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Prospective teachers enter teacher education programs with previously formed conceptions of geometry and its teaching. These conceptions help them make sense of new information about teaching, their roles as teachers, and their translation of mathematics into learning activities. The purpose of this study was to investigate the relationships among preservice teachers' conceptions of geometry, conceptions of teaching geometry and classroom practices.

Ten preservice mathematics teachers completed a card sort task with an interview. They also participated in a videotape task which consisted of viewing three experienced geometry teachers on videotape. Four of these preservice teachers were observed eight times each during their professional internship experience. All interviews and observations were videotaped and transcribed for data analysis.

Results of this study indicated a complex relationship between the preservice teachers' conceptions of geometry and conceptions of teaching geometry. The preservice teachers could not discuss their conceptions of geometry without discussing the teaching of geometry. Their conceptions about geometry and their belief that geometry was linear in nature were so strong that these views became connected with their views of teaching geometry. Clearly, the preservice teachers' conceptions of geometry influenced their conceptions of teaching geometry and the teaching of subject matter influenced the preservice teachers' conceptions of geometry as well.

The relationship between the preservice teachers' conceptions of geometry and their classroom practices was directly influenced by the textbooks used. They believed geometry was ordered according to the textbook and their classroom practices also followed the textbook.

The relationship of the preservice teachers' conceptions of geometry teaching to classroom practices indicated that what the preservice teachers said they believed and what they did in the classroom were not always consistent. Their beliefs about teaching geometry rarely emerged in their classroom practices. Finally, these preservice teachers had an overwhelming concern with classroom management. This concern governed their thinking about teaching.

Relationships Among Preservice Teachers' Conceptions of Geometry, Conceptions of Teaching Geometry and Classroom Practices

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I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my dissertation to any reader upon request.

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Janet Maria Scholz, Author

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TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

LIST OF FIGURES

LIST OF APPENDICES

Dedicated in loving memory to my father

Howard Theodore Scholz

and in love and gratitude to my mother

Jean Elizabeth Scholz

RELATIONSHIPS AMONG PRESERVICE TEACHERS' CONCEPTIONS OF GEOMETRY, CONCEPTIONS OF TEACHING GEOMETRY AND CLASSROOM PRACTICES

CHAPTER I
THE PROBLEM

Introduction

Concern for the quality of mathematics instruction has resulted in a corresponding interest in teacher preparation programs. Several organizations (Holmes Group, 1986; Carnegie Forum, 1986) recommended reforms in teacher preparation programs. The mathematics education community, for example, recommends preservice teachers examine their conceptions of mathematics and how it should be taught (National Council of Teachers of Mathematics [NCTM], 1991; Mathematical Association of America [MAA], 1991). These suggested reforms assume the preservice teachers' understanding of mathematics directly influences classroom practices, but another factor has been overlooked. Classroom practices also depend on the conceptions of mathematics held by preservice teachers. What conceptions do they bring of mathematics and teaching? Because of the effect on classroom practices, knowing the conceptions of mathematics held by preservice teachers, and its relationship to teaching, is essential. This study examines the conceptions preservice teachers hold of geometry and its teaching, and their influence on classroom practices.

Shulman (1986a, 1986b, 1987) has written extensively on what teachers must know and the importance of this knowledge in learning to teach. "Teachers must have a knowledge of the subject matter that includes a personal understanding of the content as well as knowledge of ways to communicate that understanding" (Wilson, Shulman & Richert, 1987, p. 110). Teachers need to know a distinctive body of knowledge that incorporates both mathematics subject matter and pedagogy. Shulman (1986a) labeled this body of knowledge "pedagogical content knowledge." Mathematics teachers must know the processes and underlying assumptions of the subject. They also must possess pedagogical skills from which they can select the most appropriate ways to present mathematical content to students (Clark &

Peterson, 1986; Grossman, Wilson & Shulman, 1989; Lampert, 1985; McDiarmid, Ball & Anderson, 1989; Romberg & Carpenter, 1986; Shulman, 1986a).

With the move to fifth-year teacher preparation programs as a result of The Holmes Group (Holmes Group, 1986) recommendation, subject matter requirements for admission to fifth-year graduate teacher preparation programs have increased. Program leaders assume preservice teachers possess an in-depth understanding of mathematics before admittance. However, the nature of this knowledge has not been determined, nor has the role subject matter knowledge plays in their conceptions of teaching mathematics. The requirements are as vague as a teacher's conscious or subconscious beliefs, meanings, rules, mental images, notions, ideas, concepts, assumptions, understandings and preferences about the discipline of mathematics (Thompson, 1992). For example, many prospective secondary mathematics teachers use the "vertical line test" to determine a function (Even, 1993). Use of the "vertical line test" or an explicit definition of a function reflects a particular conception.

Shulman's (1986a) model of the domains of teacher knowledge also assumes the importance of subject matter knowledge on how one learns to teach. Shulman's information represents a crucial foundation for teaching, however, preservice teachers must build upon and transform knowledge of their discipline as they become teachers. This ability to transform knowledge demands in-depth knowledge of the subject and the language of one's discipline. It requires knowledge of learners and learning, of curriculum and context, of aims and objectives, and of pedagogy.

"Cognitive science research shows that pupils' prior knowledge and beliefs powerfully influence the way they make sense of new ideas" (McDiarmid, Ball & Anderson, 1989, p. 199). Prospective teachers enter teacher education programs with previously formed conceptions of teaching mathematics. These conceptions help them make sense of new information about teaching, their roles as teachers, and their translation of mathematics into learning activities. Thus, preconceptions of subject matter and teaching are important factors to determine preservice teachers' abilities to teach. Unfortunately, previous research on preservice teacher education has focused on what these teachers need to know about subject matter, how they are prepared to teach, and what constitutes an effective teacher education program, but virtually has ignored what prospective teachers already believe about mathematics and its teaching. Researchers must address the conceptions prospective teachers bring to preparation programs, both of mathematics and its teaching (McDiarmid, 1991). Preservice programs can use the incoming conceptions of its students to direct placements in school settings, provide additional subject matter courses, reform misconceptions, and guide mathematics education coursework integrated with practical internship in order to provide more responsive teacher preparation programs.

Statement of the Problem

Much of the research on teacher's thinking has focused on how their conceptions of subject matter influence classroom practice (Thompson, 1984; Hollingsworth, 1989; Leinhardt & Smith, 1985; Putnam, Lampert & Peterson, 1990). As Thompson (1984) stated, "There is strong reason to believe that in mathematics, teachers' conceptions about the subject matter and its teaching play an important role in affecting their effectiveness as the primary mediators between the subject and the learners" (p. 105). Researchers continue to study teachers' conceptions of specific content in mathematics, such as division (Ball, 1990a, 1990b; Simon, 1993; Tirosh & Graeber, 1988, 1989), functions (Even, 1993) and rational numbers (Post, Harel, Behr, & Lesh, 1991). These studies have explored teachers' understanding of the topics of mathematics. However, virtually no research has been done on Shulman's (1986a) central assumption, that a relationship exists between knowledge of subject matter and subsequent learning to teach that subject matter.

The purpose of this study was to investigate the relationship between preservice secondary teachers' conceptions of mathematics, conceptions of teaching mathematics and their conceptions in classroom practices. Preservice teachers were chosen for this study because, although they may have conceptions about mathematics teaching, they have yet to develop a strong foundation in how to apply those conceptions in the classroom. Shulman's (1986a) assumption that subject matter knowledge influences its teaching is clearly challenged with preservice teachers. They are assumed to possess an adequate background understanding of mathematics. The question is to what extent, if any, this background influences their understanding of teaching mathematics and their classroom practices.

Geometry was chosen as the content area for this investigation because of its importance in educational reforms (NCTM, 1989). According to several studies, elementary and middle school students fail to learn basic geometric concepts and problem solving (Carpenter, Corbitt, Lindquist, & Reys, 1980; Clements & Battista, 1992; Stevenson, Lee, & Stigler, 1986). In response to these concerns, the NCTM (1989) recommends students begin to study geometry as early as kindergarten. High school geometry is the first course in which students begin to study formal mathematical thought processes. This shift considerably changes the focus of high school geometry and indicates the class today, at least in terms of reform recommendations, is not what it was in the geometry teachers' own mathematics preparation. This change requires the teacher to be knowledgeable about geometry and a variety of geometrical representations. Because of students' previous work with geometry, teachers no longer need to spend much time on shape recognition, properties of two- and three-dimensional objects, and proofs. Moreover, with the changes, geometry must be more than deductive reasoning and proofs. Teachers are encouraged to promote the development of students' skills of visualization, pictorial representation, processes, and application of geometric ideas.

Previous research has provided quantitative measures of preservice teachers' conceptions and knowledge of mathematics. Researchers have only recently begun to recognize the need for more in-depth qualitative analysis of preservice teachers' mathematical conceptions and its teaching. The goal of many more recent studies has been to determine the depth of preservice teachers' conceptual knowledge in mathematics. The goal of this study was to extend this focus and to determine if mathematical knowledge influences subsequent understanding of teaching mathematics. Specifically, this study investigated three relationships: (a) the relationship between conceptions of geometry and conceptions of teaching it, (b) the relationship between conceptions of geometry and classroom practices, and (c) the relationship between conceptions of teaching geometry and classroom practices.

Significance of the Study

The need for research on teachers' conceptions of mathematics is well documented in the literature (Ball, 1990a, 1990b; Simon, 1993; Tirosh & Graeber, 1988, 1989) and continued research is needed to investigate the influence of preservice teachers' knowledge of mathematics on their conceptions of teaching it. This study was in response to that need.

Results of the present investigation are critical for future development of mathematics teacher education programs. Teacher education has moved toward increased subject matter knowledge for admission to teacher education programs. The assumption is that increased knowledge of subject matter enhances the ability a preservice teacher will have in learning how to teach subject matter. By investigating the conceptions preservice teachers hold prior to full-time teaching, this study clarifies the importance of subject matter in learning to teach. This study focuses on geometric knowledge and its influence on teaching the subject. This study also helps teacher educators make informed, research-based decisions concerning the nature and amount of subject matter knowledge necessary for admission to teacher preparation programs as well as further study needed during the programs.

Knowledge of a specific domain in mathematics includes more than the ability to perform calculations. It also includes knowledge of the "ways, means and processes by which truths are established and . . . its (mathematics) ever-changing character" (Even, 1990, p. 527). If preservice teachers possess this somewhat different perspective on mathematics, they may envision the kinds of questions and materials that support better student understanding (McPeck, 1990). Concurrent with and essential to the development of mathematical understanding is the preservice teacher's perspective on the nature of mathematics. If preservice teachers are to help students develop appropriate knowledge and conceptions of mathematics, it is imperative these teachers examine their own conceptions. Research identifying preservice teachers' conceptions of mathematics and its teaching is only the beginning to this examination. Preservice teachers are limited by certain understandings and a vision of the information possible and appropriate to teach through the study of mathematics. This research provides feedback to teacher educators in support of helping preservice teachers examine, evaluate, and possibly reformulate their conceptions of geometry and its teaching.

Understanding preservice teachers' conceptions of geometry and its teaching and the impact of these ideas on classroom practices is critical. If these conceptions have no impact on classroom practices, further study may be futile; however, if impacts on classroom practices are identified, the results will provide a research agenda for other topics within mathematics, such as probability and statistics, and a springboard for investigations in other subject areas.

In summary, this study provides information on the relationship of preservice teachers' conceptions of geometry to their conceptions of teaching the subject matter, and the relationship of these ideas to classroom practices. The determination of preservice teachers' conceptions of geometry and its teaching will provide a basis for research-based teacher preparation programs and future studies focusing on the interactions of subject matter knowledge and learning to teach that subject matter.

CHAPTER II
REVIEW OF THE LITERATURE

Introduction

The study of teachers' and their behaviors has been pursued for many years. The process-product research projects developed instruments for determining the characteristics of effective teachers based on students' achievement. These projects were prolific in establishing a set of characteristics for effective teachers. However, no single characteristic from these research agendas significantly contributed to student achievement. Furthermore, no single teacher possessed all of the characteristics considered effective.

Several researchers, including Shulman (1986a) and Buchmann (1982) criticized the process-product research for ignoring the context of the classroom, specifically the subject matter and content studied. Because of this concern, two distinct areas of knowledge were clarified as important for teachers to possess: subject matter knowledge and pedagogical knowledge. A central assumption to education is that teachers must possess both knowledge of the subject and an understanding of general instructional methods (pedagogy). In addition to these two domains of knowledge, Shulman (1986a) suggested a third area of teacher's cognition that is related to their thoughts and classroom behaviors, as well as to students' learning and achievement. He defined pedagogical content knowledge (PCK) as:

> The ways of representing and formulating the subject that make it comprehensible to others . . . alternative forms of representation . . . an understanding of what makes the learning of specific topics easier or difficult . . . the most powerful analogies, illustrations, examples, explanations, and demonstrations . . . (Shulman, 1986a, p. 9)

Shulman's type of knowledge represents a separate kind of knowledge that transcends subject matter knowledge and pedagogical knowledge. It indicates teachers need knowledge of how to present the content they are teaching. It focuses primarily on the teacher's ability to transform his or her knowledge of the subject matter into a form students understand.

Pedagogical content knowledge is considered as that form of knowledge that makes mathematics teachers *teachers* rather than mathematicians (Gudmundsdottir, 1987).

Recently, several research projects have studied teachers' conceptions and understandings of their subject matter and how those conceptions affect classroom practices (Even, 1990; Thompson, 1984). Researchers must also investigate teachers' conceptions of teaching that subject matter and how those conceptions affect classroom practices. Furthermore, researchers must investigate how teachers' conceptions of teaching the subject matter interacts with both subject matter knowledge and pedagogical knowledge. This study investigates these interactions in the specific subject of geometry.

For the purpose of this study, **conceptions of mathematics** is defined as a teacher's conscious or subconscious beliefs, meanings, rules, mental images, notions, ideas, concepts, assumptions, understandings and preferences about mathematics (Thompson, 1992). **Conceptions of mathematics teaching** is defined as a teacher's beliefs, rules, assumptions and understandings about mathematics teaching.

Shulman's (1986a) theoretical framework for teachers' knowledge presumes that teachers make decisions based on their conceptions of the specific discipline in which they teach. In order to fully understand this framework, five significant areas were identified as critical for this literature review: (1) teachers' conceptions of mathematics, (2) teachers' conceptions of mathematics in classroom practices, (3) teachers' conceptions of mathematics teaching, (4) teachers' conceptions of mathematics teaching in classroom practices, and (5) the relationship of conceptions of mathematics to conceptions of mathematics teaching. This chapter focuses on research conducted in each area. The first set of research articles represent studies conducted on teachers' conceptual knowledge and understanding of mathematics in specific mathematical domains, while the second set focuses on how those conceptions are used in teachers' classroom practices. The third section of this literature review focuses on research describing teachers' conceptions about teaching specific mathematical topics. These studies focus on the teachers' conceptions of how they might present a lesson, or how they would plan a lesson. The fourth section reviews and discusses studies about teachers' conceptions of mathematics teaching in classroom practices. No studies were identified that addressed the relationship of teachers' conceptions of mathematics to their conceptions of mathematics teaching.

Although the present study focuses on preservice secondary mathematics teachers, studies for this review have been selected that include both inservice and preservice mathematics teachers with each section further divided between elementary and secondary. This division was made because of the different preparation at each level. Typically, the elementary teacher completes one to three courses in college mathematics, while secondary teachers complete a minimum of eight courses.

Teachers' Conceptions of Mathematics

Secondary and Elementary

As part of the author's dissertation, Ball (1990a) examined prospective teachers' conceptions of division and examined their beliefs about mathematical justifications and reasons for making statements "true" in mathematics. Ball's sample consisted of 10 prospective elementary teachers and nine prospective secondary mathematics teachers. One interview was conducted with each prospective teacher, taped and later transcribed. The prospective teachers were asked to explain or generate representations of three areas of division of fractions.

The first area involved solving and providing a "real-world" situation or story for $1\frac{3}{4} \div \frac{1}{2}$. Seventeen of the 19 participants were able to calculate and solve the problem correctly, however, only five (all mathematics majors) could provide an appropriate "real-world" situation. Five of the teacher candidates (three elementary and two secondary), showed representations that did not correspond to $1\frac{3}{4} \div \frac{1}{2}$. The most frequent error was to represent division by two instead of by one-half. Eight of the participants (six elementary and two secondary) were unable to generate any situation to represent the problem. Two prospective teachers recognized the conceptual problem that division by two was not the same as division by one-half, however, were unable to determine what division by one-half meant. The others believed that no "real-world" situation existed.

The second question asked the prospective teachers how they would respond to the situation where a student asks what seven divided by zero is and why. Of the 19 teacher candidates only one elementary and four secondary were able to explain the meaning of division by zero. These participants either explained division by zero by showing it was undefined or by showing that the quotient "explodes" as the divisor decreases. Seven of the prospective teachers, including two elementary and five secondary, explained that "You can't divide by zero." Even when probed, these students did not provide a mathematical justification for the principle that division by zero is not permitted. Five of the remaining seven prospective elementary teachers gave the wrong answer. They stated that division by zero was zero. The other two could not remember the answer to seven divided by zero. They simply stated: "I can't remember."

The last area of division asked the participants how they would respond if one of their students asked for help solving $\frac{x}{0.2} = 5$. Only one teacher candidate, an elementary major, attempted to discuss the meaning of the equation. She stated that she wanted the students to understand what they were doing first by helping them see that 0.2 has to divide into x. Fourteen of the candidates, including all of the secondary candidates, focused on the procedures of solving for the variable. Even when probed, they could not provide conceptual justification for the procedure. The remaining four elementary majors were unable to solve the equation.

"The difficulties experienced by all the teacher candidates (including those who succeeded in generating an appropriate representation) indicated a narrow understanding of division" (Ball, 1990a, p. 140). The participants' conceptions of division appeared to be based on remembering the rules for specific cases rather than on underlying meanings. Across the interviews, Ball found the teacher candidates able to give the students a correct answer, however, they were not confident in subject matter knowledge and lacked mathematical reasoning and meaning. The secondary teacher candidates were more likely able to respond to the questions with some knowledge, even though it was procedural knowledge.

Another component of Ball's (1988) dissertation considered the conceptions of division held by 217 elementary education majors and 35 secondary mathematics education majors when they entered a formal teacher education program (Ball, 1990b). The participants were administered a questionnaire at repeated intervals. The item asked the participants to choose from a set of story problems that represented the expression: $4\frac{1}{4} \div \frac{1}{2}$. Only 30% of the 252 respondents selected the correct response, but also selected one or more incorrect responses. In addition, 10% of the elementary and 6% of the secondary teacher candidates selected the "I don't know" option. Sixty percent of the prospective teachers chose an inappropriate representation from the questionnaire.

As a second source of data, a subsample of 35 prospective teachers was selected to participate in an interview. These subjects were also observed throughout their teacher education program and their first year of teaching. The subjects were first asked how they were taught to divide fractions and to show this method using $1\frac{3}{4} \div \frac{1}{2}$. They were then asked to describe a representation of the same problem. All of the participants were able to calculate the problem by using the invert and multiply rule. However, only four of the teacher candidates, all secondary mathematics majors, were able to generate an appropriate representation. Twelve of the subjects generated representations that did not correspond to the problem.

"Although few of the prospective teachers even mentioned division explicitly, the difficulties all of them experienced suggest a narrow understanding of division" (Ball, 1990b, p. 457). The author concluded that teachers need to understand mathematics themselves if they are to respond adequately to questions from students, help students extend their thoughts and ideas, and formalize students' understandings.

In a third study concerning teachers' understanding of mathematics, Khoury and Zazkis (1994) examined the reasoning strategies and arguments given by preservice teachers as they solved problems on fractions. A group of 124 undergraduate preservice teachers, including 100 elementary and 24 secondary mathematics preservice teachers, participated in the study. No further information regarding the participants was provided.

Two data sources were used for the study. The participants were first administered a written questionnaire consisting of two items:

Item 1: Is $(0.2)_{\text{three}}$ equal to $(0.2)_{\text{five}}$?

Item 2: Is the number "one-half" in base three equal to the number "one-half" in base five?

For each item the participants were asked to explain their decision, and in case of inequality, choose the larger number. The choice of items was determined by the researchers and was thought to be an unfamiliar domain to the participants. The participants' computational work and justifications were analyzed, and their arguments and reasoning strategies identified. For the second part of the study, 38 participants were selected, based on their responses to the written questionnaire, to participate in an interview. The interviews were used to validate the reasoning strategies identified from the written questionnaires and to describe the preservice teachers' conceptual understanding and explanations behind their use of the specific reasoning strategies.

Sixty-three out of 100 elementary preservice teachers, and 24 out of 24 secondary mathematics preservice teachers correctly responded to the first item on the questionnaire. These participants converted each of the number representations to a decimal fraction or to a common fraction and then compared both numbers. This conventional conversion was written:

$(0.2)_{\text{three}} = 2 \text{ X } 1/3 = 2/3$; $(0.2)_{\text{five}} = 2 \text{ X } 1/5 = 2/5$, then, $0.2_{\text{three}} > 0.2_{\text{five}}$, because $2/3 > 2/5$.

Although the students had applied a correct algorithmic conversion, an inadequate understanding of place value surfaced during the interviews. The preservice teachers tried to form an analogy with place values in base 10. For example, in base 10 the place values are 1/10, 1/100, 1/1000. . . . The preservice teachers generated such sequences as: 1/5, 1/50, 1/500. . . . Nevertheless, these interpretations did not effect the responses to the first item.

Of those preservice teachers that responded incorrectly to the first item, they either confused 1/3 with 0.3, or treated $(0.2)_{\text{three}}$ as "two out of three" or "0.2 out of three." Although these strategies may lead to a correct answer on item one, they revealed that the preservice teachers assumed the value of a base was the same as the size of the whole unit, and thus were determined to be incorrect by the researchers.

On the second item of the questionnaire, 26 out of 100 elementary preservice teachers and four out of 24 secondary mathematics preservice teachers responded correctly. The strategy used most frequently was as follows:

$(\text{one-half})_{\text{three}} = (1/2)_{\text{three}} = 1_{\text{three}}/2_{\text{three}} = 1/2$

$(\text{one-half})_{\text{five}} = (1/2)_{\text{five}} = 1_{\text{five}}/2_{\text{five}} = 1/2$

therefore, the numbers are equal.

Seventy-six percent of the preservice teachers claimed that the numbers were not equal. The error made most frequently by the preservice teachers was to change "one-half" into the decimal 0.5. For example:

$(\text{one-half})_{\text{three}} = (0.5)_{\text{three}} = 5 \text{ X } 1/3 = 5/3$ and

$(\text{one-half})_{\text{five}} = (0.5)_{\text{five}} = 5 \text{ X } 1/5 = 5/5 = 1$,

therefore, $(\text{one-half})_{\text{three}} > (\text{one-half})_{\text{five}}$.

During the interview the participants who responded in this manner were questioned: "How did you know that 0.5 is one-half in base 3?" The preservice teachers responded: "I don't know. I made an assumption." Several similar incorrect strategies were also obvious to the researchers.

The researchers concluded: "It seems like a belief in an algorithmic approach of conversion and hurrying up with applying computational skills was dominant among the mathematics education majors. The analysis of strategies indicates that preservice teachers' knowledge of place value and rational numbers is more syntactical than conceptual" (p. 203).

<u>Elementary</u>

Simon (1993) investigated prospective elementary teachers' conceptions of division. Forty-one prospective elementary teachers who had completed their required mathematics courses but had not completed an internship participated in the study.

In the first phase of data collection, 33 participants were asked to show all work and write a full explanation to a set of five open-ended questions. In the second phase, eight subjects were interviewed as they worked on three of the problems. The interviews were used to obtain a deeper understanding of the subjects' thought processes and conceptions of division.

Question one assessed the prospective teachers' knowledge of connectedness and asked the participants to write three different story problems that would be solved by dividing 51 by 4 and for which the answers would be respectively: a) $12\frac{3}{4}$; b) 13; c) 12. The results showed 76% of the participants were able to generate a story problem for part a. Only 36% were able to generate a viable story problem for part b, and 61% for part c. The second question was: "Write a story problem for $\frac{3}{4}$ divided by $\frac{1}{4}$ which would represent the operation used to solve the problem." Seventy percent of the prospective teachers were not able to create an appropriate problem. The most common error consisted of writing a story problem in which $\frac{3}{4}$ was multiplied by $\frac{1}{4}$.

The last three questions assessed the conceptions of both connectedness and fraction units of the prospective teachers. The third question asked the participants to provide two ways they could find the remainder of 598,473,947 divided by 98,762 by using a calculator. None of the 33 participants was able to generate two strategies and only 24% were able to generate one valid strategy. The fourth question was: "Serge has 35 cups of flour. He makes cookies that require $\frac{3}{8}$ of a cup each. If he makes as many such cookies as he has flour, how much flour will be left over?" Eighty-five percent of the teachers provided incorrect solutions to this problem. Most claimed that there would be $\frac{1}{3}$ of a cup of flour remaining. This answer referred to the number of cookies that could be made, not the amount of flour left. The last question asked the participants to explain what information the "multiply" step and the "subtract" step provided in a long division problem and how they contributed to arriving at an answer. None of the participants provided an explanation.

The interview data provided the researcher with a more detailed picture of the conceptions of prospective mathematics teachers. The participants were unable to generate explanations or offer more than a procedural explanation for the problems. When discussing the fifth problem regarding long division, one participant stated:

> I don't know if the reasoning was never taught to me or explained to me or whether I just forgot. . . . The only thing I can remember is to find the remainder, do this, do this, do this, put this in here, subtract that. . . . (p. 245)

The subjects appeared to be inflexible in their thinking and unable to think consciously about division. They were unable to connect the meaning of division with the symbolic representations and tended to possess primitive conceptual models based on concrete experience with whole numbers. The author concluded:

> The prospective elementary teachers in this study exhibited serious shortcomings in their understanding of division as a model of situations. They seemed to have appropriate knowledge of the symbols and algorithms associated with division, but many important connections seemed to be missing, leaving a very sparse "web of knowledge." (p. 251)

Division was explored further in two research studies conducted by Tirosh and Graeber (1988, 1989). These researchers looked explicitly at the types of beliefs that elementary teachers held about multiplication and division. A sample of 129 preservice elementary teachers participated in the first study (Tirosh & Graeber, 1988). A paper/pencil test was used to collect data.

Ninety-nine percent of the participants were able to solve multiplication problems correctly as long as the operator was greater than one. Only 59% of the preservice teachers were able to correctly solve the word problem:

> One kilogram of detergent is used in making 15 kilograms of soap. How much soap can be made from .75 kilograms of detergent?

The most common error for this problem was that participants divided .75 into 15. A similar problem occurred with the division problems. Ninety-eight percent were able to solve a word problem as long as the divisor was less than the dividend. However, only 34% were able to solve word problems that required the larger number to be the divisor.

Interviews were conducted with 33 of the preservice teachers, including 10 of the highest scorers on the written instrument and 10 of the lowest. The preservice teachers who had scored well on the written instrument tended to have more confidence in their ability. They were able to express their results, use a variety of methods, check their answers and describe their thinking. Students who scored lower on the written instrument, on the other hand, were unable to determine if their answers were "reasonable" and were not able to check their answers.

The second study by Graeber and Tirosh (1989) assessed the extent to which two common misconceptions about multiplication and division were explicitly held by preservice elementary teachers. One hundred thirty-six preservice teachers enrolled in a mathematics methods course served as subjects for the study.

The researchers wanted to determine if prospective teachers held the conceptions "multiplication always makes bigger" and "division always makes smaller." The preservice teachers were asked to label six statements related to these misbeliefs as "True" or "False" and to justify their responses. Participants were reminded of the relationship between quotient, divisor and dividend at the beginning of the statements.

Data were also collected from a computational paper-pencil test. This instrument consisted of 16 multiplication and division problems. Results from the paper/pencil instrument showed that 87% of the preservice teachers responded correctly to the first two questions concerning multiplication. Although only 13% of the preservice teachers explicitly held the misconception that "multiplication always makes bigger," the data from the interviews showed many of the prospective teachers still believed that "multiplication always makes bigger."

Seventy-one of the preservice teachers were also interviewed to obtain additional information about their conceptions of multiplication and division. The subjects were asked to write an expression for a word problem. For example, the preservice teachers were asked to solve: "The price of one bolt of fabric is $12,000. What is the cost of .55 of the bolt?" One student responded: "You want to find out what is the price of just this portion of the bolt. So you will have to divide .55 into that amount (points to $12,000) to get the portion." Forty-nine percent of the preservice teachers who were interviewed responded with a similar expression that used division to solve this problem. The authors concluded their study by stating:

> We believe that the discrepancies found among the preservice teachers' performances on different belief statements, and between their performance on computational examples and the related beliefs statement, may be explained by their reliance on procedural knowledge that dominates, or at least is not linked to, correct conceptual knowledge. (p. 92)

Zazkis and Campbell (1994) also explored preservice teachers' understanding of multiplication and division. Twenty-one preservice elementary teachers were interviewed about four sets of problems relating to multiplication and division. The interview included such questions as: (1) Is 391 divisible by 23? (2) What is the next number divisible by 23? (3) Is there a number between 12,358 and 12,368 that is divisible by 7? (4) The number 15 has exactly 4 divisors. Can you think of other numbers that have exactly 4 divisors? (5) Consider the number $M = 3^2 \times 5^2 \times 7$. Is M divisible by 7? By 5, 2, 9, 11, 15?

Data were transcribed and divided into five categories. These included: development of concepts in terms of action-process-object, relationship between division and multiplication, role of verification, use or abuse of divisibility rules, and the additive versus multiplicative structures. Each question from the interview covered one or more of these categories.

Results showed that the majority of participants were not able to discuss divisibility without actually performing the division. Responses such as "You'd have to try to see if it works" or "You cannot be sure if the result is a whole number if you don't know what the result is" were typical. The preservice teachers wanted to "work it (the problem) out to be sure" (Zazkis & Campbell, 1994, p. 6). Another major difficulty appeared when the participants were asked to reverse the tasks or check their work. When asked to give an example of a number that had exactly four divisors, all but three participants preferred to choose a number and then "check" to see if it worked. Many of them were unable to check the answers they obtained. The participants also did not demonstrate a clear understanding of the relationship between multiplication and division. Three participants claimed that 391 was not divisible by 23 because 23 was prime or because the sum of the digits of 391 was a prime number (13). The authors summarized their research by stating: "The findings of this study support the general claim that teachers' content knowledge is "weak" and teachers' conceptual understanding is "insufficient" at times to teach arithmetic even in the elementary grades" (p. 1).

A study by Post, Harel, Behr and Lesh (1991) proposed to generate profiles of elementary teachers' mathematical conceptions of rational number problems and to determine the adequacy of their explanations. Two hundred-eighteen intermediate level (grades 4-6) mathematics teachers from two districts were chosen for the study.

Data collection consisted of three instruments. Part one was composed of short answer items and one-step multiplication and division problems. Topics included ordering, equivalence, operations, estimation, and comparisons of fractions, decimals and percents. Part two of the assessment contained six problems. Teachers were asked to provide as much information as possible relative to their thought processes, solution procedures, and how each of the topics would be taught to children. Part three consisted of a two-hour structured interview related to the teachers' responses from parts one and two. Fifteen teachers were selected from each third of the distributed scores on part one.

Means and percents were calculated for the tests. In part one, 10-25% of the teachers missed items which were considered to be at the most rudimentary level. In some cases, nearly half of the teachers missed fundamental items, for example, $\frac{1}{3} \div 3$. Only 49.7% of the teachers were able to find an equivalent fraction for $\frac{8}{15}$. An even lower percentage, 37.7%, were able to decide what would happen to the value of a fraction if the denominator or numerator were increased by a certain multiple. In general, 30% of the participants scored less than 50% on the instrument in part one. The following problem was presented to the participants in part two:

> Marissa bought 0.46 of a pound of wheat flour for which she paid $0.83. How many pounds of flour could she buy for one dollar? (p. 193)

Of the 77 participants who responded to this problem, only 44.7% were able to solve it correctly. Further, only 10.5% of these participants were able to provide a coherent, rational explanation. Thirty percent of these teachers solved the problem by using rote procedures, mainly writing a proportion and solving by cross multiplication, while only 1.3% provided a conceptual explanation. Similar results were obtained for the majority of questions in part two. Results from the interviews were not provided.

Post, Harel, Behr and Lesh (1991) concluded that many elementary teachers simply do not know enough mathematics to teach mathematics.

Only a minority of those teachers who solved the problems correctly were able to explain their solutions in a clear, rational manner.

Teachers' Conceptions of Mathematics in Classroom Practices

Elementary

Little research exists that investigates the relationship of conceptions of subject matter and classroom practices even though it is usually assumed that teachers' subject matter knowledge influences instructional practices. Leinhardt and Smith (1985) explored the organization and content of fractions as a means to determine the relationship between the nature and level of teachers' subject matter knowledge and their classroom behavior. Four expert and four novice fourth-grade mathematics teachers were selected from a subsample of 12 expert and four novice teachers who had participated in a previous study by Leinhardt (1983). The expert teachers were chosen because of their fourth-grade students' consistent growth in mathematics scores over a five-year period. The novice teachers were student teachers in their last year of training and were highly recommended by their supervisors.

A card sort task was administered to the participants and analyzed to determine patterns of conceptions of fractions the teachers possessed. Noticeable differences were evident between the way the high knowledge experts and the novices performed the card sort task. The high knowledge experts ordered the card sort problems by difficulty to teach or perform. They also grouped problems by content. The novices did not note the difference in problem difficulty. They also indicated almost no internal connections among content.

Four specific questions from an interview provided similar insight into the teachers' subject matter conceptions of fractions. The first question required the participants to define a fraction. Seven participants mentioned the relationship of part to whole. The other teacher defined a fraction as being any number between zero and one. This teacher was also the only one who referred to fractions as the number line in a measurement property. The second question required the teachers to provide a definition of "equivalent." All participants defined equivalent correctly, however, when asked if

$\frac{3}{7}$ and $\frac{243}{567}$ were equivalent, only three of the experts noticed that 81 was a common factor. One of the novices stated that the fractions were equivalent, however, was not able to explain why. The other expert and two of the novices stated that the fractions were not equivalent. One of the novices did not know how to do the problem.

In response to a third item in the interview, all but one participant were able to draw pictures which represented the fractions $\frac{3}{4}$, $\frac{5}{5}$, and $\frac{5}{4}$. One of the novices represented $\frac{5}{4}$ by drawing five circles and shading one-fourth of each circle. This answer represented a direct verbal translation of the problem from five-fourths to five one-fourths. The last problem presented to the participants was whether any differences existed between a ratio and a fraction. All of the teachers stated that a fraction and a ratio were identical, similar, or stated that they did not know.

An analysis of videotapes from lessons presented by the participants on fractions revealed substantial differences in the details of the participants' presentations. "Specifically, there was considerable difference in the level of conceptual information presented as well as differences in the degree to which procedural algorithmic information was presented" (Leinhardt & Smith, 1985, p. 269). The teachers also placed emphasis in different areas and approached their presentations in a variety of ways. One teacher approached reducing fractions by using the identity element, while others used equivalent fractions. The in-depth analysis of data revealed substantial differences in knowledge of fundamental fraction concepts between the expert and novice teachers. The different levels of teachers used different approaches to teaching fractions and stressed varying aspects of the presentation.

Lehrer and Franke (1992) also chose the mathematical domain of fractions to answer several questions pertaining to the interactions of subject matter conceptions on classroom practices. They used personal construct theory to explain the content and organization of teachers' conceptions of fractions. Personal construct theory, as reported in Kelly (1955), draws from a constructivist perspective that emphasizes the individual's interpretation of meaning. According to Kelly's theory, people develop personal constructs of events by differentiating their similarities and differences. The research questions included:

1. Does personal construct psychology provide a means to elicit the various components of teacher knowledge found in other research?

2. Are there conditional relationships among the components of a teacher's knowledge?

3. Is there any relationship between the portrait of teacher knowledge obtained within the personal construct framework and teaching actions in the classroom?

Two teachers participated in the study. "Ms. Hunter" was a second-grade teacher with 17 years of teaching experience. She often posed problems to students and listened to their responses, trying to understand their thinking processes. "Ms. Gardner" was a fifth-grade teacher with five years of teaching experience. She generally followed the order of the textbook in her presentations to students. These two teachers were selected because of the clear differences in teaching practices and number of years teaching.

The teachers were presented with three fraction problems. The problems ranged from adding and subtracting fractions, identifying a fraction on the number line, to converting fractions into another form (i.e., mixed number). The teachers were asked to identify which two problems were more similar to each other, yet different in terms of the content, how students would think about the problems, and pedagogical actions they associated with each problem. This process was repeated with 10 sets of three problems.

After the presentation of all 10 triads, the teachers were shown a list of their elicited constructs (phrases). Included were a list of constructs for content, general pedagogy and pedagogical content. The teachers then rated each phrase as to the degree to which each construct was important in each of the fraction problems. This rating was done on a 10-point scale with one as unimportant and 10 as very important or relevant to the problem. In addition to the interview session, each teacher was observed once during the teaching of a lesson on fractions. Notes were taken to accompany a videotape to confirm the interview data. The videotapes and notes were used to determine the degree to which each teacher's constructs were used in the classroom.

Wide variability existed between the two teachers' responses to the fractions triads. Ms. Hunter's analysis provided a total of 33 constructs. Most of these constructs centered on pedagogy as related to the teaching of fractions. None of her constructs related to the teaching of algorithms or procedures. Instead, she tended to focus on the processes of solving fraction problems and the need to build relationships when thinking about fractions.

In her lesson, Ms. Hunter chose a context for presenting division of fractions that the students were already familiar with and related it to their everyday lives. She used the students' prior knowledge to teach them how to divide 301 pennies into different fractional parts. She asked questions which challenged the students to think about their answers and to explain or "prove" them to the rest of the class. Ms. Hunter did not provide the correct answer, she directed the students to the correct answer. The lesson was also dependent upon the use of manipulatives as concrete representations to teach the relationship between fractions. These teaching patterns were determined to be consistent with her previous constructs.

Ms. Gardner provided 18 constructs, none of which were related to how students might think about the content. She did not discuss ordering of fractions, operations of fractions, or the processes of solving fractional problems, rather centered on the procedures for solving the problems. In comparing Ms. Gardner's constructs to her lesson, it was determined that many of her actions were consistent with her constructs. She focused on fractions as parts of a whole and provided pictures to help the students visualize the concept. She attempted to provide explanations to the students, however, these explanations did not build on previous knowledge or students' level of understanding of fractions. The authors concluded, "Personal construct psychology offers a coherent and consistent framework for interpreting the interactive roles played by the multiple constructions teachers place on classroom events" (Lehrer & Franke, 1992, p. 238).

Secondary

In a study designed to investigate a mathematics teacher's subject matter knowledge, Marks (1987a) described how a teacher's conceptions of problem solving shaped planning and instruction. An experienced teacher, Sandy, volunteered to participate in the study. Sandy held a Master of Arts degree in education, was credentialed in mathematics and social science, had eight years of teaching experience, and had completed a Ph.D. in education.

He was currently teaching Algebra $\frac{1}{2}$ (pre-algebra) , Algebra 1 and Advanced Algebra at the high school level.

Data from this study were drawn from several different sources. Ten structured interviews were conducted with Sandy. Nine of these interviews addressed background, content knowledge, pedagogical knowledge, context, planning and instruction. One interview was a summary debriefing. Each interview was taped and later transcribed. Nine classroom observations of Advanced Algebra were also made. These observations were divided into three-day units so that the researcher had a better notion of the continuity of instruction. Documents such as course descriptions, unit and lesson plans, handouts and tests, and the textbook were also used as data sources.

As a last source of data, the participant was asked to complete eight experimental tasks. These tasks probed Sandy's knowledge of problem solving by asking him to solve a given math problem while thinking aloud, classify a given set of math problems, and rank a set of 40 objectives for high school mathematics according to their importance for the students.

To establish Sandy's content knowledge of problem solving, he was asked to solve the following problems:

1. How many different ways can you make change for one dollar if you have one half-dollar, four quarters, seven dimes, two nickels and eight pennies?

2. How many zeros are at the end of 50 factorial?

He solved both problems completely, using a variety of strategies in a deliberate and organized manner. He stated assumptions, chose a tree-strategy, modified his strategy, carried it out systematically, worked backwards and forwards, checked his work, and stated the final answer (which was correct). Sandy considered himself skilled at problem solving, both in his life and in mathematics, and stated that he would "be willing to put in the effort and time to figure out what background I needed, and then I would . . . rebuild my understanding from there" (p. 14). These claims were confirmed by his own use of problem solving skills. Sandy also felt that he rarely used his mathematical problem solving outside of the classroom.

Sandy believed problem solving was a critical component of mathematics teaching and believed that it should be fully integrated with the mathematics curriculum. "I think that problem solving can be taught using different methods. . . . It's not a question of whether (it can be taught) or which method (to use), but the question would be if they're done well" (p. 16).

Interestingly, Sandy's planning for integrating problem solving into his lessons was limited. Both his unit plans and his daily lesson plans dealt almost exclusively with the mathematical content. They did not include references to pedagogy or to problem solving. When asked about the lack of references to pedagogy, Sandy stated that he does plan to teach problem solving all the time, but the problem solving was embedded in the mathematics teaching and did not merit recording.

Sandy's instruction was mostly lecture format interspersed with questions. The questions asked were directed to the whole class and rarely required more than a single word or phrase for response. If a student responded incorrectly, Sandy supplied the correct answer. The students were rarely asked to explain or defend their responses. Classroom activities were teacher-centered and teacher-controlled. Student participation was highly constrained. The students' role was to listen, answer teacher questions and practice problems like those just demonstrated by the teacher. This role was usually done individually rather than in small groups.

The evaluation of student learning from the unit varied drastically from what students had done in class. Sandy gave the students a take-home quiz. All the questions on the quiz focused on concepts, called for justifications, and required higher level thinking and reasoning. Students were also allowed to work together on the quiz.

During the nine observations, Sandy used a clear set of heuristics in solving problems. He stated relevant given conditions, asked for possible approaches, and suggested alternative routes. He modeled how he might solve a problem and how he hoped the students will eventually learn to solve problems. Sandy never explicitly stated the heuristics or presented them to students. Sandy believed that "problem solving should be emphasized and an inherent part of everything" (p. 28). From Sandy's point of view, problem solving was a major feature of his class. However, the researcher felt that an observer with a different view of problem solving may say that Sandy represented problem solving minimally.

In an attempt to describe teachers' conceptions of mathematics, Steinberg, Haymore and Marks (1985) interviewed and observed four preservice secondary mathematics teachers in their first teaching experience.

The teachers (Joe, Laura, Scott and Sharon) came from two teacher training institutions and were currently either student teaching or participating in an internship. Student teaching involved working with a cooperative teacher while the internship involved having complete responsibility for one class. Two teachers were interning for one class, one teacher was student teaching in three classes, and the last teacher was interning and student teaching in two separate classes.

Teachers were interviewed six times. These interviews focused on personal characteristics, educational background, individual understanding and organization of mathematics, attraction to the teaching profession, and reflection on planning and teaching particular lessons. The teachers were asked during the interviews to solve routine and non-routine algebra problems and to explain how they would teach the concepts to their students. The participants were also asked to draw a concept map of mathematics. Each teacher was also observed twice in the classroom. Observations included a pre- and post-conference and anecdotal data from the lesson.

The teachers' knowledge of mathematics varied tremendously. Joe had completed coursework requirements for a Ph.D. in mathematics. His depictions of mathematics, as drawn in a concept map, were accurate, comprehensive and rich in interconnections. His classroom presentations were always correct and precise and he consistently challenged his students with difficult problems relating to the concept being studied.

Scott and Sharon had studied the required amount of mathematics to complete their certification requirements. Both teachers had partially developed conceptualizations of mathematics as shown by their concept maps. For the most part, their classroom presentations were accurate, but Sharon occasionally misrepresented an idea or had trouble explaining it to the students. Scott recognized his lack of understanding and described it:

> I don't think I was given really a good understanding of why things--of why we are able to do things to the equations. . . . Oh! I can get the right answer just by doing this and this. . . . I had never really thought about why we could do that. (p. 7)

Laura's concept mapping of mathematics centered on arithmetic and its applications. She made a number of mistakes in her presentations to students which often indicated a lack of understanding of fundamental ideas.

The researchers found that the participants used conceptual- versus rule-based explanations. Again, the subjects showed varying emphases on these explanations. Joe explained the derivation of the rules. He explained "why" a rule worked and tried to give the students a sense that the rules were not "magic" and he tried to stress to the students that understanding the rules was more important than memorizing the rules. Sharon questioned students to explain "why" it does not matter which equation is substituted in for the solution when discussing simultaneous equations. Her explanation was "because I want to find the point that these two lines have in common." She also demonstrated the procedure by graphing the two lines. Scott used "why" explanations in some situations. He tried to develop student's intuition about slopes and intercepts in graphing, both by having students discover the patterns using many different examples and by relating slope to the students' experiences in skiing or biking.

Laura was observed explaining "how" procedures work, even when new concepts were introduced. She introduced the idea of square root by distinguishing between taking the square root of x^2 in which both the positive and negative roots are used and the square root of a number in which only a positive root should be taken. Laura did not give any explanation as to "why" a difference existed between the two.

All four teachers were reluctant to make decisions about what should be included in the curriculum at the beginning of the year. They relied on the textbooks and most heavily on what they remembered from their own high school mathematics classes. After five months of teaching, Joe, Scott and Sharon were more willing to decide for themselves the important concepts and the order in which they should be taught.

None of the teachers made any attempt to connect the mathematics content being taught to what had previously been taught. The teachers did not relate the topics to the student's prior knowledge and rarely to the student's everyday life. The teachers mentioned the relationships between concepts in mathematics and other academic subject areas such as science or music in their interviews; however, the researchers stated that the teachers did not present these relationships in the classroom. The researchers concluded that the teachers with greater mathematical knowledge were more conceptual in their teaching and were able to engage their students in a conceptual thinking process.

Teachers' Conceptions of Mathematics Teaching

Even's (1989) dissertation focused on six aspects of secondary teachers' subject matter and teaching conceptions about functions. These included: definition of a function, different representations of functions, inverse and composite functions, functions in high school curriculum, different ways of approaching functions (i.e., point-wise, global, and as entities), and different kinds of knowledge and understanding of functions. The analysis reported in Even (1993) investigated the interrelations between teacher's conception and pedagogical content knowledge related to the concept of function. Two essential features of the concept of function were investigated, arbitrariness and univalence. The arbitrary nature of functions means that functions do not have to exhibit regularity, be described by any specific expression or particular shape of graph. Univalence of a function means that for each element in the domain only one element exists in the range.

One hundred sixty-two prospective secondary mathematics teachers from eight midwestern states agreed to participate in the study. Most of the subjects had completed their mathematics courses, but had not completed any formal teaching.

In the first phase of this study, 152 of 162 subjects completed a questionnaire administered in their mathematics methods courses. The questionnaire consisted of nine problems addressing the different aspects of teachers' subject matter knowledge about functions. For example:

1. Define function.
2. How are functions and equations related?
3. Are these graphs functions (pictures shown)?

Six items were also included in the questionnaire for the participants to analyze or respond to student incorrect solutions or misunderstandings. These included questions such as:

1. Explain what the students were thinking that could cause them to identify a function mistakenly.

2. Give an alternative version of function to help students understand.

In the second phase of this study, the remaining 10 subjects completed the same questionnaire and were interviewed in depth. Since the interview questions were developed from an analysis of the first phase data, second phase subjects were not chosen from among the first phase subjects. These 10 participants answered the questionnaire and were interviewed the next day so that they could easily recall their answers. Participants were presented with questions in the interview, which emerged as important issues from analysis of the original 152 questionnaires, and items which required longer, more thoughtful answers. The participants were also asked to reflect on, explain, and clarify their answers on their questionnaire.

The data showed that 78 of the first phase participants were able to provide a modern definition of functions, however, only 27 were able to use a modern definition when helping a student. Many of the teachers used the "vertical line test" as a rule for determining a function when explaining to a student, rather than using the language of their original definition. For example, Valerie defined a function as "a 1-1 mapping of a set of points X onto Y." When explaining to a student with difficulties with this definition, she did not use the word "mapping."

Analysis of all the participants showed that over half described a function as being "nice," "known," and represented by an equation. Some of the subjects felt that a function should be "nice" and "smooth." Eighteen first phase subjects expected the graph of a function to be continuous. Thirty-five of the participants believed an infinite amount of arbitrary functions which pass through any two points existed. The other participants stated that specific kinds of functions pass through any two points or a finite number of such functions exists. Seven second-phase participants expressed the belief that <u>all</u> functions can be represented by using a formula. Altogether, about half the participants showed signs of believing that a function must be an equation.

Many of the participants believed that univalence was one of the most important characteristics of a function; however, none of them were able to generate a reasonable explanation for the need of univalence. The interviews revealed a tendency to provide the student with the "vertical line test" as a rule to follow and get the correct answer (without a need to understand). Most did not know the importance of distinguishing between a function and a relation.

Even (1993) concluded with a discussion of the implications of this study:

> Not knowing why univalence is needed may influence pedagogical content-specific choices, by making it 'reasonable' to present students with easy procedures that overemphasize procedural knowledge without concern for meaning. (p.112)

Many of the participants in this study overemphasized the procedural knowledge when they used the "vertical line test" rule rather than providing an explanation with meaning. Teachers' pedagogical decisions (questions they ask, activities they choose) are based, in part, on their subject matter knowledge. It is therefore important that teachers develop a meaningful understanding of functions. Without this conceptual knowledge, teachers are unable to make decisions about the place of functions in the curriculum and the emphasis that should be placed on functions.

Burns and Lash (1988) interviewed nine seventh-grade mathematics teachers in a study examining teachers' conceptions about teaching problem solving. All teachers had secondary certification and were currently teaching at least one section of seventh-grade mathematics.

The teachers were interviewed, attended a three-hour workshop, and asked to plan a six-day unit on problem solving. Two semi-structured interviews were conducted. In the first interview, teachers were asked about problem solving using a set of 22 questions. This interview was conducted prior to the workshop. A second interview, conducted about a month following the workshop, consisted of 14 specific questions about the teachers' planning and methods.

The workshop was conducted by the researcher to present and discuss four problem solving skills which were deemed important from the literature. The skills included: (1) identify information necessary to solve the problem, (2) separate relevant and irrelevant information, (3) identify the intermediate step in a multiple-step problem, and (4) represent information in a table or diagram. Teachers were then asked to plan a six-day problem solving unit. The teachers could plan the unit in any manner they wished, however, they had to use the four problem solving skills as outlined in the workshop. Teachers were also provided with a workbook that gave suggested activities and problems.

The problem solving interviews suggested that the teachers in this study "had consistent pedagogical knowledge about how to teach mathematics, but had a limited pedagogical content knowledge about how to teach problem solving" (Burns & Lash, 1988, pg. 378). The teachers stated that they taught basic skills and problem solving by showing students how to do problems and then allow them to practice similar ones.

The planning interviews indicated that the teachers were not concerned with teaching problem solving, but rather collecting and organizing materials and problems for students. The teachers' planning was not any different from what they usually did to create a lesson or unit. They planned the unit in a way that was quite similar to their normal instruction.

Burns and Lash (1988) concluded the study with a discussion of the use of subject matter knowledge and pedagogical knowledge. Their research model suggested that teachers do not necessarily have to create a new delivery system for new techniques, but can integrate them into existing systems.

Teachers' Conceptions of Mathematics Teaching in Classroom Practices

In an effort to explore the relationship of teachers' conceptions of mathematics teaching to instructional practice, Stein, Baxter and Leinhardt (1990) sought to describe and analyze the teaching of functions, graphs and graphing in the elementary grades. An experienced fifth grade teacher (18 years) was selected as an experimental subject. "Mr. Gene" was recommended by his principal as being an excellent mathematics teacher and agreed to participate in the study.

A subject matter interview and card sort task were designed especially for this study. After pilot testing, the subject matter interview was administered to Mr. Gene and a mathematics educator for comparison. The interview consisted of a variety of open-ended questions regarding functions, graphing and their instruction at the elementary level. The subjects were asked to provide a definition of functions, and how, why, and when functions should be integrated into the curriculum.

The card sort task consisted of a stack of 20 cards, each of which depicted a mathematical relationship. Mr. Gene and the mathematics educator were asked to categorize the cards into groups and give a description of each group. They were asked to repeat this procedure with different groupings.

Data from videotapes of Mr. Gene's 25-lesson unit on functions and graphing were also used. The lessons were divided into activity structures, a content analysis was conducted (main concepts, procedures, and relationships), and episodes were identified linking Mr. Gene's subject matter knowledge to his instructional practice. Finally, the nature of these links was explored to determine the ways in which subject matter conceptions might have influenced instruction.

Mr. Gene defined a function as follows:

> Functions are two interrelated numbers, the value of which one depends upon the other. (For example), say you have a variable number dependent on a given number, (then) the relationship between the two changes as the variable number changes. (p. 644)

Mr. Gene continued by stating that a function was a two-computation math problem and "a function is almost like a story problem that's not a story problem" (p. 645).

The mathematics educator was also asked to define a function:

> I would define a function as a special kind of correspondence, fulfilling certain conditions . . . the correspondence being from one set to another, in which each element in the first set, which you call a domain, has one and only one corresponding element in the second set, the range. (p. 645)

The mathematics educator continued by adding that a function could have a one-to-one correspondence, but this property is not required. She also stated that a rule is not necessary to define a function.

In comparison, both the mathematics educator and Mr. Gene conveyed the need for two interrelated numbers and that one number depends upon the other. However, Mr. Gene was missing several essential features of a function. First, Mr. Gene failed to mention that one and only one element of the second set can be assigned to each element of the first set (univalence). Second, Mr. Gene failed to label the relationship as a one-to-one or a many-to-one correspondence. Last, Mr. Gene did not mention that two entities may be related to one another with or without a rule.

Mr. Gene viewed the teaching of functions as useful practice of basic math facts, providing important preparation for algebra, "fun," motivational, and as good checking devices. Furthermore, his conception of teaching functions and graphing in the same unit was that the graphs can be used to check the answers to the "function machine." The authors felt this conception of teaching functions played a significant role in his instructional practice.

The comparison of card sort tasks between Mr. Gene and the mathematics educator revealed significant differences in how they organized their knowledge of functions and graphing. When completing the card sort, Mr. Gene placed the cards in the groups: equation statements, ordered pairs, graphs, and leftovers. These categories were used by Mr. Gene because of the format in which the cards were written (i.e., pictures, tables, equations). Mr. Gene's second card sort task was not discussed. The mathematics educator grouped the cards according to functions and non-functions in both of her arrangements. She subdivided these groups into functions with a finite or an infinite set of values.

In general, it was felt by the researchers that the mathematics educator's sort was a tightly organized system and Mr. Gene's sort suggested limited knowledge and understanding of functions. His sort was described as a "surface sort," meaning that the features upon which Mr. Gene focused were related to the outward appearance of the cards rather than their deeper mathematical features. The conclusion was that Mr. Gene's subject matter knowledge was lacking some key concepts. The important concept that mathematical relationships can be represented in several different ways was missing from Mr. Gene's knowledge. He also failed to use the idea that a function maps one and only one y-value to each x-value.

Mr. Gene's subject matter conceptions were also interwoven with his conceptions of how to teach functions. He believed that functions are used as motivators to practice basic math skills. This belief regarding why and how students should learn functions and graphing did not appear to exist separately from his knowledge of the subject matter.

Four videotaped lessons were chosen from the original 25 tapes. These videotapes were chosen to represent the introduction and integration of functions with graphing. An analysis of the videotapes revealed accurate and thorough coverage of the material. In addition, the lessons contained significant amounts of material not covered by the textbook. On the other hand, some inaccuracies and underdeveloped connections were noticed. For example, Mr. Gene's classroom definition of functions was similar to his own limited definition in the interview.

Through this detailed analysis of Mr. Gene's knowledge and his classroom lessons, the researchers identified three ways in which lack of subject matter knowledge led to limited pedagogy: (1) the lack of providing groundwork for future learning in this area, (2) overemphasis of a limited truth, and (3) missed opportunities for fostering meaningful connections between key concepts and representations" (Stein, et. al., 1990, p. 659). The authors concluded, "Our findings corroborate the conclusions of other studies that have suggested that limited, poorly organized teacher knowledge often leads to instruction characterized by few, if any, conceptual connections, less powerful representations, and over-routinized student responses" (Stein, et. al., 1990, p. 659).

Eisenhart, Borko, Underhill, Brown, Jones and Agard (1993) conducted a study designed to describe and understand the novice teachers' knowledge, beliefs, thinking, and actions related to the teaching of mathematics. One preservice teacher, "Ms. Daniels," was selected from 38 cohort teachers. She had the most extensive mathematics background of any of the student teachers in the program. Ms. Daniels was placed in a sixth-grade classroom for an internship experience.

The primary source of information regarding Ms. Daniel's beliefs and knowledge was an interview administered at the beginning, middle, and end of the school year. The interviews revealed that Ms. Daniels recognized the difference between procedural and conceptual knowledge in mathematics. She was, however, able to express her ideas about teaching procedurally much better than she could express how to teach conceptually. Further, she considered her own procedural knowledge stronger than her conceptual knowledge.

During her internship experience, Ms. Daniels was observed teaching for three separate week-long visits. It was evident that Ms. Daniels believed in the importance of teaching for both procedural and conceptual knowledge. The majority of lessons reflected her ability to teach for procedural knowledge. She was able to demonstrate an algorithm and guide the students through the same. She also demonstrated a concern for teaching conceptually, however, struggled with conceptual explanations. She stated: "I just don't like saying 'Well, this is pi. Remember it . . . but where does pi come from? I just don't know" (Eisenhart, et. al., 1993, p. 18).

Ms. Daniels' university experience and public school experience were also used as an additional source of data. The university methods instructor stated he wanted the prospective teachers to focus on the conceptual aspects of teaching and learning mathematics. His goal in the methods course was to provide the prospective teachers with "survival strategies" they could use to teach conceptual knowledge. However, the prospective teachers viewed the "survival strategies" as a set of algorithms to be remembered (procedural). The district held the teachers accountable for student scores on standardized tests which reflected procedural knowledge. Teachers were encouraged to teach conceptually, however, they were not held accountable for such teaching. Thus, Ms. Daniels had little opportunity to observe teaching for conceptual understanding. The conflicting views which Ms. Daniels observed between what she believed, the university beliefs, and the public school experience, "led to an outcome in which teaching for conceptual knowledge tended to fall through the cracks" (Eisenhart, et. al., 1993, p. 37).

In a study based on the authors' dissertation, Thompson (1984) investigated the conceptions of mathematics and mathematics teaching of three junior high school mathematics teachers. The relationship between the teachers' conceptions and their instructional practice was also examined. The teachers were chosen from a group of 13 teachers who had participated in a previous pilot study. The three teachers were chosen for no specific reason or criteria other than the fact that they had more than three years teaching experience and were willing to participate.

A case study method of research was conducted. Each teacher was observed daily over a period of four weeks, however, no more than one was observed at a single time. Observations were audiotaped for later analysis. Interviews were conducted following the observed lesson during the last two weeks. The interview questions were related to specific events which occurred during the lesson for that day. Each teacher was also asked to respond in writing to six tasks. These tasks sought to reveal information about the teachers' views on: (1) the importance of various aspects of mathematics instruction; (2) how much emphasis should be given to specific objectives; (3) the importance of pedagogical practices; and (4) the evaluation of their own teaching.

Each case study and comparisons between the three teachers were presented. The differences in the ways in which these teachers conceived mathematics and its teaching, the integration of topics, and the degree to which their teaching reflected their own subject matter knowledge was discussed. For example, one participant viewed mathematics primarily as "a coherent subject consisting of logically interrelated topics" and emphasized the "mathematical meaning of concepts and the logic of mathematical procedures." Another participant regarded mathematics primarily as "a challenging subject whose essential processes were discovery and verification." The third participant indicated that mathematics was "essentially prescriptive and deterministic in nature" (Thompson, 1984, p. 119). Furthermore, these teachers' conceptions played an important role in the manner in which the content was presented to their students. The researcher concluded from this sample that teachers' beliefs, views and perceptions about mathematics and its teaching play a significant role in shaping their instructional behavior, but that other factors, such as conceptions about students or the social aspect of school, may also contribute to their instructional practices.

Conclusions and Recommendations

The studies of teachers' conceptions of mathematics were directed at specific topics in mathematics including multiplication and division, fractions, and rational number concepts. It is clear from these studies that the participating teachers had a wide range of depth in understanding the mathematical concepts presented to them. Most of the teachers who participated could perform the mathematics by a procedural method, however, possessed limited conceptual knowledge. They were unable to build upon the relationships and apply their knowledge to new situations. The participating teachers could solve the problems that were presented to them, however, they could not explain their processes or the mathematics involved (Post, et. al., 1991; Simon, 1993).

Results of these studies highlighted two important areas of concern. First, all but one of the studies focused on teachers' conceptions of division as the topic of study. Although division is an important topic in elementary mathematics, the NCTM *Curriculum and Evaluation Standards* (1989) has recommended several other areas in need of attention in mathematics education, such as geometry, probability and problem solving. Teachers need to know as much about these topics as they need to know about division. Yet, investigation of teachers' knowledge of every possible concept in mathematics would be an overwhelming task. Presumably, researchers do not have to investigate teachers' knowledge across all grade levels for every concept or process included in the mathematics curriculum. Researchers, however, need to investigate teachers' conceptions in a variety of mathematical areas, searching for similarities in the types of conceptions teachers possess.

Second, most of the studies focused on elementary teachers. Ball (1988) combined both elementary and secondary, however, she did not specifically focus on secondary teachers and did not mention the difference in the preparation these two levels of teachers receive. While it is assumed that secondary teachers possess adequate conceptions of mathematics, very little is known about what secondary teachers actually do know and understand about mathematics. Research needs to be conducted that specifically addresses the secondary mathematics teacher's knowledge of mathematics and teaching.

The studies tracing the impact of the teachers' conceptions of mathematics or its teaching on classroom practices presents a concern with the small number of classroom observations. Lehrer and Franke (1992) observed the two teachers in their study one time during their teaching of fractions and concluded that the teachers' conceptions were evident in classroom practices. However, one classroom observation provided little support for describing how or what a teacher does in the classroom. Steinberg, Haymore and Marks (1985) observed their teachers twice. Again, this small number of observations does not provide a complete view of the teachers' conceptions in classroom practices. Therefore, the conclusions drawn from these studies, that teachers were unable to represent the content and made few connections in their presentations, are not warranted. Other conclusions regarding the effect of mathematical conceptions on both the content and processes of instruction are also lacking support of classroom observations.

The major concern with research on teachers' conceptions of mathematics teaching is the lack thereof. With only two studies, one with preservice teachers and one with inservice teachers, many questions remain as to the types of conceptions that teachers hold and what factors influence these conceptions. Further research is needed to draw adequate conclusions regarding teachers' conceptions of mathematics teaching.

In summary, researchers studying teachers' conceptions of mathematics and its teaching often do not: (a) study the secondary mathematics teacher; (b) provide adequate support for stated conclusions; (c) investigate conceptions throughout mathematics and similarities in the types of conceptions teachers possess; (d) study the interactions of conceptions of mathematics, its teaching and classroom practices; or (e) study the conceptions of preservice mathematics teachers prior to teaching. Given these limitations, the need for a more careful and thorough investigation of the nature of such conceptions is evident. Virtually no research has been done to investigate the nature of teachers' conceptions of mathematics, its teaching, and the role these conceptions play in instructional practices. In particular, a description of the types of conceptions teachers possess regarding mathematics and its teaching is needed.

CHAPTER III
DESIGN AND METHOD

Introduction

Review of the literature emphasized problems associated with current research on teachers' mathematics conceptions and teaching. Teachers in research studies traditionally have been judged by the researcher to either possess or not possess appropriate knowledge of mathematics. The purpose of this research project was to extend this focus and to determine if mathematical knowledge influences subsequent understanding of the teaching of it. This study considered three interactions (a) the relationship between conceptions of geometry and conceptions of teaching geometry, (b) the relationship between conceptions of geometry and classroom practices, and (c) the relationship between conceptions of teaching geometry and classroom practices.

Traditional studies of teachers' conceptions have not been particularly successful in establishing the relationship of teachers' subject matter knowledge and instructional practice. These studies recommend the measurement of subject matter needs more sensitive evaluation procedures than grade point average in college or the number of courses taken in mathematics (Carpenter, 1989; Even, 1989; Even, 1990; Grossman, Wilson & Shulman, 1989; Romberg, 1988). In response to these recommendations, this study provides an in-depth analysis of preservice teachers' conceptions of mathematics and its teaching by collecting and analyzing extensive qualitative data. This investigation resulted in the identification and detailed descriptions of types of conceptions preservice teachers hold of geometry and its teaching. Further, this study identifies how preservice teachers' ideas about geometry affect classroom teaching.

The study was conducted in two parts: Phase I and Phase II. Phase I focused on clarifying the preservice teachers' conceptions of both geometry and its teaching. This phase employed several data sources including a card sort task, journals, and a videotape task. Phase II focused on the preservice teachers' classroom practices and consisted of classroom observations, work samples and informal interviews. Discussion of all the data instruments along with a detailed description of the implementation of Phase I and Phase

II are included in this chapter. In addition, a description of the data analysis procedures are discussed.

Subjects

The sample for this study consisted of 10 preservice secondary mathematics teachers who were enrolled in a fifth-year, graduate level teacher preparation program. While the size of this sample may not compare favorably with other studies, the descriptive nature and extensive data collection provided a more detailed picture of preservice teachers' conceptions of geometry. These preservice teachers represented the mathematics portion of 22 science and mathematics teachers enrolled in a single teacher preparation program. The fifth-year, graduate program was conducted at a medium-sized university in the western United States. Program requirements included: a bachelor's degree, academic work matched to program requirements, a 3.0 grade point average in the last 90 hours of undergraduate course work, passing scores on an approved test of basic skills, the National Teacher Exam (NTE) of mathematics content, and verification of successful experience working with youth in a school setting. The mathematics preservice teachers were required to have completed both Euclidean and transformational undergraduate geometry courses.

Types and detailed descriptions of the conceptions of geometry and of geometry teaching were obtained from the 10 preservice teachers. In order to identify the relationship of these conceptions to classroom practices, those preservice teachers teaching high school geometry were further identified. Several of the preservice teachers completed their internship at the middle school level and some of those at the high school level were not assigned to teach geometry. Although five preservice teachers were chosen for observation in the classroom, only four were observed. The fifth preservice teacher did not complete the program.

The researcher contacted subjects during Fall quarter requesting their participation in the study. A letter describing the general intent of the study, the types of data to be collected and the time commitments involved was sent to the participants (see Appendix A). This letter affirmed the confidentiality of data and was used to assess the subjects' willingness to participate in the study. The participants were asked to sign and return the letter within one week. All 10 preservice mathematics teachers agreed to participate.

Data Sources

The purpose of the study was to identify and describe preservice secondary mathematics teachers' conceptions of geometry and its teaching. To gather this information, eight types of data collection instruments were used including: a card sort task, journals, a videotape task, classroom observations, two work samples, informal interviews, additional classroom documents and admissions documents. Due to the nature of this study, the researcher was considered an important part of the data collection and analysis. Each of these data sources is discussed.

Card Sort Task

The identification of preservice teachers' conceptions of geometry included the determination of their processes, organization and structure of geometry. A card sort task was used to identify the preservice teachers' conceptions of geometry. Card sort tasks have been used in several studies to investigate teachers' conceptions of mathematics (Leinhardt & Smith, 1985; Stein, Baxter, & Leinhardt, 1990).

A card sort consists of a set of cards, each card depicting a geometrical relationship. For example, one card might have a drawing of a prism, another might have the formula for volume of a pyramid, while a third might depict a visualization of parts of a prism. Typically, participants are asked to sort the cards in a manner of their choice. The researcher then questions them as to the organization in which the cards were arranged.

A card sort was designed and developed by the researcher specifically for this study (see Appendix B). The cards were designed to provide the opportunity for participants to categorize the cards based on a variety of criteria. Usiskin's (1987) model for the dimensions of geometry was used as the framework for the development of the card sort task. In this model the dimensions included:

1. Geometry is the study of the visualization, drawing, and construction of figures;

2. Geometry is the study of the real, physical world;

3. Geometry is a vehicle for representing mathematical or other concepts whose origin is not visual or physical;

4. Geometry is an example of a mathematical system.

It was recognized that specific topics or concepts in geometry may overlap one or several of the dimensions. For instance, measurement is an example of a mathematical system and also is used in drawing and constructing figures. Because of this overlap, the cards were placed in the dimension that best matched.

Using these dimensions, the cards were designed to allow grouping in several ways. For example, a participant could group together all of the two- and three-dimensional figures or all of the relations to algebra. When preparing the cards, the researcher designed the cards so that an equal number used pictures, words, or processes to represent the topics. Previous studies utilizing card sorts have suggested the use of 20 to 40 cards (Stein, Baxter, & Leinhardt, 1990). Combining this recommendation with Usiskin's four dimensions, the card sort task consisted of 32 cards, eight cards representing each dimension. In order to establish validity, the cards were matched with the NCTM *Curriculum and Evaluation Standards* (1989) for high school level geometry through the use of a table of specifications.

The card sort task was piloted prior to use in the present study. Two geometry instructors, two mathematics educators, and two prospective teachers piloted the card sort task. First, the researcher administered the card sort task to one geometry teacher, one mathematics educator and one prospective teacher in the same manner as in this study. Following the card sort interview, the researcher asked for feedback and comments regarding the representations depicted on the cards, the types of possible organizations, and likely areas of difficulty that might arise. The card sort task was revised to represent the suggested changes that helped to clarify the card sort (i.e., including a card with circles). The card sort interview then was administered to the other geometry instructor, mathematics educator, and prospective teacher to finalize the card sort interview procedures and clarify further problems that might arise.

Journals

A journal was used to acquire information regarding the participants' conceptions of mathematics teaching. As a requirement for their part-time internship, during Fall quarter the participants completed a weekly journal of observations. Preservice teachers were asked to reflect on experiences and to share their impressions or perceptions of what was happening in their internship. They also were asked to reflect on specific aspects of teaching mathematics. Sample reflections included:

> What types of teacher behavior, attitudes, or skills make an effective mathematics teacher? Even if you have not seen good models, state what you think about what makes an effective teacher.
>
> If you were the full-time teacher, what would you change about the classroom?
>
> What types of events or things surprised you about teaching mathematics?
>
> What are your beliefs about mathematics teaching and learning?

Although the content of the journal entries did not focus specifically on teaching geometry, general conceptions emerged. Each preservice teacher completed the Fall quarter internship in a mathematics classroom; therefore impressions and conceptions expressed in the journal were specific to teaching mathematics.

Videotape Task

To obtain specific information on preservice teachers' conceptions of teaching geometry, the participants viewed three videotapes of high school teachers conducting geometry lessons. Viewing was completed individually with the researcher present. During and after the session, participants were asked to comment on the specifics of the lesson, teacher, class and content.

To obtain the videotapes, high school geometry teachers were contacted by the researcher during Fall quarter. Three teachers were chosen to obtain a variety of teaching styles (e.g., teacher-centered, learner-centered). Teachers were selected on the basis of their willingness to participate in this study.

Teachers were chosen who were not mentors for the current preservice teachers and were not teaching at any of the schools where the participants were completing their internship. Furthermore, teachers were sought from a variety of schools. An informed consent form was sent to the teachers who agreed to participate (see Appendix C) to obtain their permission to videotape and to use the data. Preservice teachers were not given the instructors' or the schools' names.

Three different videotaped geometry lessons, excluding test days, were obtained from each experienced teacher. The researcher selected one videotape from each teacher to use with the preservice teachers. Selection was based on variety of styles, clarity of tapes, and type of lesson presented. For example, one teacher used manipulatives while another used a lecture format. Since the videotapes were used to investigate the participants' conceptions of teaching geometry, the actual content of the lesson was not considered as a selection criterion. The researcher also identified at least three points on each videotape for "stopping places," in order to prompt the preservice teacher. These stopping places were chosen to represent a transition in the lesson, a different type of problem being presented or a point where the teacher made an obvious decision.

The first experienced teacher (Teacher A) had been teaching mathematics in a metropolitan high school for 10 years. In the lesson, he taught the students proofs. He presented students with the following warm-up problem to bridge to the concept of proofs:

If $y = x^2 + x + 41$ find the values of y if $x = 0, 1, 2, 3, 4, 5, 6$.

Teacher A then asked students if they noticed any patterns in the y-values. The students mentioned all the y-values were prime numbers. The teacher asked the students if they thought this problem would be true *all* the time. Many were convinced that they would get a prime number every time. Teacher A then proceeded to introduce proofs and told the students when they are proving something they can use only what they already know (i. e., postulates, definitions, and theorems). The students began by giving reasons for three individual statements. The teacher then presented several multiple-step proofs with assistance from the students. At the end of the lesson, Teacher A returned to the original warm-up problem and asked the students if they were still convinced. He asked them to try the value of 40 for x. Most students were surprised that the value did not produce a prime number. The teacher emphasized students must have the "burden of proof" before accepting absolute truth.

Teacher A kept the students involved by requiring them to take notes and to answer questions he posed. He was well-organized and prepared for the lesson. He was non-threatening, but business-like. Students seemed to understand the concept of proofs and seemed confident in their ability to complete a simple proof.

The stopping places chosen for this lesson were (1) after the introduction of the warm-up, (2) after placing the list of definitions, postulates and theorems on the board, and (3) after the teacher completed one proof. These places were chosen because they represented transitions from one part of the lesson to another. Other stopping points were determined by the preservice teachers.

The second experienced teacher (Teacher B) had been teaching at the high school level for 20 years. He was currently teaching in a private school in an urban area. For the preservice teachers, a lesson reviewing parallel lines and introducing parallelograms was chosen. He began the lesson by reading a list of vocabulary words for the students, such as, alternate interior angles and corresponding angles. He asked the students what they knew about each word and to describe the word or the concept. He then handed out a worksheet with 16 problems and solved 12 of the problems. The students were expected to write down the answers on the worksheet as the teacher helped them solve the problems. The teacher asked many questions, however, most students were unable to answer. Most of the time, male students were called on to answer questions. As a result, many of the female students just placed their heads on their desks and paid no attention. The lesson lasted 45 minutes ending with the bell in the middle of the presentation of a problem.

The stopping places selected for this lesson were (1) following the reading of the vocabulary list, (2) following the fourth problem on the worksheet, and (3) after the seventh problem. The first stopping place was chosen because it represented a transition in the lesson and the other two were chosen because of the variety of problems on the worksheet. The preservice teachers determined other stopping places.

The third teacher (Teacher C) the preservice teachers observed, had been teaching for 18 years in a small town high school. The lesson chosen for viewing had the most variety of his three lessons. Teacher C started the lesson by handing out a graded test the students took the previous day. He reviewed most of the problems and asked students why they had not done well. The students said they felt they were tested on material they did not know. Several students said they were frustrated because they did not understand what they needed for the test. The teacher seemed to refuse to accept that possibility. He suggested that perhaps the questions were worded differently. He asked the students what they would like to do about the low test scores; they decided to re-take the exam. Students asked the teacher to review the material. He said they would have three days to review on their own, but he would not take class time to review the material. He gave the students 30 minutes to review and work on the homework assignment. Instead they spent the time chatting with their neighbors. The period lasted 48 minutes.

The stopping points for this lesson were selected following the first two problems from the test, after the teacher asked the students to quiet down (after fourth review problem), and after the students told the teacher why they felt they had not done well on the test. These stopping places were chosen because the teacher had made some obvious decisions about student behavior and learning at these points in the lesson. Preservice teachers determined the other stopping places.

Work Samples

As part of the internship experience, each preservice teacher was required to complete two work samples. A work sample is a collection of data for assessing the preservice teacher's effectiveness in planning and instruction. Minimally, the work sample included 12 to 18 instructional hour units with these sections: title page, rationale for unit, unit goals, instructional objectives, prerequisite skills, materials, calendar, detailed instructional activities, resources, evaluation plan, and analysis of teaching and learning. (See Appendix D for detailed descriptions of each section.) This information provided additional data for preservice teachers' conceptions of geometry teaching.

One work sample was completed Fall quarter and one Winter quarter. As part of their internship preservice teachers were required to plan, teach, reflect and change the unit plan. The preservice teachers were also required to videotape at least two lessons to be included with their work samples. The participants chose which lessons to tape. Both work samples were collected for this study.

Classroom Observations

Since much of the previous research failed to include classroom observations, this study sought to enhance the analysis of preservice teachers' conceptions of geometry and teaching through the inclusion of classroom observations. The researcher observed four of the five preservice teachers chosen to participate in the second phase of this study. (The fifth preservice teacher did not complete the teacher preparation program.) Classroom observations determined the types of conceptions of geometry and its teaching displayed in classroom practices. Further, classroom observations allowed for an investigation into the relationship of preservice teachers' conceptions of geometry to classroom practices.

The researcher observed each preservice teacher eight times during the full-time Winter quarter internship. Eight observations of each preservice teacher provided a more detailed and comprehensive picture of their classroom practices. Three of the observations were consecutive lessons in order to describe the continuity within the classroom. These observations were prearranged with the preservice teacher. Three of the observations were unannounced. The last two observations were chosen by the preservice teacher because these lessons were required as part of the work samples. On Sunday evening at the beginning of each week, the researcher contacted preservice teachers to determine their schedules.

For each intern, all observations were of the same geometry class. The researcher attempted to observe as many different teaching strategies as possible. Lessons directly involving the teacher were of central importance for observations, as opposed to test days or student lab days.

When observing in classrooms, the researcher was as unobtrusive as possible. Initially, the preservice teacher introduced the researcher to familiarize the students with the purpose of the researcher in the classroom. The observation process was the same for each lesson. The researcher arrived before class began and asked the preservice teacher for the plan for the lesson.

The researcher asked how the students would be arranged and what was expected for the lesson. Then the researcher set up the video camera in the back of the classroom and began taping. As students entered, the researcher sat and took notes. Throughout the lesson the researcher took notes with respect to time, activities, preservice teacher actions, student behavior and any reflections or interpretations made by the researcher. Following the lesson, the video camera was stopped and the researcher left after the students. Observations lasted the entire class period.

Informal Interviews

Additional data were collected regarding preservice teachers' conceptions in classroom practices. To clarify statements or actions, informal interviews were conducted either before or after a lesson was presented. Interviews usually lasted from one to five minutes. Questions to guide the informal interviews before the lesson were:

What will you be doing in today's lesson?

What will be expected of the students in today's lesson?

How are you planning to organize the students for the lesson?

Questions to guide the informal interviews following the lesson included:

How do you feel the lesson went?

Why did you decide to (blank)?

I noticed that you said (blank). What were your thoughts at that point?

What would you change about this lesson next time you teach it?

The informal interviews were used as an additional source of data to confirm (or disprove) the researcher's interpretation of the preservice teachers' conceptions of geometry and its teaching in classroom practices. Data from the informal interviews were used to verify and describe more completely the conceptions that arose from the data analysis.

Additional Classroom Documents

After several observations, the researcher recognized additional classroom documents were needed to obtain a complete view of the preservice teachers' actions and classroom activities. Therefore, the researcher attached handouts, worksheets and quizzes given to students to field notes for each lesson. The researcher obtained a textbook from each preservice teacher and copied specific sections to accompany the field notes.

Admissions Documents

Information from admission documents was used as an additional source of data about the preservice teachers' conceptions of teaching mathematics. When applying for the program, participants were required to write an essay describing professional goals and purposes or motivation for becoming a teacher. The researcher used these essays as an additional source of the preservice teachers' conceptions of teaching.

The prospective teachers also completed an interview with three instructors from the teacher preparation program. The following questions were asked:

Why do you want to become a teacher?

What grade level do you prefer (middle school, high school)? Why did you choose this level?

Why do you want to teach mathematics?

What are your perceptions of teaching? What do you think the main goals should be? Are?

What is your perception of mathematics?

From your perception, what is your responsibility in the program?

What is your responsibility to the faculty?

Instructor notes on the responses of each prospective teacher were used as additional data regarding the preservice teachers' conceptions of teaching mathematics.

The Researcher

In a qualitative study, the researcher is the primary person to collect and analyze data. Since the researcher could be a major threat to the reliability of the data analysis, establishing possible sources of biases or misinterpretations was important. For this reason, the researcher recognized the importance of acknowledging her background, biases, beliefs and conceptions of geometry and its teaching.

The researcher holds a Bachelor's degree in mathematics education from a small college in the western United States. She has taught mathematics in a rural and a metropolitan community for eight years. She taught computer programming, computer applications, general mathematics, algebra, geometry, advanced algebra, trigonometry, pre-calculus, and calculus. While teaching at the secondary level, the researcher obtained a Master's degree in mathematics and computer education. She is currently enrolled in a doctoral program at the university in which this research was conducted.

The researcher has been involved in supervising preservice teachers, instructing pedagogy and instructional technology courses, teaching mathematics to prospective elementary teachers and working with inservice middle school teachers. As the researcher observed preservice and inservice teachers, she became interested in their knowledge of mathematics and its impact on students. After extensive review of the literature, the question guiding this study evolved.

Prior to this study, the researcher recognized the importance of establishing her own conceptions of geometry and its teaching. For this reason, the researcher also completed the card sort task. Results of the researcher's conceptions are reported and discussed to provide the reader with a better understanding of the data analysis. The card sort for the researcher is shown in Figure 1.

The researcher's organization of the cards is divided into five, nested levels. The researcher believes geometry is visualization which encompasses all of geometry and is the first level. Visualization involves being able to see shapes, properties and manipulating shapes. As a subset of visualization, history, problem solving and connections play an important role in the development of geometrical concepts and should permeate geometry at all levels. Geometry should always be set in a context such as, historically, with other areas of mathematics, or in the real world. Within this context, geometry is divided into two major areas of study. These areas include non-Euclidean geometry and Euclidean geometry and are the third level of the organization. Non-Euclidean geometry includes most of the same concepts as Euclidean geometry, however, in non-Euclidean geometry the terms and concepts may be defined differently. In Euclidean geometry, for example, parallel lines are two lines in the same plane that do not meet. In non-

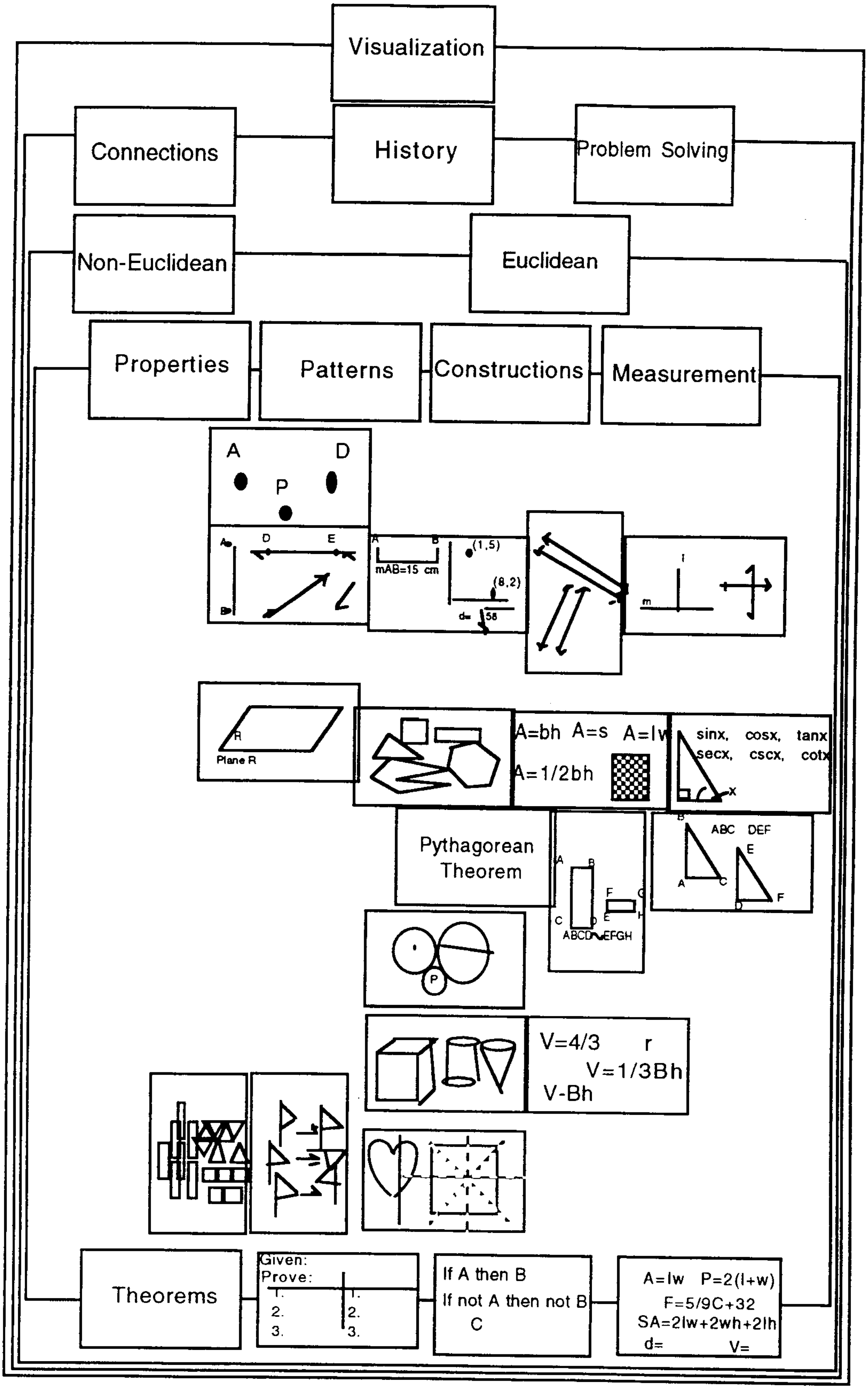


Figure 1. Researcher's card sort

Euclidean geometry, parallel lines are defined differently, depending upon the type of non-Euclidean geometry.

The fourth, nested level consists of properties, patterns, constructions, measurement, theorems, proofs, logic and formulas. These are key aspects in both types of geometry and are more concrete than the previous levels. Several of these concepts, including proofs and logic, are the first exposure to formal thought processes for high school students. These concepts are also the main tools for problem solving in geometry.

Points, lines and planes are positioned in the final level because they are central to all geometry, again, recognizing that the concepts may be defined differently, depending on the type of geometry. Without the assumption of the existence of points, lines and planes, the postulates and theorems of geometry would not be valid. The cards joined to lines are all different types of lines including segments, rays, parallel lines and perpendicular lines. The cards attached to planes were organized in a linear manner according to the dimensions. For example, polygons are two-dimensional and polyhedrons are three-dimensional. Transformations, symmetry and tessellations are some of the applications of two-dimensional or three-dimensional geometry.

For the most part, the cards in this sort represent geometry taught in high school. The researcher recognizes these cards represent just a small portion of geometry. She is also aware that many terms and concepts taught in high school geometry are missing from the set of cards represented. For example, angles, rays, segments, central angles, inscribed angles, chords, diameter, specific types of triangles and quadrilaterals, specific properties and theorems, and many other concepts are not specifically represented on separate cards. The researcher also recognizes the concepts represented are situated within geometry, however, could span several other areas of mathematics.

Method

During the Fall and Winter quarters of a four-quarter, fifth-year graduate program, data were collected. All participants had completed one quarter (Summer quarter) of the graduate program. Appendix E presents the courses completed Summer quarter and the courses in which the preservice teachers were enrolled during Fall quarter.

In late August, the preservice teachers began a part-time professional internship experience in the public schools. Each preservice teacher was placed with a mentor at either a middle school or a high school. The internship required the preservice teacher to be active in the school for at least 10 hours each week. In the last half of October, the preservice teachers were required to teach one 12 to 18 instructional hour unit in a mathematics class and participate 20 hours at the school each week.

The first quarter of the program and the first part of Fall quarter may have influenced these preservice teachers' conceptions of the teaching of geometry. Logically, however, preservice teachers' conceptions of geometry teaching are primarily influenced by their full-time internship experience or student teaching which occurs during Winter quarter. In addition all participants had the same foundation for the start of Fall quarter.

Admissions information and documents, including the essay and interview notes, were gathered prior to the start of the study. These notes were filed for future use. Preservice teachers' conceptions of geometry and its teaching were explored in two phases. In the initial phase, participants were asked to complete the card sort task, a journal and a videotape task, all of which focused on their conceptions of geometry and its teaching. The second phase included classroom observations, the work sample and informal interviews conducted before and after the preservice teachers presented lessons. This phase further supported the participants' conceptions of geometry and its teaching and these conceptions in classroom practices. These phases are discussed in the order in which they occurred. Analysis of each data source is also discussed in detail.

Phase I: Conceptions

The initial phase of data collection focused on the preservice teachers' conceptions of geometry and its teaching. During Fall quarter of the fifth-year teacher preparation program, a card sort task was administered to all 10 participants. All card sort tasks were completed within a two week period.

The researcher conducted individual interviews that lasted from 30 to 90 minutes. Each participant was reminded of the general intent of the study, the types of data to be collected and the time commitments involved in the study. The preservice teachers were also reminded the data collected would remain confidential. Prior to the card sort task, the interview focused on setting the participants at ease and helping them feel comfortable with the interview. This part of the interview focused on general questions regarding the participant's academic and professional background. The following questions guided this part of the interview:

What high school(s) did you attend?

What college(s) did you attend?

When did you graduate from college?

What have you been doing since you graduated from college?

What type of degree(s) do you have?

What undergraduate mathematics courses have you taken?

What graduate mathematics courses have you taken?

What was your most difficult mathematics course in high school? In college?

What was your easiest mathematics course in high school? In college?

When did you decide to become a teacher?

Why did you decide to be a teacher?

What type of experience have you had with students at the secondary level? At the college level? Younger?

To determine the influence, if any, the card sort task had on the preservice teachers' conceptions of geometry, a special format was used. Following the general questions, five of the participants were asked questions which included:

What is geometry?

What topics, terms or principles make up geometry?

Then the card sort interview began. The other five participants began the card sort task immediately following the general questions.

The design of the card sort interview was adapted with permission from Stein, Baxter and Leinhardt (1990). This part of the interview focused on specific information related to the card sort task. The researcher and participant discussed each card individually to clarify its meaning. The following questions guided this part of the interview:

What is meant by this card?

How would you define (blank)?

Do you have any questions about this card?

After all cards had been discussed, the participant was asked if any cards were missing or any cards should not have been included in the sort. The participants were then asked to sort the cards in any manner they chose. The participants were told to sort the cards on a large sheet of paper. They were given a pen and also told that they could change or add anything they wanted to the card sort. The sorting took between five minutes and 45 minutes. Following completion of the sort, the researcher questioned the participants regarding their card sort. They were told they might change or reorganize the cards as they described the sort. The following questions guided this part of the interview:

Describe what you have done.

What organization did you choose?

Why did you organize the cards the way you did?

Would you add any cards to this sort? What would you add? Where would you add them?

Would you take out any cards from the sort? Which ones?

Are (blank) and (blank) related?

What relationships are there between the groups that you chose?

How did you feel when you were completing the card sort task?

Have you ever thought about geometry in this manner before?

Do you think these cards are representative of geometry?

If I had asked you to do this before the program, would your sort be different? How?

If I asked you to do this sort at the end of the program, do you think it would be different? How?

The card sort interview was videotaped and later transcribed. Following each card sort interview, the researcher wrote brief notes. These notes included general impressions of the participant, reflections, descriptions, ideas, beliefs and particular patterns that arose during the interview. The card sort interview was transcribed, but not analyzed until all phases of data collection were completed. Waiting to analyze the data served to minimize researcher bias which may have affected other data collection, particularly classroom observations.

Another part of this phase occurred during Fall quarter. The ten participants completed a weekly journal of observations made in the schools as described previously. The guiding questions for the journal were suggested to the participants every other week. The journals were collected weekly, photocopied for this study, and returned to the teachers. The researcher read the photocopied journals each week and made comments in the margins of the photocopied journals for later data analysis. The comments included general impressions, thoughts, reactions, interpretations and insights into the types of conceptions of geometry teaching that began to emerge. Occasionally the researcher needed to clarify a written statement, in which case the participant was contacted by phone, electronic mail or informal meeting.

The journals served as an additional source of information regarding the conceptions preservice teachers held of geometry teaching. Final analysis of the journals was completed by the researcher after all data were collected.

Following the card sort interview, individual participants were scheduled to view three videotapes of high school geometry teachers, using a methodology adapted from Collins (1990). The schedule allowed at least two days between each interview in order to avoid any bias from a previous discussion. Each videotape was viewed by the preservice teacher with the researcher present. Before each viewing, the preservice teacher was asked to watch the tape, stop the tape at any point, make comments on any aspect of the tape and discuss important aspects of the lesson.

During the videotape task, the researcher used the planned stopping points to interrogate the participants. Questions such as the following were asked to gather information on conceptions of teaching geometry:

What do you think so far?

What do you notice about the teacher, classroom or students?

What concept is the teacher attempting to teach?

Why do you think the teacher (blank)?

What would you have done in this case?

What are some important aspects the teacher had to think about for the activity?

Why did the teacher chose to make a transition or teach the topic in this manner?

What do you think the teacher was thinking?

What other possibilities does the teacher have?

Other questions were asked as the discussion indicated.

Following the videotape viewing the researcher asked the preservice teachers to comment on the performance of each teacher in six categories. The first five categories are modifications of the core propositions of what teachers should know and be able to do, developed by the National Board for Professional Teaching Standards (1989). The sixth category was added as an overall rating of the lesson (Collins, 1990). For each category, a set of guiding questions was developed based on further evaluation of the standards. It is important to remember these questions were used as guiding questions for the post-video interview. Other questions were added as the discussion indicated. The categories with questions included:

1. The teacher attended to students and their thinking. To what extent did the teacher organize students to engage in the learning activity? Did the teacher interact with students in an appropriate manner? Was the teacher able to assess student progress throughout the lesson?

2. The teacher knew the subject matter and how to teach it. Does the teacher demonstrate an accurate understanding of the subject matter? Does the teacher select appropriate activities to teach the subject matter? What types of activities did the teacher use?

3. The teacher attended to class management and monitoring. Did the teacher maintain a classroom of equity? How? How did the teacher monitor students? What did the teacher do to maintain a classroom conducive to learning? What types of strategies did the teacher use to maintain an orderly classroom?

4. The teacher thought about and learned from activities. Did the teacher monitor and adjust teaching? Was the teacher able to reflect on the effectiveness of the learning activities? What did the teacher do to signify this reflection? Was the teacher able to get students to THINK and PROBLEM SOLVE? How?

5. The teacher participated in a learning community. Was the teacher actively involved in the classroom? What did the teacher do to show this? Were the students part of this learning community or was it a teacher-centered classroom? Was there any evidence of professional development?

6. What general impressions did you get from this lesson/teacher? What did you think of this teacher? What did you think of this lesson? What were some important aspects of this lesson? Would you teach this concept the same way?

Following the viewing of all three videotapes the participants were asked additional questions:

Which of these three teachers would/could you learn from if you were a student in the class?

Which of these three teachers would you enjoy the most?

Rank the three teachers in any manner you choose. On what basis did you rank the teachers?

All interactions between the researcher and the participant during the videotape task were videotaped. The videotapes were transcribed later and used to explore the preservice teachers' conceptions of geometry teaching. Following each session, the researcher prepared notes regarding general impressions of the viewing and patterns of conceptions which arose. Analysis of the videotaped tasks was not completed until after all phases of data collection were completed.

Phase II: Classroom Observations

The second phase of data collection occurred during Winter quarter. This phase provided information on the participants' conceptions of geometry and its teaching in the classroom. During Winter quarter, the preservice teachers participated in a full-time internship experience in the schools. Data from the four participants assigned to teach geometry during their full-time internship were gathered. Classroom observations, work samples, classroom documents and informal interviews were the data sources used for this phase.

Eight lessons of each preservice teacher were observed in geometry classrooms. All lessons were videotaped for further analysis. The researcher also collected detailed field notes during each observation. The data collection focused, specifically, on the preservice teacher. The observations, furthermore, focused on the geometry taught, presentation of the content and activities designed for the lesson. The field notes included a timed description of what the preservice teacher was doing, actions and events and interpretations that emerged. Since the researcher was unable to record every part of the lesson, the videotapes were used as an extension to the field notes.

Following each presentation the researcher formally prepared the observation, clarifying and adding any reflections. This preparation was completed within two days of each observation. The videotapes were transcribed after each observation. The videotaped transcript and researcher's field notes were merged to provide an extensive record of the lesson. Analysis of field notes was completed following all phases of data collection.

In addition to field notes, following each lesson, the researcher collected all handouts from the preservice teacher. The researcher also copied sections from the textbook the preservice teacher was using which served as additional classroom documents to make the observation more complete and to provide the researcher with additional sources of information.

Work samples served as an additional source of data in Phase II. Each work sample was collected from the preservice teachers when all changes and additions had been completed. The first work sample was completed Fall quarter and the second work sample was completed Winter quarter. Following all phases of data collection the researcher read the work samples and wrote insights, interpretations, and comments in the margins.

In addition, the researcher had informal contacts with the preservice teachers. Most of these contacts were pre- and post-lesson conversations. Prior to a lesson the researcher asked:

What are you going to be teaching today?

How will the students be organized?

What are the important aspects you had to consider when preparing this lesson?

Following the lesson, the researcher asked:

What went well in this lesson?

What would you change next time you teach this lesson?

Did the students learn what you wanted them to learn?

Why did you (blank)?

On occasion the researcher needed to clarify a statement or comment made by the preservice teacher. In this case, the researcher kept a record of each by preparing detailed notes including the date, time, content of each conversation and observer comments. If the researcher began to see discrepancies between the preservice teacher's comments (in the work sample or interview) and actions in the classroom, the researcher questioned the participant as to the proper conception. This question was phrased: "I noticed you said (blank) during the lesson. What are your thoughts on this?" Again notes were kept on all interactions between the researcher and the participants. The notes were word-processed and used as an additional source of data. Analysis of these interviews was completed after all data were collected.

Data Analysis

No single data source was used to describe specific conceptions, rather, all were used to investigate the preservice teachers' assumptions about teaching geometry. The purpose of data analysis for this study was to clarify the conceptions preservice teachers possessed of geometry and its teaching. Specifically, a detailed description of these conceptions for each preservice teacher was sought. Furthermore, this study sought to describe the relationship of such conceptions with classroom practices. In order to achieve this purpose, detailed analysis of all data sources was needed. Bogdan and Biklen (1992) described data analysis as "the process of systematically searching and arranging the interview transcripts, field notes, and other materials that you accumulate to increase your own understanding of them and to enable you to present what you have discovered to others" (p. 153).

After all data were word-processed, transcribed, photocopied and organized, the researcher continued the task of analyzing. In order to evaluate the preservice teachers' conceptions of geometry and its teaching, a method described by Denzin (1978) as triangulation, was used. Triangulation is the process of using a variety of data sources in a study to verify the analysis. Although each instrument used in this study focused on one aspect of the preservice teachers' conceptions, all phases and instruments were considered in totality during final analysis.

First, the researcher wrote detailed profiles of the professional and academic background for each teacher. The profiles were developed from the journals, initial interviews for acceptance into the teacher preparation program and essays.

Second, each preservice teachers' card sort and interview were described and general conceptions discussed. To clarify the types and descriptions of conceptions of geometry, triangulation was used between the card sort and interview. The researcher compared and refined the types of conceptions between each data source. The researcher noted patterns with respect to conceptions of geometry that began to emerge between the instruments. Furthermore, inconsistencies of conceptions also were sought in the data analysis and described.

Third, the videotape tasks were coded. This coding required each sentence to be analyzed and categorized. Initially, the codes were broad categories such as geometry, teaching, students, teachers or classrooms. After this coding was completed, the researcher read and coded the data a second time with more specific categories such as knowledge of geometry, curriculum, belief about teaching, teacher movement, student understanding, content or context.

Fourth, the preservice teachers' classroom practices were investigated through the classroom observations and classroom documents. Both videotapes and field notes were used to describe and verify or contest the conceptions. The classroom practices were reported for the four preservice teachers observed in Phase II of this study.

Lastly, the researcher developed a list of the conceptions of geometry and its teaching. For all of the preservice teachers, the relationship between the preservice teachers' conceptions of geometry and its teaching, conceptions of geometry to classroom practices, and conceptions of geometry teaching to classroom practices were examined and compared. Whenever a relationship was noticed, the researcher searched the data for non-examples of the relationship. This analysis was completed and reported for each of the 10 participants. Evidence of the relationship of these conceptions with classroom practices was also reported for the four participants in Phase II.

CHAPTER IV
ANALYSIS OF DATA

Introduction

The purpose of this study was to investigate the relationships among preservice teachers' conceptions of geometry, their conceptions of teaching geometry, and their classroom practices. Ten preservice mathematics teachers participated in this study by completing a card sort task with an interview, a journal, and interviews dealing with videotapes of three experienced geometry teachers. Four of these preservice teachers were also observed extensively by the researcher during their full-time professional internship in geometry classrooms.

All ten preservice teachers completed the card sort task and interview within a two-week time frame. Subsequently, all the preservice teachers were interviewed concerning their views of videotapes of three experienced geometry teachers. All tapes were viewed in the order: Teacher A, Teacher B, and Teacher C. All card sort tasks and videotape tasks were completed within a three-week time frame prior to the participants' entering the full-time internships.

The four preservice teachers who were observed in this study (two females and two males) were all teaching at least one geometry class during their full-time professional internship experience. They were required to teach these classes for at least nine weeks. Three different high schools were used for the internships and were located in towns with populations between 10,000 and 20,000. Each school had an enrollment between 800 and 1000 students. Two of the preservice teachers were in the same school. A third preservice teacher was in a school in the same district; thus, these three preservice teachers were teaching from the same textbook: *Geometry*, published by Houghton-Mifflin (Jurgensen, Brown & Jurgensen, 1988). The fourth preservice teacher used the textbook: *Discovering Geometry* published by Key Curriculum Press (Serra, 1989). Although this study was not designed to observe the preservice teachers teaching the same content, the researcher was fortunate to observe all four participants teaching a unit on similar figures.

To answer the questions guiding this study, it was necessary to determine the conceptions of geometry and its teaching for each participant. In order to clarify these conceptions, individual profiles of all 10 preservice teachers are described. These profiles are detailed only to the extent to which conceptions can be observed. The sources for these profiles included: admission documents, journals, card sort task with interview, and a videotape task. Each profile begins with a general description of the individual preservice teacher's academic and professional background and reasons for entering a professional teacher education program. The profile continues with a description of the participant's part-time internship and follows with analysis of the card sort interview describing the teacher's conceptions of geometry before teaching. Finally, each preservice teacher's conceptions of geometry teaching using the videotape task transcripts and the journals are described.

Profiles of the six preservice teachers who were not observed in the classroom are followed by profiles of the preservice teachers who were observed. For the preservice teachers who were observed, detailed analyses of classroom practices are described. Pseudonyms are used to assure the anonymity of the preservice teachers.

To conclude this chapter, summaries of the types of conceptions of geometry and the types of conceptions of geometry teaching that emerged are described. Triangulation of all ten preservice teachers' data supported the description of these conceptions. A summary of classroom practices is also included for the four preservice teachers observed during their full-time internship experience.

Individual Profiles

Ryan

Background. Ryan graduated with a Bachelor's degree in computer engineering in 1990. As an undergraduate, he completed three calculus courses, a linear algebra course, and a discrete mathematics course. After graduation, Ryan worked as a computer programmer for a bank. He did not like the "lack of movement" in this job and decided he would like to work with people. He wanted to "see something different" and had never been overseas, so he decided to join the Peace Corps. Ryan began working for the Peace Corps in Swaziland, Africa as a computer specialist for the Ministry of Finance. It soon became obvious to Ryan that this position would not be accommodating because it was so difficult. Ryan, therefore, began to search for another position within the Peace Corps. He was eventually placed in a rural school as a mathematics/science teacher.

With no supplies, texts, and worse yet, no common language, Ryan struggled his first year of teaching. In addition, there were as many as 60 students in his classes. He learned to improvise with lab equipment and homework assignments and soon found if he made the class fun for the students by using puzzles, games and timed assignments, he enjoyed teaching more. After two years of teaching in the Peace Corps, Ryan decided to return to graduate school "to become a good teacher."

> I want to develop teaching skills to run a classroom effectively and the understanding to plan interesting and challenging lessons for my students. [A teacher preparation program] would also give me the credentials to return overseas to teach teachers. Many third world countries suffer a chronic shortage of well-trained teachers. I would like to help by training teachers.

Ryan had not taken any courses in geometry at the undergraduate level and was required to complete both a transformational and Euclidean geometry course during the teacher preparation program.

Internship. Ryan was accepted to the teacher preparation program and was placed in a local middle school for his professional internship. His mentor teacher had been teaching middle school mathematics for 18 years and was willing to accept Ryan as an intern. Ryan began working at the middle school in the fall. He observed an algebra class, a seventh-grade mathematics class, and an eighth-grade mathematics class. He worked well with the faculty and was accepted into their school. He quickly became familiar with students' names and with classroom procedures. Ryan was surprised by the class sizes, which numbered up to 37, and also with the diversity of students.

> I haven't had too many surprises the first few weeks of school. I was a little surprised at the ethnic diversity in my mentor's

> classroom. A number of the students are from other countries and many have English as a second language. I didn't realize that students who do not have strong English skills would be thrown into math class with everyone else.

After a couple of weeks in the middle school, Ryan began to lead review sessions, go over homework and review with the students. He noticed immediately that the students were testing him. "A couple of kids were really rude. They were trying to test me so hard you could almost read it on their faces. I mostly ignored them." Ryan was also able to observe other teachers and watch their teaching styles. He stated:

> It was amazing to watch how different kids acted in each class. A kid who was passive in one class, would be active in another. One kid, who is in my mentor's class, wasn't in trouble all day until sixth period where the teacher called him down constantly.

He planned and implemented a unit on inequalities for the algebra class beginning the eighth week of the term. Ryan found in teaching his unit, the biggest problem he had was that the students were able to work at a much faster pace than he had expected.

> These kids are much more academically-oriented than kids I taught in Africa. It seems that it would be easy just to assign homework and to keep the pace going and the students would teach themselves. I have had to readjust my thinking for these kids.

Throughout the internship experience, Ryan continued to develop his philosophy on teaching and learning. He stressed the importance of the teacher establishing respect with the students. "No matter what skills or techniques a teacher may have if they don't like and respect the students they won't be effective. If a teacher doesn't respect his students, then how can he possibly expect them to respect him?"

Ryan further equated effective teaching with organization. He felt in order to be an effective teacher, one needed to be organized and prepared for the lessons. Ryan also felt an effective teacher would look for a variety of activities the students would enjoy.

> Math education should be more than learning algorithms and memorizing multiplication tables. Kids should learn that math is everywhere and how close it is related to philosophy. I think kids see math as boring because that is how it's taught. Math can be a lot of fun. There are lots of experiments that can be done and puzzles and games galore. I especially like puzzles and games because they are fun and if students enjoy something they are likely to do it outside of class. It is also important to show math as a way of thinking. When students start to think mathematically, they see math in surprising places.

Ryan believed mathematics should be fun and the best way to show students it can be fun was through the use of games and puzzles. He also believed students should be able to use and enjoy mathematics outside of school. This belief carried him through his part-time internship and through the completion of teaching his unit. He successfully completed the part-time internship and was assigned to teach algebra and eighth-grade mathematics during his full-time internship Winter quarter. He did not teach geometry so he was not observed in the classroom.

<u>Card Sort Task</u>. Ryan seemed insecure or unsure of his statements as he began the card sort interview. He was easy to talk with, but seemed to be seeking approval or confirmation of his comments. He was open to discussing his background and knowledge of geometry. He admitted he knew little about geometry, especially proofs and he had never really understood proofs, but had begun to understand them in the courses he was required to complete during the teacher preparation program.

Ryan stated geometry was the study of shapes and the patterns found in shapes. He stated several times that geometry was "looking at things pictorially instead of numerically - different ways of looking at an object rather than breaking it down into numbers." As Ryan investigated each card in the sort, he identified most of the concepts that were represented, however, was unaware of what several of the cards depicted. For example, when shown the card that represented similarity, Ryan was unable to identify the concept. He did not know what the symbol for similarity was and was unable to describe what concept was represented. Even after being told what concept the card represented, Ryan was unable to give the two conditions for assuming objects' similarity (corresponding angles congruent and corresponding sides proportional). Ryan also did not know the concept of symmetry. When shown the card representing symmetry, he finally admitted he did not know what concept was being shown and he had not seen that concept previously. Even after being told what the card represented, Ryan admitted he had not understood symmetry before as a geometric concept. Ryan also had not been exposed to tessellations. He knew tessellations were patterns, but did not know the mathematics associated with tessellations or the use of them.

Ryan defined visualization as "looking at a two-dimensional object and trying to visualize it in three-dimensions." He further explained:

> If you have a cube and you think in your mind if you turn it will it still be a cube? I guess it would also be like in drafting. You have three different sides and to be able to see all three different views in your mind's view. . . .

Ryan also stated "Euclidean geometry is based on Euclidean's rules that have been accepted for a long time and non-Euclidean breaks those rules but sets up a different set of rules." In either geometry, Ryan believed we follow those rules.

When sorting the cards, Ryan began by sorting them according to pictures or words. He noticed this himself and decided to rearrange some of the cards. Ryan's final card sort is shown in Figure 2. The group numbers were added by the researcher for clarity purposes.

Ryan explained his card sort in terms of each group he had chosen. He drew the lines to separate the groups. Group 1 was described by Ryan as mostly involving shapes or things you do with shapes, for example, tessellations, symmetry, and transformations. Group 2 contained formulas you can use.

> This group has to do with formulas. You have [sic] Pythagorean Theorem where you can find the missing side, or you can take a shape and put a numerical value on it. Then you can put numbers on the volume or the area of a shape, find the angle, find the distance between two points either by measuring or calculating it.

Ryan added perimeter and circumference with Group 2 (formulas) because he felt they were a way of finding a measure. Ryan did not choose to add any other concepts.

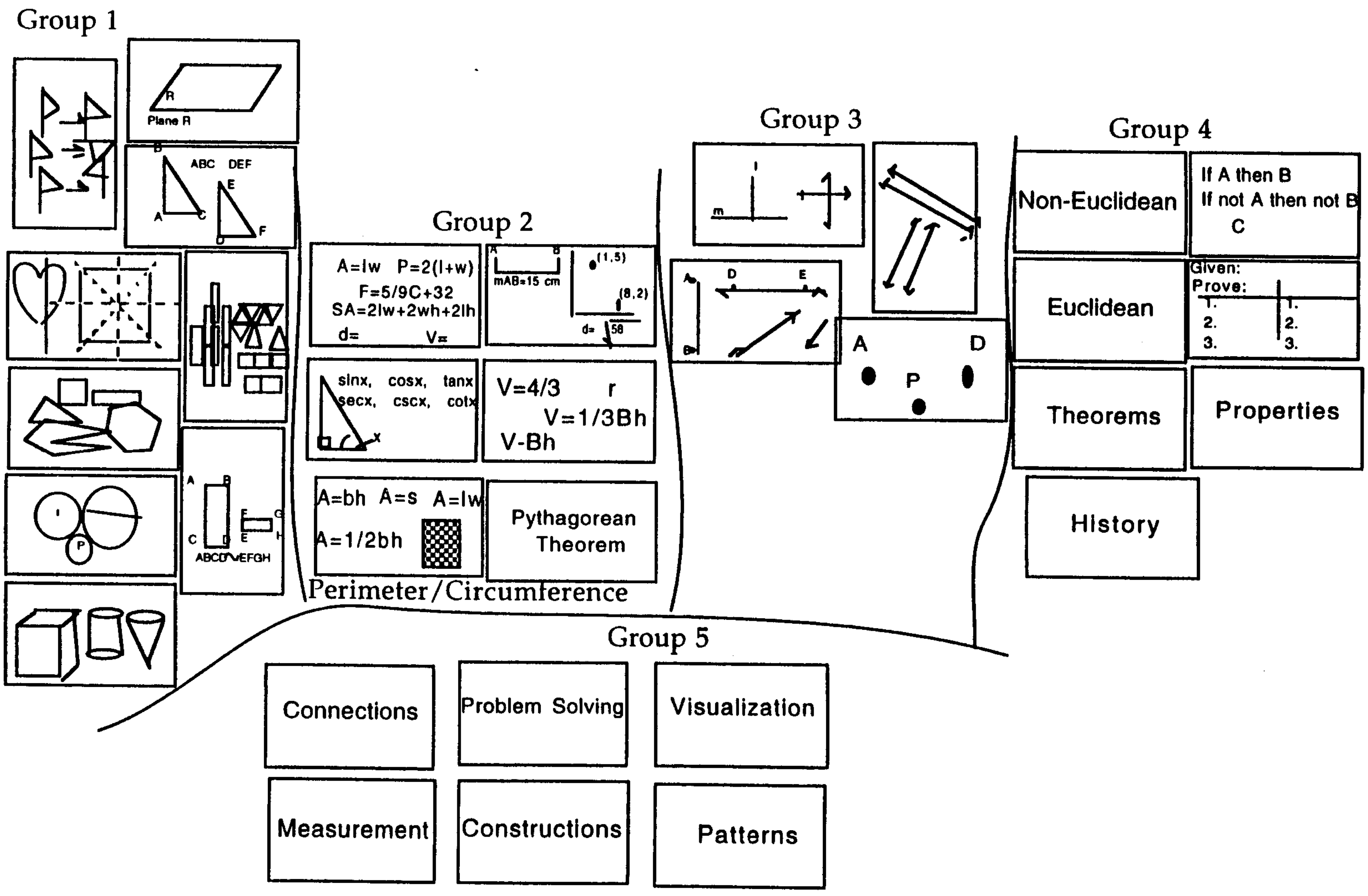


Figure 2. Ryan's card sort

The third group Ryan identified included lines, parallel lines, perpendicular lines and points. Ryan stated he would use the points to make the lines and this group represented all the different types of lines that could be made including rays and segments. Group 4, Ryan identified as proofs.

> These [Group 4] are more proofs. Euclidean and non-Euclidean are your basic overall rules that you have to stay within to do the proofs. Theorems are what you come out with [prove]. Properties are what you followed when you were doing the proof. And history is related because that is how we came up with the rules.

The last group, Ryan identified as concepts you can use with problem solving. "You use visualization to think about a problem in a different way. You use constructions; basing one shape on another shape and seeing how they are connected and then you can measure it." Ryan also felt if you had a problem you could not figure out, you could use the concepts in Group 5 to make an analogy.

When asked about the relationships between the groups, Ryan stated the first three groups were the "tools for problem solving" and the fourth group was the "backing of the tools." Ryan's explanation indicated the concepts from Group 4 were the foundation for the other groups. "The other groups would not be able to exist if you didn't have these [Group 4]." Ryan further explained his ideas by using an example:

> If you have a problem you are trying to figure out, we define some terms like parallel line and everybody accepts that and then you play by those rules. You can find another way of doing it [solving the problem] visually by using shapes, and formulas is a way of doing it numerically.

Ryan stated he felt comfortable doing the card sort and was confident in the way he chose to sort the cards. His confidence was confirmed by his actions and several other statements. He did not choose to reorganize the sort or to redo the sort. Ryan also indicated he would have sorted the cards the same six months prior to the teacher preparation program and he felt he would sort it similarly six months after the program. He was quite confident in the organization he had chosen.

Videotape Task. As Ryan watched the videotapes of the three experienced teachers, he noticed and mentioned several aspects of the classes that were important to him. Each of these aspects were mentioned by Ryan for all three teachers. First, and most often, Ryan mentioned classroom management. With all three videotapes, Ryan noticed how the students were behaving and what was expected of them. Ryan felt Teacher A had established the best classroom management. When questioned about how this teacher had established the classroom management, Ryan replied:

> He was very organized and had good transitions. There wasn't any dead time for students. His transitions were real smooth. It was easy for them to follow. I think a lot of times kids get disruptive because they don't know what is going on. He was very well organized and obviously knew his stuff, so it wasn't a case of having to wonder about that. I think he had basically established himself. He knew what he was doing and the kids respected that.

For Teacher B, Ryan seemed to waiver as to his thoughts about classroom management. At first, Ryan did not think the "teacher had any trouble keeping his class under control." Later, Ryan noticed that Teacher B had to discipline a couple of students and he was not sure if the students were being attentive. Ryan described this teacher's strategies for classroom management:

> He used names a lot. He asked a lot of questions. The students did know what was going on which helps. They were not lost. If you present something different you almost get more problems than if you stick with what they know. Kids are like anyone else, they don't like new things.

Ryan decided Teacher C had a problem with classroom management:

> A lot of times it was difficult to tell what they [students] were doing and what the expectation was. I think that would be a major problem. He doesn't seem to have any power struggles, but at the same time they didn't seem to be paying attention either.

The second aspect Ryan focused on was the teachers' behavior. Ryan was aware of the teachers' movement around the classroom. He felt all of the teachers moved around the classroom fairly well and their proximity to students helped to focus the students' attention. "He [Teacher A] seems to move around the room quite a bit, which I like. I think it is good to move around and talk to students to see where they are having problems." Ryan was also aware of the voice projection of each teacher. "He [Teacher B] moves around quite a bit which focuses your attention somewhat, but his voice is the same tone all the time." Ryan also felt all three teachers were organized. He described his view of what he felt organized meant as follows:

> I think he [Teacher B] was organized. He seemed to have all his material there. He didn't have to search on the desk for them.

Third, Ryan's beliefs about teaching mathematics were evident with these videotape tasks. For each teacher, Ryan mentioned he would use some sort of game or puzzle to teach the concepts. He could not give a specific game or puzzle he would use, but stated he would create what he needed. Ryan's reason for using puzzles and games centered around him, rather than the students.

> I am inclined to try to use a game or manipulative. I am more game-oriented. I would make it more toward puzzles or small groups so they are actually doing something and working on it. If everything is focused on the teacher, you don't have time to rest and it is more work for you.

Ryan's views of learning were also displayed with each of these interviews. Ryan believed repetition was important in helping students learn. Several statements confirmed this belief:

> A lot of time they [texts] are not repetitive enough. They don't give students enough practice, especially with your lower level students. A lot of times students just need to do something 100 times just to get it down.

> I know that if I were doing this [teaching proofs], I wouldn't jump into multiple-step proofs. I would go ahead and do maybe 20 of the one-step proofs, just because proofs are hard.

Ryan also mentioned several other aspects of the lessons he deemed important. He noticed the level of thinking two of the teachers required from their students.

> I would suspect that students were hearing the words and matching to other things, but as far as really understanding what is behind it. . . . They may be getting processes and some vocabulary, but I really doubt they are getting very in-depth learning terms or understanding the geometry.

Ryan also mentioned that setting a context for the lesson was important. "One of the things that hurts math is that it is introduced as a series of formulas or algorithms. It is reduced sometimes so much that it really doesn't have any use." Ryan felt helping students understand WHY they were learning a concept and its use, was central to teaching. "He [Teacher A] has a good idea of leading into proofs. He did a good job of saying WHY we do proofs. I remember doing proofs in high school. It was just another thing you were doing and you had no idea why."

Ryan completed the videotape tasks by ranking the three teachers in the order he had viewed them. He chose Teacher A as the "best teacher" because he had good pacing and the students would be able to follow his lesson. Ryan felt the other two teachers' lessons did not flow as well and the students seemed confused at times. However, Ryan felt these two teachers were effective because they had established respect for the students and the students respected them.

Conceptions of Geometry. Ryan held several interesting views of geometry. First, the data sources indicated his own knowledge of geometry was insufficient. He was unsure of some of the concepts in geometry and admitted geometry was his weak area. His uncertainty was obvious in the card sort and videotape tasks. Ryan was unsure of several of the concepts depicted in the card sort and he was often unsure of what the teachers were teaching in the videotapes.

Second, perhaps because of his weak geometry knowledge, Ryan avoided discussing the content in the interviews. He never talked directly about geometry, unless asked. He also never mentioned the content in viewing the videotapes. He stated he would use a game or puzzle to teach concepts, however, never actually discussed the content of the lessons in terms of the geometry being taught.

Third, Ryan believed geometry was a "toolbox" for problem solving. He felt the topics in geometry were taught to students so they would be able to use the concepts as tools for solving problems. Ryan repeated this view several times and expanded on it in when viewing Teacher A's videotape. He stated, "One possibility with proofs is to treat it like a toolbox. Give students an envelope with some theorems and definitions and have them mix and match to try and prove something."

Last, Ryan felt his organization of geometry was unchanging. He stated he would not have organized the cards any differently before the teacher preparation program or after. This permanent view again reflected his insecurity about geometry. He was not sure how he could rearrange the cards and could not imagine an alternative sort.

Conceptions of Geometry Teaching. Classroom management was of central concern to Ryan throughout this study. He was aware of what students were doing and what was expected of them. Ryan felt respect was central to teaching. He believed if the teacher showed respect for the students, they would respond by showing respect for the teacher, and thus, behave properly and ultimately cause less behavior problems. Ryan also believed by using puzzles and games, classroom management would be less of a problem.

Triangulation of the data sources clarified Ryan's beliefs about teaching and learning mathematics. He believed that learning mathematics should be fun and the best way to make it fun for the students was to have them play games or complete puzzles. Although Ryan believed the best way students learn mathematics was through games or puzzles, his reasons for using them with students was self-centered. He stressed that when students were working on games or puzzles it gave the teacher time to rest and do less work. Ryan never mentioned the effectiveness of the games or puzzles in terms of student achievement or developing understanding of the concepts.

Although Ryan stated mathematics should be more than learning algorithms, his view was shadowed by other comments he made. In general, Ryan felt mathematics was rule-bound. He consistently referred to the rules or procedures used in mathematics and how these rules and procedures must be taught to students. Ryan felt repetition was important in learning mathematics. He stated several times he would give the students more problems to practice to help them learn the concepts and rules of geometry.

Ryan's views often contradicted each other. These contradiction suggested Ryan had a view of what he wanted to believe, however, was unable to ultimately carry out this view in the classroom. Perhaps his beliefs were being formulated as he participated in the teacher preparation program and internship experience. For example, Ryan stated that helping students develop an understanding of the content and setting a context were important, but then stated he would have the students practice more problems to make sure they knew the procedures. Also, throughout the data sources, Ryan did not discuss the students in terms of their role in the classroom or their understanding of the content. Even when questioned directly about whether he felt the students were understanding the content in the videotapes, Ryan focused on the teacher and how the teacher could make teaching easier.

In summary, Ryan's views of geometry and its teaching were influenced by his perceptions for the teacher's need for survival. Ryan consistently referred to ideas or beliefs that made the teacher's job easier and avoided classroom management problems. Perhaps Ryan's own anxiety about teaching and his weak knowledge of geometry attributed to these views.

Dan

Background. Dan earned a Bachelor of Art's degree in mathematics, with minors in English and computer science in 1984. He worked as an actuary and a computer programmer for a health insurance company for seven years. After working in the business world, he studied Japanese and tutored English for a year in Japan at the Naganuma International Language School. Upon his return to the United States, Dan went back to school to earn a Master's degree in Exercise Physiology.

Dan had always been interested in education and had always thought about being a teacher, however finances delayed his further studies. While working in the business world, Dan coached high school track and cross country. "I found helping these students prepare themselves for challenges to be enjoyable and very rewarding." While completing the Master's program, he worked as a teaching assistant in human physiology and biology. He believed his academic, business, and educational experiences would enable him to "effectively and successfully assist secondary students increase their understanding of mathematics." Dan decided to enter the professional teacher preparation program because his experiences had stimulated an interest in pursuing education as a career, and he thought it would be an enjoyable profession. He stated:

> When I was working in Seattle, I coached track and cross country at the high school, and I found I enjoyed that part of my day a great deal more than working with computers in an office. I think it is enjoyable working with kids and other personalities in a school atmosphere as opposed to a business atmosphere, where people are a little more self-serving. High school students can be self-serving too, but I think it is a little more appreciable.

Although Dan had the academic background to teach biology or chemistry, he chose mathematics instead. Mathematics had been a struggle for him in high school, and he felt most teachers did not understand the struggles students have with mathematics. Therefore, he could be more understanding of students' struggles. However, since Dan did not have the required courses in geometry for entrance to the mathematics education program, he had to complete both Euclidean and transformational geometry during the teacher preparation program.

Internship. Dan was accepted into the teacher preparation program and was assigned to complete his internship at a local high school of about 1000 students. He attended the inservice workdays with the faculty and became acquainted with his mentor teacher, Mr. Jones, who had taught high school mathematics for 15 years. Dan began by observing an advanced algebra class, a pre-calculus class, and a consumer mathematics class. After the first three weeks, Dan wrote:

> I feel like a stranger when I get to school. I don't know what has been going on and don't know the students very well. I often feel that I am more in Mr. Jones' way than helping out. I am only around long enough to knock Mr. Jones off his schedule.

By the fourth week of his internship, Dan began to feel more comfortable. He was able to present some warm-up activities to the students in advanced algebra and consumer mathematics. This opportunity to be involved with the classes seemed to help Dan overcome his initial feelings of hindering his mentor teacher.

> I was much more satisfied with my internship this week. I played a larger, more active role in working with classes. It is

> much more interesting when I have a chance to see the students' response to me and my work.

Dan taught his first unit in the advanced algebra class on systems of linear equations. He completed his part-time internship at the end of Fall quarter. He was relieved when it was finished, but he was excited about teaching several classes on a full-time basis during Winter quarter.

Dan admitted some aspects of teaching surprised him. "I guess that student misconceptions and their willingness to give up without trying surprised me." He was also surprised at the students' reaction to his teaching:

> I wasn't really prepared for the angst and frustration that the students project upon the person up in the front of the room. This estrangement between the class and myself has resulted in resistance and, thus, slow progression through my material so far.

Dan also learned planning and preparation for lessons are essential. He wrote several times in his journal about not having enough content for the entire class session and the problems it caused. He was often caught with 20 to 30 minutes with nothing for the students to do. He also mentioned the importance of planning in advance instead of the previous night. Dan had consistently stayed up all night to complete or plan a lesson for the next day. He felt his own performance was affected by his lack of sleep.

Throughout his part-time internship, Dan continued to develop his philosophy about teaching and learning. Specifically, Dan believed a good teacher needed to possess some personal qualities. He felt these qualities would help the students:

> Characteristics of a good teacher include stability, consistency and all the characteristics that make one a stable and predictable force in student's lives. One of the most important traits a teacher can demonstrate to students is reliability. This allows students to know and depend upon what the teacher says and does and models behavior expected from students. Although these are very human traits, I believe that teachers need to be humans before they can reach and teach students.

Dan pointed out several qualities of his mentor teacher which confirmed this belief: "Mr. Jones is concerned, prepared, fair and responsible. These are all very important characteristics for teachers to embody."

Dan also began to develop his beliefs about mathematics teaching and learning. Dan stated his belief that mathematics should be taught to encompass the world around us.

> I believe that math teaching, as much as possible, should maintain context and connections with the rest of students' lives. I believe that many students dislike and feel alienated in math classes because the things they do and problems they work with are strange and bear no relation to anything they see and encounter anywhere else. I also think it is important to stress that math is a learned skill and not something that is inherently known or unknown. It should be emphasized that most math skills are accessible and able to be learned by *all* students.

Dan successfully completed his part-time internship and would continue working with the advanced algebra class and also the consumer mathematics class during his full-time internship. He was not assigned to teach geometry during his full-time internship and, thus, was not observed in the classroom for this study.

Card Sort Task. While completing the card sort interview, Dan stated geometry was the study of shapes and angles and the relations between them. He went on further to say that in geometry "You start with the axioms of Euclid's elements (points, lines and planes) and sort of build from there to shapes."

Several times during the interview Dan referred to his own background and limited understanding of geometry. Dan expressed his interest in geometry after completing the required Euclidean geometry course the previous Spring quarter.

> I was astounded to learn about the axioms and the postulates. That is a good place to start. I found it really interesting to have geometry grounded in the perspective of Euclid. He tried to take everything that was known and start it from as few given ideas as possible. That helped put it in perspective, and to know that he pretty much failed, that it was just too hard to define everything in terms of something else.

Dan's limited knowledge of geometry began to be evident when questioned about the differences between axioms, postulates and theorems. "I guess I would refer to them as synonyms," he said. "I use them interchangeably and I believe that is how I have seen them used." While discussing his non-Euclidean geometry experience, he said, "Even though I took a class called non-Euclidean geometry, I never actually got it." At times, Dan seemed skeptical or unsure as to how the concepts of geometry could be used and what ideas they presented. "You are *supposed* to be able to use geometry to solve things we might encounter, but I'm not sure. . . ."

As Dan organized the cards he seemed confident and comfortable and usually did not move the cards after he had placed them in a certain area. Dan's card sort is displayed in Figure 3. Dan began by drawing a line separating the top and bottom of the sort. The row of cards across the top was described as "some general mathematical things that don't have any particular relationship with geometry over other areas of mathematics. They are sort of a universal math." Dan continued to describe the top row of cards as what he had learned to incorporate throughout mathematics. "These are the connections to the problems and phenomena in the world."

Dan proceeded to describe the other cards in his sort. He stated the concepts on the cards became more difficult as he moved down the sort, and the concepts built from left to right. Dan continued to describe the lower portion of his sort:

> Euclidean, in my mind, is sort of where geometry begins. You introduce some of the basics here [left] and this is where in high school you will do some theorems, constructions, axioms, and things. You start out with the lines, points, and planes and you move to proving things about different kinds of lines, and then you head into proving things about triangles and similarity, and then you start learning more about the relationships to do some constructions.

Dan was not sure where to put circles in his sort but decided they were a bit more complex and taught after triangles, so he placed them after triangles. He finished by stating he "had never learned symmetry, transformations, and non-Euclidean geometry; so those would be on the end of the sort."

Through the interview, Dan discovered his card sort seemed to be reminiscent of his own geometry background.

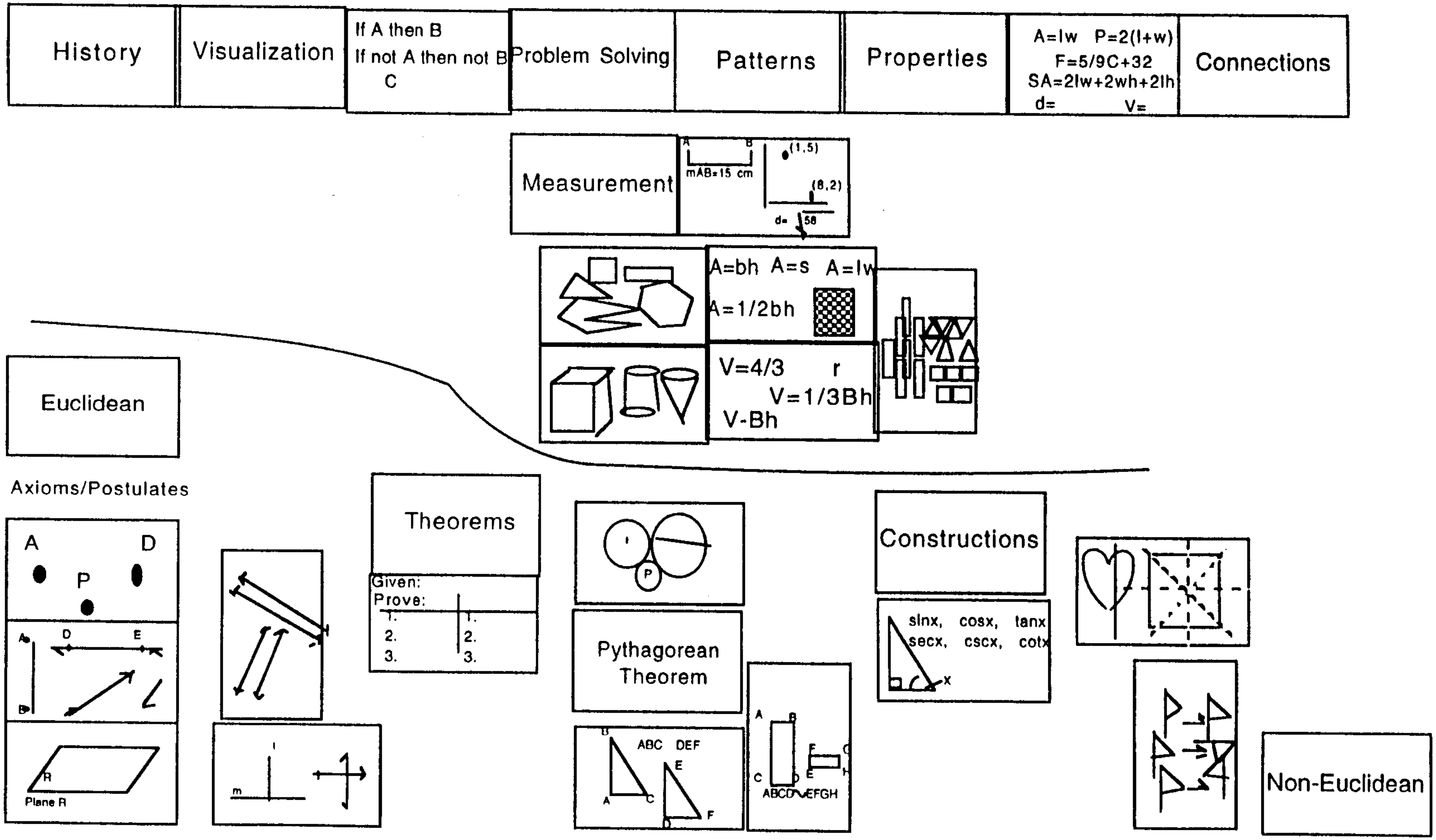


Figure 3. Dan's card sort

> As I was putting this together, I found it pretty much traces how geometry was taught to me. This is a good outline or overview of what I have experienced in geometry. You learn how to measure, you learn how to do things, you learn about shapes and you learn about equations about shapes, the two-dimensional and three-dimensional.

Dan completed the card sort by adding axioms and postulates to the left-hand side of his sort. He wanted to add more cards, but stated:

> If there were more cards to fill in these things a little better I would probably run into the same problem as Euclid did; that is, you know what comes after what, and after a while everything becomes circular.

When asked about how he felt when completing the card sort, Dan replied, "I sort of wished I'd had a [geometry] book with me so I could pinpoint exactly where I wanted to think better."

Throughout the card sort interview, Dan stressed the importance of connecting geometry to the outside world. Because of his work in the business world, Dan stated he knew it was important to show students how they might use the concepts of geometry outside of the classroom. He said, "You don't just deal with shapes and constructions in a geometry class, but maybe how they are used outside of class." Dan was not always sure how this connection could be made, but he felt it was important.

Videotape Task. When completing the videotape tasks, Dan's beliefs about geometry and its teaching continued to surface. Several aspects of importance were evident to him. First, Dan consistently and repeatedly referred to the students' understanding of the content. Regarding Teacher A's students, Dan stated:

> It would be interesting to see how many of them actually have a piece of that [proofs]. Certainly there isn't any reason to think that they don't. They haven't been doubting or complaining or anything. There is a percentage of students who, when they are not getting something, would definitely let you know. However, there is also no reason to think they *do* understand.

When watching the other two teachers, Dan noted similar concerns about students' understanding and, also the teachers' ability to assess the students. Dan doubted any of the teachers actually knew if the students understood the content. He felt Teacher B went through the material so fast he could not possibly determine if the students' understood the content. Regarding Teacher B, Dan stated:

> I didn't notice any of the students following him [Teacher B]. In fact, a couple of girls in the front put their heads down and were not paying attention. There doesn't seem to be a lot of response from the students. He does not seem to be aware of this. He is asking a lot of questions but answers his own questions without any wait-time. I would say that he is not able to assess them or know if they all understand.

Dan also thought Teacher C was unable to assess the students on their understanding of the content. He stated:

> It seemed like we were not getting an idea that any of the students knew how to go about the problems. I guess you would have to assume that he [Teacher C] had a better idea, since he worked with the students all the time.

Second, Dan referred to the teacher's planning and knowledge. He was always quick to comment on the teachers' planning, organization, and preparation.

> He [Teacher A] seemed very organized. It seemed like he had the lesson set up for some time, not only using some of these things [warm-ups], but also using some of the definitions that he had established in previous classes. I don't know whether he was following the book or just following logic, but it seemed organized.

Dan did not feel Teacher B or Teacher C had spent as much time in preparing the lesson. He noted Teacher B seemed organized because he had the worksheet prepared for the students, but he had not thought about the order in which he would present the content to the students. Teacher C, to Dan, did not seem at all organized. He stated:

> I would have to question his [Teacher C's] preparation. He obviously didn't look at the test before he handed it out to the

> kids. They had problems understanding the questions, and so did he. In fact, he even got some of the problems incorrect. He just sort of copied it and handed it out.

Dan was also well aware of the teachers' knowledge of both geometry and its teaching. Dan commented on Teacher A:

> I would say that he not only appears to understand the subject matter, but he understands the method that the students will be going through in order to understand it. The things that he is stressing, pointing out, and reinforcing are things perhaps from experience that he knows the students will have trouble with. He has a good feeling for the subject matter and feeling for how the students are understanding it.

Whereas Dan felt Teacher A had a good understanding of geometry and how to teach it, he felt Teacher B and Teacher C did not:

> It seems like he [Teacher B] is only presenting the surface level or limited aspect of the material. He is not really communicating that there is much more to this material. He does not let the students know that there is any more information that is going to be useful to them.

> I wouldn't be surprised to hear that he [Teacher C] wasn't a math teacher. He called the median "the middle thing or something like that." This cavalier attitude would imply that this was not material that he was interested in. He did the problems, as opposed to getting information on the problems from the students and having them help him. He hadn't looked at the test before he gave it to them. He seemed to think that the students did bad on the test because they couldn't understand the wording of the questions. He seems to be comfortable that he has an understanding of the problem. I am less comfortable with that, and I would question if that was the only reason.

Third, Dan consistently commented on certain aspects of the nature of mathematics and how it should be incorporated into the lessons. He felt it was important to indicate to students that, in mathematics, there can be more than one answer to a problem or more than one way to solve a problem.

> From what we see in his [Teacher C] presentation, he does not give the reasons. The fact that he got an answer for the problem,

> and the book gave another answer, and he said the book's answer must be correct, indicates that there is only one correct answer.

Dan felt it would also be important to point out to students the conventions of mathematics. Dan stated all three teachers needed to spend more time explaining to students these conventions and the idea that there can be more than one answer to a problem. Dan's comment regarding Teacher B summarizes his thoughts:

> A couple of things he [Teacher B] just said how to name them [concepts], he didn't say how that was convention; that it wasn't inherent in the subject matter. That is how people decided to name them. He also did not point out the fact that more than one answer is possible. Not to say that he didn't know that, but he just chose not to point that out.

Some other areas of importance to Dan included the level of thinking that was required during the lessons and classroom management. For example, Teacher C was not aware why the students did poorly on his test. Dan felt the students had not done well due to the level of the presentation.

> He [Teacher C] presented the material differently. He presented the material at a different level than it was tested. He presented the problems as arithmetic problems, and maybe what they need to know is *why* they can do that. It is hard to find a geometry class that would not know that if $16 = 1/2x$ then $x = 32$. These students were not following, so I would say they wanted to know WHY, as opposed to just being able to do it. Since it was a book test, they were being tested at a level that was higher than it was presented.

Dan also mentioned Teacher B presented the material to the students at a rudimentary level and did not allow the students to move beyond that level.

Another aspect of the lessons Dan noticed, but did not focus on, was classroom management. He commented on each of the teachers' ability to manage the students and how each teacher seemed to deal with problems that arose. He decided he liked Teacher B's and Teacher C's management styles, because they were non-threatening, and the teachers allowed the students to contribute their opinions. He felt Teacher A had established a "business-like" atmosphere which seemed more threatening to students.

Dan ended his videotape sessions by deciding he would be able to learn from all the teachers, but it would take a special type of student to be able to learn from Teacher B. He felt Teacher B was better suited for teaching at the college level. Dan was asked to rank the teachers in any order he chose. He ranked the teachers in the same order in which he viewed their tapes, stating personal preference for Teacher A because of his preparation and organization of the lesson.

Conceptions of Geometry. Through triangulation of data sources, Dan's conceptions of geometry began to emerge. Dan's understanding of geometry was, at best, minimum. He consistently and openly discussed his own background and knowledge of geometry and admitted there were several areas in mathematics in which he was unsure or needed to review. He said, "The students [advanced algebra] are forming functions from equations, a subject that I needed to cover myself. I have a hard time keeping up and being able to help them much."

Dan exhibited incorrect knowledge of geometry several times during the data collection. For example, when asked to supply a justification for the sum of two adjacent angles equaling 180 degrees, he replied, "Adjacent angles form a line." In fact, adjacent angles do not have to form a line. The correct response for the justification was "definition of a linear pair" (two adjacent angles whose sum is 180 degrees).

Dan also stated he had never learned symmetry, transformations, or non-Euclidean geometry. Thus, he chose to place those concepts on the end of his card sort because he would teach them last. Dan also stated he did not understand tessellations. He even chose to disregard the concept of tessellations during the card sort because he did not feel comfortable with it. He stated:

> Tessellations, I really wasn't sure. I have the feeling that there is some more complexity behind there that I really haven't been introduced to and I don't know, so I am throwing those out.

On the other hand, Dan was not always incorrect in his knowledge of geometry. He was able to provide the steps and justifications for proving alternate interior angles of parallel lines are congruent.

Conceptions of Geometry Teaching. Dan's views of geometry teaching also emerged from the data sources. First, Dan believed in the importance of incorporating geometry into the world around us. Dan stated from the beginning of his program that he felt it was important to set a context for the mathematics students were learning. Dan's own experiences in the working world seemed the foundation for this belief. He said, "Working has given me a great deal of practical experience in applying the principles of calculus, algebra, and statistics in real world business situations." Dan confirmed this belief several times during the interviews in which he participated. He suggested ways in which the teachers could have incorporated some real-world applications into their lessons, and he also stressed the need to tell or demonstrate to students *why* they were learning a concept.

Second, Dan believed mathematics is an art or skill that is not inherent and must be practiced. Repeatedly, Dan suggested that mathematics is learned rather than innate. He stated,

> Math is a learned skill and not something that is inherently known or unknown. It should be emphasized that most math skills are accessible and able to be learned by all students. With work and practice you can get it. The students that say they can't do proofs can get them with practice.

Dan confirmed this belief by discussing that he had worked to learn his own mathematics skills. His stated reason for being a teacher was to be able to "use those math skills he had worked to learn." Dan also believed students learned the mathematics skills best through practice. Regarding the videotapes, he said,

> He [Teacher A] is trying to tell them that, even if this seems foreign or difficult now, with practice everybody can do it. I liked his comment on the justifications and the fact that it was a skill, not just something that some people automatically knew, and if you didn't get it down, you would never get it.

Third, Dan was acutely aware of each teachers' preparation and planning. As discussed earlier, Dan consistently referred to the teachers' organization and preparation for their lessons. He had mentioned in his journal that planning was an area in which he wanted to improve. Perhaps Dan was aware of the teacher's planning due to his own problems with planning. He knew the importance of planning and organization, however, he never felt he was able to plan well enough for his own lessons.

Last, Dan stated he wanted to be a teacher in order to increase students' understanding of mathematics. He reiterated this view several times, especially when viewing the videotapes of the experienced teachers. Dan stated he did not think any of the teachers did a particularly good job helping the students' understand the content or assessing the students' understanding. Dan's main purpose in becoming a teacher was to help students understand the content and he continued to stress its importance throughout the teacher preparation program.

Kaylee

Background. Kaylee had wanted to be a teacher as she was growing up, but changed her mind in high school. She began undergraduate school studying marine biology and soon switched to engineering. She eventually decided to major in mathematics and graduated with a Bachelor's degree one week prior to entering the professional teacher preparation program. In her essay and interview for admittance into the teacher preparation program, Kaylee stated that she decided to become a teacher because she had always enjoyed being around young people.

> I have always enjoyed working with children and teaching them stuff. So I decided that there is a lot of value in teaching kids and it's an important job. Math has been something that has been easy for me, most of the time. Therefore, my professional goal is simple: I want to be the best mathematics teacher that I can be.

Kaylee had completed all requirements for admission to the teacher preparation program, however, had not passed the mathematics National Teacher Examination. She was accepted into the program on the condition that she would pass the exam before her full-time internship began Winter quarter.

Internship. Kaylee was placed in a middle school for her internship experience. Her cooperating teacher had been teaching for six years and had agreed to work with Kaylee. As with all the interns, Kaylee participated in the inservice for this school prior to the opening of school in September. Kaylee observed two classes of Math Concepts (seventh-grade mathematics) and two classes of Math Applications (eighth-grade mathematics). These classes were taught in a modular building adjacent to the main school. From the beginning of her internship experience, Kaylee was excited about being in the school and had a positive attitude about teaching. She was readily accepted into the school by her mentor teacher, the principal, and the other faculty members. Kaylee was encouraged to introduce herself to the classes immediately and she chose to do an activity with the students. This introduction gave the students a chance to get to know her and to accept her as an authority figure in the classroom. After the first three weeks Kaylee wrote:

> These first few weeks of my internship have been really exciting. I feel that I am learning something new about being a teacher every minute. I have realized that there is a lot more to teaching than just getting up in front of the room and teaching. I thought the newness would wear off and I would get tired of getting up early to face a bunch of kids, but so far every day is an adventure!

Kaylee immediately began to present content and activities to several classes. Usually she would teach a class with little preparation time. She learned very quickly the importance of planning ahead of time. She stated:

> I used my mentor teacher's lesson plans and did not have time to look over the activity. I hated the feeling of not knowing what they [students] were supposed to do. I realize now that I will never put myself in that position again. I will have a plan of action for each class. I do not think that was fair to the students. I guess this is a good example as to why preparation is very important.

Kaylee often filled in for her mentor teacher and the other mathematics teacher. She was always willing to help and open to teaching. Often, when a substitute was present, Kaylee was the one who presented the lessons while the substitute observed. She was utilized in the classroom and given much more responsibility than was expected for an intern. Kaylee did not have any problems living up to that responsibility.

Kaylee observed many teachers in this school who did not enjoy teaching. This particular district was in the midst of negotiations for a contract that remained unsettled for six months. The teachers were anxious about settling and, to Kaylee, seemed tense. Her philosophy about teaching was influenced by the observations Kaylee made about the teachers.

> I think the most important thing about being a teacher is wanting to be there and constantly evaluating yourself so that you can become a better teacher. As a teacher, I do not think that I will ever be able to let myself become stagnate. I would get bored too fast. To me, the challenge of teaching is always looking for a better way of teaching a concept. Teaching is a constantly changing job. Instead of stressing over change, a teacher should be able to rise to the challenge.

Kaylee was required to prepare a 12 to 18 hour unit that she would teach in one of the classes. She chose to teach a unit on area and perimeter in a pre-algebra class. Before Kaylee began teaching her unit she expressed her concerns:

> I am looking forward to starting my unit, but I am a little apprehensive. I have to teach these kids something and hope that they learn it! It is kind of scary to think that I am responsible for what these kids learn.

As Kaylee began to teach her unit she was discouraged with her planning of the material. She realized she had planned too much material for the unit and did not feel she would have enough time to cover it. Kaylee was also concerned she might have to reteach some concepts and would not have enough time to do so. She wanted to make sure the students understood a concept before moving to the next topic. Kaylee was also discouraged with the students' behavior and her inability to deal with it. She stated, "On Monday the kids were horrible. They were testing me. Afterwards, I was ready to give up teaching forever." Although quite distressed after the first few days of teaching, Kaylee soon became comfortable teaching and she felt more confident in dealing with students and their disruptions.

Kaylee continued to establish her belief about teaching mathematics as she participated in the part-time internship experience. She believed students should be given an assignment every day and be allowed to work on it during class time. Kaylee believed in establishing an environment conducive to learning and giving the students respect and responsibility in learning. She further described her philosophy:

> My philosophy of teaching is easy to describe. I believe that every student can do well and succeed. If I go into teaching believing that every one of my students can succeed, then most will rise to my expectations. I hope that I can make math fun and exciting so that all students will want to learn and participate.

Kaylee successfully completed her part-time internship and became comfortable with teaching. She was not teaching geometry at the high school level so was not observed for this study.

Card Sort Task. Kaylee participated in the card sort interview and seemed comfortable while completing it. She stated she felt like there must be a specific "answer" that made the card sort correct and she must know that answer since she would be teaching geometry some day. Kaylee was assured by the researcher there were no right or wrong answers. Kaylee's card sort is shown in Figure 4. She divided her sort into three columns by drawing lines and labeled the groups non-Euclidean, both, and Euclidean.

Kaylee showed how concepts are divided in geometry. She explained that geometry seemed to encompass two categories: Euclidean geometry and non-Euclidean geometry. Kaylee did not know what concepts she would add to the non-Euclidean group of her sort, however, she felt the concepts learned in non-Euclidean geometry were also included in Euclidean geometry.

The concepts in the middle group spanned both Euclidean and non-Euclidean geometry. Kaylee placed the cards that depicted theorems, patterns, problem solving, connections, constructions, and history in a line because "they are the NCTM *Standards* of what is supposed to be taught." The rest of the cards in the center group of Kaylee's card sort were described as being "in-between because you could use them [the concepts] in both [Euclidean and non-Euclidean geometry]." Kaylee added definitions and real-life applications to the middle group of her sort because she felt they were part of the NCTM *Curriculum and Evaluation Standards* (1989) and they are learned in both Euclidean and non-Euclidean geometry.

Kaylee had organized the cards depicting Euclidean geometry in terms of how the concepts were taught. Concerning her Euclidean group, she stated:

> I am placing points, lines and planes together because they are usually taught together. Most of these are things that are taught together because they [students] are introduced to shapes and

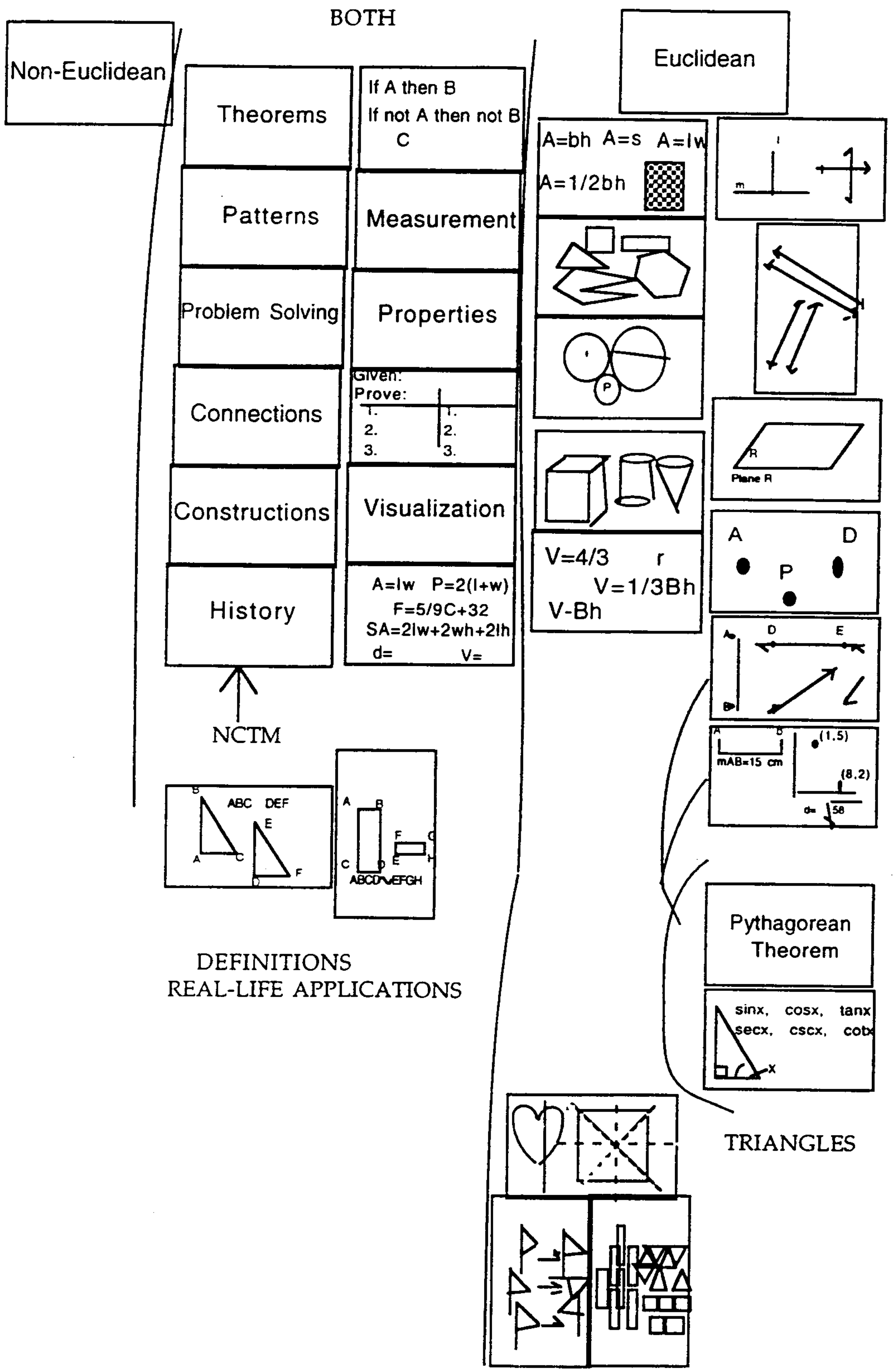


Figure 4. Kaylee's card sort

> then move into volume and area of those shapes. I guess I am doing it [card sort] in terms of how they are taught.

Kaylee also wanted to add a separate card for triangles to the Euclidean group because "triangles are usually something they [teachers] go pretty in-depth on." Kaylee further explained she had spent a considerable amount of time learning the different properties of triangles and that is why she wanted to stress them more.

Kaylee described the groups as being "all intertwined." She felt that in geometry concepts build upon each other. "You just can't start with proofs. You have to learn properties of triangles and circles and everything, and then you can do proofs."

When asked if she had ever thought about the organization of geometry, Kaylee stated:

> I don't think I have ever thought so much as defining geometry before. Usually it is just concepts that are there. Once you start teaching you have to think about all the different things. You have to think about where you are going to put them, how you are going to teach them, and in what order. But as far as breaking it down, I have never really thought about how it is organized because usually it is how your teacher does it.

Kaylee felt her card sort would have been different before beginning the teacher preparation program and would change as she began teaching. She stated she had not known about the NCTM *Curriculum and Evaluation Standards* (1989) previous to the program and so she would have arranged the cards differently. Since she was learning so much in the program and would continue to learn as she began teaching, she was certain her organization of the cards would change.

Videotape Task. When viewing the experienced geometry teachers on videotape, Kaylee remained positive about teaching. Several areas seemed important to Kaylee as she viewed the tapes. First, she consistently referred to the teachers' knowledge in two areas, geometry and geometry teaching. Kaylee was not convinced any of the teachers had a good understanding of geometry. She was critical of Teacher B because he had to read a definition from the textbook. Kaylee was surprised he could not remember the definition of scalene triangle and she did not think it was appropriate to have to read the definition from the book. She also noticed Teacher C had not answered a question correctly during class. Teacher C's content knowledge concerned Kaylee and she stated: "If he was confused then obviously he didn't teach the kids in the right way, so I don't see how he could mark it wrong if they were using what he taught them."

Kaylee felt, at times, all three teacher were not providing the students with complete conceptions of geometry. She consistently referred to the teachers not explaining to the students that there could be more than one answer to a problem.

> My concern with what he [Teacher B] said is that there is more than one scalene triangle and he is only acknowledging one. Someone might have the other scalene triangle and think they are wrong and not know why.

Kaylee also noticed all three teachers were more concerned that the students obtain "the" correct answer to a problem, rather than focusing on the processes for solving a problem.

> He [Teacher A] was looking for a specific one-step definition. That was it. He didn't really ask "How did you come up with that definition?" He is letting them know what he wants and the correct way to do it, but it could be stifling to a student's imagination. He didn't really expand on anything at all. It was all a definite answer.

Kaylee felt these naive conceptions were not consistent with the reforms in mathematics education and these teachers needed to be more open to students' suggestions for other possible answers to a problem.

Kaylee's second area discussed during the videotape tasks was related to these observations. Kaylee often mentioned the teachers' ability to represent the content to the students. She felt Teacher A did a "nice job of using what they [students] already knew and building on that." Kaylee stated it was often hard to build on what the student knew, but seemed an effective way to approach teaching. She noticed Teacher B did not seem to be able to explain the concepts so the students could understand.

> It seemed like sometimes he didn't know how to explain it. He could not think of any other way to explain it so they [students]

> could understand. He could have explained it visually or used some other technique.

Kaylee also thought Teacher C did not really have an idea of what he wanted to accomplish or what he wanted to teach and how to get the information across to students.

Third, Kaylee consistently referred to the teachers' knowledge of student understanding. Overall, she was not impressed with the teachers' assessment strategies. She did not feel any of the teachers were able to assess the students' understanding of the content, especially Teachers B and C. In fact, Kaylee felt these two teachers "had no idea" whether the students understood the concepts or not. Kaylee did, however, compliment Teacher A on one of his strategies of assessment:

> I was surprised when he went up to the board and was able to say "Most people got this, but not everybody." I thought he was just walking around to see if people were working. I didn't think he was actually checking to see if they were getting it right.

A fourth area of concern for Kaylee was with the decisions the teachers chose to make in their teaching. Kaylee often wondered why a teacher had chosen a certain problem to show the students, had chosen to structure the class in a certain manner, or had introduced the content in a certain order. She was critical of Teacher C because he had chosen to use problems for a test that were written by the textbook author. "Why didn't he take those questions and change them so they [students] would know them?" Kaylee felt Teacher C had presented an unfair testing situation to the students. She felt the teacher had presented certain types of problems during his class and then had tested students with other types of problems.

Concerning Teacher A, Kaylee wondered why he had presented the lesson in the order he did.

> I was just curious as to why he was putting up the homework at this point. If he is going through another proof, why is he putting up the homework now instead of after the proof because they [students] might start their homework instead of listening to him.

Kaylee also wondered why both Teacher B and Teacher C were asking questions of the students, but then answering their own questions. She asked if they did anything in other classes that made the students think on their own rather than the teacher telling the students.

Last, Kaylee constantly compared the classrooms she was viewing with the classroom in which she was completing her internship at the middle school. Her comparisons were made mostly in reference to student behavior and classroom management. Kaylee compared Teacher A's students to middle school students in the following statement:

> I'm surprised, just coming from the middle school, how well they are behaved. The students seem almost kind of sedated. I know there is a difference between middle school and high school students, but he hasn't had any problems with the class whatsoever.

Kaylee felt this difference in behavior was due to the types of students, not anything the teacher might have done to establish classroom management. Kaylee felt neither Teacher B or Teacher C had good classroom management. She noticed Teacher C just let the students talk while he was explaining the problems and Teacher B tried to get several students to keep quiet, but was unsuccessful. Kaylee's solution to the problem was quite clear: "Yell at them! Don't just call on them and snap your fingers. Yell at them! Tell them to shut up!"

Kaylee decided she would not enjoy being in either Teacher B's class or Teacher C's class. She ranked the teachers in the order she viewed them based on her own ability to understand the material in each teachers class.

<u>Conceptions of Geometry</u>. Although she had completed the required college courses in geometry, Kaylee's knowledge of geometry was deficient. Kaylee had a hard time passing the mathematics National Teacher Examination, but after three tries, finally succeeded. Her knowledge of geometry was lacking and she mentioned it several times as being one of her weaknesses. She admitted that non-Euclidean geometry was an area that she had not understood fully. She stated, "I didn't learn much in non-Euclidean [geometry]. The basic thing I know about non-Euclidean is that two lines meet."

When viewing the videotapes, Kaylee was often unsure of the content being taught. She asked the researcher to explain corresponding angles to her and she also could not remember vertical angles. When asked to provide reasons for three one-step proofs, Kaylee did not give any correct reasons. She excused her lack of knowledge by stating: "If I would have taught geometry I would have been able to rattle off how I would do it. I have been out of geometry for awhile so it is harder." In fact, Kaylee had completed her college courses in geometry just the previous year.

Kaylee's conceptions of geometry were naive so it was interesting when she also noted weak conceptions in the experienced teachers on videotape. Perhaps the awareness of her own deficiencies in geometry made her more cognizant to others' weaknesses.

Conceptions of Geometry Teaching. It was obvious from Kaylee's card sort that her conceptions of geometry centered around the teaching of it. She constantly referred to the order in which geometry would be taught or how geometry was taught to her. She stressed that when you are teaching geometry you must think about how to organize it. Furthermore, she believed people learn geometry as they teach it. She eluded several times that if she had taught geometry she would understand it better.

Before the teacher preparation program began, Kaylee stated her philosophy about teaching and how she would like her classroom to be:

> This is the philosophy I would like to use in my classroom: By letting your students become actively involved in what they are learning, they will feel more comfortable and able to learn with greater ease. If the students feel they are important and that they have some control over what they are learning, then I believe they will become better students.

It was obvious Kaylee continued to believe students should be actively involved and had clarified this belief throughout the present study. Kaylee believed the best way to get students involved and interested was by presenting real-life examples and activities important to students' lives. When critiquing the experienced teachers on videotape, she felt the teachers needed to get the students actively involved in the lesson, rather than the teachers telling the students about mathematics. Although she stated this belief many times, when questioned directly about how she would present a topic or concept, Kaylee had difficulty. She knew she wanted to relate the concepts to students' lives, but was unable to provide concrete examples.

Kaylee remained positive throughout her part-time internship experience. She had enjoyed the experiences and continued to reflect on how she could become an effective teacher. Kaylee realized she would continue to learn and was excited about teaching during Winter quarter.

Scott

Background. Scott attended a small college immediately after high school and graduated with both a Bachelor's and a Master's degree in mathematics. He worked as an instructor of college algebra while completing his Master's degree, and found he enjoyed sharing his knowledge of mathematics with students. He enjoyed the feeling of solving a problem for the first time and wanted to see other people get to that stage. After graduation, Scott decided he wanted to expand his teaching experience so he joined the Peace Corps. He taught algebra, arithmetic, geometry, and trigonometry in Malawi, Africa for one year. He enjoyed being able to see his students "exult when they solved elusive mathematics problems." Scott then worked as a teacher trainer for new volunteers in the Peace Corps. He felt that he was able to prepare the new teachers for their experiences in Africa, and also give them some classroom management suggestions.

Upon returning from Africa, Scott enrolled in a doctorate program and spent a year taking graduate classes. His goal was to become a mathematics education instructor at the university level, therefore, he felt it necessary to be certified and teach at the secondary level in order to be a "good college teacher." Scott applied to the Master of Arts in Teaching program and was accepted, however, he was required to take a course in non-Euclidean geometry during the program.

Internship. Scott was placed in a small rural school which included grades 1 through 12. His mentor teacher taught both mathematics and science and was actively involved in restructuring mathematics education in the state. Scott began the school year participating in the inservice days where he was introduced to the faculty. When school began, Scott spent his part-time internship observing a physical science class, an algebra class, and an advanced mathematics class which included advanced algebra, pre-calculus, and calculus students. He began to acquaint himself with the students and helped his mentor teacher by grading papers, helping individual students, and finding activities for teaching.

Scott presented several lessons and activities and started to feel comfortable in the school. After seven weeks he began to teach a unit on networks and directed graphs in a geometry class of 15 students. His mentor teacher carefully reviewed each lesson with Scott and made suggestions for improvement. The mentor also guided the first few lessons that Scott taught. Eventually, Scott taught this class by himself.

Scott reflected on several aspects of his teaching experience in his journal. One of his concerns was dealing with the students talking while he was teaching and trying to "maintain order in the classroom."

> Getting students to direct their attention to the front of the room is difficult. Next time I teach, I want to establish clear expectations and make sure that I have all students' attention before I start talking.

Scott reiterated the importance of establishing clear expectations for the students and of communicating these expectations. He felt a "structured learning environment" was a requirement for effective teaching and it must be established before students are able to learn. He stated, "If students understand the teacher's expectations they will be able to learn a subject."

Scott was also surprised by the "endless amount of planning required to make a lesson successful." He felt his management was not successful at times because he had not prepared enough material for the lesson and needed to "keep every student busy." He learned quickly the importance of planning and being prepared for a lesson. He reflected on planning:

> My teaching this week helped me to appreciate the amount of effort that is put into each class that is taught. I thought I was well prepared, but I experienced some difficult times. The review took a great deal of time and I allowed students too much time to spend on the next activity, thus they were very loud. If I had managed class time better, we would have completed what I had planned.

Scott mentioned several other instances in his journal where his lack of preparation affected the students' behavior.

Besides classroom management, Scott's main concern focused on helping the students understand the content. He felt he had been able to assess the students' knowledge and monitor their progress throughout the unit. Concerning one lesson, he stated:

> I was able to assess their [students'] performance by visiting each group and asking questions of the students. Talking to the students individually is much more effective than questioning them in class. I helped some of the students and we went over the problem to make sure they understood.

Scott suggested several other ways he was able to assess the students including questioning, student presentations, projects and testing. Scott was pleased at how well the students performed on the unit test and felt he had been successful at teaching this unit.

Scott completed his part-time internship experience and taught an advanced algebra class and a pre-calculus class during his full-time internship experience. He did not continue teaching the geometry class he started, and, therefore, was not observed teaching in this study.

Card Sort Task. Scott seemed quite comfortable when completing the card sort. He began by listing five areas he felt were not represented in the cards including coordinate geometry, simultaneous equations, modeling, matrices, and trigonometry identities. Scott talked about the cards as he initially sorted them. After completing the sort, he decided he wanted to redo the sort. His second sort is shown in Figure 5. All lines were added by Scott as he described his sort.

As Scott explained his second sort, he used convoluted sentences. He began by explaining one aspect of his sort, then jumped to another part, and frequently returned to the original statement. The statements quoted in this section are revised to represent Scott's complete thoughts.

Scott began by explaining how problem solving was the center of his card sort because it is "sort of how you analyze particular parts of geometry." History, properties, and formulas were the particular parts Scott identified. He felt one needed history to know the origin of the formulas, how to use the formulas, and how to use the shapes to apply those formulas. According to Scott, a person also needed to know the properties of shapes to be able to use the formulas, and the theorems were a way to verify the formulas.

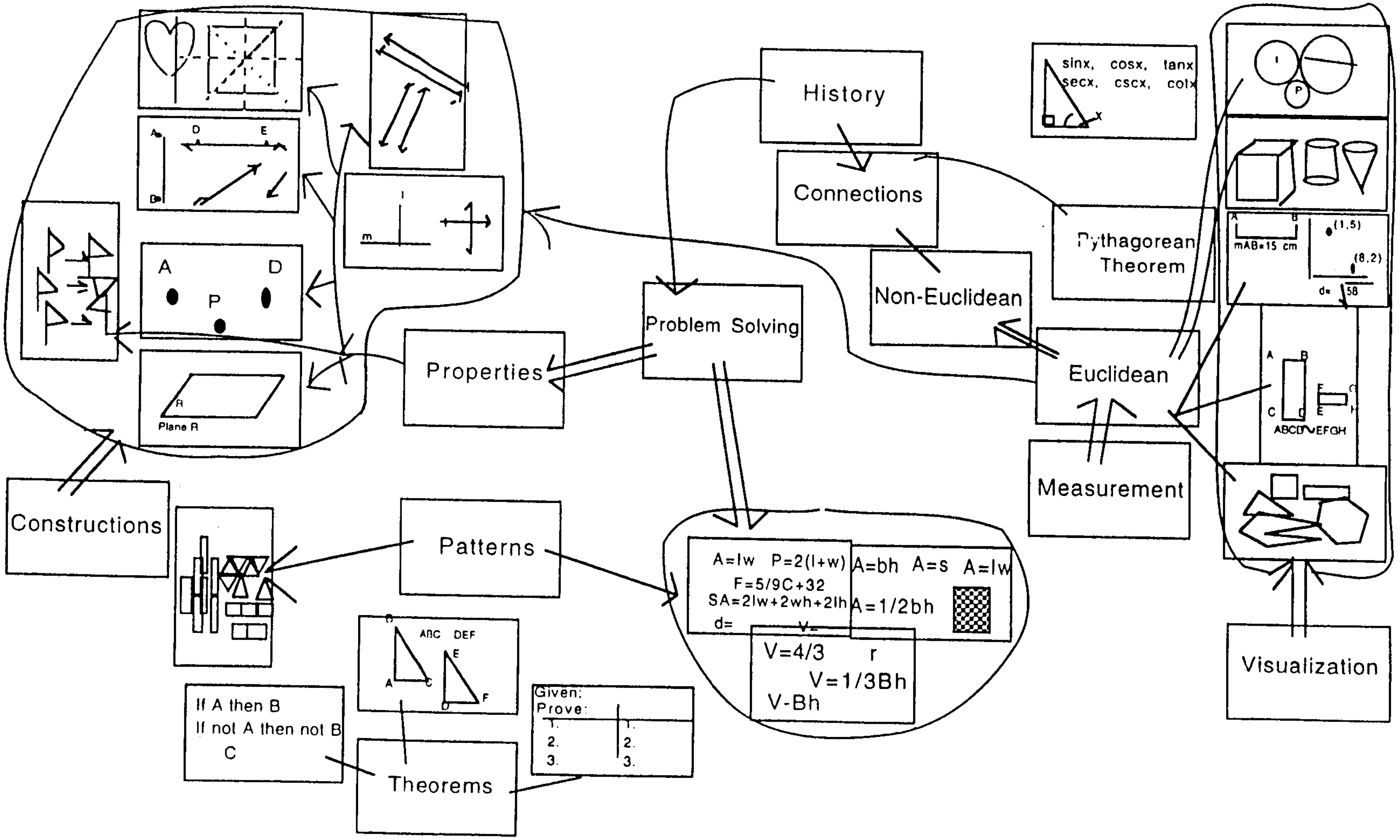


Figure 5. Scott's card sort

Scott felt the arrow between the cards representing history and connections represented a strong link. He explained:

> I think that through the history we see the connections. We see the need to make connections because math wasn't strictly a body of knowledge as it appears to be now. Before, it was used for counting or collecting objects and a way to shorten the amount of time spent doing computations or finding rules so everything doesn't have to be done by hand.

Scott explained how the five cards grouped together on the right of his sort were the foundations for Euclidean geometry and that these "skills" were needed before being able to move into non-Euclidean geometry. He stated, "To get non-Euclidean you have to have Euclidean because you need to develop visualization skills before moving to the higher level thinking of non-Euclidean geometry." He connected these cards, through the Euclidean card, to the seven cards on the left of his sort. These cards were described as being the basic properties for the shapes of Euclidean geometry.

Scott continued by describing how measurement also connected to Euclidean geometry. He stated:

> Measurement is a way of doing it [geometry]. It is hard to prove something through measurement, but it is a process you use to make objects *you need* for Euclidean geometry.

Scott asserted Euclidean geometry was more practical on a two-dimensional plane. He stated in Euclidean geometry it is assumed you are working on a flat plane, whereas, in non-Euclidean geometry you are working with three-dimensional planes. He confirmed this statement by stating that Euclidean geometry was "easier to visualize on paper." He also stated that in non-Euclidean geometry "there are not curved shapes." When questioned further on this point, Scott indicated circles were "controversial" because they do not have sides, but spheres were part of non-Euclidean geometry. Further discussion indicated that Scott did believe this difference existed between Euclidean and non-Euclidean geometry.

Scott summarized his card sort as follows:

> [This all ties together] because constructions and measurement sort of go together to form the shapes for Euclidean geometry.

> From the visualization you can find patterns and then from the patterns you can find rules. You can tie the properties and patterns together through geometry and algebra and try to come up with theorems to show that those are valid patterns that you found.

Scott stated he thought the sort was difficult to think through, but felt more comfortable with his second sort. He stated he knew where the ideas originated and how they were related, but could not figure out how to connect all of them. Scott decided he did not like to organize geometry in this manner because he felt an important aspect was missing, which was process.

Videotape Task. When viewing the videotapes, Scott was quite critical of the teachers in several areas. A predominant theme for Scott throughout the videotape tasks was the framework in which the teachers presented the lessons. He repeatedly mentioned he felt the teachers needed to "go outside the classroom" and show students where they could use the content they were learning. Scott felt that without providing this context, students would simply memorize the concepts for the test and then forget 90%. Regarding Teacher A, Scott stated:

> I think what he needs to do is to go outside. [He needs] to think about things outside the classroom. [He needs] to use the experiences the students have had, what they learned growing up. He could talk about some current issue that is really strong, like smoking in public buildings, and have the students support their positions by providing reasons.

Scott felt the teachers were working within a small setting, the textbook, and they did not move out of the classroom to help students understand the concepts. Scott's statements regarding Teacher B reiterated this theme and provided reasons for feeling so strongly about using a context other than the textbook or classroom.

> You get into the habit in the classroom, if you see it [a concept] in the book you think it is only in the book. We don't really see it anywhere else. It is just in this special place and once we shut the book, it is gone. If you take it outside, you keep reminding students that it is all over the place. Students start to analyze things automatically.

Scott felt the students would remember the concepts better if they could constantly apply them to outside situations.

Along with providing examples outside the classroom, Scott felt the teachers needed to provide reasons to the students for learning the content. Scott expressed this concern during Teacher B's presentation:

> He is not providing a purpose. He just told them "You have to remember this." But *why*? They don't really need to remember it. The only reason they need to remember it is to do their homework or take a test. Once they get out of the classroom there is no reason for them to remember it.

Scott felt the students who did not understand the purpose of learning the content did not remember it.

Another area of concern Scott mentioned during the videotapes was the type of presentations. He felt all three teachers spent their time lecturing or "telling" the students the content, rather than helping the students figure it out on their own. This type of teaching was of concern to Scott for two reasons. First, he felt the teachers were not getting the students involved in the lesson. He commented often that the students did not need to be involved or even pay attention to the lessons. One comment regarding Teacher C best expressed Scott's concerns:

> The students seemed to be able to just sit back and watch him and not engage themselves. They did not even need to be worried about having to know the material. They probably just thought, "I won't be questioned anyway so I can just sit back and watch this guy up at the board provide everything."

Second, Scott felt that by the teachers "telling" the students, they were unable to assess the students' understanding and get feedback from the students as to their progress. Scott decided all of the teachers in the videotapes were good teachers, however, their lack of assessment of students in the class "may be the heart of the problem."

> He [Teacher B] is working through the problems rather than having the students work on it and then getting their feedback. The whole time he has only been providing them with information. He has gotten very little feedback from them. He is essentially telling them everything.

Scott thought the teachers should ask more questions and make sure that most students were questioned. He also felt the teachers needed to move around the classroom more and make contact with individual students.

Scott's conceptions of classroom management became quite clear when discussing the three teachers. Scott felt all the teachers had "good" classroom management. When questioned why he thought that was true, Scott felt it was due to the students in the classes and not anything the teachers had done. He stated: "He [Teacher C] is lucky. This is a good group of students." Scott believed classroom management depended mostly on the students in the class, rather than anything the teacher had established.

Scott ended the videotape task by ranking the three teachers. He decided he would rank them according to their eye contact with the class, so he chose them in the order he had viewed them. Scott stated he would be able to learn from Teachers A and B, but not Teacher C. He thought the best thing he could learn from Teacher C was what *not* to do as a teacher.

Conceptions of Geometry. Scott's conceptions of geometry were difficult to determine. He was able to understand what each of the experienced teachers were teaching during the videotapes, and he was able to provide reasons for the proofs Teacher A presented. Scott's background in mathematics was obviously strong, however, his card sort was unorganized and the description unclear; as a result, it was difficult to determine his conceptions. Several conceptions did surface, however, through triangulation of the data sources.

First, Scott was able to relate geometry to other areas of mathematics. He knew there were many connections and interrelationships within geometry and he was specific in tying geometry to algebra. He also related many geometry concepts to other fields of mathematics. Second, he believed mathematics was created to "shorten descriptions." In geometry, Scott thought patterns were used to develop the formulas, the formulas were used to save time, and theorems were used to verify those formulas. Last, Scott had only recently completed a course in non-Euclidean geometry and had not quite assimilated this content. He believed Euclidean geometry was mainly for two-dimensions and non-Euclidean geometry was for three-dimensions.

Conceptions of Geometry Teaching. Scott had an interesting philosophy about learning mathematics. Although he had stated in his essay he believed students learned best by using manipulatives, his view was actually quite different. Scott believed the best way to learn mathematics was by "exercising the muscle of the brain" through practice and repetition. In his card sort, journal, and videotape tasks, Scott reiterated this belief. He believed students should be given timed tests often so they would be able to complete a task in a given time frame. He believed students should work on problems that were much harder than what they would see on the test so the problems on the test would seem easy. And he believed by practicing, students would not think the tasks so difficult. Scott summarized his belief in this quote:

> Just like if you are getting [students] ready for an athletic event, you want to stretch the material so students go beyond what they know. You want students to practice with timed tests or mock tests. You want to overdo it so students are ready, and when it comes to the task, it is not difficult because students have gone beyond what they know and have practiced for it.

In terms of teaching geometry, Scott felt it was important to set a context for the geometry and to provide the students with a purpose for learning the content. He discussed the importance of showing the students where and how mathematics was used outside of the classroom. He criticized the experienced teachers on the videotapes concerning this area and he continued to reiterate the importance of establishing a purpose throughout teaching his own unit.

Another concern for Scott was classroom management. He had focused strongly on classroom management during his part-time internship, and continued to focus on it when viewing the videotapes, however, this concern was limited to two aspects. Scott believed classroom management was effective because of the students who were in the class. He stated classroom management depended greatly on the types of students a teacher had in class, rather than on anything the teacher did. Scott also believed if students finished their work, the teacher should be prepared to give them more work. This belief was Scott's solution to classroom management problems. He often evaluated the success of a lesson by the students being kept busy and if they behaved.

Becky

Background. Becky graduated with a Bachelor's degree in mathematics the term prior to beginning the teacher preparation program. She had attended a community college for the first two years of her undergraduate studies and transferred to a university for her remaining studies.

Becky had always told herself she would never be a teacher. "That was the last thing I was going to do." Becky had felt there were limited rewards in teaching and students caused teachers so much "grief" that it was not worth the struggle. She stated, "You go into it [teaching] thinking you are actually going to make a difference in somebody's life, but you actually make yourself feel worse." Thus, Becky spent the first three years of her undergraduate education studying actuarial mathematics.

Becky decided she could not picture herself "sitting behind a desk for eight hours a day and studying for tests all night." She was not sure what else she could do, so decided to teach. Becky knew she liked many areas of mathematics and felt teaching provided her an avenue to use these areas. She had worked as a mathematics tutor for four years during college and enjoyed helping others learn and feel comfortable with mathematics. Becky felt her experience tutoring had taught her about motivating people to learn and she could use this knowledge in the high school classroom. She stated in her essay that she wanted to be able to "make an impact on student's lives and prepare them for college mathematics."

Becky had a strong background in a variety of areas of mathematics and a good grade point average. She had completed all requirements for the teacher preparation program and was admitted without stipulations.

Internship. Becky's internship experience was at a local high school with approximately 1100 students. Her mentor teacher had taught for eight years. Becky began her internship by observing, helping individual students, and grading papers in a college algebra course. She felt the students would be well-behaved and should be lectured to since they were college-bound and should get used to that type of teaching. She stated, "I was thinking that since they are going to college (or at least most are), then they should get used to the lecture format of college."

After several weeks of working with the students, Becky realized her views about the college-bound students were not as accurate as she had thought. Becky discovered even though these students were college-bound, they were still high school students and needed variety in the presentations. She discussed these views:

> There is something to be said about preparing students for college, but lecturing them to death is not founded in logic. They are falling asleep [during class]. If students don't get a firm background in what they will learn in college, their life is going to be much worse off. That is why I would involve them in more activities.

Becky wanted to be involved in presenting a lesson, so her mentor asked her to present a review with the students on exponential equations, logarithms, and conic sections. Becky expressed her reservations about teaching the content. She stated:

> I have very little history in these two chapters. I will basically have to teach myself these concepts to be able to review with the students. Shouldn't be too difficult though. I will cross my fingers that I won't get any of those questions like, "Why are we doing this?"

Becky also worked on planning and organizing a week-long unit for the college algebra class on matrices. Before she began to teach, Becky again expressed her reservations about teaching the content. She did not feel comfortable with the specific unit and was nervous about teaching it even though she felt prepared. She stated,

> I am worried about saying something incorrectly or just bungling a problem. Not being very adept in systems of equations and matrices in the first place might explain it. I never liked studying them, even when I could find applications. I prefer the more abstract areas like number theory and proof theory.

Although Becky had wanted to involve students, she noticed as she began to teach her unit, she spent the majority of her time working problems at the overhead and not really involving students. She stated, "I have a tendency to work out problems on the overhead and not check whether students are following me or not. I guess I just assume they are." Again, this type of teaching reflected Becky's original views about college-bound students.

Throughout her unit, Becky described in her journal several situations important to her. Most of Becky's comments expressed genuine concern for the students in terms of their feelings, understandings, and participation in the class. Becky was quite concerned with three students in her class who were not achieving at an acceptable level. She discussed these students in her journal and with her mentor teacher, and suggested possible reasons for their failure and some solutions. She was also surprised with the "learning rates" of the students in her class. Becky stated, "I was rather surprised, even in the college algebra course, at the range of learning rates--how students do perfectly well in one chapter and can seem to have total blocks of any intelligence in the next chapter." Becky continued to describe her amazement that students could graph points, but when they had to use a function to get points and then graph the points, they were unable to complete the task.

Teaching this short unit helped Becky to realize the importance of listening to students. Again, Becky's concerns for the students emerged. She discussed the importance of helping students realize their own potential and helping them learn responsibility. Becky felt an effective teacher was a teacher who was able to listen to students, get feedback from students, and return feedback to the students. Becky extended her ideas about an effective teacher:

> Effective teachers are good communicators, able to emphasize various ideas and clarify ones that students have difficulty with. They are familiar with the applications of the subject and have a good idea of what gets students motivated.

Becky continued to develop her beliefs about teaching as she completed her internship experience.

Becky completed her part-time internship successfully and continued to work with the college algebra students in her full-time internship. She also took on responsibility for a pre-algebra class and an advanced algebra class. She was not teaching in a geometry classroom, so was not observed for this study.

<u>Card Sort Task</u>. Becky did not seem particularly comfortable as she began the card sort interview. She was a bit nervous and quiet as the interview began. She felt the researcher was looking for a "correct" response, but was assured there was no intention to judge her responses as "correct" or

not. Becky became more comfortable as the interview progressed. She discussed the cards individually and decided most of the cards represented Euclidean geometry. As she began to organize the cards, Becky described herself as being "horrible at this." Again, Becky was assured that there was no "correct" answer to the card sort. Becky's card sort is shown in Figure 6.

Becky began by explaining how she had divided the sort into two main categories, Euclidean and non-Euclidean geometry. Becky made this distinction because she felt that, in teaching geometry to students, teachers were often misleading and did not explain clearly the differences between Euclidean and non-Euclidean geometry. She described one example:

> Teachers ramble off a bunch of proofs and theorems and students are supposed to think on their own whether it makes sense or not. For example, [a teacher might say:] "Parallel lines never meet." Students can believe that. But if you look down the train tracks, it looks like the rails meet. So it is kind of contradicting what the students think.

She felt Euclidean geometry was abstract and non-Euclidean was more concrete. She explained this belief further by stating, "This [Euclidean] is the chalkboard and this [non-Euclidean] is the real world." Becky used the concept of measurement to provide an example of what she meant.

> As far as measurement goes, the street is pretty flat compared to the rest of the world. As long as you are not talking about huge distances, the ideas of measurement [in Euclidean geometry] will work on a street block, but they won't work on a larger part of the world. This is when you will need the concepts of non-Euclidean geometry.

Becky explained she had organized the cards on the Euclidean side of her sort in an outline format. She marked each section with a letter down the right side and numbered each subsection. Becky described her sort as "how it was usually taught," or the order in which she had learned geometry. She felt the foundations for the study of Euclidean geometry were the main concepts of points, lines and planes. Becky stated students needed these basic ideas in order to do constructions. She included properties, parallel lines, perpendicular lines, congruence and similarity with constructions "because when you are interpreting constructions those are the properties you learn."

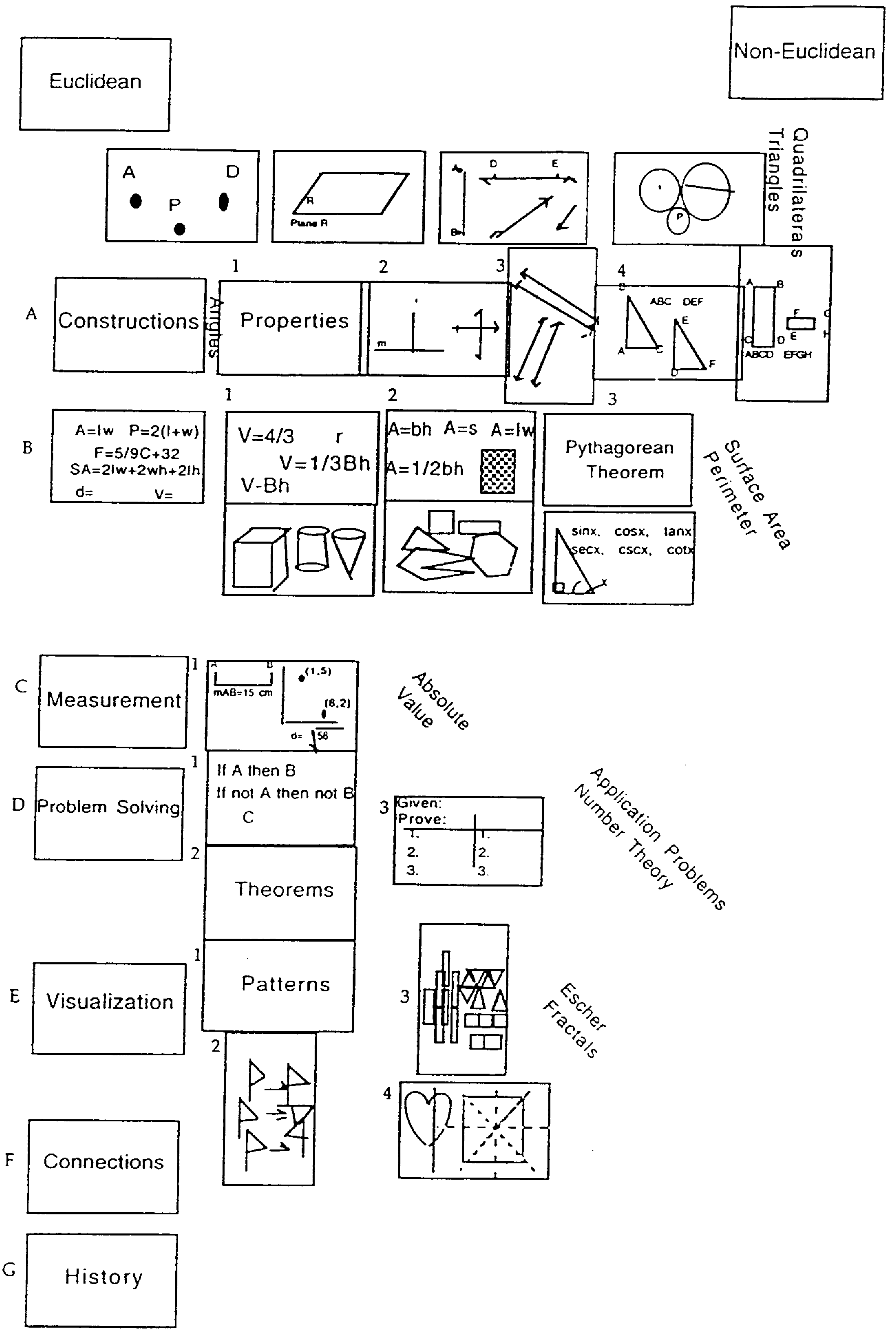


Figure 6. Becky's card sort

Becky continued by explaining that formulas and measurement were then learned by students. She felt once a teacher had introduced a figure through construction and presented some of its properties, the students needed to "have some idea of being able to measure the figure." Thus, Becky included area, volume, trigonometric functions, and the Pythagorean theorem with polygons and polyhedrons.

Becky felt students needed the first three categories (constructions, formulas and measurement) to solve problems using logic, prove theorems and complete proofs. She stated, "You have to use the formulas to solve problems. You really *get* to problem solving after you have done formulas."

Becky's next category was visualization. She felt students are usually not introduced to the topics of tessellations, symmetry and transformations until after high school geometry. Becky had not seen these concepts herself until the teacher preparation program. "I didn't really even do those in college."

Becky completed her sort of Euclidean geometry with connections and history. She decided these concepts were "all-encompassing" and they should be integrated throughout the teaching of geometry.

Becky decided to add several concepts to the Euclidean part of her sort. She included angles with constructions, but stated she could also include them with measurement. She added surface area and perimeter with the formulas, absolute value with measurement, applications and number theory with problem solving, and Escher designs and fractals with visualization. She also wanted to include all the different types of triangles that are studied in high school geometry to her sort.

In terms of the non-Euclidean part of Becky's sort, she stated there were no concepts in that category. Becky felt the cards represented mostly Euclidean geometry. She stated:

> [Euclidean geometry] is what they [high school students] learn. They don't even deal with non-Euclidean. I think non-Euclidean is probably one of *the* most important things to introduce in high school geometry, but it isn't introduced.

Becky also wanted to add several concepts to the non-Euclidean part of her sort. She listed these as: spherical, hyperbolic, the Paris problem, and Manhattan taxicab geometry.

Becky completed the card sort interview by stating she had never actually thought about organizing the concepts of geometry. Her reasons for not having thought about geometry before centered around her own experiences. She stated,

> I'm not teaching geometry this term or next term so I have not really thought about it in-depth. I remember I had a horrible time in high school geometry. It just seemed so abstract to me. I did not understand *why* we were doing the T-proofs. I think because I had a hard time I will be more sympathetic and try to make it more interesting.

Videotape Task. Consistent with Becky's comments in her journal, during the videotape tasks she expressed genuine concern for students in terms of their feelings, understandings, and motivation. Becky was concerned that students feel comfortable in the classes and with the teachers. She commented on this aspect with each of the videotapes. Becky noted that, for the most part, the students seemed comfortable in the classes and "connected" with the teachers. She felt this relationship had been established over the term through a respect the teachers showed for the students. Becky thought it "nice to have a one-on-one relationship with the students."

By feeling comfortable in the class, Becky also felt students would eventually understand the content. She was concerned with the students' understanding of the content and the teacher's ability to assess that understanding. She discussed this concern regarding Teacher A's classroom:

> I kind of cringe when I see a class like this because there are a few students who blend into the woodwork and just get forgotten. The students are just there and that bothers me. They are not included in the lesson and they may not feel comfortable in the class. The teacher checked for understanding of some of the students, but didn't for *all* of the students and did not include these students.

With Teacher C, Becky noticed several students asked questions of the teacher and he responded, but the students still did not seem to understand what he was explaining. Becky felt the students finally just gave up because they did not feel comfortable asking more questions. "I still don't think the students got it," she stated.

A predominant theme during the videotapes, for Becky, was motivation. Becky felt motivation was central to teaching. She talked about the students being motivated to learn and participate. Becky felt if students were "internally motivated" they would learn. Regarding students, she stated:

> The student who is motivated will think on their own and come up with the answer to a problem [in Teacher A's class]. The student who thinks the teacher will do the problem anyway would wonder why they should to the problem on their own.

> If you have a lot of internal motivation then you would probably do well in the class [Teacher B's]. If you don't care about being there and you just want to sleep, then sleep is all you get out of the class.

Becky commented she felt Teacher A's students "seemed fairly well-motivated." She thought his students looked motivated because "it didn't look like there were any students doing anything but listening."

Becky also discussed the teacher's role in providing motivation for the students. She felt the teacher could use several techniques to help motivate the students. These techniques included asking questions, group activities, getting students involved in the lesson, and providing positive reinforcement to the students. Regarding Teacher A's lesson, Becky remarked:

> It is hard to get students motivated doing proofs. You would think there is some better way to do this that is more motivating to students and has them actually doing something instead of just taking notes. I don't know what it is, but there should be something.

Becky felt if teachers were really interested in the students they would try to present the content so students would be motivated to learn and pay attention during class. She stated, "If the students get the idea that the teacher is trying to make the content more interesting, I think they are more apt to try harder, if they think 'this teacher actually cares that I like the subject'."

During the videotape tasks, Becky again expressed her revised views about lectures. She stated several times if teachers had taken the mathematics reforms seriously, they would avoid teacher-centered lectures and do more student-directed activities. She often suggested the teacher could have the students work in small groups to complete a task, or have the students complete a task at the board.

Becky completed the videotape tasks by ranking the three teachers in the order she had viewed them. She stated she preferred Teacher A because of his organization and planning, and Teachers B and C because of the way the students felt comfortable in their classrooms.

Conceptions of Geometry. Although Becky's conceptions of geometry were based on how it was taught to her, she seemed to have a strong view of the concepts in geometry and their interrelationships. Becky was able to discuss the differences between Euclidean and non-Euclidean geometry and provide several examples. Again, these differences centered on what was usually taught in high school geometry. She was also able to connect the concepts in geometry with other subjects in mathematics, like number theory. Considering Becky had spent so much time focusing on mathematics in her undergraduate program, it was not surprising she was able to connect the concepts. However, Becky did not connect the concepts in geometry to any areas outside of mathematics.

Becky was correct in responding to the problems posed by Teachers A, B, and C and even suggested other approaches to the problems. For example, when Teacher C was presenting the concept about the median line of a trapezoid equaling the average of the two bases, Becky suggested the teacher show the students how this concept was true using triangles. She also asked if the same idea was true for other lines, like a line that divided the sides in thirds.

Becky believed geometry was important to students as a "tool to be used in applications." She felt some basic concepts must be learned in geometry before students were able to solve problems. For example, in her card sort, she was explicit that students needed to learn constructions, measurement, and formulas before they could solve problems. There was a definite order, for Becky, as to how concepts should be presented to students.

Conceptions of Geometry Teaching. Becky entered the teacher preparation program believing that, as a high school teacher, she would prepare students for college. She believed the main goal of teaching mathematics was to prepare the students for college mathematics, and the best way to accomplish this goal was to teach much like a college professor. Becky's view of that type of teaching was basically lecture-oriented. Although Becky continued to believe in preparing students for college mathematics, she soon realized lecturing was not the most effective way of accomplishing this goal. However, in her part-time internship, she found herself still practicing this type of teaching.

Becky's main theme throughout this study was motivation. She strongly believed students must possess an internal motivation in order to learn mathematics. This belief seemed to stem from her beliefs about college-bound students. She felt college-bound students were more motivated, and thus learned the content on their own.

Becky also equated an effective teacher with motivation. She believed a good teacher was able to provide motivation for students to learn the content. Becky suggested students were motivated if the teacher seemed to care about them. She was quite concerned with students' feelings and their comfort level in the classroom. She felt students should feel comfortable with a class and with a teacher.

Becky best summarized her beliefs and philosophy about teaching mathematics:

> I think it is important to motivate students to learn mathematics in more than one way. Students should be motivated by using activities during class that the students are likely to find interesting and enjoyable, and by giving the students reasons to be interested in the particular content of the lesson. Also, teachers should make it a point to keep students involved in class, by asking questions of particular students, and keeping the class moving in the direction intended. To do all this, teachers need to be well prepared with plans, goals and ways to accomplish those goals.

Nick

Background. Nick began college studying engineering and switched to mathematics after two years. During college, he worked as a ski instructor and mathematics tutor. He graduated with a Bachelor of Art's degree in mathematics and was not sure what he should do with the degree, but decided upon teaching. He commented he had worked several jobs that were "boring" and wanted to work in a job that was not the "same thing over and over." He had enjoyed working with high school students as a ski instructor and wanted to "help young adults realize the confidence that a positive educational experience can provide." Nick felt his own love of mathematics would enable him to teach mathematical concepts to students in a positive way. He stated:

> I would like to play an active part in the development of our greatest resource: our children. I have a love for the subject matter [mathematics] and I believe I can convey an enthusiastic attitude to my students.

Nick stated he had found "great pleasure" in struggling and solving difficult mathematics problems. He had a genuine love of mathematics and wanted to share this love with high school students. He felt that observing students solving difficult problems would be rewarding for him. "I can imagine few things as satisfying as seeing a student discover they are capable of performing tasks that were once the source of fear, confusion, or frustration."

Nick stated his working experiences had helped him to realize teaching involved much more than knowing the subject matter. He felt a teacher's role was to discern what learning styles students possessed and to present the subject in manners that were effective for all students. He stated, "The teacher acts as a guide for his students, working past their fears and hopefully to an understanding of the concepts being taught."

Nick had completed all the requirements for the teacher preparation program and was accepted to begin the term after he graduated. It was recommended he spend some time with high school students in a classroom setting before he began the program; however, he chose not to do so.

Internship. Nick was matched with an experienced mathematics teacher in a local high school enrolling 1100 students. He was placed in a geometry class and an introductory algebra class for his internship. He observed several teachers at the school and commented on how the teachers' personalities made a large impact in the classrooms. "Teaching is as much, if not more, about personal interactions as it is about content." He began to learn students' names, but did not begin to help students until the third week of the term. By the fourth week of the term, Nick started to lead review sessions with the students and presented one lesson on scale drawings in the geometry class.

Nick began to plan a unit to teach on similar figures. He was concerned he would not be able to keep the students "on-task" and was worried about his own management ability. He wrote in his journal:

> This whole management thing is a little scary for me, as I have never had to manage students by myself. It seems that for experienced teachers this sort of thing comes naturally. They don't wonder what they should say or do, they just do it without thinking. On the other hand, I am not so sure what to do. I have to stop and think: "What would my mentor teacher do?" "How will the student react?" In my moment of delay, my mentor has already reacted to the situation.

Nick's stress level increased as he continued to develop lessons and began to teach his unit. Often times he spent up to 10 hours preparing a single 90-minute lesson. Nick was indecisive about what he should teach and how best to teach it. He spent hours trying to decide what to teach, hours deciding how to teach a concept, and hours creating activities or worksheets for the students. He became increasingly upset about his inability to plan and often taught without having slept the previous night. His lack of sleep and inability to plan continued to frustrate him, causing him more stress.

Nick taught his unit and felt he had presented the material clearly and students understood the concepts. Test results confirmed his beliefs that students understood the ideas of similar figures. Nick finished his teaching unit by presenting square roots to students and showing them how to simplify. Nick realized students did not understand the process and reflected on this:

> It was upsetting to see that my instruction was not sufficient. I learned something about teaching and learning. First, I realized how easy it is to inaccurately assess student understanding. I really thought these kids could do this, but I was wrong. Second, I needed to give more opportunity to practice. I should have done more in-class practice. Third, I should have given a more concrete process to solve radicals.

Nick also became increasingly disillusioned with students and their behavior. He had had positive experiences with high school students on the ski slopes and had not imagined what their actions might be like in the classroom. He was shocked that many students did not do homework, by the quality of work they did hand in, and their lack of attention during class. He wrote in his journal:

> I am continually amazed with the kids who come to class but don't pay attention at all. I want to say, "Make it easy on yourself. You have to do the homework and take the tests. It would be so much easier to pay attention." Students will complain about the homework and then they will not use the seat time to do homework unless you stand right above them. I don't get it.

Nick successfully completed his part-time internship and was assigned to teach a geometry class and an algebra applications class for his full-time internship. Nick's stress level, however, became so intense he was unable to accept the responsibility of preparing for and teaching his classes. He dropped the teacher preparation program before his full-time internship began. Nick, therefore, was not observed for this study. He had completed the first phase of data collection, so the data are reported and used in this study.

Card Sort Task. Nick began his card sort interview by describing geometry as "the study of the way shapes and the properties of shapes are related." He felt geometry was the foundation for other mathematics, like the Pythagorean Theorem and trigonometric functions. Nick also stressed the importance of teaching the vocabulary of geometry. He felt definitions were important to teach students and teaching vocabulary would help to "get kids into the habit of talking about things in the absolute terms."

Nick discussed and defined each of the cards in the sort and decided they represented geometry well and he would not add any cards. He organized the cards as shown in Figure 7. All extra lines, arrows and words were added by Nick as he explained his sort.

As Nick described his organization of the cards, he seemed disorganized in his thoughts. He often began a sentence and never finished. He was unsure of where he wanted to place several cards and moved them often. The description reported is Nick's final analysis of his card sort. Many of the quotes were paraphrased to represent Nicks' intent.

Nick isolated non-Euclidean geometry because it "is separate from all this other stuff." He stated the cards best represented Euclidean geometry and

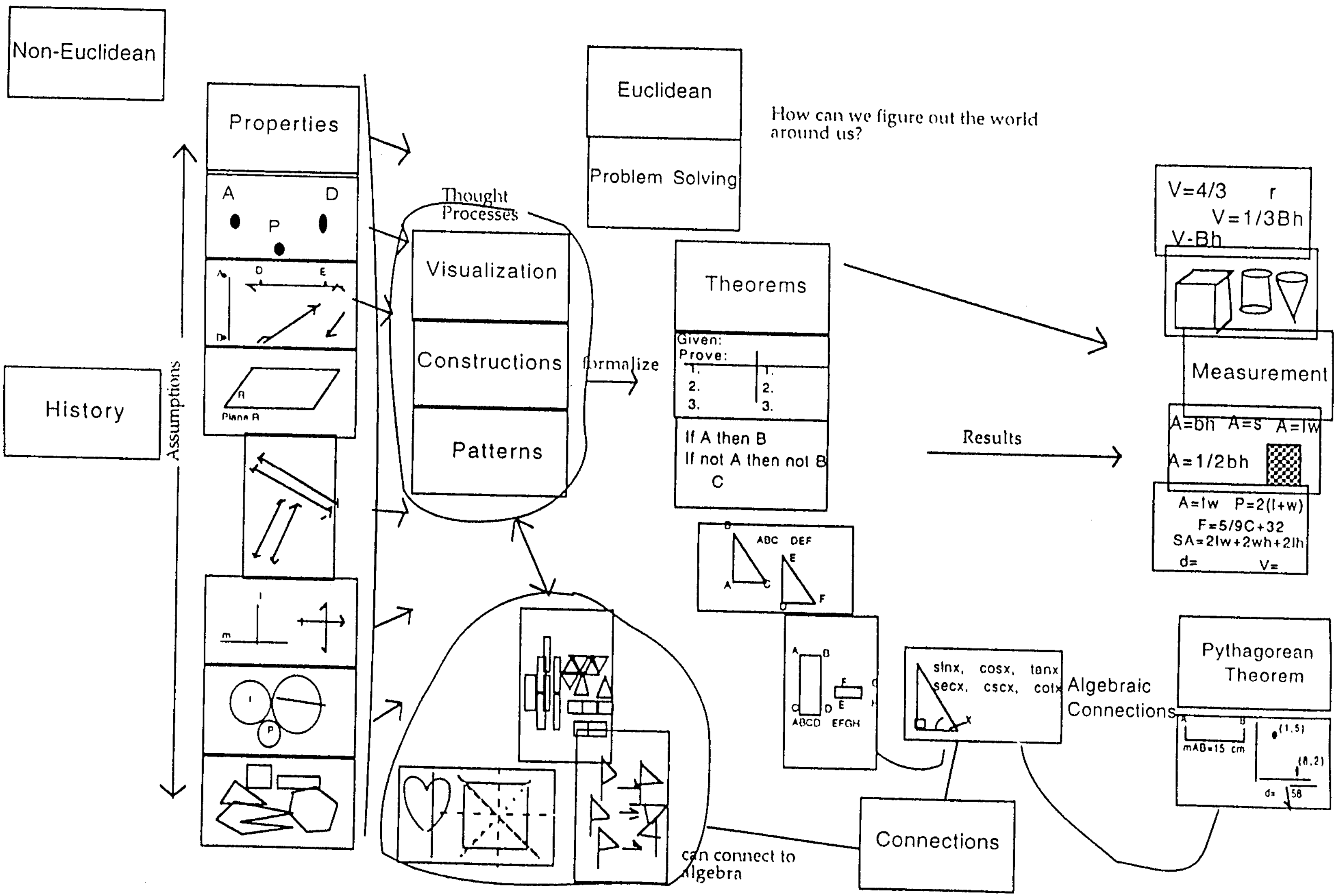


Figure 7. Nick's card sort

specifically what was taught at the high school level, however, a "parallel map" could be made with non-Euclidean geometry. Nick stated he thought Euclidean geometry's main focus was to "figure out the world around us," and that is why he placed problem solving directly under Euclidean geometry. He stated:

> When I think of geometry, I think it is to solve problems. The whole purpose of Euclidean geometry is to figure out the world around us. Geometry is rooted in practical application, it has its roots in being used in the real world. However, you can have fun with it without ever having to think about having to apply it.

Nick described the rest of his sort as moving from left to right. He had placed the card representing history on the far left-hand side of his sort. He explained that within Euclidean geometry the historical development was important. He stated:

> You have a collection of ideas in Euclidean geometry that people have been working on or known about since 400 or 500 BC. People have been looking at the way there is something special about a 90 degree angle, whether it is today or a thousand years ago. Maybe they didn't have names for the concepts, but the Egyptians had knowledge of them.

Nick placed eight cards on the left side of his sort following history. These cards represented the foundation of Euclidean geometry. He labeled them "assumptions" and referred to these concepts as being the terms that were agreed upon to be the common language of geometry. The concepts of points, lines, and planes were at the top because they are "the three main concepts."

Nick continued by explaining the connection of these concepts to the rest of the cards. Once these basic "assumptions" were defined, Nick felt a person could move to more formal thought processes. He explained the importance of visualization, constructions and patterns and referred to these cards as the "thought processes." To Nick, "there are various ways of visualization through constructions of patterns." He used tessellations, transformations and symmetry as examples of such visualizations.

By formalizing the "thought processes," Nick believed a person was able to use logic. He described proofs and theorems as examples of using logic. The cards representing congruence, similarity, and trigonometric functions were connected with this group as examples of specific concepts that needed to be proven.

Finally, Nick described the last column of cards placed on the right-hand side of the sort. These cards represented formulas that were the "results" of the assumptions, thought processes, and logic. Although Nick placed these concepts at the end of his sort, he stated his organization was not a continuum. He stressed the last column could be learned alone, however, he did not think this column exhibited a complete understanding of geometry. He stated:

> Most people don't really have a good understanding of geometry. They might know the formula for finding the area of a figure and be able to plug the numbers in, but they don't know where it came from or how to apply it to a real-world problem.

Nick finished the interview by summarizing his card sort with the following statement:

> Assuming your thinking is involving the relationship among points and lines on a perfect Euclidean plane, you would use the thought processes of visualization, constructions and patterns to formalize your thinking. Theorems, proofs, and logic are examples of that along with congruence and similarity. Then the formulas are the results of all this.

Nick was quite tentative about his sort throughout his explanation. He stated he had never thought about geometry in this manner before and he felt uncomfortable because he thought the cards were "all interrelated." He felt his sort was incomplete, but was unable to suggest ways to improve it. He stated his sort would be different the next time he organized them, however, did not want to redo the sort. He finally decided he had organized the cards in an order in which geometry might be taught.

<u>Videotape Task</u>. Nick was comfortable as he watched the videotapes. He had an awareness of what was happening in each of the classrooms, with the students and teachers. He was able to determine what each teacher was teaching and how the students were reacting in the classes. He was open

about his observations of the teachers and students, and suggested other ways to present the lessons. Several aspects of Nick's comments provided a better view of his thoughts.

First, as Nick watched the videotapes of the three experienced teachers he continued to discuss the importance of vocabulary in geometry. He felt vocabulary was a central aspect of geometry, both as a learner and a teacher, and it was essential to memorize the terms. Nick commented on vocabulary with all three teachers, stating:

> . . . especially in geometry, there is a lot of vocabulary and definitions that are built in. There are a ton of properties to know and memorize. I think a whole part of teaching a geometry class is getting students to be speaking the same language.

Nick commented several times that the teachers needed to spend more time helping the students memorize the vocabulary in geometry before the students would be able to complete proofs or solve more difficult problems.

Nick noticed the students in Teacher C's class had not performed well on the previous exam. Nick thought the students had never learned the material before the test. He suggested a possible explanation that again centered on the memorization of the vocabulary:

> This material requires a lot of memorization. Maybe he presented it, gave them [students] homework, and then assumed they knew it. I don't think they ever learned the material or vocabulary. It may have all been presented, but maybe he did not give them enough practice or model for them what they would need to do.

Second, an aspect that seemed to confuse Nick while watching the videotapes was classroom management. Although Nick only mentioned classroom management a few times, his comments indicated he was focusing on classroom management, however, he was unsure of specific strategies that might be used. He stated all the teachers seemed to have good classroom management. When asked why he thought the teachers had good management, Nick was unsure what strategies were used or why the students were well-behaved. "I don't understand that," he stated. Nick decided, "Maybe there are differences among schools as to how kids act." He never attributed the classroom management to the teacher or what the teacher had done prior to the lesson.

Third, while watching the videotapes, Nick's central focus was on the teacher. He mentioned the students, mostly regarding their feelings. He was concerned that students felt comfortable in the classes and mentioned how both Teachers A and C made their students feel comfortable and provided non-threatening environments. On the other hand, Nick felt Teacher B's students were bored and did not enjoy the class because he was "not real personable" and "seemed to be conducting business, rather than teaching students."

Nick focused on the teachers and their teaching styles. Nick felt all the teachers were "the presenters of facts and the students were the vessel to receive the information." He stated the teachers spent a great deal of time "telling" the students the information they wanted students to know, rather than allowing them to discover it or figure it out on their own. The teachers did not engage students, which Nick felt tended to lead to more management problems.

Nick also felt that, when "telling" the students the content, the teachers did not always present it in the best manner. Nick specifically commented on Teacher C's explanation about the midsegment of a trapezoid:

> Sometimes he makes jumps that students aren't quite ready to make. When he was talking about the midsegment and how that is the average of the two sides in the trapezoid he quickly made the assumptions that they understood. If he would have showed them clearly one time, he would have made it clearer for students. It doesn't seem that he was able to transfer his knowledge very well to students.

Nick stated if the teacher did not generate understanding in the students with the original explanation, then certainly a repeat of that same explanation did not help the students understand the content any better.

He noticed several times the teachers did not stress an idea as much as Nick preferred. Again, this idea centered around the vocabulary. Nick wanted the teachers to point out to students specific aspects of the vocabulary. For example, Nick commented that Teacher A needed to stress the difference between undefined terms and defined terms. He stated the teachers tended to "go through all the pieces" without making a connection to the vocabulary.

Nick ranked the teachers in the same order he had viewed them. He felt he would be able to learn from any of the teachers, however, did not think the teacher would make the difference as to whether he would learn or not. He stated,

> I feel that any math class I am in I will learn, regardless of the teacher. I've always enjoyed math and I like to do math problems. In that sense, I would enjoy it, but it would have nothing to do with the teacher.

Conceptions of Geometry. Nick's conceptions and understanding of geometry were strong. His main belief about geometry and its teaching centered around its vocabulary. Nick believed memorizing and knowing the vocabulary used in geometry was essential before a person was able to solve problems or use geometry. Nick felt learning the "language" of geometry was essential to teaching and teachers needed to help students understand definitions before they moved to more formal thought. This belief was reflected in his journal, card sort interview, and when he watched the experienced teachers on videotape. He believed the best way to learn the vocabulary was through memorization and practice. He stated:

> Practice. Practice. Practice. To learn math you have to continually practice using it. There is a line, however, when it stops being productive and students get bored with it.

Nick's knowledge of geometry was solid. He was able to provide correct answers for the proofs during the videotapes and often corrected the teachers on their content. These corrections were usually concerning the teachers' use of the vocabulary or specific wording that Nick felt was used incorrectly. Again, these corrections showed Nick's focus on the language of geometry.

Conceptions of Geometry Teaching. Nick also possessed several views about geometry teaching that surfaced during data collection. First, he believed "proof making" was important in geometry, however, it was more important to "convince" students something was true through examples, rather than through formal proof. He stated:

> In mathematics, a lot of times you don't prove something first. You look at lots of examples and you might convince yourself

> that something is true and feel a lot better about it. We can be fairly sure it is true. Later, you run the proof and show it for all cases.

Second, Nick suggested mathematics was a static discipline and had its own set of algorithms to be learned and followed. He believed that, following the vocabulary, the procedures needed to be learned in geometry. Only after the algorithms were mastered could a person solve problems. He stated:

> Students come to a conceptual or deeper understanding of material only after they have had ample "play time." In many cases this means teaching an algorithmic approach to solving certain types of problems. Equip the students with procedural knowledge or "what to do" and after they have worked with this to solve problems and get the right answer, it will be easier to get them to see the bigger picture.

From this quote and several other statements Nick made, it was clear he believed there was *one* correct answer to a problem in mathematics. He stated, "Most of the time in math there is only one answer to a problem."

Nick loved mathematics and wanted to be the perfect teacher. Before he entered the teacher preparation program, he assumed students wanted to learn mathematics and they loved mathematics as much as he did. Nick had not worked with students in a classroom atmosphere before his part-time internship. He had an idealistic view of students and their attitudes and what teaching was like. Nick believed if he were enthusiastic about mathematics, his students would automatically be enthusiastic too. His views were shattered as he started to teach. His disillusionment with students increased. He stated toward the end of his fall internship, "I thought I liked high school students, but . . ."

Nick was perceptive in each of the classrooms he observed on videotape. He was able to discuss what was happening in the classrooms and he was able to suggest changes that could be made to make the lessons more effective. Nick, however, was never able to view his own classroom with the same eye. He was critical of himself and constantly worried about being the "perfect teacher."

Robin

Background. Robin entered college studying pre-veterinary medicine, but decided to study mathematics her junior year. She completed her Bachelor's degree the Winter quarter prior to starting the teacher preparation program. Robin's parents were teachers and influenced her to become a mathematics teacher. She had participated in sports during her own schooling and felt it had taught her important lessons in responsibility, dealing with stress, and time management. She knew she wanted to coach sports and she liked mathematics. She decided the best way to take advantage of these two interests was through teaching at the high school level.

Robin had felt confident in her ability "to do math," and felt teaching helped further her own knowledge. She stated, "Mathematics has always been very interesting for me and I feel like there is no end to the amount of knowledge I can gain in the subject." Robin felt her own interest in mathematics and the fact she was female helped her to be a positive role model for female students. She wrote:

> By being a good role model for my students, I believe many of the old stereotypes about females in the fields of math and science can be changed. I plan to set an example for these students as a good role model, not only as a female mathematics teacher, but as a person.

Robin completed all the requirements for entrance into the teacher education program and was accepted into the program. She had experience in coaching high school students, but had not worked with them in a classroom setting. She was encouraged to gain experience in the classroom before the teacher preparation program, so she chose to observe a science classroom for several weeks.

Internship. Robin was placed with an experienced mathematics teacher who had taught 12 years in a school enrolling 1000 students. Her mentor was involved in mathematics reform throughout the state and educational reform at the high school level. She was a highly respected mathematics teacher and had worked with student teachers for several years. Robin observed two of her geometry classes and an algebra class with a second mentor teacher.

Robin became immediately involved with the classes in which she was placed. She began with simple tasks of taking attendance, grading papers, and designing bulletin boards. She soon started to tutor individual students and answer questions about the homework. Within three weeks, Robin had presented a short segment of a lesson and led a review session with the geometry classes.

Initially, Robin was concerned about maintaining classroom management. She was excited about teaching, but realized she might not know what to do if students misbehaved. This concern soon dissipated after watching how her mentor teacher established an orderly classroom.

While observing the two different teachers, Robin commented on several important aspects about teaching mathematics. She began to see the importance of establishing clear expectations for students. She felt by setting clear expectations and "sticking" by them, a teacher was more effective. Robin also discovered the importance of planning and organization in teaching. Regarding her mentor teacher, Ms. Smith, she commented:

> One thing that is so impressive about Ms. Smith is that she is very organized and has completely thought through what she expects from her students. So when questions start rolling, she already knows how she wants to answer them.

Robin noticed the other teacher she was observing was not as organized and felt this disorganization adversely effected the students.

Robin planned and taught a unit in the geometry classes on congruent triangles. As she began to teach, Robin reflected on this experience in her journal. She had had positive experiences working with students by coaching, but discovered it was different in the classroom. Robin was surprised at the apathy students displayed in her classes. This apathy seemed to frustrate her sometimes. Robin commented:

> I am a little surprised at the students' lack of interest and overall attitude towards school, math, and life in general. Many of them have no motivation to learn and it is like pulling teeth to actually get them to understand that, just because you are a math teacher, you are not the enemy. I didn't expect every student to bounce into class all excited about math class, but I find it really hard to deal with the students that don't want to be there and don't want to learn. I feel like I waste a lot of time dealing with

> these types of students instead of the ones who are interested in learning and want to be there.

Robin also commented on her own knowledge of mathematics and how she had learned the concepts as she taught them:

> I am so surprised with how much I didn't know when it came to mathematics. I have a Bachelor's degree in mathematics and have even taken some graduate courses and I feel like I don't know anything. Sometimes I feel like I am learning a lot of the content right along with the students.

Robin continued to establish her beliefs about how students learn mathematics. She did not believe students needed to memorize theorems and algorithms. She felt this type of teaching led the students to only think about the correct procedure they needed to use to solve the problem and they did not care if they understood how or why they did it. She wrote she believed a mathematics teacher must provide the students with applications and reasons for learning the concepts. "I think if students were given some more direct applications to real-world situations they might be more inclined to learn the concepts instead of just memorizing algorithms."

After completing her unit, Robin began to question the importance of teaching some concepts in geometry. She stated in her card sort interview:

> I understand that all the things seem like they are important, but sometimes after I look at what a student would actually use, I wonder why we teach it. Why do you need to know how to write a deductive proof? I don't understand why we can't have a class based on things that apply to what they [students] really see and do.

Robin successfully completed her part-time internship and continued to teach two geometry classes and an algebra class for her full-time internship experience. Since she was teaching geometry, Robin was observed in the classroom for the second phase of this study.

<u>Card Sort Task</u>. Robin began her card sort interview by describing geometry. She defined geometry as "the study of shapes and structures and the relationship within those shapes." She felt geometry provided more applications to real-world phenomena than other areas of mathematics.

Robin continued describing geometry by relating it to what she had learned in high school. She stated:

> What I remember from high school is that geometry is the first introduction to actually having ideas and having to prove those ideas. Before we just kind of learned how to do the math, we didn't have to know the theory behind it and give reasons for an idea.

Robin discussed each card individually and after she was finished she wanted to add cards representing deductive and inductive reasoning, proportions and ratios, postulates, and corollaries. As Robin started to organize the cards she stated she wanted to complete the sort "fast" and, in fact, it only took her about eight minutes to finish. Her final sort is displayed in Figure 8. All additional lines and statements were added by Robin as she described the sort. Group numbers were added by the researcher to clarify data analysis.

Robin's cards were organized into seven distinct groups. She described Group 1 as "problem solving." "Problem solving, constructions, and connections are ways to solve problems, either with logic or proofs." She thought problem solving could be accomplished through constructions and connections, thus needed to be taught following properties and shapes. Robin described Group 2 as all separate topics, but decided the ideas of similarity and congruence were used to elicit more concepts and theorems in geometry. The cards in Group 3 were placed together because "as you study the history of how geometry came to be, you look at the Euclidean side of it and the non-Euclidean side."

Robin's cards in Group 4 represented visualization. She felt patterns were found with tessellations, symmetry, and transformations "without having to write anything down." Robin described Group 5 as shapes. She stated:

> I didn't really know what I should do with the shapes. I just kind of blobbed them together because they were all shapes. I kind of thought of them as two-dimensional and three-dimensional shapes. They are kind of basic things you need to talk about before you go into the Pythagorean Theorem.

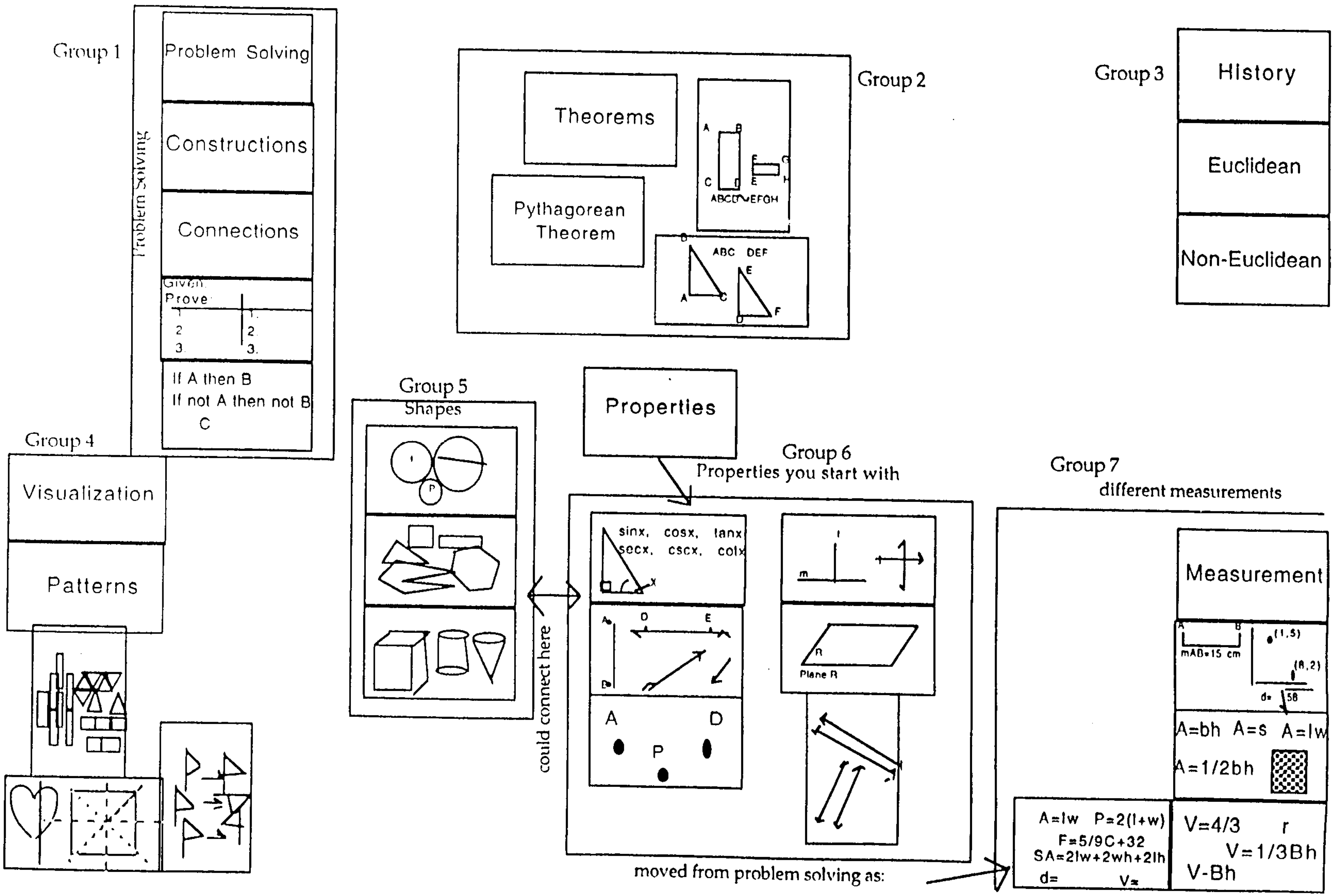


Figure 8. Robin's card sort

Group 6, to Robin, represented "all the properties of geometry, the initial things you start with." She compared this group to the teaching of geometry:

> I guess if you were going to teach it, you would want to start with the basic properties. You need to know these things in order to use the others. For example, you need to know about parallel lines to be able to say anything about congruent triangles.

Robin labeled Group 7 "different measurements," and described these cards as "ways you can find actual numbers for things."

Robin summarized her sort by confirming she had not placed the groups in a specific order. She did, however, have a definite order in which she felt the groups should be taught. She explained:

> I would see problem solving and measurement coming after you get through properties, similarity, and congruence. Visualization is more interrelated like as you are doing properties. As you are initially teaching students properties, you should at least talk about visualization, looking for patterns, and always try to relate it to the history.

Robin felt that how she had learned geometry influenced her card sort. She also felt the textbook she had been using had also influenced the sort. She completed her interview by adding deductive and inductive reasoning to the problem solving group (Group 1), ratios and proportions to Group 2, and the postulates to the basic properties (Group 6).

Videotape Task. When watching the videotapes of the experienced teachers Robin focused on several aspects. First, she felt an important aspect of teaching was knowing the students' prior knowledge and preconceived ideas about the content. She consistently mentioned the teachers' knowledge of students' preconceived ideas. With Teachers A and B, Robin discussed this knowledge in a positive manner. She felt Teacher A was aware of the students' prior knowledge and preconceived ideas about proofs. She stated:

> He had to think about where they [students] were coming from as far as what their preconceived notions were about proofs and what they weren't. He knew that a lot of them had preconceived notions about what proofs were; all kids do. Everyone talks about them, so he knows that kids hate proofs. He had to think

> about where the students were coming from so that he would not expect them to know something they had not learned before.

Concerning Teacher C, Robin indicated he was not aware of what the students' prior knowledge was and did not use their preconceptions within the lesson. "The students did not know that a square was a rectangle and if he had never talked about that before or gone through that type of thinking process, I don't think they would be able to understand what he was talking about. It wasn't anything they had ever done before." She felt Teacher C needed to spend some time determining what the students knew before he continued with the unit.

Second, Robin believed the teachers' ability to transfer their knowledge to their students was the most important aspect of teaching. She felt Teacher A was the best teacher because he could relate his knowledge to the students. Robin stated:

> I think that is the ultimate in teaching; being able to understand the book and then being able to put it in words and talk about it with the students in a way they can understand.

Robin continued by discussing Teacher A's progression through the lesson. She felt by starting with one-step proofs and "building" to more complicated proofs, Teacher A was able to help the students understand the content of the lesson. "He is very thorough and explains things real well. He is taking the time so students do not get overwhelmed."

Robin also noticed the ability to transfer knowledge with the other two teachers. She believed both of them may have had a good understanding of the content, but were unable to explain it so students could understand. "It didn't seem like he [Teacher C] was very good at breaking the content down in a way students could understand it." Robin attributed this inability to transfer the content to several areas, including lack of planning, lack of organization, and the teachers' "traditional" views. She thought both of these teachers had "probably been doing it the same way for 20 years."

Third, Robin commented on the teachers' planning. She mentioned that Teacher A was well planned and organized; Teacher B seemed planned because he was "not shuffling around his desk"; however, Teacher C was not well planned and this lack of planning showed in his presentation. She stated:

> Because he wasn't organized, didn't have very good objectives, and wasn't very well planned out, it didn't seem like he knew what he was talking about and didn't know where he was going. My impression was that he was really unorganized and had not thought about the lesson.

Robin believed the teacher's planning was essential to the lesson and provided a strong basis for the presentation of the content to the students.

Finally, she focused on the lesson as a whole, rather than the individual parts. Robin often commented on the flow of the lesson and how the individual parts "connected." She felt Teacher A had done an excellent job of planning and presenting a lesson that flowed and had good pacing. She stated:

> He has the ability to plan a lesson and link it all together. He was able to pick an example that he could use throughout the whole lesson. He started with it as a warm-up and had students think about it all the way through and then came back to it at the end.

Robin thought the lesson from Teacher B also progressed in a logical manner. "He went through all the different terms, definitions, and ideas and they kind of built on each other. He went from parallel lines to the angles formed by a transversal to some problems that used those concepts." Teacher C, to Robin, had no plan and thus the lesson seemed disjointed: "It seemed like he was ad-libbing. He just kept going back and forth. He did not emphasize one thing enough so the students could really understand it."

Robin ranked the teachers in the order she had viewed them. She chose this order because of the clarity of each presentation and the ability of the teachers to present a complete, "connected" lesson.

Conceptions of Geometry. Triangulation of data sources, including the card sort interview, the videotape tasks, classroom observations and lesson plans, displayed a set of conceptions of geometry Robin possessed. Robin admitted her own lack of content knowledge as was confirmed through the data sources. She stated she had entered teaching to gain knowledge in mathematics. Specifically, her lack of geometry knowledge was demonstrated several times throughout this study. She was unable to complete the proof in Teacher A's class and stated: "I have no idea. I can't even think." After completing the card sort, Robin stated: "When I look at this, I'm like I don't know any of this." Several times, Robin mentioned that she had not remembered a concept until she had to teach it. "I had not remembered that the median of a trapezoid was the average of the bases until I taught it last week." She also questioned some concepts: "Is the golden rectangle and other things in nature part of geometry? That is, isn't it? Kind of?"

Robin had also viewed geometry as a linear set of concepts. She conveyed that her organization of geometry was based on the order in which it was to be taught. She felt it was important to teach the properties of shapes before students were able to learn measurement or proofs. This linear organization was based on how Robin had learned geometry and had been influenced by the textbook she used.

Conceptions of Geometry Teaching. Triangulation of videotape tasks, classroom observations, and work samples supported Robin's conceptions of geometry teaching. Foremost, the importance Robin placed on students' preconceptions was evident. From the beginning of her internship experience to the end of data collection, Robin mentioned the importance of teachers knowing the students' prior knowledge and preconceived ideas about the content. She felt knowing the students' preconceived ideas was important because it helped the teacher know where to start teaching and how to "build" on those concepts. As discussed earlier, Robin focused on this aspect of teaching when viewing the experienced teachers on videotape. Robin also mentioned this aspect in her work sample:

> Every student has their own preconceived notions about math and how they learn math. Taking this into consideration, teachers must make mathematics relevant to what students already know and build from there.

This conception seemed to follow from Robin's ideas about geometry being linear and in a set order.

Robin also believed students did not need to memorize formulas or algorithms. Students needed to be able to think on their own and discover the concepts for themselves. Robin stated it helped students to remember the ideas if they had worked it on their own rather than the teacher telling them how to do a problem.

Another significant aspect of teaching geometry that emerged in Robin's data was the importance of using real world examples. Robin felt strongly that the teacher needed to provide examples of the content to the students presenting problems based on the students' lives or interests.

Classroom Practices. Robin chose her second geometry class for observations, because she felt she taught it "better the second time around." This class consisted of 25 students and met at the end of every other day for 90 minutes. She taught from the book *Geometry*, published by Houghton Mifflin (Jurgensen, Brown & Jurgensen, 1988). As stated previously, Robin's mentor teacher had established a strong set of classroom rules and the students knew what was expected of them. Robin had also worked with this group of students during Fall quarter and had established herself in the classroom.

Robin's class was observed eight times throughout Winter quarter. Each observation was videotaped and transcribed, then combined with the researchers' fieldnotes to provide a complete, detailed summary of each lesson. See Appendix F for a detailed summary of Robin's classroom observations.

Typically, Robin had 10 to 12 problems on the board as students entered the classroom. She started the lessons by having students copy and complete two to three problems intended for review. As students worked, she took attendance and reviewed her lesson, and then walked around to check on the students and answer any questions they had. She then reviewed the problems with the class. Robin completed the problems on the board, while asking the students specific questions about how to complete each problem.

After completing the reviews, Robin presented the new content to the students. Usually, she told the students the ideas she wanted them to know as they took notes. Robin then had the students complete an activity (either individually or in groups), and then she summarized the activity with the entire class. Robin reviewed several examples on the board with the students. She then had the students practice three to four problems on their own before reviewing them as a class.

Finally, Robin assigned the students homework and allowed them to work on it until the class ended. She circulated and helped individual students. Before the bell rang, Robin reviewed the content by asking the students specific questions and summarizing the important aspects of the lesson.

Throughout each lesson, Robin was able to keep students involved in the lesson by having them take notes, complete activities, answer questions, and practice problems. She was organized, well-prepared, and had thought through each of her lessons. She had prepared problems for review and relevant problems for introducing the new content. Robin followed the textbook, supplementing with additional problems and activities.
The activities she planned for the students were appropriate and helped the students gain knowledge in the content in a hands-on manner. Robin had spent time doing each problem she used in class as well as the homework problems she assigned. She stated she wanted to be prepared for any questions the students might ask and thus felt it necessary to do the problems herself. Each lesson was complete, well-planned, and well-presented.

Overall, Robin was pleasant and had established a good atmosphere for students to learn. She had a positive manner in the classroom and was always willing to help the students. She began Winter quarter a little self-conscious and nervous about classroom management, however, soon became confident in herself. This confidence allowed her to move from focusing on herself to focusing on the students and their understanding of the content. In her work sample reflections, she stated:

> I began my internship with nervousness and anxiety, but finished with confidence and pride in my accomplishments. I was happy to see students responding to me and respecting me as the person in charge. I feel like a good majority of the students were really learning the material and that is nice to know. I want them all to learn and I know they all have the capability to do so.

One of Robin's obvious attributes, as a teacher, was her sensitivity to students' prior knowledge and understanding. She never assumed the students knew the content and was aware of their level of understanding. She was able to present the concepts in a progressive manner, starting with the simple aspects and moving to the more complex. She constantly questioned students to determine if they were understanding. If students did not understand, she did hesitate to review a concept. Robin felt *all* students were capable of learning and wanted them all to learn. It frustrated her, at times, that they did not always seem interested in learning.

Although Robin was effective for a beginning teacher, there were several areas where she needed improvement. First, Robin spent a great deal of time "telling" the students the content she wanted them to know. All of her lessons were deductive in nature, where she explained or demonstrated the concepts and the students took notes from the board. Then the students practiced problems or did an activity to reinforce the ideas. Many of the concepts could have been introduced to the students in a more inductive manner. Unfortunately, this study did not provide the opportunity to question Robin about the types of presentations she made.

Second, she was good at presenting a single lesson and reviewing concepts, however, she often did not connect the lessons. For example, the first lesson observed was on scale drawings and the second lesson was on the conditions for similarity in triangles. Robin never made the connection for students as to how these two concepts were related. She discussed the condition that sides of similar triangles needed to be proportional, however did not relate that to a scale factor.

Third, she often failed to provide a purpose for the students' learning the content. Her stated purpose was often so the students could complete the homework or do well on the exam. A typical statement during the lessons was: "On the test you will be given some problems like this and you will have to decide which formula you will have to use in order to solve for the letters. This is how it will look on the test so you will have to know these."

Last, Robin used few "real-life" applications. She only related the content to the outside world during two lessons. The first lesson she compared the geometric mean to a nautilus shell and explained how the geometric mean could be seen in the shell. The second lesson she showed the angle of depression and elevation using pictures. Both times, Robin showed the students rather than having students discover or work through the idea on their own.

Jeremy

Background. Jeremy was raised in a family of teachers. His mother taught mathematics at the high school he attended and his father taught biology at several different levels. Jeremy knew he wanted to be a teacher because of their influence. He felt this influence gave him a realistic view of the lifestyle of a teacher.

Jeremy originally wanted to teach art. He began his undergraduate education studying art because he had always enjoyed drawing and thought he would enjoy using this talent. Eventually, Jeremy decided he did not like "being told what to draw" and enjoyed drawing "as a hobby, not necessarily as a career." He switched to mathematics after one year and graduated with a Bachelor's degree in mathematics prior to entering the teacher preparation program.

In addition to being influenced by his parents, Jeremy wanted to be a mathematics teacher for several other reasons. He wrote, "I have always enjoyed working with people. With this characteristic and an interest in mathematics, I have had the desire to become a teacher so I might share my knowledge and enthusiasm for this subject with others." Jeremy also wanted to be a teacher because he felt he had received a good education and wanted to "return the favor of a quality education by working with the youth of tomorrow." Finally, Jeremy wanted to be a teacher because he thought he would like the schedule. "I like the schedule that teachers work on; the same vacations as their children plus months off in the summer. After teaching for a while, I might need that."

Jeremy had worked with students at several different schools, both in the classroom and as a coach. He had observed and helped by tutoring individual students at a local high school. "When a student figures out a problem with my assistance and his or her face brightens up, I get this warm feeling of satisfaction inside. I guess this is one reason I want to teach, the satisfaction that follows successfully helping and teaching a student." He had also coached summer basketball camps for several years. These experiences provided him with more motivation for completing a teacher education program. He met all of the requirements for admission into the teacher preparation program, was interviewed and accepted into the program.

<u>Internship</u>. Jeremy was assigned to work with a cooperating teacher in a medium-sized school. Because of scheduling conflicts he also worked with a second mentor teacher. Both teachers were experienced and had previously worked with student interns. Jeremy observed both teachers' classes including a Geometry class, Algebra II class and a Math Applications class.

Jeremy began working with the students the first week of classes. He presented two lessons the second week and began to get to know the students. He also assisted his mentor teachers by taking attendance, collecting papers from students, recording homework grades, returning graded papers, and making a bulletin board. Jeremy was pleased to be participating in these activities. He stated:

> This is all great experience and assists in the process of matching names to faces and faces to names. By passing out homework, I am also more visible to students. Through continued participation in these clerical duties, not only will I get to know students and their names, they will also hopefully become more familiar and comfortable with my presence.

Jeremy planned a unit on solving systems of linear equations for the Algebra II class. He was to begin teaching the unit during the eighth week of his part-time internship. He discussed this experience in his journal:

> My anxiety level was high the night before I was to begin teaching. Did I have the necessary materials prepared? Could I present all the content I had planned in an understandable manner? I wanted students to fully understand the concepts being presented but also did not want to proceed at too slow a pace.

Jeremy soon relaxed and felt his lessons went well. He discussed the students' behavior, however, was most concerned with their understanding of the content. "The important thing: students appeared to be learning." Jeremy did realize, however, he was not always aware of the student's understanding.

> I occasionally forget I am working with high school students and erroneously believe that these students have a similar understanding of the subject matter as myself. By working with students, I am getting a better understanding of their mathematical knowledge.

Jeremy continued to learn and develop his philosophy throughout his intern experience. His attitude was always positive toward teaching and the students. He had a genuine concern for his students and felt he had developed a good rapport with them. One important aspect of teaching, for

Jeremy was the affective side of teaching. He felt it was important to learn students' names and to learn what interested them. In fact, Jeremy mentioned in his journal he felt rapport was an important characteristic of effective teaching.

> I think good rapport with the students, a positive attitude, enthusiasm toward the subject matter and learning, interest in students, and expressing confidence in all students are all characteristics of an effective teacher.

Jeremy also believed "*all* students have the capability to learn mathematics." He reiterated this belief in his journal several times. When asked to describe his philosophy of mathematics education, Jeremy replied:

> It is plain and simple: I feel all students have the ability, some more limited than others, to understand and learn mathematics. Many students just lack the confidence to do mathematics. This is where I feel it is critical for the teacher to motivate students and boost their confidence.

Jeremy felt the best way a teacher could "boost" the students' confidence was to provide a variety of learning situations for the students. Such situations included providing activities, models, and ideas where students were given an opportunity to *do* mathematics. "It is my educational belief that teachers must attempt to directly involve students in learning mathematics." Jeremy thought one way he could help students learn was through use of real world problem situations.

> By incorporating the subject matter with the reality of the real world and expressing the importance and relationship of the subject matter to their future occupational fields, I feel that I could gain the interest of most students.

Jeremy finished teaching his unit by giving the students a unit exam. He was pleased with the results and felt the students had understood the content he had taught. He successfully completed his part-time internship and continued to observe his mentor teachers throughout Fall quarter. He taught two Geometry classes and an Algebra II class for his full-time intern experience Winter quarter. He was observed eight times in one of the geometry classes for this study.

Card Sort Task. As Jeremy organized the cards he seemed to enjoy thinking about geometry and how it might be organized. He was comfortable during the interview and discussed his views openly. He stated that it was hard to remember some of the concepts and attributed his lack of memory to not having taught geometry. His card sort is displayed in Figure 9.

Jeremy started describing his sort by explaining that problem solving "encompasses the whole thing." He believed problem solving was not specific to geometry, rather included all mathematics. For this reason, he placed the problem solving card at the top of his sort and did not include it with any specific cards. "You do a lot of problem solving in math. You use formulas, theorems, constructions, in fact, you use all these cards to solve problems. I will leave it up there by itself."

Jeremy described his sort as starting with the center group and moving out from there. The center group of cards, including plane, points, lines and circles, Jeremy described as "being the basis in geometry; where you start out, then you build on those to construct things or prove things." He also stated he thought of these cards as "things you see in a plane," and "different types of relationships between lines in the plane." Jeremy then explained he associated this group of cards with the basic constructions (moving left on his sort). He felt a person needed these concepts for doing constructions, specifically, constructing congruent and similar figures.

Jeremy felt tessellations were a pattern and tessellations were constructed by using congruent shapes "over and over", so he connected those cards to the construction card. He placed the card representing connections with congruency and similarity because he felt the connections between the two were the strongest in the sort.

Moving from the center group of cards to the right, Jeremy explained the lines were used to make the polygons and the planes to make the polyhedrons and thus, he connected the cards. After the polygons are constructed, the formulas are needed to measure different aspects of the shapes. Jeremy placed all of the cards with formulas in a category he labeled measurement.

Connected to the measurement group, Jeremy placed the Pythagorean Theorem because "it was used to get the distance formula." The Pythagorean Theorem card was used as a connector card from measurement to theorems and proofs because "the Pythagorean Theorem is an example of a proof."

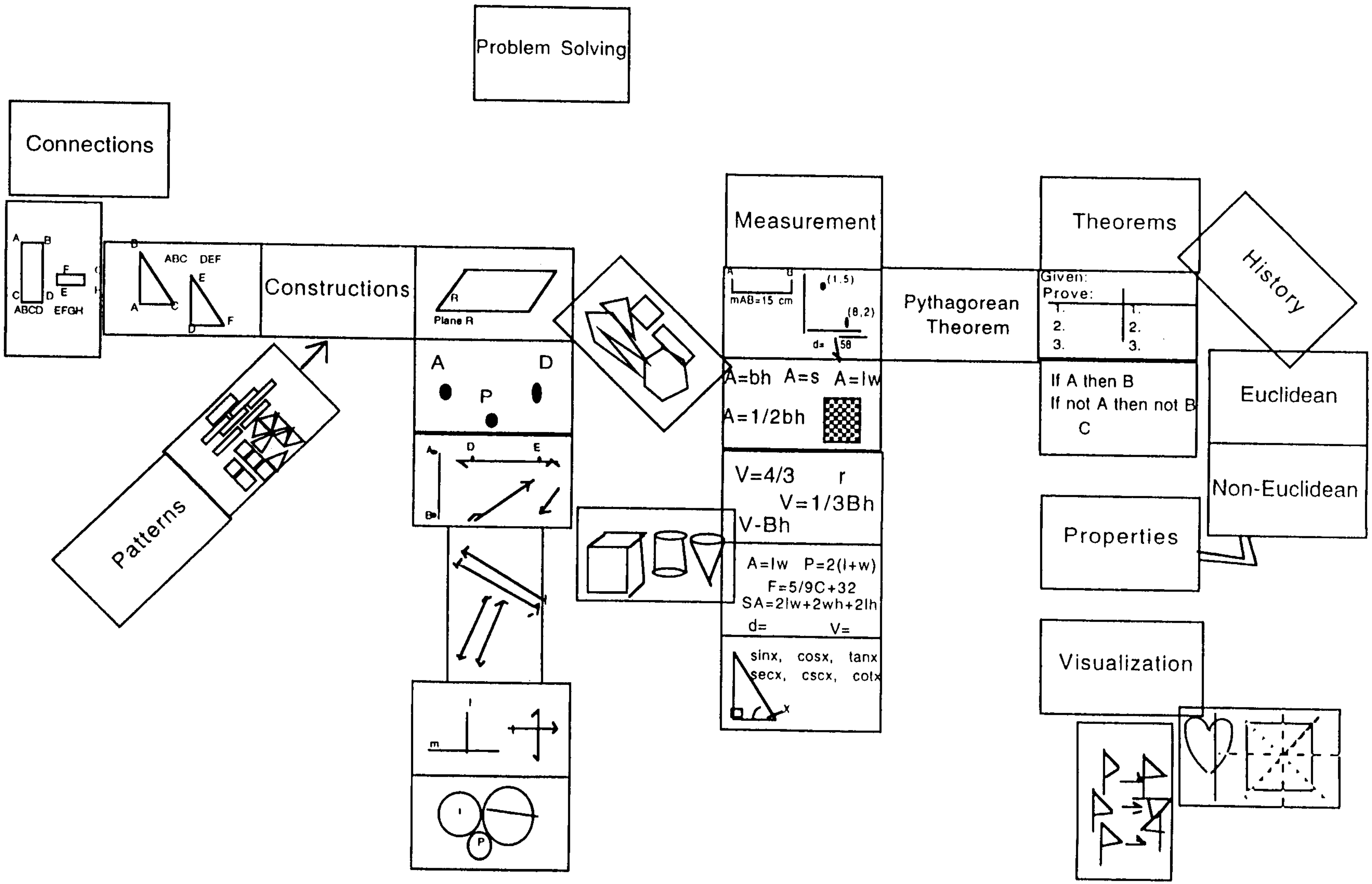


Figure 9. Jeremy's card sort

Jeremy labeled his last group of cards theorems and included proofs, logic, properties and visualization. He explained why he placed properties and visualization with the theorems:

> You use certain properties with theorems. You use some of the properties for parallel lines and perpendicular lines in proving theorems. I would also say that visualization goes with the theorems because you are dealing with pictures and diagrams when you prove something. If you can't visualize something, it is hard to prove.

Last, Jeremy connected history with the theorems because "history goes back to some famous theorems such as the Pythagorean Theorem." Euclidean and non-Euclidean geometry were placed with history because "there is a lot of history based on Euclidean and non-Euclidean geometry and how non-Euclidean geometry came about from Euclidean geometry."

Jeremy decided he could add several concepts to the set of cards in the sort. He specifically mentioned adding axioms and postulates with the theorems, triangles with the polygons, and the specific properties of parallel lines. He also decided this group of cards was a good summary of high school geometry.

Jeremy finished his interview by stating he had not thought about how geometry was organized before this task. He decided he would probably arrange the cards differently if he were to sort them again, however, "every time I would put the formulas together." Jeremy finished his card sort interview and as he left he said: "That was fun. Interesting to think about those things."

Videotape Task. When watching the videotapes of the experienced teachers, Jeremy focused on several aspects of their teaching. Four distinct themes ran through Jeremy's comments as he discussed the lessons. First, he stressed that all three teachers were able to establish a rapport with their students and provide a "comfortable" atmosphere for learning. This statement corresponds with Jeremy's previous focus on the humanistic side of teaching. Regarding Teacher A, Jeremy stated:

> I think the students are comfortable with him. He tries to relax the students about proofs and not intimidate them. He complimented the students on their ideas and made them feel comfortable talking in class. I see a mutual respect.

Jeremy felt Teacher B did not intimidate students either, but more importantly, showed a confidence in them. Jeremy also mentioned Teacher B had set high expectations for the students:

> I think he is great. I think in a way he shows confidence in the kids. He keeps working with them and doesn't give up on them. He is pretty easy-going and relaxed. He is not intimidating, but he expects kids to know and to answer his questions. I think he enjoys working with the kids.

Jeremy also stated Teacher C was "easy-going and the kids felt comfortable and free to express their opinions."

Second, Jeremy focused on the students' understanding of the content and the teachers' abilities to assess. For the most part, Jeremy focused on the questioning techniques of the teachers and how the techniques affected their assessment of student understanding. For Teachers A and C, Jeremy felt that they were unable to assess all the students because they asked few questions and did not call on a variety of students. He stated, "He [Teacher A] could have asked several people questions. Instead, he just asked one person and told them they were right. So he doesn't really know if they really understand it, except for that one student. He needs to question more."

Jeremy felt Teacher B asked good questions and called on a variety of students around the room; however, often he told the students the correct answer. "He did not get students to think because he answered his own questions and all of the questions were at the knowledge or comprehension level. Actual thinking was limited. It was mostly regurgitation."

Jeremy also mentioned the teachers' ability to present the content to the students using a variety of different representations. He felt this ability would help students understand the content more thoroughly. Concerning Teacher C, Jeremy stated:

> When he went to the board he was able to elaborate and give several different representations for each problem. He tried to show different ways to solve the problems. He even asked the students if they needed him to explain it in a different way. I think he does a good job of explaining.

Third, Jeremy focused on classroom management. He noticed Teacher A seemed to have established a set of rules prior to the lesson Jeremy was watching. "He has good classroom management established. If someone broke a rule, he had established the consequences. The students knew he would follow through. I think the kids respect him for that."

Jeremy also mentioned the other two teachers had established strategies for getting students' attention. He mentioned Teacher B used questions to get students to pay attention. "He questioned people that were not paying attention. He did a good job of using questions to keep them and bring them back in if they were talking or not paying attention." Jeremy noticed Teacher C whistled to get students' attention. "I like the little whistle he uses to get their attention. I think it is just natural when you hear a whistle you turn to it and after a while you get trained."

Last, Jeremy was critical none of the teachers had used a hands-on model for explaining the content. He felt the teachers were unable to engage the students without some type of activity that involved them. "I don't know that he [Teacher A] really engaged the students. He did not use any inductive model or any hands-on activity. He could have, at least, asked them more questions and have them provide the justifications for the proofs."

When asked to rank the three teachers, Jeremy decided he liked the second teacher the best because of his "easy-going" demeanor. He then chose the first teacher because of his preparation. Jeremy felt the last teacher gave too much time for the students to work on their homework assignment. "I could see 30 minutes of wasted time."

<u>Conceptions of Geometry</u>. Triangulation of data sources revealed a mixture of Jeremy's conceptions of geometry. He presented a linear view of geometry in the card sort, however, his explanation during the interview revealed a more integrated view of geometry. He discussed how his structure was tentative, and would probably change, except for the formulas. He also related the concepts in geometry to other areas of mathematics, including other types of geometry, algebra, and calculus.

Jeremy displayed adequate knowledge of geometry throughout the study. He was able to identify all of the cards in the sort and he was also able to provide justifications for the proofs Teacher A was doing. On the other hand, Jeremy stated he was sometimes unsure of the content and did not remember all of the concepts. He attributed this lack of knowledge to not having taught geometry yet. He stated, "I can't remember the linear pair postulates and theorems. The main theorems I do remember, but lots of them I can't remember. Not having actually taught geometry, it is hard to remember."

Jeremy believed visualization was an essential characteristic of geometry. In his card sort he discussed the importance of using pictures and visualizing a concept in order to prove a theorem. Jeremy stated, "I like geometry because there are so many different ways you can picture things." He also described how it was important in geometry to be able to visualize a concept or idea before being able to prove it.

Conceptions of Geometry Teaching. Jeremy's conceptions of teaching geometry were more clearly established. He had a strong belief that developing rapport with students was essential to effective teaching. As discussed earlier, he mentioned this aspect of teaching in his journal and he centered on this when viewing the videotapes of the experienced teachers. He stated:

> If you don't have good rapport with the students I don't think they can learn. Even if your teaching strategies are not the best, but you have good student rapport and you show that you care for the students, you can get a lot further than not being able to relate to students.

He felt if the teacher had established a good rapport, the students behaved appropriately during class.

Jeremy believed students learned best by being involved in the lesson. This involvement took two forms for Jeremy. First, he felt it was important for the teacher to ask questions of the students during class to keep them involved. Jeremy mentioned questioning with the experienced teachers from the videotapes. Second, he felt the students needed to be involved through activities and explorations. He was critical of the experienced teachers because they had not involved the students with hands-on activities. Jeremy thought the teachers also needed to use more inductive approaches to teaching in order to engage the students and allow them to discover the ideas on their own.

Jeremy also believed real world problems should be incorporated into mathematics teaching. He discussed this relationship in his card sort interview, journal, and work sample. He summarized his belief with this statement:

> Mathematics is utilized in describing our world and surroundings through its connections to and applications in the sciences. Mathematics not only applies to many facets of the real world, but so does the learning of mathematical problem solving strategies. Through acquisition of an arsenal of mathematical problem solving strategies, students will grow as independent thinkers and self-directed learners. These problem solving strategies can be used in non-mathematical situations as well. For these reasons I strongly feel mathematics education should comprise both applications of mathematics and its relationship to the real world as well as problem solving.

Classroom Practices. Jeremy was observed teaching a geometry class that met at 8:00 in the morning. This class met every other day for 90 minutes. There were 28 students in the class. He used the book: *Geometry*, published by Houghton Mifflin (Jurgensen, Brown & Jurgensen, 1988).

This geometry class was observed a total of eight times throughout Winter quarter. Each observation was videotaped and transcribed, then combined with the researchers' fieldnotes to provide a complete, detailed summary of the lesson. Appendix G summarizes Jeremy's classroom observations by listing the date, topic(s), and sequence of classroom events.

Typically, Jeremy used a warm-up on the board or overhead as students entered the classroom. The warm-up consisted of a few review problems from a previous lesson or a question the students were to answer in their journals. For example, students were to respond during one lesson to: "If I could change one thing about mathematics, it would be . . . because . . . " While students were completing the warm-up, Jeremy took attendance, handed back homework assignments, reviewed the lesson, and checked each student's homework from the previous lesson. Jeremy then reviewed the warm-up problems with the students as a class and answered any questions that arose.

After reviewing the warm-up problems, Jeremy answered any questions from the previous homework. The students were allowed to ask him to review or complete any of the problems they needed. This usually took between 15 and 35 minutes, depending on the number of questions the students had.

As the main part of the lesson, Jeremy presented the new content to the students while they took notes. Usually, he demonstrated a concept as students followed along or completed a task as a class. Jeremy had prepared the transparencies he used prior to class so he could place the problems or definitions on the overhead. He read the definition and explained it as the students copied it in their notes. Jeremy then showed students how to do several problems using the new concept. Rarely did students complete or practice a problem on their own during class.

On several occasions, Jeremy had an activity for the students to do in small groups. He walked around to check on the groups as they completed the activity. After most groups had completed the activity, Jeremy reviewed the key points of the activity with the students. Toward the end of the lesson, Jeremy assigned the homework and students were expected to work on it until the bell rang. Jeremy used this time to help individual students and check on the students who were falling behind.

Jeremy's mentor teacher had established a strong set of rules of classroom expectations. Jeremy reinforced these rules and set his own expectations for the students. Consequently, his students were well-behaved and he rarely had to deal with behavior problems in the classroom.

Jeremy was strong as a beginning teacher. Several aspects about Jeremy were worthy of discussion. First, he had established himself in the classroom and it was obvious students respected him. He had developed a good rapport with the students and talked to them or joked with them before and after class. The students often asked Jeremy how he was and if he had gone to a certain activity the previous night. Jeremy obviously enjoyed these interactions and often commented to the researcher about the importance of such interactions.

Second, Jeremy was always well-prepared. Every lesson was organized and well-planned. It was obvious he had spent a great deal of time planning and thinking through each lesson. He also had all materials prepared and ready for each lesson. However, on several occasions, Jeremy stumbled over the homework problems. He was not sure what a problem was asking, or he misinterpreted the problem. This uncertainty caused Jeremy to be confused and flustered. It was not clear if he had taken the time to review and do the problems. This type of confusion happened during four of the lessons that were observed (observations two, three, four and six).

Third, Jeremy often tried to show students where the content they were learning might be used in the real world. He provided students with a purpose for learning the material and was able to relate the content to their lives. Several times, Jeremy even asked the students to think of where or how they might use a concept.

Fourth, Jeremy's questioning techniques were good for a beginning teacher. He asked many questions as he reviewed the content and he called on a variety of students. His questioning strategies helped keep the students on track and paying attention because they did not know when or if they would be called on to answer a question. Jeremy was even able to think about the types of questions he asked. He stated: "I would like to improve my questioning in terms of asking more higher order questions, forcing students to think more deeply, and in doing so, receive higher order responses."

Last, all of the lessons observed of Jeremy's were deductive. Although he had planned several activities, he presented the concepts before the students completed the activities. For example, during observation one, Jeremy explained and defined the Golden Ratio on the overhead as students took notes, and then had students complete an activity where they measured and compared different parts of the human body. This lesson might have been more successful if Jeremy had allowed the students to discover the special ratio on their own. However, this deductive type of presentation was typical for Jeremy.

Bailey

Background. Bailey had wanted to be a teacher since the age of five. She originally wanted to teach elementary school, however, decided in high school that she enjoyed mathematics. She also decided she did not want to teach every subject, as would be required of an elementary teacher. Consequently, Bailey began her college studies majoring in mathematics and focusing on teaching at the high school level. She graduated with a Bachelor's degree in mathematics the spring term prior to the beginning of the teacher education program.

Bailey's main purpose for becoming a teacher was to make an impact on students' lives. She "had a hard life growing up." She had attended five different schools from grade school through high school.

> This made it extremely difficult to make and keep friends and to get to know my teachers. I had few teachers that I felt I could talk to when I had personal problems. This is one of my strongest reasons for wanting to become a teacher. I want to be one of those teachers that cares.

Bailey also wanted to be a teacher because she felt she would be a positive role model for students. She felt she would be able to make students feel comfortable around her and be open to her about their lives. She felt establishing a trust with the students was essential to teaching.

Bailey stated in her professional goal statement she realized the expectations of teaching. She felt aware of the hard work, stress, and low pay of teachers. "For this reason it takes a special individual to hold a career in teaching. Yet it is for these reasons that I wish to become a high school math teacher."

Bailey had completed all requirements for entrance into the teacher preparation program. She had worked with elementary students in a day-care and had tutored college-level students. She had not, however, worked with students at the high school level. She interviewed and was accepted into the program.

Internship. Bailey was placed in a high school with an experienced mathematics teacher. Throughout her part-time internship, Bailey kept a journal of her ideas, impressions, and her school activities. The journal entries were all short and usually did not express her feelings about what was happening. Thus, her development during her internship experience was difficult to trace.

Bailey began by observing a geometry class, a college algebra class, and a calculus class. She quickly learned students' names and started to feel comfortable in the classrooms. Bailey's main concern from the beginning of her internship was to be accepted and liked by the students. "It is a big deal to me to know that I am accepted in the classroom." Bailey also felt it was important to be able to joke with the students. She thought the students had been impressed with "the fact that a little ribbing doesn't bother me and that I return as much as I get."

Bailey started to work with individual students and grade quizzes during her time at the school. She also presented a mini-lesson in the geometry class. After three weeks Bailey stated, "I am amazed at how easily I could pick out those students who would do well, those who would struggle, and those who were potential disciplinary problems."

Bailey began to plan her unit that she would teach in the geometry class on constructions. She stated she was looking forward to teaching the unit, however, was not sure how the students would react to her and the activities she had planned. As she began to teach the unit she felt the lessons were going well. However, after the first quiz she realized students were not doing as well as she had thought. She stated:

> I had a period of frustration last night. I was grading quizzes and I got upset because the same students were getting the same problems wrong even though I had gone over them many times. I began to blame myself and think that it was my fault.

Bailey soon realized there were several aspects of this problem that needed to be addressed. She decided to review altitudes since most of the students had missed these problems on the quiz. She also decided she needed to encourage some students to come in for extra help. "I am still wondering if there is more I can do."

Bailey also realized how teaching a concept helped her to understand it. She felt her own understanding was an important aspect of teaching. After completing the unit, she stated:

> The teaching of this unit has given me a new perspective on constructions and their uses. It is often said that the best way to learn something is to teach it. Every time I teach, my belief in this grows stronger. Before, as a student, I knew how to perform constructions, but now, as a teacher, I know constructions.

Bailey continued to develop her beliefs about mathematics teaching and learning. She believed *all* students should learn mathematics and four years of mathematics should be required for graduation from high school. Bailey believed students should play an integral role in learning mathematics. "They should be as much a part of the teaching process as the teacher is. They learn better through self-exploration and by being a part of the learning process."

Bailey realized many students simply did not like mathematics. She attributed this dislike to the teachers. "Too often students learn to dislike math because their teachers dislike it." Bailey felt quite strongly effective teachers needed to enjoy the subject they taught. She reflected on the characteristics she felt made an effective teacher:

> An effective teacher is caring, kind, and compassionate. He or she enjoys the subject matter and working with students. An effective teacher creates enthusiasm for the subject matter and includes the students in the learning processes. He or she is not critical of the students' abilities, but helps them to expand on the abilities that they do have and learn more.

Bailey had also begun to establish her beliefs about how students learn mathematics. She felt "students learn more from doing rather than watching." She believed a teacher needed to create real world problems "that fit the curriculum and the students' every day lives."

Bailey successfully completed her part-time internship and continued to observe and teach in the classes she was to teach during Winter quarter. "I enjoyed teaching my unit and, with a little trepidation, look forward to January."

<u>Card Sort Task</u>. Bailey completed her card sort interview in 20 minutes. She was quite brief when answering the questions that were asked. She expressed her concern that the researcher was "looking for a right answer." Bailey was assured there was no specific answer the researcher was looking for and no "right" or "wrong" way to sort the cards. She still seemed hesitant about completing the sort. Bailey's card sort is shown in Figure 10. Group numbers were added by the researcher for clarity.

When she was finished organizing the cards, Bailey explained her structure. She began by explaining how she had placed history, connections and problem solving at the top because "I feel they encompass everything." She then divided the cards according to Euclidean and non-Euclidean geometry. On the Euclidean side of her sort Bailey placed the cards representing points, lines, and planes because "they are the basic elements of Euclidean geometry." Bailey explained the rest of the cards were all non-Euclidean. After some hesitation, she stated:

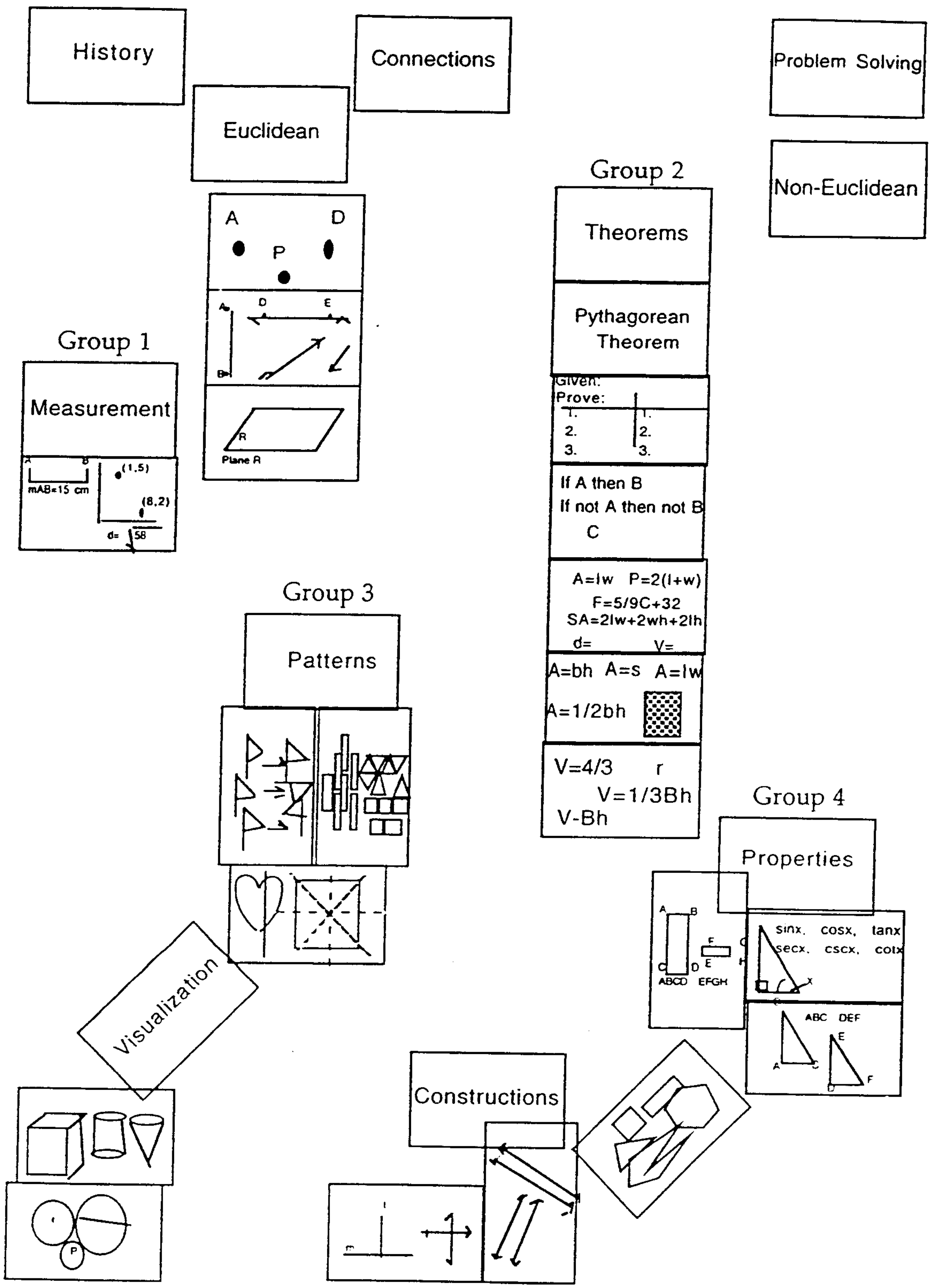


Figure 10. Bailey's card sort

> It has been so long [since taking a non-Euclidean geometry class]. There are parallel lines and distances, which measure differently. I think parallel lines is the biggest one because isn't that the axiom, or whatever, that falls apart?

Bailey stressed that the rest of the cards had not been placed in any specific order, other than being placed in groups. She described Group 1 as measurement, including distance and length. Group 2 Bailey labeled theorems and placed formulas with them because "I feel they kind of go there." She described Group 3 as representing visualization. She felt the reforms in mathematics education had focused on visualization, specifically three-dimensional figures. She continued describing this group as including patterns of symmetry, tessellations, and transformations.

Bailey described Group 4 as properties of polygons, similarity, congruence and trigonometry. She connected the properties to constructions by using the polygons. Constructions included parallel and perpendicular lines because "that is what I am teaching right now."

Bailey finished discussing her card sort by describing how each of the groups were related to each other. She stated:

> I guess they all could relate because you construct patterns and there are properties in patterns. There are properties in theorems and formulas. They are all connected. That is why they are under connections.

Much of Bailey's sort was based on how geometry might be taught. Bailey could not think of any card she would add to the sort. She stated it had been since high school since she had taken geometry and once she taught it she would remember more concepts to add. She also decided her sort would probably look about the same if she were to redo it.

When asked if she had thought about the organization of geometry before, Bailey stated, "Not myself. I think a lot of it is already organized. I know point, line, and plane are the first things students learn. The curriculum is already set out for you." Bailey finished her card sort interview by stating, "depending on the textbook you taught out of, could change the order of the cards."

Videotape Task. As Bailey viewed the experienced teachers on videotape, she was again quite brief in her comments and responses to the posed questions. Her responses had to be probed to elicit further explanations. Although her brevity made it difficult to establish her conceptions about geometry teaching, there were some conceptions that arose.

First, Bailey was aware of each experienced teachers' classroom management strategies. She commented on each teacher and how he maintained classroom order. She felt Teachers A and B had established effective classroom management strategies. Regarding Teacher A, she commented:

> He must have something set up. When I look at the variety of students, I can see where some of them *should* cause problems. They are quiet and at least pretending to pay attention. He must have set something up at the beginning of the year.

Bailey stated Teacher B did not seem to have any problems with classroom management either. She was impressed with his ability to "pull the students back on track" and thought his expectations were well communicated to the students. On the other hand, Bailey did not like Teacher C's management strategies. She stated: "He just whistled. I don't like the whistle. It makes you feel like a dog."

Second, Bailey asked "*Why*" throughout each of the teachers' lessons. This question referred to three different aspects of the lessons: (a) the purpose of the lesson, (b) why something was true or false, and (c) why the students missed a problem. Bailey felt each teacher needed to establish a purpose for the lesson he was presenting. She felt students did not know why they were learning a concept or how it applied to their lives. "It was hard to see the purpose." Bailey felt the teachers also did not tell the students *why* a concept or idea was true. Specifically, Teacher C did not establish a purpose and Bailey thought this teacher did not try to help the students understand the concept. She stated:

> I got the feeling that he was telling them what he knew. He had the attitude that if he knows and then he tells the students, then they will also know. But maybe they don't. Maybe they don't understand the process of *why* it has to be true. It is easy to tell them it is true, but *why* is it true?

She also felt the students did not know why they had missed a problem. Bailey thought Teacher B needed to ask the students to explain how they got a specific answer. She stated, "I would have them discuss the results and why they got the answer they did. I would ask them, 'Why did you choose that?' I would make them explain their answers."

Third, Bailey felt strongly all three teachers were more concerned with the students' final answer than the process they had completed. She believed it was more important to see the students' processes in solving problems than if they obtained the correct answers. She was most clear in her thoughts when discussing Teacher A regarding this belief:

> I thought it was strange that he was emphasizing the fact that you have to put definition, theorem, or postulates [in proofs]. He was looking for specific things. I guess for me I want more the *process* of how you do it rather than exact answers. If you know what you are talking about and get the process, that's better than getting all the right words.

Bailey stressed the reforms in mathematics education were "pushing for processes and higher thinking" and by looking at only the correct answer, these teachers were unable to know if students understood the content.

Last, Bailey attributed many of the aspects of the lessons to the textbooks the teachers were using. She commented the teachers may not have much choice in how they presented the material because of the organization of the books. "Different books require different things."

Bailey ranked the teachers in the reverse order she had viewed them. Although she had been critical of Teacher C's classroom management and teaching style, she decided she liked him because she felt that he was comfortable with the students and they had accepted him in the classroom. She felt that this teacher was concerned with the students and he allowed them to express themselves. "They seemed to feel free to give their opinion. She felt Teachers A and B did not give the students the freedom to express themselves.

<u>Conceptions of Geometry</u>. Bailey's conceptions of geometry were difficult to determine from her card sort interview and the videotape tasks. Several conceptions arose that were specific to geometry. Several other conceptions overlapped with her views of geometry teaching.

Bailey seemed to have an adequate understanding of geometry. She was able to supply justifications for the proofs in Teacher A's lesson and she noticed Teacher C had called a median "the middle line." She had discussed that she felt comfortable doing constructions and knew how to perform constructions. The only area Bailey admitted she was not good at was proofs. She stated she had never liked doing proofs and had struggled with them in both high school and college.

Bailey was certain about the geometry taught at the high school level. She believed there was a specific order to geometry. This order started with the basic concepts of points, lines, and planes. The rest of geometry was ordered based on the textbook the students were using.

One area of doubt in Bailey's mind became apparent during the card sort interview and developed from the differences between Euclidean and non-Euclidean geometry. She was unsure of what distinguished Euclidean and non-Euclidean geometry. Furthermore, she knew that parallel lines were involved in this difference; however, she could not state that difference.

<u>Conceptions of Geometry Teaching</u>. Triangulation of data sources including card sort and interview, videotape tasks and journals indicated Bailey believed students learn best by exploring and discovering for themselves. She stated several times that mathematics teaching should involve students in their own learning rather than the teacher telling the students the content. Furthermore, she was also critical of the experienced teachers on videotape because she felt they had spent too much time telling the students the content rather than allowing them to discover it for themselves.

Bailey's conception of teaching geometry was textbook-oriented. She had completed her card sort based on how a textbook presented the material. She had defended the experienced teachers and their teaching based on the fact they did not have any freedom since the book told them how to teach.

Bailey believed it was important to establish a good relationship with students and to use their interests in the classroom. She had entered teaching because she felt she was able to communicate and relate to students at the high school level.

Bailey stressed the importance of using real world problems in the classroom. She discussed this importance during the card sort interview and the viewing of the videotaped lessons. She believed students would be interested in and learn mathematics if they completed investigations that used real world examples.

Bailey's last area of focus was on establishing a purpose for the lessons. She believed it was important to show or tell students why they were learning a concept, and why it might be important to their future lives. She had been critical of the experienced teachers for not establishing a purpose for their lessons and not showing the students why they needed to learn the content.

Classroom Practices. Bailey was assigned to teach two geometry classes and a college algebra class for her full-time internship. She was observed in one of the geometry classes that met for 90 minutes every other day. This class was the last class of the day. Bailey had chosen this class for observation because she felt they were better behaved than the other class. There were 28 students in the class. She was teaching from the book *Discovering Geometry* published by Key Curriculum Press (Serra, 1989). A summary of Bailey's classroom observations is listed in Appendix H.

A typical lesson started with taking attendance. Most often, the students were not seated, so Bailey would ask them to sit down. After she had taken attendance, she asked the students to list which problems from the homework they wanted her to review. Bailey read the answers to the homework and solved any problems the students needed. This review usually lasted between 15 and 45 minutes.

Bailey then presented the new concepts to the students. She usually drew a picture and explained the concepts from the picture. She then wrote the definitions on the board that students copied into their notes. Bailey showed the students a couple of examples and then assigned the homework. The students were allowed to work on the homework in their assigned small groups. As students worked on the homework, Bailey walked around to help the groups or individual students. Class ended with the bell.

Several aspects of Bailey's teaching should be discussed. First, Bailey was textbook-oriented. She followed the textbook religiously. She covered every section in the book and followed the same order. Many of the examples she used were directly from the text. All of the student work was directly from the book. She required students to complete all of the problems from each section. She also had them read each section and look over the examples. This specific textbook had investigations for student to complete and Bailey had the students do these as well. She never had the students deviate from the textbook.

Second, Bailey did not seem to enjoy teaching. She often displayed annoyance at having to help students and spent a great deal of energy disciplining students. On several occasions Bailey yelled at the students to quit doing something and the more she yelled, the more students tried to annoy her. For example, during observation six, Bailey yelled at the students, "NO HELICOPTERS WITH THE RULERS! I have told you many times. They are not play toys! You are wasting your own time." Most of the students quit at this point, but several still twirled their rulers. It became a challenge for them to see what she would do.

Bailey's frustration grew with her classroom management problems. She became increasingly impatient with the students. She began to threaten students with more work if they did not stop talking. Several times, Bailey's frustrations showed when questioning students (both individually and as a class). When students asked a question, Bailey often tried to lead them to the correct answer without directly telling them the answer. Sometimes, Bailey's patience did not last long and she eventually told the students the correct answer. This type of interaction happened during five of the eight observations. The students soon stopped asking her questions.

Interestingly, Bailey seemed to have a different perception of what was happening in her classroom. She wrote in her reflections following the lessons:

> This class works relatively well in groups. They don't hesitate to ask questions, but, as with all classes, simply want the answer. They do not, however, get angry when I don't give it to them. The lesson went well and everyone seems to understand from what I observe.

Third, Bailey spent a great deal of time reviewing the homework and having students work on homework. On the average, she spent only 15 minutes explaining new material. The remainder of the 90 minutes was spent on homework or taking quizzes, leaving available time for the students to cause problems or to try to annoy the teacher.

Robert

Background. Robert had not always wanted to be a teacher. In high school he had wanted "any job making a lot of money." He originally majored in engineering in college, but decided engineering was not what he wanted to do with the rest of his life. After many different jobs, Robert decided he wanted a career that he would enjoy going to everyday. He decided to become a teacher because "it is an important job and I wanted to have something that is worthwhile that I like to do every day."

Robert decided during his sophomore year in college to become a mathematics teacher. He chose mathematics because "out of all the subjects I had studied, mathematics was the one I was good at and enjoyed the most." Robert graduated with a degree in mathematics the term prior to entering the teacher preparation program.

Robert felt his best attribute was that he liked and enjoyed kids. "More than anything else I have ever done in my life, I enjoy working with school-aged children." Robert had limited experience working with youth, but had several positive experiences. He had worked with elementary and middle school students in outdoor school and had been a camp counselor for several years. He felt those experiences had given him a view of what teaching was. He stated:

> I know that it can be incredibly difficult to reach them [students] at times and at other times they are incredibly willing to learn. This makes teaching both a challenge and a pleasure of which I would like to be a part.

Throughout his life, Robert had participated in sports and wanted to continue this interest. He felt he could have a positive influence on students outside the classroom. "I would also like to teach on a basketball court and a baseball diamond. I know I possess useful skills and knowledge that could be passed on to young athletes."

Robert had completed all the requirements for entrance into the teacher preparation program. He interviewed and was accepted into the program. Robert was encouraged to gain experience with high school students since he was interested in teaching at the secondary level. He completed a practicum in a local home for disadvantaged youths.

Internship. Robert was placed in a mid-sized high school with an experienced mathematics teacher. He observed two geometry classes and a pre-calculus class during the part-time internship. He spent the first three weeks learning students' names and observing his mentor teacher. He also helped individual students if they asked for it. "I like tutoring students and turning a totally confused student into someone who understands the concepts."

During the fourth week, Robert corrected a test with the geometry classes and answered questions from students. "It was a good experience because it helped me get used to the idea of being the central figure in the classroom and having the students look to me for guidance."

Robert believed establishing a good rapport with students was essential to effective teaching. He felt if teachers were interested in the students, they would automatically be interested in the subject area. He stated:

> I think to be an effective teacher you have to have a good rapport with the students. Many teachers are just interested in teaching and are not really interested in the students. I think if a teacher shows a genuine interest in what students do and say, the students will unconsciously have more interest in what the teacher is doing and saying.

Robert prepared a unit on congruent triangles that he taught to one of the geometry classes. Several changes were made to Robert's original plans, but for the most part, he felt prepared. He was excited about teaching his unit and started teaching the sixth week of the term.

Throughout his unit, Robert reflected on three main aspects of teaching. First, throughout this teaching experience he focused on classroom management. As stated previously, he believed by establishing a good rapport with students he would avoid management problems. Robert soon learned it took more than a good rapport with students to control his class. He did not want to "be extremely strict because it is not my style." However, he soon realized that he needed to enforce some of the previously set rules. He stated:

> To improve my teaching I need to get tougher with classroom management. I have been letting the students get away with too much. I know it is harder to get tough rather than lighten up, but I feel like I have a pretty good rapport with the students, but

> they need to know that I am serious about them behaving in class.

Second, Robert also realized that not all the students learned from verbal explanations. He felt he needed to work on helping the students to visualize the concepts. He stated, "These students need to see more diagrams with labels telling them what it means. I can't just tell the students what a concept means. I have to show them as well."

Last, Robert was concerned that the students were not always interested in learning mathematics. The students' disinterest seemed to surprise and, at times, frustrate him about teaching:

> One thing that has surprised me about teaching is how little the students remember from previous math classes and how little effort they put into learning. It makes it pretty difficult to teach concepts that build on previous concepts when students don't remember, especially when they won't do their homework or come in for extra help.

When Robert completed teaching his unit he was pleased with the students' progress. He felt the students had learned the material he had presented and were prepared to continue to the next unit.

Robert continued to establish his beliefs during his part-time internship. He believed "Everyone needs to know some mathematics." He felt high school students should be required to learn algebra, some geometry, some trigonometry and some probability because these concepts are frequently used in everyday life. For this reason, Robert believed it was essential to teach mathematics using real world examples and applications. "A lot of math teachers lecture all period long and seldom relate the topics to real life problems." He felt it was important to involve students in learning mathematics through these real life situations. In his journal, Robert wrote:

> By teaching math through interesting applications students are rarely frustrated and often challenged. For instance, if I were going to teach a unit on ratio and proportion, I might have the students build scale models. By making the concepts practical, I feel that the students will understand them better and make the connections between what they are learning and how it relates to the real world.

Robert's growth as a teacher throughout his part-time internship is best summarized by this statement:

> I think I have grown a lot as a teacher. I finally know what it is like to be responsible for a class. This is the kind of experience I have been waiting for. More than anything else that I have done, the part-time internship portion has been invaluable. I have learned how to plan better, question better, assess student learning better, and even think better.

Robert successfully completed his part-time internship and looked forward to teaching two sections of geometry and one section of pre-calculus for his full-time internship. Since he was teaching geometry, he was observed in the classroom for this study.

Card Sort Task. As Robert participated in the card sort interview he seemed relaxed. He was confident and was not afraid to express his beliefs. He began by defining geometry as "the study of shapes and the relationships that certain shapes and figures have to do with the world around us." He listed the polygons, parallel lines, perpendicular lines, and triangles as being the terms and topics of geometry. He also decided "the construction tools are a big part of geometry: the compass, protractor and that kind of stuff."

Robert organized his cards in a circular manner. He placed the card representing connections in the center and the remaining cards around that. His final card sort is shown in Figure 11. The circle, lines, and the additional words were added by Robert as he explained his organization of the cards.

Robert began by explaining that he placed connections in the middle because he wanted to connect all the different concepts in geometry. "A lot of topics in geometry build on one another so you would need to show the connection between what you have done and how it is going to relate to what you are going to do next." He continued to explain how the circle represented all of the concepts being connected. The lines through the center of the circle connected different portions to each other. "All of these are connected in some manner."

Moving clockwise, Robert explained each group of cards. Non-Euclidean was placed by itself because "I didn't really see anything to go along with it." Robert placed points, lines, planes, parallel lines, and perpendicular lines in a group he labeled Euclidean. "These are the things that jump out at me as being pretty distinctly Euclidean. I mean they are not necessarily *just*

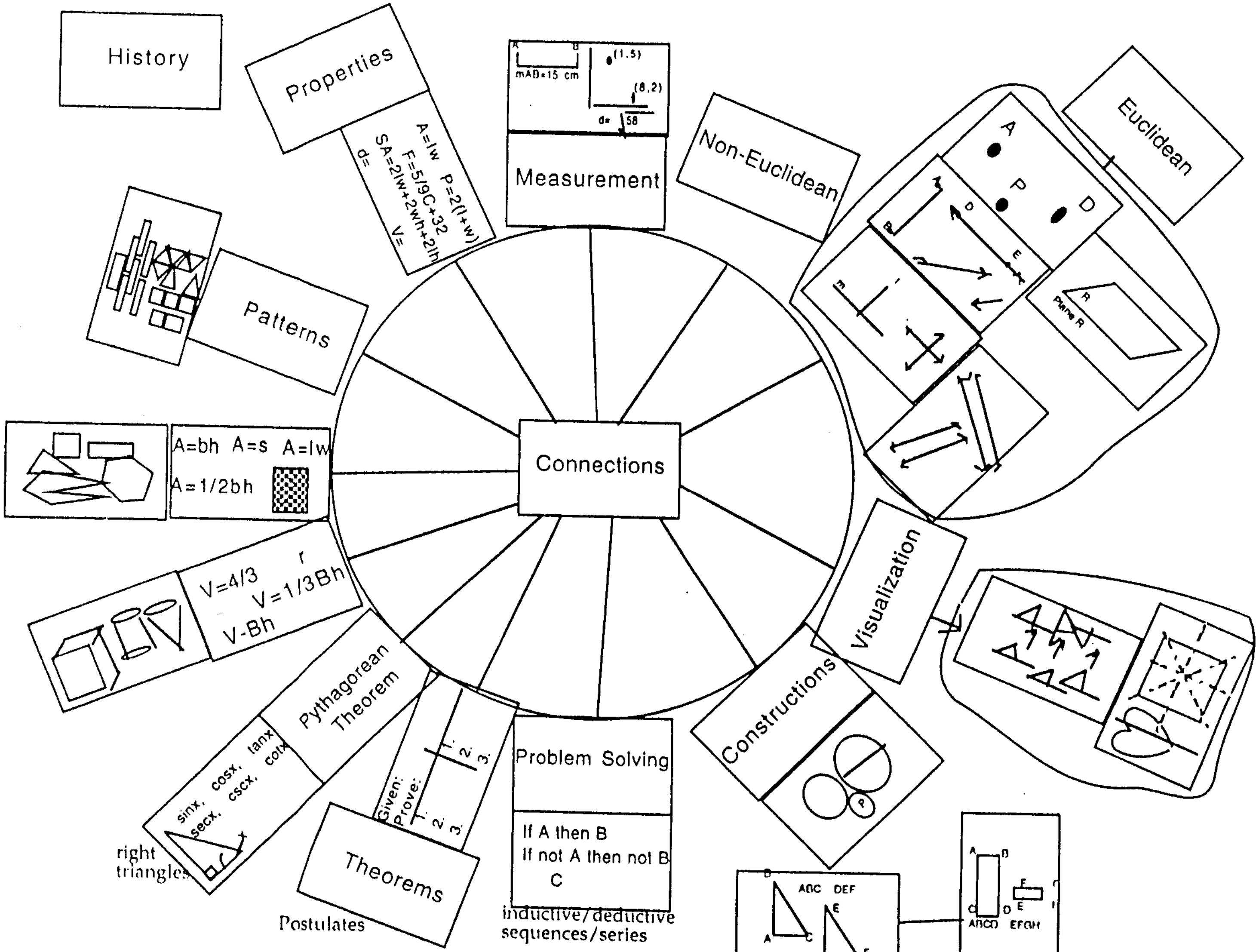


Figure 11. Robert's card sort

Euclidean, but they are characteristic of Euclidean geometry." He thought Euclidean geometry was the most common geometry and stated most people just thought of Euclidean geometry when they thought of geometry.

Robert described visualization as "being able to think of a picture in your mind." He decided transformations and symmetry were part of visualization because a person should be able to visualize what something looked like if flipped or rotated. Circles were placed with constructions because "you can't draw them freehand."

Robert matched reasoning with problem solving because he felt when problem solving, students needed to be able to think logically. As he described this group of cards, he added inductive and deductive reasoning, and sequences and series. He felt these concepts needed to be added to the set of cards to better represent geometry.

Continuing clockwise in Robert's sort, he explained theorems had to be proven so he put those two cards together. He also added postulates to this group. He put the Pythagorean Theorem with trigonometry "because they both have to do with right triangles." Volume was matched with three-dimensional shapes and area with two-dimensional shapes. "Tessellations are patterns, formulas are kind of properties, and length goes with measurement."

Lastly, Robert placed history by itself and set off from the circle. He explained this:

> That does not mean it is less important. To understand geometry I don't think you have to know a lot about the people involved in the past. To be able to pass a geometry class you wouldn't need to know who Euclid is, you just need to know what the terms are.

Robert completed his card sort interview by stating he had not really thought about his organization, but just put the cards together by "the first thing that came to my mind." Robert stated he had never thought about organizing the concepts in geometry and he had forgotten many of the concepts. "Not forgotten, but kind of shoved back with a lot of dust on it. All of the basic concepts of geometry have been coming into my mind because I have been teaching it. I think there is a lot to be said about if you can teach it, it is a lot clearer to you. I think now that I have taught this stuff I don't think it will get dusty again."

Videotape Task. Two interacting themes emerged from Robert's transcripts of the videotape tasks. First, Robert's main concern with all three teachers was the rapport they had established in their classrooms. Robert had ranked the three teachers based on their rapport with students. He felt Teacher C had better rapport and was interested in students' opinions. "I think students appreciate it more when you care about what they say. They didn't feel any threat as far as asking questions. It seemed like they were pretty comfortable in that class."

Robert did not think Teachers A and B had established good rapport with their students. Robert evaluated rapport of these teachers based on their "joking" with students. Regarding Teacher A, he stated:

> I don't know what his rapport was like with the students when it is not class time. Between class periods does he talk with them? Does he joke with them? Maybe he doesn't do any of that kind of stuff so maybe they don't have any kind of personal relationship. It seems like an impersonal class. When I am teaching I have a pretty personal relationship with almost all my students so they are a lot more apt to speak out. That's probably why there is more class management things because they feel a lot more relaxed and comfortable.

Robert made a similar comment about Teacher B. He stated, "It doesn't seem like he has much of a rapport with the students. He doesn't joke with them."

Establishing a good rapport was important to Robert and he equated rapport with classroom management. Robert's second theme when viewing the videotapes was classroom management. Robert discussed each of the experienced teachers' classroom management in terms of it being "an intangible quality." He decided he could not distinguish between the teachers' classroom management and state an exact reason why each had good management, however, he did think that "rapport had a lot to do with it." For this reason, Robert decided Teachers A and B must have strict rules and enforce them, or the students were really motivated to learn. Robert stated he thought it was easier to learn in a class like Teacher A's because "everybody is on task and there are a lot fewer distractions, but it seems a little impersonal to me. It seems to me that when there is a little bit of chatter going on in the classroom there is a better rapport with the students."

Robert felt Teacher C had a better rapport and although there might be more management problems, the trade off was having students in class who wanted to be there. He stated:

> Given, there are going to be more classroom management problems in a class like this, but at least *all* the kids want to be there. Even if they don't do anything, at least they like the class and go to class and maybe by osmosis they will learn something.

Two other, less predominant, themes arose during the videotape task. One theme was the progression of the experienced teachers' lessons. Robert felt all of the teachers had started with a simple concept and built from there to more demanding content. "I thought that he [Teacher C] would explain it in really simple terms so students could understand. It is really easy for someone who has a degree in math to start rattling off all these terms and forget that it is harder for students to understand." Robert also thought Teacher A did a good job of breaking the content down to simpler terms so the students could understand it.

Another theme Robert mentioned, was that the teachers did not relate the content to the students' interests or real life. "It didn't seem like he [Teacher B] related much of what he was talking about to real life applications or something the students could relate to more." Similar comments were made about Teachers A and C.

Robert decided he preferred to be in Teacher C's class because he was more flexible and Robert felt he was more like Teacher C. Robert believed most students did not like Teacher C, but rather Teacher A because of his organization. Robert did not find much that he liked about Teacher B.

<u>Conceptions of Geometry</u>. From analysis of the card sort interview and the videotapes, Robert's conceptions of geometry emerged. Robert's card sort was organized and he was able to explain it clearly enough that a strong knowledge of geometry seemed likely. During the videotape tasks, however, Robert was unable to provide acceptable justifications for one of the proofs Teacher A presented. He was also unable to complete the second proof. Robert admitted he did not like doing proofs and they were his area of

weakness. "Hopefully I can get students to enjoy doing proofs, but it is not likely. I hated them [proofs] too when I was in geometry."

Robert believed connections were essential to geometry. He demonstrated this belief in his card sort as he had placed the card representing connections in the center of his sort. He also explained how all the concepts in geometry connected to other concepts. He showed this connection by drawing lines on his card sort. Robert's view of geometry was limited to the concepts within geometry. He never discussed how the concepts in geometry connected to other mathematics, such as algebra, or calculus. He also never discussed how geometry connected to the outside world and how the concepts related to other subject areas.

Conceptions of Geometry Teaching. Robert's views of geometry teaching centered around the rapport he built with the students. He believed students enjoyed and learned in a classroom where the teacher had established a good rapport. He felt students achieved better if they enjoyed the class. "I am a firm believer that when students like the class they are in, they achieve better."

Robert believed students learned geometry through visualization. He had discussed the importance of visualization during his card sort interview, "I believe that being able to visualize a concept is very important." He was also critical of the experienced teachers on videotape. He felt all teachers needed to show students how the concepts can be visualized. He thought the teachers needed to draw more pictures so the students could learn the concepts "more concretely."

Robert stated he believed it was important for students to solve the problems correctly. He told the researcher he did not focus on the process that students went through to solve a problem, rather the product. "I tend to be a results kind of person, rather than the process."

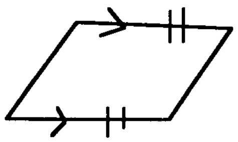
Last, Robert believed it was important for the students to see real world applications when studying geometry. He felt it important for the teacher to present the content using real world problems and allow the students to see where they might be able to use the concepts. He also felt it was important to use problems or examples which interested the students. Robert thought it was essential for the teacher to get to know the students and their interests so these interests could be used in the classroom to motivate the students.

Classroom Practices. Robert chose the observations to be done in his advanced geometry class. This class was a group of 20 freshmen. The content, textbook, and pace were the same as the regular geometry classes. He was teaching from the book, *Geometry*, published by Houghton Mifflin (Jurgensen, Brown & Jurgensen, 1988). This class met every day for 50 minutes. Eight lessons from this geometry class were videotaped and transcribed. These transcriptions were combined with the researcher's fieldnotes to provide detailed classroom practices. Robert's classroom observations are summarized in Appendix I.

Robert typically reviewed homework at the beginning of class. After the bell rang, he solicited problems the students needed to review. Robert spent time solving these problems. He drew a picture, if needed, and explained each problem. He called on students to help him or asked them questions about the problems. After he had completed his explanations he asked if the students had any questions; usually they did not.

After reviewing the homework problems, Robert had the students open their books to a specific section and begin reading. He called on individual students to read a theorem as the other students copied the theorem in their notes. After the student read the theorem from the book, Robert asked the students if they understood what the theorem was stating or had questions about it. If a student had a question Robert drew a picture on the board and explained the concept. He then called on another student to read the next theorem and continued in the same manner. For example, the following is a typical interaction (taken from observation one) during Robert's lessons:

(Teacher has figure drawn on board)



Teacher: How about theorem 4.5? Tyler?

(Student reads the theorem)

Teacher: In our same drawing, if two sides are congruent and parallel in a quadrilateral, then it has to be a parallelogram. That is pretty easy. I think everybody got that one.
Are you there? Who wants to read the next one? Theorem 4.6?

(Student reads the theorem)

Teacher: You remember what opposite sides are? You should.

Typically, five to eight theorems were presented in this manner.

After all theorems had been presented, Robert either had the students decide whether a statement was true, or he showed two or three examples. Robert sometimes had these prepared ahead of time, but typically he copied examples or statements from the book. Robert gave the students time to try these problems on their own and then discussed them as a class. Robert then assigned the homework and students worked on it if they chose. There was usually five to fifteen minutes left in class.

Throughout these lessons several observations were made of Robert's teaching. First, Robert believed he had established a good rapport with students. He was able to talk with the students and joke around with them before and after class. He showed interest in the students and they responded in a positive manner. However, Robert had an almost laissez-faire attitude about teaching. He had made the assumption with this class that since they were freshmen taking an advanced class they should be able to understand the content more easily. Unfortunately, Robert made many comments that put students at a distance in the classroom. These comments included, but were not limited to:

Teacher: These [problems] are pretty self-explanatory.

Teacher: These [problems] are pretty straight-forward.

Teacher: Are you catching this? This is not brain surgery or anything.

Teacher: Do you understand? Pretty simple stuff.

Teacher: This should be easy for you.

Robert's students did not ask many questions and these type of comments probably made them feel uneasy about asking questions.

Sometimes Robert's attitude made him appear indecisive and disorganized. Several times Robert gave the students a choice about what to do. He allowed them to run the class rather than him being in charge. This style resulted in management problems also.

Second, although he asked questions throughout the lessons, he did not call on specific students. This questioning strategy resulted in two or three students answering all of the questions. The rest of the 20 students were never called to answer questions. Robert was unable to determine if the other students understood the concepts or if they had any questions about the material.

Last, Robert's knowledge of geometry was questionable. During six of the eight observations he presented incorrect information to the students. Several times he did not use the proper vocabulary when presenting to students. For example, he called a proportion a ratio and a theorem a definition. Robert also made mistakes when solving problems. These errors included addition errors, conversion errors, and algebraic errors. For example, he converted 1.6 meters to 16 centimeters. This latter mistake resulted in an incorrect solution to a problem. When discussing ratios, he converted the ratio 2:3 to a division problem and wrote:

$$2\overline{)3}$$

Robert had switched the numbers.

Robert was also unsure how to solve several of the homework problems. On one occasion, he "proved" a problem by using what the students were originally supposed to prove (circular logic). This logic error was a major flaw in completing proofs many students have also make.

More importantly, Robert told students ideas that were not true in geometry. For example, during observation five Robert told the students ratios do not have any units. The important aspect of units in ratios is the units are the same in both numbers of the ratio. Also, during observation six, Robert discussed polygons and how to name them. He told the students there were special names for any polygon no matter how many sides it had, and polygons were not named n-gons. "If you have a 30-sided figure, you don't call it a 30-gon. There is a name for it, but who knows what it is." Actually, in geometry, this type of naming is a convention. If students do not know a specific name for a polygon, they are encouraged to use this way of naming a figure.

Robert's knowledge of geometry often limited his teaching. On several occasions he was unable to present the students with alternative explanations for concepts. Several other times he asked how to solve a problem and a student responded, but Robert wanted another way. He did not accept the student's explanation, even though it was correct.

Summary of Individual Profiles

Conceptions of Geometry

The data sources for all participants were analyzed reflecting several conceptions of geometry. As these conceptions emerged, they were confirmed through triangulation or the card sort task and interview. Many of these conceptions of geometry overlapped with the preservice teachers' conceptions of geometry teaching. Only those conceptions specific to geometry are discussed in this section.

Overall, the preservice teachers held a view of geometry as being ordered, i.e., geometry has a specific organization. Five of the ten preservice teachers believed explicitly that geometry was ordered. They described the organization of geometry as a foundation of points, lines and planes. They believed the other concepts in geometry built upon this foundation, therefore, an understanding of this foundation was essential. These preservice teachers stated a belief that, after knowledge of this foundation, the concepts in geometry have no specific order.

Two preservice teachers believed their sequence of geometry could not be altered. They both stated their organization of geometry would not change dramatically if they reorganized the card sort or after acquiring teaching experience. They believed certain concepts belonged together, therefore, should not be combined with other ideas. For example, Ryan believed all of the formulas belonged together and stated he would always place those concepts together. He did not believe the formulas could be situated with other concepts.

Three preservice teachers were tentative about their organization of the cards. They felt their organizations represented what they currently believed about geometry, however, their structures changed each time they organized the concepts. Their structures were not as linear as the first two preservice teachers, however, they still believed an order was necessary.

All the preservice teachers stated their organization of the concepts in geometry was based on three aspects. First, they believed geometry was based on how they had learned geometry. Five of the preservice teachers stated they had learned the concepts in the same order they had placed them in their card sort. Two of the preservice teachers even discarded concepts they had not learned in high school geometry. Second, the preservice teachers stated they had completed their organizations based on how geometry should be taught. Four of the preservice teachers stated they were unsure how to arrange the cards because they had not yet taught geometry. They also stated they would know the order of the cards when they started teaching geometry. Third, the preservice teachers believed their structure of geometry was not unique. Many of them thought others organized the concepts in the same manner since the organization of geometry had been determined by the experts, established curriculum, or textbooks.

All of the preservice teachers agreed the cards represented what should be taught in high school geometry. This view was not surprising since the original design of the card sort was based on the NCTM *Curriculum and Evaluation Standards* (1989). The concepts depicted on the cards were taken directly from recommendations for high school geometry content. The preservice teachers did agree the cards could be more detailed and the set could have included many more cards.

These preservice teachers also held a view of the nature of mathematics and its role in the world. Two of the preservice teachers believed mathematics was "static." These preservice teachers held the view that, in mathematics, there was one way to solve a problem. For example, Nick stated, "Usually there is one correct answer and one way to solve the problem." In contrast, the other preservice teachers were more open to different approaches to problem solving. They believed most problems can be solved in more than one way and possibly have more than one answer.

More specifically, the preservice teachers held views about the nature of geometry. Eight of the preservice teachers believed geometry was a "tool for problem solving." They thought geometry was important to learn because it provided the basis for problem solving. Three of these preservice teachers also felt geometry was "rule-bound." They thought it was important to know the "rules of geometry" in order to use them. One preservice teacher stated, "Rules had to be created to be able to shorten the descriptions of geometry."

Seven of the preservice teachers believed geometry was important because it provided a way to view and describe the world visually, rather than numerically. These preservice teachers believed geometry was designed so people are able to explain the world around them. They felt geometry, in contrast to other areas of mathematics, provided a basis for describing real world phenomena. The other three preservice teachers discussed the importance of connecting geometry to other fields of mathematics by showing students the interrelations between geometry and algebra.

A third conception that emerged was the preservice teachers' knowledge of geometry. Throughout this study, it was obvious the preservice teachers lacked confidence in their own knowledge of geometry. Nine of the ten preservice teachers admitted that they did not understand or were uncertain about various aspects of geometry. From the beginning of the card sort interview, many of the preservice teachers admitted to the researcher they did not feel their knowledge of geometry was adequate. Even the preservice teachers who seemingly possessed adequate knowledge were not confident. For example, Becky's knowledge of geometry was strong throughout the study and she was able to answer the questions posed correctly. She also had noticed some content errors with the experienced teachers on videotape. Becky provided examples of the content and extensions to other areas of mathematics. However, she lacked confidence in her knowledge of geometry. She openly discussed her apprehension about teaching because she was sure she would make mistakes with the content.

The one preservice teacher who displayed confidence in his knowledge of geometry was Nick. He felt geometry was his strong area in mathematics, therefore, he was not concerned with it. The data sources confirmed his knowledge. Nick's knowledge of geometry was strong, however he never quite conceptualized how to teach it. He left the teacher preparation program before his full-time internship began.

The lack of confidence by the preservice teachers reflected their incomplete knowledge of geometry. They displayed lack of knowledge in geometry during the card sort tasks and the videotape tasks. The most confusion was the distinction between Euclidean and non-Euclidean geometry. Two preservice teachers stated that Euclidean geometry was two-dimensional and non-Euclidean geometry was three-dimensional. Two other preservice teachers stated they were not sure of the distinction between Euclidean and non-Euclidean geometry. The rest knew the distinction had to do with parallel lines, however, were unable to delineate further.

Another area of geometry in which the preservice teachers indicated some apprehension was proofs. Four of the preservice teachers stated directly they did not like doing proofs because they were difficult. They understood the value of teaching proofs; however, their students were excused from doing them. This view seemed to stem from their own dislike of proofs. For example, Bailey had stated she did not like doing proofs and found them hard in both high school and college. She told her students they needed to try the proofs, but would not be tested on them. Five other prospective teachers did not provide correct justifications for the proofs in Teacher A's lesson. Both of the proofs were simple, three-step proofs high school students are expected to complete. Two types of problems were apparent: the preservice teachers provided inaccurate justifications or were unable to think through the proof logically. Three of the preservice teachers actually gave up trying to complete the proof. "I have no idea," stated Robin.

All of the preservice teachers tried to excuse their lack of knowledge. They stated it had been a long time since they had taken geometry and they just could not remember the concepts. Paradoxically, all of the preservice teachers had been required to complete two geometry courses prior to admittance to the teacher education program and all of them had taken at least one geometry course within the last two years.

The preservice teachers also used the excuse they had not yet taught geometry. They believed they fully understood the content only after teaching it. In fact, one of the reasons Robin decided to go into teaching was to better understand mathematics; she felt that by teaching it she could accomplish that goal. Bailey also stated she had only understood constructions after she taught them.

In essence, the preservice teachers' conceptions of geometry were varied. They held a view of geometry as ordered and built upon a foundation of the concepts of points, lines and planes. Their understanding of geometry was also incomplete leading to a lack of confidence in their knowledge portrayed throughout this study.

Conceptions of Geometry Teaching

A comparison of individual profiles produced several conceptions of geometry teaching held by the preservice teachers. The first three conceptions were specific to the teaching and learning of geometry and the last conception was generic to teaching, i.e., these views were not specific to the content area. However, they did contribute to the overall picture of the preservice teachers. These conceptions of geometry teaching were established through use of the journals and videotape tasks for all ten preservice teachers.

The first set of conceptions of teaching geometry established by the preservice teachers focused on their beliefs about the teacher's role in the classroom. Specific areas of interest to the preservice teachers included the teacher's behaviors and characteristics, knowledge of geometry, the ability to transfer that knowledge, and knowledge of student understandings. The preservice teachers continued to establish a view of what they thought a teacher should do and how a teacher should act. Two of the preservice teachers focused on the teacher's movement around the classroom. They felt Teachers B and C needed to move around the classroom instead of staying in the front of the room the entire period. Two other preservice teachers focused on the level of the teachers' voices. They thought Teacher A needed to speak louder. Three preservice teachers were concerned with students liking the teacher. They felt a good teacher who was well-liked by the students and had established a good rapport with the students would be more effective.

Teacher's knowledge of geometry was another characteristic focused on by the preservice teachers. Many of the preservice teachers noted Teacher C was unable to answer two of the problems on his own test. He also had not been able to give the students the correct answer for another problem. The preservice teachers noticed Teacher B was unable to give students a correct definition without reading it from the textbook. The preservice teachers began to hypothesize about the importance of adequate subject matter knowledge.

Related to teacher's subject matter knowledge, another attribute focused on by the preservice teachers was the ability to transfer that knowledge to students. Three of the preservice teachers specifically stated they felt good teachers were able to transfer their knowledge of geometry to students. This ability included being able to present the content in several different ways and being able to relate the content to students' lives. For example, many of the preservice teachers were critical of Teacher C because they felt he was unable to present a problem to the students in more than one way. They were also critical of Teacher B because he only accepted one answer for a problem, where there could have been more than one specific answer.

The preservice teachers also focused on the teacher's knowledge of students' knowledge. They believed it was essential for teachers to be able to assess students and be aware of the students' understanding. Six of the preservice teachers believed the experienced teachers on videotape had not assessed the students during their lessons. They felt the teachers "had no idea what the students understood." The preservice teachers were critical of the experienced teachers because of this aspect.

A second set of conceptions of geometry teaching that became apparent was the preservice teachers' established beliefs about how students learned geometry. They believed students learned geometry through hands-on explorations and discovery. They insisted teachers needed to involve the students in their own learning rather than "telling" them the content. The preservice teachers believed the best way to involve students in the lesson was to have them explore the content through hands-on activities. Overall, the preservice teachers did not think the teachers on the videotapes had involved the students in the lessons. They were critical of the experienced teachers for this aspect.

Interestingly, although the preservice teachers stated a belief that students learn geometry through hands-on explorations, other statements were contradictory. Over half of them believed it was important to memorize vocabulary and algorithms in order to learn geometry. They believed it was important for students to know certain concepts before they were able to solve problems or use higher thought processes.

The preservice teachers believed questioning played an important role in teaching and learning. They felt by asking questions the teacher was able to keep the students involved and on task. This view was central to involving the students in the lesson.

The last set of conceptions emerging from the data was the preservice teachers' established beliefs about how to teach geometry. All of them were adamant about using real-life applications in teaching geometry. They believed it was important to show students how geometry was used in the real world or how it was related to the real world. The preservice teachers also felt it was important in teaching to use examples the students were interested in or to relate the concepts to students' everyday lives. Most of this belief centered around the importance of establishing a purpose for the lesson. The preservice teachers believed teachers needed to set a context for the content they were teaching. This context included giving the students a reason for learning the content, establishing a purpose, and showing the students how the content related to their lives or to the world around them.

These preservice teachers also believed an inductive approach to teaching was important. They felt teachers needed to present their lessons in a more inductive fashion, rather than telling the students the content. The preservice teachers were critical of the experienced teachers on the videotapes because they felt these teachers had not allowed the students to explore and learn the content on their own. Rather, the teachers had told the students what they wanted them to know.

Finally, the preservice teachers were concerned with classroom management. All of them were concerned about establishing and maintaining order in the classroom. They were apprehensive because they did not know what to do if students misbehaved. This concern manifested itself throughout the data collection. This concern was mentioned when viewing the experienced teachers on videotape and also throughout their journals. The preservice teachers who were observed in the classroom had also stated an initial concern with classroom management.

Analysis of the data revealed four views about classroom management. The first view was that classroom management was based upon the specific school and the students in the class. The preservice teachers believed the specific school or district established rules for classroom behavior, therefore, students behaved. Four of the preservice teachers felt classroom management was controlled by the students in the class. They believed if the students were "more academically-oriented" or college bound, they would behave. These preservice teachers did not think the teacher had much to do with establishing and maintaining classroom order.

The second view about classroom management was the belief that planning and organization were essential to classroom management. Five preservice teachers believed by being well organized and prepared for the lesson, the teacher minimized classroom management problems. They began to realize the importance of planning in teaching during their part-time internship.

The third view about classroom management that surfaced was establishing clear expectations. The preservice teachers felt by establishing clear expectations for the students classroom management problems would be minimized. The preservice teachers began to realize, through their own experiences, that it was essential to tell students what was expected and to enforce those expectations. These expectations included giving clear directions for what students should do, setting clear rules in the classroom and for the activities, and letting students know the consequences for not following the rules. The preservice teachers were critical of two experienced teachers on videotape because they had not established clear expectations for the students. The preservice teachers noted the reason the teachers had classroom management problems was lack of clear expectations.

The most common view of classroom management held by these preservice teachers was establishing rapport with the students. The preservice teachers wanted to be accepted and liked by students. Their main concern during the part-time internship was being accepted by students. They felt it was important to be interested in students and to interact with them both during class and outside of class. Several of the preservice teachers held the view that interacting and joking with the students helped to build a good rapport. They focused much time and energy on establishing a good relationship with the students and as a result, their teaching became a second priority. The lessons or activities planned were sometimes designed for the sole purpose of "doing something the kids will like." The preservice teachers believed that, by establishing ~~a~~ rapport, the students would behave and act appropriately during class. As some of them found out, this belief was not always true.

Classroom Practices

The classroom practices of the four preservice teachers observed in geometry classrooms are summarized below. The classroom practices were established from the fieldnotes, videotapes of classroom observations, and work samples. Although the preservice teachers were different in their approaches to teaching, some common classroom practices emerged.

All four preservice teachers relied on the textbook to varying degrees. All of them followed the order of content presented in the textbook. Bailey and Robert planned their lessons directly from the textbook. They used the same examples and had students complete the problems presented in the book. Bailey required her students to complete *all* the problems from each section for homework. She did not select problems most useful for the students to complete. Robin and Jeremy also used the textbooks, but more as guides for their lessons. The examples they used in class were not those presented in the book. They also used supplementary materials and activities to help the students understand the content. When asked about supplementing the book, Robin stated, "They [students] have the book examples and the notes so that way they have more examples to look at when they get home and try the homework."

The preservice teachers observed were planned to varying degrees. This degree seemed to correspond with the degree they relied on the textbook. For example, Bailey and Robert relied so heavily on the textbooks they did not bother to review the examples or complete the problems prior to their lessons. As previously discussed, this reliance caused Robert problems when he was reviewing the homework problems with the students. He did not seem planned and often appeared disorganized. In contrast, Robin and Jeremy were well-prepared and organized. They both had prepared examples before class and knew in what order they would be presented. As part of the data collection, the researcher asked the preservice teachers before their lessons what they would be teaching, how they would teach it, and what the students would be expected to do. Jeremy and Robin were always explicit in their responses to these questions. They were able to state exactly what they would be doing during the lesson and what they expected students to be doing. Bailey and Robert usually stated the topic for the lesson, such as "Pythagorean Theorem" and stated, "the students will follow along."

Another classroom practice that emerged with the preservice teachers was their failure to teach the geometry in a context. None of them gave their students a purpose or reason for learning the content. The preservice teachers usually told their students what they would be doing during the lesson, but did not tell them *why* they would need to know the material. The lack of purpose coincided with a failure to show the students where or how they could use the content, outside of "knowing it for the test." Not only were few applications shown to the students, but only a few examples of the content applied to the world. Robin used an example of a nautilus shell, but skimmed it quickly. Jeremy was the only preservice teacher who tried to relate the content to what the students might know or have interest. He asked students to give examples of where they might need the concepts and tried to relate the concepts to the real world.

The preservice teachers observed in geometry classrooms used several types of strategies for involving students, including group work, note taking, questioning, and completing practice problems. These strategies were effective depending upon how they were implemented. For example, both Robin and Bailey used group work during their classes. Because of the expectations Robin had set, her students worked quite well within the groups. Bailey, however, had not been clear with her expectations for students. For this reason, although her students sat in groups, they did not collaborate among themselves.

Another classroom practice common among the preservice teachers was the type of presentations they used. All four used teacher-centered, deductive approaches for teaching the content. They spent a great deal of time presenting the content to their students while the students took notes or "followed along." Rarely did the students explore a concept and begin to formulate conjectures for themselves. On occasion, Robin or Jeremy had the students complete activities that could have been for exploration; however, the activities were presented in a manner that limited the students' creativity. Jeremy labeled four of his lessons as using an "inductive approach," but all were strictly teacher-centered and deductive.

Summary of Relationships

Conceptions of Geometry to Conceptions of Geometry Teaching

The relationship of conceptions of geometry to conceptions of teaching geometry was equivocal. The distinction between the preservice teachers' conceptions of geometry and how it should be taught was difficult to delineate. The overlap of the preservice teachers' conceptions was evident in three ways. First, the conceptions of geometry that surfaced during this study were linear. These conceptions were based on the preservice teachers' own experiences in high school geometry or the order in which they felt geometry should be taught. Second, both the conceptions of geometry and the conceptions of geometry teaching were textbook-bound. The preservice teachers believed geometry was organized according to the presentation by textbooks. Furthermore, the preservice teachers often discarded content they had not previously seen or was not presented in the textbooks. They viewed geometry as the content of the school curriculum.

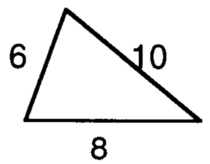
Third, as noted earlier, the preservice teachers' knowledge of geometry content was not always accurate, yet they held a strong view of the importance of subject matter knowledge. They believed an effective teacher needed to be able to present the content correctly to the students. More importantly, they believed an effective teacher was one who was able to transfer his/her knowledge of the content to the students. Furthermore, the preservice teachers discussed the need for the teacher to be able to represent the content in a variety of ways. Although the preservice teachers recognized the importance of subject matter knowledge and presenting the content correctly, they did not believe they fully understood geometry until they had taught it. This belief was used as an excuse for the preservice teachers' lack of knowledge of geometry. They believed they were unable to give examples as to how they might represent the content for the students because they had not yet taught.

Conceptions of Geometry to Classroom Practices

Although the relationship of the preservice teachers' conceptions of geometry to classroom practices was complex, their conceptions of geometry were manifested in the classrooms in several ways. First, a direct and distinct relationship between the preservice teachers' knowledge of geometry and their classroom practices existed. For example, Robert's lack of understanding of geometry emerged consistently in his classroom. He often presented the content incorrectly to the students or used improper vocabulary. His incorrect presentation confused the students and they often had to correct him during class. Robert was not aware of his lack of knowledge and when corrected by students he dismissed it as being insignificant.

The other three preservice teachers presented the content correctly, however, their lack of knowledge emerged in other, indirect ways. For example, Robin often did not notice or acknowledge various methods of solving a problem. Several times during her lessons she presented one way to solve a problem and did not allow suggestions from students for alternative methods. When she was presenting the converse of the Pythagorean Theorem, she told students that, given three sides of a triangle, they would need to test all three possibilities. One student suggested an alternative where the conversation proceeded as follows:

> Teacher: If someone gives you the length of three sides and you plug it into the Pythagorean Theorem and it works, then it is a right triangle. Let's try the next one.
>
> (Teacher has on board:)

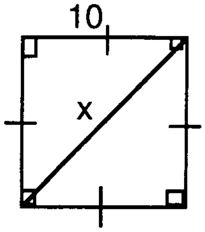


> Teacher: This is just a guess. We don't know. We are just trying it. Is 36 + 64 = 100? Yes, it is a right triangle. Even though we are correct, we still have to check all of the cases.
>
> Student: The hypotenuse is always the longest side, so you don't need to check all the sides.
>
> Teacher: You need to test all three cases.
>
> Student: Why? Have you ever come across one like that?

Teacher: No, but you might come across one like that. On your homework you will need to check all three cases to make sure it doesn't work for any of the cases.

This student had a valid point and Robin was not willing to accept it. She was limited in thinking about this problem. Perhaps her own method of solving a problem blinded her to other ways of solving that same problem.

On several other occasions, Robin was unable to visualize how the students were thinking about a problem. For example, in observation three, Robin had stressed to the students that in order to use the geometric mean, they must have a right triangle and an altitude. During observation six, Robin presented a problem as follows:



She asked students how they would solve the problem for x and they responded with "geometric mean." Robin asked, "Do you have a right triangle and an altitude? We don't have an altitude so we can't use the geometric mean." In fact, this problem does have an altitude and Robin was not able to envision why students had thought they could use the geometric mean. This exchange represented a common misconception students hold about the geometric mean and Robin did not follow the students' thought processes.

Jeremy also presented the content correctly, but because he had not reviewed the homework problems prior to reviewing them with students, he often had difficulty. On four separate occasions he became confused as he tried to explain the problems to students. Eventually, he concluded by telling students to come in after class to get more help on the problems. Jeremy realized not being able to complete the problems created difficulty and after one lesson stated, "I stumbled on a homework problem that was selected by students. I was stumped and needed more time to finish the problem. My frustration grew and I informed students to seek assistance after class."

Bailey presented the content correctly, used proper vocabulary, and was able to answer student questions. However, her own feelings about the content often influenced her classroom practices. For example, during two separate observations, the students asked her to review a proof. She told the students "not to worry about the proofs." She also did not complete the proofs as asked. She was sensitive to students' struggles with proofs and tried to ease their frustration because she also had struggled with proofs. Unfortunately, this attitude influenced the students to the extent that they decided they did not even need to try them.

A second conception of geometry that emerged in the classroom practices of the four preservice teachers was in their general view about the nature of geometry. Transcripts of card sort interviews and videotape interviews reflected a conception of geometry as being linear and occurring in a certain order. This organization was determined by the order in which the preservice teachers had learned geometry or the order in which they thought it should be taught. Furthermore, the preservice teachers held a view that geometry had a certain structure based on the specific textbook from which they would be teaching. The classroom observations of the four preservice teachers' lessons confirmed this "book-bound" conception. All of the preservice teachers' lessons were presented in the same order as organized in the text. They also made direct references to the textbook in teaching. For example, in six of the seven observations of Robin, she made some reference to the textbook. She told students, "This is the way you will see it in the book," or "The book tells you to . . . " The other preservice teachers used similar statements in their lessons.

Many of the problems the preservice teachers used in their presentations were also directly from the textbook. Both Robert and Bailey used the same examples found in the text; neither teacher chose to present the content in a different manner. Their explanations were exactly the same as the text had presented them. On several occasions, the students told both Robert and Bailey they did not understand a concept. They both took time to explain the concepts, however, their explanations were usually the same as the first. Jeremy's knowledge of geometry was also textbook-bound, however, he used a variety of explanations to help students understand. He utilized examples other than those used in the textbook and he was able to explain the content in a variety of ways. Sometimes, however, his explanations confused the students more than helped them. For example, when teaching the students to simplify radical expressions, Jeremy showed the students two different methods. Then, when showing them examples, Jeremy used the methods interchangeably. Using the two methods confused students to the extent that they began to simplify incorrectly.

Conceptions of Geometry Teaching to Classroom Practices

Often, the preservice teachers' conceptions of teaching geometry were in conflict with their implementation in the classroom. One of the conceptions of geometry teaching that emerged was the importance of using real world problems to teach the content. The preservice teachers believed it was essential for teachers to relate the geometry they were teaching to what interested the students or to something they could relate to in their lives. Robin wrote in her work sample:

> Drill and practice should become less of a focus and more problem solving and real world applications should be integrated [in geometry]. Because math connects to so many different aspects of life it is important that application is part of mathematics education. Students need to know how mathematics applies to their lives and that mathematics is a valuable and useful tool.

Interestingly, this belief did not emerge in the classroom practices of three of the four preservice teachers who were observed. Robin used few real world examples in her teaching. On the two occasions she did, the examples were shown to the students and passed over quickly. In the eight observations, Robin did not relate the mathematics to students' lives or to their interests.

Bailey also stressed the importance of presenting real world problems to students. She believed students should work with real world problems rather than "doing fifty practice problems." She also felt that these problems should be created so they "match the curriculum and are important to students' every day lives." This belief, however, did not surface during her own teaching. She did not create, present, or have students working with problems that related to their lives.

Bailey had also been critical of experienced teachers who had not established a purpose for their lessons. She stated it was essential to tell or show the students why they needed to learn the content. Otherwise, she felt students would only learn the content for the test and then forget it. Though she had stressed the importance of establishing a purpose, Bailey did not do so in any of the lessons observed. She never told or asked students about how the content could be used in their lives or where they might find it.

Robert's beliefs about connecting the content to students' interests also did not surface during the classroom observations. He never showed or used any real-world problems in his lessons. He did not give the students any purpose for learning the content and he did not use problems of interest to motivate them. He did not show relevance to the concepts within geometry or to the outside world, yet, as displayed in his card sort, connections had been central to his beliefs about geometry.

Jeremy also stressed the importance of using real world problems in teaching and had been critical that none of the experienced teachers on the videotapes had incorporated real world examples into their lessons or used the students' interests to motivate. He, on the other hand, used examples from the real world in his teaching and tried to have students think about where and how the content might be used in their lives.

Another conception of geometry teaching that emerged concerned the preservice teachers beliefs about how students learn geometry. The preservice teachers stated that students should not have to memorize formulas or algorithms. As mentioned in her journal, Robin wrote she did not believe students should have to memorize formulas or algorithms. She reiterated this belief in her work sample: "Teachers should shift away from rote memorization and having students work numerous examples." However, in her teaching, Robin regularly told students they needed to memorize a formula. She told students to memorize the basic trigonometry identities of sine, cosine, and tangent (observation seven), the Pythagorean Theorem (observations four and six), and the formulas for 30-60-90 and 45-45-90 right triangles (observations five and six). She stated:

> You will need to memorize these [45-45-90 right triangle formulas]. We figured this out, so you will never have to figure it out again. Just memorize it. Whenever you see a problem like this you will know to just fill in the formula.

The preservice teachers also believed students learned best by discovering the content for themselves, rather than having the teacher tell them. The preservice teachers believed it was essential to present lessons in an inductive manner and involve students in learning the content. For example, Robin believed students needed to think on their own and discover the content in an inductive manner. She stated:

> Mathematics education should guide students through a journey of discovery. Math education should teach students to think critically, question, and make decisions based on evidence. The traditional deductive method should be moved away from as much as possible. A more inductive approach should be taken whenever possible so that students may become more a part of their own learning.

Again, this view shared by the preservice teachers seemed to conflict with their actions in the classroom. All of Robin's and Jeremy's lessons were teacher-centered and presented in a deductive manner. Jeremy even mentioned his types of presentations in his reflections after several of the lessons. Following observation two, he stated:

> I did not really allow students to investigate the SAS Similarity Theorem on their own. This was my mistake for "doing" their investigation for them. I interfered with the students' investigations by demonstrating and leading students through most of the steps. Next time I will change it so students are doing the work instead of me.

Interestingly, in his work sample, Jeremy had labeled seven of his eight lessons as using an "inductive approach." This conflict may indicate that Jeremy had not fully assimilated an inductive approach.

All of Bailey's presentations were teacher-centered and focused on her lecturing. She had stated a belief that students learn best through exploration, however, she never had the students involved in exploratory activities. She also did not know how to present the content in a variety of ways so the students could understand. Several times Bailey presented a concept and students told her they did not understand. She took the time to explain; however, her second explanation was usually the same as the first. These explanations were also directly from the textbook she was using.

Although Robert did not directly state a belief about teaching geometry in an inductive manner, he did indicate he felt it was important to involve students in their own learning. This belief did not surface in his classroom.

He was textbook-oriented and spent a great deal of time telling the students what he wanted them to know rather than involving them.

Another conception that emerged was that of visualizing the content. The preservice teachers stressed the importance of visualizing geometry for their students. They felt it was important to show the students a picture and to use the picture to explain the content. This belief also surfaced in two of the teachers' classrooms. For example, Jeremy stressed to the students the importance of drawing a picture so they could see what was happening. During observation three, he showed the students a proof and stated:

> Use pictures. Draw the picture and mark off all the angles and sides you know. Then just look at the picture and decide what you know. Then go back and write your thoughts and steps.

Robert also believed that it was important to visualize concepts for the students. He stated this belief in his card sort interview and was critical of the experienced teachers for not showing the students a visual representation of the content they were teaching. Robert's own teaching, however, lacked this methodology. He often did not draw pictures for the students and he rarely explained a concept visually. To Robert's credit, he realized he had not visualized the concepts for students as he reflected on his own teaching. He stated:

> I need a lot of work on visualizing concepts for the students. I feel my oral explanations of ideas are good but not enough for this age level. These students need to see more diagrams with labels telling them what it means. I just can't tell the students what a concept means, I have to show them as well. I'll continue to focus on this aspect of my teaching until I feel that it is not a problem anymore. It is too important to not give it my full attention.

Another, general conception of teaching was also prevalent in the preservice teachers' classroom practices. They held a strong concern for classroom management that focused on establishing a good rapport and respecting students. All the preservice teachers observed had initially been concerned with classroom management. As they began to teach, Robin and Jeremy began to be comfortable with their classroom management. Jeremy felt establishing a good rapport with students was essential to effective teaching. He displayed a genuine concern and care for the students in his classes, in addition to establishing a comfortable atmosphere in which students could learn. Both Jeremy and Robin were successful at maintaining classroom management. Their management seemed rooted in the rules and guidelines the mentor teachers had established prior to their teaching.

Robert and Bailey, however, struggled with classroom management. Bailey wanted to be liked by her students, but her attitude put students at a distance. Robert's main concern had been establishing and maintaining a good rapport with students. He believed strongly in developing rapport and equated it with classroom management. He believed if he had established a good rapport with the students, he would be able to maintain classroom order. However, he soon found this belief was not always the case. He stated:

> When I am teaching I have a pretty personal relationship with almost all my students so they are a lot more apt to speak out. That's probably why there is more class management things [problems] because they feel a lot more relaxed and comfortable.

His problems with classroom management began from the first observation. Robert was so permissive he did not require the students to raise their hand before speaking. They began to shout out answers as he presented the lessons. Robert realized that he was beginning to struggle with maintaining classroom order. He reflected on this following one of his lessons:

> To improve my teaching I need to get tougher with classroom management. I've been letting the students get away with too much. I feel like I have a pretty good rapport with the students and they know that they can't walk all over me. Some of them try to every day, but they are unsuccessful. They need to know that I'm serious about them behaving in class.

Robert wanted the students to like him so he did not enforce the rules his mentor teacher had established. He did not encourage the students to use classroom time to complete their homework. Robert usually chatted with the students rather than helping individual students during seat work time. He also never seemed serious about dealing with inappropriate student behavior.

CHAPTER V
DISCUSSIONS AND IMPLICATIONS

Introduction

This study investigated the relationships among preservice teachers' conceptions of geometry, its teaching, and classroom practices. The complex relationships among these three domains were described in Chapter IV. Previous research focused on the interaction of preservice teachers' conceptions of their subject matter and classroom practices and assumed a linear, causal relationship. The results of this study contravene this simplistic assumption and provide a more complete description of the relationships. Seemingly, the domains are constantly influenced and being influenced by one another. Discussions concerning these relationships are addressed in the following sections. In addition, limitations of this study, implications, and recommendations for mathematics teacher education are discussed.

There have been models developed (van Hiele, 1986) that might clarify the domain of teachers' knowledge in geometry, however, the relationships among preservice teachers' conceptions of geometry, its teaching and classroom practices were grounded in cognitive science in the present study. Research in this field states that people are born with a need to organize their knowledge into psychological structures or schemata. These structures play a central role in an individual's perceptions, thoughts, and actions (Putnam, Lampert & Peterson, 1990). Piaget (1972) suggests people adapt to their increasingly complex environments by using existing schema whenever these schemata are effective and modify or add to their schemata when something new is needed(accommodation). These structures are continually combined and reorganized to become more sophisticated and useful. Piaget found that the most influential variable in development of these knowledge structures was time.

Cognitive science is a valued approach to understanding what preservice teachers do in the classroom. Three relationships were investigated in the present study: (a) between conceptions of geometry and its teaching, (b) between conceptions of geometry and classroom practices, and (c) between conceptions of geometry teaching and classroom practices. A schematic of these relationships is presented in Figure 12.

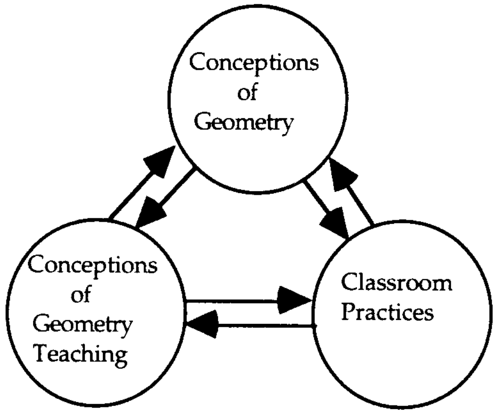


Figure 12. Interaction of Knowledge and Practice

Particular relationships require a discussion of the influence of each domain on the others and a discussion of the interaction of these domains. The first relationship investigates the preservice teachers' conceptions of geometry and their conceptions of geometry teaching. The discussion of this relationship includes the influence of the preservice teachers' conceptions of geometry on their conceptions of geometry teaching and vice versa. Also, the interaction of the conceptions of both geometry and its teaching are discussed. Similar discussions follow for the relationship of the preservice teachers' conceptions of geometry to their classroom practices and the relationship of their conceptions of geometry teaching to classroom practices.

Relationship between Conceptions of Geometry and Conceptions of Geometry Teaching

The relationship between the preservice teachers' conceptions of geometry and conceptions of teaching geometry was complex. It was clear the preservice teachers' conceptions of geometry influenced their conceptions of teaching geometry. However, the extent of this influence is not known. Shulman's rationale for his theoretical model on pedagogical content knowledge assumes that a teacher's knowledge of the subject matter influences the subsequent learning to teach that subject matter. He suggested teachers make decisions based on their conceptions of the specific discipline in which they teach. This idea assumes a directional relationship between subject matter knowledge and classroom practices, however, the results of the present study contravene this simplistic assumption. The data from the preservice teachers in this study verify Shulman's assumptions that subject matter knowledge influences its teaching, but the teaching of that subject matter also has an influence on preservice teachers' conceptions of their subject matter.

The data from this study suggest an alternative to Shulman's assumption. It was clear preservice teachers' conceptions of teaching geometry also influenced their conceptions of geometry, an idea Shulman did not speculate. The preservice teachers' conceptions of geometry in this study were clearly influenced by how they would use that knowledge, not solely conversely. The preservice teachers possessed some knowledge of geometry, but believed they would not fully understand geometry until they had been able to teach. They relied heavily on the textbooks from which they were teaching and organized their card sorts accordingly. For example, Kaylee stated she would not be able to organize the conceptions in geometry properly until she had taught. Therefore, although their subject matter knowledge influenced how they thought about teaching, the application of teaching also influenced the preservice teachers' conceptions of geometry. Thus, in contrast to Shulman's assumption, how one uses the subject matter, or the application, influences the organization of that subject matter. Other research projects have suggested a similar relationship (Gess-Newsome & Lederman, 1991, 1992; Hauslein & Good, 1989).

Cochran (1992) offered a similar explanation for how teachers organize their knowledge:

> Teachers differ from scientists, not necessarily in the quality of their subject matter knowledge, but in how that knowledge is organized and used. In other words, an experienced science teacher's knowledge of science is organized from a *teaching* perspective and is used as a basis for helping students to understand the specific concepts. A scientist's knowledge, on the other hand, is organized from a *research* perspective and is used as a basis for developing new knowledge in the field. (p. 4)

Similarly, mathematicians organize their knowledge of mathematics to develop new knowledge, whereas mathematics teachers organize their knowledge of mathematics to teach. Therefore, the preservice teachers knew they wanted to teach and had organized their knowledge of geometry from that perspective, thus making it difficult to differentiate their conceptions of each individual domain. Recognizing that preservice teachers organize their subject matter knowledge according to its use, research needs to focus on the exact influence that teaching has on preservice teachers' knowledge structures.

The idea that teaching a subject matter influences conceptions of that subject matter does not discredit Shulman's original assumption. In fact, several later projects in Shulman's Knowledge Growth in Teaching recognized preservice teachers' understanding of the subject matter might be influenced by the act of teaching (Richert, 1986). Therefore, this study complements Shulman's original assumption. Obviously, a teacher must possess a foundation of subject matter knowledge upon which to build further subject matter knowledge. Preservice teachers have developed schema of their knowledge of geometry since elementary school. The question that must be considered is: How much subject matter knowledge is needed as a foundation? Are two courses in geometry at the college level sufficient for a foundation? What should the content of the geometry courses be at that level? Furthermore, if the act of teaching geometry influences the preservice teachers' conceptions of geometry, when and how does the subject matter knowledge or structure change because of its teaching?

Research on Shulman's framework has viewed pedagogical content knowledge as that knowledge that is unique to the expert teacher. Previous research projects have established that pedagogical content knowledge develops over time as a result of teaching experience (Marks, 1987b; Steinberg, Haymore & Marks, 1985). Stein, Baxter and Leinhardt (1990) found that an experienced teacher's conceptions of functions were interwoven with his conceptions of teaching functions. The extent to which preservice teachers' conceptions of subject matter are interwoven with their conceptions of teaching that subject matter has been relatively ignored.

This study explored the interaction of preservice teachers' conceptions of geometry with their conceptions of geometry teaching. Interestingly, the preservice teachers could not discuss their conceptions of geometry without discussing the teaching of it. They held no distinctive views that were unique to either geometry or its teaching, rather intermingled the two domains.

Because the preservice teachers used geometry and its teaching interchangeably, it was difficult to delineate their conceptions of each independent domain. Their conceptions about geometry and their belief that geometry was linear in nature were so strong that these views became connected with their views of teaching geometry.

Several thoughts regarding these connected views are necessary. First, the preservice teachers had known they wanted to teach, but perhaps had not had time to develop their conceptions of teaching geometry independent of the content. Second, the question remains as to the extent that knowledge of a subject matter is actually a domain separate from knowledge of its application, in this sense, teaching that subject matter. Possibly, preservice teachers' conceptions of their subject matter and its teaching are melded together, thus indistinguishable. Shulman implied that for expert teachers, these are interwoven, suggesting two separate domains. Conceivably, Shulman's model is too simplistic and needs to be revised for preservice teachers. Lastly, the degree to which these two domains are interwoven may be useful in future evaluation of pedagogical content knowledge. Is the extent to which teachers' conceptions of their subject matter are interwoven with their conceptions of teaching that subject matter a possible indirect measure of pedagogical content knowledge? Or is it possible that the less distinguishable the domains of content and pedagogy are, the more pedagogical content knowledge teachers possess? Further research is recommended regarding these issues.

Relationship between Conceptions of Geometry and Classroom Practices

A significant conclusion of this study is the importance of improving subject matter preparation for prospective teachers. Ball (1990b) suggested preservice teachers need to possess an understanding of mathematics for teaching that includes both knowledge *of* mathematics and knowledge *about* mathematics. Knowledge *of* mathematics is the procedural and conceptual knowledge of how to do mathematics. Knowledge *about* mathematics is the understanding of how mathematics is developed, how it changes, and how truth is established.

Subst

Syntax

As indicated from the examples in this study, prospective secondary teachers' knowledge of geometry tends to be naive and incomplete. Previous research has also shown that preservice teachers do not have well-developed conceptions of mathematics (Ball, 1990a; Graeber & Tirosh, 1988, 1989; Khoury & Zazkis, 1994). Thompson (1992), Schoenfeld (1985), Lampert (1986), and Ball (1990b) have all indicated preservice teachers need to possess a stronger view *of* and *about* mathematics. This view should include historical and philosophical knowledge of mathematics and knowledge of how mathematics can be taught, independent of whether this knowledge is used in the classrooms (Thompson, 1992). Preservice teachers should know more about their subject than what they are required to teach (Mathews, 1994). A teacher must have strong subject matter knowledge to teach in a manner consistent with the conceptions of good teaching proposed by the NCTM.

This study identified both direct and indirect aspects of preservice teachers' knowledge of geometry that affected how they presented that content in the classroom. Directly, the preservice teachers made errors with the content and were sometimes unable to provide answers to students' questions. For example, Robert lacked appropriate knowledge of geometry as reflected in his classroom practices. This problem has obvious consequences: the teacher presents the content incorrectly, thus, students learn the content incorrectly.

The preservice teachers' knowledge of geometry also had indirect negative consequences in their classroom practices. They often could not answer questions about the content from assigned homework problems. They could only provide one procedure to solve a problem and often regarded students' suggestions as insignificant. Several times during her lessons, Robin presented one way to solve a problem and did not notice or acknowledge students' alternative methods of solving the problem.

Interestingly, the preservice teachers in this study stated a concern for their subject matter knowledge. They were not confident in their own knowledge of geometry and unsure of how they should present the content to the students. Brown and Borko (1992) found that preservice teachers without adequate subject matter knowledge were likely to lack confidence in their ability to teach well. Likewise, Ball (1988, 1990a) also found that preservice teachers were concerned with their own knowledge of mathematics, especially prospective elementary teachers.

The influence of the preservice teachers' classroom practices on their conceptions of geometry was less evident. Their internship experience only allowed them to teach a total of 12 weeks. They had little time to develop their classroom practices and concern for classroom management dominated their time. Therefore, the effect of the preservice teachers' classroom practices on their views about geometry had not developed. However, the interaction of their conceptions of geometry and/or the textbook and their classroom practices was convincing.

To understand how these preservice teachers' conceptions of geometry interacted with their teaching, the question arises not only what they knew *of* geometry, but also what they knew *about* geometry and the implications of this orientation for teaching (Ball, 1990b; Brophy, 1991). The preservice teachers demonstrated a limited, ordered view of geometry. Their conceptions of geometry were linear and textbook-bound. Most of them stated their conceptions of geometry were based on the order they had learned geometry in high school. Their conceptions had been established when they were students during the past 16 years in elementary through college mathematics courses. This influence has been recognized in several studies (Ball, 1988; Bush, 1983; Owens, 1987; Steinberg, Haymore & Marks, 1985). Ball (1988), Bush (1983), and Owens (1987) identified preservice teachers' conceptions as being formulated during the teachers' schooling years and shaped by their own experiences as students of mathematics.

The preservice teachers' conceptions about geometry were also based on the order in which they felt the geometry concepts should be taught. They believed the order of geometry was based on how the curriculum developers had organized the concepts in the textbooks. In addition, many of the preservice teachers believed they would not fully understand geometry until they had taught it.

Clearly, the textbook had influenced the preservice teachers' conceptions of geometry. Furthermore, what they did in the classroom was guided by the textbooks they used. The classroom observations of the four preservice teachers' lessons confirmed this "book-bound" conception. The four preservice teachers observed followed the order of the textbook and made direct references to the textbook when teaching. Both Bailey and Robert depended on the textbook for their content presentations. Steinberg, Haymore and Marks (1985) recognized similar reliance on the textbooks in their study.

Several reasons may account for the preservice teachers' reliance on the textbooks. Clearly, the preservice teachers were not confident in their own subject matter knowledge. Where or why this insecurity developed, or if they were ever secure, was not known. Whether the preservice teachers possessed this insecurity prior to the teacher preparation program remains a question of interest. Perhaps they were unsure of their geometrical knowledge prior to the teacher preparation program. It is also possible that the teacher preparation program *created* this insecurity. Because the preservice teachers were unsure of their knowledge of geometry and had no other source with which to refer, they may have depended on the textbook. They also may have relied on the textbooks because they felt confined to follow their mentor teachers' leads.

Many concerns arise because of the preservice teachers' reliance on the textbooks. First, the practice of following the textbooks can impact pupils' views of geometry. As a result, they believe, as did the preservice teachers, that geometry was structured according to the order it was presented in the textbooks. Second, both textbooks the preservice teachers were using were published before the NCTM *Curriculum and Evaluation Standards for School Mathematics* (1989) were available. As a result, the types of activities in the textbooks were not always consistent with the national reforms in mathematics education. Thus, the conceptions based on the textbooks that both teachers and students possess do not represent the integrated schema that have been suggested by the reforms. Third, the present study did not assess the consistency of the preservice teachers' conceptions of geometry with the textbooks' conceptions. Thus, the relationship between the preservice teachers' conceptions of geometry and their classroom practices are unclear because of their reliance on the textbook. The question of whether the preservice teachers' knowledge structures influenced what they did in the classroom cannot be determined because of the influence of the textbook.

The preservice teachers held a limited, static view of geometry. While their knowledge of geometry was incomplete, they also had a limited orientation of geometry. They viewed geometry as a set of established rules and procedures to be used when solving problems. Ryan viewed geometry as a "toolbox" for problem solving. Ball (1988) also noticed that prospective teachers had a limited view of mathematics. "Prospective teachers did not view mathematics as a field of human endeavor in which people argue about and discuss interpretations, problems, methods, and solutions." (Ball, 1988, p. 125) These views about what geometry is and what it means to "do" geometry held by the preservice teachers influenced their conceptions of how geometry should be taught and how they actually taught geometry. They believed geometry was a set of rules to be memorized; thus they stressed to students the need to memorize formulas or theorems. Interestingly, the preservice teachers used the upcoming test as a reason for students to memorize the formulas rather than having them understand the conceptual ideas behind the formulas.

It is consistently assumed that teachers' beliefs about mathematics are effectively conveyed by the nature in which instruction in mathematics is conducted (Steinberg, Haymore & Marks, 1985; Thompson, 1982, 1984). Researchers have accepted the view that, depending on the teachers' knowledge *about* mathematics, some students may learn that geometry is an organized body of knowledge to be understood whereas other students might learn that geometry is a collection of procedures to be memorized. Thus, the teachers' views *about* geometry affects the students' views about geometry.

Teachers' conceptions play some role in the teaching of mathematics. Since teachers are the primary mediators between the subject matter of mathematics and the students (Thompson, 1984), the conceptions they hold play some role in what they do in the classroom, and ultimately could directly impact the students. It is clear that teachers' conceptions are communicated to students through their classroom practices (Adams, 1993). The question to be considered is how those conceptions are communicated and the impact teachers' conceptions have on students and their thinking. With this communication, if a teachers' conceptions are inaccurate, the content will be presented incorrectly and students will likely develop misconceptions. However, the influence of the teachers' conceptions on their students has not been fully investigated.

The importance of subject matter knowledge lies with how that subject matter is used. Shulman and Grossman (1988) noted, "prior subject matter knowledge and background in a content area affect ways in which teachers select and structure content for teaching, choose activities and assignments for students, and use textbooks and other curriculum materials" (p. 12). The incomplete knowledge of geometry held by these preservice teachers led to inaccurate content presentation, instruction without connections, less powerful representations, limited ways of solving problems, and textbook-bound teaching. Besides being able to present the content accurately, preservice teachers need to be able to use their knowledge of the subject matter in order to supplement textbooks which may be inconsistent with the suggested reforms. They also need to be able to provide strong representations for the content and evaluate the effectiveness of such representations.

Relationship between Conceptions of Geometry Teaching and Classroom Practices

Several conceptions of geometry teaching emerged in this study including the importance of using "real world" problems to teach the content. The preservice teachers believed it was essential for teachers to relate geometry to students' interests or that which they could relate to in their lives. The preservice teachers also stated their beliefs about how students learn geometry. They stated that students should not have to memorize formulas or algorithms, rather students learned best by discovering the content for themselves through hands-on explorations. The preservice teachers also believed it was essential to present lessons in an inductive manner and involve students in learning the content.

As indicated in the relationships of conceptions of geometry and its teaching to classroom practices, what the preservice teachers said they believed and what they did in the classroom were not always consistent. Their beliefs about teaching geometry rarely emerged in their classroom practices. Previous research studies have also found similar discrepancies (Cooney, 1985; Fennema & Franke, 1992; McGalliard, 1983; Shaw, 1989; Thompson, 1982). There were many factors that may have contributed to these discrepancies. Much of what prospective teachers have learned about teaching mathematics comes from their experiences as undergraduates as well as their experiences in elementary and secondary school (Ball, 1990a; Steinberg, Haymore & Marks, 1985). Thus, the preservice teachers had developed, over the past 16 years, structures for both their knowledge of geometry and its teaching. Many of the preservice teachers were unclear about their beliefs of geometry teaching, often times stating conceptions that were contradictory. These contradictory views could be a result of being exposed to ideas about mathematics teaching in the teacher preparation program which were different from what they had experienced in their own schooling.

Although the preservice teachers had some preconceptions about geometry teaching, it was also clear their conceptions of geometry teaching were just beginning to be assimilated into their existing schema (Burns & Lash, 1988). They had not had much *time* to develop their conceptions of geometry teaching or their classroom practices. They had only taught one unit prior to data collection for this study and did not have the opportunity to integrate their conceptions about teaching geometry into their existing schemata. Therefore, when faced with teaching, the preservice teachers attempted to compensate for their limited knowledge of geometry by relying on their own schooling, textbooks, and practical experiences (Feiman-Nemser & Buchmann, 1986, 1987; Feiman-Nemser, 1990; Steinberg, Haymore & Marks, 1985). Unfortunately, the time in their internships was not long enough for them to modify their deep-rooted conceptions.

It is important to consider other factors that may have influenced what the preservice teachers believed and what they did in the classroom. Decisions teachers make are a contributing factor to these discrepancies (Shavelson, 1976). What is not known is the extent to which preservice teachers make conscious decisions as they teach (Thompson, 1984). Further, the extent to which these decisions might contradict their beliefs is also not known. Preservice teachers may believe they should involve students through hands-on explorations, however, because of the variance in student achievement and ability levels they choose to present the content in another manner (Thompson, 1984).

Although this study focused on the preservice teachers' conceptions of geometry and its teaching, these preservice teachers had an overwhelming concern with classroom management. This concern for classroom management has been documented in previous research (Doyle, 1986). For both Bailey and Robert these concerns took precedence over other views specific to the teaching of mathematics. They were concerned with maintaining order in their classrooms and were uncertain of how to react if students were misbehaving. Hollingsworth (1986) and others (e. g., Lederman & Gess-Newsome, 1991) noted that concern for management occupies so much of preservice teachers' concerns that it tends to drive how the subject matter is presented and even what pupils learn. Hollingsworth (1986) confirmed that classroom management seemed to trouble beginning teachers more than any other concern.

It is possible that these preservice teachers were so concerned with classroom management that they were unable to effectively convey the geometry they were teaching. Several research projects suggested that no matter how much subject matter knowledge preservice teachers possess, if they have not developed a routine for dealing with classroom management, they will fail to reach the point of understanding students' learning (Brown & Borko, 1992; Borko & Livingston, 1989; Hollingsworth, 1989; Feiman-Nemser & Buchmann, 1986; 1987). "General managerial routines must be in place before subject specific content and pedagogy become a focus of attention, and interrelated managerial and academic routines were needed before teachers could actively focus on students learning from academic tasks in classrooms" (Hollingsworth, 1989, p. 168).

"We believe that the transformation of subject matter knowledge is at the heart of teaching in secondary schools" (Wilson, Shulman & Richert, 1987, p. 117). Assuming preservice teachers have learned the geometry they are to teach, these preservice teachers must find ways of transforming that knowledge so that it is understood by students. In making the transition from a student of mathematics to a teacher of mathematics, preservice teachers must examine their knowledge *of* geometry and *about* geometry. In their struggle to transform their knowledge of mathematics, preservice teachers must examine their own conceptions about mathematics. Subsequently, they must develop ways of representing the content to their students. This transformation of subject matter is difficult for preservice teachers. They have not sufficiently developed their knowledge of teaching a subject to enable the construction of explanations or examples (Brown & Borko, 1992; Borko & Livingston, 1989).

Shulman's work viewed pedagogical content knowledge as partially a function of experience. This view is consistent with findings that preservice teachers do not possess an appropriate level of pedagogical content knowledge (Marks, 1990; Shulman, 1986a; Steinberg, Haymore & Marks, 1985). The preservice teachers were able to recognize the ability of the experienced teachers on videotape to transform their knowledge. However, their own ability to transform their content knowledge was weak and their own classroom practices were limited to following the textbook. The preservice teachers were able to give one explanation, but when asked to explain the concept again, they were unable to provide alternative explanations. Furthermore, the original explanations used by the preservice teachers had been the same explanations provided by the textbooks. The consequences of this practice are clear. Teachers need to be able to provide alternative explanations for concepts in order to help the variety of students in mathematics classrooms.

The preservice teachers in this study also did not attempt to connect the geometry they were teaching to previous content or to students' prior knowledge. They all discussed the importance of connecting geometry to the real world or to students' interests, yet only one of these preservice teachers did. Stein, Baxter and Leinhardt (1990) found that preservice teachers did not connect the content to students' prior knowledge or interests. McDiarmid, Ball and Anderson (1989) concluded: "The belief that academic content [mathematics] should be connected to the real world is not sufficient to enable beginning teachers to relate key dimensions of a topic to real situations that will make sense to their pupils"(p. 195). Preservice teachers need more than a belief, they need to be shown how to execute those beliefs in their classroom and have experience in implementing those beliefs effectively.

Other studies have recognized that inexperienced teachers have incomplete or superficial levels of pedagogical content knowledge (Brown & Borko, 1992; Marks, 1990). Novice teachers tend to rely on subject matter from curriculum guides and textbooks and may not have a coherent framework or perspective from which to present the information. Several researchers have found that preservice teachers' schemata for pedagogical content knowledge were limited (Borko & Livingston, 1989; Leinhardt, 1986; Stein, Baxter & Leinhardt, 1990). Without alternative representations for concepts, preservice teachers will be unable to teach effectively. They need to develop an appropriate repertoire of representations for mathematics in order to fulfill their beliefs when teaching mathematics. They also need to develop ways of evaluating the effectiveness of these representations (Thompson, 1992).

Although it may be important for preservice teachers to develop a repertoire of representations for presenting concepts, if they do not experience a "need" for alternative representations they will never be able to implement these resources. Posner, Strike, Hewson and Gertzog (1982) stated that people need to experience a reason to change their current schema before they are willing to assimilate new ideas. Furthermore, they need viable alternatives to their schema. The preservice teachers were so concerned with classroom management and their own survival that they were unable to view their own teaching objectively. Because of their overwhelming concern for classroom management and survival, the extent to which the preservice teachers experienced a *need* to change their methods of teaching or to provide alternative explanations is not known. It is also unclear whether they recognized their reliance on the textbook.

Preservice teachers have not had the experience Shulman credits to the development of pedagogical content knowledge. They often struggle with transforming and representing the content in ways that make sense to students. Grossman (1988) found that preservice teachers are concerned with presenting the content and that this concern is present even with new teachers who possess substantial subject matter knowledge prior to their teacher preparation programs.

Limitations of the Study

The present study had several limitations. First, the conceptions of geometry were acquired through a card sort task and a follow-up interview. Card sort methodologies provide teachers with a set of items that are to be included in their content maps or knowledge structures. Such restrictions in topics may bias the assessment of the organization and may influence the outcome of an investigation (Gess-Newsome & Lederman, 1993). In addition, the presentation of terms may create knowledge by acting as a stimulus for the formation of relationships among topics that have not been previously considered and may not allow the preservice teachers to demonstrate a complete, integrated structure of geometry. Thus, the card sort was limited in its ability to assess the preservice teachers' organization of their knowledge structures of geometry. Additionally, the preservice teachers in this study were asked if they had ever thought of geometry in this manner prior to this study. Most of them stated they had not, suggesting that their inexperience in using the card sort task may have contributed to the difficulty they had in organizing the cards. This idea also suggests that the teachers might learn as they are completing the tasks, thus acting as a treatment.

Second, not all of the preservice teachers were teaching in geometry classrooms. Unfortunately, only four preservice teachers were observed for the second phase of this study. Consequently, the relationships established between the conceptions of geometry and its teaching in classroom practices were based on these four teachers. Observations of additional preservice teachers would have validated these relationships.

Third, the extent of the influence of the preservice teachers' background in mathematics was unknown. Their previous experiences in geometry over the past 16 years were different. The preservice teachers, prior to participating in this study, had completed one quarter of graduate coursework, none of which was geometry. The effect of these courses on the conceptions of geometry teaching is not known. The teacher preparation program had spent a considerable amount of time focusing on state and national reforms in mathematics education. It is probable that the preservice teachers responded in the interviews based on what they had learned within the teacher preparation program.

Fourth, the preservice teachers observed in geometry classrooms were under the supervision of their mentor teachers. This study did not collect data on the mentor teachers. These mentor teachers had their own styles, rules, guidelines, and lesson structures. The preservice teachers were expected to conform to the mentor teachers' established routines and to maintain the rules previously set forth. The effect of the mentors could be central to the preservice teachers' classroom practices. This effect could account for the variance in teaching routines among the preservice teachers. Therefore, the mentor teachers' influence in unknown.

Finally, the researcher introduced several limitations. The researcher was the main instrument in collecting and analyzing the data. This study was designed to prevent as many threats to validity as possible, however, the researcher's background, experience, and biases still limited the conclusions drawn.

Implications for Mathematics Teacher Education and Future Research

In order to gain a better understanding of the relationship between preservice teachers' conceptions of geometry and conceptions of geometry teaching much more remains to be learned about such conceptions and the role these conceptions play in instructional practices. The findings of this research are of central importance to both the improvement of teacher preparation programs and future research.

As indicated from the examples in this study, prospective secondary teachers' conceptions *of* geometry tend to be incomplete. Although the preservice teachers had completed courses at the college level in both Euclidean and non-Euclidean geometry, they still possessed incomplete, and sometimes inaccurate, conceptions of geometry. Thus, researchers and teacher educators cannot assume preservice teachers have a comprehensive, well-articulated knowledge of geometry. Previous research projects have also noted that preservice teachers have incomplete knowledge and conceptions about other areas of mathematics (Ball, 1990a, 1990b; Khoury & Zazkis, 1994). In fact, researchers and teacher educators cannot assume preservice teachers have the conceptual understanding of mathematics they are required to teach.

Furthermore, the preservice teachers indicated a limited view *about* geometry. They believed geometry was a set of rules and procedures to be used when solving problems. If the preservice teachers possessed a comprehensive perspective on what geometry is, rather than a static discipline, they might be able to translate that knowledge in the classroom to improve students' understanding (Ball, 1990a). Because of the relationships discussed, any attempt to improve the quality of mathematics teaching must begin with an understanding of the conceptions held by teachers and how these conceptions are related to their classroom practices. A central concern, then, for researchers and mathematics educators should be how such conceptions are formed and what effects their conceptions have on teachers' classroom practices. Failure to recognize the role that preservice teachers' conceptions and prior knowledge play in their classroom practices is likely to result in misguided efforts to improve the quality of mathematics teacher education and instruction in the schools.

Essential to the development of the preservice teachers' understanding of geometry are their perspectives *about* geometry. Helping preservice teachers gain a perspective about geometry consistent with reforms will require designing subject matter courses in light of such reforms. The preservice teachers take two courses in geometry at the undergraduate level. These courses are related to, but not identical to the courses they may teach at the high school level. It may be too difficult for preservice teachers to envision how to teach high school geometry because of the theoretical nature of the college-level courses.

Requiring improved courses in geometry, however, does not mean changing the number of courses prospective teachers must take. Instead, efforts must be made to improve undergraduate mathematics courses which may need to be constructed differently to develop better, more conceptual understandings (Ball, 1990a; Confrey, 1990). These mathematics courses should be designed in light of the NCTM *Professional Standards for Teaching Mathematics* (1991) and *Curriculum and Evaluation Standards for School Mathematics* (1989). These improvements should facilitate the preparation of preservice teachers and have a positive effect on classroom practice.

The idea that preservice teachers' subject matter conceptions influence and are influenced by teaching that subject matter is of central importance to the improvement of mathematics teacher education programs. The implication of Shulman's (1986a) assumption regarding that influence is more complex than completing required courses in the subject matter. Yet, the current emphasis for increased subject matter knowledge for teachers is based on Shulman's assumption. Because of this assumption, teacher preparation programs now require more extensive subject matter courses for teachers. Many programs, such as the one in which these preservice teachers participated, are beginning to require prospective teachers to complete a bachelor's degree and then enroll in a teacher preparation program. The teaching of mathematics courses needs to be changed to reflect strategies recommended for effective teaching.

This study also showed that the act of teaching influenced the preservice teachers' conceptions of their subject matter. The preservice teachers' conceptions of geometry were influenced by the act of teaching geometry. The direct implication of this influence is that preservice teachers need more than just subject matter courses. Teacher preparation programs need to find ways of integrating mathematics and mathematics pedagogy. In mathematics, preservice teachers need to learn geometry, but they also need to learn a variety of ways of representing the geometry so students better understand the content. The integration of these two domains could take place at the undergraduate level as well as within teacher preparation programs. Several questions are suggested to accomplish this integration: How feasible is it to provide undergraduate courses that integrate the mathematics with its teaching? What about those undergraduate students who do not want to teach? Would individual undergraduate courses need to be provided for students interested in engineering, music, or architecture?

The idea that preservice teachers' subject matter knowledge is influenced by its use has other important implications. The preservice teachers in this study believed geometry was ordered according to experts and would not be fully understood until taught. Obviously, teachers must know something about the subject they are teaching. The question for researchers becomes: How much do preservice teachers need to know about their subject matter before beginning teacher preparation programs? Brophy (1991) suggested:

> Perhaps there is an optimal breadth and depth of subject-matter knowledge for teachers working at a particular grade level, such that additional subject-matter knowledge beyond this optimal level would be counterproductive because (1) it would never be needed for teaching the content this teacher teaches, and (2) by adding nonfunctional complexity to relevant knowledge networks, it would make it more difficult for the teacher to select appropriate content to teach to students and to transform it into pedagogical content knowledge. (p. 356)

The question remains as to the breadth and depth of mathematical knowledge preservice teachers must possess before it becomes counterproductive. Another question that arises because of the influence of teaching on subject matter conceptions is: When and how do the subject matter conceptions change because of its teaching? Research on these questions concerning the amount of subject matter knowledge for prospective teachers is suggested.

Results of this study also indicate that teacher education programs need to recognize the incoming conceptions of prospective teachers in order to understand their teaching and develop a program to better prepare them for their careers. It is critical that data be collected on preservice teachers' conceptions of geometry and other mathematical domains and their conceptions of teaching that domain prior to beginning a teacher preparation program. Furthermore, researchers and teacher educators need to question how they can help preservice teachers develop and overcome their long-held, deeply rooted conceptions of mathematics and its teaching and come to a perspective consistent with reforms. Modifying these conceptions in the short time frame of a methods course remains a major problem in mathematics teacher education.

Pedagogical content knowledge is viewed as that knowledge unique to teachers. It is clear that the level of pedagogical content knowledge possessed by a teacher is proportionally related to the expertise of that teacher. These preservice teachers recognized pedagogical content knowledge in other teachers, yet were unable to accommodate the same. Additionally, they were concerned with their ability to transform the content and communicate it effectively to students. Teacher education programs must ask how they can help preservice teachers transform their subject matter into relevant and understandable representations for students. Unfortunately, research has focused on describing teachers' pedagogical content knowledge and how it influences the teaching process (Marks, 1990). Understanding how it develops and how to enhance this knowledge in preservice programs has been relatively ignored. Teacher preparation programs must teach preservice teachers how to benefit from their experiences in the program. Preservice teachers need to learn how to reflect on their teaching experiences and develop their skills for life-long learning. Such reflection in learning to teach is of extreme importance to the improvement of preservice teacher preparation programs.

In spite of the studies (Ball, 1990a, 1990b; Khoury & Zazkis, 1994; Stein, Baxter & Leinhardt, 1990) that have identified preservice teachers' conceptions as limited and inconsistent with reforms, teacher preparation programs rarely focus on refining the knowledge or views of preservice teachers. Unfortunately, the time spent in a teacher preparation program is relatively limited. Yet, the acquisition of pedagogical content knowledge should be the primary focus of teacher education programs (Brown & Borko, 1992). Teacher preparation programs tend to focus on developing general pedagogical knowledge rather than subject matter knowledge or pedagogical content knowledge. These programs must find more effective ways of helping preservice teachers develop their knowledge of teaching the subject matter. If experience in teaching is the key to the development of pedagogical content knowledge, then preservice teachers need to spend more time teaching, without excluding campus-based courses to guide them in reflective practice. The development of pedagogical content knowledge may require extending the time spent in a teacher preparation program.

Research on learning to teach must examine how teachers learn to translate the knowledge stored in their teaching schemata into operational plans and how they learn to carry out those plans in the classroom. Research is needed on (1) how preservice teachers learn from their experiences as they interact with students and the subject matter and (2) how they might assimilate new information about mathematics, its teaching and classroom practices.

Although several research projects have observed discrepancies between preservice teachers' stated beliefs and classroom practices, many questions still arise concerning this issue. The extent to which preservice teachers are aware of inconsistencies between their conceptions and classroom practices must be considered. If they are not aware of such discrepancies, it is essential to ask: What can be done to help preservice teachers realize these discrepancies? Furthermore, if preservice teachers are aware of inconsistencies between their conceptions and their classroom practices, researchers must ask: Why do these inconsistencies exist, how do preservice teachers explain such discrepancies, how can teacher educators correct these discrepancies, and, are there ways that preservice teachers can reflect and ameliorate these discrepancies?

The present study recognized both a direct and indirect relationship between teachers' conceptions of geometry and their classroom practices. It has been suggested that a teacher's conception of mathematics is conveyed to students in a class, either explicitly or implicitly through the classroom practices (Mathews, 1994). The exact influence of teachers' conceptions of both mathematics and its teaching on students remains unknown. What remains to be studied is the effect teachers' conceptions and classroom practices have on students' conceptions.

As stated previously, it was difficult to determine the distinction between the preservice teachers' conceptions of geometry and the conceptions presented in the textbooks. Research on how teachers use the textbook is recommended. Do teachers use the textbook in manners consistent with reforms? A comparison between the teachers' conceptions and those conceptions presented in the textbooks would also clarify the relationships established.

Finally, the role of the mathematics teacher educator also must be considered. Teacher educators must find ways of helping preservice teachers examine and reflect on their conceptions of mathematics and its teaching. The preservice teachers' overwhelming apprehension about maintaining order in their classrooms influenced how the subject matter was presented. Fuller and Brown (1975) noticed that prospective and beginning teachers' primary concerns are focused on survival, suggesting that little attention is devoted to formation of understanding the content. Teacher education programs must find ways to help preservice teachers overcome such concerns and move to a concern for students and their learning.

The preservice teachers must also be helped to recognize their dependence on the textbook and be encouraged to develop their own repertoire of representations for concepts in mathematics (Cobb, Wood & Yackel, 1988; Cobb, Yackel & Wood, 1990). Preservice teachers may not be aware of how to obtain information about teaching mathematics other than from the textbook or curriculum guides. Furthermore, teacher educators need to help preservice teachers develop classroom practices consistent with the recommended reforms for mathematics teaching. Such reflection and scholarship are necessary in any teacher preparation program.

If teacher educators are expected to help preservice teachers develop and reflect on their conceptions, they must also possess those conceptions consistent with reforms. Mathematics teacher educators' must consider their own beliefs about mathematics and its teaching and learning. It is essential that teacher educators consider the extent to which their own conceptions are consistent with what they want preservice teachers to believe. Virtually no research has been conducted on teacher educators' conceptions about mathematics and its teaching. Future research is needed that will address both the conceptions of mathematics teacher educators and the affect their conceptions have on preservice teachers' conceptions.

"Teachers are the key figures in changing the ways in which mathematics is taught and learned in schools" (NCTM, 1991, p.2). Changes in the instruction of mathematics at all levels will be enhanced with more emphasis on the teachers' conceptions of mathematics, its teaching and the effect of these conceptions on classroom practices. Although preservice teachers are just beginning to learn about teaching mathematics and have some preconceived ideas about teaching, teacher educators can have a positive influence on their development. A concerted effort on the part of teacher preparation programs is needed so that preservice teachers learn the type of teaching envisioned by the reform movement.

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APPENDICES

Appendix A

Informed Consent Form
(Preservice Teachers)

Dear ____________________;

Thank you for considering participation in a research project designed to investigate the conceptions of geometry of preservice mathematics teachers prior to teaching.

Participation will be for the Fall and Winter quarter. During Fall term, a card sort task with an interview will be conducted. This interview will last approximately one hour and be videotaped. The researcher will also have access to journal entries made by the participant during Fall term. The participant will be asked to view and comment on three separate videotaped lessons. During Winter quarter, the researcher will be observing and videotaping five lessons taught by the preservice teacher. The participant will also be completing two work samples which will be used as part of the data for this study.

The researcher and major professor will be the only person with access to all data collected (card sort task, interviews, observations, work samples and field notes). Confidentiality will be maintained through use of coding. Pseudonyms will be used for the university and all subjects when reporting the results of the research. Video and audio tapes will be kept in a secure place, stored in the major professor's office until analysis is completed, at which time they will be erased.

Participation is voluntary, refusal to participate will involve no penalty or loss of benefits to which the subject is otherwise entitled. The subject may discontinue participation at any time without penalty or loss of benefits to which the subject is otherwise entitled.

Questions about the research, personal rights, or research-related injuries should be directed to: Dr. Margaret Niess at 737-1818.

Thank you for your time and participation in this research project.

Janet Scholz
737-8731

I agree to participate in this research project and understand the general intent of the study, the types of data to be collected, and the time commitments involved in the study.

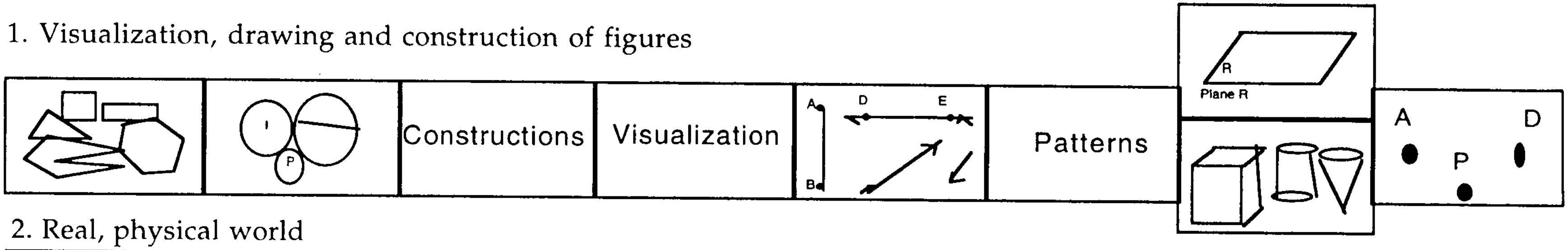
____________________________ ______________

Signature Date

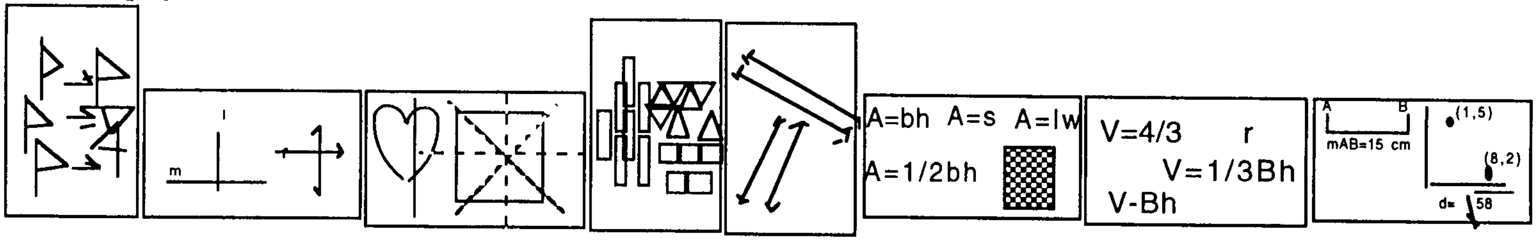
Appendix B

Card Sort Dimensions

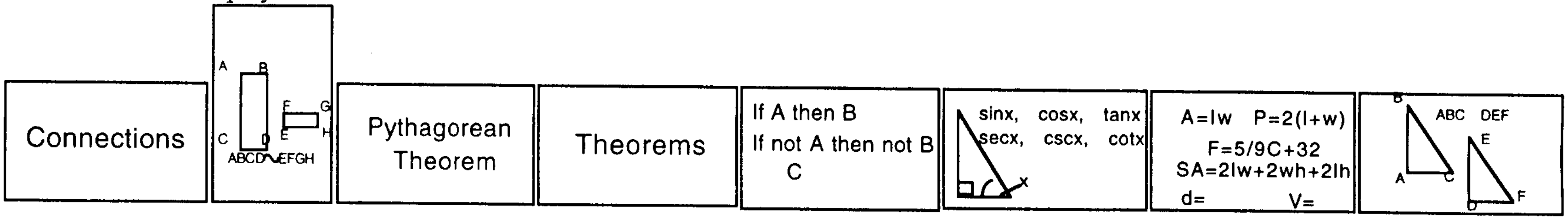
1. Visualization, drawing and construction of figures



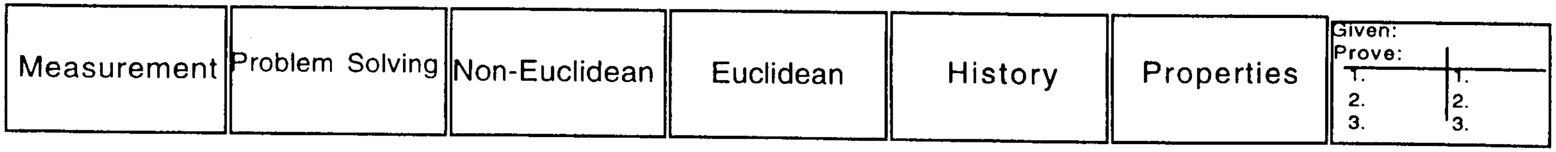
2. Real, physical world



3. Not visual or physical



4. Mathematical Systems



Appendix C

Informed Consent Form
(Geometry Teachers)

Dear ____________________;

Thank you for considering participation in a research project designed to investigate the conceptions of geometry of preservice mathematics teachers prior to teaching.

Participation for this study will be during Fall term. You will be asked to videotape three geometry lessons. You may select any lessons you want, however, please select lessons where you are teaching as opposed to "test" days. Ten preservice teachers will be viewing and commenting on these videotapes as part of this research project. This study is not designed to evaluate your teaching, rather to gain an understanding of how preservice teachers view the teaching of geometry.

To the degree possible, confidentiality will be maintained. The preservice teachers will not be given your name or the school in which you teach. Pseudonyms will be used for the university, schools and all subjects when reporting the results of the research. Video and audio tapes will be kept in a secure place, stored in the major professor's office until analysis is completed, at which time they will be erased.

Questions about the research, personal rights, or research-related injuries should be directed to: Dr. Margaret Niess at 737-1818.

Thank you for your time and participation in this research project.

Janet Scholz
737-8731

I agree to participate in this research project and understand the general intent of the study, the types of data to be collected, and the time commitments involved in the study.

_________________________ ____________

Signature Date

Appendix D

Work Sample Description

The work sample must contain all of the following sections completed with the detail as described:

Title Page	Name of developer, course for which it was developed, school where it will be taught, class in which it will be taught (including grade, period, subject, and number of students), topic of instruction, class text and/or materials
Rationale	Why is this topic included in the curriculum? Why at this time? What purpose will this knowledge serve for the learner? How about society? What is its use? In other word, address the principal reason for the study of this topic.
Unit Goals	Which of the national or state goals or standards will be met within this unit? State the goals in outcomes for the students. Words like understand, appreciate, thinks about are acceptable in this section.
Instructional Objectives	Provide the specific learning outcome (the knowledge and skills) that students should be able to demonstrate at the end of the unit. These objectives specify learning outcomes and acceptable levels of performance and will be used to evaluate instructional effectiveness. These objectives indicate the specific content that is to be taught in the unit.
Perequisite Skills/ Diagnostic Procedures	What skills must the learner bring to this new topic? How will you determine whether the student has these skills? How will you collect information for making this diagnosis?
Materials and Equipment	A list of the materials and equipment needed for this unit. This list serves as a reminder to ensure that such resources are available during the unit presentation.
Calendar	General outline for the unit. In general, this is the timetable that specifies the time allotted to the total unit and its individual activities. Include expected homework assignments. Include assessment times and strategies.

Instructional Activities	This section provides completed lesson plans for the unit and are expected to be in the formal described earlier. Be sure to include the specific instructional objectives for each lesson, time estimates, feedback strategies, method of instruction (large group, lab, etc.), strategies you will use (inductive, hands-on, deductive, etc.), questioning plans for each activity, homework assignments and plans for evaluation of student progress in meeting instructional objectives. Include any overheads, handouts, etc. that you used with the lesson. These plans should reflect the changes you believe need to be made to have a more successful lesson.
Additional Resources	This section contains additional resources that could be used in the unit but have not been utilized in the lesson plans.
Plans for Assessment and Evaluation	This section summarizes how you will assess student progress toward the goals/objectives of the unit as well as how you will assess your instruction. You need to describe alternative assessment methods (other than unit test) for assessing students, plans for both summative and formative assessment, and how the method of assessment matches with the goals/objectives being assessed.
Analysis of Teaching/ Learning	This section is where you analyze the teaching and learning in this unit. You will provide three primary pieces in this analysis:

(1) Pupil Data

This section will include data on learning gains resulting from instruction, analyzed for each pupil and summarized in relation to pupils' level of knowledge prior to instruction.

(2) Interpretation of Results

This section will provide an interpretation and explanation of assessment data. It is critical that interpretation of data be related to the unit goals and intentions. An analysis of the unit test with a reflection on item analysis and item discrimination should also be included.

(3) Future Uses of Data

This section is a description of how the data collected on students' learning gains can be used in the planning and implementation of future instruction. Finally, reflect on your growth as a teacher. What have you learned about teaching as a result of having taught this unit? What methods and strategies do you believe have the most benefit for student learning?

Appendix E

Description of Courses (Summer and Fall Quarters)

Summer quarter

Course	Course Description
Students and Teachers, Schools and Community (U)	Designed to reflect the interdependence of students, teachers, schools and community and their independent and collective impact on the educational process.
Educational Psychology, Learning and Development(U)	Designed as an opportunity to begin the transition from student to teacher. Explores the relationship between human development and learning through the life cycle.
Methods I (U)	Methods and problems in planning for mathematics instruction, selecting teaching strategies, organizing materials, evaluating student progress, and managing student behavior.
Instructional Technology (U)	Laboratory course designed to provide the preservice mathematics teacher with experience with instructional tools for teaching secondary mathematics. Instruction emphasizes the integration of computer activities with presentation of mathematical concepts.
Mathematics Pedagogy (G)	Designed to allow each student to develop pedagogical content knowledge. Specific emphasis is placed upon classroom tested instructional activities and approaches as presented by actual mathematics secondary school classroom teachers.

U=Undergraduate course
G=Graduate course

Fall quarter

Course	Course Description
Microteaching (G)	Develop, practice, and improve specific instructional skills, strategies, and modes in small-group teaching/learning situations with videotape feedback and critique by self, peers, and supervisor.
Internship (G)	Field experience in which intern will integrate academic study with classroom teaching experience to learn to function well in the context of the classroom.
Directed Activities (G)	Practicum designed to provide the preservice mathematics teacher with experience with the organization in the secondary mathematics curriculum, the students, administrative activities, and instructional activities.
Methods II (G)	Methods and problems in planning for mathematics instruction using activity and laboratory approaches. Includes selecting teaching strategies, organizing materials, evaluating student progress, and managing student behavior.

U=Undergraduate course
G=Graduate course

Appendix F

Robin's Classroom Observations

Obs. No.	Date	Topic(s)	Sequence of Classroom Events
1	1/10/95	Scale Drawings	Review worksheet on ratios and proportions; T demos how to enlarge a cartoon and make a grid; students work on enlarging a cartoon
2	2/1/95	Similarity Theorems for Triangles	Review on board; T goes through with class;T demos with pipe cleaners; students continue with worksheet as T circulates; T goes through SSS Similarity Theorem-students write in notes; T goes through SAS Similarity Theorem; T has students practice problems from board; students work individually on homework as T circulates
3	2/17/95	Geometric Mean	T hands out notes for students to fill in on geometric mean between 2 numbers; T demos geometric mean with triangles on overhead-breaks into 3 similar triangles; T shows 8 examples with 2 cases; students practice 2 problems on own; T goes over on board with class; T assigns homework with no time left.
4*	2/22/95	Pythagorean Theorem	Review with 3 problems for students to do; T goes over with class; T shows nautilus shell as an application of geometric mean; T tells students Pythagorean Theorem; T shows visual "proof"; students do activity with cutting triangle; T goes through 6 examples; T goes over converse of Pythagorean Theorem; students practice 5 problems and reviews; T gives homework

5*	2/24/95	30-60-90 Right Triangles 45-45-90 Right Triangles	T has students practice 3 problems and goes over them as review; T demos 45-45 right triangle and finds hypotenuse; T does 1 example; T demos 30-60-90 right triangle and finds sides; T goes through 3 examples; students practice 4 problems on own; T goes through with class; T assigns homework
6*	2/28/95	Review	T has students copy and complete 8 problems from board; T goes through each problem with class; T asks for other questions; T has application problem of right triangles for students to work on; T complete the problem; T assigns homework
7	3/2/95	Trig. of Right Triangles	T has students do 2 review problems; T goes over with class; T goes over sin, cos, and tan with students as they take notes; T does 2 examples; students practice 3 examples on own and T goes over with class; T assigns homework
8	3/6/95	Trig. of Right Triangles Applications	T reviews trig. of right triangles; T goes over 2 problems with angles of elevation and angle of depression; T assigns homework.

* denotes consecutive observations
T denotes teacher

Appendix G

Jeremy's Classroom Observations

Obs. No.	Date	Topic(s)	Sequence of Classroom Events
1	1/10/95	Ratios	Students complete warm-up on the board; T goes through; T defines ratios; T shows examples where ratios/scales are used; students measure/compare lengths in human body to discover Golden Ratio; T shows geometry examples; students start homework
2*	2/3/95	SAS/SSS Similarity Theorems	Warm-up on board; T goes through six homework problems from previous lesson; T demos as students follow the SAS and SSS similarity theorems; T does three examples on board; T assigns homework
3*	2/7/95	Triangle Proportionality Theorem	T has students complete 2 proofs for a warm-up; T goes through with class; T goes through 3 homework problems from previous lesson; T demos with students triangle proportionality theorem; T has students practice 10 problems; T goes over 2 and assigns the rest as homework along with a book assignment
4*	2/9/95	Angle Bisector Theorem (triangles)	Students work on warm-up; T goes through with class, asking students to provide answers; T goes through 4 homework problems from previous lesson; T explains angle bisector theorem; T does 3 examples; T assigns homework

5	2/17/95	Simplifying Radical Expressions	Warm-up on board for students; T goes through with class; T has students take notes on simplifying radicals; T has students practice 3 problems then goes over w/class; T assigns homework
6	2/22/95	Geometric Mean	Warm-up on board for students; T goes over test answers with class; T goes over homework problems from previous lesson; T goes through geometric mean as students take notes; T does 3 problems as examples; T assigns homework
7	2/24/95	Using Geo-Explorer Software Program	T explains software program and how to use; students move to computer lab and work through a worksheet (unrelated to prior content)
8	2/28/95	Pythagorean Theorem	Warm-up on overhead; T goes over with class; students construct squares off of each side of a right triangle on computer program; T does 3 examples with class; T assigns homework

* denotes consecutive observations
T denotes teacher

Appendix H

Bailey's Classroom Observations

Obs. No.	Date	Topic(s)	Sequence of Classroom Events
1*	1/9/95	Triangle Congruence Theorems	T takes attendance; T reviews SSS and SAS congruence theorems; T explains SAA Theorem; T goes over 4 examples with class; T assigns homework
2*	1/11/95	Proving Triangles Congruent/Quiz	T takes attendance; T goes over homework; T explains CPCTC Theorem and how to prove congruent triangles using it; T does 2 examples on overhead; students take quiz; T assigns homework
3*	1/13/95	Proofs with Isosceles Triangles	T takes attendance; T goes over homework with class; T has students complete an investigation from book in small groups; T walks around to help; T discusses findings with class; T assigns 2 proofs to be done before class ends; T assigns homework
4	1/30/95	Properties of Circles	T takes attendance; T has students read beginning of Chapter 6; T reviews radius, and definition of circle; T assigns 10 problems for students to work on; T goes over definitions of chord, diameter, secant, tangent, inscribed angle, central angle, arcs, intercepted arcs and concentric circles; T makes list of vocabulary on board; T assigns homework

5	2/2/95	Arc Length	T takes attendance; T goes over homework problems-specifically discusses 10 of them; T introduces arc length and writes definition on board; T shows 2 examples; T has students do 3 problems for practice then discusses with class; T assigns homework
6	2/6/95	Review/Quiz	T takes attendance; T goes over homework; students take quiz; T assigns homework
7	2/17/95	Area of Irregular Figures/Quiz	T takes attendance; T goes over homework; T shows 3 irregular figures and divides them for students; T tell students how to find the area; T assigns homework; students take quiz
8	2/22/95	Area of Regular Polygons	T takes attendance; T goes through homework problems; T has students complete investigation from book; T reviews with class; T does 2 examples; T assigns homework

* denotes consecutive observations
T denotes teacher

Appendix I

Robert's Classroom Observations

Obs. No.	Date	Topic(s)	Sequence of Classroom Events
1*	1/9/95	Properties of Parallelograms	T goes over homework with class; T has a student read a theorem; T draws picture on board and briefly explains theorem; T asks if there are any questions; T has another student read next theorem and continues in same manner with f our more theorems; T goes over classroom exercises from book with class; T assigns homework
2*	1/10/95	Properties of Quadrilaterals	T goes over homework with class; T has a student read a theorem; T asks if there are any questions; T has another students read next theorem and continues in same manner; T has students copy 5 ways to prove quadrilaterals are parallelograms from book; T puts 10 statements on overhead that students should decide if they are true or false; T has students answer on their own and then goes over with class; T assigns homework
3*	1/11/95	Special Parallelograms	T goes over homework with class; T has students draw rectangle, parallelogram and right triangle on paper; T has students read theorems; T asks if there are any questions; T goes through 4 theorems in same manner; T assigns homework
4	1/20/95	Building Towers	T hands out 10" of tape, 3 sheets of paper, scissors, and 6' of string to each pair of students; students build a tower to support a golf ball

5	1/31/95	Ratios	T introduces ratios; T goes over 3 problems from the book with class; T places 7 problems on overhead for students to answer true or false; T goes over with class; T assigns homework
6	2/2/95	Similar Polygons	T goes over homework with class; T introduces similar polygons; T writes definition on board and describes corresponding parts; T introduces scale factor; T shows 1 example; T assigns homework
7	2/6/95	SAS and SSS Similarity Theorems	T goes over homework with class; T writes SAS similarity theorem on board while students copy in notes; T does 1 example; T writes SSS similarity theorem on board as students copy in notes; T does 1 example; T goes through 2 more examples; T assigns homework
8	2/7/95	Review for test	T goes over homework with class; T plays review game with students using 15 problems from the book

* denotes consecutive observations
T denotes teacher