

## AN ABSTRACT OF THE THESIS OF

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*This thesis investigates the dynamic efficiency of emission standards and Pigouvian taxes in the regulation of pollution where research and development (R&D) and the technology level are endogenous. A key element of the analysis is a comparison of the efficiency between instances in which the regulator can and cannot commit to future regulatory levels. A partial equilibrium game theoretic model with two periods and a number of stages in each period is used. Additionally, a computer simulation of regulation is developed to examine the efficiency of the equilibrium outcome when mathematical complexity prevents finding an analytic solution. The results suggest that if innovation is deterministic, that is, a firm invests in R&D such that it succeeds with certainty or it does not invest at all, commitment to a standard will yield the optimal level of abatement and investment while taxes may not. Even in instances where innovation is stochastic instead of deterministic, emission standards, with or without commitment, can result in higher levels of welfare than using taxes as a regulatory instrument. It is also shown that non-commitment to a tax level will often yield a greater level of welfare than commitment. In general, however, whether commitment or non-commitment to future taxes will result in greater welfare is ambiguous. The results are driven in part because the Pigouvian taxation rule cannot provide both the proper ex ante and ex post incentives for pollution abatement and investment in R&D.*

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**Dynamic Pollution Regulation with Endogenous Technological Change**

by

**Michael D. Jaspin**

**A THESIS**

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Michael D. Jaspin, Author

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# Dynamic Pollution Regulation with Endogenous Technological Change

## 1. INTRODUCTION

This thesis investigates the dynamic efficiency of emission standards and Pigouvian taxes in the regulation of pollution where research and development (R&D) and the technology level are endogenous. A key element of the analysis is a comparison of the efficiency between instances when a regulator can and cannot commit to future regulation. Allowing either commitment or non-commitment by the regulator will be shown to alter the relative efficiency and optimality of taxes and standards significantly.

Pollution, such as sulfur dioxide, carbon monoxide, and nitrogen oxide, has been recognized since at least 1500 as posing a risk to human health and aesthetic values (Chambers, 1976; Freedman, 1989). While it is impossible to place a monetary value on the effects of pollution, the over \$125 billion spent annually complying with environmental regulations administered by the U.S. Environmental Protection Agency provides an indication of its significance (Jaffe *et al.*, 1995).

Economists have also long recognized the significance of pollution, and in particular, that it is a negative externality (Tietenberg, 1992). That is, the polluting activities of one actor exert a negative impact on another individual's utility outside of the price system. Because of high transaction costs, Coasian bargaining solutions are unlikely to provide a solution to the presence of pollution externalities (Coase, 1960).

A number of policy instruments have been suggested that a government or regulator may employ to resolve this market failure (or the lack of a market). For example, Pigouvian taxes have been advocated as a means of internalizing the social costs of polluting into a firm's profit function. Standards have also been suggested and used heavily to force a firm to curtail pollution to a level consistent with maximizing social welfare. The research and application of these methods have focused largely on achieving static efficiency. That is, society-wide, the marginal benefit of emissions must equal the

marginal cost of emissions and the marginal cost of abatement must be equal both across firms and within firms.

Achieving static efficiency, however, does not guarantee dynamic technological efficiency. Over-time, the technology available to abate pollution may change as the result of R&D, altering abatement costs and consequently the optimal level of abatement (or emissions). Therefore, the level at which policy instruments are set (*e.g.*, the tax level) will likely need to be altered through time. For instance, if pollution abatement becomes less expensive, the regulator should increase abatement requirements, an act known as "ratcheting." In particular, a regulator ratchets when the required level of abatement is increased (or the allowable level emissions decreased) through time in response to pollution abatement becoming less costly in order to keep the marginal cost and benefits of abatement aligned.

The incentive for firms to invest, develop, and implement new technology, however, will be dependent on the return of their investment and hence the regulations they face. As a result, firms have the ability to engage in strategic behavior. Thus, if a firm suspects the regulator will ratchet, the amount it is willing to spend on new technology will be based on the benefit accruing after ratcheting has occurred. For standards, it may be the case that a firm will not engage in R&D simply because the regulator will "penalize" the firm by tightening the regulations and actually increasing the firm's cost.

In other words, R&D should be viewed as endogenous for two reasons. The amount a firm spends on R&D is a choice variable of the firm, not an exogenous dollar amount. This point is emphasized in the R&D literature in industrial organization. Secondly, the amount invested in R&D affects the cost of pollution abatement, which in turn may affect the regulation which is enacted and vice versa. Therefore, R&D expenditures should be viewed as part of a dynamic game between firms and the regulator.

The majority of the literature regarding pollution regulation, however, treats R&D as exogenous. As was suggested above, the inclusion of R&D may result in the efficiency of various policy instruments being altered significantly. For instance Biglaiser and Horowitz (1995) show that a technology standard is needed in addition to a tax in order to

ensure efficient R&D and abatement levels. As Kneese and Schultze (1975, pg 82) write, “over the long haul, perhaps the most important single criterion on which to judge environmental policies is the extent to which they spur new technology toward the efficient conservation of environmental quality.”

It is the overriding goal of this thesis to improve our understanding of just how environmental regulations affect innovation and how regulation should be implemented to efficiently preserve environmental quality over time.

The precise objective is to investigate the dynamic efficiency of policy instruments in the regulation of pollution where technology is endogenous and a representative firm may engage in strategic behavior. The specific objectives are to:

- i) compare the socially optimal level of pollution abatement, technology, and R&D expenditures through time to the equilibrium outcome when a representative firm is regulated by Pigouvian taxes or emission standards
- ii) compare how the results from i) differ when the regulator can and cannot commit to future regulation.

The procedures to achieve these objectives are straight forward. A partial equilibrium game theoretic model with a cost minimizing representative firm and a welfare maximizing regulator is developed. The game takes place over two periods with a number of stages in each period. This model, with necessary modifications for standards, taxes, commitment, and non-commitment, is used to determine the optimal and equilibrium levels of pollution abatement and technology. Several analytic results that provide insight into the use of standards and taxes are developed using this model. Because of the mathematical complexity, the game is also analyzed with a computer simulation using GAMS.

The following section reviews the existing literature and relevant theory. The next section then develops a one period model. The two period model is then developed in the next section. At the conclusion of the section, the results from several simulation runs are presented. The Conclusion section discusses the policy implications of the research and suggests future areas for research.

## 2. THEORY AND LITERATURE REVIEW

Society's ability to abate and control pollution ultimately rests on how much it is willing to spend towards that goal along with the cost and technology that is available to abate pollution. From 1981 to 1990, the U.S. spent a constant 1.3 to 1.5 percent of its gross domestic product (GDP) on pollution abatement and control (Jaffe *et. al.*, 1995). Over the same period, France spent approximately 0.9 percent and West Germany 1.5 percent (Jaffe *et. al.*, 1995). While a number of caveats are attached to these numbers and they should rightly be viewed with skepticism, they suggest that it is unrealistic to expect society's willingness to pay for environmental improvement to increase dramatically over time. Thus, improvements in environmental quality are likely to be dependent on discovering less costly pollution control techniques.

In fact, while spending as a percentage of GDP remained constant from 1981 to 1990, emissions of the six major air pollutants (sulfur dioxide, nitrogen oxides, reactive volatile organic compounds, carbon monoxide, total suspended particles, and lead) all fell. For instance, sulfur dioxide fell by 11.9 percent and lead by 91.1 percent (Jaffe *et. al.*, 1995). The credit for these reductions goes not only to tougher standards, and phase-outs in the case of lead, but also improved abatement technologies and alterations in production processes. Moreover, while population and per capita consumption of goods and services which result in pollution has increased, significant increases in pollution been avoided in part because of improvements in technology. For example, automobiles have become more fuel efficient and thus tend to produce less pollution per mile traveled, catalytic converters for automobiles have been developed which directly reduce emissions, and today's electrical appliances are more efficient, reducing the per appliance demand for electricity. Thus, if improvements in environmental quality are linked to improvements in technology and production processes as they appear to be, it is vital that pollution regulation induces not only the proper level abatement but also technological innovation through time.

This section develops the theoretical justification for the regulation of pollution and then discusses static regulation. The shift to dynamic regulation where technological

change is possible is then made, followed by a formal review of the literature. Lastly, the niche in the literature this thesis aims to fill is described.

## 2.1 Pollution and Static Regulation

Pollution is generally considered to be an unwanted byproduct resulting from the production of goods and services by an economic actor. For instance, the production of electricity from coal-fired generating facilities produces sulfur dioxide, a precursor of acid rain and smog (Freedman, 1989). The production of pollution, however, does not necessarily imply that society is made worse off. It may be the case that the welfare generated from the consumption of the goods and services, whose production results in pollution, outweighs the loss in welfare caused by that pollution. If *Pareto-relevant externalities* exist, as is likely to be the case, the production of pollution will reduce welfare.

An externality exists whenever an individual's utility is a function of variables whose values are chosen by others without regard to the effects on the individual's welfare (Baumol and Oates, 1988). Or, as Nicholson (pg 875, 1975) writes, an externality occurs when "an affect of one economic agent on another is not taken into account by the price system." For example, let  $U_j$  be defined as the utility of individual  $j$  where

$$U_j = u_j(x_{1j}, \dots, x_{nj}, z)$$

and  $x_{ij}$  is the amount of good  $i$  consumed by individual  $j$  and  $z$  is the summation of pollution  $P$  emitted by all sources in the community. Specifically,  $z = \sum P_k$  where  $P_k$  is the pollution emitted by firm  $k$  ( $k=1, \dots, f$ ). An externality becomes *relevant* when individual  $j$  would alter the level of  $z$  if given the option. In the example of the coal-fired power plant, if individual  $j$  would prefer to reduce the level of pollution  $z$  they are forced to consume, pollution is considered a *relevant, negative externality*.

A *Pareto-relevant externality* exists only if it is possible to alter  $z$  in a manner such that the utility of individual  $j$  is improved without making firm  $k$  worse off. As Baumol

and Oates (1988) note, a necessary condition for a *Pareto-relevant externality* is that the decision-maker whose activity affects others' utility levels does not receive (pay) compensation for this activity in an amount equal in value to the resulting benefits (or costs) to others.

Pollution being a *Pareto-relevant, negative externality*, however, is not a sufficient condition for pollution to cause a reduction in social welfare and hence provide a justification for government regulation. In particular, Coase (1960) postulates that the presence of externalities does not necessarily result in inefficiencies. He argues that if bargaining is costless, an efficient allocation of resources can be achieved through reliance on bargaining among the parties involved. With pollution, the individual who is negatively impacted can partially compensate the polluting firm for reducing its pollution level, making both the firm and individual better off. This assumes, however, that the firm had the right to pollute. If it did not and the individual instead has the right not to experience pollution, the firm can bargain with the individual to compensate him for pollution. In other words, bargaining will remove the Pareto relevant portion of the externality. The initial assignment of the right, of course, may have significant distributional impacts, but a socially efficient solution will still arise.

As the number of individuals affected by pollution increases, there are likely to be costs associated with organizing individuals and bargaining. Moreover, many forms of pollution have a relatively small impact on a given individual, although large aggregate damages on society. Thus, the transaction costs to an individual are likely to exceed the value of any expected result from bargaining. The result is that transaction costs will be high enough that bargaining will not occur and hence the socially efficient outcome will also not occur.

The lack of a price system to adequately signal the costs associated with pollution, the likelihood that bargaining will not occur, and potential distributional concerns, therefore, provides an economic justification for the regulation of pollution by government. A number of policy instruments, such as standards, Pigouvian taxes, and tradable permits, have been suggested and used by government regulators to compensate for the lack of a functioning market. Standards and tradable permits are considered

quantity rules as they aim at regulating the quantity of pollution emitted, although, there are significant differences between them. Standards encompass both technology standards and emission standards. Under technology standards, the regulator specifies the technology that must be used by a firm to achieve an abatement goal. Under an emission standard, only an abatement goal or emission level is specified and the firm is free to meet the emission standard by any method it wishes. In tradable permit regulation, the regulator sets the total amount of emissions allowed in society and allocates percentages of that amount (*i.e.*, permits) to firms. The firms are then free to not only select what technology to employ to reduce emissions down to the amount they hold permits for, but may also trade permits. This implicitly places a value or opportunity cost on the right to pollute. An emission tax is considered a price rule as the regulator controls pollution emissions *via* taxes (a price on) pollution which is emitted.

Academic research and the practical application of these instruments has focused largely on achievement of static efficiency. In a simple world where there is complete information regarding the marginal cost and benefit functions and perfect competition, an optimal static regulation requires that society-wide the marginal cost of abatement equal the marginal benefit of abatement. The marginal cost of abatement must also be equal both across and within firms. Continuing in a simple world, and adding the caveat that there is only one firm, regulation by either quantity or price rules will yield the optimal outcome in terms of pollution emissions. If, however, there are multiple firms and incomplete information regarding the firm's cost of abatement exists, price rules are favored as the regulator has lower information requirements. In particular, the regulator does not need to be concerned about allocating pollution abatement among firms as a price is placed on pollution; consequently, that the regulator does not have perfect information regarding abatement costs is not as significant. (However, the regulator must still have information regarding industry-wide abatement costs or else the regulator will be unable to set a price or tax which coincides with the society-wide marginal costs and benefits of pollution being equated.) As permits implicitly have a price, are tradable, and an absolute emissions level can be set, they are considered to work as well as taxes in this

situation. An extensive literature exists on this, and Baumol and Oates (1988) and Cropper and Oates (1992) provide a good overview.

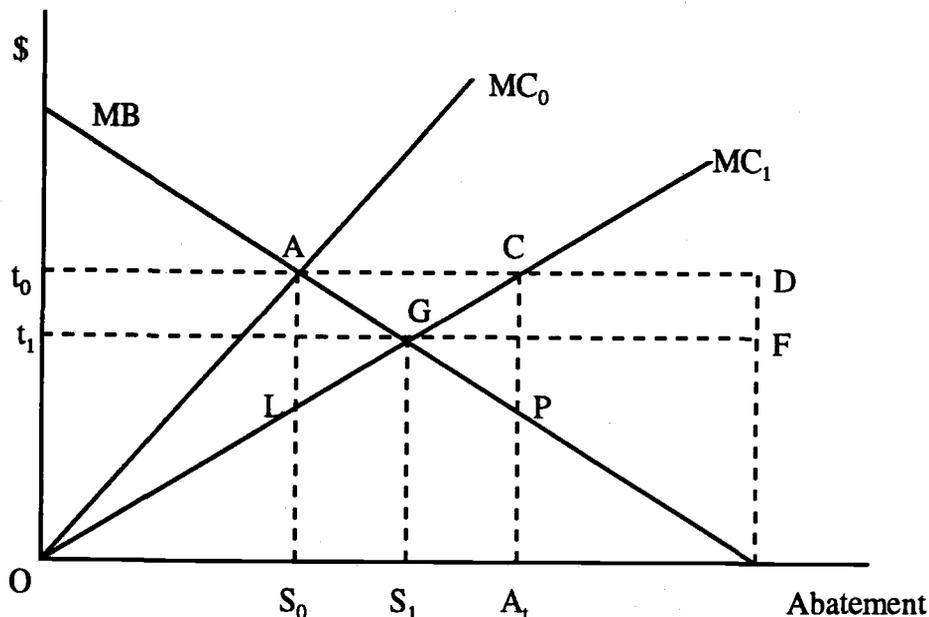
It should be noted, however, that as the assumptions of certainty, perfect information, and perfect competition are relaxed, whether regulation by price or quantity is preferred becomes increasingly ambiguous. For example, in cases of uncertainty, Weitzman (1974) shows that quantity or price preference depends on the relative slopes of the damage and cost functions. There is, however, a significant bias by economists towards policy instruments which rely on prices in the existing literature. Recently, authors such as Russell and Powell (1996) have suggested this may not be wise.

## **2.2 Dynamic Regulation**

As was suggested in the introduction, examining pollution regulation in a static setting may be inappropriate (Zerbe, 1970; Kneese and Shultze, 1975; Orr, 1976). Over time technological change is possible and firms may discover pollution abatement technologies which reduce the marginal cost of pollution abatement. As the cost of pollution abatement is reduced, the level of abatement at which the marginal cost and benefit of abatement are equated will shift towards higher levels of abatement. The level at which policy instruments are set will consequently need to be altered (a concept known as ratcheting as the regulator tightens the level of emissions allowed).

For example, in Figure 2.1  $MC_0$  represents the marginal societal cost of abatement associated with the current abatement technology and  $MB$  represents the societal marginal benefit of abatement. The abatement level  $S_0$  represents the quantity of abatement required so that the marginal cost of abatement equals the marginal benefit, point A. Therefore,  $S_0$  and  $t_0$  represent the emission standard and tax needed to maximize social welfare.  $MC_1$  represents the marginal cost associated with the discovery of a lower cost abatement technology. If this new technology is found, the new welfare maximizing level of abatement is  $S_1$ , corresponding to the marginal costs equaling the marginal benefits of

abatement at point G. Therefore, the tax must be lowered to  $t_1$  and the standard must be raised to  $S_1$  for the welfare maximizing solution to occur.



**Figure 2.1:** Ratcheting by Regulator

For expositional purposes, assume that a single pollution emitting firm bears the burden of all abatement costs but is operating in a perfectly competitive output market. Also assume that the tax or standard is set at the static efficient level so that  $MC_0 = MB$  and that no ratcheting occurs. It can be seen that even before the possibility of ratcheting is added to the model, the firm has an incentive of  $OAL$ , the reduction in abatement cost, minus the cost of innovation, to invest under regulation with an emission standard. If the firm innovates, the abatement level will remain at  $S_0$  as the regulator does not ratchet. Under an emission tax, the incentive to innovate is  $OAC$  minus the cost of innovation. If the firm is successful in innovating, the tax remains the same, but the firm will now equate the tax rate to  $MC_1$ , which occurs at point C, and abatement of  $A_t$  will occur. A regulator, who is interested in maximizing social welfare will have an incentive of  $OAG$  minus the

cost of innovation to innovate and would abate at level  $S_1$  after innovation. As Downing and White (1986) show, the standard will provide an under-incentive of ALG to invest and under-abatement if the firm innovates. On the other hand, the tax will provide an over-incentive of ACG to invest and over-abatement of  $A_t - S_1$  once the new technology is found.

Downing and White also consider the case where the regulator ratchets once the firm innovates. Once again the tax will provide an over-incentive to innovate; however, in this case the over-incentive is equal to ADFG and the *ex post* level of abatement will be optimal for the technology level. The standard will provide an incentive of OAL minus the cost of increased abatement,  $S_0LGS_1$ , minus the cost of innovation to invest in R&D. This amount will be less than the optimal incentive so the firm may not invest even when it is optimal to do so. It does this strategically in order to prevent the regulator imposing tougher regulations and raising its costs.

As aspects of dynamic regulation, such as ratcheting, where incorporated into the static model above, it should be clear that important elements of pollution regulation are dynamic in nature and not capturing them in models may produce results which are deceiving. In particular, static models will fail to capture strategic behavior between the regulator and firm over time. Of course, when there are multiple firms, there is also a possibility of strategic behavior between firms occurring. Simply put, technology and the amount spent on R&D is endogenous. Thus, examining pollution regulation in a static setting is inappropriate.

Before reviewing the literature on dynamic pollution regulation, it is worth while specifying the problem faced by the regulator and making several key points. First, the regulator must be concerned with both the *ex ante* and *ex post* regulation. That is, regulation must ensure that the marginal benefits and costs of abatement are equated both before and after so that the proper amount of abatement occurs. Moreover, the regulator's decisions will determine the incentive firms have to invest in R&D. An important aspect of this is whether the regulator can pre-commit to the regulatory policy after innovation as it alters the incentive.

It is also important to recognize that the regulator must not only be concerned with the direction or bias of the incentive given to the firm, but whether it is the *optimal incentive*. As Kennedy and Laplante (1996) emphasize, the right incentive must be given. Lastly, and also pointed out by Kennedy and Laplante, the equilibrium level of welfare resulting from abatement and investment must be considered when comparing policies. Simply put, the regulator is faced with regulating both abatement and innovation through time where there is strategic interactions among the regulator and firms.

### **2.3 Literature Review**

The literature on pollution regulation and technological change is fragmented and lacks consistency in the assumptions and models used. In particular, how technological innovation is modeled, the number and nature of firms, how a regulator responds to innovation, and whether an equilibrium analysis is used is highly variable. It is useful to keep these aspects in mind when reviewing the literature. The review is broken into three sections: basic models, uncertainty/risk, and imperfect competition. This division only serves as a crude organizational device and the literature, of course, should be viewed as a continuum.

#### **2.3.1 BASE MODELS**

Zerbe (1970) uses a graphical model of a single firm operating in a perfectly competitive output market to show the effects of emission taxes, subsidies, and direct controls on innovation. Based on allocative (abatement) and innovation merit, the ranking order of preference among regulatory controls is: a pollution damage tax, an emission tax, an input tax, subsidies, variable subsidies, and a production tax. Zerbe also emphasizes administrative costs should be considered as the transaction costs might be significant enough to alter the ranking of regulatory controls.

Likewise, Wenders (1975) quantitatively examines the effects of an emissions tax, subsidy, and emissions standard on incentives to improve abatement technology. Wenders assumes that there are  $n$  firms operating in a perfectly competitive output market. He utilizes an explicit cost function and precedes to compare how the three policy instruments affect the incentive of an individual firm to innovate. In his analysis, he discusses instances where the regulator does or does not ratchet. He concludes that: 1) technological innovation will be greater if an emissions tax is used (*assuming  $t_2 < t_1$  implying ratcheting occurs*) than an emission standard or subsidy, 2) it is possible for firms operating under emission standards or subsidies to have no incentive to innovate, and 3) firms regulated by emission standards or subsidies prefer innovation which raises the abatement level as little as possible. These results are the same as derived in Figure 2.1.

Orr's (1976) discussion of effluent charges reiterates Zerbe's and Wender's conclusion that emission taxes will spur innovation (R&D). In particular, Orr divides the basis for effluent charges into two categories: short-term allocative efficiency and long-run technological innovation. He argues, that while the two are not independent, policy emphasis should be placed on the long-run technical adaptation in the case of environmental quality. His argument for this is based on that, in the long-run, environmental quality is an essential resource which is growing scarcer. In other words, the issue is not welfare as much as growth with exhaustible resources.

Downing and White (1986) formally introduce agency response (ratcheting) and cases where the firm's decision has a marginal impact on pollution. In particular, they examine innovation in pollution control when the innovator 1) has no marginal impact on overall pollution 2) has a marginal impact on overall pollution but the regulator does not alter the incentive scheme, and 3) has a marginal impact on overall pollution and the regulator alters the incentive scheme. A single firm in a competitive output market is used where an expenditure of  $x$  will result in a lower marginal cost of abatement in the future. The innovation is specific to the firm and cannot be transferred.

The results from their Table 1 are displayed in Table 2.1 and show the incentive given to a firm to innovate under varying regulatory instruments. In the table, direct regulation is defined as an emission standard and not a technology standard. The results

	Effluent Fees	Subsidies	Marketable Permits	Direct Regulation
No change in marginal conditions	Optimal	Optimal	Optimal	Deficient
Change in marginal conditions; no ratcheting	Excessive	Excessive	Indeterminate	Deficient
Change in marginal conditions; ratcheting	Excessive	Deficient	Deficient	Deficient

**Table 2.1:** Incentives for Innovation under Various Pollution Control Arrangements

from their Table 2 are presented in Table 2.2 and show the optimality of emission levels which occur under the same instruments as in Table 2.1.

The results described earlier using Figure 2.1 are the same as shown by Downing and White in their two tables. For example, in the last row of the tables, innovation is assumed to have a marginal impact on abatement costs and that the regulator ratchets. It can be seen that under these circumstances an emission tax produces the optimal level of abatement but provides an over-incentive to invest. A standard will also produce the optimal level of abatement, but it will create an under-incentive for the firm to invest in R&D. As the results for the optimal level of emissions are not the same as those for the incentive to innovate, it can be seen that neither taxes nor standards will result in an optimal outcome in the context of dynamic optimality.

	Effluent Fees	Subsidies	Marketable Permits	Direct Regulation
No change in marginal conditions	Optimal	Optimal	Optimal	Too High
Change in marginal conditions; no ratcheting	Too Low	Too Low	Indeterminate	Too High
Change in marginal conditions; ratcheting	Optimal	Optimal	Optimal	Optimal

**Table 2.2:** Emission Levels under Various Pollution Control Arrangements

Milliman and Prince (1989) also compare the ability of differing regulatory regimes to promote technological change in pollution control. Unlike Downing and White, though, they allow for patenting of innovations and the influencing of the control agencies through lobbying and information withholding. A graphical analysis is used and the polluting industry is assumed to be competitive in the output market with a large number of  $n$  firms who emit a homogenous pollutant. Pollution abatement costs to the firm consist of direct costs, associated transfer losses, and transfer gains (from licensing of patented technology). A social welfare maximizing regulator possesses perfect information concerning the current abatement technology; however, the regulator has a time lag in the discovery of this abatement technology. Additionally, technology change occurs in a three stage process: innovation, diffusion, regulator response. Regulatory regimes are ranked on relative, ordinal rankings, based on induced cost changes. Milliman and Prince's results suggest that emission taxes and auctioned permits serve as a greater or equal incentive to investment than direct controls, free permits, and emission subsidies.

Kennedy and Laplante (1996) focus their analysis on Pigouvian emission pricing for innovation in an equilibrium setting. This is particularly important as the previous authors mentioned had neglected this aspect. They point out mistakes in a number of papers, in particular, that Milliman and Prince's analysis is incorrect. "The problem in Milliman and Prince stems from their assumption that firms anticipate no ratcheting of the emissions price in response to technology adoption even when ratcheting does occur. This assumption is not consistent with rational, forward looking behavior" (Kennedy and Laplante, 1996, pg 27). Kennedy and Laplante also point out that Downing and White's argument cannot be extended to multiple firms. They, however, do agree with their single firm findings.

In their own analysis, Kennedy and Laplante mimic the analysis of Downing and White and Milliman and Prince, correcting for the mistakes pointed out. They find that Pigouvian price rules will not always yield the efficient solution. This stems from the fact that Pigouvian taxes do not discriminate between the relative damages of different units of pollution emitted. The result also arises because "full ratcheting according to the

Pigouvian rule ensures that the emissions price is correct *ex post* but distorts incentives for technology adoption *ex ante*" (Kennedy and Laplante, 1996, abstract).

The authors discussed so far argued that policy instruments utilizing economic incentives tend to generate greater incentives to engage in R&D. This view does not always hold. Malueg (1989) uses a simple pollution emissions trading model to demonstrate that a trading program may result in decreased incentives for an *individual firm* to adopt new pollution abatement technology. The model uses a single firm in a perfectly competitive output market. The adoption of new technology results in lower marginal abatement cost; both old and new technology have the same fixed cost. A firm adopts new technology only if the reduction in marginal cost of abatement is greater than the cost of adopting new technology. A firm's position in the emission credit market is examined prior to and after adoption of new technology. The result obtained is that an "emission trading program does not necessarily increase a firm's incentive to adopt new pollution abatement technology" (Malueg, 1989, pg 56). Specifically, if the firm is a seller before and after investing, then incentive to invest is greater. If the firm is a buyer before and after investing, then the incentive to invest is less. If the firm is a buyer before and a seller after investing, then the incentive to invest ambiguous. It should be noted that Malueg ignores equilibrium conditions.

Lastly, Biglaiser, Horowitz, and Quiggin (1995) compare taxes and tradable permits in a dynamic setting with complete information. Using optimal control theory (continuous time) they show that taxes result in the first best solution and are time consistent. Permits, on the other hand, may not achieve the social optimum as tradable permit regulation will likely be time inconsistent.

### 2.3.2 UNCERTAINTY/RISK

The articles reviewed above have excluded a number of complicating factors from their models, uncertainty being among these. As Mendelsohn (1984) and Kennedy (1994) show, uncertainty may alter the optimality of policy instruments encouraging R&D.

Mendelsohn (1984) extends Weitzman's (1974) model of regulation under uncertainty to include endogenous technical change. In particular, the effect of price versus quantity regulation is studied. Assuming an average firm acts as a profit maximizing price taker, "*quantity rules tend to encourage more efficient levels of technical change*" than price regulation. It should be noted, however, that this result depends on the functional form of the cost and benefit functions. Mendelsohn also ignores the response of the regulating agencies.

Kennedy's (1994) paper examines a competitive market for emission permits when technological uncertainty exists. A dynamic model is used where the adoption of new technology is endogenous. Specifically, the model assumes there are a large number of firms who use some pollution control technology  $\theta$ . A new level of pollution control technology arrives each period and is known to always be cleaner (or at least not dirtier) than the previous technology. For pollution which is emitted, a permit (allowance) is required. The number of permits is fixed and are sold in a competitive market. The control agency does not ratchet.

The results of Kennedy's model are derived by altering the assumption of risk neutrality, beliefs about future technology, and the cost of adopting new technology. The results, from Kennedy's propositions, are: 1) if firms are risk neutral, the adoption of new technology is costless, and symmetric beliefs about new technology exist, then competitive permit trading will result in marginal abatement costs being equated across firms. 2) If future innovation is expected, the current permit price will be less than the prices without expected innovation. 3) Given a current price of permits and an expected price of permits in a future period, the demand for permits is greater when firms are risk averse than when firms are risk neutral. 4) If adoption of new technology in period  $t+1$  is costless, then marginal abatement costs are equated across firms in period  $t$ . Output and abatement level are efficient given the proportions of firms that adopt  $\theta$ . 5) If firms are risk neutral, but adoption is costly, then the equilibrium proportion of firms that adopt the new technology in period  $t$  is efficient. 6) If the efficient outcome involves partial adoption and firms are risk averse, the equilibrium proportion of firm that adopt new technology is inefficiently small. 7) If adoption of technology  $\theta_2$  in time period  $t+1$  is costly, marginal abatement

costs are not equated across firms in period  $t$ . Firms not adopting  $\theta_1$  in period  $t$  will abate excessively relative to the efficient outcome.

### 2.3.3 IMPERFECT COMPETITION

More recent research has relaxed the assumption perfectly competitive markets in pollution control technology and introduced monopoly rents<sup>1</sup> (Parry, 1994), multiple sector markets (Parry, 1994), agent interactions (Biglaiser and Horowitz, 1995), the cost of government funds (Laffont and Tirole, 1994), or some combination of these.

Parry (1994), like previous authors, examines the optimal level of a pollution tax when endogenous technological progress is possible. However, a two-stage (but 1 period), two-sector model is used where firms in both the production and research industry are competitive and homogenous. The research industry carries out research and has an increasing cost function with respect to the total number of research firms. Only one firm may make a 'discovery' which it then receives a patent for and begins behaving as a monopolist.

His paper attempts to determine whether the optimal pollution emission tax should be greater than, equal, or less than the marginal environmental damage (MED) of the emitted pollutant. Parry suggests the tax would only be greater than MED in the case of a monopoly (in production and research) and maybe when patents are weak. In other cases the tax should be less than MED because of the common pool effect of research, monopoly pricing by the patent holder, and if environmental damages are convex. The use of research prizes or contracts are suggested when patents are weak.

Biglaiser and Horowitz's (1995) research emphasizes that government regulation, rather than market forces, is an important incentive behind research, development, and adoption of pollution control technology. In particular, they use a four stage game to model the R&D and adoption process. In stage 1, firms decide whether or not to conduct

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<sup>1</sup> Milliman and Prince (1989) briefly discussed monopoly rents from patented technology but not nearly as rigorously as Parry (1994).

research with the results of any research becoming common knowledge. Stage 2 sees the government select an environmental regulation which may include emission taxes and technology standards. Firms patent and license discovered technology in stage 3 where the price of the license depends on government regulations and is endogenous. In stage 4, firms profit maximize. Key assumptions and aspects of their model are that 1) output price of firms is fixed, 2) firms are initially homogeneous, 3) licensing of technology is possible, 4) research results in technology randomly drawn from distribution which is identical for all firms, 5) actions of firms implicitly affect one another.

Results from their analysis suggest 1) that an *ex post* efficient condition will exist if emission taxes are set equal to marginal damages and some firms are required to adopt the best available control technology, as determined by the licensing fee and cost of technology adoption. 2) if regulators can commit at stage 0 to regulations conditional on research, then *stricter adoption standards or higher emission taxes will not increase research by firms* (actually, the model shows a decrease in research but the authors seem to play down the result). 3) socially optimal solutions may be achieved by the regulator awarding an innovation prize and using *ex post* optimization. 4) If the government undertakes research, private research will be displaced at a one for one ratio assuming the price charged for technology developed is the same as by private firms.

Laffont and Tirole (1994) use a two-period, principal-agent model to analyze the effects of spot and futures markets for tradable emission allowances on a polluter's compliance decision. The role of the agent's investment decision and the possibility of excessive bypass<sup>2</sup> are emphasized. In their model, pollution and production are linked, and if the agent pollutes, an allowance permit is required. By investing  $i$  in time period one, however, the agent can bypass the allowance market in period 2 as the investment is assumed to eliminate pollution. A cost of public funds ( $\lambda$ ) is included in the model, allowing excessive bypass to exist. Other aspects of the model include: 1) the agent knows his value of polluting in period 1 but does not know the value of polluting in period

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<sup>2</sup> Bypass in this context refers to the notion that when long-term investment is possible, an economic actor will take actions to reduce the use of a factor of production, thereby bypassing that factor of production. For instance, by investing in pollution control technology, a firm can bypass the amount of permits needed to pollute.

2 until the beginning of period 2 and 2) the government (regulator) does not observe the agent's valuation, only who actually pollutes. Results are: 1) in a static context, the price or number of permits should be equated to the Ramsey price. 2) Assuming the regulator can commit to second period price/quantity, the optimal emission allowance program has a permit price between the marginal cost price and the Ramsey price. The addition of a futures market in addition to spot markets reduces bypass by lowering second period price. 3) The government has an incentive to sell more permits in the second period (the allowance program is not time consistent). This may be solved by price support policy or first period giveaway of options. 4) Under "overall regulation," where the regulator knows agent's pollution level, investment, and production, the government should offer a menu of options contingent on whether the agent invests. This result is not *ex post* efficient, however. The government also taxes investment and induces less innovation than would occur under the first best solution. 5) Under pure environmental regulation, where the regulator can only regulate pollution, a more efficient allocation mechanism is to sell bundled permits to help extract rents.

#### 2.3.4 SUMMARY OF LITERATURE AND RESEARCH NICHE

If nothing else, the literature review should leave the impression that dynamic pollution regulation is complex, there is no simple optimal regulation unless the model is simplified, and that the regulator is attempting to regulate multiple actions by firms with one instrument.

A practical method to digesting the literature and formulating a new model or research niche is by asking what are the key characteristics of the problem. First, any analysis conducted must be done for equilibrium conditions. Second, the analysis should compare various policy instruments, including standards. The assumption that standards are inferior to taxes and permits may be misplaced. In fact, some of the more complex models make simplifying assumptions, such as perfect information which was part of the justification for using price rules over emission standards initially. Third, the nature of

R&D should be described more fully and made probabilistic. Fourth, the model must allow the regulator to act strategically. In particular, the regulator must be able to preemptively commit to regulation, rationally expecting innovation to occur if the proper incentives are given to the firm. Yet, the regulator must also have the ability to respond to innovation, not committing to regulation until after innovation occurs. In the previous literature, only Biglaiser and Horowitz (1995) examine both non-commitment and commitment prior to innovation.

### 3. ONE PERIOD MODEL

In this section, a one period model of pollution regulation with endogenous technological change is developed. The assumptions and characteristics of the model are first developed when the regulator can commit to the future level of either a standard or tax prior to a firm's decision to invest in R&D. The equilibrium results are then found when with either a standard or tax is used as the regulatory instrument. The key result in this analysis is that, assuming innovation is deterministic in nature, a standard will result in the first best outcome while a tax may not. The current result in the literature, which is based on non-commitment by the regulator, is then developed in a similar framework. Regulation with standards and taxes is then compared under both commitment and non-commitment.

#### 3.1 One Period Model with Commitment

Consider a one period model where a regulator and a representative firm interact over three stages. The firm is assumed to be profit maximizing and emits a pollutant that imposes a negative externality on society. The regulator attempts to maximize social welfare by controlling the level of pollution emitted *via* a Pigouvian emission tax or an emission standard. In the first stage of the model, the regulator, knowing the firm may invest in R&D and subsequently have a lower cost of abatement, commits to a standard,  $S$ , or tax<sup>3</sup>,  $t$ . The firm then decides in the second stage whether or not to invest in R&D. The firm learns the result of its R&D activities in the third stage and then abates, given the regulation set by the regulator in the first stage. The objectives of both players are common knowledge as is all the information throughout the model.

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<sup>3</sup> The tax is assumed to reflect a transfer payment only. If revenue from the tax were spent to fund other projects, such as R&D, the efficiency of taxes would be altered.

The benefit of pollution abatement is given by  $XA - mA^2$  where  $A$  is the level of abatement and  $X$  and  $m$  are constants. Taking the first derivative yields  $X-2mA$ , the marginal benefit of abatement. The downward sloping nature of the marginal benefit curve implies that each successive unit of pollution abated is less valuable to society. Conversely, it implies that the first unit of pollution emitted has little impact on social welfare while latter units of pollution have much larger impacts. For example, minute amounts of pesticides in drinking water may pose little risk to humans and thus little welfare loss. However, each additional unit of pesticide possess more significant risks because of additive impacts.

The current abatement technology available to the firm is  $\alpha_0$  which allows pollution to be abated at a cost of  $C_0A^2$ . Taking the first derivative yields a marginal cost of  $2C_0A$ . The upward sloping marginal cost curve implies that each successive unit of pollution abated is more costly than the previous unit. For example, reducing sulfur dioxide emissions from coal-fired power plants may be relatively inexpensive per unit at first as low sulfur coal may be able to be inexpensively substituted for high sulfur coal or simple scrubbers may be installed. As more units of pollution are abated, more elaborate and costly filters and changes in production processes are needed, raising the marginal cost of abatement.

The firm, however, is assumed to be capable of developing more efficient pollution control technology if it engages in R&D. If successful, the firm develops technology  $\alpha_1$  and may abate with cost  $C_1A^2$  where  $C_0 > C_1$  so the marginal cost of abating is reduced. Success in R&D is probabilistic with the probability of successfully discovering technology  $\alpha_1$  equaling  $(\alpha_1 R)^\epsilon$ . The amount the firm invests is given by  $R$  where  $0 \leq R \leq \frac{1}{\alpha_1}$  so that the probability of success lies between zero and one. The variable  $\epsilon$  is defined as greater than or equal to zero and is included in the probability of success function to allow the function to be either convex or concave with respect to  $R$ .

In particular if  $\epsilon > 1$ , the probability of success with respect to  $R$  is convex, implying increasing returns to scale in the R&D process. Therefore, each successive dollar invested in R&D will increase the probability of success by more than the previous dollar. It can be

shown that if increasing returns exist, the firm will invest such that it succeeds with certainty (a probability of one) or will not invest in R&D at all. For example, if a firm wishes to develop and implement a new production line to produce electric cars or adapt a scrubber to its plant, it will likely invest such that the new production line/scrubber will function properly with certainty. It would make little economic sense to alter the production line so it will work only part of the time or adapt half of a scrubber to a plant. In situations where  $\epsilon > 1$ , innovation, or the R&D process, will be referred to as deterministic.

Conversely, if  $\epsilon < 1$ , the probability of success with respect to R is concave, implying decreasing returns to scale in the R&D process. In other words, each successive dollar invested will result in a smaller increase in the probability of success than the previous dollar. Assuming the benefit associated with discovering  $\alpha_1$  is not very small or large, the firm will invest R such that the probability of success lies between zero and one. For example, if a firm wishes to develop a marketable electric car, the first dollars spent on research are likely to be very productive. However, as more money is spent on research, the likelihood that the additional money will increase the probability of success is small. In situations where  $\epsilon < 1$ , innovation will be referred to as stochastic.

For comparative purposes, it is useful to think of stochastic innovation as research oriented while deterministic innovation as the process of developing the findings of research into products or processes. In other words, the results of research are likely to be stochastic while the development of new products or processes from the results of research are likely to be all or nothing in nature (*i.e.*, deterministic). The important point, however, is whether there are decreasing or increasing returns in innovation and how that influences the amount a firm invests in R&D.

The variable  $\alpha_1$  is included in the probability of success function for use in the two period model to be developed later and to represent the possibility that differing technologies will be more or less difficult to discover. While this aspect could be captured largely by altering  $\epsilon$ , it will be useful to hold  $\epsilon$  constant across differing technologies, using  $\epsilon$  to make the function concave or convex and using  $\alpha_1$  as a means of altering R&D costs.

To avoid confusion, the reader should note that the subscript denotes the technology level and the value of  $\alpha$  signifies the difficulty of developing that technology.

To solve for subgame perfect equilibrium, backwards induction will be used throughout this thesis.

### 3.2 Standards with Commitment

First consider the case where a standard is used as the regulatory instrument as shown in Figure 3.1. In the figure, the marginal benefit of abatement is represented by the MB line and marginal cost by MC where  $MC_0$  denotes the marginal cost under the existing technology  $\alpha_0$  and  $MC_1$  the marginal cost under the technology  $\alpha_1$  which may be discovered.

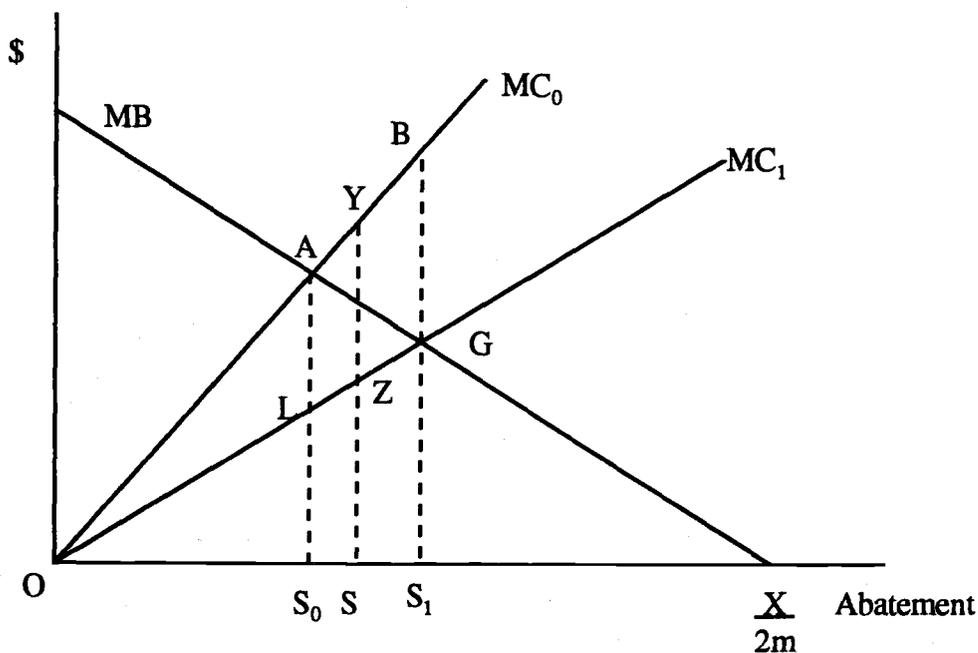


Figure 3.1: Regulation with Standards

Working backwards, when the firm makes its investment decision, it will be facing some standard  $S$  to which the regulator has already committed. Knowing this and that it may invest, the firm selects an investment level  $R$  which minimizes its expected cost:

$$\min_R \text{cost} = \min_R (\alpha_1 R)^\varepsilon C_1 S^2 + (1 - (\alpha_1 R)^\varepsilon) C_0 S^2 + R.$$

The first term in the cost function,  $(\alpha_1 R)^\varepsilon C_1 S^2$ , represents the abatement cost if technology  $\alpha_1$  is discovered multiplied by the probability that  $\alpha_1$  is discovered if  $R$  is spent on R&D. The second term,  $(1 - (\alpha_1 R)^\varepsilon) C_0 S^2$ , is the abatement cost if the new technology  $\alpha_1$  is not discovered multiplied by the probability of that occurrence if  $R$  is spent on R&D. The third term,  $R$ , is the amount the firm spends on R&D.

Taking the partial with respect to  $R$  and setting the result equal to zero, we have:

$$\begin{aligned} \frac{\partial \text{cost}}{\partial R} &= \varepsilon \alpha_1^\varepsilon R^{\varepsilon-1} C_1 S^2 - \varepsilon R^{\varepsilon-1} \alpha_1^\varepsilon C_0 S^2 + 1 = 0 \\ &= \varepsilon \alpha_1^\varepsilon R^{\varepsilon-1} S^2 (C_1 - C_0) + 1 = 0. \end{aligned}$$

Solving for  $R$ , assuming an interior solution, we find:

$$R = \left[ \frac{1}{\varepsilon \alpha_1^\varepsilon S^2 (C_0 - C_1)} \right]^{1/(\varepsilon-1)}$$

where  $R$  is the optimal R&D expenditures to minimize the firm's expected cost for any  $S$  imposed by the regulator.

Examining the second order conditions, we can see that the derivation of  $R$  will not always yield a minimum cost:

$$\frac{\partial^2 \text{cost}}{\partial R^2} = \varepsilon(\varepsilon - 1) \alpha_1^\varepsilon R^{\varepsilon-2} S^2 (C_1 - C_0).$$

Because  $\varepsilon$ ,  $\alpha_1^\varepsilon$ ,  $R^{\varepsilon-2}$ , and  $S^2$  will always be positive and  $(C_1 - C_0)$  will always be negative (as  $C_1 < C_0$ ), the sign of the second derivative depends on  $(\varepsilon - 1)$ . In particular, if:

$\epsilon > 1$ , then  $(\epsilon-1) > 0$ , so the second order condition is negative implying a maximum (cost);

$\epsilon = 0$ , then  $(\epsilon-1) = 0$ , so the second order condition is zero implying a constant (marginal cost);

$\epsilon < 1$ , then  $(\epsilon-1) < 0$ , so the second order condition is positive implying a minimum (cost).

As noted in the description of the model,  $\epsilon$  determines whether the probability of success function, and hence the cost function, is concave or convex.

### 3.2.1 $\epsilon < 1$ : STOCHASTIC INNOVATION

If  $\epsilon < 1$ , we know  $R$  will be the cost minimizing R&D expenditures for the firm and innovation will be stochastic. The regulator possesses the same information when setting the standard  $S$  and will set  $S$  such that the expected welfare is maximized:

$$\begin{aligned} \max_S \text{welfare} &= \max_S (XS - mS^2) - [(\alpha_1 R)^\epsilon C_1 S^2 + (1 - (\alpha_1 R)^\epsilon) C_0 S^2 + R] \\ &= \max_S (XS - mS^2) - \left( \frac{\alpha_1^\epsilon C_1 S^2}{(\epsilon \alpha_1^\epsilon S^2 (C_0 - C_1))^{\frac{\epsilon}{\epsilon-1}}} \right) - C_0 S^2 \\ &\quad + \left( \frac{\alpha_1^\epsilon C_0 S^2}{(\epsilon \alpha_1^\epsilon S^2 (C_0 - C_1))^{\frac{\epsilon}{\epsilon-1}}} \right) - \left( \frac{1}{(\epsilon \alpha_1^\epsilon S^2 (C_0 - C_1))^{\frac{1}{\epsilon-1}}} \right) \end{aligned}$$

Taking the partial with respect to  $S$  and setting the result equal to zero, we find:

$$\frac{\partial \text{welfare}}{\partial S} = X - 2mS - \left( \frac{\alpha_1^\varepsilon C_1 (2 - 2\varepsilon/\varepsilon - 1) S^{(1-2\varepsilon/\varepsilon-1)}}{(\varepsilon \alpha_1^\varepsilon (C_0 - C_1))^{\varepsilon/\varepsilon-1}} \right) - 2C_0 S$$

$$+ \left( \frac{\alpha_1^\varepsilon C_0 (2 - 2\varepsilon/\varepsilon - 1) S^{(1-2\varepsilon/\varepsilon-1)}}{(\varepsilon \alpha_1^\varepsilon (C_0 - C_1))^{\varepsilon/\varepsilon-1}} \right) + \left( \frac{(2/\varepsilon - 1) S^{((-2/\varepsilon-1)-1)}}{(\varepsilon \alpha_1^\varepsilon (C_0 - C_1))^{\varepsilon/\varepsilon-1}} \right) = 0.$$

Simplifying and substituting in R results in:

$$X - 2mS = \left[ 2C_0 S - \left( \frac{\alpha_1^\varepsilon (2 - 2\varepsilon/\varepsilon - 1) S^{(1-2\varepsilon/\varepsilon-1)}}{(\varepsilon \alpha_1^\varepsilon (C_0 - C_1))^{\varepsilon/\varepsilon-1}} \right) (C_0 - C_1) \right] + \left[ \frac{(-2/\varepsilon - 1)}{S} R \right].$$

Solving for S yields the equilibrium standard S that maximizes social welfare. However, this will not necessarily be the first best or optimal outcome as S, the abatement level, will not necessarily be set at the *ex post* optimal level of abatement. In general, S will be set between  $S_0$  and  $S_1$  as shown in Figure 3.1. From the above equation, though, it can be seen that the equilibrium S will be set such that the marginal benefit of pollution abatement (the LHS) will equal the expected marginal cost of abatement (the first bracketed term on the RHS) plus the marginal cost of R&D (the second bracketed term on the RHS) associated with altering S.

### 3.2.2 $\varepsilon > 1$ : DETERMINISTIC INNOVATION

If  $\varepsilon > 1$ , we know R will be cost *maximizing* and that the firm will invest such that R is a corner solution. In other words, R will be at a level which insures failure or guarantees success in the R&D process so innovation will be deterministic. Mathematically,

$$(\alpha_1 R)^e = 0 \Rightarrow R = 0$$

or

$$(\alpha_1 R)^e = 1 \Rightarrow R = \frac{1}{\alpha_1}.$$

The firm will choose to invest when the change in the equilibrium *abatement cost* is greater than or equal to the cost of R&D for some standard  $S$  imposed the regulator. Specifically, the change in abatement cost is given by  $C_0 S^2 - C_1 S^2$  or  $S^2(C_0 - C_1)$ ; therefore,

$$R = \begin{cases} 0 & \text{if } S^2(C_0 - C_1) < \frac{1}{\alpha_1} \\ \frac{1}{\alpha_1} & \text{if } S^2(C_0 - C_1) \geq \frac{1}{\alpha_1}. \end{cases}$$

Knowing the firm's criteria for investing, the regulator will set the standard  $S$  to maximize expected welfare. Before solving for the equilibrium  $S$ , though, two relevant notes must be made.

First, if the regulator sets a standard  $S$ , ignoring all incentives to the firm to invest and assuming very naively that the firm has either technology  $\alpha_0$  or  $\alpha_1$ , the optimal  $S$  would be either  $S_0$  or  $S_1$  corresponding to  $MC_0 = MB$  and  $MC_1 = MB$  as shown in Figure 3.1. These solutions are the basic static equilibrium results without the possibility of innovation and are well known in the literature (Tietenburg, 1992; Baumol and Oates, 1988). Mathematically,

$$\text{welfare} = \begin{cases} W_{S_0} = XS_0 - mS_0^2 - C_0 S_0^2 & \text{if } R = 0 \\ W_{S_1} = XS_1 - mS_1^2 - C_1 S_1^2 - \frac{1}{\alpha_1} & \text{if } R = \frac{1}{\alpha_1}. \end{cases}$$

Taking the first derivative with respect to  $S$  and setting the result equal to zero yields:

$$\frac{\partial \text{welfare}}{\partial S} = \begin{cases} X - 2mS_0 - 2C_0 S_0 = 0 & \text{if } R = 0 \\ X - 2mS_1 - 2C_1 S_1 = 0 & \text{if } R = \frac{1}{\alpha_1}. \end{cases}$$

Solving for S gives:

$$S = \begin{cases} S_0 = \frac{X}{2(m + C_0)} & \text{if } R = 0 \\ S_1 = \frac{X}{2(m + C_1)} & \text{if } R = \frac{1}{\alpha_1}. \end{cases}$$

Second, assuming commitment and that S is greater than  $S_0$ , using a standard as a regulatory instrument will create an incentive for the firm to invest in R&D which is greater than the regulator's. If the standard is set equal to  $S_0$ , the incentive for the firm to invest will equal the regulator's incentive (e.g., both the firm and regulator will have an incentive of OAL in Figure 3.1). In other words, under these conditions the change in *abatement cost* is greater than or equal to the change in *welfare*.<sup>4</sup> To see this mathematically, note that the change in welfare and abatement cost are given by:

$$\Delta \text{welfare} = \int_0^S (X - 2mA) da - \int_0^S 2C_1 A da - \left[ \int_0^{S_0} (X - 2mA) da - \int_0^{S_0} 2C_0 A da \right]$$

and

$$\Delta \text{abatement cost} = \int_0^S 2C_0 A da - \int_0^S 2C_1 A da$$

where the benefit and cost of abatement are now written in terms of areas under the marginal curves. For instance, the change in abatement cost is represented by area OYZ in Figure 3.1. Comparing the two changes,

$$\begin{aligned} \Delta \text{cost} - \Delta \text{welfare} &= \int_0^S 2C_0 A da - \int_0^S 2C_1 A da \\ &\quad - \left[ \int_0^S (X - 2mA) da - \int_0^S 2C_1 A da - \left[ \int_0^{S_0} (X - 2mA) da - \int_0^{S_0} 2C_0 A da \right] \right] \\ &= \int_{S_0}^S 2C_0 A da - \int_{S_0}^S (X - 2mA) da \\ &\geq 0 \end{aligned}$$

<sup>4</sup> The cost of R&D is not included in welfare here as the intent is to compare the incentive to invest; moreover, the cost of R&D will be the same for the firm and society.

as  $2C_0S_0 = X - 2mS_0$  and  $2C_0A$  increases while  $X - 2mA$  decreases over the region  $S_0$  to  $\frac{X}{2m}$ .

It is now possible solve for the equilibrium  $S$  the regulator will impose. Recall that under stochastic innovation, the equilibrium  $S$  was not necessarily the first best solution as the standard was not necessarily set at the *ex post* optimal level. Under deterministic innovation, however, the regulator will be able to set the first best regulation using an emission standard.

If the first best solution is to be implemented, the regulator must be able to set the standard  $S$  at either  $S_0$  or  $S_1$ , corresponding to the optimal *ex post* abatement level for technology  $\alpha_0$  or  $\alpha_1$  respectively, without altering the incentive for the firm to innovate such that a different technology level occurs than the one which the standard  $S$  is optimally set for. In other words, *the regulator must be able to set a standard prior to the firm's R&D investment decision which induces the proper level of investment in R&D but which will also result in the optimal level of abatement after R&D occurs.*

Mathematically, if  $W_{S_0} > W_{S_1}$  as defined above, then the optimal standard for the regulator to set is  $S_0$  and have the firm *not* invest. It must then be shown that *in equilibrium* that the firm will indeed not invest if  $S=S_0$ . Conversely, if  $W_{S_0} \leq W_{S_1}$ , the optimal standard for the regulator to set is  $S_1$  and have the firm invest. It must then be shown that *in equilibrium* the firm will indeed invest if  $S=S_1$ .

To see why both of these conditions hold, first suppose the regulator sets the standard at  $S_0$ , the first best level of abatement if the optimal solution is *not* to invest in R&D. The crux of the question is *does* the firm then invest in response the  $S_0$ ? To determine if it does, we check if the change in abatement cost (represented by area OAL in Figure 3.1) to the firm is greater than or equal to the expense  $R$  associated with R&D. If the change in abatement cost is indeed greater than or equal to  $R$ , then the firm's best response to the standard is to invest, resulting in a sub optimal solution. However, as noted earlier, the benefit of investing at the lower standard is the same for both the regulator and the firm, namely OAL. Thus, if the regulator sets  $S_0$  because it is optimal, the firm will not invest because the cost of R&D is greater than the change in abatement cost and a first best solution will result.

Now suppose the regulator sets the standard at  $S_1$ , the first best level of abatement if the optimal solution is to invest in R&D. Once again, the real question is *does* the firm invest in R&D in response to the standard  $S_1$ ? To answer this, we again check if the change in *abatement cost* (represented by area OBG in Figure 3.1) is greater than or equal to the cost of R&D. From the notes made above, we know the firm has an incentive to invest which is greater than the change in welfare. Thus, if the optimal solution is to set a standard at  $S_1$ , then the cost of R&D must be less the change in welfare (area OAG and excluding R&D costs); therefore, the change in abatement cost must also be less than the cost of R&D. Consequently, the firm will invest and the optimal solution can be implemented. It should be noted that there is not a concern about the firm over investing in this situation because of the deterministic nature of innovation (*e.g.*, the firm will invest until the probability of success equals 1).

Thus, if innovation is deterministic ( $\epsilon > 1$ ) and the regulator can preemptively commit to future regulation, the first best outcome can be achieved *via* an emission standard.

### **3.3 Taxes with Commitment**

Now consider the situation where a tax  $t$  is used as the regulatory instrument as shown in Figure 3.2. As in Figure 3.1, the marginal benefit and costs are represented by MB,  $MC_0$ , and  $MC_1$ .

As in the case of standards, the firm will attempt to minimize its expected cost with respect to  $R$  in response the regulator's actions. However, the firm, not the regulator, now determines the level of abatement. Since the results of R&D will be known when the firm's abatement decision is made, the abatement level will be such that the marginal cost of abatement equals the tax rate.

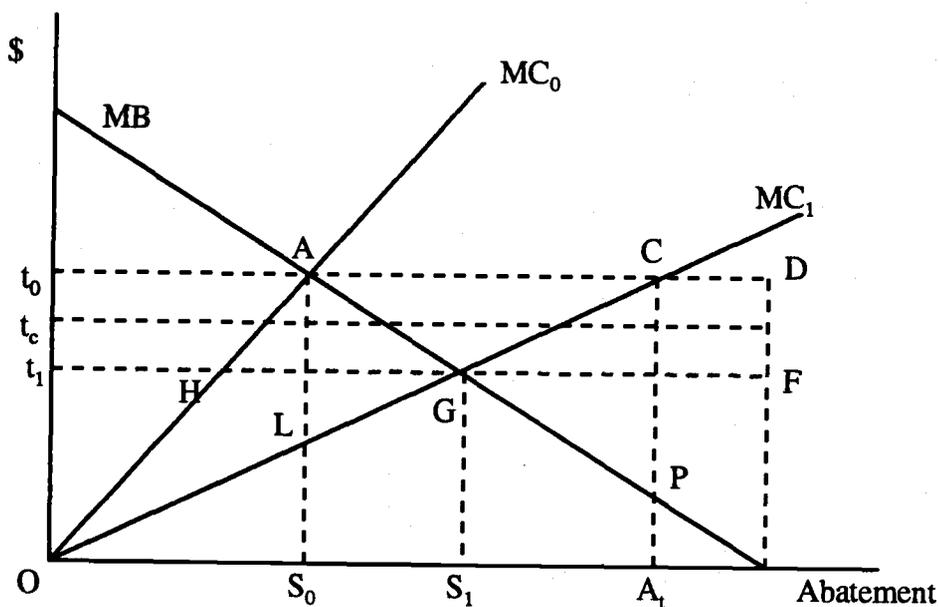


Figure 3.2: Regulation with Taxes

For instance, suppose the firm has technology  $\alpha_i$  and faces tax  $t_j$ , then:

$$\text{cost to firm} = C_1 A^2 + t_j \left( \frac{X}{2m} - A \right)$$

where the first term represents the abatement cost and the second term is the tax paid on units of pollution emitted or not abated. The constant  $C_1$  is the cost coefficient associated with technology  $\alpha_i$ . Taking the partial with respect to  $A$  and setting the result equal to zero yields:

$$\frac{\partial \text{cost to firm}}{\partial A} = 2C_1 A - t_j = 0.$$

The firm, therefore, will set:

$$A = \frac{t_j}{2C_1}.$$

Using the above information, it is now possible to determine the firm's R&D expenditures which will minimize its expected cost:

$$\begin{aligned}
 \min_R \text{ cost} &= \min_R (\alpha_1 R)^\varepsilon \left[ C_1 \left( \frac{t}{2C_1} \right)^2 + t \left( \frac{X}{2m} - \frac{t}{2C_1} \right) \right] + \\
 &\quad + (1 - (\alpha_1 R)^\varepsilon) \left[ C_0 \left( \frac{t}{2C_0} \right)^2 + t \left( \frac{X}{2m} - \frac{t}{2C_0} \right) \right] + R \\
 &= \min_R (\alpha_1 R)^\varepsilon \left[ \frac{t^2}{4C_1} + \frac{Xt}{2m} - \frac{t^2}{2C_1} \right] + (1 - (\alpha_1 R)^\varepsilon) \left[ \frac{t^2}{4C_0} + \frac{Xt}{2m} - \frac{t^2}{2C_0} \right] + R \\
 &= \min_R (\alpha_1 R)^\varepsilon \left[ \frac{Xt}{2m} - \frac{t^2}{4C_1} \right] + (1 - (\alpha_1 R)^\varepsilon) \left[ \frac{Xt}{2m} - \frac{t^2}{4C_0} \right] + R.
 \end{aligned}$$

Similar to the expected cost function under regulation using a standard, the first term,  $(\alpha_1 R)^\varepsilon \left[ \frac{Xt}{2m} - \frac{t^2}{4C_1} \right]$ , represents the simplified expression of the *tax and abatement costs* if the firm discovers technology  $\alpha_1$  multiplied by the probability of discovering  $\alpha_1$  if R is spent on R&D. Likewise, the second term,  $(1 - (\alpha_1 R)^\varepsilon) \left[ \frac{Xt}{2m} - \frac{t^2}{4C_0} \right]$ , is the simplified expression of the tax and abatement costs if the new technology  $\alpha_1$  is not discovered multiplied by the probability of that event occurring if R is spent on R&D.

Taking the partial with respect to R and setting the result equal to zero yields:

$$\begin{aligned}
 \frac{\partial \text{cost}}{\partial R} &= \varepsilon \alpha_1^\varepsilon R^{\varepsilon-1} \left[ \frac{Xt}{2m} - \frac{t^2}{4C_1} \right] - \varepsilon \alpha_1^\varepsilon R^{\varepsilon-1} \left[ \frac{Xt}{2m} - \frac{t^2}{4C_0} \right] + 1 = 0 \\
 &= \varepsilon \alpha_1^\varepsilon R^{\varepsilon-1} \left[ \frac{t^2}{4C_0} - \frac{t^2}{4C_1} \right] + 1 = 0 \\
 &= \varepsilon \alpha_1^\varepsilon R^{\varepsilon-1} t^2 \left[ \frac{C_1 - C_0}{4C_0 C_1} \right] + 1 = 0.
 \end{aligned}$$

Assuming an interior solution and solving for R, we find:

$$R = \left[ \frac{4C_0C_1}{\varepsilon \alpha_1^\varepsilon t^2 (C_0 - C_1)} \right]^{1/\varepsilon-1}$$

where  $R$  is the optimal R&D expenditures to minimize the firm's expected cost for any  $t$  imposed by the regulator.

The general form of  $\frac{\partial \text{cost}}{\partial R}$  for a tax is the same as it was for a standard.

Consequently, the conditions under which  $R$  will be cost maximizing or minimizing will be the same as for a standard.

### 3.3.1 $\varepsilon < 1$ : STOCHASTIC INNOVATION

If  $\varepsilon < 1$ , we know  $R$  will be the cost minimizing R&D expenditure for the firm and that innovation will be stochastic in nature. The regulator possesses the same information when committing to a tax  $t$  and will set  $t$  such that welfare is maximized:

$$\begin{aligned} \max_t \text{welfare} = & \max_t (\alpha_1 R)^\varepsilon \left[ X\left(\frac{t}{2C_1}\right) - m\left(\frac{t}{2C_1}\right)^2 - C_1\left(\frac{t}{2C_1}\right)^2 \right] \\ & + (1 - (\alpha_1 R)^\varepsilon) \left[ X\left(\frac{t}{2C_0}\right) - m\left(\frac{t}{2C_0}\right)^2 - C_0\left(\frac{t}{2C_0}\right)^2 \right] - R. \end{aligned}$$

Substituting in for  $R$  yields:

$$\begin{aligned} = & \max_t \alpha_1^\varepsilon \left[ \frac{4C_0C_1}{\varepsilon \alpha_1^\varepsilon t^2 (C_0 - C_1)} \right]^{1/\varepsilon-1} \left[ X\left(\frac{t}{2C_1}\right) - m\left(\frac{t}{2C_1}\right)^2 - C_1\left(\frac{t}{2C_1}\right)^2 \right] \\ & + (1 - \alpha_1^\varepsilon \left[ \frac{4C_0C_1}{\varepsilon \alpha_1^\varepsilon t^2 (C_0 - C_1)} \right]^{1/\varepsilon-1}) \left[ X\left(\frac{t}{2C_0}\right) - m\left(\frac{t}{2C_0}\right)^2 - C_0\left(\frac{t}{2C_0}\right)^2 \right] \\ & - \left[ \frac{4C_0C_1}{\varepsilon \alpha_1^\varepsilon t^2 (C_0 - C_1)} \right]^{1/\varepsilon-1} \end{aligned}$$

$$\begin{aligned}
&= \max_t \left[ \frac{\alpha_1^\varepsilon (4C_0 C_1)^{\varepsilon/\varepsilon-1}}{(\varepsilon \alpha_1^\varepsilon (C_0 - C_1))^{\varepsilon/\varepsilon-1}} \right] t^{-2\varepsilon/\varepsilon-1} \left[ \left( \frac{Xt}{2C_1} \right) - \left( \frac{mt^2}{4C_1^2} \right) - \left( \frac{t^2}{4C_1} \right) \right] \\
&\quad + \left( \frac{Xt}{2C_0} \right) - \left( \frac{mt^2}{4C_0^2} \right) - \left( \frac{t^2}{4C_0} \right) \\
&\quad - \left[ \frac{\alpha_1^\varepsilon (4C_0 C_1)^{\varepsilon/\varepsilon-1}}{(\varepsilon \alpha_1^\varepsilon (C_0 - C_1))^{\varepsilon/\varepsilon-1}} \right] t^{-2\varepsilon/\varepsilon-1} \left[ \left( \frac{Xt}{2C_0} \right) - \left( \frac{mt^2}{4C_0^2} \right) - \left( \frac{t^2}{4C_0} \right) \right] \\
&\quad - \left[ \frac{(4C_0 C_1)^{1/\varepsilon-1}}{(\varepsilon \alpha_1^\varepsilon (C_0 - C_1))^{1/\varepsilon-1}} \right] t^{-2/\varepsilon-1}.
\end{aligned}$$

Taking the partial with respect to  $t$  and setting the result equal to zero:

$$\begin{aligned}
\frac{\partial \text{welfare}}{\partial t} &= \left[ \frac{\alpha_1^\varepsilon (4C_0 C_1)^{\varepsilon/\varepsilon-1}}{(\varepsilon \alpha_1^\varepsilon (C_0 - C_1))^{\varepsilon/\varepsilon-1}} \right] \left[ \left( \frac{(1-2\varepsilon/\varepsilon-1)Xt^{-2\varepsilon/\varepsilon-1}}{2C_1} \right) - \left( \frac{(2-2\varepsilon/\varepsilon-1)mt^{1-(2\varepsilon/\varepsilon-1)}}{4C_1^2} \right) \right] \\
&\quad - \left( \frac{(2-2\varepsilon/\varepsilon-1)t^{1-(2\varepsilon/\varepsilon-1)}}{4C_1} \right) \\
&\quad + \left( \frac{X}{2C_0} \right) - \left( \frac{mt}{2C_0^2} \right) - \left( \frac{t}{2C_0} \right) \\
&\quad - \left[ \frac{\alpha_1^\varepsilon (4C_0 C_1)^{\varepsilon/\varepsilon-1}}{(\varepsilon \alpha_1^\varepsilon (C_0 - C_1))^{\varepsilon/\varepsilon-1}} \right] \left[ \left( \frac{(1-2\varepsilon/\varepsilon-1)Xt^{-2\varepsilon/\varepsilon-1}}{2C_0} \right) - \left( \frac{(2-2\varepsilon/\varepsilon-1)mt^{1-(2\varepsilon/\varepsilon-1)}}{4C_0^2} \right) \right] \\
&\quad - \left( \frac{(2-2\varepsilon/\varepsilon-1)t^{1-(2\varepsilon/\varepsilon-1)}}{4C_0} \right) \\
&\quad - \left[ \frac{(-2/\varepsilon-1)(4C_0 C_1)^{1/\varepsilon-1} t^{(-2/\varepsilon-1-1)}}{(\varepsilon \alpha_1^\varepsilon (C_0 - C_1))^{1/\varepsilon-1}} \right] = 0.
\end{aligned}$$

Simplifying and substituting in R results in:

$$\frac{(-2/\varepsilon - 1)}{t} R = \left[ \frac{(\alpha_1 R)^\varepsilon}{4C_0^2 C_1^2} \left[ (2(1 - 2\varepsilon/\varepsilon - 1)XC_0 C_1(C_0 - C_1)) + (2 - 2\varepsilon/\varepsilon - 1)mt(C_1^2 - C_0^2) \right] \right. \\ \left. + (2 - 2\varepsilon/\varepsilon - 1)tC_0 C_1(C_1 - C_0) \right] \\ + \left( \frac{X}{2C_0} \right) - \left( \frac{mt}{2C_0^2} \right) - \left( \frac{t}{2C_0} \right).$$

Solving for  $t$  yields the equilibrium tax  $t$  that maximizes the expected social welfare. However, it will not necessarily be the first best as the tax  $t$  will not be set at a level which necessarily guarantees the *ex post* optimal level of abatement. While the simplified equation above is not particularly clear, the equilibrium  $t$  will be set such that the marginal cost of R&D (the LHS) equals the expected net benefit of pollution abatement and reduction in tax payments to the firm (the RHS).

### 3.3.2 $\varepsilon > 1$ : DETERMINISTIC INNOVATION

If  $\varepsilon > 1$ , we know  $R$  will be cost *maximizing* and that the firm will invest such that  $R$  is a corner solution or that innovation will be deterministic in nature. As was the case with a standard,  $R=0$  if the firm does not invest and  $R = \frac{1}{\alpha_1}$  if the firm does invest.

The firm will choose to invest when the change in the abatement and tax cost is greater than or equal to the cost of R&D for some tax level  $t$  imposed by the regulator. Specifically, the change in cost is given by:

$$\Delta \text{cost} = \left[ C_0 \left( \frac{t}{2C_0} \right)^2 + t \left( \frac{X}{2m} - \frac{t}{2C_0} \right) \right] - \left[ C_1 \left( \frac{t}{2C_1} \right)^2 + t \left( \frac{X}{2m} - \frac{t}{2C_1} \right) \right] \\ = \left[ \frac{Xt}{2m} - \frac{t^2}{4C_0} \right] - \left[ \frac{Xt}{2m} - \frac{t^2}{4C_1} \right] \\ = \left[ \frac{t^2}{4C_1} - \frac{t^2}{4C_0} \right].$$

Therefore, the firm will choose:

$$R = \begin{cases} 0 & \text{if } \left[ \frac{t^2}{4C_1} - \frac{t^2}{4C_0} \right] < \frac{1}{\alpha_1} \\ \frac{1}{\alpha_1} & \text{if } \left[ \frac{t^2}{4C_1} - \frac{t^2}{4C_0} \right] \geq \frac{1}{\alpha_1} \end{cases}$$

Knowing the firm's criteria for investing, the regulator will set the tax  $t$  to maximize the expected welfare. Unlike was the case for a standard, the first best outcome will not always be achievable using an emission tax when innovation is deterministic. As before, several relevant notes must be made before solving for the equilibrium  $t$ .

To begin, if the regulator sets a tax  $t$ , ignoring all incentives created for the firm to invest and assuming that either technology  $\alpha_0$  or  $\alpha_1$  exists, the optimal tax would be either  $t_0$  or  $t_1$  corresponding to  $MC_0=MB$  and  $MC_1=MB$  in Figure 3.2. As before, these solutions are the basic static equilibrium result in the literature. Mathematically,

$$\text{welfare} = \begin{cases} W_{t_0} = \frac{Xt_0}{2C_0} - m \left( \frac{t_0}{2C_0} \right)^2 - C_0 \left( \frac{t_0}{2C_0} \right)^2 & \text{if } R = 0 \\ W_{t_1} = \frac{Xt_1}{2C_1} - m \left( \frac{t_1}{2C_1} \right)^2 - C_1 \left( \frac{t_1}{2C_1} \right)^2 - \frac{1}{\alpha_1} & \text{if } R = \frac{1}{\alpha_1} \end{cases}$$

Simplifying welfare results in:

$$\text{welfare} = \begin{cases} W_{t_0} = \frac{Xt_0}{2C_0} - \frac{m t_0^2}{4C_0^2} - \frac{t_0^2}{4C_0} & \text{if } R = 0 \\ W_{t_1} = \frac{Xt_1}{2C_1} - \frac{m t_1^2}{4C_1^2} - \frac{t_1^2}{4C_1} - \frac{1}{\alpha_1} & \text{if } R = \frac{1}{\alpha_1} \end{cases}$$

Taking the first derivative with respect to  $t$  and setting the result equal to zero yields:

$$\frac{\partial \text{welfare}}{\partial t} = \begin{cases} \frac{X}{2C_0} - \frac{2mt_0}{4C_0^2} - \frac{2t_0}{4C_0} = 0 & \text{if } R = 0 \\ \frac{X}{2C_1} - \frac{2mt_1}{4C_1^2} - \frac{2t_1}{4C_1} = 0 & \text{if } R = \frac{1}{\alpha_1}. \end{cases}$$

Solving for  $t$  gives:

$$t = \begin{cases} t_0 = \frac{XC_0}{(m+C_0)} & \text{if } R_1 = 0 \\ t_1 = \frac{XC_1}{(m+C_1)} & \text{if } R_1 = \frac{1}{\alpha_1}. \end{cases}$$

Secondly, setting the tax at  $t_0$  results in the incentive given to the firm to invest being greater than that of the regulator. In Figure 3.2, this over incentive is represented by area ACG and occurs because the emission tax must be lowered if innovation occurs for an *ex post* optimal level of abatement to occur. Because the regulator commits to a higher tax level associated with no innovation, however, the firm has an additional incentive to invest to avoid the emission tax which will not be lowered even if innovation occurs. This occurrence is well known in the literature (Wenders, 1975; Downing and White, 1986). Conversely, at the lower tax level  $t_1$  corresponding to the firm innovating, the incentive given to the firm to innovate is less than the regulator's by area HAG. This occurs as committing to the lower tax rate before the firm invest lowers the incentive as the firm will receive the benefit of lower taxes even if it does not innovate.

Using this information, it is now possible to solve for the equilibrium  $t$  the regulator will impose. As was the case for a standard, the optimal solution can only be implemented if the regulator is able to set a tax prior to the firm's R&D investment decision which results in the *ex post* optimal level of abatement while still providing the proper incentive to innovate.

First suppose the regulator sets the tax at  $t_0$ , the tax level which results in the first best level of abatement if the optimal solution is for investment *not* to occur. As was pointed above, at this tax level the firm has a greater incentive than the regulator to invest. Consequently, it is possible that when the optimal outcome is for no investment to occur,

the firm will nevertheless invest. In particular, the firm will invest if the change in *abatement and tax cost* (denoted by  $\Delta \text{cost}_{t_0, \text{high}}$  and represented by area OAC in Figure 3.2) is greater than or equal to the cost of R&D. Thus, if the change in cost is greater than or equal to the cost of R&D, the firm's best response to the tax  $t_0$  is to invest and a first best outcome *cannot* be achieved as over-investment and abatement occurs.

Conversely, if the change in cost is less than the cost of R&D, the firm's best response to the tax  $t_0$  is to *not* invest and a first best outcome can be achieved. In other words, in this situation the best strategy for the regulator is to set the tax at the tougher level  $t_0$  as the over-incentive created is not large enough to cause it the firm to invest.

Now suppose the regulator sets the tax at  $t_1$ , the tax level corresponding to the first best level of abatement if the optimal solution is to invest in R&D. In this situation, the firm always has a *smaller* incentive to innovate than the regulator. As a result, it is possible that when the first best outcome is for investment to occur, the firm will have an insufficient incentive to innovate. In essence, the argument from above is reversed. The firm will innovate only if the change in the cost of abatement and tax payments (denoted by  $\Delta \text{cost}_{t_1, \text{low}}$  and represented by area OHG in Figure 3.2) is greater than or equal to R&D expenses. Thus, if the change in cost is greater than or equal to the cost of R&D, the firm's best response to the tax  $t_1$  is to invest and a first best outcome *can* be achieved. This occurs as the under-incentive to invest is not large enough to cause the firm not to invest. Conversely, if the change in cost is less than the cost of R&D, the firm's best response to the tax  $t_1$  is to *not* invest and a first best outcome *cannot* be achieved as under-investment occurs.

From the preceding discussion, we now know that the first best outcome will be achievable if  $\Delta \text{cost}_{t_0, \text{high}} < \frac{1}{\alpha_1}$  or  $\Delta \text{cost}_{t_1, \text{low}} \geq \frac{1}{\alpha_1}$ . In other words, when the under or over-incentive given to the firm to innovate is not sufficient to cause the firm to act differently than the regulator would prefer, the optimal solution is possible. However, if  $\Delta \text{cost}_{t_0, \text{high}} \geq \frac{1}{\alpha_1}$  and  $\Delta \text{cost}_{t_1, \text{low}} < \frac{1}{\alpha_1}$ , the first best optimal solution cannot be achieved. In particular, a situation exists where it is impossible for the regulator to set a tax which

results in the optimal level of innovation and the *ex post* optimal level of pollution abatement. In these instances, the welfare maximizing equilibrium (though not optimal) policy will be for the regulator to set the tax 1) just equal to the cost of R&D so the firm innovates, or 2) just below the cost of R&D so the firm *does not* innovate.

To see why the regulator's best strategy is to set the tax such that the firm is given just enough incentive to innovate or not, let  $t_c$  in Figure 3.2 be the critical tax rate at which the cost of R&D equals the change in cost to the firm. Assuming an optimal solution cannot be implemented, the regulator is faced with two options: 1) set a tax which will result in no innovation and lead to under-abatement, or 2) set a tax which will induce technology  $\alpha_1$  to be developed, but result in over-abatement. If a tax is to be set which will not induce innovation, then the regulator will want to set the tax a small amount,  $\theta$ , below  $t_c$  so that innovation will not occur. Setting the tax lower than this will not be welfare maximizing because the lower tax will result in more under-abatement than will occur at tax level  $t_c - \theta$ . Setting a higher tax will result in technology  $\alpha_1$  being induced. Conversely, if a tax is to be set which will induce the technology  $\alpha_1$  to be developed, the regulator will want to set the tax at  $t_c$ . A higher tax level will result in additional over-abatement while a lower level will not provide a sufficient incentive for the firm to innovate. In other words, the regulator will want to set the tax at or just below  $t_c$  to control the technology level which is developed but not at any other point as that would result in unnecessary welfare losses attributable to additional over or under-abatement. The regulator will choose to induce innovation when the welfare resulting from innovation is greater than the welfare with no innovation.

Mathematically, the critical tax level  $t_c$  at which innovation does or does not occur is found by noting that:

$$\Delta \text{abatement and tax costs (to firm)} = \left[ \frac{t_c^2}{4C_1} - \frac{t_c^2}{4C_0} \right] \quad (\text{from above}).$$

Setting the change in cost equal to R and solving for  $t_c$  yields:

$$\left[ \frac{t_c^2}{4C_1} - \frac{t_c^2}{4C_0} \right] = \frac{1}{\alpha_1}$$

$$t_c^2 = \frac{4C_0C_1}{\alpha_1(C_0 - C_1)}$$

$$t_c = \sqrt{\frac{4C_0C_1}{\alpha_1(C_0 - C_1)}}$$

The two possible levels of welfare are given by:

$$\text{welfare} = \begin{cases} W_{t_c \text{ low}} = \frac{Xt_c}{2C_0} - \frac{mt_c^2}{4C_0^2} - \frac{t_c^2}{4C_0} & \text{where } t_c \text{ such that } \Delta \text{cost} + \theta = R \quad \theta \rightarrow +0 \\ W_{t_c \text{ high}} = \frac{Xt_c}{2C_1} - \frac{mt_c^2}{4C_1^2} - \frac{t_c^2}{4C_1} - \frac{1}{\alpha_1} & \text{where } t_c \text{ such that } \Delta \text{cost} = R \end{cases}$$

where  $W_{t_c \text{ low}}$  is the welfare associated with the firm not investing as the regulator has lowered the tax rate to discourage innovation to improve overall welfare. And,  $W_{t_c \text{ high}}$  is the welfare associated with the firm investing as the regulator has raised the tax rate to induce innovation to improve overall welfare. For future reference, let  $t_{c \text{ low}}$  be the tax associated with welfare  $W_{t_c \text{ low}}$  and  $t_{c \text{ high}}$  be the tax associated with welfare  $W_{t_c \text{ high}}$ .

The complete subgame perfect equilibrium strategy set for the regulator, defining the possible tax rates the regulator may set, may now be written as:

$$t = \begin{cases} \frac{XC_0}{(m + C_0)} & \text{if } \Delta \text{cost}_{t_{c \text{ high}}} < \frac{1}{\alpha_1} \\ \sqrt{\frac{4C_0C_1}{\alpha_1(C_1 - C_0)}} - \theta & \text{if } \Delta \text{cost}_{t_{c \text{ low}}} < \frac{1}{\alpha_1} \text{ and } \Delta \text{cost}_{t_{c \text{ high}}} \geq \frac{1}{\alpha_1} \text{ and } W_{t_c \text{ low}} > W_{t_c \text{ high}} \\ \sqrt{\frac{4C_0C_1}{\alpha_1(C_1 - C_0)}} & \text{if } \Delta \text{cost}_{t_{c \text{ low}}} < \frac{1}{\alpha_1} \text{ and } \Delta \text{cost}_{t_{c \text{ high}}} \geq \frac{1}{\alpha_1} \text{ and } W_{t_c \text{ low}} \leq W_{t_c \text{ high}} \\ \frac{XC_1}{(m + C_1)} & \text{if } \Delta \text{cost}_{t_{c \text{ low}}} \geq \frac{1}{\alpha_1} \end{cases}$$

The analysis in this section indicates that for regulation using an emission tax, when the regulator can commit to the future level of the tax, the first best outcome can be achieved when the over or under-incentive given to the firm does not cause the firm to deviate from the optimal technology level (e.g., the tax rate can be set at  $t_0$  or  $t_1$ ). However, if the firm deviates, the regulator will not be able to induce both the proper technology level and optimal *ex post* abatement level. Instead, the regulator is forced to balance over and under abatement and investment to maximize welfare, resulting in a second best solution. The issue of tradeoffs between abatement and investment will be discussed more thoroughly in Section 3.7.

### **3.4 One Period Model without Commitment**

Using the same general model as above, now assume the regulator *cannot* commit in the first stage to the level of the standard or tax that will be in place after innovation occurs. Consequently, we can think of the interaction between the firm and regulator in the following manner. In the first stage, the firm invests in R&D knowing that the regulator will base the regulation on the outcome of R&D. In the second stage, the firm and regulator learn the results of R&D at which time the regulator sets either the tax or standard level. The firm abates in the third stage.

### **3.5 Standards without Commitment**

First consider the situation in which a standard is used as the regulatory instrument. Using the approach of backwards induction, the firm will abate at the standard level  $S$  set by the regulator. The regulator, knowing the results of R&D when the standard is set, in essence makes the regulatory decision in a static situation. Consequently, the standard will be set such that the marginal benefit of abatement equals the marginal cost of abatement. Let  $S_0$  be the standard if no innovation occurs and  $S_1$  be

the standard if innovation does occur. Note that these standards will be the same as those set with commitment when innovation was deterministic ( $\epsilon > 1$ ).

The firm possesses the above information when making its R&D investment decision and will set  $R$  such that its expected costs are minimized:

$$\min_R \text{cost} = \min (\alpha_1 R)^\epsilon C_1 S_1^2 + (1 - (\alpha_1 R)^\epsilon) C_0 S_0^2 + R.$$

Taking the partial with respect to  $R$  and setting the result equal to zero, results in:

$$\begin{aligned} \frac{\partial \text{cost}}{\partial R} &= \epsilon \alpha_1^\epsilon R^{\epsilon-1} C_1 S_1^2 - \epsilon R^{\epsilon-1} \alpha_1^\epsilon C_0 S_0^2 + 1 = 0 \\ &= \epsilon \alpha_1^\epsilon R^{\epsilon-1} (C_1 S_1^2 - C_0 S_0^2) + 1 = 0. \end{aligned}$$

Solving for  $R$ , assuming an interior solution, we find:

$$R = \left[ \frac{1}{\epsilon \alpha_1^\epsilon (C_0 S_0^2 - C_1 S_1^2)} \right]^{1/(\epsilon-1)}.$$

The general form of  $\frac{\partial \text{cost}}{\partial R}$  for the standard is the same as it was for both standards and taxes with commitment. Consequently,  $R$  will be cost maximizing or minimizing under the same conditions (*i.e.*, the value of  $\epsilon$ ). It should be noted that while the bracketed term in  $\frac{\partial \text{cost}}{\partial R}$  is not the same as under commitment, it will still be negative unless  $C_1 S_1^2 > C_0 S_0^2$ , in which case the firm does not invest as discovering  $\alpha_1$  will result in higher abatement costs due to the more stringent standard  $S_1$ .

In situations where  $\epsilon > 1$  so that innovation is stochastic, the regulator will be able to impose the *ex post* optimal level of abatement as the technology level is known when the regulatory decision is made. However, a first best outcome cannot be guaranteed as the firm may not invest in order to decrease the probability that the regulator will ratchet, for example, when  $C_1 S_1^2 > C_0 S_0^2$ .

In situations where  $\epsilon > 1$ , the firm invests either  $R = \frac{1}{\alpha_1}$  so that success in R&D is assured or it does not invest at all. The firm will choose to invest when the change in the equilibrium abatement cost is greater than or equal to the cost of R&D. Specifically, the change in abatement cost is given by  $C_0 S_0^2 - C_1 S_1^2$ . Therefore, the firm will choose:

$$R = \begin{cases} 0 & \text{if } (C_0 S_0^2 - C_1 S_1^2) < \frac{1}{\alpha_1} \\ \frac{1}{\alpha_1} & \text{if } (C_0 S_0^2 - C_1 S_1^2) \geq \frac{1}{\alpha_1}. \end{cases}$$

While this solution looks similar to standards with commitment and deterministic innovation, it will not guarantee the first best outcome. This result occurs because, even though the abatement level will be *ex post* optimal for the technology at hand, there is an *ex ante* under-incentive to invest in R&D. This under-incentive arises because the regulator's ratcheting of the standard if innovation occurs raises the firm's abatement cost. As the firm moves first, it can prevent ratcheting from occurring by not investing, thereby creating an incentive for it not to invest. This point was addressed in section 2.2 and is well documented in the literature (Downing and White, 1986; Milliman and Prince, 1989). It is important to note that the only difference between this case and the with commitment case was the order of play. When the regulator moved first, an optimal outcome occurred. By reacting to innovation after it occurs, however, the regulator is not able to ensure the first best outcome.

### **3.6 Taxes without Commitment**

Now consider the situation in which a tax is used as the regulatory instrument. As in the case of taxes without commitment, the abatement level will be determined by the equation  $A = \frac{t_j}{2C_i}$  where  $A$  is the abatement level,  $t_j$  is the tax level in effect, and  $C_i$  is the cost coefficient associated with the current level of technology. As the regulator will

know the technology level when setting  $t$ , it will be set such that marginal benefits and costs of abatement are equated. In particular,

$$\begin{aligned} \text{if } \alpha_0, \text{ then } t_0 &= \frac{XC_0}{(m+C_0)} \Rightarrow \text{abatement } A_0 = \frac{X}{2(m+C_0)} \\ \text{if } \alpha_1, \text{ then } t_1 &= \frac{XC_1}{(m+C_1)} \Rightarrow \text{abatement } A_1 = \frac{X}{2(m+C_1)}. \end{aligned}$$

Note that both  $t_0$  and  $t_1$  are the same as the taxes set under commitment when innovation was deterministic ( $\epsilon > 1$ ).

The firm possesses the same information when making its R&D investment decisions and will select  $R$  such that its expected cost is minimized:

$$\begin{aligned} \min_{\mathbf{R}} \text{cost} &= \min_{\mathbf{R}} (\alpha_1 R)^\epsilon \left[ C_1 \left( \frac{X}{2(m+C_1)} \right)^2 + \left( \frac{XC_1}{(m+C_1)} \right) \left( \frac{X}{2m} - \frac{X}{2(m+C_1)} \right) \right] \\ &\quad + (1 - (\alpha_1 R)^\epsilon) \left[ C_0 \left( \frac{X}{2(m+C_0)} \right)^2 + \left( \frac{XC_0}{(m+C_0)} \right) \left( \frac{X}{2m} - \frac{X}{2(m+C_0)} \right) \right] + R \\ &= \min_{\mathbf{R}} (\alpha_1 R)^\epsilon \left[ \frac{C_1 X^2}{4(m+C_1)^2} + \frac{C_1 X^2}{2m(m+C_1)} - \frac{C_1 X^2}{2(m+C_1)^2} \right] \\ &\quad + (1 - (\alpha_1 R)^\epsilon) \left[ \frac{C_0 X^2}{4(m+C_0)^2} + \frac{C_0 X^2}{2m(m+C_0)} - \frac{C_0 X^2}{2(m+C_0)^2} \right] + R \\ &= \min_{\mathbf{R}} (\alpha_1 R)^\epsilon \left[ \frac{C_1 X^2}{2m(m+C_1)} - \frac{C_1 X^2}{4(m+C_1)^2} \right] \\ &\quad + (1 - (\alpha_1 R)^\epsilon) \left[ \frac{C_0 X^2}{2m(m+C_0)} - \frac{C_0 X^2}{4(m+C_0)^2} \right] + R. \end{aligned}$$

Taking the partial with respect to  $R$  and setting the result equal to zero, we have:

$$\begin{aligned} \frac{\partial \text{cost}}{\partial R} &= \epsilon \alpha_1^\epsilon R^{\epsilon-1} \left[ \frac{C_1 X^2}{2m(m+C_1)} - \frac{C_1 X^2}{4(m+C_1)^2} \right] - \epsilon \alpha_1^\epsilon R^{\epsilon-1} \left[ \frac{C_0 X^2}{2m(m+C_0)} - \frac{C_0 X^2}{4(m+C_0)^2} \right] + 1 = 0 \\ &= \epsilon \alpha_1^\epsilon R^{\epsilon-1} \left[ \frac{C_1 X^2}{2m(m+C_1)} - \frac{C_1 X^2}{4(m+C_1)^2} - \frac{C_0 X^2}{2m(m+C_0)} + \frac{C_0 X^2}{4(m+C_0)^2} \right] + 1 = 0. \end{aligned}$$

Let  $\lambda$  equal the negative of the bracketed term (which represents the change in abatement and tax costs). Then, solving for R yields:

$$R = \left[ \frac{1}{\epsilon \alpha_1^\epsilon \lambda} \right]^{\frac{1}{\epsilon-1}}.$$

The general form of  $\frac{\partial \text{cost}}{\partial R}$  for the tax is the same as it was for the standard. Consequently, R will be cost maximizing or minimizing under the same conditions (*i.e.*, the value of  $\epsilon$ ). The bracketed term in  $\frac{\partial \text{cost}}{\partial R}$ ,  $\lambda$ , will always be negative. Intuitively, this occurs because the abatement and tax costs under the lower tax will always be less than with the higher tax. Conversely, in the case of standards, it was seen that the tightened standard may result in either increased or decreased abatement costs.

Regardless of whether innovation is stochastic ( $\epsilon < 1$ ) or deterministic ( $\epsilon > 1$ ), the regulator will be able to impose the *ex post* optimal level of abatement, but there will be an *ex ante* over-incentive to innovate so an optimal outcome cannot be assured. This is the standard result in the literature (Downing and White, 1986; Milliman and Prince, 1989; Kennedy and Laplante, 1995).

### **3.7 Comparison of Taxes and Standards and Commitment vs. Non-Commitment**

As was indicated in the literature review section, the general conclusion from simple models of pollution regulation where technological innovation is possible is that standards will create an under-incentive for a firm to invest in R&D relative to the optimal. Taxes, on the other hand, are viewed to provide an over-incentive to invest in R&D

(Wenders, 1976; Downing and White, 1986; Milliman and Prince, 1989; Kennedy and Laplante, 1996). An implicit assumption in these models is that regulator does not commit to the future regulation. These models are equivalent to the non-commitment models developed above.

The simple model of regulation with commitment developed in this thesis arrives at a much different result than the current literature. In particular, it was shown that committing to a standard when innovation is deterministic will yield the first best outcome whereas committing to taxes may or may not result in the first best. The reason for this divergence from the literature is simple: allowing the regulator to commit enables the regulator to act preemptively to counter strategic behavior by the firm. In the case of standards, committing allows the regulator to prevent the firm from reducing R&D expenditures in order to prevent the regulator from tightening standards and increasing abatement costs (*e.g.*, ratcheting). Committing to taxes, however, may lower the incentive to the point where the firm does not invest, but not committing may create an over incentive to invest.

This subsection more rigorously compares standards, taxes, and commitment vs. non-commitment based upon the models developed above. Comparisons are made in the situation where  $\epsilon > 1$  (*i.e.*, innovation is deterministic) and Figure 3.3 is used throughout. The differences between a tax, standard, and commitment or non-commitment are much clearer in this situation; furthermore, it is extremely difficult to make comprehensive comparisons when  $R$  is not 0 or  $\frac{1}{\alpha_1}$  (*i.e.*, innovation is stochastic). The issue of stochastic innovation will be briefly addressed in the summary of this section and in the simulation runs in the two period model.

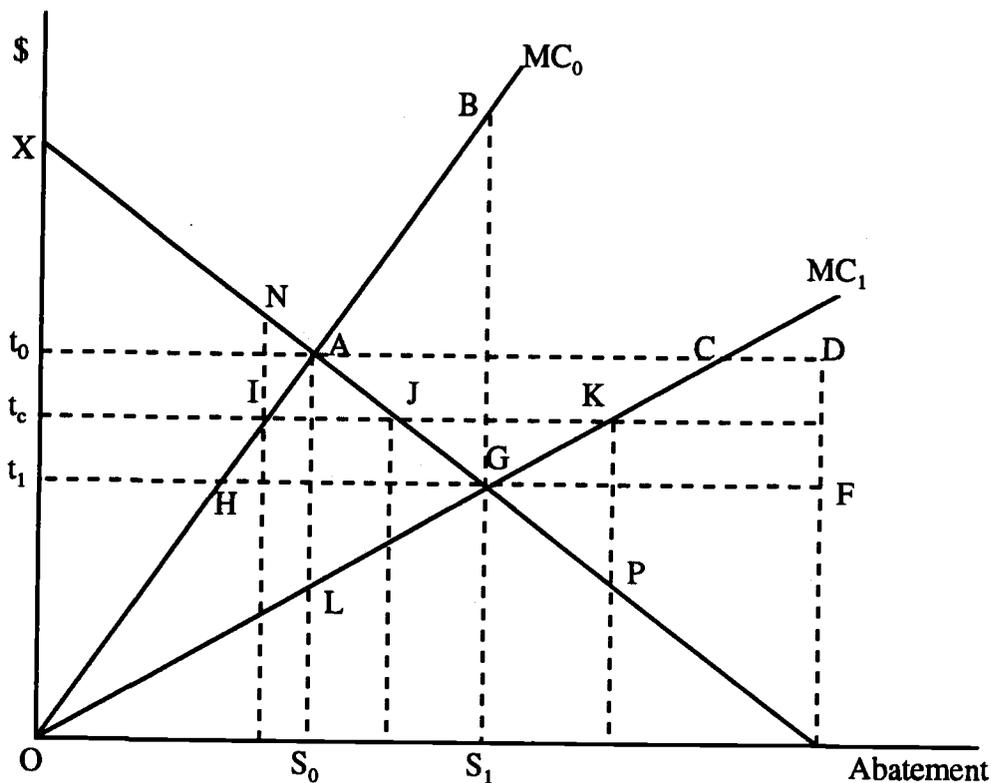


Figure 3.3: Comparison of Standards, Taxes, and Commitment vs. Non-Commitment

### 3.7.1 STANDARDS WITH COMMITMENT VS. STANDARDS WITHOUT COMMITMENT

As was shown previously, standards with commitment will yield the first best result. Not committing to a standard, however, will not always yield the first best. To see this graphically, note that in Figure 3.3 the net benefit accruing to the regulator from innovation is area  $OAG - \frac{1}{\alpha_1}$ . This may also be written as  $OAL + ALG - \frac{1}{\alpha_1}$ . The incentive for the firm to innovate is given by  $OAL - S_0LGS_1 - \frac{1}{\alpha_1}$ . Subtracting the firm's incentive from the regulator's incentive, it can be seen that the firm will have an under-incentive to invest of  $S_0AGS_1$ . Thus, whenever  $OAL - \frac{1}{\alpha_1} < S_0AGS_1$ , the firm will not invest. If  $OAG > \frac{1}{\alpha_1}$ , the regulator favors innovation and will set  $S_1$  to induce the firm to innovate (if

commitment were possible). In these situations, the under incentive causes non-commitment to be sub-optimal. In some situations, non-commitment will be optimal. For instance, if  $\frac{1}{\alpha_1}$  is greater than OBG, the optimal action for both the regulator and the firm is to not invest, and under both commitment and non-commitment, innovation will not occur. As a standard with commitment always yields the first best outcome and a standard without commitment will known, attention will now be focused on comparing taxes with and without commitment and comparing a standard with commitment to taxes with and without commitment as this yields more insightful results.

### 3.7.2 TAXES WITH COMMITMENT VS. TAXES WITHOUT COMMITMENT

If taxes with commitment are used as a policy instrument, it was shown above that there are four possible levels at which the regulator may set the tax:  $t_0^c$ ,  $t_{c\text{ high}}^c$ ,  $t_{c\text{ low}}^c$ , and  $t_1^c$ . The superscript c has been added to indicate commitment. If the regulator cannot commit, it was demonstrated that  $t_0^{w/o}$  and  $t_1^{w/o}$  are the levels at which the tax may be set, where the "w/o" superscripts indicate the non-commitment of the regulator.

Comparing taxes with commitment to taxes without commitment reveals there are eight possible combinations of tax levels which may occur. For example, in a given situation the regulator may set  $t_{c\text{ low}}^c$  if he commits or  $t_1^{w/o}$  if he does not. Of these, only four are feasible given the framework of the model. Each of the four feasible combinations and the situations in which they occur are examined below and summarized in Table 3.1.

Case #	Situation	Tax	Incentive to invest	Incentive Comparison	Welfare	Policy Preference
1a	$\frac{1}{\alpha_1} \geq \text{OADFG}$	$t_0^c$	OAC	$\frac{1}{\alpha_1} \geq \text{OADFG} > \text{OAC}$	OXA	$t_0^c$ or $t_0^{w/o}$
		$t_0^{w/o}$	OADFG		OXA	
1b	$\text{OADFG} \geq \frac{1}{\alpha_1} > \text{OAC}$	$t_0^c$	OAC	$\text{OADFG} \geq \frac{1}{\alpha_1} > \text{OAC}$	OXA	$t_0^c$
		$t_0^{w/o}$	OADFG		$\text{OXG} - \frac{1}{\alpha_1}$	
2	$\text{OAC} \geq \frac{1}{\alpha_1} > \text{OHG}$ and $\text{OXA-INA} \leq \text{OXG-GKP-OIK}$	$t_{c \text{ high}}^c$	OIK	$\text{OADFG} > \text{OIK} = \frac{1}{\alpha_1}$	$\text{OXG-OIK} - \text{GKP}$	$t_1^{w/o}$
		$t_1^{w/o}$	OADFG		$\text{OXG-OIK}$	
3	$\text{OAC} \geq \frac{1}{\alpha_1} > \text{OHG}$ and $\text{OXA-INA} > \text{OXG-GKP-OIK}$	$t_{c \text{ low}}^c$	OIK	$\text{OADFG} \geq \frac{1}{\alpha_1} > \text{OIK}$	$\text{OXA-INA}$	if $\text{INJ} \leq \text{JKG}$ then $t_{c \text{ low}}^c$
		$t_1^{w/o}$	OADFG		$\text{OXG-OIK}$	if $\text{INJ} > \text{JKG}$ then $t_1^{w/o}$
4	$\frac{1}{\alpha_1} \geq \text{OHG}$	$t_1^c$	OHG	$\text{OADFG} > \text{OAC} \geq \frac{1}{\alpha_1}$	$\text{OXG} - \frac{1}{\alpha_1}$	$t_1^c$ or
		$t_1^{w/o}$	OADFG		$\text{OXG} - \frac{1}{\alpha_1}$	$t_1^{w/o}$

**Table 3.1:** Taxes with Commitment vs. Taxes without Commitment

### 3.7.2.1 Case 1a: $t_0^c$ vs. $t_0^{w/o}$ and $\frac{1}{\alpha_1} \geq \text{OADFG}$

This situation occurs when the cost of innovation,  $\frac{1}{\alpha_1}$ , is greater than the incentive to invest for any tax level which may be imposed. Graphically, the incentive to invest under  $t_0^c$  is area OAC which is less than the incentive to invest under  $t_0^{w/o}$ , area OADFG. As the firm does not invest, OADFG must be less than  $\frac{1}{\alpha_1}$ . Under taxes with and without commitment, welfare is given by area OXA, so both tax schemes result in an equivalent level of welfare.

### 3.7.2.2 Case 1b: $t_0^c$ vs. $t_0^{w/o}$ and $\text{OADFG} \geq \frac{1}{\alpha_1} > \text{OAC}$

This situation occurs when the cost of innovation is greater than OAC, the incentive to invest under commitment to a high tax rate. However,  $\frac{1}{\alpha_1}$  is less than

OADFG, the incentive to invest under non-commitment. As the benefit to society of innovation is OAG, no innovation is preferred. Yet, under non-commitment, innovation occurs and welfare is  $OXG - \frac{1}{\alpha_1}$  and reduction in welfare occurs as-over investment occurs.

### 3.7.2.3 Case 2: $t_{c\text{high}}^c$ vs. $t_1^{w/o}$

Recall from the taxes with commitment case that the regulator sets  $t_{c\text{high}}^c$  if a high tax  $t_0$  induces innovation but a low tax  $t_1$  does not cause the firm to invest and the welfare benefit from innovating more than offsets the cost of innovation and over-abatement. Graphically, the cost of innovation,  $\frac{1}{\alpha_1}$ , equals OIK as  $t_{c\text{high}}^c$  was set so that innovation just occurs when tax  $t_{c\text{high}}^c$  is imposed. Notice that this is greater than area OHG so committing to a low tax  $t_1$  will not cause innovation but less than OAC so committing to high tax  $t_0$  will induce innovation. The resulting welfare from the firm innovating is given by  $OXG - GKP - OIK$ , where GKP is the loss attributable to over-abatement. If the regulator does not commit, the incentive to innovate is OADFG which is greater than the cost of innovation, OIK. The firm will innovate and the regulator will subsequently impose  $t_1^{w/o}$ . The resulting welfare will be  $OXG - OIK$ . Thus, taxes without commitment will be the better policy for the regulator to pursue as committing results in the additional loss of welfare associated with over-abatement, GKP.

For clarity, note that the situation where  $t_{c\text{high}}^c$  is selected by the regulator instead of  $t_{c\text{low}}^c$  occurs when area GKP, the welfare loss from over-abatement, plus OIK, the cost of R&D, is less than INA plus OAG, where INA is the under abatement loss if the firm does not innovate and OAG is the reduced cost of abatement and benefit of increased abatement. In other words, the costs must be less than the benefits.

### 3.7.2.4 Case 3: $t_{c\text{low}}^c$ vs. $t_1^{w/o}$

This case is similar to case 2, but the welfare benefit from innovating is now less than the cost of innovation and over-abatement. Graphically,  $OXA - INA > OXG - OIK - GKP$ . The regulator will set  $t_{c\text{low}}^c$  instead of  $t_{c\text{high}}^c$ . The cost of innovation is just above

OIK as  $t_{c\text{ low}}^c$  is set just low enough so that innovation will not occur. Committing to a low tax  $t_1$  will still not cause innovation as OHG is less than OIK, but OAC is still greater than OIK so committing to high tax  $t_0$  will induce innovation. The resulting welfare from the firm not innovating but the regulator lowering the tax to prevent innovation is given by OXA-INA, where INA is the loss attributable to under-abatement. If the regulator does not commit, the incentive to innovate is OADFG. The firm will innovate and the regulator will subsequently impose  $t_1^{w/o}$ . The resulting welfare will be OXG-OIK.

Whether commitment or non-commitment to a tax regulation will result in the greater welfare is not readily transparent. To determine which is better, first note that we are in the situation where  $t_{c\text{ low}}^c$  is greater than  $t_{c\text{ high}}^c$ . Therefore,

$$\begin{aligned} \text{OXA} - \text{INA} &> \text{OXG} - \text{GKP} - \text{OIK} \\ 0 &> \text{OAG} - \text{OIK} + \text{INA} - \text{GKP} \\ 0 &> \text{IAJ} - \text{JKG} + \text{INA} - \text{GKP} \\ 0 &> \text{INJ} - \text{JKG} - \text{GKP} \end{aligned}$$

Now, assume that commitment to a tax level results in a higher welfare level than non-commitment. Consequently,

$$\begin{aligned} \text{OXA} - \text{INA} &> \text{OXG} - \text{OIK} \\ 0 &> \text{OXG} - \text{OXA} + \text{INA} - \text{OIK} \\ 0 &> \text{OAG} + \text{INA} - \text{OIK} \\ 0 &> \text{INJ} - \text{JKG} \end{aligned}$$

Unfortunately, this is not the same condition as for  $t_{c\text{ low}}^c$  to result in a higher welfare than  $t_{c\text{ high}}^c$  so  $t_{c\text{ low}}^c$  does not imply  $t_{c\text{ low}}^c$  will result in a greater level of welfare than  $t_1^{w/o}$ .

Thus, if  $\text{INJ} < \text{JKG}$  commitment will result higher welfare while if  $\text{INJ} \geq \text{JKG}$ , non-commitment will result in higher welfare.

There is a very intuitive reason why this is so. First break INJ into INA and IAJ. Recall that INA is the under-abatement which occurs if  $t_{c,low}^c$  is imposed and subtracting IAJ from JKG results in the over-investment by the firm. Therefore, the criteria for whether commitment or non-commitment is better could be stated in the following manner: is the cost of under-abatement associated with committing to tax  $t_{c,low}^c$  less than or greater than the cost of not committing and the firm over-investing. Carrying this a step further, it is possible to see why the fact that  $t_{c,low}^c$  insures a higher welfare than  $t_{c,high}^c$  does not guarantee commitment to be better than non-commitment. The term  $-GKP$  in  $0 > INJ - JKG - GKP$  means that there is a penalty, in the form of over-abatement, for using  $t_{c,high}^c$  instead of  $t_{c,low}^c$ . As a result, there are instances when  $t_{c,low}^c$  is set because, even though innovation is preferred, the penalty of over-abatement is greater than the loss due to under-abatement. When comparing commitment and non-commitment, however, there is no over-abatement penalty, so in instances where innovation is preferred, non-commitment results in a larger welfare as the incentive to invest remains high but there is no cost associated with the higher incentive.

### 3.7.2.5 Case 4: $t_i^c$ vs. $t_i^{w/o}$

This case is the opposite extreme of case 1. If a high tax was imposed, the incentive to the firm under both  $t_i^c$  and  $t_i^{w/o}$  is sufficient to induce innovation. If the lower tax (as if the firm invests) is imposed, the firm will still invest, so the lower tax is appropriate. Graphically, the incentive to invest under  $t_i^c$  is area OHG which is greater than  $\frac{1}{\alpha_1}$ . Under  $t_i^{w/o}$  the incentive to invest is OADFG which is greater than both OHG and  $\frac{1}{\alpha_1}$ . The welfare under both taxes is given by area OXG so commitment and non-commitment result in the same level of welfare.

### 3.7.3 COMPARISON OF STANDARDS AND TAXES

In the preceding section, it was demonstrated that there are five distinct situations where differing levels of taxes and commitment or non-commitment are needed to maximize social welfare. (*e.g.*, the four cases presented, remembering that case three there are two possible tax policies which may be needed.) In this section, *standards with commitment* (which will always yield the optimal outcome) are integrated into the various situations to complete the comparison. The comparison continues to be based upon Figure 3.3 and the results are summarized in Table 3.2.

#### 3.7.3.1 Case 1a:

Under either tax policy  $t_0^c$  or  $t_0^{w/o}$ , the firm does not invest as  $\frac{1}{\alpha_1} > OADFG$ . In determining which standard to set in this situation, the regulator can see that the benefit of innovation, area OAG, is less than OADFG so innovation is not welfare improving. Consequently, the standard  $S_0$  is set and innovation does not occur. As the resulting welfare is OXA, the standard yields the same welfare as  $t_0^c$  and  $t_0^{w/o}$ .

#### 3.7.3.2 Case 1b:

In this case, the regulator selected to commit to tax level to  $t_0^c$  over imposing  $t_0^{w/o}$  without commitment. This was done as committing reduced the over-incentive to innovate sufficiently that the firm does not innovate. As committing does not result in under-abatement in this case, the regulation was optimal, resulting in welfare OXA. The standard  $S_0$  will also prevent innovation while causing the optimal level of abatement.

#### 3.7.3.3 Case 2:

The regulator selected  $t_1^{w/o}$  in this case over  $t_{c\text{high}}^c$  as  $t_1^{w/o}$  induced innovation without causing over-abatement, results in a welfare loss of GKP. In this case the regulator will want to impose  $S_1$  to induce innovation, but without causing additional costs. Graphically, the regulator will desire innovation only if the welfare associated with

Case #	Situation	Best Tax	Tax Incentive to Invest	Standard	Standard Incentive	Standard Welfare	Tax Welfare	Policy Preference
1a	$\frac{1}{\alpha_1} \geq \text{OADFG}$	$t_0^c$	OAC	$S_0$	OAL	OXA	OXA	$t_0^c, t_0^{w/o}, \text{ or } S_0$
		$t_0^{w/o}$	OADFG				OXA	
1b	$\text{OADFG} \geq \frac{1}{\alpha_1} > \text{OAC}$	$t_0^c$	OAC	$S_0$	OAL	OXA	OXA	$t_0^c$ or $S_0$
2	$\text{OAC} \geq \frac{1}{\alpha_1} > \text{OHG}$ and $\text{OXA-INA} \leq \text{OXG-GKP-OIK}$	$t_1^{w/o}$	OADFG	$S_1$	OBG	OXG-OIK	OXG-OIK	$t_1^{w/o}$ or $S_1$
3a	$\text{OAC} \geq \frac{1}{\alpha_1} > \text{OHG}$ and $\text{OXA-INA} > \text{OXG-GKP-OIK}$ and $\text{INJ} \leq \text{JKG}$	$t_{c \text{ low}}$	OIK	$S_0$	OAL	OXA	OXA-INA	$S_0$
3b	$\text{OAC} \geq \frac{1}{\alpha_1} > \text{OHG}$ and $\text{OXA-INA} > \text{OXG-GKP-OIK}$ and $\text{INJ} > \text{JKG}$	$t_1^{w/o}$	OADFG	$S_0$	OAL	OXA	OXG-OIK	$S_0$
				$S_1$	OBG	OXG-OIK		$t_1^{w/o}$ or $S_1$
4	$\frac{1}{\alpha_1} \geq \text{OHG}$	$t_1^c$	OHG	$S_1$	OBG	$\text{OXG} - \frac{1}{\alpha_1}$	$\text{OXG} - \frac{1}{\alpha_1}$	$t_1^c, t_1^{w/o}, \text{ or } S_1$
		$t_1^{w/o}$	OADFG				$\text{OXG} - \frac{1}{\alpha_1}$	

**Table 3.2:** Comparison of Standards and Taxes

$S_1$  is greater than with  $S_0$ , or area  $OXA+OAG-\frac{1}{\alpha_1} \geq OXA$ . Note that  $\frac{1}{\alpha_1}$  equals OIK as the situation faced was originally developed when  $t_{c\text{ high}}^c$  was used and the tax was set just high enough to cause innovation. The criteria for the regulator desiring innovation can be reduced to  $IAJ-JKG \geq 0$ . Area IAJ represents the under-incentive to innovate if  $t_{c\text{ high}}^c$  was imposed and JKG represents the over-incentive. The criteria for setting the standard is now in the framework of taxes, making comparisons relatively simple. To see that  $IAJ-JKG$  will be greater than zero, recall that the firm invests under  $t_1^{w/o}$ . In order for this to occur, the change in abatement cost must be greater than the cost of R&D. Graphically, this means  $OAG \geq OIK$ . This expression can be rewritten as  $OIG+IAJ \geq OIJG+JKG$ . Simplifying,  $IAJ-JKG \geq 0$ ; consequently, the regulator will set  $S_1$  and the firm will innovate. The resulting welfare is  $OXG-OIK$  so both  $S_1$  and  $t_1^{w/o}$  will result in the equivalent outcome which is optimal.

#### 3.7.3.4 Case 3a:

The regulator selected  $t_{c\text{ low}}^c$  in this case over  $t_1^{w/o}$  and  $t_{c\text{ high}}^c$  as the cost of under-abatement was less than the cost associated with either over-abatement if  $t_{c\text{ high}}^c$  was set or the over-investment created by  $t_1^{w/o}$ . In essence, the regulator selected the lesser of three inefficiencies, thus, it should not be surprising the standard will improve welfare. In particular, the imposition of  $S_0$  will result in a higher level of welfare. To see why the regulator selects  $S_0$ , note that  $t_{c\text{ low}}^c$  was selected over  $t_1^{w/o}$  so  $0 > INJ-JKG$ . Rewriting this yields  $IAJ+INA < JKG$  which implies  $IAJ < JKG$ . Therefore,  $IAJ-JKG \geq 0$  does not hold and regulator will not favor innovation. A standard  $S_0$  is set, and welfare  $OXA$  results. This is greater than  $OXA-INA$ , the welfare under  $t_{c\text{ low}}^c$ .

The key result from this case is that taxes may be poor at constraining innovation. Not committing causes excess innovation while committing results in either under or over-abatement.

### 3.7.3.5 Case 3b:

In this case, the regulator selected  $t_1^{w/o}$  over  $t_{c,low}^c$ . Precisely why depends on the parameters of the model.  $t_1^{w/o}$  may have been selected over  $t_{c,low}^c$  if the over-investment associated with  $t_1^{w/o}$  was less than the under-abatement cost associated with  $t_{c,low}^c$ . Or,  $t_1^{w/o}$  may not cause over investment in which case it would be selected over  $t_{c,low}^c$  results in under-abatement. The difference is subtle but the first case implies a standard  $S_0$  should be set while the second that  $S_1$  should be set. As  $t_1^{w/o}$  was preferred to  $t_{c,low}^c$  we know  $OXG-OIK \geq OXA-INA$ . Rewriting this results in  $INA+IAJ-JKG \geq 0$ . This does not give sufficient information to indicate whether  $IAJ-JKG \geq 0$ , so we do not know whether innovation is favored by the regulator or not. *If  $IAJ \geq JKG$* , then the regulator prefers innovation and will set  $S_1$  resulting in welfare  $OXG-OIK$ . This is the same welfare as if  $t_1^{w/o}$  was set so it is equivalent to  $S_1$ . Intuitively this situation arises because  $t_{c,low}^c$  was preferred over  $t_{c,high}^c$  as under-abatement was less costly than over-abatement. However, if  $t_1^{w/o}$  could be used in a similar situation, it is preferred as over-investment is not an issue in this situation and avoids the complexities of under/over-abatement. *If  $IAJ < JKG$* , the regulator does not want innovation to occur and subsequently sets  $S_0$ . The resulting welfare is  $OXA$  which is greater than the welfare  $OXA-OIK$  under  $t_1^{w/o}$ . The intuitive reason for this result is that  $t_1^{w/o}$  does result in over investment but it is less than both the over and under-abatement caused by  $t_{c,high}^c$  and  $t_{c,low}^c$ , respectively. If allowed to impose a standard (with commitment), the regulator can effectively discourage innovation while still setting the efficient abatement level.

### 3.7.3.6 Case: 4

In this situation, under either  $t_1^c$  or  $t_1^{w/o}$ , the firm will innovate as  $OHG \geq \frac{1}{\alpha_1}$ . Thus, if the regulator sets a standard  $S_1$ , the incentive for the firm to innovate is  $OBG$  which is greater than  $OHG$ , so the firm will innovate. Moreover, the incentive for the regulator to innovate, area  $OAG$ , is greater than  $OHG$  so the regulator will indeed set  $S_1$ . The

resulting welfare is  $OXG - \frac{1}{\alpha_1}$  which is the same as for  $t_1^c$  and  $t_1^{w/o}$  so the three policy instruments will yield equivalent welfare results.

### 3.8 Summary

This section developed a one period model with three stages to examine pollution regulation where R&D is endogenous. The model was then used to study the subgame perfect equilibrium welfare arising under regulation by taxes, standards, and various assumptions of commitment.

The most significant result found was that if innovation is deterministic in nature and the regulator commits to a standard, the first best outcome can be achieved. In other words, the regulator can set the optimal *ex post* standard while still providing the *ex ante* optimal incentive to innovate. The regulator is able to do this because preemptively committing to a future standard prevents the firm from not investing in order to avoid prevent the regulator from ratcheting. Additionally, the deterministic nature of innovation prevents over-investment from occurring.

Using a tax as a regulatory instrument when innovation is deterministic in nature, regardless of commitment or non-commitment by the regulator, was found not to guarantee the optimal outcome. In particular, it was demonstrated here, as well as in the literature, that non-commitment to a tax causes an excessive incentive to innovate. Thus, there are instances in which the optimal solution would be for no innovation to occur but the firm invests anyway.

If the regulator commits to a future tax level, there may be either an over or under-incentive to innovate. Consequently, there are causes where the optimal solution is for innovation to occur, but committing to a lower tax rate associated with the *ex post* optimal abatement level results in an insufficient *ex ante* incentive for the firm to innovate. The reverse is also possible. If the optimal outcome is for no innovation to occur and the regulator commits to a high tax corresponding to the optimal *ex post* abatement level, the

*ex ante* incentive given to the firm may be such that the firm innovates, resulting a sub-optimal outcome.

In Table 3.1, the relative efficiency of taxes with and without commitment was compared when innovation is deterministic. The general result was that neither tax scheme is always preferred to the other. Committing to a tax tended to be preferred when innovation was not desired as it provides a smaller over-incentive to invest than non-commitment. Additionally, by being able to adjust the tax rate over a larger range than with non-commitment, the regulator is better able to balance the over incentive to innovate with the under or over-abatement which will occur. Not committing to a tax tended to be preferred when innovation was desired. This result occurred because non-commitment provides a very powerful incentive to innovate and *ex post* optimal abatement levels. Whether taxes with commitment is preferred over taxes without commitment, though, is ultimately dependent the relative magnitudes of the cost of over/under abatement and the loss/benefit from innovation. In general, the difficulty with taxes rests in the fact that a tax cannot be imposed which is both *ex ante* and *ex post* optimal with regard to abatement and investment. This result also holds when innovation is stochastic in nature.

The comparison of taxes and standards made in Table 3.2 was relatively simple under deterministic innovation. The standard was always optimal and it tended to be the case that one of the tax policies would also be optimal. Under stochastic innovation, very little can be said because of the nature of innovation. However, it should be stressed that there is no *a priori* reason to believe that a tax will necessarily result in higher level of welfare than a standard. This assertion will be borne out in the simulation model in Section 4.

#### 4. TWO PERIOD MODEL

In the one period model, the firm and regulator each had an opportunity to act once – the firm selecting an investment level and the regulator setting a standard or tax. In reality, however, the regulator and firm will make a series of investment and regulatory decisions. In particular, over a short time horizon, the regulator will likely be able to commit to a specific regulatory policy, but over a longer horizon will be unable to commit. At the end of the time period over which the regulator committed, the regulator will reconsider the level at which regulatory instruments are set and commit to another regulatory policy for some time period. For instance, many of the environmental acts passed by Congress must be reauthorized after a set period of time. States often issue pollution discharge permits which are valid for a fixed period of time after which they must be renewed. Thus, it is proper to think of pollution regulation, investment decisions, and abatement occurring in a series of discrete time units. The firm's investment decision today, therefore, is not only dependent on yesterday's regulations, but also on the expected future regulations. As a result, the regulator's ability to commitment to inter-period regulations implicitly affects the incentives given to the firm to invest in new, less costly pollution control technologies.

The purpose of this section is to formally capture this inter-period dynamic between the regulator and firm. A two period model will be constructed based upon the single period *with commitment* model from the preceding section. Because of the mathematical complexity of the solutions, it will be analyzed, for the most part, through a computer simulation using GAMS. The assumptions of the model are first developed, followed by a formal solution for the standards with and without commitment. Taxes with and without commitment are then presented. Lastly, results from several simulation runs are presented.

#### **4.1 Two Period Model Assumptions**

Consider the one period commitment model developed in Section 3.1. Assume now that at the conclusion of this one period game, a similar one period game is played. In particular, if technology  $\alpha_1$  was found in the first period, the firm is able to engage in R&D once again to attempt to find technology  $\alpha_2$ , an improvement or development upon  $\alpha_1$ . The probability of discovering  $\alpha_2$  is given by  $(\alpha_2 R)^\epsilon$  where  $0 \leq R \leq \frac{1}{\alpha_2}$  and  $\epsilon \geq 0$ . If  $\alpha_2$  is found in the second period, the cost of abating pollution is  $C_2 A^2$  where  $C_0 > C_1 > C_2$ . If  $\alpha_1$  was not discovered in period one, the period one game is played again (*i.e.*, the firm can invest in R&D with probability of success  $(\alpha_1 R)^\epsilon$ ). It is assumed the benefits of pollution remain the same over both time periods.

If the regulator can commit to the second period regulation in period one, the game begins by the regulator setting  $S_1$  and  $S_2$  or  $t_1$  and  $t_2$  where the subscripts refer to the period. The firm then makes its first period investment decision, denoted by  $R_1$ , knowing the level of regulation that will be in place in both periods. The first period concludes by the firm learning the results of R&D and abating. In period two, the firm makes another R&D decision, investing  $R_{2a}$  if  $\alpha_1$  was found in period one or  $R_{2b}$  if  $\alpha_1$  was not found. The results of R&D are learned and the firm abates according to the regulation  $S_2$  or  $t_2$  as announced by the regulator at the beginning of the first period. If the regulator cannot commit the second period regulation, only  $S_1$  or  $t_1$  is set at the beginning of the first period.  $S_{2a}$ ,  $S_{2b}$ ,  $t_{2a}$ , or  $t_{2b}$  is then set at the beginning of period where 2a represents the standard or tax if  $\alpha_1$  was found in period one and 2b the standard or tax if  $\alpha_1$  was not found.

#### **4.2 Standards with Commitment**

First consider the situation where the regulator uses a standard and commits to the second period regulation at the beginning of the first period. Working backwards, the firm

will begin period two with either pollution control technology  $\alpha_1$  or  $\alpha_0$  (the original). Since the standard  $S_2$  has already been set for period two, the firm selects R&D expenditures  $R_{2a}$  or  $R_{2b}$ , corresponding with whether or not technology  $\alpha_1$  was found, to minimize its second period cost. Using the same solution process from the one period model:

$$R_{2a} = \left[ \frac{1}{\epsilon \alpha_2^\epsilon S_2^2 (C_1 - C_2)} \right]^{1/(\epsilon-1)}$$

$$R_{2b} = \left[ \frac{1}{\epsilon \alpha_1^\epsilon S_2^2 (C_0 - C_1)} \right]^{1/(\epsilon-1)}$$

where  $R_{2a}$  and  $R_{2b}$  are the optimal R&D expenditures to minimize the firm's second period expected cost for any  $S_2$  imposed by the regulator if  $\alpha_1$  was or was not found in period one, respectively.

As in the one period model, R will be cost minimizing or maximizing depending on the value of  $\epsilon$ .

#### 4.2.1 $\epsilon < 1$ : STOCHASTIC INNOVATION

If  $\epsilon < 1$ , we know  $R_{2a}$  and  $R_{2b}$  will be cost minimizing R&D expenditures in period two. In period one, the firm knows the optimal  $R_{2a}$  and  $R_{2b}$  as the regulator has already announced  $S_2$ . The firm can use this information and that  $S_1$  has also been set already to find the optimal  $R_1$  to invest in period one which will minimize its expected total cost:

$$\min_{R_1} \text{ total cost} = \min_{R_1} (\alpha_1 R_1)^\epsilon [C_1 S_1^2 + (\alpha_2 R_{2a})^\epsilon C_2 S_2^2 + (1 - (\alpha_2 R_{2a})^\epsilon) C_1 S_2^2 + R_{2a}] +$$

$$(1 - (\alpha_1 R_1)^\epsilon) [C_0 S_1^2 + (\alpha_1 R_{2b})^\epsilon C_1 S_2^2 + (1 - (\alpha_1 R_{2b})^\epsilon) C_0 S_2^2 + R_{2b}] + R_1$$

where  $R_{2a}$  and  $R_{2b}$  are the optimized R&D expenditures for the second period found above. Let the first bracketed term equal  $[x]$  and the second bracketed term equal  $[y]$ .

Taking the partial with respect to  $R_1$  and setting the result equal to zero, yields:

$$\begin{aligned}\frac{\partial \text{total cost}}{\partial R_1} &= \varepsilon \alpha_1^\varepsilon R_1^{\varepsilon-1} [x] - \varepsilon \alpha_1^\varepsilon R_1^{\varepsilon-1} [y] + 1 = 0 \\ &= \varepsilon \alpha_1^\varepsilon R_1^{\varepsilon-1} [x - y] + 1 = 0.\end{aligned}$$

Solving for  $R_1$ , assuming an interior solution, we find:

$$R_1 = \left[ \frac{1}{\varepsilon \alpha_1^\varepsilon [y - x]} \right]^{1/(\varepsilon-1)}$$

where  $R_1$  is the optimal first period R&D expenditure to minimize total expected cost for any  $S_1$  and  $S_2$  imposed by the regulator.

The general form of  $\frac{\partial \text{cost}}{\partial R_1}$  is the same as the one period model and  $\varepsilon < 1$ , so  $R_1$  will be cost minimizing. There is one exception to this. Taking the second derivative of total cost with respect to  $R_1$  yields:

$$\frac{\partial^2 \text{total cost}}{\partial R_1^2} = \varepsilon(\varepsilon - 1) \alpha_1^\varepsilon R_1^{\varepsilon-2} [x - y].$$

If  $[x]$  is greater than  $[y]$ , the second order condition will then be negative, implying  $R_1$  *maximizes* cost. This situation occurs when the second period cost,  $[x]$ , associated with discovering  $\alpha_1$  in period one is greater than the second period cost,  $[y]$ , if no innovation occurred. In other words, the firm's marginal benefit from investing in period one will always be less than its marginal cost of investing. If this occurs, the firm will not invest and  $R_1$  will equal zero; if not,  $R_1$  will be cost minimizing and the firm will invest according to the formula for  $R_1$  previously presented.

Since the regulator knows how the firm will act and imposes regulation first, welfare can be written as a function of  $S_1$  and  $S_2$  only (*i.e.*, the firm's reaction can be completely expressed in terms of  $S_1$  and  $S_2$ ). The regulator maximizes total expected welfare with respect to  $S_1$  and  $S_2$ :

$$\begin{aligned} \max_{S_1, S_2} \text{ total welfare} = & \max_{S_1, S_2} (XS_1 - mS_1^2) + (XS_2 - mS_2^2) - [(\alpha_1 R_1)^\epsilon [C_1 S_1^2 + \\ & (\alpha_2 R_{2a})^\epsilon C_2 S_2^2 + (1 - (\alpha_2 R_{2a})^\epsilon) C_1 S_2^2 + R_{2a}] + \\ & (1 - (\alpha_1 R_1)^\epsilon) [C_0 S_1^2 + \\ & (\alpha_1 R_{2b})^\epsilon C_1 S_2^2 + (1 - (\alpha_1 R_{2b})^\epsilon) C_0 S_2^2 + R_{2B}] + R_1] \end{aligned}$$

where the R's are the optimized R&D expenditures for the firm in each period.

This expression becomes the objective function in the computer simulation

#### 4.2.2 $\epsilon > 1$ : DETERMINISTIC INNOVATION

As  $\epsilon > 1$ ,  $R_{2a}$  and  $R_{2b}$  will be cost *maximizing*, and the firm will invest such that  $R_{2a}$  and  $R_{2b}$  are corner solutions. In other words,  $R_{2a}$  and  $R_{2b}$  will be at levels which ensure failure or guarantees success in the R&D process. The firm will choose to invest only when the change in period two abatement cost is greater than or equal to the cost of R&D for some  $S_2$ . From the one period model, we can write  $R_{2a}$  and  $R_{2b}$  as:

$$R_{2a} = \begin{cases} 0 & \text{if } S_2^2 (C_1 - C_2) < \frac{1}{\alpha_2} \\ \frac{1}{\alpha_2} & \text{if } S_2^2 (C_1 - C_2) \geq \frac{1}{\alpha_2} \end{cases}$$

$$R_{2b} = \begin{cases} 0 & \text{if } S_2^2 (C_0 - C_1) < \frac{1}{\alpha_1} \\ \frac{1}{\alpha_1} & \text{if } S_2^2 (C_0 - C_1) \geq \frac{1}{\alpha_1} \end{cases}$$

The firm knows the optimal  $R_{2a}$  and  $R_{2b}$  as the regulator has already announced  $S_2$ . The firm can use this information and that  $S_1$  has already been set to find the optimal  $R_1$  to invest in period one which minimizes its expected total cost:

$$\min_{R_1} \text{total cost} = \min_{R_1} (\alpha_1 R_1)^\epsilon \left[ C_1 S_1^2 + (\alpha_2 R_{2a})^\epsilon C_2 S_2^2 + (1 - (\alpha_2 R_{2a})^\epsilon) C_1 S_2^2 + R_{2a} \right] \\ (1 - (\alpha_1 R_1)^\epsilon) \left[ C_0 S_1^2 + (\alpha_1 R_{2b})^\epsilon C_1 S_2^2 + (1 - (\alpha_1 R_{2b})^\epsilon) C_0 S_2^2 + R_{2b} \right] + R_1$$

where  $R_{2a}$  and  $R_{2b}$  are the optimized levels of R&D expenditures in period two. Taking the partial with respect to  $R_1$ :

$$\frac{\partial \text{total cost}}{\partial R_1} = \epsilon \alpha_1^\epsilon R_1^{\epsilon-1} \left[ C_1 S_1^2 + (\alpha_2 R_{2a})^\epsilon C_2 S_2^2 + (1 - (\alpha_2 R_{2a})^\epsilon) C_1 S_2^2 + R_{2a} \right] - \\ \epsilon \alpha_1^\epsilon R_1^{\epsilon-1} \left[ C_0 S_1^2 + (\alpha_1 R_{2b})^\epsilon C_1 S_2^2 + (1 - (\alpha_1 R_{2b})^\epsilon) C_0 S_2^2 + R_{2b} \right] + 1.$$

Then, taking the second order conditions yields:

$$\frac{\partial^2 \text{total cost}}{\partial R_1^2} = \epsilon(\epsilon-1) \alpha_1^\epsilon R_1^{\epsilon-2} \left[ \begin{array}{l} (C_1 S_1^2 - C_0 S_1^2) \\ + [(\alpha_2 R_{2a})^\epsilon C_2 S_2^2 + (1 - (\alpha_2 R_{2a})^\epsilon) C_1 S_2^2 + R_{2a}] \\ - [(\alpha_1 R_{2b})^\epsilon C_1 S_2^2 + (1 - (\alpha_1 R_{2b})^\epsilon) C_0 S_2^2 + R_{2b}] \end{array} \right].$$

As was the case with  $R_{2a}$  and  $R_{2b}$ ,  $R_1$  will maximize total cost if  $\epsilon > 1$ . To see this, note that the term in front of the large bracket is positive if  $(\epsilon-1) > 1$ ; consequently, if and only if the bracketed term is negative will  $R_1$  be cost maximizing. The top expression in the brackets will always be negative as  $C_0 > C_1$ . To determine the sign of the lower two terms, recall that  $R_{2a}$  and  $R_{2b}$  will always be such that the probability of success will be 0 or 1. The result is that there are four possible combinations of the lower two bracketed terms (e.g., combinations of investment decisions in R&D). For instance, the firm invests in period two if  $\alpha_1$  is found in period one but will not invest if  $\alpha_1$  is not found. In this case, the summation of the lower bracketed terms will be  $C_1 S_2^2 - (C_1 S_2^2 + R_{2b})$ . The four possibilities are:

$$1) C_1 S_2^2 - (C_1 S_2^2 + R_{2b}) < 0$$

$$2) C_1 S_2^2 - C_0 S_2^2 < 0$$

$$3) C_2 S_2^2 + R_{2a} - (C_1 S_2^2 + R_{2b}) = C_2 S_2^2 + (C_1 S_2^2 - C_2 S_2^2) - (C_1 S_2^2 + (C_0 S_2^2 - C_1 S_2^2)) < 0 \\ = C_1 S_2^2 - C_0 S_2^2 < 0$$

$$4) C_2 S_2^2 + R_{2a} - C_0 S_2^2 = C_2 S_2^2 + (C_1 S_2^2 - C_2 S_2^2) - C_0 S_2^2 < 0 \\ = C_1 S_2^2 - C_0 S_2^2 < 0.$$

The first and second cases are clearly less than zero. The third and fourth can also be seen to be less than zero by substituting for the minimum incentive needed to induce  $R_{2a}$  and  $R_{2b}$  to be invested (e.g., the firm invests  $R_{2a}$  if  $C_1 S_2^2 - C_2 S_2^2 \geq R_{2a}$ ).

Thus, the firm's investment  $R_1$  in R&D will be either 0 or  $\frac{1}{\alpha_1}$ . The firm's decision to invest will be based on whether or not the change in period one abatement cost and period two cost is greater than or equal to the cost of R&D. The change in cost is defined by:

$$\Delta\text{cost} = \begin{cases} (C_0 S_1^2 - C_1 S_1^2) + (C_0 S_2^2 - C_1 S_2^2) + \left( C_1 S_2^2 - C_2 S_2^2 - \frac{1}{\alpha_2} \right) & \text{if } (C_1 S_2^2 - C_2 S_2^2) \geq \frac{1}{\alpha_2} \\ (C_0 S_1^2 - C_1 S_1^2) + (C_0 S_2^2 - C_1 S_2^2) & \text{if } (C_1 S_2^2 - C_2 S_2^2) < \frac{1}{\alpha_2} \end{cases}$$

where the first parenthesis term represents the change in period one abatement costs. The second parenthesis term represents the second period change in abatement costs, and the third term represents an additional cost reduction in abatement minus the R&D costs, if the firm invests in period two.

It follows that  $R_1$  can be defined as:

$$R_1 = \begin{cases} 0 & \text{if } \Delta\text{cost} < \frac{1}{\alpha_1} \\ \frac{1}{\alpha_1} & \text{if } \Delta\text{cost} \geq \frac{1}{\alpha_1} \end{cases}$$

for some general  $S_1$  and  $S_2$  imposed by the regulator.

Knowing the firm's criteria for investing, the regulator sets  $S_1$  and  $S_2$  to maximize the expected welfare over both periods. To solve for  $S_1$  and  $S_2$ , begin by noting that at the end of period one, either technology  $\alpha_1$  or  $\alpha_0$  will exist. Thus, if the analysis is restricted to just the second period, a simple one period model of regulation with a standard under commitment exists. From previous arguments, we know the regulator can set the optimal standard  $S_2$  to maximize expected welfare for any technology level which exists at the beginning of the period. Thus, if  $\alpha_1$  was found in period one, then the standard  $S_2$  should be set at either  $\frac{x}{2(m+C_1)}$  or  $\frac{x}{2(m+C_2)}$  depending on whether the second period welfare would be greater with the discovery of technology  $\alpha_2$  than without. Let  $S_{2a}$  be the second period standard set by the regulator if  $\alpha_1$  existed at the beginning of the period. If  $\alpha_1$  was not found in period one, the optimal  $S_2$  would be identical to the one period with commitment case. Let this be denoted as  $S_{2b}$ . It should be reiterated that both  $S_{2a}$  and  $S_{2b}$  will result in the optimal outcome if either  $\alpha_1$  or  $\alpha_0$  exists at the beginning of period two. Let welfare $_{S_{2a}}$  be the second period welfare associated with the regulator imposing  $S_{2a}$  and welfare $_{S_{2b}}$  be the second period welfare if  $S_{2b}$  is imposed in period two.

Returning to the two period situation, total welfare can be written as:

$$\text{welfare} = \begin{cases} W_0 = XS_1 - mS_1^2 - C_0 S_1^2 + \text{welfare}_{S_{2b}} & \text{if } R_1 = 0 \\ W_1 = XS_1 - mS_1^2 - C_1 S_1^2 - \frac{1}{\alpha_1} + \text{welfare}_{S_{2a}} & \text{if } R_1 = \frac{1}{\alpha_1} \end{cases}$$

where the maximized welfare in period two has been added to the first period welfare, assuming the regulator is able to impose either  $S_{2a}$  or  $S_{2b}$ . *Since the regulator can commit to the second period standards in period one*, the reasoning to solve for  $S_1$  in the two period model is the same as in the one period case *and* will result in the *optimal* level of welfare. Intuitively, as long as the proper technology is developed in period one, it is possible to set the optimal second period standard. By committing to the second period

regulations in period one, however, any period two incentives or disincentives to invest in period one are perfectly transferred to period one. As the regulator could optimally control investment before the second period was added, the addition of an optimal incentive scheme (with commitment) from the second period does not alter the regulator's ability to effectively control investment in period one. Therefore, the proper technology level will be developed in period one, the first period *ex post* optimal abatement will occur, and the optimal second period regulation can be set in period one.

Consider the following example. Suppose that if the firm innovates in period one, the regulator will ratchet the second period standard, thereby raising the firm's cost above the benefit of innovation. The firm, however, will then intentionally not invest to prevent ratcheting. The regulator by committing to the period two standard before period one R&D occurs, however, has in effect ratcheted already so the firm does not have the option of not investing to prevent the tougher regulation from being imposed. Moreover, the regulator avoids over abatement in period one because the tougher standard will not take place until the following period, after the firm has had an opportunity to innovate. If the regulator did not commit, then the second period incentive to innovate would not have been perfectly transferred to period one, and the firm could have not invested, thereby avoiding the tighter standard.

Formally,  $S_1$  can be defined as:

$$S_1 = \begin{cases} \frac{X}{2(m + C_0)} & \text{if } W_0 < W_1 \\ \frac{X}{2(m + C_1)} & \text{if } W_0 \leq W_1 \end{cases}$$

where  $W_0$  and  $W_1$  are the total welfare levels defined above.

### 4.3 Standards without Commitment

Now assume that the regulator cannot commit in period one to the level of the standard in period two. For the most part, the solution to this situation is the same as in the commitment case. The significant difference is that the regulator now sets the second period standard after the results of period one R&D are known. Consequently, there are two possible levels,  $S_{2a}$  and  $S_{2b}$ , for the second period standard corresponding with whether or not  $\alpha_1$  was discovered.

Working backwards,  $R_{2a}$  and  $R_{2b}$  will be of the same general form as when the regulator commits. However,  $S_2$  is replaced by either  $S_{2a}$  or  $S_{2b}$  in the formula where 2a and 2b represent whether or not technology  $\alpha_1$  was discovered in period one.  $R_{2a}$  and  $R_{2b}$  will still be cost minimizing if  $\epsilon < 1$  or cost maximizing if  $\epsilon > 1$ .

#### 4.3.1 $\epsilon < 1$ : STOCHASTIC INNOVATION

As the regulator does not commit, the second period portion of the model is identical in nature to the one period with commitment model. Thus, the values for  $S_{2a}$  and  $S_{2b}$  can be solved in a similar manner as is Section 3.2.1. If the firm did not innovate in period one, technology  $\alpha_0$  exists and the solution for  $S_{2b}$  will be identical to that for the one period model. The solution for  $S_{2a}$  will be of the same general form also, but 1 must be added to all the subscripts to note the higher level of technology and new cost of abatement coefficients.

When making its first period investment decision, the firm knows the optimal  $R_{2a}$ ,  $R_{2b}$ ,  $S_{2a}$ , and  $S_{2b}$  as found above. It will invest such that total expected cost is minimized with respect to  $R_1$ :

$$\begin{aligned} \min_{R_1} \text{ total cost} = & \min_{R_1} (\alpha_1 R_1)^\epsilon [C_1 S_1^2 + (\alpha_2 R_{2a})^\epsilon C_2 S_{2a}^2 + (1 - (\alpha_2 R_{2a})^\epsilon) C_1 S_{2a}^2 + R_{2a}] \\ & + (1 - (\alpha_1 R_1)^\epsilon) [C_0 S_1^2 + (\alpha_1 R_{2b})^\epsilon C_1 S_{2b}^2 + (1 - (\alpha_1 R_{2b})^\epsilon) C_0 S_{2b}^2 + R_{2b}] + R_1. \end{aligned}$$

This is the same general form for total cost as in the two period commitment case. The only difference is that the  $S_2$ 's have been replaced by  $S_{2a}$  or  $S_{2b}$ . From this point on, the analysis is the same as in the two period commitment case and is not repeated. It should be noted, though, that the results of the model will not necessarily be the same as  $S_2 \neq S_{2a} \neq S_{2b}$  and therefore not necessarily optimal.

#### 4.3.2 $\epsilon > 1$ : DETERMINISTIC INNOVATION

Once again, the solution process is similar to the commitment case except for several simple, but significant differences. Changing  $S_2$  to either  $S_{2a}$  or  $S_{2b}$ ,  $R_{2a}$  and  $R_{2b}$  are defined the same as under the two period commitment case. As the regulator does not commit, the second period portion of the model is identical to the one period with commitment model. Thus, the values for  $S_{2a}$  and  $S_{2b}$  can be arrived at through a similar process and will be optimal.

When making its first period investment decision, the firm knows the optimal  $R_{2a}$ ,  $R_{2b}$ ,  $S_{2a}$ , and  $S_{2b}$  as found above. It will invest such that total expected cost is minimized with respect to  $R_1$ :

$$\begin{aligned} \min_{R_1} \text{ total cost} = \min_{R_1} & (\alpha_1 R_1)^\epsilon [C_1 S_1^2 + (\alpha_2 R_{2a})^\epsilon C_2 S_{2a}^2 + (1 - (\alpha_2 R_{2a})^\epsilon) C_1 S_{2a}^2 + R_{2a}] \\ & + (1 - (\alpha_1 R_1)^\epsilon) [C_0 S_1^2 + (\alpha_1 R_{2b})^\epsilon C_1 S_{2b}^2 + (1 - (\alpha_1 R_{2b})^\epsilon) C_0 S_{2b}^2 + R_{2b}] + R_1. \end{aligned}$$

This is the same general form for total cost as in the two period commitment case. The only difference is that the  $S_2$ 's have been replaced by  $S_{2a}$  or  $S_{2b}$ . This difference is significant and alters the solution process. When second order conditions are examined, as was done on page 60-61 for the two period with commitment case, it is impossible to sign the expression.

The reason this occurs is simple and has been seen several places in this thesis. Let  $[x]$  equal the first bracketed term in total cost and  $[y]$  the second bracketed term. Taking the partial with respect to  $R_1$  and setting the result equal to zero, yields:

$$\begin{aligned}\frac{\partial \text{total cost}}{\partial R_1} &= \varepsilon \alpha_1^\varepsilon R_1^{\varepsilon-1} [x] - \varepsilon \alpha_1^\varepsilon R_1^{\varepsilon-1} [y] + 1 = 0 \\ &= \varepsilon \alpha_1^\varepsilon R_1^{\varepsilon-1} [x - y] + 1 = 0.\end{aligned}$$

Solving for  $R_1$ , we find:

$$R_1 = \left[ \frac{1}{\varepsilon \alpha_1^\varepsilon [y - x]} \right]^{1/(\varepsilon-1)}.$$

Taking the second derivative gives:

$$\frac{\partial^2 \text{total cost}}{\partial R_1^2} = \varepsilon(\varepsilon - 1) \alpha_1^\varepsilon R_1^{\varepsilon-2} [x - y].$$

Thus if  $[x]$  is greater than  $[y]$ , the sign of the second order condition will be positive, implying a minimum cost. Examining  $[x]$  and  $[y]$  reveals that  $[x]$  is the cost if innovation occurs in period one and  $[y]$  is the cost if innovation does not occur. Consequently, in situations where  $[x]$  is greater than  $[y]$ , the firm will not invest to avoid the higher costs associated with innovation, and  $R_1$  will equal zero. In all remaining cases, the second order condition will be negative, ensuring a corner solution. Thus, the firm's investment  $R_1$  in R&D will always be 0 or  $\frac{1}{\alpha_1}$ .

The solution process from this point forward is similar to the with commitment case except for the fact that the definitions for the change in cost and welfare levels must be altered to account for the control the firm has over second period costs as the regulator cannot commit. In particular, the change in abatement costs will now be equal to the change in the first period abatement cost, as before, plus the difference between the second period costs if technology  $\alpha_1$  exists or not entering the second period. More precisely, the difference between the second period costs will be cost  $\alpha_0$  - cost  $\alpha_1$  where cost  $\alpha_0$  refers to the second period abatement and investment costs if no innovation occurs in the first period and cost  $\alpha_1$  is the second period abatement and investment costs if innovation does occur.

In this non-commitment situation, the incentives from the second period are not perfectly transferred to the first period. In particular, the firm's ability to alter the second period regulation allows it to effectively circumvent more costly regulation. The resulting welfare from regulation, investment, and abatement, therefore, will not be optimal in instances where the firm does not innovate in order to avoid future regulation. If the firm does not do this, the optimal welfare will be achieved.

#### 4.4 Taxes with Commitment

Working backwards, the firm will begin period two with either pollution control technology  $\alpha_1$  or  $\alpha_0$ . As the tax  $t_2$  has already been set for period two, the firm selects R&D expenditures  $R_{2a}$  and  $R_{2b}$  to minimize its expected second period cost. Using the same solution process as in the one period model with commitment,  $R_{2a}$  and  $R_{2b}$  can be written as:

$$R_{2a} = \left[ \frac{4C_1C_2}{\varepsilon \alpha_2^\varepsilon t_2^2 (C_1 - C_2)} \right]^{\frac{1}{\varepsilon-1}}$$

$$R_{2b} = \left[ \frac{4C_0C_1}{\varepsilon \alpha_1^\varepsilon t_2^2 (C_0 - C_1)} \right]^{\frac{1}{\varepsilon-1}}$$

where  $R_{2a}$  and  $R_{2b}$  are the optimal R&D expenditures in period two to minimize the firm's expected second period cost for any  $t_2$  imposed by the regulator if  $\alpha_1$  was or was not found in period one. As in the one period model,  $R_{2a}$  or  $R_{2b}$  will be cost minimizing or cost maximizing depending on the value of  $\varepsilon$ .

#### 4.4.1 $\varepsilon < 1$ : STOCHASTIC INNOVATION

Given that  $\varepsilon < 1$ ,  $R_{2a}$  and  $R_{2b}$  will be second period, cost minimizing R&D expenditures. When making its first period investment decision, the firm is able to determine the above relationships and knows the level of  $t_1$  as the regulator has already committed to it. The firm can use this information in finding the optimal  $R_1$  to invest in period one (*i.e.*, the firm minimizes its expected total cost with respect to  $R_1$ ):

$$\min_{R_1} \text{total cost} = \min_{R_1} (\alpha_1 R_1)^\varepsilon \left[ \begin{aligned} & C_1 \left( \frac{t_1}{2C_1} \right)^2 + t_1 \left( \frac{X}{2m} - \frac{t_1}{2C_1} \right) \\ & (\alpha_2 R_{2a})^\varepsilon \left[ C_2 \left( \frac{t_2}{2C_2} \right)^2 + t_2 \left( \frac{X}{2m} - \frac{t_2}{2C_2} \right) \right] \\ & + (1 - (\alpha_2 R_{2a})^\varepsilon) \left[ C_1 \left( \frac{t_2}{2C_1} \right)^2 + t_2 \left( \frac{X}{2m} - \frac{t_2}{2C_1} \right) \right] + R_{2a} \end{aligned} \right] \\ + (1 - (\alpha_1 R_1)^\varepsilon) \left[ \begin{aligned} & C_0 \left( \frac{t_1}{2C_0} \right)^2 + t_1 \left( \frac{X}{2m} - \frac{t_1}{2C_0} \right) \\ & (\alpha_1 R_{2b})^\varepsilon \left[ C_1 \left( \frac{t_2}{2C_1} \right)^2 + t_2 \left( \frac{X}{2m} - \frac{t_2}{2C_1} \right) \right] \\ & + (1 - (\alpha_1 R_{2b})^\varepsilon) \left[ C_0 \left( \frac{t_2}{2C_0} \right)^2 + t_2 \left( \frac{X}{2m} - \frac{t_2}{2C_0} \right) \right] + R_{2b} \end{aligned} \right] + R_1.$$

Let the first large bracketed term equal  $[x]$  and the second large bracketed term equal  $[y]$ .

Taking the partial with respect to  $R_1$  and setting the result equal to zero yields:

$$\frac{\partial \text{total cost}}{\partial R_1} = \varepsilon \alpha_1^\varepsilon R_1^{\varepsilon-1} [x] - \varepsilon \alpha_1^\varepsilon R_1^{\varepsilon-1} [y] + 1 = 0$$

$$= \varepsilon \alpha_1^\varepsilon R_1^{\varepsilon-1} [x - y] + 1 = 0.$$

Solving for  $R_1$ , assuming an interior solution, we find:

$$R_1 = \left[ \frac{1}{\varepsilon \alpha_1^\varepsilon [y - x]} \right]^{\frac{1}{\varepsilon - 1}}$$

where  $R_1$  is the optimal R&D expenditures in period one to minimize the firm's total expected cost for any  $t_1$  and  $t_2$  imposed by the regulator.

The general form of  $\frac{\partial \text{cost}}{\partial R_1}$  is the same as in the one period model and  $\varepsilon < 1$ , so  $R_1$  will be cost minimizing. Note that unlike in the two period model with commitment for standards,  $[x]$  cannot be greater than  $[y]$ .

Since the regulator possesses the same information the firm does and imposes taxes first, welfare can be written as a function of  $t_1$  and  $t_2$ . The regulator maximizes total expected welfare with respect to  $t_1$  and  $t_2$ :

$$\begin{aligned} \max_{t_1, t_2} \text{welfare} = \max_{t_1, t_2} & \left[ (\alpha_1 R_1)^\varepsilon \left[ X\left(\frac{t_1}{2C_1}\right) - m\left(\frac{t_1}{2C_1}\right)^2 - C_1\left(\frac{t_1}{2C_1}\right)^2 \right] \right. \\ & + (\alpha_2 R_{2a})^\varepsilon \left[ X\left(\frac{t_2}{2C_2}\right) - m\left(\frac{t_2}{2C_2}\right)^2 - C_2\left(\frac{t_2}{2C_2}\right)^2 \right] \\ & \left. + (1 - (\alpha_1 R_{2a})^\varepsilon) \left[ X\left(\frac{t_2}{2C_1}\right) - m\left(\frac{t_2}{2C_1}\right)^2 - C_1\left(\frac{t_2}{2C_1}\right)^2 \right] - R_{2a} \right] \\ & + (1 - (\alpha_1 R_1)^\varepsilon) \left[ X\left(\frac{t_1}{2C_0}\right) - m\left(\frac{t_1}{2C_0}\right)^2 - C_0\left(\frac{t_1}{2C_0}\right)^2 \right. \\ & \left. + (\alpha_1 R_{2b})^\varepsilon \left[ X\left(\frac{t_2}{2C_1}\right) - m\left(\frac{t_2}{2C_1}\right)^2 - C_1\left(\frac{t_2}{2C_1}\right)^2 \right] \right. \\ & \left. + (1 - (\alpha_1 R_{2b})^\varepsilon) \left[ X\left(\frac{t_2}{2C_0}\right) - m\left(\frac{t_2}{2C_0}\right)^2 - C_0\left(\frac{t_2}{2C_0}\right)^2 \right] - R_{2b} \right] - R_1. \end{aligned}$$

where the  $R$ 's are the optimized R&D expenditures.

This expression becomes the objective function in the computer simulation.

#### 4.4.2 $\epsilon > 1$ : DETERMINISTIC INNOVATION

If  $\epsilon > 1$ ,  $R_{2a}$  and  $R_{2b}$  will be cost *maximizing*, and the firm will invest such that  $R_{2a}$  and  $R_{2b}$  are corner solutions. The firm will choose to invest when the change in its second period abatement and tax costs are greater than or equal to the cost of R&D under some  $S_2$ . From the one period model,  $R_{2a}$  and  $R_{2b}$  can be written as:

$$R_{2a} = \begin{cases} 0 & \text{if } \left[ \frac{t_2^2}{4C_2} - \frac{t_2^2}{4C_1} \right] < \frac{1}{\alpha_2} \\ \frac{1}{\alpha_2} & \text{if } \left[ \frac{t_2^2}{4C_2} - \frac{t_2^2}{4C_1} \right] \geq \frac{1}{\alpha_2} \end{cases}$$

$$R_{2b} = \begin{cases} 0 & \text{if } \left[ \frac{t_2^2}{4C_1} - \frac{t_2^2}{4C_0} \right] < \frac{1}{\alpha_1} \\ \frac{1}{\alpha_1} & \text{if } \left[ \frac{t_2^2}{4C_1} - \frac{t_2^2}{4C_0} \right] \geq \frac{1}{\alpha_1} \end{cases}$$

In the first period, the firm is able to determine the optimal  $R_{2a}$  and  $R_{2b}$  as the regulator has already committed to  $t_2$ . The firm can use this information and that  $t_1$  has also been set to find the optimal  $R_1$  to invest in the first period to minimize its total expected cost:

$$\begin{aligned}
\min_{R_1} \text{ total cost} &= \min_{R_1} (\alpha_1 R_1)^\epsilon \left[ \begin{aligned} &C_1 \left( \frac{t_1}{2C_1} \right)^2 + t_1 \left( \frac{X}{2m} - \frac{t_1}{2C_1} \right) \\ &(\alpha_2 R_{2a})^\epsilon \left[ C_2 \left( \frac{t_2}{2C_2} \right)^2 + t_2 \left( \frac{X}{2m} - \frac{t_2}{2C_2} \right) \right] \\ &+ (1 - (\alpha_2 R_{2a})^\epsilon) \left[ C_1 \left( \frac{t_2}{2C_1} \right)^2 + t_2 \left( \frac{X}{2m} - \frac{t_2}{2C_1} \right) \right] + R_{2a} \end{aligned} \right] \\
&+ (1 - (\alpha_1 R_1)^\epsilon) \left[ \begin{aligned} &C_0 \left( \frac{t_1}{2C_0} \right)^2 + t_1 \left( \frac{X}{2m} - \frac{t_1}{2C_0} \right) \\ &(\alpha_1 R_{2b})^\epsilon \left[ C_1 \left( \frac{t_2}{2C_1} \right)^2 + t_2 \left( \frac{X}{2m} - \frac{t_2}{2C_1} \right) \right] \\ &+ (1 - (\alpha_1 R_{2b})^\epsilon) \left[ C_0 \left( \frac{t_2}{2C_0} \right)^2 + t_2 \left( \frac{X}{2m} - \frac{t_2}{2C_0} \right) \right] + R_{2b} \end{aligned} \right] + R_1 \\
&= \min_{R_1} (\alpha_1 R_1)^\epsilon \left[ \begin{aligned} &\frac{t_1 X}{2m} - \frac{t_1^2}{4C_1} + (\alpha_2 R_{2a})^\epsilon \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_2} \right] \\ &+ (1 - (\alpha_2 R_{2a})^\epsilon) \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_1} \right] + R_{2a} \end{aligned} \right] \\
&+ (1 - (\alpha_1 R_1)^\epsilon) \left[ \begin{aligned} &\frac{t_1 X}{2m} - \frac{t_1^2}{4C_0} + (\alpha_1 R_{2b})^\epsilon \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_1} \right] \\ &+ (1 - (\alpha_1 R_{2b})^\epsilon) \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_0} \right] + R_{2b} \end{aligned} \right] + R_1.
\end{aligned}$$

Taking the partial with respect to  $R_1$  yields:

$$\frac{\partial \text{total cost}}{\partial R_1} = \varepsilon \alpha_1^\varepsilon R_1^{\varepsilon-1} \left[ \begin{aligned} & \left[ \frac{t_1 X}{2m} - \frac{t_1^2}{4C_1} + (\alpha_2 R_{2a})^\varepsilon \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_2} \right] \right. \\ & \quad \left. + (1 - (\alpha_2 R_{2a})^\varepsilon) \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_1} \right] + R_{2a} \right] \\ & - \varepsilon \alpha_1^\varepsilon R_1^{\varepsilon-1} \left[ \begin{aligned} & \left[ \frac{t_1 X}{2m} - \frac{t_1^2}{4C_0} + (\alpha_1 R_{2b})^\varepsilon \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_1} \right] \right. \\ & \quad \left. + (1 - (\alpha_1 R_{2b})^\varepsilon) \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_0} \right] + R_{2b} \right] + 1. \end{aligned} \right. \end{aligned} \right]$$

Then, taking the second order conditions results in:

$$\frac{\partial^2 \text{total cost}}{\partial R_1^2} = \varepsilon(\varepsilon - 1) \alpha_1^\varepsilon R_1^{\varepsilon-2} \left[ \begin{aligned} & \left[ \frac{t_1^2}{4C_0} - \frac{t_1^2}{4C_1} \right. \\ & \quad \left. + (\alpha_2 R_{2a})^\varepsilon \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_2} \right] + (1 - (\alpha_2 R_{2a})^\varepsilon) \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_1} \right] + R_{2a} \right. \\ & \quad \left. - \left[ (\alpha_1 R_{2b})^\varepsilon \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_1} \right] + (1 - (\alpha_1 R_{2b})^\varepsilon) \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_0} \right] + R_{2b} \right] \right] \end{aligned} \right]$$

We can see, as was the case with  $R_{2a}$  and  $R_{2b}$ , that  $R_1$  will maximize total cost if  $(\varepsilon - 1) > 1$ . To see this, note that the term in front of the large brackets is positive if  $(\varepsilon - 1) > 1$ ; consequently, if and only if the bracketed term is negative will  $R_1$  be cost maximizing. The first term in the large brackets will always be negative as  $C_0 > C_1$ . To determine the sign of the second and third terms, recall that  $R_{2a}$  and  $R_{2b}$  will always be 0 or 1. The result is that there are four possible combinations of the second and third terms. For instance, the firm does not invest if  $\alpha_1$  is found in period one but does invest  $R_{2b}$  if  $\alpha_1$  was not found in period one. The four possibilities are:

$$\begin{aligned}
1) \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_1} \right] - \left( \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_1} \right] + R_{2b} \right) &= \frac{t_2^2}{4C_1} - \frac{t_2^2}{4C_1} - R_{2b} < 0 \\
2) \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_1} \right] - \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_0} \right] &= \frac{t_2^2}{4C_0} - \frac{t_2^2}{4C_1} < 0 \\
3) \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_2} \right] + R_{2a} - \left( \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_1} \right] + R_{2b} \right) &= -\frac{t_2^2}{4C_2} + R_{2a} + \frac{t_2^2}{4C_1} - R_{2b} \\
&= -\frac{t_2^2}{4C_2} + \left( \frac{t_2^2}{4C_2} - \frac{t_2^2}{4C_1} \right) + \frac{t_2^2}{4C_1} - \left( \frac{t_2^2}{4C_1} - \frac{t_2^2}{4C_0} \right) \\
&= \frac{t_2^2}{4C_0} - \frac{t_2^2}{4C_1} < 0 \\
4) \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_2} \right] + R_{2a} - \left( \left[ \frac{t_2 X}{2m} - \frac{t_2^2}{4C_1} \right] \right) &= -\frac{t_2^2}{4C_2} + R_{2a} + \frac{t_2^2}{4C_1} \\
&= -\frac{t_2^2}{4C_2} + \left( \frac{t_2^2}{4C_2} - \frac{t_2^2}{4C_1} \right) + \frac{t_2^2}{4C_1} \\
&= \frac{t_2^2}{4C_0} - \frac{t_2^2}{4C_1} < 0
\end{aligned}$$

The first and second cases are clearly less than zero. The third and fourth cases can be shown to be negative by substituting for the minimum incentive needed to induce  $R_{2a}$  and  $R_{2b}$  to be invested (e.g., the firm invests  $R_{2a}$  if  $\left[ \frac{t_2^2}{4C_2} - \frac{t_2^2}{4C_1} \geq \frac{1}{\alpha_2} \right]$ ).

Thus, the firm's investment  $R_1$  in R&D will be such that the probability of success will be either 0 or 1. The firm's decision to invest will be based on whether or not the change in period one tax and abatement costs and period two cost is greater than or equal to the cost of R&D. The change in cost is defined by:

$$\Delta \text{cost} = \begin{cases} \left( \frac{t_1^2}{4C_1} - \frac{t_1^2}{4C_0} \right) + \left( \frac{t_2^2}{4C_1} - \frac{t_2^2}{4C_0} \right) + \left( \frac{t_2^2}{4C_2} - \frac{t_2^2}{4C_1} - \frac{1}{\alpha_2} \right) & \text{if } \left( \frac{t_2^2}{4C_2} - \frac{t_2^2}{4C_1} \right) \geq \frac{1}{\alpha_2} \\ \left( \frac{t_1^2}{4C_1} - \frac{t_1^2}{4C_0} \right) + \left( \frac{t_2^2}{4C_1} - \frac{t_2^2}{4C_0} \right) & \text{if } \left( \frac{t_2^2}{4C_2} - \frac{t_2^2}{4C_1} \right) < \frac{1}{\alpha_2} \end{cases}$$

where the first parenthesis represents the change in period one abatement and tax costs. The second parenthesized term represents the second period change in abatement and tax

$$\Delta\text{cost} = \begin{cases} \left(\frac{t_1^2}{4C_1} - \frac{t_1^2}{4C_0}\right) + \left(\frac{t_2^2}{4C_1} - \frac{t_2^2}{4C_0}\right) + \left(\frac{t_2^2}{4C_2} - \frac{t_2^2}{4C_1} - \frac{1}{\alpha_2}\right) & \text{if } \left(\frac{t_2^2}{4C_2} - \frac{t_2^2}{4C_1}\right) \geq \frac{1}{\alpha_2} \\ \left(\frac{t_1^2}{4C_1} - \frac{t_1^2}{4C_0}\right) + \left(\frac{t_2^2}{4C_1} - \frac{t_2^2}{4C_0}\right) & \text{if } \left(\frac{t_2^2}{4C_2} - \frac{t_2^2}{4C_1}\right) < \frac{1}{\alpha_2} \end{cases}$$

where the first parenthesis represents the change in period one abatement and tax costs. The second parenthesized term represents the second period change in abatement and tax costs, and the third term represents an additional cost reduction in abatement and taxes, minus the cost of R&D for  $\alpha_2$ , if the firm invests in period two.

It follows that  $R_1$  can be defined as:

$$R_1 = \begin{cases} 0 & \Delta\text{cost} < \frac{1}{\alpha_1} \\ \frac{1}{\alpha_1} & \Delta\text{cost} \geq \frac{1}{\alpha_1} \end{cases}$$

for some general  $t_1$  and  $t_2$  imposed by the regulator.

Knowing the firm's criteria for investing, the regulator sets  $t_1$  and  $t_2$  to maximize welfare over period one and two. In order to solve for  $t_1$  and  $t_2$ , recall that it was just shown that the firm will invest in an all or nothing manner. This means that the regulator will be able to determine the exact investment and abatement levels by the firm for any tax scheme imposed. If just the second period is examined, it is known that the firm will begin the period with either technology  $\alpha_0$  or  $\alpha_1$ . For argument sake, let  $t_{2b}$  be  $t_2$  if  $\alpha_0$  is the technology existing at the beginning of period two, and  $t_{2a}$  be  $t_2$  if  $\alpha_1$  is the technology existing at the beginning of period two. From the one period tax model with commitment, it is known what the welfare maximizing  $t_{2a}$  and  $t_{2b}$  should be. In particular,  $t_{2b}$  will be identical to that found in Section 3.3.2 and  $t_{2a}$  will be:

$$t_{2a} = \begin{cases} \frac{XC_1}{(m+C_1)} & \text{if } \Delta \text{cost}_{t_1, \text{high}} < \frac{1}{\alpha_2} \\ \sqrt{\frac{4C_1C_2}{\alpha_2(C_2-C_1)}} - \theta & \text{if } \Delta \text{cost}_{t_2, \text{low}} < \frac{1}{\alpha_2} \text{ and } \Delta \text{cost}_{t_2, \text{high}} \geq \frac{1}{\alpha_2} \text{ and } W_{t_2, \text{low}} > W_{t_2, \text{high}} \\ \sqrt{\frac{4C_1C_2}{\alpha_2(C_2-C_1)}} & \text{if } \Delta \text{cost}_{t_2, \text{low}} < \frac{1}{\alpha_2} \text{ and } \Delta \text{cost}_{t_2, \text{high}} \geq \frac{1}{\alpha_2} \text{ and } W_{t_2, \text{low}} \leq W_{t_2, \text{high}} \\ \frac{XC_2}{(m+C_2)} & \text{if } \Delta \text{cost}_{t_2, \text{low}} \geq \frac{1}{\alpha_2} \end{cases}$$

where the general form is the same but the subscripts on  $\alpha$  and the cost coefficients have been changed to reflect the higher level of technology entering the second period.

Returning to the two period situation, the welfare maximizing  $t_1$  for the regulator to impose will be determined along the same lines as the one period case. However,  $\Delta \text{cost}_{t_0, \text{high}}$  will now be defined as:

$$\Delta \text{cost} = \begin{cases} \left( \frac{t_1^2}{4C_1} - \frac{t_1^2}{4C_0} \right) + \left( \frac{t_{2b}^2}{4C_2} - \frac{t_{2b}^2}{4C_0} \right) - \frac{1}{\alpha_2} & \text{if } \left( \frac{t_{2b}^2}{4C_2} - \frac{t_{2b}^2}{4C_1} \right) \geq \frac{1}{\alpha_2} \\ \left( \frac{t_1^2}{4C_1} - \frac{t_1^2}{4C_0} \right) + \left( \frac{t_{2b}^2}{4C_1} - \frac{t_{2b}^2}{4C_0} \right) & \text{if } \left( \frac{t_{2b}^2}{4C_2} - \frac{t_{2b}^2}{4C_1} \right) < \frac{1}{\alpha_2} \end{cases}$$

to represent the change in cost to the firm if it invests and discovers  $\alpha_1$  but the regulator imposed the optimal tax based upon technology  $\alpha_0$  (as if the firm does not invest). The tax  $t_{2b}$  is used for  $t_2$  in this situation as, if  $\alpha_0$  is not discovered in period one,  $t_{2b}$  is the welfare maximizing tax for the second period.  $\Delta \text{cost}_{t_1, \text{low}}$  will be defined in an analogous situation but for the regulator setting the first period tax as if the firm invests, and  $t_{2a}$  will be used to represent  $t_2$ .

If either of the extreme case holds, in which the firm and regulator favor or disfavor innovation, the imposed regulation will be optimal. If neither of the extreme cases hold, then the solution process is similar to the one period situation, with the adjustment described above to account for the second period. There is also an additional

adjustment to the solution process that must be made. The ratio of the rate of change in cost to the firm (incentive to innovate) and rate of change in welfare must be equal across the two time periods. If this condition does not hold, it is possible for the regulator to improve welfare by altering the level of taxes in each period. For instance, if the regulator is trying to prevent innovation in period one, then lowering the tax in period two will also lower the incentive to innovate in period one. If only the first period tax is lowered, at some point, the regulator gives up a large amount of welfare relative to the decrease in incentive. Yet, at the same point, lowering the second period tax will result in a smaller loss in welfare but a relatively larger decrease in the incentive to innovate. Of course, the amount the tax may be lowered or raised in either period is bounded by the point at which the technology shifts to a different level. To solve for the optimal  $t_1$ , all possible combinations of investment and technology levels must be checked. As before, the regulator will select the one which yields the highest level of welfare.

To summarize, the solution processes for the two period model with commitment where taxes are used as the regulatory instrument and innovation is deterministic has just been described. If the cost of innovation is very large or small, the regulator can impose the optimal regulation as the incentive to the firm and regulator is similar enough (in part because of deterministic innovation) that the regulator does not need to make a trade off between innovation and abatement. If the cost of innovation is such that the regulator cannot impose a first period tax that will provide an *ex ante* optimal incentive for the firm to invest and an *ex post* optimal abatement level, then a more complex solution process is involved. Because the regulator commits in period one to second period regulation, not only must the first period tax be altered to maximize welfare when making tradeoffs between abatement and innovation, but the second period tax must be altered as well to ensure the cost of tradeoffs between periods are equal. It should be noted this implicitly occurs in the computer simulations for stochastic innovation.

## 4.5 Taxes without Commitment

Now assume that the regulator cannot commit in period one to the level of the tax in period two. For the most part, the solution to this situation is the same as in the commitment case. The significant difference is that the regulator now sets the second period tax after the results of period one R&D are known. Consequently, there are two possible levels,  $t_{2a}$  and  $t_{2b}$ , for the second period tax corresponding with whether or not  $\alpha_1$  was discovered.

Working backwards,  $R_{2a}$  and  $R_{2b}$  will be of the same general form as when the regulator commits. However,  $t_2$  is replaced by either  $t_{2a}$  or  $t_{2b}$  in the formula.  $R_{2a}$  and  $R_{2b}$  will still be cost minimizing if  $\epsilon < 1$  or cost maximizing if  $\epsilon > 1$ .

### 4.5.1 $\epsilon < 1$ : STOCHASTIC INNOVATION

The solution process is analogous to that of the two period model of standards without commitment and stochastic innovation. The standards must simply be changed to taxes and the objective function for the regulator is the same as given in section 4.4.1 but  $t_2$  is changed to  $t_{2a}$  or  $t_{2b}$ .

### 4.5.2 $\epsilon > 1$ : DETERMINISTIC INNOVATION

Once again, the solution process is analogous to the two period standards without commitment and deterministic innovation. It should be noted, though, that as the regulator does not commit in the first period to second period regulation, there is no need to worry about the ratio of trade off between cost and welfare between periods as was the case when the regulator committed.

## 4.6 Simulation

In order to compare the welfare resulting from the various policy instruments, a computer simulation was developed based upon the preceding two period model. Below, the results from running the simulation under two situations or sets of parameters are presented. In each situation, the optimal level of welfare and investment is presented along with the expected equilibrium welfare arising under standards and taxes with and without commitment between periods and for deterministic innovation. For each situation, the simulation is also run using the same parameters for stochastic innovation.

### 4.6.1 THRESHOLD INNOVATION

In this case, the simulation was parameterized to mimic the situation in which the cost of innovating in the first period is greater than the benefit of first period innovation (from the regulator's perspective). Yet, if innovation is successful in the first period, the cost of investing in the follow-up technology  $\alpha_2$  is small enough, and the benefits large enough that the benefit of being able to innovate in period two outweighs all first period losses. In other words, the optimal solution is for investment in technology  $\alpha_1$  and  $\alpha_2$ . This situation may occur in the real world when there is some threshold of benefits which must be reached for innovation to occur. For instance, the development of electric automobiles requires a high cost, low benefit developmental period. If this period is gone through, the benefits from mass production and reduced emissions in the second period are likely to be large; however, the costs are likely to be relatively low (*i.e.*, altering production lines will be a small cost in the second period relative to the development costs in the first period).

The parameters of the model are set at  $\alpha_1=.0018$ ,  $\alpha_2=.0025$ ,  $C_0=.75$ ,  $C_1=.60$ ,  $C_2=.25$ ,  $X=75$ , and  $m=.375$ . Innovation will be deterministic in nature as  $\epsilon$  is set equal to 2. The results from the simulation are presented in Table 4.1. In the table, the probability of each path is given by  $P(\alpha_i, \alpha_j)$  where  $\alpha_i$  is the technology at the end of the first period

	Expected Welfare	Probability of Each Technology Path				Investment		
		$P(\alpha_1, \alpha_2)$	$P(\alpha_1, \alpha_1)$	$P(\alpha_0, \alpha_1)$	$P(\alpha_0, \alpha_0)$	$R_1$	$R_{2a}$	$R_{2b}$
Optimal	2736	1	0	0	0	555	400	0
Std. w/ Commit	2736	1	0	0	0	555	400	0
Std. w/o Commit	2500	0	0	0	1	0	400	0
Taxes w/ Commit	2693	1	0	0	0	555	400	0
Taxes w/o Commit	2736	1	0	0	0	555	400	0

**Table 4.1:** Threshold Innovation with Deterministic Innovation

and  $\alpha_j$  is the technology at the end of the second period. For instance  $P(\alpha_1, \alpha_2)$  is the probability of the firm discovering technology  $\alpha_1$  and then finding  $\alpha_2$  in the second period.

Under deterministic innovation, the optimal expected welfare is 2736 and for technology  $\alpha_1$  and  $\alpha_2$  to be developed. Both standards with commitment and taxes without commitment will yield the optimal welfare and technology levels. This result should not be surprising. If the firm does not innovate when the regulator commits to more stringent standards in the future, it will face significant abatement costs in the second period as it will be abating with relatively inefficient abatement technology. Likewise, if the firm does not develop technology  $\alpha_1$  and  $\alpha_2$  when the regulator does not commit to future tax levels, the regulator will not lower the tax rate. Consequently, the firm will be left paying such a large amount in taxes that the cost of innovation could have been more than offset by the tax payments.

Standards without commitment yields a lower expected welfare of 2500. This result occurs as the firm does not invest in period one in order to prevent a higher standard, which will raise its costs, in period two. *A priori*, it might have been expected that taxes with commitment would have performed poorly as committing to the future tax level would have reduced the incentive given to the firm to innovate to the point where it does not. However, while taxes with commitment will not be optimal, the firm will still invest, develop technology  $\alpha_2$ , and welfare will be 2693, only slightly below the optimal level. In this case, the regulator raises the second period tax to create an additional incentive to invest. Even though this causes under-abatement once  $\alpha_2$  is found, the loss in welfare is relatively small compared to the loss in welfare of not innovating.

	Expected Welfare	Probability of Each Technology Path				Investment		
		$P(\alpha_1, \alpha_2)$	$P(\alpha_1, \alpha_1)$	$P(\alpha_0, \alpha_1)$	$P(\alpha_0, \alpha_0)$	$R_1$	$R_{2a}$	$R_{2b}$
Optimal	2787	.698	0	.0520	.25	270	400	16.6
Std. w/ Commit	2736	1	0	0	0	555	400	131
Std. w/o Commit	2500	0	0	0	1	0	0	0
Taxes w/ Commit	2685	.533	.043	.04	.384	183	342	5.03
Taxes w/o Commit	2677	.819	.181	0	0	555	268	18

**Table 4.2:** Threshold Innovation with Stochastic Innovation

Table 4.2 presents the results if stochastic innovation occurs ( $\epsilon=.5$ ). The optimal policy would now result in an expected welfare of 2787 and the probability of reaching  $\alpha_2$  is relatively large at .698. It is interesting to note that the concave nature of R&D research means period one research expenditures are reduced dramatically (from 555 to 270). Yet, if  $\alpha_1$  is found, the benefit of finding  $\alpha_2$  is high enough that R&D expenditures are such that success in discovering  $\alpha_2$  is guaranteed. None of the policy instruments yields an optimal outcome as the incentive to invest and abate are not identical to the regulator's. Standards with commitment works relatively well as the regulator favors innovation and is able to provide a proper incentive while being able to set an abatement level that will be close to the *ex post* optimal level. Standards without commitment will work poorly as under deterministic innovation. This occurs because, if the firm succeeds in innovation, the more stringent standards will result in increased abatement costs that are larger than any potential reduction in abatement cost on units currently abated.

Taxes with and without commitment perform similarly in terms of welfare. However, without commitment the firm invests heavily to assure itself the opportunity of having taxes lowered by the regulator in response to innovation, resulting in over-investment in period one. Commitment to taxes, though, results in the regulator setting a tax higher than *ex post* optimal to induce innovation, resulting in over-abatement if innovation is successful. However, excessive investment does not occur as under without commitment. In fact, the expected welfare loss from over-abatement under commitment is low enough that committing to future tax levels now results in higher welfare than without commitment, whereas in the case of deterministic innovation, non-commitment yielded the

optimal solution. Lastly, note that if commitment did not occur within individual periods, the incentive to innovate under taxes without commitment would be very high. In particular, the firm would increase R&D in period two to 400 from 268, causing a reduction in welfare greater than the gain in welfare from the of increased probability of successful innovation.

#### 4.6.2 NO INNOVATION OPTIMAL

The simulation is now parameterized in a manner such that no innovation is preferred by the regulator. In other words, the costs of innovation outweigh the benefits. The parameters are set similar as before but  $\alpha_1=.002$  and  $\alpha_2=.0062$  to reflect the new cost structure (difficulty) of innovation.

	Expected Welfare	Probability of Each Technology Path				Investment		
		$P(\alpha_1, \alpha_2)$	$P(\alpha_1, \alpha_1)$	$P(\alpha_0, \alpha_1)$	$P(\alpha_0, \alpha_0)$	$R_1$	$R_{2a}$	$R_{2b}$
Optimal	2500	0	0	0	1	0	0	0
Std. w/ Commit	2500	0	0	0	1	0	0	0
Std. w/o Commit	2500	0	0	0	1	0	0	0
Taxes w/ Commit	2500	0	0	0	1	0	0	0
Taxes w/o Commit	2387	1	0	0	0	500	161	0

**Table 4.3:** No Innovation Optimal with Deterministic Innovation

First consider the case where innovation is deterministic ( $\epsilon=2$ ). From Table 4.3, the optimal and all policy instruments except taxes without commitment will result in an expected welfare of 2500 and no innovation. Taxes without commitment, however, will result in discovering technologies  $\alpha_1$  and  $\alpha_2$  and a lower welfare of 2387. This result occurs because without commitment, the firm has a powerful incentive to innovate in order that the regulator will lower the future tax rates. Committing to tax rates, on the other hand, allows the regulator to reduce the incentive. In this case, by committing not

to lower the tax rate, the regulator reduces the potential savings to the firm arising from paying a lower tax rate on units of pollution it would have emitted under any circumstance. The key lesson from this is that not committing when tax regulation is used results in a powerful over-incentive for the firm to invest in R&D. Consequently, the firm may develop technologies which are unwanted. While this model allows only one technology to be found each period, this result should suggest in more complex models the firm will pursue different technology paths than the regulator because of the excessive incentive given to firms to innovate by taxes without commitment.

	Expected Welfare	Probability of Each Technology Path				Investment		
		$P(\alpha_1, \alpha_2)$	$P(\alpha_1, \alpha_1)$	$P(\alpha_1, \alpha_0)$	$P(\alpha_0, \alpha_0)$	$R_1$	$R_{2a}$	$R_{2b}$
Optimal	2601	.209	.2	.114	.478	83.3	42.1	18.4
Std. w/ Commit	2582	.16	.231	.121	.489	76.3	26.9	19.5
Std. w/o Commit	2545	.05	.048	.158	.744	4.80	42.0	15.2
Taxes w/ Commit	2593	.227	.18	.107	.486	82.7	50.1	16.1
Taxes w/o Commit	2560	.344	.334	.064	.258	229	41.4	19.9

**Table 4.4:** No Innovation Optimal with Stochastic Innovation

If  $\epsilon$  is set equal to .5, innovation is stochastic and the results will be much different as seen in Table 4.4. The optimal expected welfare will be 2601 and all policy instruments will perform similarly in terms of welfare. Taxes without commitment now performs significantly better than when innovation was deterministic, but it is interesting to note that it still provides an excessively large incentive to innovate. For instance, the firm would spend 229 on R&D in period one but the regulator's optimal expenditure is 83.3. On the other hand, standards without commitment produces too little incentive, the firm spending on 4.8 on first period R&D. The reason for this is the same as encountered through out this thesis: the firm does not invest to avoid tougher standards in the future which will raise its costs. When the regulator commits to standards, the result is much different. By raising the firm's future costs, the firm invests to reduce its costs instead of not investing

to avoid them. In this particular situation, committing to the standard results in the R&D expenditures being close to the optimal levels.

#### 4.7 Summary

This section has developed a two-period model where the regulator commits within periods but inter-period commitment is left open. It was shown that if innovation is deterministic and standards with commitment (between periods) is used as the regulatory instrument, the optimal can be achieved. Of course, there is no *a priori* reason why other policy instruments will not yield the optimal if the model is parameterized properly, but it cannot be guaranteed.

The results of several simulation runs were then presented. A key result was that standards performed well, although non-commitment to a standard performed well only in instances where no innovation was preferred. Taxes with commitment were also shown to perform better than non-commitment to taxes. The reason for this result is that non-commitment to taxes creates an excessive incentive to innovate. Taxes with commitment will not always be better than non-commitment, though. Commitment reduces the incentive to invest but in some instances does so at the expense of under or over-abatement. In these instances, non-commitment may be preferred. It should be noted that the simulations were designed to show instances where controlling innovation is important and hence situations in which standards may perform well.

Biglaiser, Horowitz, and Quiggin (1995) show that taxes will result in the optimal outcome through continuous time even when innovation is possible. However, when discrete time is used, as in this thesis, taxes do not necessarily result in optimal solutions. Their work, therefore, should be tempered by the fact that innovation and pollution regulation tend to be discrete and moves in a "jerky" fashion. Consequently, a model such as the one presented here should be considered when evaluating the efficiency of taxes (although preferably with more than one firm). Assuming that innovation and regulation is discrete and that the marginal benefits of pollution abatement is downward sloping, taxes

will invariably not be optimal. In fact, with sophisticated models, it is difficult to imagine any regulation being optimal. Thus, as Kennedy and Laplante (1996) suggest, it is important that the equilibrium welfare should be examined to compare alternative policies. In the simple model develop here, standards often resulted in a higher level of welfare than taxes. So while answers regarding which policy instrument will result in a higher level of welfare through time will likely remain ambiguous, standards should not be disregarded.

## 5. CONCLUSIONS

This thesis developed a simple model of dynamic pollution regulation with endogenous technological change to compare the efficiency of emission standards and Pigouvian taxes. The model was developed in a manner to allow for probabilistic R&D, commitment or non-commitment by the regulator to future regulation, and was solved for the equilibrium social welfare. A particularly important aspect of the model is that commitment to a *regulatory level* after innovation may be made prior to the actual decision by the firm to invest in R&D, an element typically ignored in the literature. Including this aspect resulted in several significant results, such as committing to a standard yielding the first best outcome when innovation is deterministic in nature.

R&D, of course, is an exceedingly complex process. This model, as with all models, does not capture this complexity in its entirety. Of particular significance is that the model includes only one firm, eliminating the possibility of firm interactions and issues regarding how to regulate multiple firms. As was suggested earlier, however, a one firm model may be appropriate in circumstances where firms are regulated independently, not an uncommon occurrence. Another significant assumption is that perfect information exists. In reality, it may well be the case that the regulator does not possess perfect information regarding firms' abatement and R&D probability functions. Laffont and Tirole (1994) go so far as to model second period cost functions as unknown to the firm itself in the first period. The model developed in this thesis also assumes that there is only one technology which may be developed each period. A more realistic model would have a continuum of technologies. There are, of course, other simplifications; nevertheless, the simplicity of the model allows the basic process of dynamic pollution regulation to be understood and provides a basis for future research. Most importantly, it provides several fundamental insights and offers guidance to policy-makers.

## **5.1 Policy Implications**

From a policy-maker's perspective, the results of this thesis have a number of implications regarding the selection of policy instruments and the level at which they are set. Among these is that the regulator should recognize the nature of innovation. The nature of innovation includes not only whether innovation is deterministic or stochastic, but also how the relative costs and benefits associated with innovation are related to each other through time. To see how these can affect the efficiency of policy instruments, recall that if innovation is deterministic, then committing to a standard will yield the optimal solution. This is a powerful result which occurs because the regulator is able to control both the investment and abatement level optimally. As another example, in the threshold innovation model that was solved using the computer simulation, no innovation was preferred in the first period and standards and taxes with and without would have been able to perform equally well. The addition of the second period with large benefits and low innovation costs, however, made first period innovation desirable and altered the relative efficiency of policy instruments. In other words, the longer time horizon altered the optimal technology level and relative efficiency of regulatory instruments.

The results of this thesis also suggest that emission standards, especially in situations when innovation is deterministic, should not be discounted as much as proponents of taxes suggest. It was demonstrated repeatedly that emission standards with commitment, in this albeit simple model, resulted in equilibrium welfare levels which were greater or equal to those of taxes.

This result is driven largely because the model allows the regulator to act preemptively, recognizing her impact on innovation and subsequent abatement levels. Previous models, for the most part, have assumed the regulator reacted to innovation. Allowing the regulator to act first is an extremely important change in the assumptions. There is no realistic constraint, however, to the regulator moving first in many real world situations. For instance, California has committed to a series of more stringent automobile emission standards well past the year 2000. Part of the justification made for this action is to give firms the incentive to invest in R&D activities.

For regulation with standards, moving first is critical and, in general, will yield a larger welfare than not committing. For the case of taxes, whether the regulator should move first by committing or not to a tax level in the future is ambiguous. Consequently, it is important for policy-makers to recognize when to commit. As was shown in the one period model with deterministic innovation, non-commitment tends to provide an over-incentive to innovate. Yet, it is preferred when the benefits of innovation are larger than the cost of innovation and any under or over-abatement which would result if the regulator did commit. Committing to a lower tax rate on the assumption that the firm will innovate must be done with caution as the lowering of the rate may reduce the incentive to the firm below the optimal level.

This result, whether the regulator commits or not, raises an interesting point. There is a real-world tendency to set a lower tax rate over a higher rate because of the large transfer payments involved and subsequent political pressure. Of course, there should be a counter-vailing pressure from environmental groups to raise the tax; however, this opposition appears to be much smaller than one might expect. A possible explanation may be that taxes are politically so unpopular that arguing for them would cause environmental groups to lose public support. Consequently, they may have focused their lobbying efforts on ensuring standards for environmental quality are not lowered, an approach that the public may be more sympathetic too. This may also help to explain why standards are rarely lowered.

Regardless of cause, this tendency to set lower taxes may lead to too large of a reduction in the incentive to invest in R&D, actually reducing innovation. Even if the regulator does not commit to a tax, lowering it only if innovation occurs, there may be a belief that the tax will be lowered in the future (*e.g.*, a shift in the political alignment of Congress). If this belief exists, the firm may reduce its investment in R&D. This raises the question of time consistency and reputation in regulation. In particular, if the regulator has a reputation of committing to regulation but then altering it latter, firms may not believe the regulator has truly committed and therefore behave differently. The model developed here assumed that if the regulator committed to future regulation, the firm believed fully that the regulation would not be altered. Relaxing this assumption will

affect the efficiency of both standards and taxes as suggested above. Thus, policy makers should be aware that the effectiveness of committing is partially tied to their reputations, or the belief firms' hold regarding their actions.

Lastly, it should be reiterated that the evaluation of policy instruments should be based on the combined, equilibrium efficiency of pollution abatement and investment in new technology and production processes over time. When this is done, the results of this thesis suggest that committing to standards can result in higher levels of welfare than committing or not committing to Pigouvian taxes.

## **5.2 Further Research**

As was indicated at the beginning of the conclusions section, the model developed in this thesis does not capture the complete complexity of pollution regulation and R&D. It does, however, provide a basis upon which more complex models may be built.

One of the most important extensions of the model is the relaxation of complete and symmetric information. In reality, it is likely that the regulator does not possess perfect information regarding firms' abatement and R&D probability functions. Allowing a situation where asymmetric information exists means that the firm may have an opportunity to collect information rents; consequently, the regulator would be faced with attempting to control abatement, innovation, and information rents simultaneously.

The incorporation of multiple firms and interactions between them would also seem to be a next step in the model's development. Firm interactions have the potential to greatly affect the strategies of both the firms and regulator. For instance, firms may engage in R&D as a means of not only lowering their own cost structure, but as a means of increasing their rival's costs by forcing the regulator to impose tighter abatement standards. Along the same lines, one firm innovating and reducing its cost structure may force another firm to preemptively innovate in response to this strategy. There is also the possibility that firms may collude in not innovating to prevent the imposition of tighter regulations.

Another important avenue for future research is the nature of innovation. In particular, the incorporation of multiple technologies and paths would likely alter the investment decisions of all actors involved. This would allow the possibility of sunk costs preventing firms from switching to new technology paths over time. If incorporated with multiple firms, it would also allow for the possibility that firms would invest in less costly and significant R&D research than the regulator. In particular, the regulator has an incentive equal to the societal improvement in welfare to engage in R&D, however, an individual firm only has an incentive equal to the benefits it receives from innovating. Therefore, if there are many firms, the benefit of innovating to a single firm may be small and it will choose to invest in a more modest R&D project. The regulator, on the other hand, having an incentive derived from society-wide benefits, may have a much larger incentive to innovate than the firm and undertake larger R&D projects aimed at different technologies. Learning from past research could also be incorporated into the model.

The use of multiple regulatory instruments is another area in which more research is needed. As seen throughout this thesis, the regulator struggled to set efficient *ex ante* and *ex post* regulations to cover multiple actions by a firm. This difficulty will only increase as more sophisticated models are developed. The logical response would be to give the regulator a large portfolio of instruments which may be used.

As a concluding note, this thesis allowed the regulator to act preemptively and made comparisons between quantity and price rules in a subgame perfect equilibrium setting. Future research that builds from the model developed here or other models should include these aspects. If they do not, the results should be interpreted with caution. Models not using an equilibrium analysis may be deceiving and allowing the regulator to act in a more strategic manner has been shown to dramatically affect the results. Moreover, there is not sufficient evidence at this time to disregard quantity rules, especially in settings where any set of regulations imposed is likely to be second-best.

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