AN ABSTRACT OF THE THESIS OF
Fey Frentress Egan for the degree of Honors Baccalaureate of Science in Forest Engineering presented on August 12th 2011. Title: Exploring the Differences between Lagrangian and Eulerian Reference Frames for Stream Temperature Data.

Abstract Approved: ____

Marv Pyles

Stream temperature research relies on reference frames in which to project data. An important decision in the project design is which frame of reference to use. This aspect of research planning is not always given the consideration of thought that would lead to the best decision. In this thesis, two frames of references (Eulerian and Lagrangian) are compared in order to garner a better understanding of whether the choice of one reference frame over the other leads to a difference in the interpretation of the stream temperature data.

The Eulerian and Lagrangian reference frames were compared through a series of graphs in which the data was projected on both reference frames. In two dimensions of space and temperature, the interpretation between the Eulerian and Lagrangian reference frames are fairly similar. When the third dimension of time was graphed alongside temperature and space, the interpretations of the data differed between reference frames. All three dimensions should be considered when choosing a reference frame for research projects. The appropriate reference frame to use depends on the statements that need to be made about the data at the conclusion of the study.

Key Words: Reference frame, Eulerian, Lagrangian, Hydrology, stream temperature

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Exploring the Differences between Lagrangian and Eulerian Reference Frames for Stream Temperature Data

By
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I understand that my project will become part of the permanent collection of Oregon State University, University Honors College. My signature below authorizes release of my project to any reader upon request.

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Fey Frentress Egan, Author

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Contents

List of Figures and Tables

Figures

Tables

Exploring the Differences between Lagrangian and Eulerian Reference Frames for Stream Temperature Data

Introduction

The interpretation of stream temperature data gives rise to environmental management plans and silvicultural prescriptions. These interpretations can also affect whether the stream is classified as having thermal pollution under the Clean Water Act. Thus, it is important that the interpretations of the stream temperature data are appropriate. This study examines how the use of two different frames of references to interpret stream temperature can affect the conclusions about the stream.

In general, "the Eulerian reference frame is based on measuring the flux of objects through or within a spatially bounded area" (Doyle and Ensign 2009). Hydrologists use an Eulerian frame of reference to interpret stream temperature data because it is simple and straightforward. For example, one study on response of stream temperature to partial-harvest logging in riparian buffers of boreal mixedwood forest watersheds used the weekly average of maximum daily temperatures to interpret the impacts of logging. The author supported this decision by writing, "We focused on daily maximum temperatures as the metric of interest because other studies … have shown daily or weekly maximum temperatures to be more sensitive to logging impacts than daily average temperatures" (Kreutzweiser, et al., 2009). The authors cite two other studies (Brownlee, et al., 1988, Wilkerson, et al., 2006) as examples of this type of measurement reporting. Even the alternative metric considered by the authors, daily average temperatures, used an Eulerian reference frame.

In this study, the Eulerian coordinate system represents the data as an observation of stream temperature measured from a specified location through time. The water moves past the measuring point which is fixed in the frame of reference. The Lagrangian frame of reference is an alternative that can be used for stream temperature. However, "it is perhaps fair to say that the Lagrangian approach is used only in a relatively small fraction of research papers … and, except for the idea of the material derivative, hardly mentioned in general textbooks on fluid mechanics" (Yeung, 2002). A Lagrangian interpretation of the data represents a parcel of water as it travels downstream through time. The Lagrangian frame of reference "is based on an observer following the trajectories of fluid particles" (Yeung, 2002) The water is stationary in the Lagrangian frame of reference, while the stream channel's physical features move in the negative direction within the coordinate system.

The Eulerian and Lagrangian methods are not purely separate from one another in this study. Here, a Lagrangian coordinate system tracks the temperatures of a *control volume* of water through time and position downstream, instead of a molecule. Within the control volume, water molecules are lost and gained through natural processes, such as evaporation and advection of water into and out of the ground (Yearsley, 2009), and are influenced by solar and long wave radiation. By creating this spatially dependent object of study, the model becomes one of an Eulerian object projected onto a Lagrangian reference frame. A model of what is occurring to the water in terms of temperature fluxes, position, and time in the Lagrangian reference frame represents the moving frame of reference in terms of inputs and outputs in a confined Eulerian space. The opposite would be true if stream temperature were to be modeled in an Eulerian reference frame. From one set location to another, the velocity, along with all the inputs and outputs of the system, would have to be known. The system would again be defined as a certain volume of water. The Eulerian reference frame depends on the Lagrangian movement of the water to inform the model about how much the inputs and outputs affect stream temperature. The difference in the two frames of reference is the perspective in which the information is viewed. The data required is the same for both. Either frame of reference can be an acceptable representation of the truth. However, many studies use empirical data alone to make causal statements about water temperature. Eulerian data often excludes information that is crucial to understanding whether one observation is caused by or related to another. Though researchers usually attempt to justify why the information can be left out, no study has examined what differences in interpretation of the data can occur.

A major drawback to the Eulerian reference frame is that with missing information, the bigger picture cannot be inferred. For example, two temperature probes are placed in a stream 50 feet apart and each is set to record temperatures every 15 minutes. Assume that at 2:45 PM the upstream probe records a maximum daily temperature of 75°F and at 3:00 PM, the downstream probe records a maximum daily temperature of 77°F. The interpretation can *not* be made that a common volume of water ran through the stream and warmed during that 50 feet of stream. Since the velocity of the stream is not known, the inference that the 75 degree temperature upstream warmed to 77 degrees downstream is not valid.

One might be able to draw such a conclusion with a Lagrangian frame of reference. Since, in theory, the Lagrangian frame of reference will track a specific parcel of water as it moves down the stream, the recorded temperatures could give an accurate account of the temperatures as they rise and fall through time and space. This could lead to a better understanding of the factors that influenced the stream temperature. A drawback to the Lagrangian frame of reference is that it requires water velocity data, which can be difficult or time consuming to collect.   Another drawback is that that in the Lagrangian frame of reference, a volume of water is tracked rather than individual water molecules. Because water molecules continually mix within a stream, the volume of water being traced continually changes. Accounting for these changes would be nearly impossible because of their complex nature.

Thus, choosing a reference frame for a research project is an important preparatory step. "The reference frame adopted in a study constrains not only how data are collected, but also what data are collected and, most important, what questions are asked" (Doyle and Ensign 2009). Depending on the reference frame, different questions can arise from the study, and different conclusions can be drawn. Lagrangian reference frames lead to questions that pertain to individual molecules or volumes, such as "How does the temperature of a particular volume of water change through a stretch of stream?" The Eulerian reference frame elicits questions such as, "How high are the maximum stream temperatures along the stream?"   Keeping the goal in mind, to compare these two reference frames for significant differences in interpretation of the data, the following background, data, and methods lay the foundation for a discussion of the objective statement. The objective of this project is to investigate whether or not the frame of reference influences the interpretation of stream temperature data.

Background

Reference Frames

Euler

Leonhard Euler was a Swiss mathematician who lived from 1707 to 1783. His publications on mechanics introduced some of the first concepts on rigid body mechanics and relative motion. While studying motion, Euler recognized the importance of time for motion from one place to the next, stating, “a motion cannot be immediate, but to need time for the body to arrive at some place from some previous place” (Euler 1736). In his publication of 1736 entitled “Mechanica”, he comes to grasp the ideas leading up to his proposal of the most convenient frame of reference to describe motion. He discovers three important clues to motion, as described by Giulio Maltese in *On the Relativity of Motion in Leonhard Euler’s Science*, “1) absolute motion is a mere mathematical concept; 2) consequently, it is pointless to take up work on it, since the same laws of motion rule both absolute and relative motion; 3) this means that one can always transform a problem by changing the frame of reference” (Maltese 2000). Maltese explains that Euler used the “idea of reference frame and of rectangular Cartesian coordinates” to express relative motion. Cartesian coordinates use a reference point, or origin, and coordinates to describe the time, distances, and, in this case, temperature, in relative space. Relative space is a finite space in which the movement of the space itself is ignored. Euler used Earth as an example. The Earth itself moves, but the objects on the Earth can be said to be static until they themselves change location upon the Earth (Euler 1736).

Lagrange

Lagrange, however, dealt with the movement of space. Joseph-Louis Lagrange, an Italian-born mathematician, lived in Italy, Prussia, and France over the span of his life, from 1736 to 1813. Though offered a prestigious post at the Berlin Academy prior to 1766, Lagrange would not accept, stating “It seems to me that Berlin would not be at all suitable for me while M Euler is there” (Vinter 2010). In the mathematics circle, “it is well known that Lagrange only reluctantly gave Euler’s contributions to mechanics the credit they deserved” (Maltese 2000). When Euler stepped down in 1766, Lagrange replaced him as the director of mathematics at the Berlin Academy. While there, he wrote his famous “*Mécanique Analytique*” (Analytical Mechanics) in which he describes a reference frame in which to track a particle in space from the perspective of the particle. His formulation of the ideal reference frame was one in which no particular coordinate system was used, but rather “any convenient variables could be used to describe the system, and these variables, or generalized coordinates, could be any independent variable of the system” such as the “distance along a river channel (Doyle and Ensign 2009).

The Study Areas

Four streams were used in this study; Hinkle creek, Clay creek, West Scheele, and Tribulation. Hinkle and Clay creeks are located in the North and South Forks of Hinkle Creek. These watersheds are located in the foothills of the Cascade Mountains, about 30 miles northeast of Sutherlin, Oregon. Hinkle Creek flows into Calapooya Creek, which then drains into the Umpqua River five miles north of Sutherlin. Mean annual precipitation in the Hinkle watershed ranged from 140 cm to 190 cm. The watersheds are privately owned and actively managed for solid wood products.

West Scheele and Tribulation are located in western Oregon. West Scheele is located in Benton county at approximately 44-35'09'' N and 123-31'44'' W. West Scheele is just over 600 feet in elevation. Tribulation is classified as a small, non-fish bearing stream. In the spring of 2003, most of the overstory vegetation upstream of Tribulation was removed through a harvesting operation (Gauger and Skaugset, 2004).

Data

Four parameters were required to carry out the analysis: stream temperature, time of record for the stream temperature, position along the stream where the temperature was recorded, and the longitudinal velocity of the stream. Longitudinal velocity refers to the modal velocity of a volume of water averaged between locations along the stream. With the longitudinal velocity, the volume of water is tracked rather than the individual water molecules and the complicated mixing of the water molecules within that volume of water is ignored. Vemco Minilog recording loggers were used to record stream temperature and time. These were placed at 25 foot increments along each stretch of stream. Slug tests were used to determine longitudinal velocities for all streams.

Sodium chloride (NaCl) and two Yellow Springs Instrument (YSI) specific conductance probes and dataloggers were used for the slug tests. First, the stream was divided into sections with similar characteristics. Next, a sodium chloride solution was released into the creek, while the specific conductance was measured at the downstream end of each section and recorded with the data logger. Breakthrough curves from these tests were used to generate a longitudinal velocity for each section of the stream (Kilpatrick, 1989). Data for Tribulation were analyzed in 2003 and longitudinal velocities were determined. Hinkle Creek, Clay Creek, and West Scheele data were compiled and longitudinal velocities were analyzed in 2011.

Stream temperature data was often collected for more than one day for each stream. The discharge data was collected in late July or early to mid August, depending on the stream. At summer times of low flow, the discharge of the stream did not vary appreciably.

Methods

Office

It is difficult to collect data on individual molecules of water to satisfy the needs of a Lagrangian frame of reference. Therefore, a volume of water was measured instead. Various methods of floating temperature probes downstream were tried, but they failed each time. When the water flowed around a bend, the probe would catch and tangle in debris. This was not an accurate way to measure a volume of water because when the probe became snagged, the water would pass it by. An alternative was to use existing stream temperature data to trace a volume of water down a stream. Data were collected at fixed positions along the stream, and the temperature and the time it was recorded was collected at each position. The longitudinal velocity of the stream was also known. The data was collected from stationary temperature probes along each stream, nominally 10 meters apart, with temperatures recorded every 5 minutes.

A starting temperature within the data was necessary to begin the data analysis. This starting location was the first data point for the Lagrangian graph of stream temperature verses location along the stream. Along with stream temperature and location, this point represents the third dimension of time and is the time at which the temperature was recorded. For each successive stream location, temperature was calculated based on the amount of time required for the volume of water to move to the next location in the stream. Any arbitrary point can function as a starting location, but the choice of a maximum or minimum daily temperature simplifies this process. The focal point of this analysis is maximum daily temperature. Maximum daily temperature is often used for cause and effect analysis and maximum temperatures fluctuate more dramatically. The exaggerated nature of maximum daily temperatures more clearly illustrates stream temperature patterns along the stream.

The maximum temperatures and their associated time of day were determined for each day and position along the stream for each stream temperature dataset. The Eulerian frame of reference was represented by a graph of these data. A maximum temperature could occur more than one time and place during the day. When this happened, the high temperature recorded earliest along with the earlier time was chosen as the data point for that location along the stream. Data for each stream spanned one to two weeks and one day was chosen for each stream. Two factors were considered when choosing that day: the date with the highest daily temperature and the proximity of that day to when the longitudinal velocity data was collected.

Other Eulerian projections of the data include a ‘snapshot’ and a surface graph. The snapshot is a graph of the temperature at each position along the stream at one particular point in time. The surface graph is a three dimensional plot of all the data, location, time and temperature, for one day. It is an Eulerian representation because the data is spatially bound.

Three Lagrangian graphs were produced for comparison with the Eulerian graphs. The starting point for the first graph was the maximum temperature at the upstream end of the study reach. The starting point for the second graph was the maximum temperature at the downstream end of the study reach. The starting point of the third graph was when the maximum temperature was at the middle study reach. These three different plots were graphed to check whether the starting location mattered for the Lagrangian trace of the data.

The two dimensional line plots for the Lagrangian reference frame were created using all three parameters of temperature, location, and time, as well as the longitudinal velocity of the volume of water. The longitudinal velocity was used to determine the travel time of the volume of water between temperature probes, or thermistors. The travel time was added to the time at the starting point. This new time is the time at which the volume of water arrived at the next thermistor downstream. The database was searched for the closest match to this new time. An exact match usually did not exist in the database because the dataloggers were programmed to record every five minutes. This means that the calculated time and the next closest time in the database could be off by two and a half minutes at the most. The closest time in the database and the temperature for that time were recorded for each thermistor location. To check for error in the database search, the difference between the closest time in the database and the time calculated from the longitudinal velocity was determined. If the difference was less than two and a half minutes, then human errors were unlikely to exist.

Field—Slug Tests

The longitudinal velocity of the water in the study streams was determined using a tracer dilution technique called a slug test. In the slug test, a tracer solution was used to track the longitudinal velocity of individual water particles from one point to another. Sodium chloride (NaCl) was used as a tracer and specific conductance (SC) was used as the parameter to measure the tracer. First, the SC of the stream was measured. Next, the tracer solution was made by mixing NaCl with stream water. SC was measured with probes that were connected to data loggers.

The dataloggers were programmed to record a value of SC every five seconds. A fixed and known volume of the tracer was discharged into the stream at a known location. Two SC dataloggers recorded SC versus time. Figure 1 shows a representative example of SC versus time for a slug test.

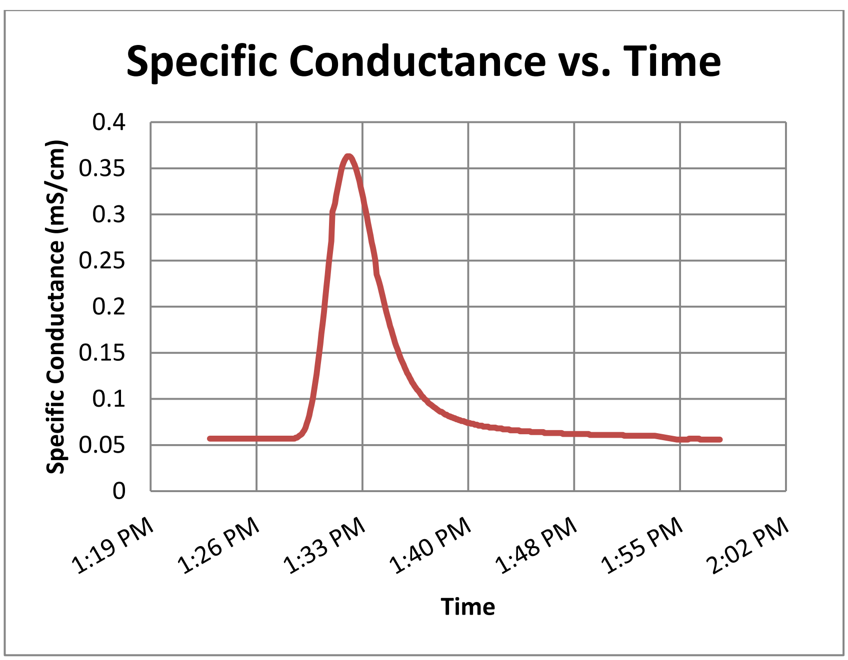

Figure 1 A typical graphical result of a slug test with SC versus time for the tracer solution NaCl

The highest value of specific conductance is the modal value of time for the tracer molecules took to pass from the injection point to the SC probe. The longitudinal velocity of the water between the injection point and either SC probe is calculated by dividing the distance between the points by the time it took for the mode of the water molecules to reach the downstream SC probe. In order to calculate the longitudinal velocity between two dataloggers, the length between them is known, but the time the water molecules took needed to be calculated. The equation used for this longitudinal velocity calculation is below (see appendix for derivation):

$$t_x = \frac{(x_2 - x_1)^2}{\left[\frac{x_2{}^2}{t_2} - \frac{x_1{}^2}{t_1}\right]} \quad \text{(Equation 1)}$$

Where:

t_x is the time it takes the mode of the water molecules to pass from datalogger1 to datalogger2

t_1 is the time it takes the mode of the water molecules to pass from the starting point to datalogger1

t_2 is the time it takes the mode of the water molecules to pass from the starting point to datalogger2

x_1 is the distance between the starting point and datalogger 1

x_2 is the distance between the starting point and datalogger 2

The longitudinal velocity for the stretch of the stream between datalogger1 and datalogger2 (V_x) would then be:

$$V_x = \frac{x_{1\to 2}}{t_x} \quad \text{(Equation 2)}$$

Where:

$x_{1\to 2}$ is the distance between datalogger1 and datalogger2

Results and Discussion

The results of the comparison between the two frames of reference for the stream temperature for the four study streams are illustrated in a series of graphs. These graphs include two dimensional line plots that show stream temperature versus position in the stream and three dimensional graphs that combine stream temperature, position, and time. The two dimensional line graphs that show temperature versus thermistor location for the four streams are shown in Figures 2-4. The "Eulerian Maximums" are the maximum daily temperature for each thermistor location. The three other lines trace the temperatures of a volume of water along the stream and was initiated by a daily maximum temperature at one of the three locations. Those locations are the beginning of the study reach (0'), the middle of the study reach (400' or 500'), or the end of the study reach (800' or 1000'). From this initial point of maximum daily temperature and location, the longitudinal velocity of the stream was used to track the volume of water through the rest of the study reach. Temperatures were reported every 25 feet at the thermistor locations. The latter three lines represent the Lagrangian reference frame.

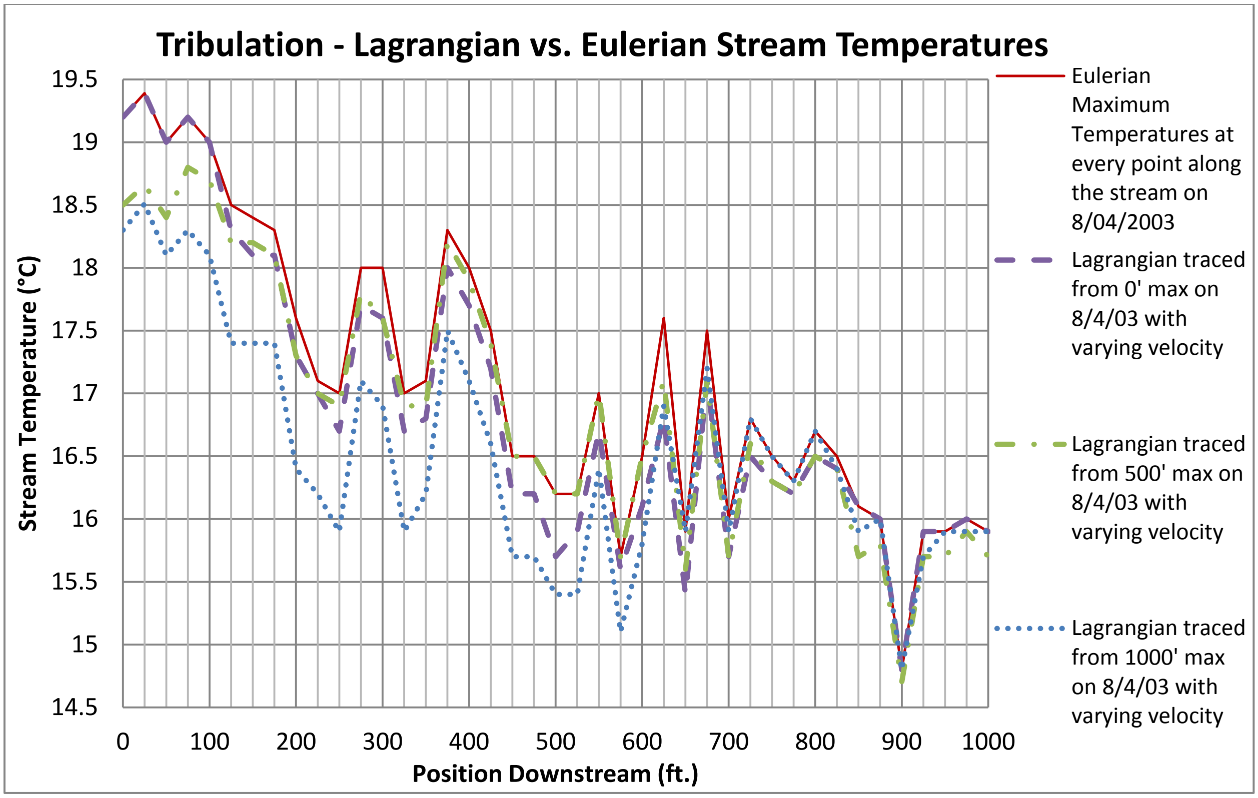

Figure 2 Three line plots in a Lagrangian frame of reference and one line plot of maximum daily temperatures in an Eulerian frame of reference for Tribulation on 8/4/2003

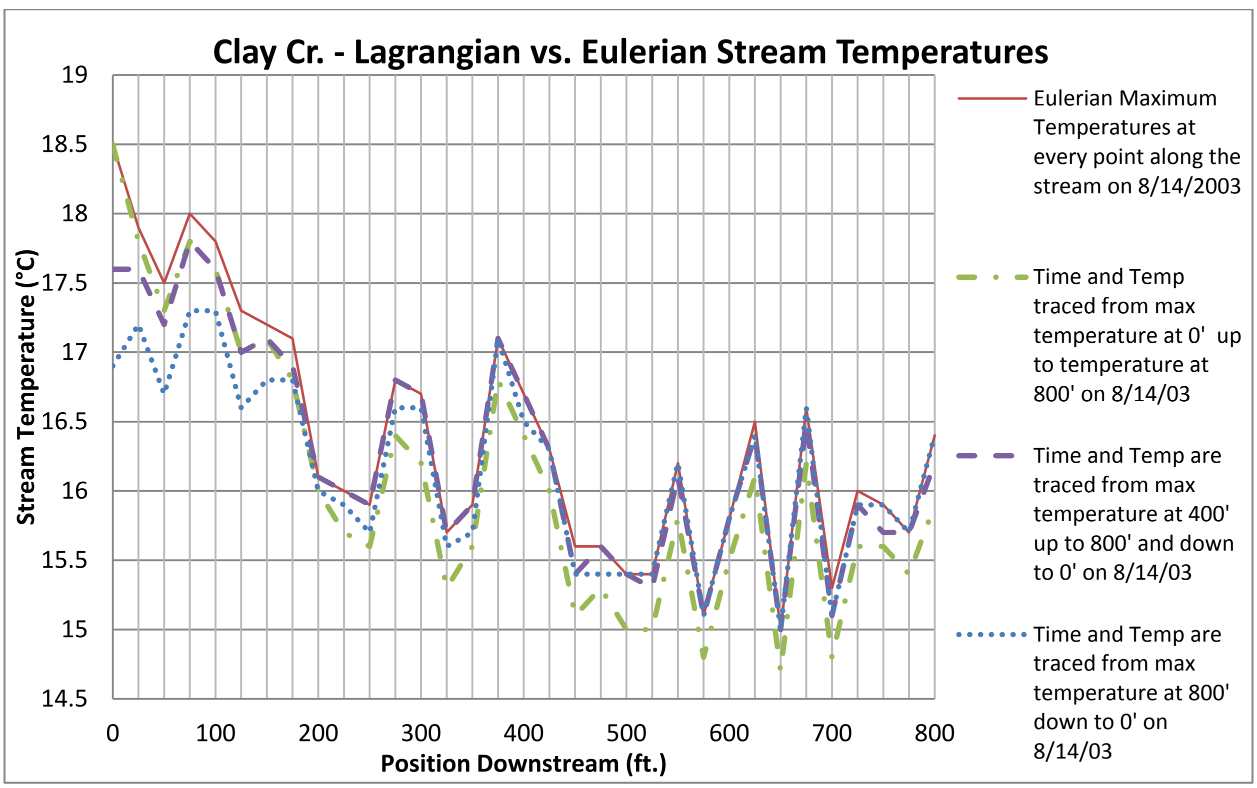

Figure 3 Three line plots in a Lagrangian frame of reference and one line plot of maximum daily temperatures in an Eulerian frame of reference for Clay Creek on 8/14/2003

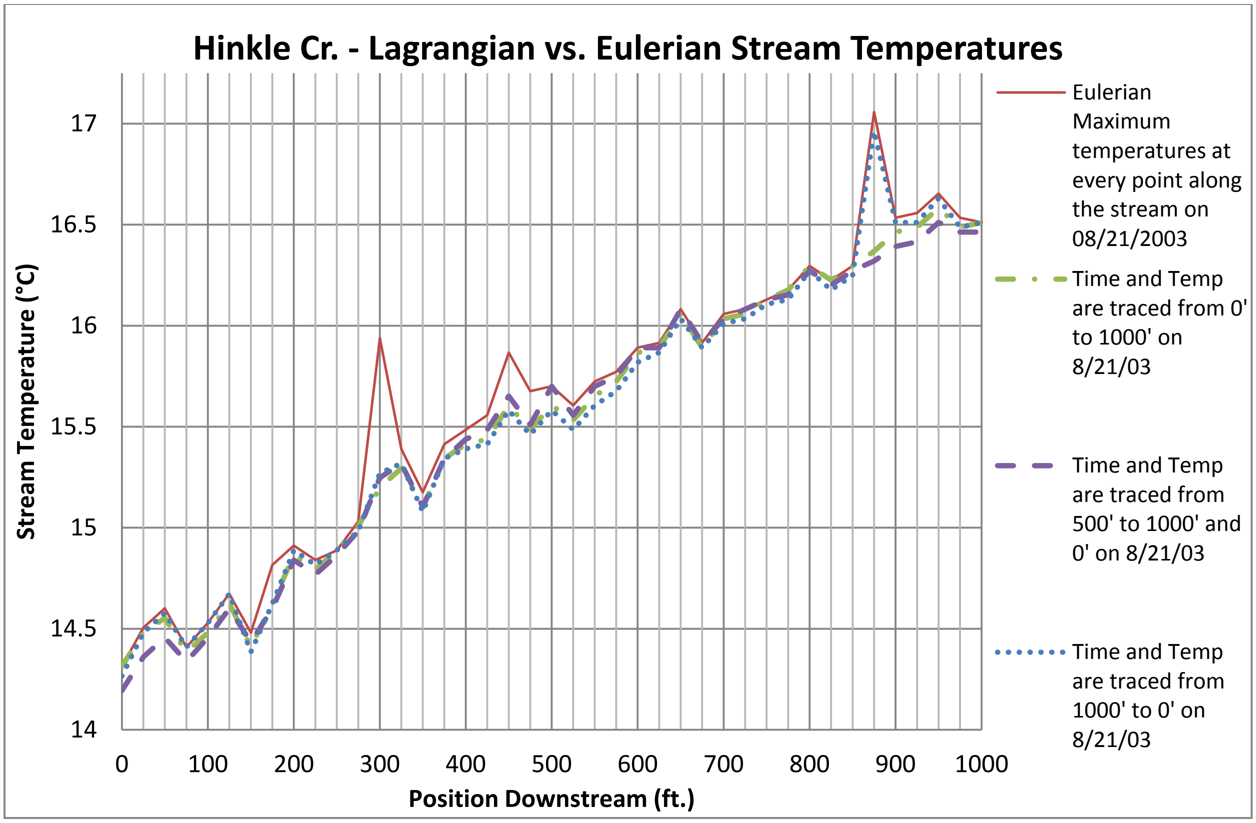

Figure 4 Three line plots in a Lagrangian frame of reference and one line plot of maximum daily temperatures in an Eulerian frame of reference for Hinkle Creek on 8/21/2003

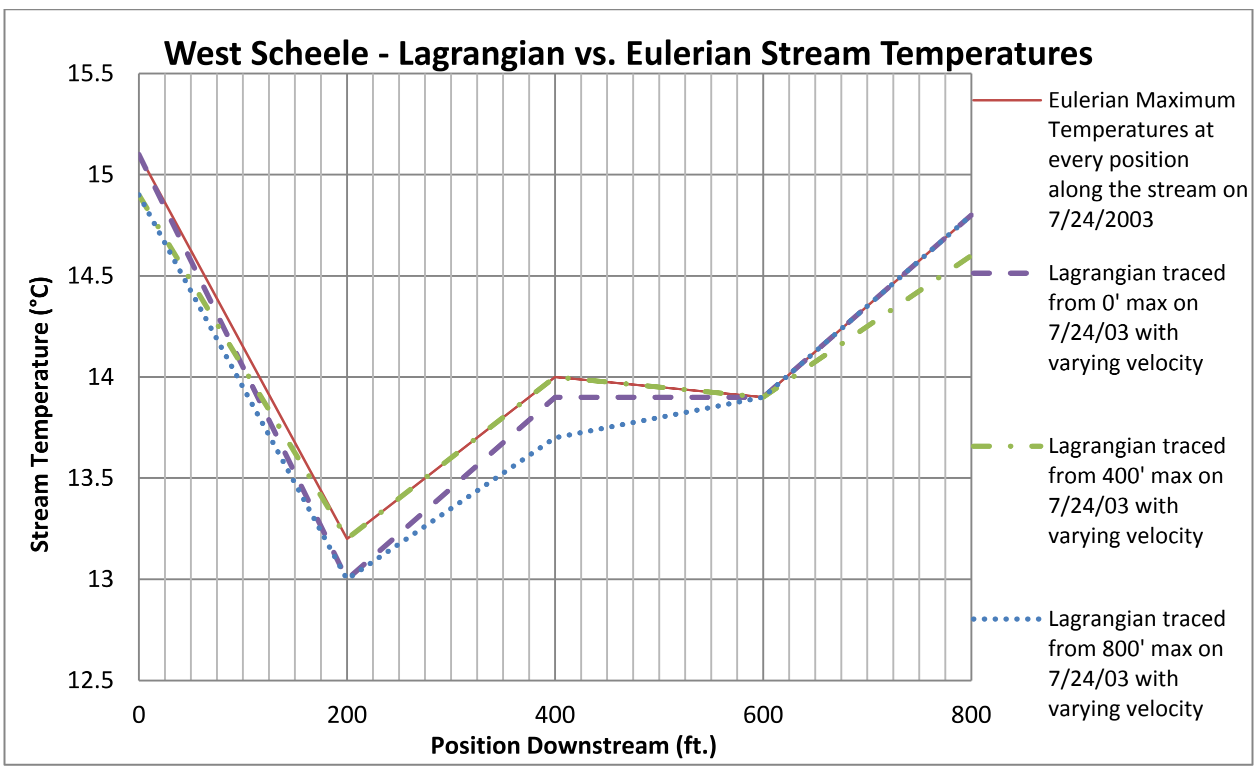

Figure 5 Three line plots in a Lagrangian frame of reference and one line plot of maximum daily temperatures in an Eulerian frame of reference for West Scheele on 7/24/2003

The first evident difference between the Eulerian and the Lagrangian reference frames is that the Eulerian reference frame reports temperatures at each point which are higher than or as high as the temperatures in the Lagrangian reference frame at each point. This is how the Eulerian line is defined as the line that connects all maximum daily temperatures. Data graphed in the Eulerian reference frame could also consist of all temperatures along the stream recorded at one point in time or all temperatures at one point in time, with all record times for one day. Again, the Eulerian graph of maximum temperature at each point along the stream is compared to the Lagrangian graphs because it is commonly used in hydrology.

Looking at the Hinkle Creek graph, the data in the Eulerian reference frame experiences some spikes in temperature along the stream. At 300 feet, the temperature spikes to nearly 16 degrees Celsius, significantly higher than the Lagrangian temperature data at that point. Temperatures leading up to this point are recorded from 2:30 pm to 4:50 pm with nine temperatures recorded between 2:30 pm and 2:45 pm and three temperatures recorded between 3:05 pm and 4:50 pm. The recording at 300 feet downstream was recorded at 1:00 pm, which is a much earlier time of day. The table below shows the spike in the data compared to the times the temperatures occurred. Note the temperature and time of recording at 300 feet (outlined) compared to other records.

Table 1 An example of the data from 0' to 300' for an Eulerian reference frame for Hinkle Creek illustrates the stochastic time data

Position (ft.)	Temperature (Degrees Celsius)	Date and Time
0	14.3	08/21/03 2:55 PM
25	14.5	08/21/03 2:45 PM
50	14.6	08/21/03 2:45 PM
75	14.4	08/21/03 2:45 PM
100	14.5	08/21/03 2:45 PM
125	14.7	08/21/03 2:50 PM
150	14.5	08/21/03 4:50 PM
175	14.8	08/21/03 2:30 PM
200	14.9	08/21/03 2:55 PM
225	14.8	08/21/03 2:50 PM
250	14.9	08/21/03 3:05 PM
275	15.0	08/21/03 4:40 PM
300	15.9	08/21/03 1:00 PM

This difference in temperature does not occur in the data graphed in the Lagrangian frame of reference. The data from the same section of stream traced from the maximum temperature at position 0 feet to position 300 feet is shown below. Note the time at which the data is recorded. This consistent time differs from the data above because the Eulerian reference frame for this example is organized by position and high temperature and disregards time of day.

Table 2 An example of the data from 0' to 300' for a Lagrangian reference frame for Hinkle Creek illustrates the time relationship between data points

Position (ft.)	Temperature (Degrees Celsius)	Date and Time
0	14.3	8/21/03 2:55 PM
25	14.5	8/21/03 3:00 PM
50	14.6	8/21/03 3:05 PM
75	14.4	8/21/03 3:10 PM
100	14.5	8/21/03 3:15 PM
125	14.6	8/21/03 3:20 PM
150	14.4	8/21/03 3:25 PM
175	14.6	8/21/03 3:30 PM
200	14.9	8/21/03 3:35 PM
225	14.8	8/21/03 3:40 PM
250	14.9	8/21/03 3:40 PM
275	15.0	8/21/03 3:45 PM
300	15.2	8/21/03 3:50 PM

The spikes in temperature in the Eulerian data in Figure 4 are mostly "smoothed out" in the Lagrangian projections of the data. Hinkle Creek had a few points that might be considered outliers from the general upward trend that the temperature data exhibits. Merely comparing the daily maximum temperatures may make for a dramatic argument about stream temperature behavior, but it may also lead to erroneous interpretations of the data. For example, if a high temperature of 18.5 degrees Celsius is recorded one day, and a high temperature of 17 degrees Celsius is recorded the next day, it should not be inferred that the stream has cooled over the course of a day. The whole picture may look surprisingly different. Suppose that on the first day, the stream had high temperatures around 16 or 17 degrees Celsius, and suppose on the second day, the stream had consistent high temperatures of about 17 degrees Celsius. These two days are actually very similar in temperature, but the daily high temperatures would suggest otherwise. Outliers are most likely to occur in a daily or weekly high or low temperature than anywhere else. By definition, outliers exist on the outer extremes of the data, and should perhaps be left out of an analysis if they do not lie within an acceptable range of values that follow the general trend of the data. Comparing daily outliers can lead to incorrect interpretations of the data.

In observing the four line graphs, other noticeable disparities immerge. The Lagrangian graph that pulls the maximum temperature from the middle of the study reach and traces it to the beginning and end of the study reach shows the least deviance from the Eulerian than any of the other Lagrangian lines. At each thermistor location along the study streams, the difference between the Eulerian temperature and each Lagrangian temperature was found. Then for all streams, these data points were summed to determine a total temperature difference between the Eulerian line and each Lagrangian line. These total temperature differences for all the streams for each Lagrangian line are summarized below.

Table 3 The sum of the differences between time at each position on each stream between line plots graphed in the Lagrangian reference frame and line plots of maximum daily temperatures graphed in the Eulerian reference frame

Total Difference Between Eulerian and Lagrangian Temperatures for All Streams	
Lagrangian Graph	**Total Temperature Difference (°C)**
Beginning to End Trace	22.88
Middle to Beginning and End Trace	17.07
End to Beginning Trace	35.77

The total temperature differences likely varied because each Lagrangian line started at a time determined by the Eulerian time at a particular position along the stream. In Tribulation, the Lagrangian line that started at 1000 feet also started rather earlier in the day than any of the other Lagrangian lines. Figure 6 shows the pattern in stream temperatures for Tribulation from 11 AM to 9 PM. Note that temperatures increase rapidly early in the day, then increase more slowly after 3 PM.

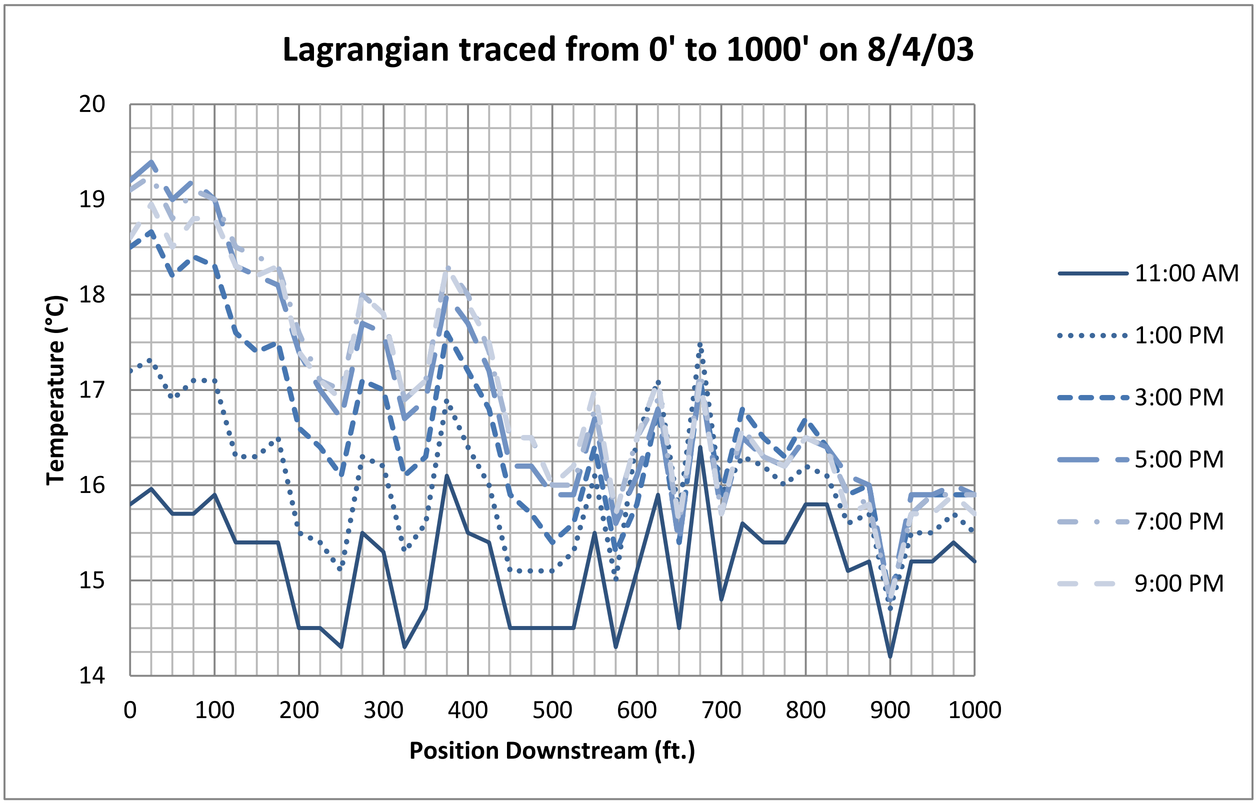

Figure 6 Line plots graphed every two hours starting at 11 AM in the Lagrangian reference frame traced from 0' to 1000' on 8/4/03 for Tribulation

Differences in stream temperature exist between the streams due to location, size of stream, vegetation, and other variables. The temperatures at West Scheele are lower than for the other three streams and range from 13.0°C to 15.1°C. The temperatures at Hinkle Creek increase steadily instead of decreasing downstream, as in the case of both Clay Creek and Tribulation. Clay Creek and Tribulation have similar temperature patterns. Both exhibit steadily decreasing temperatures in a downstream direction. This can also be observed in the pattern for daily maximum temperatures, which is similar for each Lagrangian trace of data for Tribulation and Clay Creek. See Figures 7 and 8.

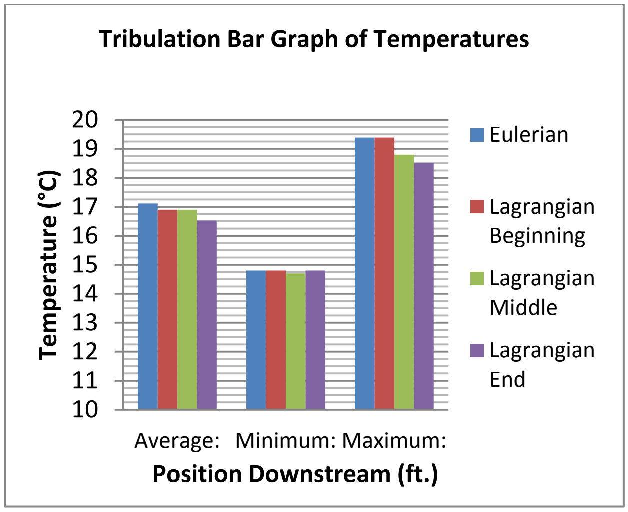

Figure 7 Tribulation bar graph of average, minimum, and maximum temperatures for 8/4/03

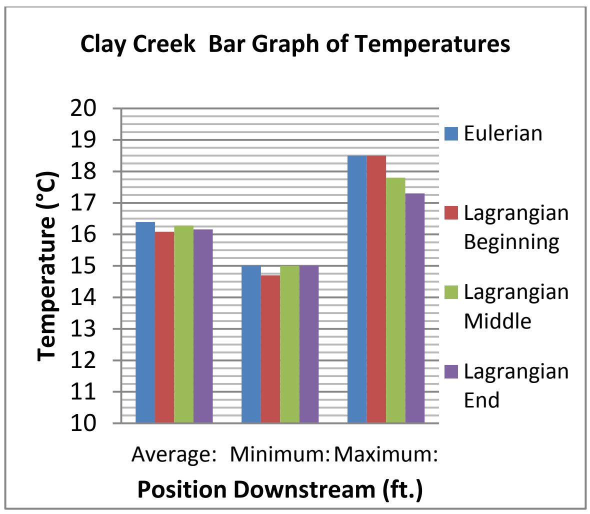

Figure 8 Clay Creek bar graph of average, minimum, and maximum temperatures for 8/14/03

Each line showing a Lagrangian frame of reference differs from the other lines, but they share a common trait that is different from the Eulerian frame of reference. For example, the data for the Lagrangian and Eulerian frame of reference is graphed three dimensionally with time added to the two dimensional plots of thermistor position and temperature as previously graphed. This three dimensional graph illustrates that although the streams exhibit the same pattern for stream temperature when viewed in two dimensional space, with the added dimension of time, the graphs of the Eulerian and Lagrangian frames of reference look starkly different. Figure 9 shows an oblique view of the data while Figure 10 shows a bird's eye view of the data. Figure 11 is similar to the two dimensional graphs, and shows the stream temperature and position downstream plane.

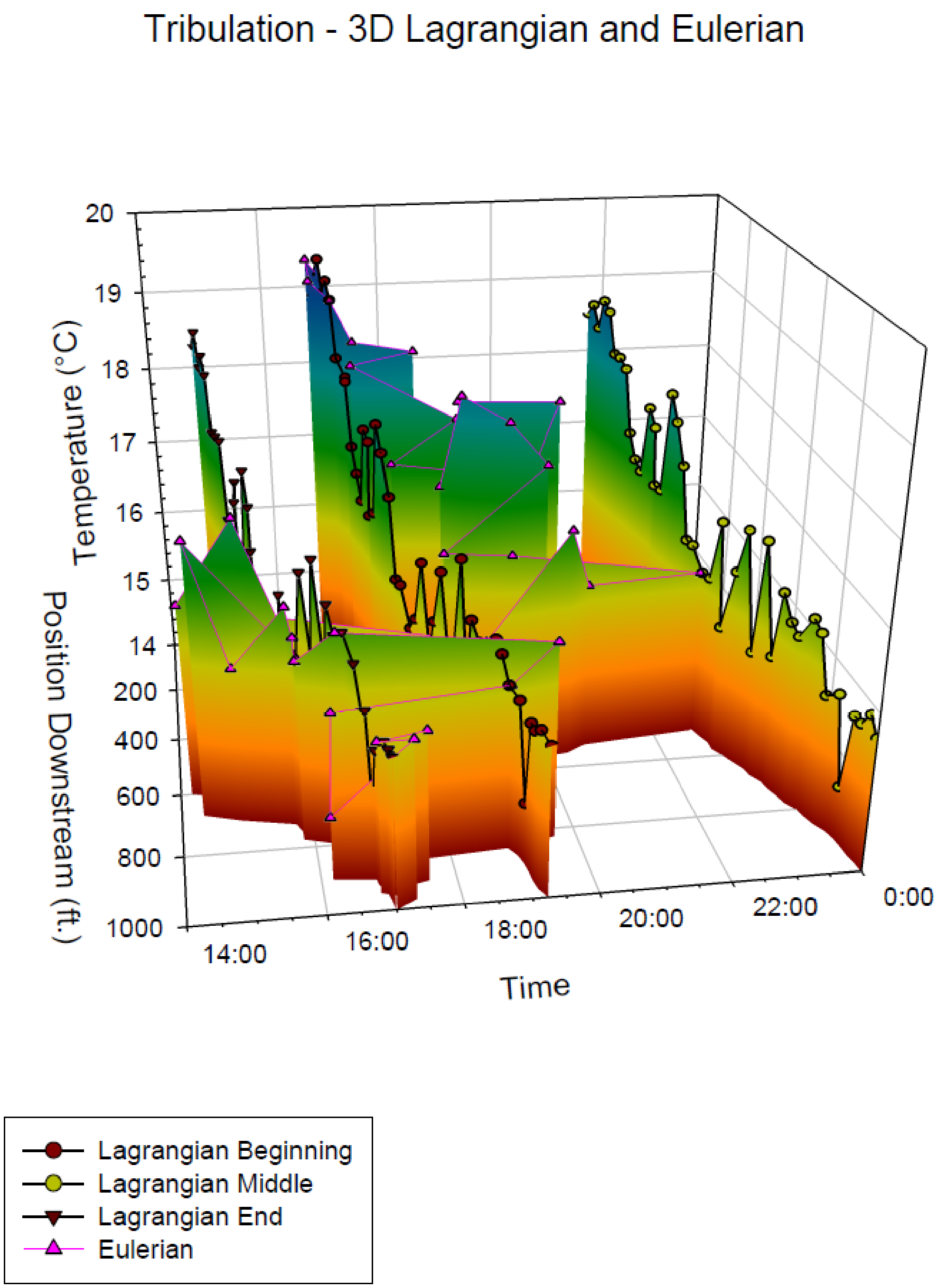

Figure 9 An oblique, three dimensional view of three plots in the Lagrangian reference frame and one plot in the Eulerian reference frame for Tribulation on 8/4/03

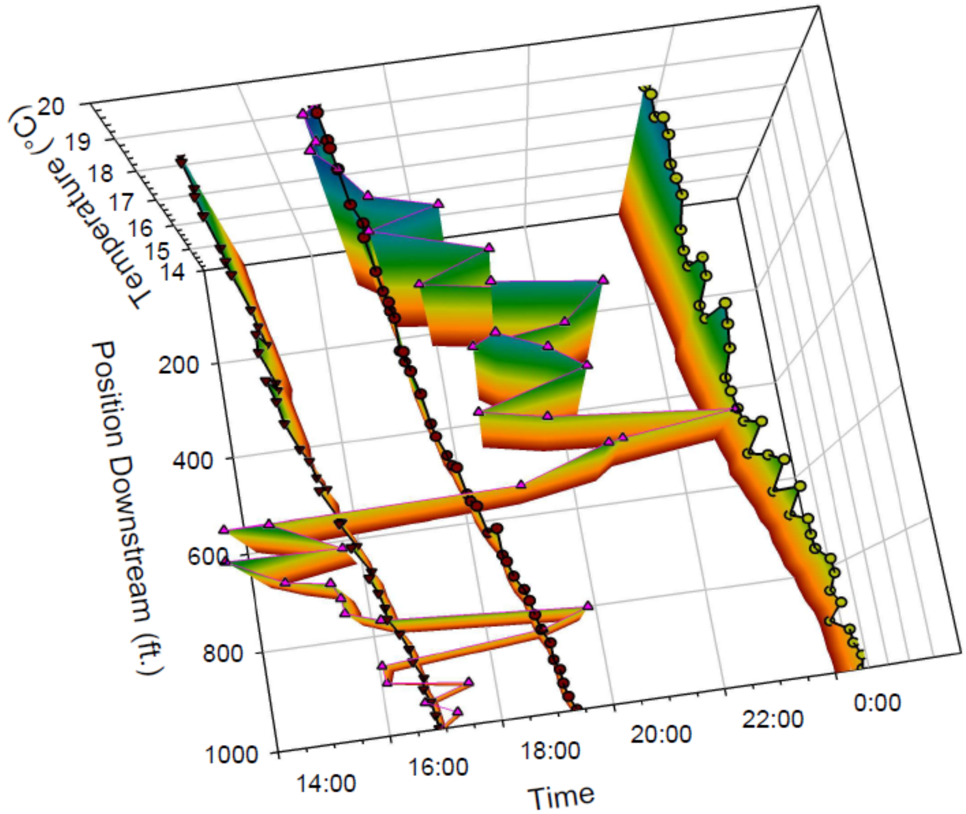

Figure 10 A bird's eye, three dimensional view of three plots in the Lagrangian reference frame and one plot in the Eulerian reference frame for Tribulation on 8/4/03

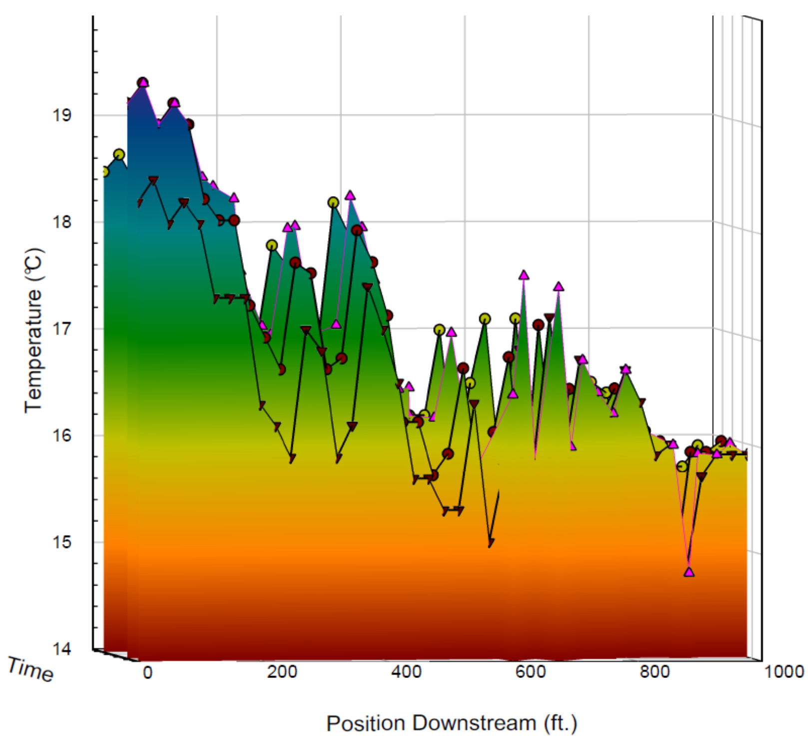

Figure 11 A planar, three dimensional view of three plots in the Lagrangian reference frame and one plot in the Eulerian reference frame for Tribulation on 8/4/03

Looking at the bird's eye view, it is apparent that none of the lines are truly "straight" in the position downstream and time plane. The Lagrangian lines would be straight in this plane if the velocities were constant, rather than varying with different stretches of the stream. Regardless of the slight curving, the Lagrangian lines are obviously and distinctly different from the more stochastic Eulerian lines on the time axis. As the Eulerian line meanders through the Lagrangian lines, the Lagrangian lines show the same pattern in stream temperature while exhibiting a more consistent change in time from point to point.

One way to graph data using an Eulerian reference frame, without such irregular times is to graph temperatures from all positions at only one point in time. That is, to take a snapshot of the stream at one particular time. An example of this is shown in Figure 12.

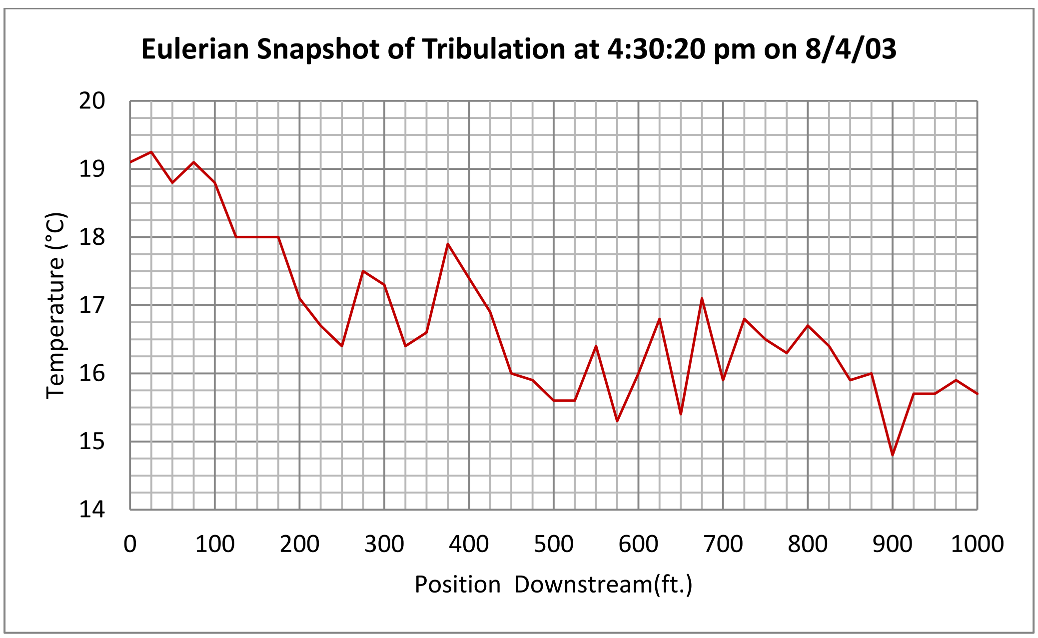

Figure 12 An Eulerian snapshot of Tribulation at 4:30:20 on 8/4/03 showing the stream temperatures for all locations at one point in time

Comparing this to a Lagrangian line starting at the same point in time and tracing the temperature from position 0 feet to 1000 feet, the graphs are remarkably similar.

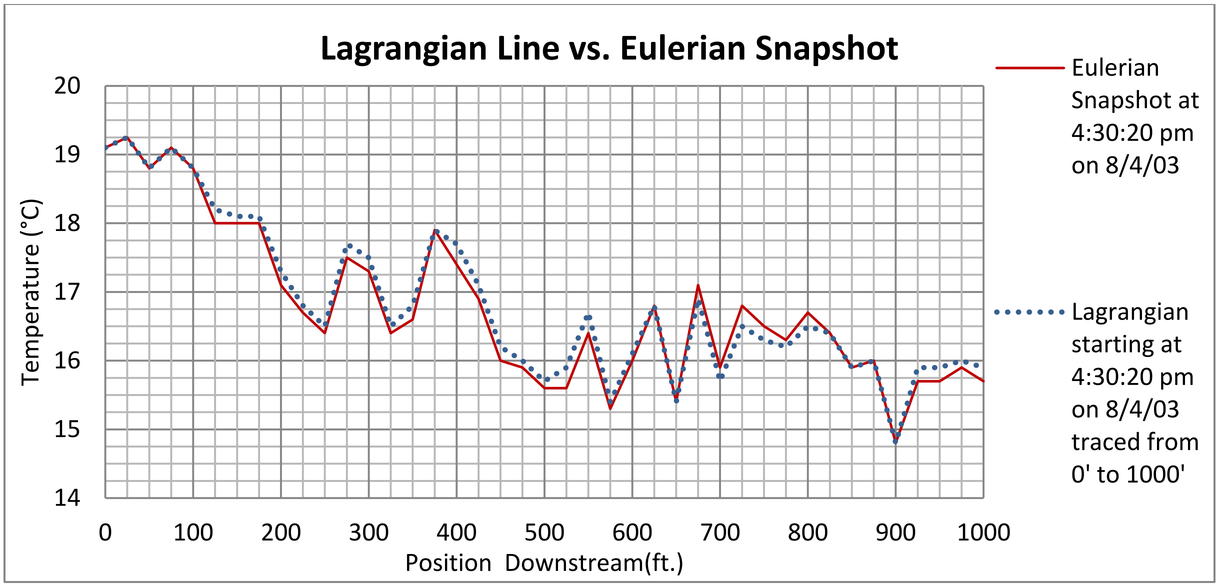

Figure 13 An Eulerian snapshot of Tribulation at 4:30:20 on 8/4/03 showing the stream temperatures for all locations at one point in time and a line plot of Tribulation graphed in the Lagrangian reference frame starting at 4:30:20 on 8/4/03

Without the third dimension of time, it appears that for these short stretches of stream, the Eulerian graph of stream temperature at one point in time is so similar to the Lagrangian graph, in which time varies, that the same interpretations might be made. Figure 14 illustrates the actual differences in time between the snapshot and the Lagrangian line from Figure 13 in three dimensions. The same pattern can be observed in the Eulerian graphs in which the maximum temperatures at each position along the stream were plotted with the Lagrangian graphs starting at one of those maximum temperatures. Both were similar enough that the interpretations would likely be the same but for the difference in time, with the Eulerian graph differing wildly in time from point to point, and the Lagrangian differing only slightly in time in a manner consistent with the longitudinal velocity of the stream.

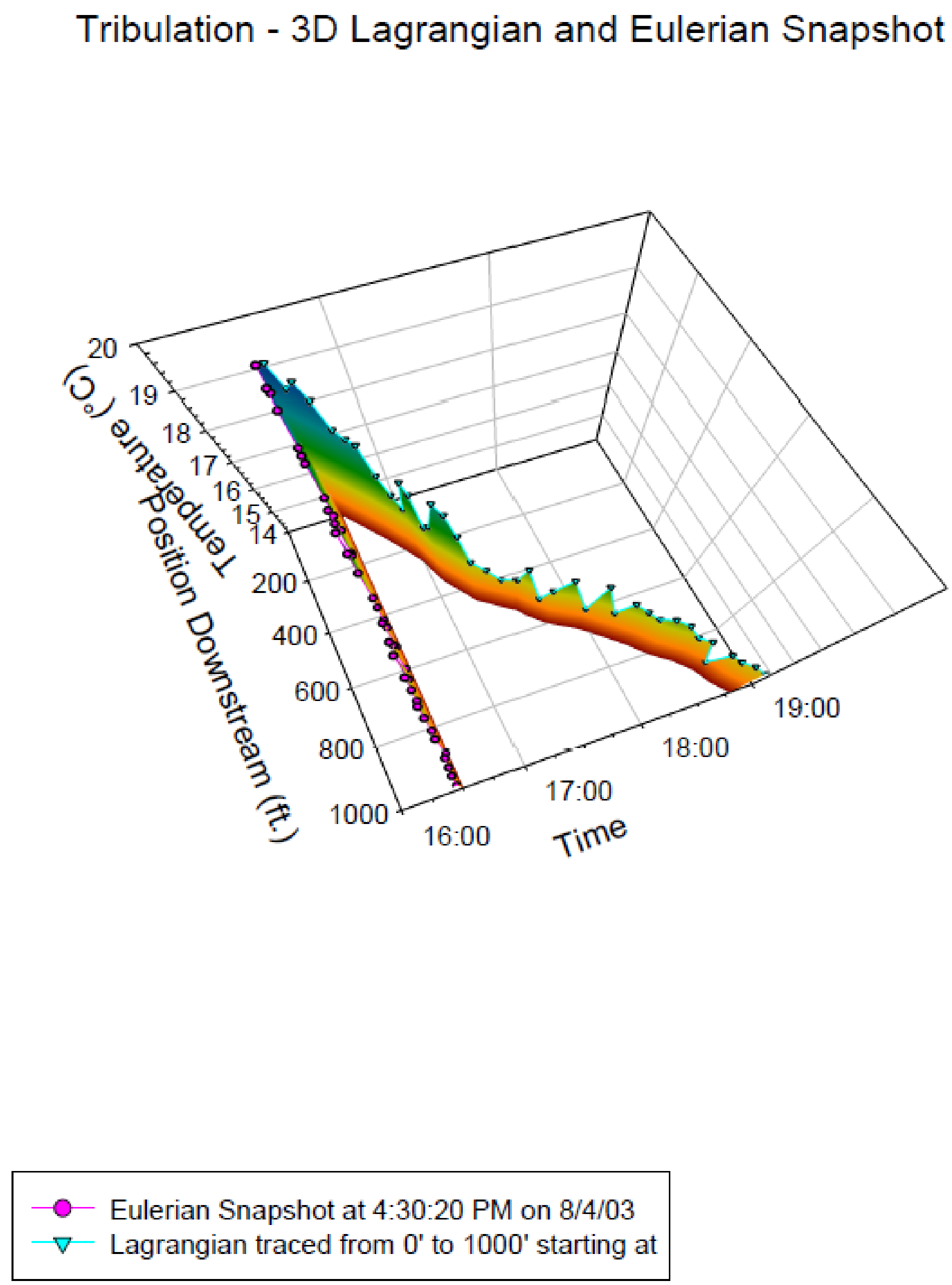

Figure 14 A three Dimensional graph of stream temperature for Tribulation that compares the difference in time between a Eulerian snapshot and Lagrangian line plot on 8/4/03

Another way to visualize an Eulerian set of data would be to plot all the data points to produce a three dimensional surface that represents all the temperatures in one day along the entire stretch of stream. Figure 15 and 16 are examples of this for Tribulation data collected on August 8th, 2003. The three Lagrangian lines that start from the beginning, middle and end of

Tribulation as well as the Eulerian data points of maximum temperatures along the stream are overlaid for comparison.

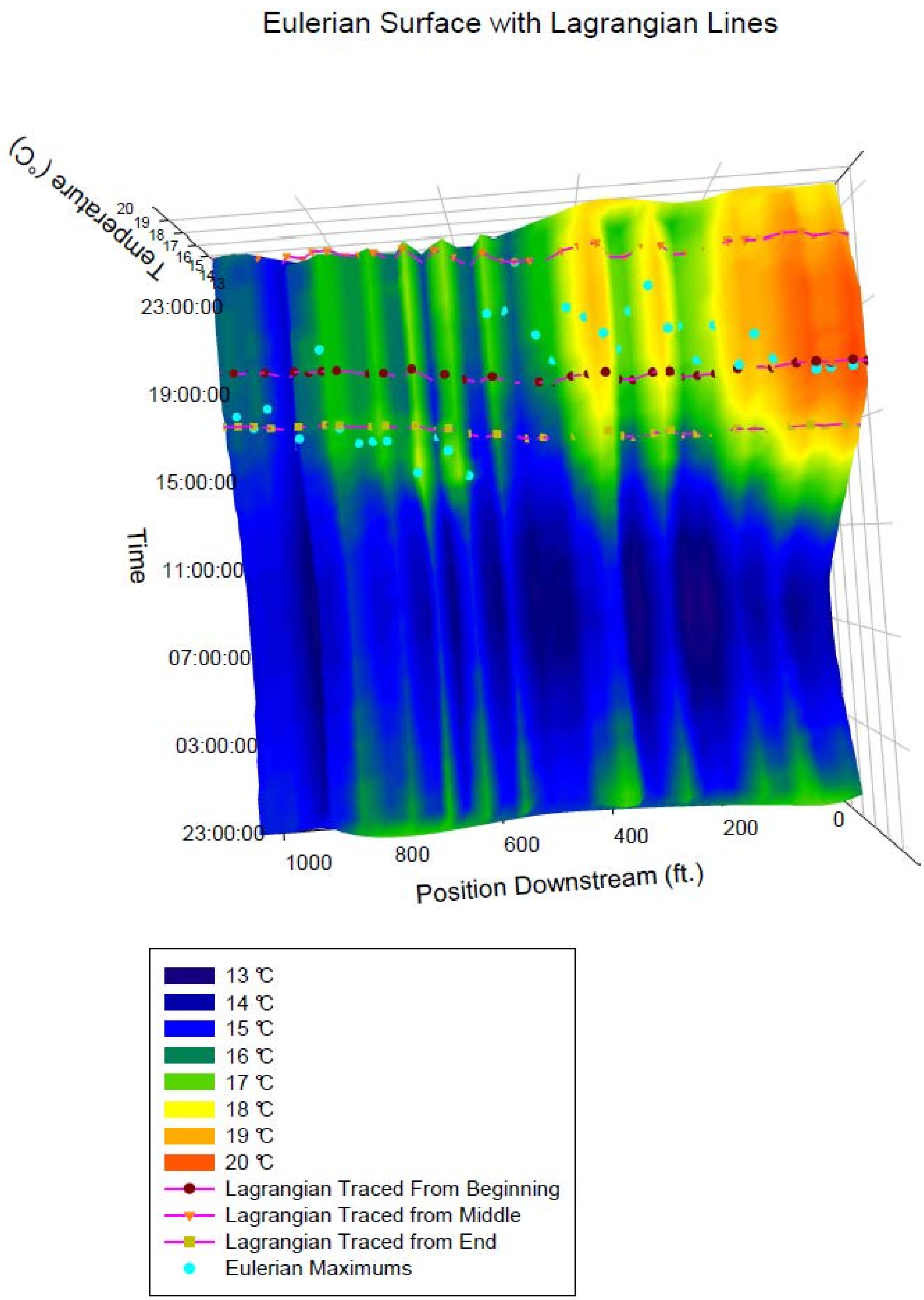

Figure 15 A bird's eye view surface graph with a plot of maximum daily temperatures in an Eulerian reference frame and three line plots in the Lagrangian frame of reference for Tribulation data on 8/4/03

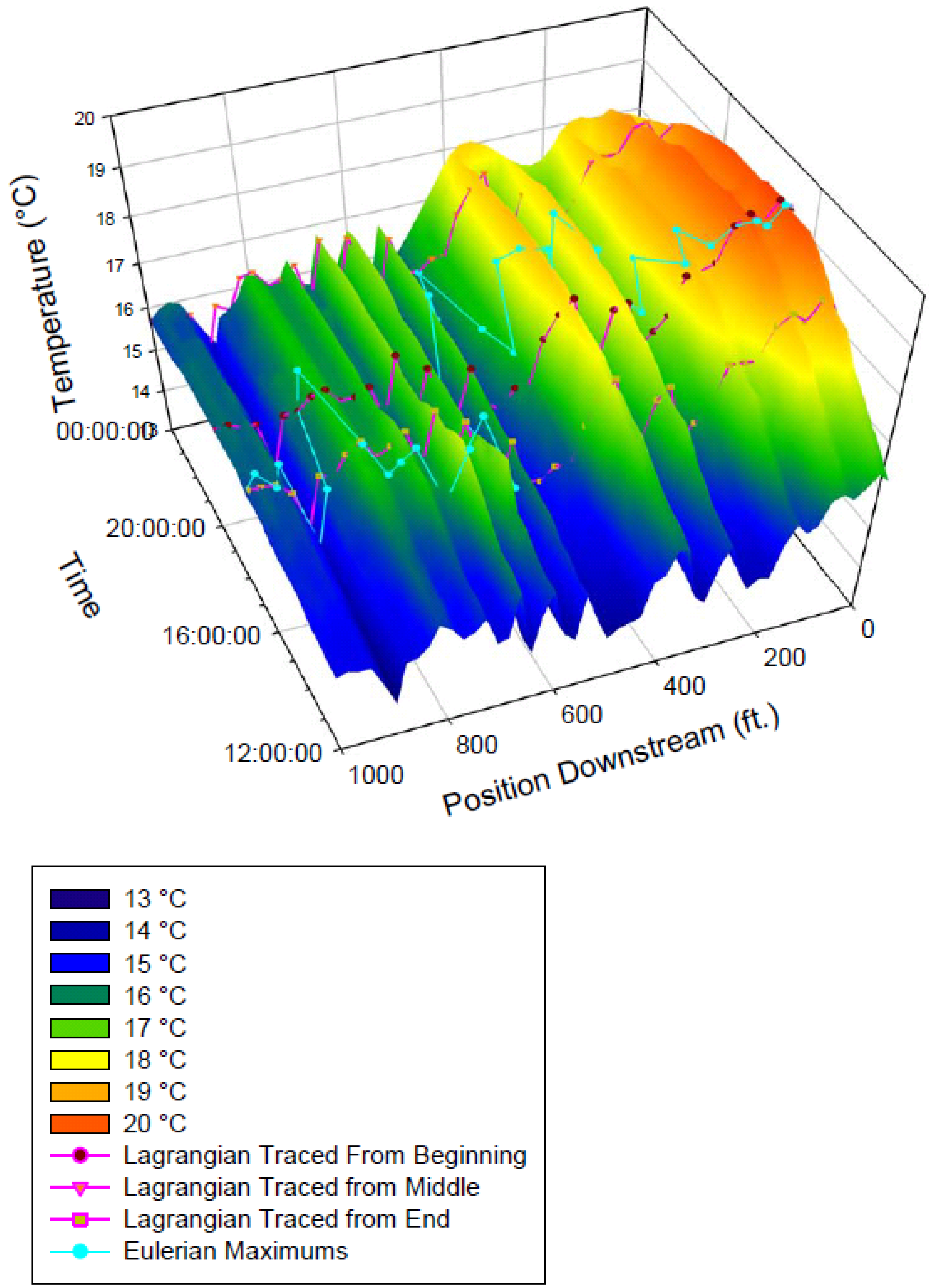

Figure 16 An oblique surface graph of Figure 15, zoomed in to the hotter part of the day with a plot of maximum daily temperatures in an Eulerian reference frame and three line plots in the Lagrangian frame of reference for Tribulation data on 8/4/03

Conclusion

Persons using an Eulerian analysis to make cause and effect statements should take care to consider all dimensions of the data. After studying the Eulerian and Lagrangian frames of reference on four sets of data, little difference was found between them while in the two dimensional space of temperature and position downstream. Yet, in three dimensions, the shapes of the graphs diverge into two distinctly different projections. Therefore, appropriate statements that are not linked to time are acceptable to make from interpretations of the Eulerian data. A statement such as "The 24 hour average temperature of the stream increased in the downstream direction" could be an appropriate statement. Another appropriate statement could be, "The maximum daily temperatures along the stream will not result in any harmful effects to the fish based on the acceptable temperature range for the species."

It is also acceptable to speculate that the Eulerian data may convey the same information about temperature as the Lagrangian. However, before forming any cause and effect statements, these speculations should be verified. The question of time must be addressed at some point. Perhaps an Eulerian set of data could act as a quick pre-test to garner an idea of the future results. If a set of Eulerian data looks promising, longitudinal velocity data should be collected.

The significance of this study lies in the similarities between the two frames of references. Although an Eulerian projection of stream data may sometimes lead to the same interpretation of the data as the Lagrangian projection, this is not always the case. Because they can frequently result in similar patterns in stream temperature, it may be mistakenly assumed that the two reference frames are both acceptable for all situations. This is not so.

The adoption of a reference frame for the data for any research project should be based on what questions are to be examined and what manner of statements must be articulated at the end of the study.

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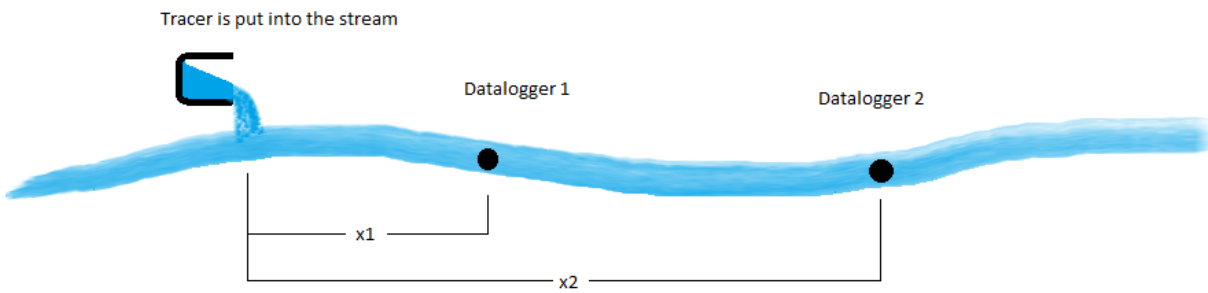
Appendix

Derivation of Equation 1



$$V_2 = \frac{x_2}{t_2}$$

$$\frac{x_2}{t_2} = \frac{\frac{x_1^2}{t_1} + \frac{(x_2 - x_1)^2}{t_x}}{x_2}$$

The equation is weighted by the length of the stream between x1 and x2

$$\frac{{x_2}^2}{t_2} = \frac{x_1^2}{t_1} + \frac{(x_2 - x_1)^2}{t_x}$$

$$\frac{{x_2}^2}{t_2} - \frac{x_1^2}{t_1} = \frac{(x_2 - x_1)^2}{t_x}$$

$$t_x = \frac{(x_2 - x_1)^2}{\frac{{x_2}^2}{t_2} - \frac{x_1^2}{t_1}}$$

Using this time, the longitudinal velocity can be determined (see equation 2)

Where:

t_x is the time it takes the mode of the water molecules to pass from datalogger1 to datalogger2

t_1 is the time it takes the mode of the water molecules to pass from the starting point to datalogger1

t_2 is the time it takes the mode of the water molecules to pass from the starting point to datalogger2

x_1 is the distance between the starting point and datalogger 1

x_2 is the distance between the starting point and datalogger 2