AN ABSTRACT OF THE THESIS OF

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(Name) (Degree)

in INDUSTRIAL ENGINEERING presented on October 1, 1970
(Major) (Date)

Title: DECISION PROCEDURES FOR CAPITAL RATIONING UNDER CONDITIONS OF RISK

Abstract approved: Redacted for Privacy
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The problem of deciding exactly how to allocate a limited amount of capital resources among competing investment alternatives in order to provide the greatest economic benefit to the business enterprise has been a subject of interest for nearly a century. Many decision models and methods for evaluation and comparison of alternatives have been described in the various business, economic, accounting, and engineering journals.

The purpose of this thesis is twofold: First, to review the significant literature concerning capital budgeting written during the past 15 or 20 years, with the intent on highlighting and organizing the material. Second, to select an area within the field where little work has yet been done, or where the treatment has been less than satisfactory.

In regards to the second part of the over-all objective of the thesis, the development of a game board for simulating the risk

surrounding possible investment payoffs is described. A capital rationing problem containing essential elements of real-world situations is developed and described in detail. The main thrust of the present paper is a new approach to finding the "best" solution to this type of capital rationing problem encountered under conditions of risk. Drawing on basic principles of cardinal utility theory, a quantitative investment policy is presented. The investment criterion used is the before-taxes percentage rate of return; decision rules are based on an evaluation of the risk-return profile curve for each alternative under consideration. Parameter values for probability coefficients and for critical rate of return values are arbitrarily established for a hypothetical risk-seeker, risk-avoider, and risk-insensitive. The investment decisions based upon their individual risk preferences is presented for data developed in the sample problem.

Finally, the results of an investigation as to how individuals actually make decisions (that is, what decision-rules they use) in problems of this type are given, and suggestions for further research are made.

Decision Procedures for Capital Rationing Under Conditions of Risk

by

David William Nebergall

A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Master of Science

June 1971

APPROVED:

Redacted for Privacy

Professor of Industrial Engineering
in charge of major

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Date thesis is presented October 1, 1970

Typed by Mary Jo Stratton for David William Nebergall

TO MY PARENTS,

With Love

ACKNOWLEDGMENT

An undertaking on the magnitude of a thesis simply cannot be the accomplishment of a single individual. Rather, a finished thesis is the achievement of many persons, each contributing his own special skills and talents. This author wishes to acknowledge his heartfelt gratitude and indebtedness to some of the people who made the completion of this thesis possible.

First of all, to Dr. James L. Riggs, my major advisor, for helpful counsel and advice during all my years of undergraduate and graduate work at Oregon State University, and in particular for his inspiration and guidance in regards to this thesis. Without his faith in me, I could never have undertaken this work.

To the staff of the Oregon State University Kerr Library for their patient and helpful assistance in tracking down sources and references for the literature survey. To Jack Kellogg, engineering mechanician, for his suggestions and help in the design and construction of the three-dimensional display board. To Mrs. Sharon Lough, secretary for the Department of Industrial Engineering, for typing material for the initial drafts. To the many authors and publishers whose efforts are highlighted in this thesis for their cooperation. To the subjects who volunteered for the experimental study for their interest and participation. And to Mary Jo Stratton I must declare my appreciation for her superb job of typing the final copy, and to Type-Ink of

Corvallis for their cooperation in fulfilling all the necessary requirements and meeting the deadlines.

Finally, I owe a special debt of gratitude to my wife Sharon for patiently enduring the many household chores and family affairs that were neglected or postponed so that work on the thesis could continue.

TABLE OF CONTENTS

		Page
I.	INTRODUCTION	1
	Objective of the Study	1
	Scope of the Study	2
	Historical Development	3
II.	SIMPLIFIED METHODS OF COMPARING ALTERNATIVES	6
	"Seat of the Pants" Method	6
	Total Annual Cost Method of Comparison	7
	Capital Recovery Period Method	7
	Rate of Return of the Investment Method	8
III.	THE APPROACH OF THE ENGINEERING ECONOMISTS	10
	The Time Value of Money; the Three-Phase Analysis Approach	10
	Present Worth Comparison	12
	Equivalent Annual Amount Comparison	13
	Capitalized Comparison	14
	Rate of Return Comparison	15
	Service Life Comparison	16
IV.	TRADITIONAL CAPITAL BUDGETING TECHNIQUES	18
	Introduction: Assumptions of Independence and Certainty	18
	Present Worth Method	20
	Rate of Return Method	22
	Present Worth vs. Rate of Return	33
	Capital Planning	39
V.	OPERATIONS RESEARCH MODELS FOR EVALUATING CAPITAL INVESTMENT PROPOSALS	41
	Assumptions of Certainty and Independence	41
	Assumptions of Certainty and Project Interdependence	51
	Concepts of Risk and Uncertainty	55

		Page
	Simulation, Utility, and Other Models	66
	Allocating Resources Other than Capital	77
VI.	A MODEL FOR SIMULATING RISK	78
	The Need to Consider Risk and Uncertainty in Capital Budgeting Problems	78
	The Development of the Model	85
VII.	THE DEVELOPMENT OF A QUANTITATIVE INVESTMENT POLICY	101
	Introduction and Theory	101
	Evaluation Criteria and Decision Rules	108
	Solution to the Sample Problem	111
VIII.	AN EXPERIMENTAL STUDY	116
	General Experimental Procedure and Game Format	116
	Results of the Investigation	121
IX.	CONCLUSION	128
	Summing Up	128
	Suggestions for Further Research	130
BIBLIOGRAPHY		133
APPENDIX		139

LIST OF FIGURES

Figure		Page
VI-1	Photographs of the risk simulation model.	88
VI-2	Payoff graphs for Program #1.	93
VI-3	Payoff graph for Program #2.	94
VI-4	Payoff graphs for Programs #3 and 5.	95
VI-5	Payoff graphs for Program #4.	96
VI-6	Risk-return profile curves.	99
VII-1	Regions of payoff maximization and risk minimization.	103
VII-2	Risk-seeker's aspiration line.	106
VII-3	Risk-avoider's aspiration line.	107
VII-4	Gambler's investment decision.	109
VII-5	Conservative's investment decision.	110
VII-6	Investment decision-making flowchart.	113

LIST OF TABLES

Table		Page
VI-1	Probabilities of one or more marbles occupying given zones.	89
VI-2	Program lifespan and total dollar outlay requirements.	90
VI-3	Program investment pattern requirements.	91
VI-4	Possible dollar payoffs by zone for each channel.	92
VI-5	Program listing of frequency of net return and rate of return.	98
VII-1	Parameter values.	108
VIII-1	Original program total dollar outlay requirements.	116
VIII-2	Expanded capital outlay possibilities and investment decisions based on quantitative investment policy.	117
VIII-3	Breakdown of investment allocations by decision-maker.	122
VIII-4	Breakdown of decision-makers by allocation.	123
VIII-5	Decision-maker's preference for game board or payoff distribution graphs.	123
VIII-6	Actual vs. theoretical investment decisions.	124

DECISION PROCEDURES FOR CAPITAL RATIONING UNDER CONDITIONS OF RISK

I. INTRODUCTION

Objective of the Study

The development of formal procedures for determining the manner in which a business firm should acquire funds for long-term capital investment and the manner in which it should use these funds has been a topic of interest for nearly a century. During this period, various aspects of the capital budgeting problem have been treated in the business, economic, accounting, and engineering journals. These streams of literature have, for the most part, been entirely independent of one another. Within the last 20 or so years, during which time interest in the topic has been particularly great, there has been some "cross-fertilization" of ideas. As a result, two additional problems have developed: 1) the significant articles are very thinly distributed among the large and growing number of journals, and 2) there is a large amount of confusion with regard to terminology, assumptions, limitations, and errors of the various proposed methods for capital investment planning (7, p. 201).

It is this latter problem which is more serious and more difficult to correct. It is this problem to which this study is addressed. Our objective is to review some of the significant literature in the field during the last 15 or 20 years with an emphasis on organizing and highlighting the material. We shall not be so concerned with the details of the various methods, techniques, and models as we shall be with summarizing the basic principles involved and identifying the relevant concepts and ideas. We shall indicate how the various models are interrelated and how they are built upon similar fundamental assumptions. In addition, we shall attempt to select an area in the field where little work has heretofore been done and to contribute some original thoughts regarding this area, thereby (hopefully) advancing the "state of the art" one step further.

Scope of the Study

To keep the paper to a reasonable length, we are limiting the scope of our study to decision models for the allocation of a limited amount of capital resources among several potential investment projects. The capital budgeting problem where there is an upper limit on available funds is referred to as capital rationing. Capital rationing is a more realistic approach to the problem than is the situation where resources are unlimited. The terms "capital budgeting" and "capital rationing" are often used interchangeably.

Also, we are restricting ourselves to the selection of one or more well-defined projects or proposals, such as research and development projects or equipment investment alternatives. There has been some material written on portfolio selection and stock market investment (see, for example, Markowitz (46) and Naslund (50)), but these areas are not considered here. Nonetheless, certain of the models for capital rationing can be modified and extended to include these areas.

Historical Development

As we have indicated, the volume of capital budgeting material evident in numerous articles in the technical as well as the popular journals and in the increase in the number of textbooks on the subject indicates widespread interest in investment decision-making on the part of both academic and business audiences. Concurrent with this heightened concern with capital budgeting there has occurred developments in the methodology of operations research, where progress has been made in the application of mathematical models and digital computers to complex business problems. Prior to the middle 1960's, however, this progress had made little impact on the majority of the work being done in the capital budgeting field. The picture today and the outlook to the future are much brighter, and as we shall presently see, headway is being made.

Historically (i.e., prior to World War II), theorists on the economics of capital focused their attention on such ancient and fundamental issues as the nature and functions of interest rates.

There was implied in much of this discussion, and more expressly stated by the mathematical economists, a model of optimum investment by the firm. This model, like much of economic theory at the time, made use of such simplifying assumptions as continuity and differentiability of investment functions, certainty of future, and perfect capital markets. The model proposed that under an optimum investment program, investment would be carried by the firm to the point where the marginal internal rate of return equaled the market rate of interest. (We shall define the "internal rate of return" of an investment project as that interest rate for which the net present value of all streams of cash flows associated with the project sums to zero. See Chapter III.)

This theory and its underlying assumptions were satisfactory when applied to the very limited class of problems with which the economists were then primarily concerned. In fact, crude as the model was, it did at least consider the element of time; much literature on the management of investment planning and actual business practice has either neglected the time dimension entirely or treated it in an even more grossly simplified way (74, p, 1-2).

Our survey of the literature begins with some of the simplified techniques for producing "ball park" estimates in the comparison of alternatives. These simplified versions of more refined techniques do not consider the effects of interest, taxes, and other important factors.

Chapter III introduces the concept of the "time value of money" and outlines the several methods given by the engineering economists for comparing investment proposals. The next chapter reviews the two primary methods developed during the fifties by business and economic analysts -- the present worth method and the rate of return method. These two methods and the assumptions they were built upon are termed the "traditional" means for determining capital investment programs. The most recent developments in the application of operations research techniques and mathematical models to capital budgeting problems is given in Chapter V.

II. SIMPLIFIED METHODS OF COMPARING ALTERNATIVES

"Seat of the Pants" Methods

There are many investment decision situations where a detailed analysis is either unnecessary, uneconomical, or too time-consuming. Such situations might occur, for example, when decisions must be made in very short order or when it is possible to manipulate the costs and unquantifiable criteria into some form which will make possible a convenient and meaningful comparison of the alternatives. The simplified, so-called "seat of the pants" methods described below are for the selection of the "best" single investment proposal. The effects of interest, taxes, and other important factors are ignored.

These simplified versions of three of the common methods of quoting the relevant costs in a comparison of alternatives will be illustrated in an example given by Krick (39). Horngren (35) defines "relevant cost" as being a) an expected future cost, and b) a cost differing among alternatives. The relevant cost data for our example is as follows:

Alternative	Initial Cost (IC)	Annual Operating Cost (OC)	Expected Useful Life of Equipment (N)
P (present method)	$ 0	$38,000	5 years
A	$16,000	$31,000	7 years
B	$11,000	$34,000	6 years

Krick evaluates these alternatives by estimating a) the total annual cost of each alternative, b) the capital recovery period (payback period), and c) the rate of return of the investment.

Total Annual Cost Method of Comparison

To compare the alternatives in our example in this manner, it is necessary to convert the initial investment to an annual basis and then add this prorated figure to the annual operating cost to obtain the total annual cost (TAC). Thus, TAC = OC + IC/N. The total annual costs for our three alternatives are calculated below:

$$(TAC)_P = \$38,000 + \$0/5 = \$38,000 \text{ per year}$$

$$(TAC)_A = \$31,000 + \$16,000/7 = \$33,286 \text{ per year}$$

$$(TAC)_B = \$34,000 + \$11,000/6 = \$35,833 \text{ per year}$$

Under this method of comparison, alternative A offers the lowest total annual cost, which takes into account both the annual operating cost and the initial investment prorated to a yearly basis.

Capital Recovery Period Method

By this method, calculations are made to estimate the period of time required for the cumulative savings in operating costs offered by a proposal to equal the initial cost of that proposal, or in other words, to recover or pay back the initial investment. The savings here are the difference in operating costs of the present and proposed methods.

A simplified estimate of the capital recovery period (CRP) is given by

$$(CRP) = \frac{(IC)_a}{(OC)_p - (OC)_a}$$

where $(OC)_p$ is the operating cost of the present method and $(OC)_a$ is the operating cost of the alternative under consideration. For our example:

$$(CRP)_A = \frac{\$16,000}{\$38,000 - \$31,000} = 2.28 \text{ years}$$

$$(CRP)_B = \frac{\$11,000}{\$38,000 - \$34,000} = 2.75 \text{ years}$$

It appears that proposal A offers to recover its own initial cost in a shorter period than proposal B. Of course, the decision-maker must decide whether or not the 2.28 year capital recovery period of proposal A makes the investment worthwhile in the light of other possible uses of capital. This will depend on company policy regarding such matters. Some firms have the policy that a proposed investment must have a capital recovery period less than or equal to the maximum acceptable period, say, one to three years for small projects and three to five years for major investments, in order for the alternative to be approved.

Rate of Return of the Investment Method

This procedure estimates the percentage of the initial investment that will be recovered per year through the resulting savings in operating costs. This annual rate of return of the investment (not to be confused with the internal rate of return to be described in Chapter III) is given by the reciprocal of the capital recovery period:

$$(R/I) = \frac{(OC)_p - (OC)_a}{(IC)_a}$$

Substituting the figures from our example yields:

$$(R/I)_A = \frac{\$38,000 - \$31,000}{\$16,000} \times 100 = 43.75\% \text{ per year}$$

$$(R/I)_B = \frac{\$38,000 - \$34,000}{\$11,000} \times 100 = 36.40\% \text{ per year}$$

Thus for alternative A it is expected that the initial investment will be paid back at a rate of 43.75 percent of that investment per year, offering a 7.35 percent advantage over alternative B.

By any one of these three simplified or short-cut versions we see that alternative A is superior to both the present method and alternative B. It should be pointed out that the choice as to which method of comparison to use in a given situation is up to the decision-maker; the methods discussed in this chapter are equivalent in effect, since they all lead to the same conclusion.

III. THE APPROACH OF THE ENGINEERING ECONOMISTS

The Time Value of Money; the Three-Phase Analysis Approach

In all but the very simplest and routine capital investment decisions, the so-called "time value of money" is an important consideration. This time value of money is termed "interest," and is used to designate a fee for the use of money. The charging of interest is a practice dating back to the time of man's earliest recorded history. The ethics and economics of interest have been a subject for discussion by philosophers, theologians, statesmen, and economists throughout the ages. Within the last century, the standard business and engineering economics textbooks have developed interest formulas and extensive tables of interest factors. It is these formulas and factors which allow a more meaningful comparison of potential investments, since the usual engineering economy analysis is concerned with alternatives, each of which will involve expected outputs and inputs at specific points in time. These outputs and inputs are then converted into monetary receipts and disbursements, insofar as this conversion is possible. The resulting quantitative information may be manipulated mathematically for comparison. As before, with the simplified methods, we are looking for the "best" single investment alternative. Only in this case, we are taking the time value of money into consideration.

In this paper we shall assume the use of the discrete model of cash flows for computing interest. As Riggs (63) points out, both discrete and continuous models of cash flows are approximations of actual transactions. Cash flows do not always follow a uniform pattern, nor are they always continuous, nor do they always occur as periodic end-of-period payments. They are often irregular in timing and amount. Tables of interest factors have been prepared for both models. However, we focus our attention on discrete rather than continuous compounding of interest because it is more commonly understood, accepted, and used by industry.

According to Bullinger (11), engineering economy analysis is done in three phases, separately and in sequence. These three phases are described in part as follows (p. 36-37):

> The economy analysis... is aimed at finding out what it will cost to set up a project in good working order, what it will cost to keep it in good operating condition, what probable income will be received from it, and whether or not it is worthwhile doing anyway.
>
> The financial analysis... is to determine the amount of money necessary to finance the project and to determine which of several methods of financing is most desirable.
>
> The intangible analysis is an analysis of the facts concerning the project which have been found incapable of expression in concrete figures or money values.

In a review of the work done in the field of engineering economy, Radnor (56) says: "A serious criticism of the previously mentioned three-layer analysis is the notion that each of the analyses can be made separately. This must lead to a static suboptimal approach which can hardly be saved by reference to the intangibles" (p. 137).

Bernhard (8) briefly describes the model of Weingartner (74) to show where the separation of analyses into phases does not lead to a suboptimal answer as claimed by Radnor. The Radnor assertion would, however, be correct if it were modified to say that the three-phase analysis may lead to a suboptimal answer rather than saying it must.

The most common bases for comparing investment alternatives which take into account the time value of money are given by Thuesen and Fabrycky (70) and include the present worth amount, the equivalent annual amount, the capitalized amount, the rate of return, and the service life. These bases are briefly described in the sections below; for a more detailed discussion the reader is referred to any standard engineering economy text.

Present Worth Comparison

The present worth basis of evaluation is an outgrowth of property valuation practices and techniques mainly developed since the turn of the century. Through the use of the interest formulas and factors, the present worth of either a single or a series of equal future monetary values is easily computed and readily understood.

When the decision is either to accept or reject a single investment alternative, the net present worth of the alternative is computed. The net present worth is defined as the present worth of the positive cash flows (receipts) associated with the alternative discounted through time at an appropriate interest rate less the present worth of the negative cash flows (disbursements). If the net present worth is positive in sign, the alternative is accepted; otherwise the proposal is rejected.

When comparing more than one alternative, the objective is to maximize the net present worth. In other words, the decision-maker selects the alternative with the greatest net present worth.

The present worth basis of comparison often results in amounts that are quite large in comparison with the amounts encountered in the alternatives being compared. A slight change in the interest rate may result in large changes in present worth amounts. This method of comparison is somewhat cumbersome in comparing alternatives spanning different periods of time.

Equivalent Annual Amount Comparison

The development of this basis of comparison is associated with the development and general use of cost accounting procedures, which necessitated consideration of costs on an annual basis and therefore consideration of depreciation as an annual expense. A distinct advantage of the equivalent annual amount comparison is that it conforms to thought patterns that are used in accounting practices and are therefore familiar to most people in business. A second important advantage of this method is that the period under consideration is always a unit of time, namely, one year. The amounts determined with it are actually a summation of receipts and disbursements per unit of time. Since equivalent annual amounts are based on a common unit, they are easier to comprehend and to compare.

Ordinarily the first step in this calculation is to find the present worth of receipts and disbursements for each investment proposal being considered. Next, these quantities are converted to an annual basis using the equal-payment-series capital-recovery interest factor. The decision rules for evaluating a single alternative or comparing multiple alternatives are the same as for the present worth method, except here we are maximizing net annual cost.

Capitalized Comparison

The capitalized amount basis is best suited for evaluation and comparison of long-term alternatives with stable income and interest costs that are high in relation to subsequent costs. Public works projects such as tunnels, bridges, and dams are examples of this type of long-term project.

On a capitalized basis, the income and disbursements will be calculated as though they will continue forever. Hence, this basis of evaluation is not ordinarily suitable for investment proposals of short life. The method basically consists of finding a single amount in the present whose return at a given rate of interest will be equivalent to the net difference between receipts and disbursements assuming the given patterns of receipts and disbursements were repeated in perpetuity. Actually, this basis of comparison is rarely used, but is mentioned here for the sake of completeness.

Rate of Return Comparison

The rate of return is a widely accepted measure of success. It serves equally well both for the comparison of an alternative with other alternatives that are delineated and for an alternative with other opportunities which are believed to exist but are not delineated. The expected rates of returns from different types of ventures are fairly well established and thus provide a "standard" for comparing proposals that are being considered.

The rate of return on an investment, also known as the internal rate of return, is a value i such that the present worth of the flow of receipts from the investment less the present worth of the flow of disbursements required by the investment equals zero when the interest rate equals i.

This internal rate of return method, as well as the relative merits of this and the present worth method, are discussed further in the next chapter.

Service Life Comparison

It is generally conceded that the longer the life of the proposed investment, or the greater the planning horizon, the greater is the uncertainty the decision-maker must face. (Problems of uncertainty and risk are considered in Chapter V.) One merit of the service life comparison is that it specifically directs attention to the length of life embraced by an alternative. The hazard of an opportunity may often be pointedly revealed by calculating results for service lives less than that originally estimated. The service life method of evaluation is useful as a supplement to the other methods discussed in this chapter.

Service life is usually determined on the basis of an interest rate of zero.[1/] The service life is then the period of time required for the difference between operating receipts and disbursements to equal the capital cost of the investment at zero interest. In other words, service life is equal to the time period required for an asset to "pay for itself" from net operating returns. When interpreted as a payback period, the method gives a break-even point that is useful in decision-making. Finally, this basis of comparison is especially well accepted as a means for evaluating the desirability of purchasing capital equipment and other productive assets.

[1/] Accountants define service life as the period over which the benefits of the asset are expected to flow.

Again it should be pointed out that all of these methods are comparable, and the selection or preference of one over another is strictly the policy of the firm or is left to the discretion of the decision-maker. Also, these are not the only techniques available for evaluation and comparison of investment alternatives. Other procedures have been developed for specialized application in specific industries, such as benefit-to-cost ratios used by many governmental agencies and the Gillette method used in the petroleum industry. Others, like the capitalized cost method, are seldom used at all anymore.

IV. TRADITIONAL CAPITAL BUDGETING TECHNIQUES

Introduction: Assumptions of Independence and Certainty

Nearly all of the decision-making procedures described in this chapter are based upon the simplifying assumptions of independence of investment proposals and certainty of the future. Lorie and Savage (42) indicate that investment proposals can be termed "independent" when the worth of an individual proposal is not significantly affected by the acceptance of others. Weingartner (75) generalizes this statement by saying that investment alternatives are independent in the sense that the acceptance of any set of them does not affect the feasibility or profitability of accepting any different set. This is really an extreme assumption since in reality true independence is never complete.

Most of the writers reviewed in this chapter also assume that present and future market interest rates as well as all future cash flows (both revenues and expenses) of the investment proposals are known with certainty. Again, while not very realistic, the assumption of certainty does make things easier to deal with. The concepts of risk and uncertainty and the means for handling these concepts are discussed in the next chapter.

Heller (29) describes a survey of manufacturing firms in the Minneapolis/St. Paul area conducted in 1950. The survey revealed that most companies in the area at that time used the "payoff period" calculation to determine proposal savings. Managers tended to resist use of "fancy formulas" and preferred to rely on the judgment of the operating personnel. They felt that computations were misleading since there were so many intangible factors that could not be evaluated. Of the firms that used other methods such as rate of return, there was wide variation in the minimum acceptable rate of return for projects; Heller concludes that this variation was most likely due to the many different methods of calculation used.

This chapter describes and compares two of the most widely advocated capital budgeting procedures in the literature -- the present worth method and the rate of return method. The present worth method is also known as the present value method; Bierman and Smidt (10) list the following terms as defining the concept of rate of return:

- interest rate of return
- internal rate of return
- return on investment
- present value return on investment
- discounted cash flow
- profitability index
- investor's method
- marginal efficiency of capital
- yield of an investment

It is easy to see why there has been so much confusion over the terminology used in the literature. For the requirements of this paper, we shall define two rates of return for computational purposes, depending on whether or not cash flows exist which must be discounted. Let us call the first the rate of return on investment, defined as net return (i. e., revenues less disbursements for the period) divided by the investment required, expressed as a percentage.

$$\% \text{ Rate of return on investment} = \frac{\text{Net return}}{\text{Investment}} \times 100$$

Let us call the second the internal rate of return, defined in Chapter III as a value i such that the net present worth (revenues less disbursements) of the cash flows associated with the investment proposal equals zero when the discount interest rate equals i.

Present Worth Method

We saw in the last chapter that the present worth method for comparing alternatives was one of several methods suggested by the engineering economists. The present worth method is rated preferable by several authors over the other methods, and over the rate of return method in particular, on a number of counts.

A paper by Hirshleifer (33) reviews the principles of Irving Fisher's famous work The Theory of Interest (New York, MacMillan Co., 1930) to see what light they shed on the present value rule and the rate of return rule. The author concludes that the present value rule is correct and superior to the internal rate of return rule in a wide variety of cases where investments are assumed to be independent and where the interest rates for borrowing and lending are equal and constant over time. The rate of return method will only give correct answers in general if restricted to two-period comparisons.

More important is Hirshleifer's conclusion that the Fisherian approach stands over all the proposed formulas as the general theoretical solution to the problem of investment decisions under certainty, because, from a Fisherian viewpoint, none of the formulas proposed is universally valid.

Pye (54) presents an algorithm for determining the present value of a proposed investment in an imperfect market (that is, where the interest rate for borrowing funds does not equal the interest rate for lending): For N, the total net cash flow (receipts less disbursements) from an investment, let the maximum amount obtainable now from N be called its realizable present value and let the minimum amount obtainable now from N be called its generating present value.

For a "regular" income stream (negative components followed by some positive ones), $PV_{realizable}$ will be positive if and only if the rate of return is greater than r_{borrow}; similarly, $PV_{generating}$ will be negative if and only if the rate of return is less than r_{lend}. Thus, an investor will undertake a regular, independent investment if and only if the rate of return is greater than r_{borrow}; it will not be undertaken if and only if the rate of return is less than r_{lend}.

In the case of capital rationing where no borrowing is allowed, the situation can be handled by setting r_{borrow} equal to infinity.

The rules derived in this paper will permit most projects to be accepted or rejected independently, thus greatly reducing the necessary calculations (there are, for n projects, 2^n conceivable combinations of projects).

Rate of Return Method

In the Introduction we described the position of the majority of theoretical economists prior to World War II. It was Joel Dean who finally improved the situation by pointing out to business managers and accountants deficiencies in their capital budgeting procedures. He provided them with an alternative approach based upon the economic theory of investment.

According to Dean (17), practical rationing of capital requires not only a ranking of investment projects according to profitability, but it also requires a rejection standard to separate projects that are not sufficiently profitable to merit funds from those that are. Theoretically, this minimum cut-off rate of return is automatically determined by the intersection of the capital supply and demand curves; usually in practice, however, cut-off rates are set by management.

The rejection rate has three uses in the administrative control of capital budgeting: 1) to provide a tentative forecast of return expectancies for the coming period's budget program, 2) to weed out projects that have too low a profitability to justify further attention at either the divisional or top-management levels, and 3) to implement a long-run capital budgeting plan that seeks to avoid making marginal investments of low productivity in times of slack investment demand.

Conceptually, four forms of the rejection rate of return can be identified: 1) a fluctuating effective rate of return that may move up and down with phases of the business cycle and that will determine the cut-off point for normal projects at any one time, 2) a basic minimum rate of earnings that sets a normal floor for any projects in any phase of the cycle, 3) a stable long-run rate that is frozen as the cut-off rate for all phases of the business cycle, and 4) exception rates of return that differ for different kinds of investments to accommodate disparities in risks and variations in management strategies. The effective rate and the minimum rate can be used together for short-term budgeting. Alternatively, the stable long-run rate is best for the long planning horizon. The exception rates can be used with either of these capital rationing schemes.

Dean's presentation is based upon a number of simplifying assumptions, including the following:

1) The assumption that the objective of the firm is to maximize profits in the narrow and calculable sense of the term.

2) The assumption of perfect foresight into all the opportunities for investing capital within the firm.

3) The assumption that the prospective rate of return on each proposal can be projected with precision.

4) The assumption that all risks of all projects either are equal or have been made equal by suitably adjusting the rates of returns on the various projects.

5) The assumption that the company has access to the necessary capital markets for raising equity money and debt money, and that the rates for each can be determined.

Although Weingartner (74) acknowledges that Dean's approach was an important step forward, he criticizes the fact that it was a literal, although simplified, version of the economic theory of investment. This fact meant that it could guarantee an optimum decision-making policy only in those situations which conformed to the underlying assumptions -- perfect certainty, perfect capital markets, continuous and continuously differentiable investment functions, and strict independence of investment projects. Unfortunately, one or more of the assumptions will frequently not be satisfied in actual business problems. Thus, not only will the procedure not yield an optimum, but in some cases it is not even clear how Dean's prescribed procedure could be implemented at all.

The shortcomings of Dean's original proposals came under close examination and were the subject for much discussion in a major portion of the subsequent literature on capital budgeting. Particularly influential has been a well-known paper by Lorie and Savage, where the authors show clearly why and how Dean's rate of return criterion must fail whenever 1) the projects being evaluated are not independent, 2) the sum total capital expenditures is constrained or limited in more than one time period, and 3) the cash flows of a project contain alternations in sign.

Lorie and Savage (42) discuss three problems related to the rationing of capital which are outlined below:

1) Given a firm's cost of capital and a management policy of using this cost to identify acceptable investment proposals, which group of "independent" investment proposals should the firm accept? That is, how should the firm's cost of capital be used to distinguish between acceptable and unacceptable investments?

2) Given a fixed sum of money to be used for capital investment, what group of proposals should be undertaken? With a policy of fixing the dollar upper limit of the capital budget, without explicit cognizance of, or reference to, its cost of capital, how can the firm best allocate that sum among competing investment proposals?

3) How should a firm select the best among "mutually exclusive" alternatives? In other words, when management develops more than one plausible way of investing money in conformance with the budget, how can it select the "best" way?

It is necessary to distinguish between the terms "independent" and "mutually exclusive." Investment proposals can be termed "independent" when the worth of an individual proposal is not significantly affected by the acceptance of others. Although the degree of independence is never complete, such independence is markedly greater for sets of "mutually exclusive" proposals. Acceptance of one proposal in such a set renders all others in the same set clearly unacceptable or impossible.

In a review of the work by Lorie and Savage, Weingartner (74) states that the authors were able to develop reasonably satisfactory procedures for handling the situations where the projects being evaluated are not independent and where the stream of returns on a project contain alternations in sign. However, they were not particularly successful in dealing with the important case where the sum total of capital expenditures is constrained or limited in more than one time period. The trial and error procedure they describe does not guarantee a solution within a finite number of steps, nor will it work when there are substantial indivisibilities or interrelations in the investment alternatives being considered.

The experiences of Continental Oil Company in their use of the discounted cash flow method are described by McLean (48). His article answers the questions of why return-on-investment figures are preferable to years-to-payback figures, why the discounted cash flow procedure is likely to yield better results, and what techniques and assumptions will help executives who want to make practical use of the discounted cash flow method.

The advantages of this method be summarized as follows: 1) it gives the true rate of return offered by a proposal, rather than an approximate value; 2) it gives figures which are meaningful in relation to those used throughout the financial world in quoting interest rates on borowed funds, yields on bonds, and for various other purposes, thus permitting direct comparison of the projected returns on investments with the cost of borrowing funds--which is not possible with other procedures; and 3) it makes allowance for differences in the time at which investments generate their income, and it can discriminate among investments that have a) a low initial income which gradually increases, b) a high initial income which gradually decreases, or c) a uniform income throughout the life of the investment.

Continental Oil found the discounted cash flow technique to be a powerful management tool, easier to introduce and apply than they had anticipated.

Reul (62) outlines the profitability index method--a single, simple, accurate, and dependable guide which is capable of straightforward, impartial comparisons of rates of return on any and all types of investments. Although the concept is not new, it has not in previous articles been developed into a complete working system.

The PI yardstick reflects all four basic factors involved in an investment:

1) total amount to be invested.

2) when the increments of investment are to be expended.

3) total amount of the net cash flow-back.

4) when the increments of cash flow-back are anticipated.

In addition the yardstick takes into account the time value of money; it is a consistent plan for the return of the original investment.

The author defines the profitability index as that interest rate for which the present worth of all expenditures of the investment equals the present worth of all receipts. This definition is a restatement of the definition of internal rate of return given earlier. Two simplifying assumptions built into the present worth factors used in the PI calculations are: 1) continuous compounding of interest, and 2) continuous receipt and expenditure of funds.

Reul summarizes the advantages of the PI as:

1) there is no need for prior agreement on a satisfactory interest rate.

2) any kind of investment proposal can be evaluated.

3) the time value of money is accurately taken into consideration.

4) no arbitrary assumptions are made regarding the charging off of the investment.

5) the calculations are simple and can be performed quickly.

Beranek (5) discusses four principle decision criteria for the rationing of resources over multiple periods: 1) maximizing the rate of return on investment, 2) minimizing the payback period, 3) maximizing the present value of the resources required, and 4) maximizing the internal rate of return. The author states that while these criteria are consistent with each other and with the goal of maximizing stockholder wealth, under frequently encountered conditions their application will yield different choices. Some of the conditions are described in the next section.

Under the maximum internal rate of return criteria, projects are ranked by order of magnitude of their internal rates of return, Starting with the top-ranked project, projects are selected until one is reached whose internal rate of return is equal to the firm's cost of capital. The primary difficulties with the criterion are 1) the possibility that some projects will yield multiple internal rates of return, 2) it is necessary to assume that each project's payoffs can be reinvested at a rate of return equal to each project's respective internal rate of return, and 3) it is difficult to obtain a satisfactory "cut-off" rate.

Bernhard (7) deals with this last problem in a paper which clarifies some of the confusion in the literature surrounding choice of a cut-off rate by developing a general model of which the several different cut-off rate procedures used previously are special cases. He concludes that despite criticisms of the internal rate of return as a device for appraising the merits of a capital investment project, under some special assumptions it has been shown to be a useful index.

Hirshleifer (34) was one of the earliest writers to deal with the problem of uncertainty regarding future investments. He points out that, in the actual world of uncertainty, one finds that the yields realized on alternative investments differ drastically. One school of thought maintains that the divergences of observed yields conceal an underlying harmony of the capital markets. This view is basically a programmatic hypothesis; those who hold it feel that the search for a consistent structure amidst the seeming confusion of observed yields will ultimately be rewarded.

In a world of uncertainty, the equilibrating market forces can only work on the prospective returns on investments. There are two potential sources of difficulty here: 1) prospects or anticipations are not ordinarily observable, and 2) in a world of uncertainty the anticipated returns are multi-valued (usually expressed as a probability distribution).

The author then develops and compares two alternative formulations of the investor's attitude towards multi-valued returns: 1) preferences derived from the probability distribution of returns, and 2) preferences which depend upon the point in time and the specified state of the environment. Hershleifer suggests that time-state preferences have an advantage over probability distribution preferences in showing promise toward harmonizing the bewildering diversity of market yields.

A paper by Baldwin (2) critical of the rate of return method begins by pointing out a significant fallacy in a major assumption upon which most procedures for determining the estimated internal rate of return on a proposed investment using present value principles are based.

The assumption is this: That the future receipts and payments are reduced to their present value by discounting them at the same rate as that which the proposed investment is estimated to provide. In reality, this is simply not true; only by coincidence would the two rates be equal.

Baldwin then describes a procedure for determining the "real" rate of return. Four basic elements of data are essential:

1) Net investment -- the total dollars to be invested, less any reduction due to trade-in, salvage value, or to other reasons.

2) Cash income -- the incremental after-tax cash receipts directly resulting from the investment.

3) Economic life -- the period of time over which the investment must be justified.

4) Value of money -- the composite annual return that the company can expect to gain on its invested funds as a percentage of the investment.

Previous methods made the highly unrealistic assumption that funds reserved for or originally returned by an investment would be earning the same rate while being used for general company purposes as the investment is earning. The proposed method accounts for earnings more realistically. The difference in results from the two methods will be even greater as the rates get farther away from the value of money to the company. The proposed method requires only one series of straightforward calculations, rather than a number of trial and error calculations required by the previous methods. Baldwin's proposed formula accounts for the real cash flow resulting from the investment. It is then an easy job to determine what rate of compound interest, when applied to the investment, would yield an equivalent cash flow. This interest rate is the real rate of return on the proposed investment, according to Baldwin, one which is consistent with the actual business operations of the firm. The return is not based on a discount rate that bears no relation to the financial performance of company operations.

Two common errors made by many users of the rate of return method are pointed out in a paper by Fleischer (23). These errors are described as the "ranking error" and the "error of preliminary selection." The author demonstrates that it is incorrect to rank alternatives in descending order of rates of return and to conclude that the alternatives having the highest rates are superior to the others. What should be done is to determine the incremental rate of return between pairs of investment proposals. The second type of error occurs either when alternatives are removed from consideration in a preliminary screening at the lower levels of the organization or when they are excluded for technical reasons, prior to becoming a contender for limited capital funds. This error can be overcome, according to Fleischer, by evaluating each of the various investment "packages." For n investment alternatives, there are 2^n investment packages, each of which is mutually exclusive. Once these packages have been determined, the selection of the optimum package follows in a straightforward manner using any of the evaluation methods we have discussed which takes into account the amount and timing of cash flows and the time value of money.

Present Worth vs. Rate of Return

If we assume that all the costs and returns from all conceivable investment projects can be correctly measured with certainty, then without a doubt the simplest criterion for proposal evaluation has been the payback period. This method has, since the mid-50's, been replaced in popularity by the methods discussed in the previous two sections and advocated by economists -- the present value criterion and the internal rate of return criterion. According to Turvey (71), these two criteria are one in the same when the appropriate discount rates equal the market rate of interest at which funds are freely available. This author maintains that use of present value does, and use of internal rate of return does not, involve a discount rate representing the relative evaluation of current and proposed returns and costs. If the concern is for the futurity of gains and losses, then the internal rate of return is not the criterion to use. The present value criterion can only be dispensed with if we are totally indifferent to the relative degree of futurity of costs and revenues. The essential feature of the internal rate of return criterion is that it enables us to choose between investments without using a discount rate, either explicitly or implicitly. Hence, that criterion is the wrong one to use so long as the decision-maker continues to value an immediate gain more than an equal but remoter gain.

In their book, the Lutzes (43) show that the four investment comparison criteria they list may lead to different results except in those cases where there is equality between the marginal interest rate on borrowed funds, the outside lending rate, the maximized rate of return on the firm's own capital, and the maximized average internal rate of return (which is also equal to the marginal internal rate of return). The authors conclude that in all the practically important cases, maximizing V - C both with respect to scale and with respect to technique is a correct criterion to follow. Here the quantity V - C is defined as the present value of the future gross revenue stream (V) less the present value of the future cost stream (C) of the proposal. The optimum "scale" of investment is that which maximizes the rate of return k on the firm's own capital and which also maximizes V - C, provided the firm has unrestricted access to the market for borrowed funds. The optimum "technique" is that investment period which gives the maximum rate of return on the firm's own capital over the whole period for which the decision-maker has it available; this will again coincide with the technique which maximizes V - C.

An article by Bernhard (6) points out some of the shortcomings of the internal rate of return method and clarifies procedures for surmounting these shortcomings. The author makes the usual assumptions concerning independence and certainty.

Bernhard indicates that numerous books and articles in the literature of the late 50's and the early 60's suggest or explicitly state that the present worth and the internal rate of return methods are one in the same. He then proceeds to demonstrate that they are not identical, and that in general the internal rate of return method is not a meaningful procedure for comparing the benefits of alternative proposals.

Lorie and Savage (42) pointed out that the internal rate of return, ρ^*, for the proposed project need not have a unique value in the required interval $(-1 \leq \rho^* \leq \infty)$. If one or more of the net incremental returns Q_s (for $s = 1, 2, \ldots, n$) from the project is negative, the project may have no internal rate of return or more than one unique value.

Also, in comparing the relative desirability of two mutually exclusive investment projects, A and B, it is possible to construct examples such that at a given interest rate, i,

$$PW_A > PW_B \text{ while } IRR_A < IRR_B.$$

The reasons for these disagreements are discussed, and Bernhard suggests the use of the average rate of return method to resolve the conflict. This method allows for the possibility that intermediate receipts from a project may be reinvested either in other projects or in the outside market. Assuming Q_0 is negative for the project, the average rate of return is defined as that interest rate on the initial investment which would give the same total wealth at the end of n periods as would be given by the net incremental returns Q_s (for s = 1, 2, . . . , n) from the project, together with the assumed reinvestment. For the assumption of two mutually exclusive projects, A and B, both requiring the same initial investment $-Q_0$, and both lasting the same number of periods (n), the average rate of return method gives the same ranking as the present worth method.

The claim is made in an article by Klein (38) that, although the rate of return method for selecting investment proposals is the criterion usually preferred by managers and engineers, the net present worth method is a superior measuring and selecting tool. Economic evaluation of a potential investment may be used in two ways: 1) to decide whether the project would bring in more than the cost of the capital invested in it, and 2) to rank projects competing for investment. The minimal interest rate used for discounting in the net present worth method is usually the firm's cost of capital, but sometimes a cut-off rate is established by management policy.

Klein indicates that it is possible to use the rate method "cautiously" to make proper decisions. However, since the method is based upon "unrealistic reinvestment assumptions," it is questionable whether one should even try, in actual practice, to base decisions on this rule.

The net present worth rule gives a realistic value to each project. In the rare situation of unlimited capital available for investment, selection by this method gives the optimal investment program. In the case of limited capital and mutually exclusive projects, the optimization criterion becomes net present value per dollar invested. In case of many projects and restrictive conditions, linear programming based on straight maximization of net present value determines the best investment scheme. (Linear programming and other operations research models are discussed further in the next chapter).

The areas where the rate of return method fails are summarized by Klein as follows:

1) unconventional cash flow patterns (e. g., alternating positive and negative flows); here one may encounter
   a) nonexistence of an internal rate of return, or
   b) several different values for rates, or
   c) unrealistic values for rates (either very high or very low).

2) analysis of mutually exclusive projects.

3) comparison of projects with varying lives.

Although the use of the discounted cash flow method of capital budgeting has been explained and its use encouraged in numerous papers, Lerner and Rappaport (41) admit that the approach is not followed in many companies. The authors feel there is good reason for its rejection: The DCF method tends to favor projects which, though most profitable in the long run, would produce erratic earnings from year to year. They therefore propose an alternative approach which employs present value subject to the condition that reported earnings must rise at a stipulated annual rate.

This earnings growth constraint changes the approach to capital budgeting in two important ways: 1) It makes income flows as well as expense flows relevant to the investment decision; it therefore results in a portfolio of accepted projects that has a lower present value than the unconstrained method allowed. 2) More importantly, it raises a policy question of what planning horizon the company should use in preparing its capital budget.

Finally, Henrici (30) stresses that no mathematical method for investment decision-making can substitute for judgment. Rather, the methods serve to guide the application of such managerial judgment. In the preparation stages of decision-making, one must determine the validity of the various methods by ensuring that there is no uncertainty about the answers to such questions as the following:

1) What is the effective amount of capital outlay?

2) Have consequent outlays been considered?

3) Are the projected returns realistic? Do they allow for future costs?

4) Is the discounting rate relevant to the firm's financial characteristics?

Henrici concludes that managerial judgment, experience, and knowledge must still operate in the decision-making environment; the need for qualitative analysis is very real.

Capital Planning

This chapter concludes with the highlights of a study by Marglin (45) of the "when" of capital expenditure planning -- the timing of investment projects in relation to changes over time in the value of the goods and services they provide.

The author distinguishes between static planning rules and dynamic planning rules. Static planning rules are rules which plan investments to be undertaken at any time in terms of the payoffs (present values) for investment at that time alone; they thus determine present investment plans solely in terms of the payoffs for present investment. Dynamic planning rules, on the other hand, are rules which plan investment at different times to reflect changes in payoffs occasioned by undertaking projects sooner or later.

The word "dynamic" is supposed to convey the idea that time enters the situation in an essential way; this is precisely the sense in which dynamic planning rules differ from static ones.

Whereas the concern of the traditional capital budgeting literature has been with the investment decision of the firm, Marglin's emphasis and concern is with the planning of public investment (specifically, investment in water resource systems).

The author makes the following assumptions: 1) interest rates are independent of and constant over time, 2) all present and future benefits and costs are known with certainty, 3) projects are independent in the sense that each project's benefits and costs do not depend on whether or when other projects in the program are undertaken, and 4) projects are indivisible -- that is, each project can be constructed to only one scale, each must be constructed at one time rather than in stages, and each can provide only one product mix. Thus, for an indivisible project the investment decision reduces to the choice of the time at which to undertake it.

V. OPERATIONS RESEARCH MODELS FOR EVALUATING CAPITAL INVESTMENT PROPOSALS

Assumptions of Certainty and Independence

Perhaps the most exciting developments in the application of operations research models and techniques to the solution of complex business problems have been in the area of capital budgeting. We saw in the last chapter that the traditional capital budgeting techniques were incapable of dealing with such problems as uncertainty of the future and project interactions. This chapter highlights some of the more important operations research applications to problems of capital budgeting that have appeared in the literature within the last eight years or so. The first section again makes the simplifying assumptions of certainty and independence, but subsequent sections show how such aspects of the capital budgeting problem as risk and uncertainty, project interdependence, simulation, utility, and others can be handled.

To begin, Fleischer (23) suggests that the problem of optimizing the firm's total present worth may be solved by evaluating the present worth of each feasible "budget package," where a package is a set of projects that constitutes a potential solution to the budgeting problem. For example, if A, B, and C are independent projects, then the budget packages are (do nothing), A, B, C, AB, BC, AC, and ABC; in general, for n independent projects, there are 2^n packages, not all of which may be feasible (i. e., do not violate budget constraints). Although this procedure is theoretically sound, it is impossible computationally for a large number of projects (e. g., for $n = 50$, one must evaluate more than 10^{15} packages!).

Fortunately, as Nemhauser points out (52), there are more efficient ways of solving the problem. Weingartner (75) contains a review of various methods including Lagrange multipliers, integer and dynamic programming. The Lagrange multiplier approach was first proposed by Lorie and Savage (42) and extended by Kaplan (36) and others. Weingartner (74) has applied integer programming, and Cord (16) and Weingartner (75) have applied dynamic programming. All of these techniques are concerned with maximizing the present worth of the investment proposals being evaluated.

Another model dealing with present worth is given in an article by Reisman and Buffa (58). They present a mathematical model describing the most general case in equipment investment policy. The general model reduces to their respective present worths all disbursements and receipts involved in the possession and operation of a succession of equipment having varying initial costs, life spans, salvage values, income and cost functions.

The technique of linear programming is brought to bear upon the problem of efficient allocation of capital resources within a company in a paper by Charnes, Cooper and Miller (14). According to these authors, the rational allocation of funds requires simultaneous consideration of many closely related questions, including the following three:

1) Given the structure of a firm's assets, what operating program -- in the sense of plans for purchases, production, and sales over the relevant planning period -- will yield the greatest net returns in light of profit and other objectives? Linear programming techniques have been successfully applied to a number of planning problems, particularly in the production area, and there is no reason to doubt that the same techniques can be used in financial planning.

2) What is the "yield" to the firm of each of the various possible changes in its asset structure, assuming that these assets are employed to maximum advantage? Linear programming offers a way of bypassing some of the technical difficulties which have been encountered in connection with attempts to evaluate projects on the basis of their rates of return. In addition, with a programming formulation, some of the harder parts of the task of tracing through the interactions of proposed investments with each other and with existing facilities can be left to the mathematics.

3) What is the opportunity cost of funds in the firm, in the sense of the prospective rate of yield on an increment of funds committed to the enterprise and optimally employed during the planning interval? Knowledge of this opportunity cost is required for determining, among other things, whether the "yield" of a proposed investment is sufficient

to justify its undertaking. Under some conditions this opportunity cost of funds can be determined in advance and independently of the actual operating and investment decisions. On the other hand, when the firm is practicing capital rationing (only a fixed amount of funds available), the use of some predetermined external yield rate for purposes of internal allocations is not, in general, appropriate or feasible. Modifications of the standard linear programming formulations can incorporate the funds components of the profit objective directly among the constraining relations on programs. This procedure provides a measure of the marginal internal yield of funds and one which takes account of the feedback between operations and finance when funds are limited. It can also be made to yield evaluations of some of the "qualitative" restrictions which are features of many actual financing arrangements.

The "warehousing model" of linear programming is used in the discussion by these authors because with suitable adjustments it contains all the essentials which are required for the proposed yield analyses in extremely simple form: fixed facilities, inventory carrying charges, and transactions over time which involve payables and receivables.

In a paper by Teichroew, Robichek and Montalbano (68), certain properties of the present and future values of a sequence of cash flows which have applications in the theory of capital budgeting are proved. The same authors in a subsequent paper (69) present a detailed examination of the relevance and implications of these results. The properties of the decision rules based on discounted present value and internal rate of return are studied for the class of projects described by a finite sequence of cash flows. This class of projects is broken down into three types: 1) pure investment projects, where the firm has money invested during every period; 2) pure financing projects, where the firm owes money to the project during every period; and 3) mixed projects, where the firm has money invested during some periods and owes money to it during other periods. The authors derive the necessary and sufficient conditions under which the decision rules lead to unique solutions. Where the decision rule does not provide a unique solution, two rates are defined: 1) the project financing rate is defined as the rate which the project "receives" when the balance of the project is positive (i.e., the project is a net financing source), and 2) the project investment rate is defined as the rate which the project "earns" when the balance of the project is negative (i.e., the project is a net investment). The extension of the project analysis in terms of these two rates permits the derivation of unambiguous decision rules for all projects.

A simple yet interesting application of linear programming to the problem of the distribution of a certain fixed amount of capital resources among the various productive processes or departments of an enterprise, so as to obtain a maximum profit improvement, is given by Bertoletti (9).

The model is formulated as follows:

$$\text{MAXIMIZE:} \qquad Z = \sum_{j=1}^{n} \gamma_j X_j$$

$$\text{SUBJECT TO:} \quad \sum_{j=1}^{n} a_{ij} X_j \leq b_i + \beta_i Y_i \qquad \text{for } i \quad 1, 2, \ldots, m \text{ (sections)}$$

$$\sum_{i=1}^{m} \alpha_i Y_i = C \; ; \; X_j \geq 0 \qquad \text{for } j \quad 1, 2, \ldots, n \text{ (products)}$$

where a_{ij} = time used per unit of product j when it is produced through use of equipment and facilities of section i.

b_i = total time available for use of equipment and facilities of section i for a given period of time.

γ_j = contribution of the unit product j to fixed cost plus earnings; this is the marginal income per unit of product, equal to sales price minus variable cost.

X_j = number of units of product j to be made in the period of time selected.

α_i = price of each machine or production center in section i.

β_i = quantity of time added to b_i for each machine or production center added to section i.

C = available capital for investment.

Y_i = number of machines added to section i.

Z = total marginal income in the period of time considered.

Z_1 = total marginal income before investing C.

Z_2 = optimum marginal income after investing C.

We want to maximize the return on investment, namely, maximize

$Z/C = (Z_2 - Z_1)/C$. The problem can be solved in a straightforward manner either graphically or by use of the Simplex method.

Using a different approach, Everett (20) shows that the Lagrange multiplier method is especially useful for solving problems requiring the optimal allocation of a limited (constrained maximum) number of resources to a number of independent proposals, where the total payoff is given by the sum of the payoffs that accrue from each proposal. The resource allocation problem thusly stated becomes essentially a cell problem, and as such the Lagrange method reduces the problem to a series of independent unconstrained maximization problems, one for each cell. (The cell problem may be described as a situation where there are a number, say m, of independent areas into which available resources may be committed, and for which the over-all payoff that accrues is simply the sum of the payoffs that accrue from each independent project (cell).) It can be shown that the solution to the maximization problem of the <u>unconstrained</u> Lagrangian function is also a solution to the <u>constrained</u> cell problem whose constraints are, in fact, equal to the resource levels expended in maximizing the Lagrangian function.

Everett comments that the technique is not in general certain to produce solutions for all constraint levels of interest. It is, however, "fail-safe" in the sense that any solution which it does produce is a true optimum. There are a number of modifications which often succeed in regions where the basic method fails or is not applicable.

The Lagrange multiplier techniques presented in this article are particularly well-suited to use with computers, where the "trial and error variation" of the multipliers, as well as the maximizations within the individual cells of a cell problem, can be programmed to be rapidly and automatically executed.

A specific type of all-integer integer programming problem which occurs frequently in the area of capital budgeting involves a zero-one restriction on the values of the problem variables. That is, the problem variables are the investment proposals being evaluated; the variable takes on a value one if the proposal is accepted and a value zero if the proposal is rejected. Kaplan (36) discusses approaches to solutions to such problems by use of the General Lagrange Multiplier (GLM) method. This procedure, because of its relative simplicity, has certain advantages over other direct methods such as the use of integer programming algorithms, especially in cases where the constraints are not binding to the degree indicated by the problem statement. In contrast to such methods as integer programming and dynamic programming, which by their very nature produce unique solutions (assuming such solutions exist), the GLM method is capable of displaying a variety of solutions to problems of interest.

Fisher (22) introduces the idea of a stochastic investment problem. Such a problem occurs when a decision-maker with capital assets to invest is confronted by a time sequence of investment opportunities characterized by probability distributions of returns. Given an investment opportunity, the decision-maker must compare his present gain (the investment opportunity) with his future expectation (projected future opportunity for investment) and on some basis decide whether to accept or reject the investment opportunity.

A procedure is developed for determining optimum decision rules for the class of stochastic investment problems which employs the functional equation of dynamic programming. Two sub-classes of problems are considered: 1) the assets of the decision-maker are fixed at some point in time and his interest is in investing them over some future interval, and 2) the assets of the decision-maker are augmented through time.

Fisher reviews the numerous procedures for developing optimum decision rules for investment behavior discussed in the literature. As we have seen, these procedures can generally be grouped into the present value method (see, for example, Hirshleifer (33) and Baldwin (2)) or the capital supply and demand method (see, for example, Dean (17) and Lutz (43)). Both of these methods can be criticized, according to Fisher, as incomplete either on the basis that they disregard the future investment opportunities available to the decision-maker or on the basis that the decision-maker's available assets are ignored. There are other factors that can and should be made explicit in the development of decision criteria for investment. These additional variables include the estimated future investment opportunities and the amount of disposable assets available to the decision-maker. As developed in this paper, the decision making problem consists of two distinct stages: First, the decision-maker is interested in determining optimal decision rules for accepting or rejecting investment opportunities. Second, given these optimal decision rules, the decision-maker is interested in whether he should hold the disposable assets he currently has or whether these disposable assets should be transferred to some activity where they will be more productive. The problem of determining the maximum amount of assets the decision-maker should hold is also examined.

To summarize, Fisher's analysis is different from previous approaches in that 1) the problem is formulated as a stochastic process, 2) the analysis is carried out using the functional equation of dynamic programming, and 3) additional variables are brought into the development of optimal decision rules.

By now one should realize that the usefulness of mathematical modeling as a tool of capital budgeting hinges on the development of an efficient solution technique. In this regard Lawler and Bell (40) developed an algorithm which greatly facilitates the solution of integer linear programs with zero-one decision variables, using a technique best described as partial enumeration. However, their algorithm is not general enough to handle situations where the objective function is not linear but rather quadratic. An extension of the algorithm is given by Mao and Wallingford (44) so that it can be applied to both integer linear and integer quadratic problems. In addition they report on their computational experiences using sample capital budgeting decision problems. A number of these test problems were solved, some involving as many as 15 projects and 15 constraints. None of the linear integer problems required more than four seconds of execution time on an IBM 7090 computer. Most of the problems required less than one second, which compares favorably with alternative solution techniques available currently.

Assumptions of Certainty and Project Interdependence

Our review thus far of the capital budgeting literature indicates that the majority of writers restrict themselves to the extreme assumption of independent or mutually exclusive investment proposals, as defined by Lorie and Savage (42). However, as Reiter (60) points out, investment decisions sometimes involve choice among available investment projects that are <u>not</u> mutually exclusive. That is, the payoff to any one project may be contingent upon the other projects undertaken with it. The standard techniques for analysis of investments do not deal with the interdependencies existing among available projects. For this reason it is common practice to reduce all problems to the case of independent projects by combining those projects among which interdependencies seem strong, and ignoring any remaining interdependencies. The extent to which this practice leads to good decisions has been obscured by the lack of an analysis technique capable of finding good decisions in specific instances.

In simplest terms, the problem Reiter considers is that in which the total payoff to the joint undertaking of several projects may be analyzed into the sum of contributions made by each pair of projects undertaken. The problem confronting the decision-maker is, out of the 2^n possible combinations of n projects or investment programs he might select, to choose a program which makes the total payoff as large as possible. In certain cases, the decision-maker may be willing to settle for a combination that comes "close" to maximizing the total payoff.

In an earlier paper, Reiter and Sherman (61) formulate this problem as an integer linear programming problem. The authors confess that the polyhedron developed in the problem is rather complicated, and, as it turns out, it has many extreme points whose coordinates are not integers. While the formulation does not seem to be a promising one for computing optimum allocations, it does lead to an interesting interpretation of the direct (primary) and dual programming problems in terms of the economic organization. This interpretation shows that it is possible to partition decision-making and information in such a way that each decision-maker knows only a part of the total information of the problem and controls only part of the total decision. This interpretation provides a theoretical basis for constructing a decentralized organization capable of solving this problem.

Weingartner (75) contends that as capital budgeting problems become more complex, the decision procedures developed must allow for more aspects of the real world. His paper surveys the techniques available to handle the important and generally neglected problem of project interrelationships such as mutual exclusion and interdependencies. The techniques utilized are linear and integer programming, dynamic programming, and the discrete optimizing procedure outlined by Reiter (60). Project interrelationships arising from randomness of outcomes and nonlinear utility functions are analyzed using these procedures; additional interrelationships arising in the context of research and development budgets are analyzed.

The work of Weingartner and others is extended in a recent paper by Nemhauser and Ullmann (53) in their development of dynamic programming algorithms for the optimum allocation of capital subject to budget constraints. Treated in this article are multilevel projects, reinvestment of returns, borrowing and lending capital, capital deferrals, and project interactions. Because the optimum investment returns are increasing monotonic step functions, the authors are able to handle dynamic programming models with several state variables.

Reisman (57) states that an optimal budgeting policy is one which will drive the enterprise in such a way that the enterprise response through time will be in close proximity of the organizational goals established. The factors that need to be identified in defining organizational goals and objectives can be grouped into three categories: 1) tangibles -- present and future assets and present and future revenues and disbursements; 2) pseudotangibles -- enterprise future potential and enterprise flexibility; and 3) intangibles -- enterprise "image" from the point of view of customers, suppliers, employees, and governmental regulating bodies. Reisman identifies the two primary problems in capital budgeting as 1) the recognition of the major enterprise objectives and 2) the determination of the magnitude and timing of financial allocations.

In a subsequent article Reisman, Rosenstein and Buffa (59) propose a methodology for determining the allocation of a fixed amount of resources among competing sectors of an enterprise, the operations of which may be interrelated, where the decision has to be made under conditions of uncertainty, and where intangible factors predominate. The procedural steps to be followed in applying this methodology are listed below:

1) Selection of criteria.
2) Selection of a functional scale between each criterion and per-unit allocation.

3) Selection of a weighting function for each criterion.
4) Selection of the most relevant or indispensable needs for each investment proposal.
5) Application of Lagrange's method of undetermined multipliers.
6) Selection of the relative weights for the criteria.
7) Determination of the final allocations.

Concepts of Risk and Uncertainty

To begin this section on the literature's treatment of the concepts of risk and uncertainty in capital budgeting programs, Farrar (21) stresses the need to distinguish between the two terms "risk" and "uncertainty." Although both refer to a situation in which future outcomes are imperfectly known, the situation is said to be characterized by risk only if the probabilities of alternative, possible outcomes are known. To qualify as a risk situation, an experiment must be repetitive in nature and must possess a frequency distribution from which observations can be drawn and about which inferences can be made by objective, statistical procedures.

Uncertainty, on the other hand, is said to be present when the experiment in question cannot be carefully replicated. The situation is therefore unique and its frequency distribution cannot be objectively specified.

The usual and most obvious extension of the traditional capital budgeting procedures when uncertainty enters the picture consists simply of revising the usual certainty model so that expected return or expected profit becomes the object to be maximized.

According to Grayson (24) the distinction between risk and uncertainty has little meaning in any business decision. He categorizes people into three classifications -- risk seekers, risk averters, and risk ignorers. Some of the common methods which he discusses for handling risk are listed below:

1) payback rules -- payback period, present value method, and internal rate of return method.

2) risk adjusted discount rate -- a composite discount rate adjusted to allow for both a time preference and a risk preference.

3) conservative forecasts -- a forecasted return adjusted to a more conservative level.

Each of these methods has certain merits and does represent an attempt to make risk allowances, but each also has one or more of the following disadvantages:

1) The procedure contains built-in assumptions with which the decision-maker would not necessarily agree if he knew them.

2) The procedure concentrates on a single "best estimate" without giving explicit recognition to other possibilities.

3) The riskiness of an individual project is not considered in the procedure but is assumed similar to all other investments or is lumped into a risk class.

4) A true description of risk estimation and risk preferences becomes lost because of the lumping together of many factors into simplified procedures.

Some of the newer statistical tools and analysis techniques available for handling risk and uncertainty are described by Grayson as follows:

1) probability assignments -- a more specific estimation of risk through an explicit probability distribution.
2) expected monetary value -- an explicit "calculated risk."
3) decision trees -- a decision is literally mapped out in the form of branches of a tree; the various paths that the decision-maker and nature can take are diagrammed.
4) computer simulation models -- to imitate repeatedly the various ways that all of the decision variables could combine as the complex future unfolds.

Two of the statistical models which can be used for incorporating risk preferences include 1) utility functions -- a numerical index describing a person's personal preference in risky situations which can be useful as a guide to consistent decision-making; and 2) indifference curves -- designed to reflect a tradeoff that an investor might make between various risks and returns.

As Hillier points out (32), the amount of risk involved is often one of the most important considerations in the evaluation of proposed investments. For example, a reasonably safe investment with a certain expected rate of return will often be preferred to a much more risky investment with a somewhat higher expected rate of return. This is especially true when the risky investment is so large that the failure to achieve expectations could significantly affect the financial position of the firm. Hillier's paper indicates how, under certain assumptions, such information (explicit, well-defined, and comprehensive information that is essential for the accurate appraisal of a risky investment) in the form of probability distributions for the internal rate of return, present worth, or annual cost of a proposed investment can be derived.

Hillier goes on to comment that although the capital budgeting literature prior to the early 60's had not given much consideration to the analysis of risk, such procedures as had been suggested for dealing with risk tended to be either quite simplified or somewhat theoretical. These procedures tended either to provide management with only a portion of the information required for a sound decision or they assumed the availability of information which was almost impossible to obtain. The author proposes a procedure that only requires that, in addition to an estimate of the expected value of a prospective cash flow, the inexactitude of the estimate be described by an estimate of the standard deviation. Such a technique should complement, rather than supercede, most of the traditional procedures for evaluating investments.

In a comment on Hillier's paper, Rothkopf (65) states that in order for the expected value of the rate of return to be well defined when probabilistic information is used in the evaluation of projects, the rate of return must exist with probability one. This condition is unlikely to be met in practice. Secondly, the probability distribution and the expected value of the rate of return of a project may be extremely misleading if there is a correlation between the size of the project and its profitability. For example, if a project with two possible outcomes was equally likely to either be a $1,000,000 - 20%

return project or a \$10,000 - 0% project, the expected rate of return would be a misleadingly low 10%. Thus, in place of the expected value of the rate of return, the author suggests use of the rate of return, r', that satisfies

$$\text{expected present value} \equiv \bar{p}(r') = 0.$$

While r' may not always exist or be unique, it will often do so when r, the usual rate of return, does not do so with probability one. In addition, the usefulness of r' is not impaired by correlation between project size and profitability.

According to Canada and Wadsworth (12), the greatest drawbacks to Hillier's procedures are the extreme difficulties of estimation and computation, expecially when a large number of projects are under consideration. These authors present a simplified method for considering the risk of proposed capital investment projects under a set of assumptions commonly realistic for such economic evaluations. The risk is measured by the probability distributions of present value measures of merit when several of the most important quantities to be estimated are subject to variation. The set of simplifying assumptions is listed below:

1) The model for the present worth of an individual project assumes that a) There is only one lump investment required, occurring at the beginning of project life; b) the net annual cash flow is constant throughout each year of the project life; and c) any salvage value occurs directly at the end of project life.

2) The equation for computing the exact expected value of a project assumes that the net annual cash flow and the salvage value are each independent of project life.

3) The equation for approximating the variance of a project assumes mutual independence of all projects, but it can be modified to reflect the effect of non-independence of any pair of projects.

4) The formulas for determining the mean and variance of the distribution of the difference between two projects are satisfactory except where the distributions of individual project outcomes are extremely skewed.

The technique proposed in a paper by Cord (16) assumes 1) the use of the interest rate of return method of ranking investments, 2) the independence of project cash flows, and 3) a single budgetary period, with all net outlays occurring in this period.

Cord defines the interest rate of return to be that rate of interest which makes the present value of the cash proceeds expected from an investment equal to the initial cost of the investment.

If the interest rates of return on individual investment projects were known with certainty, the best allocation of available funds is found by adopting the traditional convention of ranking projects in the order of these rates. Projects would be chosen in rank order, up to a point where either a) the selection of one additional project will exceed the investment funds available or b) the interest rate of return on the proposed project is less than the return the firm can earn by investing the funds outside the firm.

If the interest rates of return are known without certainty, the decision rule above can lead to a result which management finds objectionable. Thus, in the case of uncertainty, the following objective is chosen: To maximize the total return on investment, subject to the constraints imposed, namely, the total funds available for investment will not be exceeded, and the average variance for the total investment will not exceed some preassigned level.

The author formulates the model in terms of both linear and dynamic programming. With the recurrence relationship of dynamic programming, the capital budgeting problem is viewed as a multistage allocation problem. Each investment project is considered as a separate stage, and at each stage, the available funds are varied incrementally from zero to the total amount available, with a maximum return computed for each incremental value.

Dyckman (18) makes the following points in regards to Cord's paper: First, the solution to a capital budgeting problem should ignore neither non-quantifiable factors nor the impact of future methods of implementing or continuing a given project. Second, although theoretically investment project returns may be independent, such returns are not independent of the financing costs. Simultaneous solution of these interdependent aspects of the problem is needed. Third, the relevant analysis should be a sensitivity analysis to both the budget and variance constraints. Finally, a complete model must incorporate the relevant information concerning the productive use outside the firm of non-allocated budget funds.

Dyckman also feels that Cord's discussion implies either equal project lives or reinvestment in projects with similar characteristics.

He suggests that this assumption could be relaxed and present values used without basically altering the solution.

A mathematical model for rational, multi-period investment in the stock market is given in a paper by Naslund (50). The decision rules are optimal in the sense that they generate the highest expected return on investment, while honoring risk constraints in the form of inequalities.

The model presented can be extended to deal with the general problem within the business organization of allocating limited resources among several competing investment proposals. In this context the dual variables of the model reflect how the expected value varies as the firm alters its views on the degree of risk to which it is willing to expose itself. The firm thus may be willing to alter the constraints to obtain a more preferred balance between expected return and risk taking. The dual variables are also useful to the investor in deciding whether to seek additional capital for investment by providing a measure of the highest interest rate he should be willing to pay.

In a subsequent article, Naslund and Whinston (51) list several of the ways in which risk and uncertainty can be taken into account: 1) by substituting expected return as a deterministic equivalent, 2) by increasing the interest rate for more risky projects, 3) by subtracting a certain amount from the mean future inflows associated with the projects, 4) by shortening the planning horizon for more risky projects, or 5) some other form of sensitivity analysis.

The authors introduce risk into the model proposed by Weingartner (74) for the certainty case by allowing a_{tj} (the flow of money associated with project j in period t) to be a normally distributed random variable. By allowing for randomness the model becomes one of mathematical programming under risk. There are three major methods suggested in the literature for dealing with this class of problems: 1) stochastic linear programming, 2) mathematical programming under uncertainty, and 3) chance-constrained programming.

Naslund and Whinston follow the third method, because it does not assume that it is possible to numerically specify the effects of constraint violations. In addition, the method is more flexible in the sense that it makes it possible to adjust to a desired degree to outside events.

The problem is formulated such that the maximization of the value of the firm at the horizon must be made subject to probability constraints -- one for each year -- reflecting the fraction of time that the investment plan is allowed to be violated. These probability constraints can be transformed to indicate the financial reserves (excess capacity) needed to obtain the above-mentioned restriction on constraint violations. These reserves cannot be reinvested and thus represent an opportunity cost.

By using this formulation for the investment problem it is possible to analyze some effects of economic policy such as variations in the interest rate in both perfect and imperfect capital markets.

A framework by which combinations of risky investments may be evaluated for capital budgeting purposes through the use of probability distributions is given by Van Horne (72). The method allows cash flows for various future periods generated from an individual investment to be treated as being dependent in nature. Of particular importance is the recognition of covariance among investments and the fact that total variance must take account of existing investment projects as well as proposals under consideration. Risk cannot be analyzed in isolation; rather, the interrelation of risk among all investments must be assessed. Through the proper diversification of investments, a firm is able to obtain the most desirable combination of expected net present value and risk. The selection of the most desirable combination of investments will depend upon the utility preferences of the company with respect to expected net present value and variance, assuming that such a preference function does indeed exist. The most desirable combination of expected net present value and risk is the point of tangency between the efficient combination line and one of the family of indifference curves.

Van Horne in a later paper (73) proposes a method for providing information by which the resolution of uncertainty over time for a single new product and for the entire product mix of the firm may be analyzed. If the firm has existing products, it is important to evaluate the marginal impact of a new product decision on the resolution of uncertainty for the firm as a whole. This impact can be judged by studying the differential pattern of expected uncertainty resolution for the firm's over-all product mix with and without the addition of the new product. Information concerning the resolution of uncertainty is valuable in planning for new products and in balancing the risk of the firm over time, as well as in evaluating any risky investment. To ignore the implications of uncertainty resolution may result in unwanted fluctuations in the net present value of the firm and sub-optimal planning for future new products. The framework proposed in the paper allows management to choose new products that best fulfill corporate objectives with respect to expected return, the variance about this return, and the resolution of uncertainty over time.

We conclude this section with a warning from Quirin (55) that one cannot apply corporate standards on risk to individual project analysis because corporate riskiness does not necessarily vary directly with project risk and may, in fact, vary inversely with it. Any scheme for selecting a capital investment program which seeks to take explicit account of risk in a satisfactory manner must consider the correlation between project value and the value of the company's existing operations. The model the author develops is a heuristic adaptation of Markowitz's security portfolio selection model (46), which is not directly applicable to the capital budgeting problem. The formula for computing the variance of the net present value of the firm contains a covariance term which indicates the magnitude of the correlation between the net present values of the firm's existing operations and the proposed addition.

Simulation, Utility, and Other Models

The use and advantages of a computer simulation model in the analysis of capital budgeting problems are described in a paper by Salazar and Sen (66). They extend Weingartner's model (74) to include the cases of risk and uncertainty. Their approach utilizes the concepts of decision trees, simulation, and stochastic linear programming. Expected cash flows from investment proposals are treated as random variables, subject to two kinds of uncertainty. The first type is the effect of future variations in significant economic and competitive variables which are likely to influence subsequent cash flows. The second type of uncertainty is in the estimation of the cash flows themselves.

The model extended is Weingartner's Basic Horizon Model. Instead of maximizing the firm's present worth, the BHM maximizes its value as of some future time period called the horizon. Interdependencies between projects can be represented in the programming model by additional sets of constraints. Mutually exclusive projects and contingent projects are the two types of interdependencies discussed.

The output of the model's simulation is a risk-return curve of various project programs. Knowing the nature of the risk-return program curve, the decision-maker can choose that program which is most compatible with his personal risk-return utility function.

The chief advantage of the simulation analysis presented in this paper is the range of choices it provides for management. A heuristic ranking scheme is used to generate several project programs. The risk and return for each program are calculated. If desired, management can set acceptance limits on the programs' risks and returns.

In general, any investment decision should be concerned with a choice among the available alternatives, and it is always subject to an unknown future environment. Actual future costs, markets, and prices will inevitably differ from any single set of assumptions used as a framework for weighing proposals. Moreover, as we have thus far seen, a variety of criteria -- payback period, average annual return, net present value, internal rate of return on investment -- may be used as yardsticks for evaluating proposals. Yet, despite much theoretical discussion, it has been hard for management to guess what difference, if any, the choice of a particular yardstick would make in actual long-term dollars-and-cents results.

According to Hertz (31), any investment policy, if it is to guide management's choices among available investment alternatives, must contain two components: 1) one or more criteria by which to measure the relative economic attributes of investment alternatives, and 2) decision rules -- which may or may not seek to take uncertainty into account -- for selecting "acceptable" investments.

The research results in Hertz's paper were obtained through computer analysis and simulation; they show that the development of a good investment policy involves four requirements:

1) The determination of risk profiles for all investments.

2) The use of a discounting measure (either discounted internal rate of return or an equivalent net present value) for assessing the merit of an investment proposal.

3) The establishment of alternative screening rules for investment proposals.

4) The determination of risk boundaries for alternative policies.

The use of Monte Carlo simulation is discussed in an article by Clarke (15). He concludes that the primary advantage in using this simulation technique to evaluate capital investment is that it overcomes the principle drawbacks of the traditional methods of appraising investments under uncertainty and turns them into advantages, such as:

1) The forecasted probability density function does not have to meet any particular shape or mathematical equation.

2) The full range of data is considered in the analysis of the estimated funds' flow.

3) Management can evaluate the risks involved in alternative investments.

Clarke concludes that the most important measure of any simulation model is its predictive ability; this points out the need for accurate historical data. The accuracy and validity of the assumptions made in quantifying the system variables must be considered. Simulation does offer the opportunity for sensitivity analysis; it also has unique advantages in evaluating investment risk.

The concept of utility and utility functions is exceedingly complex; it includes many subjective and non-quantifiable factors. Utility is not constant over time. Nonetheless, English (19) feels that a decision to invest in one alternative in preference to some other should be based on judging which offers the greater expected payoff or return, in terms of some utility function. The author lists the important factors which differ among alternatives as: 1) the utility of time (the time value of money), 2) the length of time for the investment committment, and 3) the utility associated with the risk involved.

The method proposed by English provides a means for taking into account a varying rate of discount for future monies, a utility associated with the opportunity to reconsider a decision at a later time, and a weighting of the size of the capital committment. Although these elements involve judgment and subjective evaluation, the author feels that their consideration should result in improved investment decisions.

In his chapter on uncertainty, the French scholar Massé (47) concludes:

> In the present state of economic analysis, the most consistent theory, and hence the most satisfactory to the mind, despite some objections, consists in taking as one's standard of choice the mathematical expectations of utility, properly defined. It does, however, sometimes conflict with the practical difficulty that we cannot define very clearly the utility function of an enterprise (p. 243).

It is often more realistic to consider the optimum behavior of the enterprise as a compromise between profit maximization and risk minimization; this would lead us to work with mathematical expectations of money returns, corrected upwards or downwards according to the required margin of safety. The correction factor for the safety margin should increase 1) the less perfectly the conditions of the law of large numbers are satisfied, and 2) the more uncertain are the probability estimates.

The tools of mathematical programming are used in a paper by Baumol and Quandt (4) to explore the choice of optimal investment project combinations and show how a number of problems left unsolved in the previous literature can be handled. In doing so, their article brings together two strands of economic literature: the neo-classical theory of capital and the mathematical analysis of capital budgeting.

In this chapter we have reviewed a number of mathematical programming models which have been designed for investment project selection. Although these models have proven to be extremely powerful tools, their authors left serious gaps by failing to come to grips with the discount rate problem. Baumol and Quandt show that it is possible to determine an appropriate set of discount rates, rates which rely heavily on the shadow prices of the dual programs. Fortunately, as it turns out, the dual program agrees with the classical theory.

The basic approach taken by most programming models of investment project selection under capital rationing involves maximization of an objective function expressing the discounted present value of the positive and negative cash flows corresponding to any combination of investment projects selected. This maximization is carried out under the limitations imposed by a number of constraints. At least some of them are budget constraints which represent the distribution of funds among competing uses available during each period in which the supply of cash is expected to be limited.

There are a number of limitations of the simple model described above. Three minor problems may be listed as follows: 1) There exists the possibility that funds may also be used for investment outside the firm or for consumption by the capitalist or stockholder, and the simple model has made no explicit allowance for these alternatives; 2) funds unused during the period can normally be retained for use during later periods and the model should provide for this possibility; and 3) there is an apparent relationship between the revenue and expense of a project, since whenever an investment yields a positive (or negative) cash flow it should add to (or reduce) the amount of money available during that period for use in other investments.

The most serious difficulty of all lies in the assumption that we are given an appropriate discount rate $1/(1 + i)$, the cost of capital. The manner in which the literature has dealt with the determination of the discount rate for such a model must be considered casual and unsatisfactory.

Baumol and Quandt construct a model for capital rationing which, while it makes use of a subjective utility index, for the first time also provides an objective measure of the discount rate. The distinction is made between the subjective discount rate U_t/U_0 and the objective internal production opportunity rate ρ_t/ρ_0, which is the ratio of the dual prices corresponding to the budget constraints for periods t and 0.

For the more realistic case where funds may be carried over from one period to the next, the model is formulated as follows:

<u>Primal:</u> MAXIMIZE: $\Sigma U_t W_t$

SUBJECT TO: $-\Sigma a_{jt} x_j + W_t + C_t - C_{t-1} < M_t$

$x_j \geq 0 \quad W_t \geq 0 \quad C_t \geq 0$ and $C_0 = 0$

<u>Dual:</u> MINIMIZE: $\Sigma \rho_t M_t$

SUBJECT TO: $-\Sigma a_{jt} \rho_t \geq 0$

$$\begin{array}{ll} & \vdots \\ \rho_t \geq U_t & \\ \rho_0 - \rho_1 \geq 0 & \rho_{t-1} - \rho_t \geq 0 \\ \rho_1 - \rho_2 \geq 0 & \rho_t \geq 0 \\ \vdots & \end{array}$$

where U_t = Fixed utility of a dollar in period t.

W_t = Sum to be used for consumption in period t.

a_{jt} = Net cash flow obtained from a unit of project j during period t.

x_j = Number of units of project j to be constructed.

M_t = Amount of cash made available from outside sources during t.

ρ_t = Dual price corresponding to the <u>tth</u> budget constraint.

We have already mentioned Farrar (21) and the distinction he makes between the terms "risk" and "uncertainty." His approach to decision-making under uncertainty is a unique model which can be characterized as: 1) a direct descendant of classical utility theory, 2) linked to the certainty equivalence tradition of risk aversion, and 3) a mathematical programming approach.

Farrar's model relies on only two assumptions: 1) an investor's utility of money function is positively sloped and concave downward, and 2) his investment strategy is the maximization of expected utility.

The proposed model fits nicely with the main body of literature on economic theory. Its demands for information are limited to specifying a single parameter, namely, the coefficient of risk aversion. As a quadratic programming model, it lends itself easily to the solution of actual numerical problems. When expanded to multiple dimensions it can explain diversification as a conscious and rational investment policy.

Gupta and Rosenhead (25) describe a sequential investment decision as one where later decisions in the sequence can be postponed until more up-to-date information is available. Where future variation in uncontrollable factors may significantly affect the outcome of an investment plan, it is inappropriate to aim the plan at the achievement of what appears, on the basis of current information, to be the best end-state for the system. In doing so the initial implemented decisions may unreasonably restrict the possibilities remaining for the subsequent decisions, so that little use can be made of the more recent information then available.

The alternative approach given by these authors is, where there are a number of possible end-states whose outcomes (on the basis of current information) are not much inferior to that of the "best" end-state, to make initial decisions which permit the achievement of as many of these end-states as possible. The authors term such initial decisions "robust" (that is, flexible). Where there are several robust decisions, an appropriate discriminatory factor is "stability" -- the ability of the system (as amended by the initial decision or decisions) to perform well should the subsequent stages of the investment plan be delayed or cancelled.

The argument for the criteria of robustness and stability does not depend on reaching a best end-state of a particular problem, but rather on the advantage in the long run of the maintenance of flexibility. This can be especially important in many industrial situations, particularly competitive ones, where uncertainty is considerable.

According to Hanssmann (27), most stochastic investment models with a probability-oriented investment criterion usually assume that each investment alternative requires a deterministic investment and yields a stochastic return. The following criteria have been proposed for the analysis of investment alternatives:

1) maximization of expected return,

2) maximization of expected return subject to a constraint on the variance of return, and

3) maximization of expected utility.

Only the first and second possibilities can be considered operational, Hanssmann says. Weingartner (74) has observed that the first criterion adds little beyond the deterministic case, as deterministic returns are merely replaced with expected returns. The second approach was introduced in a classical treatise by Markowitz (46). His book was subsequently extended by Cord (16) and others.

Hanssmann's article views the decision problem in a different way. He assumes that the investing firm is primarily concerned with achieving a specified minimum return that is critical to its economic survival. Thus, an effort is made to maximize the probability of exceeding the specified minimum return. This decision criteria is applied in the context of three static investment models similar to the Markowitz model.

To conclude this section, an interesting model is presented by Hartmann and Moglewer (28) for the allocation of company resources to research proposals. The approach is based on game theory and focuses upon the uncertainties of contract funding and their effect on company policy. A general solution is derived for the game in which the payoff function for each proposal exhibits a diminishing return. An algorithm is given to obtain quick numerical solutions using only a slide rule. Data taken from the literature for the proposed activity of an aerospace company over a period of three years show good agreement between actual and calculated expenditures. Finally, the minimax strategy based on two-person games is compared with policies based upon other decision criteria and is shown to have an important place in the planning process.

Allocating Resources Other than Capital

We conclude this chapter on a review of the operations research models available for evaluating capital investment proposals with a paper by Asher (1) which was the only article discovered in our library search not concerned with the allocation of capital resources. Asher's paper considers the optimum utilization of another scarce resource, namely, professional manpower, among several alternative research projects. Other parameters and restrictions of the model include: the economic value of a successful project, the probability of success, the man-hours required per test or screen per project, the total available man-hours, the cost per man-hour, and the available raw materials. These factors are used to construct a linear programming model. The solution indicates the optimum allocation of professional manpower over the most attractive projects required to maximize the return to the firm.

VI. A MODEL FOR SIMULATING RISK

The Need to Consider Risk and Uncertainty in Capital Budgeting Problems

In the previous chapters we have attempted to highlight a portion of the significant literature in the field of capital budgeting. Our intent has been to describe some of the many and varied decision rules and models for allocating capital resources and for evaluating and comparing economic investment alternatives. Our objective has been to organize and clarify this material. It should be recognized that this survey is by no means exhaustive. Additional references are available in the extensive bibliographies given by Charnes and Cooper (13) and by Hanssmann (26). The latter is especially concerned with operations research applications to capital allocation problems; the emphasis of the former is on general linear programming algorithms.

We wish to turn now to an area within the capital budgeting field where the treatment in the literature has been less than complete and has certainly been less than satisfactory. This area is concerned with the problem of risk and uncertainty. In this section the need to and importance of considering risk and uncertainty will be established. In the following section we will trace the development of a game board for simulating risk.

Some operational definitions of risk and uncertainty are given by Barish (3). He states that risk is a variation from the average or expected value which occurs in a random chance pattern. The larger the variation, the larger the risk. The individual factors causing these random variations are numerous and are not important enough to be recognized separately. They interact with each other statistically and produce a distribution of values for the random variable. Uncertainty, on the other hand, is caused by errors in forecasting one or more factors which are significant in determining future values of the random variable or by the complete absence of such a forecast. Differences between expected values and actual values caused by uncertainty can be assigned to factors which have been incorrectly forecast or for which no forecast exists. The author goes on to indicate that the assignable factors which cause uncertainty in economic decision-making are affected by many circumstances.

Morris (49) describes two types of uncertainty: The first type deals with uncertainty with regards to future contingencies, or "states of nature," which are beyond the control of the decision-maker but which affect future outcomes. The second type of uncertainty is concerned with variation in the possible outcomes themselves. The three-dimensional game board described in the next section simulates the type of risk described by Barish and the second type of uncertainty described by Morris, namely, random variation in the possible outcomes. Thus, for the purposes of this thesis, we shall from here on use the terms risk and uncertainty interchangeably.

According to Swalm (67), the best available evidence indicates that very few businessmen use approaches such as those described by Hertz (31) and others which explicitly take risk into consideration. In general, Swalm says, the approach by today's businessmen can be termed in modern phraseology as "models under assumed certainty." In other words, in their approaches to capital budgeting decision-making, probability statements seldom appear directly.

Morris (49) postulates that consideration of risk is suppressed because the human mind has limited information handling capabilities. Thus, any decision-making procedure consists of conceptual simplifications of the real-world situation. One such simplification is a denial of ignorance of the future and evaluation of a decision-making problem by acting as if only one possible future state will occur. This is not to say that one knows the future with certainty, but is simply a matter of simplification, and allows us to answer the question: "If this particular set of circumstances were to occur, what would be the proper course of action to take?" If a decision-maker does undertake such a course of action, he may be thought of as acting as if the particular circumstances in question were sure to occur.

Swalm concludes that businessmen do not attempt to optimize their expected dollar outcome in a risk situation involving, what to them are, large dollar amounts. As a result, methodologies which assume they do tend to be rejected by such businessmen. The author's primary conclusion is that cardinal utility theory, which attempts to measure the "value" or preference an individual has for different dollar amounts, offers a reasonable basis for judging the internal consistency of a series of decisions made by persons dealing with risk situations. The concept of utility offers a relatively simple means of classifying decision-makers as being risk-seekers (gamblers), risk-avoiders (conservatives), or risk-ignorers (insensitives).

The shortcomings of the traditional methods of dealing with uncertainty are discussed by Klausner (37). The most common of these traditional methods of risk analysis include the following:

1) Assumed certainty. Here single values are estimated for those elements of an investment which determine its outcome. Users of this deterministic method generally feel that since they have used "best estimates" (that is, most-likely values), the resulting measure of merit is an approximation of the most-likely outcome of the investment. The uncertainty which surrounds the determining elements of the investment is incorporated only subjectively.

2) Payback period. This method is used in conjunction with the assumed certainty approach. The payback period method is perhaps the most crude of all. It reflects the intuitive desire of the decision-maker to realize a return on his investment early rather than late in the project life. Because of the failure to take into consideration the effect of cash flow patterns and because it fails to value earnings

beyond the payout period, the payback period calculation is not an acceptable method for evaluating an investment's worth, according to this author. In addition, it cannot give a meaningful indication of investment risk.

3) Conservative adjustments. This technique is a variation on the best estimate approach described above. Some or all of the investment elements are adjusted to make them more conservative in order to reflect the uncertainty associated with them. There are many weaknesses in this approach to investment evaluation under uncertainty. The principle weakness is that the conservative adjustments are entirely subjective in nature and involve a determination of just how much of an adjustment to make for the amount of risk felt to exist. This leads to a second weakness -- inconsistency. Conservative adjustments cloud the investment picture and can lead with successive inconsistent adjustments to a completely false picture of the investment.

4) Sensitivity analysis. This method is also used with the assumed certainty approach. Sensitivity analysis seeks to determine the effect on the outcome of over- or underestimating an element's value and significantly highlights the relative importance of accurately estimating each element. Although an effective tool, this method of dealing with uncertainty lacks both conciseness and comprehensiveness. It is frequently used, for example, to determine how much

variation in an investment element would be necessary to reverse the decision based upon best estimate values. It does not take into consideration, however, the likelihood of such variation, which must be left to subjective analysis.

5) Adjustment of the discount rate. This method of compensating for risk is also based on the best estimate technique. It adjusts the minimum acceptable rate of return to reflect the degree of uncertainty felt about the investment income. The less certain the investment data values, or the greater the stakes involved, the higher the minimum acceptable rate of return. Uncertainty allowances for different types of investments will vary from industry to industry and from firm to firm because such allowances are basically determined by the technical and managerial skills as well as the resources of the firm. As a consequence, the specification of the appropriate interest rate for a particular investment becomes a matter of subjective judgment; herein lies the technique's fundamental weakness.

6) Multiple cases. The purpose of this method is to test an investment's outcome to various assumptions regarding the basic data. It differs from sensitivity analysis in that the values of several elements are changed rather than a single element each time the measure of merit is calculated. Frequently the multiple case approach is used to determine the effect of generally optimistic and pessimistic estimates of the investment elements in order to establish the limits

of probable outcome. This is in addition to the best estimate derived under the assumed certainty approach.

Klausner concludes that each of these traditional means of handling risk starts from a basic evaluation of an investment using the most-likely data values. While some of these approaches have features which are useful for dealing with uncertainty, all are inadequate in several critical respects; their deficiencies can be summed up as follows:

1) Judgment is usually applied to the results of the analysis, rather than to the underlying assumptions.

2) There is no over-all indicator of outcome variability generated.

3) There is no accounting for investment element interaction.

4) All these methods result in a qualitative "gut feeling" rather than a quantitative assessment of risk.

5) The methods do not provide a consistent analysis framework either from project to project or from individual to individual.

By now the need to consider risk and uncertainty when evaluating capital investment alternatives must be acknowledged and accepted. The deterministic or assumed certainty approach, using a single best estimate value for determining an investment's outcome, has the serious weakness of failing to allow for other possibilities that might occur. As a result, the decision-maker obtains no indication of likely variation around the investment's expected outcome. Furthermore, neither the relative uncertainty associated with each of the investment variables nor the significance of this uncertainty can be determined. Consequently, the assumed certainty approach cannot give a clear picture -- and in fact may give an inaccurate or misleading picture -- of an investment opportunity.

In the next section we shall describe a model for simulating the risk associated with different investment alternatives.

The Development of the Model

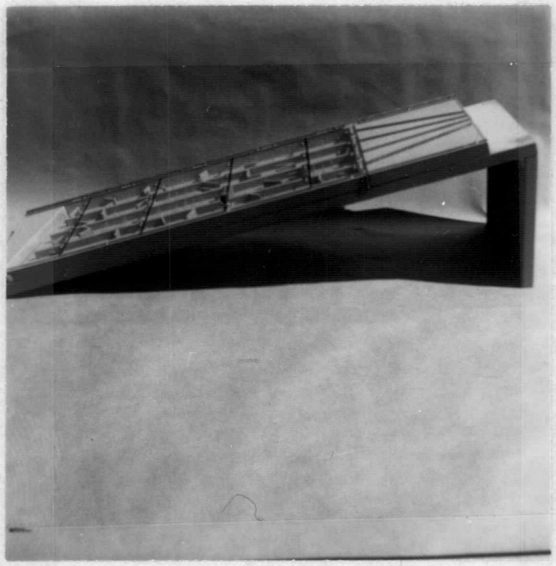
Several articles have appeared in the recent literature which deal with computer simulation of risk and uncertainty (see, for example, Hertz (31), Salazar and Sen (66), and Klausner (37)). It was our desire to develop some alternative means for satisfactorily simulating risk in capital budgeting. This section describes the development and application of our alternative model. It must be emphasized that we are not suggesting that our model is superior to the other models which have been proposed; we are only saying that there are several models available for simulating risk and we are offering another one. Naturally, we modestly hint that our model has certain advantages over the computer simulation models, such as simplicity of operation, value as a learning device, graphic portrayal of random variation of outcomes, flexibility of operation, and is in general an interest-getting and "fun" learning device.

First of all, our approach can be termed bi-directional, in that it has a quantitative and a qualitative aspect. The quantitative aspect deals with the development and solution of a capital rationing problem that authentically simulates real-world situations. The qualitative aspect is concerned with a small-scale investigation of how individuals actually make decisions (i.e., what decision rules they use) in problems of this type. The remainder of this chapter summarizes the construction and operation of the simulation model, as well as the preparation of basic data for a sample capital budgeting problem. This problem will be solved quantitatively using the investment policy developed in the next chapter. Chapter VIII reports on the results of the qualitative investigation.

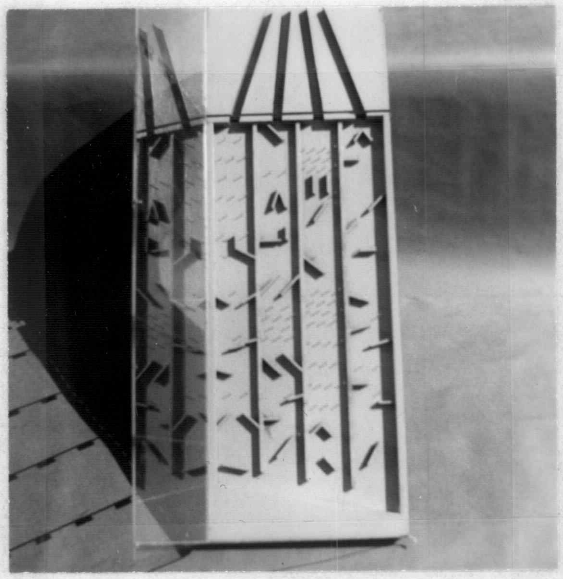
Our model can be described as a 1" by 12" board approximately 36" long which rests on a 9" incline. The board is divided into four channels, each channel is an obstacle course constructed of small nails and pieces of balsa wood. Marbles are inserted into four chutes at the top of the board. When a release bar is pushed, the marbles fall through the channels. Most of the marbles are trapped in the obstacle course along the way, but some make it all the way to the bottom. The board is fitted with a hinged plexiglass cover, which is divided into five areas or zones by 1/8" wide black tape. On the cover also is the possible dollar payoff to each zone printed on labelmaker tape. See the next page for photographs of the board.

Each channel corresponds to one or more potential investment programs having a certain specified "risk-profile" curve. (The concept of a risk-profile curve has been developed by Hertz (31). The modification and adaption of his concept to our model will be presented shortly.) Marbles are used to represent capital. A limited number of marbles is available to the decision-maker, and there is a limit on the capacity of each channel. The zone(s) that the marble(s) occupies for a given investment program determines the payoff for the investment. The payoff is subject to uncertainty because it is not exactly known in advance of actually playing the game which zone(s) the marble(s) will become trapped in.

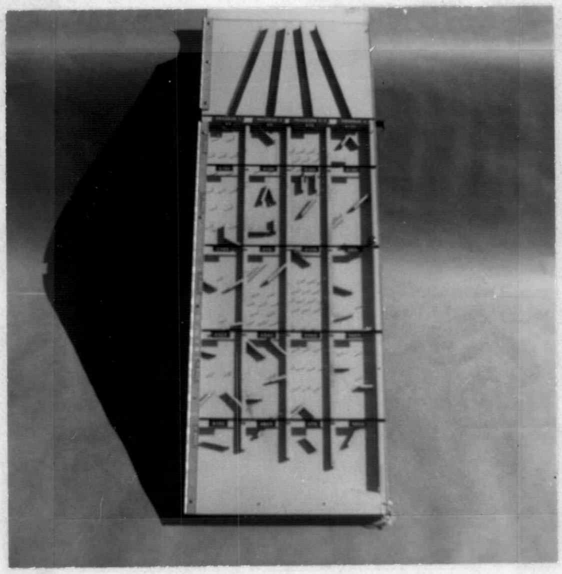
It is possible, however, to determine the relative frequency, or probability, that a given number of marbles will occupy a given zone of a given investment program. These are _a priori_ probabilities, because they are determined in advance of, or prior to, actually running the experiment -- that is, playing the simulation game. As a compromise of convenience, we arbitrarily set the capacity of each channel at five marbles. Then we chose to run 50 trials each of one, two, three, four, and five marbles through each of the four channels and recorded the zones into which the marbles were trapped. The tally sheets can be found in the Appendix; the results of the tabulation are summarized



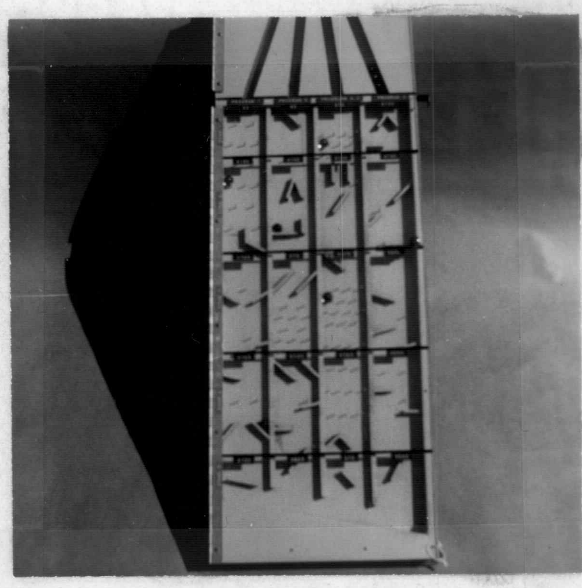
Side view.



Front view with cover open.



Front view with cover closed.



Front view showing marbles occupying zones.

Figure VI-1. Photographs of the Risk Simulation Model.

in the table below.

Table VI-1. Probabilities of one or more marbles occupying given zones.

	Number of marbles allocated	Probability Channel 1	Channel 2	Channel 3	Channel 4
Zone 1	1	.08	.00	.36	.04
	2	.32	.00	.66	.08
	3	.44	.00	.84	.12
	4	.64	.00	.86	.34
	5	.48	.00	.98	.32
Zone 2	1	.52	.36	.38	.42
	2	.74	.66	.54	.64
	3	.96	.90	.80	.80
	4	.94	1.00	.76	.84
	5	1.00	.94	.76	.90
Zone 3	1	.04	.26	.14	.24
	2	.18	.42	.26	.38
	3	.22	.46	.38	.48
	4	.36	.58	.60	.66
	5	.30	.64	.70	.80
Zone 4	1	.26	.36	.10	.10
	2	.38	.44	.20	.24
	3	.38	.48	.28	.52
	4	.52	.60	.26	.54
	5	.80	.78	.30	.66
Zone 5	1	.10	.02	.02	.20
	2	.14	.00	.08	.48
	3	.26	.02	.06	.56
	4	.20	.06	.12	.56
	5	.32	.10	.14	.72

The reader may observe that, when allocating more than one marble, the cumulative probabilities when summed over all five zones are greater than one. This occurs because some marbles have been counted more than once in giving the probability of one or more marbles occupying a given zone. No difficulty is presented however; the table is only intended to show a summary of results.

We turn our attention next to the development of the basic problem information. It is our desire to present a problem situation containing several elements of realism. The following three elements were considered to be essential:

1) A number of potential investment alternatives are to be evaluated and compared.
2) Limited capital resources are available for allocation.
3) There is uncertainty concerning investment payoffs.

The basic problem information is given below:

1) There are five investment programs being considered. The anticipated lifespan in years and the total dollar investment required for each program are given in the table below.

Table VI-2. Program lifespan and total dollar outlay requirements.

Program number	Expected lifespan in years	Total dollar investment required
1	3	$400,000
2	3	300,000
3	2	500,000
4	4	400,000
5	1	200,000

Each of the programs is assumed to be independent. In addition, it is assumed known that the annual capital outlays for each year of a given program's life will take the pattern shown in the table below.

2) There are only ten marbles available to the decision-maker for use with the game board. Each marble is worth $100,000, so that the total available resources equal $1,000,000. The $1,000,000

Table VI-3. Program investment pattern requirements (000's omitted).

Program number	Year 1	2	3	4
1	$100	$100	$200	---
2	100	100	100	---
3	300	200	---	---
4	400	0	0	0
5	200	---	---	---

represents the maximum amount the firm is able to earmark for investment at the present time. No additional funds are anticipated within the near future, at least as far as the investments now under consideration are concerned. It is assumed that any net profits resulting from the investments will be put to work within the firm. Also, it is assumed that the investment payoffs occur at the end of the year in which the outlay is made, with the exception of Program #4, where the payoff occurs at the end of year four. Company policy has established the minimum acceptable rate of return on investment at 10% per annum.

3) The uncertainty surrounding investment payoffs is simulated by the three-dimensional game board. It is assumed that Program #5 has a risk profile curve for a $200,000 investment similar to that for Program #3.

Next, the possible payoffs are determined for each zone of each channel. This is a trial and error procedure to give expected payoffs and expected rates of return greater than the minimum acceptable limits. Table 4 below shows the results of the trial and error manipulation.

Table VI-4. Possible dollar payoffs by zone for each channel (000's omitted).

Zone number	Possible dollar payoff Channel 1	Channel 2	Channel 3	Channel 4
1	$000	$000	$ 75	$100
2	150	100	125	150
3	125	75	175	200
4	125	150	125	250
5	100	225	75	250

After determining the possible payoffs for each program, it is then a matter of "busy work" to calculate the a priori payoffs by referring back to the tally sheets for the a priori probabilities. The results are shown in the payoff graphs for each program given on pages 93 through 96. The relative frequency of each of the payoffs is expressed as a percentage.

The next step is to compute the net return and the rate of return on investment for each required capital outlay and for each program. Recalling that in Chapter IV we defined a simple rate of return (i. e., no cash flows which must be discounted) as net return divided by investment required, namely,

$$\text{Rate of return} = \frac{\text{payoff - investment}}{\text{investment}}$$

$$= \frac{\text{net return}}{\text{investment}} .$$

The calculation of the rate of return for Program #4 requires use of a method given by Riggs (63), since the payoff is deferred until the end

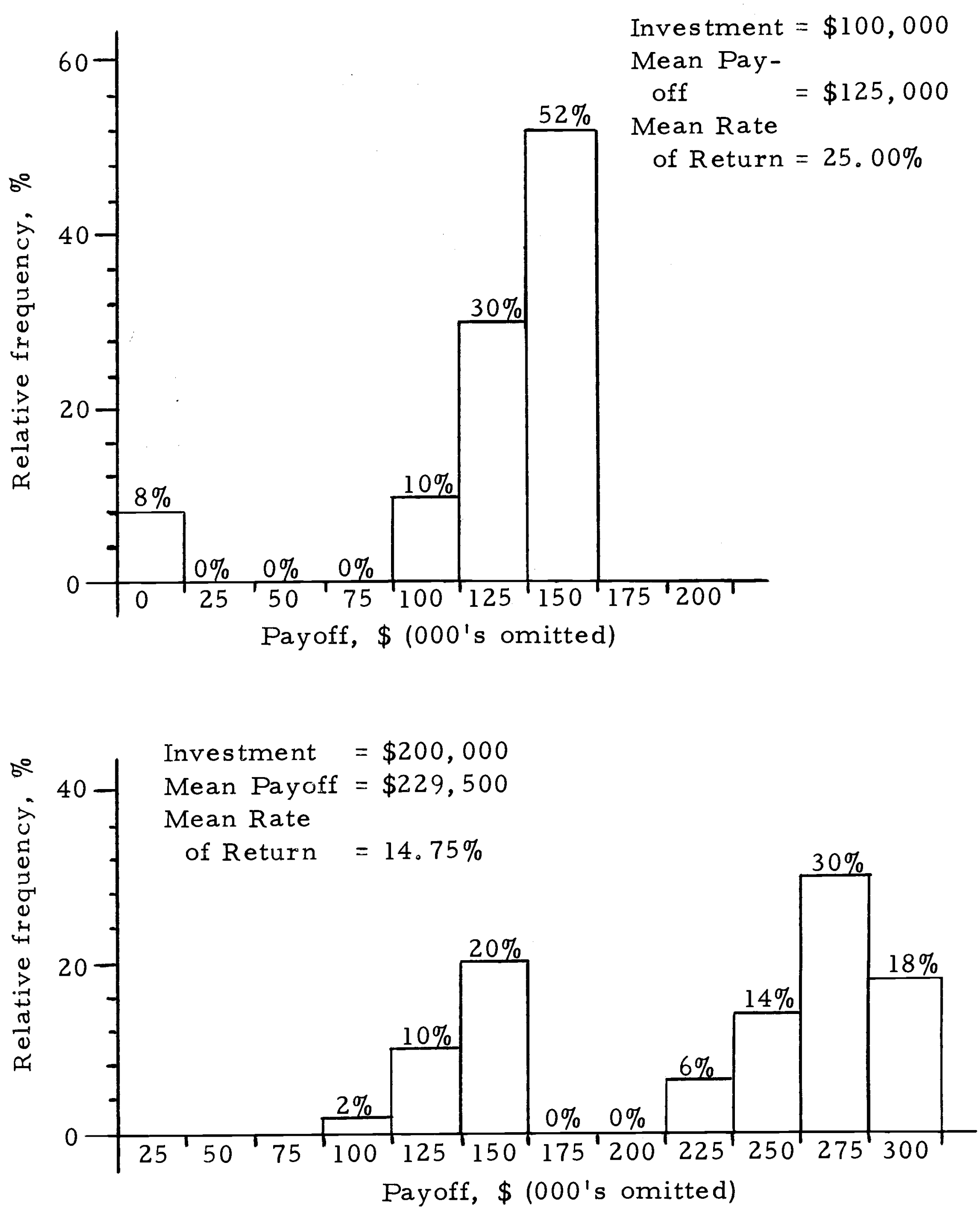


Figure VI-2. Payoff graphs for Program #1.

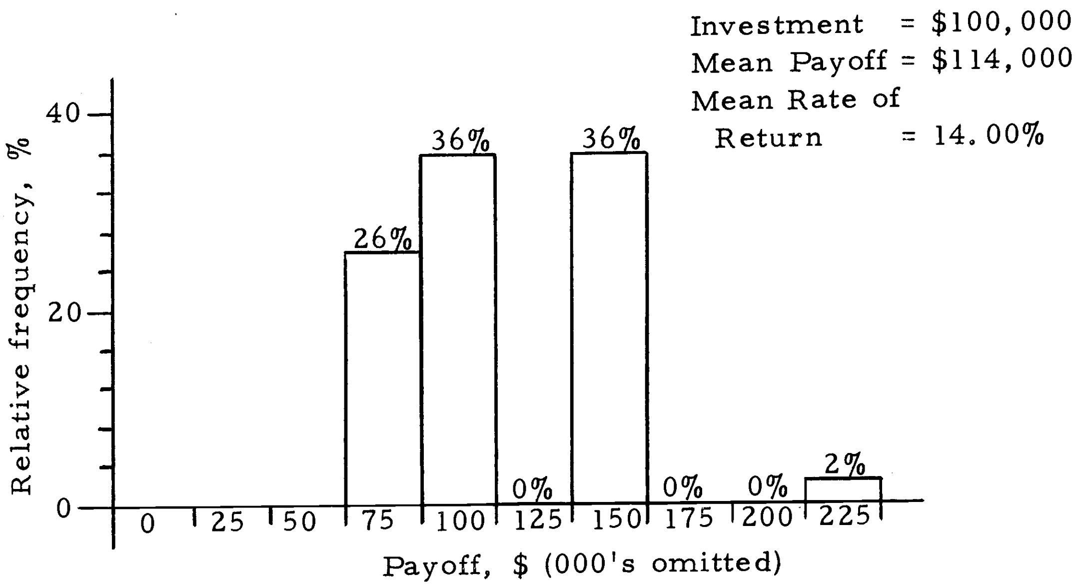
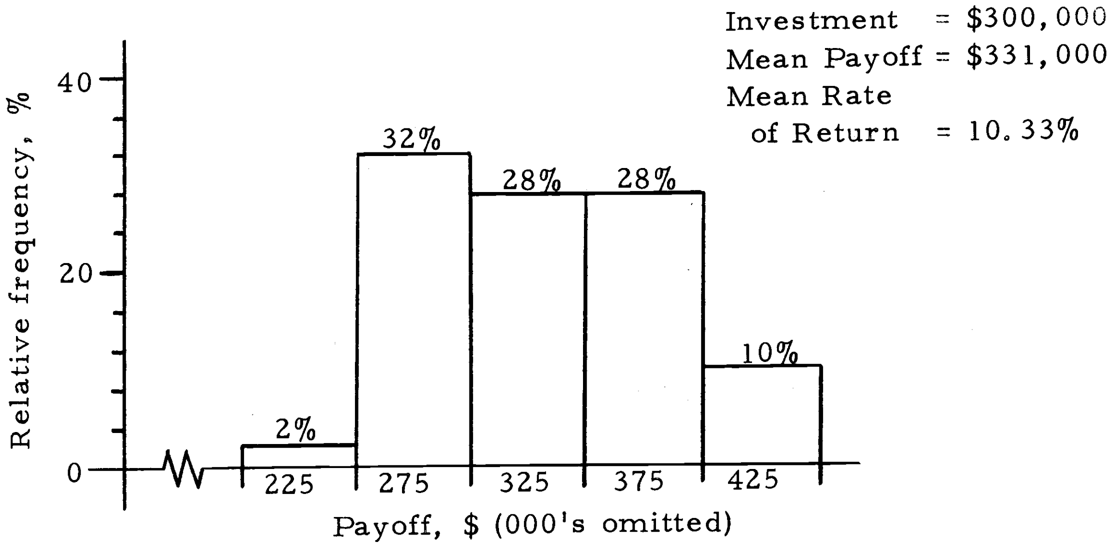


Figure VI-3. Payoff graph for Program #2.



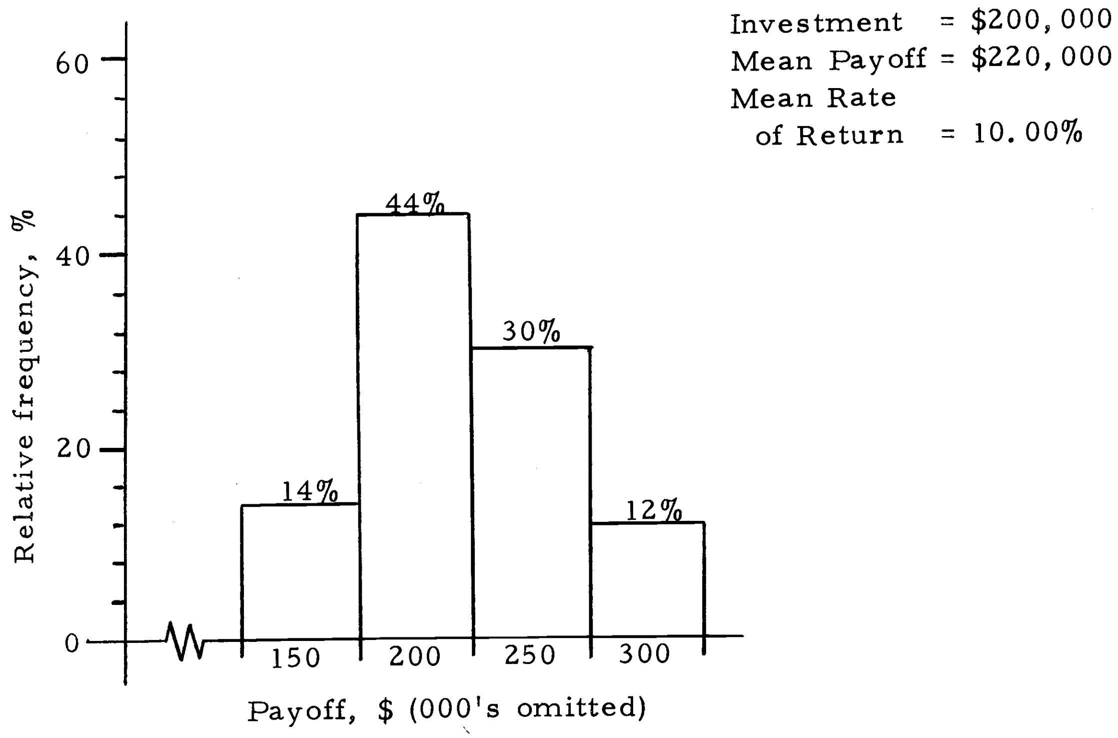
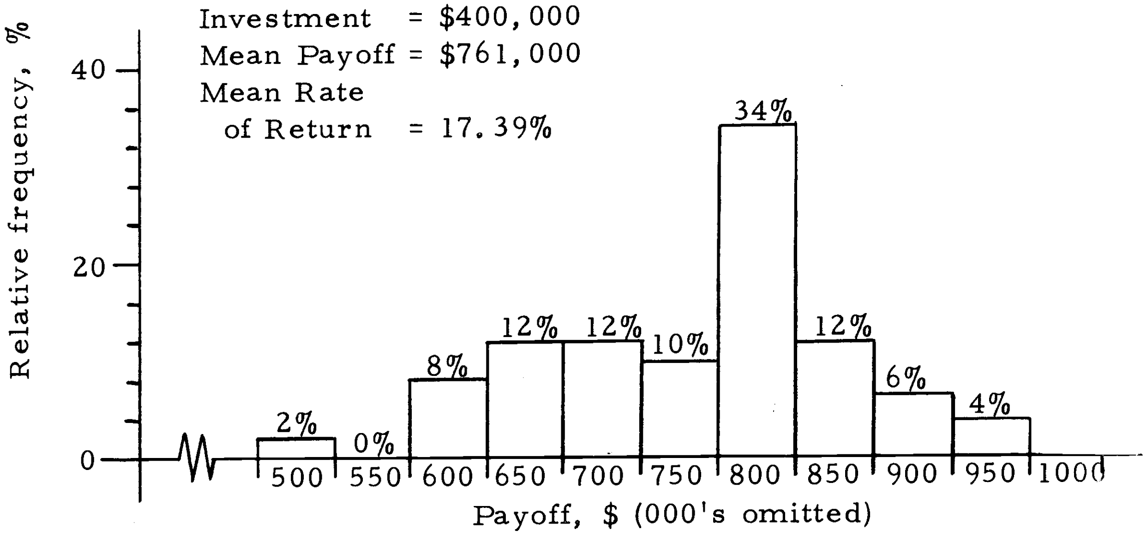


Figure VI-4. Payoff graphs for Program #3 and 5.



Note: Payoffs for Program #4 occur at end of year 4 only, rather than at the end of each year as with the other programs.

Figure VI-5. Payoff graph for Program #4.

of year four. First, the "future worth -- present amount" factor was figured by dividing the given payoff by the investment of \$400,000. Then, the answer was determined by interpolation in the tables of interest factors. A sample calculation is shown below:

$$\text{Let } \frac{\text{payoff}}{\text{investment}} = (f/p)_4^i$$

$$\text{Then } i = \frac{(f/p)_4^i - (f/p)_4^{15}}{(f/p)_4^{20} - (f/p)_4^{15}} \times 5 + 15.00$$

$$\text{Example: } (f/p)_4^i = \frac{\$761,000}{\$400,000} = 1.9025$$

$$i = \frac{1.9025 - 1.749}{2.074 - 1.749} \times 5 + 15.00$$

$$= 2.39 + 15.00 = 17.39\ \%\ \text{rate of return}$$

The results of these rate of return calculations are given in the table on page 98.

Following the calculation of the possible rates of return on the various capital outlays for the five potential investment programs, it is then an easy task to plot the "risk-return profile" curve for each program. The resulting curves are combined into one graph shown on page 99. The percentage cumulative probability is shown on the vertical axis; this is the probability that the actual rate of return for the investment will be greater than or equal to a specified rate of return. This probability is a measure of the "riskiness" of an

Table VI-5. Program listing of frequency of net return and rate of return (000's omitted).

	Payoff	Probability of payoff	Net return	Rate of return
Program #1	$000	.08	$-100	-100.00%
	100	.10	000	0
$100,000 outlay	125	.30	+ 25	+ 25.00
	150	.52	+ 50	+ 50.00
	100	.02	-100	- 50.00
	125	.10	- 75	- 37.50
	150	.20	- 50	- 25.00
$200,000 outlay	225	.06	+ 25	+ 12.50
	250	.14	+ 50	+ 25.00
	275	.30	+ 75	+ 37.50
	300	.18	+100	+ 50.00
Program #2	$ 75	.26	$- 25	- 25.00%
	100	.36	000	0
$100,000 outlay	150	.36	+ 50	+ 50.00
	225	.02	+125	+125.00
Program #3	$225	.02	$- 75	- 25.00%
	275	.32	- 25	- 8.33
$300,000 outlay	325	.28	+ 25	+ 8.33
	375	.28	+ 75	+ 25.00
	425	.10	+125	+ 41.67
	150	.14	- 50	- 25.00
	200	.44	000	0
$200,000 outlay	250	.30	+ 50	+ 25.00
	300	.12	+100	+ 50.00
Program #4	$500	.02	1.250*	+ 5.74%
	600	.08	1.500	+ 10.65
$* = (f/p)^i_4$	650	.12	1.625	+ 12.88
	700	.12	1.750	+ 15.01
$400,000 outlay	750	.10	1.875	+ 15.94
	800	.34	2.000	+ 18.86
	850	.12	2.125	+ 20.79
	900	.06	2.250	+ 22.71
	950	.04	2.375	+ 24.63
Program #5	$150	.14	$- 50	- 25.00%
	200	.44	000	0
$200,000 outlay	250	.30	+ 50	+ 25.00
	300	.12	+100	+ 50.00

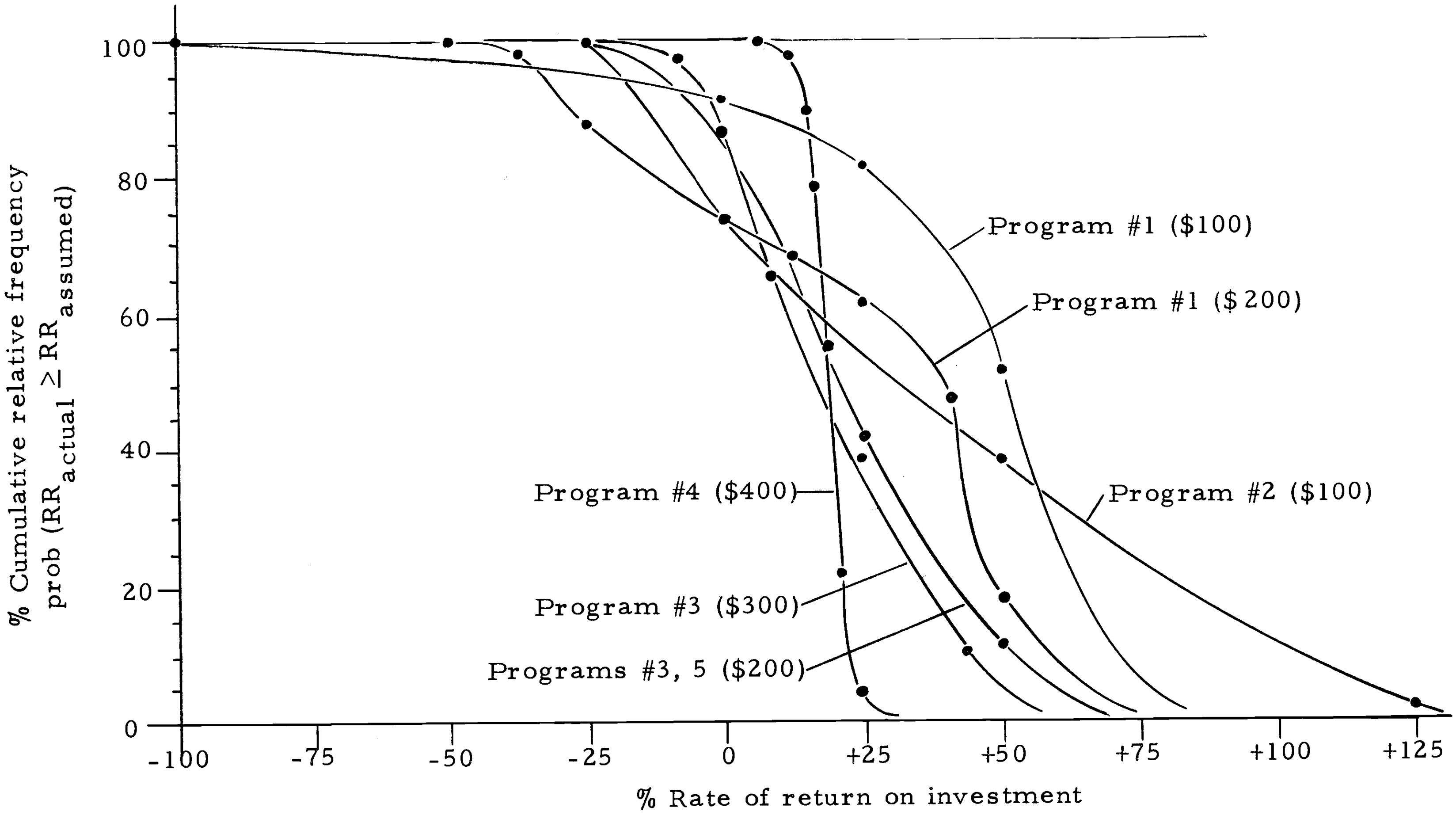


Figure VI-6. Risk-return profile curves.

investment. The percentage rate of return on the investment is plotted along the horizontal axis.

At this point, the only step remaining involves the derivation of a quantitative decision-making procedure. The development of such an investment decision-making policy for capital budgeting problems is described in the next chapter.

VII. THE DEVELOPMENT OF A QUANTITATIVE INVESTMENT POLICY

Introduction and Theory

Now that we have derived the basic data for our capital budgeting problem and have established the operational groundrules (that is, the framework within which the problem must be solved), the only question remaining is, just what sort of quantitative investment policy will lead us to the "best" solution to the problem? This chapter describes the development of such a policy for determining the optimum allocation of a limited amount of capital resources among several competing investment proposals. The decision-making procedure is applicable to the general case of capital budgeting problems illustrated by the sample problem situation of the previous chapter.

As Hertz (31) points out, any investment policy must include two essential elements: 1) one or more criteria by which to measure the worth of an investment, and 2) a series of decision rules to screen the various investment proposals and ultimately lead to the desired decision. These two elements constitute an objective means for evaluating and comparing alternatives. The criterion and decision rules for our investment policy will be described shortly.

In his article on utility theory, Swalm (67) suggests that decision-makers can be classified as risk-seekers, risk-avoiders, or risk-insensitives on the basis of an evaluation of their individual utility functions. Risk-seekers, or gamblers, are persons who are willing to take a chance on winning big stakes; they look for the investments having big payoffs, even if the chances of getting these payoffs are fairly remote. In addition, gamblers are characterized by the ability to tolerate the possibility of a sizable loss on the investment. Risk-avoiders, on the other hand, are conservative individuals who want to be reasonably certain of their investments yielding at least a minimum return. These conservatives wish to minimize their risk; they insist that the probability of losing all or a portion of their investment will be quite small. Finally, risk-insensitives are the type of decision-makers characterized by Morris (49) as acting as if a future state were certain to occur. The ignorer who assumes certainty will be considered at the conclusion of this chapter.

It is our contention that, given the two classifications of decision-makers (as risk-seekers and risk-avoiders), it is entirely possible to establish two points on the probability-rate of return grid which reflect the desire of the decision-maker to 1) maximize the chances of a big payoff or return, and 2) minimize the risk of a sizable loss. Figure VII-1 on the next page shows these regions of payoff maximization and risk minimization associated with a typical risk-return profile curve. These are the regions that the decision-maker, whether he be a gambler or a conservative, will pay particular attention to.

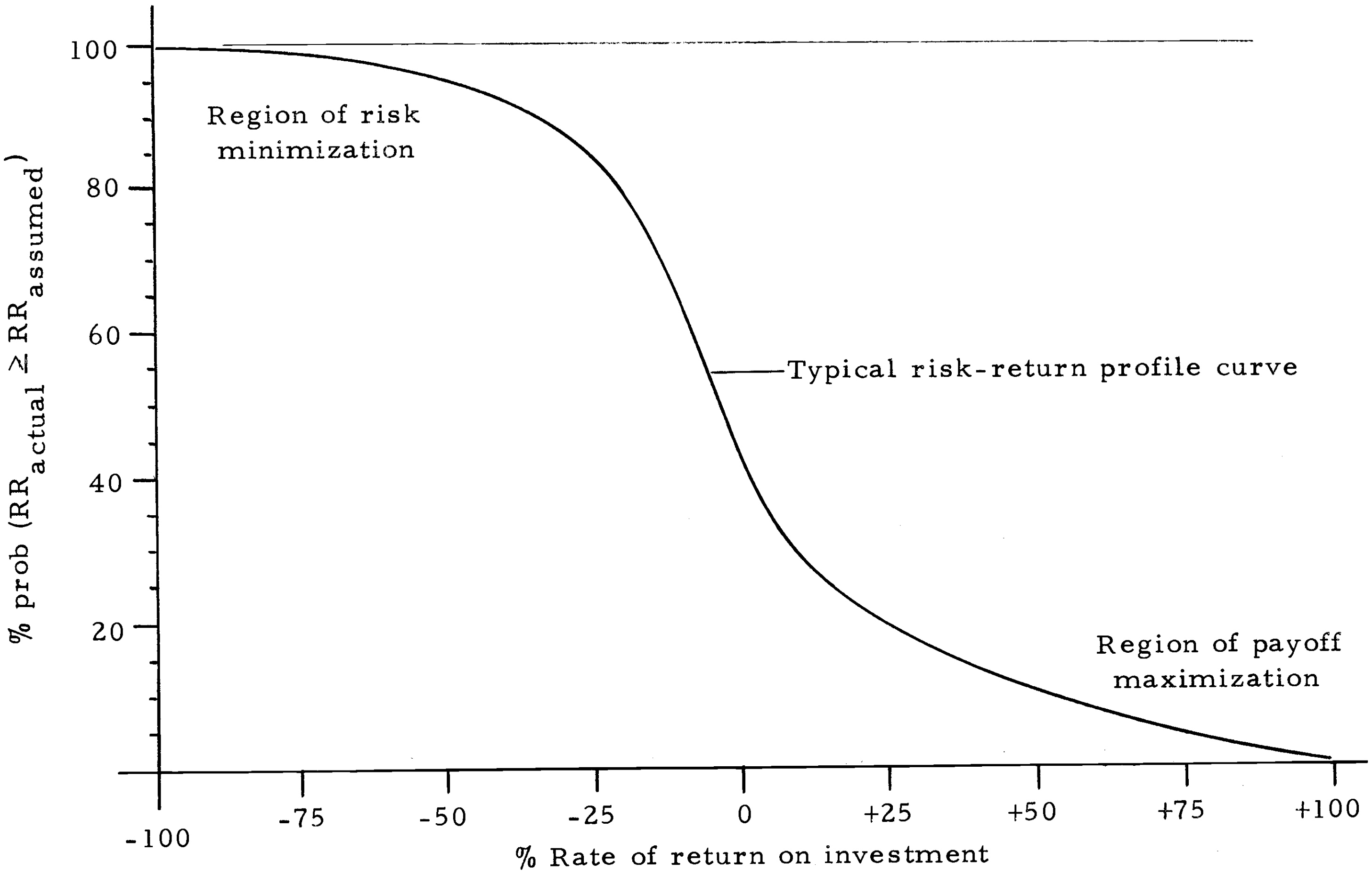


Figure VII-1. Regions of payoff maximization and risk minimization.

The point in the region of payoff maximization and the point in the region of risk minimization are located as follows: For the gambler and for the conservative two probability coefficients must be determined. These coefficients can in reality be set, for example, by management policy or by a study of past investments of a similar nature. The first of these coefficients is termed α, the risk coefficient; α reflects the amount of concern the decision-maker has regarding possible investment losses. The second probability coefficient is β, the payoff coefficient; β similarly is a measure of the intent on the part of the decision-maker to maximize his payoff. One pair of α and β coefficients must be determined. For the gambler these are termed α_g and β_g; for the conservative these are designated α_c and β_c.

Associated with these risk and payoff coefficients are certain critical values in the possible range of rates of return for the investments being considered. Again, these critical values must be set following an evaluation of the decision-maker as a gambler or a conservative. These critical rates of return are designated as follows: RR_{gcr} is the gambler's critical value to minimize risk; RR_{gcp} is the gambler's critical value to maximize payoff. Similarly, RR_{ccr} is the conservative's critical value to minimize risk; RR_{ccp} is the conservative's critical value to maximize payoff. RR_{gcr} and RR_{gcp} are associated with α_g and β_g, respectively; RR_{ccr} and RR_{ccp}

are associated with α_c and β_c, respectively.

The two points of interest are determined by the intersection of the perpendicular lines which pass through the probability coefficient and the associated critical value when these parameters are located on the coordinate axes of the probability-rate of return grid. These two points are labeled P_1 and P_2 on Figures VII-2 and VII-3 shown on the next two pages. The straight line connecting P_1 and P_2 is called the "aspiration" line ("aspiration" is a term borrowed from utility theory). In this paper we shall assume that the aspiration line is linear; however, we do not deny the possible existence of other line configurations.

Figure VII-2 shows a possible aspiration line for a gambler. Having determined his particular aspiration line, he will accept only those investments having risk-return profile curves lying entirely to the right of the line; any investments with a curve which crosses the aspiration line at some point or lies entirely to the left of the line will automatically be rejected. Those investments which are acceptable are allocated capital resources in rank order of maximizing return until 1) funds are exhausted or 2) the minimum acceptable rate of return is reached. The rank order of investments is determined by extrapolating the risk-return profile curves to the horizontal axis (i.e., probability = 0) and reading from right to left. Figure VII-3 shows one possible aspiration line for a conservative investor.

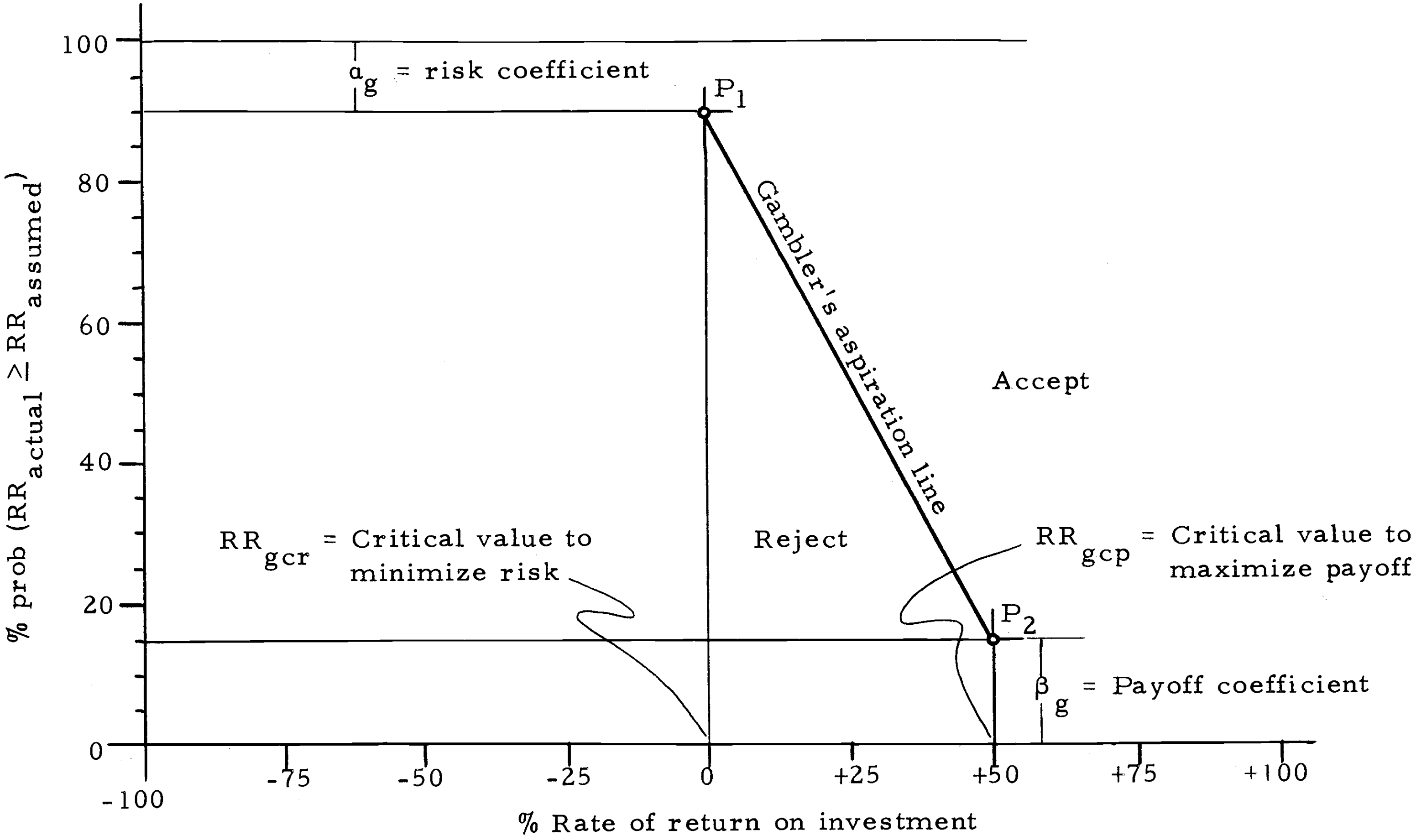


Figure VII-2. Risk-seeker's (gambler's) aspiration line.

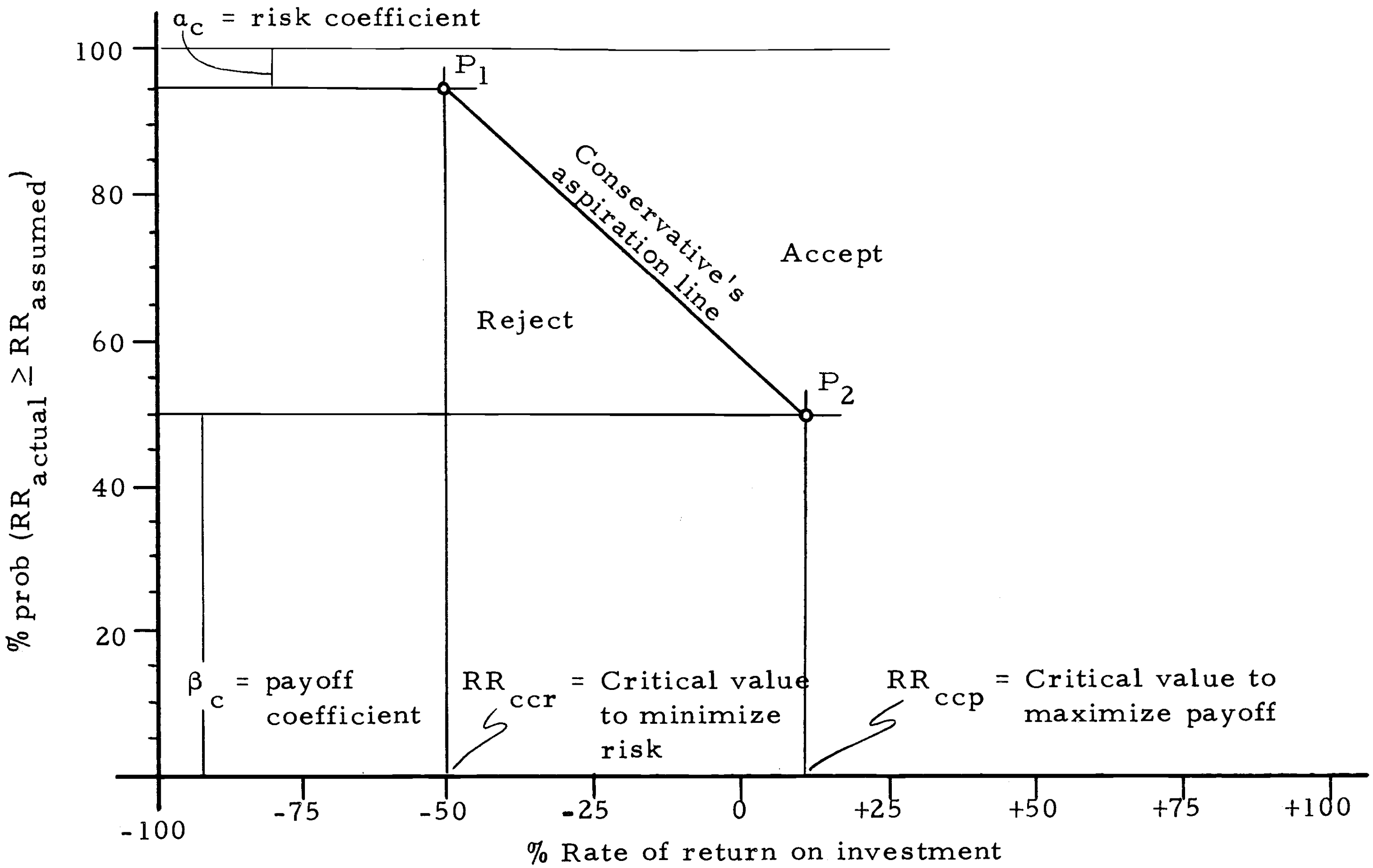


Figure VII-3. Risk-avoider's (conservative's) aspiration line.

Evaluation Criteria and Decision Rules

For our particular capital budgeting problem, the investment policy is given as follows: 1) The criterion used to measure investment worth is the before-taxes percentage rate of return on investment, defined as before; 2) the decision rules used to screen potential investments are based on an evaluation of the risk-return profile curves for the alternatives under consideration, as developed in the preceding section.

Values for the probability coefficients and the critical value parameters have been arbitrarily set for a hypothetical risk-seeker and risk-avoider. These values are given in Table VII-1 below. The investment decisions for our hypothetical gambler and conservative are shown graphically on the next two pages in Figures VII-4 and VII-5.

Table VII-1. Parameter values.

Decision-maker	Probability coefficients		Critical values	
	α	β	RR_{cr}	RR_{cp}
Risk-seeker (gambler)	5%	10%	-50%	+50%
Risk-avoider (conservative)	10%	50%	0%	+15%

These parameter values allow us to establish the two points necessary to determine the aspiration line which divides the risk-return grid into acceptance and rejection regions for the given type of

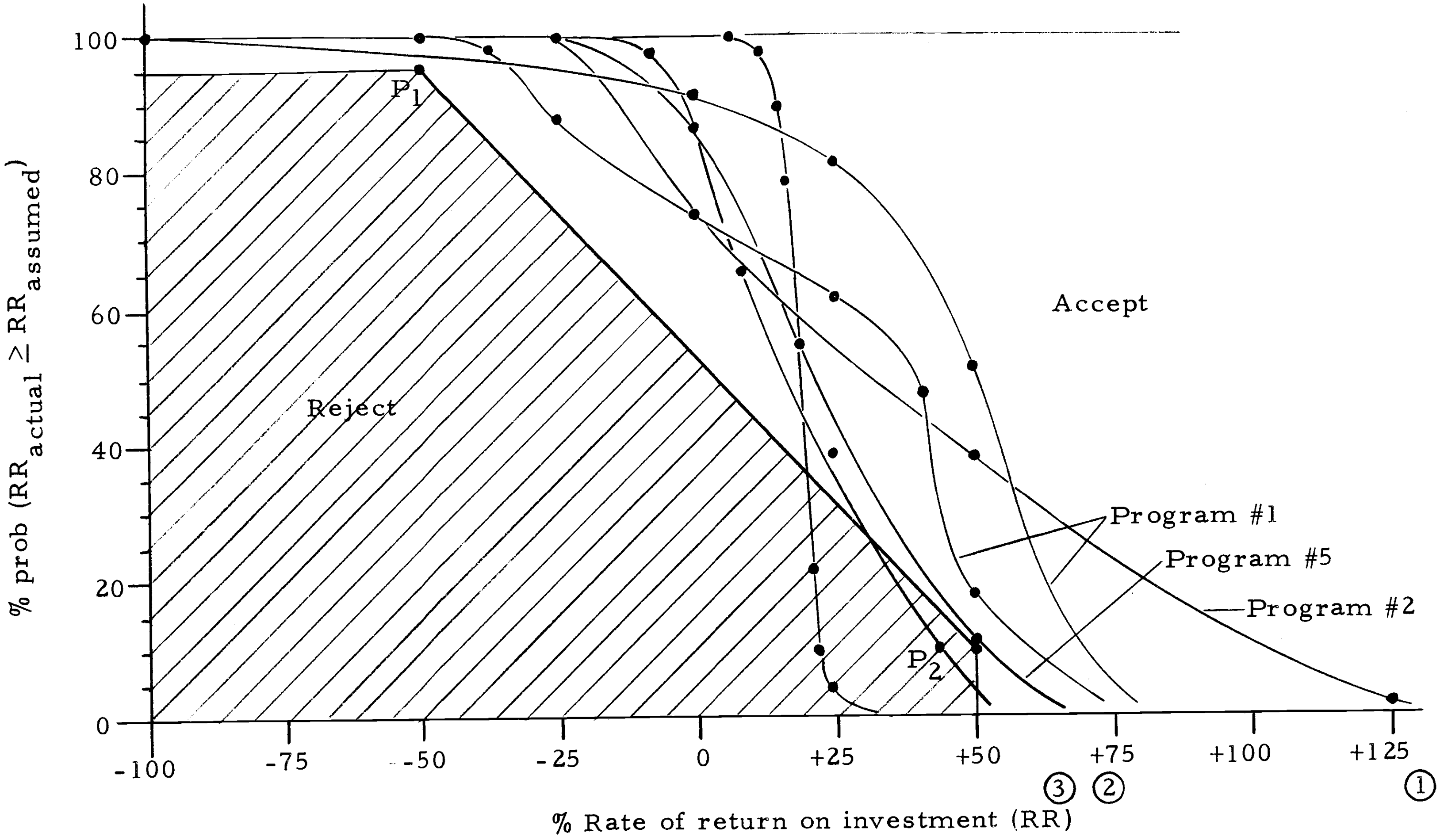


Figure VII-4. Gambler's investment decision.

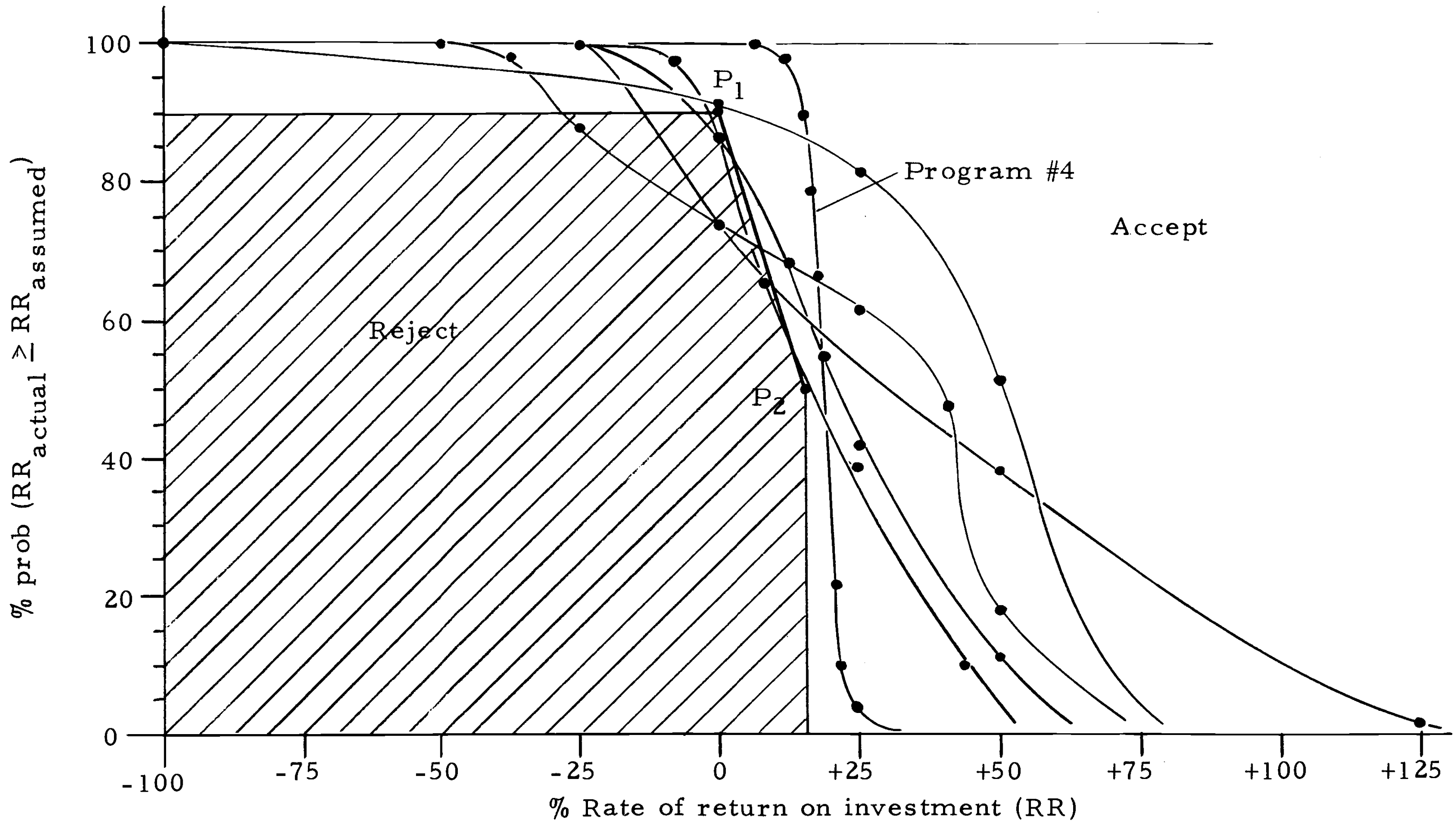


Figure VII-5. Conservative's investment decision.

decision-maker.

Solution to the Sample Problem

As can be seen from the graphs of Figures VII-4 and VII-5, the gambler and the conservative would not make the same investment decision. Based upon the decision rules given above, the gambler would make a total dollar investment of $900, 000 in Programs 1, 2, and 5. The conservative, meanwhile, would allocate $400, 000 only to Program 4.

To complete the picture, we return to the third type of decision-maker -- the risk-insensitive. Recall that the risk-insensitive individual assumes certainty. Thus, in this case any of the decision rules discussed in the previous chapters and based on the assumption of certainty would be appropriate. We suggest ranking the expected values of the distribution of rates of return for the various investments, and allocating the available funds in order of greatest expected rate of return until the resources are depleted or until the minimum acceptable rate of return is reached. Using this method, the insensitive would invest $800, 000 in Programs 1 and 4. The method described by Fleischer (23) may be used alternatively to compute the incremental rate of return if it is desired to avoid the two types of errors he describes.

The quantitative investment policy described in this chapter is summarized in the investment decision-making flowchart shown in Figure VII-6. The flowchart breaks the decision procedure down into a series of sequential steps and decision points. The description accompanying each step should be self-explanatory.

The next chapter presents the general procedure to follow when a decision-maker actually uses the risk simulation game board to determine investment program payoffs. The chapter also reports the results of a small-scale investigation of how real decision-makers do make decisions in problems of this type.

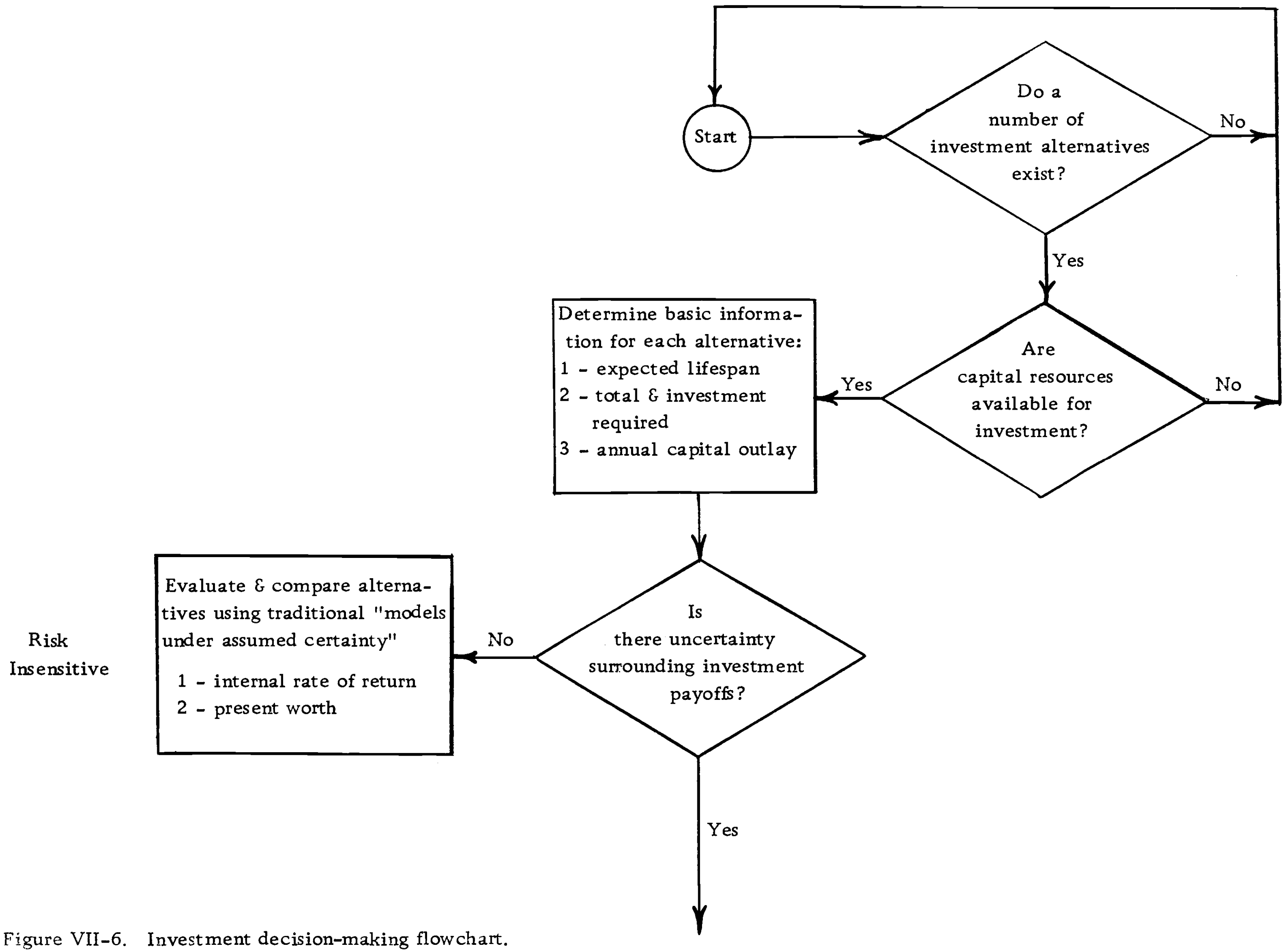


Figure VII-6. Investment decision-making flowchart.

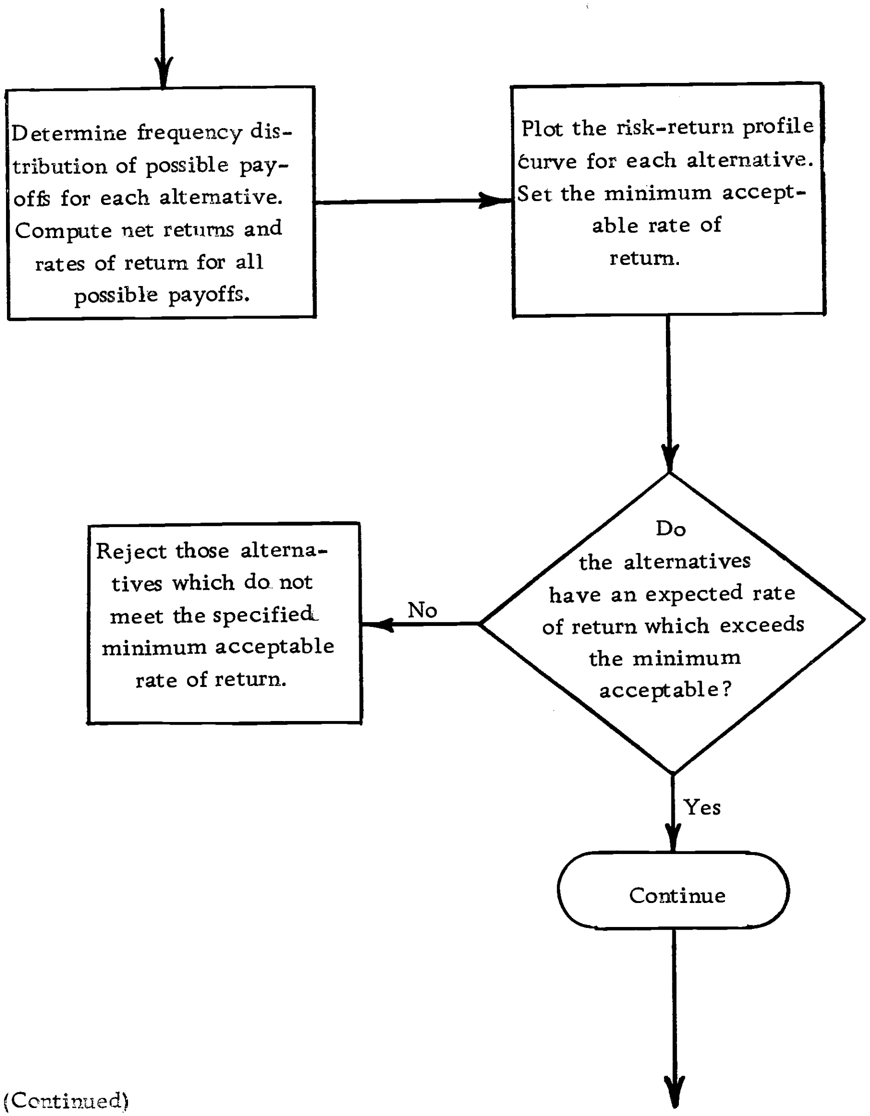


Figure VII-6. (Continued)

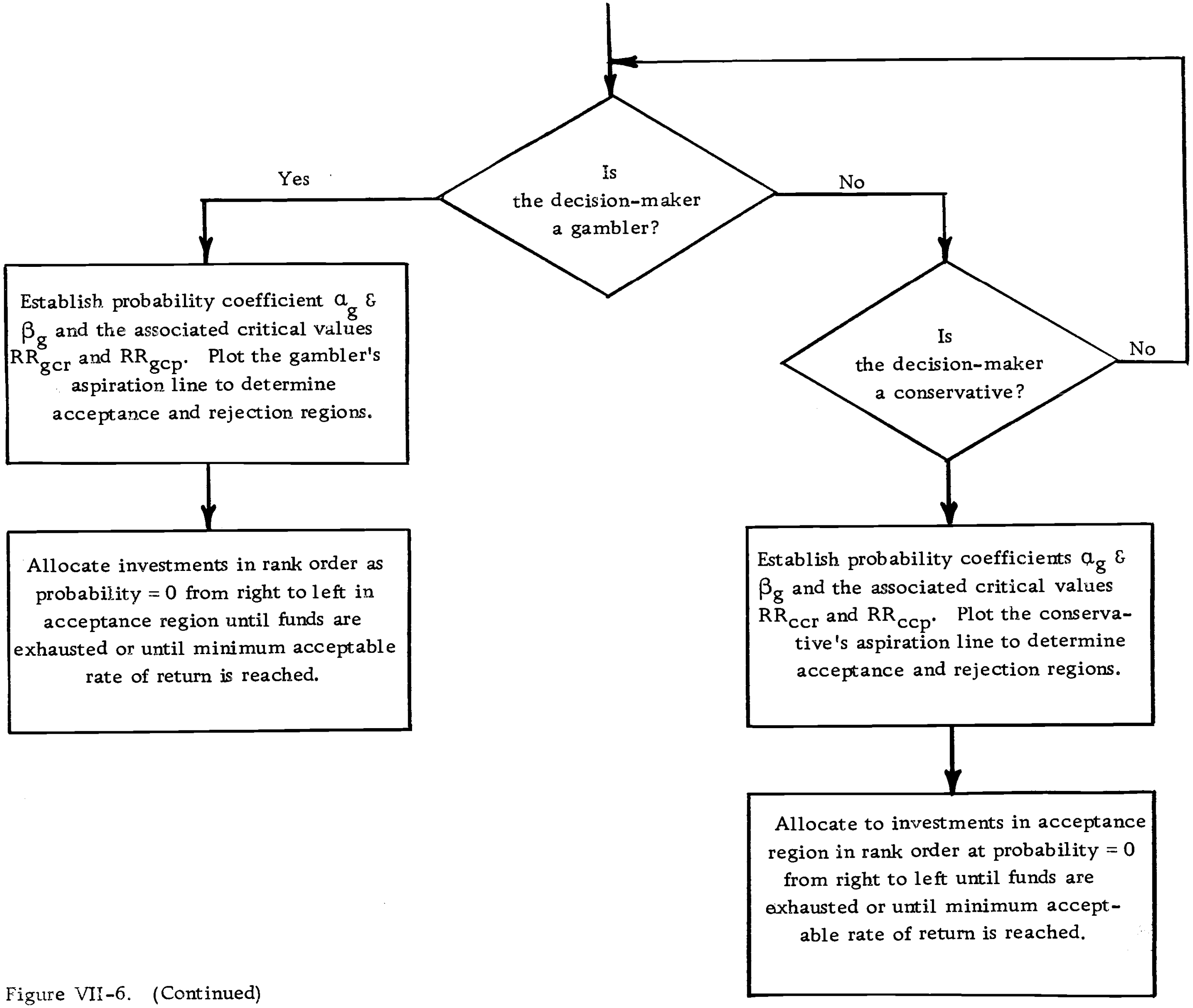


Figure VII-6. (Continued)

VIII. AN EXPERIMENTAL STUDY

General Experimental Procedure and Game Format

Chapter VI presented arguments stressing the importance of the need to recognize and evaluate the uncertainty surrounding investment outcomes. The chapter then presented certain basic problem information developed in conjunction with the three-dimensional model or game board. Five capital investment outlays were presented, one for each of the five investment programs represented on the game board. These outlays may be summarized as follows:

Table VIII-1. Original program total dollar outlay requirements.

Program number	Total dollar investment required
1	$400,000
2	300,000
3	500,000
4	400,000
5	200,000

For the purposes of investigating just how real decision-makers actually make decisions in problems of this type, a study was conducted using a small sample drawn from a cross-section of decision-makers. The sample was composed of students, professors, and businessmen with a wide range of age, educational background, and business experience. The purpose of this chapter is to report the results of the investigation.

To make the game board more interesting and flexible for the decision-makers, other possible investment outlays were developed. Table VIII-2 below is an extension of the figures in Table VIII-1, and shows the other investment requirements formulated. The table also summarizes the decision for each program and allocation amount using the investment policy presented in the last chapter for the hypothetical gambler and conservative.

Table VIII-2. Expanded capital outlay possibilities and investment decisions based on quantitative investment policy.

Program number	Capital outlay	Theoretical decision based on investment policy for gambler	for conservative
1	$100	Accept	Accept
	200	Accept	Reject
	400	Reject	Reject
2	100	Accept	Reject
	300	Reject	Reject
3	200	Accept	Reject (barely)
	300	Reject	Reject
	500	Reject	Reject
4	100	Reject	Accept
	200	Reject	Accept
	400	Reject	Accept
5	200	Accept	Reject

NOTE: For the purposes of the study, the total investment requirements in thousands of dollars were changed to capital outlays in hundreds of dollars, in order to give the decision-makers participating in the game figures more easily grasped.

The full text of the introduction to the general problem and instructions for playing the game as presented to the participants is given below.

Would you classify yourself as a risk-seeker, a risk-avoider, or a risk-insensitive? That is to say, in problem situations where the outcome is uncertain, do you tend to be a gambler, playing for the long shot and going for the big payoff? Or, do you favor cautious conservatism, attempting to minimize your losses? Or thirdly, do you prefer to largely ignore risk, acting as if one outcome were certain to occur? Think about this for a few minutes as we describe a problem situation using the game board in front of you. We shall be testing to see if your decision-solution is consistent with your self-image as a risk-seeker, a risk-avoider, or a risk-insensitive.

Here are ten marbles representing capital resources available for investment in any or all of the five investment programs represented on the game board. Each marble is worth $100. To see how the game board works, load up the chutes at the top of the board with one or more marbles. Push the release bar and observe which zones the marbles become trapped in. These occupied zones determine the payoff for the particular amount of invested capital. Note that the payoff per $100 of investment is given on the plexiglass cover for each zone and program. These are the payoffs that would occur during the same time period that the investment is made, with the exception of Program #4. Program 4 is unique in that its payoffs are deferred four years after the investment is made, as shown by the gold labels (on the plexiglas cover). The present worth of the payoffs is shown above the deferred amount in green tape; this present worth figure represents the payoff you could receive right now per $100 of investment. These present worth values are given to put Program #4 on a comparable basis with the other four programs. Also note that Program 5 is shown with Program #3. This is done because it is assumed they have similar risks.

Now load up the chutes again. Follow the marble capacities shown. It may not be possible to allocate all ten marbles each time, but this is all right. The object of the game is to allocate as many of the ten marbles as you desire among any or all of the five programs so as to yield the greatest payoff. Keep this objective in mind as you repeat the game play ten times. For each trial your investment and payoff score will be recorded.

(PAUSE WHILE PARTICIPANT REPEATS GAME PLAY TEN TIMES.)

On the basis of your game play, decide now what seems to be the best allocation of capital in order to receive the maximum payoff.

(PAUSE WHILE DECISION IS MADE AND RECORDED.)

Here are payoff frequency distribution graphs for the various programs and amounts of invested capital. These graphs are based on a large number of trials. This means for a given program if you repeated your allocation many times, you could expect your payoff to approximate the distribution shown by the graph. Feel free to comment if this additional information would affect your decision in any way.

A sample scorecard is given in the Appendix. In addition to the participant's initial decision (i. e., programs selected and amounts allocated) recorded on the scorecard, the participant's name, present occupation, number of years of college, number of years of business or work experience, and self-image as a risk- seeker, a risk-avoider, or a risk-insensitive were recorded. Following the game play, the decision-maker was given the opportunity to study the payoff frequency distribution graphs for all possible investment allocations. He was then asked if, based on his study of the graphs, he would change his original decision in any way. The individual was also asked whether the game board or the payoff frequency distribution graphs were more helpful in making the investment decision. The responses to these questions were recorded in spaces provided on the scorecard. The decision-makers were given the opportunity to make any comments or suggestions regarding the game.

Results of the Investigation

A total of 25 individuals provided the data used in the final analysis. Several other participants contributed their time and suggestions during preliminary trial runs of the game. Table VIII-3 below summarizes the information gathered about the subjects themselves (self-image, number of years of college, number of years of work experience) and the investment decisions they made (programs selected and amounts allocated). The 25 subjects are not arranged in any particular order. It should be noted that, based on the descriptions of the three types of decision-makers given, nine of the subjects rated themselves as gamblers (36% of the total), 12 rated themselves as conservatives (48%), and four considered themselves to be insensitive to risk (16%).

The various possible capital outlays, or investment decisions, for each program are broken down in Table VIII-4 according to the type of decision-maker making a given allocation. For example, the first row of figures in the table indicates that five out of the nine gamblers (or 55% of the decision-makers who classified themselves as gamblers) plus two out of the 12 conservatives (or 17%) plus one out of the four insensitives (or 25%) for a total of eight out of 25 (or 32% of the total) made the decision to allocate \$100 to Program 1 as a part of their solution to the capital rationing problem. The other table entries are interpreted in a similar manner. In summary, it

Table VIII-3. Breakdown of investment allocations by decision-maker.

Subject number	Self-image*	Number of years of college	Number of years of work experience	Investment allocations to programs 1	2	3	4	5
1	C	4	29	$200		$200	$400	$200
2	C	4	0	400	$100		400	
3	C	6	14 1/2		100	500	400	
4	G	4	12	100	300	200	400	
5	C	4	28			300	400	
6	C	4	19	400			400	200
7	G	3 1/2	2 1/2	400			400	200
8	C	4	30	400			400	200
9	C	3 1/2	1				400	200
10	G	3	6			300	400	
11	C	3	27	100	100	500	100	200
12	C	6	3	100		300	400	200
13	G	5	5	200	100	300	400	
14	G	4	2	100	100		200	200
15	G	4 1/2	6	100	300		400	200
16	G	7	15	100	300		400	200
17	I	2	30	100		500	400	
18	G	4	20	100		500	400	
19	C	4	2				400	200
20	G	4 1/2	2 1/2		100	200	400	
21	I	4	20	200		200	400	
22	C	3 1/2	1 1/2				400	200
23	C	4	19	400		200	400	
24	I	4	25	400		200	400	
25	I	4	34	200			400	

*G = Gambler
C = Conservative
I = Insensitive

Participants in the study:

	Number	%
G	9	36
C	12	48
I	4	16
	25	100

can be seen that 76% of the participants made some allocation to Program 1, 36% to Program 2, 56% to Program 3, 100% to Program 4, and 48% to Program 5.

Table VIII-4. Breakdown of decision-makers by allocation.

Program number	Capital outlay	Breakdown of decision-makers making allocation							
		# G	% G	# C	% C	# I	% I	Total	% T
1	$100	5	55	2	17	1	25	8	32
	200	2	22	1	8	2	50	5	20
	400	1	11	4	33	1	25	6	24
2	100	3	33	3	25	0	0	6	24
	300	3	33	0	0	0	0	3	12
3	200	2	22	2	17	2	50	6	24
	300	2	22	2	17	0	0	4	16
	500	1	11	2	17	1	25	4	16
4	100	0	0	1	8	0	0	1	4
	200	1	11	0	0	0	0	1	4
	400	8	89	11	92	4	100	23	92
5	200	4	44	8	67	0	0	12	48

In response to the question of which tool the subjects found most helpful in making their investment decision (the game board vs. the payoff distribution graphs), the tally ran about three to one in favor of the game board. Table VIII-5 shows the exact figures.

Table VIII-5. Decision-makers' preference for game board or payoff distribution graphs.

Most helpful	Number of subjects	%
Game board	19	76
Payoff distribution graphs	6	24
	25	100

Using the quantitative investment policy developed in Chapter VII, a "theoretical investment decision" was determined for each of the three types of decision-makers. Recall that parameter values were arbitrarily established for a hypothetical gambler and conservative. These theoretical allocations are summarized in Table VIII-6 below, along with the percentage of subjects actually making the theoretical allocation. For the gambler, the theoretically "ideal" investment "package" would be to invest \$400 in Program 1 over three periods, \$300 in Program 2 over three periods, and \$200 in Program 5 in one period, for a total investment of \$900. The ideal investment package for the conservative was determined to be \$400 for one period in Program 4. Using traditional methods, the resulting ideal package for the risk-insensitive decision-maker was \$400 in Program 1 over three periods plus \$400 in Program 4 for one period, for a total investment of \$800.

Table VIII-6. Actual vs. theoretical investment decisions.

Type of decision-maker	Theoretical allocation			Percentage of decision-makers actually making theoretical allocation
	Program	Total \$	\$ Each period	
Gambler	1	400	100-100-200	78% (\$100, 200 or 400)
	2	300	100-100-100	67% (\$100 or 300)
	5	200	200	44% (\$200 only)
Conservative	4	400	400	92% (\$400 only)
Insensitive	1	400	100-100-200	100% (\$100, 200 or 400)
	4	400	400	100% (\$400 only)

Study of Table VIII-6 indicates that, with one exception, the majority of a given type of decision-maker made some allocation to one or more of the programs in the theoretically ideal investment package. However, the important fact to recognize is that no single participant made an investment decision equivalent to his ideal allocation decision. This conclusion is derived from Table VIII-3.

Other information can be inferred from Tables VIII-3 and VIII-6. For instance, it is noted that identical investment decisions were made by pairs of subjects having differing self-images (see, for example, subject numbers 5 and 10, 23 and 24, 17 and 18). It can also be seen in this table that Program 4 was a unanimous investment choice. Apparently the restriction that payoffs for this program were deferred four years did not affect the attractiveness of the program. In addition, no significant relationship is apparent from the data between the decision-maker's allocation and his number of years of college or work experience.

Another interesting fact that can be drawn from the results of the experimental study is that 84% of the participants indicated they would take into account the variation in outcomes for the given investment alternatives. This is contrary to our notion that investment decisions might be based on a comparison of average or expected value for the outcomes. One should also not ignore the possibility that decisions might well be based mostly upon an intuitive or "gut" feeling for the worth of an investment.

In any event, the primary conclusion that can be derived from this small sample of decision-makers is that, although the investment decisions made by the subjects appear to be consistent with their self-image as a gambler, conservative, or insensitive, these decisions are nonetheless sub-optimal. The principle reason behind the decision-makers' less-than-ideal investment allocations apparently lies in their failure to take into consideration all of the relevant factors affecting the investment alternatives. Some of these relevant factors include the number of alternatives being evaluated and compared, the amount and timing of the capital outlays for each alternative, and the possible pay-offs and the probabilities associated with each payoff for the alternatives under consideration. The decision-maker simply cannot maintain all of the relevant factors in proper balance at a given point in time and for a given problem. It is a mental impossibility. The decision-maker is forced to make a number of conceptual simplifications to reduce the problem to more manageable proportions. And the result is a decision that is less than optimal.

While a detailed analysis of the role and importance of these relevant factors affecting an investment decision is beyond the scope of this paper, it would seem worthwhile to investigate further some of the questions raised by this study using data collected from a much larger sample of decision-makers. Suggested areas for further research regarding risk and uncertainty in capital rationing is the topic of the next and final chapter.

IX. CONCLUSION

Summing Up

Before we indicate some possible directions for further study, let us briefly review our efforts to this point. First of all, we examined numerous articles, papers, and textbooks dealing with various aspects of the capital budgeting field. In particular the highlights of a number of decision models and procedures for the evaluation and comparison of potential investment alternatives were presented. An attempt was made to organize and clarify the key points and the basic assumptions of the various methods and techniques.

It soon became apparent that the "models under assumed certainty" did not adequately represent real-world situations. Their so-called conceptual simplifications significantly affected their value as solution techniques. Several authors have made proposals for incorporating risk and uncertainty into the analysis. Included in these proposals is the use of computer simulation to randomly sample the elements which determine an investment's outcome.

What we offered in this thesis was the simplicity and the flexibility of a "fun" learning device -- a three-dimensional game board which uses marbles falling through obstacle courses to simulate the uncertainty surrounding an investment's payoff. The game board is presented as an addition to, and not as a replacement for, the other models for simulating risk. To demonstrate the worth of our three-dimensional model, a problem situation containing essential portions of realism was devised. Three key elements in the problem were: 1) a number of investment alternatives are being evaluated and compared, 2) limited capital resources are available for allocation, and 3) there is uncertainty surrounding investment payoffs.

The principle contribution of this thesis was a new approach to finding the "best" solution to problems of this type. Drawing on the fundamentals of cardinal utility theory, a quantitative investment policy was presented. The policy was composed of a criterion used to measure the worth of an investment and a series of decision rules to screen the various potential investments and ultimately lead to the desired solution. The criterion we used was the before-takes percentage rate of return on investment; the decision rules were based on an evaluation of the risk-return profile curve for each alternative under consideration. Values for probability coefficients and for the critical rate of return values were arbitrarily selected for a hypothetical gambler and conservative. The investment decisions based on their risk preferences were presented.

At this point we were curious to learn just how individuals actually make decisions, or what decision rules they follow, in problems of this type. So we put our problem situation and our game board to the test, and asked a number of people to play our game. We also wanted to know if decision-makers make decisions which are consistent with their self-image as a gambler, a conservative, or an insensitive. The results of our small-scale investigation indicated that, although the decisions made by the participants appeared to be consistent with their self-image, these decisions were nonetheless less than optimal. This sub-optimality apparently was caused by the failure of the decision-maker to simultaneously take into consideration all of the relevant factors affecting the investment alternatives.

Suggestions for Further Research

In studying the details of the investment policy developed to quantitatively solve our capital rationing problem, particularly the plot of the risk-return curves for each proposed investment and the establishment of a pair of upper and lower points on the aspiration line for each type of decision-maker, we were struck by the similarity of this concept to that of the "operating characteristic" curves given in quality control manuals. (See, for example, Acheson J. Duncan. Quality control and industrial statistics. 3d ed. Homewood, Ill., Irwin, 1965. p. 147-160.) We feel it would be extremely worthwhile to investigate the degree of this similarity. Such a study should focus on the correspondence between the parameters $Prob_1$, $Prob_2$, RR_1, and RR_2 given in our investment policy and the parameters α (producer's risk), β (consumer's risk), AQL (acceptable quality level), and p_t' (the lot tolerance fraction defective) associated with the operating characteristic curves. It would also be of interest to determine the possibility of developing tables of values similar to those given in quality control for the design of acceptance sampling plans for the purpose of constructing a risk-return profile curve in a given capital budgeting problem.

We have already indicated in several locations throughout this paper our dependence on the work of utility theory in establishing the classifications of decision-makers and the notion of an "aspiration line" separating the risk-return grid into regions of acceptance and rejection. Additional research in this area may reveal just how strong this relationship between capital rationing and utility theory really is. To illustrate, this research might explore the idea of applying "aspiration level curves" from utility theory to capital rationing problems. Other aspects of utility theory may also provide further insight and application into our own problem.

This paper has dealt primarily with the efficient allocation of <u>capital</u> resources. But what about other prime resources, such as manpower, time, and equipment? More work needs to be done to develop practical decision models and analysis techniques for problem situations involving these scarce resources.

Although a great deal of material has been written in regards to the manifold aspects of the capital budgeting problem, as we have seen, it appears that managers and other decision-makers from business and industry have been slow in their acceptance of many of the models and techniques advocated. Thus, we can only look to the future and hope that a more realistic and workable means for handling the problem can be formulated. The groundwork has already been laid. The potential worth is tremendous. The challenge is now.

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APPENDIX

Program: 1 2 3 4

No. of Marbles: 1 2 3 4 5

Results:

Area	1	2	3	4	5	
Probability	.11	.57	.06	.18	.08	Σ = 1.00

Trial No. \ Area	1	2	3	4	5
1		//		//	/
2	/	///	/		
3	/	////			
4	//	//		/	
5		////		/	
6		///			//
7		///	/		/
8	/	///		/	
9		////		/	
10		////		/	
11	/	//	/	/	
12	/	//		/	/
13		///		/	/
14	/	///		/	
15	/	//		/	/
16		///	/	/	
17	/	//	/	/	
18		//		/	//
19		//	/	//	
20	/	//	/	/	
21		///		/	/
22		///		/	/
23	/	///		/	
24		////			/
25		//	/	/	/
totals	12	70	8	22	13

Trial No. \ Area	1	2	3	4	5
26		//	/	//	
27	//	//		/	
28	/	//	/	/	
29		/////			
30		//	/	/	/
31		////		/	
32		////		/	
33		/////			
34	/	/	//		/
35	//	//		/	
36	/	//		//	
37		////		/	
38		////	/		
39	/	///		/	
40		///	/	/	
41		////			/
42		////		/	
43	/	///		/	
44	/	///		/	
45	//	//		/	
46	/	//		/	/
47	/	///		/	
48		//		/	//
49	/	///		/	
50	/	//	/	/	
	16	73	8	22	6
	20 + 4×2 = 28	1 + 20×2 + 16×3 + 11×4 + 2×5 = 143	14 + 1×2 = 16	36 + 4×2 = 44	13 + 3×2 = 19

NAME: ______________________________

CURRENT TITLE or OCCUPATION: ______________________

NUMBER OF YEARS OF COLLEGE: _____
OF WORK EXPERIENCE: _____

SELF-IMAGE: (check one)
- ☐ Risk-seeker (gambler)
- ☐ Risk-avoider (conservative)
- ☐ Risk-insensitive (assumes certainty)

DECISION: (circle program number and indicate amount of allocation)

Program 1 $_____
2 $_____
3 $_____
4 $_____
5 $_____

QUESTIONS: Would you make any changes in your initial decision after careful study of the payoff distribution graphs? (check)
- ☐ Yes
- ☐ No

If yes, indicate these changes below:

____

____

In retrospection, which was more helpful to you in making your decision? (check one)

- ☐ Game board
- ☐ Payoff distribution graphs

COMMENTS: ______________________________________

____

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