

AN ABSTRACT OF THE THESIS OF

Deing Wang for the degree Doctor of Philosophy  
(Name) (Degree)

in Mechanical Engineering presented on November 18, 1986  
(Major department) (Date)

Title: EXPERIMENTAL INVESTIGATION OF NATURAL CONVECTION  
FROM AN ARRAY OF UNIFORMLY HEATED HORIZONTAL CYLINDERS TO  
MERCURY

Abstract approved: *Redacted for Privacy*

An experimental investigation was conducted to better understand the heat transfer process with natural convection from an array of uniformly heated horizontal cylinders to mercury.

Five assembly geometries were studied, these being the single horizontal cylinder, two cylinders with spacing ratios  $S/D = 2, 3$  and  $4$ , ( $S$  represents the center-to-center spacing between cylinders of diameter,  $D$ ), and three cylinders with a single spacing ratio of  $S/D = 2$ . All cylinders were arranged in the same vertical plane.

A scale analysis was employed to predict the relationship between the average Nusselt number and the Boussinesq number. In the case of the single cylinder good agreement was achieved between scale analysis and experimental results. The Nusselt number is a function of Boussinesq number only in the single cylinder case.

These findings were extended to cases with two and three cylinders. Correlations of Nusselt number with

different Boussinesq numbers, various ratios of spacing and diameters of cylinders have been established for these multiple cylinder cases.

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EXPERIMENTAL INVESTIGATION OF NATURAL CONVECTION  
FROM AN ARRAY OF UNIFORMLY HEATED HORIZONTAL CYLINDERS  
TO MERCURY

by

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A THESIS

submitted to

Oregon State University

in partial fulfillment of  
the requirements for the  
degree of

DOCTOR OF PHILOSOPHY

completed November 18 1986

commencement June 1987

APPROVED:

*Redacted for Privacy*

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November 18, 1986

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## SYMBOLS

### English Letter Symbols:

A	area, ft <sup>2</sup>
B	constant
Bo	Boussinesq number
B'	constant
Cp	heat capacity, Btu/lbm·F
D	diameter, ft
d	diameter, ft
Gr	Grashof number
g	local acceleration due to gravity, ft/sec <sup>2</sup>
h	heat transfer coefficient, Btu/(hr·ft <sup>2</sup> ·F)
k	thermal conductivity, Btu ft/(ft <sup>2</sup> ·hr·F)
L	significant length, ft
M	mass, lbm
N	number of cylinders in an array
Nu	Nusselt number
P	power, Btu/sec
P	Pressure, lb/ft <sup>2</sup>
Pr	Prandtl number
Q	heat flux, Btu/(hr·ft <sup>2</sup> )
Ra	Rayleigh number
S	center-to-center spacing between cylinders, ft
T	temperature, F
T <sup>s</sup>	surface temperature, F
T <sub>∞</sub> <sup>s</sup>	environmental temperature, F
V <sub>r</sub>	velocity in r direction, ft/sec
V <sub>θ</sub>	velocity in θ direction, ft/sec

### Greek Letter Symbols:

α	thermal diffusivity, ft <sup>2</sup> /sec
β	coefficient of thermal expansion
Δ	change in a quantity
δ <sub>t</sub>	boundary layer thickness, ft
θ	angle, degrees
μ	dynamic viscosity, lbm/ft·sec
ν	kinematic viscosity, ft <sup>2</sup> /sec
ρ	mass density, lbm/ft <sup>3</sup>
σ	standard deviation
φ	angle measured from lower stagnation point, degrees

EXPERIMENTAL INVESTIGATION OF NATURAL CONVECTION  
HEAT TRANSFER FROM AN ARRAY OF UNIFORMLY HEATED  
HORIZONTAL CYLINDERS TO MERCURY

CHAPTER I

INTRODUCTION

1.1 Description of this work

The work described herein is part of an ongoing experimental investigation of natural convection heat transfer to liquid metals at Oregon State University. A number of studies have been conducted related to natural convection in mercury during the past 12 years at OSU. This effort is motivated by problems associated with nuclear power. This topic has received considerable attention in recent years because low Prandtl number fluids are known to have high coefficients of heat transfer in comparison with "ordinary" fluids such as air or water. For this reason liquid metal free convection-cooling is finding increasing applications, e.g. in nuclear power reactors and in certain types of heat exchangers.

White, Welty, and Hurt (45) began the OSU program by looking at laminar temperature and velocity fields next to both a uniformly-heated and an isothermal vertical plate in a quiescent pool of mercury. White reported heat transfer correlations for these conditions. Colwell and Welty (8)

investigated the same problem with a more complex geometry, which was the vertical, open, heated channel. Colwell developed some heat transfer correlations and plate spacing optimization criteria. Humphreys (16) extended the results of Colwell to include the transition and turbulent regimes. Welty, and Peinecke (44) reported velocity measurements for buoyancy-induced flow in mercury adjacent to a vertical single cylinder. Dutton and Welty (11) reported research on natural convection for an array of uniformly heated vertical cylinders in mercury. Kim (20) investigated the single horizontal cylinder case.

The objective of this study is to investigate natural convection heat transfer from an array of horizontal cylinders to mercury. Heat exchangers, in general, involve large numbers of tubes. Thus, a knowledge of the heat transfer performance of tube arrays is important for the evaluation of heat exchanger performance.

The replacement of a single tube by a set of tubes or a bank of a certain arrangement introduces a high degree of complexity in the heat transfer process. The pattern of flow around a tube in a vertical bank is greatly influenced by its neighboring tubes. Therefore, when the buoyancy-induced fluid flow from one of the cylinders encounters others in an array, the affected cylinders can no longer be assigned heat transfer coefficients characteristic of a single cylinder. In addition such information is hard to

find for liquid metals.

This work is concerned with horizontal cylinders situated one above another in the same vertical plane. Two and three cylinder cases are considered. There are two effects on the upper cylinders. First, the fluid flowing past the upper cylinders has its motion already driven by the buoyancy generated from the lower cylinder. Second, the temperature of the fluid reaching the upper cylinders is higher than that of the ambient. Motion of the fluid tends to increase the heat transfer coefficient and the higher temperature tends to decrease the heat transfer coefficient for the upper cylinders. For the two cylinder case, separation distances  $S$  (center to center) of 2, 3, and 4 times the cylinder diameter  $D$  were considered. For the three cylinder case, a single spacing,  $S=2D$ , was used. Operating conditions included upper cylinder Rayleigh numbers between  $5 \times 10^5$  and  $1.062 \times 10^7$ . Since these experiments were performed with mercury as the heat transfer fluid, properties such as density, thermal conductivity, and thermal expansion coefficient were evaluated at the reference temperature  $T_r = 0.7T_s + 0.3 T_\infty$ .

## 1.2 Application of results

Interest in liquid metals as heat transfer media is attributable to growing needs in the nuclear and space programs throughout the world. Severe operating conditions

and the need to maximize temperature while maintaining a low vapor pressure is one of the main reasons for using liquid metals in nuclear reactors. Some reactors require coolants with low neutron absorption characteristics and many liquid metals are suitable.

A comparison between liquid metals and conventional fluids as heat transfer media follows:

1. Thermal conductivity: (See values listed in table (1-1). The conductivity of mercury is much higher than that of water (12.3/0.67). Sodium displays a still higher value of  $k$  than mercury.

2. Boiling temperature: Heat transfer processes at high operating temperatures can be achieved with liquid metals at relatively low pressures. Again, from table 1-1, the ratio of boiling temperatures of mercury and water is seen to be 2.5:1. Sodium, again displays a still higher value.

3. Corrosive effects on construction materials: Corrosive effects of liquid metals on conventional materials are less than those of water.

From this comparison liquid metals have many advantages. These advantages must be considered in comparison with the enormous health and safety problems, and with the cost of equipment and other facilities necessary to achieve an acceptable standard of safety to personnel. Mercury was chosen as the best liquid metal for performing this study due to its ease of handling. Mercury displays properties

characteristic of liquid metals.

TABLE 1-1 Comparison of physical properties of Hg and Na with water (saturated @ p=1 atm)

Property	Water	Mercury	Sodium
Melting point ( °C )	0	-39	97.5
Boiling point ( °C )	100	256.9	887.8
Latent heat of vaporization ( kJ/kg )	2256	293	1662
Density ( kg/m <sup>3</sup> )	801	12720	741
Thermal conductivity ( W/m*k )	0.67	12.3	55.4
Viscosity (cp)	0.30	0.89	0.15

## CHAPTER II

## SCALE ANALYSIS

## 2.1 Background

Convective heat transfer occurs in general between a solid body and a fluid. The heat transfer is affected by a combination of molecular conduction within the fluid and energy transport resulting from the motion of fluid particles. Convection can be divided into the categories of forced convection and natural convection. In forced convection the circulation is produced by mechanical means. In natural convection fluid motion is caused by density variations which result from a nonuniform temperature distribution.

For representing the heat transfer coefficient,  $h$ , for general sets of conditions, a dimensionless parameter should be used. In the case of natural convection from a heated object to an adjacent fluid, the variables are listed in table 2.1 ( Welty (43)).

The velocity does not appear as a variable since it is a result of other effects associated with energy transfer. Formal dimensional analysis techniques will yield, as one parameter, the Grashof number, 
$$Gr = \frac{g\beta\rho^2 \Delta TD^3}{\mu^2}$$

Table 2.1 Dimension of the important variable

Variable	Symbol	Dimensions
Significant length	L	L
Fluid density	$\rho$	M/L <sup>3</sup>
Fluid viscosity	$\mu$	M/Lt
Fluid heat capacity	Cp	Q/MT
Fluid conductivity	k	Q/LtT
Fluid coefficient of thermal expansion	$\beta$	1/T
Gravitational acceleration	g	L/t <sup>2</sup>
Temperature difference	$\Delta T$	T
Heat-transfer coefficient	h	Q/L <sup>2</sup> tT

The Grashof number has been interpreted physically, as the ratio of buoyant to viscous forces in the convective system. Bejan (2) has expressed an alternate approach which will be considered later in this chapter.

Another frequently-used parameter, formed by the product Gr and Pr, is called the Rayleigh number,

$$Ra = Gr Pr = \frac{g\beta D^3 \Delta T}{\nu \alpha}$$

Yet another combination is the product Ra Pr, the Boussinesq number,

$$Bo = Ra Pr = \frac{g\beta D^3 \Delta T}{\alpha^2}$$

Dimensional analysis suggests as one possible form for correlating natural convection data, the expression,

$$Nu = f(Gr, Pr)$$

The problem, which remains, is "what is the function f?"

## 2.2 Scale analysis of natural convection from circular cylinders

Steady state, two dimensional, laminar, natural convection from horizontal cylinders is governed by three conservation principles.

For a two-dimensional cylindrical coordinate system, there are three equations describing the three fundamental principles. Effects along the axial direction are not included since they are not of interest in the case of horizontal cylinders. The equations which apply are the following:

Conservation of mass

$$\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_\theta}{r \partial \theta} = 0 \quad (2.1)$$

Conservation of momentum, r-direction,

$$\rho \left( V_r \frac{\partial V_r}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_r}{\partial \theta} - \frac{V_\theta^2}{r} \right) = - \frac{\partial P}{\partial r} + \mu \left( \frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r} \frac{\partial V_r}{\partial r} + \frac{1}{r^2} \frac{\partial^2 V_r}{\partial \theta^2} - \frac{V_r}{r^2} - \frac{2}{r^2} \frac{\partial V_\theta}{\partial \theta} \right) - g \rho \beta (T_s - T_\infty) \cos \theta \quad (2.2)$$

Conservation of momentum,  $\theta$ -direction

$$\rho \left( V_r \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_\theta}{\partial \theta} - \frac{V_r V_\theta}{r} \right) = - \frac{1}{r} \frac{\partial P}{\partial \theta} + \mu \left( \frac{\partial^2 V_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial V_\theta}{\partial r} + \frac{1}{r^2} \frac{\partial^2 V_\theta}{\partial \theta^2} - \frac{V_\theta}{r^2} + \frac{2}{r^2} \frac{\partial V_r}{\partial \theta} \right) + g \rho \beta (T_s - T_\infty) \sin \theta \quad (2.3)$$

Conservation of energy,

$$C_p \left( V_r \frac{\partial T}{\partial r} + \frac{V_\theta}{r} \frac{\partial T}{\partial \theta} \right) = k \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right) \quad (2.4)$$

We next consider the conservation of mass, momentum,

and energy in the thermal boundary layer region, where the heating effect of the cylindrical surface is manifested. In the thermal boundary layer the characteristic length in the  $r$  direction is  $\delta_t$ ; the significant streamwise length for the cylinder is  $D$ . At steady state heat conducted from the wall radially into the fluid is moving along the surface of the cylinder as an enthalpy stream. The energy equation (2.4) expresses a balance between convection and conduction terms.

The convection terms are of the order

$$V_r \Delta T / \delta_t \text{ and } V_\theta \Delta T / D$$

The conduction terms are of order

$$\alpha \frac{\Delta T}{\delta_t^2} \quad \alpha \frac{\Delta T}{D \delta_t} \quad \text{and} \quad \alpha \frac{\Delta T}{D^2}$$

As discussed in the last section  $T_s - T_\infty$  is of order  $T_o - T_\infty$

which can, thus, be expressed as  $\Delta T$ .

We know  $\frac{\partial V_r}{\partial r}$  is of order  $\frac{\partial V_r}{\delta_t}$

$$\frac{V_r}{r} \text{ is of order } \frac{V_r}{D}$$

and since

$$\delta_t \ll D, \quad V_r / D \text{ tends to } 0,$$

$$\frac{1}{r} \frac{\partial V_\theta}{\partial \theta} \text{ is of order } \frac{V_\theta}{D}$$

where the  $r \partial \theta$  is of order  $D$ . From mass conservation in the boundary layer equation (2.1) yields

$$\frac{V_r}{\delta_t} \approx \frac{V_\theta}{D}$$

$$\text{So, } \frac{V_r \Delta T}{\delta_t} \approx \frac{V_\theta \Delta T}{D} \quad \text{Also} \quad \frac{\Delta T}{\delta_t^2} \gg \frac{\Delta T}{D \delta_t} \gg \frac{\Delta T}{D^2}$$



and for the  $\theta$ -direction

$$\frac{V_{\theta}^2}{D} + v \frac{V_{\theta}}{\delta_t^2} \sim g\beta\Delta T \sin\theta \quad (2.7)$$

Inertia                      Friction                      Buoyancy

where the inertia term is of the same order as the pressure term. If  $Pr \gg 1$ , the friction force  $\gg$  inertia force.

Squaring both sides, and adding, the friction and buoyancy balances become:

$$v^2 \frac{V_r^2 + V_{\theta}^2}{\delta_t^2} \rightarrow g^2 \beta^2 \Delta T^2 (\sin^2\theta + \cos^2\theta)$$

since  $V_{\theta} \gg V_r$ ,  $\sin^2\theta + \cos^2\theta = 1$ , and we get the result that

$$v \frac{V_{\theta}}{\delta_t^2} \rightarrow g\beta\Delta T$$

Dividing both sides by  $g\beta\Delta T$ , and recalling that

$$V_{\theta} \sim \alpha \frac{D}{\delta_t^2}$$

we obtain

$$\left( \frac{\alpha D}{\delta_t^2} \right) \left( \frac{1}{\delta_t^2} \right) \left( \frac{1}{g\beta\Delta T} \right) \sim 1 \quad (2.8a)$$

For

$$Ra_D = \frac{g\beta\Delta T D^3}{\alpha v}$$

$$\text{or} \quad \frac{\alpha v}{g\beta\Delta T D^3} \frac{D^4}{\delta_t^4} = \left( \frac{D}{\delta_t} \right)^4 \frac{1}{Ra_D}$$

Then from (2.8a) we obtain

$$\left( \frac{D}{\delta_t} \right)^4 Ra_D^{-1} \sim 1$$

or

$$\left( \frac{D}{\delta_t} \right) \sim Ra_D^{\frac{1}{4}} \quad (2.8)$$

Recalling that

$$h\Delta T \sim k \frac{\partial T}{\partial r} \sim k \frac{\Delta T}{\delta_t}$$

The Nusselt number becomes 
$$Nu = \frac{h D}{k} \sim \frac{k D}{\delta_t k} = \frac{D}{\delta_t}$$

that is 
$$Nu = \frac{h D}{k} \sim \frac{D}{\delta_t} \sim Ra_D^{\frac{1}{4}} \quad (2.9)$$

If  $Pr \ll 1$ , the friction force will be small and there is, thus, a balance between the inertia force and the buoyancy force. With the same treatment as before we obtain

$$v_\theta^2 / D \rightarrow g \beta \Delta T$$

Dividing both sides by  $g \beta \Delta T$  and recalling equation (2.5)

we get

$$\frac{\alpha^2 D^2}{D \rho g \beta \Delta T \delta_t^4} \sim 1$$

Writing

$$Ra_D = \frac{g \beta \Delta T D^3}{\alpha \nu}$$

$$Pr = \frac{\mu C_p}{k} \quad \text{and} \quad \alpha = \frac{k}{\rho C_p}$$

the above expression may be expressed as

$$\left(\frac{D}{\delta_t}\right)^4 Ra_D^{-1} Pr^{-1} \rightarrow 1$$

or

$$\left(\frac{D}{\delta_t}\right) \rightarrow Ra_D^{\frac{1}{4}} Pr^{\frac{1}{4}} \quad (2.10)$$

The Nusselt number is thus seen to be related as

$$Nu = \frac{h D}{k} \rightarrow \frac{D}{\delta_t} \rightarrow Ra_D^{\frac{1}{4}} Pr^{\frac{1}{4}} \quad (2.11)$$

If  $Pr$  is near 1, equations (2.9) and (2.11) are effectively the same. Equation (2.9) is recommended for simplicity.

### 2.3 Discussion

From the previous section the Prandtl number is seen to play an important role in heat transfer correlations. The Prandtl number, like the viscosity and the thermal conductivity, is a material property and it thus varies from fluid to fluid. To understand the importance of Pr its physical interpretation is necessary.

The molecular diffusivity of momentum and energy have been defined as

$$\begin{aligned} \text{momentum diffusivity: } \nu &= \frac{\mu}{\rho} \\ \text{thermal diffusivity: } \alpha &= \frac{k}{\rho C_p} \end{aligned}$$

That these two are designated similarly would indicate that they must also play similar roles in their specific transfer modes. This is indeed the case. They have the same dimensions  $L^2/t$ . The ratio of these two is the Prandtl number which is, of course, dimensionless.

Fluids with high Prandtl numbers obviously show greater viscous spreading; small Prandtl number fluids favor thermal spreading.

When  $Pr \ll 1$ , the momentum diffusivity is much smaller than the thermal diffusivity. The balance of inertia and buoyancy forces in a layer of thickness  $\delta_t$  is expected, from

equation (2.10), to be

$$\left(\frac{D}{\delta_t}\right) \sim Ra_D^{\frac{1}{4}} Pr^{\frac{1}{4}}$$

where  $Ra \cdot Pr$  is called the Boussinesq number  $Bo$ . We see that  $Bo$  is related to the ratio of cylinder diameter to the thermal boundary layer thickness for  $Pr \ll 1$ .

These dimensionless numbers,  $Ra$  and  $Bo$ , raised to the  $\frac{1}{4}$  power can be thought of as ratios of some significant lengths.

From this scale analysis we have learned that

$$\begin{aligned} Nu &\sim Ra_D^{\frac{1}{4}} && \text{if } Pr \gg 1 \\ Nu &\sim Ra_D^{\frac{1}{4}} Pr^{\frac{1}{4}} = Bo^{\frac{1}{4}} && \text{if } Pr \ll 1 \end{aligned}$$

Both relations have been supported by more precise analyses and numerous laboratory measurements which are listed in the next chapter.

## CHAPTER III

## LITERATURE REVIEW

## 3.1 Natural convection from a horizontal cylinder

For solving the problems of natural convection from an array of uniformly heated cylinders to mercury, pertinent literature generally is in two areas.

1. Natural convection from a single cylinder or a bank of cylinders to fluids in general-including liquid metals.

2. Natural convection in liquid metals for other geometries

Natural convection heat transfer from a single horizontal cylinder has been intensively investigated in the past; the subject is presented in an extensive review by V.T.Morgan (31). It can be seen from Table 3-1 that several correlations have been developed, the simplest having the form (30),  $Nu = B*(Ra)^m$  where Nu and Ra are based on diameter.

Table. 3.1 Literature concerning Natural convection heat transfer from a single horizontal cylinder.

Author	Ref.	Fluid	Ra		B	m	
			from	to			
Ayrton & Kilgour	[1]	A	$10^{-4}$	$3 \cdot 10^{-2}$	1.61	0.141	
Petavel	[32]	A	0.1	$3 \cdot 10^2$	1.05	0.14	
Mason & Boelter	[30]	A	$5 \cdot 10^4$	$5 \cdot 10^6$	0.74	0.225	A
Davis	[10]	AGL	$10^{-4}$	$10^6$	0.47	0.25	
Rice	[33]	A	$4 \cdot 10^3$	$6 \cdot 10^6$	0.97	0.203	B
Schuring	[37]	AL	$2.7 \cdot 10^3$	$8.2 \cdot 10^5$	0.57	0.24	E
King	[21]	AL	$10^3$	$10^6$	0.53	0.25	
			$10^6$	$10^{12}$	0.13	0.33	
Lauder	[24]	GL	$10^3$	$10^7$	0.49	0.25	
			$10^8$	$10^9$	0.12	0.333	
Jakob	[18]	AL	$10^{-7}$	$10^{-2}$	1.0	0.1	
			$10^4$	$10^8$	0.52	0.25	C
			$10^8$	$10^{12}$	0.126	0.333	
Cheng	[3]	A	$2.7 \cdot 10^2$	$1.2 \cdot 10^3$	0.70	0.23	
McAdams	[28]	AGL	$10^{-4}$	$10^{-2}$	0.948	0.0768	
			$10^{-2}$	1	1.08	0.104	
			1	$5 \cdot 10^2$	1.08	0.145	
			$10^4$	$10^9$	0.525	0.25	
			$10^9$	$10^{12}$	0.129	0.333	
Lemlich	[26]	A	$6 \cdot 10^2$	$6 \cdot 10^3$	0.45	0.22	
Kays & Bjorklund	[19]	A	$2.8 \cdot 10^5$	$7.5 \cdot 10^5$	0.53	0.25	
Fand & Kaye	[15]	A	$10^4$	$4 \cdot 10^4$	0.485	0.25	
Etemad	[14]		$1.2 \cdot 10^5$	$1.3 \cdot 10^6$	0.456	0.25	D
Elenbaas	[13]	A	$10^8$	$10^9$	0.49	0.25	

A - Air; G - Gases; L - Liquid; SH - Sodium hydroxide;

(The Letter in the last column corresponds to the line of Fig 3.1)

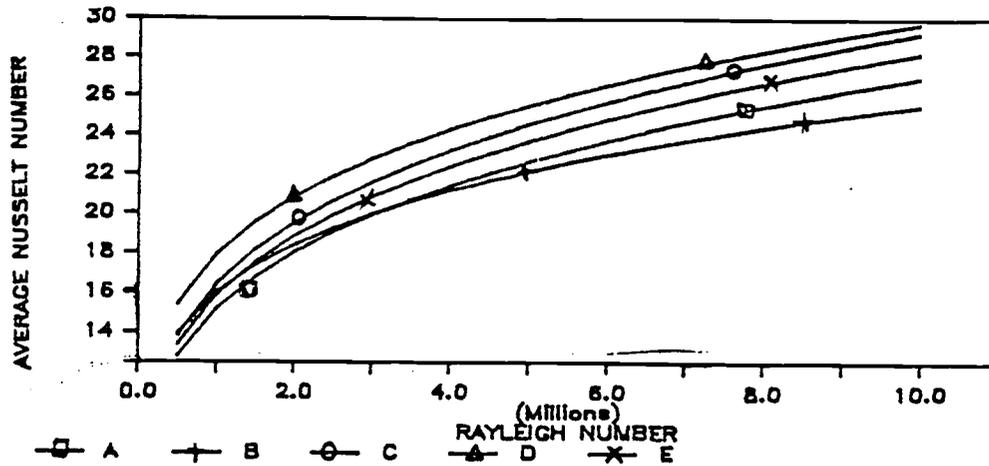


Figure 3.1 Nusselt number for a single horizontal cylinder from the literature

Morgan (31) proposed values for the correlation between Nusselt and Rayleigh numbers for horizontal smooth circular cylinders given in Table 3.2a and Table 3.2b

Table 3.2a Proposed correlation for Natural Convection from Horizontal cylinders

Range of Ra		Nu = B * ( Ra ) <sup>m</sup>	
From	To	B	m
10 <sup>-10</sup>	10 <sup>-2</sup>	0.675	0.058
10 <sup>-2</sup>	10 <sup>2</sup>	1.02	0.148
10 <sup>2</sup>	10 <sup>4</sup>	0.850	0.188
10 <sup>4</sup>	10 <sup>7</sup>	0.48	0.25
10 <sup>7</sup>	10 <sup>12</sup>	0.125	0.333

Table 3.2b Proposed Nusselt numbers for different Rayleigh numbers

Ra	Nu	Ra	Nu
$10^{-10}$	0.178	10	1.43
$10^{-9}$	0.203	$10^2$	2.02
$10^{-8}$	0.232	$10^3$	3.11
$10^{-7}$	0.265	$10^4$	4.80
$10^{-6}$	0.303	$10^5$	8.54
$10^{-5}$	0.346	$10^6$	15.2
$10^{-4}$	0.396	$10^7$	27.0
$10^{-3}$	0.452	$10^8$	58.0
$10^{-2}$	0.516	$10^9$	125
$10^{-1}$	0.726	$10^{10}$	269
$10^0$	1.02	$10^{11}$	580
		$10^{12}$	1250

### 3.2 Natural convection from a horizontal cylinder to media with small Pr

All of the correlations listed are valid for air, and some are valid for liquids. None are correct for liquid metals.

A correlation for liquid metals is reported from Fedynskii's experiments (17) with mercury. The tests were conducted with horizontal cylinders having diameters of 25, 65, and 85 mm. The results of these tests are shown in Figure 3-2. The line corresponds to the Lorentz equation of the form:

$$\text{Nu} = 0.54 \text{ Ra}^{\frac{1}{4}} \quad (3.2)$$

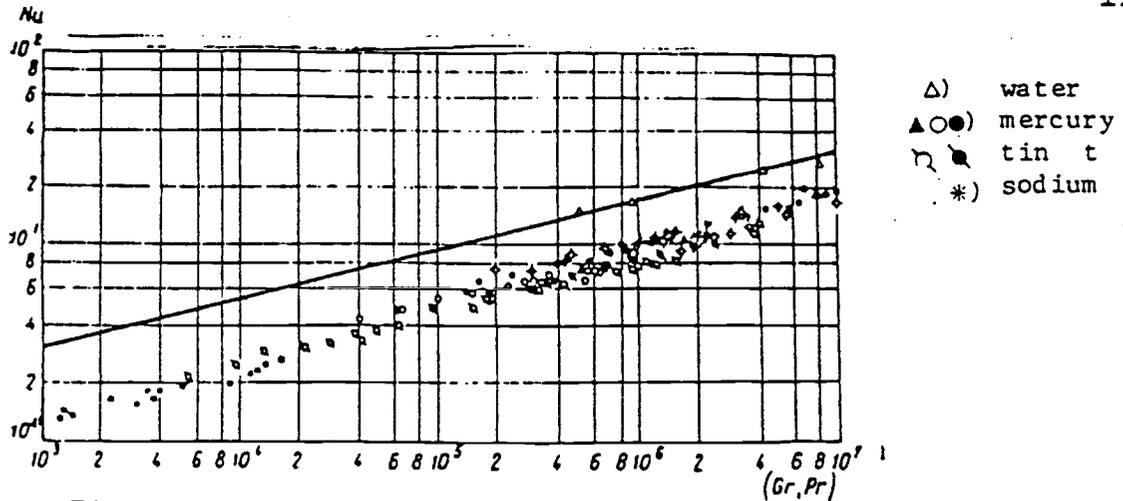


Figure 3.2 Experimental data with metallic liquids

but the coefficient 0.54 is for media with  $Pr \geq 1$ . From the plot one may observe that Nusselt Number data for natural convection from a cylinder to mercury are below those for media with  $Pr \geq 1$ , although having almost the same slope. From scale analysis we know that the Nusselt number is of order  $Ra^{\frac{1}{4}}$  when  $Pr \geq 1$ . For  $Pr \ll 1$  the Nusselt number is of order  $Ra^{\frac{1}{4}} * Pr^{\frac{1}{4}}$  or  $Bo^{\frac{1}{4}}$ . For mercury at room temperature  $Pr = 0.023$ , thus the coefficient is 0.6925. A line representing  $0.6925 Ra^{\frac{1}{4}}$  fits the data quite well. Fedyskii recommended the equation

$$Nu = 0.53 * Gr^{\frac{1}{4}} * (Pr^2 / (0.952 + Pr))^{1/4} \quad (3.3)$$

Kutateladze (21) also suggests the following two equations

$$Nu = 0.67 * ((Pr^2 * Gr) / (1 + Pr))^{\frac{1}{4}} \quad \text{for } 10^2 < Gr < 10^8 \quad (3.4)$$

$$Nu = 0.16 * ((Pr^2 * Gr) / (1 + Pr))^{0.16} \quad \text{for } Gr > 10^8 \quad (3.5)$$

The equations are good for  $Pr \ll 1$  or  $Pr = 1$ .

Some additional equations have been developed from numerical methods using boundary-layer theory solutions for cylinders. Saville and Churchill(35) have shown these solu-

tions to be quite accurate for moderate Rayleigh numbers for which the wake is confined to a small region at the rear of the cylinder. For asymptotic cases with  $Pr \rightarrow \infty$  and 0, Lefevre(25) and Saville and Churchill(35), respectively, have derived solutions for the mean Nusselt number expressed as

$$Nu = 0.518 * Ra^{\frac{1}{4}} \quad \text{as } Pr \rightarrow \infty \quad (3.6)$$

$$Nu = 0.599 * Ra^{\frac{1}{4}} * Pr^{\frac{1}{4}} \quad \text{as } Pr \rightarrow 0 \quad (3.7)$$

Saville and Churchill (33) have computed numerical values for  $Pr=0.7$  and have presented the following expression for all  $Pr$ :

$$Nu = 0.518 * (Ra / [1 + (.599/Pr)^{9/16}])^{16/9} * Pr^{1/4} \quad (3.8)$$

It may be noted that the exponent,  $1/4$ , is identical to that derived by Churchill and Ozoe (6) for free convection from a vertical flat plate.

Equation (3.8) would be expected to become invalid as  $Ra$  increases sufficiently increasing the development of the wake region and also as  $Ra$  decreases sufficiently, owing to thickening of the boundary layer relative to the diameter of the cylinder. Tsubouchi and Masuda (42) proposed an empirical limiting value of 0.36, based on the analysis of several sets of data. Combining this thinking and equation (3.8) results in the following best expression for the entire laminar regime.

$$Nu = 0.36 + 0.518 * ( Ra / [1 + (0.599 / Pr)^{9/16}]^{16/9} )^{1/4} \quad (3.9)$$

More recently, Saville and Churchill(32) studied the case for  $Ra \rightarrow \infty$ , (the completely turbulent boundary layer and wake change characteristic of heat flow). They suggested the expression

$$Nu = A * Ra^{1/3} * f(Pr) \quad \text{as } Ra \rightarrow \infty \quad (3.10)$$

where the A is a constant and  $f(Pr)$  is a function, which approaches unity for  $Pr \rightarrow \infty$  and is proportion to  $Pr^{1/3}$  for  $Pr \rightarrow 0$ .

They combined equations (3.9) and (3.10) and used for  $f(Pr)$  the function  $f(Pr) = [1 + (.599 / Pr)^{9/16}]^{16/9 * (-1/3)}$ , to obtain the correlation

$$Nu^{1/2} = 0.6 + .387 * (Ra / (1 + (.599 / Pr)^{9/16})^{16/9})^{1/6} \quad (3.11)$$

Mean coefficients for a horizontal cylinder using these expressions are improvements on prior graphical and empirical correlations both in accuracy and in convenience.

For uniform heat flux the numerical solution of Wilkes has been utilized by Churchill(5) to construct the correlation

$$Nu = 0.579 * (Ra / [1 + (0.442 / Pr)^{9/16}]^{16/9})^{1/4} \quad (3.12)$$

The values of Nu on which this correlation is based were apparently obtained by averaging local values. Applying the factor which Sparrow and Gregg(38) used in the vertical plate case yields values in much closer agreement with those for uniform wall temperature. As an approximation for equation (3.12), the results are very close to equation

(3.7), except when Pr is very small. Assuming the same limiting value as for equation (3.9)

$$\text{Nu} = 0.36 + 0.521 * (\text{Ra} / [1 + (0.442 / \text{Pr})^{1/6}]^{1/9})^{1/4} \quad (3.13)$$

Churchill and Chu tested equation (3.13). In the turbulent region equation (3.11) is recommended as a correlation both for uniform heating and for uniform surface temperature. Equation (3.13) is, possibly, a more accurate expression for the laminar region and for small Pr.

For the present case, equation (3.7) seems to be the best choice.

### 3.3 An array of horizontal cylinders

In most investigations reported in the literature attention has been restricted to a single cylinder. Only a few investigators have attempted to study interactions between two or more horizontal cylinders.

In 1948 Eckert and Soehngen (12) carried out a limited study of natural convection with heated cylinders which interacted. Three cylinders (0.878 in. dia) were arranged in a vertical array, and the measured Nusselt numbers showed that the upper cylinders exhibited increasingly higher temperatures. It was also found that, for the upper cylinder, the Nusselt number decreased as additional tubes were added. For two horizontal tubes the upper tube Nusselt number was 87% of the value for the lower tube, which displayed the same Nusselt number as for a single tube.

With a third tube added the Nusselt number of the lowest tube remained the same, the Nusselt number of the second tube decreased to 83% of the value of the lowest tube and  $Nu$  for the third tube was 65% of that for the lowest tube.

With tubes arranged in a staggered array - for the case with the middle tube offset by a half tube diameter - the middle tube Nusselt number was increased above that of the lowest tube by 3%. The uppermost tube displayed a Nusselt number which was 86% of the value of the bottom tube. In this study solid copper cylinders with a length-to-diameter ratio of 13 and  $Gr = 34300$  were used for the vertical arrays (based on diameter). For staggered arrays the value,  $Gr=14650$ , was used.

In explaining these results, Eckert and Soehngen stated that the induced temperature and velocity fields had opposite influences on the downstream tubes. With tubes directly in line with each other the temperature differences between the tubes and fluid in their immediate vicinity was less due to the warm wake from lower tubes. This caused a reduction in the heat transfer from the upper tubes and a corresponding lower value for the Nusselt number. The uppermost tube of the staggered arrangement is not completely in the wake of the second tube, thus there is a strong velocity effect resulting in an increase in the Nusselt number, but, at the same time, the warm fluid wake from the lowest tube tends to decrease the Nusselt number.

The net result of these two effects on higher tubes varies depending on the distance of separation.

In 1969 Lieberman and Gebhart(27) studied the interactions of heated wires arranged in a vertical array. Their experiments involved an array with ten cylinders of .127 mm dia. They concluded their work with the statement that there is an optimum spacing for each array configuration to achieve a maximum Nusselt number. The highest average Nusselt number for 16 different combinations occurred at a spacing of 75 diameters and an array angle of 60 degrees.

In 1971 Marsters (29) reported his experimental work. He suggested that the cylinder temperature in a vertical array is a function of spacing and position for a given dissipation rate. For close spacings, the temperature rises, generally, but experiences a decrease if the Grashof number, based on cylinder distance from the bottom of the array, exceeds a certain critical value. For wide spacings the temperature decreases monotonically. This behavior is consistent with the properties of wakes above line sources of heat.

In 1981 Sparrow and Niethammer (40) carried out experiments on the heat transfer from a pair of heated cylinders( $d=3.787\text{cm}$ ) in a vertical line and reported the effect of vertical separation distance and the cylinder-to-cylinder temperature imbalance on the upper cylinder Nusselt number. They found that the upper cylinder

Nusselt number had a maximum value as a function of separation distance (from two to nine dia.).

In 1983 Tokura, Saito, Kishinami, and Muramoto (40) reported their empirical work on natural convective heat transfer from a cylinder array arranged in a vertical line. An empirical formula to predict the average Nusselt number for the upper cylinder in a two-cylinder array was obtained. A recommendation was made for designing a heat exchanger with a single line of cylinders. The formulas are as follows:

$$Nu = 0.261 \cdot (x/d)^{\frac{3}{4}} \cdot [1 - \exp\{-2.22 / ((x/d)^{\frac{3}{4}} - 1)\}] \cdot Ra^{\frac{1}{4}} \quad (3.14)$$

where  $x$  = center-to-center cylinder separation distance measured from the bottom cylinder, and  $d$  is the cylinder diameter. He predicts that, for  $x/d=1$  to 15, at  $Gr=1.2 \cdot 10^5$ , the error will be within 9%. The formula for predicting the average Nusselt number of an array is as follows:

$$Nu = \sum_{i=0}^n Nu_i / N = 0.41 \cdot Ra^{\frac{1}{4}} \cdot \ln\left[\left(\frac{b}{d}\right) / 1.3\right]^{0.055n} + 0.434 \quad (3.15)$$

The parameter  $N$  is the number of cylinders in the array. Equation (3.15) represents experimental data within  $\pm 10\%$  in the ranges of  $[(b/d)/1.3] = 0.7$  to 1.2 and  $Gr = 4 \cdot 10^4$  to  $4 \cdot 10^5$ .

Equations (3.14), (3.15) apply for the constant-surface-temperature boundary condition. Closely related to this work, but still not suitably described, is the case of cylindrical arrays in liquid metals. Some comparisons between

these relationships and the present work will be presented  
in Chapter 5.

## CHAPTER IV

## EXPERIMENTAL APPARATUS AND PROCEDURE

## 4.1 Test section and thermocouple probe

A photograph of the entire equipment setup is shown in Figure 4.1 and a closeup of the mercury vessel with the test section is shown in Figure 4.2. Individual items of the apparatus used in this experiment are :

a data acquisition system, HP-3054; a micro-computer, HP-85; a Vernier x-y positioner mounted on a test section; a Sorensen DCR 300-351 DC power supply; a Haake model E 51 proportional controller; a mercury vessel contained in a water tank; and two electric-motor-driven stirrers.

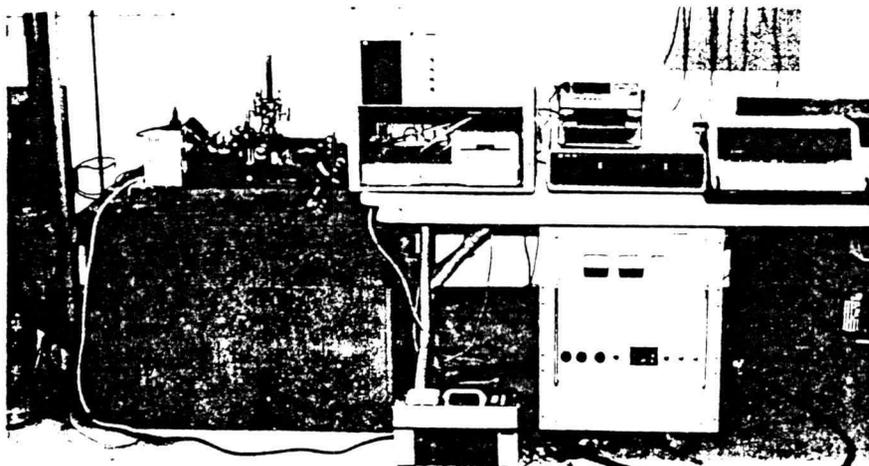


Figure 4.1 Overall system arrangement

The geometry used for the test section in this work was an array of cylinders, mounted with their axes horizontal and in a vertical plane. Five different cases were studied:

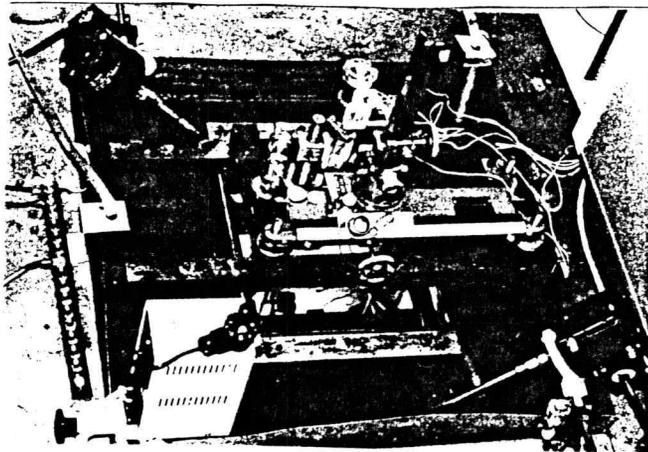


Figure 4.2 Closeup of mercury vessel and test apparatus

1. A single cylinder which was used to obtain data for comparison with prior correlations and for base line data in later studies with cylinders in an array.
2. Two cylinders with a separation distance of  $2*D$  .
3. A pair of cylinders with  $s = 3*D$ .
4. A pair of cylinders with  $s = 4*D$
5. Three cylinders with  $s = 2*D$ .

The distance of separation,  $s$ , is the center-to-center spacing between cylinders.

In actual heat exchangers cylinder diameters are typically in the range from 0.5 to 2 in. In this work 1.365 in. diameter cylinders were used.

The heated cylinders were constructed by attaching an electrical resistance heating unit to an acrylic core. The three-cylinder case is shown in Figure 4.3.

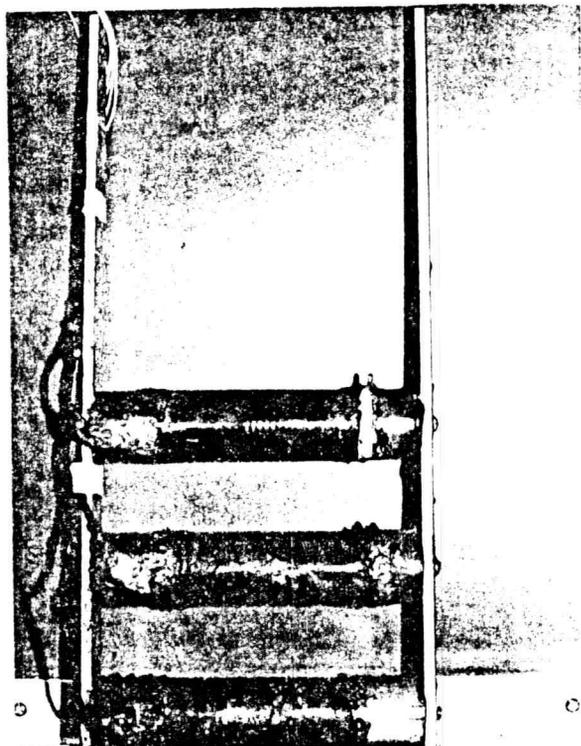


Figure 4.3 Three cylinders mounted in a vertical plane

The heater element was fabricated by Electrofilm Inc. and consisted of a chemically etched 0.001 inch thick nichrome foil bonded on both sides to fiberglass reinforced silicone rubber insulation as shown in Figure 4.4.

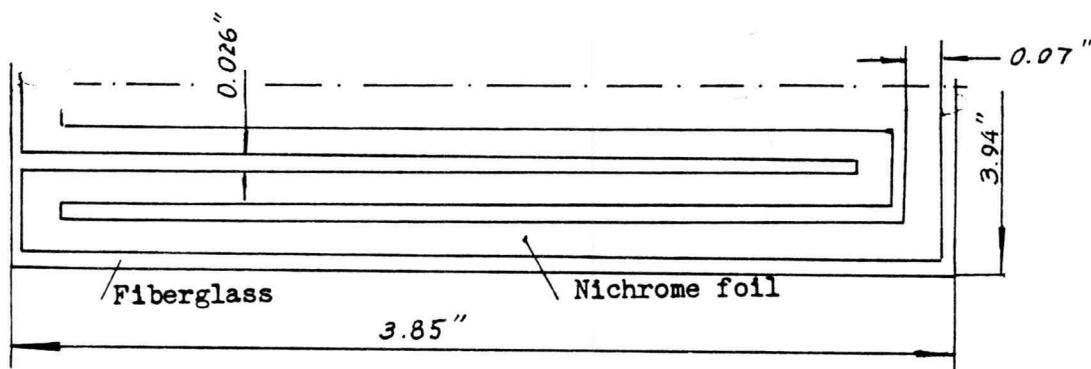


Figure 4.4 Typical heater unit details

The heater unit was placed on a 1.25-inch-diameter cylindrical die and heat treated to form the cylindrical shape.

The teflon-insulated lead wires emerged from a 0.75x0.575 inch rectangular tab at the top of each heater. Figure 4.2 shows the details and dimensions. The normal thickness was 0.045 inches and the height was 3.85 inches . The nichrome elements covered approximately 75% of the heater area. The electrical resistivity of this chemically etched nichrome foil is extremely constant so that the heat flux generated by these heaters was quite uniform.

The heater units, as received from Electrofilm, were placed on a ten inch long cast acrylic cylindrical core. The section of the cores where the units were placed were turned to a diameter of 1.25 inches so that the heaters closed snugly at the seam. The acrylic core at either end of the heater section was machined to a diameter of 1.365 inches which closely matched the average diameter of the finished heater section.

Three layers of two-sided tape ( 1 inch wide and 0.003 inches thick) were wrapped tightly around the heaters to hold them to the cores. Cylindrical steel cover plates, 0.01-inches thick, of the same size as the heaters were pressed onto the tape by means of a tightening jig. Then another core with diameter of 1.365 inches was attached to the outside surface of the heater to form the complete heating unit in which the heated part is in the middle, acrylic cores are at both ends and all parts are the same diameter ( 1.365 inches).

The final step in the fabrication process was to seal carefully both edges of the heater and seams with silicone rubber and epoxy. When the heater was placed into the mercury, care was necessary to prevent electrical short circuits caused by mercury penetrating under the steel plates.

Temperatures were measured with a thermocouple probe which was epoxied to a frame of 0.125 inches diameter. The frame was shaped so as to place the probe nearly perpendicular to the surface of the cylinder. A sketch of this arrangement is shown in Figure 4.5.

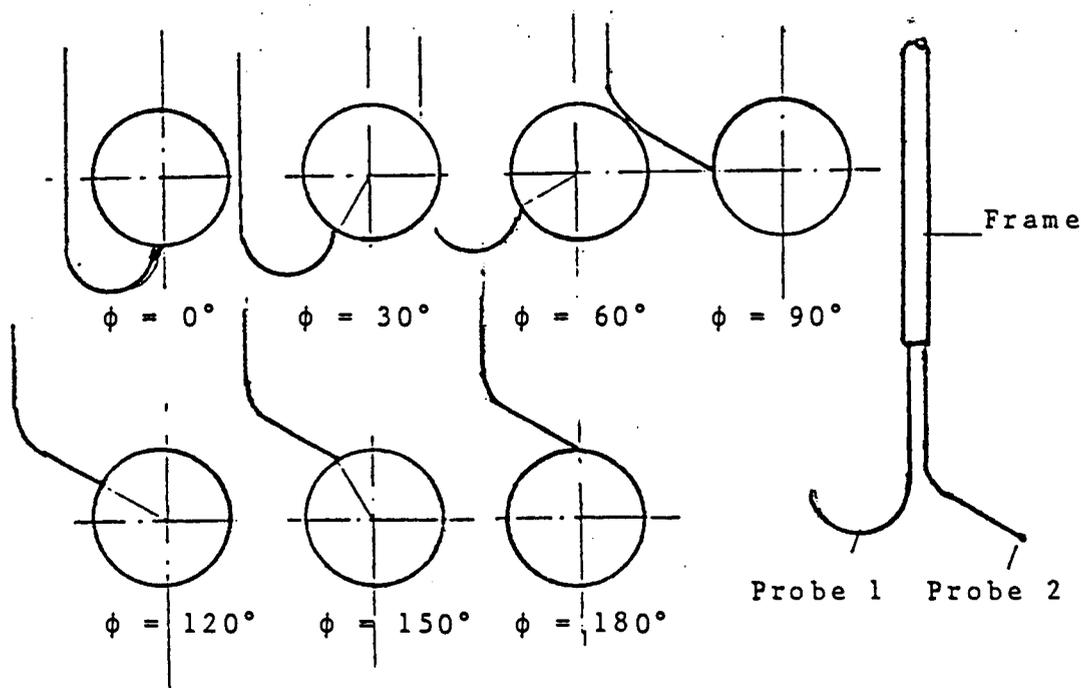


Figure 4.5 The thermocouple probe and frame.

The thermocouple was a 0.01-inch-diameter sheathed, copper-constantan unit manufactured by Omega Engineering. A

schematic of the thermocouple assembly is shown in Fig.4.4. The assembly of the thermocouple, as shown in Figure 4.6, consisted of 0.001 inch diameter copper and constantan wires which were inside a stainless steel sheath and insulated from the sheath and each other by refractory material. The extremely small size of the resulting junction allowed essentially point temperatures to be recorded.

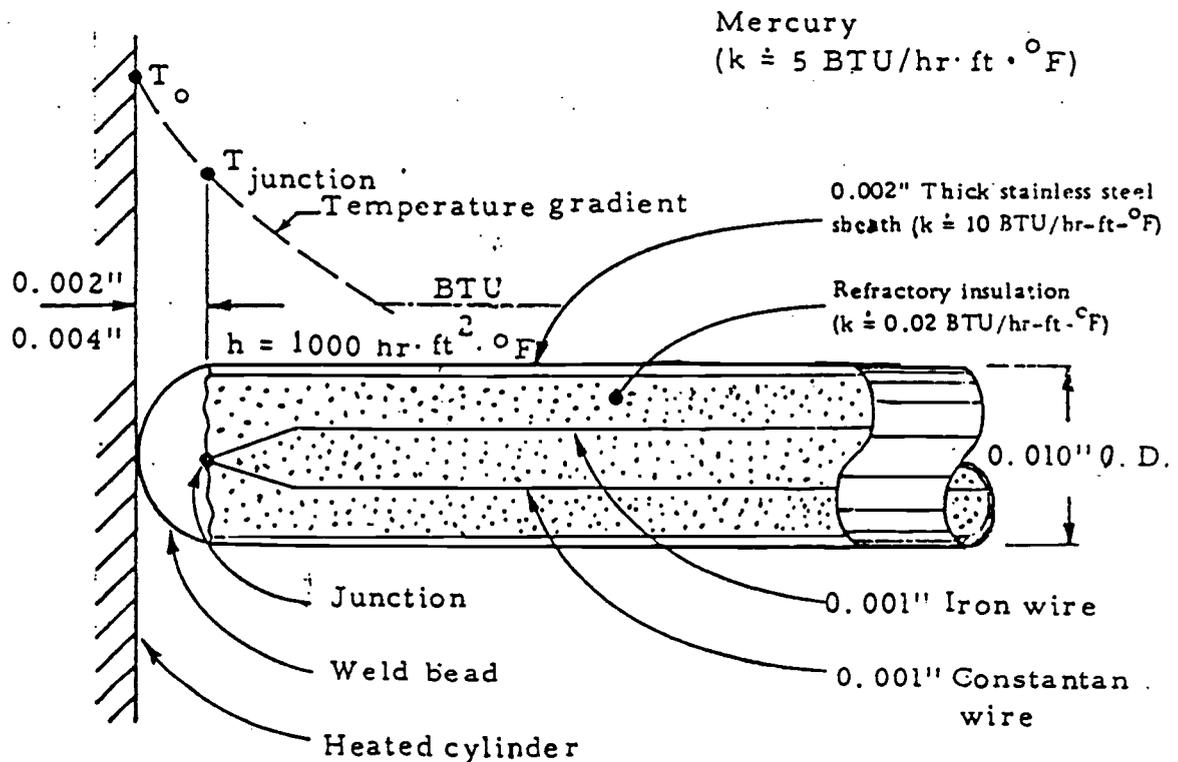


Figure 4.6 The thermocouple probe assembly

The low thermal resistance at the sheath-mercury interface and negligible thermal resistance of the stainless steel sheath led to the conclusion that the sheath is essentially at the same temperature as the mercury at any given location along the probe.

#### 4.2 Mercury vessel and thermal control

The mercury pool was contained in a 12 x 12 x 16-inch-deep stainless steel tank, with a stainless steel angle-iron lip at the top which was used for mounting the probe and traverse assembly. The tank held about 1,000 lb of mercury.

The tank was placed on pads for isolating the test section from the environment, both for vibration and heat exchange.

Surrounding the tank was a mild steel box which contained an isothermal water bath. The water was kept in circulation with two electric-motor-driven stirrers and a pump which was a part of the temperature controller. An outside box containing insulation held the entire apparatus.

The water temperature was controlled by a Haake model E 51 proportional controller, which included a 1 kW heater. Running water was used to remove heat from the water bath through two immersed coils made of 3/8 inch copper tubing. The water bath contained about 35 gallons of water.

#### 4.3 Probe traverse mechanism and position control

A vernier x-y positioner capable of +0.001-inch accuracy was used for positioning the probe. The device had a

leveling bubble and four adjustable legs to keep the probe motion exactly along the horizontal and vertical directions.

Measurements were made at 30 degree intervals around the test cylinder. Before each test sequence the probe position relative to the test cylinder was calibrated and stored in the HP-85 computer. For each measurement the computer specified the x and y coordinates. The probe was then placed at the proper position by the traverse mechanism.

The coordinates of each measuring position at the surface of the test cylinder were checked in air. Care was taken to make sure that the thermocouple touched the surface of the test cylinders at the innermost radial position for each angular position.

#### 4.4 Power supply circuit

Power was supplied to the electrical heating units by a Sorensen DCR 300-351 DC power supply. Fluctuations in the power supply were less than 2% during the experiment.

The heat flux generated in the test section was measured electrically by reading the voltage. The electrical resistance of the heater had been carefully measured previously with an ESI model 300 PVB resistance bridge with a specified accuracy of  $\pm .02\%$ . A simple application of Ohm's law yields for the power dissipation in the heater:

$$P = V^2/R_i$$

The power dissipation per unit area then was calculated for voltages of 29.72, 42.03, 51.50, 72.83, 89.21, and 126.15V. Corresponding heat fluxes were 333, 666, 1000, 2000, 3000, and 6000 Btu/hr·ft<sup>2</sup>.

#### 4.5 Procedure

The first thing required in the process was the establishment of thermal equilibrium. After starting the cooling water flow and turning on all of the equipment, the program "start" was initiated. This program set the data acquisition system to read 5 environmental temperatures every 5 minutes after the first hour. The results were recorded on the HP-85 . When the variation in the average temperatures was less than 0.2% for 5 minutes steady-state conditions were considered to be established. The system was then ready for recording temperature data.

Even in a condition considered to be a steady state, fluctuations in the temperature of the heated cylinder were too large to be neglected. For the lowest cylinder the change in temperature was approximately  $\pm 0.02^\circ\text{F}$ , and, at the upper cylinder, the temperature variation was approximately  $\pm 0.2^\circ\text{F}$ . Therefore an average value was used. For every data point, 50 measurements were obtained to establish the average value. The standard deviation was recorded for each mean temperature. If a standard

deviation value was considered too large a measurement was repeated.

## CHAPTER V

## HEAT TRANSFER RESULTS

## 5.1 Basic study

The primary goal of this study is to find the relationship among the fundamental natural convection variables such as heat transfer rate, surface and ambient temperatures, fluid properties, and surface geometry for cylindrical arrays in liquid metals.

From scale analyses the Nusselt number for a single uniformly heated horizontal cylinder for natural convection in mercury has been shown to be of the order  $Ra^{\frac{1}{4}} * Pr^{\frac{1}{2}}$ . The functional relationship can be written as

$$Nu = B * Ra^{\frac{1}{4}} * Pr^{\frac{1}{2}}$$

where  $Ra = Gr * Pr$ ,  $Pr = \frac{\mu * C_p}{k}$  ; B is an unknown constant which must be obtained from experiment. The Grashof number for the cylinder is:

$$Gr = \frac{g \beta \Delta T \rho^2 D^3}{\mu^2} \quad (5.1)$$

The temperature dependence of  $k, \mu, C_p$ , and  $\rho$  were accounted for by evaluating them at an appropriate reference temperature. Properties of mercury were evaluated from the following expressions :

$$k = 4.47924 + 8.30958 \times 10^{-3} T_r - 3.80163 \times 10^{-6} T_r^2$$

(Btu/hr. ft. °F)

$$C_p = 3.3462 \times 10^{-3} - 3.93353 \times 10^{-6} T_r + 3.44649 \times 10^{-9} T_r^2$$

(Btu/lb. °F)

$$\mu = 4.3462 - 9.91162 \times 10^{-3} T_r + 1.7906 \times 10^{-5} T_r^2 - 1.27524 \times 10^{-8} T_r^3$$

(lb<sub>m</sub>/ft hr)

$$\rho = 851.514 - 8.6488 \times 10^{-2} T_r + 9.86194 \times 10^{-6} T_r^2 - 5.92566 \times 10^{-9} T_r^3$$

(lb<sub>m</sub>/ft<sup>3</sup>) (5.2)

where  $k$  is the thermal conductivity,  $C_p$  is heat capacity,  $\mu$  is the coefficient of viscosity,  $\rho$  is the density, and  $T_r$  is the reference temperature. Sparrow and Gregg (36) suggested the appropriate reference temperature to be

$$T = 0.7 T_o + 0.3 T_\infty .$$

In the temperature range of this work, the property which varied the most was the viscosity; its variation was less than 2%. Physical property changes were, clearly, not widely variant.

The coefficient of thermal expansion,  $\beta$ , and gravity,  $g$ , were effectively constant, with the following values:

$$\beta = 0.000101 \text{ (1/}^\circ\text{F)}$$

$$g = 32.174 \text{ (ft/sec}^2\text{)}$$

By calculating  $Gr$  and  $Pr$  for each set of data the constant  $B$  was obtained at each different condition and the exponent,  $1/4$ , could also be verified.

## 5.2 Single cylinder case

A set of heat transfer data was taken for the uniformly heated single horizontal cylinder. Four different heat fluxes were controlled by adjusting the heater voltage. The cylinder diameter was 1.365" - the same as in the array configurations.

The primary reason for studying this case was to verify the data taking process and to check the related equipment set up. Also, a basic correlation for the single horizontal cylinder, obtained in this investigation, could be compared with other work.

After evaluating the average surface temperature (50 data points for each position) and the average Nusselt number (7 positions and 30 degrees each) six mean Nusselt numbers for  $q'' = 333, 666, 1000, 2000, 3000, \text{ and } 6000$  Btu/hr ft<sup>2</sup> were obtained. From  $B = Nu / (Ra * Pr)$  it was found that the constant, B, compared very closely with results obtained by Lefevre, Saville and Churchill(6). Results are list in Table 5.1, and plotted in Figure 5.1.

Table 5.1 Constant B for the single cylinder case

Heat flux ( Btu/ hr ft <sup>2</sup> )	Nu	B	$\sigma$
333	6.57	0.609	1.429
666	7.46	0.581	1.499
1000	8.25	0.581	1.925
2000	10.33	0.612	2.599
3000	11.22	0.599	3.257
6000	13.58	0.610	4.014

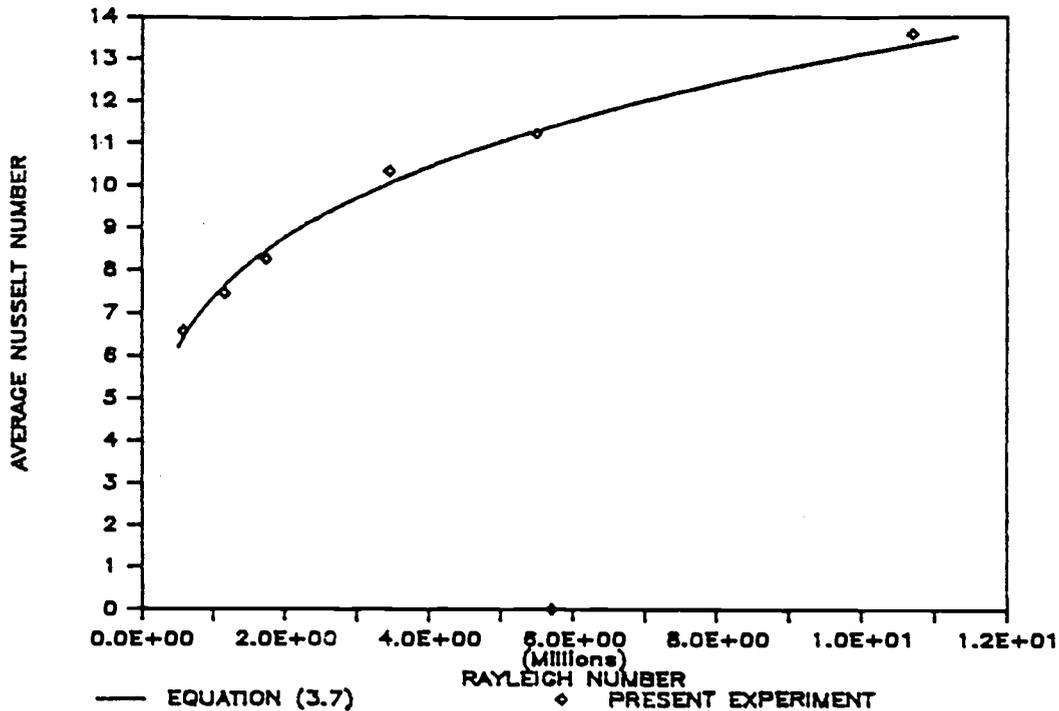


Figure 5.1 Comparison of single cylinder result

The curve in Figure 5.1 represent equation (3.7) which is

$$Nu = 0.599 * Ra^{\frac{1}{4}} * Pr^{\frac{1}{4}} \quad Pr \rightarrow 0 \quad (3.7)$$

The fact that the correlating equation for the single cylinder case compares well with data indicates that equation (3.7) is a good representation for  $Pr=0.0235$  with Rayleigh numbers in the range from  $5 \times 10^5$  to  $1.06 \times 10^7$ . Agreement between the data and this particular relationship from the literature lends strong support to the results of scale analysis, as well as to this experimental study.

### 5.3 The two horizontal cylinder case

Attention will now be directed to the Nusselt number for an interacting pair of horizontal cylinders of the same

diameter. Three different spacings with four different heat flux values ( $q''=333, 666, 1000, 2000, 3000, \text{ and } 6000$  Btu/hr·ft<sup>2</sup>) were tested. The center-to-center distance between the lower cylinder and the upper cylinder,  $s$ , was set at 2, 3, and 4 cylinder diameters.

Data reduction was carried out for both cylinders. The Nusselt number values for the lower cylinder were found to be very close to those obtained for a single cylinder with the same heat flux and same ambient temperature. The upper cylinder will thus be considered in greater detail.

The data show the Nusselt number of the uniformly heated upper cylinder for natural convection not to be just a function of the Rayleigh and Prandl numbers, It is also a function of the ratio of the separation distance to cylinder diameter.

This observation leads to two questions: 1) Can the correlation expressed as  $Nu = B' * Ra^{\frac{1}{4}} * Pr^{\frac{1}{4}}$  be used to describe this configuration? If so, what is the relationship between  $B/B'$  and  $S/D$ ? The results shown in Table 5.3 and Fig 5.2 suggest an affirmative answer to the first of these questions.

Table 5.2 Nusselt number and the constant B' for the upper-cylinder at given Heat flux.

S/D	Heat flux (Btu/hrft <sup>2</sup> )	Nu	B'	$\sigma$
2	333	6.172	0.572	1.347
	666	7.026	0.547	1.334
	1000	7.863	0.553	1.500
	2000	10.282	0.609	2.343
	3000	11.456	0.612	2.967
	6000	13.422	0.603	3.624
3	333	6.264	0.589	1.364
	666	7.286	0.567	1.492
	1000	8.121	0.572	1.471
	2000	10.705	0.634	2.314
	3000	12.206	0.652	2.875
	6000	14.715	0.661	3.617
4	333	6.436	0.596	1.319
	666	7.465	0.581	1.546
	1000	8.378	0.589	1.460
	2000	10.816	0.640	2.383
	3000	12.232	0.654	2.994
	6000	14.956	0.672	3.726

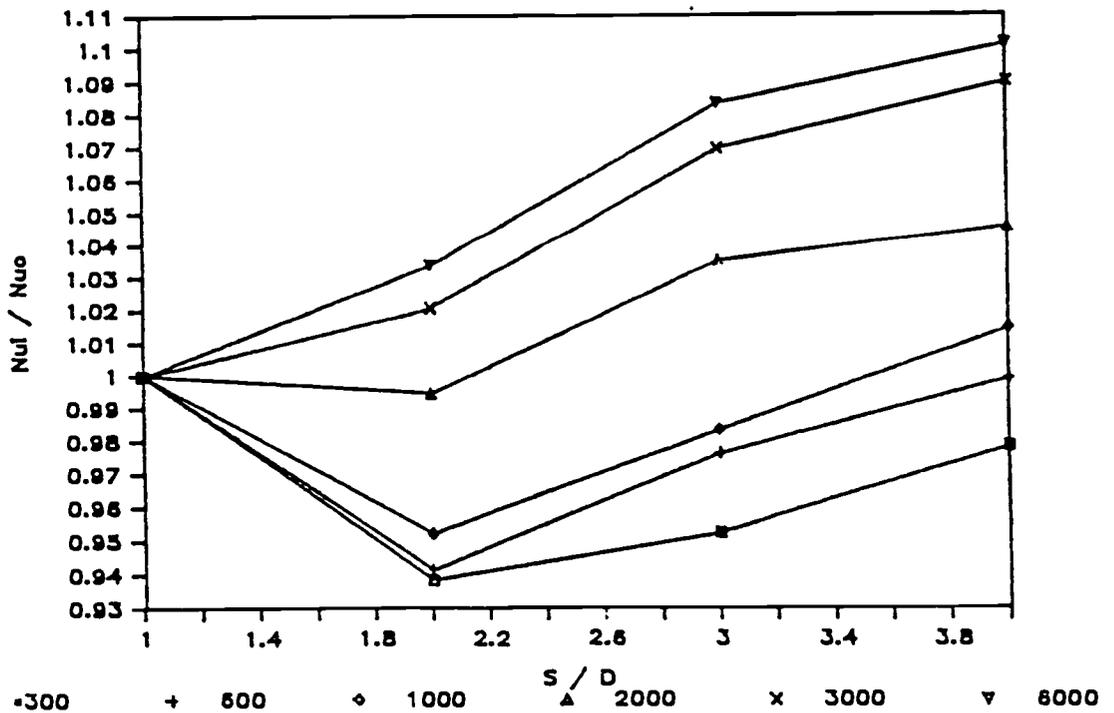


Figure 5.2 Effect S/D on Nu of upper cylinder For two-cylinder case

From these data the constant  $B'$  is clearly a function of the separation distance,  $S$ . From physical considerations, it is clear that the mercury arriving at the upper cylinder has initial motion due to the buoyancy created by the lower cylinder. Therefore, the presence of the lower cylinder should enhance the heat transfer coefficient at the upper cylinder relative to that for the single cylinder, and similarly the Nusselt number. Consequently, the empirical formula to predict the average Nusselt number for the upper cylinder should involve the functional relation  $Nu/(Ra^{\frac{1}{4}} Pr^{\frac{1}{4}}) = f(s/D)$ . When  $S/D$  changes, the velocity of mercury around the upper cylinder changes, and  $Nu$  for the upper cylinder must also change.

On the other hand, heat transfer at the lower cylinder tends to raise the temperature of the mercury arriving at the upper cylinder to a value higher than that of ambient mercury. Thus, the lower cylinder acts as a preheater. This preheating effect will decrease the heat transfer coefficient for the upper cylinder compared with that which would exist if the lower cylinder were absent. The empirical formula should, thus, consider another functional relation  $Nu/f(S/D) = f'(Ra, Pr)$ .

Referring to Table 5.3, the Nusselt number for the top cylinders is related to the Nusselt number for the bottom cylinder with each experimental condition. The remaining problem is to determine the functions  $f(S/D) = Nu/Ra^{\frac{1}{4}} * Pr^{\frac{1}{4}}$  for

each case. Using the data for  $S/D=2,3,$  and  $4$  and assuming the simplest equation that satisfies these data, the following expression can be obtained.

$$Nu = 0.65 * Ra^{\frac{1}{4}} * Pr^{\frac{1}{4}} * ((S/d)^{0.06} - .126) \quad (5.3).$$

A comparison between the equation (5.3) experimental work is shown in Figure 5.3 and Figure 5.4.

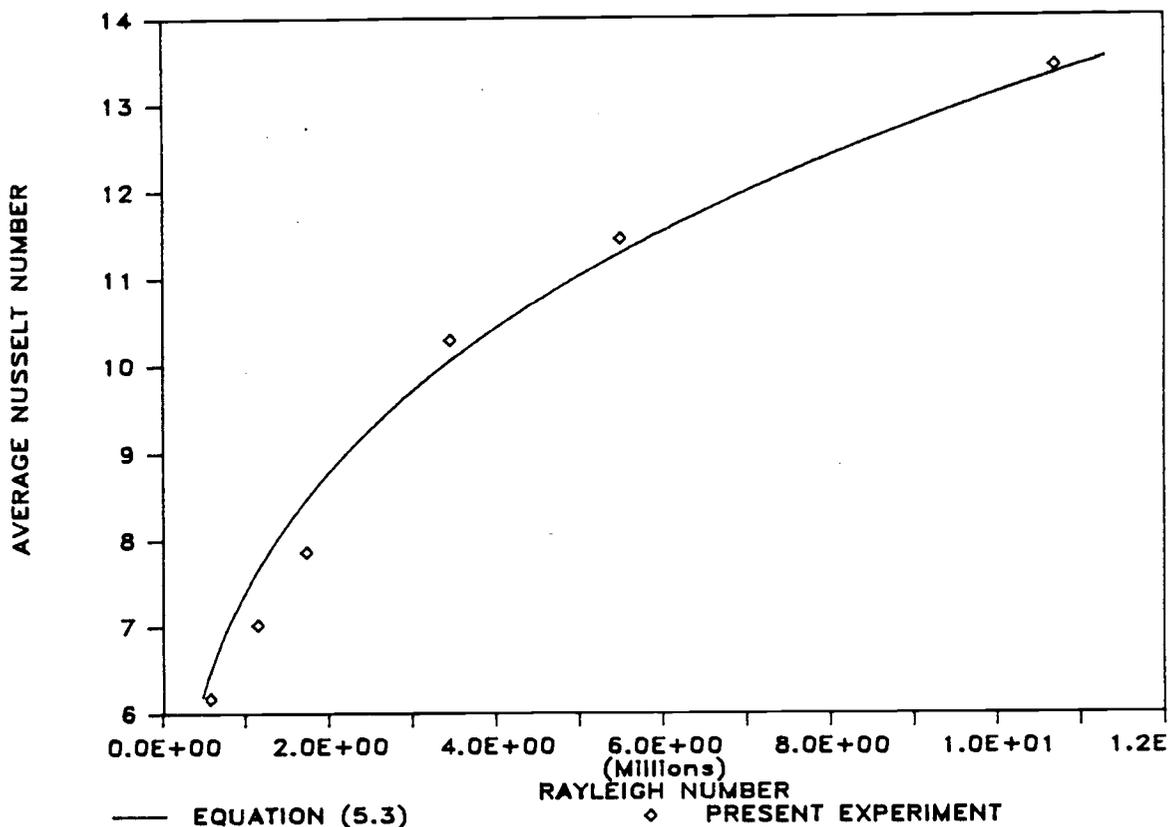


Figure 5.3 Comparison for the upper cylinder in two-cylinder case ( $S/D=2$ )

Differences between equation (5.3) and with data from the experiment are less than 8% for pairs cylinders with  $S/D$  from 2 to 4 and values of Rayleigh number from  $5 \times 10^5$  to  $1.062 \times 10^7$ .

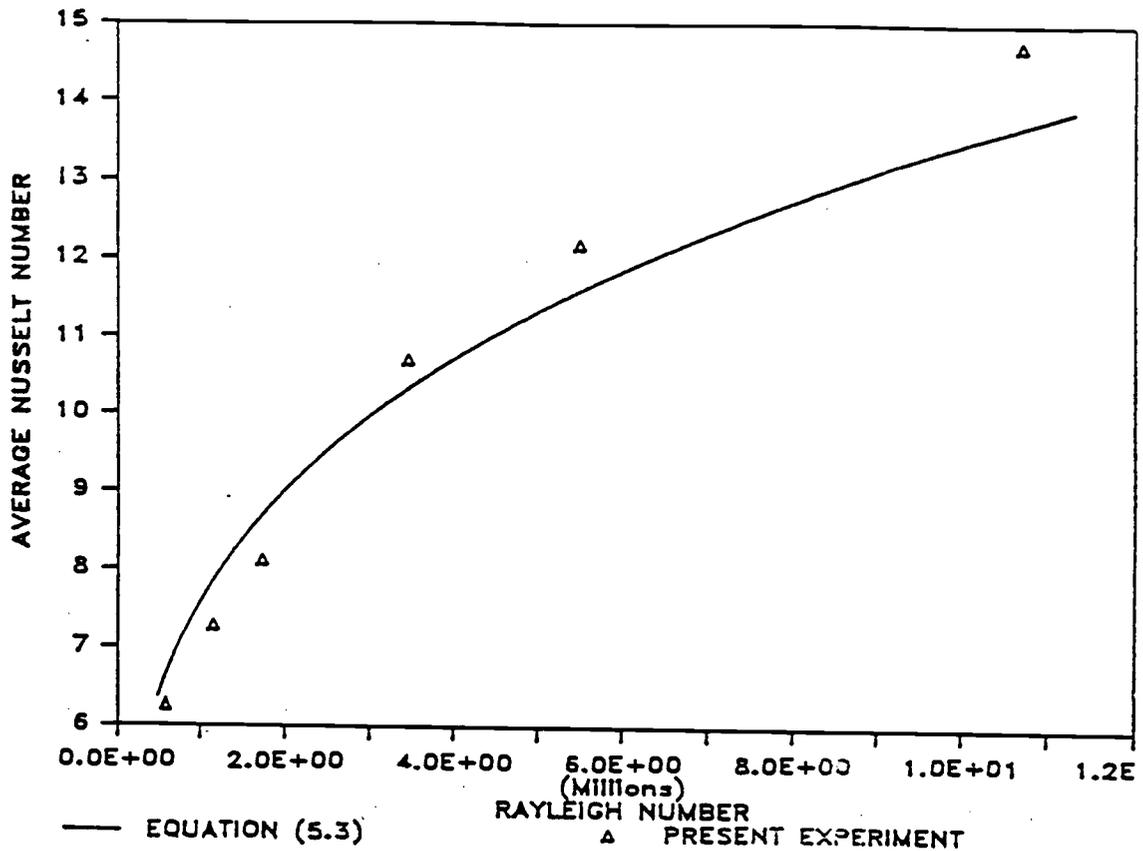


Figure 5.4 Comparison for the upper cylinder in two-cylinder case ( $S/D=3$ )

#### 5.4 Three cylinder case

In the three cylinder case, the fluid arriving at the top cylinder is in motion induced by the two lower ones. The top cylinder in this case is situated in a stronger forced convection flow than the top cylinder in the two cylinder case with the same conditions. Consequently, the presence of two lower cylinders should enhance the heat transfer at the top cylinder relative to that for the two cylinder case. At the same time, however, heat transfer at the

lower cylinders tends to raise the temperature of the mercury which is adjacent to the top cylinder to a value higher than that of the two cylinder case. Thus this pre-heating effect, as mentioned in discussing the two cylinder case, tends to decrease the heat transfer coefficient for the top cylinder compared with the two cylinder case.

The experiments for the three cylinder case with  $S/d = 2$  were performed. The cylinders diameters were the same as in the single and two cylinder cases. The heat fluxes, were the same as before ( $q'' = 333, 666, 1000, 2000, 3000, \text{ and } 6000 \text{ Btu/hrft}^2$ ). It is clear from Table 5.4 and Figure 5.5 that the bottom cylinder behaves almost as if the other cylinders were not present. The middle cylinder shows obvious interference effects. It is located in the wake region of the bottom cylinder and it, in turn, induces a wake which affects the upper cylinder. As in the two cylinder case the Nusselt number of the middle cylinder is a strong function of separation distance.

Table 5.3 Nusselt number ratio for three cylinder arrays

cylinder	Heat flux (Btu/hrft <sup>2</sup> )	Average Nusselt number	Nui/Nuo
Bottom	333	7.340	1.0
	666	7.459	1.0
	1000	8.263	1.0
	2000	10.344	1.0
	3000	11.745	1.0
	6000	13.614	1.0
Middle	333	6.342	0.864
	666	6.699	0.898
	1000	7.809	0.945
	2000	10.282	0.994
	3000	11.721	0.998
	6000	13.600	0.999
Upper	333	5.969	0.807
	666	6.491	0.870
	1000	7.504	0.908
	2000	9.630	0.931
	3000	11.041	0.942
	6000	12.906	0.948

From this table the Nusselt number is seen to actually decrease for the top and middle cylinders. This is due to the small separation distance  $S/D = 2$ . Rayleigh numbers are in the range from  $5 \times 10^5$  to  $1.062 \times 10^7$ . Compared with the two cylinder case we can expect the result for the three cylinder case to display the same tendency, i.e., the upper cylinders will likely display reduced Nusselt numbers at close spacing and enhanced Nusselt numbers at larger spacings.

By taking the arithmetic mean of the Nusselt numbers for an array, the heat transfer can be represented in the same functional form as is used for a conventional single

cylinder. For the limited conditions of this experiment it can only be used in the three cylinder case with  $S/D=2$ . The following formula predicts the average Nusselt number of an array with  $S/d$  close to 2. that is

$$Nu = \frac{\sum_{i=1}^n Nu_i}{N} = 0.62 * Ra^{\frac{1}{4}} * Pr^{\frac{1}{4}} * (\ln(Ra)/20 + 0.224) \quad (5.4)$$

The errors of this equation are within 10% in the range between  $Ra=5 \times 10^5$ , and  $1.062 \times 10^7$ .

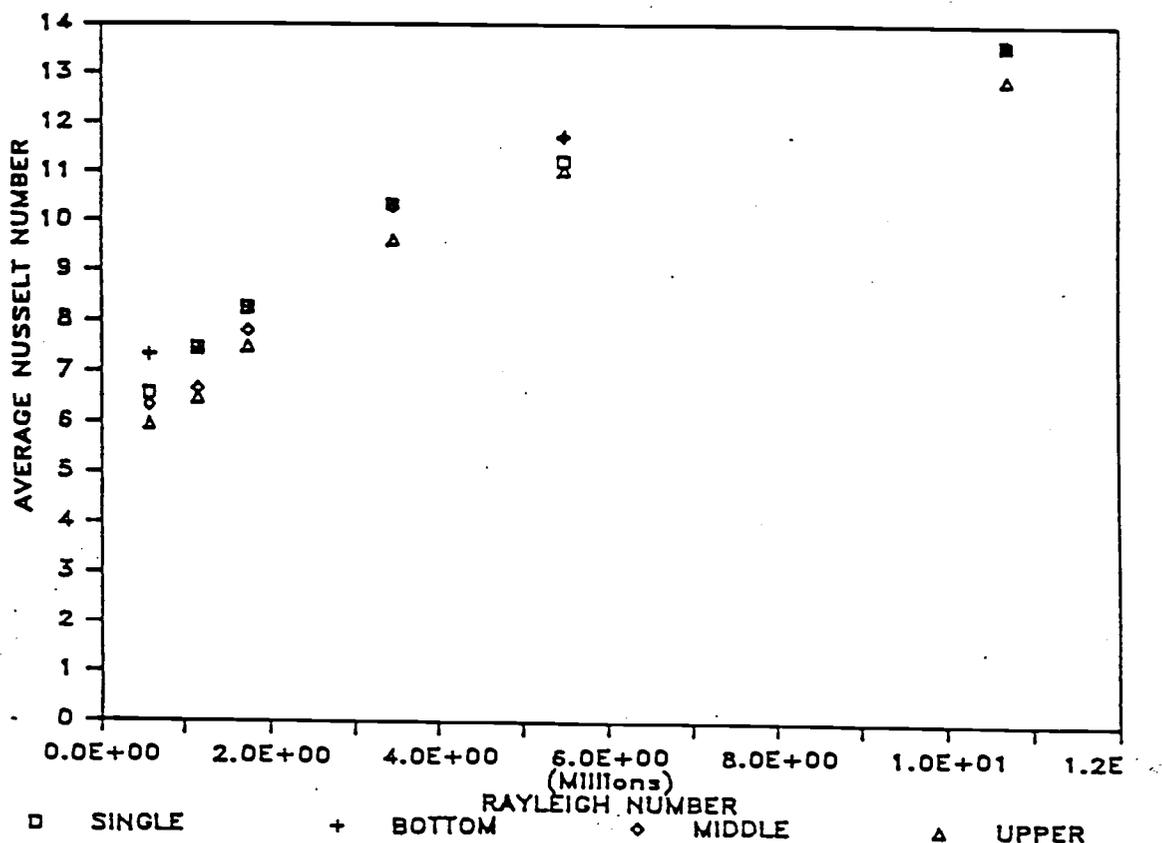


Figure 5.5 Comparison of single and three cylinder cases

## CHAPTER VI

## LOCAL HEAT TRANSFER AND LOCAL TEMPERATURE MEASUREMENT

## 6.1 Local heat transfer

Heat transfer correlations, such as those presented in the preceding chapter, can be used quantitatively in specific design applications. Characteristics of the liquid metal temperature field are also important. A knowledge of these characteristics allows a qualitative conclusion to be drawn concerning the validity of assumptions made in analytical work.

In Fig.6.1 curves are presented showing the local Nusselt number at the surface of a single cylinder, around which mercury and air are flowing. It is evident from this

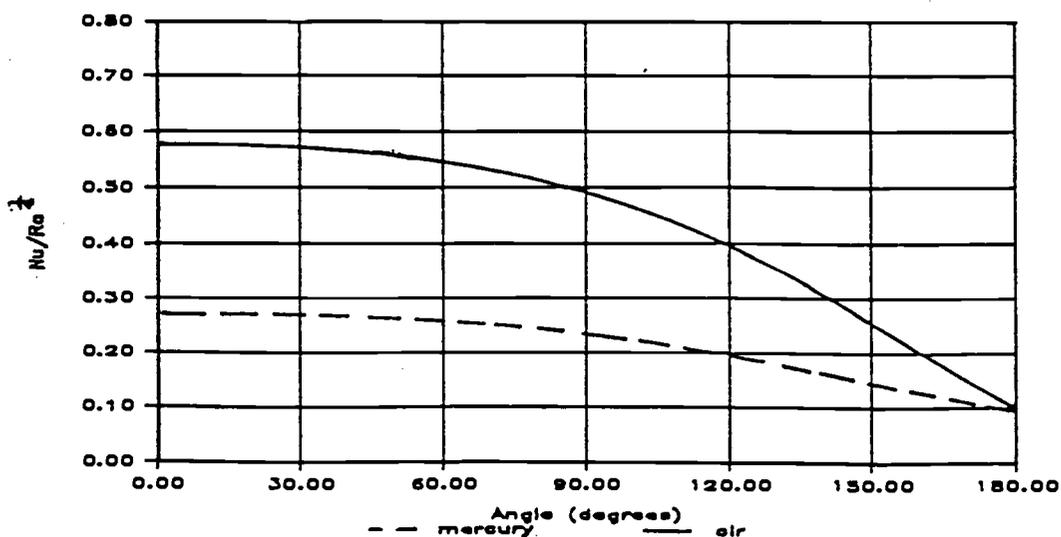


Fig. 6.1 Local Nusselt numbers around the perimeter of a cylinder with natural convection in 1) air [Kuehn and Goldstein (22)] and 2) mercury (present experiments)

figure that the higher conductivity of the mercury and smaller Pr number, which reduces the intensity of turbulence, substantially alter the heat transfer distribution for the cylinder compared with the cases of the flow of a gas or nonmetallic liquid around a cylinder.

Local Nusselt numbers for cylinders in an array are quite different from the case of a single horizontal cylinder due to mutual interaction between the cylinders. In the following discussion the variation of the local Nusselt number for cylinders will be described as a function of angular position,  $\phi$ , measured from the lower stagnation

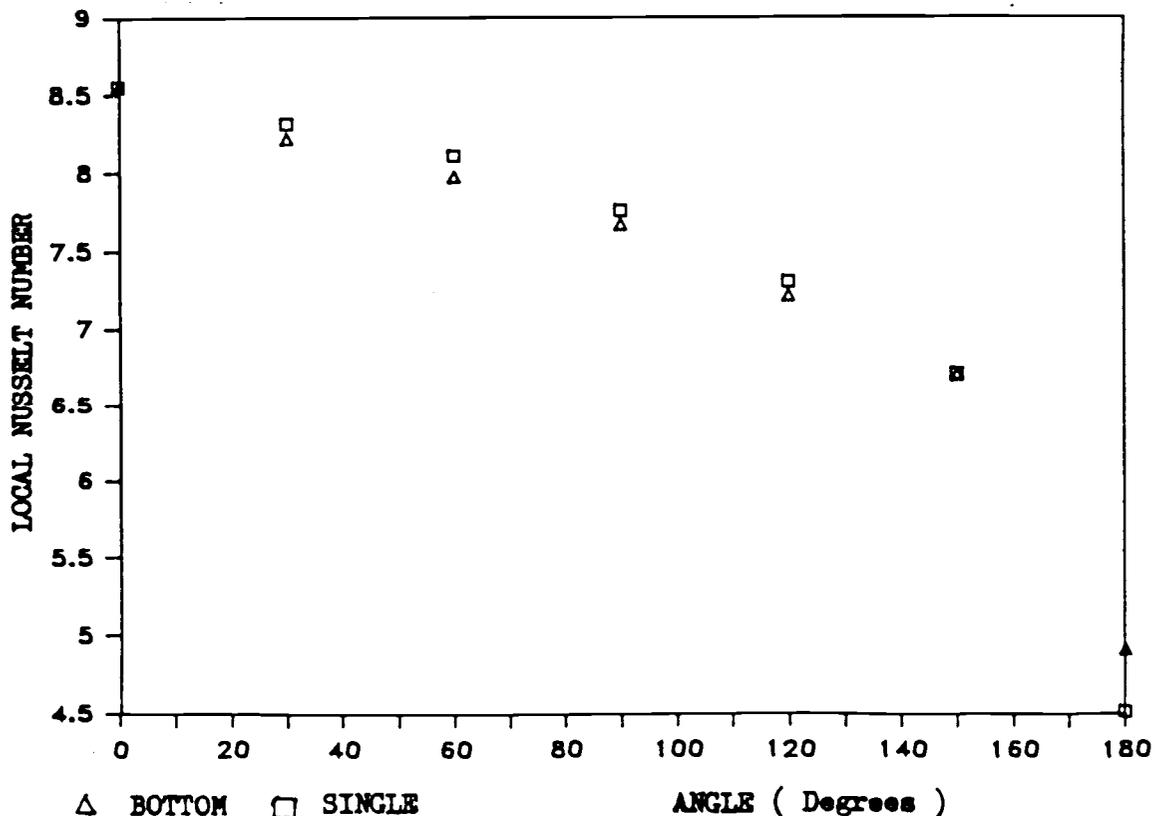


Figure 6.2 Comparison of local Nusselt number between the bottom-most cylinder and single cylinder (two-cylinder case  $S/D = 2$ )

point. The local Nusselt number for the bottom-most cylinder is almost the same as that of a single cylinder except for the region near  $\phi=180$  degrees. In this region the Nusselt number varies with the spacing. For this experiment with spacing,  $S=2D$ , the Nusselt number was observed to increase slightly as in Figure 6.2.

The local Nusselt numbers for the second cylinder show obvious interference effects since its lower surface is affected by the wake flow from the first cylinder and the upper surface interacts with the third cylinder. Local

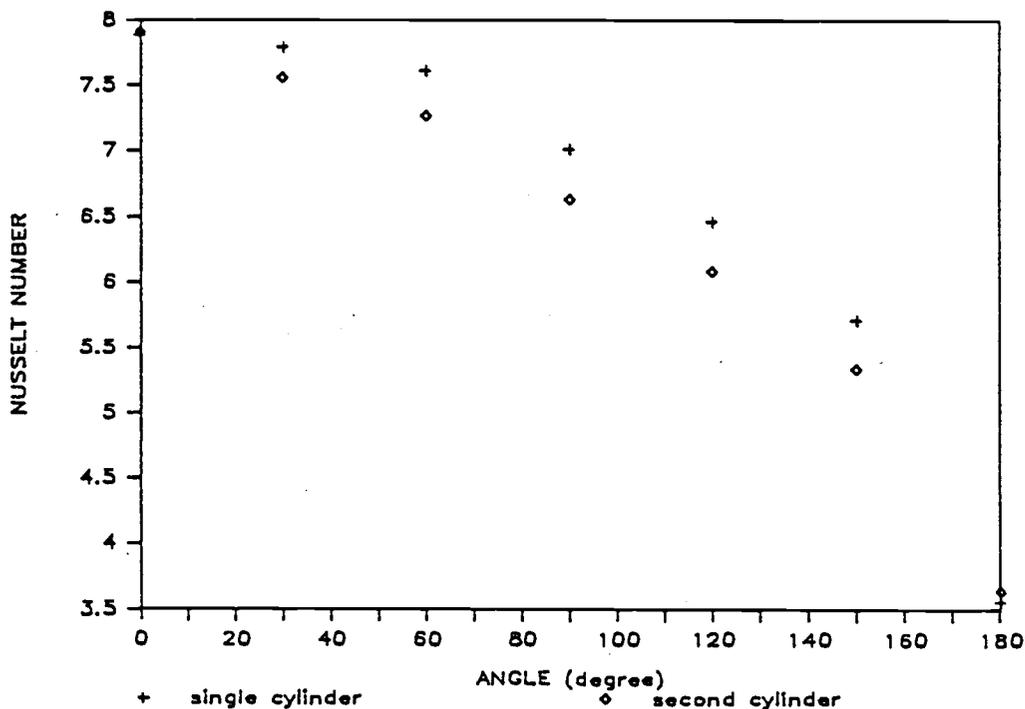


Figure 6.3 The local Nusselt number for the second cylinder (two-cylinder case  $S/D=2$ )

Nusselt numbers for the middle cylinder were observed to increase slightly near  $\phi=0$  and  $\phi=180$  degrees, when  $s=2D$ . If the spacing is much smaller than  $s=2D$  Nusselt number values for the second cylinder would significantly decrease. In the case where the spacing is very small, the buoyant flow around a cylinder cannot penetrate the narrow space between cylinders and "dead regions" form.

## 6.2 Local temperature measurement

Two methods have been used for local temperature measurements. One of them was described in chapter 3. Because of the possible effect on the flow patterns caused by the presence of the probe and its support a fixed probe was mounted under the surface of the heated cylinders. This arrangement, although eliminating the possibility of an external probe influencing the flow, does present another effect which must be considered, viz. that due to the temperature gradient at the steel cover sheet and the air gap between the steel cover and the heater. After considering these factors both methods are considered adequate for this local temperature measurement. (for details see Appendix A3.)

## CHAPTER VII

## CONCLUSIONS AND RECOMMENDATIONS

## 7.1 Conclusions

The experimental investigation described in this thesis has provided new information concerning natural convection phenomena in liquid metals. While the results may be used quantitatively in certain practical applications, an additional contribution is the evidence concerning how certain important parameters affect the heat transfer for this configuration. Some empirical formulas have been proposed.

Experiments have shown that the heat transfer characteristics exhibited by arrays of heated horizontal cylinders are not predicted by simple superposition of single cylinder behavior in the case of liquid metals. For closely spaced arrays such as  $S = \text{one diameter}$ , individual tube Nusselt numbers are found to be smaller than for a single cylinder. For wider spacings, however, such as  $S = 3D$  or  $4D$ , Nusselt numbers are higher. Thus array spacing, Rayleigh number, and Prandtl number are the important factors for determining the overall Nusselt number of an array. Both scale analysis and experiments predict the Nusselt number of a horizontal, uniformly-heated cylinder in a liquid metal to be proportional to the 1/4th power of the product of Rayleigh number and Prandtl number (Boussinesq number).

Other main conclusions reached in this study may be summarized as follows:

1. The average Nusselt number of the bottom cylinder in an array is almost the same as for a single horizontal cylinder, regardless of the number of cylinders in the array, except when the cylinders are extremely close (less than  $S/D = 2$  ).

2. The empirical formula predicting the mean Nusselt number for the upper cylinder in a two-cylinder system has a form given by equation ( 5.3).

3. The mean Nusselt number for an array of three cylinders is expressed by equation (5.4).

4. Local Nusselt numbers for cylinders in an array are quite different from those of a single horizontal cylinder due to mutual interaction between the cylinders. The variation in local Nusselt number for cylinders can be described as a function of angular position,  $\phi$  , measured from the lower stagnation point. The local Nusselt number for the bottom cylinder in an array is almost the same as that for a single cylinder except for the region near  $\phi = 180$  degrees. In this region Nusselt numbers vary with the spacing. For cylinders in the wake of others the local Nusselt numbers in the regions where surfaces face another cylinder also depend on the spacing. In general, for the spacings used in this experiment, local Nusselt numbers in these regions slightly increase. If the spacings were very small they would

significantly decrease.

## 7.2 Recommendations

Work which would extend the results obtained here is recommended as follows:

1. An investigation of velocity distributions would be highly desirable. These results would allow conclusions to be drawn on flow regimes, influences on heat transfer due to the flow, and temperature distribution etc. These would be helpful in establishing proper conditions for analytical solutions.

2. To complete the picture regarding heat transfer results, investigations of other spacings, cylinder diameters, numbers of cylinders in arrays, and heated length are needed. In these investigations a highly stable environment is desired, and outside circulation for the mercury constant temperature control is suggested to avoid possible influence of the heat generation of the temperature control system.

3. Considering differences of effects at different separation distances, it is possible that the presence of lower cylinders might enhance the upper cylinder Nusselt number. There is a limit for this enhancement and the possibility exists that an optimum separation distance exists where the Nusselt number is a maximum.

There are several other issues which have arisen and stand unresolved, such as when the heat flux is high ( $q=6000$

Btu/hr·ft<sup>2</sup>) the Nusselt number distribution near the stagnation point is seem to be different than in the other cases; Also in industrial heat exchangers cylinders are frequently placed in channels. The effect of channel walls have not been considered in this work. There are some changes which involve a variation in the pressure distribution with influences on the velocity distribution outside the boundary layer and in the velocity gradient induced by the bottom cylinder. Some other studies have suggested that the average Nusselt number for the first cylinder in an array between plates greatly increases in comparison with that in the free space. This results in an increase in the average Nusselt number for an array of 10 to 15 percent in comparison with the case of open space. These effects might be studied in further research activities.

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**APPENDIX**

## Part 1 Computer Programs for the experiment

## A.1 START

```
10 ! PROGRAM START
15 ! THIS PROGRAM IS USED FOR THE ESTABLISHMENT OF THERMAL
20 ! EQUILIBRIUM.
25 ! THIS PROGRAM CAN BE RUN AT HP-85 AND HP-3497A DATA
30 ! ACQUISITION SYSTEM OR HP-86 AND HP-3497A WITH A
35 ! LITTLE CHANGE. (DEVICE 709 CHANGES TO 609)
40 ! LIST OF VARIABLES
45 !     V - VOLTAGE MEASURED FROM THERMOCOUPLES
50 !     T - TEMPERATURE CALCULATED FROM V
55 !     C1, C2, C3, C4 - COEFFICIENTS OF THERMOCOUPLE
60 !     TYPE "T"
65 !     T1(I), T2(I), T3(I), T4(I), AND T5(I) ARE
70 !     TEMPERATURES MEASURED AT ENVIROMENT POINTS 1,
75 !     2, 3, 4, AND 5, A1(J), A2(J), A3(J), A4(J) AND
80 !     A5(J) ARE THEIR AVERAGES.
85 ! THIS PROGRAM SET THE DATA AQUISITION SYSTEM TO READ 5
90 ! ENVIROMENTAL TEMPERATURES EVERY 5 MINUTES AFTER THE
95 ! FIRST HOUR. WHEN THE VARIATION IN AVERAGE
100 ! TEMPERATURES WERE LESS THAN 0.2% FOR 5 MINUTES STEADY
105 ! STATE CONDITIONS WERE CONSIDERED TO BE ESTABLISHED.
110 DIM T1(50)
115 DIM T2(50)
120 DIM T3(50)
125 DIM T4(50)
130 DIM T5(50)
135 DIM A1(50)
140 DIM A2(50)
145 DIM A3(50)
150 DIM A4(50)
155 DIM A5(50)
160 !
165 ! INPUT COEFFICIENTS OF T TYPE THERMOCOUPLE
170 !
175 C1=0.0256613
180 C2=-.0000000619549
185 C3=2.21816E-11
190 C4=-3.55009E-16
195 !
200 ! MEASURE THE TEMPERATURES
201 !
205 FOR J=1 TO 12
210 FOR I=1 TO 50
215 CLEAR 709
220 OUTPUT 709 ;"A1"
225 ENTER 709 ; V
230 GOSUB 700
235 T1(I)=T
240 V=0
245 CLEAR 709
```

```
250 OUTPUT 709 ; "AC2"
255 ENTER 709 ; V
260 GOSUB 700
265 T2(I)=T
270 T=0
275 CLEAR 709
280 OUTPUT 709 ; "AC3"
285 ENTER 709 ; V
290 GOSUBE 700
295 T3(I)=T
300 T=0
305 CLEAR 709
310 OUTPUT 709 ; "AC3"
315 ENTER 709 ; V
320 GOSUBE 700
325 T4(I)=T
330 T=0
335 CLEAR 709
340 OUTPUT 709 ; "AC3"
345 ENTER 709 ; V
250 GOSUBE 700
355 T3(I)=T
360 T=0
365 NEXT I
366 !
367 ! CALCULATE THE AVERAGE TEMPERATURES
368 !
370 O1=0
375 O2=0
380 O3=0
385 O4=0
390 O5=0
395 FOR I=1 TO 50
400 O1=O1+T1(I)
405 O2=O2+T2(I)
410 O3=O3+T3(I)
415 O4=O4+T4(I)
420 O5=O5+T5(I)
425 NEXT I
430 A1(J)=O1/50
435 A2(J)=O2/50
440 A3(J)=O3/50
450 A4(J)=O4/50
455 A5(J)=O5/50
460 IF J=1 THEN GO TO 520
465 K1=ABS(A1(J)-A1(J-1))
470 K2=ABS(A2(J)-A2(J-1))
475 K3=ABS(A3(J)-A3(J-1))
480 K4=ABS(A4(J)-A4(J-1))
485 K5=ABS(A5(J)-A5(J-1))
490 IF K1<0.002 THEN GOTO 495 ELSE 520
495 IF K2<0.002 THEN GOTO 500 ELSE 520
```

```
500 IF K3<0.002 THEN GOTO 505 ELSE 520
505 IF K4<0.002 THEN GOTO 510 ELSE 520
510 IF K5<0.002 THEN GOTO 515 ELSE 520
515 GOTO 600
520 WAIT 200000
530 DISP " THE EQUILIBIUM HAS NOT BEEN ESTABLISHED"
540 DISP " NOW NEXT MEASUREMENT BEGIN."
545 DISP J+1
550 NEXT J
600 FOR I=1 TO 10
605 BEEP 200,100
610 NEXT I
620 DISP " THE EQUILIBIUM HAS NOT BEEN ESTABLISHED"
630 DISP " NOW PROGRAM TEMPERATURE CAN BEGIN"
640 STOP
650 END
700 T=V*(C1+V*(C2+V*(C3+V*C4)))*1000000
705 RETURN
```

## A.2 TEMPERATURE

```

10 ! PROGRAM TEMPERATURE
15 ! THIS PROGRAM IS USED FOR COLLECTING AND STORE DATA
20 ! FROM EXPERIMENT OF NATURAL CONVECTION FROM AN ARRAY
22 ! OF CYLINDERS TO MERCURY
25 ! THIS PROGRAM CAN BE RUN AT HP-85 AND HP-3497A DATA
30 ! ACQUISITION SYSTEM OR HP-86 AND HP-3497A WITH A
35 ! LITTLE CHANGE. (DEVICE 709 CHANGES TO 609)
40 ! LIST OF VARIABLES
45 !     V - VOLTAGE MEASURED FROM THERMOCOUPLES
50 !     T - TEMPERATURE CALCULATED FROM V
55 !     C1, C2, C3, C4 - COEFFICIENTS OF THERMOCOUPLE
60 !     TYPE "T"
65 !     T1(I,J) WHERE I ARE REPRESENT THE INDEX OF
70 !     TEMPERATURES MEASURED AT ENVIROMENT POINTS 1,
75 !     2, 3, 4, AND 5, AND J REPRESENT THE HOW MANY
77 !     TIME IT MEASURED,
78 !     T2(N,L,I) - SURFACE TEMPERATURE IN EACH CASE
81 !     A1(N,L) - AVERAGE ENVIROMENTAL TEMPERATURE
82 !     A2(N,L) - AVERAGE SURFACE TEMPERATURE
83 !     R1(N,L) - RAYLEIGH NUMBER FOR THIS CASE
84 !     N1(N,L) - NUSSELT NUMBER FOR THIS CASE
85 ! THIS PROGRAM SET THE DATA AQUISITION SYSTEM TO READ 5
90 ! ENVIROMENTS L TEMPERATURES AND THE TEMPERATURE AT
95 ! THE PROBE WHICH TOUCH THE SURFACE OF THE CYLINDER
100 ! AT 7 POSITION FOR EACH CYLINDER AND WITH 6 DIFFERENT
105 ! HEAT FLUXES.
110 DIM T1(5,50)
115 DIM T2(6,7,50)
120 DIM A1(6,7)
125 DIM A2(6,7)
130 DIM S(6,7)
135 DIM R1(6,7)
140 DIM N1(6,7)
150 DISP " INPUT HEAT FLUX Q "
151 INPUT Q
153 DISP " INPUT THE NUMBER OF THE PROGRAM REPEAT N "
155 INPUT N
160 !
165 ! INPUT COEFFICIENTS OF T TYPE THERMOCOUPLE
170 !
175 C1=0.0256613
180 C2=-.000000619549
185 C3=2.21816E-11
190 C4=-3.55009E-16
195 !
200 ! MEASURE THE TEMPERATURES
201 !
205 FOR L=1 TO 7
210 FOR I=1 TO 50
215 CLEAR 709

```

```
220 OUTPUT 709 ; "AC1"
225 ENTER 709 ; V
230 GOSUB 700
235 T1(1,I)=T
240 V=0
245 CLEAR 709
250 OUTPUT 709 ; "AC2"
255 ENTER 709 ; V
260 GOSUB 700
265 T1(2,I)=T
270 T=0
275 CLEAR 709
280 OUTPUT 709 ; "AC3"
285 ENTER 709 ; V
290 GOSUB 700
295 T1(3,I)=T
300 T=0
305 CLEAR 709
310 OUTPUT 709 ; "AC4"
315 ENTER 709 ; V
320 GOSUB 700
325 T1(4,I)=T
330 T=0
335 CLEAR 709
340 OUTPUT 709 ; "AC5"
345 ENTER 709 ; V
250 GOSUB 700
355 T1(5,I)=T
360 T=0
361 CLEAR 709
362 OUTPUT 709 ; "AC6"
363 ENTER 709 ; V
364 GOSUB 700
365 T2(N,L,I)=T
365 NEXT I
366 !
367 ! CALCULATE THE AVERAGE TEMPERATURES A2(N,L)
368 ! AND STANDARD DERIVATION S(N,L)
368 !
370 O1=0
375 O2=0
380 O3=0
385 FOR J=1 TO 5
390 FOR I=1 TO 50
395 O1=O1+T(J,I)
400 NEXT I
405 NEXT J
410 A1(N,L)=O1/250
415 FOR I=1 TO 50
420 O2=O2+T2(N,L,I)
425 O3=O3+T2(N,L,I)^2
425 NEXT I
```

```

430 A2(N,L)=O2/50
435 S(N,L)=SQR(O3/50-A2(N,L)^2
436 IF S(N,L)>0.2 THEN GOTO 210
438 !
440 ! CALCULATE THE NUSSELT NUMBER FOR THIS POSITION
445 !
450 U=(A2(N,L)*1.8+32)*0.7+(A1(N,L)*1.8+32)*0.3
460 K=4.479+8.30958*U*.001-3.80163*.000001*U^2
465 V1=4.3462-9.91162*.001*U+1.7906*.00001*U^2-1.2752*E-8*U^3
470 C=3.3462*.01-3.93353*.000001*U+3.44649*E-9*U^2
475 O=851.514-8.6488*.01*U+9.86194*E-6*U^2-5.92566*E-9*U^3
480 G=32.174*.000101*O^2*(A1(N,L)-A2(N,L))*(1.365/12)^3*3600^2
/V1^2
490 P1=V1*C/K
495 R1(N,L)=P1*G
500 N1(N,L)=Q*(1.365/2)/(K*(A1(N,L)-A2(N,L))*1.8)
505 NEXT L
510 IF N<7 THEN GOTO 150
515 !
520 ! OUTPUT THE RESULTS: 1. SURFACE TEMPERATURE
525 !                       2. ENVIROMENT TEMPERATURE
530 !                       3. STANDARD DERIVITION
535 !                       4. RAYLEIGH NUMBER
537 !                       5. NUSSELT NUMBER
539 !
540 OPTION BASE 1
545 ASSIGN #1 TO "DATA2:D701"
550 FOR J=1 TO 6
555 FOR L=1 TO 7
560 PRINT# 1 ;A1(N,L),A2(N,L),S(N,L)
565 PRINT# 1 ;R1(N,L)
570 PRINT# 1 ;N1(N,L)
575 NEXT L
580 NEXT J
590 ASSIGN# 1 TO *
595 STOP
600 END
700 T=V*(C1+V*(C2+V*(C3+V*C4)))*1000000
705 RETURN

```

## PART 2 OTHERS

## A3 LOCAL TEMPERATURE MEASUREMENT

For local temperature measurement two methods have been used; one of them was described in chapter 3. The use of a fixed probe, mounted under the surface of heated cylinder, is another method. Using this second method readings were higher for the cylinder in all three cases (single, two or three cylinder case). The temperature readings and the local Nusselt numbers are tabulated in Table A3.1 for probe 1 (p1 fixed probe) probe 2 (p2 moving probe located downstream of from the measuring point):

Table A3.1 Local temperature for the single cylinder case using probe 1 and 2 ( $Q = 333, 666, 1000, 2000$  Btu/hr-ft<sup>2</sup>).

Q.	Probe *	$\phi = 0^\circ$	$30^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$150^\circ$	$180^\circ$
333	p1 T *	27.95	28.25	28.40	28.59	28.91	29.25	31.29
	p2 T *	27.65	27.75	27.85	28.02	27.94	28.13	28.65
666	p1 T *	29.11	29.13	29.18	29.34	29.40	29.63	33.49
	p2 T *	28.45	28.27	28.31	28.39	28.51	28.64	29.28
1000	p1 T *	29.51	30.04	30.07	30.19	30.35	30.56	36.60
	p2 T *	28.45	28.67	28.71	28.78	28.90	29.14	30.15
2000	p1 T *	31.28	32.26	32.23	32.76	33.08	33.36	44.21
	p2 T *	29.30	29.34	29.49	29.66	29.82	30.42	32.23

To evaluate the effect of an air gap on heat transfer by free convection across an annulus with its axis horizontal an equivalent thermal conductivity term  $k$  is used. This term includes the effects of both convection and

conduction in the gas. The heat transfer rate through a cylindrical annulus can be calculated per unit length :

$$Q/L = (2 \pi k_c / \ln(D2/D1)) * (T_1 - T_2) \quad (1)$$

where D1 is the inside diameter and D2 is the outside diameter as illustrated in Figure A3.1. As the width of the annulus  $(D2-D1)/2$  becomes smaller the effect of the free convection decreases and becomes negligible when the width is reduce to 0.118 in. according to Todd and Ellis (1). Heat is transferred across narrow annuli by thermal conduction through the gas.

For this case the temperature of the heater is  $T_1$ , the temperature of the inner face of the cover sheet is  $T_2$ , the external surface of the cover sheet is  $T_3$ . Equation (1) was used with  $Q$ ,  $L$ ,  $k$ ,  $D1$ ,  $T_3$  known, and  $Q/A$  values of

333, 666, 1000, 2000 Btu/hr ft where  $A=L \cdot \pi \cdot D$ . Other property values used were  $k_c = k$  for air at 300K is 0.01516 Btu/hrftF,  $k_c$  (for steel at 300k) = 31.28 Btu/hr.ft.F,

$$D1=1.3566 \text{ in.}$$

$$D2=1.3575 \text{ in.}$$

$$D3=1.365 \text{ in.}$$

$$T2-T1 = q * \ln(D2/D1) * D1 / (2 * k) \quad (2)$$

and

$$T3-T4 = (q * \ln(D3/D2) * D2) / (2 * k) \quad (3)$$

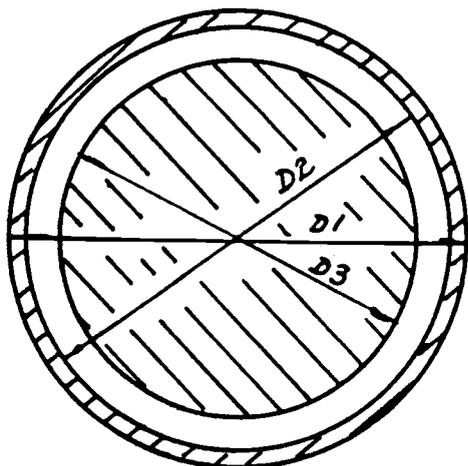


Figure A1.1 The cross-section of the cylinder

The results for  $T$  are listed in table A3.2

Table A3.2. Temperature difference for each  $q$

$q$	333	666	1000	2000
T1-T2	.4603	.9206	1.3809	2.7617
T2-T3	.00185	.0037	.0055	.0111
T1-T3	.46215	.9243	1.3864	2.7728

Table A3.3 gives the results for surface temperature by using probe 1 and by using T1-T3 added to the temperature measured by probe 2.

From Table A3.3, we see that 21 out of 28 of the calculated values agree with the temperature measured by probe 1. The errors are less than 0.5%. The maximum errors occurred at top of the cylinder ( $\phi=180$  deg.). This error may be due to the difference in the gap size or possibly to turbulence in this region. Another possibility is the change

Table A3.3. Comparason of the surface temperature (using probe 1 and probe 2 plus T1-T3 which is calc.1)

Q	probe *	0	30	60	90	120	150	180
333	probel	27.95	28.25	28.40	28.59	28.91	29.25	31.29
	probe2	27.65	27.75	27.85	28.02	27.94	28.13	28.65
	calc.1	28.11	28.21	28.31	28.48	28.8	28.59	29.11
666	probel	29.11	29.13	29.18	29.34	29.40	29.63	33.49
	probe2	28.45	28.27	28.31	28.39	28.51	28.64	29.28
	calc.1	29.37	29.19	29.23	29.31	29.43	29.56	30.20
1000	probel	29.51	30.04	30.07	30.19	30.35	30.56	36.60
	probe2	28.45	28.67	28.71	28.78	28.90	29.14	30.15
	calc.1	29.83	30.05	30.09	30.16	30.28	30.52	31.53
2000	probel	31.28	32.26	32.23	32.76	33.08	33.36	44.21
	probe2	29.30	29.34	29.49	29.66	29.82	30.42	32.23
	calc.1	32.06	32.10	32.25	32.42	32.58	33.18	34.99

in flow direction caused by wake effects. In general both probe arrangements are satisfactory for this local temperature measurement.