AN APPARATUS FOR MEASURING INTERNAL FRICTION AND FATIGUE STRENGTH OF CORE MATERIALS USED IN SANDWICH CONSTRUCTION

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AN APPARATUS FOR MEASURING INTERNAL FRICTION
AND FATIGUE STRENGTH OF CORE MATERIALS
USED IN SANDWICH CONSTRUCTION

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Abstract

Apparatus and techniques are described for measuring the energy absorbed by sandwich core materials subjected to rapidly cycled shear stress.

Exploratory data are given on three commercial aluminum honeycomb cores with foil thicknesses of 0.002, 0.003, and 0.004 inch.

Introduction

Internal friction is the mechanism by which energy of deformation in a material is converted to heat. When a material is caused to vibrate, internal friction may have a large effect on the power required to maintain vibration, on the amplitude of vibration under given conditions of excitation, and on the rate of decay of free vibrations.

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2 Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

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The structure of jet-powered aircraft is subject to strong vibration-exciting forces from the jet noise, as the noise is of high intensity and includes a wide distribution of frequencies. Therefore, internal friction in materials used for construction of jet airplanes may be of importance in limiting the amplitude of vibration of parts of the aircraft.

It follows then that internal friction may be an important property of structural sandwich, as this form of material is used frequently in modern aircraft.

The intent of the work reported here is to develop equipment and techniques for measuring internal friction in metal foil honeycomb cores of the type used in structural sandwich, when subjected to shear stress; and then to evaluate the relative internal friction of existing cores.

The experimental method described here is, briefly, to use a specimen of core material to connect two masses, and to excite vibration in this mass-specimen system. The system is driven at its natural frequency, so that large stresses can be induced in the core material with relatively low driving force. The driving force and resulting amplitude of vibration are measured, from which the energy input per stress cycle can be computed. Energy losses outside the specimen are made as small as possible, and it is assumed that all measured energy is absorbed in the specimen.

Energy per cycle absorbed by the specimen is measured as a function of stress amplitude and number of stress cycles. Throughout the report, stress amplitude is the term applied to the absolute value of the maximum stress experienced by the core specimen. The average stress is in all cases zero, and the stress cycles are almost exactly sinusoidal with time. Thus, the specimen experiences a complete reversal of stress during each cycle.

The total number of stress cycles and the frequency of vibration are determined by an electronic counter and timer. Stress amplitude is determined from the measured frequency and amplitude of vibration, using the known value of the masses attached to the specimen. The stress amplitude is held constant throughout the test of a specimen, the number of stress cycles required to fail the specimen is recorded, and the energy per cycle absorbed by the specimen is recorded at intervals throughout a test.

**Experimental Methods**

**Specimens**

A complete specimen uses two samples of core material, each 1/2 by 2 by 6 inches. These are combined with three 1/2-inch-thick steel plates to form a
5-layer sandwich. The outside plates are 2 by 6 inches, and the middle plate is 4 by 6 inches. The two outer plates are ground flat on one side, and this side is drilled and tapped to receive bolts for mounting the specimen in the vibration apparatus. The unground side is bonded to the core samples. The edges of the middle plate are drilled and tapped to receive bolts for supporting the apparatus. An edge view of a specimen is shown in figure 1.

The specimen is assembled, using a suitable adhesive. Results reported here are on specimens assembled with an epoxy adhesive, prepared as specified by the manufacturer. A special jig holds the specimen parts in correct position during assembly and curing of the adhesive. Particular care is taken when spreading the adhesive to assure reasonably uniform fillets and bond thickness between specimens.

Apparatus

General. --The apparatus consists of a mechanical vibrating system (of which the core specimen is the spring), an electro-mechanical transducer for driving the vibrating system, devices for measuring the driving force and resulting displacement of the system, and a cathode-ray oscilloscope and camera for recording the measured quantities. A variable-frequency audio generator and power amplifier supply energy to the driver, and an electronic counter and timer records the total number of stress cycles and their frequency. The counter has been modified by adding switches to disable the auto reset and display time circuits.

Vibrating System. --The vibrating system is made of steel and consists of two similar complex forms, of equal mass, mechanically connected through the specimen. The form of the masses is designed to locate the mass center of each at the geometric center of the specimen when the vibrating system is assembled. A cross section of the arrangement is shown in figure 2.

Each mass, or half of the vibrating system, is composed of two main plates, rigidly connected by two side plates. On each half, the main plates are 12 inches square, one 2 inches thick and the other 1 inch thick. The side plates are 3/8 by 5-3/4 by 10 inches. To maintain symmetry, the side plates are connected to the main plates at diagonally opposite positions. This can be seen in figures 3 and 4.

The 2-inch-thick main plates are bolted to the specimen. The area of contact was surface ground to assure positive connection of specimen and plate. The side plates are designed to hold the 1-inch-thick main plates at the correct position so each half of the system is balanced around a plane through the middle of the specimen.
With this design, there is no tendency for the two major masses to rotate as the system vibrates in the desired mode, which is with shear deformation of the core specimen. Further, the stress on the specimen is nearly pure shear, the moments being reduced to the minimum by the design of the system. A residual moment probably exists because the shear stress is applied across the finite thickness of the specimen, but with the thickness only 1/12 of the length in the stressed direction, the stresses other than shear are negligible except in the neighborhood of the end of the specimen.

The vibrating system is supported by steel wires through bars attached to the middle plate of the specimen. This middle plate is a plane of dynamic symmetry of the system, and therefore has no first-order tendency to move as the system vibrates. In this way energy loss through the support is minimized. The dead weight of the system is carried through the specimen, one half being in tension and the other in compression. Each half of the vibrating system totals 142.2 pounds weight; when spread over 12 square inches of specimen area, this weight results in a static stress of just under 12 pounds per square inch. This stress in the cell direction of existing metal foil honeycombs is negligible.

Part of the total mass of the vibrating system are the superstructures for mounting the driving and displacement measuring devices. These and other details of the system can be seen in figures 3, 4, and 5.

Driving Device. --The driver, seen in figure 5, is a modified high-power loudspeaker. The original voice coil suspension was replaced by flexible paper spiders to permit large excursions of the voice coil with small force. A brass tube, 1/4 inch in diameter and with a wall thickness of 0.005 inch, was bonded to the diaphragm and used to transmit force to the vibrating system.

By nature, the driver delivers relatively small forces over a relatively large distance, while the force required to drive the vibrating system is relatively large but acts over a small distance. This amounts to a mechanical impedance mismatch and is corrected by a simple lever, seen in figure 5. This lever is analogous to an impedance-matching transformer and has a ratio of 10:1.

Force-Measuring Device. --The force delivered by the driver, multiplied by 10 by the lever, is applied to the vibrating system through a small load cell. This is a steel tube 3/16 inch in diameter with a wall thickness of 0.0033 inch. Two resistance wire strain gages are bonded to the tube wall, and the force transmitted through the tube is thereby measurable in terms of strain in the tube. The load-strain characteristic of the load cell was determined by using a precision testing machine. The load cell, with the connecting shielded wire, is visible in figure 5.

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The strain gages are connected in series with a stable 500-ohm resistor and a 25-volt battery. An amplifier with a constant gain of 1,770 is connected through a 0.1 microfarad capacitor across the two strain gages, and thus amplifies the alternating component of the voltage across the strain gage. The amplified voltage is applied to the vertical input of the oscilloscope.

**Displacement-Measuring Device.** Amplitude of vibration, or displacement of one half of the vibrating system relative to the other, is measured with a capacitance displacement gage.

A flat copper surface about 1 inch square, oriented to be perpendicular to the direction of motion of the system as it vibrates, is fastened rigidly through a nonconducting support to one half of the vibrating system. Parallel to this surface is a circular surface, 3/4 inch in diameter, mounted on the spindle of a micrometer head that is rigidly fastened to the other half of the vibrating system. These details can be seen in figure 4.

The insulated surface and the circular surface on the micrometer spindle form a capacitor whose capacitance is a function of the relative position of the two halves of the vibrating system. This capacitor is in the frequency-determining circuit of a nominally 5-megacycle oscillator. The oscillator output, amplified to a constant level by a limiting amplifier, drives a frequency discriminator. The output of the discriminator thus indicates the instantaneous relative position of the two halves of the vibrating system. This voltage is applied to the horizontal input of the oscilloscope.

The use of a micrometer head to hold one plate of the displacement-measuring capacitor provides a convenient means of adjusting the spacing of the capacitor, and also permits direct calibration of the displacement gage in thousandths of an inch.

**Operation of Apparatus.** A block diagram of the apparatus is shown in figure 6.

The voltage signals for indicating force and displacement are applied to the vertical and horizontal inputs, respectively, of the oscilloscope as described above. When the driving force is periodic at the natural frequency of the vibrating system, the system is in resonance and the displacement is in quadrature with the driving force.

If both force and displacement are sinusoidal and the system is in resonance, the oscilloscope will trace an ellipse. The vertical size of the ellipse indicates the driving force, the horizontal size indicates the displacement, and the area of the ellipse indicates the energy absorbed by the specimen during each cycle of vibration. The elliptical trace is photographed after prescribed
accumulations of stress cycles, the resonant frequency and total number of cycles being recorded for each photograph. From these data can be derived the desired information, as will be described later. A typical series of photographs, showing the progressive increase in energy per cycle, is shown in figure 7. The displacement, as indicated by the horizontal size of the ellipse, is shown as increasing in successive parts of the figure in order to keep the stress amplitude constant as the resonant frequency of the system decreases. This also will be explained in later sections of the report.

Calibration. --Individual elements of the apparatus that require calibration are the force-measuring device, displacement-measuring device, oscilloscope, and the equipment for making measurements from the photographs of the oscilloscope screen.

Calibration of the force gage involves determination of the load-strain function of the thin-wall tube used for the load cell, determination of the voltage gain of the amplifier used, calibration of the voltmeter used for measuring the voltage of the strain gage supply battery, and checking the standard 500-ohm resistor used in the strain gage circuit.

The thin-wall tube, with the resistance strain gages attached, was set up for compression loading in a precision testing machine. Loads, measured to 0.01 pound, were applied in 0.5-pound steps to a maximum of 10 pounds, and corresponding strains in the tube read from the attached strain gages with a standard bridge-type strain indicator. The load-strain function was found to be 586 pounds per 1 percent strain.

The amplifier was calibrated for sinusoidal signals by comparing the input and output amplitude, using an oscilloscope as a voltmeter. The oscilloscope was equipped with precision attenuators that were compared with a precision voltage calibrator. The voltage gain of the amplifier was found to be 1,770 for output signals with an amplitude of 5 volts or less, and essentially constant for a frequency range of 30 to 100,000 cycles per second. The phase shift in the amplifier also was checked and found to be negligible over a frequency range of 100 to 2,500 cycles per second.

The voltmeter was checked by comparing it with a vacuum tube voltmeter that was standardized with a standard cell. The voltmeter was found to be accurate to the limits of readability.

The 500-ohm resistor was compared with the resistance of the strain gages, the latter being accurate to about 0.25 percent. When carrying the prescribed current in use in the strain gage circuit, the 500-ohm resistor was found to be within 0.5 percent of 500 ohms.

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The displacement-measuring device, or displacement gage, is calibrated before each test. As it tends to drift, the displacement gage is calibrated periodically during tests of long duration. Calibration is accomplished by moving the circular surface in 0.001-inch steps, using the micrometer head and recording the corresponding displacement of the oscilloscope spot. The horizontal sensitivity of the oscilloscope is therefore not calibrated separately, but is included in the overall calibration of the displacement gage.

The vertical sensitivity of the oscilloscope is quite stable and rarely needs adjustment. However, it is routinely checked by comparison with a precision voltage calibrator before each run.

Details of design, construction, and calibration of the equipment are recorded in Appendix I.

Experimental Procedures

Design of Experiment

The information to be gained through the experiment is primarily the energy absorbed by the specimen per stress cycle as affected by stress amplitude, number of stress cycles, foil thickness and cell size of the core, direction of loading, and any other controllable variable that may be shown to be important. Also obtained are the dynamic shear modulus of the core and the relation between stress amplitude and number of stress cycles to failure.

Five specimens are made of each core to be tested. Each specimen is run at a different stress amplitude, selected to provide as wide a range as practical in the number of stress cycles to failure.

If there are available static and fatigue strength data on the core being studied, an estimate is made of the stress amplitude that should produce failure in about 1,000,000 cycles. This is the stress amplitude at which the first specimen is tested, and subsequent tests are made at stress amplitudes selected on the basis of the performance of the previous specimens. If fatigue data are not available, the first specimen is tested at a stress amplitude of 25 percent of the static strength.

For a given test on one specimen at one stress amplitude, the energy per cycle is obtained by photographing the oscilloscope trace at intervals that depend primarily on the observed trend in resonant frequency and energy absorption. For all specimens the oscilloscope trace is photographed within the first 5,000 cycles and subsequently as needed. Resonant frequency and total number of cycles are recorded at the time of each photograph.
When the stress amplitude is low and the specimen is enduring a large number of stress cycles, the energy photographs are made at intervals of about 1,000,000 cycles.

**Test Procedure**

With the specimen installed in the apparatus, the stress amplitude selected, and the force and displacement calibrations obtained, a low level of excitation is applied to the driver. The driving frequency is adjusted until the oscilloscope shows an ellipse with axis horizontal. The system is then at resonance, and the resonant frequency is obtained from the electronic counter and timer. Knowing the resonant frequency, the displacement can be calculated that will give the desired stress amplitude. A nomograph has been prepared to obtain this value easily, and its derivation is given in Appendix I.

The counter is then reset to zero and set to read total cycles, and the driving power is quickly brought to a level that gives the desired stress amplitude. The system is maintained at resonance by adjusting the driving frequency and, after running approximately 5,000 cycles, the oscilloscope trace is photographed. Immediately after the photograph is made, the driving power is shut off and the total number of stress cycles read from the counter. The frequency is then determined by setting the counter for a standard gate time of 1 or 10 seconds, and the driving power is switched on for one complete standard period. The number of cycles in the standard period is the frequency (with due regard to decimal place). The small number of cycles occurring outside the standard gate period, and therefore not added to the total, is considered negligible. The counter is then switched back to unlimited gate time (gate remains open), and the driving power is applied and maintained at the correct level and frequency until the next photograph of the oscilloscope trace is to be made.

As the specimen approaches failure, as indicated by a decreasing resonant frequency and larger energy absorption, the photographs are made more frequently. When resonance can no longer be maintained, the specimen is considered to be failed.

**Reduction of Data**

All measured quantities except frequency and total number of stress cycles are derived from the oscilloscope photographs.

The total excursions of the trace in both the horizontal and vertical directions are measured with dividers, using the centimeter grid on the photographs as a scale. For convenience, the unused channel of the oscilloscope is displayed.
giving a line whose length is easily measured and equal to the horizontal excursion of the elliptical trace. These measurements, using the oscilloscope sensitivities and appropriate calibrations, yield the total range of the driving force and displacement of the system at the time of the photograph.

If the traces were perfect ellipses, their areas could be calculated easily from their horizontal and vertical extremes. Slight imperfections in the driving linkage and perhaps other disturbances, however, distort the shape somewhat, making this method less accurate than other methods of area measurement.

Therefore, area measurements are made with a planimeter designed to read to 0.01 square inch. The planimeter was standardized by measuring the area of 48 squares on a photograph of the oscilloscope grid. Since the object-to-image size ratio of the camera is fixed, this standardization is constant. The areas, measured in square inches, are reduced to energy per cycle in inch-pounds through the use of the planimeter standardization, oscilloscope sensitivities, and force and displacement calibrations.

Results

Test Material

Exploratory data were obtained on three core materials. These were type 3003-H19 aluminum foil honeycombs with 3/8-inch cells corrugated from 0.002-, 0.003-, and 0.004-inch foils. The honeycombs were bonded with a thermosetting vinyl-phenolic adhesive. Five specimens were made from each core, cut for loading the core perpendicular to the ribbons.

These cores had been studied in detail previously, and important mechanical properties determined. Repeated-stress fatigue data at a stress repetition rate of 15 cycles per second had been obtained for one of these cores.4

Vibration Tests

On the basis of previously obtained fatigue data for the one core, it was estimated that a stress amplitude of 40 percent of static strength would fail these


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cores after about 5,000,000 stress cycles. This stress amplitude was therefore selected for the first specimen in a series.

It was found, however, that failure occurred in these cores much sooner than would be predicted from low-frequency fatigue data and, at stress amplitudes of about 40 percent of static strength, failure occurred after about 100,000 cycles. The plan was therefore revised and initial runs were made at a stress amplitude of about 25 percent of static strength.

Test Results

The data obtained are plotted in figures 8 through 19 and summarized in table 1.

Figures 8, 9, and 10 show the energy absorbed by a core material as a function of stress amplitude. The data plotted here were obtained before any specimen had endured more than 10,000 stress cycles, and therefore the data apply to essentially undamaged material. These data are plotted on logarithmic coordinates because the energy per cycle is expected to vary directly as a power of the stress, and thus should give a linear plot on such paper. The data show that the energy per cycle is only approximately proportional to a power of the stress and that the power ranges from 2.3 to 9.0 for a single curve.

Figures 11, 12, and 13 show the energy per cycle absorbed by the three cores at different stress amplitudes as a function of the number of stress cycles experienced by the material. For each curve the stress amplitude was held constant within experimental limits. The general result is that the energy per cycle at one stress amplitude does not change until the specimen approaches failure, although some data show definite deviations from this.

Figures 14, 15, and 16 are standard S-N curves showing the relation between stress amplitude and number of stress cycles to failure. Curves for all core materials are similar; for comparison, the S-N curve for the 0.004-inch foil honeycomb, obtained with a repeated stress at a rate of 15 cycles per second, is shown as a dashed line. The present data were obtained with reversed stress at frequencies near 200 cycles per second.

Figures 17, 18, and 19 show the effect of cyclic loading on the shear modulus of the core materials. There is a slight decrease in modulus throughout the test with a catastrophic decrease as the specimen approaches failure. Also indicated by the data is the tendency for the shear modulus to decrease as the stress increases. This effect was very definitely observed during the tests, as the resonant frequency was higher at low stress than at high stress for every specimen.

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Table 1 lists some physical properties of the three cores used in the exploratory tests. Of particular interest are the comparison of static and dynamic shear moduli, comparison of high- and low-frequency fatigue limits, and the energy per cycle absorbed by each core at a stress equal to its fatigue limit.

The dynamic shear modulus values average 10 to 20 percent higher than the static, and the fatigue limits under high-frequency reversed stress are lower than those under low-frequency repeated stress.

Core densities were obtained previously\(^3\), and recorded here for convenience.

Approximate weight of adhesive as a percent of total core weight was obtained by weighing a large number of samples of cores with foils 0.002, 0.003, 0.004, and 0.005 inch thick. The average weight of core of each foil was obtained. It was assumed that the weight of foil was proportional to the nominal foil thickness, and that the absolute amount of adhesive in a given core volume was approximately constant for all foil thicknesses. Thus, for each core, an equation of this form could be written:

\[
 t_k W_a = W_t
\]

where \( t \) is foil thickness, \( W_a \) is weight of adhesive, \( W_t \) is total weight, and \( k \) is the proportionality factor between foil thickness and weight.

Using the method of least squares, a value of \( k \) was found, assuming that \( W_a \) is constant. With this value of \( k \), individual values of \( W_a \) could be calculated for each core. This method is only approximate, as it depends upon the values of \( W_a \) being nearly constant and on the actual foil thickness being proportional to the nominal thickness. However, the results of this calculation were checked by estimating the total amount of aluminum in the core samples from cell size and foil thickness. The weight of foil estimated by the two methods differed by no more than 5 percent. The high percentage of adhesive in the core of 0.002-inch foil is therefore probably significant, and further it agrees with values previously found by chemical analysis.\(^3\)

The values of fatigue limit were estimated from the S-N relations plotted in figures 14, 15, and 16; the energy absorbed per cycle at the fatigue limit was obtained by extrapolating the energy-stress amplitude curves (figs. 8, 9, and 10) to the fatigue limit.
Experimental Errors

The experimental errors intrinsic in the data given here are of three general types. First are the normal errors in calibration and reading of the various instruments. The second type are errors due to imperfect control of the test conditions, such as stress amplitude and resonant frequency. Finally, there are also errors due to unknown external modification of the measured quantities, of which energy loss through the supports of the vibrating system is probably the most important example.

Accuracy of Measurements.--The estimated accuracy of the system for measuring driving force is ±3 percent. Errors are almost entirely due to limitations in measurement of the oscilloscope trace and uncertainty in the gain of the amplifier used. Errors in the calibration of the load cell and voltmeter are less than 1 percent.

Accuracy of the displacement gage is limited by measurements from the oscilloscope screen and by readings of the micrometer for calibration. Also, the gage is assumed to be linear when actually this is not precisely true. Deviation from linearity is about ±3 percent over the usual range; so, combined with the normal errors in reading the oscilloscope records, the overall error is probably within ±6 percent.

Measurements of the area of the oscilloscope photographs are made with a planimeter that can be read to ±0.005 square inch. The areas to be measured are usually no smaller than 0.20 square inch, so usual area measurements are, by this limitation, in error by no more than ±2.5 percent. With additional errors in tracing the photograph with the planimeter, total area error would be in the order of ±5 percent.

Errors in the resonant frequency are, in all cases, less than 0.25 percent, as measured by the counter. The overall errors in measurement thus fall roughly within the limits of ±13 percent, but as addition of all maximum errors in one direction is unlikely, the estimated error in measurements and calibration is roughly ±6 percent.

Accuracy of Control.--Control of the test conditions involves keeping the vibrating system precisely at resonance and maintaining the amplitude of vibration at the correct value to provide the desired stress amplitude. These conditions are controlled manually so no standard estimate of their accuracy can be made. When the specimen is far from failure, and thus is changing slowly, the test conditions can be maintained quite accurately. Resonant frequency can be controlled and measured to an error of less than 1 percent, and the correct
amplitude, derived from the resonant frequency and a nomograph, can be set to an accuracy limited primarily by ability to read from the oscilloscope screen. This is about ±3 percent for average conditions.

When the specimen is changing rapidly, as when near failure, maintaining the correct stress amplitude is more difficult; in general it cannot be controlled to ±3 percent. The average resonant frequency over any 1- or 10-second interval can still be obtained to 1 percent accuracy and the required amplitude computed, but as the amplitude is being adjusted, the resonant frequency changes. The error introduced by this difficulty depends, of course, on how fast the frequency is changing, the largest errors being in the final readings of a test where high accuracy is not vital.

Errors Due to Spurious Effects. --In measuring energy absorption in a test specimen, the assumption is made that all of the measured energy injected into the vibrating system is dissipated in the specimen. While this is obviously a false assumption, if it is approximately true the test results will be useful.

The most important potential source of error in the category of spurious effects is the loss of energy from the vibrating system through its support. The apparatus used in this investigation was designed to minimize such loss, first by supporting the vibrating system at a plane of dynamic symmetry so the support ideally would tend to remain stationary in space as the system vibrates. Further, since this ideal is never realized, the supporting structure itself was hung by flexible arrangements to permit the support to move with minimum energy loss.

Early forms of the support structure involved two large steel yokes that were hung on knife edges fastened to small steel rods. In several cases, unreasonable increases in measured energy absorption of the system were traced to resonant vibration of these yokes or the steel rods. These spurious vibrations could be suppressed by application of clamps to the vibrating part, with apparent correction of the fault, but the correction was possibly imperfect.

Removal of all secondary resonant systems from the support structure would probably be impossible, but it is feasible to reduce the structure to a form where all spurious resonances would be at frequencies far from the range covered in vibration tests. This apparently has been achieved by the present design, where the support structure consists of two steel bars that hang from steel wires. The bars are resonant in the longitudinal direction at about 3,000 cycles per second, and the steel wires are resonant at about 250 cycles per second. In no test using this arrangement has there been any indication of energy loss due to resonant vibration of the support structure.

In one test using the improved support system, spurious energy losses occurred through resonant vibration of the driver supports. Subsequently, a
plate weighing about 5 pounds was mounted rigidly on the driver, which re-
duced its tendency to vibrate in resonance with the main vibrating system.

Errors in the energy absorption data due to spurious energy absorption have
probably been reduced to a minimum. Remaining is an error due to an un-
known but reasonably constant fraction of the input energy being dissipated
through the support of the vibrating system. This error is certainly not zero,
as vibration of the supporting structure can be felt. However, a large part of
the total energy dissipated in the support may come directly from the driver,
as it is fastened rigidly to the support. As such, this energy is not measured
and therefore introduces no error.

Overall Accuracy.--The general conclusion of these considerations on
accuracy is that the overall estimated error of measurement and control is
less than ±10 percent; the additional error from spurious energy loss, while
unknown, is not large enough to make the results valueless. However, it is
likely that some early data reported here were distorted by the spurious
resonances mentioned above and, in particular, some of the peaks and valleys
in the plots of energy versus number of stress cycles may be due to these
effects.

Energy-Stress Relations

If a harmonically vibrating system is described by simple theory, where the
damping is assumed to be either viscous (damping force proportional to and
in phase with velocity) or hysteretic (damping force proportional to displace-
ment and in phase with velocity), the energy per cycle absorbed in the system
will increase as the square of the amplitude of vibration. In practice, stress
and displacement amplitudes are nearly proportional, so the energy is then
proportional to the square of the stress amplitude. Other forms of damping
result in other functional relationships between energy and stress amplitude.
Lazan has found that, for a wide variety of metals subject to low frequency
bending fatigue, the energy per cycle increases approximately as the 2.4
power of the stress amplitude when the latter is below the fatigue limit. The
exponent is generally larger for stress amplitudes above the fatigue limit and
depends upon the number of stress cycles. Interesting exceptions to this are

5--Bishop, R. E. D. The Treatment of Damping Forces in Vibration Theory.

6--Lazan, B. J. Fatigue Failure Under Resonant Vibration Conditions.
two types of steel that showed, at stress amplitudes near but below the fatigue limit, an abrupt increase in the exponent to values of 10 or more after about 1,000 cycles.

At stress amplitudes at or below the fatigue limit, the aluminum foil honeycombs tested here show energy absorption approximately proportional to the square of the stress amplitude and therefore exhibit behavior described approximately by simple theory. At stress amplitudes above the fatigue limit, the energy stress relations, as shown in figures 8, 9, and 10, become steeper. If the energy is treated as proportional to a power of the stress, maximum value of the exponent ranges from 4 to 9 for stress amplitudes near 50 percent of static strength. The departure of the exponent from 2 indicates that damping mechanisms not explained by elementary theory are involved, but the nature of these complex mechanisms of energy absorption is not fully known.

The energy per cycle absorbed by the three cores at the fatigue limit, shown in table 1, is somewhat erratic. It might be expected that the core of 0.002-inch foil would absorb the least energy because it has the least volume, increasing to the largest energy loss per cycle occurring in the core of 0.004-inch foil. Such a trend is not observed. Furthermore, if the volume of stressed material is assumed to be proportional to the foil thickness, the energy per cycle per volume of stressed material is nearly 1.5 times as high for the 0.002-inch foil as it is for the 0.004-inch foil, and twice that for the 0.003-inch foil. An interesting possibility is that this is due to the unusually large percentage of adhesive in the core of 0.002-inch foil, the adhesive in this case serving as an effective energy absorber.

**Energy-Cycles Relation**

As the number of stress cycles increased, the energy per cycle absorbed by most specimens at a given stress amplitude remained practically constant until the specimen approached failure, and then increased rapidly. Data contrary to this general trend are from early tests and may have been distorted by spurious energy losses as discussed in a previous section.

The constant energy absorption before failure indicates that no major macroscopic changes occurred in the specimen as a result of the repeated stressing during this period. Nevertheless it is clear that some modification of the material occurs under cyclic stressing because after a more or less definite number of stress cycles at a certain stress amplitude the specimen fails. These subtle effects of cyclic stressing do not, however, seem to alter the internal friction in the material until they have actually caused localized failures. The true nature of these subtle modifications in a material subject to cyclic stress remains unknown.
The catastrophic increase in energy as the specimen approaches failure is probably due primarily to the energy absorbed in producing and extending cracks in the cell walls of the honeycomb and, in addition, energy is absorbed by the slipping of material on opposite sides of a fracture.

**S-N Relations**

The relation of stress amplitude to number of stress cycles to failure (S-N relations) of the core materials tested shows typical fatigue behavior. Of particular interest is the comparison of the fatigue strength of the material tested here with similar material tested under different conditions. The dashed curves in figures 14, 15, and 16 show fatigue results on honeycomb of 0.004-inch foil of 3003-H19 aluminum tested at 15 stress cycles per second and with the average stress at about 61 percent of the stress amplitude. Present tests were made at about 200 cycles per second and with completely reversed stress (average stress zero). The material tested at 15 cycles per second endured in the order of 100 times as many stress cycles as did the same material at the same stress amplitude tested at 200 cycles per second. The fatigue limit, expressed as a fraction of static strength, was nearly twice as high when the material was tested at the lower frequency. These differences are probably due to the difference in frequency of stress cycling and to the different method of applying and cycling the stress. Further, there may be differences due to different criteria of failure, but it is doubtful that these are important.

**Shear Modulus-Stress Cycles Relation**

The information revealed by the relation between shear modulus and number of stress cycles is primarily that the shear modulus is practically unaffected by repeated stress reversals until the honeycomb approaches failure. The decrease in shear modulus is probably due to local failures in the cell walls. A crack in the cell wall virtually eliminates the shear stiffness of that wall from the total stiffness. The decrease in shear modulus coincides with the increase in energy absorbed by the specimen, which strengthens the hypothesis that the increase in energy absorbed by the specimen near the end of its useful life is due primarily to the production of shear failures and the slipping of material on opposite sides of a failure.

While the shear modulus is practically constant with repeated stress reversals before failure is approached, a very slight but definite decrease in modulus was observed in tests running to 100,000 cycles and more.
Finally, there is a small but definite decrease in effective shear modulus of the honeycombs as the stress is increased, as indicated by a lower resonant frequency with higher stress. There is a reduction in resonant frequency due to damping, but it can be shown that for viscous damping and for the average conditions of these tests -- where the energy loss per cycle is in the order of 1 percent of the stored energy per cycle -- the effect of damping on the resonant frequency is less than 1 part per million. The observed reductions in resonant frequency are in the order of 1 percent or more, depending on the stress amplitude. This may be due to a nonlinearity in the stress-strain relation of the honeycomb, the shear modulus becoming smaller at higher strains.

Summary

The apparatus described can measure successfully the internal friction in core materials used for sandwich construction, and stress amplitudes can be achieved that are high enough to fail many core materials in less than a minute. Functions relating internal friction to stress amplitude and number of stress cycles, and relations between stress amplitude and number of cycles to failure, can be determined.

Exploratory data were obtained on three cores of 3003-H19 aluminum foil. The cores were honeycombs with 3/8-inch cells of 0.002-, 0.003-, and 0.004-inch foils. The cores were subjected to reversed cyclic shear strain in the WT plane.

When subjected to stress amplitudes below the fatigue limit, the cores showed energy absorption per cycle that was nearly proportional to the square of the stress and independent of the number of stress cycles. Above the fatigue limit, the energy absorption increased more rapidly with stress. Considering the energy to be proportional to a power of the stress, the exponent ranged from 2 to 9 for stress amplitudes up to 50 percent of the static strength of the material. At stress amplitudes where the energy showed tendencies toward rapid increase, either with increasing stress amplitude or number of stress cycles, the expected endurance of the specimen was no more than a few thousand cycles.

The fatigue strengths of the three cores under the conditions of test were similar, the fatigue limits ranging from 20 to 23 percent of static strength. The fatigue strength at 100,000 cycles averaged about 40 percent of static strength. One of these cores tested under stress repeated in one direction at 15 cycles per second showed a fatigue strength at 100,000 cycles of about 70 percent of static strength.

Report No. 1866
APPENDIX I

Details of Equipment Design, Operation, and Calibration

Driver and Force Gage

The transducer for obtaining the driving force from electrical power is a modified high-power loudspeaker. Because the original mounting of the voice coil was a stiff diaphragm, it was removed and the mount replaced by flexible paper spiders. A tube 1/4 inch in diameter and made of 0.005-inch-thick brass shim stock was fixed to the diaphragm, using an epoxy adhesive and aluminum gussets. The paper spiders were fastened to this tube and the end of the tube was provided with bearings for delivering force to the driving lever. The driver is of a type rated for 50 watts continuous power over the audible range of frequencies, and 100 watts at over 300 cycles per second. The vibration tests are usually at around 200 cycles per second, and peak power input to the driver is about 70 watts. At this power the driver becomes hot, but continues to give reasonable service.

The mechanical impedance-matching lever consists of two thin-wall box beams for rigidity and low mass. The beams are 1/4 by 1/8 inch in cross section and are 2 inches between pivot centers. Pivots are 60 degree cone bearings that can be adjusted for free movement with virtually no looseness. One end of the lever pivots around a point fixed to one-half of the vibrating system, and 3/16 inch from this pivot is the pivot attachment to the load cell. The other end of the load cell is fastened through a cone pivot to the other half of the vibrating system, and the other end of the lever is connected through cone bearings to the driver.

The load cell is a steel tube of 3/16 inch inside diameter with 0.0033-inch walls. Two 120.5-ohm resistance strain gages are fixed to opposite sides of the tube. As described in the section on calibration, the load-strain characteristic of the tube was determined to be 586 pounds for 1 percent strain.

The strain gages are connected in series with a 500-ohm resistor and a battery of voltage $E$. The voltage across the strain gages is designated as $e$, and the relationship between $e$ and other quantities is given by:

$$e = E \left( \frac{R}{R + R_s} \right)$$  \hspace{1cm} (1)
where $R$ is the resistance of the strain gages (241 ohms), and $R_s$ is the 500-ohm standard resistor. The effect of $R$ on $e$ is given by the derivative of equation (1):

$$de = \frac{ER_s}{(R + R_s)^2} \, dR$$

Using the values for $R$ and $R_s$:

$$de = 0.000911 \, E \, dR$$

The strain gages have a gage factor of 1.97, which means that the relative change in resistance of the gage is 1.97 times the strain. That is:

$$dR = 1.97 \, R \, dS$$

where $dS$ is the increment of strain. But from the load calibration of the thin-wall tube, the force is related to strain by:

$$df = 58,600 \, dS$$

Combining equations 3, 4, and 5 gives:

$$df = \frac{135,600}{E} \, de$$

where $f$ is force in pounds through load cell, $E$ is applied voltage, and $de$ is the voltage change across the strain gages as a result of $df$.

Because the measured voltage is amplified by a factor of 1,770 by the amplifier mentioned previously, we have:

$$df = \frac{135,600}{1,770} \, \frac{de_m}{E} = \frac{76.6}{E} \, de_m$$

where $de_m$ is measured voltage changes, $E$ is applied battery voltage, and $df$ is the measured increment in driving force.

**Stress Calculations and Nomograph**

The stress on the specimen is the direct result of the acceleration of the mass of the vibrating system. If we assume the motion of the system is...
sinusoidal, which it very nearly is, we have:

\[ s = s_0 \sin \omega t \]

where \( s \) is the instantaneous displacement of the mass from rest, \( s_0 \) is the amplitude of vibration, and \( \omega \) is \( 2\pi \) times the frequency of vibration. Differentiating twice gives:

\[ s'' = -s_0 \omega^2 \sin \omega t \]

and the maximum acceleration is therefore:

\[ s'''_{\text{max}} = -s_0 \omega^2 \]

(8)

The maximum stress, or stress amplitude, then is:

\[ S = -\frac{m s_0 \omega^2}{A} \]

where \( S \) is stress amplitude, \( m \) is the mass of one-half of the vibrating system, and \( A \) is the area of the specimens.

It is convenient to measure the total displacement of one half of the vibrating system in relation to the other. The system is in effect two independent systems that vibrate exactly in opposite phase, and the measured total displacement is the sum of the total movement of each of the two half-systems. Therefore, the measured displacement is four times the amplitude of vibration of each half of the system.

Therefore,

\[ S = -\frac{m s_t \omega^2}{A} \]

(9)

where \( s_t \) is the measured total displacement, and the minus sign indicates that the stress is directed opposite to the displacement.

For this apparatus, using a system of units where length is in inches, force is in pounds, and time is in seconds, \( m \) equals \( \frac{142.2}{386.4} \) mass units, \( A \) equals 12 square inches, and \( s_t \) is in inches.

Therefore,

\[ S = \frac{(142.2)(2\pi)^2 t^2 s_t}{(12)(386.4)(4)} \]
\[ S = 0.30 f^2 s_t \quad (10) \]

where \( S \) is stress in pounds per square inch.

This is the stress amplitude to which the specimen is subjected. It depends upon both frequency and displacement and must be held constant as the frequency changes; therefore, a nomograph is used for rapid determination of the correct displacement at which to operate in order to maintain the stress at the correct level.

The form of the relation between stress, displacement, and frequency produces a family of straight-line plots on logarithmic coordinates. Such a plot is a satisfactory chart from which to read displacements necessary to obtain a given stress amplitude at a given frequency. For this application, displacement is plotted against frequency for constant stress amplitudes, giving a straight line for each value of stress amplitudes. Corresponding displacement and frequency values can thus be read by following along the line corresponding to the desired stress.

The nomograph may be read somewhat faster. On a vertical logarithmic scale, displacements and frequencies are plotted parallel a few inches apart. The ranges of displacement and frequency plots are those expected in these tests. Between the vertical plots of frequency and displacement is constructed a plot of stress, arranged so that the values of stress, frequency, and displacement at the intersections of any straight line through the three plots satisfy equation (10). The stress plot is also logarithmic but, in general, on a different scale than the frequency and displacement plots.

**Shear Modulus Calculation**

The displacement measured is the total of one half of the vibrating system relative to the other and thus is four times the amplitude of each half of the system. As each half of the complete honeycomb specimen is a sample 1/2 inch thick, the shear strain in each specimen part is given by:

\[ d = \frac{s_t}{2} \]

and the shear modulus, \( G \), is therefore:

\[ G = 0.60 f^2 \]
Table 1.--Physical properties of 3003-H19 aluminum-foil honeycombs with 3/8-inch cells

<table>
<thead>
<tr>
<th>Foil thickness (nominal)</th>
<th>Core density</th>
<th>Approximate weight of adhesive</th>
<th>Average shear modulus</th>
<th>Static-shear strength</th>
<th>Approximate fatigue limit</th>
<th>Energy per cycle at fatigue limit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In.</td>
<td>Lb. per cu. ft.</td>
<td>Percent of total</td>
<td>1,000 P.s.i.</td>
<td>1,000 P.s.i.</td>
<td>P.s.i.</td>
<td>Percent of static strength</td>
</tr>
<tr>
<td>0.002</td>
<td>3.1</td>
<td>25.0</td>
<td>12.8</td>
<td>15.0</td>
<td>79</td>
<td>23</td>
</tr>
<tr>
<td>.003</td>
<td>4.0</td>
<td>9.0</td>
<td>18.6</td>
<td>21.1</td>
<td>118</td>
<td>22</td>
</tr>
<tr>
<td>.004</td>
<td>5.2</td>
<td>9.0</td>
<td>25.6</td>
<td>26.2</td>
<td>172</td>
<td>20</td>
</tr>
</tbody>
</table>

1 All tests were made with stress perpendicular to core ribbon direction.
Figure 1. --Edge view of vibration specimen, showing typical failures in cell walls.
Figure 2.--Cross section of vibrating system. Parts labeled A comprise one vibrating mass, or half of the system, and those labeled B, the other.
Figure 3.--General view of apparatus for measuring internal friction in metal foil honeycombs.

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Figure 5. --Driving mechanism and load cell. D is the driver, L the lever, and C is the load cell.
Figure 6.--Block diagram of apparatus.
Figure 7. -- Typical energy records, showing increase in area (energy absorbed by specimen) as the specimen approaches failure.
Figure 8. -- Energy-stress amplitude curve for honeycomb of 0.002-inch foil of 3003-H19 aluminum, stressed perpendicular to the core-ribbon direction.
Figure 9.--Energy-stress amplitude curve for honeycomb of 0.003-inch foil of 3003-H19 aluminum, stressed perpendicular to the core-ribbon direction.
Figure 10. -- Energy-stress amplitude curve for honeycomb of 0.004-inch foil of 3003-H19 aluminum, stressed perpendicular to the core-ribbon direction.
Figure 11.--Energy-number of cycles curves at various stress amplitudes for honeycomb of 0.002-inch foil of 2001-T49 aluminum, stressed perpendicular to the core-ribbone direction.
Figure 12. - Energy-number of cycles curves at various stress amplitudes for honeycomb of 0.003-inch foil of 3003-H14 aluminum, stressed perpendicular to the core-ribbon direction.
Figure 11. --Energy-number of cycles curves at various stress amplitudes for honeycomb of 0.004-inch foil of 2003-H19 aluminum, stressed perpendicular to the core-ribbon direction.
Figure 14. - S-N curve for honeycomb of 0.002-inch foil.
Figure 15.—S-N curve for honeycomb of 0.003-inch foil.
Figure 16. -- S-N curve for honeycomb of 0.004-inch foil.
Figure 17. Shear modulus-number of cycles curves at various stress amplitudes for honeycomb of 0.302-inch foil. The dashed line is the static shear modulus.
Figure 18.—Shear modulus-number of cycles curves at various stress amplitudes for honeycomb of 2.023-inch foil. The dashed line is the static shear modulus. The curve at 50 pounds per square inch indicated by ○ was obtained from a specimen previously subjected to $10^7$ cycles at 28 pounds per square inch. The increased fatigue life resulting from pre-fatiguing at low stress has been observed generally, and is termed "coating."
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