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Abstract approved: _____

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Reliable controller designs have been developed in this thesis for a number of finite-horizon and infinite-horizon problems with possibly non-zero initial conditions. These reliable controllers assure that system stability and system performance will be maintained despite certain system faults. The performance measure used in this thesis is an “ H_∞ -like norm”, which is an induced two-norm from all exogenous signals and initial conditions to the regulated output and final states. Controller designs and existence conditions are presented for a reliable controller for faults in any pre-selected subset of actuators or sensors. Also, controller designs and an existence condition are presented for a reliable controller for any single sensor or actuator fault using sensor and actuator redundancy.

RELIABLE CONTROLLER DESIGN FOR SYSTEMS WITH TRANSIENTS

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RELIABLE CONTROLLER DESIGN FOR SYSTEMS WITH TRANSIENTS

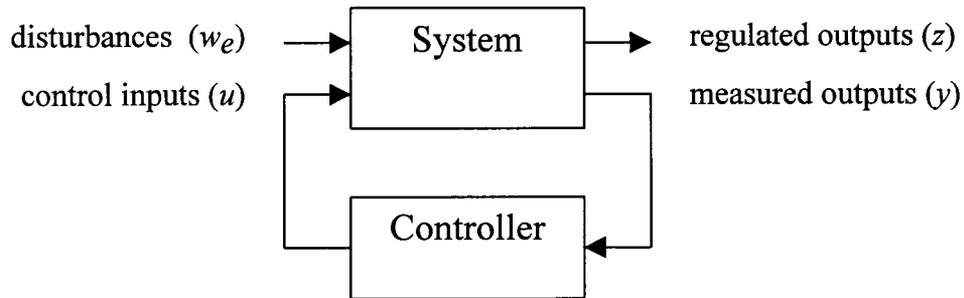
CHAPTER 1 INTRODUCTION

Reliable controller design, as discussed in this thesis, is the design of feedback controllers that guarantee closed-loop system stability and performance during the occurrence of certain sensor and actuator faults. Consider the control of aircraft. When faults occur in sensors and actuators, it is important that the aircraft keep flying.

H_∞ control theory is about the design of control systems to satisfy an H_∞ norm performance requirement. It was developed to address the robust control problem – how to design a controller that is H_∞ optimal for the worst case “disturbance” in some pre-specified set.

Given a nominal system description, we can design a controller that meets a closed-loop stability requirement. This is classical control theory. However, in most cases, we can not model the system accurately. Furthermore, some parameters of the system will change because of environmental and other unpredictable factors. The closed-loop system may not meet the performance requirements when the parameters change. Classical control theory can not solve this kind of problem. Robust control theory is required to solve this problem.

When system parameters may change or the system may experience small disturbances, we use robust control theory to design the controller. (See the following figure.)



A robust controller is one that satisfies some performance criteria for a whole range of parameter values and/or a set of disturbance signals. If the H_∞ norm is used to define the performance criterion, then this is an H_∞ control problem. The H_∞ control problem is to design a controller such that the H_∞ norm of the closed-loop transfer-function matrix from w_e to z is less than a given bound.

We can solve the H_∞ control problem either in the frequency domain or in the state-space domain. In the H_∞ control problem, we always assume that the variation in the parameter value or the disturbance signals in the system is bounded. Many papers have been published in this area. Francis described how to solve the H_∞ problem systematically in frequency domain [1]. He used operator theory and complex analysis. However, the algorithm to solve the problem in the frequency domain is very complex and is not easily realized on a computer. In 1989, Doyle, Glover, Khargonekar and Francis published their now-famous paper on how to solve the H_∞ problem using state-space methods [2]. The problem addressed was to design a feedback controller for which the closed-loop system is stable and the H_∞ norm of the closed-loop transfer matrix, from the disturbance input to the regulated output, is less than some pre-selected constant for all disturbances in some

set. The controller design algorithm in the state-space domain is easily implemented. It requires solving two algebraic Riccati equations.

The set up for the H_∞ problem is somewhat limiting. The only failures considered are those that can be modeled as part of the disturbance input. The effect of any non-zero initial conditions of the state variables on the system is neglected. (The initial conditions are assumed to be zero.) A critical assumption of this method is that the total energy of the disturbance input, integrated over all time, is bounded.

Veillette, Medanić and Perkins found that using the general H_∞ controller design algorithm from [2] can not guarantee the stability of the closed-loop system and the H_∞ performance requirement when there exist faults in sensors or actuators. They introduced the H_∞ reliable control concept to address this type of problem [3].

Consider again the control of an aircraft. Faults could occur in an entire set of actuators or sensors, if they are due to either damage to some part of the aircraft or to a problem in an electronics subsystem. Faults caused by aging of a component would likely occur in a single sensor or actuator.

Veillette, Medanić and Perkins proposed a design of a reliable controller that maintains closed-loop stability and an H_∞ norm bound when there are outages in a pre-selected subset of sensors and actuators [3], [4], [5]. They discussed both centralized and decentralized cases for continuous-time systems. The algorithm to solve this reliable control problem is not very difficult to implement. It involves solving two Riccati-like equations. Paz and Medanić analyzed the reliable control problem for discrete-time systems [6]. They had numerical difficulties with their solution of the discrete-time

problem. Shor, Perkins and Medanić developed a unified formulation for discrete-time and continuous-time reliable controller design, providing a numerical method to solve the reliable control problem [7], [8], [9].

For faults resulting from aging or failure of the sensor or actuator components themselves, one would expect there to be only one sensor or actuator outage at a time. It could occur in any sensor or actuator – not just a pre-selected subset. Medanić proposed a different H_∞ reliable controller design using redundant control elements for robustness to any single sensor or actuator fault [10].

All of these early H_∞ reliable controller designs treated only sensor or actuator faults that could be modeled as complete outages, or complete signal losses, in input or output channels. They provided observer-based controller designs and gave sufficient conditions for the existence of observer-based controllers guaranteeing reliable H_∞ performance. Shor and Kolodziej proposed a reliable controller design for actuator faults modeled as partial signal losses [11]. Veillette proved the necessary and sufficient condition for the existence of an H_∞ reliable state-feedback controller [12].

In all of this past work, permanent failures of sensors and actuators were treated and the problem was set up as an infinite-horizon problem with zero initial conditions. The H_∞ performance measure was used.

The new results presented in this thesis remove many of the restrictions in the earlier H_∞ reliable control problem. The author, Feng, and her advisor, Shor, presented an earlier version of these results in 1994 [13].

Failures may occur during the operation of a system, not only before it is started, and failures can change from time to time. The system does not always start out in a relaxed state with zero initial conditions. The initial condition could be either zero or nonzero. It could be an arbitrary (unknown) value. In such cases, the transient behavior is important. When the transient response is important, we can not use the earlier reliable control results, since those reliable controller designs only considered permanent failures of sensors and actuators, setting the problem up as an infinite-horizon problem with zero initial conditions.

For aircraft control, extensive checks are conducted before take-off. One would not fly the aircraft if the entire system were not in working order in advance. Reliable control is needed to address faults that begin during the flight, and the system will not start in a relaxed state.

In order to address failures that occur or change during operation of the system, the transient behavior can be weighted in the performance measure. The performance can be measured using an induced two-norm from all exogenous signals *and* initial conditions to the regulated output signals *and* final states [14]. This induced two-norm may be taken over either a finite horizon or the infinite horizon. Khargonekar, et. al, treated the general “ H_∞ -like” control problem with transients, using this “ H_∞ -like norm” [14]. Necessary and sufficient conditions were proved for the existence of controllers meeting these performance requirements.

In this thesis, new reliable controller designs are presented for the cases when the (possibly non-zero) initial conditions are unknown, for both finite and infinite horizon

problems, generalizing the previous continuous-time reliable results in [3], [4], [10], [11], and [12]. For the case when the faults must lie within a pre-specified subset of actuators or sensors, the actuator and sensor faults and fault combinations are permitted to change from time to time during the horizon of the problem, and both partial and whole actuator or sensor signal losses are treated. The necessary and sufficient conditions are given for the existence of a state-feedback reliable controller and sufficient conditions are given for the existence of dynamic output-feedback reliable controllers robust to whole or partial signal losses in a pre-selected subset of actuators or sensors. It is shown here that the sufficient conditions for the existence of reliable controllers given by [3] and [12] for zero-initial conditions, infinite horizon and permanent complete outages also guarantee robustness to partial signal losses in a pre-specified subset of sensors or actuators. The requirement in [3] for the actuator outage case that the reliable controller be open-loop stable is eliminated here. For the case when any single sensor or actuator fault may occur, sufficient conditions are presented for the existence of reliable controllers for both finite or infinite horizon problems with unknown possibly nonzero initial conditions.

The thesis is organized as follows. In Chapter 2, theorems for the general “ H_∞ -like” control problem with transients given by Khargonekar, et al., [14] are reviewed. A result from Doyle, et al., [2] and the bounded real lemma from Veillette, et al., [3] are also recalled. In Chapter 3, new reliable controller designs for actuator faults are given. In Chapter 4, new reliable controller designs for sensor faults are presented. In Chapter 5, reliable controller designs for single sensor or actuator faults are given, with sufficient

conditions for their existence, for both finite- and infinite- horizon problems with unknown initial conditions. Chapter 6 is the conclusion.

CHAPTER 2 BACKGROUND

This chapter will review the results on “ H_∞ -like” control with transients given in [14] and the basic state-space approach to H_∞ control theory. A very important lemma, which is the basis for reliable control, is also shown here.

Consider the finite-dimensional linear time-invariant system Σ :

$$\frac{dx}{dt} = Ax + Bu + Gw_0, \quad x(0) = x_0$$

$$y = Cx + w$$

$$z = \begin{bmatrix} Hx \\ u \end{bmatrix}$$

$$w_e = \begin{bmatrix} w_0 \\ w \end{bmatrix}$$

The variables x, w_e, u, z , and y denote the state, exogenous input, control input, regulated output, and measured output. It is assumed throughout this thesis that the initial state $x(0)$ is unknown and possibly nonzero. The triples (A, B, H) and (A, G, C) are assumed in [14] and in this thesis to be both stabilizable and detectable in all infinite horizon problems.

The “ H_∞ control with transients” problem, addressed in [14], is to design a closed-loop controller such that, for a fixed final time $T > 0$, symmetric positive semi-definite matrix S and symmetric positive-definite matrix R , the following “ H_∞ -like” worst-case closed-loop performance measure is bounded by γ :

$$J(\Sigma_{cl}, R, S, T) := \sup \left\{ \left(\frac{\|z\|_T^2 + x'(T)Sx(T)}{\|w_e\|_T^2 + x_0'Rx_0} \right)^{1/2} \right\} < \gamma.$$

Here, Σ_{cl} represents the closed-loop system. The supremum is taken over the set of all $x(0) = x_0 \in \mathfrak{R}^n$ and $w_e \in L_2[0, T]$ for which $\|w_e\|_T^2 + x_0'Rx_0 \neq 0$. The signal norm $\|f\|_T$ is defined as $\left(\int_0^T \|f(t)\|^2 dt \right)^{1/2}$. In this definition, the final T is allowed to be ∞ , in which case $S=0$.

Necessary and sufficient conditions are given in [14] for the existence of a closed-loop controller that satisfies this “ H_∞ -like” performance requirement. Formulas for the controller are given separately for the finite and infinite horizon cases.

For the full state-feedback case, we assume that we can measure the state accurately, which means for the system Σ that $y = x$, or matrix $C = I$ and $w = 0$. The controller can then be designed by state feedback.

Consider first the state-feedback problem for the finite-horizon case.

Theorem 2.1 [14, Theorem 2.1] *Consider the system Σ . For the finite-horizon case, let R and S be given symmetric matrices such that S is positive semi-definite and R is positive definite.*

(a) *There exists an admissible state-feedback controller such that $J(\Sigma_{cl}, R, S, T) < \gamma$ if and only if there exists a (unique) symmetric matrix function $P(t)$, $t \in [0, T]$, such that*

$$-\dot{P}(t) = A'P(t) + P(t)A + P(t) \left(\frac{1}{\gamma^2} GG' - BB' \right) P(t) + H'H$$

$$P(T) = S \text{ and } P(0) < \gamma^2 R$$

(b) In this case, the control law $u(t) = -B' P(t)x(t)$ achieves $J(\Sigma_{cl}, R, S, T) < \gamma$.

For the infinite-horizon case, the matrix $P(t)$ will be a constant P . The next result addresses the state-feedback problem in the infinite-horizon case.

Theorem 2.2 [14, Theorem 2.2] Consider the system Σ . Let R be a given positive-definite matrix.

(a) There exists an admissible state-feedback controller such that $J(\Sigma_{cl}, R, 0, \infty) < \gamma$ if and only if there exists a unique symmetric matrix P such that

$$A' P + P A + P \left(\frac{1}{\gamma^2} G G' - B B' \right) P + H' H = 0,$$

with $A + \left(\frac{1}{\gamma^2} G G' - B B' \right) P$ asymptotically stable, and $0 \leq P \leq \gamma^2 R$.

(b) In this case, the control law $u(t) = -B' P x(t)$ achieves $J(\Sigma_{cl}, R, 0, \infty) < \gamma$.

In practice, we are rarely able to measure the state directly. We can only measure the output with noise. In the following theorems, dynamic output feedback is used to design the controller.

For the dynamic output-feedback problem on a finite horizon, the following theorem is given.

Theorem 2.3 [14, Theorem 2.3] Consider the system Σ . Let R and S be given symmetric matrices such that S is positive semi-definite and R is positive definite.

(a) There exists an admissible dynamic output-feedback controller such that

$J(\Sigma_{cl}, R, S, T) < \gamma$ if and only if the following three conditions hold:

(i) *There exists a symmetric matrix function $P(t)$ such that*

$$-\dot{P}(t) = A'P(t) + P(t)A + P(t)\left(\frac{1}{\gamma^2}GG' - BB'\right)P(t) + H'H$$

$$P(T) = S \text{ and } P(0) < \gamma^2 R$$

(ii) *There exists a symmetric matrix function $Q(t) > 0$ for all $t \in [0, T]$ such that*

$$\dot{Q}(t) = AQ(t) + Q(t)A' - Q(t)\left(C'C - \frac{1}{\gamma^2}HH'\right)Q(t) + GG'$$

$$\text{with } Q(0) = R^{-1}, \text{ and}$$

(iii) $\rho\left(\frac{1}{\gamma^2}P(t)Q(t)\right) < 1$ for all $t \in [0, T]$, where $\rho(\cdot)$ denotes the maximum

singular value

(b) *If these conditions are met, then one controller that achieves $J(\Sigma_{cl}, R, S, T) < \gamma$ is given by*

$$\begin{aligned} \dot{\xi}(t) &= (A + BK + GK_d)\xi(t) + L(y - C\xi), \quad \xi(0) = 0, \\ K &= -B'P(t), \quad K_d = \frac{1}{\gamma^2}G'P(t), \quad L = (1 - \gamma^{-2}Q(t)P(t))^{-1}Q(t)C' \\ u(t) &= -B'P(t)\xi(t). \end{aligned}$$

The next result is a solution to the output-feedback problem in the infinite horizon case for linear time-invariant systems.

Theorem 2.4 [14, Theorem 2.4] *Consider the system Σ . Let R be a given positive definite matrix.*

(a) *There exists an admissible dynamic output-feedback controller such that*

$J(\Sigma_{cl}, R, 0, \infty) < \gamma$ *if and only if the following three conditions hold:*

(i) There exists a (unique) symmetric matrix P such that

$$A'P + PA + P\left(\frac{1}{\gamma^2}GG' - BB'\right)P + H'H = 0,$$

with $A + \left(\frac{1}{\gamma^2}GG' - BB'\right)P$ asymptotically stable, and $0 \leq P < \gamma^2 R$.

(ii) There exists a symmetric bounded matrix function $Q(t) > 0$ for all $t \geq 0$ such that

$$\dot{Q}(t) = AQ(t) + Q(t)A' - Q(t)\left(C'C - \frac{1}{\gamma^2}HH'\right)Q(t) + GG'$$

with $Q(0) = R^{-1}$, and the unforced linear time-varying system

$$\dot{p}(t) = (A - Q(t)(C'C - \gamma^{-2}H'H))p(t)$$

is exponentially stable.

(iii) The function $\rho\left(\frac{1}{\gamma^2}P(t)Q(t)\right) < 1$ for all $t \geq 0$ and is bounded.

(b) Moreover, if $Q(t)$ with the above properties exists for all $t \geq 0$, then $\lim_{t \rightarrow \infty} Q(t)$ exists

and equals Q_∞ , where Q_∞ is the unique symmetric matrix such that

$$AQ_\infty + Q_\infty A' - Q_\infty (C'C - \gamma^{-2}H'H)Q_\infty + GG' = 0,$$

$A - Q_\infty (C'C - \gamma^{-2}H'H)$ is asymptotically stable, and $Q_\infty \geq 0$.

(c) If the conditions above are met, then one controller that achieves $J(\Sigma_{cl}, R, 0, \infty) < \gamma$

is given by

$$\begin{aligned}\dot{\xi}(t) &= (A + BK + GK_d)\xi(t) + L(y - C\xi), \quad \xi(0) = 0, \\ K &= -B'P, \quad K_d = \frac{1}{\gamma^2}G', \quad L = (1 - \gamma^{-2}Q(t)P)^{-1}Q(t)C' \\ u(t) &= -B'P\xi(t).\end{aligned}$$

When R is sufficiently large, then the following corollary holds:

Corollary 2.1 [14, Corollary 2.5] *Let the conditions of Theorem 2.4 be satisfied, and let R be such that $TR^{-1} < Q_\infty$. Then the linear time-invariant controller of the form given in Theorem 2.4 with Q_∞ replacing $Q(t)$ achieves $J(\Sigma_{cl}, R, 0, \infty) < \gamma$.*

The above result is very similar to the general H_∞ control problem.

For the infinite-horizon case, when the initial condition $x(0)$ is zero, the “ H_∞ -like” performance measure in this thesis reduces to the usual H_∞ norm, $J = \sup \left(\frac{\|z\|_2}{\|w_e\|_2} \right)$.

Doyle, et al., in [2] gave the following theorem for the infinite-horizon case:

Theorem 2.5 [2, Theorem 2.3] *Consider the system Σ with the initial condition $x(0)=0$.*

(a) *There exists an admissible output-feedback controller such that $J(\Sigma_{cl}, R, 0, \infty) < \gamma$ if and only if the following three conditions hold:*

(i) *There exists a (unique) symmetric matrix P such that*

$$A'P + PA + P \left(\frac{1}{\gamma^2} GG' - BB' \right) P + H' H = 0,$$

with $A + \left(\frac{1}{\gamma^2} GG' - BB' \right) P$ asymptotically stable, and $P \geq 0$.

(ii) *There exists a symmetric matrix $Q \geq 0$ such that*

$$AQ + QA' - Q(C' C - \gamma^{-2} H' H)Q + GG' = 0,$$

with $A - Q_\infty(C' C - \gamma^{-2} H' H)$ asymptotically stable.

(iii) *The function $(1 - \rho(\gamma^{-2} QP))^{-1} < 0$.*

(b) If the conditions above are met, then one controller that achieves $J(\Sigma_{cl}, R, 0, \infty) < \gamma$ is given by

$$\begin{aligned}\dot{\xi}(t) &= (A + BK + GK_d)\xi(t) + L(y - C\xi), \quad \xi(0) = 0, \\ K &= -B'P, \quad K_d = \frac{1}{\gamma^2}G', \quad L = (1 - \gamma^{-2}QP)^{-1}QC' \\ u(t) &= -B'P\xi(t).\end{aligned}$$

The development in the following chapters will be based directly on the necessary and sufficient conditions given above and the following bounded lemma, from [3].

Lemma 2.1 [3, Theorem 2.3] *Let $T(s) = H(sI - F)^{-1}G$, with (F, H) a detectable pair. If there exists a real matrix $X \geq 0$ and a positive scalar γ such that*

$$F'X + XF + \gamma^{-2}XGG'X + H'H \leq 0,$$

then F is Hurwitz, and $T(s)$ satisfies $\|T\|_\infty \leq \gamma$.

The condition in this lemma is only sufficient. We may have more freedom to design a controller such that the closed-loop system satisfying these conditions than this lemma suggests.

CHAPTER 3 RELIABLE CONTROLLER DESIGN WITH RESPECT TO ACTUATOR FAULTS

This chapter will present reliable controller designs for the case of actuator faults for both the finite and infinite horizon cases for systems with transients and unknown initial conditions using the “ H_∞ -like” performance measurement. It extends the reliable controller designs for actuator faults for systems with zero initial conditions in [3], [12] to the case with unknown initial conditions. We consider not only whole signal losses because of sensor or actuator outages, but also partial signal losses. For the reliable controller design, we consider the case when the faults must be in a pre-selected subset of actuators or sensors. The output-feedback and state-feedback cases will be discussed separately.

Define the subscript Ω to be the index set of actuators (or sensors in later chapters) susceptible to faults, Ω' to be the complement of the index set of Ω , ω' to be the index set of actuator (or sensor) signals experiencing no losses, and ω to be the complement of the index set of ω' . Assume that the actuators are ordered $(u_1, u_2, \dots, u_{m\Omega}, \dots, u_m)$, where m is the number of actuators and the first $m\Omega$ actuators are susceptible to faults.

Let $\Delta = \text{diag}(\Delta_\Omega, I)$, a diagnostic matrix of dimension $\{m \times m\}$, where $\Delta_\Omega = \text{diag}(\delta_1, \delta_2, \dots, \delta_{m\Omega})$ and $\delta_i \in [0, 1]$ is the degree of attenuation of the actuator susceptible to faults, where 0 corresponds to a complete outage and 1 corresponds to no outage. We use $B\Delta u$ to represent the signals that remain during the fault. When there is no actuator fault, $\Delta_\Omega = I$. When $\Delta_\Omega = 0$, all actuators in subset Ω are experiencing complete

outages. When $0 \leq \Delta_\Omega \leq I$ and Δ_Ω is neither the null matrix 0 nor the identity matrix I , then at least some actuators in subset Ω experience partial signal losses.

The signal losses are not included in the regulated output variables, and hence are not weighted in the performance measure. Let $z_{\omega'}$ represent the regulated output with actuator signal losses during $[0, T]$. The performance requirement is then

$$J_{\omega'}(\Sigma_{cl}, R, S, T) = \sup \left\{ \left(\frac{\|z_{\omega'}\|_T^2 + x'(T)Sx(T)}{\|w_e\|_T^2 + x_0'Rx_0} \right)^{1/2} \right\} < \gamma,$$

where $z_{\omega'} = \begin{bmatrix} Hx \\ \Delta u \end{bmatrix}$.

We will design a controller that will make the closed-loop system stable and will meet the above performance requirement. We consider both the finite horizon case, $0 < T < \infty$ and the infinite horizon case, $T = \infty$.

3.1 Reliable State-Feedback Controller Design

Consider first the state-feedback reliable controller design with transients. The finite-horizon and infinite-horizon cases will be proved separately. We will prove the necessary and sufficient conditions first for the finite-horizon case then for the infinite-horizon case.

Theorem 3.1 (Finite-horizon case) *Let R and S be given symmetric matrices such that S is positive semi-definite and R is positive definite.*

(a) There exists an admissible state-feedback controller such that $J_{\omega'}(\Sigma_{cl}, R, S, T) < \gamma$ for all combinations of actuator faults in the pre-selected subset Ω if and only if there exists a (unique) symmetric matrix function $P(t), t \in [0, T]$, such that

$$-\dot{P}(t) = A' P(t) + P(t) A + P(t) \left(\frac{1}{\gamma^2} G G' - B B' \right) P(t) + H' H$$

$$P(T) = S, \text{ and } P(0) < \gamma^2 R$$

(b) In this case, the control law $u(t) = -B' P(t)x(t)$ achieves $J_{\omega'}(\Sigma_{cl}, R, S, T) < \gamma$, for all combinations of actuator faults in Ω and all levels of attenuation Δ_{Ω}

Proof

For simplicity, the proof is only given for $\gamma = 1$. The proof method is the same for $\gamma \neq 0$.

Sufficiency:

We let the actuator attenuation values in Δ change a finite number of times in any finite interval. Suppose subset $\omega_i \subseteq \Omega$ of actuator faults occurs during $[t_{i-1}, t_i)$, for $i = 1, \dots, n$, where $0 = t_0 < t_1 < t_2 < \dots < t_n = T$. Differentiating $x' P(t)x$ along the trajectories of Σ for the $P(t)$ given in Theorem 3.1, and after some algebraic computation, we have

$$\begin{aligned} d(x' P x) / dt &= (G w_0)' P(t)x + (B \Delta u)' P(t)x - x' P(t) G G' P(t)x + x' P(t) B_{\Omega} B_{\Omega}' P(t)x \\ &\quad - x' H' H x + x' P(t) B \Delta u + x' P(t) G w_0 \\ &= -(w_0 - G' P(t)x)' (w_0 - G' P(t)x) + (\Delta u + B' P(t)x)' (\Delta u + B' P(t)x) \\ &\quad - (B_{\Omega}' P(t)x)' (B_{\Omega}' P(t)x) + w_0' w_0 - z_{\omega_i}' z_{\omega_i} \end{aligned}$$

Here, Δu are the actuator signals remaining. By integrating from t_{i-1} to t_i we obtain

$$\begin{aligned} &x'(t_i) P(t_i) x(t_i) - x'(t_{i-1}) P(t_{i-1}) x(t_{i-1}) \\ &= \|w_0\|_{[t_{i-1}, t_i]}^2 - \|z_{\omega_i}\|_{[t_{i-1}, t_i]}^2 - \|w_0 - G' P(t)x\|_{[t_{i-1}, t_i]}^2 - \|B_{\Omega}' P(t)x\|_{[t_{i-1}, t_i]}^2 \end{aligned}$$

Note that $d(x'Px)/dt$ is finite everywhere and is discontinuous only on a set of measure zero, and $x(t)$ and $P(t)$ are both continuous everywhere. (We are modeling faults as sudden changes of actuator or sensor function.) Taking the integral from 0 to T , we obtain

$$\begin{aligned} \int_0^T \{d(x'Px)/dt\} dt &= \sum_{i=1}^n \int_{t_{i-1}}^{t_i} \{d(x'Px)/dt\} dt \\ &= x'(T)P(T)x(T) - x'(0)P(0)x(0) \\ &= \|w_0\|_T^2 - \sum_{i=1}^n \|z_{\omega_i}\|_{[t_{i-1}, t_i]}^2 \\ &\quad - \|w_0 - G'Px\|_T^2 - \sum_{i=1}^n \|B_{\Omega_i}'Px\|_{[t_{i-1}, t_i]}^2 \end{aligned}$$

Since $P(T) = S$,

$$\begin{aligned} \|w_0\|_T^2 + x'(0)Rx(0) - \sum_{i=1}^n \|z_{\omega_i}\|_{[t_{i-1}, t_i]}^2 - x'(T)Sx(T) \\ = \|w_0 - G'Px\|_T^2 + \sum_{i=1}^n \|B_{\Omega_i}'Px\|_T^2 + x'(0)(R - P(0))x(0). \end{aligned}$$

If $R > P(0)$, then $\|w_0\|_T^2 + x'(0)Rx(0) - \|z_{\omega_i}\|_T^2 - x'(T)Sx(T) > 0$, where $\|z_{\omega_i}\|_T^2$ is defined as

$$\sum_{i=1}^n \|z_{\omega_i}\|_{[t_{i-1}, t_i]}^2. \text{ Thus, } J_{\omega_i}(\sum_{cl}, R, S, T) < 1.$$

Necessity:

The complete loss of all the susceptible actuators is an admissible contingency. Therefore, a reliable state feedback exists only if the performance can be achieved using the non-susceptible actuators $B_{\Omega'}$ alone. According to Theorem 2.1, this condition is equivalent to the existence of a solution of the Riccati equation in Theorem 3.1.

-Q.E.D.-

For the infinite-horizon case, we must consider the stability of the closed-loop system. The next theorem is about reliable state-feedback control in the infinite horizon case with unknown initial conditions. Recall that we assume for all infinite horizon problems that the triples (A, B, H) and (A, G, C) are both stabilizable and detectable

Theorem 3.2 (Infinite horizon case) *Consider the same system Σ . Assume that $(A, B_{\Omega'})$ is stabilizable. Let R be a given positive definite matrix. There exists an admissible state-feedback controller such that $J_{\omega'}(\Sigma_{cl}, R, 0, \infty) < \gamma$ for any combinations of actuator faults in the pre-selected subset Ω if and only if there is a unique symmetric matrix P such that*

$$A'P + PA + P\left(\frac{1}{\gamma^2}GG' - B_{\Omega}B_{\Omega}'\right)P + H'H = 0,$$

with $A + \left(\frac{1}{\gamma^2}GG' - BB'\right)P$ asymptotically stable, and $0 \leq P < \gamma^2 R$.

In this case, the control law $u(t) = -B'Px(t)$ achieves $J(\Sigma_{cl}, R, 0, \infty) < \gamma$.

The proof of this theorem is different from the finite-horizon case. It involves the construction of an augmented system based on the nominal system. We consider the loss of actuator signals as the disturbance from the outside.

Proof

Sufficiency:

Replacing G by $G_+ = [G \ \gamma B_{\Omega} \Delta_{\Omega}^{1/2}]$ and B by $B\Delta^{1/2}$ in system Σ , construct the augmented system Σ_a with the fictitious disturbance input w_f

$$\dot{x} = Ax + B\Delta^{1/2}u_1 + [G \gamma B_\Omega \Delta^{1/2}] \begin{bmatrix} w_0 \\ w_f \end{bmatrix}, \quad x(0) = x_0$$

$$z_a = \begin{bmatrix} Hx \\ u_1 \end{bmatrix},$$

The condition

$$A'P + PA + P\left(\frac{1}{\gamma^2}GG' - B_\Omega B_\Omega'\right)P + H'H = 0,$$

is equivalent to

$$A'P + PA + P\left(\frac{1}{\gamma^2}G_+G_+' - B_\Omega \Delta B_\Omega'\right)P + H'H = 0,$$

The condition that $A + \left(\frac{1}{\gamma^2}GG' - B_\Omega B_\Omega'\right)P$ be asymptotically stable is equivalent to

$A + \left(\frac{1}{\gamma^2}GG' - B\Delta B'\right)P$ being asymptotically stable. Since (A, B_Ω) is stabilizable,

$(A, B\Delta^{1/2})$ is stabilizable. According to Theorem 2.2, the control law

$u_1(t) = -(B\Delta^{1/2})'Px(t)$ achieves $\gamma^2(\|w_0\|^2 + \|w_f\|^2 + x'(0)Rx(0) - \|Hx\|^2 - \|u_1\|^2) > 0$ and

the closed-loop system is stable.

Since $\Delta \leq I$ and $w_f \in L[0, \infty)$, the above inequality still holds with u_1 replaced by $u_\omega = -(B\Delta)'Px(t)$ and $w_f = 0$. In this case, the augmented system is the original system with actuator outage in B_Ω and control signal $u = -B'Px(t)$ is replaced by u_ω .

Necessity:

The complete loss of all the susceptible actuators is an admissible contingency.

Therefore, a reliable state-feedback control exists only if the performance can be achieved

using the non-susceptible actuators Ω' only. According to Theorem 2.2, this condition is equivalent to the existence of a solution of the algebraic Riccati equation in Theorem 3.1

and the additional properties that $A + \left(\frac{1}{\gamma^2} GG' - B_{\Omega'} B_{\Omega'}' \right) P$ be asymptotically stable and

$0 \leq P < \gamma^2 R$. In order to use Theorem 2.2, we require that $(A, B_{\Omega'})$ be stabilizable.

-Q.E.D.-

If the state of the system can not be measured directly, we can not use state feedback. Instead, we will use dynamic output feedback to design a reliable controller. The following section will discuss these dynamic output feedback designs.

3.2 Output-Feedback Reliable Control

The theorems proved next about dynamic reliable output-feedback control are different from the state-feedback case. They are more complex, and furthermore, all the conditions about existence of dynamic output feedback reliable controllers are sufficient conditions, not necessary and sufficient conditions, such as those obtained for state-feedback controllers.

Like the state-feedback problem, we will also discuss two cases for dynamic output-feedback - the finite-horizon case and the infinite horizon case.

Let us first consider the finite horizon case and prove the following theorem.

Theorem 3.3 (Finite horizon case) *Consider system the Σ , and let $R > 0$ and $S \geq 0$ be as in Theorem 2.3. There exists a reliable dynamic output-feedback controller for all possible combinations of actuator faults in the pre-selected subset Ω if the three*

conditions of Theorem 2.3 hold with G replaced by $G_+ = [G \ \gamma \ B_\Omega]$ and K_d replaced by $K_{d+} = \gamma^{-2} G_+ ' P(t)$.

Proof

Replacing G by $G_+ = [G \ \gamma \ B_\Omega]$ and K_d by K_{d+} in system Σ , construct the augmented system Σ_a with the fictitious disturbance input w_f

$$\begin{aligned} \frac{dx}{dt} &= Ax + Bu + [G \ \gamma \ B_\Omega] \begin{bmatrix} w_0 \\ w_f \end{bmatrix}, & x(0) &= x_0 \\ y &= Cx + w \\ z &= \begin{bmatrix} Hx \\ u \end{bmatrix} \end{aligned}$$

According to Theorem 2.3, the controller that satisfies the system performance requirement is as follows:

$$\begin{aligned} \dot{\xi}(t) &= (A + BK + G_+ K_{d+})\xi(t) + L(y - C\xi), \quad \xi(0) = 0, & (3.3) \\ u(t) &= -B' P(t)\xi(t). & (3.4) \end{aligned}$$

If the condition in Theorem 2.3 holds with G replaced by $G_+ = [G \ \gamma \ B_\Omega]$ and K_d by K_{d+} , then $J(\Sigma_a, R, S, T) < \gamma$, i.e., for all $w_e, w_f \in L_2[0, T]$ and $x_0 \in \mathfrak{R}^n$,

$$\gamma^2 (\|w_0\|_T^2 + \|w_f\|_T^2) + x'(0)Rx(0) - \|z_a\|_T^2 - x'(T)Sx(T) > 0 \quad (3.5)$$

If particular, this inequality holds if $w_f = [0 \ \gamma (I - \Delta_\Omega B_\Omega ' P(t))\xi(t)]$. (More generally, we can allow $w_f = [0 \ \gamma (I - \Delta_\Omega B_{\omega_i} ' P(t))\xi(t)]$ for $t \in [t_{i-1}, t_i]$.) The augmented system Σ_a is then the original system Σ with only the actuator signals $B\Delta u$ remaining.

Inequality 3.5 reduces to

$$\gamma^2 (\|w_e\|_T^2 + x'(0)Rx(0)) - \|Hx\|_T^2 - \|(2\Delta_\Omega - \Delta_\Omega^2)^{1/2} u_\Omega\|_T^2 - \|u_\Omega\|_T^2 - x'(T)Sx(T) > 0.$$

Since $(2\Delta_\Omega - \Delta_\Omega^2) \geq \Delta_\Omega^2$, then

$$\gamma^2 (\|w_e\|_T^2 + x'(0)Rx(0)) - \|Hx\|_T^2 - \|\Delta_\Omega u_\Omega\|_T^2 - \|u_\Omega\|_T^2 - x'(T)Sx(T) > 0$$

Thus, $J_{\omega'}(\Sigma, R, S, T) < \gamma$.

-Q.E.D.-

The next two results give sufficient conditions for the existence of reliable dynamic output-feedback controllers for actuator faults in the pre-selected subset Ω for the infinite horizon case with zero initial conditions and with unknown initial conditions, respectively.

Theorem 3.4 (Infinite horizon case with zero initial conditions) *Consider system Σ with zero initial conditions $x(0) = 0$. Then there exists a reliable dynamic output-feedback controller with $J_{\omega'} < \gamma$ for all possible combinations of actuator outages in the pre-selected subset Ω if the conditions of Theorem 2.5 hold with G replaced by $G_+ = [G \ \gamma B_\Omega]$ and K_d replaced by $K_{d+} = \gamma^{-2} G_+' P$ and the additional requirement that $Q > 0$.*

Proof

Replacing G by $G_+ = [G \ \gamma B_\Omega]$ and B by $B\Delta^{1/2}$ in system Σ , construct the augmented system Σ_a with the fictitious disturbance input w_f

$$\frac{dx}{dt} = Ax + B\Delta^{1/2}u_1 + [G \ \gamma B_\Omega \Delta^{1/2}] \begin{Bmatrix} w_0 \\ w_f \end{Bmatrix}, \quad x(0) = x_0$$

$$y = Cx + w$$

$$z_a = \begin{bmatrix} Hx \\ u_1 \end{bmatrix}$$

If the conditions in Theorem 2.5 hold with G replaced by $G_+ = [G \ \gamma \ B_\Omega]$ and K_d by K_{d+} , then we have

$$A'P + PA + \gamma^{-2}P[G \ \gamma \ B_\Omega \Delta_\Omega^{1/2}][G \ \gamma \ B_\Omega \Delta_\Omega^{1/2}]^T P - PB\Delta B' + H'H = 0,$$

with $P \geq 0$ and $A + \gamma^{-2}[G \ \gamma \ B_\Omega \Delta_\Omega^{1/2}][G \ \gamma \ B_\Omega \Delta_\Omega^{1/2}]^T P - B\Delta B' P$ Hurwitz,

$$AQ + QA' + \gamma^{-2}QH'HQ - QC'CQ + [G \ \gamma \ B_\Omega \Delta_\Omega^{1/2}][G \ \gamma \ B_\Omega \Delta_\Omega^{1/2}]^T = -\gamma^{-2}B_\Omega(I - \Delta_\Omega)B_\Omega' \leq 0$$

with $Q \geq 0$, $A - QC'C + \gamma^{-2}QH'H$ Hurwitz, and $\sigma_{\max}(QP) < \gamma^2$.

Suppose the output-feedback controller is

$$\begin{aligned} \dot{\xi}(t) &= (A + B\Delta^{1/2}K_\Omega + G_{+\Omega}K_{d+\Omega})\xi(t) + L(y - C\xi), \quad \xi(0) = 0, \\ u_1(t) &= -\Delta^{1/2}B'P(t)\xi(t) = K_\Omega \xi(t) \end{aligned} \quad (3.6)$$

where $G_{+\Omega} = [G \ \gamma \ B_\Omega \Delta_\Omega^{1/2}]$ and $K_{d+\Omega} = \gamma^{-2}G_{+\Omega}'P$.

After some algebraic computations, the closed-loop system satisfies Lemma 2.1.

Then the augmented closed-loop system will be stable and

$$\gamma^2 (\|w_e\|^2 + \|w_f\|^2) - \|z_a\|^2 \geq 0,$$

The above controller (3.6) is simply

$$\begin{aligned} \dot{\xi}(t) &= (A + BK + G_+K_{d+})\xi(t) + L(y - C\xi), \quad \xi(0) = 0, \\ u_1(t) &= -\Delta B'P(t)\xi(t) = \Delta K \xi(t) \end{aligned} \quad (3.7)$$

for the original system.

Since $\Delta \leq I$, the above performance inequality still holds with u_1 replaced by $u = -\Delta B'P\xi(t)$ and with $w_f = 0$. This is just the case of the original system with actuator outage in the pre-selected subset B_Ω . The remaining actuator signals are $B\Delta u$.

-Q.E.D.-

We note that this theorem does not require the reliable controller to be open-loop stable, which is a requirement of [3].

The existence of a reliable controller for the infinite horizon case with unknown initial conditions is proved as follows.

Theorem 3.4 (Infinite horizon case with unknown initial conditions) *Consider system Σ with unknown initial conditions and let $R > 0$ as in Corollary 2.1. Then there exists a reliable output-feedback controller with $J_{\omega} < \gamma$ for all possible combinations of actuator outages in the pre-selected subset Ω if the conditions of Corollary 2.1 hold with G replaced by $G_+ = [G \ \gamma B_{\Omega}]$ and K_d replaced by $K_{d+} = \gamma^{-2} G_+^T P$.*

Proof

Replacing G by $G_+ = [G \ \gamma B_{\Omega}]$ in system Σ , We have the augmented system Σ_a with the fictitious disturbance input w_f

$$\frac{dx}{dt} = Ax + Bu + [G \ \gamma B_{\Omega}] \begin{bmatrix} w_0 \\ w_f \end{bmatrix}, \quad x(0) = x_0$$

$$y = Cx + w$$

$$z_a = \begin{bmatrix} Hx \\ u \end{bmatrix}$$

If the conditions in Corollary 2.1 hold with G replaced by $G_+ = [G \ \gamma B_{\Omega}]$, then the controller

$$\begin{aligned} \dot{\xi}(t) &= (A + BK + G_+ K_{d+}) \xi(t) + L(y - C\xi), \quad \xi(0) = 0, \\ u_1(t) &= -B^T P(t) \xi(t) = K \xi(t) \end{aligned} \quad (3.7)$$

will make the closed-loop system stable and

$$\gamma^2 (\|w_e\|^2 + \|w_f\|^2 + x'(0)Rx(0)) - \|z_a\|^2 \geq 0.$$

Let $w_f = [0 \quad \gamma^{-1}(I - \Delta_\Omega)B'P\xi(t)]$ for the augmented system. This is just the case that the original system experiences actuator faults in the pre-selected subset Ω . (The actuator signals remaining are $B\Delta u$.) According to Theorem 4.2 in [3], the closed-loop system will stay stable after the actuator faults in B_Ω . Since $w_e \in L_2[0, \infty)$, then $w_f \in L_2[0, \infty)$. Substituting the specific w_f and with some algebraic computation, we have

$$\gamma^2 (\|w_e\|^2 + x'(0)Rx(0)) - \|z_w\|^2 \geq 0$$

-Q.E.D-

This chapter discussed the reliable controller design with transients for possible actuator faults. This part of work is the extension of the work by Veillette et al. [3], [12].

CHAPTER 4 RELIABLE CONTROLLER DESIGN WITH RESPECT TO SENSOR FAULTS

For reliability with respect to sensor faults in the pre-selected subset, we have theorems similar to those for reliability with respect to actuator faults.

Define the subscript Ω to be the index set of sensor set susceptible to faults, Ω' to be the complement of the index set Ω , ω' to be the index set of sensor signals experiencing no losses, and ω to be the complement of the index set ω' . Order the sensors susceptible to faults first in the measurement vector $y = (y_1, y_2, \dots, y_{r\Omega}, \dots, u_r)$, where r is the number of sensors and $r\Omega$ is the number of sensors susceptible to faults. Let $\Delta = \text{diag}(\Delta_\Omega, I)$, $\Delta_\Omega = \text{diag}(\delta_1, \delta_2, \dots, \delta_{r\Omega})$, be a diagonal matrix of dimension $m \times m$, where $0 \leq \delta_i \leq 1$ is the degree of attenuation of the sensors susceptible to faults. When there is no sensor fault, $\Delta_\Omega = I$. When $\Delta_\Omega = 0$, all sensors in subset Ω are experiencing complete outages. When $0 \leq \Delta_\Omega \leq I$ and Δ_Ω is neither 0 nor I , then some of the sensors in subset Ω are experiencing partial signal losses.

Introduce the decomposition $C = \begin{bmatrix} C_\Omega \\ C_{\Omega'} \end{bmatrix}$.

When a sensor experiences a signal loss or attenuation, then the corresponding disturbance w is assumed to experience the same loss or attenuation. The signal losses are not weighted in the performance measure. The performance bound is thus

$$J_{\omega'}(\Sigma_{cl}, R, S, T) = \sup \left\{ \left(\frac{\|z\|_T^2 + x'(T)Sx(T)}{\|w_{\omega'}\|_T^2 + x_0'Rx_0} \right)^{1/2} \right\} < \gamma$$

where $w_{e\omega'} = \begin{bmatrix} w_0 \\ \Delta w \end{bmatrix}$. Similar to reliable controller design with respect to actuator faults,

we will consider the reliable controller design with respect to sensor faults separately for the infinite horizon and finite horizon cases.

Theorem 4.1 (Finite horizon case) *Consider system Σ and let $R > 0$ and $S \geq 0$ be as in Theorem 2.3. There exists a reliable output-feedback controller for all possible combinations of sensor faults in the pre-selected subset Ω if the three conditions of*

Theorem 2.3 hold with H replaced by $H_+ = \begin{bmatrix} H \\ \gamma C_\Omega \end{bmatrix}$. One reliable dynamic output-feedback controller is the controller given in Theorem 2.3 with H replaced by H_+ .

Proof

By replacing H in system Σ with H_+ , constructed the augmented system $\Sigma_{a'}$:

$$\frac{dx}{dt} = Ax + Bu + Gw_0, \quad x(0) = x_0$$

$$y = Cx + w_1$$

$$z_{a'} = \begin{bmatrix} Hx \\ \gamma C_\Omega x \\ u \end{bmatrix}$$

$$\dot{\xi}(t) = (A + BK + G K_d)\xi(t) + L(y - C\xi), \quad \xi(0) = 0,$$

$$u_1(t) = -B' P(t)\xi(t) = K \xi(t)$$

with K, K_d and L as in Theorem 2.3.

If the conditions in Theorem 2.3 hold with H replaced by H_+ , then

$J_{\omega'}(\Sigma_{cl}, R, S, T) < \gamma$, i.e.,

$$\gamma^2 (\|w_0\|_T^2 + \|w_1\|_T^2 + x'(0)Rx(0)) - \|z_{a'}\|_T^2 - x'(T)Sx(T) \geq 0.$$

In particular, the inequality above still holds if

$$w_1 = \begin{bmatrix} -(I - \Delta_\Omega)C_\Omega x + \Delta_\Omega w_\Omega \\ w_\Omega \end{bmatrix}.$$

Then the augmented system Σ_a will be the original system with sensor faults in subset Ω .

The remaining sensor signals are ΔCx and

$$\gamma^2 (\|w_{e\omega'}\|_T^2 + x'(0)Rx(0)) - \|Hx\|_T^2 - \|(2\Delta_\Omega - \Delta_\Omega^2)^{1/2} C_\Omega x\|_T^2 - \|u\|_T^2 - x'(T)Sx(T) > 0$$

Thus

$$\gamma^2 (\|w_{e\omega'}\|_T^2 + x'(0)Rx(0)) - \|z\|_T^2 - x'(T)Sx(T) \geq 0.$$

This means $J_{\omega'}(\Sigma, R, S, T) < \gamma$

-Q.E.D.-

Next, we consider the dynamic output-feedback reliable controller design problems in the infinite horizon case with zero initial conditions and with unknown initial conditions. First, let us consider the case with zero initial conditions.

Theorem 4.2 (Infinite horizon case with zero initial conditions) *Consider system Σ with zero initial conditions, $x(0) = 0$. There exists a reliable output-feedback controller for all possible combinations of sensor faults (partial or whole signal losses) in the pre-selected subset Ω if the three conditions of Theorem 2.5 hold with H replaced by*

$$H_+ = \begin{bmatrix} H \\ \gamma C_\Omega \end{bmatrix}, \text{ one reliable dynamic output-feedback controller is the controller given in}$$

Corollary 2.1 with H replaced by H_+ .

Proof

Please refer to the proof of Theorem 4.1 in the work of Veillette, et al. [3]. Just replace C_ω in F_e with $(I - \Delta)C$ and L_ω in $G_e G_e'$ with $L_\Omega \Delta$, the rest of the proof is the same as the proof Theorem 4.1 in [3] and is omitted.

-Q.E.D.-

Theorem 4.3 (Infinite horizon case with unknown initial conditions) *Consider system Σ with unknown initial conditions and let $R > 0$ as in Corollary 2.1. Then there exists a reliable dynamic output-feedback controller with $J_\omega < \gamma$ for all possible combinations of sensor faults (partial or whole signal losses) in the pre-selected subset Ω if the conditions of Corollary 2.1 hold with H replaced by $H_+ = \begin{bmatrix} H \\ \gamma C_\Omega \end{bmatrix}$. One reliable dynamic output-feedback controller is the controller given in Corollary 2.1 with H replaced by H_+ .*

Proof

The proof is similar to the finite-horizon case and is omitted.

-Q.E.D.-

CHAPTER 5 A RELIABLE CONTROLLER DESIGN USING REDUNDANT CONTROL ELEMENTS

In this chapter, we will consider the reliable controller design with transients (unknown initial conditions) for the case when only one sensor or actuator fault may occur at a time. We assume that any single sensor or actuator fault may occur at any given time. This requires the use of redundant control elements. We will define redundancy later.

We first prove two lemmas that are very useful in this chapter.

Consider the finite dimensional linear system Σ'

$$\begin{aligned}\frac{dx}{dt} &= Ax + Gw, \\ z &= Hx,\end{aligned}$$

Lemma 5.1 (finite-horizon case) *Consider the linear system Σ' . Let R and S be given symmetric matrices such that S is positive semi-definite and R is positive definite. If there exists a symmetric matrix function $Q(t) > 0$, $t \in [0, T]$ such that*

$$\dot{Q}(t) = AQ(t) + Q(t)A' + \gamma^{-2}Q(t)H'HQ(t) + GG' + F, Q(0) = R^{-1}, \gamma^2Q^{-1}(T) > S$$

where F is any positive semi-definite matrix, then the performance $J(\Sigma', R, S, T) \leq \gamma$.

Proof

Note that

$$\frac{d(x'Q^{-1}x)}{dt} = \dot{x}'Q^{-1}x + x'Q^{-1}\dot{x} - x'Q^{-1}\dot{Q}Q^{-1}x.$$

It is easy to see that

$$\frac{d(x'Q^{-1}x)}{dt} = w'w - \gamma^{-2}x'H'Fx - [w - G'Q^{-1}x][w - G'Q^{-1}x] - x'Q^{-1}FQ^{-1}x.$$

Integrating the above equation from 0 to T , we have

$$\begin{aligned} & x'(0)Rx(0) + \|w\|_T^2 - \gamma^{-2}(\|z\|_T^2 + x'(T)Sx(T)) \\ &= x'(T)(Q^{-1}(T) - \gamma^{-2}S)x(T) + \|w - G'Q^{-1}x\|_T^2 + \|Q^{-1}F^{1/2}x\|_T^2 \geq 0 \end{aligned}$$

This shows that $J(\Sigma', R, S, T) \leq \gamma$.

-Q.E.D.-

Next, we consider the infinite-horizon case for system Σ' .

Lemma 5.1 (infinite horizon case) *Consider the linear system Σ' . Let R be a given symmetric positive definite matrix. If there exists a symmetric matrix Q such that*

$$AQ + QA' + \gamma^{-2}QH'HQ + GG' + F = 0, \quad Q > R^{-1}$$

where F is any positive semi-definite matrix. Then the performance $J(\Sigma', R, 0, \infty) < \gamma$.

Proof

In this case, Q is not a function of time. Therefore,

$$\begin{aligned} \frac{d(x'Q^{-1}x)}{dt} &= \dot{x}'Q^{-1}x + x'Q^{-1}\dot{x} \\ &= -\gamma^{-2}x'H'Hx - x'Q^{-1}GG'Q^{-1}x \\ &\quad - x'Q^{-1}FQ^{-1}x + w'G'Q^{-1}x + x'Q^{-1}Gw \end{aligned}$$

According to Lemma 2.1, Σ' is an asymptotically stable system. Integrating from

0 to ∞ , we obtain

$$-x'(0)Q^{-1}x(0) = \|w\|^2 - \gamma^{-2}\|z\|^2 - \|w - G'Q^{-1}x\|^2 - \|Q^{-1}F^{1/2}x\|^2.$$

This means that

$$\begin{aligned} & x'(0)Rx(0) + \|w\|^2 - \gamma^{-2}\|z\|^2 \\ &= x'(0)(R - Q^{-1})x(0) + \|w - G'Q^{-1}x\|^2 + \|Q^{-1}F^{1/2}x\|^2 \end{aligned}$$

Since $Q > R^{-1}$, the above equation is greater than or equal to zero. Thus,
 $J(\Sigma', R, 0, \infty) < \gamma$.

-Q.E.D.-

Next, we will introduce the redundancy concept from [10]. This concept is very important here.

We use the notation

$$B = [B_1 \dots B_{i-1} B_i B_{i+1} \dots B_m]$$

$$C^T = [C_1^T \dots C_{i-1}^T C_i^T C_{i+1}^T \dots C_r^T]$$

and

$$B^i = [B_1 \dots B_{i-1} B_i B_{i+1} \dots B_m]$$

$$(C^i)^T = [C_1^T \dots C_{i-1}^T C_i^T C_{i+1}^T \dots C_r^T]$$

Definition 5.1.3 *The collection of sensors associated with the output matrix C is called single contingency redundant if rank of C is $r-1$ and strictly redundant if, in addition, the rank of C^i is $r-1$, $i=1, 2, \dots, r$.*

Definition 5.2.3: *The collection of sensors associated with the output matrix C is called single contingency redundant if the rank of B is $m-1$ and strictly redundant if, in addition, the rank of B^i is $m-1$, $i=1, 2, \dots, m$.*

In this chapter, we also assume that (A, C^i) is detectable and (A, B^i) is stabilizable.

Given strictly redundant actuators and strictly redundant sensors, there exist real numbers $0 < \omega_B < 1$, $0 < \omega_C < 1$ such that $\omega_B BB' \geq B_i B_i'$, for all $i = 1, \dots, m$ and $\omega_C C' C \geq C_i' C_i$, for all $i = 1, \dots, r$.

Here, we only consider when any single sensor or actuator may experience a fault. We no longer consider pre-selected subsets of sensors or actuators, as we did in early chapters.

For the finite-horizon case, we have the following theorem.

Theorem 5.1 Consider system Σ' with B and C strictly redundant, so that conditions

$$\omega_B BB' \geq B_i B_i', \quad i = 1, \dots, m$$

$$\omega_C C' C \geq C_i' C_i, \quad i = 1, \dots, r$$

hold. Let $R > 0$ and $S \geq 0$ be as in Theorem 2.3. There exists a reliable output-feedback controller for any single sensor or actuator fault if the three conditions of Theorem 2.3

hold with G replaced by $G_+ = [G \sqrt{\omega_B} \gamma B]$, H replaced by $H_+ = \begin{bmatrix} H \\ \sqrt{\omega_C} \gamma C \end{bmatrix}$, and K_d

replaced by $K_{d+} = \gamma^{-2} G_+' P(t)$.

Proof

Replace G by G_+ , H by H_+ and K_d by K_{d+} in system Σ , and construct the augmented system Σ_a with the fictitious disturbance input w_f

$$\frac{dx}{dt} = Ax + Bu + [G \sqrt{\omega_B} \gamma B_\Omega] \begin{bmatrix} w_0 \\ w_f \end{bmatrix}, \quad x(0) = x_0$$

$$y = Cx + w$$

$$z_a = \begin{bmatrix} Hx \\ \sqrt{\omega_c} \gamma Cx \\ u \end{bmatrix}$$

The controller is

$$\dot{\xi}(t) = (A + BK + G_+ K_{d+})\xi(t) + L(y - C\xi), \quad \xi(0) = 0, \quad (5.1)$$

$$u(t) = -B'P(t)\xi(t). \quad (5.2)$$

If the conditions in Theorem 2.3 hold with G replaced by G_+ , H replaced by H_+ and K_d replaced by K_{d+} , then $J(\sum a, R, S, T) < \gamma$, i.e., for all $w_e, w_f \in L_2[0, T]$ and $x_0 \in \mathfrak{R}^n$, and

$$\gamma^2 (\|w_0\|_T^2 + \|w_f\|_T^2) + x'(0)Rx(0) - \|z_a\|_T^2 - x'(T)Sx(T) > 0 \quad (5.3)$$

Since $\|z_a\| \geq \|z\|$, the above inequality still holds with $\|z_a\|$ replaced by $\|z\|$.

If $w_f = 0$, then the performance bound is satisfied. This is the case if the original system is without any sensor or actuator fault.

Consider what happens if there is a single sensor fault. Let $w_f = 0$. Then, $w = [0, 0, \dots, -\alpha C_i x, \dots, 0, 0]^T + w_{\omega'}$, where $\alpha \in [0, 1]$. Then the inequality reduces to

$$\gamma^2 (\|w_{\omega'}\|_T^2 + x'(0)Rx(0)) - \|z\|_T^2 + (1 - \alpha \omega_c) \|C_i\|_T^2 - \omega_c \|C_i\|_T^2 - x'(T)Sx(T) > 0$$

Since $\omega_c C' C \geq C_i' C_i$, then $(1 - \alpha \omega_c) \|C_i\|_T^2 - \omega_c \|C_i\|_T^2 \leq 0$. Thus,

$$\gamma^2 (\|w_{\omega'}\|_T^2 + x'(0)Rx(0)) - \|z\|_T^2 - x'(T)Sx(T) > 0.$$

This means $J_{\omega'}(\sum, R, S, T) < \gamma$. This is the case when the original system experiences a fault in a sensor.

Next, consider a single actuator failure for the original system. Let us construct the following augmented system Σ_a

$$\frac{dx}{dt} = Ax + B'u + [G \bar{B}] \begin{bmatrix} w_0 \\ w_f \end{bmatrix}, \quad x(0) = x_0$$

$$y = Cx + w$$

$$z_a = \begin{bmatrix} Hx \\ \sqrt{\omega_c \gamma} Cx \\ u \end{bmatrix}$$

with the controller

$$\dot{\xi}(t) = (A + BK + \bar{G}_+ \bar{K}_{d+}) \xi(t) + L(y - C\xi), \quad \xi(0) = 0, \quad (5.4)$$

$$u(t) = -B' P(t) \xi(t). \quad (5.5)$$

where $\bar{G}_+ = [G \bar{B}]$, $\bar{B}' \bar{B} = \omega_B \gamma^2 BB' - \gamma^2 B_i B_i' > 0$ and $K_{d+} = \gamma^{-2} \bar{G}_+' P(t)$. $P(t)$ and $Q(t)$ are symmetric matrices satisfying the following equations:

$$-\dot{P}(t) = A' P(t) + P(t) A + P(t) \left(\frac{1}{\gamma^2} \bar{G}_+ \bar{G}_+' - \bar{B} \bar{B}' \right) P(t) + H_+' H_+$$

with $P(T) = S$ and $P(0) < \gamma^2 R$, and

$$\dot{Q}(t) = A Q(t) + Q(t) A' - Q(t) \left(C' C - \frac{1}{\gamma^2} H_+ H_+' \right) Q(t) + G_+ G_+' + B_+ B_+'.$$

Where $Q(t) > 0$ for all $t \in [0, T]$ and $Q(0) = R^{-1}$, and $\rho \left(\frac{1}{\gamma^2} P(t) Q(t) \right) < 1$ for all $t \in [0, T]$.

The above two equations are the same as the equations in Theorem 2.3 with G replaced by G_+ , H replaced by H_+ and K_d by K_{d+} separately. For simplicity, we assume that $\gamma = 1$.

Differentiating $x'P(t)x$ and then integrating it from 0 to T , we obtain

$$\|w_1\|_T^2 + x'(0)Rx(0) - \|z\|_T^2 - x'(T)Sx(T) = x'(0)(R - P(0))x(0) + \|w_1 - \bar{G}_+'Px\|_T^2 - \|u + B'Px\|_T^2$$

Define $e = x - \xi$, $v = u + B'Px$, we have the following dynamic system Σ_b in the new error variable e

$$\begin{aligned}\dot{e} &= (A - LC + \bar{G}_+\bar{K}_{d+})e + \bar{G}_+w_1 - \bar{G}_+\bar{K}_{d+}x - LCw \\ v &= -B'P(t)e(t)\end{aligned}$$

Let $Z(t) = (I - Q(t)P(t))^{-1}Q(t)$. We have

$$\begin{aligned}\dot{Z}(t) &= (A + \bar{G}_+\bar{G}_+'P(t))Z(t) + Z(t)(A + \bar{G}_+\bar{G}_+'P(t))' + Z(t)P(t)B'B'Z(t) \\ &\quad - Z(t)C'CZ(t) + G_+G_+' + (I - Q(t)P(t))^{-1}B_iB_i'(I + P(t)Z(t))\end{aligned}$$

with $Z(0) = (R - P(0))^{-1}$.

Since

$$\begin{aligned}F &= (I - Q(t)P(t))^{-1}B_iB_i'(I + P(t)Z(t)) \\ &= (I - Q(t)P(t))^{-1}B_iB_i'(I - P(t)Q(t))^{-1} \geq 0\end{aligned}$$

According to Lemma 5.1, $J(\Sigma_b, R - P(0), 0, T) \leq 1$. Thus

$$x'(0)(R - P(0))x(0) + \|w_1 - \bar{G}_+'Px\|_T^2 - \|u + B'Px\|_T^2 \geq 0$$

This is the same as

$$J(\Sigma_a, R, S, T) < 1$$

When $w_f = 0$, the augmented system Σ_a is the original system with an actuator fault. Thus, $J(\Sigma, R, S, T) < 1$

-Q.E.D.-

The next result gives a sufficient condition for the existence of a reliable controller for the infinite horizon case with unknown initial conditions.

Theorem 5.2 Consider system Σ with B and C strictly redundant, so that

$$\omega_B BB' \geq B_i B_i', i = 1, \dots, m$$

$$\omega_C C' C \geq C_i' C_i, i = 1, \dots, r$$

hold. Let $R > 0$ as in Corollary 2.1. There exists a reliable output feedback controller with $J(\Sigma, R, 0, \infty) < \gamma$ for any single sensor or actuator fault if the three conditions of

Corollary 2.1 hold with G replaced by $G_+ = [G \sqrt{\omega_B} \gamma B]$, H replaced by

$$H_+ = \begin{bmatrix} H \\ \sqrt{\omega_C} \gamma C \end{bmatrix} \text{ and } K_d \text{ replaced by } K_{d_+} = \gamma^{-2} G_+' P. \text{ One reliable output-feedback}$$

controller is the same as the controller for Corollary 2.1 with G replaced by

$$G_+, H \text{ replaced by } H_+, \text{ and } K_d \text{ replaced by } K_{d_+}.$$

Proof

According to [10], the closed-loop system will stay stable after one actuator or sensor outage. Based on this result and Lemma 5.2, we can prove this theorem with the same method as the finite horizon.

-Q.E.D.-

CHAPTER 6 CONCLUSION

The reliable controller designs for finite-horizon and infinite-horizon cases with possibly non-zero initial conditions have been developed in this thesis. The performance measure used in this thesis is an “ H_∞ -like norm”, which is an induced two-norm from all exogenous signals and initial conditions to the regulated output and final states.

In Chapter 2, the H_∞ control with transients is reviewed. The performance measure for the control with transients is not the H_∞ norm. It is a kind of “ H_∞ -like” norm. It is not easy to compute it numerically. However, it has a significant contribution to theory. The general H_∞ control problem is a special case of the H_∞ control with transients. The H_∞ control with transients is considered for both the finite- and the infinite-horizon cases with unknown initial conditions. The general H_∞ control problem is also reviewed in this chapter.

In Chapter 3, the existence conditions for the reliable controller for actuator faults are presented. The state-feedback and output-feedback cases are both discussed. For the state-feedback reliable controller design, the necessary and sufficient conditions for the existence of the reliable controller are proved for both finite- and infinite-horizon cases. For the observer-based reliable controller design, the sufficient conditions for the existence of the reliable controller are proved for both finite- and infinite-horizon cases. These conditions are also suitable for the general reliable controller design. This eliminates the restriction that the controller must be open-loop stable required by Veillette, et al., in [3].

In Chapter 4, the existence conditions for reliable controller design with respect to sensor faults are presented. Both the finite- and infinite-horizon cases are considered.

All of the reliable controller designs make the system robust to any combinations of whole or partial signal losses in the sensors or actuators. In some cases, the sensor or actuator fault subset will change from time to time and the design method guarantee robust performance for the resulting transient response as well. For the case of zero initial conditions, the performance measure used reduces to the usual H_∞ norm.

In Chapter 5, the existence condition is presented for the reliable controller for any single sensor or actuator fault. Both finite- and infinite-horizon cases are considered. The redundancy of the sensors and of the actuators is used.

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