

AN ABSTRACT OF THE THESIS OF

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↓  
(Dr. Stephen J. Hawkes)

The study of the obstructive factor by Hawkes (11) showed a relation between the obstructive factor and velocity using a continuous method. Later, Bowers and Hawkes (12) developed a similar method with some improvements in the apparatus and found out that the obstructive factor was not flow dependent. This study uses Bowers and Hawkes' apparatus with some changes and restudies the relationship between the obstructive factor and velocity with unretained gases on glass beads. The dependence of the obstructive factor on velocity was verified. The effects of diffusion coefficients and the particle-to-column diameter ratios on the obstructive factor-velocity relationship were investigated. The results are consistent with Hawkes' hypothesis (11) that values of  $\gamma$  at low velocities are averages over tightly packed and loosely packed domains while at high velocities they are weighted in favor of the loosely packed domains where there is more flow. However, no explanation has been found for Bowers and Hawkes' results.

The Obstructive Factor

in

Gas Chromatography

by

Pornpan Thumneum

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### LIST OF SYMBOLS

|                  |   |  |
|------------------|---|--|
| $A$              | = | eddy diffusion contribution to plate height                    |
| $B, \bar{B}$     | = | longitudinal molecular diffusion term in plate height equation |
| $C_g, \bar{C}_g$ | = | gas phase resistance to mass transfer term                     |
| $C_l$            | = | liquid phase resistance to mass transfer term                  |
| $D, D_m, D_g$    | = | diffusion coefficient in mobile fluid                          |
| $D_{go}$         | = | diffusion coefficient in carrier gas at outlet pressure        |
| $d$              | = | diameter of a local domain                                     |
| $d_c$            | = | column diameter  |
| $d_p$            | = | particle diameter  |
| $f$              | = | Giddings compressibility factor                                |
| $H$              | = | plate height   |
| $\hat{H}$        | = | apparent plate height  |
| $j$              | = | James - Martin compressibility factor                          |
| $L, l$           | = | length of a column   |
| $l_T$            | = | length of a tortuous path                                      |
| $P_i$            | = | inlet pressure   |
| $P_o$            | = | outlet pressure  |
| $r$              | = | column radius  |
| $t_d$            | = | diffusion time   |
| $t_m$            | = | time spent by molecule in mobile phase                         |
| $t_r$            | = | retention time   |
| $v$              | = | linear velocity  |

LIST OF SYMBOLS (continued)

|                   |   |  |
|-------------------|---|--|
| $\bar{v}$         | = | average linear velocity                    |
| $v_o$             | = | outlet velocity                            |
| $W_{\frac{1}{2}}$ | = | width at half height of a recorded peak    |
| $\gamma$          | = | obstructive factor                         |
| $\sigma_t$        | = | standard deviation of zone, in time unit   |
| $\sigma_t^2$      | = | variance of zone, in time unit             |
| $\sigma_x$        | = | standard deviation of zone, in length unit |
| $\sigma_x^2$      | = | variance of zone, in length unit           |
| $\omega$          | = | ill-defined geometric constant             |

## THE OBSTRUCTIVE FACTOR IN GAS CHROMATOGRAPHY

### I. INTRODUCTION

J. Calvin Giddings (1) has written a part about the obstructive factor in chromatography; it said "---  $\gamma$  is essentially a structural constant and thus independent of the particular mobile phase or solute being considered. However, there is very likely a slight velocity dependence to  $\gamma$ ----. The theory of  $\gamma$ 's velocity dependence has not been worked out; as things stand now, we must assume that the velocity effect is negligible."

There are two experimental methods to determine the value of the obstructive factor:

- Static method and
- Dynamic method

When a static method or theoretical model is used to determine the value of the obstructive factor,  $\gamma$ , in a bed of randomly packed solid spheres, the value of  $0.60 \pm 0.02$  is obtained (2-5). If a dynamic method is used, the value of  $0.73 \pm 0.02$  is obtained for the value of  $\gamma$  (6-10).

Because of the discrepancy between the values of  $\gamma$  obtained by using a dynamic and a static method, the flow dependence of the obstructive factor was investigated by Hawkes (11). He found that the obstructive factor is clearly flow dependent. For the column packed with beads, the  $\gamma$  has a value of  $0.60 \pm 0.02$  at zero velocity (by extrapolation) and  $0.73 \pm 0.02$  at higher velocities.

Later, Bowers and Hawkes (12) repeated the work on flow dependence of the obstructive factor. They improved the system to solve the problems on the back diffusion and significantly skewed peaks which Hawkes had experienced in his previous work (11). No effect of gas flow or velocity on the variation of  $\gamma$  was found.  $\gamma$  remained constant at  $0.74 \pm 0.02$  over a velocity range from 0.1-8.0 cm/sec.

The value and interpretation of  $\gamma$  are still doubtful. The theory of  $\gamma$ 's velocity dependence has not been worked out. In books or calculation, generally they just assume that the velocity effect is negligible and use the value of 0.60 for most non-porous densely packed materials and use the value of 0.74 for porous materials.

To obtain some more information on the relationship between  $\gamma$  and velocity, I compare the two experiments which showed the obstructive factors are flow sensitive by Hawkes (11) and the obstructive factors are not always flow dependent (12) one by one, as follows:

| <u>flow dependent</u>               | <u>not always flow dependent</u>    |
|-------------------------------------|-------------------------------------|
| - column length 730 cm              | - column length 736.6 cm            |
| - column diameter                   | - column diameter                   |
| 1/4 in. O.D.                        | 1/8 in. O.D.                        |
| 3/16 in. I.D.                       | 1/18 in. I.D.                       |
| - diameter of glass beads           | - diameter of glass beads           |
| 50/60 mesh                          | 60/80 mesh                          |
| - $d_p/d_c = 0.06$ or               | - $d_p/d_c = 0.15$ or               |
| $d_c/d_p = 17$                      | $d_c/d_p = 7$                       |
| - nitrogen (N <sub>2</sub> ) sample | - methane (CH <sub>4</sub> ) sample |
| - helium (He) carrier gas           | - helium (He) carrier gas           |
| - at 50° C                          | - at 50° C                          |
| - obtained significantly            | - used post- and pre- columns       |
| skewed peaks                        | No skewed peaks                     |
| - varied velocity from 0.1-         | - varied velocity from 0.1-         |
| 8.0 cm/sec                          | 8.0 cm/sec                          |

I conclude from the above comparison that the samples and the particle-to-column diameter ratio might have some effect on the  $\gamma$  - velocity relationship. My experimental system was set up like Bowers and Hawkes' system (12) because it produced unskewed and accurate peaks, with some changes, and I was able to obtain good results. The  $\gamma$  was determined by a dynamic and a static method while varying velocity, sample and particle-to-column diameter ratio.

## II. THEORY

### A. BACKGROUND

In gas chromatography, when an infinitely thin slice of a gas B spreads by diffusion into another gas A the concentration profile of B in A after a finite time (t) is gaussian and the rate of spreading is given by the equation 1.

$$d \sigma_x^2 / dt = 2D_g \quad (1)$$

where  $\sigma_x$  is the standard deviation of the gaussian concentration profile, measured as a distance.  $D_g$  is the diffusion coefficient of gas B in A. Equation 1 holds for axial diffusion in a uniform empty tube.

But if the tube is packed with obstructing, but not sorbing, material, diffusion is hindered and the rate of spreading is given by the equation 2.

$$d \sigma_x^2 / dt = 2 \gamma D_g \quad (2)$$

where  $\gamma$  is called the "Obstructive Factor" (1) and  $\gamma$  is less than unity.

The obstructive factor is the factor that indicates the degree to which diffusion is hindered by the packed material. When  $D_m$  is the diffusion coefficient for solute in the mobile phase and  $D$  is the effective diffusion coefficient

$$\gamma = D/D_m \quad (3)$$

The value of  $\gamma$  probably ranges from 0.4-0.9 when all types of chromatographic materials are taken into account.

Generally, in practice, the measurement of  $\sigma_x^2$  as a function of length is not possible. An elution method must be used to obtain the time variance,  $\sigma_t^2$  of the eluted band rather than the length variance  $\sigma_x^2$ . Since  $\sigma_x/\sigma_t = v$ , a constant linear velocity, diffusional spreading is given by

$$d \sigma_t^2 / dt = 2D_g / v^2 \quad (\text{empty tube}) \quad (4)$$

$$d \sigma_t^2 / dt = 2\gamma D_g / v^2 \quad (\text{packed tube}) \quad (5)$$

### B. THE INTERPRETATION OF $\gamma$

When the solute moves along the packed column, diffusion will then occur in each of the interstitial flow channels.

Giddings (1) has explained that these channels have two main characteristics which hinder diffusion:

1. The channels zig-zag through the material in a tortuous manner. This prevents the diffusion of molecules along the shortest direct path. See figure I.

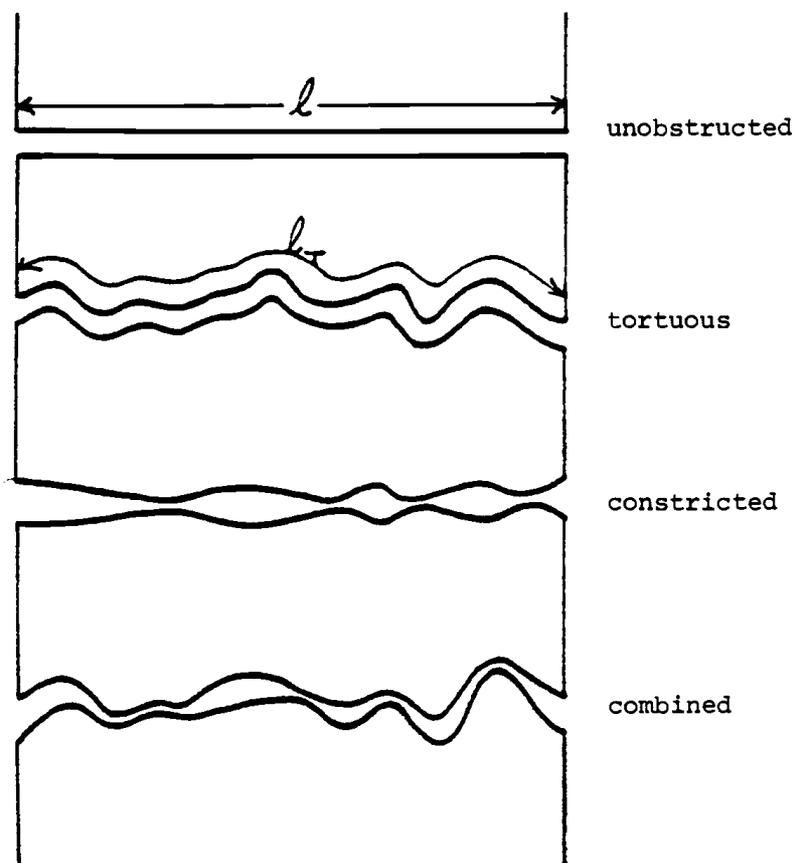
2. The channels alternately vary from wide to narrow. The narrow constrictions slow diffusion. See figure I.

Hence, there are two factors which hinder diffusion through chromatographic materials:

#### Tortuosity and Constriction

The tortuous path slows diffusion by increasing path length. The constricted path creates "traffic bottlenecks" at random points, that slow diffusion. The combined path provides both kinds of hindrances.

In a packed bed the situation at first appears more complex than figure I since there are no channels with clearly defined walls. However, the concepts of tortuosity and constriction are still valid if we define a diffusion path in an analogous way to a stream line in a flow-



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FIGURE I: The nature of tortuous and constricted paths in a chromatographic packing.

ing mobile phase. We define a "diffusion path" as a path through the column which is always in the direction of the maximum concentration gradient and consequently normal to successive surfaces of equal concentration. These diffusion paths are analogous to stream lines which are normal to isobaric surfaces. But stream lines will generally pass around porous particles, diffusion paths may pass through them as well as around them. The diffusion paths in a packed bed will be tortuous and any bundle of such paths will be constricted and expanded to that their envelope will be similar to figure I.

### C. EQUATIONS FOR CALCULATIONS

The height equivalent to a theoretical plate has been given by Van Deemter, Zuiderweg, and Klinkenberg (13) in a form that reduces to

$$H = A + \bar{B}/\bar{v} + \bar{C}_g \bar{v} + C_1 \bar{v} \quad (6)$$

where A is the eddy diffusion term,  $\bar{B}$  is the longitudinal diffusion term at the average pressure in the column,  $\bar{C}_g$  is the gas phase resistance to mass transfer term at the average pressure in the column,  $C_1$  is the liquid phase resistance to mass transfer term, and  $\bar{v}$  is the average carrier gas velocity at the average pressure in the column.

In gas chromatography, working with uncoated glass beads, the  $C_1 \bar{v}$  term can be excluded. Thus,

$$H = A + \bar{B}/\bar{v} + \bar{C}_g \bar{v} \quad (7)$$

Giddings (14-15) has proposed the alternative form of the equation in which the eddy diffusion term A, is coupled with part of the gas phase resistance to mass transfer,  $\bar{C}_g$

$$H = \frac{1}{1/A + 1/\bar{C}_g \bar{v}} + \bar{B}/\bar{v} + \bar{C}_g \bar{v} \quad (8)$$

Consider the term  $1/(1/A + 1/\bar{C}_g \bar{v})$ ; rapid diffusion in the gas phase probably makes the term  $\bar{C}_g \bar{v}$ , where  $\bar{C}_g$  is inversely proportional to the gaseous diffusion coefficient, significantly smaller than A at velocities near the optimum. This term,  $1/(1/A + 1/\bar{C}_g \bar{v})$ , then, approaches  $\bar{C}_g \bar{v}$  in value. With this approximation

$$H = \bar{B}/\bar{v} + \bar{C}_g \bar{v} \quad (9)$$

The plate height H of a uniform column is proportional to variance  $\sigma^2$ .

$$H = \frac{\sigma^2}{L} \quad (10)$$

From equation 9, we will first consider the contribution to the plate height made by longitudinal molecular diffusion. The spreading of a zone due to molecular diffusion in the mobile phase can be calculated from the equation

$$\sigma^2 = 2Dt_d \quad (11)$$

where  $t_d$  is the diffusion time of the time period over which diffusion occurs, D is the diffusion coefficient.

If the solute molecules are migrating through the column with the distance L, traveling with the average mobile phase velocity  $\bar{v}$  then the time spent in reaching L is

$$t_m = L/\bar{v} \quad (12)$$

From equation 11

$$\sigma^2 = 2Dt_d$$

So, 
$$\sigma^2 = 2D t_m \quad (13)$$

From equation 12 and 13

$$\sigma^2 = 2D L/\bar{v} \quad (14)$$

where  $D_m$  is the diffusion coefficient in the mobile phase.

In packed columns, there are the tortuosity and constriction factors, and an obstructive factor  $\gamma$  is applicable. Thus,

$$\sigma^2 = 2\gamma D_m L / \bar{v} \quad (15)$$

From equation 10 and 15

$$H = 2\gamma D_m / \bar{v} \quad (16)$$

Second we will consider the contribution to the plate height made by the gas phase resistance to mass transfer term. The plate height is always (except for the transcolumn effect) approximately proportional to the particle size  $d_p$  squared and to velocity  $\bar{v}$ , and inversely proportional to the mobile phase diffusion coefficient  $D_m$ .

$$H = \omega d_p^2 \bar{v} / D_m \quad (17)$$

where  $\omega$  is essentially constant. Combining two terms to obtain the equation of plate height,

$$H = 2\gamma D_m / \bar{v} + \omega d_p^2 \bar{v} / D_m \quad (18)$$

or

$$H = 2\gamma D_{g0} / v_o + \omega d_p^2 v_o / D_{g0} \quad (19)$$

where  $D_{g0}$  is the diffusion coefficient of sample in carrier gas at outlet pressure,  $v_o$  is the outlet velocity.

#### D. THE ROLE OF PRESSURE GRADIENTS

In a gas chromatographic column, the gas flows in the narrow interstices between the particles of packing, which offer resistance to flow and thus, because of the finite viscosity of the gas, there is a pressure gradient along the length of the column. Any gas packet within a column containing significant pressure gradients will physically expand as it passes down the column by an amount over and above that due to ordinary plate-height spreading.

Giddings, Seager, Stucki and Stewart (14) have analysed this effect and corrected the equation of plate height. The final result is

$$H = 2\gamma D_{go} j f / \bar{v} + \omega d_p^2 \bar{v} f / D_{go} j \quad (20)$$

where  $j = 3((p_i/p_o)^2 - 1) / 2((p_i/p_o)^3 - 1)$

$$f = j^2((p_i/p_o)^2 + 1) / 2$$

$$v_o = \bar{v} / j$$

$\bar{v}$  is the average velocity,  $p_i$  is the inlet pressure,  $p_o$  is the outlet pressure.

### III. EXPERIMENTAL WORK

The gas chromatograph consisted of a coiled copper column with 1/8-inch outside diameter and 1/18-inch inside diameter packed with glass beads. To ensure a consistent environment, post- and pre- columns 182.9 cm in length packed with the same size glass beads as in the column were placed between the reference side of the detector and the injection port, and after the sample side of the detector to atmosphere as was done by Bowers and Hawkes (12). The instrument was placed in a  $50 \pm 0.5^\circ$  C oil bath, and a 762 cm empty tube coiled copper column was placed before the detector to ensure the constant temperature of  $50^\circ$  C of the helium carrier gas before the gas entered the column. A sample was injected through an injection port of minimal dead volume with the Hamilton gas-tight syringe. The detector was a Carle microkatharometer detector and peaks were recorded on an Esterline-Angus EL10S recorder. A general scheme of the apparatus used in this study is shown in figure II.

The helium gas from the gas tank flowed through the empty tube, the gas pressure was measured by the gas pressure gauge. Before entering the temperature equilibrium column, the helium carrier gas passed through a two-way tap. The two-way tap was used to alternate the helium carrier gas flow to the temperature equilibrium column or a dummy column. The dummy column consisted of an empty copper tube with an adjustable valve. The temperature equilibrium column was connected to the reference side of the detector. The helium gas at  $50^\circ$  C from the temperature equilibrium column would pass through the pre-column and

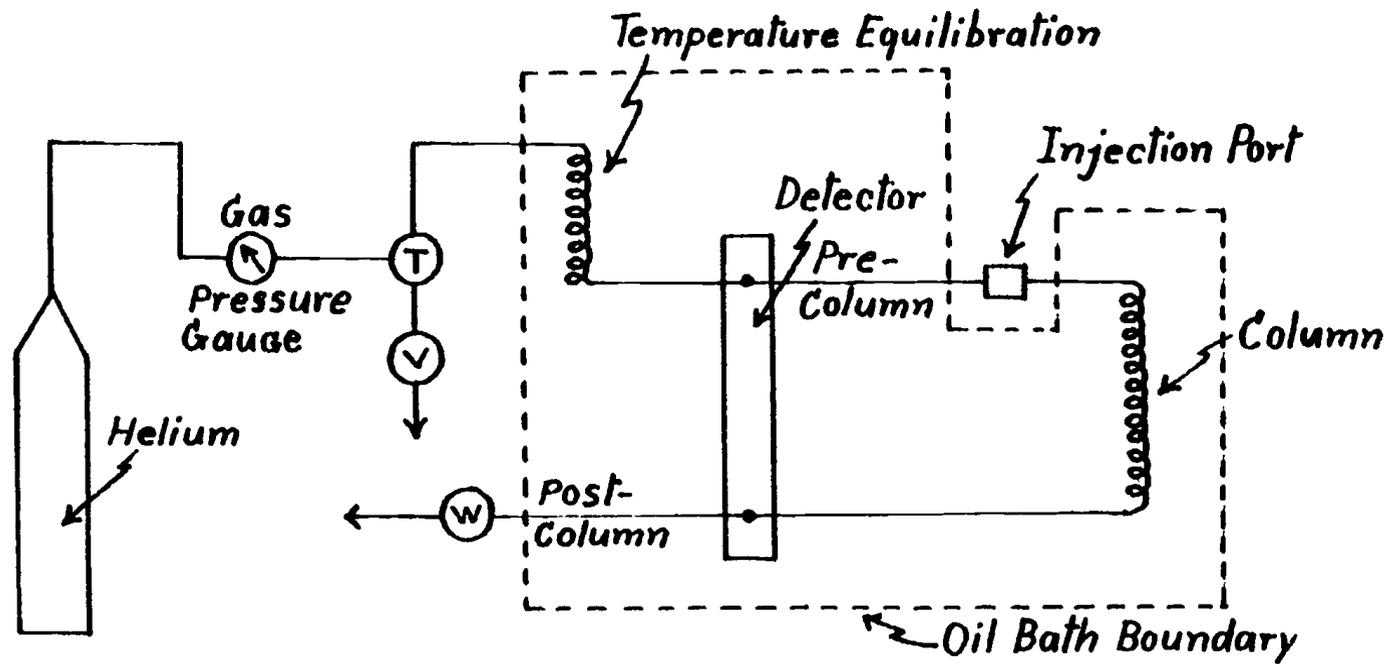


FIGURE II: Diagrammatic scheme of the apparatus

T = two way tap, V = needle valve,

W = one-way tap.

injection port, samples were swept through the column, detected by the detector, passed the post-column and on to atmosphere. The dummy column and the one-way tap on the post-column were used in Knox and McLaren's arrested elution method.

The column was packed by bouncing and vibrating method and terminated with gauze screens to avoid undesirably large instrumental spread and peak asymmetry. The post- and pre- columns were placed with the same size of glass beads as in the column to avoid peak asymmetry. The light paraffin oil was used in the bath instead of water because it is less corrosive liquid. The O-ring between the detector and the adaptor is Teflon, therefore, it does not soften in warm solvent.

#### A. CONTINUOUS ELUTIONS

A typical experiment was carried out as follows. A gas sample was injected into the column and eluted in the normal way without arresting gas flow. The time of passage along the column,  $t_r$ , is the total elapsing between injection and detection. The average velocity was obtained from  $\bar{v} = L/t_r$ ,  $L$  is the length of the column.  $W_{1/2}$  was measured from recorded peaks.  $H$  was calculated from  $H = LW_{1/2}^2/5.546t_r^2$ . The obstructive factor,  $\gamma$ , was calculated from equation 20.

$$H = 2\gamma D_{go} j f / \bar{v} + \omega d_p^2 f \bar{v} / D_{go} j$$

The inlet pressure was varied to obtain velocities from 0.1-17 cm/sec (0.1-25 cm/sec in one case). The value of  $\omega$  was determined using high velocities as in the previous work (11). To study the effect of the diffusion coefficient on the obstructive factor - velocity relationship, methane and argon gases were used as samples. The diffusion coefficient of methane in helium at 50° C is 0.7682 cm<sup>2</sup>/sec (16). The

diffusion coefficient of argon in helium at 50° C was calculated to be 0.8453 cm<sup>2</sup>/sec using the value of 0.7335 cm<sup>2</sup>/sec at 25° C and correcting for temperature by proportion to  $T^{1.77}$  (17). To study a particle-to-column diameter ratio effect on the obstructive factor-velocity relationship, the 1/8-inch O.D., 1/18-inch I.D. copper columns, post- and pre- columns were packed with 60-80 mesh glass beads, 25-35 mesh glass beads and 80-100 mesh glass beads. In this case the sample was methane only.

#### B. ARRESTED ELUTIONS

To study static spreading, Knox and McLaren's arrested elution technique (2) was used. A band of methane was eluted about half way along the column (figure II), by turning the two-way tap, T. After a delay of 1 to 20 minutes the flow was reconnected to the column and the peak eluted.

According to equation 5, a plot of  $\sigma_t^2$  against delay time  $t$  should be a straight line with the slope of  $2\gamma D_{go}/v_o^2$ . The  $\sigma_t^2$  was obtained from  $\sigma_t^2 = W_{1/2}^2/5.546$ . The outlet velocity,  $v_o$ , was obtained from  $\bar{v}$  by applying the James-Martin pressure correcting factor, knowing the pressure drop across the column,  $v_o = \bar{v}/j$ . The obstructive factor,  $\gamma$  was calculated from the slope of  $\sigma_t^2$ - $t$  plot.

#### C. CORRECTION FOR INLET AND OUTLET PRESSURE OF THE COLUMN

To correct the effect of pressure gradient on the plate-height spreading, the James-Martin compressibility factor  $j$  and the Giddings compressibility factor  $f$  were applied to the equation of plate height (equation 20). This corrected the effect of pressure gradient along the column. But in this experimental system there were pre- and post-columns which were packed with the same size glass beads as the column,

thus there were pressure gradients in the pre- and post- columns. Hence, the gas pressure measured with the gauge before the gas entered the pre-column must be corrected to give the inlet pressure of the column, and the atmospheric pressure that the post-column was leading to must also be corrected to the outlet pressure of the column. Littlewood (18) has derived an equation for the pressure at any point in the column.

$$\frac{x}{l} = \frac{p_i^2 - p^2}{p_i^2 - p_o^2} \quad (21)$$

$x$  is the length at any point in the column,  $l$  is the total length of the column,  $p_i$ ,  $p_o$  are the inlet and outlet pressure of the column respectively,  $p$  is the pressure at the length  $x$ .

Applying equation 21 to the pre- and post- column problems in order to calculate the inlet and outlet pressure of the column, the following equations were obtained:

$$\frac{L_{pre}}{L_{pre} + L_{col} + L_{post}} = \frac{p_m^2 - p_i^2}{p_m^2 - p_{atm}^2} \quad (22)$$

$$\frac{L_{pre} + L_{col}}{L_{pre} + L_{col} + L_{post}} = \frac{p_m^2 - p_o^2}{p_m^2 - p_{atm}^2} \quad (23)$$

$L_{pre}$ ,  $L_{col}$ ,  $L_{post}$  are the length of the pre-column, column itself and post-column respectively.  $p_m$  is the pressure measured at the pressure gauge.  $p_{atm}$  is the atmospheric pressure. The inlet pressure of the column,  $p_i$  and the outlet pressure of the column,  $p_o$  were calculated from equation 22 and 23 respectively.

To check the deviation of calculated  $p_i$ ,  $p_o$  from the real measured  $p_i$ ,  $p_o$  of the column, the system at the injection port was removed and

a manometer was connected directly to the inlet of the column. Let the gas flow and keep the oil bath at 50° C, read the real inlet pressure from the manometer. The average percentage of deviation of calculated  $p_i$  from measured  $p_i$  at several gas flows was  $-(2.00 \pm 0.01)\%$ . If the measured  $p_o$  was assumed to be smaller than the calculated  $p_o$  about 2%, the ratios of  $p_i/p_o$  were the same in both calculated and measured pressures. This resulted in no change in the values of  $H$  and  $\gamma$  on both cases. In this experiment, the calculated  $p_i$ ,  $p_o$  were used because it was more convenient.

#### IV. RESULTS AND DISCUSSION

##### A. PERFORMANCE CHECK - OPEN TUBULAR COLUMN

The performance of the system was evaluated using an open tubular column, for which plate height could be calculated directly from theory (19, 20). A 731.5 cm long, 1/8-inch O.D., 1/18-inch I.D. copper tube was used as a column in this study. Methane is a sample and helium as a carrier gas.

From the Golay theory of capillary column performance (19), which gives, for an unretained component,

$$H = 2\gamma D_{go}/v_o + r^2 v_o/24D_{go} \quad (24)$$

where  $r$  is the radius of the tube,  $D_g$  is  $0.7682 \text{ cm}^2/\text{sec}$  at  $50^\circ \text{ C}$  for  $\text{CH}_4$ .

With the expressions derived by Giddings (14) the modification of equations (24) is as follows:

$$\hat{H}/f = 2\gamma D_{go} j/\bar{v} + r^2 \bar{v}/24D_{go} j \quad (25)$$

where  $\hat{H}$  is the apparent plate height obtained from the emergent peak width and retention time, and  $f$  is Giddings compressibility factor.

Performance data for the open tubular column are shown in table I. The value of  $\gamma$  agrees with the theory that the value of the obstructive factor of the open tubular column is unity.  $\gamma$  value seems to be stable over the 4-21 cm/sec velocity range.

##### B. THE OBSTRUCTIVE FACTOR AND VELOCITY

The variation of obstructive factor with velocity was studied on 735.4 cm long copper coiled column, 1/8-inch O.D., 1/18-inch I.D.,

TABLE I: Performance data on open-tubular column, 731.5 cm long, 1/8-inch O.D., 1/18-inch I.D., He carrier, CH<sub>4</sub> sample.

|                    |          |          |         |         |
|--------------------|----------|----------|---------|---------|
| $p_i$ (p.s.i.)     | —        | 16.5655  | 18.4384 | 21.4234 |
| $p_o$ (p.s.i.)     | —        | 15.0884  | 15.4384 | 16.2657 |
| $p_i/p_o$          | —        | 1.0979   | 1.1718  | 1.3171  |
| $j$                | —        | 0.9527   | 0.9189  | 0.8578  |
| $f$                | —        | 1.0007   | 1.0021  | 1.0062  |
| $t_r$ (sec)        | 172.7500 | 100.4375 | 58.7500 | 34.7844 |
| $\bar{v}$ (cm/sec) | 4.2344   | 7.2831   | 12.4511 | 21.0301 |
| $W_{1/2}$ (sec)    | 9.0751   | 4.0381   | 1.8273  | 0.7357  |
| $H$ (cm)           | 0.3640   | 0.2132   | 0.1276  | 0.0702  |
| $\gamma$           | 1.0001   | 1.0498   | 1.0907  | 1.0076  |

CH<sub>4</sub> sample, He carrier gas. The value of  $\omega$  was determined to be 1.2

(11) The relation is shown in figure III. The obstructive factor is clearly flow dependent. This result agrees with Hawkes' work (11). The obstructive factor at zero velocity, achieved by extrapolation, is 0.60 in agreement with the static determinations (2,3,4,5). The highest value of  $\gamma$  is 0.74 in agreement with the dynamic determinations (7,8,9,10).

Hawkes (11) had explained that 'This dependence lies in the fact that the lowest flow resistance is offered by gaps or wide paths in the packing structure, especially at the walls, where  $\gamma$  is greatest. Accordingly more sample flows through regions of high  $\gamma$ , and the value of  $\gamma$  is weighted by this effect. At low or zero flow this is offset by diffusion between domains of low and high  $\gamma$ .' To quantitatively compute the maximum of  $\gamma$ , he used the consideration that the maximum value of  $\gamma$  will be obtained when escape from a domain by flow is faster than by diffusion. This may be roughly estimated when

$$\frac{\bar{v}d}{D} > \frac{4}{3} \quad (26)$$

where  $\bar{v}$  is the velocity that 1/3 of the material at a point source at the center of domain diffuses out of it before the source has moved 1/3 of the distance to the domain boundary.  $d$  is the diameter of domain.  $D$  is the diffusion coefficient of the sample in mobile fluid. If the domain is the column cross-section  $d = 1/18$  inch, the velocity at which  $\gamma$  will reach the maximum value would be 7 cm/sec. From the experiment, the flattening point on the  $\gamma - \bar{v}$  curve is about 7 cm/sec, this agrees very well with the estimated velocity.

Another part of the experiment to study the  $\gamma$  - velocity dependence, Knox and McLaren's arrested elution method (2) was applied at each velocity step. The data is shown in table II.

Column: 1/8-in. O.D., 1/18-in. I.D., 735.4 cm long

Packing: 60-80 mesh glass beads

Sample: CH<sub>4</sub>

Carrier gas: He

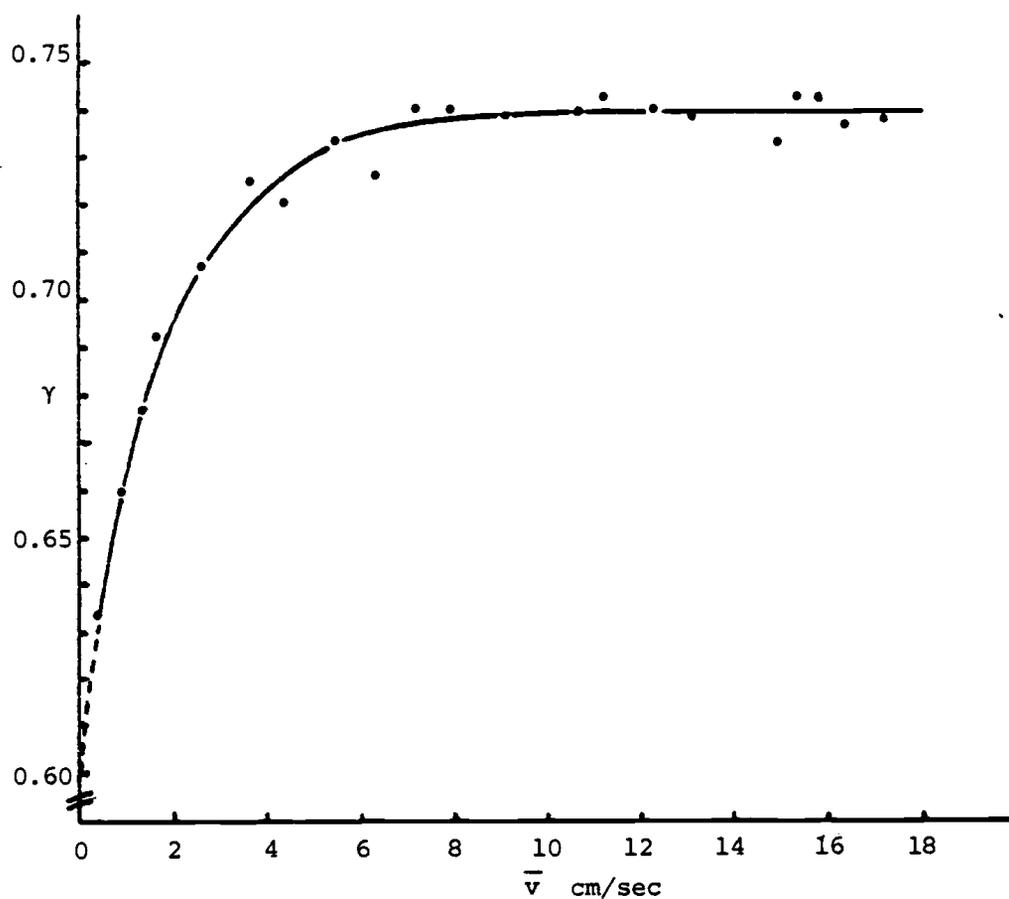


FIGURE III: Variation of obstructive factor with velocity,  $\bar{v}$ .

TABLE II: Arrested elution; column 736.6 cm long, 1/8-in. O.D., 1/18-in. I.D., sample-CH<sub>4</sub>, carrier gas-He, D<sub>g0</sub> of CH<sub>4</sub> = 0.7682 cm<sup>2</sup>/sec at 50° C, particles-60-80 mesh glass beads.

| $\bar{v}$ (cm/sec) | $v_o$ (cm/sec) | $\gamma$ |
|--------------------|----------------|----------|
| 2.2840             | 2.5085         | 0.6300   |
| 3.0591             | 3.4507         | 0.6320   |
| 3.9908             | 4.6011         | 0.6064   |
| 4.8070             | 5.6413         | 0.6087   |
| 6.8771             | 8.3693         | 0.3207   |
| 8.8084             | 10.9482        | 0.2905   |

From the data it seems to be that the static method could be applied when the velocity was low or the pressure drop across the column during the elution was low. With large pressure drop the gas velocity did not reach its stationary value before the peak was eluted, thus  $\sigma_t$  was overestimated and an excessively low value of the obstructive factor was obtained. But at low velocity the obstructive factor value agrees with the value from the continuous elution.

#### C. THE OBSTRUCTIVE FACTOR AND DIFFUSION COEFFICIENT

Methane (D<sub>g</sub> = 0.7682 cm<sup>2</sup>/sec at 50° C) and argon (D<sub>g</sub> = 0.8453 cm<sup>2</sup>/sec at 50° C) samples were studied on the same column and packing; 735.4 cm long, 1/8-in. O.D., 1/18-in. I.D., 60-80 mesh glass beads packing, He carrier gas,  $\omega$  value was 1.2, using the continuous elution method.  $\gamma$  was calculated using equation 20.

Plots of  $\gamma$  and  $\bar{v}$  of both samples are shown in figure IV. At zero or very low velocities,  $\gamma$  values of both samples are

Column: 1/8-in. O.D., 1/18-in. I.D., 735.4 cm long

Packing: 60-80 mesh glass beads

Samples: CH<sub>4</sub>, Ar

Carrier gas: He

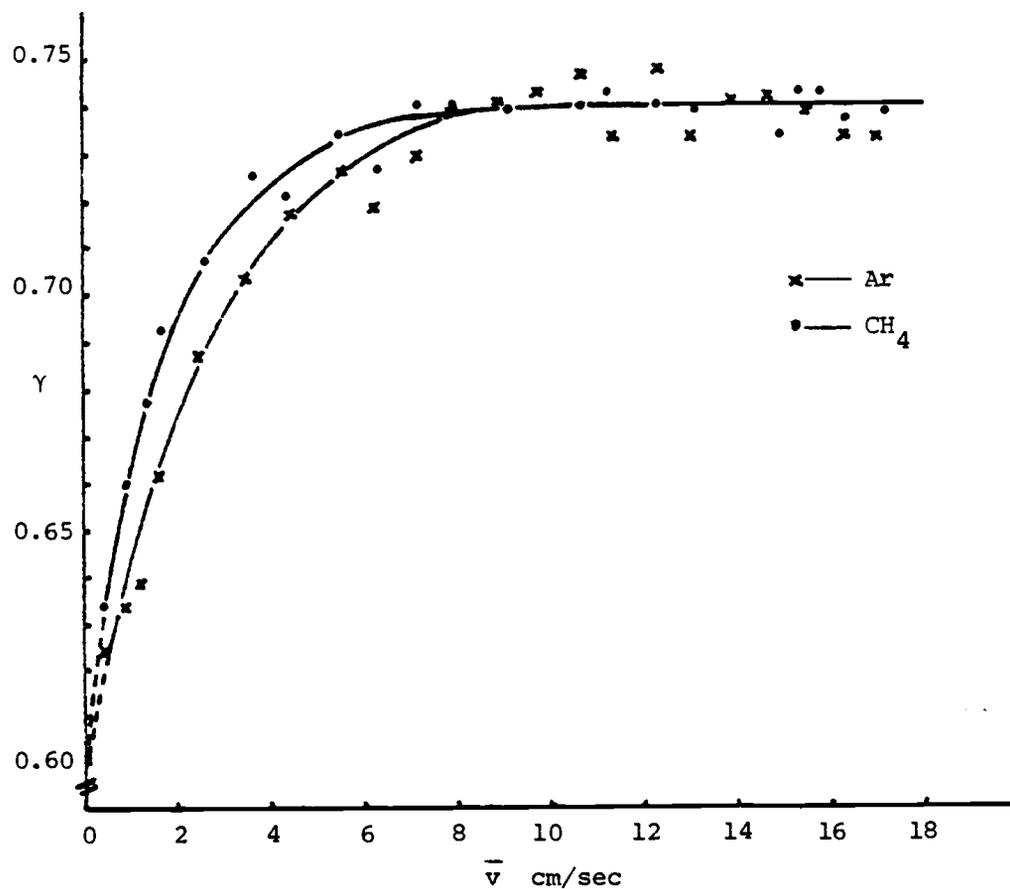


FIGURE IV: Variation of obstructive factor with velocity,  $\bar{v}$ .  
Compare between CH<sub>4</sub> and Ar samples.

about 0.60, and at high velocities are about 0.74. By using  $\bar{v}d/D > 4/3$  (equation 26), the maximum value of  $\gamma$  will be obtained at  $\bar{v} = 8$  cm/sec, this agrees well with the experimental flattening point. But between the minimum and maximum values of  $\gamma$ , there is difference in  $\gamma$  on argon's and methane's curves. At the same velocity, the value in the case of argon as a sample is smaller than in the case of methane as a sample. This can be explained that at the same velocity, with higher diffusion coefficient, sample has more chance to diffuse into more tightly packed regions of the column before it will be swept through the gas flow.

#### D. THE OBSTRUCTIVE FACTOR AND THE PARTICLE-TO-COLUMN DIAMETER RATIO

Three columns were studied in this case; 1/8-in. O.D., 1/18-in. I.D. columns packed with 25-35 mesh ( $\omega$  value was 1.3), 60-80 mesh ( $\omega$  value was 1.2) and 80-100 mesh glass beads ( $\omega$  value was 1.3), methane sample, He carrier gas. The particle-to-column diameter ratios ( $d_p/d_c$ ) were varied from 0.1 to 0.4. See table III.

TABLE III:

| column | column diameter<br>cm. | packing mesh<br>range | particle diameter<br>cm. | diameter ratio<br>$d_p/d_c$ |
|--------|------------------------|-----------------------|--------------------------|-----------------------------|
| 1      | 0.1411                 | 25-35                 | 0.061                    | 0.43                        |
| 2      | 0.1411                 | 60-80                 | 0.021                    | 0.15                        |
| 3      | 0.1411                 | 80-100                | 0.016                    | 0.12                        |

The relation between the obstructive factor and velocity at each particle-to-column diameter ratio is shown by figure V.

The minimum value and maximum value of  $\gamma$  are the same for all particle-to-column diameter ratios.  $\gamma_{\min.} = 0.60$  and  $\gamma_{\max.} = 0.74$ . But the  $\gamma$  values in between were different at the same velocity.

It is generally accepted that a direct proportionality exists between plate height and particle diameter (22, 23). In 1964 Sternberg and Poulson (21) studied the particle-to-column diameter ratio effect on band spreading. According to the modified Van Deemter equation, equation 7

$$H = A + \frac{B}{\bar{v}} + C_g \bar{v}$$

they found that the effect of the particle-to-column diameter ratio on the eddy diffusion term, A, is negligible within experimental error. On the longitudinal diffusion term, B, the smaller particle-to-column diameter ratio causes the lower value of B. This agrees with Kieselbach's work (24) and Norem's work (20) (the higher  $d_p/d_c$  causes the higher B value). The B coefficient covers a range of only 15%, the variations in the particle-to-column diameter ratio affect only the gas phase mass transfer,  $C_g$ .

Theoretical treatments (1,25) predict a quadratic dependence of  $C_g$  upon the particle diameter,  $d_p$ .

$$C_g = \omega d_p^2 / D_g \quad (27)$$

where  $\omega$  is the geometric constant. Sternberg and Poulson have found that the values of  $\omega$  are strongly dependent upon the particle-to-column diameter ratio; for glass beads, this dependence extends well into the range of particle and column diameters normally employed

Column: 1/8 in. O.D., 1/18-in. I.D.,

653.3 cm, 735.4 cm, 728.2 cm long.

Packing: 25-35 mesh, 60-80 mesh, 80-100 mesh glass beads.

Sample: CH<sub>4</sub>

Carrier gas: He

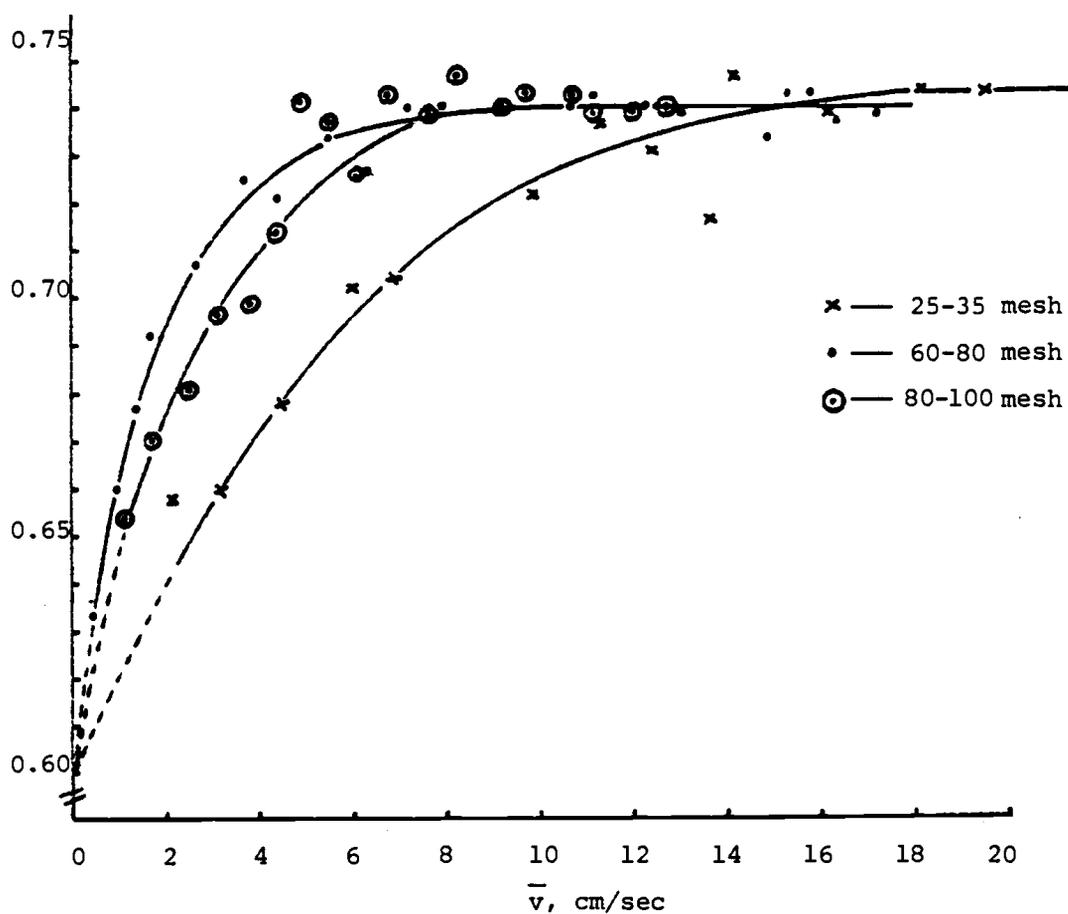


FIGURE V: Variation of obstructive factor with velocity,  $\bar{v}$ , among different  $d_p/d_c$  ratios.

( $d_p/d_c$ ; 1/5 to 1/50). With particles relatively large for the column,  $\omega$  becomes as small as 1/10 of its value for normal particle-to-column diameter ratios. Sternberg and Poulson evaluated the value of  $\omega$  using the Van Deemter equation for plate height, expressed in the form

$$H/f = A + B/\bar{v} + C_g \bar{v} \quad (28)$$

The A value was negligible, the equation can be rewritten as

$$H\bar{v}/f = B + C_g \bar{v}^2 \quad (29)$$

and  $H\bar{v}/f$  is a linear function of  $\bar{v}^2$ , with a slope  $C_g$ . From equation 27,  $\omega$  value was calculated. For glass beads packing and  $d_p/d_c = 0.423$ ,  $\omega$  value was 0.054. If this  $\omega$  value of 0.054 (instead of the  $\omega$  value of 1.3 obtained by Hawkes' consideration (11)) was used to calculate the value of  $\gamma$  from equation 20, the values of  $\gamma$  at high velocities were greater than unity which is absurd. Hence, in this experiment the  $\omega$  value of 1.3 was used to calculate the  $\gamma$  values.

A possible explanation for the low value of Sternberg and Paulson's  $\omega$  are that there might be coupling between  $\omega$  and the A term. And this  $\omega$  value was the value over the long range of velocities. An explanation for the higher  $\omega$  value ( $\omega = 1.3$ ) using Hawkes' consideration at high velocities is that  $\omega$  value was determined at the velocities which were not high enough to support the coupling theory.

From figure  $V_s$  for  $d_p/d_c = 0.12$  and  $d_p/d_c = 0.15$  (within the range normally employed) the points at which the  $\omega$  value reached its maximum are about at  $\bar{v} = 7$  cm/sec. This agrees with the previous explanation (equation 26) that it is more likely a long range effect, the domain is the column cross section. For  $d_p/d_c = 0.43$ , the particle is about three times larger than in the other two columns. The

experimental flattening point is about 17 cm/sec. It seems to be that the domain is about the size of a single particle ( $d_p$ ), then the velocity would be 17 cm/sec. This agrees well with the experimental data.

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TABLE IV (Appendix) Individual Run Data

Column: 1/8-in. O.D., 1/18-in. I.D., 735.4 cm long

Packing: 60-80 mesh glass beads

Samples: CH<sub>4</sub> Carrier gas: He

Continuous elution

|                    |         |         |         |         |         |         |         |         |         |         |
|--------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| H (cm)             | 2.1888  | 1.0196  | 0.7115  | 0.5302  | 0.3373  | 0.2349  | 0.1862  | 0.1509  | 0.1299  | 0.1146  |
| $\bar{v}$ (cm/sec) | 0.4217  | 0.8996  | 1.2693  | 1.6804  | 2.5829  | 3.6204  | 4.4435  | 5.4798  | 6.2754  | 7.2209  |
| j                  | 0.9466  | 0.9011  | 0.8620  | 0.8287  | 0.7855  | 0.7380  | 0.7146  | 0.6881  | 0.6745  | 0.6587  |
| f                  | 1.0003  | 1.0031  | 1.0059  | 1.0086  | 1.0133  | 1.0193  | 1.0223  | 1.0263  | 1.0280  | 1.0307  |
| H (cm)             | 0.1045  | 0.0929  | 0.0890  | 0.0810  | 0.0786  | 0.0735  | 0.0706  | 0.0651  | 0.0650  | 0.0642  |
| $\bar{v}$ (cm/sec) | 7.9503  | 9.0685  | 9.8176  | 10.7199 | 11.1981 | 12.2710 | 13.0738 | 14.9700 | 15.3059 | 15.6885 |
| j                  | 0.6502  | 0.6402  | 0.6346  | 0.6279  | 0.6241  | 0.6194  | 0.6167  | 0.6132  | 0.6113  | 0.6087  |
| f                  | 1.0317  | 1.0336  | 1.0344  | 1.0356  | 1.0362  | 1.0370  | 1.0375  | 1.0380  | 1.0384  | 1.0388  |
| H (cm)             | 0.0627  | 0.0617  | 0.0593  | 0.0609  | 0.0607  | 0.0607  |         |         |         |         |
| $\bar{v}$ (cm/sec) | 16.3422 | 17.1272 | 20.1736 | 32.7804 | 36.9032 | 41.1214 |         |         |         |         |
| j                  | 0.6072  | 0.6052  | 0.5981  | 0.5952  | 0.5938  | 0.5866  |         |         |         |         |
| f                  | 1.0390  | 1.0393  | 1.0407  | 1.0411  | 1.0413  | 1.0426  |         |         |         |         |

Arrested elution

$\bar{v} = 2.2840$  cm/sec,  $j = 0.8290$

| Delay time, t (min)              | 0       | 3        | 5        | 10       | 15       | 20       |
|----------------------------------|---------|----------|----------|----------|----------|----------|
| $\sigma_t^2$ (sec <sup>2</sup> ) | 85.2980 | 117.2467 | 131.4461 | 196.3577 | 228.8389 | 284.9013 |

$\bar{v} = 3.0591$  cm/sec,  $j = 0.7859$

| Delay time, t (min)              | 0       | 3       | 5       | 10      | 15       | 20       |
|----------------------------------|---------|---------|---------|---------|----------|----------|
| $\sigma_t^2$ (sec <sup>2</sup> ) | 32.1353 | 42.6236 | 53.6535 | 88.2646 | 109.7828 | 129.9896 |

$\bar{v} = 3.9908 \text{ cm/sec}, j = 0.7523$

|                                     |         |         |         |         |         |         |
|-------------------------------------|---------|---------|---------|---------|---------|---------|
| Delay time, t (min)                 | 0       | 3       | 5       | 10      | 15      | 20      |
| $\sigma_t^2 \text{ (sec}^2\text{)}$ | 13.6477 | 18.0761 | 22.0662 | 36.6142 | 49.7611 | 59.6439 |

$\bar{v} = 4.8070 \text{ cm/sec}, j = 0.7261$

|                                     |        |        |         |         |         |         |
|-------------------------------------|--------|--------|---------|---------|---------|---------|
| Delay time, t (min)                 | 0      | 3      | 5       | 10      | 15      | 20      |
| $\sigma_t^2 \text{ (sec}^2\text{)}$ | 6.9555 | 8.5612 | 12.5530 | 19.1756 | 27.6128 | 34.4782 |

$\bar{v} = 6.8771 \text{ cm/sec}, j = 0.6752$

|                                     |        |        |        |        |        |        |
|-------------------------------------|--------|--------|--------|--------|--------|--------|
| Delay time, t (min)                 | 0      | 3      | 5      | 10     | 15     | 20     |
| $\sigma_t^2 \text{ (sec}^2\text{)}$ | 1.9414 | 2.5356 | 3.5381 | 5.2396 | 7.5998 | 9.5184 |

$\bar{v} = 8.8084 \text{ cm/sec}, j = 0.6473$

|                                     |        |        |        |        |        |        |
|-------------------------------------|--------|--------|--------|--------|--------|--------|
| Delay time, t (min)                 | 0      | 3      | 5      | 10     | 15     | 20     |
| $\sigma_t^2 \text{ (sec}^2\text{)}$ | 0.9905 | 1.2424 | 1.5230 | 2.6380 | 3.5907 | 5.0410 |

column: 1/8-in. O.D., 1/18-in. I.D., 735.4 cm long

Packing: 60-80 mesh glass beads

Sample: Ar. Carrier gas: He

Continuous elution

|                    |        |        |        |         |         |         |         |         |         |         |
|--------------------|--------|--------|--------|---------|---------|---------|---------|---------|---------|---------|
| $\bar{H}$ (cm)     | 2.4481 | 1.1702 | 0.7784 | 0.6024  | 0.3753  | 0.2576  | 0.2049  | 0.1641  | 0.1416  | 0.1232  |
| $\bar{v}$ (cm/sec) | 0.4086 | 0.8288 | 1.2046 | 1.5539  | 2.4777  | 3.5176  | 4.4118  | 5.4550  | 6.2059  | 7.2164  |
| j                  | 0.9466 | 0.9011 | 0.8620 | 0.8287  | 0.7855  | 0.7380  | 0.7146  | 0.6881  | 0.6745  | 0.6587  |
| f                  | 1.0003 | 1.0031 | 1.0059 | 1.0086  | 1.0133  | 1.0193  | 1.0223  | 1.0263  | 1.0280  | 1.0307  |
| $\bar{H}$ (cm)     | 0.1138 | 0.1023 | 0.0949 | 0.0882  | 0.0826  | 0.0790  | 0.0746  | 0.0722  | 0.0703  | 0.0677  |
| $\bar{v}$ (cm/sec) | 7.9155 | 8.8937 | 9.7203 | 10.6435 | 11.3438 | 12.3047 | 13.0629 | 13.8960 | 14.5601 | 15.4872 |
| j                  | 0.6502 | 0.6402 | 0.6346 | 0.6279  | 0.6241  | 0.6194  | 0.6167  | 0.6132  | 0.6113  | 0.6087  |
| f                  | 1.0317 | 1.0336 | 1.0344 | 1.0356  | 1.0362  | 1.0370  | 1.0375  | 1.0380  | 1.0384  | 1.0388  |

|                    |         |         |         |         |         |         |
|--------------------|---------|---------|---------|---------|---------|---------|
| $\bar{H}$ (cm)     | 0.0655  | 0.0643  | 0.0624  | 0.0623  | 0.0624  | 0.0624  |
| $\bar{v}$ (cm/sec) | 16.3593 | 16.9881 | 20.0098 | 32.3598 | 40.9699 | 48.0310 |
| j                  | 0.6072  | 0.6052  | 0.5981  | 0.5952  | 0.5938  | 0.5866  |
| f                  | 1.0390  | 1.0393  | 1.0407  | 1.0411  | 1.0413  | 1.0426  |

Column: 1/8-in. O.D., 1/18-in. I.D., 653.3 cm long

Packing: 25-35 mesh glass beads

Sample: CH<sub>4</sub> Carrier gas: He

Continuous elution

|                    |        |        |        |        |        |        |         |         |         |         |
|--------------------|--------|--------|--------|--------|--------|--------|---------|---------|---------|---------|
| $\bar{H}$ (cm)     | 0.4679 | 0.3328 | 0.2529 | 0.2125 | 0.1965 | 0.1715 | 0.1697  | 0.1686  | 0.1675  | 0.1722  |
| $\bar{v}$ (cm/sec) | 2.1914 | 3.1704 | 4.5031 | 5.9637 | 6.7986 | 9.8670 | 11.3126 | 12.4772 | 13.5874 | 14.0495 |
| j                  | 0.9846 | 0.9768 | 0.9667 | 0.9569 | 0.9509 | 0.9302 | 0.9215  | 0.9131  | 0.9060  | 0.9049  |
| f                  | 1.0001 | 1.0002 | 1.0004 | 1.0006 | 1.0008 | 1.0015 | 1.0019  | 1.0024  | 1.0027  | 1.0028  |

|                    |         |         |         |         |         |         |         |
|--------------------|---------|---------|---------|---------|---------|---------|---------|
| $\bar{H}$ (cm)     | 0.1777  | 0.1873  | 0.1933  | 0.2158  | 0.2407  | 0.3588  | 0.3904  |
| $\bar{v}$ (cm/sec) | 16.1833 | 18.0657 | 19.5306 | 22.8205 | 26.2176 | 40.3145 | 43.3487 |
| j                  | 0.8888  | 0.8782  | 0.8702  | 0.8369  | 0.8150  | 0.7664  | 0.7507  |
| f                  | 1.0038  | 1.0046  | 1.0051  | 1.0079  | 1.0101  | 1.0155  | 1.0175  |

Column: 1/8-in. O.D., 1/18-in. I.D., 728.2 cm long

Packing: 80-100 mesh glass beads

Sample: CH<sub>4</sub> Carrier Gas: He

Continuous elution

|                    |        |        |        |        |        |         |         |         |         |         |
|--------------------|--------|--------|--------|--------|--------|---------|---------|---------|---------|---------|
| $\bar{H}$ (cm)     | 2.2909 | 1.4665 | 0.4878 | 0.3164 | 0.2550 | 0.2002  | 0.1748  | 0.1613  | 0.1430  | 0.1244  |
| $\bar{v}$ (cm/sec) | 0.3860 | 1.1033 | 1.6856 | 2.5024 | 3.0924 | 3.8453  | 4.4453  | 4.9068  | 5.4816  | 6.1794  |
| j                  | 0.9016 | 0.8297 | 0.7867 | 0.7396 | 0.7164 | 0.6901  | 0.6766  | 0.6609  | 0.6526  | 0.6425  |
| f                  | 1.0030 | 1.0086 | 1.0131 | 1.0189 | 1.0221 | 1.0259  | 1.0279  | 1.0303  | 1.0316  | 1.0332  |
| $\bar{H}$ (cm)     | 0.1163 | 0.1031 | 0.0970 | 0.0873 | 0.0830 | 0.0770  | 0.0733  | 0.0692  | 0.0667  | 0.0636  |
| $\bar{v}$ (cm/sec) | 6.7499 | 7.6069 | 8.1899 | 9.1095 | 9.6895 | 10.5659 | 11.1659 | 12.0135 | 12.6256 | 13.4420 |
| j                  | 0.6371 | 0.6304 | 0.6266 | 0.6219 | 0.6193 | 0.6159  | 0.6139  | 0.6113  | 0.6099  | 0.6072  |
| f                  | 1.0341 | 1.0352 | 1.0358 | 1.0366 | 1.0370 | 1.0376  | 1.0379  | 1.0383  | 1.0386  | 1.0390  |

|                         |         |         |         |         |         |
|-------------------------|---------|---------|---------|---------|---------|
| $\overline{H}$ (cm)     | 0.0817  | 0.0550  | 0.0541  | 0.0531  | 0.0500  |
| $\overline{v}$ (cm/sec) | 14.0541 | 16.9106 | 17.7267 | 18.1345 | 20.7869 |
| j                       | 0.6052  | 0.5981  | 0.5952  | 0.5938  | 0.5866  |
| f                       | 1.0393  | 1.0407  | 1.0411  | 1.0413  | 1.0426  |