22 is: if z_1, z_2, or z_3 is the outcome of the experiment, choose a_2; z_4 is the observation, choose a_3. That is, if z_4 is the result of the experiment, the firm should adopt a policy of moving toward supplying 40 percent of its raw material internally. Otherwise, the firm should have a policy of supplying only 20 percent of its wood internally. In other words, if the firm neglects the influence of federal income taxation in its decision-making, it will adopt a policy of supplying considerably less than the optimum amount of its raw material internally.

A Priori Probability Distribution

Unlike federal taxation, the *a priori* probability distribution is not an explicit component of the model. That is, tax considerations are governed by very definite regulations, while the prior distribution is, by its very nature, subjective. As indicated, the *a priori* distribution must be consistent with the decision-maker's feelings and information regarding the future. However, an individual decision-maker may feel equally comfortable with a range of distributions. This range may, however, be quite narrow. Therefore, it is worthwhile to examine some additional prior distributions to gain insight into the stability of the one used. The decision-making model for the raw material supply problem was recomputed using four additional a priori distributions:

1. (1/6, 1/6, 1/6, 1/6, 1/6, 1/6) Each state of nature is equally likely to occur.
2. (.10, .15, .25, .25, .15, .10) The distribution is normal in shape.
3. (.05, .05, .15, .20, .25, .30) The distribution is skewed toward higher external wood costs.
4. (.30, .25, .20, .15, .05, .05) The distribution is skewed toward lower external wood costs.

The Bayesian decision rule and implied degree of internal raw material supply were computed (Table 23).[9]

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[9] Implied degree of internal raw material supply is a discrete level of internal supply equivalent to that implied by the strategy. For example, a strategy of: if z_1 or z_2 is observed, choose a_3; if z_3 or z_4 is observed, choose a_4, is equivalent to: (.0675) (.40) + (.2300) (.40) + (.4250) (.60) + (.2775) (.60) = 54 percent internal supply.

Table 23. Bayesian strategies and implied internal supply by various *a priori* probability distributions

Distribution	Observation z_1	z_2	z_3	z_4	Internal supply
					Percent
(.05, .15, .30, .25, .20, .05) (Original)	a_4	a_4	a_4	a_5	65
(1/6, 1/6, 1/6, 1/6, 1/6, 1/6)	a_3	a_4	a_4	a_5	64
(.10, .15, .25, .25, .15, .10)	a_4	a_4	a_4	a_5	65
(.05, .05, .15, .20, .25, .30)	a_4	a_5	a_5	a_5	79
(.30, .25, .20, .15, .05, .05)	a_3	a_3	a_4	a_5	60

Examination of Table 23 indicates that, unless the prior distribution is very drastically skewed, the original Bayesian strategy remains at least near optimum. This would imply that the decision-maker should devote his efforts to improving components of the model rather than the *a priori* distribution.

Summary

In order to emphasize the importance of the various components used to implement the example Bayesian model for decision-making under uncertainty, certain components of the example were modified. The modified components were discount rate, planning period, land value, federal income taxation, and the *a priori* distribution. Additional components, such as rotation length, could have been modified.

It is possible to compare the original solution with the modified solutions. The original solution indicated that the firm should adopt a policy of moving toward 60 percent internal raw material supply, if z_1, z_2, or z_3 was the result of the experiment. If z_4 was the result, the firm should move toward 80 percent internal supply. z_1, z_2, and z_3 occur with a combined frequency of .0675 + .2300 + .4250 = 72.25 percent; z_4 occurs with a frequency of 27.75 percent. These frequencies imply moving toward supplying (72.25) (.60) + (27.75) (.80) = 65 percent of raw material internally. Implied internal supply may be similarly computed for other solutions (Table 24).

Table 24. Implied degree of internal raw material supply for various solutions

Solution	Internal raw material supply
	Percent
Original	65
30 year planning period	64
40 dollar land value	25
No capital gains taxation	25
Eight percent discount rate	0

No general conclusions can be drawn from Table 24 except that each bit of information used to implement the decision-making model should be scrutinized very closely. Modifications in the Bayesian model, such as illustrated in this chapter, can serve to provide some sort of stability test. Then, those components which are most unstable for a given model can be very carefully considered. For example, in the illustrated model, the solution is quite stable over the two planning periods and various *a priori* distributions. On the other hand, discrete levels of discount rate and land value require detailed justification.

VI. CRITIQUE

A study which introduces a concept to a discipline or relies upon a hypothetical example to support its conclusions is likely to receive some criticism. The purpose of this critique is to anticipate and reply to possible criticism as part of the study.

Evaluation of The Example

The example decision-making problem developed in Chapter IV is a realistic problem. Every wood-using firm must decide how much raw material will be supplied from internal sources. It is also obvious that the problem is one of decision-making under uncertainty. However, some of the assumptions employed in developing the example might receive criticism.

Specifically, the example is meant to illustrate forest management decision-making under uncertainty but a number of factors are assumed certain. For example, raw material consumption, discount rate, and timber yield are assumed known. Only future internal and external wood costs are allowed to remain uncertain. This might appear to assume away the problem or imply that decision theory can consider only one uncertain factor. Actually, there is no definite limit to the number of uncertain factors which can be incorporated into a decision-making problem. The inclusion of more uncertain factors in the example appeared more confusing than helpful. An illustration will indicate how decision theory can systematically consider more than one source of uncertainty.

Suppose a forest manager must decide whether or not to prune a tree. He knows the present cost of pruning and, if the discounted future revenue, which results from pruning, is greater than the cost, he will prune. If the discounted future revenue is less than the pruning cost, he will not prune. There are three uncertain factors which influence the decision: future price, future yield, and the discount rate. For simplicity, assume there are two possible levels of each factor. That is, price is either p_1 or p_2, yield is either y_1 or y_2, and discount rate is either r_1 or r_2. Therefore, the eight states of nature which can affect the decision are:

$s_1 = p_1, y_1, r_1$ $\qquad$ $s_5 = p_2, y_1, r_1$

$s_2 = p_1, y_1, r_2$ $\qquad$ $s_6 = p_2, y_1, r_2$

$s_3 = p_1, y_2, r_1$ $\qquad$ $s_7 = p_2, y_2, r_1$

$s_4 = p_1, y_2, r_2$ $\qquad$ $s_8 = p_2, y_2, r_2$

If the following a priori probabilities are assigned to the above factors,

$P(p_1) = .5$ $\qquad$ $P(p_2) = .5$

$P(y_1) = .4$ $\qquad$ $P(y_2) = .6$

$P(r_1) = .3$ $\qquad$ $P(r_2) = .7$

the states of nature have the following *a priori* probabilities.

$s_1 = .06$	$s_5 = .06$
$s_2 = .14$	$s_6 = .14$
$s_3 = .09$	$s_7 = .09$
$s_4 = .21$	$s_8 = .21$

The above method of assigning the *a priori* probabilities to the states of nature implies the three factors are independent. Two factors, A and B, are considered independent when $P(A) = P(A|B)$. For a discussion of statistical independence, see Schlaifer (1959, p. 170) or Hoel (1962, p. 11).

Once the states of nature are defined and an *a priori* probability distribution assigned to them, no matter how many uncertain factors contribute to the states, the problem formulation and solution proceeds according to the general model. The more uncertain factors that are considered, the more complex the mechanics of the decision-making problem become. For this reason, some of the factors which could have been considered uncertain in the example of Chapter IV were assumed known. To have considered all possible factors as uncertain would have distracted from the objective: to illustrate the systematic consideration of uncertainty in forest management.

Some of the other assumptions in the example may receive unwarranted criticism. For instance, the assumption of a constant raw material requirement over the planning period might be disputed on the following grounds. The pulp and paper industry's raw material consumption has been increasing. Therefore, even though consumption is to be assumed known, it should be assumed known to increase at a certain rate rather than remain constant. There are no conceptual differences between assuming raw material consumption will increase at a known rate and assuming it will remain constant; the only differences are arithmetic.

The discussion developing the cost of internal wood at the mill implies a perfectly competitive land market. This is unrealistic but it detracts from neither the example nor the model. An actual firm defining the same type of decision-making problem would either know the type of land market and ownership pattern surrounding its mill; or, would have to determine these factors before finalizing problem formulation.

Evaluation of Decision Theory in Forest Management

The theory for decision-making under uncertainty is of fairly recent origin. The theory is still concerned with evaluating various criteria for decision. Nevertheless, it is well enough developed to have proved useful in some disciplines, primarily business.

Uncertainty is quite obviously present in forest management. Further, like others faced with the task of making decisions under uncertainty, forest managers must choose among a set of possible alternatives and the desirability of these alternatives depends upon a set of possible future events. The application of the theory may be more complex in forestry than in many other disciplines, primarily because of the time element in forest production, but this does not detract from the value of the theory.

Grayson (1960, p. 235) summarized the role of decision theory as follows:

> Actually, all that statistical decision does is to formalize the many mental assumptions, facts, and goals that go into making up a complex decision under uncertainty. But by so doing, and by following certain logical principles to guide action, the chances for error and inconsistent action are reduced.

When the above statement is analyzed, it says quite a lot. If the systematic format provided by decision theory is followed, the possibility of taking the wrong decision is reduced. When dealing with uncertainty, this is about all that can be expected. There is currently no accepted procedure in forest management that provides the same function as decision theory. The advantages to be gained by applying decision theory in forest management appear to greatly outweigh the difficulties that might be encountered in the application.

One final point should be made concerning the place of decision theory in forest management. The above discussion is not to be construed as an argument that forest managers <u>should</u> consider uncertainty in their decision-making. This is obvious, forest managers have always known they should consider uncertainty. The above discussion does mean that forest managers now <u>can</u> consider uncertainty in their decision-making. Further, they can consider it in a systematic manner which leads to an optimal decision, given: their preferences among consquences, their opinions concerning the future, and the state of information regarding the decision-making problem.

VII. SUMMARY AND RECOMMENDATIONS

It is not difficult to establish the existence of uncertainty in forest management decision-making. It is also not difficult to illustrate that systematic consideration of uncertainty in forest management has not been adequate. However, it is not enough to point to omissions. Significant contributions in this area of forest management will come through development and adoption of decision-making models capable of systematically coping with uncertainty. The primary objective of this study was to illustrate and evaluate a model for the systematic consideration of uncertainty in forest management decision-making.

To present the model, Bayesian decision theory, within the context of forest management, an example of a policy decision within the southern pulp and paper industry was developed. The example decision-making problem was concerned with the optimum level of forest land ownership by a hypothetical member firm of the industry.

A number of models for decision-making under uncertainty were considered, the Bayesian model was chosen for two major reasons. First, the other models (minimax, minimax regret, Hurwicz index, and Laplace) all inherently imply complete ignorance about the states of nature affecting the decision-making problem. Decision-makers, such as forest managers, are not completely ignorant about their problems. Granted, the knowledge they possess may be subjective or fragmentary; in many cases it may be nothing more than a "feel" for the situation. Nevertheless, regardless of the amount of information available, it is doubtful that experienced decision-makers will endorse a concept of complete ignorance.

The second reason the Bayesian model was chosen is that it is a logical, consistent, systematic procedure which results in a decision based upon the criterion of maximum (minimum) expected utility (loss, profit, etc.). The decision resulting from the Bayesian model is (1) consistent with the way in which the decision-maker evaluates consequences and (2) consistent with the way the decision-maker appraises the future.

The Bayesian approach to decision-making under uncertainty constitutes a general model. The components of the general model are:

1. the states of nature
2. the possible decision alternatives
3. the set of consequences which associate each alternative to each state
4. an _a priori_ probability distribution on the states
5. an experiment, the outcome of which has predictive value on the states.

When the above components or sets are defined within the context of a particular problem, the general model is restricted and pertains only to that particular problem. However, various decision-makers may quantify the restricted model in various ways, especially with respect to the prior distribution. Therefore, the same restricted model can lead to different decisions for different decision-makers.

To illustrate the application of the Bayesian model to forest management decision-making, the components of the general model were defined as follows:

1. The decision alternatives were possible policies of obtaining specified levels of raw material from internal sources.
2. The states of nature were the changes in external wood cost relative to internal wood cost over the firm's planning period.
3. The consequence for each alternative and state was the present value of obtaining all raw material over the firm's planning period for that alternative and state.
4. The prior probability distribution on the states was a combination of the frequencies with which the states occurred in the past and the decision-maker's opinion of how well these frequencies reflected the future.
5. The experiment was a body of information which was shown to have predictive value concerning the relative changes in

internal and external wood cost.

With the above definitions, the Bayesian model for decision-making under uncertainty was defined for the problem of optimum level of forest land ownership for the hypothetical firm. The solution indicated that a policy of moving toward either 60 or 80 percent internal raw material supply, depending on the outcome of the experiment, was optimal.

The actual numerical solution for the hypothetical firm is not the primary contribution of the study. The underlying implications to similar forest management decision-making problems are more important. First, a model has been presented which, if it can be quantified, will produce a decision which is optimum given the level of available information and the decision-maker's knowledge of the situation. Second, using published data and assuming certain indexes to reflect other data, the model was quantified for a hypothetical firm. The quantification resulted in a decision which provided an explicit basis for corporate forest policy. As additional information becomes available or as different experiments can be performed, the model may be requantified and the decision revised as necessary. Most important, at any one time the firm is basing its policy upon as good a decision as it can obtain. Naturally, as new information becomes available, it may be necessary for the firm to revise its policy. The important point is: changes in policy, whether large or small, will be based on explicit data produced by a logical, systematic decision process, not on whim.

To illustrate the sensitivity of the developed model to certain quantified components, a number of modifications were made. The modifications were concerned with the discount rate, planning period, land value, federal income taxation, and the *a priori* probability distribution. The modifications indicated that choice of the appropriate discount rate and land value are very important decisions, while the optimum solution was not very sensitive to planning period or prior distribution. The influence of federal income taxation is so important that under no circumstances can it be ignored in private forest management planning.

The introduction mentioned the relationship of the study to forest management theory. Most important in this respect is that forest managers do make decisions similar to the example developed for this study. As indicated, almost all forest management decisions having any degree of significance over time contain inherent uncertainty. Present theoretical concepts of forest management decision-making contain inadequate recognition of uncertainty. If forest managers are going to make decisions in which uncertainty is an integral factor, and they *must* make such decisions, the theory underlying their decision-making processes should be capable of recognizing and systematically considering uncertainty. Among the contributions of this study is the fact that a theoretical decision-making model has been presented which:

1. recognizes and systematically considers the uncertainty inherent in forest management decision-making, and
2. forces the focusing of attention upon the information actually needed to make a specific decision because the general model defines the sets of information needed and each set must be expressed quantitatively.

The developed model has a useful place in industrial forest management. The need for considering uncertainty and the theoretical review which led to the model have apparent application in forestry. However, whether or not the model developed in this study has practical application can only be inferred, not proven, by means of a hypothetical example. Therefore, the next logical research effort should be an empirical application of the Bayesian in forest management. Forest managers and forest management researchers should be alert for opportunities to apply the Bayesian model in their decision-making processes.

It should be reemphasized that defining the decision-making problem is a critical aspect of decision-making. Unless the problem is defined in a manner which is compatible with a decision-making model, that particular model cannot be used to solve the problem. There may have been a tendency in this study to minimize the difficulties in problem definition in favor of model emphasis. The very real difficulties of defining forest management problems in a manner compatible with the Bayesian or other decision theory models should be the subject of subsequent research.

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