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Simple models of transpiration, e.g., the Penman-Monteith equation, treat atmospheric conditions as driving variables. In fact, transpiration modifies temperature and humidity throughout the convective boundary layer, creating feedbacks that stabilize the water use of vegetation. This thesis concentrates on the new empirical relationships proposed by Monteith (1995), for developing simple models of feedback, and then applies these relationships to data from the Oregon Cascades. Monteith showed that there is strong laboratory evidence to support a linear relationship between leaf transpiration rate and leaf conductance. If this relationship holds for vegetation in the field, simple models to explain the diurnal variation of canopy conductance can be developed. When this model was applied to data from a Douglas fir forest, canopy conductance changed in response to transpiration rate, rather than to saturation deficit, as has been previously assumed. Monteith also reanalyzed data from McNaughton and Spriggs (1989) which explored the dependence of the Priestley-Taylor coefficient alpha on surface parameters. He showed that there is a linear relationship between alpha and surface conductance. By combining this "demand function" with the physiological "supply function" described earlier, the PMPT model is developed in which evaporation rate depends on physical feedbacks in the convective boundary layer and physiological feedbacks within plants. The thesis will focus on the results of the research done using this model. The PMPT

model will then be compared with other simple models of transpiration in order to determine its applicability.

Transpiration and the Atmospheric Boundary Layer:
Progress in Modeling Feedback Mechanisms

by

Faith Ann Heinsch

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I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

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Faith Ann Heinsch, Author

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Roman Alphabet

a variable used in calculations

A CO_2 assimilation rate (mol m^{-2} s^{-1})

b variable used in calculations

B wet bulb depression

B_0 wet bulb depression once air has reached equilibrium with the underlying surface

c also variable used in calculations

C sensible heat flux (W m^{-2})

c_e external CO_2 concentration in Equations 2.5 (µmol mol^{-1})

c_s mole fraction of intercellular CO_2 at the leaf surface (µmol mol^{-1})

c_p specific heat at constant pressure (temperature dependent) (J kg^{-1} °C^{-1})

c_u "consumptive use factor" (W m^{-2} °C^{-1}), obtainable from engineering literature. It is the amount of water needed to meet the evaporation needs of a vegetated area. It is empirically developed using a crop coefficient K (which varies by crop and climate and can be found in the FAO Report on evaporation (Doorenbos and Pruitt 1977)).
$c_u = K * f = K (D_l * (T/100))$

d zero plane displacement (generally proportional to vegetation height) (m)

D vapor pressure deficit between the surface and the atmosphere (Pa or kPa)

D_l fraction of daylight hours occurring during a month (dimensionless), based on latitude (This information is can be obtained from literature on radiation or in the FAO Report on evaporation (Doorenbos and Pruitt 1977).)

D_0 saturation vapor pressure deficit of the air in equilibrium with the surface (Pa)

D_s humidity deficit at the leaf surface (kPa)

E evaporation (mm day^{-1})

e_a actual vapor pressure of the air (Pa)

e_c vapor pressure of the surface (Pa)

E_m maximum evaporation (mm day^{-1})

e_s saturation vapor pressure of the air (Pa)

$f(u)$ some wind function (m s^{-1})

g_a aerodynamic conductance of vegetation (mm s^{-1})

g_c canopy conductance (mm s^{-1})

g_e entrainment conductance (mm s^{-1})

g_l leaf conductance (mm s^{-1})

g_{lm} maximum leaf conductance (mm s^{-1})

g_m maximum stomatal conductance obtained by extrapolating the stomatal conductance to zero transiration (mm s^{-1});
$g_m = [k*A / (c_s - \Gamma)]$

g_0 stomatal conductance as $A \rightarrow 0$ (when leaf irradiance $\rightarrow$ 0) (µmol mol^{-1})

g_{sw} stomatal conductance to water vapor (m s^{-1})

G soil heat flux (W m^{-2})

h_s relative humidity at the leat surface (%/100)

H net heat available from radiation and any change of heat stored in the system (W m^{-2})

H_m maximum available energy (W m^{-2})

I heat index, summed over twelve months, used in Equation 1.2
$I = \sum(T_n/5)^{1.514}$

k von Karman's constant; k = 0.41 (dimensionless);
In Equations 2.1 - 2.4, k (dimensionless) is an empirical constant representing the composite sensitivity of g_{sw} to A, h_s and c_s.

L daylight and month factor (dimensionless), tabulated in Thornthwaite's paper (a copy of the table can be found in Table 1.1)

PE monthly potential evapotranspiration (cm month^{-1})

r_a aerodynamic resistance of vegetation (s m^{-1})

r_c canopy resistance (s m^{-1})

r_l leaf stomatal resistance (s m)

LAI leaf area index (m^2 m^{-2})

R_n net radiation (W m^{-2})

S incoming solar radiation (W m^{-2})

T air temperature (°C)

T_a actual mean air temperature for each month (°C)

T_c surface temperature (°C)

T_{mean} daily mean air temperature of the month of consideration (°C)

T_n climatological normal temperature for each of the twelve months (°C)

$u(z)$ wind speed at height z (m s^{-1})

w_e entrainment velocity (mm s^{-1})

x a variable used in the calculations of Equation 1.2;
$x = (0.675*I^3 - 77.1*I^2 + 17920*I + 492390) * 10^{-6}$

z height of interest within a canopy (m)

z_0 roughness length of vegetation (generally proportional to vegetation height (m))

Greek Alphabet

α ~1.26, empirically-derived Priestley-Taylor coefficient (dimensionless)

α_m maximum Priestley-Taylor coefficient, varies with vegetation type (dimensionless)

β Bowen ration (dimensionless); It is the ratio of sensible heat flux to latent heat flux

Γ CO2 compensation point (Pa Pa^{-1}) ; This is the point where the carbon lost by the plant through respiration and photorespiration equals the carbon gained by the plant through photosynthesis.

γ psychometric constant (Pa °C^{-1})

Δ slope of the saturation vapor pressure curve (Pa °C^{-1})

λE latent heat flux (in this thesis, it analogous to the transpiration rate) (W m^{-2})

λE_{eq} equilibrium latent heat flux (W m^{-2})

λE_{imp} imposed latent heat flux (W m^{-2})

λE_m maximum latent heat flux (or transpiration rate), which is obtained by linearly extrapolating the transpiration rate to zero conductance (W m^{-2})
$\lambda E_m = 1.6 * D_0 * g_m$ (in mol m^{-2} s^{-1}); in this case, D_0 is an empirical constant (Leuning 1995)

ρ density of dry air (The value of ρ is dependent upon pressure and temperature.) (kg m^{-3})

ψ leaf water potential (J m^{-2})
Ω decoupling coefficient (dimensionless)

DEDICATION

This thesis is dedicated to my parents for everything they've done for me.

...what happens in the sky is caused by the heat of the sun; for, when the moisture is drawn up out of the sea, the sweet part, which is distinguished by its fine texture, forms a cloud, and drips out as rain by compression like that of felt, and the winds spread it around.

Xenophanes (ca. 570-460 B.C.)

Transpiration and the Atmospheric Boundary Layer: Progress in Modeling Feedback Mechanisms

Chapter 1- Introduction

The phenomena of evaporation and transpiration have fascinated scientists in various disciplines for decades. With 97.5% of the planet's water located in the oceans, and an additional 2% locked up in the polar icecaps, there is only a small amount of water available to support terrestrial life. However, this small amount of water is vital to the survival of terrestrial ecosystems. In addition, "latent heat accounts for about 90% of the annual heat transfer from the ocean to the atmosphere and about 50% of the transfer to the atmosphere from land" (Monteith 1981). Thus, the evaporation and transpiration of water, and the energy it transfers as it changes phase, are of great importance when studying terrestrial ecosystems.

Written studies of evaporation existed as early as 1687 when Halley published his account entitled "An estimate of the quantity of vapor raised out of the sea by the warmth of the sun, derived from an experiment shown before the Royal Society" (Budyko 1963). While this experiment is of historical interest, it has no practical application to current studies of evaporation. In fact, according to Budyko (1963), any work done on quantifying evaporation prior to 1910 is merely of historical interest. One of the first papers analyzing evaporation published after this date was written by Leighly in 1937. In this paper he describes the process of evaporation but does not attempt to explain the phenomenon mathematically. Thornthwaite (Thornthwaite 1940) associated measurements of rainfall (in the form of precipitation indices - tables that list the average precipitation for a given area) and river runoff with the study of evaporation (Thornthwaite 1940), but he ignored the underlying physics of the processes involved . Thornthwaite soon published another article, in which he described the concept of "potential evaporation" (explained later in this

chapter), but did not attempt to quantify his ideas (Thornthwaite 1944). Quantitative analysis of the process of evaporation began primarily in 1948 when Penman first published his paper entitled "Natural evaporation from open water, bare soil and grass" (Penman 1948). In this paper Penman developed the first physical model for calculating evaporation, i.e., a model based on sound laws of physics, rather than empirical correlations. The concept of evaporation continued to be developed around the world, with research particularly active in the United States, Europe, Australia and Canada. In 1948 Budyko published a book on the ideas being independently developed in the Soviet Union. This book was smuggled out of the U.S.S.R. in 1963 and translated into English (Budyko 1963). Because of his limited access to research being done in the rest of the world, Budyko developed and tested his own equations to describe evaporation. These equations were very similar to those being developed in the Western world at the same time. The success of his methods lies not in their development, however, but in their execution. Budyko was able to apply his methods both across broad regions and in areas where there was little data available to calculate evaporation (Thornthwaite and Hare 1965). In 1965 Monteith published his now-famous work "Evaporation and Environment," in which he modified Penman's original equation in order to include vegetative effects (Monteith 1965). This work is reviewed further in the section entitled "The development of the Penman-Monteith model." In 1972 Priestley and Taylor wrote an article in which they developed a model of regional evaporation based upon the idea of equilibrium evaporation, which was partially a physical model and partly a model of statistical "best fit" (Priestley and Taylor 1972). And, later, Shuttleworth and Wallace (1985) published their own energy-balance equation of evaporation, which attempted to model areas of sparse vegetation, i.e., those areas that fall somewhere between bare soil and closed canopies. They were able accomplish this by including a new parameter defining the

resistance of the substrate in addition to that of the vegetation (Shuttleworth and Wallace 1985). While this paper is interesting, it is a framework for development of such equations and not a solution to the problem of sparse canopies.

All of these models have been extensively tested, and while each of them has merit, none *fully* reflects what is occurring in nature. This is primarily because none of these models include the feedbacks between a vegetated surface and the atmosphere that serve to stabilize evaporation. In the past fifteen years, a number of scientists have begun to develop models of the vegetation-atmosphere system which do include feedbacks. Some of these schemes are part of Global Circulation Models (GCM's) and must of necessity be simplifications that are rapidly computed (e.g., the OSU one-dimensional model developed by Mahrt and Ek (1991)). Others are one- and two-dimensional models designed to explore physical and physiological processes, such as Forest-BGC developed by Running (1994). It is this second class of model that this thesis considers. In particular, Monteith (1995a) recently published a novel approach to studying these feedbacks, and these ideas will be examined here. In particular his method will be applied to forests in addition to the agricultural and grassland areas which he originally studied. First, however, the concepts and ideas behind evaporation and transpiration will be reviewed. Several approaches to estimating evaporation will be examined in detail in order to ascertain their applicability in the model developed by Monteith, and gaps in current knowledge will be reviewed. Finally, results from the model will be discussed, and areas of future research will be identified.

1.1 The Water Cycle over Vegetation

In reviewing the process of evaporation, this thesis is primarily concerned with the special case of transpiration. Transpiration is evaporation of water that has moved

internally through vegetation. Particularly, it is the water transfer that occurs through stomata and, to a small part, through the cuticular tissue of the plants. Stomata (or stomates) are tiny pores (on the order of 10 μm in diameter) in the leaves of many plants that, when open, allow the uptake of carbon dioxide by the plant and, therefore, water vapor to leave the plant. The opening and closing of a stomate is controlled by guard cells which surround it. If the plant begins to lose water more rapidly than it can be withdrawn from the soil, the plant must close its stomata to prevent further water loss. This closure also stops uptake of carbon dioxide by the plant. Plants have evolved over time in order to adapt to their climate regime and the tradeoff between obtaining the necessary carbon dioxide and the loss of water.
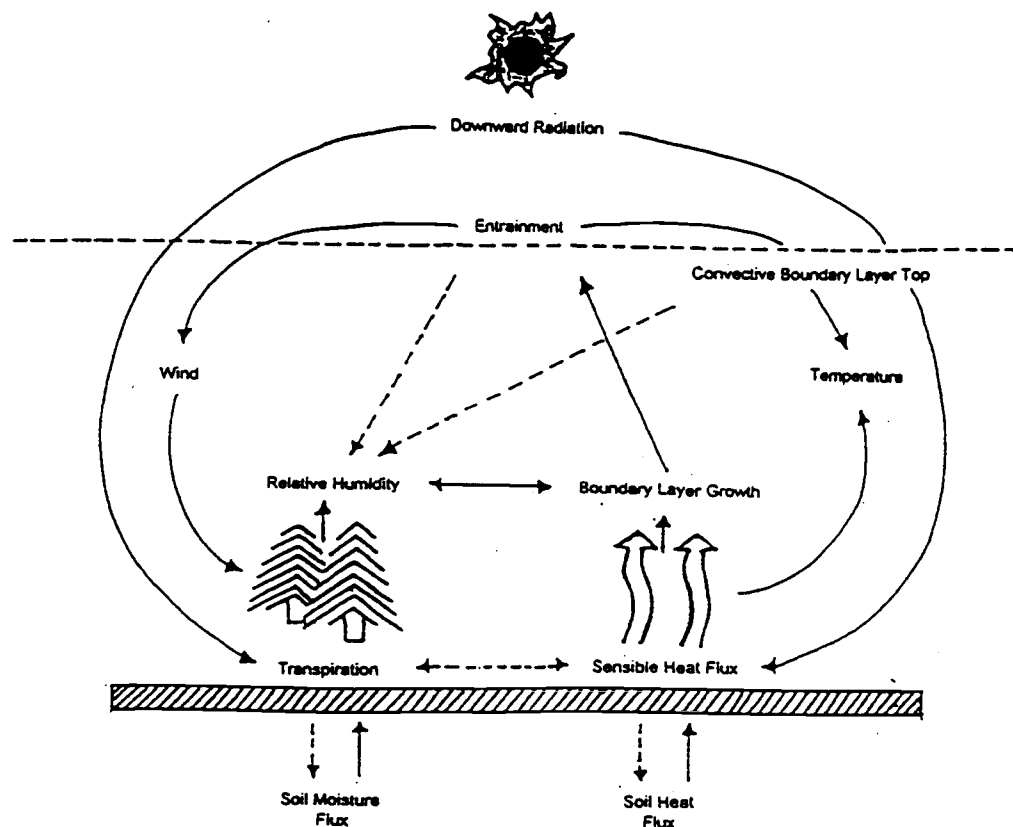
Figure 1.1 The water cycle over vegetation

In order to understand transpiration, it is necessary to understand the water cycle above a vegetated system. Figure 1.1 shows such a system. Net absorbed solar and longwave radiation increase the temperature at the surface and drive the system. As this radiation is absorbed by vegetation, it heats up the leaves. As the temperature difference between leaves and the air increases, convection from the leaves increases, and more heat is returned to the atmosphere. This increase in leaf temperature also increases the water vapor pressure deficit between leaves and the air, and transpiration is increased. To attain an energy balance the heat lost by convection (sensible heat), evaporation of water (latent heat) and conduction (often negligible) must balance the net radiation. If the flow of water through the soil/plant system cannot be sustained, the leaves begin to dry out and the stomata begin to close. As the stomata close, and less energy is lost as latent heat flux (evaporation/transpiration), more energy must be lost as sensible heat flux, and the atmosphere heats up in response. The increase in air temperature and hence in atmospheric vapor pressure deficit, increases the atmospheric demand for evaporation. This positive feedback continues until the stomata have fully closed, and transpiration is shut down.

Rising convective air causes entrainment from the usually drier atmosphere above the convective boundary layer. If the sensible heat flux from the ground increases, entrainment increases because of an increase in temperature at the surface, and therefore an increase in rising convective air plumes. These plumes generally reach the top of the Atmospheric Boundary Layer (ABL), which will be defined later in this chapter. Because of buoyancy and upward momentum, these currents are able to punch through the inversion at the top of the ABL, encouraging the entrainment of the drier air from above. This serves to further increase the vapor pressure deficit, and the cycle continues. The feedbacks within this cycle, such as entrainment of drier air and the

impact of sensible heat flux on the system are not modeled in current calculations of evaporation. To develop more realistic models, valid at regional scales, we must begin to include such feedbacks in our models. Unfortunately, complex methods of calculation are currently necessary in order to apply such feedbacks to models, and most models do not include a rigorous treatment of both the atmosphere *and* vegetation. These complex calculations are excellent ways to estimate the effects of the feedbacks on the system. The problem with employing these schemes at the larger scale, such as in models of the Atmospheric Boundary Layer, is the computation time required. Thus, less complex (i.e., quicker) calculations are needed in order for models of the feedbacks to be useful in regional or global models. Monteith has done this by combining the ideas of a "big-leaf" treatment of vegetation, first developed by Penman and Monteith (Monteith 1965) with the regional evaporation concept of Priestley and Taylor (1972). He does this by defining the interaction between the Atmospheric Boundary Layer and vegetation in terms of water vapor (latent heat flux) and heat (sensible heat flux) exchange. Thus, transpiration rate and saturation deficit can be estimated as functions of surface conductance and net radiation. In order to understand this work, however, it is necessary to review various models and the principles behind them. It is also important to compare the chosen models to others in order to determine the best models for use in the development of a model of feedback mechanisms.

1.2 The Concept of Evaporation

In order to understand the models of evaporation, one must first understand evaporation itself. Evaporation is defined as the change in the state of water from liquid to vapor (Shuttleworth and Wallace 1985). It is the main process responsible for water transfer between the surface and the atmosphere. It can only occur if (1) the vapor

pressure of the ambient air is less than that of the surface, (2) if there is a source of energy to drive the exchange (i.e., solar radiation) and (3) if there is an aerodynamic mechanism to remove the vapor from the air directly above the surface (i.e., wind). There are three types of surfaces that are important in the return of water vapor to the atmosphere: (1) open water, snow and ice surfaces from which evaporation takes place directly, (2) bare ground, from which water evaporates at the moist soil-air interface (which may be below ground level), and (3) vegetation, on which the stomates act as sources of water vapor (Penman 1948).

Evaporation of water from the soil can be a very important part of the water balance of a system. The transfer of water through the soil is dependent primarily upon the water potential of the system. Water potential is a measure of the free energy of the water in a soil (i.e., the capacity of the water to do work) as compared to the free energy of pure water. Water tends to move from areas of high free energy to areas of low free energy. Total water potential is composed of matric potential, osmotic potential, pressure potential and gravitational potential. Matric potential is a measure of the reduction in free energy caused by the attraction of water molecules to the surrounding soil particles. Osmotic potential is a measure of the reduction in free energy caused by solutes within the water itself. Pressure potential is the hydrostatic pressure of the soil-water system. Gravitational potential is a measure of the force of gravity upon water within the soil. All of these potentials are combined to determine the direction and movement of water in the soil. Total water potential is generally negative, resulting in a net downward movement of water through the soil. Another important component of soil water evaporation is capillary rise, or the movement of water from the water table to the surface against gravity. The rate of capillary rise depends upon the distance from the water table to the surface and the total soil water potential. Estimates of capillary rise can vary from one to five

millimeters per day. Water vapor located in soil pore spaces is also important in soil evaporation. Water vapor is located in all soil pores not filled with water. This is only true, however, as long as there is some water in the soil. This water vapor is important to plants. Once the pore space is at saturation water vapor density, roots will no longer lose moisture by evaporation. Capillary rise is very sensitive to temperature and is therefore a very dynamic property of soil water potential. Evaporation from the soil surface depends upon the moisture content of surface soil, because this will in part determine the water potential of the soil. For example, if the surface is wet, it will evaporate in the same manner as a free water surface. This continues until the surface water film retreats into the soil. But, evaporation can only continue if water can move from lower soil levels to the surface. As the soil becomes drier, secondary effects, such as the thermal nature of the surface and the nature of the surface wind, become much more important in determining the rate and occurrence of evaporation (Meidner and Sheriff 1976).

Evaporation from a vegetated surface can have all three components: (1) direct evaporation of water lying on the leaves and other structural elements of the plants, (2) transpiration (i.e., water which moves from the soil through plants and to the atmosphere via the stomata) and (3) evaporation from the soil beneath the vegetation. Vegetation is of particular importance to the water cycle because "plants can draw on moisture throughout a considerable depth of soil, transfer vapor through a considerable thickness of air and restrict transfer to daylight hours" (Penman 1963). In a dense forest, or an area in which the ground in completely shaded and the area is relatively shielded from wind eddies, evaporation from the soil surface is often negligible (Jones 1992).

One approach to modeling total evaporation from canopies is to treat the canopy as a series of layers. The energy balance of the canopy can then be estimated from the energy balance of each layer and, in addition, the energy balance of the soil beneath the

canopy (Sellers et al 1997). This approach requires knowledge of the distribution of radiation throughout the canopy, the windspeed profile within the canopy and characteristics of the foliage (leaf angle, leaf area, leaf shape and leaf size). A second approach is to treat the canopy as a single unit. This method is know at the "big-leaf" concept, and will be discussed in Section 1.4.

When vegetation is plentifully supplied with water, the rate at which water is transpired is dictated primarily by weather conditions, with plant and soil characteristics playing only a minor role (Penman 1963). Conversely, if the vegetation is poorly supplied with water, the plant and soil characteristics dominate the exchange, with weather conditions playing only a minor role. It is the combination of the two extremes that is interesting to study and which is common in nature - when both weather conditions and plant/soil factors play a role in evaporation. The type of vegetation is also an important consideration. For example, if a forest and a grassland are exposed to the same weather conditions, the rate of transpiration may be very different from the forest than from the grassland for reasons which are discussed later.

1.3 The Penman Approach vs. More Empirical Approaches

The first equations of evaporation that will be reviewed are the Blaney-Criddle equation, the Thornthwaite equation and the Penman equation. The Blaney-Criddle and Thornthwaite equations are empirical, statistical models, while the Penman equation is based on the physics of evaporation and water vapor transfer.

The Blaney-Criddle model, developed in 1950, was designed to provide estimates of daily evaporation from agricultural crops, averaged over a month-long period of time. Its physical basis depends upon the fact that both the "radiation" and "aerodynamic" terms that drive evaporation have some relationship to temperature (Shuttleworth 1983).

Since the "radiation" term is often the more important of the two terms for agricultural crops and soils, its correlation with temperature is more important. This was the aspect studied by Blaney and Criddle in the development of their model. The model was originally developed by Blaney and Morin, who correlated pan evaporation (a measure of evaporation from pans of water that can be monitored and used to estimate the evaporation at an area of interest) with monthly mean temperature, relative humidity, and percentage of total yearly daylight hours for each month. The correlations were later used to develop an empirical consumptive use index, employed in the most current developments of the equation (Rosenberg 1974). Later, these climate data were combined with precipitation and length of growing season and then correlated to the crop consumptive use of water in order to obtain an equation useful for vegetation. The final equation is as follows:

$$\lambda E = c_u D_l * (0.46 T_{mean} + 8) \tag{1.1}$$

This equation has several problems. While it is easy to apply, it is not applicable to areas dissimilar to the agricultural areas for which it was developed (Doorenbos and Pruitt 1977). This equation does not attempt to account for the time lag between net radiation and temperature. An accounting of this time lag is vital, especially as one moves to middle and higher latitudes. While maximum net radiation occurs in July, maximum temperatures don't occur until August. Therefore, an equation based on temperature alone will inaccurately estimate the radiation load of an area. In addition, this equation is empirical, and therefore cannot take into account any abnormalities that occur in the weather. Finally, it gives monthly mean evaporation, which has limited usefulness for most applications. Its usefulness lies in the fact that it is easy to use and can give a rough estimate of evaporation, which may be accurate enough for irrigation purposes in

Table 1.1 The duration of sunlight, in units of twelve hours
(Thornthwaite and Hare 1965).

Duration on the 15th day of:												
Latitude (°N)	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
10	0.97	0.98	1.00	1.03	1.05	1.06	1.05	1.04	1.02	0.99	0.97	0.96
20	0.92	0.96	1.00	1.05	1.09	1.11	1.10	1.07	1.02	0.98	0.93	0.91
30	0.87	0.93	1.00	1.07	1.14	1.17	1.16	1.11	1.03	0.96	0.89	0.85
40	0.80	0.89	0.99	1.10	1.20	1.25	1.23	1.15	1.04	0.93	0.83	0.78
50	0.71	0.84	0.98	1.14	1.28	1.36	1.33	1.21	1.06	0.90	0.76	0.68

humid or semiarid regions.

The Thornthwaite equation, developed in 1955, is also empirical. It is based on rainfall and runoff measurements for river basins throughout the United States. The formula is based on the concept of "potential evapotranspiration". Potential evapotranspiration is defined as "water loss from a moist soil tract completely covered by vegetation (without specifying the type) and large enough for oasis effects to be negligible" (Thornthwaite and Hare 1965). The formula derived by Thornthwaite is as follows (Thornthwaite and Hare 1965):

$$PET = 1.6L * (10 * \frac{T_a}{I})^x \qquad (1.2)$$

As can be seen from the formulas involved, the equation is completely empirical. It is a linear regression of evaporation for the U.S. drainage basin data used in its development, thus leaving many of the factors of the equation with strange dimensions. Since much of the data required for the model is published in commonly available tables such as those found in Thornthwaite's own work, it has "the obvious advantage that it yields an answer in a matter of minutes" (Thornthwaite and Hare 1965). Thornthwaite has also developed

a nomograph which allows the user to graphically determine the value of potential

evaporation easily (See Figure 1.2). The Thornthwaite equation is an improvement over

the Blaney-Criddle model because the difficulty involved in the relationship between

evaporation and temperature is removed by the use of the tables. It derives its success

from the fact that the mean temperature does, to a large extent, serve as an indicator of

the net radiation cycle. Unlike the Blaney-Criddle, it includes a parameter within the heat

index "I" for the time lag between net radiation and temperature. It has been severely

criticized by many, even Thornthwaite himself. He writes, "This mathematical

development is far from satisfactory....It is completely lacking in mathematical elegance.

It is very complicated and without nomograms and tables as computing aids would be
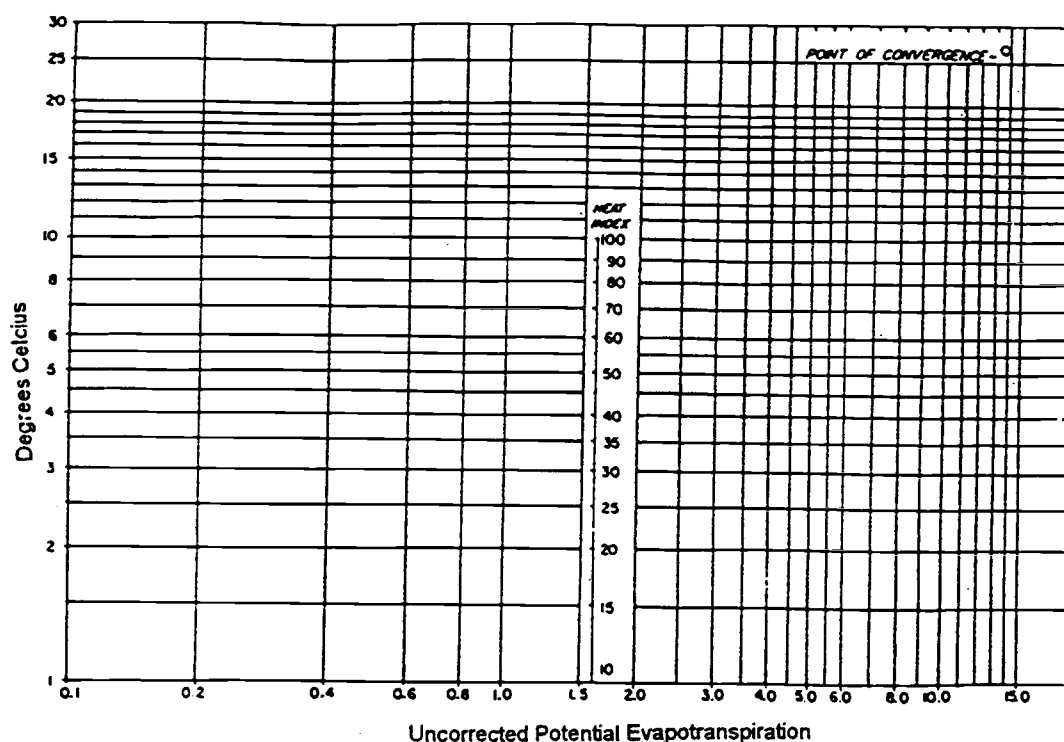
Figure 1.2 A sample nomogram solving Thornthwaite's Equation (Equation 1.2). The curve for unadjusted Potential Evaporation passes through the point of convergence of all curves and the point on the heat scale corresponding to the "I" value computed in the equation (reprinted with permission from Thornthwaite and Hare 1965).

quite unworkable" (as quoted in Monteith 1994). His formula is not applicable in areas where the climate is markedly different from the well-watered drainage basins of the eastern United States in which it was developed. This equation is still useful, particularly to estimate evaporation from satellite radiometry, in which the easily-obtained surface radiative temperature is often used to estimate the radiation load of an area (Monteith 1994).

In contrast to the previously-mentioned models, the Penman model is based on the physics of water vapor transfer. In fact Penman (1948) writes, "To the physicist, the use of the energy balance sheet to interpret the water balance sheet is one of the important weapons." Penman combines an energy balance approach (in terms of net radiation) with an aerodynamic approach (in terms of a wind function) in order to solve the evaporation puzzle. In current terminology, the equation is:

$$\lambda E = \frac{\Delta H + f(u)\rho c_p(e_s - e_a)}{\Delta + \gamma} \tag{1.3}$$

In order to fully understand this equation it is necessary to review its derivation, which can be found in Appendix A. This derivation is quite simple, and includes many of the components necessary to accurately predict evaporation from an area. The strength of this equation lies in its physics, which can be proven and repeated for different climatic regions. In addition, this equation includes the radiation balance of an area, the vapor pressure deficit of the atmosphere and an aerodynamic function (based on wind speed), thus accounting for all three of the factors known to determine evaporation. Because of these reasons, the Penman equation is a vast improvement over both of the previously-mentioned equations, and is therefore considered the most likely candidate for further studies of evaporation. This equation does have its limitations, however. The next

section of this chapter is devoted to these limitations and subsequent improvements to the model.

1.4 The Development of the Penman-Monteith Equation

When Penman developed his equation, he recognized many of the limitations of the assumptions upon which the equation was based. The most problematic limitation was the amount of data needed (i.e., air temperature, relative humidity, wind speed and net radiation) in order to solve the equation. Another limitation of this equation was the assumption of the wind function. The wind function developed by Penman systematically underestimated the aerodynamic effects for both crops and forests (Thom and Oliver 1977). This problem arises from the fact that Penman's equation requires aerodynamic information at only one level within the canopy, called the effective canopy roughness. If the effective canopy roughness underestimates the true canopy roughness, as it often does, the aerodynamic function will be underestimated (Thom and Oliver 1977). In order to improve the accuracy of the equation, a well-defined aerodynamic function needed to be developed. Finally, Penman's equation included the notion that water was never the limiting factor in the evaporation of an area. Thus, the results were accurate only in areas that were well-watered. In an attempt to correct these deficiencies, Monteith added physiology and introduced an Ohm's Law analogy to Penman's model. By combining resistances associated with the physiology of the surface and with the turbulent structure of the atmosphere, Monteith was able to provide science with a more accurate model of local evaporation, as is described in the following paragraphs.

When an area is not well-supplied with water, the evaporation rate is much less than the potential evaporation predicted by the Penman equation. This difference is caused in two ways. First, there is much less water evaporated directly from the soil

surface as the surface dries out. Second, there is a limit to the amount of water that can be withdrawn by plants once the soil begins to dry. However, since plants can draw water from deep within the soil, it is possible that transpiration can continue after evaporation from the soil surface has ceased. In order to protect themselves when they are not well-supplied with water, plants will eventually close their stomata in order to restrict the movement of water from the plant to the atmosphere. Monteith (1965) developed the concept that the stomatal control of plants could be described as a resistance to water vapor transfer which is similar to an electrical system, and thus can be described by Ohm's Law. This is known as the "big leaf" concept. The "big leaf" concept treats a vegetative surface as ONE BIG LEAF, which has the CANOPY properties of conductance. In order to do this, one must assume that the overall effect of the whole canopy on the energy fluxes above the canopy can be reasonably approximated by assuming that all elements making up the vegetation are exposed to the same microclimate. Each leaf within a vegetative canopy has its own aerodynamic, or boundary layer, resistance based upon its shape and orientation, and its own leaf resistance (the inverse of conductance) to water vapor transfer based upon its physiology (i.e., the density and size of its stomata). The canopy aerodynamic resistance, r_a, replaces the wind speed function in Penman's model. This resistance governs aerodynamic transfer between a reference height in the atmosphere and a level within the canopy considered the bulk source for water vapor and sink for momentum transfer. The aerodynamic resistance is dependent upon the height and roughness of the vegetation in question, and on the horizontal wind speed. For the special case when the wind speed increases logarithmically with height (valid for uniform horizontal surfaces in neutral stability) the equation is:

$$\frac{1}{r_a} = g_a = \frac{k^2 * u(z)}{[\ln((z-d)/z_0)]^2} \qquad (1.4)$$

Thus, wind speed and the height of the vegetation play an important role in determining how much water can be evaporated from a canopy based on its general characteristics. In general, the canopy aerodynamic resistance is the summation of individual leaf boundary layers (determined primarily by leaf shape) within the canopy as well as the boundary layer from the top of the canopy to some reference height. This total resistance is parameterized by the terms z_0 and d described earlier. The aerodynamic conductance is then the inverse of this sum.

The second part of the resistance analog developed by Monteith involves the physiology of the plant itself. The canopy (physiological) resistance, r_c, consists of some combination of the individual leaf stomatal resistances. It is often roughly estimated as $r_c = [\sum(LAI_i / r_{li})]^{-1}$, i.e., assuming that the leaf resistances (r_i) can be combined in parallel. The leaf area index (LAI) is the amount of leaf area per square meter of ground. By using these assumptions, the "big leaf" can act as a single unit with **canopy** properties of reflectivity (albedo) and aerodynamic and canopy (physiological) conductance.

While this may seem to be an effective way of simplifying the complex transfer paths of a vegetation canopy, there are several problems with its development. It is difficult to separate values of g_c and g_a unless the apparent source of heat and water vapor is at the exact level of the apparent sink for momentum, which is not usually the case. In fact, the sources for heat and water vapor are often lower in the canopy than the sink for momentum. Since the "big leaf" model assumes a uniform horizontal canopy, there are problems with estimating the canopy conductance of a complex canopy (such as a forest) because of the effects of shading, uneven heating and the mix of species. These differences can lead to varying degrees of stomatal opening within the canopy. The more complex the canopy, the larger the error in assuming a single canopy conductance may become. Also, the assumption of the "big leaf" concept can only be

applied to a continuous, homogeneous canopy. This assumption causes problems when the canopy is sparse, because the equation does not consider the transfer processes from an understory or the soil. And, if the canopy is heterogeneous, it is difficult to determine the "average" leaf conductance or the leaf area index that should be applied in the equation.

On the other hand, the equation has strengths, which make it very useful. It is a simple way of including surface parameters in atmospheric models, especially in determining the partitioning of energy into sensible and latent heat. It also provides a link between plant physiology and atmospheric science without requiring detailed calculations of plant-water relations (although an understanding of the dependence of g_c on soil moisture and seasonal changes in physiology remains a problem). The "big leaf" model provides an adequate level of detail for testing atmospheric models and their responses to land surface changes. Its assumptions are valid for many regions, such as the Pacific Northwest, that contain large areas of relatively uniform vegetation. Finally, it is based on the physiological properties of particular canopies, i.e., it doesn't generalize to all vegetation types. This allows users to employ the model in many regions by making allowances for climate and vegetation type.

While Penman and Monteith are the scientists most intimately involved with the development of the Penman-Monteith equation, other scientists have also made improvements to the model. In fact, there have been several milestones in its development since the equation was first created in 1948. Some of these milestones are summarized in Table 1.2. These milestones serve to improve understanding of evaporation as well as improve the accuracy and applicability of the model itself. One of the most important milestones in furthering understanding of evaporative processes was the partitioning of evaporation into equilibrium and imposed evaporation. Equilibrium

Table 1.2 Milestones in the development of the Penman-Monteith Equation

1948	Penman develops the Penman equation, which is applicable for wet surfaces
1965	Monteith improves the model, making it applicable for dry surfaces and vegetation. While the equation was developed principally for agricultural crops, he realizes its potential for forests.
1967	Rutter applies the Penman-Monteith equation to forested areas.
early 1970s	Stewart and Thom develop the idea of "rapid evaporation" of intercepted water from canopies, thus describing an important factor causing differences between evaporation over grassland and forest.
early 1980s	Jarvis begins his own detailed study of the influence of vegetation structure on energy partitioning.
1983	McNaughton and Jarvis improve upon Jarvis' idea by developing the concept of "coupling" between vegetation and the atmosphere.

evaporation occurs when the saturation deficit of a reference level in the atmosphere equals that of the surface. This can occur when an air mass travels over a large homogeneous surface. It is the evaporation that can be calculated based upon the radiation load of an area. The surface does not have to be wet, nor does the air have to be saturated for this event to occur. When equilibrium evaporation dominates, the heat and mass transfer from the surface to the atmosphere is inefficient (Jones 1992). Underlying vegetation is not well-coupled to the atmosphere; this typically occurs over short grass. Imposed evaporation, on the other hand, is controlled by the physiology of the vegetation. It can be considered the evaporation caused by the lack of equilibrium between the vegetation and the atmosphere. In this case, leaf temperature approaches air temperature; evaporation occurs at a rate proportional to leaf conductance; and there is an efficient transfer of heat and water vapor. This case is important when the vegetation is well-coupled to the atmosphere, such as in forested regions. In an attempt to apply the Penman-Monteith equation at a larger scale, McNaughton and Jarvis (1984)

expanded these concepts into an equation using a "decoupling coefficient" (Ω), which expressed the degree to which vegetation was NOT coupled to the atmosphere. In this way they could easily partition evaporation into equilibrium and imposed evaporation. More will be said about this approach in the next section.

The most common form of the Penman-Monteith equation is as follows:

$$\lambda E = \frac{\Delta(R_n - G) + \rho c_p (e_s - e_a) g_a}{\Delta + \gamma (1 + \frac{g_a}{g_c})} \tag{1.5}$$

This is the equation which will be used in the development described in Chapter 2. The Penman-Monteith equation does have a few problems. Values of the canopy conductance are not unique to a canopy unless the sources and sinks of latent and sensible heat within the canopy have the same spatial distribution. Also, as with the Penman equation, it is impossible to completely separate the value of canopy conductance from that of aerodynamic conductance unless the sources and sinks of latent and sensible heat are found at the same level within the canopy. In spite of these objections, the Penman-Monteith equation has proved useful because it "provides a convenient framework for discussion of transpiration and evaporation from different kinds of vegetation such as tall, rough forest, arable crop, heathland and grassland" (McNaughton and Jarvis 1984). It also "gives a one-dimensional description of the inter-relationships between the main weather and vegetation variables at an appropriate level of complexity" (McNaughton and Jarvis 1984). The applicability and ease of use of the Penman-Monteith equation make it ideal for diagnostic purposes. While its use as a predictive tool is problematic, primarily because of the difficulty in estimating the canopy conductance, this thesis examines a way to combine this equation with others to provide an improved method for evaporation estimation.

1.5 The Priestley-Taylor Approach vs. Other Regional Approaches

The Penman-Monteith equation does not include the feedback terms that are a necessary part of the hydrologic cycle. Slatyer and McIlroy picked out this weakness (McIlroy and Angus 1964). They determined that the Penman-Monteith equation was driven by a prescribed set of atmospheric conditions defined by the user, not by large-scale atmospheric processes. This cannot be correct. In fact, large-scale atmospheric processes, local scale meteorology and local physiological variables all play a role in the evaporation process. In an attempt to correct for this weakness, Slatyer and McIlroy derived a form of the Penman equation in which the adiabatic term (the second term) is obtained by assuming that a parcel of an initial wet bulb depression B makes contact with the surface and cools adiabatically until it reaches equilibrium with the surface in terms of temperature and vapor pressure (Monteith and Unsworth 1990). At this time, it has a smaller wet-bulb depression B_0. B (the initial wet bulb depression) is related to D (the vapor pressure) by the equation $D \sim B(\Delta + \gamma)$. This draws attention to the fact that there is a lack of equilibrium between the state of the atmosphere at the reference height and the state of the air which is in equilibrium with the surface. This leads to a large-scale Penman-Monteith equation derived by Slatyer and McIlroy:

$$\lambda E = \frac{\Delta R_n + \rho c_p (D - D_0) g_a}{\Delta + \gamma} \tag{1.6}$$

In this equation D_0 (and thus B_0) depends not only on the surface resistance to vapor transfer, but also on prevailing weather. This equation is not used often, because it is very difficult to obtain information on the actual equilibrium vapor pressure at the surface.

In a different attempt to apply the Penman-Monteith equation at the larger scale, McNaughton and Jarvis further expanded the idea of equilibrium evaporation (see Table 1.2). In their work, they separate evaporation into two parts, equilibrium (E_{eq}) and

imposed evaporation (E_{imp}). McNaughton and Jarvis also defined a decoupling coefficient (Ω), which partitions the evaporation between the two parts. Their equation is as follows:

$$\lambda E = (\Omega)\lambda E_{eq} + (1 - \Omega)\lambda E_{imp} \qquad (1.7)$$

Typical values of Ω are 0.1-0.2 for forests, implying that they are highly *coupled* to their environment and imposed evaporation dominates, and 0.8-0.9 for grass, implying that grass is highly *decoupled* from its environment and equilibrium evaporation plays a dominant role. Ω is an empirically derived term, and has good potential for linking larger scale processes to evaporation equations.

There is also a slightly different way of looking at equilibrium evaporation. In this case, equilibrium evaporation is a special case of heat transfer from a fully wet surface into a saturated atmosphere (Monteith 1994). As mentioned already, the second term of the Penman-Monteith equation, the adiabatic term, can be thought to represent the lack of equilibrium between the state of the atmosphere at a reference height and the state of the air which is in equilibrium with the surface (Monteith and Unsworth 1990). Priestley and Taylor extended the reasoning of Slatyer and McIlroy to suggest that air moving over an extensive area of uniform wetness should come into equilibrium with the surface (i.e, $D - D_o = 0$), so that the equation becomes (Monteith and Unsworth 1990):

$$\lambda E_{eq} = \frac{\Delta R_n}{\Delta + \gamma} \qquad (1.8)$$

Priestley and Taylor determined that over land the "local value of net radiation controls evaporation and heat flux into air" (Priestley and Taylor 1972). After an exhaustive review of measurements over open water and well-watered vegetation, they determined that actual evaporation exceeded equilibrium evaporation by 20-30%. Thus, they developed

their own equation for regional evaporation in which

$$\lambda E = \alpha \frac{\Delta R_n}{\Delta + \gamma}.$$ (1.9)

It now appears that actual evaporation exceeds equilibrium evaporation because the size of the "constant" α apparently depends upon the way in which the convective boundary layer receives dry air from above by entrainment and increases in depth during the day due to heat input from above and below (Monteith and Unsworth 1990). The Priestley-Taylor equation works well in temperate climates and was thought at one time likely to supersede versions of Penman's equation, since it appeared to be more convenient and equally reliable (Monteith 1994).

However, there are some problems with the model. The research done since it was first proposed has revealed that there are a wide variety of values for α for different surfaces and under different atmospheric conditions, particularly for very rough canopies and for dry conditions. This is because the equation takes no account of the aerodynamic or physiological properties of the underlying vegetation. It also assumes that the canopy is well-watered, thus ignoring conditions in drier areas. It does not make sense to adjust the equation by incorporating a variable value for α, because such adjustment would be the same as developing an inferior version of the Penman equation (Monteith 1994). While the equation may be very effective in estimating potential evaporation across a large area where detailed measurements are difficult to make, it is not very effective at the local scale. It is an improvement over the Slatyer-McIlroy equation in that it considers the large scale atmospheric processes, yet is easy to use in the computation of evaporation. Therefore, it is the best regional model of evaporation to consider in any further development.

1.6 Convective Boundary Layer Models

There have been many models developed that have attempted to estimate the fluxes within the Atmospheric Boundary Layer. There are numerous definitions of the Atmospheric Boundary Layer; in this thesis the term refers to the layer of air within the troposphere which is closest to the earth's surface, in which the effects of the surface, such as friction, heating and cooling, have direct influence on time scales shorter than a day. The major fluxes of momentum, heat and matter are carried away from the surface through turbulent motions with length scales that are on the order of the depth of the boundary layer or less (Garratt 1992). These turbulent motions can be caused by wind shear (mechanical turbulence) or heating (thermal convective turbulence). The Atmospheric Boundary Layer varies in height from about 50 meters to 5 kilometers depending upon the characteristics of the air above it and the amount of diurnal heating or cooling that it receives. It is limited in depth by the development of a capping inversion above it caused by buoyancy forces and by the surface of the earth below it. A special case of the Atmospheric Boundary Layer is the Convective Boundary Layer. This layer occurs when strong surface heating causes thermal instability or convective plumes. In these conditions, the capping inversion is well-defined by its stability. Turbulence from the convective plumes erode the base of the capping inversion, causing the entrainment of air from above the inversion.

In 1976 McNaughton developed a boundary layer model of evaporation. Perrier followed with a similar model in 1980. Both McNaughton's and Perrier's models considered an atmospheric boundary layer model of constant height. However, there are problems with this assumption because radiation is continuously supplied to the surface without allowing the heat to escape through turbulence or boundary layer growth. If the boundary layer is not allowed to grow within the model, the heating can lead to

unrealistically high modeled temperatures within the Convective Boundary Layer. This error leads to incorrect development of the surface fluxes of heat, mass and momentum, and such models do not explain why the Priestley-Taylor coefficient can be found to be greater than unity over wet surfaces and less than unity over dry surfaces.

Thus, De Bruin (1983) coupled a more rigorous Atmospheric Boundary Layer model to the Penman-Monteith equation in order to obtain a physically realistic model. He wanted to analyze the behavior of α (the Priestley-Taylor coefficient) and to determine its sensitivity to entrainment from above. De Bruin's model allows the Convective Boundary Layer to be variable in height. It also allows the entrainment of drier air from above as the Atmospheric Boundary Layer grows. De Bruin's model also includes the dynamics of a well-mixed boundary layer. By assuming a well-mixed boundary layer beneath the capping inversion, he is able to avoid a mathematical description of the turbulence within a boundary layer that is not well-mixed. By coupling an existing Atmospheric Boundary Layer model to the Penman-Monteith equation, De Bruin developed a physically realistic model. Unfortunately, the resulting equations proved too time-consuming for use in modeling. As a simplification to allow rapid solution of the model, De Bruin assumed that the flux of water vapor at the top of the Atmospheric Boundary Layer was proportional to surface evaporation. He also chose to confine his model to complete vegetation coverage and typical summer weather conditions in temperate latitudes (i.e., clear and sunny). The assumption of proportional vapor fluxes is unlikely to be generally true, but the assumption worked in this case because of the vegetation and weather conditions chosen in running the model (De Bruin 1983). His model neglects several items, which he viewed as unimportant for his analysis. They are: the surface layer dynamics (the surface layer is small when compared to the height of the boundary layer), the effects of horizontal advection, large scale vertical motions and the

divergence of radiation (i.e., radiation absorbed by aerosols and gases in the Convective Boundary Layer).

With his model De Bruin was able to obtain a good statistical fit between the model results and observed data from studies such as those by Shuttleworth and Calder (1979) and Gash and Stewart (1975) over many types of vegetation from grasses to forests. The values of α that he calculated agreed with those in the literature. He determined that α is dependent upon wind speed, surface roughness and entrainment of dry air, but that the surface resistance is the primary determinant of the value of α on clear days (i.e., α is low when the surface conductance is low). De Bruin remarked that while α decreases with increasing wind speed, this observation is insignificant in comparison to the effects of surface conductance. Thus, De Bruin determined that the dependence of α on wind speed could be largely ignored. The value of α did not appear to be strongly dependent on entrainment from above the Atmospheric Boundary Layer. De Bruin's (1983) analysis supports the use of a Priestley-Taylor-like parameterization for surface fluxes in cases where advection can be ignored (i.e., on the larger scale). De Bruin explained that his model is a model for predicting α and is not a complete model of the Atmospheric Boundary Layer or of surface fluxes.

In 1986 and 1989 McNaughton and Spriggs published results from a Convective Boundary Layer model that they had developed. Their model is also a mixed-layer model to estimate regional evaporation. They stated that while α is based on a plausible physical development and that it has had some empirical success, it cannot be derived entirely from physical principles, leaving doubts about it reliability and applicability (McNaughton 1989). The McNaughton-Spriggs model is very similar to that of De Bruin. They include a method of boundary layer growth and entrainment at the upper level of the Atmospheric Boundary Layer. In their model they assume that the major characteristic of

the Convective Boundary Layer is that its daily growth causes erosion at the base of the capping inversion. This erosion enables the boundary layer to incorporate warmer, drier air into itself. Unlike the De Bruin model, which includes a humidity budget equation to estimate entrainment, the McNaughton-Spriggs model contains a physically-derived entrainment term (McNaughton and Spriggs 1986). Like De Bruin, they assume complete vegetation coverage, thus assuming that transpiration is the only component of evaporation (McNaughton 1989). McNaughton and Spriggs include a term which explains the sharp gradients of temperature and humidity in the "thin" layer at the surface. This is a term ignored by De Bruin as being inconsequential. The model used by McNaughton and Spriggs allows the interpretation of effects of large scale conditions on evaporation rates.

The model developed by McNaughton and Spriggs also lends support to the use of the Priestley-Taylor coefficient α at the larger scale. While they show that the Priestley-Taylor equation is not an exact relationship but rather an empirical one because α varies widely around the assumed value of 1.26, they determine that the factor 1.26 is adequate to determine the potential evaporation of a region. This model has also been presented as an improvement to De Bruin's model. McNaughton and Spriggs (1986) state, however, that while their model is a more physically sound model, they are unable to state with conviction that their predictions of evaporation are better, because the results are similar. Both models improve scientific understanding of the use of the Priestley-Taylor coefficient by providing a more stable physical basis for its development. Monteith uses the results of both of these models to combine the Penman-Monteith equation with the Priestley-Taylor equation to include feedbacks within the atmosphere.

One of the newest Atmospheric Boundary Layer models to be developed is that of Ek and Mahrt (1991). This model closes the system of equations by including a simplified

vegetation and soil scheme within a detailed boundary layer model. This model includes turbulent mixing, and therefore does not require the assumption of a well-mixed atmosphere. The model allows a surface exchange of sensible and latent heat fluxes between the surface layer of the Atmospheric Boundary Layer and the underlying surface. This model also has the capability to model clouds within the boundary layer. This model is currently in use in many research offices throughout the United States. Further work with this model appears promising. Currently, however, the development of the equations further used in this Chapter will rely on the work done by De Bruin and furthered by McNaughton and Spriggs.

Chapter 2 - Development of the "PMPT" Model

Recent research by Monteith (1995a,b) suggests ways in which physiological and physical feedbacks between vegetation and the atmosphere interact to stabilize transpiration rates from vegetation. He develops this model by assuming that leaf conductance is a function of transpiration, arguing that this relationship is supported by the literature and leads to simpler coupled vegetation-atmosphere models. In this section, the "PMPT" model, based on Monteith's research, will be developed. The two primary evaporation models employed in this development are the Penman-Monteith equation, an approach combining the surface energy balance with the aerodynamic and physiological properties of vegetation, and the Priestley-Taylor equation, a large-scale approach to evaporation containing no vegetative characteristics. Two new relationships, based on the Convective Boundary Layer models described previously, will be derived as well. Monteith's method of combining the new relationships with the evaporation equations in order to develop a coupled land-atmosphere model will be reviewed. Finally, the model, named PMPT here, will be modified for forest canopies; first by considering how to parameterize a coniferous forest as a land surface, and second by coupling the land surface to the Atmospheric Boundary Layer.

Monteith's recent research derives from his longstanding interest in fully understanding the water balance of vegetation and the interaction between vegetation and the atmosphere. The system in which plants survive has four main regimes: the root zone, the foliage, the atmospheric surface layer and the mixed layer. There is a supply of water to the plant from the soil (the root zone) and a demand for water from the atmosphere (the mixed layer). The plant must balance this supply and demand, and combine it with its own requirement for nutrients obtained from both the soil and the

atmosphere. A complex process has developed within vegetation to meet these challenges. Through this process, plants accommodate and affect both the soil and the atmosphere. For example, as an area of vegetation begins to run short of water, the stomata begin to close. This reduces the transpiration rate, but increases the input of sensible heat into the Convective Boundary Layer to preserve the energy balance (the total amount of energy in the system must remain constant). The increase of sensible heat causes the Convective Boundary Layer to become warmer and drier. The boundary layer depth grows faster and the boundary layer becomes even drier because of increased entrainment from above the inversion. Thus, the demand for water increases. Plants respond by closing their stomata even further, creating a positive feedback.

In order to illustrate Monteith's model of vegetation-atmosphere interaction over grassland and agricultural crops, this chapter will concentrate on two ideas: (1) the parameterization of surface conductance dependence on environmental factors and (2) the dependence of Convective Boundary Layer behavior on surface conductance (Monteith 1995a). Then, the ability of this model to accurately parameterize forest evaporation will be examined and the forest parameters useful for the model will be determined.

In order to understand the new relationships that will be developed later in the chapter, it is necessary to explain the current knowledge in the field. Many researchers have considered the relationship between leaf conductance and vapor pressure deficit. It has been well-established that leaf conductance decreases when leaves are exposed to dry air. For example, Bunce (1985) shows this relationship for three separate species of plant. This relationship can also be seen in Jarvis' work for a forest canopy; it holds both before and after the thinning of a forest (Jarvis 1993). It has been commonly assumed that models of leaf conductance should contain a function of vapor pressure deficit or

relative humidity. This suggests that as the vapor pressure deficit of the air increases and the air becomes drier, plants will begin to close their stomata in order to decrease water loss to the air. Stomatal closure leads to a lower conductance to water vapor loss. Less water vapor is lost to the atmosphere and the atmosphere becomes drier as a result. One of the models developed to attempt to explain this process is the Ball-Berry model of stomatal mechanisms.

2.1 The Ball-Berry Model of Stomatal Response

Ball et al (1987) were unable to establish a concise description of stomatal response to various combinations of environmental factors. Therefore, they decided to link a model of stomatal conductance with a model of carbon dioxide assimilation (as a function of intercellular CO_2). By doing this, they were able to predict the distribution of the flux control between the stomata and the leaf "biochemistry" under the conditions found in a gas exchange cuvette (Ball et al 1987). Several patterns began to emerge as they tested the model under varying conditions of CO_2 and relative humidity (all other factors were held constant). They found that as CO_2 concentration increases, stomatal conductance tends to decrease and CO_2 assimilation tends to increase. They also noticed that stomata tend to close (i.e., stomatal conductance increases) with decreasing air humidity, while the assimilation rate (or rate of photosynthesis) tends to remain the same. With this information, they developed an empirical model which can account for the variations in stomatal conductance as several environmental factors change at the same time. The model is:

$$g_{sw} = kA\frac{h_s}{c_s} \tag{2.1}$$

According to Ball et al (1987), this model is able to give new fundamental insights

into the nature and interaction of stomatal responses, especially to humidity. Their model can also be expanded in order to include additional components of the plant system, such as the Atmospheric Boundary Layer or plant biochemistry. The model can be developed to illustrate all of the interactions among the components of the system.

Unfortunately, this model has several problems. First, as stated in Chapter 1, there is still no known mechanism within plants for sensing the relative humidity of the atmosphere or the mole fraction of CO_2 at the leaf surface. Therefore, Leuning (1995) improved upon the physics of the model by employing values of atmospheric CO_2 concentration at the leaf surface rather than intercellular CO_2 concentration at the leaf surface, thus obtaining more accurate results. Leuning also replaced the relative humidity of the equation with the vapor pressure deficit between the leaf surface and the atmosphere, such as can be seen in Equation 2.2.

$$g_{sw} = g_0 + \frac{kA}{(c_s - \Gamma)(1 + \frac{D_s}{D_0})} \qquad (2.2)$$

Leuning showed that this change improved the ability of the model to describe observed data. Equation 2.2 also allows the transpiration rate to decrease with increasing vapor pressure deficit.

Another problem with the Ball-Berry model was examined by Aphalo and Jarvis (1993). They showed that A (the CO_2 assimilation rate) and g_{sw} (the stomatal conductance to water vapor) are interdependent, e.g., the value of one cannot be determined without also determining the value of the other. The model, therefore, contains two unknowns but only one equation. In employing this model to determine g_{sw} as a function of A, therefore, Ball and Berry are using it as a causal model, when it is in fact a functional relationship *between* the assimilation rate (A) and the stomatal

conductance (g_{sw}). Alternatively, if one looks at the ratio of g_{sw} to A, one can begin to make mechanistic interpretations of the results. Such a model can be seen in Equation 2.3, which is very similar to Equation 2.1.

$$\frac{g_{sw}}{A} = k\frac{h_s}{c_s} \qquad (2.3)$$

In summary, while the Ball-Berry model is fairly successful in describing the responses of the stomata of many plant species to the variations of such variables as CO_2 assimilation, vapor pressure deficit and CO_2 concentration at the leaf surface, it has several fundamental problems. First, it includes the response of g_{sw} to relative humidity, even though no mechanism has ever been found within plants to explain the response to relative humidity. Second, it is not a model of g_{sw}, but rather a model of the interdependent relationship between g_{sw} and A, and therefore cannot define a causal relationship between the two. The model is also limited to a restricted set of conditions: those in which g_{sw} increases with increasing temperature. The model's correlation to assimilation rate (A) is seemingly simple, but in actuality, a complex model is needed to simulate the value of A if the model is to be driven by environmental variables alone. A better understanding of the coupling behavior between the CO_2 assimilation rate, stomatal conductance and water stress is needed in order to improve upon the Ball-Berry model. Therefore, while such a model cannot be used in determining the mechanistic behavior of plants, it can be useful in *empirically* describing the variation of stomatal behavior over a range of stomatal conditions.

2.2 The Dewar Model

One of the problems in using the Ball-Berry model is that there is no mechanistic interpretation of stomatal response. Dewar (1995) proposed a mechanistic interpretation

of the stomatal response to the environment in terms of the guard cell function. The guard cells are those cells surrounding the stomatal pore. Changes in the turgor (or pressure) of the guard cell causes changes in the stomatal pore aperture. When the turgor is high, the guard cells force the stomatal pore to open, but if the turgor is low, then the guard cells deflate, similar to a balloon, and the stomatal pore will close. There are two reasons why guard cell turgor can change: (1) there can be a change in the total water potential of the guard cell (i.e., a change in the supply of water to or loss of water from the cell) or (2) there can be an active change in the osmotic potential of the guard cell. The first change is based upon environmental conditions, while there may be several causes for the second change, including a response to photosynthesis. An interpretation of this type of response is useful because it supports the current empirical models of stomatal conductance. Dewar used the equations employed by Ball and Berry (based in part upon relative humidity) and Leuning (essentially the Ball-Berry equation, replacing relative humidity with vapor pressure deficit). In particular, the success of the Leuning equation (see Equation 2.2) in well-watered plants gave Dewar the insight to attempt to gain a better understanding of the underlying mechanisms of the stomata to the guard cell function. The stomatal responses to vapor pressure deficit and leaf surface CO_2 concentration found in Leuning's equation cannot be interpreted directly in terms of guard cell mechanisms (i.e., there is still no mechanism within plants to explain a stomatal response to humidity). However, Mott and Parkhurst (1991), in an experiment explained in the next section of this chapter, have demonstrated that stomata do respond directly to transpiration *rate*. And, prior to this particular experiment, Mott had shown that stomata respond to intercellular CO_2 (Mott 1988). Dewar then recast Leuning's equations, replacing the vapor pressure deficit with the evaporation rate and the leaf

surface CO_2 concentration with the intercellular CO_2. Dewar's equation is

$$g = g_m (1 - \frac{\lambda E}{\lambda E_m})$$
(2.4)

This equation defines a negative-linear relationship between the stomatal conductance, g, and transpiration rate, λE. This is only true if the assimilation rate, A, (and hence, λE_m) does not change as the humidity varies. This relationship is actually only approximately linear, because A will eventually begin to decrease as the vapor pressure deficit increases and the stomatal conductance decreases. It becomes extremely non-linear if the transpiration rate exceeds λE_m and may even begin to decline with any further increase in the vapor pressure deficit (known as a feed-forward response), creating a hyperbolic curve. However, the linear assumption is appropriate because the relationship between the stomatal conductance and the transpiration rate is linear over much of the range of interest.

This new equation gives the clearest insight developed thus far into the nature of the mechanisms underlying the relationship among stomatal conductance, CO_2 and vapor pressure deficit. The mechanical response of stomatal aperture to the relative turgor of the guard cells and the epidermal cells demonstrates that stomatal movements (i.e., opening and closing) result from changes in the relative turgor of these cells. If we assume uniformity of stomatal response across a leaf surface, we can easily scale from stomatal aperture to leaf conductance. Dewar also examined an empirical relationship in which g_m and E_m are directly proportional to the osmotic gradient between the guard cells and the epidermal cells. This osmotic gradient develops as both types of cells compete for the same available water supply. The regulation of this osmotic gradient is not yet well-understood, but Dewar's research suggests that the guard cell itself drives the

action, varying with light and CO_2. More research needs to be done in this area to fully understand the mechanisms involved in stomatal responses.

2.3 Mott and Parkhurst - Experimental Verification that Stomata Respond to Transpiration Rate

Mott and Parkhurst designed an experiment in order to determine whether plants do in fact respond to humidity, as has been previously discounted, or to transpiration rate. In order to distinguish experimentally between a plant's responses to the two factors, a method needed to be developed in which the factors could be varied independently of each other while keeping the leaf temperature constant. They could not study their plants in ambient air without varying the leaf temperature, because the diffusional loss of water by a plant in ambient air is uniquely related to the mole fraction of water vapor at a given stomatal conductance. Consequently, Mott and Parkhurst placed their plants in a helium and oxygen mixture known as helox (to which CO_2 and water vapor were added in order to match the values found in the ambient air sample). In helox, molecular diffusion occurs 2.33 times faster than in ambient air due to differences in the molecular weight of the two gases, while the mole fraction of water vapor remains the same as in ambient air. By using helox and ambient air, the researchers were able to keep the leaf temperature constant and vary the transpiration rate and humidity independently of each other. In order to use helox, Mott and Parkhurst (1991) also assumed that helium would have no other effects on the physiology of the plant, an assumption which has been validated experimentally.

Two experiments were performed on the plants that were held in the helox mixture. In the first experiment the leaf was brought to a steady state in ambient air. Then, helox was substituted for that air. The leaf temperature, intercellular CO_2 and *water vapor concentration* were kept constant at the values found in the ambient air.

During the adjustment period, the transpiration rate and the stomatal conductance both increased to more than two times their ambient air values. When equilibrium was achieved in the helox, the transpiration rate and stomatal conductance were still slightly higher than those found in air, but the relative stomatal aperture was only one-half that of the leaf in ambient air. Once the leaf was returned to ambient air and equilibrium was achieved, the values were roughly the same as they were prior to the exposure to the helox mixture. Therefore, in this experiment, while there was no change in the vapor pressure deficit, there was a noticeable change in transpiration rate (due to the change in the diffusivity of water vapor). This led to a reduction in stomatal aperture.

In the second experiment the leaf was brought to a steady state in ambient air. When the helox mixture was added, the leaf temperature and intercellular CO_2 were again held constant, but the vapor pressure deficit was lowered until the *transpiration rate* was the same as in ambient air. In this case, the stomatal conductance increased, but the stomatal aperture (opening) remained unaffected. In order to determine if the stomata were truly responding to the transpiration rate, Mott and Parkhurst then verified that the response of stomatal aperture to the vapor pressure deficit of the air was the same as the response to 2.33 times the vapor pressure deficit of the helox. There was no difference in the response when this test was performed, leading to the conclusion that stomatal aperture and stomatal conductance are truly determined by the rate of transpiration and not by either relative humidity or vapor pressure deficit.

In light of these findings, Mott and Parkhurst (1991) concluded that any model similar to the Ball-Berry model that relies on humidity or vapor pressure deficit in order to calculate stomatal conductance is completely empirical and cannot describe the mechanisms behind such a relationship. The work done by Dewar (1995) and described in Section 2.2 supports this research done by Mott and Parkhurst. In its defense, the

Ball-Berry model includes both photosynthesis and CO_2 concentration and is uniquely able to integrate the response of stomata to both temperature and water vapor concentration. The model is therefore still useful for predicting stomatal conductance in fluctuating environmental conditions.

2.4 Monteith - Interpretation of Leaf Data

The surface conductance of vegetation plays a major role in determining how the net supply of energy to a system is partitioned between latent heat flux and sensible heat flux. This surface conductance, g, is often estimated by inverting the Penman-Monteith equation to solve for it. It is also estimated by measuring the stomatal conductance of individual leaves within a canopy and then extending these measurements to the canopy scale. Both types of estimates are problematic, but in order to proceed, we must assume that canopy conductance is determined primarily by stomatal behavior at various levels within the canopy and that the bulk behavior of these stomata is similar to the behavior of a single leaf (the "big-leaf" principle), despite the wide variations found within the canopy. If these assumptions hold, then the environmental factors governing stomatal behavior in the lab should be the same factors that govern stomatal behavior in the field.

Many researchers have attempted to model stomatal behavior in the lab and in the field. Jarvis et al (1976) has suggested that the environmental factors can be treated individually by writing the stomatal conductance, g_l, of a single leaf as

$$g_l = g_{lm} f_1(S) f_2(T) f_3(D) f_4(\psi) f_5(c_e) \tag{2.5}$$

here f_1, f_2, ..., f_5 are functions of incoming solar radiation, air temperature, vapor pressure deficit, leaf water potential and external CO_2 concentration. This equation was derived from independent sets of laboratory data and major tests of its validity in the
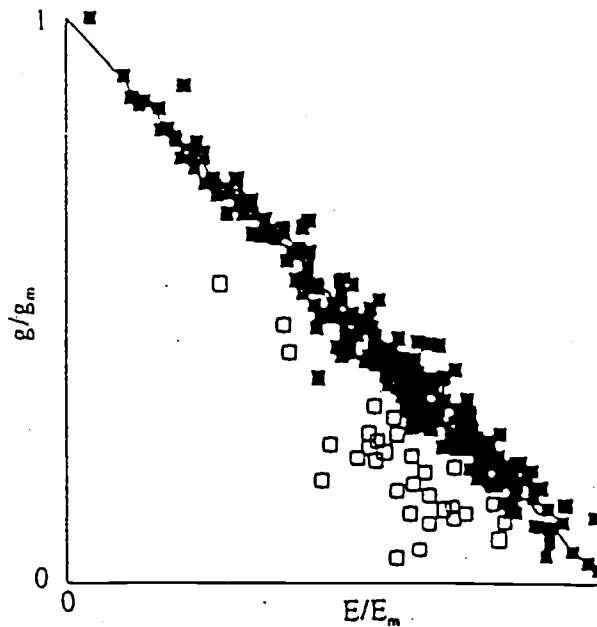
Figure 2.1 The apparent dependence of leaf stomatal conductance (g) on transpiration rate (λE). These data are from laboratory measurements on single leaves (reprinted with permission from Monteith 1995a).

field have been confined to individual stands of vegetation. In this case, the leaf

conductance (g_l) becomes the canopy conductance (g_c) and the leaf water potential

becomes the average soil water content. The external CO_2 concentration is omitted from

the canopy level equation because it is approximately constant at this level during daylight

hours. The major weakness of this equation is its assumption that all of the involved

parameters operate independently. While the equation has not been rigorously tested,

work done with the equation shows great consistency. Currently, however, the equation

is largely empirical, but containing a sound mechanistic basis for most of the parameters

involved.

Equation 2.5 employs a function of vapor pressure deficit, even though it has already been stated that there is no physiological sensor for such a relationship. In fact, almost every set of measurements showing a non-linear relation between leaf conductance, g_l, and vapor pressure deficit, D, can be reinterpreted as a linear relation between g_l and evaporation, E, in the form

$$g_l = a - bE \tag{2.6}$$

(Monteith 1995). The constant a is the maximum conductance that can be achieved when transpiration is zero. It can therefore be written as g_{lm}, and described as in Equation 2.5. As g_l decreases and reached a theoretical limit of zero, transpiration increases to a maximum of $\dfrac{a}{b}$, also written as E_m. Equation 2.6 can then be rewritten as

$$g_l = g_{lm} - bE \tag{2.7}$$

The equation can be rewritten non-dimensionally and in terms of transpiration rate as

$$\frac{g_l}{g_{lm}} = 1 - \frac{\lambda E}{\lambda E_m} \tag{2.8}$$

This equation, based upon synthesis of laboratory measurements, is identical to Equation 2.4, which was derived theoretically by Dewar. Further evidence of its validity is found in Figure 2.1. Figure 2.1 is a collection of the measurements that originally purported a non-linear relationship between the leaf conductance and the vapor pressure deficit. As stated earlier, these points were all replotted by Monteith (1995) as a linear relationship between leaf conductance and transpiration rate. The anomalous points on this graph (□) are points at which the transpiration had reached its maximum prior to the time at which $g_l \rightarrow g_{lm}$. At this point, any further increase in the vapor pressure deficit is accompanied by a decrease in both stomatal conductance and transpiration rate. This anomalous effect could be caused by patchy closure of stomata within the canopy in response to stress.

Equation 2.8, like the Jarvis model mentioned earlier (Equation 2.5), can be used to explore the response of canopy conductance to environmental factors if the leaf conductance is replaced by the canopy conductance, g_c. In this case, the maximum canopy conductance, g_m, is a function of solar radiation, temperature and CO_2 concentration, and the equation becomes

$$\frac{g_c}{g_m} = 1 - \frac{\lambda E}{\lambda E_m} \qquad (2.9)$$

All of the species for which Equations 2.8 and 2.9 were originally developed were agricultural crops and shrubs. In order for these equations to be applied to conifers, its validation among conifers must be determined.

There is no direct evidence that Equations 2.8 and 2.9 apply to conifers, but there is strong circumstantial evidence to suggest that it is applicable. All of the species considered in Monteith's graphical development of the equation are C_3 plants. In photosynthesis, C_3 plants fix CO_2 with the enzyme RuBP carboxylase. The first stable product formed during this process is phosphoglyceric acid (PGA), which has a skeletal structure consisting of three carbon atoms. Alternatively, C_4 plants fix CO_2 using the enzyme PEP carboxylase to form four-carbon acids such as malate or asparate. Photosynthesis then proceeds in exactly the same way in both C_3 and C_4 plants. The CO_2 compensation point (the point at which the CO_2 concentration is high enough to support net photosynthesis) can be used to distinguish between C_3 and C_4 plants (Barbour et al 1987). Physiologically, all C_3 plants behave in the same general way in response to environmental factors. Therefore, since conifers are C_3 plants, the equation should be applicable. Also, one must further consider the "independent" development of the equation by Dewar. His equation is based on the physiological response of stomata to transpiration rate. His work is based on that of Leuning (1995) and Mott and Parkhurst

(1991) (explained earlier in Chapter 2). Their work was also performed on various C_3 plants. This fact also suggests that the equation can be applied to conifers with little reservation. Before considering the validity of this equation for C_4 plants, however, more research will need to be done.

2.5 The Modified Penman-Monteith Equation

In the previous section, it was determined that leaf conductance is proportional to the transpiration rate. The first major assumption in the modification of the Penman-Monteith equation is that this proportionality of conductance to transpiration rate holds at the canopy level (i.e., for canopy conductance) as well. This is a huge "leap of faith". However, there is good reason to believe that it is a valid assumption. Many different properties are accurately scaled from the leaf to the canopy in the development of the "big leaf" model (explained in Chapter 1). Therefore, there is reason to believe that the scaling should hold for the conductance/transpiration relationship as well. In addition, forests are well coupled to the atmosphere, and thus, the transpiration rate is sensitive to the value of canopy conductance (and vice versa). The second half of the Penman-Monteith equation (from Equation 1.6), containing the canopy conductance, usually dominates the equation when applied to forests. Since the assumption of scaling has been proven to work for grasses and short crops, which are not well-coupled, it should work effectively for more coupled systems such as forests. The more strongly coupled a canopy is, the more similar its environmental responses are to those of a leaf in a cuvette (the system used for laboratory studies of stomatal conductance).

The next consideration in the development of the Modified Penman-Monteith Equation is the type of forests to which it would be applicable. It would easily apply to a dense forest, where the canopy conductance is a good indicator of the total evaporation.

Sparse forests are more problematic because canopy conductance is then only the conductance from the tallest vegetation. The understory will have a completely different conductance, and soil evaporation will begin to play an important role in total evaporation and can no longer be ignored as being insignificant. The forest also needs to be homogeneous, so that the "big leaf" assumption applies. Thus, the following development will only be considered for a homogeneous, dense coniferous forest.

In order to modify the Penman-Monteith, it is combined with Equation 2.9. Again, the Penman-Monteith equation is

$$\lambda E = \frac{\Delta H + \rho c_p g_a D}{\Delta + \gamma (1 + \frac{g_a}{g_c})} \tag{2.10}$$

Writing $N = \Delta H + \rho c_p g_a D$ in Equation 2.10, g_c can be eliminated between Equations 2.9 and 2.10. Solving for $\frac{\lambda E}{\lambda E_m}$, a quadratic equation in x (substituted for $\frac{\lambda E}{\lambda E_m}$) is obtained:

$$0 \, (Zero) = ax^2 + bx + c \tag{2.11}$$

where: $a = g_m (\Delta + \gamma) E_m$
$b = -(a + c + \gamma g_a E_m)$
$c = (g_m N) / \lambda$, where N is defined above

The appropriate root of this equation is

$$\lambda E = \frac{\lambda E_m [-b - (b^2 - 4ac)^{\frac{1}{2}}]}{2a} \tag{2.12}$$

Equation 2.12 enables the user to develop relationships between the canopy conductance and the transpiration rate for a range of values of solar radiation and vapor pressure deficit.

2.6 Numeric Values for a Coniferous Forest

In order to apply this and subsequent models to a coniferous forest, values of the

Priestley-Taylor coefficient (α), the aerodynamic conductance (g_a), the maximum canopy

conductance (g_m) and the maximum latent heat flux (λE_m) will have to be developed. A

search of the literature available on the subject resulted in the values for various conifers

listed in Table 2.1. Some of these values will be used in Chapter 3 to calculate the

canopy conductance as a function of the transpiration rate. They will also be used to

mathematically use and test the theory developed in this section. This forest model can

only be applied to a homogeneous, dense coniferous forest for the reasons listed at the

beginning of this section.

2.7 Graphical Analysis of the Modified Penman-Monteith Equation

Figure 2.2 illustrates this type of relationship for a Douglas fir forest in the

Cascade Mountain Range. In this figure, canopy conductance is considered only in

relation to solar radiation. As a working relationship it is considered proportional to solar

radiation up to a value of 20 mm s^{-1} when the solar radiation is 400 W m^{-2}, after which it is

Table 2.1. The list of values for coniferous forests to be used in the Modified Penman-Monteith and the PMPT Models. These data were collected from Kelliher et al (1993), Kelliher et al (1995) and Schulze et al (1994). The values for short vegetation (Monteith 1995) were added for purposes of comparison.

Vegetation type	α	g_a (mm s^{-1})	g_m (mm s^{-1})	λE_m (W m^{-2})
Douglas fir	1.05	17	17	350
Sitka spruce	1.25	20	25	365
Norway spruce	1.25	20	25	365
Scots pine	1.49	20	25	255
Short vegetation (grass)	1.1	5	25	500

taken as constant. However, the linear relationships that g_m has with both temperature and carbon dioxide could be added to this relationship if necessary. λE_m is assumed to be 500 W m^{-2}, a value supported by many experimental measurements. Figure 2.2 has several benefits. First, the equation upon which it is based is simple to solve. Second, the model includes changes in climatological conditions, illustrating the sensitivity to each change. Finally, the model can apply for a range of physiological and atmospheric conditions based upon known factors. Figure 2.3 shows how a series of points, shown in Table 2.2, may be represented on the graph. At 7 am, the "physiological" day begins as the stomata open when the solar radiation nears 100 W m^{-2}, and the vapor pressure deficit is very small. By 8 am, the solar radiation has increased, and the vapor pressure deficit increases slightly. This causes a small change in the latent heat flux (horizontal axis), but a significant change in the canopy conductance (vertical axis). At 9 am, g_m is maximized and any further increase in solar radiation will not increase its value. This limit is expressed on the graph as a line giving maximum values where E_m and g_m where it crosses the x- and y- axes respectively. By 10 am, vapor pressure deficit (D) has increased by 0.3 kPa, and transpiration has increased considerably, as has canopy conductance. Later, transpiration would be expected to increase in response to increasing solar radiation, but this tendency is offset by a decrease in stomatal conductance (i.e., a closing of the stomata), such that the physiological maximum is not exceeded. The vapor pressure deficit increases, the latent heat flux increases more slowly, and the stomata continue closing in response to the increasing transpiration rate. In the evening (around 6 pm), as the sun begins to set and the solar radiation decreases g_m decreases again. The vapor pressure deficit also begins to decrease, and the latent heat flux and canopy conductance decrease in response. Finally, at 7 pm, the solar
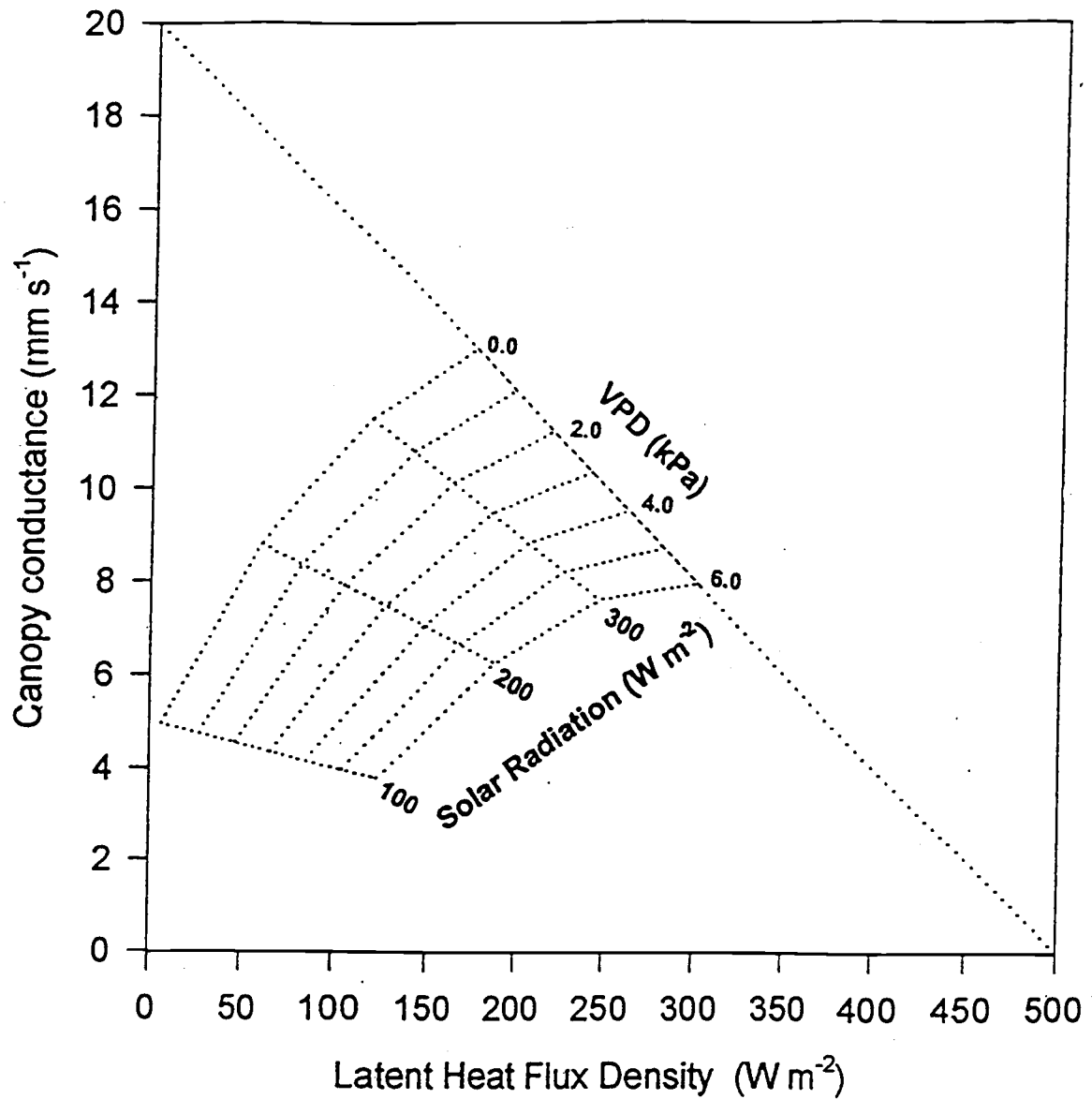
Figure 2.2 A sample graph of the Modified Penman-Monteith Equation, as used over a Douglas fir forest.

Table 2.2 Data from a Douglas fir forest used in creating
Figure 2.3 (Law and Waring 1994).

| Date	Time	Solar Radiation	Vapor Pressure Deficit
920824	Midnight	0	1.26
920824	1:00 am	0	1.19
920824	2:00	0	0.96
920824	3:00	0	0.99
920824	4:00	0	0.93
920824	5:00	0	0.95
920824	6:00	15	0.90
920824	7:00	125	1.04
920824	8:00	300	1.25
920824	9:00	500	1.47
920824	10:00	650	1.80
920824	11:00	775	2.24
920824	Noon	850	2.62
920824	1:00 pm	825	2.96
920824	2:00	815	3.20
920824	3:00	700	3.40
920824	4:00	575	3.42
920824	5:00	400	3.38
920824	6:00	215	3.17
920824	7:00	50	2.82
920824	8:00	0	2.56
920824	9:00	0	2.40
920824	10:00	0	2.27
920824	11:00	0	2.11
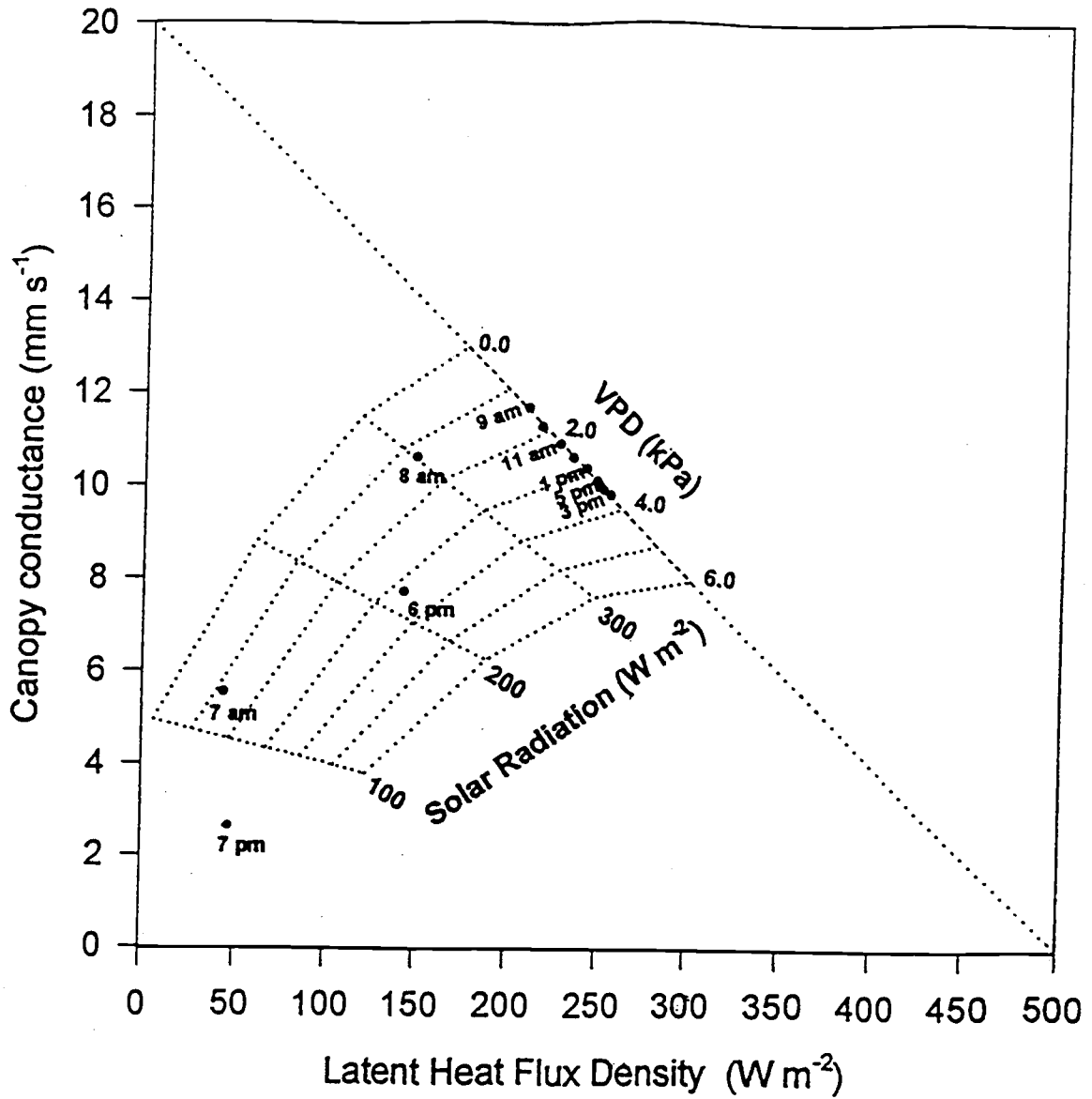
Figure 2.3 A graphic solution of the Modified Penman-Monteith Equation using data points from a Douglas fir forest (data from Law and Waring 1994).

radiation and vapor pressure deficit have fallen off sharply, resulting in a sharp decline in latent heat flux. This is followed by a decrease in canopy conductance as the stomata begin to shut down for the night. Finally, Figure 2.4 shows the time series of the modeled canopy conductance found from the graph. This time series is remarkably similar to those taken from field measurements (see Figure 2.5).

2.8 The Coupled Penman-Monteith-Priestley-Taylor (PMPT) Model

The model developed in the last section is still not coupled to the atmosphere. Thus, it is in truth no more accurate than the equations discussed in Chapter 1. In order to couple this model to the atmosphere, feedbacks between vegetation and the atmosphere must be added. This is done by relating α to g_c and combining that relationship (Equation 2.15) with the Priestley-Taylor equation (Equation 1.10).

To recap, the Priestley-Taylor equation is

$$\lambda E = \alpha \frac{\Delta R_n}{\Delta + \gamma} \tag{2.13}$$

Previous research using the Priestley-Taylor equation has shown a dependence of the Priestley-Taylor coefficient α on the canopy resistance (plotted as $\log(r)$) in a complex sigmoid relationship (McNaughton 1989). Figure 2.6a shows this relationship. However, as Monteith (1994) showed, if α is plotted against $1/r$ (i.e., against the canopy conductance, g), a simpler hyperbolic relationship is revealed (see Figure 2.6b). Both of the above cases have been studied in detail over grasslands only. Therefore, the validity of this assumption over a forest must be examined.

In this development as in the development of Equation 2.12, the concept of coupling between vegetation and the atmosphere can be considered. Research shows that if the canopy conductance is above 20 mm s^{-1}, transpiration (and, therefore, α) is
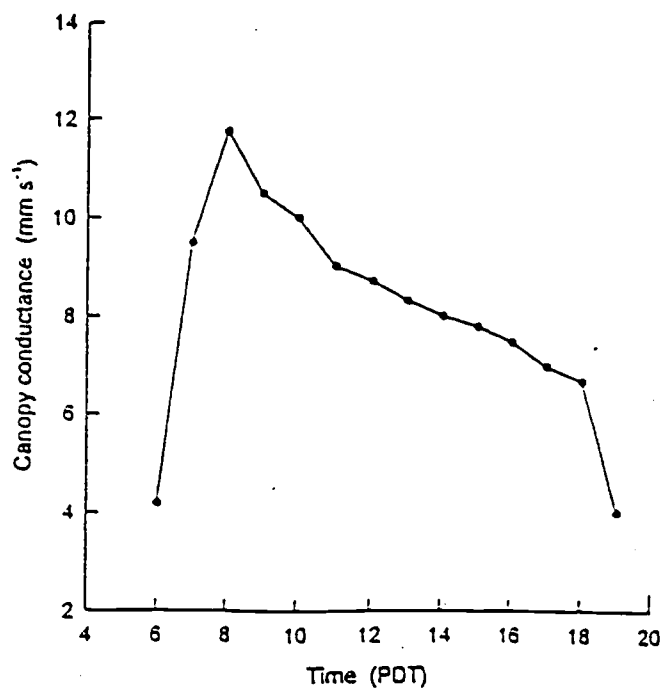
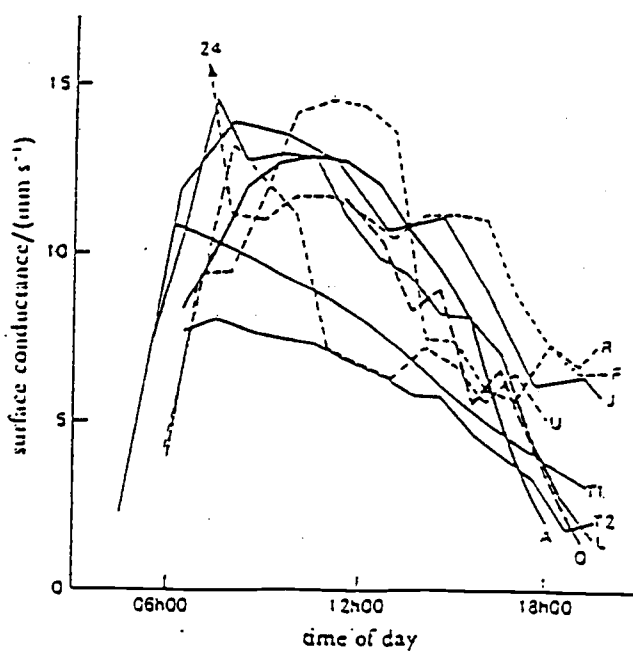**Figure 2.4** A time series of the data from Figure 2.3.

Figure 2.5 Daily variation in surface conductance for eight forest canopies (A, Amazonian rain forest; F, Sitka spruce; J, Scots pine; L, Maritime pine; O, Oak-hickory; R, Sitka spruce; T1 and T2, Scots-Corsican pine; U, Douglas fir) (reprinted with permission from Shuttleworth 1989).

insensitive to small changes in canopy conductance (e.g., as for grasses). However, if

the canopy conductance is below 20 mm s^{-1}, such as is the case in many forests, then a

hyperbolic relationship is formed between the two variables (α and g_c), thus showing that

the hyperbolic assumption from Figure 2.6b holds for forests as well as grasslands.

Further support for the form of the relationship between α and the canopy

conductance, g_c, is found in Figure 2.7. Figure 2.7 actually shows the relationship

between the Bowen ratio, β, and canopy resistance. The Bowen ratio is defined at the

ratio of sensible heat flux (C) to latent heat flux (λE) but, α is related to β by the formula

$\dfrac{1}{\alpha} = (\beta + 1)(\dfrac{\Delta}{\Delta + \gamma})$ and g_c is related to the canopy resistance, r_c by the formula

$g_c = \dfrac{1}{r_c}$. Figure 2.7 also shows the relationship between these variables in different

climate regimes. It illustrates that there is no single relationship, but that the climate of a

region (i.e., marine humid or dry continental) makes a difference in the relationship. The

Priestley-Taylor approach is used in large-scale estimate. Therefore, it is not valid to

apply it to most coastal-maritime forests because they are too small for equilibrium
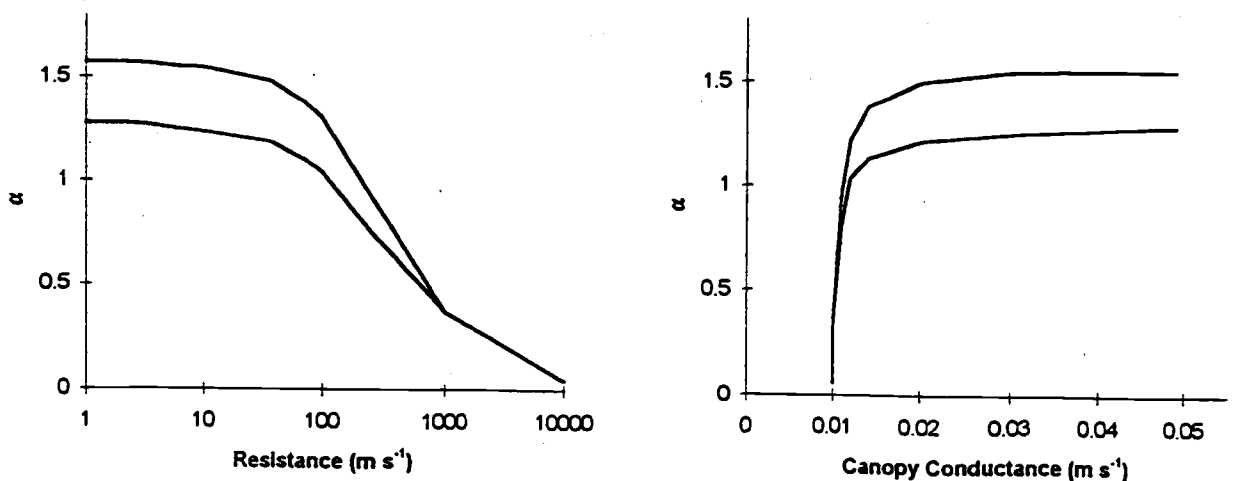
Figure 2.6 The dependence of the Priestley-Taylor coefficient α on surface resistance r
is shown in the left-hand graph (a), while the hyperbolic relationship
comparing α to g ($1/r$) is shown in the right-hand graph (b).

evaporation to be achieved. However, if we assume the scale is large enough that we can apply it (i.e., if the forest is large enough), then the coastal-maritime forest would be represented by the steepest curve on the graph, while a forest in the center of a continent would tend to have a shallower relationship between β and r_c, and therefore a different relationship between α and g_c. In order for the hyperbolic assumption to hold, therefore, we will assume an extensive well-watered forest.

The examples above lead to the conclusion that a hyperbolic relationship between the Priestley-Taylor coefficient and the canopy conductance can be developed. In order to do this, we first combine the Penman-Monteith with a Convective Boundary Layer Model (as has been done previously by Dewar (1995) and McNaughton and Spriggs (McNaughton 1989)). McNaughton and Spriggs (McNaughton 1989), developed a
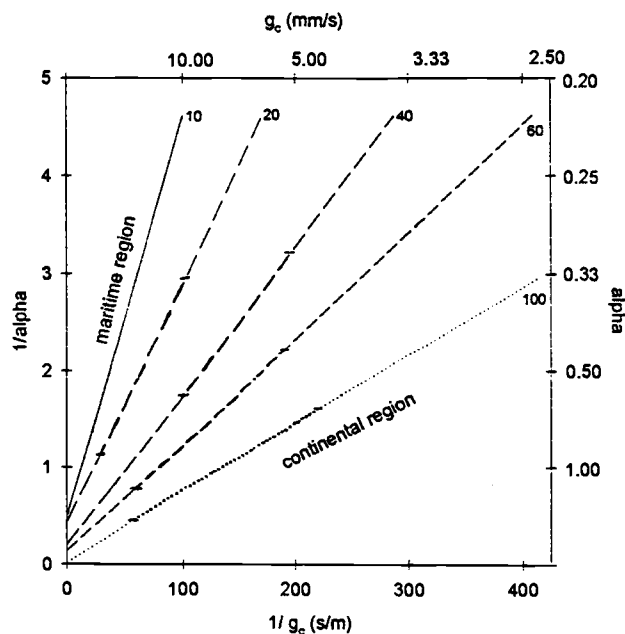
Figure 2.7 The relationship between 1/α and the inverse of canopy conductance (1/g_c) when the available energy is 500 W m^{-2}, the aerodynamic conductance is 200 mm s^{-1} and the temperature is 25°C (adapted from Jarvis et al 1976). Values of α and g_c are also shown for convenience. The numbers beside the lines represent values of the climatological resistance (in s m^{-1}), which is proportional to the vapor pressure deficit of the atmosphere divided by the available energy. The heavy lines indicate appropriate ranges of α and g_c at the different sites.

sigmoid relationship between $\frac{\lambda E}{H}$ and $\ln(r_c)$. Monteith (1995a) discovered that a much easier relationship could be developed between $\frac{\lambda E}{H}$ and g_c ($1/r_c$) by incorporating an entrainment conductance, g_e, which is a parameterization of the physical processes at the top of the boundary layer. This equation is

$$\frac{\lambda E}{H} = \frac{[\frac{\lambda E_m}{H_m}]}{[1 + \frac{g_e}{g_c}]} \qquad (2.14)$$

In boundary layer meteorology, the concept of an entrainment velocity has been well-established. As the surface heats up, thermals develop, which then rise into the lower atmosphere. Once in the atmosphere, the thermals continue to rise, as they are warmer than the surrounding air. They gain momentum as they ascend, and upon reaching the top of the Convective Boundary Layer, have enough momentum to penetrate the inversion. The thermal then sinks back into the CBL. During this interaction, some of the air from the free atmosphere above the Convective Boundary Layer is pushed into the CBL, a process called entrainment (Stull 1988). The volume of air entrained into the CBL per unit time per unit area is know as the entrainment velocity and has units of mm s^{-1}. Typical values of the entrainment velocity range from 10-20 mm s^{-1} (Stull 1988).

In developing the entrainment conductance, Monteith (1995a) observed a wide range of data from grasslands and agricultural crops. From these data, he concluded that 5 mm s^{-1} was an appropriate value for the entrainment conductance. In principle, the entrainment conductance must depend upon conditions at the top of the boundary layer, such as the strength of the temperature inversion and the mixing ratio gradient across the inversion. It is not clear why Monteith's estimate of entrainment conductance is smaller than the values quoted by Stull, but the discrepancy is quite small and may be related to the differences in convective conditions between the data sets they were analyzing. For

example, entrainment conductance is likely to be very small during weak convection early and late in the day. There have been no detailed analyses of entrainment conductance over forests at this stage. In the absence of further information, the value of entrainment conductance over a coniferous forest has also been assumed to be 5 mm s^{-1}. As a test of the validity of this assumption, the relationship between g_e and λE has been explored by solving the "PMPT" equation (Equation 2.18) for g_e under typical forest conditions. It was found that g_e varies from approximately 4.7-5.2 mm s^{-1}, depending upon the value chosen for α_m. α_m is the maximum Priestley-Taylor coefficient found over a given vegetation type. Thus, 5 mm s^{-1} appears to be a good estimate of the actual value of the entrainment conductance.

In order to develop a relationship between α and g_c, we simply multiply Equation 2.14 by $\dfrac{\Delta + \gamma}{\Delta}$ to get

$$\alpha = \frac{\alpha_m}{[1 + \dfrac{g_e}{g_c}]}$$

(2.15)

where: $\alpha \to \alpha_m$ as $g_c \to \infty$
$\quad\quad g_e = 5$ mm s^{-1} (assumed)

Now if $\dfrac{g_c}{g_m} = 1 - \dfrac{\lambda E}{\lambda E_m}$ (Equation 2.9) is assumed valid for a uniform cover of vegetation as for single leaf and if $\alpha = \dfrac{\alpha_m}{[1 + \dfrac{g_e}{g_c}]}$ (Equation 2.15) has a mechanistic

basis in conventional CBL models, then these two equations can be combined by eliminating the g_c from both of them, obtaining

$$\alpha = \frac{\alpha_m}{1 + [(\dfrac{g_e}{g_m})(1 + \dfrac{\lambda E}{\lambda E_m})]}$$

(2.16)

Or, they can be combined by eliminating λE from both equations, by taking (2.15) and

multiplying through by the available energy (*H*)

$$\lambda E = \frac{\lambda E'}{1 + \frac{g_e}{g_c}}$$

(2.17)

where: $\lambda E' = (\alpha\ \Delta\ H) / (\Delta + \gamma)$

Then, combining Equation 2.17 with $\frac{g_c}{g_m} = (1 - \frac{\lambda E}{\lambda E_m})$ (Equation 2.9), in the form $\lambda E = \lambda E_m (1 - \frac{g_c}{g_m})$ gives

$$\lambda E = \lambda E_m (1 - \frac{g_c}{g_m})$$

(2.18)

where: $\lambda E_m = \dfrac{\lambda E'}{[(1 + \frac{g_e}{g_c})(1 - \frac{g_c}{g_m})]}$

This equation is the "PMPT" or "Penman-Monteith-Priestley-Taylor" model. It is based on models of the surface and of the Convective Boundary Layer, thus closing the model system and including the feedbacks between vegetation and the atmosphere. Examples of results from this equation will be shown in Chapter 3. If Equation 2.18 is then combined with the Penman-Monteith equation, then values of the vapor pressure deficit, in this case a result of canopy conductance, not a cause, can be obtained. The equation is

$$D = \frac{\lambda E[\Delta + \gamma(\frac{g_a}{g_c})] - \Delta H}{\rho c_p g_a}$$

(2.19)

Results from this equation will also be examined in Chapter 3. Both of these equations are easily solved numerically. An example of the program used in developing the results of Chapter 3 is found in Appendix B.

Chapter 3 - Results

Equations 2.12 and 2.14 were solved numerically, and the results are reviewed in this chapter. The first section explores the uncoupled model of transpiration (Equation 2.12), and the second analyses the coupled model (Equation 2.14). In the final section, the coupled model is extended over a wider range of net radiation and vapor pressure deficit values.

3.1 The Uncoupled Model - A Sensitivity Analysis

This section is concerned with the examination of how the physiological control exerted by plants (through the controlled opening and closing of the stomata) affects values of latent heat flux and canopy conductance and the sensitivity of the uncoupled model developed in Equation 2.12. Figure 3.1 shows a series of four graphs. On the top row, the maximum conductance (g_m) is 20 mm s^{-1}, just as it was in Figure 2.2. On the bottom row, the maximum conductance is set at 10 mm s^{-1}, i.e., ½ of the above maximum conductance. These values were chosen to examine how the physiological control exerted by the plants (through controlled opening and closing of the stomata) affects the values of transpiration and canopy conductance. Also, the two graphs on the right have a maximum latent heat flux of 1000 W m^{-2}, while the graphs on the left have a maximum latent heat flux of only 250 W m^{-2}. These values were chosen to examine the effect that the supply of water would have on the transpiration rate and canopy conductance. A high maximum latent heat flux is synonymous with a large supply of water to the plant and/or a root system that is adept at obtaining that water (i.e., a well-watered landscape). A small maximum latent heat flux is synonymous with a small supply of water and/or a root system that is unable to obtain soil water (i.e., an arid landscape).
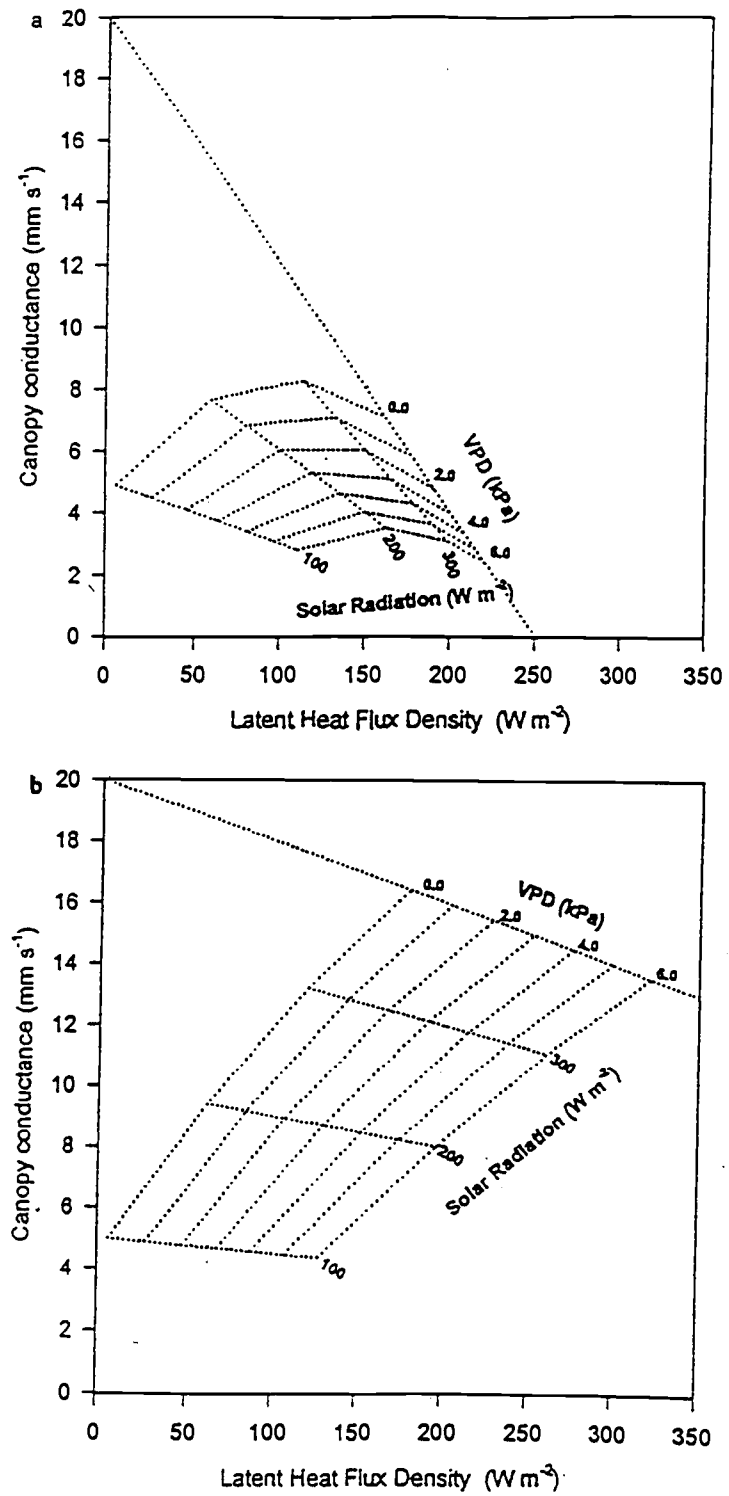
Figure 3.1 The results of the Modified Penman-Monteith Equation considering changes in E_m and g_m.
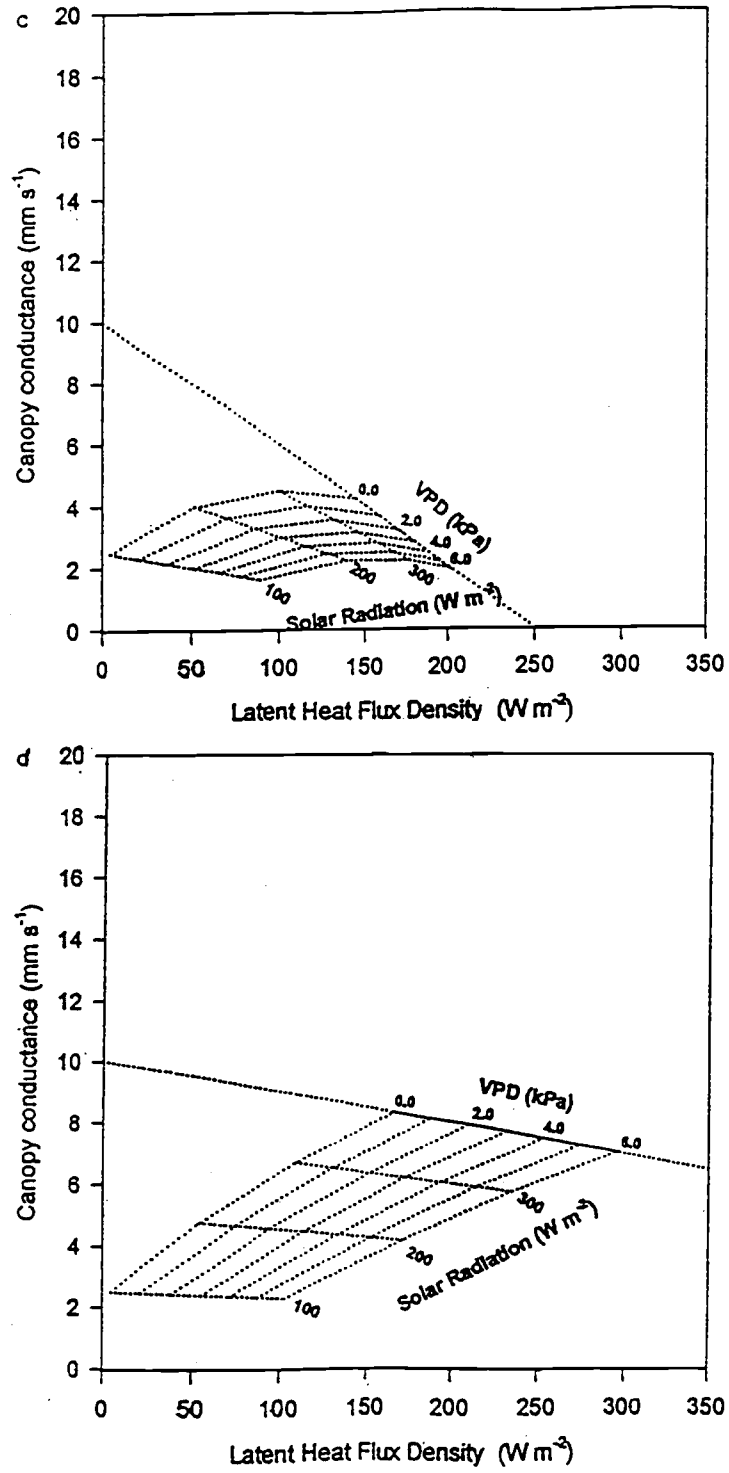
Figure 3.1 (continued) The results of the Modified Penman-Monteith Equation considering changes in E_m and g_m.

By first examining the canopy conductance at the two points mentioned above, it can be seen that while the canopy conductance of the top row is roughly twice that of the bottom row, there does not appear to be a significant increase in the latent heat flux of the system. However, if the maximum latent heat flux is increased from 250 Wm^{-2} to 1000 Wm^{-2}, the relationship between latent heat flux and canopy conductance becomes much more linear at higher solar radiation values. Thus, there is an increase in the latent heat flux at a given canopy conductance, becoming much more pronounced as the vapor pressure deficit (the atmospheric "demand") increases. These results indicate that the supply of water to the system plays a dominant role in determining the transpiration rate (latent heat flux density), especially when there is also a potentially large canopy conductance. Such values might be found in well-watered regions. There does not appear to be a significant change in the latent heat flux from the system at low canopy conductances, such as those values found in arid regions. A canopy that has a large supply of water can afford to have "leaky" leaves (i.e., support a large canopy conductance) without adversely affecting the vegetation. However, a canopy that is not well-supplied with water will exert physiological control over its water loss by closing its stomata. This affects the amount of water being transpired into the atmosphere and thus, the water balance of the canopy. It also affects the carbon balance of the canopy, since if the plant's stomata are partially closed to conserve water, the plants will be unable to obtain as much carbon dioxide as those plants whose stomata can remain open.

As a result of the results described above, it was decided to determine what effect, if any, a doubling of maximum canopy conductance, maximum latent heat flux or both would have on the theoretical system illustrated in Figure 2.2. The results are summarized in Figure 3.2, where the maximum canopy conductance doubles by
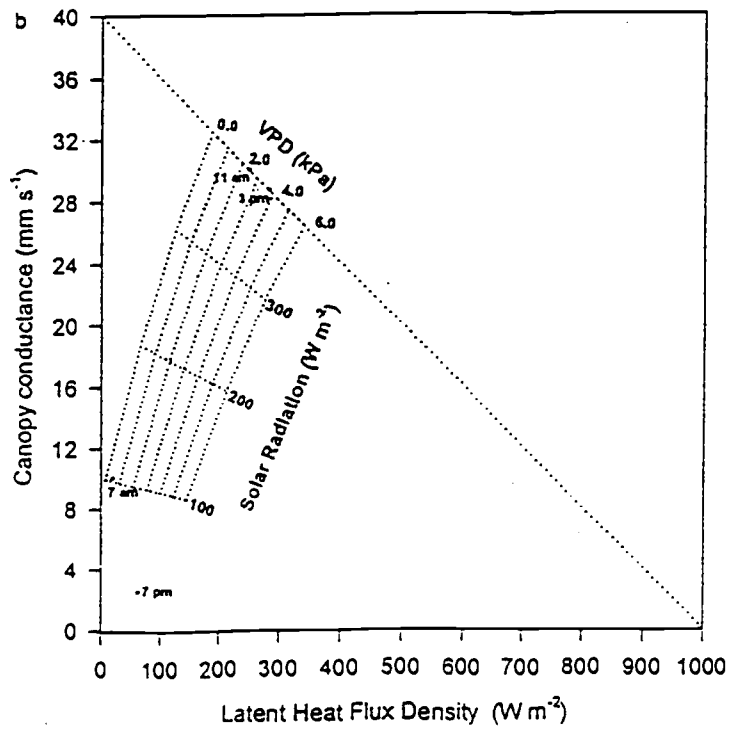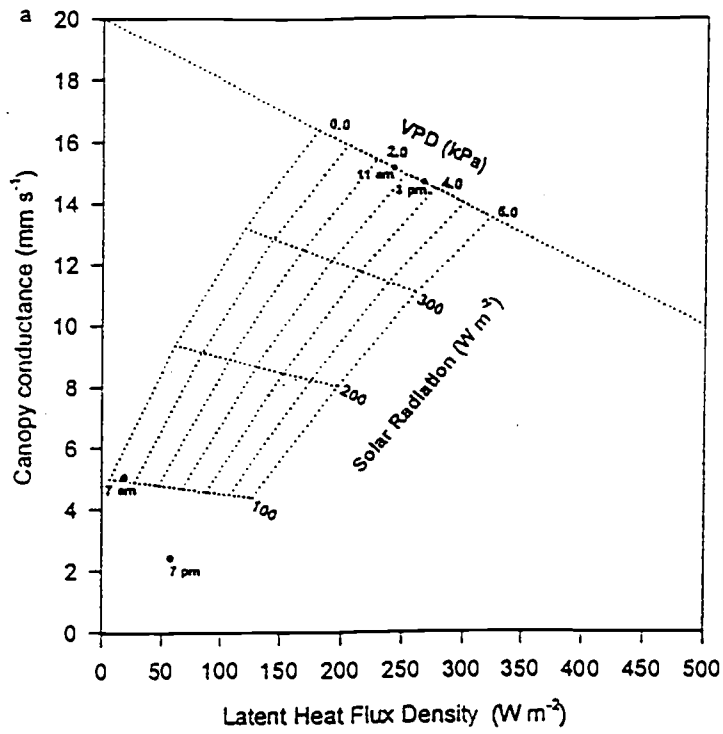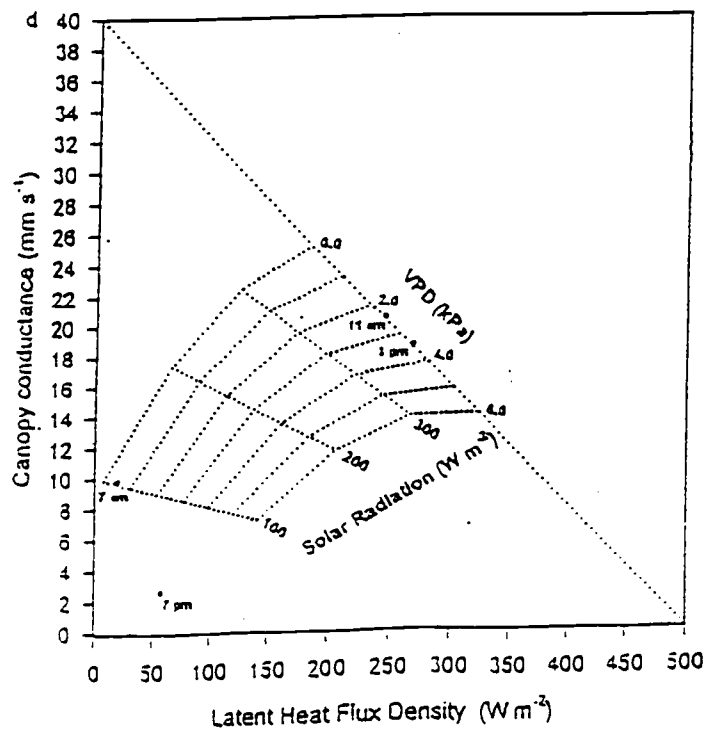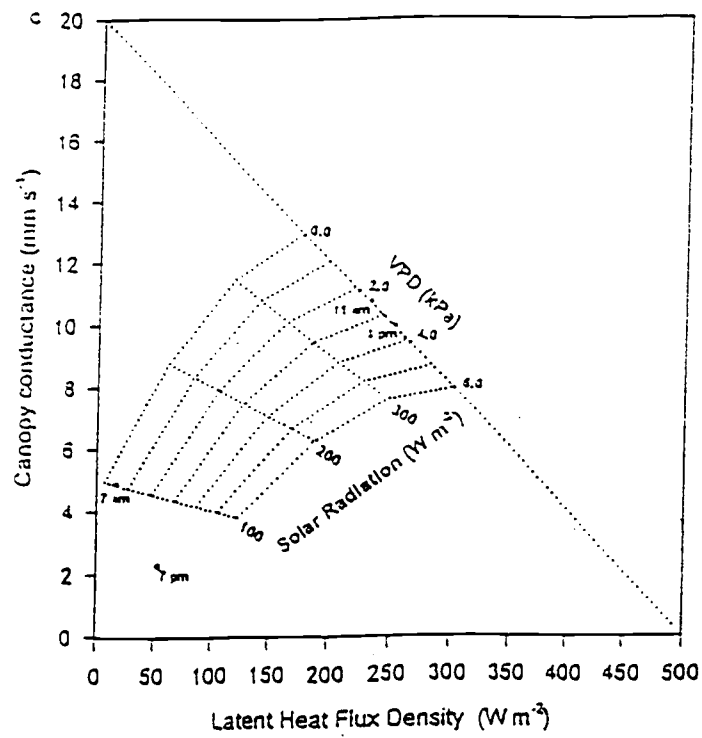
Figure 3.2 The results of the Modified Penman-Monteith Equation considering a doubling of E_m and g_m.

Figure 3.2 (continued) The results of the Modified Penman-Monteith Equation considering a doubling of E_m and g_m.

comparing 3.2a and 3.2c with 3.2b and 3.2d. The maximum latent heat flux doubles by comparing 3.2c and 3.2d with 3.2a and 3.2b. Several points, chosen from Table 2.2, are shown on the graph in order to examine the consequences of the differences for the daily water use cycle. Comparing first Figures 3.2c with 3.2b, the values of the latent heat flux remain much the same, even though the canopy conductance that it is able to support almost doubles in response to a doubling of the maximum canopy conductance. These results imply that the effect of doubling the potential supply of water in a moist system will not affect the water balance of the system. However, the carbon balance of the system will again be affected because the stomata are open wider for a longer period of time in a system of doubled maximum canopy conductance, thus gaining an increase in carbon for the vegetation.

3.2 The Coupled Model vs. Other Models of Transpiration

The second part of this chapter will focus on the coupled model of transpiration, as developed in Section 2.8. Data are currently unavailable to make a statistical analysis of the model. Therefore, in order to test the sensitivity of the model, typical values of the variables were chosen as shown in Table 3.1. The model is then compared to common models of transpiration in order to determine its suitability in future studies.

Figures 3.3 - 3.6 show results from several models of transpiration. To aid comparison, all of the graphs begin at the same point: the canopy conductance is 3.58 mm s^{-1}, the latent heat flux is 215 W m^{-2} and the vapor pressure deficit is 1.54 kPa. The net radiation for each model was set at 250 W m^{-2}. In this way the developments within the models are comparable. For ease of comparison, the graphs should be read from right to left. Figure 3.3 demonstrates the Priestley-Taylor Approach to transpiration. This equation, as shown in Equation 1.10, does not include a measure of canopy
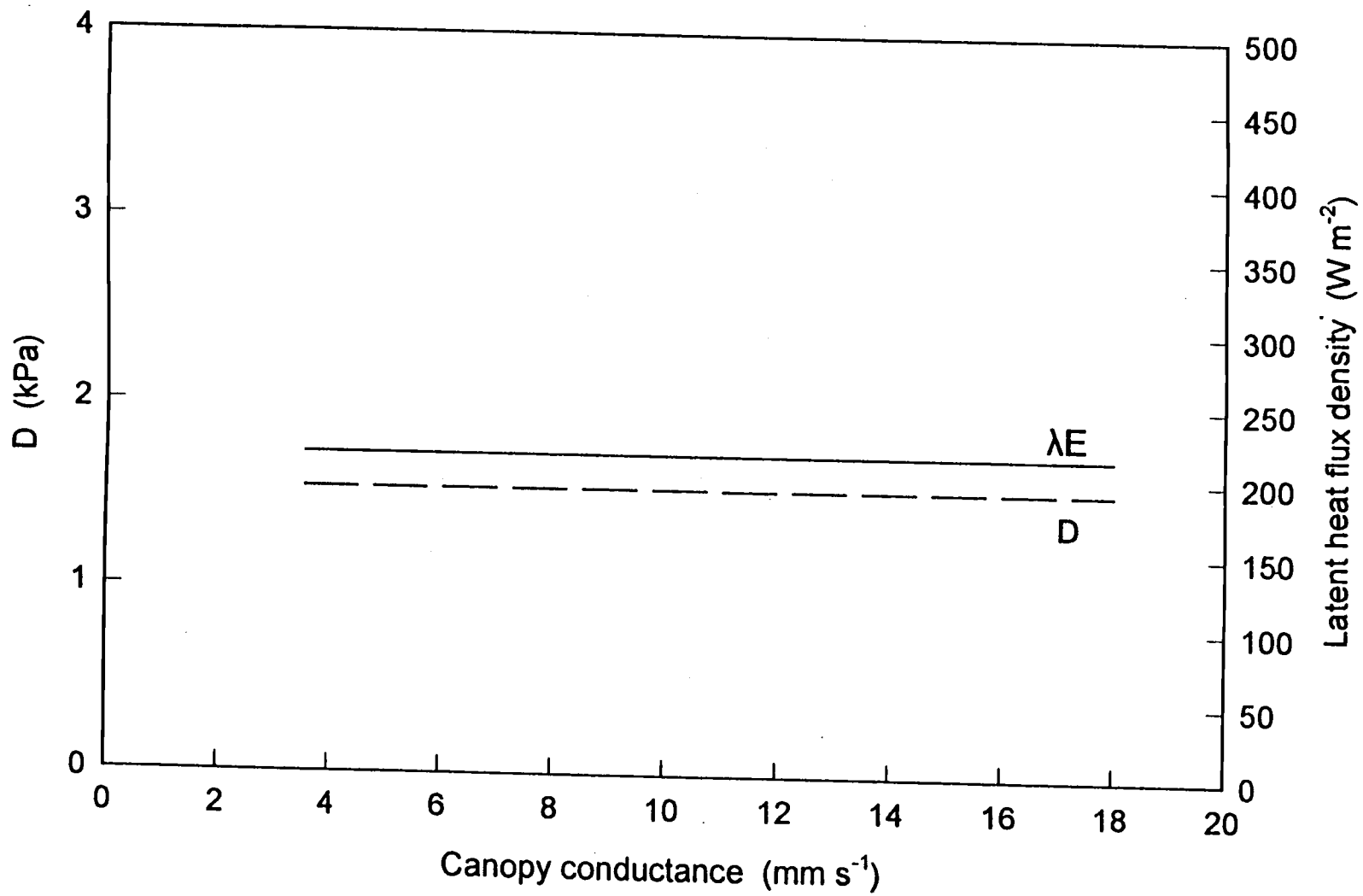
Figure 3.3 The Priestley-Taylor Approach to transpiration
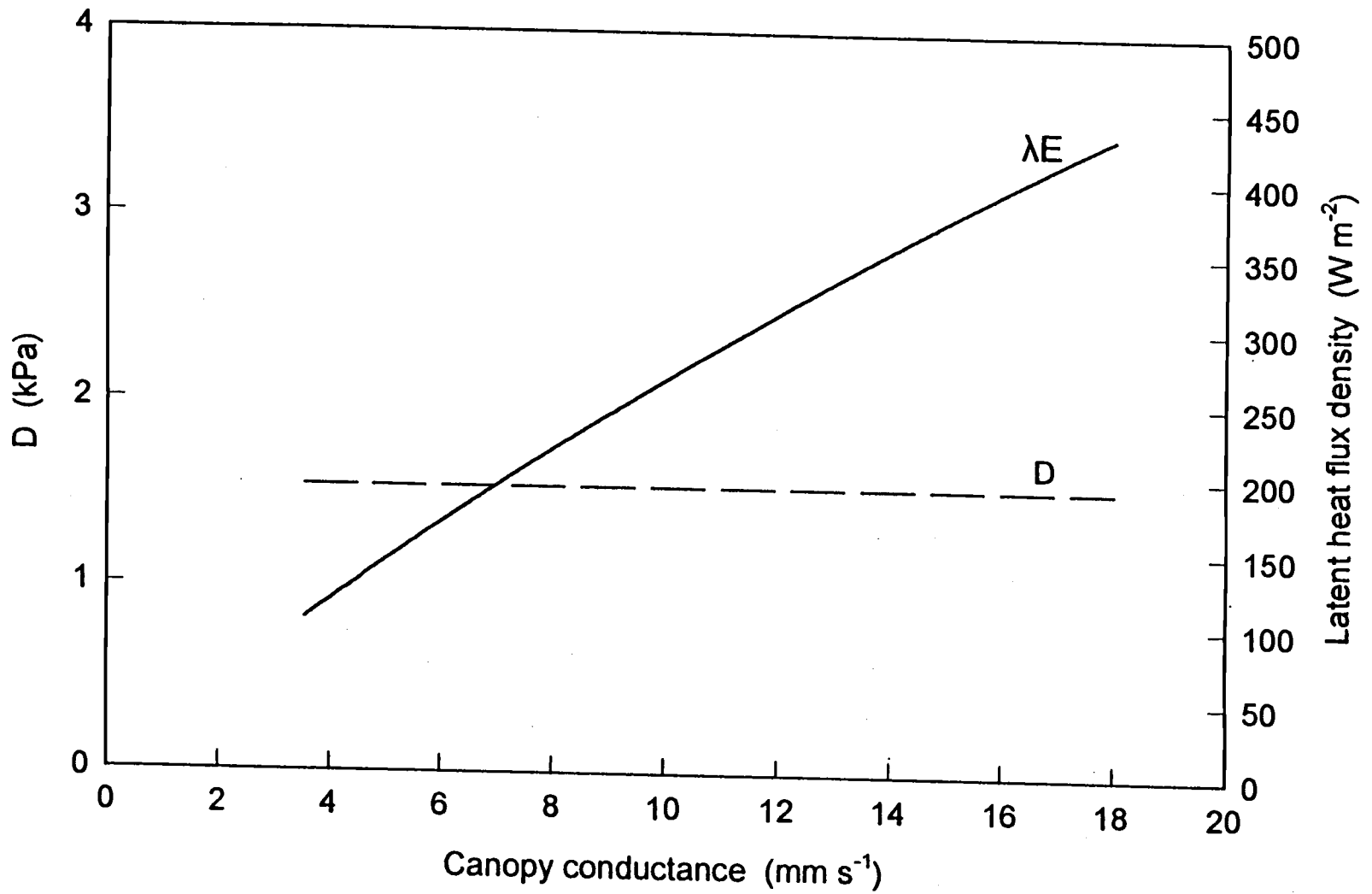
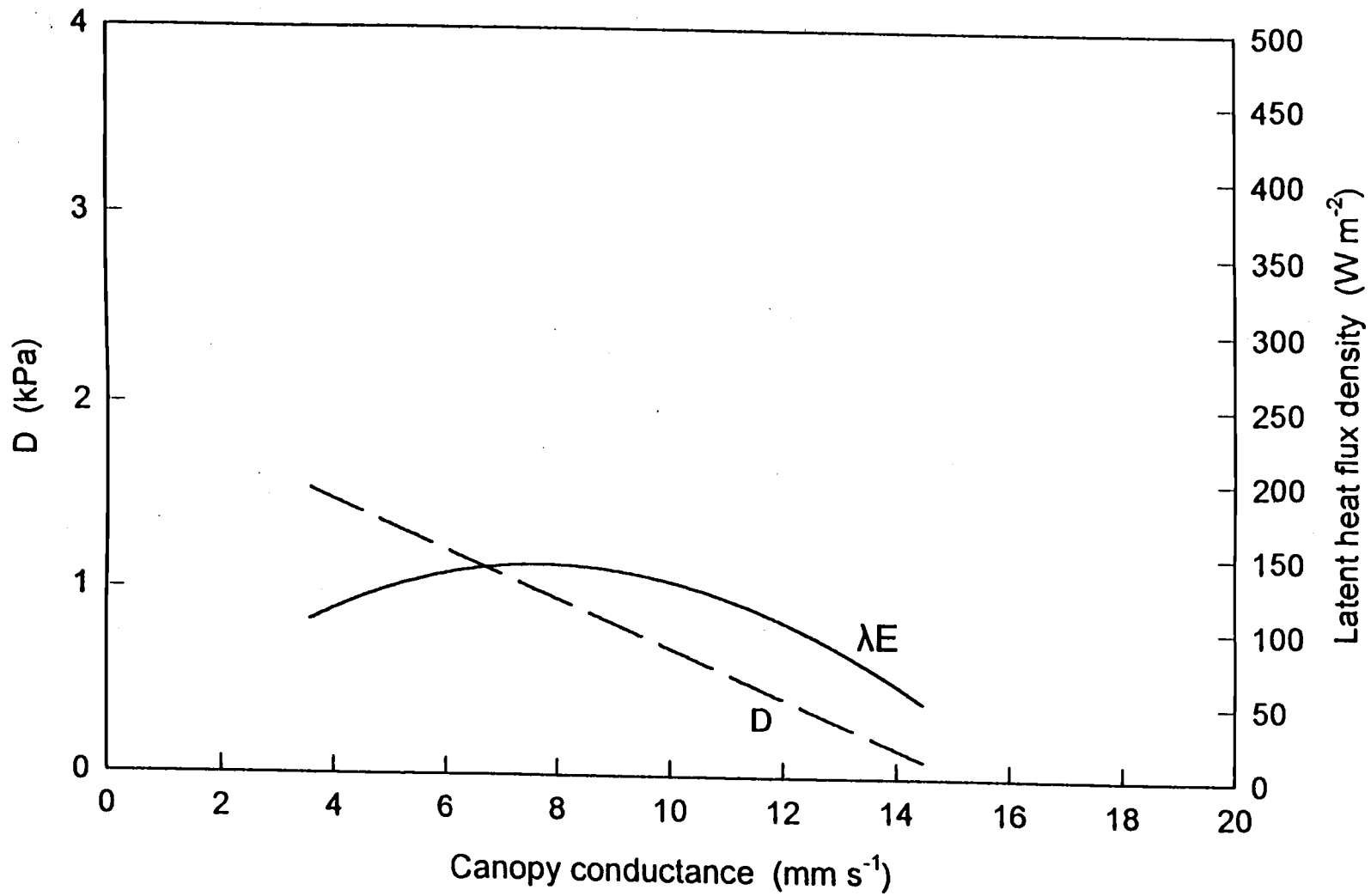**Figure 3.4** The Penman-Monteith Approach to transpiration

Figure 3.5 The Penman-Monteith Approach to transpiration, employing the Jarvis Approach to canopy conductance
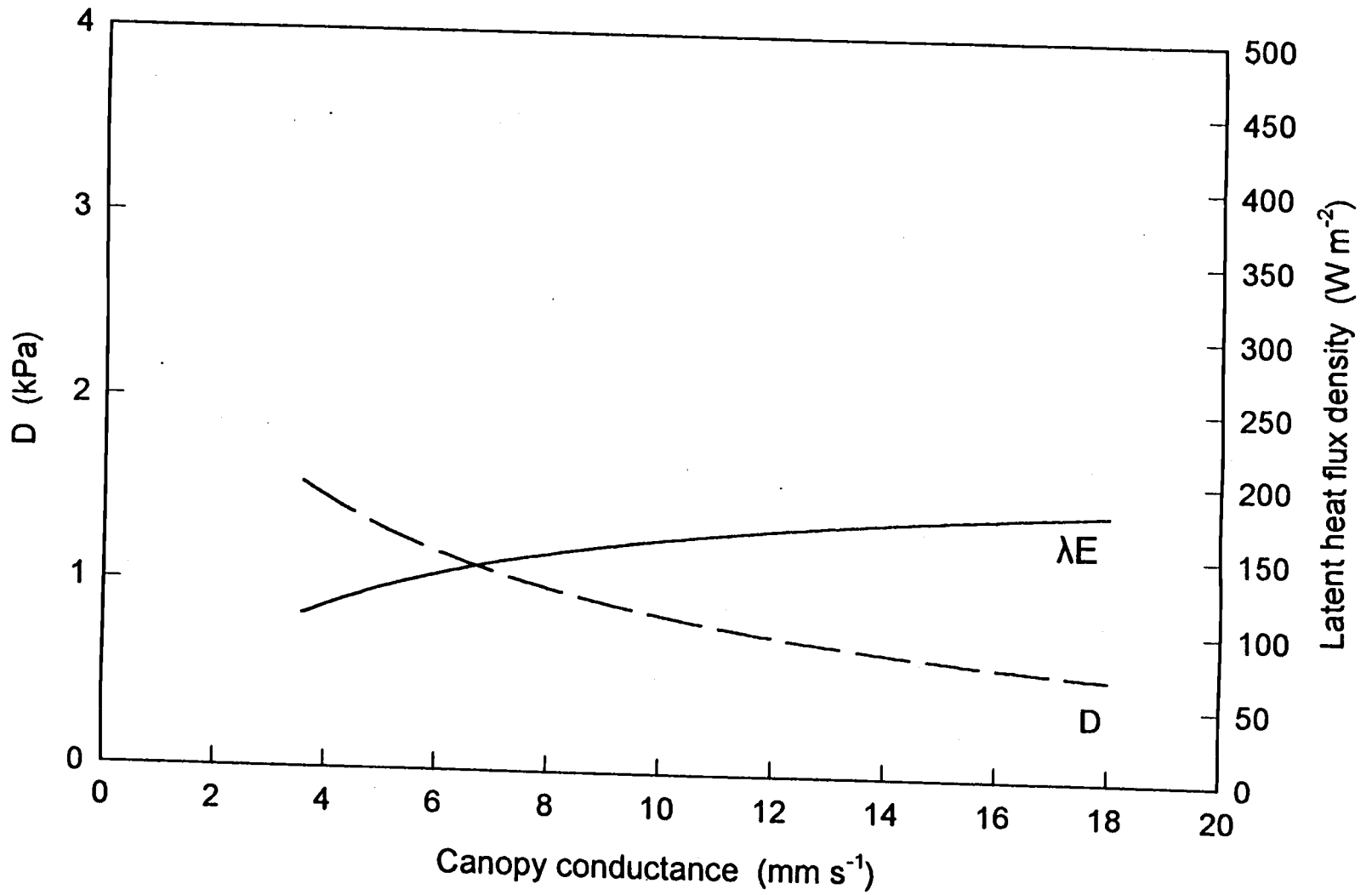
Figure 3.6 The PMPT Approach to transpiration

conductance. The value of latent heat flux depends only on the net radiation, and the value of vapor pressure deficit is independently imposed. Therefore, once an initial vapor pressure deficit and latent heat flux have been established, they will not change with changing canopy conductance. For this reason, the Priestley-Taylor model is not valid over areas small enough for researchers to be concerned about changing values of canopy conductance.

Figure 3.4 shows the results of the Penman-Monteith model if the canopy conductance decreases steadily until it reaches the value 3.58 W m^{-2}. The vapor pressure deficit is an input variable in this case and is not altered by changing canopy conductance. The latent heat flux begins at an unrealistically high value considering the net radiation available to drive the system. This could only be supported if advected energy was available. The latent heat flux then decreases almost linearly with decreasing canopy conductance. This model is unrealistic because a decrease in latent heat flux must be accompanied by an increase in sensible heat flux to satisfy the energy balance. If there is an increase in sensible heat flux, then the atmosphere will become hotter and the vapor pressure deficit will increase. The model is incapable of including these feedbacks between the vegetation and the atmosphere, so cannot be entirely correct either.

The third model under consideration, shown in Figure 3.5, is the Penman-Monteith model employing the Jarvis approach to canopy conductance (see Equation 2.5, modified for a canopy). The Jarvis approach is an empirical formula which relates canopy conductance to a number of environmental factors, assumed to act independently. In this case as the vapor pressure deficit increases, the canopy conductance decreases. The latent heat flux initially responds to the vapor pressure deficit by increasing to a maximum at a canopy conductance of 9 mm s^{-1}. Once this maximum has been reached, the effect

of the decreasing canopy conductance begins to overwhelm the response of the latent heat flux to the vapor pressure deficit and the latent heat flux decreases as a result. There are several problems with this method. First, the Jarvis method is an empirically-derived equation, whose values are unlikely to be representative of the test site (Shuttleworth 1983). Second, this equation states that the canopy conductance is a function of the vapor pressure deficit. This concept was examined in Chapter 2, and evidence exists to refute such a claim. The fact that the vapor pressure deficit is a driving variable in this equation is a weakness that perpetuates throughout the assumptions involved in the equation.

The last equation under consideration is the "PMPT" equation. The results are shown in Figure 3.6. In this equation, the canopy conductance is used to determine the latent heat flux which is then used to determine the vapor pressure deficit, including a numerical description of entrainment from above the Convective Boundary Layer. As the canopy conductance decreases, the latent heat flux slowly decreases as a result. In response, the atmospheric vapor pressure deficit begins to increase, responding to the accompanying increase in the sensible heat flux. In this equation, the variables are modified through feedbacks between the vegetation and the atmosphere. The latent heat flux is therefore never able to reach the unrealistically high values produced by the Penman-Monteith equation (Figure 3.4) because the flux is continually modified by changes in the vegetation and the environment. The numbers found from this model appear to be much more realistic than those of the other equations, adding confidence to the theoretical development in Chapter 2 that this approach is indeed an improvement on existing models.

3.3 Extension of the "PMPT" Model

Finally, the influence of net radiation on values of latent heat flux and vapor pressure deficit was investigated in the PMPT model. The PMPT model was run with the same vegetation parameters as before, but for net radiation values ranging from 200 - 600 W m^{-2} at 25 Wm^{-2} intervals. Figure 3.7 shows the results for the latent heat flux.

At the local scale, the aerodynamic conductance determines the division of evaporation into equilibrium evaporation and imposed evaporation (McNaughton and Jarvis 1984). As the aerodynamic conductance of an area tends to zero (0), the evaporation tends to equilibrium evaporation (evaporation dependent upon available energy). At the other end of the spectrum, as the aerodynamic conductance tends to infinity (∞), the evaporation tends to imposed evaporation (evaporation dependent upon the canopy conductance and vapor pressure deficit). McNaughton and Jarvis (1991) have suggested that there is a large (regional) scale coupling with the canopy conductance analogous to the local scale coupling of evaporation to aerodynamic conductance. In this case, as the canopy conductance tends to zero (0), the canopy conductance and vapor pressure deficit of the atmosphere will drive the water balance and the evaporation is analogous to that of the imposed evaporation above. But, if the canopy conductance becomes large (i.e., tends to infinity [∞]), the vegetation becomes decoupled from the synoptic scale weather patterns and the system is driven only by the available energy. In this case, the evaporation acts like the equilibrium evaporation just described and the Priestley-Taylor coefficient α becomes 1.26. When this analysis is applied to Figure 3.7, it provides support for the complicated analysis of McNaughton and Jarvis (1991). Figure 3.7 shows that if the canopy conductance is large, there is a steep, almost linear relationship between latent heat flux and net radiation. If, however, the canopy conductance is small, the latent heat flux is strongly influenced by the canopy
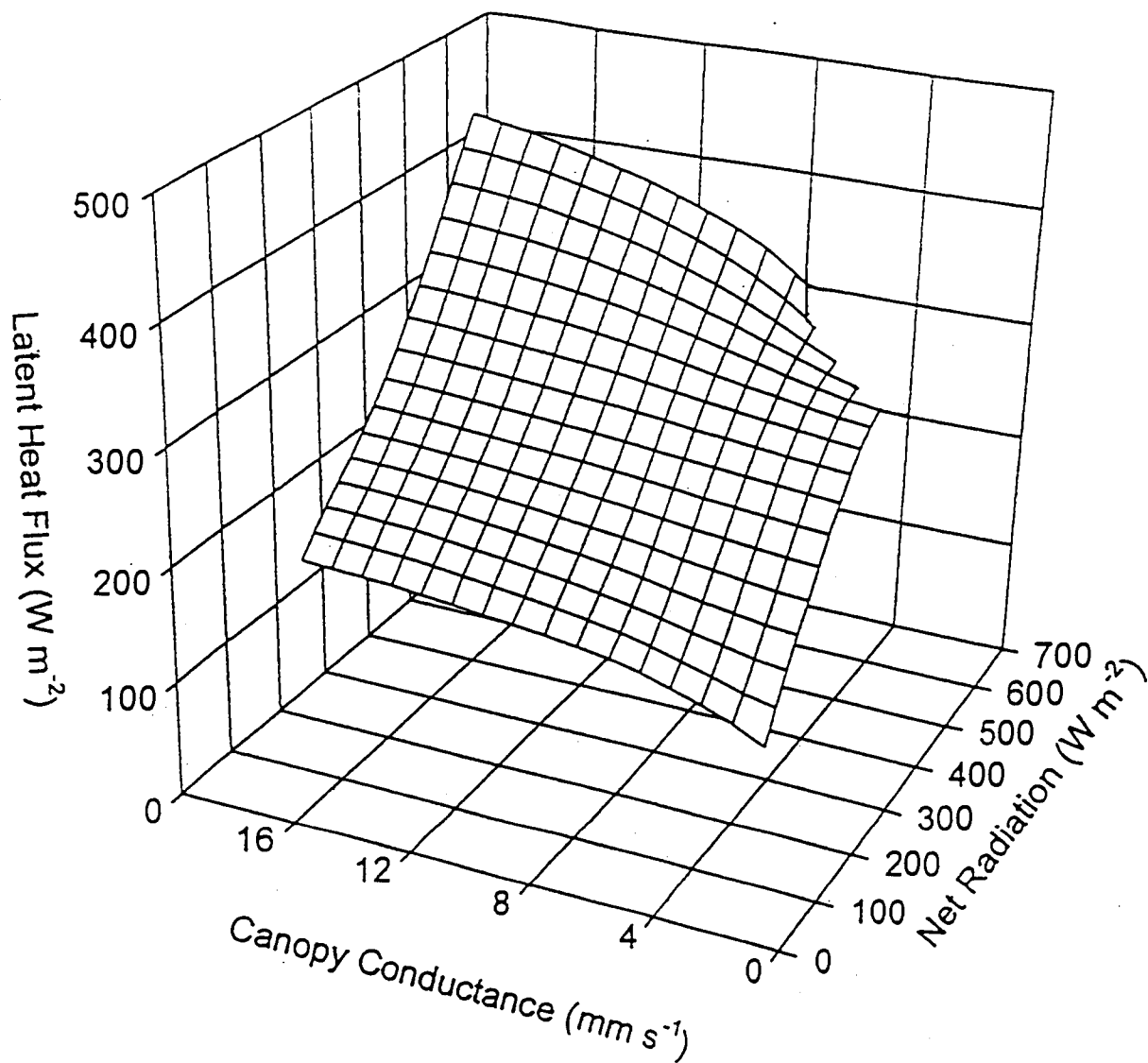
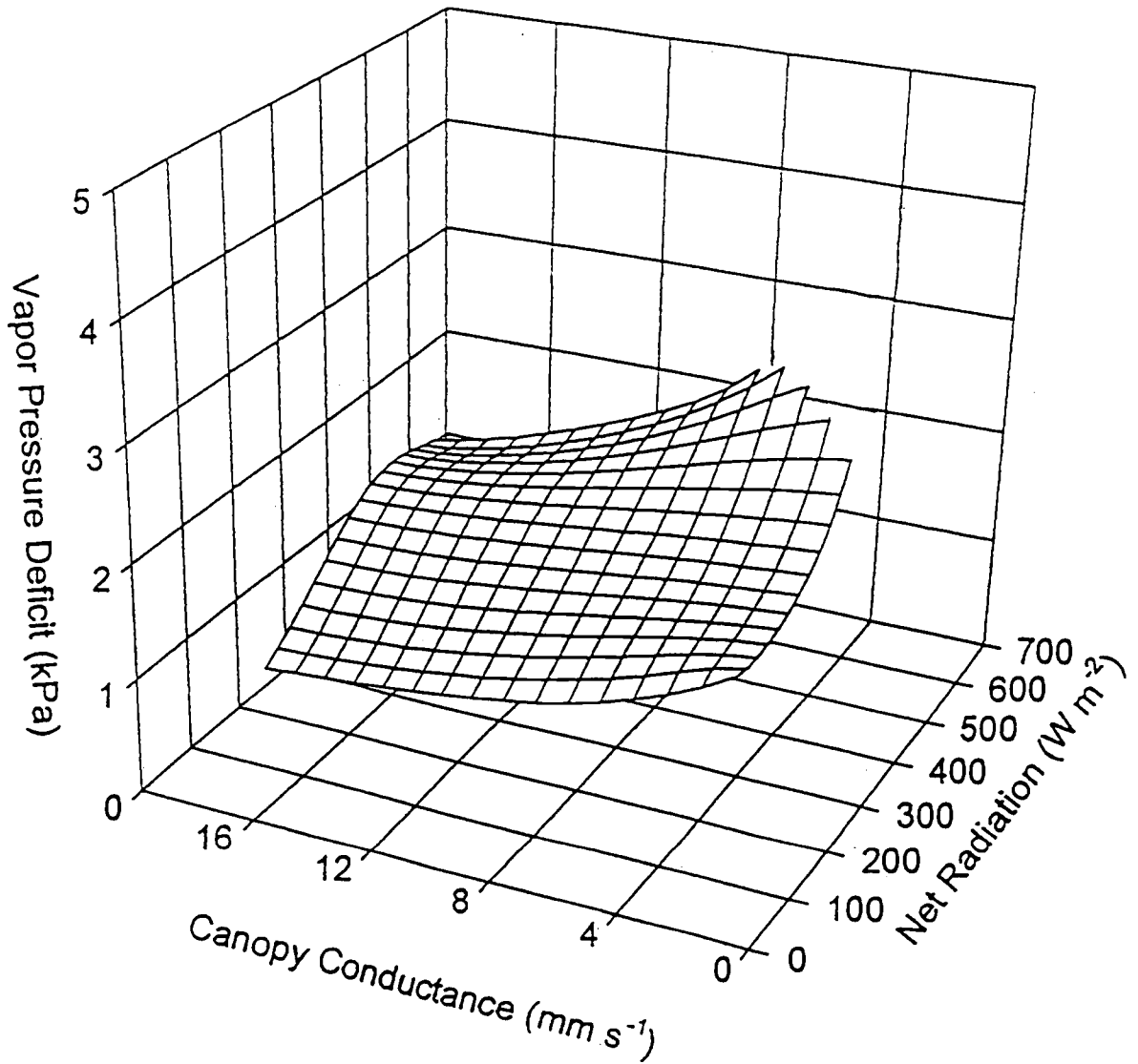**Figure 3.7** Latent heat flux results from the Extended PMPT Model

Figure 3.8 Vapor pressure deficit results from the Extended PMPT Model

conductance.

Figure 3.8 reveals that if the canopy conductance is large, the vapor pressure deficit of the atmosphere will not change very much. This occurs because while moisture is being added to the atmosphere, heat is also being added to the atmosphere. Therefore, there is little net change in the vapor pressure deficit. If the canopy conductance is small, the vapor pressure deficit increases as the net radiation increases. Since the canopy conductance is small, much of the net radiation is partitioned into sensible heat flux, causing an increase in the boundary layer height, thus, the vapor pressure deficit increases due to the entrainment of drier air from above the boundary layer.

Figures 3.7 and 3.8 are based largely on the limits chosen in parameterizing the model. Primarily, the limits are determined by the fact that the canopy conductance is a linear function of net radiation to a value of 400 W m^{-2}, after which the canopy conductance remains constant. Therefore, if the canopy conductance is low and the net radiation is moderate to high, then the latent heat flux is strongly related to the canopy conductance and vapor pressure deficit. This can be better seen in Figure 3.6. In this figure, at a small canopy conductance, the vapor pressure deficit "checks" the latent heat flux through regulation of the transpiration rate. At large values of canopy conductance, the latent heat flux and the vapor pressure deficit are independent of each other, leading to the expected equilibrium evaporation discussed previously.

The results mentioned in this chapter appear reasonable when considering the physics of the water balance of the system. They demonstrate that the PMPT model is a simple way of exploring the sensitivity of latent heat flux and vapor pressure deficit on the regional scale to surface and entrainment conditions. More complex coupled models are needed in order to better understand how entrainment is influenced by the system, but

this simple illustration is useful for regional models which cannot use the complexity of a truly mesoscale latent heat flux model.

Chapter 4 - Conclusion

It has been shown in this thesis that although there are many simple models of transpiration, none of them *fully* reflect what is occurring in nature. This is primarily because none of these models include a full treatment of the feedbacks between a vegetated surface and the atmosphere that serve to stabilize evaporation rates. The PMPT model has been developed to include such feedbacks. This model serves as a simple way of exploring the partitioning of available energy into latent and sensible heat fluxes at the surface. At the same time this model incorporates the physics of evaporation at the surface and a simple parameterization of entrainment at the top of the convective boundary layer.

There are several reasons why the PMPT model has been developed. Current models of transpiration assume that the canopy conductance responds to vapor pressure deficit. As Mott and Parkhurst (1991) have shown, however, canopy conductance responds to transpiration rate. Thus, a model that depends upon vapor pressure deficit in order to calculate stomatal conductance is empirical and cannot accurately describe the mechanisms behind such a relationship. For this reason, the PMPT model employs a mechanistic relationship between canopy conductance and transpiration rate. This relationship has been developed independently by Dewar (1995) and Monteith (1995a). Monteith derived the equation through the synthesis of the results of laboratory studies on leaf conductance. Dewar, on the other hand, has provided the mechanistic definition behind the relationship, lending support to Monteith's results. Another important concept leading to the development of the PMPT model in this thesis is the assumption that the model originally developed for short agricultural crops or grassland (Monteith 1995a) will hold for a forest canopy. This assumption is supported by the fact that the crops and

grasses used in the original model, and Douglas fir used in the current development, are C_3 plants and therefore physiologically similar. Aerodynamic differences between the canopies can be accounted for by well-established relationships between aerodynamic conductance, roughness length and zero plane displacement.

Most importantly, the PMPT model is the first simple model to incorporate feedbacks between the vegetation and the atmosphere. The feedbacks between vegetation and the atmosphere were described in Section 1.1 and are important in regulating the transpiration from vegetation. By incorporating feedbacks between vegetation and the atmosphere, the PMPT model is able to combine both surface and atmospheric conditions to more accurately describe the latent heat flux from a canopy. This development is possible because of the concept of entrainment conductance (Monteith 1995a). The entrainment conductance is a measure of the ability of the Convective Boundary Layer to entrain air from above the inversion layer, and it furnishes a parameterization of the top of the boundary layer. This parameterization provides a way to combine a surface layer transpiration equation, such as the Penman-Monteith equation, with an atmospheric transpiration equation, such as the Priestley-Taylor equation. These developments have led to the successful development of a closed-system transpiration model.

In order to assess the utility of the PMPT model, its sensitivity to the maximum canopy conductance and maximum latent heat flux parameters was studied. The results of this analysis led to further insights into the response of a canopy to the two parameters. It is shown that a doubling of the value of the maximum canopy conductance does not appear to affect the latent heat flux of the system. However, at a given canopy conductance, latent heat flux increases with increasing net radiation. If both the maximum canopy conductance and the maximum latent heat flux are doubled,

there does not appear to be a significant change in the water balance of the system. However, the carbon balance will be affected because the stomata can remain open for a longer period of time, gaining an increase in carbon for the vegetation.

In order to further assess the utility of the PMPT model, it was compared to several other models of transpiration. It was shown that this model leads to more realistic relationships among canopy conductance, transpiration rate and vapor pressure deficit. Further analysis of the PMPT model was done by employing a three-dimensional development of the model. This development allows for the examination of the coupling between a canopy and the atmosphere at different canopy conductances and radiation loads. The results reveal that if the canopy conductance is large, there is a steep, almost linear relationship between the latent heat flux and net radiation, and the vapor pressure deficit of the atmosphere remains relatively constant. On the other hand, if the canopy conductance is small, the latent heat flux is strongly influenced by the canopy conductance and the vapor pressure deficit increases as net radiation increases. These results can be interpreted using the concept of coupling between vegetation and the atmosphere. For example, in a system well supplied with water, the vegetation is poorly coupled to the atmosphere and transpiration is thus driven by available energy. If the system has a poor water supply, the vegetation is well-coupled to the atmosphere. In this case, values of canopy conductance drive transpiration.

While the PMPT model leads to realistic relationships among canopy conductance, transpiration rates and vapor pressure deficit, there are areas of the model that need improvement. First, a better understanding of the entrainment conductance is needed. Research and a literature search has shown that its value does not appear to vary much, but this does not make intuitive sense. If there is entrainment of drier air from above the boundary layer, it would seem that this would increase the vapor pressure

deficit of the Convective Boundary Layer and, thus, the transpiration rate. However, De Bruin (1983) has shown that transpiration is not sensitive to entrainment. More research needs to be done in order to determine the importance of the entrainment conductance. For example, should the entrainment conductance be a function of the rate of change of boundary layer height? It would seem that rapid growth of the boundary layer would require a high value of entrainment conductance. Also, how does the entrainment conductance depend upon the strength of the temperature inversion and the mixing ratio gradient across the inversion (as stated in Section 2.7)? Is it indeed synonymous with the entrainment velocity? How are the temperature inversion and the mixing ratio gradient related to larger scale synoptic conditions? This relationship could explain why the surface conditions may not change from one day to the next, while the entrainment at the top of the boundary layer changes dramatically.

Another important aspect to consider is the theory that if leaf conductance is proportional to the transpiration rate, the canopy conductance is also proportional to transpiration rate. There is substantial theoretical evidence that this theory is true, but all of the experiments that have been done to date have been performed on leaves. Experiments at the canopy level have not yet been done because of the expense and difficulty involved, but they will need to be done in order to truly test this theory at the canopy level. A further improvement could be confirming the applicability of the equation $\dfrac{g_c}{g_m} = 1 - \dfrac{\lambda E}{\lambda E_m}$ to conifers and quantifying the terms g_m and E_m. This equation is likely to be applicable because of the physiological similarity between the two types of plants, but conifers have shallow roots. Therefore, the maximum latent heat flux is likely to be smaller than the values set for agricultural crops and grasses, which have been selected to have deeper root systems. A study of the relationship among maximum latent heat flux, rooting depth and soil water dynamics will need to be done to find a value of

maximum latent heat flux that is typical for conifers and other plant species. Finally, field data are needed in order to perform a full numerical test of the model and its parameterization.

There are also several ways in which the model can be improved to make it more generally applicable. First, the impact of soils and beneath-canopy dynamics should be included in the model. The influence of these variables is often negligible with dense canopies, but, in order for the model to be versatile, these variables should be included. In particular, evaporation from sparse canopies needs to be investigated. Sparse canopies are more complex. Soil evaporation and understory transpiration become very important in sparse canopies and need to be included in a full model of transpiration.

Finally, the model needs to be compared with more complex models of evaporation that take into account the mixed vegetation and complex topography found in many ecosystems. In order for a simple model of transpiration, like the PMPT model, to be universally useful, it must be parameterized so that it can produce results similar to those of the more complex models.

In its current form, the PMPT model is very useful for studying the partitioning of available energy into sensible and latent heat flux. It may also be useful for regional models which cannot employ the complexity of mesoscale latent heat flux models. With further study, the model should prove even more beneficial, providing science with a more complete, closed-system model of transpiration.

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APPENDICES

Appendix A - The Derivation of the Penman Equation

First, the available energy (H) of an area is equal to the difference between the net radiation (R_n) and the heat storage of the soil (G) at the surface.

$$H = R_n - G \tag{A-1}$$

In addition, the available energy (H) is also equal to the sensible heat flux (C) and the latent heat flux (λE) into the atmosphere.

$$H = C + \lambda E \tag{A-2}$$

By virtue of the law of the conservation of energy, one can combine (A-1) and (A-2) to get

$$R_n - G = C + \lambda E \tag{A-3}$$

By definition, the sensible heat flux is a function of wind speed and the difference in temperature between the surface and the atmosphere.

$$C = f(u)^*(T_c - T_a) \tag{A-4}$$

The latent heat flux is a function of wind speed and the difference between the vapor pressure between the surface and the atmosphere.

$$\lambda E = f(u)^*(e_c - e_a) \tag{A-5}$$

Looking at Figure A.1, one can see that e_c is actually the saturation vapor pressure at temperature T_c,

$$e_c = e_s(T_c) \tag{A-6}$$

and, thus,

$$e_c - e_a = e_s(T_c) - e_a \tag{A-7}$$

Now, according to Figure A.1, one can also see that the difference between e_s (T_c) and e_a is actually the vertical distance from e_a to $e_s(T_a)$ and the horizontal distance from T_a to T_c, which can be related be the slope of the line between the two temperature values (called Δ). Thus, beginning at the saturation value of $e_s(T_a)$ and traveling a distance along the slope line from T_a to T_c, results in the equation:

$$e_c - e_a = [e_s(T_a) - e_a] + [\Delta(T_c - T_a)] \tag{A-8}$$

This can be simplified to

$$e_c - e_a = D + \Delta(T_c - T_a) \tag{A-9}$$

where D is the vapor pressure deficit between the surface and the atmosphere.

Now, substituting Equation (A-9) into Equation (A-5), one gets

$$\lambda E = [f(u)]^*[D + \Delta(T_c - T_a)] \tag{A-10}$$

But, now substitute Equation (A-4) into equation (A-10)

$$\lambda E = [f(u)]^*[D + \frac{\Delta C}{f(u)}] \tag{A-11}$$

Now, substitute Equation (A-2) into (A-11)

$$\lambda E = [f(u)]^*[D + \frac{\Delta(H-\lambda E)}{f(u)}] \tag{A-12}$$

Simplifying, gives the equation

$$\lambda E = f(u)^*D + \Delta H - \Delta(\lambda E) \tag{A-13}$$

which becomes

$$\lambda E^*(1 + \Delta) = f(u)^*D + \Delta H \tag{A-14}$$

Finally, substituting, equation (A-1) into Equation (A-14) gives the Penman equation

$$\lambda E^* (1 + \Delta) = f(u)^*D + \Delta(Rn - G) \tag{A-15}$$

$$\text{or,} \quad \lambda E^* (1 + \Delta) = \frac{\Delta(R_n - G) + f(u)*D}{\Delta + 1} \tag{A-16}$$
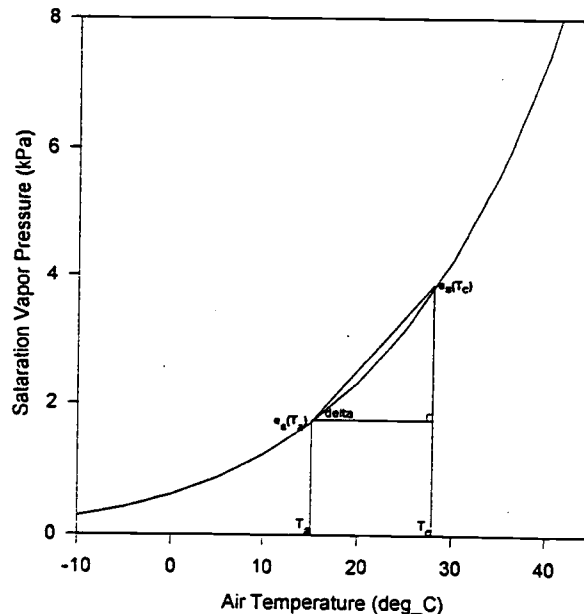

Figure A.1 The Basic Geometry of the Penman Equation
(adapted from Monteith and Unsworth 1990)

Appendix B - The "PMPT" Computer Model

The Main Program, employing the Functions and Constants listed afterward

```c
/* Programmed by Faith Ann Heinsch */

/* This program is designed to calculate evaporation and surface conductance
 * based on an article by John Monteith "Accommodation between transpiring
 * vegetation and the convective boundary layer" in Journal of Hydrology
 * (1995) 166:251-263.
 */

#include <stdio.h>
#include <math.h>
#include "pgm.h"


#define M 101  /* number of rows */

#define wind_speed 5        /* wind speed in m/s          */

#define albedo 0.1          /* albedo for vegetation      */

#define ge 0.005            /* entrainment conductance in m/s */

#define m_alpha 1.281       /* Priestley-Taylor coefficient   */


main()
{

                  /**** INITIALIZING THE VARIABLES ****/

    int i = 0,                  /* variable controlling the loops        */
        j = 0;                  /* variable controlling the loops        */

    float air_temp[M],          /* air temperature at site (deg_C)        */
        D[M],                   /* calculated vapor pressure deficit (kPa)  */
        evap[M],                /* evaporation rate based on latent heat flux */
        g[M],                   /* canopy conductance (m/s)               */
        ga[M],                  /* aerodynamic conductance (m/s)          */
        gm[M],                  /* maximum conductance (m/s)              */
        latheat[M],             /* latent heat flux based on PM-PT (W/m^2) */
        maxlatheat[M],          /* maximum latent heat flux (W/m^2)       */
        primelatheat[M],        /* Monteith E' (from fax)                 */
        rad[M];                 /* incoming solar radiation (W/m^2)       */

    float dnlwrad = 0.0,        /* net longwave radiation (W/m^2)         */
```

```
maxevap = 0.0;          /* maximum evaporation rate (g/m^2 s)    */

float c1 = 0.0,         /* variable for calculation               */
      c2 = 0.0,         /* variable for calculation               */
      temp1 = 0.0,      /* variable for calculation               */
      temp2 = 0.0;      /* variable for calculation               */
```

/**** INITIALIZING THE ARRAYS ****/

```
for(i = 0; i < M; i++)
{
    air_temp[i];
    D[i] = 0.0;
    evap[i] = 0.0;
    g[i] = 0.0;
    ga[i] = 0.0;
    gm[i] = 0.0;
    latheat[i] = 0.0;
    maxlatheat[i] = 0.0;
    primelatheat[i] = 0.0;
    rad[i] = 0.0;
}
```

/**** CALCULATION OF VARIABLES ****/

```
g[0] = 0.0;

for(i = 0; i < M; i++)
{
    if(i != 0)
        g[i] = g[i-1] + 0.0002;

    ga[i] = aero_cond(wind_speed);
    gm[i] = 0.0001 * NETRAD(rad[i], albedo, air_temp[i]);

    temp1 = 1 + (ge/g[i]);
    temp2 = 1 - (g[i]/gm[i]);


    primelatheat[i] = (m_alpha * DELTA(air_temp[i]) * NETRAD(rad[i], albedo,
        air_temp[i])) / (DELTA(air_temp[i]) + GAMMA(AIR_PRESS, air_temp[i]));

    maxlatheat[i] = primelatheat[i] / (temp1 * temp2);

    maxevap = maxlatheat[i] / LAMBDA(air_temp[i]);
```

```c
    latheat[i] = maxlatheat[i] * (1 - (g[i]/gm[i]));

    evap[i] = latheat[i] / LAMBDA(air_temp[i]);


    temp1 = temp2 = 0;

    c1 = AIR_DENSITY * CP_AIR * ga[i];
    c2 = DELTA(air_temp[i]) + (GAMMA(AIR_PRESS, air_temp[i]) * (1 + (ga[i]/g[i])));
    D[i] = (((latheat[i] * c2) - (DELTA(air_temp[i]) * NETRAD(rad[i], albedo, air_temp[i])) /
        c1) / 1000);

    c1 = c2 = 0;
}



printf("\n\n");
printf("    g      VPD    Latent Heat Flux\n");
printf(" (mm/s)  (kPa)       (W/m^2)      \n");
printf("---------------------------------------\n");

for(i = 0; i < M; i++)
    printf(" %5.2f  %5.4f     %6.2f \n", g[i] * 1000, D[i], latheat[i]);


}
```

Functions and Contants used in the main program above - "pgm.h"

```c
extern double aero_cond(double veg_height, double windspeed, double ht_interest, char
        class);

extern double latheat(double air_temp, double air_press, double rad, double albedo,
        double canopy_cond, double PT_alpha);

extern double evaporation(double air_temp, double latent_heat);

extern double vap_press_def(double latent_heat, double air_temp, double air_press,
        double rad, double albedo, double ga, double canopy_cond);


#define SCALING_COND 0.012      /* scaling conductance in m/s              */

#define AIR_DENSITY 1.204       /* density of air in kg/m^3)               */

#define CP_AIR 1.005e3          /* specific heat of air at constant pressure in
                                   J/ kg/deg)                              */
```

```
#define STEFBO5.67032e-8          /* Stefan-Bolzmann constant (W m-2 deg-4)     */
#define LAMBDA(temp) (2501 - (2.3727 * (temp)))

#define GAMMA(press,temp) \
        ((press * CP_AIR) / (0.622 * LAMBDA(temp)))

#define CONSTANT_A 0.61375
#define CONSTANT_B 17.502
#define CONSTANT_C 240.97

#define SAT_VAPOR_PRESS(temp) \
    (1000 * CONSTANT_A * exp((CONSTANT_B * (temp)) / (CONSTANT_C + (temp))))

#define DELTA(temp) \
    (((CONSTANT_B * CONSTANT_C) / pow(CONSTANT_C + (temp), 2.0)) * \
            SAT_VAPOR_PRESS(temp))

#define DN_LW_RAD(temp) (213 + (5.5 * (temp)))

#define NET_RAD(rad,albedo,temp) \
    ((rad) - ((albedo) * (rad)) + DN_LW_RAD(temp) - (STEFBO * pow(temp, 4.0)))
```