AN ABSTRACT OF THE THESIS OF

Porntip Swangrojn for the degree of Master of Science in Mathematics Education

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Abstract Approved: ______________________
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The purpose of this study was to investigate solution strategies that Thai second graders used to solve addition and subtraction word problems. Fifty-eight children participated in this study, 40 children took a written test, and 18 children participated in an interview. In addition, two classrooms were selected for observation, and six second-grade teachers completed a questionnaire regarding their understanding of mathematical problem solving.

The results indicated that Thai children in this study were successful in solving addition and subtraction word problems. For addition word problems, the compare problem and the join problem were the easiest problems. The combine problem was moderately difficult. For subtraction word problems, the compare problem and the separate problem were easy while the combine problem was difficult. The join problem was the most difficult subtraction word problem. Since word problems used in this study contained mostly two-digit numbers, Thai children in this study used knowledge such as borrowing, carrying and regrouping to solve two-digit addition and subtraction word problems. Most Thai children in this study used counting strategies with fingers to solve

both addition and subtraction word problems. Counting on strategies were most often used for addition word problems and counting up strategies were most often used for subtraction word problems. Counting all strategies were not used by Thai children in this study. Other strategies that were not based on counting strategies were also found. Those strategies were using tallies, using a known number fact, using an invented fingers model, and using a base-ten strategy.

The strategies that Thai children used to solve word problems were not different from those used by children in United States and other countries such as Korea. Most children used three basic strategies to solve word problems: counting strategies; using a known number fact; and using a base-ten strategy. However, the base-ten strategy was not usually used by children in United States. The base-ten strategy was used by Thai children in this study and in Asia countries such as Korea. Moreover, the findings showed that Thai children in this study used mostly fingers to represent counting sequences while children in the United States used other physical objects such as cubes and counters.

In observing Thai classroom instruction and having Thai teachers complete the questionnaire, the results suggested that Thai teachers in this study viewed problems as routine word problems and viewed problem solving as solving routine word problems. Teachers did not emphasize problem solving, reasoning, and thinking skills. As a result, children developed memorizing skills rather than thinking and reasoning skills. Children had never explored different types of word problems and non-routine problems, so when the children were faced with problems that differed from their school mathematics instruction, for the most part, children were unable to solve the problems.

Problem Solving Strategies of Thai Second Graders for Addition and Subtraction Word Problems

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Dean of Graduate School

I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

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Porntip Swangrojn, Author

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This thesis is dedicated to the loving memory of my grandparents,

Mr. Pair and Mrs. Powng Swangrojn

Mr. Ta and Mrs. Khumsaj Chaisith

Problem Solving Strategies of Thai Second Graders for Addition and Subtraction Word Problems

CHAPTER I

THE PROBLEM

Introduction

In the new millennium, society expects schools to insure that all students have an opportunity to become mathematically literate workers. A literate worker is defined as a worker who has mathematical competency in several basic skills including problem solving. To have competency in problem solving, young students should have an ability to set up problems with an appropriate operation and be able to use a variety of techniques to approach and work on problems. Moreover, students should see the applicability of mathematical ideas to common and complex problems and should prepare to be able to confront open-ended problem situations (National Council of Teachers of Mathematics [NCTM], 1989). To meet the needs of this societal expectation, schools should provide more mathematical opportunities for students to use a variety of strategies to become capable of solving problems that require reasoning strategies in addition to procedural routines.

Krulik and Rudnick (1988) stated that children of all ages see little connection between what happens in real life and what happens in school. For instance, Saxe (1988) investigated mathematical ability of children who were candy sellers in an urban center in northeastern Brazil. He compared mathematical understanding in children's out of school

activity as candy sellers with their school mathematics. The tasks for being candy sellers involved estimating what candy types were most in demand and coordinating those considerations with possible comparative pricing at different wholesale stores.
The findings showed that sellers with little or no schooling developed expertise in their practice with comparative pricing. In contrast, when these sellers were in school and were asked to read and compare 20 multidigit numerical values—values that were within the range that they addressed in their candy selling—they identified virtually no values correctly and their performance on these tasks remained at relatively low levels with school mathematics.

The importance of problem solving is that it can diminish the gap between the real world and the classroom world and therefore set a positive mood in the classroom (Krulik & Rudnick, 1988). Moreover, problem solving encourages children to see connections between what happens in real life and what happens in schools. Because of the importance of problem solving, teachers need to provide opportunities for children to develop their ability to use several strategies and techniques for solving problems.
There is ample evidence that young children, even pre-schoolers, can solve simple arithmetic word problems before being exposed to formal instruction in problem solving (e.g., Adetula, 1989; Carpenter, Franke, Jacobs, Fennema, & Empson, 1997; English, 1998; Fuson & Kwon, 1992; Fuson, Wearne, Hiebert, Murray, Human, Olivier, Carpenter, & Fennema, 1997). In addition, Piaget (Grossnickle, Reckzeh, Perry, & Ganoe, 1983) identified that children at school age (7-11 years) can understand mathematical structure if it is introduced using appropriate manipulatives. Therefore, it is not necessary to wait until children enter middle or high school to engage in problem-

solving tasks. The teaching and learning of problem solving processes should start as soon as children enter school, and continue throughout their entire school experience. An elementary school teacher has the responsibility for beginning this learning and so laying the foundation for the children's future problem solving experiences (Krulik & Rudnick, 1988).

Statement of the Problem

Word problems are one component of the elementary school problem solving curriculum and most curricular programs clearly assumed that word problems are difficult for children at all levels of mathematics, and that children must learn symbolic addition and subtraction operations before they will be able to solve even simple word problems. National Assessment of Educational Progress (NAEP) results (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980) presented some evidence to the view that young children were poor at problem solving. In addition, the results and from the NAEP (Campbell, Reese, O'Sullivan, & Dossey, 1996) trends in academic progress showed that problem solving performance of children remains low. Most children failed to solve complex problems.

However, there is a growing body of research clearly showing that young children could use their informal arithmetic knowledge to analyze and solve addition and subtraction word problems before they received formal arithmetic teaching (e.g., Carpenter, Ansell, Franke, Fennema, & Elizabeth, 1993; Carpenter et al., 1997; Gibb, 1956; Groen & Resnick, 1997). In a cross-cultural study (Ginsburg, Posner, & Russell, 1981; Posner, 1978), researchers reported that pre-school children and older children in the Ivory Coast who were not attending school were capable of solving concrete addition problems. The study of Maurer (1987) showed that children tried inventively to deal with problems and usually followed their process and built a solution, although it was sometimes wrong. Bebout (1990) indicated that first-graders could learn to represent word problems with open number sentence forms that reflect problem structure. The three-year longitudinal study of Cognitive Guided Instruction project (CGI) (Carpenter et al., 1997) provided an existence proof that children could invent strategies for adding and subtracting multidigit numbers.

Moreover, there is evidence that schooled and unschooled children could build methods for adding and subtracting multidigit numbers without explicit instruction (Carpenter & Fennema, 1992; Hiebert & Wearne, 1996; Nunes, 1992). Studies from several countries such as Holland (Decorte & Verschaffel, 1987), Korea (Fuson & Kwon, 1992, 1992b), Japan (Hatano, 1982), Israel (Nesher, 1982), Netherlands (Beishuizen, 1993), Australia (English, 1993), and Nigeria (Adetula, 1989) supported that children could solve simple addition and subtraction word problems by using their informal knowledge. The data from the Third International Mathematics and Science Study (TIMSS, 1997) indicated that children at third and fourth grade in many countries (e.g. Singapore, Korea, Japan, Ireland, & Slovenia) performed well in solving mathematics problems. Evidence from cognitive development indicated that children at school age have an ability to solve mathematics problems.

According to Piaget, school-age children (7-11 years) in the concrete operational stage can begin to think logically in a consistent way, but only with regard to real and concrete features of their informal knowledge (Berger, 1994). Children can think through the steps in a problem and move forward or backward in their problem-solving to reexamine earlier assumptions or return to the beginning (Berger, 1994; Berk, 1993). For example, Lizzie understood that addition and subtraction were reversible operations. In other words, when Lizzie added 7 plus 8 to get 15, then she could tell that 15 minus 8 must be 7 (Berk, 1993). Bruner (1968) indicated that children do not solve problems in a vacuum. They use available tools to help them. Some of these tools, such as spoken language, written language, and mathematics, are extremely prevalent. These tools help children solve many problems. Siegler (1991) showed that children create their own symbolic tools to solve problems, as well as using ones given to them. For example, 7 to 11 years old can generate informal maps to guide solving problems.

The evidence above supports the fact that young children initially have an ability to solve addition and subtraction word problems even though they have limited knowledge. Additionally, the question of how children process their thinking in solving these problems has been investigated over the years in the mathematics education literature (e.g., Carpenter & Moser, 1984; Carpenter et al., 1997; Riley, Greeno, & Heller, 1983; Hiebert, 1982; Franke & Carey, 1997). These studies, mostly carried out in the United States, investigated the strategies schooled children use to solve addition and subtraction word problems.

However, there is little knowledge about solution strategies used by Thai children. In particular, it is not clear whether Thai children follow the same developmental sequence procedures constructed by children in the United States (e.g., Bebout, 1990; Carpenter et al., 1997) or whether Thai children show different sequence of solution procedures that is supported in their culture or schools. Consequently, this study will investigate and describe strategies that elementary school children in Thailand use to solve addition and subtraction word problems. This study will seek to answer the following questions:

(a) How successful are Thai children in solving addition and subtraction word problems?

(b) Which strategies are used by Thai children to solve addition and subtraction word problems?

Significance of the Study

There are many studies on children's strategies for addition and subtraction word problems that have been done in the United States. For instance, Carpenter et al. (1993) studied kindergarten children's problem solving processes. Hiebert (1982) investigated the position of the unknown set and children's solutions of verbal arithmetic problems. Hiebert and Wearne (1996) studied the instruction, understanding, and skill in multidigit addition and subtraction. In particular, most of the studies on young children's addition and subtraction problem solving in recent years were done by Carpenter and Hiebert and their colleagues.

The results from these studies help teachers understand basic cognitive skills of young children and help teachers plan how to teach addition and subtraction problem solving more effectively. The results also enable teachers to make instructional decisions based on the identified strengths and weaknesses of the children. In addition, many other countries such as Japan (Haltano, 1982), Korea (Fuson & Kwon, 1992), and Australia (English, 1993) conducted studies on problem solving. For example, in Korea, Fuson and

Kwon (1992) studied Korean children's ability to solve addition problems with sums of 10, single-digit addition problems with sums between 10 and 18, and single-digit subtraction problems with minuends between 10 and 18.

In Thailand, problem solving was introduced into the school curriculum after National Council of Teachers of Mathematics (NCTM, 1980) pronounced that problem solving must be the focus of school mathematics, but Thai teachers and educators still may not understand the teaching and learning of problem solving. Although educators in Thailand have reformed the mathematics curriculum since 1990 by focusing more on thinking mathematically and using mathematics to solve daily life problems, children still have low achievement on arithmetic word problems (TIMSS, 1997). There is not much research on problem solving in Thailand. In the past, the Elementary Education Department, Thailand (1958) studied mathematics achievement of first graders. The results showed that children could not solve addition and subtraction problems. For example, they answered 6 + 4 equal 64 or 46 instead of 10. That was because they did not understand the concept of zero in base-ten number system. In addition, although sixth-graders could do simple arithmetic problems, they could not solve mathematical problems that have complex wording. In 1969, the Educational Technique Department in Thailand showed that sixth-graders had low mathematical achievement, particularly, in solving mathematical problems. Recent data from TIMSS (1997) still showed that children in primary grades (third and fourth grade) in Thailand have much lower mathematics achievement than the international average. The problem might be that teachers do not have much knowledge in teaching and learning mathematics and do not understand mathematical concepts (Malenee, 1986). Teachers only asked questions about mathematical knowledge superficially. This may partially explain why children in Thailand do not understand mathematical concepts very well, have low mathematical achievement, and often have negative attitudes toward mathematics.

Therefore, the present study was done in hope that the results of the study could provide teachers with a better knowledge and understanding of problem solving and how Thai children solve addition and subtraction word problem. This may in turn enable teachers to assist children to learn to solve mathematics word problems with a better understanding of arithmetic concepts. It is hoped that the findings of this study may have an impact on education policy makers, teacher education, and educators to change the mathematics curriculum in Thailand and to provide needed staff development for elementary teachers.

CHAPTER II

REVIEW OF LITERATURE

Introduction

In 1980, the National Council of Teachers of Mathematics (NCTM) published An Agenda for Action: Recommendations for School Mathematics of the 1980s. One of the Council's recommendations was that "problem solving be the focus of school mathematics in the 1980s." Since that time, problem solving has been an important component of the mathematics curriculum, and indeed, lies at the heart of mathematical activities (Moses, Bjork, & Goldenberg, 1990; Brown & Walter, 1993; Silver, 1994). Moreover, many articles, research studies, and books have been published specifically about problem solving since 1980.

Published literatures in this area demonstrate that articles and studies have focused on many different aspects of problem solving. Much of this literature has provided teaching strategies to improve student achievement in problem solving (Cobb, Yackel, Wood, & Wheatley, 1988; Davis-Dorsey, Ross, & Morrison, 1991; May, 1989; Thompson & Yancey, 1989). Other articles focused on the cognitive processing of problems solving (Bebout, 1990; Beishuizen, 1993; Carpenter et al., 1993; English, 1993; Frank, 1988; Malloy & Jones, 1998; Pepper & Hunting, 1998). Another line of research focused on the connection between reading skills and learning to solve word problems (Thomas, 1988). This chapter is divided into two sections, which review articles in two areas of problem solving. The first section is the review on problem solving in general which sets a context for this study. The second section is the review on children's problem solving strategies for addition and subtraction word problems (e.g., Carpenter et al., 1993; Houlihan & Ginsburg, 1981) which is the context for this study.

Problems and Problem Solving

"If we are going to talk about problem solving, it would be good to have a clear understanding of what a problem is and what problem solving is" (Van De Walle, 1994). A problem involves a situation in which a person wants a resolution to a problematic situation and does not know immediately how to get the solution (Reys, Suydam, & Lindquist, 1989). If a problem is so easy that children know how to get the answer immediately, there is really no problem at all. Additionally, Polya (1980) stated that there is no problem unless the individual has the desire to find a solution. From this definition, the implication is that what a problem is to one person will not be perceived as a problem by another. In mathematics, problems always involve finding answers to questions that cannot be obtained by a habitual response (Grossnickle et al., 1983).

On the other hand, problem solving is a process by which the choice of an appropriate strategy enables an individual to proceed from what is given in a problem to its solutions (Grossnickle et al., 1983). The answer is often the least important part of the problem solving process. The ideas used in the process are often much more valuable than the answer. Moreover, Polya (1980) defined problem solving as: ". . . finding an unknown means to a distinctly conceived end . . . To find a way when no way is known offhand, to find a way out of a difficulty, to find a way around an obstacle . . .."

Problem Solving in the Mathematics Curriculum

Particular methods of solving problems have a long history in mathematics curriculum. An example is from the Social Utility Arithmetic textbook by Upton (1939). Upton tried to make children think about the process of solving a problem by presenting the problem without numbers. For example, if you know the cost of a coat, a hat, and a suit, how do you find the cost of all of them? However Upton did not go on to discuss what one can learn from such problems. The National Council of Supervisors of Mathematics (NCSM, 1977, p.20), stated that "Learning to solve problems is the principle reason for studying mathematics." Stanic and Kilpatrick (1989) stated that sets of word problems have long been a part of the mathematics curriculum. "Primarily within the last century, discussions of the teaching of problem solving has moved from advocating that students simply be presented with a problem or with rules for solving particular problems to developing a more general approach to problem solving."

Problem solving plays an important role in the study of mathematics and has been the theme of the 1980s and 1990s. In addition, the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) supported that a primary goal for students is "that they become mathematical problem solvers." As such, problem solving is a primary goal of all mathematics teaching and an integral part of all mathematical activities.
Problem solving is not a clearly defined topic, but problem solving is a process that should permeate the entire mathematics program and provide the context in which concepts and skills can be learned. These statements above attest to the high regard mathematics educators have for problem solving as an instructional goal. In fact, problem solving is a popular topic on the agenda of every conference for teachers of mathematics,

and problem solving has been the focus of innumerable journal articles and books since the 1980s (e.g., Beishuizen, 1993; Carpenter & Moser, 1981; Fuson & Kwon, 1992; Jones, 1998; Malloy & Secada, 1991; NCTM, 1980, 1989).

Problem Solving in Early Elementary Grades

During the preschool years, children show a natural curiosity concerning quantitative events and circumstantially build an informal mathematics (Ginsburg & Baron, 1993). For example, Adam answered the written problem 66 + 4 with 610. When asking him if he had 66 stamps in his stamp book and he put in 4 more, "How many stamps do you have now?", he proceeded to count and provided the correct answer: "70." Children also use informal strategies such as making marks, counting, and using concrete subjects to model and solve problems such as when Adam used a counting strategy above (Bardody, 1987).

Children learn because they want to know in order to extend their knowledge and because their minds are made to learn. Children are normally exposed to physical and social environments rich in mathematical concepts before and after they enter school. They confront quantity in the physical world, counting numbers in the social world, and mathematical ideas in the literary world. Everyday situations afford a rich quantitative environment, so children have a strong motivation to learn mathematical concepts (Ginsburg & Baron, 1993). Children enter school with a great deal of informal knowledge of mathematics (Ginsburg, 1977; Resnick & Ford, 1981). Riley et al. (1983) indicated that one area where children's informal or invented strategies demonstrated a remarkable degree of insight is in solving simple word problems. Franke and Carey

(1997) reported that first graders perceived of mathematics as a problem-solving endeavor in which many different strategies were considered viable and communicating mathematical thinking is an integral part of the task. The children recognized and accepted a variety of solution strategies, with many children who valued all solutions and assumed a shared responsibility with the teacher for their mathematics learning. Children begin to develop an intuitive idea of adding as early as the age of 2 or 3 (Ginsburg & Baron, 1993).

The study of Gelman and Gallistel (1978) supported that most young children, from 2 to 4 years of age, improved an intuitive notion of things being added or taken away. For example, young children were frequently shown a set of three objects. After the children learned to name the set with the arbitrary label "winner", the set was hidden and an object was secretly added to it. Upon seeing the set again, the children said that it was no longer the "winner" since something was added to it. Asked how the set could be "fixed", the children replied that something must be taken away. In this case, the children did not calculate in order to decide the exact number of the set, but clearly understood something about the fact that addition and subtraction can alter quantities.

By the age of 4, children begin to calculate in concrete addition situations (Ginsburg & Baron, 1993). Ginsburg and Russell (1981) reported a study in which children were presented with a task in which three objects presented in one group were to be added to four objects showed in another group. The children normally did the calculation accurately by means of the strategy "counting all." The children usually interpreted addition as the act of combining and counting separate sets. Addition involved operating on sets in order to get a new set. As children grow up, they spontaneously develop efficient approaches to calculation (Groen & Resnick., 1977). The children abandon "counting all" for a more advanced and easier approach, namely "counting on", usually from the larger number. It is as if they get bored with counting all and find that it is easier simply to count on from the larger number.

Finally, concrete supports are no longer needed. Children calculate only on a mental level. The children can now add four plus three "in their head." Often this is accomplished through the use of clear mental imagery. The children picture four objects, and three objects in their mind, and counts the images, again usually from the larger number. Other children make the calculation by means of spoken numbers only, without visualization of the objects (Ginsburg & Baron, 1993). The cross-cultural research (e.g. Ginsburg & Russell, 1981; Saxe, Guberman, & Gearhart 1987) stated that although mathematical thinking is not identical across cultures, children from various cultures, literate and preliterate, rich and poor of various racial backgrounds, gain the basics of informal addition in a similar development progression. The following section will present more about strategies that young children use to solve addition and subtraction word problems.

Solution Strategies for Addition and Subtraction Word Problems

Word problems are one component of the elementary school problem-solving curriculum. Word problems are tasks that require the integration of linguistic and arithmetic processing skills. In word problems, a situation is described in which there is some modification, exchange, or combination of quantities. Mathematical word problems, or story-problems, have long been familiar features of school mathematics

(Gerofsky, 1996; Heller & Greeno, 1978; Karrison & Carool, 1991). From previous studies (Carpenter, Hiebert, & Moser, 1981; De Corte, Verschaffel, & DeWin, 1985; Gibb, 1956; Lindvall & Ibarra, 1980), four different types of word problems that represent addition and subtraction are defined in Table 1. Some problems are difficult for students to solve and some problems are easy (Stockdale, 1991). Results of the National Assessment of Educational Progress showed that although elementary students could solve very simple addition and subtraction word problems, other word problems are difficult (Kouba, Brown, Carpenter, Lindquist, Silver, & Swallord, 1988; Campbell et al., 1996).

Table 1. Description of the problem of each type

Problem Types	Description
Addition	
Combine	In combine problems, there are two distinct amounts, considered in combination, as in the following example: "Jame has 3 marbles, Joe has 5 marbles; How many marbles does he have altogether?"
Compare	Compare problems involve two compared quantities and the difference between them, such as in this problem: "Joe has 3 marbles; Jame has 5 more marbles than Joe; how many marbles does Jame have?"
Join	Join problems relate to situations in which one set is joined to another, as in the following example: "Jame has 3 marbles; Joe gives him 5 more marbles; how many marbles does Jame have now?"
Subtraction	
Combine	In combine problems, there are two distinct amounts, considered separately, as in the following example: "Joe has 8 marbles; 5 are blue and the rest are green; how many green marbles does Joe have?"
Compare	In compare problems, two quantities are compared in order to find out how much greater one quantity is than another, as in this problem: "Jame has 8 marbles; Joe has 5 marbles; how many more marbles does Jame have than Joe?"
Join	Join problems relate to situations in which some event increases or decreases the value of a quantity, for example: "Joe has 5 marbles; how many more marbles does he have to put with them so he has 8 marbles altogether?"
Separate	Separate problems involve a situation in which one set is removed or separated from another, for example: "Joe has 8 marbles, then he gave 5 marbles to Jame; how many marble does Joe have now?"

To solve word problems, students must employ reading, language comprehension, problem solving, and mathematics computation skills almost simultaneously (Reutzel, 1983). Pellegrino and Goldman (1987) indicated that, generally, students use the following steps in solving word problems:

1. Reading the problem.
2. Selecting the necessary computation.
3. Deciding what information is to be manipulated.
4. Performing the computation to read a solution.
5. Determining the appropriate units for the solution.

For many students, these steps become somewhat automatic processes with practice, but some children have difficulty approaching problem-solving tasks. (Karrison & Carool, 1991). Some students may experience fear and anxiety, both of which are detrimental to reaching a clear and logical solution, when faced with a word problem. They may also find word problems too abstract to attempt a solution. Many previous studies on addition and subtraction word problems provided a reasonably coherent view of how children solve addition and subtraction word problems (e.g., Bebout, 1990; Carpenter & Moser, 1984). This section reviews research on children problem solving strategies for addition and subtraction word problems.

This section is divided into two parts. The first part is the review of studies on children's single-digit addition and subtraction methods, include single-double digits and double-single digits. The second part is the review of the studies on children's multidigit addition and subtraction methods.

Children's Single-Digit Addition and Subtraction Methods

There are many studies in the past two decades on arithmetic word problems which attempt to uncover children's solutions involved in performing addition and subtraction word problems (e.g., Carpenter et al, 1993). This part deals with eight studies which have examined children's solution strategies for addition and subtraction word problems. The first article deals with kindergarten children. Carpenter et al. (1993) investigated problem-solving processes of kindergarten children. The other seven articles deal with children's solution strategies for addition and subtraction word problems of first through third graders. The study of Carpenter et al. (1981) focused on first-grade children's ability to solve certain verbal problems. Houlihan and Ginsburg (1981) examined the addition strategies of first- and second- grade children. Hiebert (1982) designed the study to examine the effect of the position of the unknown set on first-grade children's representation and solution processes for verbally presented addition and subtraction problems. In 1984, Carpenter and Moser studied children's solutions to simple addition and subtraction word problems followed from grades 1 through 3. Adetula (1989) studied problem-solving strategies used by schooled and unschooled Nigerian children to solve simple addition and subtraction word problems. Bebout (1990) investigated the theory that children who reflect the structure of word problems with concrete models will be successful in learning to symbolically represent problems with structure-based open number sentences. Finally, Fuson and Kwon (1992) studied Korean first-graders' ability to solve addition word problems.

Carpenter et al. (1993) investigated problem-solving processes of kindergarten children who had spent a year in kindergarten classes in which they had an opportunity to explore a range of problem situations. The subjects of this study were 70 kindergarten children who returned parent permission forms. Throughout the year, children were taught to solve a variety of different problems. The teachers generally presented the problems and provided the children with counters that the children could use to solve the problems, but the teachers typically did not show the children how to solve a particular problem. Children regularly shared their strategies for solving a given problem with the class or a small group, so a child might have learned a particular strategy by watching other children used it.

When children had completed almost eight months of kindergarten, the children were individually interviewed by three trained interviewers. Each interviewer had observed in the kindergarten classes on at least four occasions. Each child was asked to solve nine problems and each problem was read to the child by the interviewer. The interviewer reread the problem as many times as the child wished or when a child asked specific questions. Counters and paper and pencil were available on the table. The entire interview was audiotaped and the interviewer also coded children's responses.
The interviewer asked the child to explain when the interviewer could not understand what the child had done.

The results indicated that overall the children in this study showed remarkable success in solving word problems. Children could solve a wide range of problems, including multiplication and division situations, early in the primary grades. Kindergarten children's strategies could be identified as representing or modeling the action or relationships described in the problems. For example, to solve 13 – 6, children directly modeled the action in the problem by making a set of 13 counters and removing 6 of them. To solve 7 + ? = 11, children directly modeled by making a set of 7 counters and adding on counters until there was a total of 11. To solve 3×6, children modeled the problem by making three sets with six counters in each set. To solve 19 + 5, children constructed three sets with 5 counters in each and a final set of 4 counters. This study provided an existence proof that many kindergarten children can learn to solve problems by directly representing or modeling the action or relationships related in the problems, and they can apply this ability to a reasonably wide range of problems.

The results also suggested that instruction did encourage the use of direct modeling to solve problems. The children in this study were about as successful in solving multiplication and division problems as they were in solving addition and subtraction problems. This suggested that much more challenging problems involving a range of operations can be introduced early in the primary grades as long as contexts are provided for the problems. In addition, the results suggested that if specific multiplication and division schemata are required, these schemata are sufficiently well developed in many kindergarten children that they can solve multiplication and division problems by representing the action and relationships in the problems. The results indicated that modeling seems to be a basic process that comes relatively naturally to most primary grade children. Teacher could help children build this intuitive modeling skill for developing problem-solving abilities in children in primary grades.

In the study of Carpenter et al. (1981) with first grade children, the researchers focused on how successful children were at solving different types of addition and subtraction problems prior to formal instruction. This study also focused on strategies children used to solve problems and factors that lead to the selection of different strategies. The subjects of this study were 43 first graders from a parochial school. At the time the children were tested, no formal instruction in symbolic representation of addition and subtraction had been given. The children were asked to solve ten different addition and subtraction problems and then were interviewed to identify the strategies they were using to solve each of the problems. Each problem was read to the children by one of two experimenters. Problems were reread in their entirely as often as necessary. A set of about 40 Unifix cubes was made available to the children. When the child could not solve one problem, the experimenter went on to the next problem. Children were asked to describe how the answer was found. When the solution was not clear, the experimenter continued the questioning until it was clear what strategy the children used.

The results from this study showed that the children used three basic counting models identified by Groen and Parkman (1972) in solving addition problems. The three strategies were counting all, counting on from the smaller number and counting on from the larger number (see Table 2).

Table 2. Addition strategies

Strategies	Description
Counting all	Both set are represented and the union of the two sets is counted.
Counting-on from the smaller number (first)	The counting sequence begins with the first (smaller) given number in the problem and continues the number of units represented by the second number. The answer is the final number in the sequence.
Counting-on from the larger number	The counting sequence begins with the larger (second) number given in the problem and continues the number of units represented by the smaller number. The answer is the final number in the sequence.

The counting all strategy can be carried out using cubes or fingers or by counting mentally. Both sets are represented and then the union of the two sets is recounted beginning with one if cubes are used. If counting is done mentally or with fingers, the counting sequence begins with one and ends the number representing the total of the two given quantities. The counting on from the smaller number may be done mentally, or by using cubes or fingers to keep track of the number of steps in the counting sequence. Children also used several strategies that were not based on counting to solve addition problems. The researchers reported that children gave an answer with the explanation that it was the result of knowing some basic number facts. In addition, children generated solutions from a small set of known basic facts. These strategies were usually based on doubles or numbers whose sum is 10. For example, to solve 6 + 8 = ?, a child responded that 6 + 6 = 12 and 6 + 8 is just 2 more than 12 is 14, or alternatively 6 + 4 = 10 plus 4 more is 14.

However, the children were not quite as successful with the subtraction problems as they were with addition problems. Four basic subtraction strategies were used. The four strategies were separating, separating to, adding on, and matching (see Table 3). The results from this study suggested that children were successful both in modeling action or relationships implied in problems and in using other appropriate models of addition and subtraction. The findings also suggested that children posed different strategies required solving word problems directly before children had any instruction. It is frequently assumed that children must master computational skills before they can apply them to solve problems. However, the results from this study indicated that word problems may be an appropriate context in which to introduce addition and subtraction operations. Word problems also provide different interpretation of addition and subtraction, interpretations that are important for children to understand. In conclusion, the researchers suggested that by introducing operations based on verbal problems and integrating verbal problems throughout the mathematics curriculum, children will develop their natural ability to analyze problem structure and will develop conceptions of basic operations.

Table 3. Subtraction strategies

Strategies	Description
Separating (take away)	The larger quantity is represented, and the smaller quantity is removed from it. The remaining objects are counted to find the answer.
Separating to	The larger quantity is represented, and objects are removed until the smaller quantity remains. The removed objects are counted to find the answer.
Adding on	A set of smaller number is constructed. Objects are added to this set until there is a total of larger number. The added objects are counted to find the answer.
Matching	The two sets are matched one-to-one. The unmatched objects are counted to find the answer.
Counting up from given	A forward counting sequence starts with a smaller number and then incremented by ones until a larger number is reached. The number of counting words spoken gives the answer.
Counting down from	The separating action is represented by counting backward. For example, to solve the problem 5 - 2, the child would count, "5 [pause], 4, 3. The answer is 3".

The study by Houlihan and Ginsburg (1981) focused on the procedures used by first- and second- grade children in solving addition problems. The subjects for this study were 56 children, 25 first graders and 31 second-graders. The children in each grade came from the same classroom. The textbooks for both grades presented place value concepts but did not discuss place value in connection with addition problems involving double-digit addends. From the teachers' reports, it did not appear that the children in this study had worked with double-digit addends in class. Two sets of six problems were used. Each child was asked to solve six addition problems. On three of the problems each child was asked to describe the solution procedure. Of the three problems, one contained two single-digit addends (the S-S digit problem); another, one single and one double-digit addend (the S-D digit problem); and the third, two double-digit addends (the D-D digit problem). The problems were presented orally to half the subjects and in writing to the other half.

The child was asked to explain the solution for the second, fourth, and sixth problems. The questions used were drawn from a pool of questions developed through pilot testing. Children's responses to the interview were recorded verbatim. The responses then were coded into 11 categories suggested by the previous research and the pilot data (see Houlihan & Ginsburg, 1981, p. 98 – 99, for more details). Before any analysis of each grade's performance was done, the Fisher Exact Probability Test (FEPT) was used to see if there were any significant differences between either grade's Problem set 1 and Problem Set 2 groups or between either grade's Oral Presentation and Written Presentation groups. The FEPT produced nonsignificant differences between each grade's Set 1 and Set 2 groups. With one exception, the FEPT also produced nonsignificant differences between the Oral and Written Presentation groups. Thus, the data for Problems Sets 1 and 2 for the Oral and Written Presentation groups were pooled for further analysis.

In general, the results showed that children from both grades were able to apply appropriate methods to problems of a larger size than those with which they were experienced. But they were often unsuccessful in their attempts. For the first graders, the S-D problem was more difficult than the S-S problem, but not more difficult than applying an appropriate method. The results indicated that the first graders used counting methods to solve each problem. On the S-S and the S-D digit problems the first graders who used appropriate methods were about equally divided among four different counting methods: (a) count from 1 with concrete aids, (b) count from 1 without concrete aids, (c) count on with concrete aids, and (d) count on without concrete aids. The first-grade teacher reported that the children had been taught to add by counting from 1. Thus, it was possible that the children who used counting on methods developed these methods themselves. On the D-D digit addend problem, only nine children of the first graders used appropriate methods and the most frequently used methods were counting on from the larger addend.

Unlike the first graders, the second graders used both counting and noncounting methods on each of the three interview problems. The results showed that the second graders had one dominant counting method on all three problems, counting on with concrete aids. Many of those who counted, counted on from the larger addend. The second graders' noncounting methods varied with addend size: memory methods on the S-S digit problem, memory and place value methods on the S-D digit problem, and place value only on the D-D digit problem. The predominant second-grade noncounting method on the S-D and the D-D digit problems was addition by place value. Both the teacher's report and the contents of the textbooks suggested that the children had worked with the concept of place value, particularly in regard to notation, but had not studied place value in addition. The explanations of the children who used place value methods suggested that the children developed their methods themselves. The result from this study suggested that some second graders could apply the knowledge of place value to addition problems, thus teacher should lead children to discover place value notions through an examination of their own invented strategies. The results also showed that children in the same class did not used the same addition methods. Some children used methods as taught in class while others use their own methods. Teachers should observe the addition methods that children use in addition problems and used this observation to evaluate children's understanding of addition. The data from this study also showed that children used appropriate methods but the children did not use them accurately. Thus, tests and evaluation should attempt to measure not only correct responses but also students' methods for solving problems.

Hiebert (1982) studied children's modeling behavior on different types of problem and their solution process for each problem. This study was done by systematically manipulating the position of the unknown set in verbal addition and subtraction problems. The subjects for this study were 47 first graders from three classrooms. At the time of testing, the children had not received formal instruction in solving verbal problems or in using concrete objects to represent or model problem situations. The problems used in this study were six verbal arithmetic problems of similar semantic structure—three joining problems and three separating problems. The six different problems were generated by placing the unknown in each of the three positions in the associated number sentence (see Table 1 of Hiebert, 1982, p. 343, for more details).

The six problems were read to each child in an individual interview. Small cubes were available, and the children were told they could use the cubes to help them to solve the problems. The interviewer continued questioning until it was clear what strategy the child used or until it was clear that no explanation was forthcoming.

The results from this study showed that the children modeled the situations in the problem by using cubes. No children in this study used their fingers to model the problems. Most of the children used strategies based on counting. For addition problems, two counting strategies were found—counting all and counting on. Both of these strategies could be used with or without a physical representation of the problems. More complete descriptions of these two strategies can be found in Table 2 in the study.
For subtraction problems, four basic counting strategies were used—separate, separate to, adding on, counting down. The first two depend on modeling the problem with physical objects. The results also showed that children used several strategies that were not based on counting. The strategies were a known fact and a derived fact. Some children used inappropriate strategies on the problems such as adding when subtraction was needed.

The findings showed that the position of the unknown set had a substantial effect on children's modeling behavior. The results indicated that the problems with the unknown in the first position were difficult to model and difficult to solve. It was clear that the children from this study could add and subtract verbal problems of the form $a \pm b = ?$. In summary, the results from this study indicated that the position of the unknown set in a verbal problem determines to a substantial degree whether or not the problem can be modeled successfully by first grade children.

Carpenter and Moser (1984) studied children's solutions to simple addition and subtraction word problems in a three – year longitudinal study that followed children from grades 1 through 3. The participants who participated over the 3 years were 88 children from three elementary schools. The children were followed from a point before they had received any formal instruction in addition and subtraction to a point when they were expected to have learned addition and subtraction algorithms. The three schools used the same mathematics program. This program placed a much greater emphasis on problem solving and the instruction on addition and subtraction included a great deal of attention to addition and subtraction problem situations. Six basic types of addition and subtraction problems were used in this study. The six types of problems were administered under six different conditions over the course of the study (see Carpenter & Moser, 1984, p. 184-185, for more details). Clinical interviews were used to identify the processes that children used. The interviews were conducted by trained interviewers. The word problems were read to the child by the interviewer.

The results revealed that the children were able to solve addition and subtraction problems by using a variety of modeling and counting strategies even before they received formal instruction. For addition problems, the results from this study clearly showed that children solved the problem with a counting all strategy and that this strategy gives way to counting on. It seemed reasonable that children who could count on from the larger number chose to do so rather than using the less efficient counting on from first strategy. According to this study, children initially modeled the subtraction problems directly using the adding on strategy, separating from strategy and matching strategy. Later, children use the more efficient strategies: counting up from given strategy and counting down from strategy. However, children tended to avoid the counting down from the larger number strategy. Children in this study also used derived facts for subtraction problems. Many of the derived subtraction facts were based on addition. For example, to explain how to find 14 - 8, the child said, "7 and 7 is 14; 8 is 1 more than 7; so the answer is 6." To solve 13 – 7, many children responded that they just knew that 7 + 6 = 13. Some children used number combinations. This solution was based on doubles or numbers whose sum is 10. For example, to solve 6 + 8, one child responded, "6 + 6 = 12 and 6 + 8 is just 2 more than 12, so it was 14." To solve 4 + 7, one solution involved the following analysis: "4 + 6 = 10 and 4 + 7 is just one more than 10, so it was 11." Children in this study were classified into five levels. Level 0, the children were unable to solve any addition or subtraction problems. Level 1, the children were limited to direct modeling strategies. Level 2, the children used both modeling and counting strategies. Level 3, the children relied primarily on counting strategies. The highest, level 4, the children solved addition and subtraction problems using number facts.

Adetula (1989) studied the problem strategies used by schooled and unschooled Nigerian children to solve simple addition and subtraction word problems. The purpose of this study was to investigate (a) whether the advanced strategies that children used to solve simple addition and subtraction word problems could be acquired without schooling, and (b) whether children were more successful and used more advanced strategies to solve problems presented in their first language compared to problems presented in their second language. The subjects for this study consisted of 48 schooled children and 47 unschooled children. The schooled subjects were randomly drawn from the Yoruba-speaking groups in grades 1 through 4 of a Nigerian university staff school in

Zaria. The unschooled subjects were drawn from Zaria, a town in which the staff school is located. Some of the children in this group had no formal schooling but some of them had been in school 6 months before financial circumstances had forced them to withdraw.

Adetula used individual interviews to identify the process that children used to solve each of the problems. During the interviews, the schooled children were asked to solve 15 addition and subtraction word problems in English and 15 in Yoruba. Children were allowed to use counters for only 11 of 15 problems in each language. For each unschooled child, similar procedures were used. All problems used in this study involved numbers with sums between 19 and 20. The responses were audiotaped and coded by the investigator using a strategy-coding system developed by Carpenter and Moser (1984). The mean and standard deviation of the associated strategy scores for each cluster at each age level were calculated.

The results showed that unschooled children became as successful in solving word problems as schooled children, although the success generally came at an older age. The results also indicated that simple join, combine, and simple separate were the easiest problems. There were few qualitative differences in the strategies used by schooled and unschooled children. However, schooled children used more advanced strategies than unschooled subjects of equal age. For the effect of language, the schooled children answered more problems correctly and used more advanced strategies when the problems were easy. The results indicated that, in both groups, the compare problems and the start-unknown problems were the most difficult, with the combine subtraction problems slightly more difficult. Three levels of performance were classified. In the first level, the children solve the simple join, the simple separate, and the combine addition problems by modeling the action or relationships described in the problems with physical objects. In the second level, the children began to solve a wider range of problems and develop the counting strategies. For example, the counting all with counters strategy was replaced with counting on from larger using counters. The third level involved the use of number-facts recall and derived facts to solve a wide range of problems. Most schooled children attained this level by the fourth grade but unschooled children attained this level by age 13. One major conclusion of this study was that the language of problem presentation had an effect on children's performance in both skills and strategies. For the difficult problems, this study showed that schooled children performed better when problems were presented in their native language then in the English language. Lastly, the development of addition and subtraction problem-solving skills of Nigerian children was not so different from that of Western cultures, except that the abstract counting strategies that Western children had demonstrated were less pronounced for children in this study.

Bebout (1990) investigated informal strategies for solving addition and subtraction word problems of first graders. The children were taught to write canonical and noncanonical open number sentences to symbolically present the structure of eight types of change and combine word problems. The sample of this study consisted of 45 first graders. The sentence-writing tests were administered to children before and immediately after instruction to determine their performances in symbolically representing and solving different types of word problems. Each child was interviewed before instruction to determine children's choice of strategy and success in concretely representing and solving word problems.

The interviewer read each problem and asked the child to use concrete items to represent and solve problems. From the results of the individual interviews, the first graders in this study were very successful in learning to solve symbolic representation actions of change and combine problems. Their successes were due to instruction that was designed to capitalize and link children's informal insights into problem structure with open number sentence forms that reflect problem structure. The children were categorized into three level performances: basic level, direct modeling level, and representation level. The basic level included children who were successful in concrete representing only the most primary addition and subtraction problems. The direct modeling level included children who in general attempted to directly represent the structure of problems with their concrete strategies and used an adding on strategy. The representing level included children who tried to ignore the structure of problems by using concrete representations that transformed problems into canonical solution forms and used counting forward and number fact strategies. The results from this study suggested that young children's informal insights into the structure of addition and subtraction word problems presents a strong rationale for teaching children that the symbols of mathematics can reflect problem structure.

Korean children's ability to solve word problems was studied by Fuson and Kwon in 1992. This study focused on Korean children's ability to solve addition problems with sums of 10, single-digit addition problems with sums between 10 and 18, and single-digit subtraction problems with minuends between 10 and 18. The subjects for this study were 18 children from two schools in Seoul, Korea. Six children were drawn at random from each of three classrooms in each school. Children were interviewed by the second author that can speak Korean. Each child was told that the interviewer was interested in how children solve addition and subtraction problems. Children were asked to solve the problems after the interviewer read them. Sheets of paper and a pencil were available to the child. The interviewer suggested some solutions when the child could not solve a problem. Children's verbatim responses and any visible solution method were recorded in Korean by the interviewers.

The recording sheets were translated into English by a Korean teacher living in the United States. The first author then classified each child's response to each of the problems after the translation. The second author also classified each child's response but in the Korean protocol. Each solution was given a descriptive code agreed on by the two coders. The categories for addition were counting all, finger patterns, counting on, recomposition and, a known fact. The categories for subtraction were separate, recomposition, and a known fact (see Fuson & Kwon, 1992, for more details on each category).

The observation in which Korean children used their fingers to show sums was focused on this study because finger patterns were important for understanding possible developmental sequences of solution procedures for Korean children. The method was called the contiguous folding/unfolding method (see Fuson & Kwon, 1992, for more details). However, these finger methods were not used by many children in this study because these children were so advanced. The results showed that the majority of Korean first graders were able to solve addition word problems. For the addition with sums to 10, almost three-fourths of the children used a known fact solution. Of the remaining solutions about half involved finger patterns or counting on, and about half involved a procedure that was not clear to the interviewer and was not explained adequately by the child. For the addition with sums over 10, the children in this study used counting all rarely, even counting on. The most common solutions were recomposition solutions, and most of these solutions were mental recompositions. Only a few of the children used folded/unfolded fingers to show the recomposition methods.

The result on subtraction solution procedures was not clear because Korean children refused to attempt more subtraction than addition combinations. As a result, fewer correct answers were obtained in subtraction than in addition. The most common solutions involved the subtraction recomposition procedures. Twice the recomposition procedures were down-over-ten and subtract-from-ten procedures. A few children carried out these solutions using Korean folding/unfolding fingers, but most of the solutions were done mentally and then described to the interviewer.

The results from this study suggested that Korean first graders showed remarkable competence at solving the more difficult single-digit addition and subtraction combinations with sum between 10 and 18. Most of the solution procedures the children used were advanced solutions that involved known facts or recomposition methods structured around ten, and addition and subtraction had similar combined percentages for these two solution categories.

From the eight reviewed studies above, it can be concluded that young children are able to solve addition and subtraction word problems prior to having any formal instruction. Children began solving problems by modeling the actions or relationships in the problems, then they used more advanced strategies such as counting, or a known number fact to solve problems. However, children still used visible objects to represent counting sequences. The results from these studies also showed that young children could solve many difficult problems include multiplication and division problems even before they had received any instruction on multiplication and division.

Children's Multidigit Addition and Subtraction Methods

This part reviewed two articles on children's solution procedures of multidigit addition and subtraction situations. The first article was a report of the progression of children from 4 projects with a problem-solving approach to teaching and learning multidigit number concepts and operations (Fuson et al., 1997). The second article reported mental strategies for addition and subtraction up to 100 in Dutch second grades (Beishuizen, 1993).

Recent research by Fuson et al. (1997) reported methods of multidigit addition and subtraction used by children from four different projects: Cognitive Guided Instruction (CGI), the Conceptual Based Instruction project (CBI), the Problem Centered Mathematics Project (PCMP), and the Supporting Ten-Structured Thinking projects (STST) (see Fuson et al., 1997, p. 133 – 136, for more details on each project). All four projects take a problem-solving approach to teaching multidigit number concepts and operations. The learning of multidigit concepts and procedures in these projects is perceived as a conceptual problem-solving activity rather than as the transmission of established rules and procedures. The children were allowed to work out their own procedures and then to share and discuss strategies for solving addition and subtraction problems and tasks involving place-value meanings of numbers.

The teacher played an active role in the classroom by posing the problems, coordinating the discussion of strategies, and joining the students in asking questions about strategies.

Single-Digit Addition and Subtraction.

There were three development levels, described by Fuson and Kwon (1992b), that children in all four projects used to solve single-digit addition and subtraction problems.

Level 1. Children constructed addition or subtraction situations by using physical objects. These models were used to model directly the addition or subtraction operation given in the situation. Children counted all the objects to add, and they took away and counted the remaining objects to subtract.

Level 2. Children could simultaneously consider all three quantities in an addition or subtraction situation by embedding the addends within the total and considering objects as being simultaneously part of the addend and part of the total. To add, they counted on from one addend word while keeping track of the other addend words counted on, or they counted on. To subtract, they counted back from the total, kept track of the addend counted back; counted back from the total to an addend; or counted up from the known addend to the total, keeping track of how many were counted up.

Level 3. The addends no longer have to be embedded within the total but exist outside in a numerical triplet structure in which the two addends are seen as equivalent to the total. A given numerical triplet can be recomposed in a related triplet of known facts. These solutions commonly use doubles (a + a) in the United States. For example, 7 + 6 = 6 + 6 + 1 = 12 + 1 = 13. In Asian countries (e.g., Japan, Korea) children learn to recompose numbers into ten-structured triplets (Fuson & Kwon, 1992b). For instance, 7 + 6 = 7 + 3 (to make ten) + 3 = 13. This method was used much less frequently in the United States.

Two-Digit Addition and Subtraction.

In children's two-digit addition and subtraction methods, children used many different methods. Children in Level 1 who could count above 10 could use a unitary multidigit concept to add two 2-digit numbers by making objects for each number and counting all of the objects. They could subtract by making objects, taking away from those objects, and counting the remaining objects. Children at Level 2 could count on by ones, add on objects by ones or verbally count all by ones to add. To subtract, they could count back or count up to by ones. Moreover, children in the project classrooms with sequence tens or separate tens used many different methods for adding and subtracting two-digit numbers. Some of these methods were carried out with objects, some were done verbally, and some were done with written numerals on paper to record. The methods children used were categorized into four kinds: Begin-With-One-Number Methods, Mixed Methods, Change-Both-Numbers Methods, and Decompose-Tens-and-Ones Methods (see Table 1 of Fuson et al., 1997 p. 147-149, for complete data).

Fuson et al. (1997) reported that the strategies for solving three-digit addition and subtraction problems were extended from two-digit addition and subtraction methods. The begin-with-one-number is straightforward but very cumbersome to do by counting on or down or up if one carries along the whole sequence of values as one count. The method involving adding or subtracting hundreds, tens, and ones separately and regrouping when necessary are simple extensions of the two-digit methods (see Table 2 of Fuson et al., 1997, p. 155-156, for complete data).

For addition or subtraction of four-digit and larger numbers, children in the CGI project, who worked with numbers of four digits and more, generally abandoned left-to-right methods and moved from the right-to-left methods. The sequence of counting methods that begin with one number would seem to get quite cumbersome with large numbers if done verbally without recording, because one would have carry along a whole multidigit number. The separate multiunits methods generalize easily to larger numbers. Some children in all projects did pose and solve larger problems, but systematic data on these methods are not yet available.

Within each project, many children used varied solution methods across different problems and problem settings. Children in Problem Centered Mathematics Project (PCMP) frequently began with one number and mixed methods for two-digit addition and subtraction. For three-digit problems, adding up to make tens and hundreds was popular. Children in the Conceptually Based Instruction (CBI) project most frequently used methods in which multiunits were added or subtracted separately. Children in the Cognitively Guided Instruction (CGI) project varied considerably from one classroom to another and within classrooms, with many children using methods in which multiunits were added or subtracted separately. Sequence methods that began with one number were also often used for two-digit problems. In the early Supporting Ten-Structured Thinking (STST) projects, children predominantly used methods in which multiunits were added or subtracted separately.

In the Netherlands, mathematics programs emphasize mental addition and subtraction in the lower grades (Beishuizen, 1993). Beishuizen reported that Dutch second graders used two strategies for mental addition and subtraction of two-digit numbers between 20 and 100: the 10 + 10 strategy (1010) and N + 10 strategy (N10). Both strategies have in common that they handle the tens first before the units (left to right). The 1010 strategy starts with splitting off the tens from both numbers and adding or subtracting them separately (e.g., 46 + 23 is determined by taking 40 + 20 = 60, 6 + 3 = 9, and 60 + 9 = 69). Meanwhile, the N10 strategy starts directly with jumping by tens from the first unsplit number (e.g., 46 + 23 is determined by taking 46 + 20 = 66, then 66 + 3 = 6 9). In Dutch, the 1010 is called *split method* while N10 is called *jump method* (e.g., Wolters, Beishuizen, Broers, & Knoppert, 1990). In the United States literature (e.g., Resnick, 1983; Fuson & Kwon, 1992), the 1010 strategies is called the decomposition or regrouping procedures, but the N10 strategy is less well known and barely described.

Conclusion

This chapter reviewed literature on problem solving and solving addition and subtraction word problems. A problem is a situation in which a person wants something and does not know how to do to get a solution (Reys et al., 1989), while problem solving is a process by which the choice of an appropriate strategy enables an individual to proceed from what is given in a problem to its solutions (Grossnickel et al., 1983). Particular methods of solving problems have a long history in mathematics curriculum. Upton (1939) tried to make children think about the process of solving a problem by presenting the problem without numbers. Problem solving has been introduced to school mathematics in the past two decades since the National Council of Teachers of Mathematics (NCTM, 1980) declared that problem solving should be the "focus of school mathematics."

Mathematical word problems, or story-problems, have long been familiar features of school mathematics (Gerofsky, 1996; Karrison & Carool, 1991). Word problems are one component of the elementary school problem-solving curriculum. Some problems are difficult for students to solve and some problems are easy (Stockdale, 1991). To solve word problems, students must employ reading, language comprehension, problem solving, and mathematics computation skills almost simultaneously (Reutzel, 1983). Young children can use their informal knowledge to analyze and solve simple addition and subtraction word problems before they receive a formal arithmetic instruction (e.g., Ginsburg & Baron, 1993; Carpenter et al., 1981). In addition, children enter school with a great deal of informal knowledge of mathematics such as using concrete objects to model and solve problems (Ginsburg, 1977; Resnick & Ford, 1981). Children also use informal strategies such as making marks, counting, and using concrete subjects to model and solve problems (Bardody, 1987).

A number of studies in the past two decades indicated that young children are successful in solving word problems even before they receive any formal instruction (e.g., Carpenter et al., 1981, 1997; Hiebert, 1982; Bebout, 1990; Adetula, 1989; Fuson & Kwon, 1992). The findings of those studies showed that children showed remarkable success in solving word problems. Children could solve a wide range of problems, including multiplication and division situation, early in the primary grades. Kindergarten children's strategies could be identified as representing or modeling the action or relationships described in the problems (e.g., Carpenter et al., 1993). Then, children developed to use counting models identified by Groen and Parkman (1972) in solving addition and subtraction problems. Children also used several strategies that were not based on counting to solve addition and subtraction word problems such as knowing a number fact.

It is frequently assumed that children must master computational skills before they can apply them to solving problems. However, the results from previous studies suggested that it is not necessary to wait until children master computational skills and then introduce word problems. Teachers might introduce addition and subtraction word problems to teach children how to add and subtract. Moreover, the results from many studies showed that children were not quite as successful with the subtraction problems as they were with addition problems.

In summary, it appears that children, from many studies of children at primary grades including unschooled children, have the capability to solve a variety of simple addition and subtraction word problems before they have had any formal instruction. The results from previous studies revealed that there were three strategies that children most frequently used to solve addition and subtraction word problems. These strategies were strategies based on direct modeling with fingers or physical objects, strategies based on the use of counting sequences, and mental strategies that involve the use of number facts either at the recall level or derived from other number facts. These strategies may vary in children from different cultures. Some children used base-ten number strategies to solve word problems.

The results suggest that by introducing operations based on verbal problems and integrating verbal problems throughout the mathematics curriculum, children will develop natural ability to analyze problem structure and will develop conceptions of basic operations.

There are additional studies that were done between 1990 and 1998 about word problems. However, those studies focused on particular topic such as the study of difficulty of arithmetic word problems involving the comparison of sets (Stern, 1993). Several articles also focus on teaching strategies to improve student ability in solving word problems (Adetula, 1996; Mwangi & Sweller, 1998; Engelhardt & Usnick, 1991). Thus, these studies are not included in this literature review.

CHAPTER III

METHOD

Introduction

Problem solving is a complex mental process involving visualization, imagination, manipulation, abstraction, and the association of ideas (May, 1974). However, teachers often do not have insight into mental processes that children bring to solving problems. It is true that teachers cannot see the mental processes that children use in solving problems. Teachers can only make assumptions about what is in children's minds. This study was designed to investigate the processes that Thai children used to solve addition and subtraction word problems. This study sought to answer the following questions:

a) How successful are Thai children in solving addition and subtraction word problems?

b) Which strategies are used by Thai children to solve addition and subtraction word problems?

This study relied on individual interviews with 18 children in the first semester of second grade to identify the strategies that children used to solve addition and subtraction word problems. Six children from two rural schools were interviewed in an initial study, and 12 children from one urban school were interviewed in a main study.

The first section of this chapter contains a description of the participants who participated in the initial study and the main study. The next section deals with data collection, and the last section deals with data analysis.

Participants

Total participants in this study were 58 second-graders from three elementary schools in Thailand. Fifty-two children were from school A, three children were from school B, and another three children were from school C. School A is an urban school and the children in this school were from middle-income families. School A is a private school with 40 to 45 children in each classroom. Schools B and C are rural schools and the children in these two schools were from low-income families. School B is a medium sized school with 10 to 15 children in each classroom. School C is a small sized school with seven to eight children in each classroom. Schools B and C are private schools. All participants received permission from their parents to participate in the study. The primary focus of the study was on the six rural children from schools B and C, and 12 children from school A. At the time of the study, children had received lessons on solving two-digit addition and subtraction where the sum of the two numbers was not greater than 100.

First, 40 children were selected from one classroom in school A. These 40 children took a written test and the results from this test were used to see whether children at this level could read and understand word problems requiring two-digit addition and subtraction. The results provided information which assisted in developing the seven word problems used in the initial and main study. Next, six second-graders from schools B and C were selected to participate in the initial study. Three children—low, average, and high achievement—were chosen from each school. Finally, 12 children were drawn from five second-grade classrooms in school A, excluding the classroom that contained 40 children who participated in the written test described above.

The 12 children—six boys and six girls—were selected by their mathematical achievement as recommended by their teachers: low, average, and high. Two boys and two girls were selected from each achievement level (see Table 4 and 5). Additionally, six second-grade teachers of the children participating in this study from school A participated in a questionnaire study about their understanding of problem solving. All teachers were female and held a Bachelor's Degree in Education.

Table 4. Summary of participants in the initial study
N = 6

Schools	B		C		Total
Sex and Achievement	**Boys**	**Girls**	**Boys**	**Girls**	
High	1			1	2
Average		1	1		2
Low		1	1		2
Total	**1**	**2**	**2**	**1**	**6**

Table 5. Summary of participants in the main study
N=12

Classroom	1		2		3		5		6		Total
Sex and Achievement	**Boys**	**Girls**	**Boys**	**Girls**	**Boys**	**Girls**	**Boys**	**Girls**	**Boys**	**Girls**	
Low		1	1		1			1			4
Average	1						1	1		1	4
High	1		1			1				1	4
Total	**2**	**1**	**2**	**-**	**1**	**1**	**1**	**2**	**-**	**2**	**12**

Data Collection

This section provides a description of each phase of the data collection. Phase I describes the development of seven word problems used in this study. Phase II presents data collection of six children from schools B and C. Phase III presents data collection of 12 children from school A, the main study. Phase IV is a description of classroom observations and a questionnaire study about Thai teachers' understanding of problem solving. This section includes information on the researcher who did this study.

Phase I: Problem Development

Seven addition and subtraction word problems were developed for this study. The seven word problems were selected from categories representing different semantic structures: two joining problems, two comparison problems, two combining problems, and one separating problem (see Table 1). Appendix A presents problems of each type used in this study.

These seven word problems were chosen because an earlier study (Carpenter et al., 1981) indicated that these types of problems would elicit a variety of solution strategies. The problem set was mixed with both difficult and easy problems. The reason for having both easy and difficult problems was to probe children's thinking strategies at a variety of difficulty levels. It was desirable to have a variety of problems so that all children could solve some problems, but also be challenged by others.

The seven word problems were translated into Thai language by the researcher so Thai children could solve them. To determine the validity of the seven problems, six knowledgeable Thai elementary teachers from school A read these seven problems to determine the match of the problems to the objectives of the curriculum as well as the readability and difficulty of the problems. These teachers had taught in an elementary school for many years. The seven word problems were then revised to match the curriculum and the objectives of second grade mathematics in Thailand. The numbers used in the seven word problems were mostly two-digit numbers where the sum of the two numbers was not greater than 100.

The seven word problems then were tried in a written test with 40 children from classroom four in school A. The purpose of the written test was to see whether children in this study could read and understand the seven word problems developed above. The test was also used to measure and determine difficulty and discrimination of each problem. Forty children were given seven word problems in the Thai language in a booklet (see Appendix A). The test took place in the classroom during math period. Manipulatives were not given. The initial time used in this test was 50 minutes. However, 50 minutes was not sufficient for students to complete the test. Therefore, 30 more minutes were given for children to finish the test. The test took one hour and 20 minutes rather than the expected 50 minutes because the children wasted time drawing lines in the test booklet. Children could not write without lines. When the children were not satisfied with the lines they had drawn, they erased and drew them again. Therefore, it took a great deal of time to finish the test. The researcher observed and took notes about what the children did while they were solving problems.

The results from the test with 40 children indicated that children in this study were generally successful in solving addition and subtraction word problems.

The majority of children could solve Problems 1, 3, 4, 5 and 7. Problems 2 and 6 were difficult for children to solve. However, Problems 2 and 6 were not eliminated from this study because the researcher was curious about how six children from the initial study and 12 children from the main study solved these two difficult problems. By calculating the difficulty of each item, the results show that Problems 1 and 3 were easy but Problem 6 was difficult (see Table 6). Discrimination shows the measure of how well an item differentiates between high achievers and low achievers. Table 6 shows that Problems 1 and 3 have low discrimination value, below 0.20. It would explain that Problems 1 and 3 were easy so these two problems did not discriminate between high achievers and low achievers. It means that both high and low achievers could solve these problems.

In contrast, Problem 2, 4, 5, 6, and 7 were discriminating items. It means that these five problems can differentiate between high achievers and low achievers.

Table 6. Item of difficulty and item of discrimination
N = 40

Problem	Item Difficulty	Item Discrimination
Addition		
1. Compare	0.95	0.15
2. Combine	0.48	0.46
3. Join	0.92	0.08
Subtraction		
4. Compare	0.65	0.54
5. Combine	0.45	0.69
6. Join	0.18	0.38
7. Separate	0.58	0.54

Phase II: The Initial Study

The primary purpose of the initial study was to practice interview questions with children from schools B and C prior to interviewing 12 children from school A and after the test with 40 children. The results from the initial study were also used in addition to results from the main study to answer the two research questions. The researcher selected six second-graders from schools B and C. The six children brought signed parental permission forms. The setting of the initial study was the same as the interview described in the main study. The interview with these six children took place in the library of each school. Each child was called to the library individually and was asked to read and solve each problem. The entire interview was audiotaped for later study. The prepared questions used in the interview were:

- What did the problem ask you to find?
- How did you know that?
- Could you describe your solution to the problem and how you found it?
- How did you decide whether to add or subtract to find the answer?
- How did you count on your fingers? (When a child used fingers.)

Other questions were asked to clarify individual students' problem solving procedures. The questions began with the following: "Could you describe your solution to the problem and how you found it?" or "What did the problems ask you to find?" Tell me how you knew that." Depending on the child's response to the initial question, the interviewer used several further questions like, "Tell me how you decided whether to add or subtract to find the answer." When the children said that they used some kind of counting method, the interviewer would say, "Tell me out loud how you counted," or

"Show me how you counted." When the child indicated the use of a mental strategy, the interviewer would ask, "Tell me out loud what you did in your mind. Did you imagine counters in your mind or remember number facts?"

Phase III: The Main Study

The purpose of the main study was to interview 12 children from school A to determine the success of Thai children in solving addition and subtraction word problems, and to determine strategies that Thai children used to solve addition and subtraction word problems. The results from this section were also used to determine whether Thai children solved addition and subtraction word problems differently from children in the United States. Children were interviewed by the researcher in Thai language during school periods in the English Sound Lab room of school A. Each child was told that the interviewer was interested in how children solved addition and subtraction word problems. Each child was asked to read and solve seven addition and subtraction word problems, one problem at a time. Each interview took 30 to 45 minutes. In the interview, the children did not need to draw lines on paper because the interviewer provided lined papers for them, so the time needed for the interview was less than the time that 40 children used to solve seven word problems in the written test form.

The interviewer and the child sat on the same side of a small desk with the interviewer to the right of the child. On the table, paper and counters were available to the child as well as the interview sheets. In order to put the children at ease prior to the interview, the interviewer talked with the children about their lives. The interviewer explained to each child what she would do during the interview and told the child that she would use a tape recorder in order to help her remember everything that was said. The tape recorder was in plain view of the child.

All interviews used the same interview format and asked the same initial follow-up questions. The interview questions used in the main study were similar to the questions developed from the initial study. Each problem was presented in Thai language on a card (see Appendix A). Each child was asked to read and solve each problem. When the child could not read the problem, the researcher read the problem to the child. While the child was solving each problem, the interviewer observed and took notes about what the child was doing. The interviewer did not give feedback about the correctness of any response.

The interviewer coded responses as the child solved each problem on the observation card (see Figure 1). After the child finished solving each problem, the interviewer asked the child to show and explain the solutions out loud. The interviewer continued questioning until it was clear what strategy the child was using. When a solution strategy that a child used was obvious or the child could not explain how he or she completed a problem, the interviewer went on to the next problem. The researcher was the only interviewer, so the reliability in giving the interview was assumed. Students' written work on each problem was also collected. Children's verbatim responses were recorded in Thai language using an audiotape. The recordings were then translated into English by the researcher.

Problem-solving Observation Card

Student's name: ____________________ Date: ______________

Response:

Figure 1. An observation card.

Phase IV: Classroom Observation and Questionnaire Study

The purpose of the classroom observation was to explore children's behavior, teacher's instruction, and classroom environments in Thailand. The data from this observation provides readers with a picture of what happens in Thai classrooms and what teachers and children do during mathematics instruction. The classroom observations were done after the test with 40 children and the interview with six children in the initial study and with 12 children in the main study. Two of six classrooms in school A were observed during math period. The same researcher conducted all the observations, and took notes during the lesson. All observed sessions were videotaped and audiotaped. Also, the classroom environments were pictured.

The observer was in the back of the classroom to avoid interfering in classroom activities. However, some children were not on task because they were interested in the video camera. This was not a big problem for the majority of the class but it was a problem for two or three children at the back of the classroom who were very annoying in class. These children were looking at the camera and did not pay attention to the lesson. However, when the teacher told them to stop being annoying, they did stop for a moment. But they continued looking at the video camera during the rest of the class.

In order to gain an understanding of teacher's perception of mathematical problem solving and teachers' instruction in mathematics in classrooms, the researcher developed a questionnaire (see Appendix B). Questions were open and were placed in a booklet easy for teachers to read with space to respond to each question. Six teachers of the 52 children from school A completed the questionnaire after the interview with 12 children. The questionnaire began with a question about how teachers usually teach mathematics in the classroom, how they specifically teach addition and subtraction, and how they taught mathematical problem solving (see Appendix B for more details).

The Researcher

The primary instrument for data collection and analysis in this study was the researcher. The researcher administered the test, the interview, and the observations. The researcher earned a bachelor degree in mathematics education in 1995 at Chiang Mai University in northern Thailand. She had taught secondary school mathematics for eight months, six months as a student teacher and two months as a teacher. Following this brief experience as a teacher, the researcher came to Oregon State University as a scholar from Thailand. She has studied for her Masters degree in mathematics education since Fall 1996.

However, this study did not involve mathematics at the secondary school level. Thus, it is necessary to explain why the researcher became interested in problem solving at the elementary level. The researcher had a chance to visit a second grade classroom several times in a small town in Oregon. The researcher observed elementary student teachers teaching problem solving. She was surprised that the children had different ways to approach and solve problems and had the ability to explain their own reasoning and thinking. This problem solving was different from what the researcher had experienced in her own school experience.

Moreover, the researcher had a chance to interview a second grader in that small town Oregon school regarding number concepts. The researcher learned from this interview that this child's understanding of number concepts was complex. From both the teaching and the interview, the researcher became curious and interested in children's problem solving performance. The researcher would like to know what children do when they solve problems, especially Thai children who are raised in a different culture from United States children. Therefore, in this study the researcher was the person who collected the data about how second graders in Thailand solve addition and subtraction word problems. Furthermore, the researcher observed and questioned the teachers regarding their understanding about problem solving.

Data Analysis

This section describes how the data obtained from each phase of this study were analyzed in both quantitative and qualitative ways based on two research questions stated at the beginning of this chapter.

The data obtained from Phase I were used to determine problem difficulty and validity of seven word problems used in this study. To determine validity, the seven word problems were analyzed and revised by Thai teachers based on Thai mathematics curriculum and objectives. The revisions were for wording, number choice, and readability. The revision problems then were tested with 40 children from school A to determine the difficulty, readability, and discrimination of the seven word problems. The difficulty was analyzed by using the formula (Doran, 1980):

$$P = \frac{R}{N}$$

where R represents the number of children who get correct answer on each problem, and N is the total of children in the study (in this study, N = 40). The discrimination was also analyzed by considering on how well children in this study solved each problem. The discrimination was analyzed by using the formula (Doran, 1980):

$$D = \frac{(H - L)}{\frac{N}{2}}$$

The scores were in order from top to bottom, H is the number of children in the half top who get a correct answer on each problem, whereas L is the number of children in the half of the bottom who get a correct answer on each problem. N is the total of children in the study. The result from the Phase I of this study was interpreted in both descriptive and numerical methods.

The data obtained from Phase II was used to develop interview questions used in this study. The transcriptions from the initial study were analyzed based on how well each interview question stimulated children to describe the solutions to the problems clearly. The follow up questions that were developed during the initial study were also recorded and used in the main study. The data obtained from both Phase II and Phase III of this study were used to answer two research questions. The data were analyzed in both qualitative and quantitative ways. In order to grade children's work, the researcher developed criteria to score children's work (see Table 7). The observation notes were analyzed to determine characteristics of each child in the initial study and in the main study.

Table 7. Criteria used to score children's work

Scores	Characteristics
3 points	• The students show complete understanding of the problem and select an appropriate solution strategy. The correct answer is given.
2 points	• The students show complete understanding of the problem and select an appropriate solution strategy. However, there is a copying or computational error. Thus, an incorrect answer is given.
1 point	• The students show no understanding of the problem and implement an incorrect solution strategy. However, the students show no misconceptions or errors in the calculation.
0 points	• The students show no understanding of the problem and implement an incorrect solution strategy. The students also show misconception of the calculation. • The paper is blank.

To answer research question number one (How successful are Thai children in solving addition and subtraction word problems?), individuals' work on seven word problems were graded by using the criteria in Table 7. The child was classified as successful if he or she used an effective solution strategy that would led to a correct answer and the child implemented the strategy without any errors or misconceptions in calculations. The child was classified as somewhat successful if he or she used an effective solution strategy that would lead to a correct answer, but the child calculated incorrectly. The child who could not solve the problem or did not attempt to solve the problem was defined as unsuccessful on that problem. The results were reported using graphs and descriptive methods.

To answer research question number two (Which strategies are used by Thai children to solve addition and subtraction word problems?), data obtained from children's work, transcripts from individual interviews, and observations during both the initial and main study were analyzed to determine strategies that children used to solve addition and subtraction word problems. Solution strategies on addition and subtraction word problems were categorized into counting strategies and non-counting strategies. The counting strategies were counting all, counting on either from smaller or larger numbers, counting up, and counting down. The non-counting methods were: use of number fact, use place value, and use of base-ten system. Additional methods were also recorded. The results on solution strategies of the children in the initial study and the children in the main study were interpreted in both descriptive and numerical methods.

Data obtained from Phase IV of this study were analyzed to see what types of mathematics lessons were taught in Thai classrooms. Demonstrations, explanations, questions, and responses between teachers and children were analyzed by searching for patterns. Teachers' responses to questionnaires were also analyzed to reveal Thai teacher's understanding of problem solving. No initial criteria were established. The discussion of these findings will follow in Chapter IV.

CHAPTER IV

RESULTS

Introduction

The results from the analysis of two sets of data are reported in this chapter: data from the interview with six children from the initial study in Phase II; and data from the interview with 12 children from the main study in Phase III. The data were analyzed based on two research questions: (a) How successful are Thai children in solving addition and subtraction word problems?; and (b) Which strategies are used by Thai children to solve addition and subtraction word problems?

This chapter contains five sections. The first section describes characteristics of each child who participated in the initial study and the main study. The second section reports success of Thai children in solving word problems. The third section reports strategies that Thai children used to solve addition and subtraction word problems. The fourth section is about Thai classroom environment. The last section reports Thai teachers' understanding about problem solving.

Characteristics of Each Child

This section provides information regarding characteristics of each child who participated in the initial study and the main study. A correctly solved problem meant the child used an effective solution strategy and got a correct answer to a problem.

An effective solution strategy meant the child used a strategy that could have generated a correct answer.

Initial Study

The participants from the initial study were six children—two boys and four girls—from two rural schools (schools B and C) in Thailand. They were categorized by their academic achievement level as recommended by their teachers: high, average, and low.

High Achievement.

P-Boy 1 (PB1): He was the only child in this study who solved all seven problems correctly. He read and understood all of the problems quickly. From the observation, the boy did not use paper, he calculated mostly in his mind. The boy scored 21 points.

P-Boy 2 (PB2): Like PB1, the boy solved problems mostly in his mind. He solved six problems correctly. However, he solved Problem 6 incorrectly. He used addition instead of subtraction. He scored 18 points.

Average Achievement.

P-Girl 3 (PG3): She used effective solution strategies to solve six problems, but she got correct answers to five problems. She could not solve Problem 6 because she thought it required addition. The girl did the calculation in a row form. She scored 17 points.

P-Girl 4 (PG4): The girl used effective solution strategies and got correct answers to five problems. She could not solve Problem 2 and Problem 6 because she misunderstood the keyword in the problems. She used knuckles to show the counting sequence. The girl scored 15 points.

Low Achievement.

P-Girl 5 (PG5): The girl should be in third grade by now but she could not read and write, so she was still in second grade. She could not read well, but she attempted to read and understand the context in the problems. The interviewer did not read the problem to her. It took a lot of time for her to solve each problem. She used effective solution strategies to solve four problems, but she got correct answers for only two problems. She scored 9 points.

P-Girl 6 (PG6): The girl used effective solution strategies to solve three problems but she could not get a correct answer for any of the problems. She did not have a problem in reading but she had a misconception about calculation in subtraction. The girl always subtracted the smaller digit from the larger digit, for example, 38 – 19 = 21. She scored 6 points.

Main Study

The participants from the main study were 12 children from an urban school in Thailand, school A. Six boys and six girls were selected from three different achievement levels as recommended by their teachers: high, average, and low.

High Achievement.

Boy 1 (B1): The boy read all seven problems without assistance from the interviewer. He used effective solution strategies and got correct answers for five problems. He could not solve Problems 2 and 6 because he misunderstood the keyword in the problems. While he read each problem, he wrote the numbers in the problem on the sheet of paper, then placed the operation. He scored 15 points.

Boy 2 (B2): The child could read all seven problems. At first, he could not solve Problem 5 because he did not understand the context in the problem and he did not want to do it, so we went to the next problem. After the child finished solving Problem 7, we went back to Problem 5 and the child finally solved Problem 5 correctly. He used effective solution strategies to solve five problems but he got correct answers to four problems. He could not solve Problems 2 and 6. The boy scored 14 points.

Girl 3 (G3): The girl did not use paper and pencil in calculation. She calculated in her mind. The girl said that she counted by picturing fingers or counters in her mind. The girl used effective solution strategies to solve four problems. The girl could not solve Problems 2, 5 and 6 because she misunderstood the keyword in the problems. She scored 12 points.

Girl 4 (G4): The girl used effective solution strategies and got correct answers to six problems. She could not solve Problem 2 because she misunderstood the keyword in the problem. She spent a lot of time on Problem 5 because she did not understand the context in the problem but she could get a correct solution. She scored 18 points.

Average Achievement.

Boy 5 (B5): He read all seven problems without assistance from the interviewer. He used effective solution strategies and got correct answers to six problems. He could not solve Problem 2 because he misunderstood the keyword in the problem. He scored 18 points.

Boy 6 (B6): The child read all problems without help from the interviewer but his explanation to the solution strategies was not clear. He used effective solution strategies and got correct answers to four problems. The child could not solve Problems 2, 5, and 6 because he misunderstood the keyword in the problems. He scored 12 points.

Girl 7 (G7): The girl used effective solution strategies and got correct answers for five problems. She could not solve Problems 2 and 6. She was capable of reading seven problems but she was not clear in giving explanations. The girl scored 15 points.

Girl 8 (G8): The girl was able to read all seven problems. She solved problems mentally. She used effective solution strategies to solve five problems but she got correct answers to only four problems. She could not solve Problems 5 and 6. She scored 14 points.

Low Achievement.

Boy 9 (B9): The boy read each problem slowly but he understood the problems. For some problems, the boy got a correct answer but he could not explain how he got an answer. The interviewer had to ask him step by step to get the explanation. From the observation, it seemed that this boy was not willing to solve problems or he did not like solving problems. He used effective solution strategies and got correct answers for four problems. He could not solve Problems 3, 4, and 6. He scored 12 points.

Boy 10 (B10): The boy had difficulty in reading. He tried to read all seven problems but it was very slow. He had to spell out some difficult words (Thai words). Therefore, the interviewer read some problems to him. He did the calculation in a row form rather than a column form. He used effective solution strategies to solve five problems but he got correct answers for four problems. The boy could not solve Problems 5 and 6. He scored 14 points.

Girl 11 (G11): The girl did not explain clearly how she did the problems. She used effective solution strategies for five problems but she did not get correct answers for any problems. She could not solve Problem 2 and Problem 6. She scored 8 points.

Girl 12 (G12): The girl had difficulty in reading. The interviewer read two problems to her. From the observation, when she solved problems, she wrote the number from the problem on the paper and then she reread the problem and decided whether to add or subtract. As a result, she took about 7 to 8 minutes to solve each problem. She used effective solution strategies and got correct answers for five problems. She could not solve Problems 2 and 6. She scored 15 points.

Success of Thai Children

In general, Thai children in this study were successful in solving addition and subtraction word problems. Figure 2 showed that 15 out of the 18 children in the interview used effective solution strategies and got correct answers for Problems 1 and 3. In contrast, the results showed that Problems 2 and 6 were the two most difficult problems for the 18 children to solve, as predicted by the test given in classroom in Phase I of the study. Six out of 18 children (33.3 %) could solve Problem 2 and three out of 18 children (16.6%) could solve Problem 6 correctly. Twelve out of 18 children were able to solve Problems 4 and 7 while 11 out of 18 children were able to solve Problem 5.

Addition Word Problems

Addition word problems used in this study were Problems 1, 2, and 3. The two easiest problems for children to solve were: (a) Problem 1 (Combine problem; classroom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes?) and (b) Problem 3 (Join problem; Suda has 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now?). The compare problem (Problem 2: Manee has 23 apples. Mana has 15 more apples that Manee does. How many apples does Mana have?) was the most difficult addition word problem for children to solve (see Figure 2).

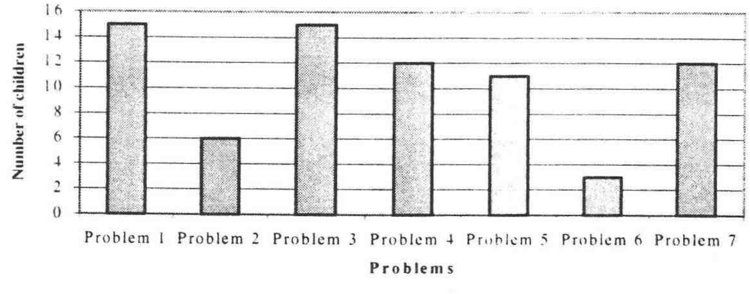


Figure 2. The number of children who used effective solution strategies and got correct answers for both addition and subtraction word problems (N = 18).

Initial Study.

Figure 3 shows that five of the six children from the initial study solved both combine and join problems correctly (Problems 1 and 3). Only three children of the six children solved the compare problem correctly (Problem 2). One child (PG6) from this group could not solve any of the addition problems. The girl used wrong operations for Problem1 and Problem 2. She used a correct operation for Problem 3 but she did not carry and regroup. For example, to solve 58 + 15, she gave 63 as the answer instead of 73.

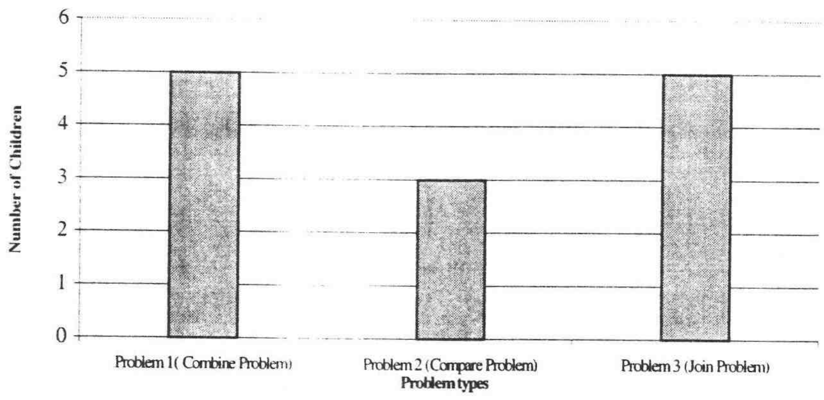


Figure 3. The number of children who used effective solution strategies and got correct answers for addition problems (Initial Study, N = 6).

Main Study.

The results indicated that 10 of the 12 children from the main study solved the combine problem and the join problem correctly. Only three of the 12 children solved the compare problem correctly (see Figure 4).

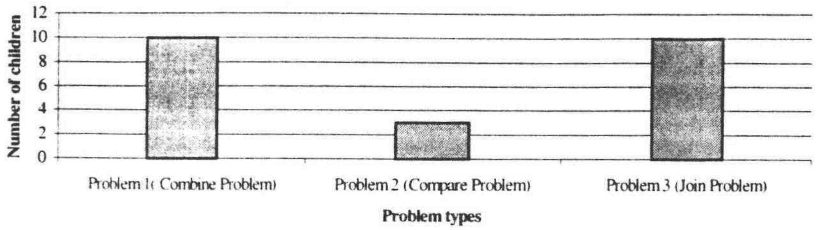


Figure 4. The number of children who used effective solution strategies and got correct answers for addition problems (Main Study, N = 12).

In summary, for the addition problems, the majority of Thai children in this study were not as successful in solving compare problems as they were successful in solving combine and join problems.

Subtraction Word Problems

Subtraction word problems used in this study were Problems 4, 5, 6, and 7. The results showed that the join problem (Problem 6: Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 balloons altogether?) was the most difficult problem for children in this study to solve. Only three of 18 children used effective solution strategies and got correct answers for the join problem (see Figure 2).

Initial Study.

Figure 5 showed that three of the six children from the initial study correctly solved the combine problem (Problem 4: There are 42 chickens. 25 are male. How many chickens are female?). Four of the six children correctly solved the compare problem (Problem 5: Suda's pencil is 20 centimeter long. Manee's pencil is 9 centimeter long.

How much longer is Suda's pencil than Manee's pencil?) and separate problems (Problem 7: The farmer has 38 cows. He sells 19 cows. How many cows are left?). Only one child (PB1) correctly solved the join problem (Problem 6: There are 42 chickens. 25 are male. How many chickens are female?).

Main Study.

Figure 6 shows that nine of 12 the children from the main study correctly solved the combine problem (Problem 4: There are 42 chickens. 25 are male. How many chickens are female?) and separate problems (Problem 7: The farmer has 38 cows. He sells 19 cows. How many cows are left?). Seven of 12 the children correctly solved the compare problem (Problem 5: Suda's pencil is 20 centimeter long. Manee's pencil is 9 centimeter long. How much longer is Suda's pencil than Manee's pencil?). Two of the 12 children in the main study solved the join problem correctly (G4 and B5).

In summary, Thai children in this study were not as successful in solving the join problems as they were solving combine, compare, and separate subtraction problems.

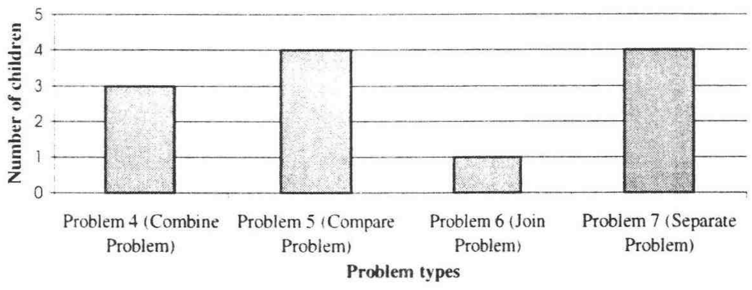


Figure 5. The number of children who used effective solution strategies and got correct answers for subtraction problems (Initial Study, N= 6).

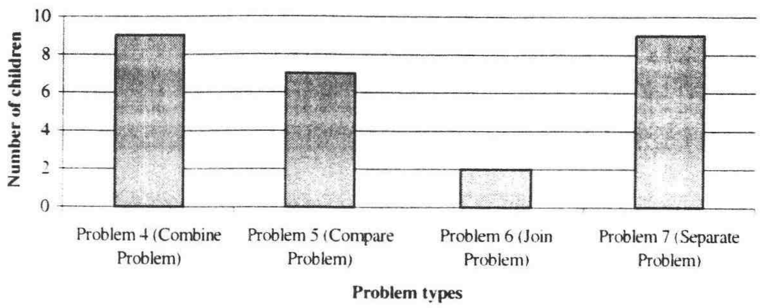


Figure 6. The number of children who used effective solution strategies and got correct answers for subtraction problems (Main Study, N = 12).

Summary

The results indicated that Thai children in this study were generally successful in solving both addition and subtraction word problems. For addition problems, children were not as successful in solving the compare problem as they solved combine and join problems. For subtraction problems, the majority of Thai children in this study solved combine, compare, and separate problems correctly. However, the majority of the children were not successful in solving the subtraction join problem. Explanations for these successes and difficulties will be discussed in the next section.

The results, however, showed that the teachers in schools B and C accurately described high, average, and low achievers as indicated by these addition and subtraction word problems (see Figure 7). Interestingly, the scores did not discriminate among high, average and low achievers in school A in the same way as the teachers did. High, average, and low achieving children in school A all scored in the same range on the interview (see Figure 8).

Although no data were collected to attempt to explain this situation, one could conjecture that class sizes might effect this situation. Teachers in school A had large class sizes, while teachers in schools B and C had small class sizes. Thus, teachers in schools B and C might more accurately described high, average, and low achievers than teachers in school A. However, since the questions were not written with the intent to discriminate between children, but rather to explore students solution strategies, it is not possible to draw any conclusions from this interesting detail.

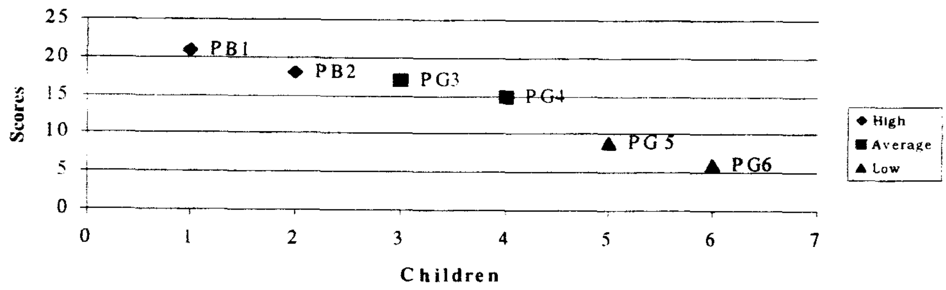


Figure 7. Children's scores compared with their achievement level (Initial Study, N= 6).

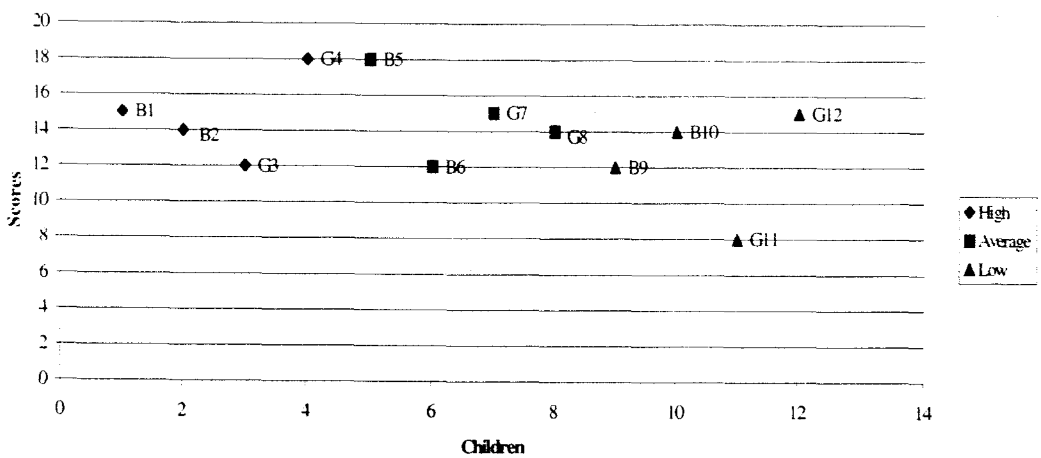


Figure 8. Children's scores compared with their achievement level (Main Study, N= 12).

Thai Children's Solution Strategies

The primary focus of this study was to examine strategies that Thai children used to solve addition and subtraction word problems. The results in this section were analyzed from six children from the initial study and 12 children from the main study. This section presents solution strategies that Thai children used to solve addition and subtraction word problems. The children were categorized into three groups. Children in Group1 were defined as children who used an effective solution strategy that would result in a correct answer and was applied without any errors or misconceptions (Successful methods). In Group 2, children used an effective solution strategy that would result in a correct answer but calculation errors occurred (Partially successful methods). The last group, Group 3, children used an ineffective solution strategy and thus gave an incorrect answer (Unsuccessful methods).

Addition Strategies

The numbers in each problem were two-digit numbers, therefore children in this study who were successful in solving problems used carrying and regrouping in the tens column (see Figure 9).

The solution strategies were observed when the children did the calculation in the ones and the tens column. The three basic counting models identified by previous studies (e.g. Carpenter et al., 1981) were also found in this study as well as some additional strategies. With the counting strategies, fingers were often used to represent the addends and the counting sequence. Several identified strategies that were not based on counting were: a known number fact, and a base-ten strategy. The strategies that were found are summarized in Table 8.

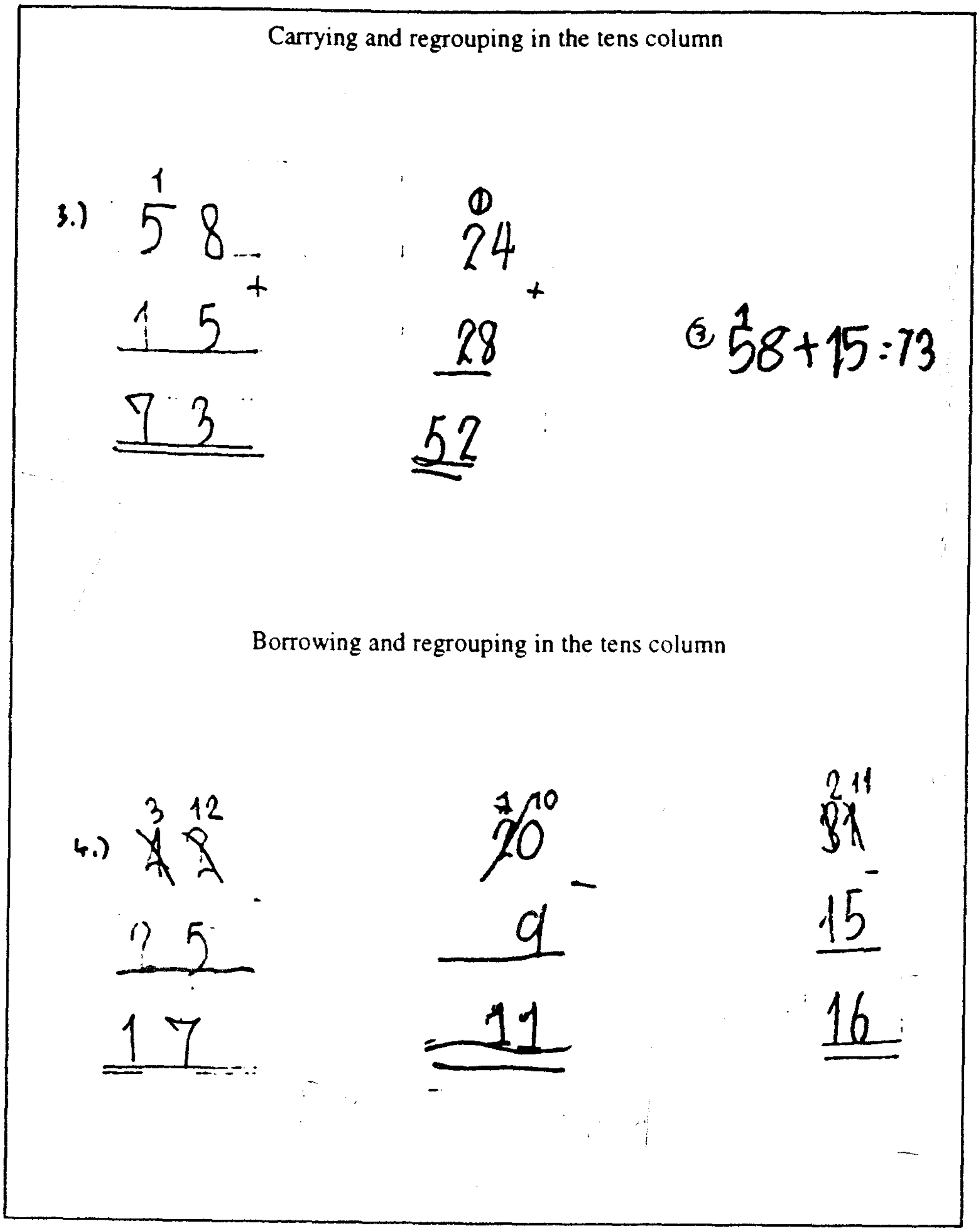


Figure 9. Carrying, borrowing, and regrouping in the tens column.

Table 8. Addition strategies.

Strategies	Description
Counting on from the larger number	In this counting method, children count with or without fingers. Some children count in their minds by imagining counters or fingers. Most of them used folded finger methods to show the sequence of counting.
Counting on from the smaller number	Children count with or without fingers from the smaller number given in the problems. However, this method was used less by these children than in other studies.
Identifying a know number fact	Children give an answer with the justification that they remember the number facts.
Using knowledge of base 10 number system	Children try to make sets of tens first and then added to another number. For example 8 + 5: the child said "8 have 5, 5 plus 5 equals 10, and another 3 so it is 13".

In the counting on from the larger number strategy, children began counting forward with the larger number in the problem. For example, to solve 24 + 28, the child would find the sum in the ones column first by counting "8 [pause], 9, 10, 11, 12. The answer is 12." If the child used fingers, four fingers would be folded to represent the number 4 (see Figure 10). The child then wrote down two and carried one to the tens column, then added all values at the tens column together to yield the final answer. The counting on from the smaller number strategy is identical except that children began counting forward with the smaller of the two addends. To solve 24 + 28, the child would solve the ones column first by counting, "4 [pause], 5, 6, 7, 8, 9, 10, 11, 12. The answer is 12." The child then put the number 1 at the top of the tens column and put the number 2 at the answer space of the ones column. The child added the number in the tens column together to yield the answer.

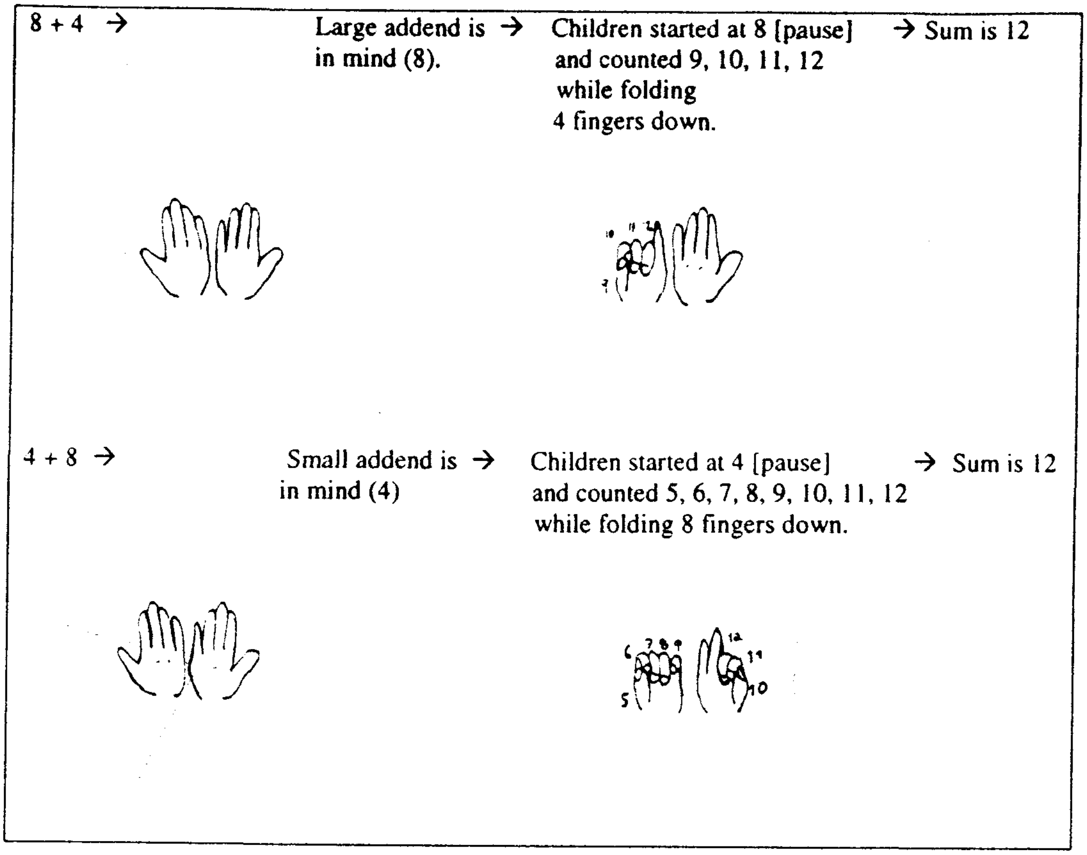


Figure 10. Addition by counting on using the folding fingers method.

Children's solutions strategies to addition word problems were not limited to counting strategies. Children also used a known number fact and applied this knowledge to solve problems. For example, to solve 23 + 15, one child remembered that 3 + 5 is 8 and 2 + 1 is 3, so he suddenly gave 38 as an answer. The children also used a combination of ten strategy to derive solutions for problems. For instance, to solve 58 + 15, one child would say, " eight has five. Put five together so it is 10. Plus three more so it is 13. Carrying one to the tens column, five plus one is six. Plus one more from the carrying so it is seven. The answer is 73." Table 9 shows the number of children who correctly solved each problem and the number and kind of an effective solution strategy the children used.

Table 9. Number of children correctly solving each type of addition problems and the number and kind of effective strategies used (N= 18).

Effective Strategies	**Problems**		
	1. Combine	**2. Compare**	**3. Join**
Counting on from the larger number			
• Mental	1	0	1
• Using fingers	9	0	12
Counting on from the smaller number			
• Mental	0	1	0
• Using fingers	1	0	0
Identifying a known number fact	3	5	1
Using knowledge of base 10 number system	1	0	1
Total	15	6	15

Problem 1: Combine Problem.

Classroom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes? Of the 18 children in this study, 15 children were categorized into Group 1 (Successful methods), two children were categorized into Group 2(Partially successful methods), and one child (PG 6) from the initial study were categorized into Group 3 (Unsuccessful methods).

Group 1: Successful Methods. A total of 15 children used a successful method for the combine problem. Ten children used counting on from the larger number, one child (PB1) counted by picturing counters in his mind and nine children kept track of the number of counts on their fingers. One child used counting on from the smaller number by using his fingers. Three children used a known number fact. One child got a correct answer for the problem but he could not explain how he solved the problem.

Group 2: Partially Successful Methods. Two children from the main study (G8 and G11) used an effective solution strategy that would result in a correct answer but they miscopied the number from the problem to the calculation. For example, to solve 24 + 28, one child copied 21 instead of 24 and 22 instead of 28.

Group 3: Unsuccessful Methods. One girl (PG6) from the initial study could not solve the compare problem. There was no explanation to why she used subtraction instead of addition. One might conjecture that the girl guessed the operation to get the answer.

Problem 2: Compare Problem.

Manee has 23 apples. Mana has 15 more apples than Manee does. How many apples does Mana have? Of the 18 children in this study, six children were categorized into Group 1 (Successful methods), one child was categorized into Group 2 (Partially successful methods), and 12 children were categorized into Group 3 (Unsuccessful methods).

Group 1: Successful Methods. Six children used a successful method for the compare problem. One child (B9) used counting on from the smaller number by calculating in his mind. Five children used a known number fact. For example, to solve 25 + 13, the child said that he knew that three and five was eight, and two and one was three, so the answer for this problem was 38.

Group 2: Partially Successful Methods. One child (G11) used an effective solution strategy for this problem but she miscopied the number from the problem to the calculation. For example, to solve 23 + 15, she copied 24 instead of 23 and then she got 39 for the answer.

Group 3: Unsuccessful Methods. Twelve children in this study could not solve the compare problem because they did not understand the context in the problem. These 12 children used subtraction instead of addition. The children were mislead by the keyword *more than* in the problem. The children assumed the keyword more than implied addition. These difficulties will be discussed more in the next chapter.

Problem 3: Join Problem.

Suda had 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now? Fifteen of the 18 children in this study were categorized into Group 1 (Successful methods). Two children were categorized into Group 2 (Partially successful methods), and one child (B9) was categorized into Group 3 (Unsuccessful methods).

Group 1: Successful Methods. Fifteen children used a successful method for the join problem. Thirteen children used counting on from the larger number. Of the 13 children who used counting on from the larger number, one child counted in his mind (PB1), while 12 children counted by keeping track of the counting sequences on their fingers. One child used a known number fact; and one child used a base-ten strategy.

Group 2: Partially Successful Methods. Two children implemented an effective solution strategy but they both got an incorrect answer. One girl (PG 6) from the initial study had a misconception about calculation. She did not know how to carry and regroup. For example, to solve 58 + 15, she got 63 instead of 73 for the answer. Girl 11 (G11) implemented an effective solution strategy but she got an incorrect answer. There was no clue why the incorrect answer was given. She did 58 + 15 = 74 instead of 73. However, one might conjecture that the girl used guessing to get a reasonable answer or perhaps she could have counted on incorrectly.

Group 3: Unsuccessful Methods. One boy (B9) could not solve this problem. He used subtraction instead of addition. The boy did not give an explanation to why he used subtraction. However, one might conjecture that the boy guessed at the operation to be used, or perhaps the boy chose the operation whose meaning fit the problem as he understood it.

Subtraction Strategies

Three basic levels of subtraction strategies analyzed from this study have been identified: strategies based on the use of a counting sequence, a known number fact, a base-ten strategy and other strategies. With the counting strategies, fingers were used to represent each of the subtrahend. Other strategies were also found such as using tallies and using knuckles. Since the numbers in each problem were two-digit numbers, children used borrowing and regrouping in the tens column (see Figure 9). The children did the calculation from right-hand side to the left-hand side. The solution strategy was observed when children did the calculation in the ones and the tens column. The subtraction strategies were summarized in Table 10.

Table 10. Subtraction strategies.

Strategies	Description
Counting up from a given number	Children count from the smaller given number in the problem with fingers until it reaches the large number given in the problem. Children count folded fingers to yield the answer.
Counting down to the smaller number	Children count backward from larger number given in the problem to the smaller number given in the problem. Children use fingers to keep track of the number. Children count folded fingers to yield the answer.
Identifying a known number fact	Children give an answer with the justification that they remember or recall the number facts.
Using knowledge of base 10 number system	These methods usually are based on the number 10. For example, to solve a problem representing 42 – 25, a child responds that 2 cannot subtract 5, borrow one ten from 4 so it is 12. 12 – 5 = ?, the child responds that 10 – 5 = 5 and 5 + 2 = 7 so 12 – 5 = 7.
Others	Children use strategies that can not be categorized in the above categories. These strategies are using a tally and using knuckles to model the problems.

In the counting up from the given number strategy, the child started a forward counting strategy beginning with the smaller given number in the problem. The sequences ended with the larger given number in the problem. By keeping track of the number of counting words in the sequence, the child determined the answer. For example, to solve 42 – 25, one child would say that two cannot subtract five, borrow one from the tens column so it become 12. The child then counted " 5 [pause], 6, 7, 8, 9, 10, 11, 12. Counting the numbers after the pause, then put seven at the ones column. One was borrowed from four so three is leftover. Three minus two is one. Put one at the tens column so the answer is 17." If the child used his fingers, he would fold each finger while he was counting. Then the child counted seven folded fingers for the answer (see Figure 11).

The counting down to the smaller number strategy is represented by counting backward from the larger number to the smaller number given in the problem. The sequences ended with the smaller number given in the problem. By keeping track of the number of counting words in the sequence, the child determined the answer. Similar to the example above, to solve 12 – 5, the child would count, "12 [pause], 11, 10, 9, 8, 7, 6, 5." The child got seven for the answer by counting seven folded fingers (see Figure 11).

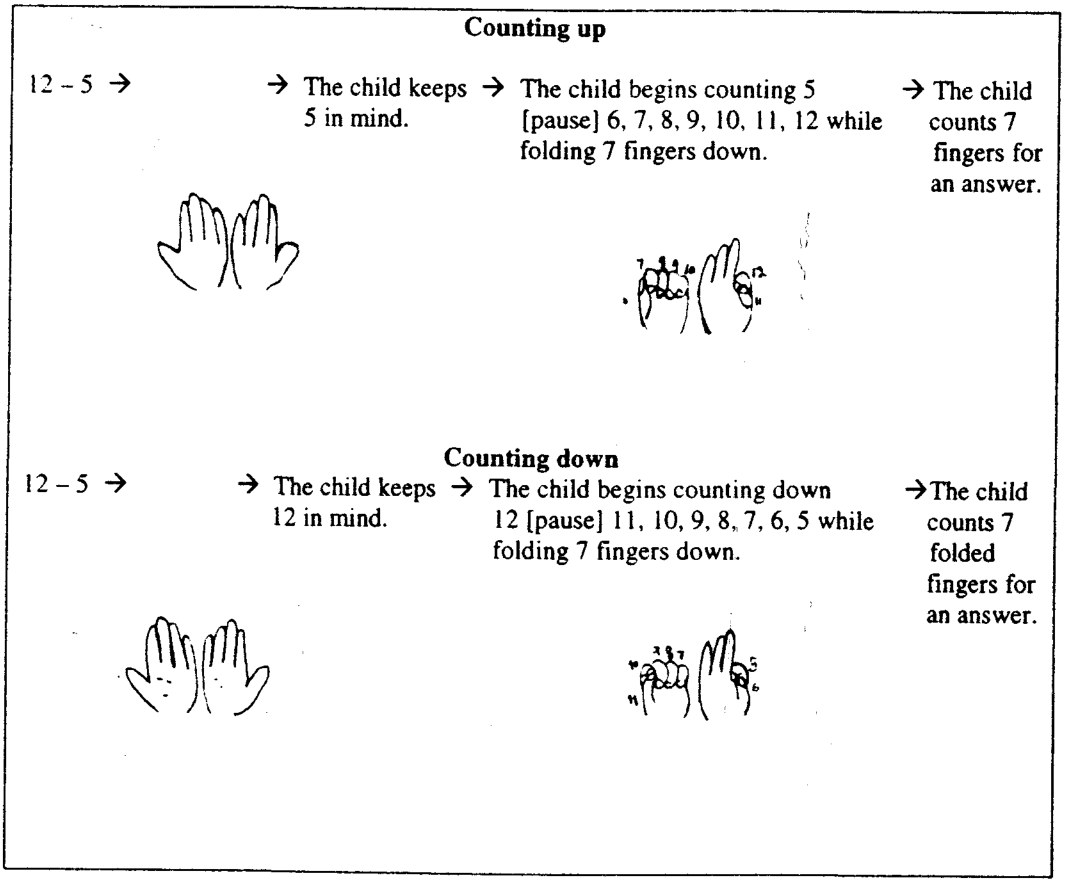


Figure 11. Subtraction by counting up and counting down using the folding fingers method.

Children's solutions strategies for subtraction word problems were not limited to counting strategies. Children were able to apply advanced strategies to the solution of subtraction word problems. The most predictable strategy was a known number fact. For example, to solve 20 – 9, one child would tell "Zero could not subtract nine, borrow one from two, so it is 10. I know that ten minus nine is one. Since one is borrowed from two so one is leftover. The answer is 11." In the base-ten strategy, the child solved problems based on the number 10. For example, to solve 12 – 5, one child responded that 10 – 5 = 5 and 5 + 2 = 7 so 12 – 5 = 7. Another strategy used was tallies. For example, to solve 15 + ? = 31 by using tallies, the child would tally and count from 15 until it reached 31. The child then counted the total tallies for the answer. One girl used knuckles to solve subtraction word problems (see Figure 13). Table 11 shows the number of children who correctly solved each problem and the number and kind of an effective strategy they used.

Problem 4: Combine Problem.

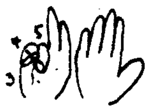
There are 42 chickens. 25 are male. How many chickens are female? Twelve of the 18 children in this study were categorized into Group 1 (Successful methods). Four children were categorized into Group 2 (Partially successful methods) and two children (B9 and G11) were categorized into Group 3 (Unsuccessful methods).

Group 1: Successful Methods. Twelve children used a successful method for solving the combine problem. Seven children used counting up from the smaller number. Six of the children who used counting up from the smaller number strategy counted by using their fingers to keep track of the counting sequences, and one of them counted in her mind by picturing counters or fingers. Two children used counting down to the smaller number by using fingers (B10 and G12). Another two children used a base-ten strategy. One child (B6) used his invented strategy by modeling with fingers. For example, to solve 42 – 25, the child first borrowed one from the number 4. To solve 12 - 5, the child respond by showing ten unfolded-fingers, then counted [pause] 3, 4, 5. While the child was counting 3, 4 and 5, he folded three fingers for 3, 4 and 5. There were seven unfolded fingers left. The child counted the unfolded fingers to yield the answer. Then, the child did the calculation in the tens column by remembering that three minus two is one. This strategy works because it is based on the number 10. Figure 13 shows how this strategy works.

Table 11. Number of children correctly solving each type of subtraction problems and the number and kind of effective strategies used (N = 18).

Effective Strategies	**Problems**			
	4. Combine	**5. Compare**	**6. Join**	**7. Separate**
Counting up from a given number				
• Mental	1	0	0	1
• Using fingers	6	2	2	7
Counting down to the smaller number				
• Mental	0	0	0	0
• Using fingers	2	0	0	2
Identifying a known number fact	0	8	0	0
Using knowledge of base 10 systems	2	0	0	2
Others				
• Using knuckles	0	1	0	0
• Using a tallies	0	0	1	0
• Using an invented fingers model.	1	0	0	1
Total	12	11	3	13

12 – 5 → The child count 2 [pause] 3, 4, 5 while folding three fingers down. → The child counts seven unfolded fingers for an answer.



42 - 25 → The child borrow one tens from four at the tens column. → 42 - 25 → By trying to make the number at the ones column to be the same, the child might add 3 more to the number 2 to make the number become 5. The child might realize that he used 3 from the number 10 to add to the number 2, so 7 is leftover. The number 7 is put in the answer space of the ones column.

Figurre 12. Another invented finger model and how it works.

Group 2: Partially Successful Methods. Four children used an effective solution strategy that would result in a correct answer but they had a misconception about calculation or made errors. One child (PG5) got an incorrect answer because she miscopied the number from the problem to the calculation. She copied 24 instead of 42. PG 3 implemented an effective solution strategy but there was no explanation for why she got an incorrect answer. Two children (PG 5 and B2) used an effective solution strategy but they had a misconception in calculation. They subtracted the smaller digit from the larger digit, for example, to solve 42 – 25, they got 23 where 17 was needed. They did not know how to borrow the number from the tens column. They just subtracted the smaller from the larger digits given in the problem, so two from five was three, and two from four was two. This strategy yields correct answers only when each digit in the larger number are larger than the digits in the smaller number.

Group 3: Unsuccessful Methods. Two children from the main study (B9 and G11) could not solve this problem. They used addition instead of subtraction. These children give no explanation to why they used subtraction. One might assume that these two children guessed which operation to be used to get the reasonable answer.

Problem 5: Compare Problem.

Suda's pencil is 20 centimeter long. Manee's pencil is 9 centimeter long. How much longer is Suda's pencil than Manee's pencil? Eleven of the 18 children in this study were categorized into Group 1 (Successful methods). One child was categorized into Group 2 (Partially successful methods). Six children could not solve this problem and was categorized into Group 3 (Unsuccessful methods).

Group 1: Successful Methods. Eight of the 11 children who used a successful method used a known fact to derive an answer to this problem. For example, to solve 20 – 9, the children said that they remembered the number fact that 20 – 9 = 11. Two children used counting up from a given number by using fingers to keep track of the counting sequences. One child (PG4) solved this problem by using knuckles (see Figure 13). The girl said that her teacher taught this strategy in class.

Group 2: Partially Successful Methods. One child (Girl 11) got an incorrect answer because she did not understand the concept of place value. She identified a correct operation but she did not know how to borrow and regroup at the tens column. She subtracted the smaller number from the larger number instead. For instance, to solve 20 – 9, she subtracted zero from nine and got 29 for the answer.

Each finger has three knuckles. **Each finger has value of 4, include fingertip. Therefore, each hand has value of 20. By using two hands, a child can subtract the numbers from 1 to** 40.

20 -9 → The child would use one hand → The child knows that two fingers contain 8, so the child would fold two fingers down and keep in her mind that one fingertip of the next finger was added to 8 so it was 9. Then, the child counted unused-knuckles and fingertips for the answer. The answer is 11.

One fingertip

Two folded fingers and one fingertip represent the number 9.

Figure 13. Counting strategy by using knuckles and fingertips for subtraction.

Group 3: Unsuccessful Methods. Six children could not solve this problem. They used addition instead of subtraction because they misunderstood the meaning of the keyword *much longer* in the problem. They thought that *much longer* required addition.

Problem 6: Join Problem.

Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 balloons altogether? Only three of the 18 children in this study were categorized into Group 1 (Successful methods). No child in this study was categorized into Group 2 (Partially successful methods). Fourteen children were categorized into Group 3 (Unsuccessful methods). One child (B2) did not attempt to solve this problem.

Group 1: Successful Methods. Three children (PB1, G4 and B5) used a successful method for this problem. One child from the initial study (PB1) used tallies to derive the solution to the problem. To solve 15 + ? = 31, the boy would keep 15 in mind, then the boy tallied from 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31 (////////////////). The boy then counted the tally to yield the answer, the answer was 16. Two children from the main study (G4 and B5) used counting up from a given number to derive the solution to the problem.

Example 1: Girl 4

"[At the ones column.] One cannot subtract five, borrow one tens from three. One becomes 11. Then, counted up from five to 11. 5 [pause, then fold each fingers while counting], 6, 7, 8, 9, 10, 11 [Six fingers are folded, the child counts six folded fingers for the answer.], get 6. [At the tens column.] Three is borrowed one, so two is leftover. Two minus one is one. The answer for this problem is 16."

Example 2: Boy 5

"15 has no addend, 15 [Pause, then fold each fingers while counting.]16, 17, 18, 19, 20, 21,..., 31. [Count fingers] The answer is 16. Because 15 had no addend so it has to find an addend."

Group 3: Unsuccessful Methods. Fifteen out of 18 Thai children in this study could not solve this problem because they misunderstood the meaning of the keyword *altogether* in the problem. The keyword *altogether* meant addition for most of these children. The children did not attempt to decipher the meaning of the entire context of the problem. The discussion of this difficulty will follow in the next chapter.

Problem 7: Separate Problem.

The farmer has 38 cows. He sells 19 cows. How many cows are left? Thirteen of the 18 children in this study were categorized into Group 1 (Successful methods). Five children were categorized into Group 2 (Partially successful methods). No child was categorized into Group 3 (Unsuccessful methods).

Group 1: Successful Methods. Thirteen children used a successful method for the separate problem. Eight children used counting up from the smaller number. Seven of the eight children who used counting up from the smaller number counted by using fingers to keep track of the counting sequences, and one child counted in his mind (PB1). Two children used counting down to the smaller number strategy by using fingers to keep track of the counting sequences. Another two children used a base-ten strategy. For example, to solve 38 – 19, at the one columns, one child responded that "Eight cannot subtract nine so borrow one tens from three. Eight becomes 18 and two is leftover from three." To solve 18 – 9, the child said that "ten minus nine is one and eight minus one is nine. In the tens column, two minus one is one. The answer is 19." One child (B6) used his own strategy by modeling with his fingers. For example, to solve 38 - 19, the child first borrowed one from the number 3, so eight became 18. To solve 18 - 9, the child respond by showing ten unfolded-fingers, then counts 8 [pause] 9. While the child was counting from nine, he folded one finger for nine. There were nine unfolded fingers left. The child counted the unfolded fingers to yield the answer (see Figure 13). Then, the child did the calculation at the tens column by remembering that two minus one is one.

Group 2: Partially Successful Methods. Five children used an effective strategy for this problem but they got an incorrect answer. Three children (PG5, PG6, and B10) made errors in calculation because they did not know how to borrow the number from the tens column. They subtracted the smaller number from the larger number. For example, to solve 38 – 19, the children did 38 – 19 = 21 instead of 19. Girl 11 (G11) used a correct operation but she could not calculate. She did 38 – 19 = 2_, going from right to left, and she could not continue. She stopped at 8 –9. It might be because she did not know how to subtract nine from eight. Girl 8 (G8) implemented a correct operation but there was no clue why she got an incorrect answer. She did 38 – 19 = 16 instead of 38 – 19 = 19. One might conjecture that the girl used guessing to give the most reasonable answer or miscounted while using her fingers.

Summary

Most Thai children in this study used counting strategies with fingers to solve both addition and subtraction word problems. Counting on strategies were most often used for addition word problems and counting up strategies were most often used for subtraction problems. Counting all strategies were not used by Thai children in this study. Other strategies that were not based on counting strategies were also found. Those strategies were using tallies, using a known number fact, using an invented fingers model, and using a base-ten strategy.

Moreover, the findings showed that Thai children in this study used mostly fingers to represent counting sequences while children in the United States used other physical objects such as cubes and counters.

Thai's Classroom Environment

This section describes two second-grade classroom observations in school A. The observations were focused on exploring the interaction between teachers and children, the classroom environment, and mathematics problem solving. The two observations took place during mathematics periods. The observations were done after the student interview had been conducted. Each classroom report is divided into two parts. The first part consists of a description of the classroom environment. The second part consists of a description of classroom activities.

An Observation of Ms. A' Math Class

Classroom Environments.

Figure 14 shows a drawing of Ms. A's classroom. There were five bulletin boards in Ms. A's classroom. The mathematics board and Thai language board were located in the front of the classroom. The three boards at the back of the classroom were about general science, Thai culture, and home economics. The mathematics bulletin board showed the time tables from 2 to 5. There were small plants around the classroom. Forty children were in Ms. A's class. The children were sitting in five groups around tables. Each group consisted of 8 children. The teacher's desk was at the back of the classroom. The classroom was very small, so there was little space for the teacher to walk to individual children.

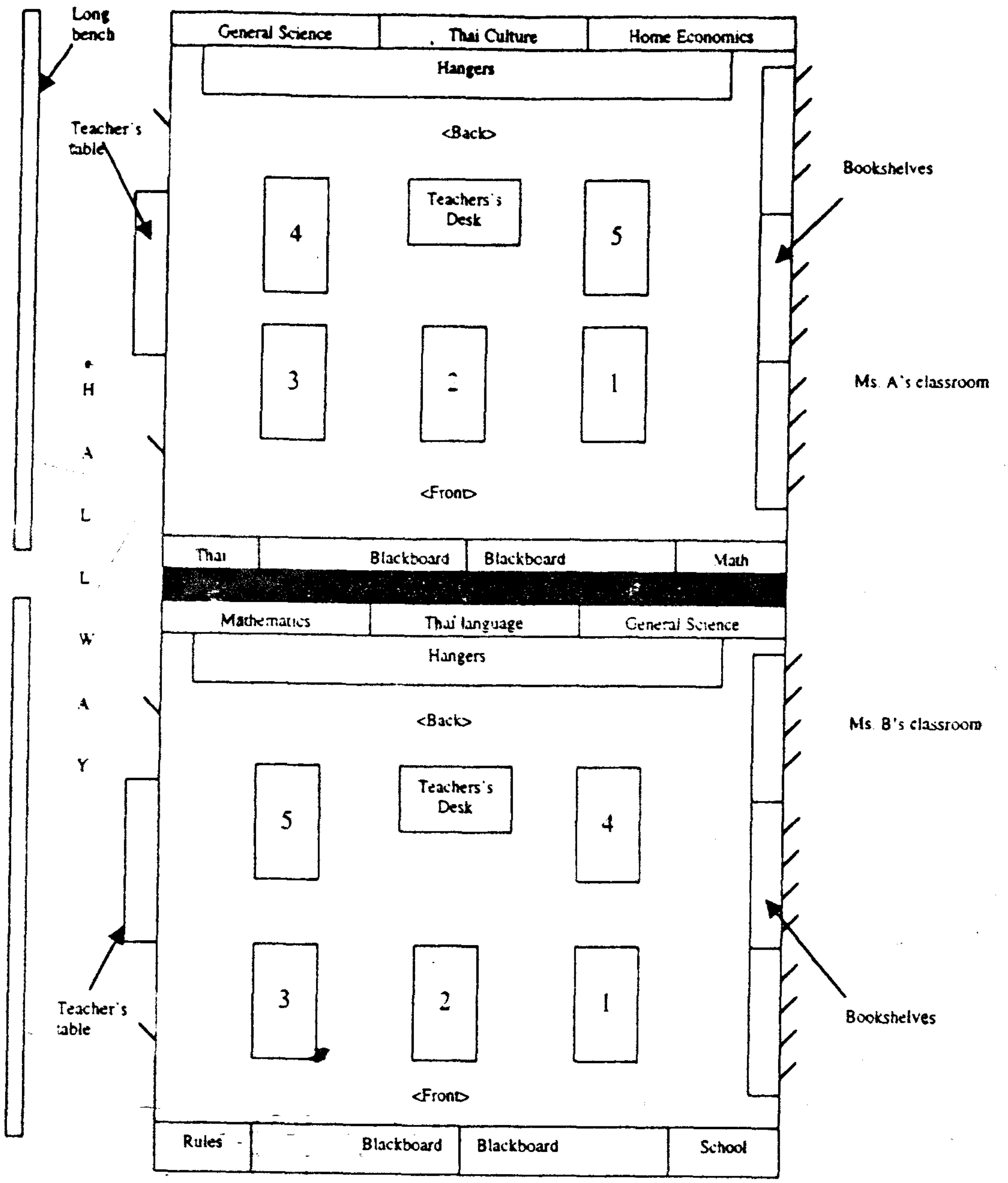


Figure 14. Sketch of Ms. A and Ms. B's classroom.

On the observation day, the lesson was subtraction of three-digit numbers in column form without borrowing and regrouping.

Classroom Activities.

The way in which learning activities were carried out in Ms. A's classroom is illustrated by an episode which focuses on subtraction of three-digit numbers without carrying and regrouping in column form. Ms. A worked along with the children by showing a number card (see Figure 15) and asked the children to say the number on the card. Then she reviewed subtraction of one and two-digit numbers. Next, Ms. A showed the children how to subtract three digit numbers by using hundred-squares, ten-squares, and one-squares (see Figure 16) to represent each number.

T: [Write 231 on the blackboard.] How many hundred-squares will we use?

Ss: Two (see Figure 17).

T: [Pick two hundred-squares and put on the board (see Figure 18).] Yes, two. OK., how many ten-squares will we use?

Ss: Thirty.

T: Well, one ten-squares represents ten, so how many of ten-squares will we use?

5 Ss: Three (see Figure 17).

T: [Pick and count three ten-squares and put on the board (see Figure 18).]

T: OK.,and how many one-squares will we use?

Ss: One (see Figure 17).

T: Good [Pick one one-squares and put on the board (see Figure 18).], how much do we have to subtract from 231?

10 Ss: 120.

T: Take away 120. How many hundred-squares do we have to remove from 231 squares?

Ss: One (see Figure 17).

T: Good [Remove one hundred-squares from the board (see Figure 18).], and how many ten-squares we have to remove from 231 squares?

Ss: Two (see Figure 17).

15 T: [Remove one ten-squares from the board (see Figure 18).] What is the number at the ones column?

Ss: Zero.

T: Do we have to remove one-squares from 231 squares?

Ss: No.

T: How do we say 231?

20 Ss: Song-Roy-Sam-Sib-Ed (Two hundred-thirty-one)

T: Well, we subtract 120 from 231, how much is left?

Ss: [Look at the squares.] One hundred-eleven (see Figure 17).

T: Good job, applaud yourself

Ss: [Applaud]

Ms. A's role was that of a leader, and the children's role was that of followers. In attempt to cooperate with the teacher, children respond as the teacher asked them, and replied to known-answer questions. The teacher asked the children questions but never asked for the reason.

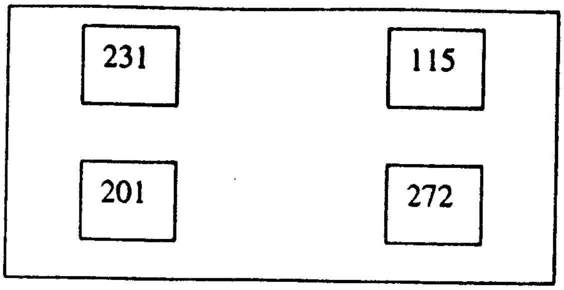


Figure 15. Number cards.

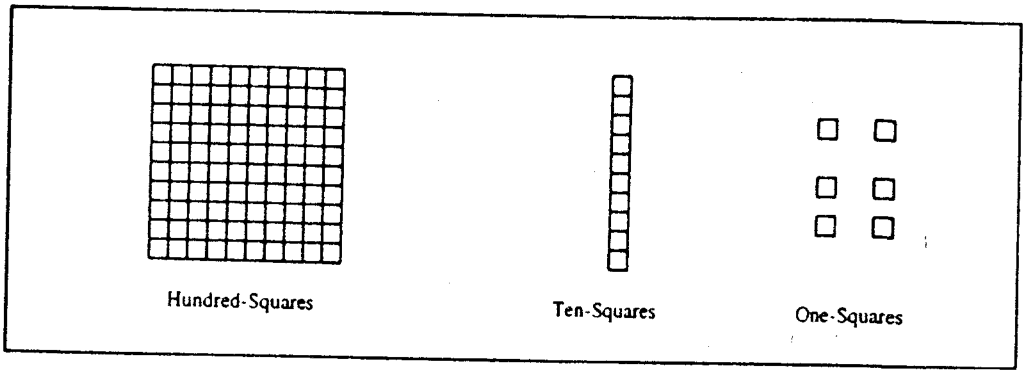


Figure 16. Hundred-squares, ten-squares, and one-squares.

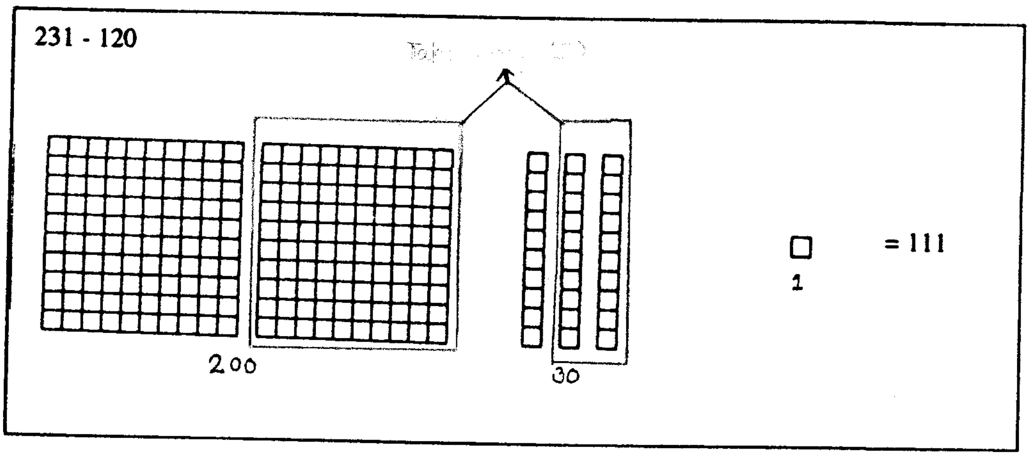


Figure 17. Subtraction by using hundred-squares, ten-squares, and one-squares.

Figure 18. Behind the teachers is the board that the teachers put hundred-squares, ten-squares, and one-squares to represent numbers.

It seemed that the teacher wanted only a correct answer to the questions she posed. For example, the teacher likely intended to ask the question in line 13 only for the answer, not for reasoning. When the teacher got the correct answer, she went on the next question. This could be confusing for children who did not know why the teacher took away one hundred-squares and thus these children may not follow the lesson. Unfortunately, the observer did not observe this situation with individual students. On the other hand, the teacher did use a base-ten model to develop children's understanding of subtraction concept quite clearly. From the episode above, the teacher always praised the children when they got a correct answer. From the observation, when children got praise from the teacher, the children looked happy and learned actively.

In the following episode, Ms. A asked for a volunteer to do the subtraction by using the squares in Figure 16 in front of the class. After the volunteer finished doing the work, Ms. A asked the whole class to check the work that the volunteer did. It can be noted again that children were not expected to explain their thinking and reasoning. The teacher only asked the children about how they did the problem. The teacher did not ask for reasons. For example, line 13 and 14, when the children gave the answer, Ms. A did not ask the children why they did not remove one-squares from 372 squares.

T: Every one looks at the board. Does your friend complete it correctly?

Ss: [Look on the board and make noise.]

T: The problem is 372 minus 220. How many hundred-squares does your friend use?

Ss: Three.

5 T: How many ten-squares does your friend use?

Ss: Seven

T: How many one-squares does your friend use?

Ss: Two.

T: We have to subtract 220 from 372, how many hundred-squares does your friend remove from 372 squares?

Ss: Two.

T: How many ten-squares does your friend remove from 372 squares?

Ss: Two.

T: Good, and how many one-squares does your friend remove from 372 squares?

Ss: None.

T: What is the answer?

Ss: 152.

T: Is it correct?

Ss: Yes [Loud noise.]

T: Applaud your friend.

Ss: [Applaud.]

After that, Ms. A showed the class 18-bead computing abacus (see Figure 19). Ms. A then asked children to locate value of each three places of the abacus. Ms. A showed the number 433 on a card and asked the children to read this number. Ms. A then demonstrated how to used the abacus to represent each number and demonstrated how to subtract by using the abacus. She made the abacus to represent 433 by using four red beads, three yellow beads, and three blue beads.

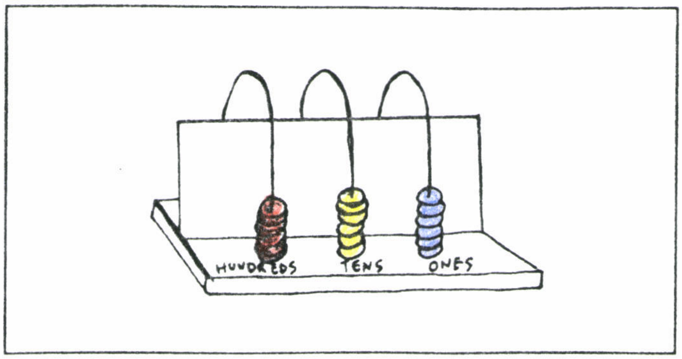


Figure 19. The 18-bead computing abacus.

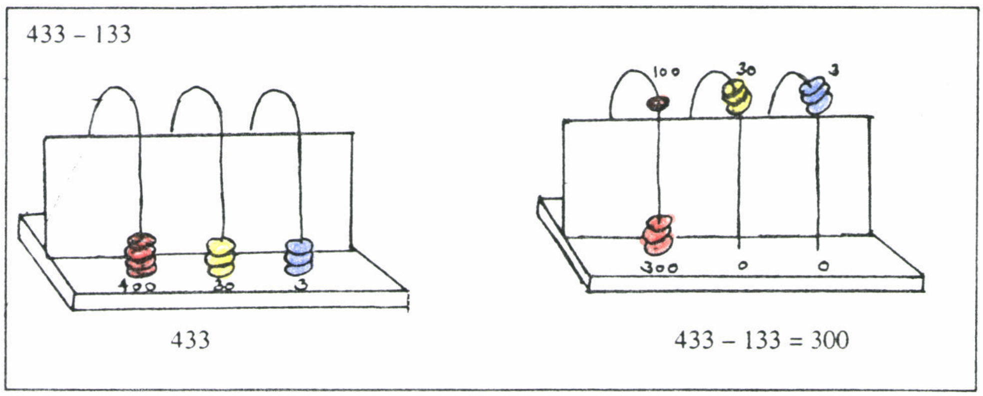


Figure 20. Subtraction by using 18 bead – computing abacus.

T: [Hold the abacus on her hand.] This is 433 on the abacus. We will subtract 433 by 133. [Remove one red bead, three yellow beads and three blue beads from 433.]

T: How many beads were left in the hundreds place?

Ss: Three (see Figure 20).

T: How many beads were left in the tens place?

Ss: None (see Figure 20).

T: How many beads left in the ones place?

Ss: None.

T: What is the answer, then?

Ss: Three hundred. [Sam Roy.]

T: Great! Applaud yourself.

Ss: [Applaud.]

Next, Ms. A let the children work in groups. During group activities, children had a chance to think without assistance from the teacher. In the activity, the task was to construct two three-digit numbers and subtract using these two numbers that had been constructed. The teacher cooperated with children by answering children's questions and giving explanations when children had problems. The children were very noisy and busy doing the task. Although the children had a chance to practice thinking skills, the teacher still did not ask for reasons or explanations from the children. The teacher only asked the children to explain how they solved a problem and she judged the response as correct or incorrect. Consider the following episode.

Group 1 (G1): 311 minus 110

[The child put three hundred-squares, two ten-squares and one ones-squares to represent the number 311 on the board. Then, he removed one hundred-squares, one ten-squares, and one ones-squares from 311 squares. He then, looked at the table and gave the answer.]

G1: The answer is 201.

T: Is Group 1 right, class?

Ss: Yes.

T: Applaud Group 1.

Ss: [Applaud.]

In summary, the instruction did not explicitly emphasize problem solving and reasoning skills. However, the teacher did provide opportunity for children to think by working in groups and to do an explanation of the task they were assigned. The teacher usually encouraged children by praising them when they did good work. The next lesson would be subtraction of three-digit numbers without borrowing and regrouping by using distribution and place value tables (see Figure 21). Subtraction of three-digit numbers with borrowing and regrouping would be introduced after that.

<u>An Observation of Ms. B's Math Class</u>

Classroom Environments.

There were 5 bulletin boards in Ms' B's classroom (see Figure 14). Two boards in the front of the classroom were about school and the rules of the classroom. Three boards in the back were general science, Thai language, and mathematics. The mathematics board was about comparisons between numbers (less than, more than, and not equal). Ms. B's classroom contained 41 students. On the observation day, the students were sitting in groups. There were five groups, 8 children in each group—one child was absent at the time of the observation. Like Ms. A' classroom, the teacher's table was in the back of the classroom. There were many plants around the classroom. The math period began at 8:30 and ended at 9:30. The lesson on the observation day was solving three-digit number addition word problems without carrying and regrouping.

Classroom Activities.

Ms. B began the lesson by telling children to count from 100 to 200. She then showed number cards (see Figure 15) to children and asked the children to say the number on the card out loud. After that, Ms. B asked for a volunteer from each group to come to the front of the class. She then read the number and told the volunteered children to write numbers on the blackboard. The purpose of this activity was to practice saying and writing three-digit numbers, and practice giving the value of the number in each place. Next, Ms. B asked for a volunteer to write an addition number sentence with carrying and regrouping on the blackboard. The following is an excerpt from the beginning of the lesson.

T: [Show the number 115.] How do we say this number?

Ss: One hundred-fifteen [Nuang Roy-Sib-Ha].

T: What is the number at the ones place?

Ss: Five.

T: What is the value at the hundred place?

Ss: One-hundred.

T: What is the value of a 1 at the tens column?

Ss: Ten

T: [Show the number 201.] How do we say this number?

Ss: Two hundred – one [Song Roy – Ed].

T: I underlined the number 1 to remind you that when one is at the ones place, we will say 1 as what?

Ss: Ed

T: How do we spell Ed (Thai language)?

Ss: [Spell the word 'Ed' in Thai language.]

T: What are the numbers that end with Ed?

Ss: 201, 301, 231, 321.

Children practiced saying and writing three-digit numbers because they often misspelled or misread numbers. Children in this class completed an activity that used thinking skills. The teachers asked for a volunteer to set up an addition number sentence, with carrying and regrouping.

Then, Ms. B let children completed the activity in groups. The task was to set up a word problem that contained the number assigned by the volunteer. Each group had to report their work in the front of the class after they finished. The type of the word problems that children set up were mostly combine problems.

The following episode exemplifies the word problems that the children from each group proposed.

Student from G1: "Hi my name is Akachai. I will read the problem. I have 771 Baht. My father gives me 229 Baht more. How much money do I have altogether?"

Student from G3: "I have 771 Baht. My mother gives me 229 Baht more. How much money do I have?"

Student from G5: "I have 771 Baht. My mother gives me 229 Baht more. How much money that I have altogether?"

Student from G4: "My mother gives me 771 flowers. Dad gives 229 flowers. How many flowers do I have altogether?"

Student from G2: "Dad buys a fan for 771 Baht. He buys a shirt for 229 Baht. How much money does he spend?"

Three out of five problems contained the keyword *altogether*. This supports the results from the interview section of this study that Thai children usually remembered keywords in word problems instead of trying to understand the context in problems. However, children seemed to have a connection between mathematics and every day life situations but used only one type of problem in this example, combine problem.

Ms. B then selected one out of five problems to teach the children how to solve three-digit number addition word problems. The teacher used direct instruction by asking children questions. The children responded by replying to those questions. The teacher taught the children how to solve word problems step-by-step by asking children to answer her questions. She did not call on individuals to answer her questions. Consider the following episode:

T: [Read a problem.] 'I have 771 Baht. My father gives me 229 Baht more. How much money do I have altogether?'

T: What does the problem tell us?

Ss: 'I have 771 Baht.'

T: Correct, what's next?

Ss: 'Dad gives 229 Baht more.'

T: OK., what else?

Ss: Nothing.

T: Good, what did the problem ask us to find.

Ss: 'How much money do I have altogether?'

T: The problem wanted to know how much money there is altogether, right?

Ss: Yes.

T: From this problem, how do we find the answer?

Ss: Use addition.

T: OK, we will do this problem together on the blackboard.

T: What is the first sentence?

Ss: I have 771 Baht. [Ms. B wrote this sentence on the blackboard.]

T: Second?

Ss: My father gives 229 Baht more [Ms. B wrote this sentence on the blackboard.]

T: Third?

T: Money that I have altogether. [Ms. B wrote this sentence on the blackboard.]

T: When you add, which column do you add first?

Ss: The ones column.

T: How much in the ones column?

Ss: Ten.

T: Ten, put 0, and how much do we have to carry to the tens column?

Ss: One.

T: [At the tens column.] Seven plus two? Keep 7 in mind and count?

Ss: 8, 9.

T: Plus one more so it is?

Ss: Ten.

T: Put 0 here, carry?

Ss: One.

T: What is seven plus two?

Ss: Nine.

T: Plus one more so it is?

Ss: Ten.

T: So, how much money?

Ss: 1,000

The teacher taught solving word problems by using step-by-step procedures rather than emphasis on problem solving skills. As in Ms. A' class, the teacher did not use questions that required higher level thinking and reasoning skills. The teacher asked only the answer for the questions posted. The teacher ended the lesson that day by having the children completed exercises in textbooks. The next lesson would be subtraction of three-digit numbers, without borrowing and regrouping.

Summary

The results from observation suggested that teachers in this study did not emphasized student's reasoning and thinking skills. As a result, children developed memorizing skills rather than thinking and reasoning skills. A start of a sequence of instruction is common, although procedural step-by-step routines were quickly developed. Word problems chosen for working in class were largely routine, procedural problems, rather than problems that could be solved in a number of ways that are not immediately know to the child. However, teachers in this study did use manipulatives in teaching such as 18-bead computing abacus, and number cards.

Thai Teachers' Understanding of Problem Solving

The participants in this section were six Thai second-grade teachers from school A. All of them were female. They all had Bachelor's degrees in Education. Five of them have been teaching more than 20 years. One of them had been teaching for only four years. This section reports about Thai teachers' mathematics instruction and their understanding of problem solving.

Mathematics Instruction in Class

From the observations and the questionnaire, teachers used different ways to teach mathematics. Teachers might not use the same method to teach the same topic but they used direct instruction techniques. Three ways were classified which were either observed by the researcher or described by the teachers.

First, teachers taught by using real things, which could be seen in everyday life. For example, to learn measurement, the teachers might give notebooks, pencils, erasers, and rulers to children to estimate and measure lengths. To learn about money, teacher might use bank notes, and coins to show the children different types of bank notes and coins. If it is difficult to find things from the real world, teachers might use pictures of real things.

Second, when teachers taught about solving word problems, they might use games or role-playing. The reason for using games is that children have an opportunity to use their own thinking and practice cooperation. Role-playing can help children see a problem more concrete. For example, teachers let children act as a seller and a buyer to learn addition and subtraction word problems. Some teachers taught by telling a story or using pictures and then insert addition and subtraction. Another examples was from the observation which Ms. B taught solving word problems by having children make a story problem by using the given number and operations.

Third, teacher manuals are sold in bookstores in Thailand. Manuals guide teachers about how to teach each topic, which teaching aids should be used, and how to evaluate students' learning. The manual provides ready-to-use lesson plans for teachers. Therefore, teachers often adapt from the manuals or follow the lesson in the manual for teaching mathematics lessons.

Addition and Subtraction Instruction

Teachers in this study relied mostly on the method in the teacher's manual to teach addition and subtraction with concrete aids. The concrete aids are counters, squares (base-ten and base-hundred cards, see Figure 16), 18-bead computing abacus (see Figure 19), bundles of ten, soda corks, ice cream sticks and drinking straws, problem cards, and exercises in textbooks. Three ways that teachers used to teach addition and subtraction were:

(a) add and subtract by using distribution (see Figure 21)

(b) add and subtract by using place value tables (see Figure 21),

(c) add by using carrying and regrouping, and subtract by using borrowing and regrouping.

Problems and Problem-Solving

The six teachers explained several meanings for a problem and problem solving.

Teacher 1: "A problem is a thing that needs a solution. Problem solving is an ability to analyze a problem."

Teacher 2: "A problem is a question or a message that wants an answer or needs a correct solution. Problem solving is a process for finding an answer to a problem."

Teacher 3: "A problem is a thing that needs a correction. Problem solving is an ability to analyze a problem."

Teacher 4: "A problem is a thing that needs a correct answer from analyzing the problem. Problem solving is an analysis of a problem: what was given in the problem, and what the problem asked you to find. Problem solving is an analysis to see whether to add or subtract."

Teacher 5: "A problem is a question that needs an answer. Problem solving is a process of finding an answer to a problem or a technique for finding an answer to a problem."

Teacher 6: "A problem is a question. Problem solving is the process for finding an answer to a problem."

In summary, teachers in this study defined a problem as a question that needs a solution or an answer or a correct solution. The teachers understood a problem as a meaning of word problems in mathematics rather than a situation in which a person wants a mathematical resolution to a problematic situation and does not know how to get it (Reys et.al, 1989). Teachers defined problem solving as an ability to analyze a problem, or a process or techniques for finding an answer to the problem. Teachers understood problem solving as a step-by-step process of solving routine word problems, where a procedure was determined and readily carried out (see Figure 22).

Addition

232 + 421 =

(a) Distribution

200 + 30 + 2
\+
400 + 20 + 1
600 + 50 + 3 = 653

(b) Place Value Tables

Hundreds	Tens	Ones
2	3	2
4	2	1
6	5	3

= 653

Subtraction

241 - 120 =

(a) Distribution

200 + 40 + 1
\-
100 + 20 + 0
100 + 20 + 1 = 121

(b) Place Value Tables

Hundreds	Tens	Ones
2	4	1
1	2	0
1	2	1

= 121

Figure 21. Addition and subtraction by using distribution and place value tables.

Problem 7: The farmer has 38 cows. He sells 19 cows. How many cows are left?

Step 1: Read and Understand a problem to find what is in the problem and what the problem asks you to find.

The problem has 3 sections: (1) The farmer has 38 cows. (2) He sells 19 cows (3) how many cows are left?

Step 2: (Write in a notebook) Write a mathematical sentence

Mathematical sentence 38 − 19 = □

Step 3: Write the process

Solutions (Solution presented here is in the Thai's grammar)

(From section 1) The farmer has cow 38 Toua (Thai's pronoun of animals)

(From section 2) He sells the cows 19 Toua

(From section 3) The cows are left Toua

Step 4: Perform calculation by using the process in Step3

Step 5: Writing the answer

Answer The cows are left 19 Toua.

Figure 22. Step-by-step process of solving routing word problems.

Problem Solving in Classroom

Teachers talked about solving word problems in classroom more than talking about problem solving skills. However, the teachers explained how they taught solving routine word problems as follows. Teachers gave easy word problems that usually are seen by children in everyday life to children and let children think in their mind and provide the answer. For example, Suda has \$10. She got \$5 more from her mother. How much money does Suda have altogether? Teachers then provided more difficult word problems and taught children how to analyze the problem step-by-step. That is, reading the problems and find out what the problem gave us and what the problem asked for. Then, determine a procedure for finding the solution and write a number sentence for the problem. After that, the teachers will teach children how to write the solution to the problem step-by-step.

If the children cannot analyze the problem, the teachers will use role-playing. Teachers often used sets of word problems when they teach solving word problems but teachers do not analyze problem by types (as indicated in Table 1 in Chapter II). Moreover, teachers did not use different type of word problem in classroom. In fact, from analyzing the teachers' manual, there are games available that can mediate children's problem solving skills. However, teachers viewed games were inappropriate classroom activities and they did not often use games in teaching.

How Teachers Connect Classroom Mathematics to the Real World

Teachers had three different ways to help children see the connection between classroom mathematics and mathematics in the real world.

First, teachers taught students to set up a problem by themselves. The problem that might be seen in the real world by using numbers that teacher assigned.

Secondly, teachers told students how important mathematics is and how mathematics is used in everyday life. For instance, using money, buying things, and looking at a clock. Teachers rarely let children do activities such as working in groups to solve nonroutine problems, or playing mathematics games to improve children's thinking skills.

Characteristics of Successful Children in Problem Solving

According to the six teachers, children who were successful in solving routine word problems, in general, had good achievement in reading, writing and calculation. The children should understand and analyze problems correctly. Teachers from this study emphasized that these skills—reading, writing, calculation, understanding, and analyzing—were very important. All children should have these skills to become successful in solving routine word problems. Most teachers agreed that children should do many exercises, both in textbooks and from every day life. One of the teachers said that successful children must love to learn mathematics and must pay attention when they were learning. The minority of the teachers emphasized an ability to write number sentences.

Characteristics of Unsuccessful Children in Problem Solving

All teachers in this study agreed that unsuccessful children had low ability in reading, writing, understanding, and calculation. As a result of an inability to read, children could not analyze problems and therefore could not solve problems. Examples that support these data were from the children's interview session where some of the low achievers who could not read or were slow readers were not successful in solving word problems. In contrast, some children were successful when given additional time or reading help. In addition, one teacher suggested that unsuccessful children do not have much experience in solving problems in everyday life. The children like to play more than to learn and therefore do not always pay attention to learning. However, the teachers gave no data to support this idea.

Why Some Children are not Successful in Problem Solving

There were different reasons for why children were not successful in solving routine word problems according to their teachers. The majority of teachers in this study indicated that children were not successful in solving routine word problems because the children were not capable of analyzing problems and did not have experience in solving word problems. Two teachers suggested that children were unsuccessful because they did not like learning and they had low IQs. Only one teacher pointed that ineffective teaching could make children unable to understand how to solve routine word problems and thus, children were not always successful in solving routine word problems.

Which is Important, An Answer or A Process for Finding The Answer

Two teachers in this study did not answer this question. Four teachers who respond to this question suggested that the process for finding the answer is important because teachers will know how children think and how they process their work before they get the answer. The teachers indicated that thinking skills and solving word problems are related. Teachers in this study said they did not rely on the correct answer but they emphasized individual thinking skills that is more important than the answer. "Children might have different ways to think to derive the same answer to the same question," one teacher said. From the observation, however, teachers did not much use children's thinking and reasoning in their classroom instruction as they indicated in the questionnaire. Krathwohl (1998) suggested that nonverbal behavior is often a give away that contradicts verbal behavior.

Skills that Children Should have in Solving Routine Word Problems

Two teachers did not answer this question. Four teachers who responded to this question agreed that children should have an ability to read and comprehend. If the children could read, they could analyze a problem into a particular type and then understand the meaning of the problem. Children should also have an ability to do calculation. Calculation is a basic skill that children should have.

Summary

Thai teachers in this study viewed problems as routine word problems and viewed problem solving as solving routine word problems. Teachers relied mostly on textbooks and the teacher's manual for preparation of lesson plans. According to these teachers, children who will successful in solving problems should have ability in reading, writing, understanding, and calculation skills.

Conclusion

In conclusion, I would like to emphasize six points. The six points are: (a) problem solving skills of Thai children; (b) success of Thai children in solving addition and subtraction word problems; (c) Thai children's strategies for addition and subtraction word problems; (d) danger of focusing on keywords in word problems; (e) difficulties of Thai children with subtraction; (f) and Thai's classroom environment and Thai's teacher understanding of problem solving.

First, problem-solving skills of young Thai children in this study were mostly step-by-step algorithms to solve the problems (see Figure 22). The children had been taught to solve word problems by using step-by-step algorithms. From the observation during the interview, the children usually stopped doing problems if they could not go through each step. Step-by-step procedural skills did not help some children solve some problems because they used memorized routines instead of thinking and reasoning skills.

Second, the results indicated that Thai children in this study were successful for the most part in solving addition and subtraction word problems. The instances of failure were relatively rare. The children did better in addition problems than subtraction problems. However, not all children are good at solving all types word problems. Nevertheless, when given the opportunity, some children demonstrated good problem solving thinking.

Third, counting strategies were a good model used by most Thai children in this study to solve addition and subtraction word problems. Counting on from the larger number was used in solving addition word problems and counting up from the given numbers was used in solving subtraction word problems. The sequences of counting were often represented by fingers. None of the children in this study used other physical objects such as counters. Thai children also used other strategies, two based on counting and two was not based on counting. The two strategies based on counting was the use of tallies and counting. The children also used invented strategies, using an invented fingers model and base-ten strategy.

Fourth, keywords in problems seemed to effect children's representation of problems. Keywords were sometimes helpful for children to do problems more quickly, but often dependence upon keywords are an obstacle for children's success in solving word problems. Although keywords such as *more* and *altogether* are typically associated with addition, and keywords such as *less* and *leftover* are typically associated with subtraction, the meaning of these words is context dependent. These words can mislead children into representing a problem incorrectly if they do not consider the entire context of the problem.

Fifth, a few of the children in this study had misconceptions about regrouping and borrowing in subtraction word problems. Instead, these children subtracted the smaller digit from the larger digit. For example,

$$\begin{array}{r} 42 \\ -\ \underline{25} \\ \underline{23} \end{array}$$

From this example, the ones is subtracted and recorded, then the tens is subtracted and recorded. The child obviously considered each position as a separate subtraction problem. The child likely did not think of the number 42 and 25, but only of two and five, and four and two. The child might think of the larger of the two numbers as the number of the set, and the smaller as the number to be removed from the set. Another child could not complete a subtraction problem that she solved from left to right. For example, to solve 38 – 19, one girl did 3 – 1 = 2 at the ten columns first and she could not continue to do 8 – 9. She then stopped doing the calculation and gave the reason that she could not do it.

Finally, Thai classrooms usually contain approximately 40 children, although some schools have 55 to 60 children in one class. Children stay with their primary (homeroom) teacher during most of each school day. Classes involved a mixture of subjects, and completed various activities for groups of students to work on together. There are seven periods per day in Thai classes. Most classrooms in Thailand were laid out with combinations of group tables, paired or joined desks, and bulletin boards. School days included some time at desks, sometime in the playground, and sometime in the lab room (e.g. English Sound Lab and Computer Lab). Thai classes involved questions mostly from the teachers. Thai teachers showed an authoritative and direct instructional style in interactions with students.

According to Thai Buddhist precepts, children are reared to be nonaggressive, obedient, and respectful of others, particularly authority such as teachers and others who are older than they are. The children showed respect to the teachers at the beginning and the end of the period. Their respect is showed by bowing with hands pressed together in a prayerful position—with which social interaction in Thailand begins and ends. In Thai society, teachers are accorded special honor. Children are taught to obey and honor their teacher. Thai people use honorific terms (e.g., archan) to refer to teachers and the entire profession is celebrated on National Teachers Day (Weisz, Chaiyasit, Weiss, Eastman, & Jackson, 1995). As a result of special honor to teachers, Thai children rarely talk with the teachers or give their reasons unless the teacher asks for input. From the observation above; although the children did the activity or answered the questions, the teachers never asked them for reasons. The lesson relied mostly on teachers' questions.

In summary, two classroom teaching episodes did not emphasize development of problem solving skills. The teachers did emphasize the development of step-by-step procedural routines, but the teacher did not require student explanation of their reasons. However, the teachers from both classrooms were very good at encouragement by giving praise when children did a good work and did attempt to teach new concepts using a variety of representations for the purpose of developing student understanding prior to developing the procedural algorithms. Nevertheless, it must be noted that two observations only provide a snap shot of teaching in school A, not the whole picture. Besides, the teaching of more difficult mathematics topics were not observed.

The results from the observation and questionnaire study suggested that Thai teachers in this study viewed problems as routine word problems and viewed problem solving as solving routine word problems. Teachers taught problem solving by emphasizing step-by-step skills such as determining the problem type, then writing the mathematical sentence, and finally completing the computation. Teachers relied mostly on textbooks and teacher manuals when they did the lesson plans. Word problems chosen for work in class are largely routine, procedural problems, rather than problems that could be solved in a number of ways that are not immediately known to the child. Different types of problems were not explicitly included in the classroom instruction. Teachers did not emphasize student's reasoning and thinking skills. As a result, children developed memorizing skills rather than thinking and reasoning skills. Children had never explored different types of word problems and non-routine problems, so when the children face the problems that differed from a textbook or teachers have taught, they could not do it (e.g., Problem 2 and Problem 6 in this study).

CHAPTER V

DISCUSSION AND CONCLUSION

Introduction

The primary focus of this study was to investigate solution strategies that Thai second graders used for solving addition and subtraction word problems. This study examined the following questions: (a) How successful are Thai children in solving addition and subtraction word problems (b) Which strategies are used by Thai children to solve addition and subtraction word problems? This section contains a summary and discussion of the main findings, a dicussion of strategies that Thai children used to solve addition and subtraction word problems compared with children in the United States and in other countries. Finally, this chapter presents limitations of the study, implications of the study for teacher education, and suggestions for future research.

Summary and Discussion of the Main Findings

The results indicated that most Thai children were successful in solving addition and subtraction word problems. Additionally, children were more successful in solving addition word problems than subtraction word problems. This finding was consistent with the result from previous research that indicated subtraction word problems were more difficult than addition word problems for children to solve (e.g., Bebout, 1993; Carpenter et al., 1993; Fuson et al., 1997). Moreoever, this diffculty became more pronounced when borrowing was required.

The results also suggest that a minority of children in this study could not solve addition compare problems (Manee has 23 apples. Mana has 15 more apples that Manee does. How many apples does Mana have?). Results were similar to other empirical studies which indicated that of the three basic categories of addition and subtraction word problems (change, combine, and compare), compare problems were the most difficult (DeCorte, Verschaffel, & Verschueren, 1982; Pauwels, 1987; Reiley & Greeno, 1988).

Two possible explanations for why Thai second graders in this study solved the addition compare problem incorrectly are discussed next. First, Thai second-grade mathematics textbooks do not include many addition compare problems in exercises. For addition problems, mathematics textbooks in Thailand present join problems and combine problems the most. Addition compare problems are mostly found in addition of three-digit numbers. However, at the time of the interview, children in this study had not been taught addition of three-digit numbers yet, so the children had no opportunity to solve addition compare problems. This might be one explanation for why children in this study were not successful in solving compare problems requiring addition.

Second, children were mislead by keywords in the problem. In the addition compare problem used in this study (Problem 2), keyword *more than* led children in using subtraction instead of addition. Children rearranged the relation and the subjects in the problems. For example, "*Manee* has 23 apples. *Mana* has 15 *more* apples *than* Manee does. How many apples does Mana have?" might be transformed into "*Manee* has 23 apples. *Mana* has 15 apples. How many *more* apples *Manee* have *than Mana* have?", which required subtraction.

Although the addition compare problem was difficult, the results from the main study showed that average and low achievers, as indentified by their teachers, correctly solved the compare problem requiring addition correctly more oftem than high achievers did. One possible explanation why high achievers could not solve the addition compare problem is that high achievers might not be careful in reading the whole context of the problem because they might interpret the problem as very easy. In class, when high achievers used keywords to solve word problems in exercises, they generally got correct answers. Thus, when the high achievers could locate a keyword in the addition compare problem used in this study, they then used that keyword to decide whether to add or subtract wihout reading the rest of the problem or trying to understand context in the problem. As a result, high achievers misunderstood the context in some of the problems and thus solved the problem incorrectly. It can be said that the high achievers had high confidence in their ability because they got correct answers most every time when they used keywords in classes, and thus the high achievers believed that the answer they got was correct.

In contrast to high achievers, low achievers were more careful in reading the problems because they were afraid of making mistakes. A possible explanation is because low achievers thought that they had low ability in doing mathematics (Kloosterman & Cougan, 1994), they must try their best to do math problems. When low achievers carefully read the problem, they probably took time to understand the context in the problem and thus were not mislead by keywords in the problems. Consequently, low achievers were able to solve the problem correctly.

However, the results from the initial study indicated that all high achievers from rural schools (schools B and C) solved the addition compare problem correctly. This might be inconsistent with the results and the discussion above. The possible explanation that could be made was that high acheivers in rural schools displayed higher ability in doing mathematics than high achievers in urban schools who participated in the main study. The researcher had a conversation with one of education officers in the area that the researcher did this study (Phunthawee, personal communication, December 4, 1998). Three assumptions were made from the conversation.

First, the classroom size in two schools are different. In urban areas in Thailand, schools are big and there are many students in each classroom. Thus, it is difficult for teachers to take care of each individual student and talk about misconceptions that occurred in learning and doing mathematics. From the obervation, while children did the exercises 10 minutes before the class ended, the teachers sat at the desk and checked children's homework rather than looking at what the children did in the exercises. On the other hand, schools in rural areas are small and there are at most 20 students in each classroom. For example, one classroom in school B in this study had 15 children. The education officer suggested that since there were not too many students in classroom, perhaps teachers had less work to do and thus had a chance to give individual attention to every student and talk with them about their learning and doing mathematics.

Second, Ministry of Education in Thailand rewards teachers who do best in teaching every year. Perhaps, teachers who get the reward have creativity in teaching which can encourage students to use their own reasoning and thinking skill. Teachers also create manipulatives to use in the classroom and engage students in doing variety of activities. Moreover, teachers should participate in several meetings or seminars for improving their teaching. As a result of having this reward, teachers are enthusiastic about improving their teaching and thus benefit their children. Since schools in rural areas are not big, teachers have more time to improve their teaching, do activities, and create materials. In contrast, teachers in urban areas might not enthusiastic to get this reward because teachers have work to do beside teaching. As told by the education officer, the teachers in urban schools do work such as filling data base for their students, checking students' homework for each subject, do school activities other than teaching, and do community activities. Therefore, teachers in urban areas do not have much time to improve their teaching and thus the effective teaching may be somewhat lower (Education Research Bureau, Ministry of Education, 1996, Thailand).

Third, rural children may have more enthusiasm to learn more children in urban areas because children in rural areas are mostly from low-income families and do not have a chance to study in schools because parents do not have enough money for support schools. Thus, when children in rural areas enter school, they might study harder than children in urban areas because they want to get knowledge to go to college and get a good job in the future. Moreover, children in rural areas have been working since they were young to help their parents, so children in rural areas might face many real-life problems than children in urban areas. Therefore, the children in rural areas might have much more experience in solving problems than children in urban areas. In contrast to the children in rural areas, children in urban areas might not worry about their families' incomes because parents have enough money for them to go to schools, so children in urban areas do not have to work. Thus, the children in urban areas have less experience in solving problems than children in urban areas (Phutawee, personnal comminication, December 4, 1998; Educational Research Bureau, Ministry of Education, 1996, Thailand). These are all conjectures about possible explanations why children in rural areas displayed high ability in solving addition and subtraction word problems in this study, and require future research.

For subtraction problems, the join problem (Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 ballons altogether?) was the most difficult for the majority of Thai children in this study to solve. As conjectured by teachers from school A, the majority of the children in this study solved this problem incorrectly because they were not used to solving this type of problem. By analyzing Thai second-grade mathematics textbooks, the type of problems seen in subtraction word problems were compare problems, separate problems, and combine problems. Subtraction join problems were mostly seen in the context of three-digit numbers that the children in this study had not been taught yet at the time of the interview. Therefore, children in this study did not have a chance to learn how to solve subtraction join problems and had not practiced them very often. Thus, the children solved the subtraction join problem incorrectly. When the researcher had a conversation with teachers about this situation, teachers suspected that if children were given join problems orally, they could solve them. However, the reseacher did not test this assumption.

A second explanation for why children in this study solved the subtraction join problem incorrectly was because children misunderstood the meaning of a keyword in join problem. The keyword *altogether* in the subtraction join problem is consistent with required addition operations for most word problems studied by these Thai children. This keyword, thus, lead children to a wrong operation, addition instead of subtraction.

However, the two children who could solve the subtraction join problem correctly demonstrated some skills in problem solving. These two children tried to get an answer by counting up from the number 15 until it reached the total 31. Then they counted the numbers in counting sequence to yield the answer. Perhaps, when these two children figured out that the subtraction join problem could not be solved by using step-by-step procedures, they tried to add up more balloons by counting up until it reached the total number given in the problems. Thus, these two children were able to apply problem solving strategies.

A third explanation might be that Thai children in this study were not encouraged to understand context in word problems. From the observations, teachers emphasized writing step-by-step solution procedures and then calculated numbers in problems for the answer. When these children solved the subtraction join problem which could not be solved by using step-by-step solution procedures, the children did not read the problem for understanding, but instead solved it incorrectly.

Strategies of Thai Children Compared with Children in the United States and Other Countries

This section presents a discussion of strategies that Thai children used to solve addition and subtraction word problems compared with children in the United States and other countries as reported in the literature. It does not focus on which country does better than another.

The results showed that Thai children used many of the same solution strategies in solving addition and subtraction word problems as children in the United States and other countries such as Korea and Nigeria. In this study, both addends in word problems were two-digit numbers, children added or subtracted the digits that have the same place value by using any of these methods—for example, counting on from the larger number, counting on from the smaller number, counting up from the given number, counting down to the smaller number, or a known number fact—described in the preceding section. For example, the child solved 24 + 28 by first solving 4 + 8 by counting on from eight and then did the carrying to the tens column, then added all the number in the tens column (2 + 2 + 1) by remembering the relevant addition fact. To solve 32 – 17, the child solves by first solving 12 – 7 (borrowed one ten from the tens column by counting up from seven and then solve 2 – 1 (at the tens column) by remembering the subtraction fact.

The results from this study were similar to the results from the study of Houlihan and Ginsburg (1981). The results showed that second graders in the United States used place value method on solution of two double-digit addend problems. For example, the child solved 23 + 16 by first solving 3 + 6 by counting on from six and then solved 2 + 1 by remembering the relevant addition fact. In addition, Fuson et al. (1997) reported that one method children in the United States used to add and subtract two-digit numbers was decomposed-tens-and-ones method in which the tens and the ones were added or subtracted separately from each other. This method was not different from the place value method that Thai children in this study used to solve two-digit addition and subtraction word problems. Moreover, the results from Korea (Fuson & Kwon, 1992) also showed that Korean children always used place value methods on addition and subtraction of two or more digit numbers.

By considering when children did the calculation in the ones and the tens column, children in this study used three strategies to solve addition problems. The three strategies were counting on from either smaller or larger numbers given in the problem, a known number fact, and a base-ten strategy. The results indicated that Thai children used five different strategies to solve subtraction problems. The five strategies were counting up from a given number, counting down to the smaller number, a known number fact, a ten-based strategy, and other two strategies such as counting tallies and knuckles.

Addition Strategies

The following part discusses addition strategies that Thai children in this study used to solve addition word problems compared with addition strategies that children in the United States and other countries used to solve addition word problems.

Counting On Strategy.

The results showed that the majority of Thai children in this study used mostly counting on from the larger number strategy. Counting was done mentally or by using fingers to keep track of the number of steps in the counting sequence. For example, to solve 24 + 28, the child would solve the ones column first by counting "8 [pause], 9, 10, 11, 12. The answer is 12." When the child used fingers, four fingers were folded to represent the number 4 (see Figure 6). The child then carried one to the tens column and added all values at the tens column together to yield the final answer. Consistent with previous research, counting on from the larger number strategy was mostly used by children in the United States (Carpenter & Moser, 1984; Fuson & Kwon, 1992; Fuson et al., 1997; Hiebert, 1982) and children in other countries such as Korea (Fuson & Kwon, 1992, 1992b) and Nigeria (Adetula, 1989). The results from these studies indicated that children deliberately entered the counting sequence at the larger number and then counted forward as many times as the smaller addend. For example, to solve 3 + 5, the child would count "5 [pause], 6, 7, 8." The answer was eight. The results from those studies also showed that children may count with or without concret aids such as fingers, cubes, or physical objects. Fuson and Kwon (1992) indicated in the study with Korean children that counting on was very natural and many, even most, children eventually invented it for themselves. Korean children also used the counting on strategy with folding/unfolding fingers to keep track of second addends.

Moreover, a few Thai children in this study used counting on from the smaller number strategy to solve addition word problems. For instance, to solve 24 + 28, the child would solve the ones column first by counting, "4 [pause], 5, 6, 7, 8, 9, 10, 11, 12. The answer is 12." The child then put the number 1 at the top of the tens column and put the number 2 at the answer space of the ones column. The child added the number in the tens column together to yield the answer. This result was similar to findings from Carpenter and Moser (1984) indicating that children in the United States used counting on from the smaller number to solve addition problems. The study of Houlihan and Ginsburg (1981) also supported that the children in the United States used counting on strategies to solve addition problems. The results showed that counting on could be done from either the smaller number or the larger nmber with a concrete aid. For example, given 6 + 4 the child started counting with the number 4 and went from there. Fuson et al. (1997) also indicated that, to add, children counted on from addend word while keeping track of the other addend word counted on.

The results from this study and from the previous studies (e.g., Adetula, 1989; Carpenter & Moser, 1984; Fuson & Kwon, 1992; Fuson et al., 1997; Houlihan & Ginsburg, 1981) showed that counting on from the smaller number given in the problem and counting on from the larger number given in the problem were the dominant counting strategies for children in many countries. However, Thai and Korean children kept track of counting sequences by using fingers while children in the United States kept track of counting sequences by using cubes, or other physical objects. Although, there was evidence that children in the United States had been taught to use one-handed finger patterns to add, the processes of using fingers were different from finger methods that Thai and Korean children used. In the United States, finger methods was complex (Fuson, 1987).

A Known Number Fact.

The results from this study showed that Thai children developed advanced strategies to solve addition problems by using a known number fact. For example, to solve 23 + 15, the child remembered that 3 + 5 was 8 and 2 + 1 was 3, so he suddenly gave 38 as an answer. Consistent with the study of Hiebert (1982) that showed that children in the United States also used a known number fact to solve addition problems. The children gave an answer with the justification that it was the result of knowing some basic addition fact. For example, $7 + 6 = 6 + 6 + 1 = 12 + 1 = 13$. The children knew that $6 + 6 = 12$. This strategy was also used by children in other countries such as Korea (Fuson & Kwon, 1992) and Nigeria (Adetula, 1989). Fuson and Kwon (1992) reported that the majority of Korea children used a known number fact strategy for solving addition problems of the sum to ten.

A Base-Ten Strategy.

In this study, Thai children used combinations of ten to derive an answer for a problem. This strategy is taught in school. For instance, to solve 58 + 15, the child would say, " eight has five. Put five together so it is 10. Plus three more so it is 13. Carrying one to the tens column, five plus one is six. Plus one more from the carrying so it is seven. The answer is 73." This result is consistent with the study of Fuson and Kwon (1992b) that children in Asian countries such as Japan, Korea, China, and Taiwan learned to recompose numbers to ten-structured triplets. For example, $7 + 6 = 7 + 3$ (to make ten) $+ 3 = 13$.

However, this strategy was used less frequently in the United States. There was some evidence from previous studies in the United States indicating that children used base-ten knowledge to solve addition problems (Carpenter et al., 1981, 1993; Houlihan & Ginsburg, 1981). The results indicated that children calculated based on numbers whose sum is 10. For example, to solve 5 + 7, the child split the seven into five and two and solved the problem by doing $5 + 5 = 10$ by memory and then $10 + 2 = 12$, also by memory.

Most Chinese, Japanese, Korean, Taiwanese, and Thai language use the tens and ones to make multiple models. For example, 12 is said "ten two", 58 is said "five ten eight", and so on, while the number in English language does not support the structure around ten such as 12 is twelve, 58 is fifty eight and so on. According to the lack of explicit naming of the tens in English, it is more difficult for English-speaking children to use base-ten strategies to add (Fuson, 1992).

In Asian countries, children are taught to use base-ten systems when adding. To add, the children are taught to break the number into the number that will make ten with the other addend and the leftover: 8 + 6, for example, is thought of as "eight plus two from the six is ten plus the four leftover from the six is ten four." The answer can be found just by saying ten and the part of the second addend that is left after making ten.

In English culture, this method is more difficult because the English ten words are not automatically given by finding the left-over part, and many United States second graders did not know that ten plus any number is; they must count on to find that ten plus four is fourteen (Steinberg, 1984).

Summary.

Counting on strategies were mostly used by Thai children and children in the United States and in other countries for solving addition word problems. The results from the United States (e.g., Carpenter & Moser, 1984; Fuson et al., 1997) showed that some children in the United States used counting all strategy by using physical objects or fingers to represent a problem. However, none of Thai children in this study used counting all strategy.

Subtraction Strategies

The following part contains the discussions of subtraction strategies used by Thai children, children in the United States and in other countries.

Counting Up from A Given Number.

Thai children in this study frequently used this strategy. The counting was done mentally or by using fingers to represent the counting sequences. For example, to solve 42 – 25, the child would say that two cannot subtract five, borrow one from the tens column so it become 12. The child then counted " 5 [pause], 6, 7, 8, 9, 10, 11, 12. Put seven in the ones column. Four is borrowed one so three is leftover. Three minus two is one. Put one in the tens column so the answer is 17." When the child used fingers, he folded each finger while he was counting. Then the child counted seven folded fingers for the answer (see Figure 7). Children in the United States also used this strategy to solve subtraction problems (Carpenter & Moser, 1984; Fuson et al., 1997; Hiebert, 1982). The results from these studies showed that children counted up from the known addend to the total, keeping track of how many are counted up. For example, to solve 3 + ? = 8, the child counts, " 3 [pause], 4, 5, 6, 7, 8." The answer was eight. Carpenter and Moser (1984) found in their longitudinal study that counting up appeared before counting down from and that more children used counting up than counting down. The results from the United States also indicated that children used fingers to keep track of the counting sequences. Some of them used objects such as counters, cubes, as the tracking device. Others displayed no use of objects or fingers, but kept track in their minds (e.g., Carpenter & Moser, 1984). However, the counting up from a given number was used less by children in Nigerian (Adetula, 1989) and in Korea (Fuson & Kwon, 1992).

Counting Down to the Smaller Number.

The findings from this study indicated that a minority of Thai children used counting down to a smaller number in the problem with fingers to solve subtraction problems. The counting down to represented counting backward from the larger number given in the problem. The sequences ended with the smaller number in the problem. Sequence-counting down to the smaller given in the problem in order to solve 12 – 5 is "12 [pause], 11, 10, 9, 8, 7, 6, 5" where the words stop when the 5 is said and some method of keeping track of how many words are between 11 and five is used to find the answer. By keeping track of the number of counting words in the sequence by using fingers, the child got seven for the answer by counting seven folded fingers.

The counting down that Thai children used was different from the counting down that children in the United States used. Children in the United Stated often used counting down from the larger number. Carpenter and Moser (1984) reported that a child started counting from the larger number in the problem and then decrement many times as the smaller number in the problem and then the last word spoken was the answer. Sequence-counting down from the larger number in order to solve the same problem is "12 [pause], 11, 10, 9, 8, 7" where some methods of keeping track of five words counted from 12 is used and the last word tells the answer, 8. However, this strategy was less used by Korean children (Fuson & Kwon, 1992), Nigerian children (Adetula, 1989), and Dutch children (Beishuizen, 1993).

A Known Number Fact.

The results from this study showed that Thai children were able to apply advanced strategies to the solution of subtraction problems. One strategy was a known number fact. For example, to solve 20 – 9, the child would tell "Zero could not subtract nine, borrow one from two, so it is 10. I know that 10 – 9 is 1. Since two is borrowed, one is leftover. The answer is 11." The results was consistent with previous studies (e.g., Fuson et al., 1997; Hiebert, 1982; Houlihan & Ginsburg, 1981). The studies indicated that children responded by recalling the particular number facts. For example, 3 – 2 = 1. This strategy was also used by Korean children (Fuson and Kwon, 1992, 1992b), Nigerian children (Adetula, 1989), and Dutch children (Beishuizen, 1993).

A Base-Ten Strategy.

Like addition strategies, some Thai children in this study used the combination of the number ten to derive the solutions to the problems. In the base-ten strategy, the children solved problems based on the number 10. For example, to solve 12 – 5, the child separated 12 to ten and two and responded that 10 – 5 = 5 and 5 + 2 = 7 so 12 – 5 = 7. To solve 20 – 9, the child would say that there were two tens. One ten was subtracted by nine so one was left. Another tens was added to the 1 so the answer was 11. Similarly, Fuson and Kwon (1992) indicated that most Korean children used base-ten structure to subtract. For example, to solve 13 – 6, one child responded that "six is from three plus three.

Take three, ten remain. Ten take away three again. It becomes seven." This strategy was called down-over-ten strategy. Another strategy was subtract-from-ten strategy. For example, to solve 13 – 6, one child responded "ten take away six leaves four, and there is three more. Add together, three and four make seven."

In Asian countries, children are taught to use this base-ten system. However, this strategy was used much less frequently by children in the United States, though some children used them, especially for an addend of 9 (e.g., Steinberg, 1984, 1985). As indicated in the addition strategies, children in United States lack cultural supports for ten-structured methods.

Other Strategies.

Some children in this study used strategies that were different from the listed categories. The strategies found were using tallies, using knuckles, and using fingers (different from folded fingers). For example, to solve 15 + ? = 31 by using tallies the child would tally and count from 15 until it reached 31. The child then counted the total tallies for the answer. The example of using knuckles was showed in Figure 8. To use fingers to solve 38 - 19, the child first borrowed one from the number 3, so eight became 18. To solve 18 - 9, the child respond by showing ten unfolded-fingers, then counts 8 [pause] 9. While the child was counting from nine, he folded one finger for the number 9. There were nine unfolded fingers left. The child counted the unfolded fingers to yield the answer (see Figure 7). Then, the child did the calculation at the tens column by remembering that two minus one is one. These three strategies were not found in any studies that have been done in the United States and in other countries (e.g., Adetula, 1989; Carpenter & Moser, 1984: Carpenter et al., 1993; Fuson & Kwon, 1992; Fuson et al., 1997; Houlihan & Ginsburg, 1981).

Summary.

Counting up from a given number were a dominant strategy for Thai and the United States children and some children from other countries in solving subtraction word problems. The counting down strategy was also found in both Thai and United States children. However, the counting sequences in counting down strategy were not similar. Thai children used counting down to the smaller number in the problem whereas United States children used counting down form the larger number in the problem. The children from both countries also used a known number fact to derive an answer to a problem. Children in the United States used a base-ten strategy less than children in Asian countries. There were three strategies that were not found in previous studies that have been done in the United States and in other countries. The three strategies were using fingers in different ways, using knuckles, using a base-ten strategy and using tallies.

Problem Difficulties and Misconceptions

Research on problem types conducted in the United States (e.g., DeCorte, Verschaffel, & Verschueren, 1982; Mwangi & Sweller, 1998; Riley et al., 1983; Riley & Greeno, 1988; Stern, 1993; Verschaffel, DeCorte, & Pauwel, 1992; Verschaffel, 1994) showed that schooled children experience a great difficulty in representing and solving compare word problems—both addition and subtraction. The results from this study similarly showed that Thai children had difficulty in solving compare problems, particularly addition problems. The reason was that the keyword *more* in the problem misleads children into representing a problem incorrectly. Moreover, the results from this study showed that the majority of Thai children could not solve join problems requiring subtraction because the meaning of the keyword *altogether* mislead children into representing the problem incorrectly. The study of Stockdal (1991), and Karrison and Carool (1991) supported results that a keyword was not associated with only one arithmetical operation. The strategy of memorizing a list of cue words and the arithmetical operation could not be recommended as the sole method of helping children become better solvers of word problems. However, the result from previous studies (e.g., Carpenter et al., 1981, 1993) showed no evidence that United States children had difficulty in solving join problems requiring subtraction.

The misconception that was found with Thai children in this study was calculation misconception with subtraction. The majority of the children in this study, in general, set up the problem properly but some calculated incorrectly. The children did not know how to borrow tens and instead subtracted the smaller ones digit from larger number. For example, to solve 38 – 19, they calculated 9 - 8 = 1 and thus gave 21 as the answer. This result was also found with the children in the United States (e.g., Cebulski & Bucher, 1986; Engelhardt & Usnick, 1977; Haneghan, 1990; Knifong & Boyd; 1976), and in other countries such as Netherlands (Beishuizen, 1993) and Korea (Fuson & Kwon, 1992, 1992b). The results showed that children must have read and understood the problem but they solved problems with calculation misconceptions.

For example, 91 - 36 = 65. In this case, the child would think of the larger of the two numbers as the number of the set and the smaller as the number to be removed from the set. Cox (1975) found that 83 percent of the children with calculation misconceptions used this particular procedure.

Limitations of the Study

The findings of this study are limited in the research designed and methods. The representativeness of the sample of students and teachers were small. The number of word problems used in the study were also small. The relationship between researcher and students participated in this study was rare.

The observations of teacher's instruction were done mostly in one school. Thus, we could not conclude that elementary mathematics instruction in Thailand is not concerned with developing student's problem solving abilities. Some schools might emphasize problem solving and some schools might not emphasize problem solving. In addition, teachers' understanding about problem solving was limited to only solving routine word problems. Again, we could not assume that all teachers at all grade levels in other schools do not have knowledge about problem solving. Indeed, some teachers in different grade levels or in different schools might know about problem solving. More research should be done in this area.

A small sample of problems were used in this study, thus, the problem set might not be complete to test children's knowledge of solving word problems. The set of problems used in this study should be revised and tested before being used in future studies. Another limitation of this study is the time that the interviewer spent with children in this study. The children in this study were not used to having stranger in their classroom. The children were shy to talk with a stranger. When the interviewer asked children to explain solutions to problems, the children would explain their thinking to the interviewer only after prompting. Thus, some explanations given by children in this study were not clear. This suggested that the interviewer should spend more time with the children in classroom, by either teaching them or doing activities with them, to get acquainted with children before doing the interview with them. Therefore, the children might feel more comfortable when talking with the interviewer and might give more in depth explanations.

The sample size of this study was small. Thus, we did not see other strategies that might be interesting. In addition, since the number of children participated in this study were small and from only three schools in one region of Thailand, we cannot conclude that all young Thai children solved addition and subtraction word problems in the way indicated in this study. Moreover, this study showed that children in rural areas scored higher than children in urban areas. However, we shall not conclude that children in rural areas did better than children in urban areas because the sample size was too small to compare. The results from the main study showed that children in all levels—high, average, and low—scored in the same range. Thus, we expect that the different achievement categories as assigned by teachers were not able to predict student's success on the addition and subtraction tasks in this study.

The results from this study showed that individual children used different of strategies in solving addition and subtraction word problems. However, we should not conclude that second grade children had learned how to solve word problems only from their second-grade teachers. The children might have learned strategies for solving word problems from their parents, their first-grade teachers, their tutors, or from other math books.

Implications for Instruction

The preceding discussion of the study's main findings provided a number of recommendations for mathematics instruction regarding teaching and learning mathematics and curriculum development in elementary school level.

The results indicated that Thai children generally remembered keywords to decide whether to add or subtract rather than attempting to understand the context of the problems. This suggests that teachers should explain to children or provide activities that demonstrate keywords do not always help deciding about whether to add or subtract when solving word problems. Teachers might have children do different kinds of problems that contain keywords that often mislead them to a wrong operation.
Then, teachers should let children discuss and explain why a simple interpretation of the keyword works in this problem but why the same keyword interpretation does not work in other problems. Teachers should emphasize to children that they should try to understand the context in word problems rather than simply looking for keywords.
As indicated, children relied mostly on step-by-step routine procedural skills rather than attempting to understand the context of the problems. When children were confused about the problem, they could not proceed to other steps. This might indicate that children feel hopeless and give up in solving problems. Teachers should implement more

nonroutine word problems in the classroom to help children practice thinking and reasoning skills rather than relying only on step-by-step procedures.

Problem solving, as understood by teachers in this study, was a step-by-step procedure for solving word problems. Indeed, teachers used mathematics games that can improve problem-solving skills, but the teachers did not usually use mathematics games in classrooms. There are games in mathematics that Thai teachers often post on the board outside the classroom for children to do. If children get correct answers, they are rewarded. In Thailand, many games such as card games, domino games, and puzzle games are prohibited during class time. Children can only play those games in activity rooms and in activity times. Teachers might bring mathematics game puzzles to the classroom for children to play. The purpose for bringing games into the classroom is not just playing games for fun but using games to practice thinking and reasoning skills, and to help children see the connection and usefulness of mathematics in real life.

In addition, educators should conduct programs for teachers to develop knowledge about problem solving. Teachers should be introduced to Polya's problem solving steps that is often used in problem solving (Polya, 1980). The program such as Cognitively Guided Instruction program (CGI) might be useful to introduce teachers about student's problem solving. The results of introducing CGI program for teacher in primary schools (1-6) in Thailand indicated that CGI program could be implemented effectively to some degree in a primary school in Thailand (Komalabutr, 1995).
If teachers understand about problem solving and problem solving heuristics, as well as children's thinking, they then can assist children's development of problem solving skill and disposition rather than simply solving routine word problems.

The results showed that some children had misconceptions about calculation. Teachers, together with their colleagues, should discuss this problem and find ways to help children overcome the problem. One misconception found in this study was in subtraction. For example, to solve 20 – 9, the children answered 29. This happened because children did not know how to borrow and regroup. Thus, the children subtract smaller from larger, 9 – 0 = 9. In order to alleviate this problem, teacher might emphasize the concepts of place value, and borrowing and regrouping. Teachers might begin from the smaller set of number such as 11 – 2, 13 – 5, 21 - 2, and 32 – 7 by using physical objects such as counters or cubes or base 10 blocks and then carefully develop the concept of place value. Teachers might use calculators to teach subtraction that require borrowing by having children punch the calculator to find the different between 21 – 2 and then let children discuss the answer and work out how to calculate by hand. This might encourage children to find out how to calculate 21 – 2 by hand. However, educators in Thailand thought that calculators were not suitable for young children because children might lack thinking skills and might rely on calculators when doing even simple calculations. However, we do not mean to teach children to use calculators for calculations and ignore calculation by hand. We tend to use calculators to build children's enthusiasm to learn, to discover, and to think by themselves. It was difficult to teach children to solve subtraction problems that need borrowing and regrouping. Thus, teachers should be prepared for this misconceptions because previous studies also showed that subtraction problems were more difficult when borrowing is required (Cox, 1975; Ellis, 1972).

From the observations, two problems about teaching and learning of mathematics were found. First, classroom size was too large with too many students in one classroom so it was difficult for teachers to look after each child and to implement activities so that all individual understand. School policy makers should consider reducing students in each classroom to a smaller number. However, it might difficult because there are not enough teachers for the numbers of students who are increasing each year. Thus, schools accept teachers who do not have knowledge about teaching and thus are ineffective in maximizing children's learning. One suggestion to overcome this problem is to supervise those teachers several times during their instruction and have them observe other teachers teach.

Second, teachers' instruction was teacher-centered rather than student-centered and the instruction was sometimes abstract for young children. Teachers should build knowledge from what children already know and put more variety of activities in the classroom. Young children naturally want to know everything around them, teachers might use activities that are interesting to children and embody concepts of mathematics.

Moreover, from the observation of the two classrooms, mathematics instruction relied mostly on teachers' talk. Children rarely talked or initiated questions or discussions with teachers. Therefore, this type of instruction might effect children's thinking and reasoning skills and ability to apply their mathematical knowledge to new problems. This suggests that problem-solving skills should be taught in classrooms and teachers should provide variety of problem types. These problem types should not only routine word problems but also nonroutine word problems and more difficult problems for children to solve. Teachers might ask children individually to explain their thinking on a regular basis. Furthermore, teachers cold let children work in groups to discuss and exchange their ideas or communicate their thinking and reasoning when working together. These are all areas requiring future research.

The curriculum for elementary school needs to be changed in Thailand The current curriculum was developed by qualified people and experts in various fields. Thus, the content and learning process tended to be difficult for the users and placed too much pressure on children. To change the curriculum, the processes of child development should be considered. In addition, social context and the national situation must also be considered. Teachers from different schools in different areas should be part of curricula development, not only experts in various fields but also teacher educators and teachers. The curriculum should emphasize thinking and reasoning skills, problem solving rather than solving routine exercises and word problems. Besides, Thai second-grade mathematics textbooks should be analyzed and improved to provide variety of problems for children to practice problem-solving skills.

Future Research

The preceding discussion of this study provided a number of recommendations for future research in mathematics education and teacher education.

Future studies should focus on children's finger counting methods. It might be compared with finger methods used by children in the United States. Thai children's solution strategies on compare problems should be investigated. More difficult problems and more nonroutine problems should be included in continued studies. Large sample sizes of children should also be considered. The future study might also focus on children misconception in addition and subtraction word problems. It would be important to future research to study kindergarten and first graders for their thinking processes in solving addition and subtraction word problems. The investigation for why children in the United States used less a ten-based strategy than children in Asia countries would be interesting.

The results from this study showed that some children had difficulty in reading and thus effected their ability to solve word problems unless the problem was read to them. This suggests that the research on reading ability and mathematics ability to solve word problems should be studied.

Action research should be introduced to teachers because teachers can do a small research with their colleagues in the same field. Teachers should do action research on children's thinking. The small research might help teachers improve their teaching and thus giving an idea in developing mathematics curriculum for young children. Moreover, future research should investigate how teachers teach addition and subtraction in elementary grade and compare with students' understanding of addition and subtraction. The result might help teachers to understand children's learning processes and to improve their instruction.

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APPENDICES

APPENDIX A
Verbal Problems, A Test Booklet, and Test Cards

Verbal Problems used in this study

Structures	Problems
<u>Addition</u>	
1. Combine	Classroom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes? In Thai wording, classroom 1 has 24 students. Classroom 2 has 28 students. How many students in two classes are altogether? In Thai: ชั้น ป. 2/1 มีนักเรียน 24 คน ชั้น ป. 2/2 มีนักเรียน 28 คน นักเรียนทั้งสองห้องมีทั้งหมดกี่คน
2. Compare	Manee has 23 apples. Mana has 15 more apples than Manee does. How many apples does Mana have? In Thai: มานีมีแอปเปิ้ล 23 ผล มานะมีแอปเปิ้ล มากกว่ามานี 15 ผล มานะมีแอบเปิ้ลกี่ผล
3. Join	Suda had 58 Baht*. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now? In Thai: สุดามีเงิน 58 บาท แม่ให้อีก 15 บาท สุดามีเงินทั้งหมดเท่าไหร่
<u>Subtraction</u>	
4. Combine	There are 42 chickens. 25 are male. How many chicken s are female? In Thai wording, there are 42 chickens. 25 are male. How many female chickens are left over? In Thai: เลี้ยงไก่ไว้ฝูงหนึ่ง 42 ตัว เป็นไก่ตัวผู้ 25 ตัว ที่เหลือเป็นไก่ตัวเมียกี่ตัว
5. Compare	Suda's pencil is 20 centimeter long. Manee's pencil is 9 centimeter long. How much longer is Suda's pencil than Manee's pencil? In Thai: ดินสอของมานียาว 20 เซนติเมตร ดินสอของมานะยาว 9 เซนติเมตร ดินสอของมานียาวกว่ากี่เซนติเมตร
6. Join	Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 balloons altogether? In Thai: มานะมีลูกโป่ง 15 ลูก มานะต้องการลูกโป่งอีกกี่ลูกจึงจะมีลูกโป่งทั้งหมด 31 ลูก
7. Separate	The farmer has 38 cows. He sells 19 cows. How many cows are left? In Thai wording, the farmer has 38 cows. He sells 19 cows. How many cows are leftover? In Thai: ชาวนามีวัว 38 ตัว ขายไป 19 ตัว เหลือวัวกี่ตัว

*Thai currency

การวัดและประเมินผลวิชาคณิตศาสตร์
ชั้นประถมศึกษาปีที่ 2
MathematicalTest for Second Graders

กระบวนการในการแก้โจทย์ปัญหาการบวกและลบเลขสองหลัก
Problem solving strategies for two-digit addition and subtraction word problems.

ชื่อ(Name)____________________ ชั้น (Class)________________

คำสั่ง ให้นักเรียนเขียนประโยคสัญลักษณ์และแสดงวิธีทำ

Direction For each problem, write a number sentence and show a solution

1. ชั้น ป. 2/1 มีนักเรียน 24 คน ชั้น ป.2/2 มีนักเรียน 28 คน นักเรียนทั้งสองห้องมีทั้งหมดกี่คน (Clasrrom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes?)

2. มานีมีแอปเปิ้ล 23 ผล มานะมีแอปเปิ้ลมากกว่ามานี 15 ผล มานะมีแอปเปิ้ลกี่ผล (Manee has 23 apples. Mana has 15 more apples then Manee does. How many apples does Mana have?)

3. สุดามีเงิน 58 บาท แม่ให้อีก 15 บาท สุดามีเงินทั้งหมดเท่าไหร่ (Suda had 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now?)

4. เลี้ยงไก่ไว้ฝูงหนึ่ง 42 ตัว เป็นไก่ตัวผู้ 25 ตัว ที่เหลือเป็นไก่ตัวเมียกี่ตัว (There are 42 chickens. 25 are male. How many female chickens are leftover?)

5. ดินสอของมานียาว 20 เซนติเมตร ดินสอของมานียาว 9 เซนติเมตร ดินสอของมานียาวกว่ากี่เซนติเมตร (Suda's pencil is 20 cemtimeter long. Manee's pencil is 9 cemtimeter lone. How much longer is Suda's pencil than Manee's pencil?)

6. มานะมีลูกโป่ง 15 ลูก มานะต้องการลูกโป่งอีกกี่ลูกจึงจะมีลูกโป่งทั้งหมด 31 ลูก (Mana has 15 ballons. How many more balloons does he have to put with them so he has 31 ballons altogether?)

7. ชาวนามีวัว 38 ตัว ขายไป 19 ตัว เหลือวัวกี่ตัว (A farmer has 38 cows. He sells 19 cows. How many cows are left?)

Test cards

1. ชั้น ป. 2/1 มีนักเรียน 24 คน ชั้น ป.2/2 มีนักเรียน 28 คน นักเรียนทั้งสองห้องมีทั้งหมดกี่คน (Clasrrom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes?)

2. มานีมีแอปเปิ้ล 23 ผล มานะมีแอปเปิ้ลมากกว่ามานี 15 ผล มานะมีแอปเปิ้ลกี่ผล (Manee has 23 apples. Mana has 15 more apples then Manee does. How many apples does Mana have?)

3. สุดามีเงิน 58 บาท แม่ให้อีก 15 บาท สุดามีเงินทั้งหมด
เท่าไหร่ (Suda had 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now?)

4. เลี้ยงไก่ไว้ฝูงหนึ่ง 42 ตัว เป็นไก่ตัวผู้ 25 ตัว ที่เหลือเป็นไก่ตัวเมียกี่ตัว
(There are 42 chickens. 25 are male. How many female chickens are leftover?)

5. ดินสอของมานียาว 20 เซนติเมตร ดินสอของมานียาว 9 เซนติเมตร ดินสอของมานี ยาวกว่ากี่เซนติเมตร (Suda's pencil is 20 cemtimeter long. Manee's pencil is 9 cemtimeter lone. How much longer is Suda's pencil than Manee's pencil?)

6. มานะมีลูกโป่ง 15 ลูก มานะต้องการลูกโป่งอีกกี่ลูกจึงจะมีลูกโป่งทั้งหมด 31 ลูก (Mana has 15 ballons. How many more balloons does he have to put with them so he has 31 ballons altogether?)

7. ชาวนามีวัว 38 ตัว ขายไป 19 ตัว เหลือวัวกี่ตัว (A farmer has 38 cows. He sells 19 cows. How many cows are left?)

APPENDIX B
Set of Questions Used in Questionnaire Study

แบบสอบถามทางคณิตศาสตร์เกี่ยวกับทักษะการแก้โจทย์ปัญหา (Problem Solving).

1. Personal information (รายละเอียดเกี่ยวกับตัวท่าน)

   Gender (เพศ)______________

   Education (วุฒิการศึกษา)______________

   Teaching Experience (ประสบการณ์ในการสอน)______________

2. How do you teach mathematics in classroom? Explain
   ท่านสอนคณิตศาสตร์ในห้องเรียนอย่างไร โปรดอธิบาย

   ____

   ____

   ____

   ____

   ____

   ____

   ____

3. How many methods do you use to teach addition? How do you teach and what kinds of manipulatives do you use in teaching?
   ท่านมีวิธีการสอนการบวกเลขกี่วิธี อย่างไรบ้าง และท่านใช้สื่ออะไรในการช่วยสอน

   ____

   ____

   ____

   ____

   ____

   ____

   ____

   ____

   ____

4. How many methods do you use to teach subtraction? How do you teach and what kinds of manipulatives do you use in teaching?
ท่านมีวิธีการสอนการลบเลขกี่วิธี อย่างไรบ้าง และท่านใช้สื่ออะไรบ้างในการช่วยสอน

Since this study is concerning about Problem-Solving, the following questions will ask you about your beliefs and understanding of problem-solving.
กเนื่องจากแบบสอบถามนี้มุ่งเน้นเรื่องทักษะในการแก้ปัญหา (Problem Solving)
จึงอยากทราบว่าท่านมีความคิดเห็นอย่างไรเกี่ยวกับเรื่องนี้

5. Please give the definition of Problems.
จงให้คำจำกัดความของคำว่า ปัญหา (Problems)

6. Please define the definition of Problem-Solving.
จงให้คำจำกัดความของคำว่า ทักษะการแก้ปัญหา (Problem solving)

7. Have you ever heard Polya's four steps in solving problem? If yes, please explain each step. ท่านเคยได้ยินกระบวนการแก้ปัญหา 4 ขั้นตอนของ Polya หรือไม่ ถ้าเคย โปรดระบุกระบวนการแก้ปัญหา 4 ขั้นตอนนั้น

8. How do you teach problem-solving in your classroom?
ท่านมีวิธีการสอน ทักษะการแก้ปัญหาแก่นักเรียนอย่างไรบ้าง

9. How do you connect mathematics in your classroom to mathematics in real-life?
ท่านมีวิธีสอนอย่างไรบ้างที่ช่วยกระตุ้นให้นักเรียนเห็นความสัมพันธ์ของคณิตศาสตร์ในห้องเรียนกับคณิตศาสตร์ในชีวิตประจำวัน

10. Please describe characteristics of a successful child in problem-solving.

อธิบายคุณลักษณะของนักเรียนที่ประสบผลสำเร็จในการแก้ปัญหา

11. Please describe characteristics of an unsuccessful child in problem-solving.

อธิบายคุณลักษณะของนักเรียนที่ไม่ประสบผลสำเร็จในการแก้ปัญหา

12. Explain why some children are not successful in problem-solving.

จงให้เหตุผลว่า ทำไมนักเรียนบางคนจึงไม่ประสบผลสำเร็จในการแก้ปัญหา

13. In problem-solving, which is important: an answer to a problem or a process to derive an answer. Explain your own opinion.

ท่านให้ความสำคัญกับคำตอบของปัญหาที่นักเรียนคิดได้ หรือ

ให้ความสำคัญกับกระบวนการคิดหาคำตอบของนักเรียน (โปรดอธิบาย)

14. What skills should elementary children have in problem-solving?

ท่านคิดว่า นักเรียนในระดับนี้ควรมีทักษะอะไรบ้างในการแก้โจทย์ปัญหา

APPENDIX C
Narrative from the Interview

This appendix describes each child's solution on each problem. This includes the narratives between an interviewer and each child on the seven problems. The interviewing took place after the child finished solving each problem. This section will begin from high achievement children to low achievement children. The children in this study did not use counters that were provided. They used fingers to model the problems and they did not look back for checking how they solve the problems.

High Achievement

Boy 1 (B1)

This boy read all seven problems without assistance from the researcher. He used effective solution strategies and got correct answers to five problems. He could not solve Problem 2 and Problem 6 because he misunderstood the keyword in the problems. He scored 15 points. While he read each problem, he wrote the numbers in the problem on the sheet of paper, then placed the operation.

Problem 1: Classroom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes?

Interview (I): Why do you decide to use addition to solve this problem?

Boy 1(B1): Well, because the word "altogether" in the problem. Altogether means addition.

I: How do you get 52?

B1: 4 plus 8 equals 12, right. 12, put 2 at the bottom of the ones column, Trade 1 to the tens column. [At the tens column.] 2 plus 2 equals 4, plus 1 from the trading, so it is 5.

I: Tell me how you get 12.

B1: Well, keep 8 in mind [Put his hand on his chest.] and I count 8 [Pause, show another hand and fold 4 fingers while counting.], 9, 10, 11, 12. And, I know that 2 plus 2 is 4 because it is easy and I remember it.

Problem 2: Manee has 23 apples. Mana has 15 more apples than Manee does. How many apples does Mana have?

I: How do you know it is subtraction?

B1: Right here, Manee has more apples than Mana. It should be subtraction if it much more or less than.

I: Tell me how you get 12.

B1: [At the ones column.] 3 minus 5 is 2. [At the tens column.] 2 minus 1 is 1.

I: Do you have other solution for this problem?

B1: No, I don't. I used to use only this solution.

The child did wrong for this problem, he subtracted rather that added because he thought that the word much more or less than always mean subtraction. He did not use borrowing and regrouping.

Problem 3: Suda had 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now?

I: Do you add or subtract for this problem?

B1: Add

I: How do you know it is addition?

B1: Because her mother gives her more 15 Baht, so I decide to add.

I: How do you get the answer?

B1: [At the ones column.] Well, 8 has 5, right. Put 5 together and has three so it is thirteen. Carrying 1 to the tens column, 5 plus 1was 6, plus 1 from the carrying so it is 7. The answer is 73.

I: Do the teacher teach this method in classroom?

B1: No, I invented myself.

The child use ten-based method to find the solution. He invented this solution by himself. He did very well for this problem.

Problem 4: There are 42 chickens. 25 are male. How many are female?

I: Could you describe your solution to this problem?

B1: Yes, I use subtraction.

I: How do you know?

B1: There are 42 chicken, right. The problem gave the total of male chicken and wanted to know how many female chickens were, right. Therefore, I use subtraction to find the answer.

I: Tell me how you solve this problem?

B1: [At the ones column.] 2 cannot subtract 5, borrow 1 from four, so 2 become 12. [At the tens column] 12 subtract 5 is 7. 4 is borrowed 1, so it is 3 leftover. 3 minus 2 equals 1.

I: Tell me how you find 12 – 5 = 7.

B1: I use fingers.

I: Show me how you count.

B1: Keep 5 in mind [Show tens fingers and count.] 5 [pause, then folding each finger while counting], 6, 7, 8, 9, 10, 11, 12 [Seven fingers are folded. The child counts seven fingers for the answer.]

The child used counting up from method to find the answer. He counted by using fingers.

Problem 5: Suda's pencil is 20 centimeter long. How much longer is Suda's pencil than Manee's pencil?

I: How did you decide it is subtraction?

B1: Well, from "how much longer" in the problem, so it should be subtraction.

I: OK., and how do you get 11?

B1: 0 could not subtract 9, borrow 1 from 2, so it is 10.

10 minus 9 is 1. 1 at the tens column does not have subtrahend, so pull it down for the answer. The answer is 11.

Problem 6: Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 balloons altogether?

I: What the problem asked you to find?
B1: How many more balloons that Mana wanted.
I: Do you add or subtract to find the solution?
B1: Add.
I: Why do you decide to add?
B1: Because the problem asked how many more balloons Mana wanted. Therefore, to find how many more is to add.

The child was not successful in solving this problem because he understood that it was addition.

Problem 7: The farmer has 38 cows. He sells 19 cows. How many cows are left?

I: Do you add or subtract?
B1: Subtract
I: How do you know it is subtraction?
B1: Well, the farmer has cows, right. He sold the cows, so it must be subtraction.
I: How do you get 19?
B1: [At the ones column.] 8 cannot subtract 9, borrow 1 from 3 so it is 18. 18 minus 9 is nine. [At the tens column.] 3 is borrowed 1, so it is 2 leftover. 2 minus 1 is 1.
I: How do you calculate 18 – 9?
B1: Keep 9 in mind and count 9 [Pause, then fold each fingers while counting.] 10, 11, 12, 13, 14, 15, 16, 17, 18. [The child counts 9 folded fingers for the answer.]

<u>Boy 2 (B2)</u>

The child could read all seven problems. At first, he could not solve Problem 5 because he did not understand the context in the problem and he did not want to do it, so we went on to the next problem. After the child finished solving Problem 7, we went back to the problem 5 and the child finally solved the Problem 5 correctly. He used effective solution strategies to solve five problems but he got correct answers to four problems. He could not solve Problem 2 and Problem 6. This boy scored 14 points.

Problem 1: Classroom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes?

The narrative for this Problem was missing but there was some information from the observation. For this problem, this child counted on from larger number by using fingers. This child used unfolded fingers method. The child was carrying and regrouping at the tens column.

Problem 2: Manee has 23 apples. Mana has 15 more apples than Manee does. How many apples does Mana have?

I: Tell me how you solve this problem.
Boy 2 (B2): I use subtraction.
I: How do you know it is subtraction?
B2:??
I: Where in the problem that help you decide to subtract?
B2: [Think.] The word "more than."
I: OK., so you know it is subtraction by this word, right?
B2: Yes.
I: How do you get 12?
B2: Umm, [At the ones column.] 3 minus 5 equals 2. [At the tens column.] 2 minus 1 equals 1. The answer is 12.
I: Why 3 minus 5 is 2?
B2:??

Problem 3: Suda had 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now?

I: Could you describe your solution to this problem?
B2: I use addition.
I: How do you know it is addition?
B2: Because mother gave 15 Baht more, so I decide to add.
I: Tell me what is your answer;
B2: 73
I: Could you explain to me out loud how you get 73?
B2: [At the ones column.]First, 8 plus 5 is 13. Now I get 13. Then, put 1 on five. [At the tens column.] 5 plus 1 equals 6 and plus one more equals 7. So the answer is 73.

This child used counting on from larger method. He counted in his mind.

Problem 4: There are 42 chickens. 25 are male. How many are female?

I: How do you solve this problem?
B2: Subtract
I: How do you know it is subtraction?
B2: I know from the problem that asked for how many female chickens were leftover.
I: What is your answer?
B2: 23
I: Tell me how you get the answer.
B2: 2 cannot subtract 5, so 5 minus 2 is 3. [At the tens column.] 4 minus 2 is 2. The answer is 23.

The child knew the solution to the problem but he had error with the calculation. The child did not use borrowing.

Problem 5: Suda's pencil is 20 centimeter long. How much longer is Suda's pencil than Manee's pencil?

I: What did the problem asked you to find?
B2: How much longer Suda's Pencil is.
I: Is it addition or subtraction?
B2: Subtraction.
I: How do you know it is subtraction?
B2: From the world "longer" in the problem.
I: OK., what is your answer?
B2: 11
I: Could you explain out loud how you got 11?
B2: Yes, 0 cannot subtract 9, borrow 1 from 2. It is 10. [At the ones column.] 10 minus 9 is 1. [At the tens column.] 2 is borrowed 1, so 1 is leftover. The 1 has nothing to add, so pull the 1 down. The answer is 11.

The child did this problem by counting mentally. He remembered that 10 – 9 = 1 so it did not take too long for him to get the answer.

Problem 6: Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 balloons altogether?

This child could not do this problem.

Problem 7: The farmer has 38 cows. He sells 19 cows. How many cows are left?

I: Tell me how you find the solution to this problem.
B2: I use subtraction and I get 19 for an answer.
I: Well, how do you know it is subtraction?
B2: I read the problem and it asked how many leftover cows were. From the word "leftover", I decide to use subtraction.
I: How do you get 19?
B2: [At the ones column.] 8 cannot subtract 9, borrow 1 from 3, so it is 18. Then, count from 9 to 18 by using fingers.
I: Tell me how you count your fingers.
B2: 9 [Pause, then begin folding each finger while counting.] 10, 11, 12, 13, 14, 15, 16, 17, 18 [9 fingers are folded, the child then counted 9 folded for the answer.] It is 9.
I: What's next?
B2: [At the tens column.] 2 is leftover. 2 minus 1 is 1. The answer for this problem is 19.

The child used counting up from with fingers to subtract.

<u>Girl 3 (G3)</u>

This girl did not use paper and pencil in calculation. She calculated in her mind. The girl said that she counted by picturing fingers or counters in her mind. This girl used effective solution strategies to solve four problems. The girl could not solve Problem 2, 5 and 6 because she misunderstood the keywords in the problems. She scored 12 points.

Problem 1: Classroom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes?

I: How do you solve this problem so you get 52?

G3: [At the ones column.] 4 plus 8 is 12. Put one above the tens column. [At the tens column.] 2 plus 2 is 4, plus one more so it is 5.

I: Tell me how you get 12.

G3: I count mentally.

I: Tell me out loud, what do you do in your mind?

G3: I count from 8 [Pause, then begin folding each finger while counting.], 9, 10, 11, 12. [The child peaks the last number word in a counting sequence for the answer.]

I: I don't see you use fingers, counters, and paper and pencil. How do you do without these things?

G3: Well, I count and remember in my mind. I picture fingers in mind.

The child understood the problem. She used correct solution by using counting mentally.

Problem 2: Manee has 23 apples. Mana has 15 more apples than Manee does. How many apples does Mana have?

I: How do you solve this problem?

G3: Subtract

I: How do you know it is subtraction?

G3: I read the problem.

I: Where in the problem told you that it is subtraction?

G3: Right here, "Mana has more 15 apples than Manee"

I: So, you decide to subtract from this sentence, right.

G3: Yes

I: OK, tell me how you find the answer.

G3: [At the ones column.] 3 cannot minus 5, borrow 1 ten from 2. 3 become 13. 10 minus 5 is 5. Then, add 3 with 5, so it is 8.

I: What is next?

G3: [At the tens column.] 2 that is borrowed 1, 1 is leftover. 1 minus 1 is 0. The answer for this problem is 8.

The child misunderstood the context in the problem. When she saw the word "more than", she thought that it must be subtraction. However, she knew when to borrow and regroup. She also used ten-based method in subtraction.

Problem 3: Suda had 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now?

I: How do you know it is addition?

G3: Right here, "mother gives 15 Baht more."

I: OK, what is the answer?

G3: 7 3

I: Explain to me, how you get 73.

G3: [At the ones column.] Add 8 with 5, get 13, three carrying one. [At the tens column.] 5 plus 1 is 6, plus one from the carrying so it is 7. The answer is 73.

I: and, how do you get 13.
G3: 8 plus 5.
I: How?
Child: Add mentally. Keep 8 in mind and picture five fingers and count from 8. 8 [Pause, begins folding each fingers while counting.], 9, 10, 11, 12, 13. [Five fingers are folded, the child speaks the last number word in the counting sequence for the answer.]

The child used counting on from smaller. She counted mentally.

Problem 4: There are 42 chickens. 25 are male. How many are female?

I: Could you describe the solution to this problem for me?
G3: I use subtraction.
I: How do you know that?
G3: I read from the problem, from the word "leftover" right here [Points at the problems.]
I: How do you get 17?
G3: [At the ones column.] 2 cannot subtract 5, borrow 1 ten from 4. 10 minus 5 is 5. 5 plus 2 is 7. [At the tens column.] 3 minus 2 is 1. Thus, the answer is 17.

The child used ten-based method to subtract.

Problem 5: Suda's pencil is 20 centimeter long. How much longer is Suda's pencil than Manee's pencil?

I: Tell me how you solve this problem.
G3: I use addition.
I: Why do you decide to add?
G3: Right here, the word "much longer". The problem wantsed to know how much longer Suda's pencil is, right, so I use addition to find the answer.
I: OK, and what is the answer?
G3: 29.
I: How do you get 29?
G3: [At the one column.] 0 plus 9 equals 9. [At the tens column.] 2 did not have addend so it is 2. The answer is 29.

The child did not understand the context in the problem very well. She decided that the word 'much longer' meat addition.

Problem 6: Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 balloons altogether?

I: Do you add or subtract?
G3: Add
I: Tell me why you decide to add.
G3: Because the problem wants to know how much more balloons that Mana wants. The word "much more" makes me decide to add.
I: OK, what is your answer?
G3: 46

The child did not understand the context in the problem. She added because the word 'much more'.

Problem 7: The farmer has 38 cows. He sells 19 cows. How many cows are left?

I: Tell me how you find the solution to this problem, add or subtract?

G3: Subtract.

I: Where in the problem that helps you decide to subtract?

G3: The word "sell" and "leftover".

I: What is the answer?

G3: 19

I: Could you tell me how you get 19?

G3: [At the ones column.] 8 cannot subtract 9, borrow 1 ten from 3. 10 minus 9 is 1, one plus eight is nine. [At the tens column.] 3 is borrowed 1, so 2 is leftover. 2 minus 1 is 1. The answer is 19.

The child used the word in the problem to help her decide whether to add or subtract. She used ten-based method to subtract.

Girl 4 (G4)

This girl used effective solution strategies and got correct answers to six problems. She could not solve Problem 2 because she misunderstood the keywords in the problems. She scored 18 points. She spent a lot of time on Problem 5 because she did not understand the context in the problem. She read the problem 5 many times to try to understand the context in the problem and finally she solved the problem 5 correctly.

Problem 1: Classroom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes?

I: Tell me your solution to the problem.

Girl 4 (G4): Plus 24 with 28. [At the ones column.] 4 plus 8 is 12. Put 2 at the ones column and carry 1 to the tens column. [At the tens column.] 2 plus 2 is 4, plus 1 from the carrying so it is 5.The answer is 52.

I: How do you get 12?

G4: I remember it.

The child could remember the number fact

Problem 2: Manee has 23 apples. Mana has 15 more apples than Manee does. How many apples does Mana have?

I: Could you describe your solution for this problem?

G4: Yes, subtraction.

I: Why do you subtract?

G4: Because the problem told that Mana has more 15 apples than Manee does, so I decide to subtract.

I: Any reasons?

G4: No

I: Tell me how you get the answer.

G4: I get 8. [At the ones column.] 3 cannot subtract 5, borrow 2. 2 is in the tens column, so I borrow 1 ten from 2. 3 becomes 13. 13 minus 5, I count from 5 [Pause, then fold each fingers while counting.] 6, 7, 8, 9, 10, 11, 12, 13 [8 fingers are folded, the child counts 8 folded fingers for the answer. I get 8. [At the tens column.] 2 is borrowed 1 so 1 is leftover. 1 minus 1 is 0. The answer is 8.

The child thought that the word "more than" means subtraction. However, she used counting up from method to solve the problem.

Problem 3: Suda had 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now?

I: Tell me your solution to this problem.
G4: Well, I used addition and I get 73.
I: How do you get 73?
G4: [At the ones column.] 8 plus 5 is 13, trade one ten to the tens column. [At the tens column.] 1 plus 5 is 6, plus one from the trading so it is 7. The answer is 73.
I: So, how do you get 13?
G4: Count mentally from 8. 8 [Pause, then fold each finger while counting.] 9, 10, 11, 12, 13 [5 fingers are folded, the child speaks the last number word in the counting sequence for the answer.] I get 13.

The child used counting on from larger number given in the problem. She counted mentally without using counters or fingers.

Problem 4: There are 42 chickens. 25 are male. How many are female?

I: Tell me how you find the answer for this problem.
G4: [At the ones column.] 2 cannot subtract 5, borrow 1 ten from 4. 2 becomes 12. 12 minus 5, count from 5 [Pause, then fold each finger while counting.] 6, 7, 8, 9, 10, 11, 12 [7 fingers are folded, the child counts 7 folded fingers for the answer.] [At the tens column.] 4 is borrowed 1, so 3 is leftover. 3 minus 2 is 1. The answer is 17.

The child counted up from the smaller number to the larger number in the problem. She counted by using fingers.

Problem 5: Suda's pencil is 20 centimeter long. How much longer is Suda's pencil than Manee's pencil?

I: Do you subtract or add for this problem?
G4: Subtract.
I: How do you know it is subtraction?
G4: Because of how much longer, so I decide to subtract.
I: What is the answer?
G4: 11
I: How do you get 11?
G4: [At the ones column.] 0 cannot subtract 9, borrow 1 ten from 2. 10 minus 9 is 1. [At the tens column.] 2 is borrowed 1, so 1 is leftover. The answer is 11.

Problem 6: Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 balloons altogether?

I: How do you know it is subtraction?
G4: From how many more balloons that Mana wanted.
I: What is the answer?

G4: 16
I: Could you explain to me how you get 16?
G4: Yes, [At the ones column.] 1 cannot subtract five, borrow 1 ten from 3. 1 becomes 11. Then, count from 5 to 11. 5 [Pause, then fold each fingers while counting.] 6, 7, 8, 9, 10, 11 [6 fingers are folded, the child counts 6 folded fingers for the answer.], get 6. [At the tens column.] 3 is borrowed 1, so 2 is leftover. 2 minus 1 is 1. The answer is 16.

The child used counting up from with fingers.

Problem 7: The farmer has 38 cows. He sells 19 cows. How many cows are left?

The interview of this problem was missing. The child could do this problem by using an appropriate strategy. She used counting up strategy with fingers.

Average Achievement

Boy 5 (B5)

He read all seven problems without assistance from the interviewer. He used effective solution strategies and got correct answers to six problems. He scored 18 points. He could not solve Problem 2 because he misunderstood the keywords in the problems.

Problem 1: Classroom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes?

I: How do you know it was addition?
Boy 5 (B5): From the problem, how many students in the two classrooms are altogether.
I: Tell me what is the answer?
B5: 52
I: Tell me how you get 52.
I: How much in the ones column?
B5: 4 plus 8 is 12.
I: How do you get 12?
B5: Count from 4 [Pause, then fold each fingers while counting.] 5, 6, 7, 8, 9, 10, 11, 12 [8 fingers are folded, the child speaks the last number word in the counting sequence for the answer.]
I: Good, what is next?
B5: Put 1 at the top of the tens column and put 2 in the answer space of the ones column. Adding all the number at the tens column, 2 plus 1 is 3, plus 2 is 5. The answer is 52.

The child counted on from smaller number by using fingers.

Problem 2: Manee has 23 apples. Mana has 15 more apples than Manee does. How many apples does Mana have?

I: Why do you subtract?

B5: I read the problem. At the sentence "Mana has more apples", so I decide to subtract.

I: Tell me how you get 8.

B5: [At the ones column.] 3 cannot subtract 5 so take 1 from 2 and put it with 3 to make 13. 13 minus 5 is 8.

I: Could you explain to me how you get 8?

B5: Well, 10 minus 5 is 5. There is 3. 3 plus 5 is 8.

I: OK. What do you do in the tens column?

B5: 2 is borrowed 1 so 1 is leftover. 1 minus 1 is 0. The answer is 8.

The child did not understand the context in the problem. He decided to make subtraction because of the word "more than". However, he used ten-based method to subtract at the ones column.

Problem 3: Suda had 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now?

I: What do you do with this problem, add or subtract?

B5: Add.

I: How do you know it is addition?

B5: Because Suda had 58 Baht, mother gave her 15 more, so it must be addition.

I: What is your answer?

B5: 73.

I: Tell me how you get 73.

B5: Well, [At the tens column,] 8 plus 5 five is 13. Put 1 at the top of the tens column and put a 3 in the answer space of the ones column. Then, [At the tens column.] 5 plus 1 is 6, 6 plus 1 more is 7. Put a 7 in the answer space of the tens column. Therefore, the answer is 73.

I: At the ones column, tell me how you get 13.

B5: Well, I count my fingers. Keep eight in mind plus 5 more. 8 [Pause, then fold each finger while counting.] 9, 10, 11, 12, 13 [5 fingers are folded, the child speaks the last number word in the counting sequence for the answer.

At the ones column, the child used counting on from larger with fingers to find the answer. He counted by folding his fingers down.

Problem 4: There are 42 chickens. 25 are male. How many are female?

I: Tell me how you solve this problem.

B5: I use subtraction.

I: How do you know that it is subtraction?

B5: From the problem, how many female chickens were leftover.

I: What is the answer?

B5: 17.

I: What is your solution?

B5: I use fingers. [At the ones column.] 2 has no value. Take 1 from 4 and put it with 2 to make 12. Then 12 minus 5 is 7. Put a 7 in the space of the ones column.

[At the tens column.] 4 is borrowed 1, so 3 is leftover. 3 minus 2 is1. Put a 1 in the answer space of the tens column. The answer is 17.
I: At the ones column, how did you calculate 12 minus
B5: Well, I used fingers. 10 minus 5 is 5, 5 plus 2 here, so it was 7.
The child used ten-based method to subtract in the ones column. He knows how to trade and to regroup the numbers at the tens column. However, he misunderstood that 2 had no value.

Problem 5: Suda's pencil is 20 centimeter long. How much longer is Suda's pencil than Manee's pencil?
I: Do you add or subtract for this problem?
B5: Subtract
I: What is your answer?
B5: 11
I: How do you get 11?
B5: Well, 0 has no value, borrow 1 ten from 2, put it with 0 to make 10. 10 minus 9 is 1. Pull 1 ten at the tens column down. The answer is 11.
The number in this problem was easy so the child solve this problem quickly.

Problem 6: Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 balloons altogether?
I: How do you solve this problem?
B5: 15 has no addend, 15 [Pause, then fold each finger while counting.] 16, 17, 18, 19, 20, 21, 21, ..., 31. The answer is 16. Because 15 has no addend so it has to find an addend.
I: You try to find the number to add to 15 to make 31, right?
B5: Yes.
I: How do you know it is 16?
B5: I count from 15 to 31, and count fingers for the answer.

Problem 7: The farmer has 38 cows. He sells 19 cows. How many cows are left?
I: Could you describe your solution to this problem?
B5: I use subtraction.
I: Where in the problem told you that it is subtraction?
B5: [The child read.] "The farmer has 38 cows, he sell 19 cows. How many cows are leftover?" The word "sell" and "leftover" told me that it is subtraction.
I: What is the answer?
B5: 19
I: Tell me how you get it.
B5: [At the ones column.] 18, 10 take away 9 is 1, plus 8 here so it is 9. [At the tens column.] 2 minus 1 is 1. The answer is 19.
I: Where is 18 from?
B5: Well, 8 cannot subtract 9, take 1 ten from 3, put it with 8 to make 18.
The child used ten-based method to subtract in the ones column. The number in the tens column was easy so the child knew how the result was.

Boy 6 (B6)

The child read all problems without help from the interviewer but the explanation to the solution strategies was not clear. He used effective solution strategies and got correct answers to four problems. The child could not solve Problem 2, 5, and 6 because he misunderstood the keywords in the problems. He scored 12 points.

Problem 1: Classroom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes?

I: Tell me your solution to this problem.
Boy 6 (B6): I use addition.
I: How do you know that it is addition?
B6: Because it is the combination of two classrooms.
I: What is the answer?
B6: 52
I: Tell me how you get 52.
B6: I use carrying.
I: Could you explain to me how you do it?
B6: 8 cannot plus 4, have to trade. Count 8 [Pause, then fold each finger while counting.], 9, 10, 11, 12 [4 fingers are folded, the child speaks the last number word in the counting sequence for the answer.], get 12. Then trade 1 to the tens column. 2 plus 2 is 4, plus 1 more so it is 5. The answer is 52.

The child used count on from larger with fingers to add the numbers at the ones column.

Problem 2: Manee has 23 apples. Mana has 15 more apples than Manee does. How many apples does Mana have?

I: Which operation did you use, addition or subtraction?
B6: Subtraction
I: Tell me how you solve it.
B6: 3 cannot subtract 5, take 1 ten from 2 to make 3 become 13. 13 minus 5 is 8, then 1 minus 1 is 0.
I: Could you tell me how you get 8?
B6: Count 3 [Pause] 4, 5 [2 fingers are folded and count 8 unfolded fingers for the answer.]

The child did not understand this problem. However, he knew how to borrow the numbers and regroup them. This child has different method for finding the solution.

Problem 3: Suda had 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now?

I: What is the answer for this problem?
B6: 73
I: How do you get 73?
B6: 8 plus 5 is 13, 5 plus 1 is 6, plus 1 from trading is 7.
I: Tell me why 8 plus 5 equals 13.

B6: Count 8 [Pause, then fold each finger while counting.], 9, 10, 11, 12, 13 [5 fingers are folded, the child speaks the last number word in the counting sequence for the answer.]

The child used counting on from larger with fingers to find the answer for the ones column.

Problem 4: There are 42 chickens. 25 are male. How many are female?

I: Could you tell me why you decide to subtract?

B6: Because the word "leftover" in the problem.

I: What is the answer?

B6: 17.

I: Tell me how you find it.

B6: Count fingers.

I: Tell me how you count.

B6: 12 minus 5. 2, [Pause, then fold each fingers while counting.] 3, 4, 5 [The child folds three fingers and counts seven unfolded fingers for the answer.], so it is 7. 3 minus 2 is 1.

The child used counting up from given.

Problem 5: Suda's pencil is 20 centimeter long. How much longer is Suda's pencil than Manee's pencil?

I: Which operations do you use?

B6: Addition.

I: How do you know?

B6: The word "longer than" and I get 29 for the answer.

I: Tell me how you get 29.

B6: [At the ones column.] 9 plus 0 is 9. [At the tens column.] 2 does not have addend, so pull it down. The answer is 29.

The child used addition instead of subtraction.

Problem 6: Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 balloons altogether?

I: How do you solve this problem?

B6: I use addition.

I: Tell me how you know it is addition.

B6: [Read the problem again.] Umm, the sentence "how many more balloons does he want" in the problem.

I: What is the answer?

B6: 46

I: Could you explain how you get 46?

B6: [At the ones column.] 5 plus 1 is 6. [At the tens column.] 3 plus 1 is 4. The answer is 46.

The child used addition instead of subtraction because the word "many more" in the problem. The solution for the problem was easy because the child used to add those numbers before.

Problem 7: The farmer has 38 cows. He sells 19 cows. How many cows are left?

I: Do you use addition or subtraction?

B6: Subtraction.

I: How do you know it was subtraction?

B6: The word "leftover" in the problem.

I: What is your answer?

B6: 19.

I: How do you get 19?

B6: [At the ones column.]18 minus 9 is 9. [At the tens column.]3 is borrowed 1, so 2 is leftover. 2 minus 1 is 1.

The child used the same strategy as in the Problem 4 to subtract the number in the ones column.

Girl 7 (G7)

This girl used effective solution strategies and got correct answers to five problems. She could not solve Problems 2 and 6. The girl scored 15 points. She was capable of reading seven problems but she was not clear in giving explanations.

Problem 1: Classroom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes?

I: Tell me how you solve this problem?

Girl7 (G7): Add

I: How do you know it is addition?

G7: At the sentence "how many students in the two classrooms are altogether."

I: What is the answer?

G7: 52

I: How do you get 52?

G7: I use my hands.

I: Tell me how you use your hand.

G7: [At the ones column.] Keep 8 in mind, count 8 [Pause, then fold each finger while counting.], 9, 10, 11, 12 [The child speaks the last number word in the counting sequence for the answer.], get 12, trade 1. Add all the tens column. 2 plus 2 is 4, plus 1 from the trading so it is 5. The answer is 52.

The child used counting on from smaller by using fingers.

Problem 2: Manee has 23 apples. Mana has 15 more apples than Manee does. How many apples does Mana have?

I: How do you solve this problem?

G7: Subtraction

I: How do you get 8?

G7: [At the ones column.] 3 cannot subtract 5, borrow 1 from 2. 2 now is 1. Then, count from 5 up to 13. 5 [Pause, then fold each finger while counting, 6, 7, 8, 9, 10, 11, 12, 13 [8 fingers are folded, the child counts 8 folded fingers for the answer.] [At the tens column.] 1 minus 1 is 0. The answer is 8.

The child used subtraction for this problem. The child gave reason that then word "more...than" in the problem helped her to decide to add.
The child used counting up from given with fingers to find the answer at the ones column.

Problem 3: Suda had 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now?

I: Which solution do you use to solve this problem, addition or subtraction?
G7: Addition
I: Where in the problem tell you that it is addition.
G7: At "mother give her 15 more". More means increase. Increasing is addition.
I: What is the answer?
G7: 73
I: How do you get 73?
G7: Count by using fingers.
G7: [At the ones column.] 8 plus 5 is 13. 8 and 9, 10, 11, 12, 13 [Count by folding 5 fingers and the last number word in the counting sequence is the answer.], get 13 trade 1. [At the tens column.] 5 plus 1 is 6. 6 plus 1 from the trading is 7. The answer is 13.

Problem 4: There are 42 chickens. 25 are male. How many are female?

I: For this problem, do you add or subtract?
G7: Subtract
I: How do you know it is subtraction?
G7: Because [The child read the problem again.], from the question "how many female chickens are leftover" The word "leftover" means subtraction.
I: What is the answer?
G7: 17
I: Explain to me how you get 17.
G7: [At the ones column.] Count fingers. 2 cannot subtract 5, borrow 1 from the tens column. 4 is borrowed 1 so 3 is leftover. 2 become 12. 12 minus 5 is 7. [At the tens column.] 3 minus 2 is 1. The answer is 17.
I: Why 12 minus 5 equal 7?
G7: 5 [pause], 6, 7, 8, 9, 10, 11, 12 [The child folds 7 fingers while counting, then counts 7 folded fingers for the answer.

The child used counting up from given with fingers for finding the result at the ones column.

Problem 5: Suda's pencil is 20 centimeter long. How much longer is Suda's pencil than Manee's pencil?

I: For this problem, do you add or subtract?
G7: Subtreact
I: Where in the problem tell you that it is subtraction?
G7: At the word "longer than" in the problem.
I: What is your solution?
G7: 11

I: How do you get 11?
G7: Use brain.
I: Could you tell me what are you doing in your brain?
G7: Use fingers.
I: Could you explain to me how you use fingers?
G7: OK., [At the ones column.] 0 cannot subtract 9, borrow 1 from the tens column so 1 become 10. 10 minus 9 is 1. [At the tens column.] 2 is borrowed 1 so 1 is leftover. 1 has no subtrahend so it is 1. The answer is 11.

The child used mental strategy to solve this problem because the numbers in this problem is easy.

Problem 6: Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 balloons altogether?

I: Do you add or subtract?
G7: Add
I: Why do you use addition?
G7: Well, from how many more balloons does Mana wanted.
I: What is the answer?
G7: 46
I: How do you get 46?
G7: I remember 5 plus 1 is 6 and 3 plus 1 is 4, so the answer is 46.

The child could not do this problem because she thought that if someone needs something more, you have to add. Actually, this idea is also right for this problem. If the child implemented correctly by using counting up from 15 to 31, it would not be wrong and the child would know from the sequence in counting number.

Problem 7: The farmer has 38 cows. He sells 19 cows. How many cows are left?

I: For this problem, do you add or subtract?
G7: Subtract
I: Why do you subtract?
G7: [Look and Read the problem again.] from "how many leftover cows are" so I decide to subtract.
I: What is the answer?
G7: 19
I: Tell me how you find it.
G7: [At the ones column.] 8 cannot subtract 9, borrow 1 from the tens column so 8 becomes 18. 18 minus 9 is 9. [At the tens column.] 3 in the tens column is borrowed by 1 so 2 is leftover. 2 minus 1 is 1. The answer is 19.
I: How do you solve 18 minus 9?
G7: Count fingers. 9 [Pause, then fold each finger while counting.] 10, 11, 12, 13, 14, 15, 16, 17, 18 [9 fingers are folded, the child counts 9 fingers for the answer.]

The child used counting up from 9 to 18 by using fingers. She counted fingers the same as she counted in the Problem 4 but only different in numbers.

Girl 8 (G8)

This girl was able to read all seven problems. She solved problems mentally. She used effective solution strategies to solve five problems but got correct answers to only four problems. She could not do Problem 5 and 6. She scored 14 points.

Problem 1: Classroom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes?

I: For this problem, do you add or subtract?

Girl 8 (G8): Add and get 71.

I: Do you solve mentally or use fingers?

G8: Mentally

The girl understood what the problem asked to find. She used correct strategy but she miscopied the number from the problem, so she got an incorrect answer. When I asked her about how she knew it was addition, she did not answer although I changed several questions.

Problem 2: Manee has 23 apples. Mana has 15 more apples than Manee does. How many apples does Mana have?

I: Do you add or subtract for this problem?

G8: Add

I: How do you know?

G8: Because this one is 8. [I have no idea what she wanted to tell, she did not give more explanation for this.]

I: OK., what is the answer?

G8: [At the ones column.] 38, 3 plus 5 is 8. [At the tens column.] 2 plus 1 is 3. The answer is 38"

The girl could understand this problem and got a correct answer but she did not give a clear explanation.

Problem 3: Suda had 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now?

I: Do you add or subtract for this problem?

G8: Add

T: How do you know it is addition?

G8: The problem told "Manee has 58 Baht and her mother gave her 15 Baht more", and the problem asked how much money Suda has, so I decide to add.

I: Well, what is the answer?

G8: 73

I: Could you tell me how you get 73?

G8: [At the tens column.] 8 plus 5 is 13, trade 1 to the top of the tens column. [At the tens column.] 5 plus 1 is 6, plus 1 from the trading is 7. The answer is 73.

I: How do you solve 8 plus 5?

G8: 8 plus 5, and I count mentally. Count from 8 and continue counting 5 more. 8 [pause], 9, 10, 11, 12, 13, so I get 13 [The child speaks the last number word in the counting sequence for the answer.]

Problem 4: There are 42 chickens. 25 are male. How many are female?

I: Tell me how you solve this problem?

G8: Subtract

I: How do you know it is subtraction?

G8: I know from the word "leftover" in the problem.

I: What is the answer?

G8: 17

I: Where is 17 from?

G8: Borrowing 1 from 4, so 2 become 12 and 4 has 3 leftover. [At the ones column.] 12 minus 5 is 7. [At the tens column.] 4 minus 2 is 1..oh! 3 minus 2 is 1. The answer is 17.

I: Why do you have to borrow?

G8: Because 2 is not enough to subtract 5.

I: How do you get 7 in the ones column?

G8: I calculate mentally.

I: Could you tell me how?

G8: Count up from 5 to 12 and count the increasing. 5 [pause], 6, 7, 8 ,9 10, 11, 12, so it is 7.

The child used counting up from by counting mentally.

Problem 5: Suda's pencil is 20 centimeter long. How much longer is Suda's pencil than Manee's pencil?

I: Tell me how you solve this problem.

G8: Add

I: Why do you know it is addition?

G8: Because the word "longer than" in the problem.

I: What is the answer?

G8: 29

I: How do you get 29?

G8: I can think immediately because it is easy. It is a tens number.

The child used addition instead of subtraction because of the keyword that she read from the problem.

Problem 6: Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 balloons altogether?

I: For this problem, do you add or subtract?

G8: Add and I get 46.

Problem 7: The farmer has 38 cows. He sells 19 cows. How many cows are left?

I: Do you add or subtract for this problem?

G8: Subtract

I: How do you know it is subtraction

G8: [Read the problem again.] The word "leftover" in the problem.

I: What is the answer?

G8: 16
I: How do you get 16?
G8: [At the ones column.] 8 cannot subtract 9, borrow 1 from 3 so 8 becomes 18. [At the tens column.] 2 is leftover from 3. 2 minus 1 is 1.
I: Where is 6 from?
G8: 8 minus 9 is 6
I: Why do you get 6?
G8: Because 18 minus 9 is 6.
I: Do you count up from 9?
G8: Yes

She used counting up from without using fingers. She had error in calculation.

Low Achievement

Boy 9 (B9)

This boy had difficulty in reading but he understood the problems. For some problems, the boy got a correct answer but he could not explain how he got an answer. The interviewer had to ask him step by step to get the explanation. From the observation, it seemed that this boy was not willing to solve problems or he did not like solving problems. He used effective solution strategies and got correct answers for four problems. He could not solve Problem 3, 4, and 6. He scored 12 points.

Problem 1: Classroom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes?

I: Do you add or subtract for this problem?
Boy 9 (B9): Add
I: What is the answer?
B9: 52
I: How do you get 52?
B9: I don't know.
I: Well, could you tell me where 52 from?
B9: I don't know.

The child got a correct answer but he could not tell or did want to tell how he get the answer.

Problem 2: Manee has 23 apples. Mana has 15 more apples than Manee does. How many apples does Mana have?

I: How do you solve this problem?
B9: I use addition.
I: How do you know it was addition.
B9: At the word "more apples than" in the problem.
I: What is the answer?
B9: 38

I: Could you explain to me how you get it?
B9: [At the ones column.] Count from 3. 3, [pause]4, 5, 6, 7, 8 [The child speaks the last number word in the counting sequence for the answer.], get 8. [At the tens column.] Count from 2. 2, [pause], 3, get 3. The answer is 38.

The child used count on from first (either smaller or larger number).

Problem 3: Suda had 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now?

I: What is the answer?
B9: 43
I: Could you explain to me how you get 43?
B9: ??
I: Well, tell me how you get 3 in the ones column.
B9: Count 8, [pause] 7, 6, 5, so I get 3 [The child folds 3 fingers and counts 3 folded fingers for an answer.]
I: What about 4 in the tens column, how do you find it?
B9: Well, I count down from 5, [pause] 4, 3, 2, 1, so I get 4 [The child folds 4 fingers and counts 4 folded fingers for the answer.]

For this problem, the child used subtraction rather than addition. He used counting down strategy for finding the answer in the ones and the tens column.

Problem 4: There are 42 chickens. 25 are male. How many are female?

I: Tell me how you solve this problem?
B9: Add
I: What is the answer for this problem?
B9: 67
I: Where is 7 in the ones column from?
B9: 2 plus 5. Count from 2 [pause], 3, 4, 5, 6, 7 [Fold five fingers], so get 7 [The child speaks the last number word in the counting sequence for the answer.]
I: Where is 6 in the tens column from?
B9: 6 is from counting 4 [Pause], 5, 6 [Fold 2 fingers, and speaks the last number word in the counting sequence for the answer.]

The child used addition instead of subtraction.

Problem 5: Suda's pencil is 20 centimeter long. How much longer is Suda's pencil than Manee's pencil?

I: Do you add or subtract for this problem?
B9: Subtract
I: How do you know that it is subtraction?
B9: ??
I: Where in the problem tell you that it is subtraction?
B9: The word "longer than" in the problem.
I: What is the answer?
B9: 11
B9: [At the ones column.] Borrowing 1 from 2, so 1 becomes 10. 10 minus 9 is 1. [At the tens column.] 2 is borrowed 1 so 1 is left over. The answer is 11.

Problem 6: Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 balloons altogether?

I: Do you add or subtract?
B9: Add
I: What is the answer?
B9: 46
I: Where 6 in the ones column from?
B9: 5 and 1 is 6.
I: And where is 4 from?
B9: 1 and 3
I: Do you count or remember ?
B9: I count on from 5 and one more, 5 [pause] 6, and count on from 3 and one more, 3 [pause], 4.

The child could not do this problem. He used addition rather subtraction. He used counting on from first to find the answer in both columns.

Problem 7: The farmer has 38 cows. He sells 19 cows. How many cows are left?

I: Tell me how you solve this problem.
B9: I use subtraction.
I: How do you know?
B9: At the word "sell" in the problem.
I: What is the answer?
B9: 19
I: Could you explain to me why you get 19?
B9: 3 is borrowed 1, so 8 becomes 18. 18 minus 9 is 9.
I: Where is 1 from?
B9: ??
I: Ok. How did you solve 18 minus 9
B9: Count down from 18.

The child used counting down from.

<u>Boys 10 (B10)</u>

This boy had difficulty in reading. He tried to read all seven problems but it was very slow. He had to spell out some difficult words (Thai words). Therefore, the interviewer read some problems to him. He did the calculation in a row form rather than column form. He used effective solution strategies to solve five problems but he got correct answers for four problems. The boy could not solve Problems 5 and 6. He earned 14 points.

Problem 1: Classroom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes?

I: Do you add or subtract?
Boy 10 (B10): Add
I: Why do you decide to add?

B10: The problem wanted to know how many students were in the two classrooms.
I: Could you explain to me how you get 52?
B10: 8 plus 4.
I: Umm...how you find 8 plus 4 equals 12?
B10: I calculate mentally.

I could not continue the interview with this boy because he did not response for many questions that I asked him. He was quite and was not willing to answer any questions.

Problem 2: Manee has 23 apples. Mana has 15 more apples than Manee does. How many apples does Mana have?

I: Tell me how you solve this problem?
B10: Add
I: Where in the problem let you know that it is addition?
B10: The word "have" in the problem.
I: What the answer do you get?
B10: 38
I: How do you find 38?
B10: [At the ones column.] 3 plus 5 is 8. [At the tens column.] 2 plus 1 is 3.
I: Do you count your fingers?
B10: No, I calculate mentally.

Problem 3: Suda had 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now?

I: Do you add or subtract?
B10: Add
I: Where in the problem telling you that it is addition?
B10: At "mother give her 15 Baht more" in the problem.
I: What is the answer?
B10: 73. 3 is from 8 plus 5, count on mentally from 8 [pause], 9, 10, 11, 12, 13, trade 1 to the tens row. Add all the tens together, so the answer is 73.

This boy counting on mentally from the smaller number.

Problem 4: There are 42 chickens. 25 are male. How many are female?

I: Do you add or subtract?
B10: Subtract
I: Where in the problem telling you it is subtraction?
B10: 25 chicken were male and how many female chicken were leftover.
I: What is the answer?
B10: 17
I: How do you get 17?
B10: ??
I: Where is 7 in the ones column from?

B10: 2 cannot subtract 5, borrow 1 from 3 so 2 become 12. Then count down from 12 to 5. 12, [Pause, then fold each finger while counting.] 11, 10, 9, 8, 7, 6, 5 [The child counts 7 folded fingers for the answer.], so it is 7.

I: And, where is 1 in the tens column from?

B10: 3 is borrowed 1, so 2 is leftover. 2 minus 1 is 1, so I get 1.

The child used counting down from.

Problem 5: Suda's pencil is 20 centimeter long. How much longer is Suda's pencil than Manee's pencil?

I: Tell me how you solve this problem?

B10: Add

I: Why do you add?

B10: Because of the word "longer than" in the problem.

I: What is the answer?

B10: 29

I: How do you get 29?

B10: 9 cannot add 0, so it is 9. 2 does not have an addend, so it was 2. The answer is 29.

The child used addition instead of subtraction because the word "longer than" in the problem.

Problem 6: Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 balloons altogether?

I: Do you add or subtract?

B10: Add

I: How do you know that it is addition?

B10: At how many more balloons do Mana wanted to have 31 balloons.

I: What is the answer?

B: 46

I: Tell me how you get 46.

B: 6 is from 5 plus 1, 4 is from 1 plus 3.

The child used addition rather than subtraction.

Problem 7: The farmer has 38 cows. He sells 19 cows. How many cows are left?

I: Do you add or subtract?

B10: Subtract

I: Where is the problem let you know that it is subtraction?

B10: At the word "sell" in the problem.

I: What is the answer?

B10: 21

I: Where is 1 from?

B10: 1 is from 8 minus 9, and 2 is from 2 minus 1.

The child understood the problem and used a correct solution but he had error in calculation. He did not use trading.

Girl 11 (G11)

The girl did not explain clearly how she did the problems. She used effective solution strategies for five problems but she did not get correct answers for any problems. She could not solve Problem 2 and Problem 6. She scored 8 points.

Problem 1: Classroom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes?

I: How do you get 43?

Girl 11 (G11): 3 is from 1 plus 4. 4 is from 2 plus 2.

The girl knew that it was addition but she miscopied the number from the problem. When asking 2 + 1 = 3, she said it was not right. It must be 1 + 2 = 3.

Problem 2: Manee has 23 apples. Mana has 15 more apples than Manee does. How many apples does Mana have?

I: How you solve this problem?

G11: Add

I: What is the answer?

G11: 39

I: How do you get 39?

G11: Count on from 3 to 5 more and count on from 2 to 1 more.

Problem 3: Suda had 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now?

I: Do you add or subtract?

G11: Add

I: What is the answer?

G11: 74

I: Where is 4 from?

G11: 4 is from 8 plus 5. Count on from 8 to 5 more, get 14. Put 4 at the bottom of the ones column and trade 1 to the tens column"

I: Where is 7 from?

G11: From 4 plus 1 from the trading, plus 2 more so it is 7.

The child knew how to add but she calculated wrong.

Problem 4: There are 42 chickens. 25 are male. How many are female?

I: Which operations do you use to solve this problem?

G11: Addition

I: How do you know?

G11: At there were 25 male chickens and the problem wanted to know how many female chicken were.

I: How do you get 70?

G11: [At the ones column.] 5 plus 5 is 10, trade 1. [At the tens column.] 4 plus 2 is 6, plus 1 from the trading so it is 7.

The girl used addition instead of subtraction because she did not understand the problem. She also miscopied the number from the problem.

Problem 5: Suda's pencil is 20 centimeter long. How much longer is Suda's pencil than Manee's pencil?

I: Do you add or subtract?
G11: Subtract
I: How do you know that it was subtraction?
G11: From Mana's pencil.
I: Could you tell me more how do you know?
G11: I don't know.
I: What is the answer?
G11: 29
I: How do you get 29?
G11: 2 has no subtrahend, so it is 2. 0 cannot subtract 9, so pull 9 down. The answer is 29.

The child knew it was subtraction but she had error in calculation. She always subtract the larger number from the smaller number. She did not know how to borrow the number from the tens column.

Problem 6: Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 balloons altogether?

I: Do you add or subtract?
G11: Add
I: How do you know?
G11: From how many more balloons that Mana wanted.
I: What is the answer?
G11: 46
I: Tell me how you get 46.
G11: [At the ones column.] 5 plus 1 is 6. [At the tens column.] 1 plus 3 is 4.

The child used addition instead of subtraction.

Problem 7: The farmer has 38 cows. He sells 19 cows. How many cows are left?

I: Do you add or subtract?
G11: Subtract
I: How do you know?
G11: I don't know.
I: What is the answer?
G: I can't do it. [Speaks hopelessly.]

The child knew it was subtraction but she cannot perform the calculation.

<u>Girl 12 (G12)</u>

The girl had difficulty in reading. The interviewer read two problems to her. From the observation, when she solved problems, she wrote the number from the problem on the paper and then she reread the problem and decided whether to add or subtract. As a result, she took about 7 to 8 minutes to solve each problem. She used effective solution strategies and got correct answers for five problems. She could not solve Problems 2 and 6. She scored 15 points.

Problem 1: Classroom 1 has 24 students. Classroom 2 has 28 students. How many students are in the two classes?

I: Do you add or subtract?
Girl 12 (G12): Add
I: How do you know that it is addition?
G12: The problem asked to find the total of students in two classes.
I: What is the answer?
G12: 52
I: How do you get 52?
G12: [At the ones column.] 4 plus 8 is 12, trade 1. [At the tens column.] 2 plus 2 is four, plus 1 from the trading, so it is 5.
I: How do you get 12?
G12: I think mentally.

Problem 2: Manee has 23 apples. Mana has 15 more apples than Manee does. How many apples does Mana have?

I: Do you add or subtract?
G12: Subtract
I: How do you know it was subtraction?
G12: From how many apples that Mana has.
I: What is the answer?
G12: 8
I: Tell me how you get 8.
G12: [At the ones column.] 3 cannot subtract 5, so borrow 1 from 2. Count from 5, [Pause, then fold each finger while counting.] 6, 7, 8, 9, 10, 11, 12, 13, get 8 [The child counts 8 folded fingers for the answer.] [At the tens column.] 2 is borrowed 1 so 1 is leftover, 1 minus 1 is 0. The answer is 8.

Problem 3: Suda had 58 Baht. Her mother gave Suda 15 Baht more for her birthday. How much money does Suda have now?

I: Do you add or subtract?
G12: Add
I: Why do you add?
G12: From how much money that Suda has.
I: How do you get 73?
G12: Count on from 8 to 5 more. 8 [Pause, fold each finger while counting.], 9, 10 , 11, 12, 13 [The child speaks the last number word in the counting sequence for the answer.], get 13 and trade 1 to the tens column. Add all the number in the tens column. The answer is 73.

Problem 4: There are 42 chickens. 25 are male. How many are female?

I: Do you add or subtract?
G12: Subtract

I: How do you know it is subtraction?

G12: The problem ask to find how many female chicken, so I use subtraction.

I: What is the answer?

G12: 17

I: How did you get 17?

G12: [At the ones column.] Counting down from 12. 12, [Pause, then fold each finger while counting.], 11, 10, 9, 8, 7, 6, 5 [7 fingers are folded, the child counts 7 folded fingers for the answer.], get 7 at the ones column. [At the tens column.] 4 is borrowed 1, so 3 is leftover. 3 minus 2 is 1. Therefore, the answer is 17.

Problem 5: Suda's pencil is 20 centimeter long. How much longer is Suda's pencil than Manee's pencil?

I: How do you know it is subtraction?

G12: From how much longer of Manee's pencil.

I: What is the answer?

G12: 11

I: Could you tell me how you get 11?

G12: [At the ones column.] I remember that 10 minus 9 is 1. [At the tens column.] 2 is borrowed 1, so 1 is leftover. Put a 1 down at the bottom of the tens column, so the answer is 11.

Problem 6: Mana has 15 balloons. How many more balloons does he have to put with them so he has 31 balloons altogether?

I: Do you add or subtract?

G12: Add

I: What is the answer?

G12: 46

I: How do you get 46?

G12: [At the ones column.] 3 plus 1 is 4. [At the tens column.] 5 plus 1 is 6.

Problem 7: The farmer has 38 cows. He sells 19 cows. How many cows are left?

I: How do you solve this problem?

G12: I use subtraction.

I: How do you know it is subtraction?

G12: Because the farmer sold the cows.

I: What is the answer?

G12: 19

I: Explain to me how you get 19.

G12: [At the ones column.] 8 cannot subtract 9, borrow 1 from 3 so 8 become 18. Count down from 18 [Pause, fold each finger while counting], 17, 16, 15, 14, 13, 12, 11, 10, 9 [The child counts 8 folded fingers for the answer.], so it is 9. [At the tens column.] 3 is borrowed 1, so 2 is leftover. 2 minus 1is 1. The answer is 19.