# MODULUS OF ELASTICITY OF WOOD DETERMINED BY DYNAMIC METHODS 

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In Cooperation with the University of Wisconain

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## Sumnary

Tests at the Forest Products Iaboratory prior to World War II showed that the modulus of elasticity of wood could be measured by use of sound waves. This report presents data from more recent studies made to compare results obtained by sonic (dynamic) methods with *hose from static tests. Samples were tested in compression parallel to grain and in static bending according to American Society for Testing Materials Standard DI43-50; they were also given both longitudinal and transverse sonic tests.

The effect of shear deformation in both the static and sonic transverse tests reduced observed values of modulus of elasticity by about 10 percent. After a correction was made for this effect the modulus of elasticity as determined by sonic methods was about 11 percent higher than that obtained by static methods. The reason for the higher values is not wholly clear, but they may be caused by the difference in the amounts of deformation of the wood brought about by the two methods; the deformations with sonic methods are minute compared to those with static methods.

## Introduction

The determination of modulus of elastic放y by dyamic methods is not new. The elastic properties of gases were determined many years ago by vibration methods that measured the velocity of sound in air and in gases. There are many methods of obtaining dynamic moduli. These include slow flexural and torsional vibrations, usually below the

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frequency of sound, obtained by mountin $n_{i}$ the specimens as flexural or torsional pendulums; the vibrating-reed type of test using small specimens mounted for electrical excitation; longitudinal and transverse vibrations of bars and beams in resonance induced by electrical means; and the measurement of the velocity of sound transmission through the specimen using short pulses of ultrasonic waves.

Ide, ${ }^{2}$ working at Harvard University's Cruft Laboratory, determined the dynamic modulus for several solids. This work was done particularly in connection with the travel of sound through terrestrial materials in geological studies. The agreement between his values and those found in handbooks and other accepted tables is very close.

In 1943 icBurney 3 showed fairly close agreement between the modulus of elasticity determined by sonic methods ( $1,616,000$ pounds per square inch) and that determined by the standard static compressive test parallel to grain (1,679,000 pounds per square inch) from tests on a series of Sitka spruce specimens. The specimens had an average specific gravity of 0.382 and were tested at an average moisture content of 12.5 percent.

In the sonic tests, the specimen was placed in a horizontal position and "supported at its two nodal points, 0.227 of the length from each end, by means of knife edges." The accurate location of the true nodal points in such a test is difficult because of the uncertainty of the proportion of the deflection that results from shear.
Hearmon ${ }^{4}$ in 1948 reported studies using both the transverse and longitudinal methods on wood. His studies included slow flexural and torsional vibration as well as electrically induced flexural and longitudinal vibrations in the sonic range. This work indicates dynamic values from 10 to 12 percent above static values. He considers the fact that the dynamic tests are adiabatic and the static tests are isothermal. However, over a wide range of frequencies Hearmon reports that the theoretical difference between the isothermal and the adiabatic conditions amounts to less than 0.1 percent for the longitudinal modulus of elasticity. The difference is less than 1 percent for moduli of elasticity perpendicular to grain and is zero for modulus of rigidity.
$\underline{Z}_{\text {Ide, }}$ John Li. Some Dynamic Eethods for Determination of Young's inodulus, Review of Scientific Instruments 6(10):296-298. Oct. 1935.
3McBurney, R. S. Evaluation of Fiodulus of Elasticity of Sitka Spruce by the Sonic liethod. U. S. Forest Products Laboratory. Advance Progress Report on Forest Products Laboratory Wood, Plywood, and Glue Research and Development Program Prepared for the Ninth Conference of the ANC Executive Technical Subcomittee on ?equirements for :Nood Aircraft and Wood Structures in Aircraft. Madison, Wisconsin June 14-18, 1943. Madison, U. S. Forest Service, FPL, June 1, 1943. pp 32-34.
4 Hearmon, R. F. S. Elasticity of Wood and Plywood. Great Britain Department of Science and Industrial Research, Forest Products Research Laboratory Special Report 7. London. 1948

In 1948 Kuenzi ${ }^{5}$ used Iow-frequency vibration methods for determining elastic moduli of thin sheets of veneer, plywood, and aluminum, using the torsion pendulum and the vibrating cantilever. He concluded that the modulus of rigidity may be determined fairly accurately by means of the torsion pendulum test, provided the length to width ratio is greater than 10 and the specimen is thin. He also concluded that the modulus of elasticity of plywood beams can be determined by using vibrating cantilevers without end weights.
In 1949 Leslie, $\frac{6}{}$ in reporting on the pulse vibration technique using ultrasonic waves for the testing of concrete, mestioned preliminary results on tests of sound and rotted wood.
Horio and others, 7, 8, 2 (1951) reported studies made with forced vibration of reeds as a method of determining viscoelastic properties of several htgh polymers and of pulp and paper in the audio-frequency range.

McSwain and Kitazawa 10 (1951) report results of tests on 25 specimens of assorted herdwoods and softwoods with average values 8 percent higher for dymamic than for static bending moduli on 2-x 2-x 30-inch test beams. Their results show also that the longitudinal resonant modulus exceeds the transverse bending modulus by an average of 16 percent, the transverse modulus being uncorrected for deflection due to shear.

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Kuenzi, E. W. Low Frequency Vibration Methods for Determining Elastic Moduli. Proj. L246-1AJ2. Unpublished U. S. Forest Products Laboratory Report, April 29, 1948.
G-Leslie on Fulse Techniques Applied to Dynamic Testing。 American Society for Testing Naterials Special Tech. Pub. No. 101, June 1949.

Horic, M., Onogi, $\mathrm{S}_{.}$, Nakayama, C., and K. Yamamoto, Viscoelastic Properties of Several High Polymers. Journal of Applied Physics 22(7):966-970, July 1951.
${ }^{8}$
-Horio, Mo, and Onogi, S. Dynamic Measurements of Fhysical Properties of Pulp and Paper by Audiofrequency Sound. Journal of Applied Physics 22(7):971-977, July 1951.
2
-Horio, Mo, and Onogi, S. Forced Vibration of Reed as a Method of Determining Viscoelasticity. Journal of Applied Physics 22(7): 977-981, Juㅍy 1951.
10
NcSwain, George, and Kitazawa, George. Application of Ultrasonic and Sonic Vibrations for Improvement and Testing of Wood. Teco Proj. No. B-33, Aug. 1951.

Report No, 1977

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In most cases reported and in the resuits presented in this paper, Young's moduli determined by dynamic methods exceed those determined statically by approximately 8 to 15 percent. No completely satisfactory explanation of this difference has been made, but it has been found to exist over a very wide range of frequencies. Ide has suggested that the "dynamic modulus is obtained for minute alternating stresses, far below the elastic limit, which do not give rise to complex 'creep' effects or elastic hysteresis."

## Scope of Study

The experiments discussed in this report cover the determination of longitudinal and transverse moduli of elasticity by resonant vibration in the audio-frequency range on 40 specimens of secoud-growth Douglas-fir that had been prepared for standard strength tests, and on a few specimens of other species. The specimens were caused to vibrate either transversely or longitudinaily, and the resonant or fundamental frequencies were determined. These data were obtained in connection with other wort and are submitted as further evidence of the possible usefulness of this type of test procedure. They indicate the accuracy that can be expected when compared to the conventional static test methods.

Equipment

The initial work on transverse vibration was done with apparatus arranged as shown in figure 1 . The signais produced by the audiofrequency oscillator were delivered to the audio-frequency amplifier, where they were amplified and delivered to the recording head at a meximum level of 15 watts. The recording head moved in a lateral direction and imparted to the specimen a lateral vibration. The recording head was placed near one end of the specimen, which was mounted on two pieces of sponge rubber placed approximately at the quarter points of the specimen. A variable-reluctance cartridge commonly used for the pickup in a record player was used as a vibration detector. The output of the cartridge was fed to a one-tube amplifier, and then to an oscilloscope and a vacuum-tube voltmeter in parallel. The amplitude of oscillation of the specimen, as indicated by the variable-reluctancem cartridge piclup, was observed on both the oscilloscope and the vacuumtubevolimeter. As the audio-frequency oscillator was varied through a considerable range of frequencies, the amplitude observed on the oscillos cope and yoltmeter reached its maximum when the oscillator frequency became equal to the natural frequency of vibration of the specimen. This is called its fundamental frequency of vibration. Nost specimens showed a pronounced fundamental-frequency response. There were some, however, in which it was rather difficult to select the true fundamental frequency.

Apparatus used in the study of longitudinal wave motion of wood specimens differed from the transverse set-up only in the means of exciting the vibrations. Figure 2 shows how the output of the audio amplifier was fed through a condenser to the aluminum plate that acted as the high-voltage electrode of the exciting condenser, the other side of which was held at ground potential. A direct-current voltage was also fed to the aluminum plate through a 2 -megohm resistor. In this manner both the variable audio frequency and the direct-current voltage were applied to the exciting condenser. The wood specimens were stood on end on the high voltage electrode. The excitingecondenser arrangement was taken from drawings in Ide's article. 11 The forces established between the two plates of the condenser when the voltage was applied were sufficient to set the specimen into vibration in a longitudinal direction. The pickup was placed against the side of the specimen near its top. The output of the pickup was fed to a preamplifier, and the axplified wave was observed on the oscilloscope screen and its amplitude on the vacuumtube voltmeter. The dielectric used was glass cloth impregnated with resin.

## Determination of Modulus of Elasticity by Transverse Vibration

The value of modulus of elasticity of a wood specimen when vibrating transversely was calculated by use of the following equation: $=$

$$
\begin{equation*}
E_{T R}=0.002443 W \frac{8^{3}}{a b^{3}} f^{2} \tag{1}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathbb{B}_{\mathrm{TR}}=\begin{array}{c}
\text { modulus of elasticity, when vibrating transversely, } \\
\text { either in a tangential or radial direction }
\end{array} \\
& \mathrm{W}=\text { weight of specimen in pounds } \\
& \ell=\text { length in inches } \\
& \mathrm{A}=\text { width in inches } \\
& \mathrm{b}=\text { depth in inches (in the plane of vibration) } \\
& \mathrm{f}=\text { frequency in cycles per second }
\end{aligned}
$$

Ide, Review of Scientific Instruments.
12 Den Hartog, Jacob Peter. Mechanical Vibrations. Ed. 3. $478 \mathrm{pp}$. , illus. New York. 1947.

Report No. 1977

The effects of shear deformations and the rotation of the beam are not included in the formula.

Table 1 gives results on 40 specimens representative of approximately 250 specimens of second-growth Douglas-fir tested by transverse vibration in the tangential direction at a moisture content of 12 percent, and their Young's moduli determined by formula (1). These same specimens were also statically tested as centrally loaded beams on a 28-inch span. Further, when the specimens were originally cut from the timbers, additional specimens 2 by 2 by 8 inches were cut and end-matched to them. These addjtional specimens were tested in end compression by using a Lamb's roller extensometer of G-inch gage length to obtain a value of the modulus. The modulii of elasticity resulting from the three types of test -- the transverse vibration test, the static bending test, and the static compression test (computed by means of formula (1) for the vibration test and by the usual engineering formulas for the static tests) -- are given in table 1. Neither the data from the static bending tests nor that from the vibration tests were corrected for shear deformations.

Values of modulus of elasticity at resonant frequencies were determined also for 6 specimens of various species including southern yellow pine, Sitka spruce, and oak, all at a moisture content of 9 percent. The modulus of elasticity was obtained also by statically testing these same specimens as centrally loaded beams and using the usual engineering formula. The span used in each case was 2 inches shorter than the length of the specimen. Table 2 gives the test data and the ratios between the dynamic and static moduli. Neither the dyamic values nor the static values were corrected for shear.

Determination of Modulus of

## Elasticity by Longitudinal Vibration

When the wood specimen was vibrating longitưdinally, modulus of elasticity was calculated by using the following equation:13

$$
\begin{equation*}
E_{L}=0.010352 \mathrm{~W} \frac{\ell}{A} f^{2} \tag{2}
\end{equation*}
$$

## 13 <br> Ibid.

$$
\begin{aligned}
& E_{L}=\text { modulus of elasticity when vibrating longitudinally } \\
& W=\text { weight of specimen in pounds } \\
& \ell=\text { length in inches } \\
& A=\text { area in cross section in square inches } \\
& \vec{X}=\text { frequency in cycles per second }
\end{aligned}
$$

Equation 2 comes directly from the expression for the velocity of sound
in an elastic medium, $V=\sqrt{\frac{E}{P}}$. Velocity is equal to frequency times wave length; the weve length, in the case of longitudinal vibration, is equal to twice the length of the specimen. Another equation was set up that included Poisson's ratio. It was found that a correction for Poisson's ratio would increase the results, as determined by equation (2), by about I percent. This correction factor was not used, however, because correct values for Poisson's ratio are difficult to obtain. A differential equation was also set up by Hermen W. Narch of the Forest Products Labcratory to include a frictional force within the wood specimen. The frictional force was assumed to be proportional to the velocity of motion of a particle within the specimen. It was found that the correction for internal friction was negligible.

Thirteen of the specimens of second-growth Douglas-fir that were used to obtain the data given in table I were vibrated in the longitudinal direction, and their resonant frequency for longitudinal vibration was determined. The modulus of elasticity, calculated by the use of formula (2), was determined for each specimen by means of the resonant frequency in longitudinal vibration and by two static tests for compression parallel to the grain. One of these static tests was made on the same specimens that were vibrated longitudinally. These specimens were 30 inches long and the strains were measured by means of a Lamb's roller extensometer having a 6 -inch gage length mounted at the center of the specimen. The other static test was made on end-matched 8 -inch specimens as previously described. The moisture content of the specimens was 12 percent at the time of both static and dyamic tests. The results are given in table 3.

## Discussion

In a beam bent transversely, deflections result from shear deformations within the beam as well as from deformations due to longitudinal stress. The ordinary engineering formulas relating deflection and modulus of
elasticity do not take into account the effects of shear strains in producing deflection. The value of modulus of elasticity as calculated from static bending tests is, therefore, not the true modulus.

The proportion of the deflection of a beam that is due to shear is dependent upon the relation between modulus of rigidity and modulus of elasticity and upon the ratio of span to beam depth. For a span-depth ratio of 14, such as was used in the static bending tests reported here, the correction to values of modulus of elasticity for shear deformations are about 10 to 11 percent. The average difference of moduli of elasticity from static tests in compression and bending of 11.5 percent as shown in table $I$ is in general conformity to this relationship. As was indicated earlier, the formula for computing modulus of elasticity from dynamic transverse tests neglects the offects of shear deformations. The computed sonic modulus values for transverse specimens, like those for static bending, are accordingly in error by about 10 percent.

A number of references quoted in the introduction report values of dynamic modulus somewhat higher than the static modulus, the difference usually being on the order of 10 percent. This is in agreement with the average difference of 10 percent shown in table 1 .

The average values of dynamic modulus from bending tests and of static modulus from compression tests as show in the tables are approximately the same, numerically. This apparent similarity of values should not be taken to indicate that moduli determined by the two methods are the same. Shear deformations have not been taken into account in computing the tabulated values of dynamic modulus. True values of modulus, computed by taking into account shear deformations in the transverse vibration tests will be about 10 percent higher than those tabulated, and will no longer be similar to the static modulus from the static compression test.

The trend quoted above for Douglas-fir is confirmed by limited tests of three additional species (table 2). The data indicate a small difference between ratios of dynamic to static modulus depending upon whether the deflection was in the radial or the tangential direction. It is doubtful that this difference is significant. It is based on only 6 tests and results from the difference, in the 2 directions, of the static data; the dynamic data are essentially the same in the 2 directions.

The Sitke spruce specimen tested shows dynamic moduli less than static moduli. The other data, however, show the dynamic moduli to average about 10 percent higher than the static moduli.

Vibration in the longitudinal direction gave results (table 3) similar to those resulting from fibration in the transverse direction. Based on the static tests on the 30 -inch specimen, the dynamic moduli average about 9 percent higher than the static moduli. One specimen showed the reverse of the general trend. If its values be excluded, the dynamic modulus is about 10 percent higher than the static modulus.

Table 4 permits direct comparison of specimens that have been tested both transversely and longitudinally, The similarity of individaal ratios of dymamic to static moduli in both bending and compression, with a few exceptions, is evident. On the average, the two ratios are very nearly the same,

## Method of Inducing Vibrations

The excitation of vibrations by means of the vibrating electrode of the condenser offers greater possibilities than does exciting by a recorder head because of the larger energy output of the condenser electrode. Although, in the experiments reported here, the electrode was used only to excite longitudinal vibrations, it can be used also to excite transverse vibrations. The large energy output obtainable from a condenser exciter suggests the possibility of testing relatively large timbers. Although the data presented were determined on specimens not over 2 by 2 by 30 inches in size, pieces up to 6 by 6 inches by 6 feet, have been tested in other experiments. The limiting dimensions of timbers that might be tested by this method have not been determined.

## Application of Dynamic Testing

Figures 3 and 4 present, in graphical form, data from tables 1 and 3 respectively. Both figures indicate a general trend with specific gravity. In both, however, two points deviate considerably from the trend. In each figure, the two points represent specimens 41-N6-a and 41-W6-a. Thus, both the transverse and the longitudinal vibration test methods have picked out the two specimens in which the modulus values were especially low for their specific gravity. Thus, for applications where modulus of elasticity is important, both methods offer an opportunity, when used in connection with specific gravity determinations, of excluding low-line pieces.

## Conclusions

Based on a limited number of static and dynamic tests, the following conclusions may be drawn:
(1) Young's modulus for wood may be determined from tests in which the specimens are caused to vibrate, either longitudinally or transversely. The data required in the evaluation are the resonant frequency of vibration and the weight and dimensions of the specimen.
(2) The value of Young's modulus determined from vibration tests will be about 10 percent higher than that determined from a comparable static test.
(3) The value of Young's modulus determined from a longitudinal. vibration test will be somewhat higher than the apparent value determined from a transverse vibration test, but when the effect of shear deformation in the transverse test is taken into account, the values are of the same order.
Table 1. - Results of individual tests on Douclas-fir to evaluate the modulus of elasticity by trans-
verse vibration and by static mothods (moisture content about 12 percent)

Table $l_{0}-$ Results of individual tests on Douglas-fir to evaluate the modulus of elasticity by trans-


[^0]Table 1.-Results of individual tests on Douglas-fir to evaluate the modulus of elasticity by trans-
verse vibration and by static methods (moisture content about 12 percent) (continued)

| $\begin{gathered} \text { Speci- : } \\ \text { men } \\ \text { number: } \end{gathered}$ | Dimensions | :Weight :Specific:Resonant : <br> : : :frequency: |  |  | Modulus of elasticity |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | : | : gravity |  | : Static : | $\therefore$ Trans- | Static |  | ansve |  | at |
|  |  | : | : | : | :bending: | : verse | :compres |  | ibration |  | pressive |
|  |  | : | : | : | : test : | :vibration | : sive |  | test | : | test |
|  |  |  | : | ! | : : | : test | : test | : | Static | : | Static |
|  |  | : | : | : | : | : ${ }^{\text {c }}$. |  | : | bending |  | bending |
|  |  | : | : | : | : : | : |  |  | test | : | test |
| : | Inches | : Pounds | : | C.p.s | $: 1,000$ : | : 1,000 | $: 1,000$ |  | Ratio | : | Ratio |
| : |  | : | : | : | : p.s.i. | : p.s.i. | : p.s.i. |  |  | : |  |
| : |  | : | : | $:$ | : | : ${ }^{\text {a }}$ | : |  |  | : |  |
| 21-W5-1: | $0.996 \times 2.004 \times 30.06$ | $: 1.2114$ | : 0.559 | 467 | : 1,975 | 2;187 | : 2,107 |  | 1,11 | : | 1.07 |
| 26-N5-b: | $1.507 \times 2.001 \times 30.04$ | : 7.8535 | : .567 | 502 | : 2,262 | 2,562 | : 2,492 |  | 1.13 | : | 1.10 |
| 26-W4-b: | $2.003 \times 2.002 \times 30.08$ | :2.4455 | . 563 | 501 | : 2,268 | : 2,524 | : 2,568 |  | 1.11 | : | 1.13 |
| 41-E4-S: | $2.004 \times 2.000 \times 30.04$ | :2.0818 | :-. 479 | 405 | : 1,266 | : 1,411 | :1,506 | : | 1.11 | : | 1.19 |
| 41-S4-s: | $2.004 \times 2.001 \times 30.05$ | :1.9923 | . 458 | 415 | : I,272 | : 1,417 | : 1,370 | : | 1.11 | : | 1.08 |
| : |  | : | : 6 | : | : |  | : |  |  |  |  |
| 41-N6-a: | $2.000 \times 2,000 \times 30,06$ | :2.3657 | . 621 | 396 | : 1,373 | 1,539 | : 1,083 |  | 1.12 | : | . 79 |
| 41-E7-a: | $2.001 \times 1.999 \times 30.05$ | :2.7045 | . 623 | : 428 | : 1,844 | : 2,055 | : 1,732 |  | 1.11 | : | . 94 |
| 41-N3mr: | $2.003 \times 2.003 \times 30.04$ | :2.0517 | . 471 | : 426 | : 1,349 | : 1,532 | : 1,493 |  | 1.14 |  | 1.11 |
| 41-57-a: | $1.504 \times 2.007 \times 30.05$ | :1.8841 | . 575 | : 446 | : 1,868 | : 2,043 | : 1,996 | : | 1.09 | : | 1.07 |
| 41-W6-a: | $2.011 \times 2.007 \times 30.06$ | :2.5641 | : .583 | 398 | : 1,488 | 1,658 | : 1,474 | : | 1.11 | : | . 99 |
| : |  | : | : | : | - | : | : ${ }^{1}$ |  |  | : |  |
| Average: |  |  |  | :........ | . 2,046 | : 2,259 | : 2,294 | : | 1.099 | : | 1.115 |
| $\underline{-}$ |  | : | : | : | : | . |  | : |  | : |  |

[^1]Table 2.--Results of individual tests on several species to evaluate the modulus of elasticity


- Based on weight and volume at time of test, approximate values.
${ }^{2}$ Vibrated in the tangential direction.
Rept. No. 1977
Rept. No. 1977
Table 3.--Results of individual tests on Douglas-fir to evaluate modulus of elasticity by longi-

Fikration and by atatic mathois (moisture content about 12 percent)

${ }^{\text {I Based on weight and volume at time of test. }}$
Report Ho. 1977
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Figure 1.--Diagram of apparatus nsed in the transverse-vibration testa.

Figure 2. -- Diagram of apparatus used in Iongitudinal-vibration teste.


Figure 3. --Relation between specific gravity and modulus of elaaticity determined by tranaverae
z K 96183 I vibration.


Figure 4, --Relation between epecific gravity and modulus of elasticity determined by longitudinal vibration.


[^0]:    Rept. No. 1977

[^1]:    ${ }^{-}$Based on weight and volume at time of test. 2This 3 This value is b in formula (1).

