Modulation Transfer Function of Sea Water*

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The modulation transfer function (MTF) of a simple submarine viewing situation is computed as a function of range and sea-water properties. For the cases considered, we found that the MTF follows the simple exponential law, $\exp(-\omega/\omega_0)$, where ω is the spatial frequency in lines/deg and where ω_0 is a complicated function of the range, scattering albedo, and shape of the scattering phase function. A set of graphs summarizes the dependence of ω_0 on the inherent optical properties c and $\beta(\gamma)$.

Index Headings: Oceanography; Water; Scattering; Modulation transfer; Spread function; Image formation; Luminance.

The modulation transfer function (MTF) of an optical system specifies the contrast transmitted by the system as a function of image size, and is determined by the inherent optical properties of the system. Because many features of an optical image-transmitting system are summarized by its modulation transfer function, we have computed the MTF of sea-water viewing paths in order to predict the underwater performance of image-receiving equipment. Since our computations are based on the equation of transfer, we must assume that sea water acts as a linear optical system with respect to contrast and that the image-carrying light is incoherent. These assumptions are acceptable in most underwater viewing situations. It is also necessary to devise an image-transmitting geometry which is mathematically tractable, yet physically realistic.

For most image-transmission systems, the MTF is dependent on the wavelength, and sea-water systems are no exception. For reasons of notational convenience the explicit dependence of the various optical properties upon wavelength is not shown in the discussions which follow. However, the reader should keep this dependence in mind and realize that the MTF is not defined for a nonmonochromatic case. We also have restricted our attention to the case of unpolarized light. Note also that the MTF of a sea-water system is unaffected by the presence or absence of scattered daylight (space light). However, this space light may affect the performance of the final image-detecting equipment. For this reason, it is useful to compute the penetration and the scattering of submarine daylight in the same water types for which the MTF's were calculated.[1]

A previous method of predicting underwater visibility[2] requires only two measurements of the optical properties of sea water, the beam-extinction coefficient c and the diffuse-attenuation coefficient $K(+)$. With these two values, the range of detectability of a target of known size and contrast can be determined. It is not possible to apply this method in its present form to optical systems other than the human eye. This method is also unable to provide information on the resolution of image details. Both of these shortcomings are eliminated when the MTF is known.

THEORY

In order to justify the use of MTF theory, we have assumed that a sea-water system is linear in contrast, that it is invariant with respect to position in the object plane, and that sea water is a passive system. O'Neill has shown[3] that any image is given by the convolution of the object and the point-source response function (impulse response). The image can also be found by taking the inverse Fourier transform of the product of the Fourier transform of the object and the Fourier transform of the impulse response. Thus the Fourier transform of the impulse response gives the contrast transmitted by the system as a function of object wavenumber. Although a complete description requires the use of a two-dimensional impulse response (which in itself depends on the location of the impulse in the object space), the assumption of rotational symmetry permits the description of an optical transmission system by either the line-source response function or its Fourier transform. This Fourier transform is in fact the MTF as used in this paper. As a result of the assumptions made earlier, the derivative of the edge-gradient response is the line-source response function. This fact is used as a basis for the calculations that follow.

The principles of radiative-transfer theory have been applied by several investigators to the problem of determining the penetration of sunlight into a horizontally homogeneous layered atmosphere[4–6] and have recently been extended to the case of natural hydrosols.[7,1] One of these techniques[1] yields detailed information on the structure of the radiance field as a function of the depth and the inherent optical properties of the

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[2] W. E. K. Middleton, *Vision through the Atmosphere* (University of Toronto Press, Toronto, 1952), p. 103.

[3] E. L. O'Neill, *Introduction to Statistical Optics* (Addison-Wesley Publ. Co., Inc., Reading, Mass., 1963).

[4] S. Chandrasekhar, *Radiative Transfer* (Clarendon Press, Oxford, 1950).

[5] V. A. Ambartsumian, *Theoretical Astrophysics* (Pergamon Press, Inc., New York, 1958).

[6] V. V. Sobolev, *A Treatise on Radiative Transfer* (D. Van Nostrand Co., Princeton, N. J., 1963).

[7] R. Preisendorfer, *Radiative Transfer on Discrete Spaces*, (Pergamon Press, (Ltd.), Oxford, 1965).

TABLE I. Values of the scattering albedo b/c and the scattering indicatrix parameters ef and eb used in the numerical calculations.

b/c	ef	eb
0.80	0.95	0.70
0.20	0.80	0.50
0.05	0.65	0.30

hydrosol. This same technique is used herein to compute the MTF of a sea-water system. The calculations correspond to the model of the underwater viewing situation described below.

The hydrosol occupies a volume extending from $z=0$ to $z=+\infty$; $-\infty<x<\infty$; $-\infty<y<\infty$. The observing system is located on the z axis at $z=+z_0$. The target is effectively located at $z=-\infty$. One half-plane is occupied by a Lambert's-law source, the other half-plane by a dark field. We recognize that this viewing situation is not often obtained in the field. However, the MTF computed for this geometry should provide insight into the MTF expected under more realistic conditions.

The pertinent optical properties of sea-water are the beam extinction coefficient c, the total scattering coefficient b, and the shape of the scattering indicatrix $\beta(\gamma)$. In this paper the scattering indicatrix $\beta(\gamma)$ is approximated by

$$\beta(\gamma)=\beta_0/(1-ef\cos)^4(1+eb\cos)^4, \tag{1}$$

where ef and eb are parameters of the forward and backward lobes, respectively, and β_0 is a normalizing constant such that

$$\int_c^{4\pi}\beta(\gamma)d\Omega=b, \tag{2}$$

where Ω is the solid angle. The parameters in $\beta(\gamma)$ and the value of b/c may be varied to represent different water conditions. Some typical values of b/c and comparisons of $\beta(\gamma)$ with representative data are given in a previous paper.[1]

A direct solution of the equation of radiative transfer, in a situation where multiple scattering is important, is extremely difficult if the scattering indicatrix shows pronounced forward scattering. Since this is the case in sea water and in most other natural hydrosols, an alternate method must be used to solve for the radiance field. The method adopted in this case was to divide the hydrosol conceptually into slabs of thickness z and to determine the point spread function for each slab, with the assumption that multiple scattering within the slab can be ignored if the slab is sufficiently thin. The spread function is a function of both object and image coordinates. The scattered portion of the transmitted radiance field is the convolution of the respective point spread functions with the entering radiance field, whereas the direct portion of the transmitted radiance is simply the entering radiance attenuated exponentially. This operation is denoted by $T[\cdot]$; details are given in Beardsley and Zaneveld.[1]

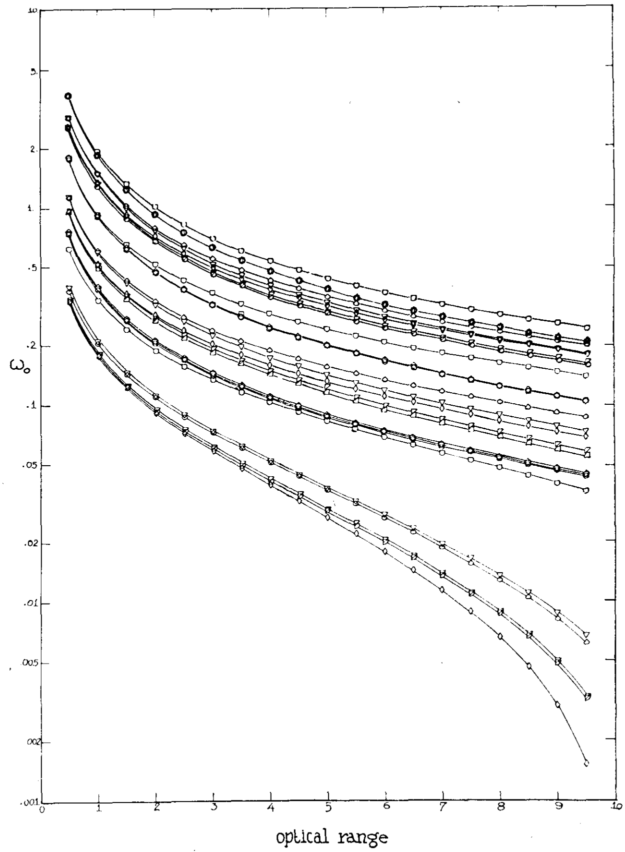

FIG. 1. The constant ω_0 of the modulation transfer function as a function of optical range (in units of $1/c$), for a scattering albedo of 0.05; (top 7 curves); for a scattering albedo of 0.20 (next 7 curves), and for a scattering albedo of 0.80 (last 9 curves), with values of ef and eb shown in Table I.

A strict application of the slab method to the radiative-transfer process requires that a reflection operator $R[\cdot]$ be derived in a similar way. The true radiance is then given by the solution to the system

$$\begin{aligned} N_j^-(\theta,\phi)&=T[N_{j-1}^-(\theta',\phi')]+R[N_j^+(\theta',\phi')],\\ N_j^+(\theta,\phi)&=T[N_{j+1}^+(\theta',\phi')]+R[N_j^-(\theta',\phi')], \end{aligned} \tag{3}$$

where N^-, N^+ refer to penetrating and escaping radiances, and j is the number of layers from the surface. However, b/c is always less than unity for natural hydrosols, so that $T[\cdot]$ and $R[\cdot]$ are always lossy operators. Thus it is possible to obtain a series solution to the exact equations, in which

$$N_j^-(\theta,\phi)T\{T[\cdots T[N_0^-(\theta',\phi')]\cdots]\} \tag{4}$$

is the leading term.

For the ranges of parameters used in the computations described below, only the leading term was retained, as computation of the higher term in a few cases resulted in a change of the radiance less than 1%.

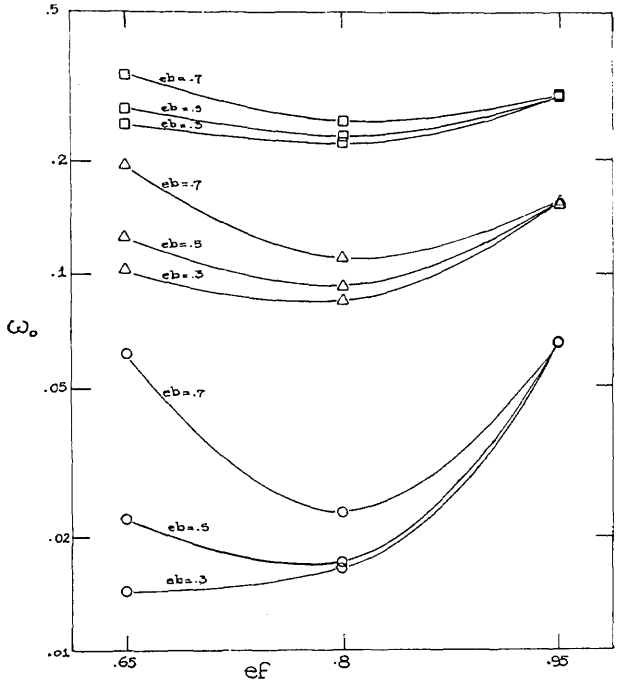

FIG. 2. The constant ω_0 of the modulation transfer function as a function of the scattering indicatrix parameters ef and eb, and the scattering albedo b/c, for an optical range of 6.5 units.

The edge-gradient response of the system of slabs is the intersection of the image radiance solid with the $\phi=0°$, 180°, plane when the object radiance $N_0^-(\theta,\phi)$ is given by

$$N_0^-(\theta,\phi)=N_0;\ 0°<\theta\leq 90°,\ -90°<\phi<+90°$$
$$O;\ 0°<\theta\leq 90°,\ +90°<\phi<+270°. \quad (5)$$

The edge-gradient response may be closely approximated in the region $-20°<\theta<20°$ by an arc-tangent function. The Fourier transform of the derivative of this arc-tangent function is the MTF and is found analytically to be

$$\mathrm{MTF}=\exp\{-\omega/\omega_0\}, \quad (6)$$

where ω is the angular frequency, and ω_0 is given by

$$\omega_0=(1/\theta)\tan\left[\pi\left(\frac{N_j(\theta)-N_j(0)}{2N_j(0)}\right)\right], \quad (7)$$

where θ is a convenient angle.

CALCULATIONS AND RESULTS

The method just outlined has been reduced to a numerical calculation with the parameters ef, eb, b/c and Δz. The routine also requires a specification of the incoming-radiance field $N_0^-(\theta,\phi)$. Using this radiative transfer method, we can find the radiance field in the hydrosol when the incoming radiance represents the edge-gradient target.[1]

Since the MTF can be expressed as a function of a single parameter ω_0, graphs of ω_0 vs the various optical parameters are sufficient to summarize the behavior of the MTF. The summary graph which follows are based on calculations made at the 27 possible combinations of the values of the parameters listed in Table I. Some typical results for parameters most closely simulating sea water are shown in Fig. 1. At each optical depth given by $j\Delta zc$, the value of ω_0 may be plotted as a function of the parameters ef, eb, and b/c. Figure 2 summarizes this dependence at the near-asymptotic depth of 6.5 units.

CONCLUSIONS

Although the MTF's obtained from the present theory are applicable only to the simple viewing situation discussed, we feel that the behavior of the MTF's in other viewing situations will be similar in functional form to the ones computed.

It would be desirable to compare the theoretically obtained MTF's with some MTF's obtained by experimental means. Unfortunately, none of the known experimental measurements are suitable for direct comparison, because the critical optical properties c and $\beta(\gamma)$ of the hydrosol were not determined. However, the measurements described by Replogle,[8] indicate that the MTF's fit the exponential law $\mathrm{MTF}(\omega)=\exp[-\omega/\omega_0]$ within the accuracy of the data.

It is clear from these studies that further experimental investigations of the MTF of sea water must include accurate measurement of c, and of $\beta(\gamma)$ if the results are to be properly interpreted.

[8] F. W. Replogle, *Seminar Proceedings: Underwater Photo-Optics* (Society of Photo-Optical Instrumentation Engineers, Santa Barbara, 1966), p. A-V-1.