FORECASTING DEMAND IN A SEASONAL INDUSTRY

by

LOUIS HOWARD HILDEBRANDT

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APPROVED:

[redacted]

Professor of Industrial Engineering

in Charge of Major

[redacted]

Head of Department of Industrial Engineering

[redacted]

Chairman of School Graduate Committee

[redacted]

Dean of Graduate School

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INTRODUCTION

> "What is chance, but the rude stone
> which receives its life from the
> sculptor's hand? Providence gives
> us chance --and man must mould it
> to his own design."

Schiller's lines are as appropriate today as in 1767 when Don Carlos was released. The element of chance enters every facet of man's endeavors. How he uses his resources to "mould it to his own design" influences his whole world. Today, more than ever before, man has at his command techniques for calculating and minimizing risk --the risk associated with the uncertainty of the future. Wherever chance exists, he has applied statistical and probability theory to improve his decisions. Not only do better decisions under uncertainty contribute directly to a firm's profit, but also to the efficient use of our nation's resources.

Producing for an unknown demand is a problem facing almost every industrial endeavor. For, unless the firm is a monopoly in an unsaturated market, the demand for the product is subject to considerable uncertainty. The scheduling of production is characterized by a formidable array of interrelated variables, but the very heart of the whole problem is the demand for the finished product. Someone must forecast the sales for the future --the anticipated demand. Only when the future requirements are known can optimum decisions be made in ordering raw materials, scheduling machines, overtime, lay offs, hiring, finished goods inventory, and order backlog. When these elements can be decided with a degree of certainty, their costs can be minimized by appropriate planning.

The elements affecting demand are not fully understood. The individual effect of such factors as competition, advertising, service and substitutes illustrate this point. Yet, it is not necessary to forecast the individual factors. It is their cumulative effect that is of concern. This is analogous to the behavior of an individual neutron in an atomic pile. Its movement is random and cannot be predicted. But, because of the very fact that there is randomness, the aggregate of neutrons has very stable characteristics. The factors that have affected demand in the past are not new to the future. Also, in any given system, factors can appear which haven't occurred in the past. The terms "forecast" and "predict" describe the two types of prognostication. The forecast is the projection of previous experience into the future. The prediction is a skilled "hunch" as to the future, the predictor utilizing as many factors as he can obtain and assimilate. Only the mathematical forecast is considered herein.

The cooperation of Sawyer's, Inc, Progress, Oregon in furnishing sales and inventory data on a seasonal product makes this paper possible. The sincere efforts of Mr. Ray Vanderzanden of that company have been especially helpful. Sawyer's activities are in the photographic field, particularly stereo cameras, viewers, and stereo pictures. Their peak season is the fall months -- the Christmas business.

SUMMARY OF PRESENTATION

The presentation herein is made in three major categories, developed in the following order: (1) The nature of forecasting and its importance in industry; (2) Theory of forecasting systems; (3) Application of two forecasting systems to Tru-Vue Stereo Film Cards, a product of Sawyer's, Incorporated.

The forecasting techniques applied are of two types. The first is the exponential smoothing factor method as developed by Brown (3, p. 45). The variations in this method are (1) random pattern, and (2) cyclic pattern. The second major technique is called simplified seasonal pattern as suggested by the title of the article in which the method was originally described (10, p. 62). The exponential smoothing technique entails a monthly calculation with the changes in the most recent demand history as the basis for the forecast. This method involves calculations for determining the trend, change in trend and correction for lag. The simplified seasonal pattern utilizes data from the preceding two years. Its use is suggested in preparation of annual or semi-annual forecasts, although nothing prevents its monthly use. The exponential smoothing technique is sufficiently involved to justify the use of a computer. The use of the Oregon State College Mathematics Department digital computer (ALWAC III-E) in the computations is described in the third section of this paper. The simplified seasonal pattern method is quite easily handled with a desk calculator.

THE NATURE OF FORECASTING AND ITS IMPORTANCE IN INDUSTRY

Definition and Aims of the Forecast

The forecast of demand is a projection of requirements on a given system into the future. In the typical inventory situation, the forecast is the anticipated sales of units from stock during a given future period. The term demand makes the statement more general. Whereas a sale involves a buyer and a seller, demand implies a drain or requirement on the system. This can be from raw material inventory, from parts stocks, or the most common consideration --the customer demand on the finished product.

The word forecast is used to imply a beforehand calculation of the future as opposed to a prediction. The difference is described in the text on the subject by Robert G. Brown (3, p. 3). In describing the term forecast, he writes, "literally, the word means to 'throw ahead' to continue what has been happening. 'Prediction' or 'saying beforehand' will be reserved for management's anticipation of changes and of new factors affecting demand". Thus, the forecast is a systematic method for estimating future demand. The fact that it is systematic permits management to delegate the work of forecasting.

The demand for a given article is affected by many factors, more notably, competition, advertising, weather, service, and substitutes. Most of the factors affecting demand are not new to the future. The composite affect of these elements is remarkably stable. The influence of a given element tending to change demand is offset by changes of opposite nature in other elements. However, occasional new factors enter the picture. A promotional scheme might send sales up, or a competitor's action might temporarily change the sales pattern. These are changes that management must predict. But, often, these are predictable, and the forecast can provide the steady base for planning.

The accuracy of the forecast has a direct relationship to the cost of an inventory system. The optimum situation would be to have an exact knowledge of demand at any future time. Since this is usually impossible, allowances must be made for unknowns. A certain amount of inventory called safety stock is carried to allow for variations in demand. The alternative is to permit orders to backlog, with the consequence that the customer's need is not immediately satisfied. Either way (safety stock or backlog) is more expensive than inventory costs associated with the theoretical situation wherein future demand is known exactly. Safety stock requires capital for materials, labor, space, insurance, taxes, obsolescence, etc. An unsatisfied customer can, on the other hand, cause immediate loss of sales as well as a long term or permanent loss.

There is also the intangible of good will which may be affected. The better the forecast, the more favorable the costs associated with all of these elements.

In the usual industrial endeavor, the forecast is the base for a host of planning activities. Schedules of men, machines and materials stem from the forecast. Considerable literature has been devoted to the subject of scheduling. Mathematical programing has become a powerful tool in this field, and it is still developing. However, the forecast is the base for the entire process. Here, again, accuracy and costs are related. A good forecast will permit a good schedule of manpower, machines and material. The costs of hiring, training and layoffs are minimized. Machines may be utilized most efficiently, thus realizing the most on the capital invested. Materials can be procured in the most economical manner.

An integral element of forecasting and inventory control is lead time. The lead time is the time required to bring an item into inventory. It starts when the decision is reached to order and terminates when the order is in stock. Lead times are usually subject to variation. This variation stems from many variable delays from the time that preparation of the order starts until the unit is transported into the inventorying warehouse. To be of value, the forecast must reveal demand far enough into the future to allow for a reasonable variation in lead time. Past experience best indicates the base for such allowances.

Once the lead time is established, a decision must be reached as to the time period beyond the lead time that the forecast should encompass. If an item has a two month lead time, a forecast on June 1st would have to be for three months, if one month's supply (August) were to be ordered. However, the situation is often encountered where one month's supply is not the most economical quantity to fabricate or purchase. Management is interested in a forecast that will include sufficient periods ahead to permit economical order sizes. It might be advantageous to know the forecast for multiples of lead time --two, four and six months. This asks more of the forecasting system.

Techniques of Estimating Future Demand

Every industrial endeavor utilizes some method of estimating future demand. The correlation analysis made by large corporations are very complex. Many smaller concerns do not have any systematic effort in forecasting. But, whenever a decision has been made, some method has been utilized, whether it is consciously recognized or not. MacNiece (8, p. 113) describes five techniques of estimating future demand. These are: (1) Opinion; (2) Estimates by salesmen; (3) Statistical sampling; (4) Historical background and statistical projection; and (5) Regression or correlation analysis. This paper is concerned only with the fourth method --the statistical projection of past demand. However, a good system of forecasting will utilize more than one technique. Thus, an opinion of future sales might be quite erroneous if not based on the facts. But a seasoned management working with data obtained by statistical sampling or projection may be able to make a realistic prediction of overall future sales. However, the average executive is usually not familiar enough with details to break this projection down into sales of individual units or even product lines (8, p. 113). Such a breakdown is the main aim of the statistical projection of demand developed in this paper. The systematic estimation of future demand for each stock unit permits more accurate forecasts of the entire line.

THEORY OF FORECASTING SYSTEMS

Forecasting future demand by statistically projecting past demand has not received a great deal of attention. The text by Brown (3) is a recent effort devoted entirely to this subject. Many authors have recognized the prevalent use of simplifying assumptions in this area. In the text devoted to the mathematical studies of inventory and production, Scarf comments, "Most authors who have written on the subject of inventory control have made the assumption either that future demand is known precisely, or that it may be described by a definite probability distribution" (1, p. 201). The few forecasting systems that eliminate these assumptions are developed from the theoretical viewpoint in this section.

<u>The Forecast is Based on Past Demand</u>

The simplest forecast is obtained by taking demand for a past period as the best estimate for a future period. Thus, next month's sales will be the same as last month's, next year's, the same as this year's, etc. While such a rudimentary forecast may be of value in a few situations, most demand patterns contain random fluctuations which would make such a forecast inaccurate. These fluctuations are analagous to noise in an electrical circuit, and this term will be used with this implication. To improve the forecast, the noise, or fluctuations, could be averaged out by considering demand over several periods in the past. Next month's sales might be estimated by taking one-twelfth of last year's sales. Or the average of the sales in the last quarter could be used for next month's sales. Proper selection of past demand periods will dampen the noise element.

Often the periodic demand fluctuations will camouflage a trend. A gentle rising or falling sales pattern may go undetected for some time because of the noise element. At the same time, the process of averaging out the noise for the forecast will conceal a trend. The more past demand periods that are averaged to smooth out the noise element, the less will be the influence of the most recent periods wherein the trend is occurring. The forecasting system should detect and project a trend. One method would be to plot the period by period demand, fit a straight line to these points and continue the line on the graph into the future. Fitting of straight lines can be accomplished by the method of least-squares. The theory of this method is developed in the text by Bowker and Lieberman (2, p. 245).

While the method of least-squares provides a technique for projecting past performance into the future, further improvements are available. In his text on the subject, Brown developes a system which he denotes exponential smoothing (3, p. 13). This system permits the analyzer to vary the weight given to the most recent period as compared to previous periods when calculating the forecast. This, in effect, permits systematic variation of the sensitivity of the forecast to the immediate past. An analogy can be made by comparing this sensitivity (or weighting) factor to the volume control on a radio. A larger factor makes the system more sensitive to the most recent period. Compare this to turning up the volume on a radio. The system also becomes more sensitive to random fluctuations --noise. As the sensitivity is increased, the emphasis is placed on only the most recent demand. Conversely, a low weighting factor puts more emphasis on older data, and is less likely to give a noise distorted forecast.

Exponential smoothing has two principal advantages over the ordinary least-squares method. First, the amount of data keeping is reduced. When using least-squares, the demand for each past period in the base series must be available. Exponential smoothing requires only the most recent month's demand figure. Second, ability to easily vary the sensitivity of the system, has advantages not inherent in other forecasting systems. If management anticipates a change in demand, the sensitivity factor (which Brown designates alpha) can be temporarily increased. Note that management does not have to say in which direction the change will occur. The higher sensitivity or alpha factor will weight the most recent demand data more heavily, thus weighting the forecast accordingly. A change to a larger alpha value can also be made when it appears that the system is not keeping up with a swing in the demand pattern. The change need only be temporary. Once the sustem is in control, a lower sensitivity can be resumed, thus decreasing the affect of noise.

Formula For Exponential Smoothing

Following the method developed by Brown (3, p. 47), the basic formula for exponential smoothing is obtained as follows. Assigning symbols to the known quantities, let D_0 represent the most recent actual demand; D_1 the demand one period (one month) back; D_2 the demand two periods back; and so on up to D_k, being the demand in the k-th period back. A new estimate is prepared using the D_i and the old estimate. The old estimate was the new estimate one period back. With the "smoothing constant" designated by the Greek letter alpha, the new estimate is expressed as follows:

$$\text{new estimate} = \text{old estimate} + \alpha\,(D_0 - \text{old estimate})$$

If the "period" being considered is one month, then the above equation is interpreted as follows:

1. The new estimate is the next month's estimate (the unknown in the equation).
2. The old estimate is the estimate prepared one month ago for the month that has just passed.
3. D_o is the actual demand for the month that just passed.
4. Alpha is assigned a value between zero and one depending on the weighting to be given D_o.

The equation can be rearranged to read:

$$\text{new estimate} = \alpha(D_o) + (1 - \alpha)(\text{old estimate})$$

Note that an alpha value of one considers only the new demand (D_o), and an alpha value of zero places all of the weight on the past. Being a dynamic situation the old estimate was prepared one period back from the same equation; but with the actual demand D_1 (using the notation that was started above).

$$\text{estimate one period back} = \alpha(D_1) + (1 - \alpha)(\text{estimate prepared two periods back})$$

Substituting the right hand side of this equation for the old estimate in the equation that has been developed, causes it to read as follows.

$$\text{new estimate} = \alpha(D_o) + (1 - \alpha)\left[\alpha(D_1) + (1 - \alpha)(\text{estimate prepared two periods back})\right]$$

Following this procedure back for k periods, it can be seen that D_1 is multiplied by $(1 - \alpha)$

for the first time, D_2 for the second time, etc. Using the demand terms, D_1, D_2, ----, D_k, the expression becomes:

$$\text{new estimate} = \alpha\Big[D_o + (1-\alpha)\,D_1 + (1-\alpha)^2\,D_2 + (1-\alpha)^3\,D_3 + \text{----} + (1-\alpha)^k\,D_k + (1-\alpha)^k\,(\text{estimate made } k \text{ months ago})\Big]$$

Note that the weight given to older data declines exponentially. The number of periods, (k), does not have to be very large to make the affect of the estimate k periods ago negligible. Also, it can be shown that the sum of the weighting factors adds up to one.

$$\alpha + \alpha(1-\alpha) + \alpha(1-\alpha)^2 + \text{----} + \alpha(1-\alpha)^k + \text{----} = \alpha\,\frac{1}{1-(1-\alpha)} = 1$$

Therefore, the expression is an average. The basic rule for exponential smoothing is then:

$$\text{new average} = \alpha\,(\text{new demand}) + (1-\alpha)(\text{old average})$$

Two more elements are introduced into the computation: (1) A correction for trend; and (2) A correction for lag. The latest figure for the average trend is designated "new trend" and it is computed in two steps:

(1) Current trend = new average - old average

(2) New trend = α (current trend)+(1 -α)(old trend)

Brown states that "this method of computing the trend is in fact the least-squares estimate of the trend, if the weights given to the demand in each previous month are the same as those used in computing the average" (3, p. 50). The expected demand, with a correction for lag[1], is:

$$\text{expected demand} = \text{new average} + \frac{1 - \alpha}{\alpha}\,(\text{new trend})$$

This, then, is the expected demand for next month. To obtain the demand over the lead time, it is necessary to multiply by the number of months (or periods) in the lead time. It is also possible to extrapolate the trend. Thus, demand in the second month will equal the present expected demand plus twice the trend. Demand in the third month will equal the present expected demand plus three times the trend, etc. Judgement must be exercised and usually there should be a definite reason for a trend before extrapolating.

Seasonal Demand

To this point the problem of seasonal or cyclic demand patterns has not entered the discussion, except in the introductory remarks. Once the fact has been established that demand for a product has a seasonal pattern, this knowledge must be used. To use exponential smoothing for

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1. The mathematical development of this factor is treated by Howard (7, p. 139-142).

forecasting under cyclic conditions a new element must be introduced. This is the actual pattern of demand through the year. The pattern can be expressed in the form of a base series. This base is usually the monthly average of demand for several years in the past. A ratio of the latest current demand to the equivalent month in the base series is the new element to be considered. This ratio is subject to changes and trend, and can be handled by exponential smoothing in the same manner as the average demand in the previous discussion. Following previous development, the first element calculated is the new average ratio.

$$\text{the average ratio} = \alpha(\text{new ratio}) + (1-\alpha)(\text{old average})$$

If R_0 is the new ratio, R_1 the ratio one period back, R_2 the ratio two periods back, etc., then, the expression for the average ratio becomes:

$$\text{the average ratio} = \alpha\left[R_0 + (1-\alpha) R_1 + (1-\alpha)^2 R_2 + (1-\alpha)^3 R_3 + \text{----} + (1-\alpha)^k R_k + (1-\alpha)^k (\text{estimate made k months ago})\right].$$

The remarks made in the previous section concerning the development of the average demand are still applicable. The weighting factor given to the older data declines exponentially. The sum of the weighting factors adds up to one.

Therefore, the expression is an average (ratio).

The change in the average is the new average minus the old average. The new trend is found by the expression:

$$\text{new trend} = \alpha\,(\text{change in average ratio}) + (1-\alpha)(\text{old trend})$$

Since the element being forecast is a ratio, the equation which carried the expression for correction of lag will be:

$$\text{expected ratio} = \text{new average ratio} + \frac{(1-\alpha)}{\alpha}\,(\text{New trend})$$

The expected demand for any particular month in the future will be the expected ratio times the particular value of the base series for that month.

The base series is the period by period expected demand pattern. As mentioned in the preceding paragraph, it can be the average of a particular month's sales for several years. Or, it can simply be the sales for the corresponding period one year ago. When the demand period is one month, it often happens that the seasonal pattern will shift slightly from year to year. For instance, the sale of seeds to home gardeners in the spring can be early if good weather occurs and late if there is a late spring. In this situation, a moving average is usually the best. The moving average is a simple average of past demand in the periods before, after and including the base period. Usually, three months are used. Thus, a moving average value of demand for July would be the average of June, July and August demand. Similarly, the moving average for August would be the average of July, August and September demands.

Choosing the base series becomes a matter of trial and error. The alternative bases can be compared by observing their respective ratios to the actual demand. Thus, a month by month ratio for each base would be obtained from the following:

$$\text{demand ratio} = \frac{\text{demand in the current month}}{\text{value of the base series for the current month}}$$

Plotting successive monthly values of the demand ratio for each proposed base series depicts the relative steadiness of their patterns. The base which yields the most stable pattern would be the best to use.

Detecting Changes

The errors in the forecast can be used to indicate changes in trend. Brown shows how the forecast error can be used to obtain a control chart with upper and lower limits (3, p. 102). The forecast error used in this analysis is the forecast for one month (one month lead time). On this basis, the forecast error for a given month is called the current deviation and is obtained by the following (3, p. 93):

$$\text{current deviation} = \begin{array}{l}\text{(Expected value of demand calculated}\\ \text{last month) minus (actual demand for}\\ \text{the current month)}\end{array}$$

The next step is to find the mean absolute deviation (3, p. 93):

$$\text{new mean absolute deviation} = \alpha\,\left|\text{(current deviation)}\right| + (1-\alpha)\,\text{(old mean absolute deviation)}$$

Note that the absolute value of the current deviation is used (sign is disregarded). Rather than estimate a standard deviation for the mean absolute deviation and plot it in traditional control chart form, a simpler control system as developed by Brown has been used. A running sum of current deviation is maintained (with regard to sign). Each month this value of the cumulative deviation is divided by the new mean absolute deviation. If the value lies between plus and minus four, the system is in control. These values are approximately equivalent to the conventional three sigma limits (3, p. 102). When the quotient exceeds these values, it is an indication that the forecast is lagging behind a change in demand. The correction would be to temporarily raise the smoothing factor (alpha) to make the system more sensitive to change. This larger alpha value can be used for several months, after which time the lower value can be resumed. The cumulative sum of the current deviation is set back to zero when the alpha value is increased.

Simplified Seasonal Pattern Forecast

The forecasting ~~technique described by Semon (10, p.~~ 86 ~~62), which he calls "A Simple Seasonal Pattern", is of interest in view of its simplicity.~~ A month by month forecast for the ensuing twelve months is obtained on the basis of three factors: (1) Annual demand for the past two years; (2) Historic (or expected) monthly change in demand for each calendar month; (3) An estimate of the total demand for the next twelve months. The second factor (monthly demand changes) can be obtained from historic demand for the specific unit, or the pattern for an entire line can be used. The expected demand for next year can simply be the demand last year, or, if a trend is indicated, this may be placed in the estimate. 12

Knowledge of the month by month expected changes in demand could yield a forecast in the following manner. If the last month for which actual demand is known is December, then the January forecast is obtained by adding the change in expected demand between December and January to the actual December demand. The February forecast could be obtained in the same manner from the January forecast, and so on through the year. The principal discrepancy in such a system is that the whole forecast is based on one month's demand, (in this case December). If the actual December demand had deviated from the average pattern, the entire forecast would be influenced accordingly.

Instead of using actual demand for the most recent month of known demand, this technique of forecasting developes a "normalized" demand (10, p. 62). The normalized demand is obtained in two steps, namely:

(1) The most recent 12 month's demand and the month by month expected changes in demand are utilized to obtain an expected demand for the base month one year previous.

(2) The average monthly change in trend in the past two years is added to the expected demand from above to obtain the "normalized" demand. 7

To develop these statements further, let C_1 represent the expected monthly change in demand for the first month, C_2 the change between the first and second month, and so on to C_{12} to give the entire year's changes on a month by month basis. If D_o is the demand for the base month then the expected demand for a whole year is:

$$(C_1 + D_o)+(C_2 + C_1 + D_o) + ---- + (C_{12} + C_{11} + ---- + C_2 + C_1 + D_o)$$

Which simplifies to:

$$12\,D_o + 12\,C_1 + 11\,C_2 + ---- + 2C_{11} + C_{12}$$

Let R represent the sum of the products obtained from the monthly changes.

$$R = 12\ C_1 + 11\ C_2 + \text{----} + 2\ C_{11} + C_{12}$$

If A represents the actual demand for the past 12 months, then the expected demand for the base month, 12 months previous, is:

$$\text{Expected } D_o = \frac{A - R}{12}.$$

If Z represents the actual demand for the year immediately before A, then the average monthly change becomes

$$\frac{A - Z}{12}.$$

Letting M represent the normalized demand for the last month in year A, the above expressions combine to give

$$M = \frac{A - R}{12} + \frac{A - Z}{12}.$$

This simplifies to

$$M = \frac{2A - R - Z}{12}.$$

The implication at this point is that to find the January forecast, the seasonal change is added to the value of M calculated for December. The February forecast is obtained by adding the February seasonal change to the

January forecast, etc. However, if a forecast has been made for the entire year as indicated in the first paragraph of this discussion, a constant factor must be added each month to obtain the annual estimate. Since the constant added in January will be amplified 12 times, and in February 11 times, etc., the constant appears 78 times (12 + 11 + 10 + ---- + 1) in the year's forecast, and k is obtained from

$$k = \frac{F - 2A + Z}{78}$$

where F is the estimated annual sales for the ensuing year.

Starting with M, the forecast for January is

M + (January seasonal change) + (k)

The forecast for each succeeding month is

(Forecast last month) + (this month's seasonal change)+(k). 12

APPLICATION OF THE FORECASTING SYSTEMS TO TRU-VUE FILM CARDS

This section of the report serves to demonstrate the application of the techniques described in the previous section as well as to compare their relative accuracies. Actual monthly demand data for a product of the Sawyer's Corporation was obtained. The data covers the period from January, 1954 through August, 1959. In order to maintain the confidential nature of the information, all data in this paper contains a constant error.

Description of Sawyer's

The Sawyer's Corporation is a young company whose principal endeavor is concerned with stereo photographs and equipment. Items which are manufactured at the Progress, Oregon plant include: Viewmaster Stereo Reels, Tru-Vue Film Cards, stereo cameras, stereo projectors and a line of stereo and monodimensional viewers. Facilities include photographic equipment for exposing and processing color film, equipment for producing the stereo reels and cards; plastic injection molding machines for producing camera projectors and viewer bodies; lens grinding equipment, and miscellaneous metal working equipment. The plant is serviced by the Oregon Electric Railroad as well as all the Portland truck lines.

Figure 1 shows the Tru-Vue Film Card. It consists of seven sets of stereo photographs mounted in a special card to fit the Tru-Vue viewer. This paper considers only the Tru-Vue Film Cards, however, the entire line produced by Sawyer's follows a similar demand pattern. The forecasting techniques developed herein for the Tru-Vue Cards are applicable to any other item in the line.

Demand History of Tru-Vue Film Cards

The sales of Tru-Vue Cards follow a seasonal pattern with a low in June and July, and a high in October and November. Figures 2 and 3 show the average demand for ten cards selected at random from the cards being sold between 1954 and 1957.[1] Figure 2 depicts the monthly demands for the years 1954 and 1955, and figure 3 for the demands of 1956 and 1957. Note that the 1957 demand, although lower than the three preceding years, still follows the general pattern.

In searching for a base series to represent the demand pattern, several approaches can be made. Since the demand pattern for the ten random selections shows consistency, the average for the four years is of interest. This is depicted in figure 4. The demand for the third year (1956)

1. The cards selected were catalog numbers DA-1, FA-1, TA-1, C-1, C-3, D-1, D-3, F-2, F-3, T-2.

Figure 1. True-Vue Film Card

is exactly average, when the demand for the entire four year period is considered. The demand for 1954 is 114% of the average of the four years. The demand for 1955 is 119% of the average, and for 1957, 65.4% of the average. The relative levels of the various years are not the main concern. Rather, it is the consistency of the pattern obtained, and its ability to represent future patterns that is of interest. With this objective, the average values were chosen as the most representative since they tend to "smooth out" the random fluctuations.

An alternative base series for the same set of data is the month by month average about the quarter. This series is also plotted in figure 4. The broadening or smoothing of the fluctuations in the demand pattern by use of the average about the quarter is readily apparent. This brought the question as to which would yield the best base series --straight average or average about the quarter.

The Sawyer's people expressed concern that the year 1954 might not be representative, since this was the first year for Tru-Vue Cards. With this thought in mind, four possible base series were examined, namely: (1) The four year (1954-1957) straight average, (2) The four year (1954-1957) average about the quarter, (3) The three year (1955-1957) straight average, and (4) The three year average about the quarter. The procedure for comparing the

Figure 2. Tru-Vue Demand
Monthly Averages - Ten Selections
1954 - 1955

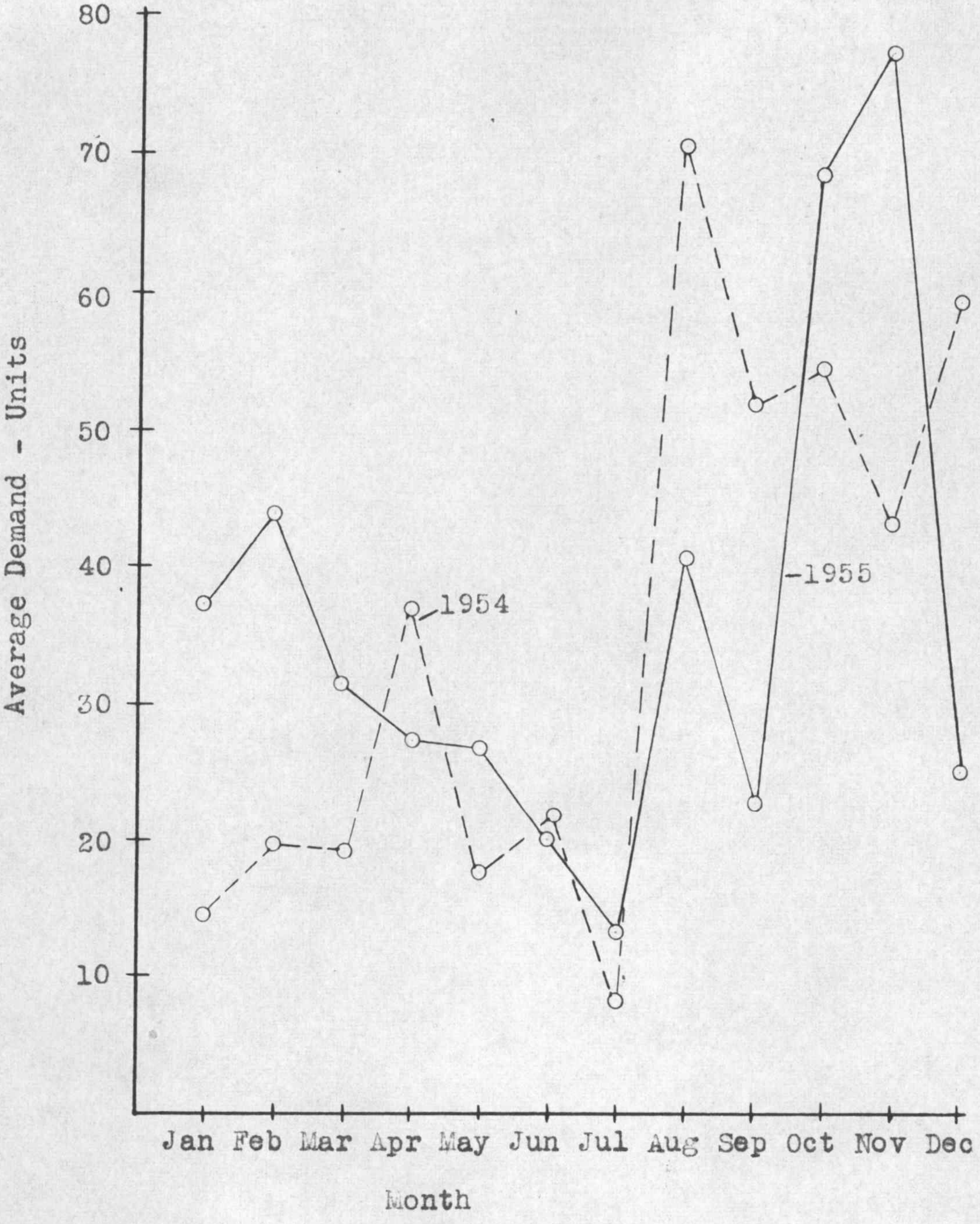


Figure 3. Tru-Vue Demand

Monthly Averages - Ten Selections
1956 - 1957

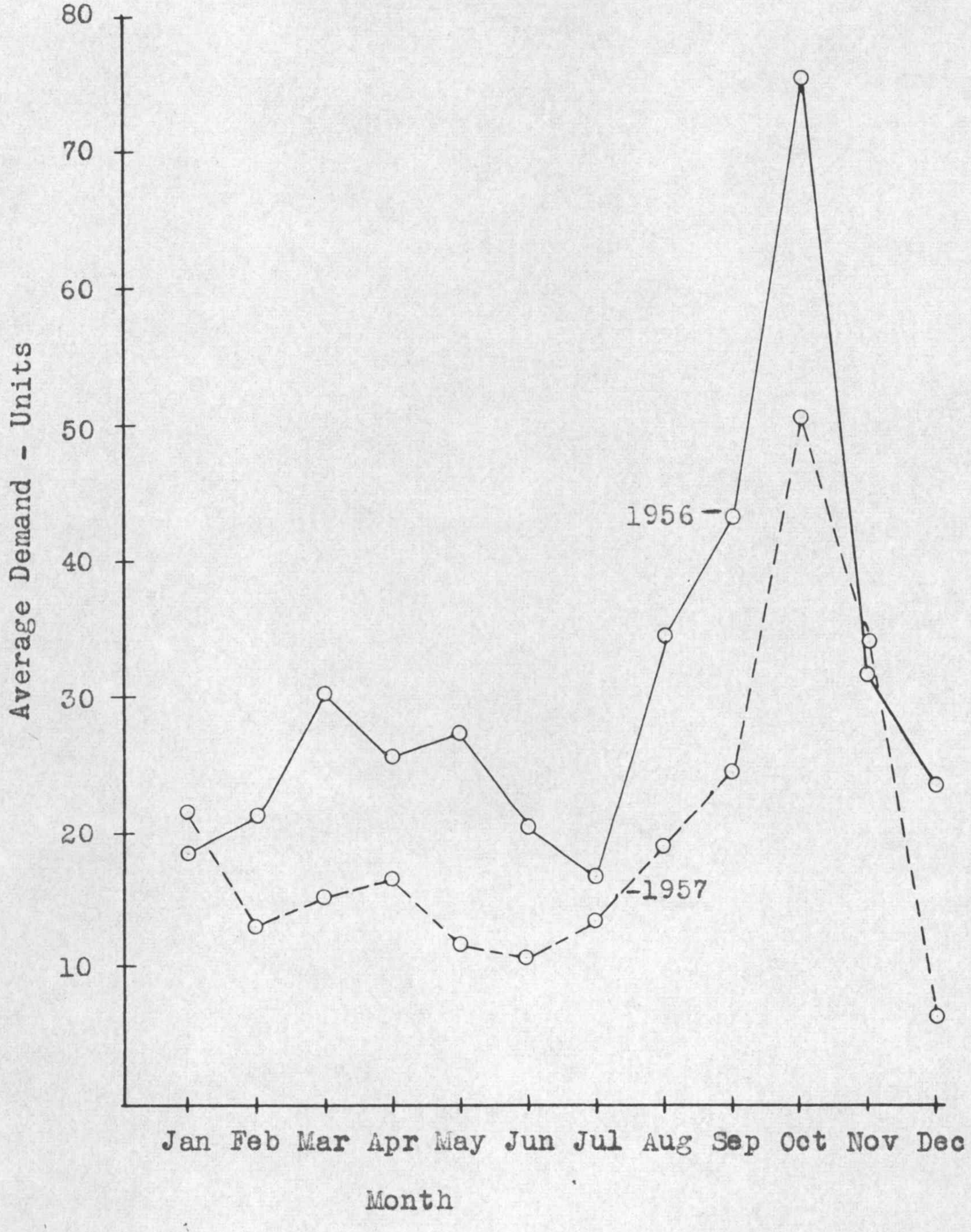
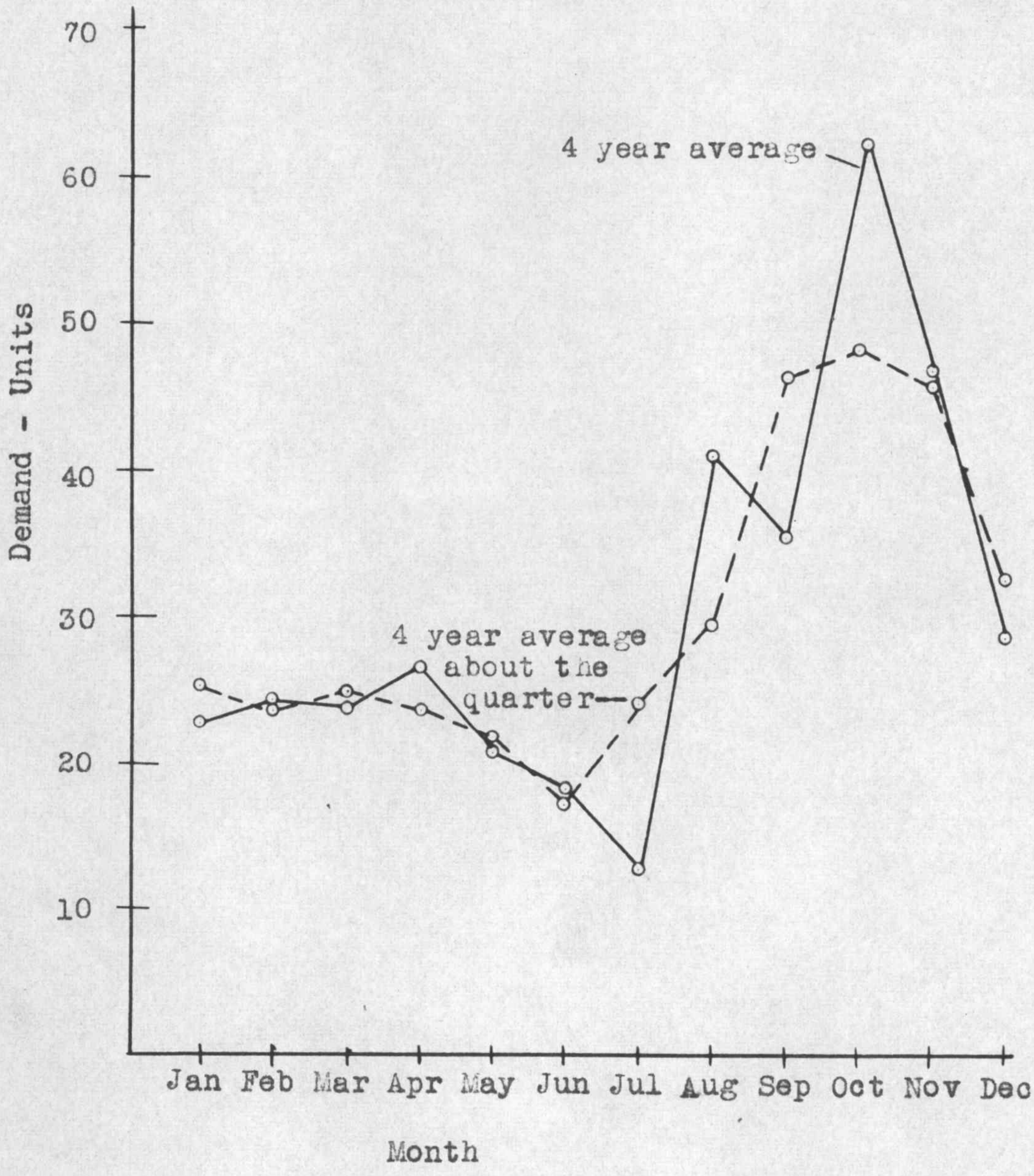


Figure 4. Tru-Vue Demand
Monthly Averages - Ten Selections
Four Year Average (1954 - 1957)
And Average About The Quarter



four proposed base series is developed in the next paragraph.

As a test of the possible base series, the ratio of actual demand of a specific Tru-Vue Card to the proposed base was calculated. Two cards were chosen, which were not in the ten selections composing the base series.[1] For each month a ratio was obtained as follows:

$$\text{Demand ratio} = \frac{\text{demand for given month}}{\text{value of proposed base series for given month}}$$

The most desirable base series would yield successive values of the demand ratio close to unity. Plotting the data in graph form wherein successive ratios are connected by straight lines permits interpretation. Figures 5 through 8 depict the demand ratios for the four proposed base series. The ratios for Tru-Vue Card F-4 are somewhat smoother than for card D-5. Differences between the various bases are almost negligible. The base obtained from the weighted average of the four years stays a little closer to unity, although this is hardly discernible. On the basis of these graphs (figures 5 through 8), the four year average about the quarter was chosen as the most representative demand pattern. These values are listed in table 1. They are, in essence, the expected demands for a Tru-Vue Film Card through the year. They are the base for

1. Cards chosen were catalog numbers D-5 and F-4.

Figure 5. Tru-Vue F-4

Ratio of Actual Demand to Three Year Base Series

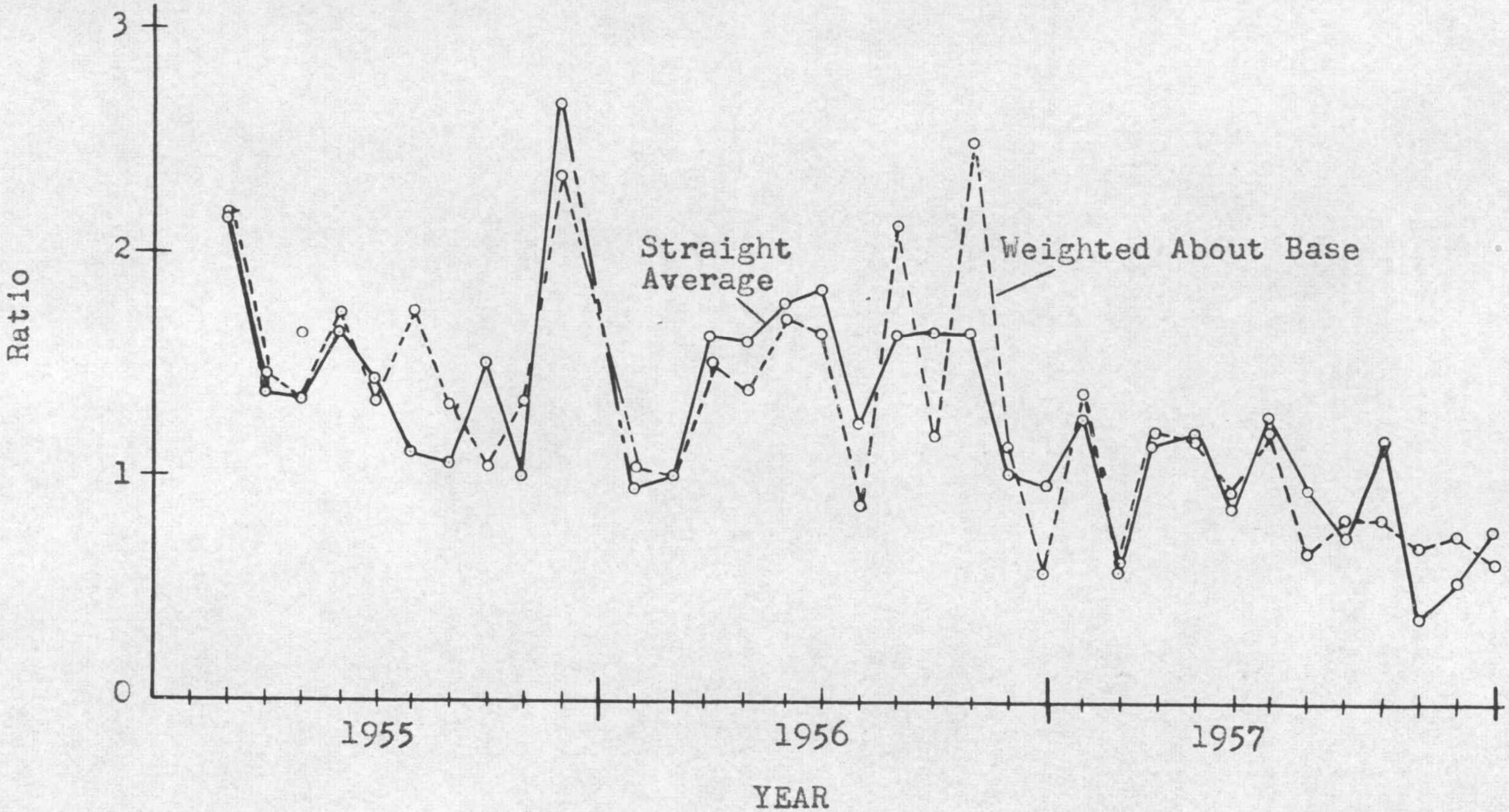


Figure 6. Tru-Vue F-4

Ratio of Actual Demand to Four Year Base Series

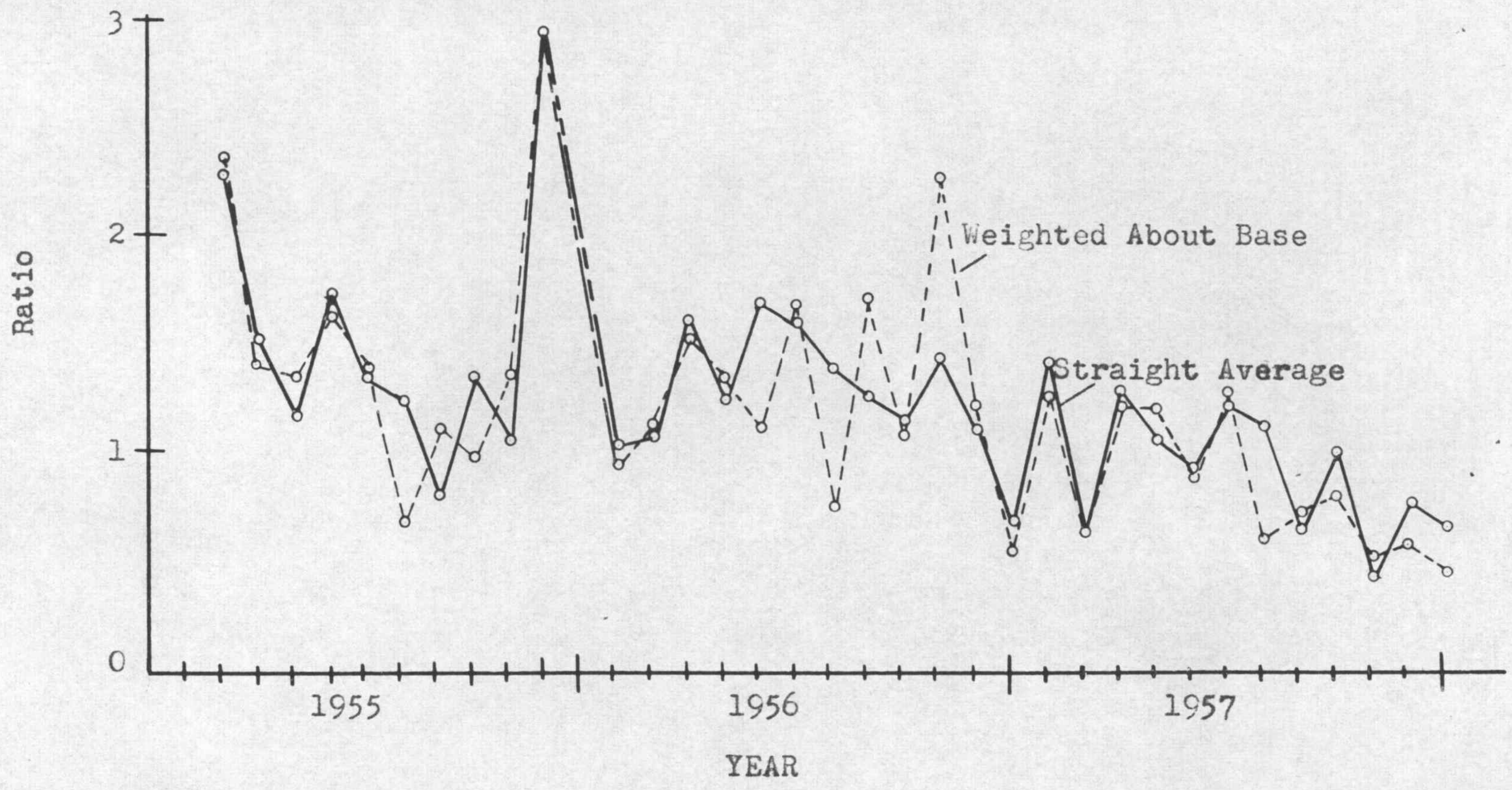


Figure 7. Tru-Vue D-5

Ratio of Actual Demand to Three Year Base Series

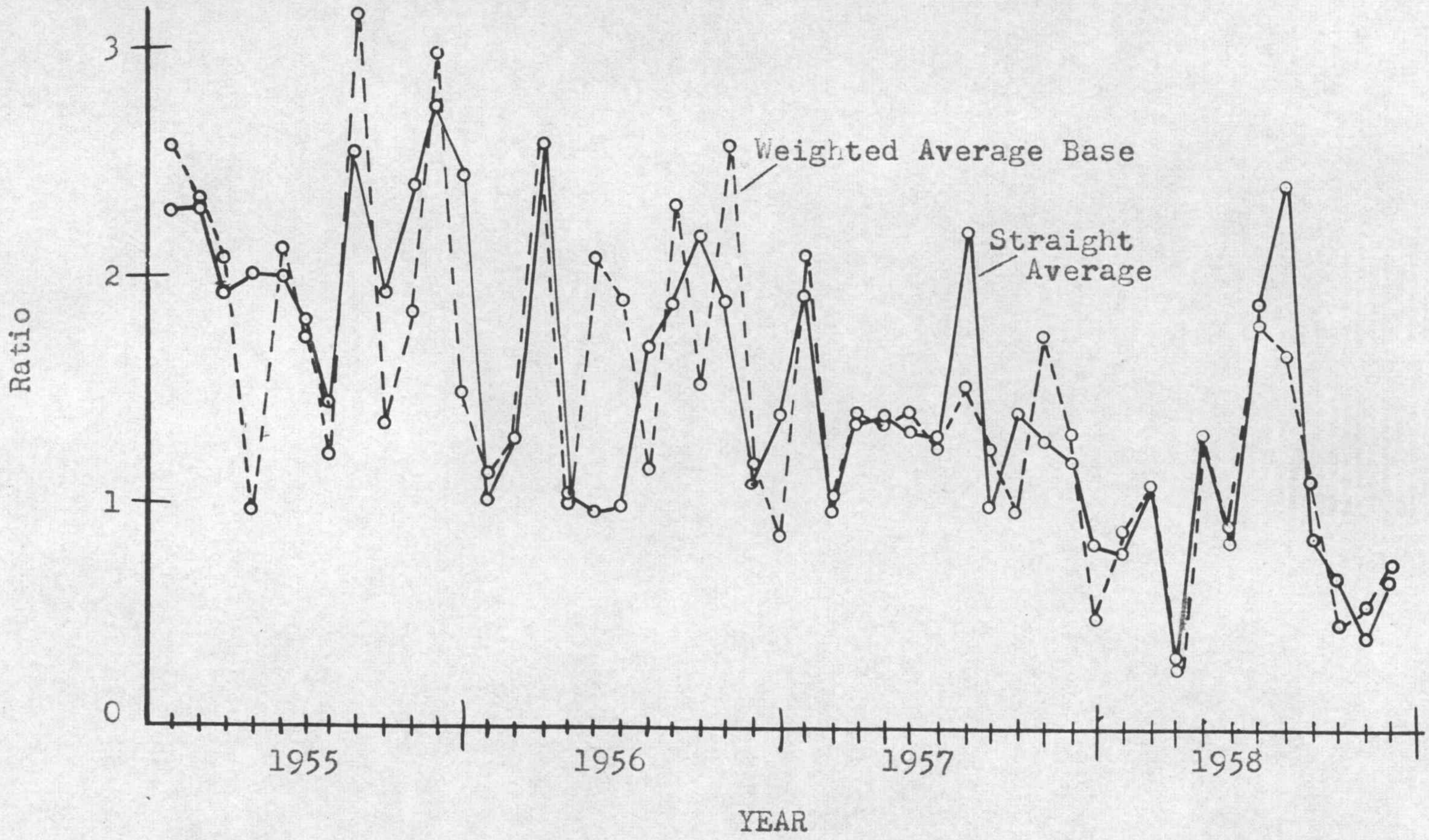
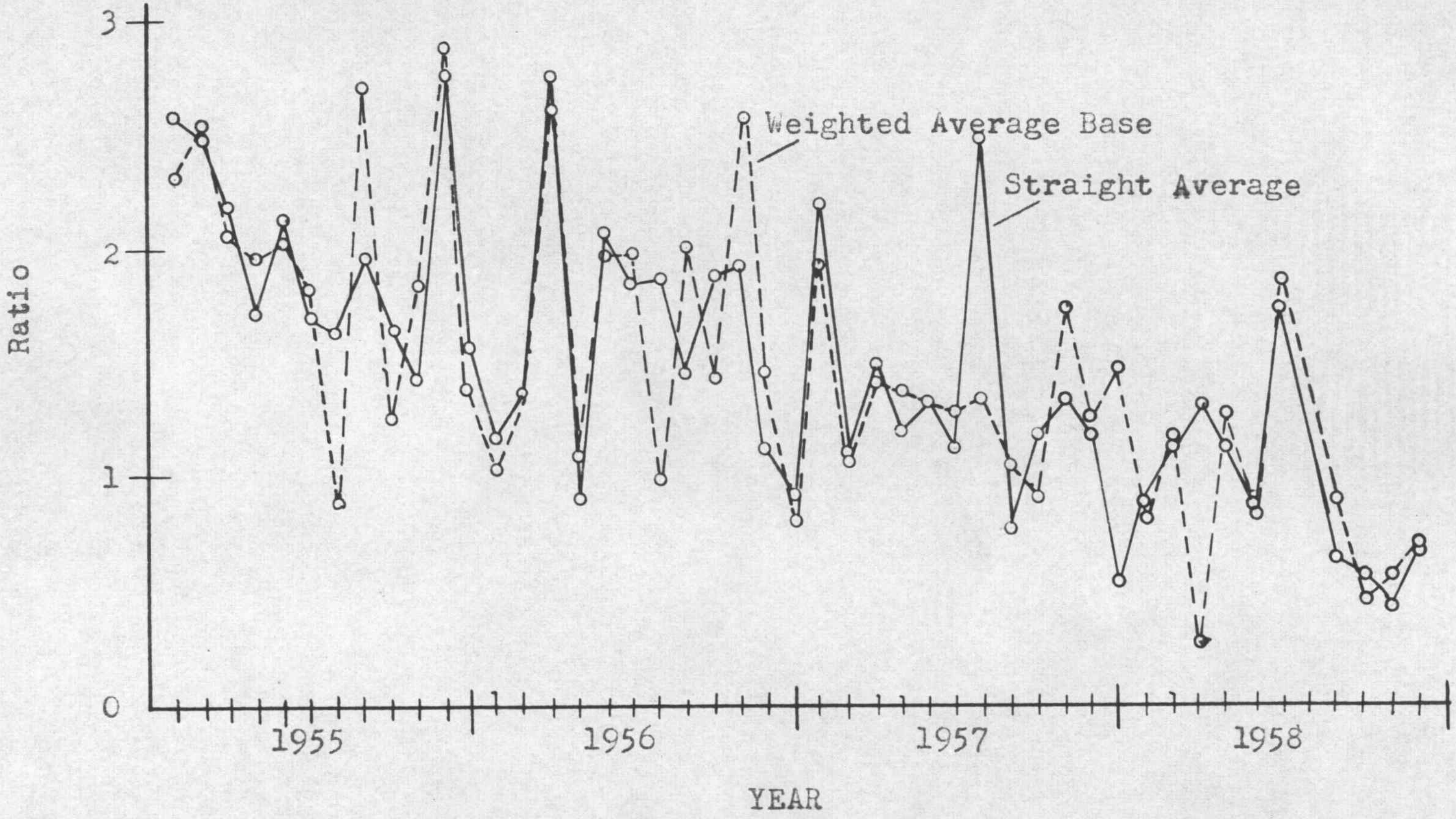


Figure 8. Tru-Vue D-5

Ratio of Actual Demand to Four Year Base Series



the forecast, and are used in the exponential smoothing formulas in the next section as well as in the simplified seasonal pattern in the succeeding section.

TABLE 1

BASE SERIES - TRU-VUE DEMAND

(From ten random selections - years 1954 through 1957)

Averages weighted about the quarter

Month	Base	Month	Base
January	25.3	July	24.1
February	23.7	August	29.8
March	24.9	September	46.3
April	23.7	October	48.2
May	21.9	November	45.9
June	17.3	December	32.7

<u>Application of Exponential Smoothing With Seasonal Pattern</u>

The initial calculation for the forecast of demand by exponential smoothing requires establishing certain initial quantities. These elements enter where the expression $(1 - \alpha)$ (old value) occurs in the formula. Specifically, the average ratio, the trend, and the mean absolute deviation require initial values. The values choosen for the first calculations proved satisfactory, and they have been used in all the later calculations. The initial value for the average was taken as one, the trend as zero, and the mean absolute deviation as 20. The values for average and trend are obvious choices since the respective values of one and zero dictate the base series as the current best estimate of demand. The choice of the initial value of the mean deviation is discussed in the next paragraph. With these initial values, the system reaches stability very quickly, and has indicated little affect on the forecast after the first few periods.

The value of the mean absolute deviation is proportional to the standard deviation of demand. Brown (3, p. 93) derives the ratio as 1/1.25 or 0.8. A knowledge of the standard deviation will yield an initial value for the mean absolute deviation. For Tru-Vue Cards in the 1954-1957 base period, the demand standard deviation of several random selections was found to be near 25. The mean absolute deviation, being 0.8 of the standard deviation, is found to be 20.

The initial procedure was to forecast demand over several years at a constant value of alpha. Forecasts for Tru-Vue Cards D-5, F-1 and F-4 were prepared at various alpha values from .075 to 0.4. This permitted a comparison and indicated the alpha value that gave the smallest deviation in the forecast. The method and results of this effort will be discussed further.

The sale of Tru-Vue Card D-5 started in 1955. The demand for January of that year was 58 cards. Using this value, forecasting by exponential smoothing proceeds in the following manner. From table 1, the base series for January is 25.3. Using $\alpha = 0.1$, we have:

1. The demand ratio $= \dfrac{\text{current month's demand}}{\text{value of base series for this month}}$

$$= \frac{58}{25.3} = \underline{2.29}$$

2. Average ratio $= \alpha\,(\text{new value}) + (1 - \alpha)\,(\text{old value})$

   (using an initial value of one for the average ratio, the new value is)

$$= (0.1)\,(2.29) + (0.9) = \underline{1.129}$$

3. Change = (new average ratio) - (old average ratio)

   (again using an initial value of one for the average ratio, the change is)

$$= 1.129 - 1.00 = \underline{0.129}$$

4. Trend $= \alpha\,(\text{new change}) + (1 - \alpha)\,(\text{old value of trend})$

   (using an initial value of zero for the trend, the current trend is)

$$= (0.1)\,(0.129) + (0.9)\,(0) = \underline{0.0129}$$

5. Expected ratio $= \text{current average} + \dfrac{(1 - \alpha)}{\alpha}$ current trend

$$= \text{item 2} + \frac{(1 - \alpha)}{\alpha}\,(\text{item 4})$$

$$= 1.129 + \frac{0.9}{0.1}\,(0.0129) = \underline{1.245}$$

6. Expected demand in Nth month

$= \text{(Expected ratio) (Value of base series for Nth month)}$

Expected demand in February = (1.245) (23.7) = $\underline{29.5}$

7. Expected demand for Two, Four, and Six months in future

Expected demand for next N months

= (Expected ratio) (Sum of base series for next N months) These values are given in table 2.

Expected demand for next two months

$= (1.245)\ (23.7 + 24.9) = (1.245)\ (48.6) = \underline{60.5}$

Expected demand next four months

$= (1.245)\ (23.7 + 24.9 + 23.7 + 21.9)$

$= (1.245)\ (94.2) = \underline{117.3}$

Expected demand next six months

$= (1.245)\ (23.7 + 24.9 + 23.7 + 21.9 + 17.3 + 24.1)$

$= (1.245)\ (135.6) = \underline{168.8}$

As soon as the actual demand for the next month is known, the normalized cumulative deviation can be calculated for control purposes. The demand for Tru-Vue D-5 in February 1955 was 60 cards. The forecasting system controls are calculated in the following four steps (8 through 11).

8. Error = (demand forecast) - (actual demand)

$= 29.5 - 60 = \underline{-30.5}$

TABLE 2

EXPECTED DEMAND FOR VARIOUS LEAD TIMES

(Obtained from 1954-1957 Base Series)

	Lead Time			
	One Month	Two Months	Four Months	Six Months
January	25.3	49.0	97.6	136.8
February	23.7	48.6	94.2	135.6
March	24.9	48.6	87.8	141.7
April	23.7	45.6	87.0	163.1
May	21.9	39.2	93.1	187.6
June	17.3	41.4	117.5	211.6
July	24.1	53.9	148.4	227.0
August	29.8	76.1	170.2	228.2
September	46.3	94.5	173.1	222.1
October	48.2	94.1	152.1	200.7
November	45.9	78.6	127.6	176.2
December	32.7	58.0	106.6	152.2

9. Cumulative error = running sum of errors = $\underline{-30.5}$

10. Mean absolute deviation = α |(error)| + (1 - α) (old mean absolute deviation)

(using an initial value of 20, this calculation for card D-5 in February 1955 becomes)

$$= (0.1)\ (30.5) + (0.9)\ (20) = \underline{21.05}$$

11. Normalized cumulative error = $\dfrac{\text{cumulative error}}{\text{mean absolute deviation}}$

$$= \frac{-26}{20.6} = \underline{-1.261}$$

The eleven steps applied above to obtain the forecast from one month's demand require 21 arithmetic operations. There are eight multiplications, five additions, five subtractions, and three divisions. The average time on the first few forecasts was approximately 20 minutes, even though a desk calculator was used. The main reason for this excessive time was due to the occasional error. In preparing month by month forecasts from 12 months demand data, 252 arithmetic operations are involved. The probability of an error becomes significant, and finding an error once it has been made becomes time consuming. This discussion points to the desirability of the digital computer, wherein the computation and printing out of one month's forecast takes 20 seconds. The computer does make errors, but always of the gross type that are immediately evident. The use of the computer in the operation is discussed further near the end of this section.

Choosing the Most Effective Alpha Value

The accuracy of the forecast is influenced by the value of alpha. Choosing the best alpha value for a particular demand history is a matter of trial and error. The demand history for a given item over several years can be used as a base. A forecast prepared for the item over the same period and at a fixed value of alpha will yield a certain set of forecast errors. The alpha value can be changed, another forecast prepared, and a new set of forecast errors are obtained. This process is repeated until the forecast errors go through a minimum. The measuring stick used is the standard deviation of the errors.[1]

The demand histories for Tru-Vue Cards F-1, D-5, and F-4 were used to find an optimum alpha value. Demand forecasts were made for lead times of one month, two months, four months and six months. The forecasts were calculated monthly over the period for various alpha values, and their respective errors obtained by comparing to actual demands.

---

1. The standard deviation is a measurement of the average deviation within a set of data --the deviation from the average or mean value of the data. The sample standard deviation (S_x) is given by:

$$S_x = \sqrt{\frac{1}{N - 1}\left[\Sigma X^2 - \frac{(\Sigma X)^2}{N}\right]}$$

See (6, p. 135) for mathematical proof.

Tru-Vue Cards D-5 and F-4 were studied for 1955 and 1956. The demand history for card F-1 was studied for three years --1955 through 1957.

For each card, (D-5, F-1, F-4), four forecasts were prepared each month. These are: (1) The forecast of demand for next month (one month lead time); (2) The forecast of demand for the next two months (two months lead time); (3) The forecast of demand for the next four months (four months lead time); and (4) The forecast of demand for the next six months (six months lead time). There are 144 individual forecasts for card F-1, for each alpha value. The four alpha values used resulted in 576 forecasts for card F-1. Similarly, 184 forecasts were prepared on card F-4, and 192 forecasts for card D-5. Each of these 952 forecasts had to be compared with the actual demand for the period forecasted to obtain the 952 individual forecast errors. The computation of standard deviations from these errors is summarized in tables 3 and 4.

The standard deviation of the forecast errors for the various alpha values serves as the basis for comparison. For each alpha value there is a standard deviation of errors for each lead time. Thus, for card F-1 and an alpha of 0.4, the standard deviations of the forecast errors are 22.1, 48.9, 67.9 and 87.0 for one, two, four, and six month lead times respectively. For comparison purposes, the

TABLE 3

STANDARD DEVIATIONS OF FORECAST ERRORS TRU-VUE CARD F-1
FROM EXPONENTIAL SMOOTHING FORECASTS WITH SEASONAL PATTERN
FOR YEARS 1955 THROUGH 1957

N = 36 months

Alpha	Lead Time Months	Sum Errors	Sum $(\text{Errors})^2$	Standard Deviation
0.075	one	10.1	11,864.7	18.4
0.075	two	69.2	25,250.9	26.8
0.075	four	196.7	49,417.3	37.2
0.075	six	373.3	75,498.1	58.9
0.1	one	32.3	12,994.1	19.2
0.1	two	64.5	25,376.9	26.7
0.1	four	211.1	49,334.3	37.1
0.1	six	372.7	73,564.3	44.6
0.2	one	62.1	13,794.8	19.8
0.2	two	84.6	30,945.9	29.6
0.2	four	146.9	66,033.2	43.2
0.2	six	282.4	102,899.2	53.6
0.4	one	27.4	16,670.0	22.1
0.4	two	37.3	83,833.1	48.9
0.4	four	332.1	164,360.0	67.9
0.4	six	360.4	268,477.0	87.0

$$\text{Standard deviation} = \sqrt{\frac{N \quad \left[\sum (\text{errors})^2\right] - \left[\sum \text{errors}\right]^2}{N\ (N - 1)}}$$

TABLE 4

STANDARD DEVIATION OF FORECASTS ERRORS FOR

TRU-VUE CARDS D-5 AND F-4

FROM EXPONENTIAL SMOOTHING FORECASTS WITH SEASONAL

PATTERN FOR YEARS 1955-1956

N = 47 months

Alpha	Lead Time Months	Sum Errors	Sum $(\text{Errors})^2$	Standard Deviation
0.1	one	-1.7	26,416.6	24.0
0.1	two	90.6	73,530.1	39.3
0.1	four	260.7	123,410.8	51.5
0.1	six	652.1	173,148.5	59.7
0.3	one	-17.9	34,244.0	27.3
0.3	two	-63.7	66,266.0	37.9
0.3	four	-136.4	152,958.0	57.8
0.3	six	346.4	245,543.0	72.7

$$\text{Standard deviation} = \sqrt{\frac{N\left[\Sigma(\text{errors})^2\right] - \left[\Sigma\ \text{errors}\right]^2}{N(N-1)}}$$

forecast errors for cards D-5 and F-4 are considered together. The data are presented in graphical form in figures 9 and 10. Note that an alpha of 0.1 yields slightly less errors than a value of 0.2. The standard deviation of the errors obtained with an alpha of 0.075 was checked on card F-1. It is noted that this alpha value is not as good as 0.1 for lead times greater than one month.

An optimum alpha value of 0.1 is in agreement with the text on statistical forecasting. The author makes the following comments:

> "In practice I have found that α = 0.1 is a satisfactory compromise between a very stable system that fails to track real changes and a 'nervous' system that fluctuates with demand. Prediction of changes in the pattern of demand can help improve the compromise (3, p. 54)."

The same "measuring stick" applied to the forecast errors can be used to measure the noise element in the actual demand. The standard deviation of actual monthly demand can be compared to the forecast error deviations. The comparison alluded to can be stated in the form of a question --can a forecast be prepared with an error pattern of greater stability than the fluctuations in actual demand?

Determining the standard deviation of actual demand requires a different approach than that used in the computation of the error deviation. This is necessitated by the

Figure 9. Standard Deviation of Forecast Errors
Tru-Vue F-1 (1955 - 1957)
(Forecasts Made by Exponential Smoothing
Using A Four Year Weighted Average)

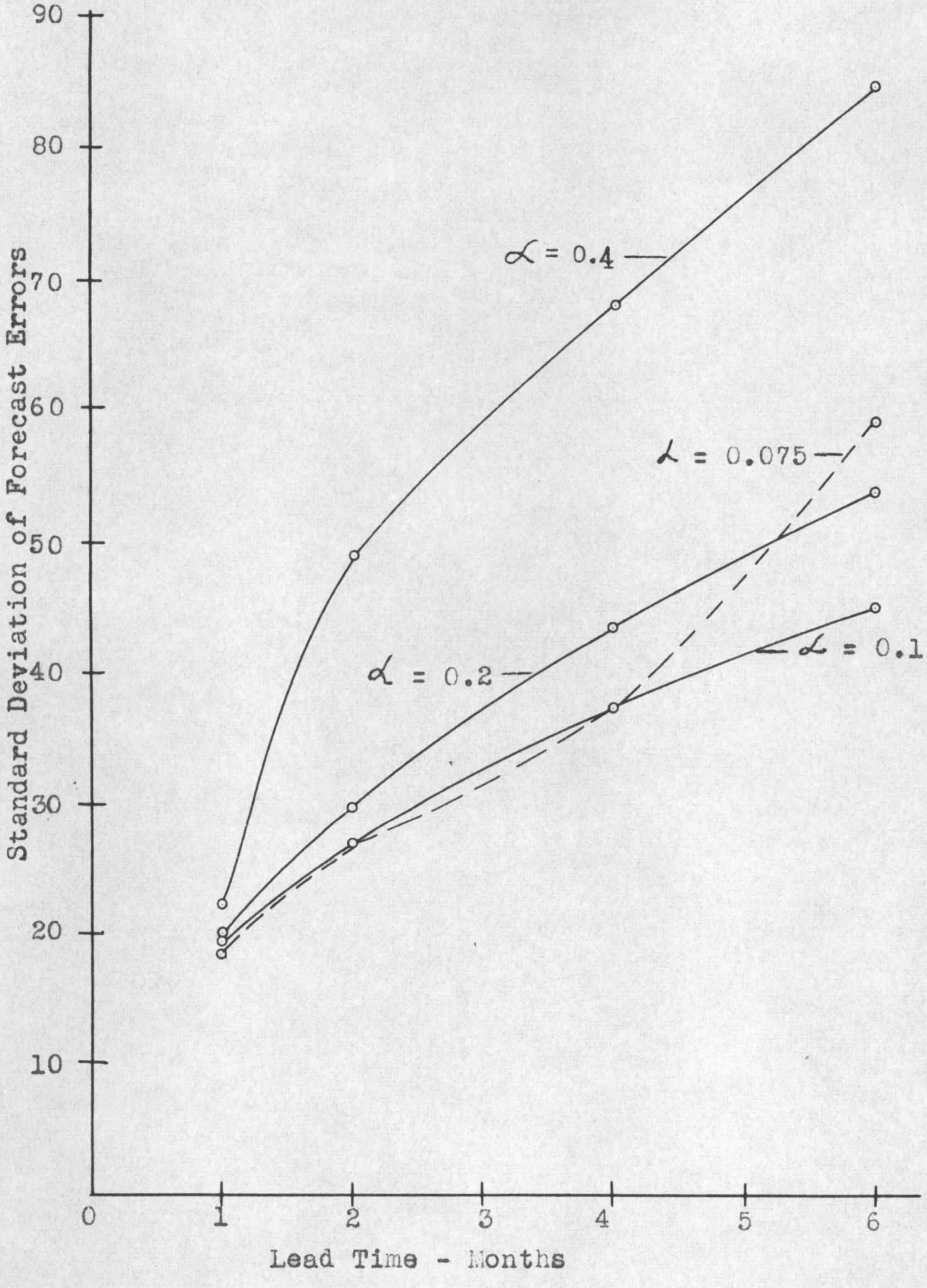
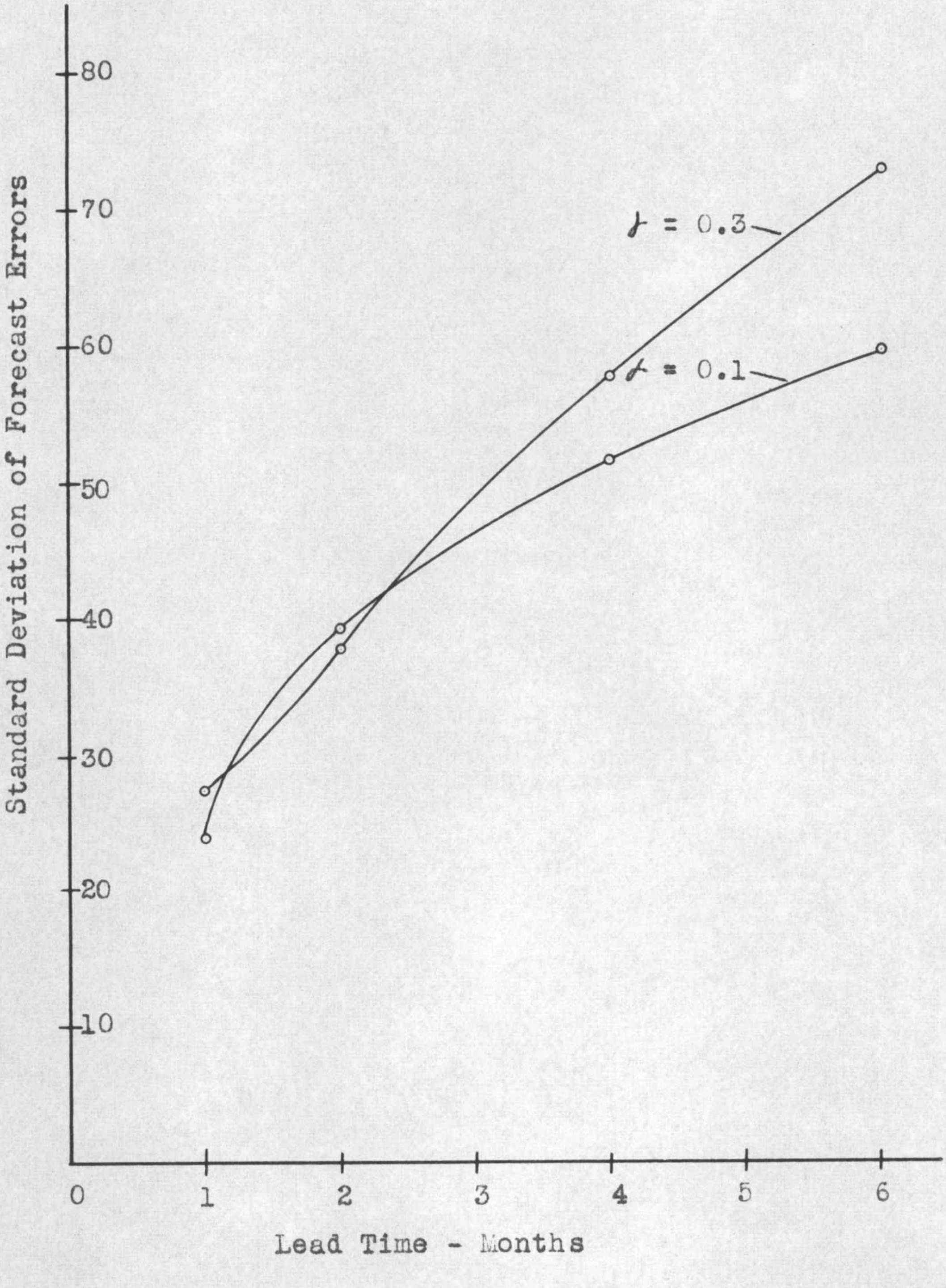


Figure 10. Standard Deviation of Forecast Errors
Tru-Vue Cards F-4 & D-5
(1955 - 1956)
(Forecasts Made by Exponential
Smoothing Using Four Year Weighted Average)



change in demand due to the seasonal pattern. The "measurement" should be made so that the seasonal changes in demand are not included. This can be accomplished by examining a given month's demand history over a period of several years. Thus, for card F-1, the actual demands for January, 1955, January, 1956 and January, 1957 yield a statistic independent of the seasonal pattern. The demand history for the three Februarys, and each of the other calendar months, can be examined. The relationship between the range[1] and the standard deviation can be used. The ratio of the average range to the standard deviation is designated d_2. Values of d_2 for various sample sizes are tabulated in Cowden's text (4, p. 691).

The actual demands for card F-1 are presented in table 5 along with the monthly range of values for the years 1955 through 1957. Similar tabulations for Tru-Vue D-5 and F-4 are given in table 6. The standard deviation is computed at the foot of each table from each month's range. The value for Tru-Vue Card F-1 is 14.3. For cards D-5 and F-4 together, it is 17.3. It is noted that this is a lower value than the forecast errors standard deviation depicted in figures 9 and 10. Thus, this system of forecasting does not yield an error pattern as stable as the fluctuations in actual demand. The point is discussed

1. The range is the difference between the highest and lowest values in a set of data.

further in the section which compares the various forecast methods.

Application of Exponential Smoothing Without Seasonal Pattern

With the downward trend in demand in 1958, the seasonal pattern seems to disappear. The average 1958 monthly demands for nine Tru-Vue Cards are shown in table 5. The 1954-1957 base is presented for comparison. Note that the 1958 average monthly demand was lowest in December, with March and September ranking in that order from the low position. Abandoning the seasonal pattern which proved so consistent from 1954 through 1957 is hardly warrented by one or two years of erratic behavior, particularly when the downward demand trend may be masking the seasonal pattern. However, this apparent change offers the chance to investigate the method of exponential smoothing forecasts without the seasonal pattern, and to compare the results with the other methods.

In order to start the exponential smoothing forecast, two initial values must be established. These are the initial values for the average and the trend. Each of these factors is used as the "old estimate" in the "rule" for exponential smoothing.

$$\text{new estimate} = \alpha \text{ (new value)} + (1 - \alpha) \text{ (old estimate)}.$$

TABLE 5

TRU-VUE CARD F-1 MONTHLY DEMAND HISTORY AND RANGE VALUES

1955-1957

Month	Demand 1955	Demand 1956	Demand 1957	Range
January	47	27	12	35
February	37	36	29	8
March	28	30	26	4
April	31	34	23	11
May	21	32	20	12
June	17	18	28	11
July	60	54	29	31
August	44	77	34	43
September	69	124	95	55
October	97	50	63	47
November	32	28	7	25
December	26	29	17	9

Calculation of standard deviation of demand with seasonal fluctuations eliminated.

$$\overline{R} = \frac{35 + 8 + \text{----} + 9}{12} = \frac{291}{12} = 24.25$$

For samples of three, $d_2 = 1.693$ reference (4, p. 691).

$$\text{Standard deviation} = \frac{24.25}{1.693} = 14.3$$

TABLE 6
TRU-VUE CARDS D-5 AND F-4 MONTHLY DEMAND HISTORIES AND RANGE VALUES 1955-1956

MONTH	D-5 Demand 1955	D-5 Demand 1956	D-5 Range	F-4 Demand 1955	F-4 Demand 1956	F-4 Range
January	60	33	27	--	--	--
February	51	65	14	36	38	2
March	46	24	22	31	33	2
April	44	43	1	36	35	1
May	31	34	3	24	29	5
June	21	24	3	16	18	2
July	80	59	21	33	51	18
August	58	66	8	45	50	5
September	88	123	35	65	109	44
October	131	52	79	135	51	84
November	45	26	19	-4	18	22
December	26	49	23	24	32	8

Calculation of standard deviation of demand with seasonal fluctuations eliminated.

$$\bar{R} = \frac{2 + 2 + 1 + \text{----} + 8 + 27 + \text{----} + 23}{23}$$

$$= \frac{448}{23} = 19.48$$

For samples of two, $d_2 = 1.128$ (4,p.691)

$$\text{Standard deviation} = \frac{19.48}{1.128} = 17.27$$

TABLE 7

COMPARISON OF 1958 TRU-VUE DEMAND TO

1954 - 1957 BASE

Month	1954-1957 Base	1958 Average[1]	Ratio To Base
January	25.3	18.5	0.732
February	23.7	28.0	1.18
March	24.9	12.0	0.483
April	23.7	26.7	1.125
May	21.9	18.7	0.855
June	17.3	17.7	1.02
July	24.1	18.0	0.748
August	29.8	29.3	0.985
September	46.3	14.9	0.322
October	48.2	16.7	0.346
November	45.9	26.4	0.575
December	32.7	4.3	0.1315

1. Average of nine selections (Cards D-5, D-6, F-1, F-2, D-4, F-5, D-23, T-20, and T-23).

The value of the 1954-1957 base series for the month immediately before the starting month of the forecast was taken as the initial value for the average. The initial trend value was set at zero. Using the formulas given on pages 14 to 16, monthly forecasts were prepared for Tru-Vue F-1 starting with October,1957, and terminating with the August, 1959 forecast. Forecasts were prepared using alpha values of 0.8, 0.5, 0.3, 0.1, 0.005, and 0.001. The forecast errors for one month lead time were determined by subtracting the actual demand for a given month from the forecasted demand made in the preceding month. For each alpha value, the standard deviation of the forecast errors for the 22 months in the forecast was calculated. The results of these calculations are presented in figure 11.

Since it is desirable to have the smallest standard deviation as possible, an alpha value of zero is indicated. However, this puts no weight on the newest demand and 100% on the past. This is equivalent to taking the initial value of the average as the forecast for every month in the future. In order to have the system "track" the real changes, an alpha value of at least 0.1 is required. The comments made in the previous section on an optimum value of 0.1 are just as applicable here. The forecasts obtained by using an alpha of 0.1 are presented in table 8 along with the actual demands. The standard deviation of the

Figure 11. Standard Deviation of Forecast Errors
Tru-Vue F-1 (Oct. 1957 - Aug. 1959)
(Forecasts made by exponential smoothing without seasonal pattern)
One Month Lead

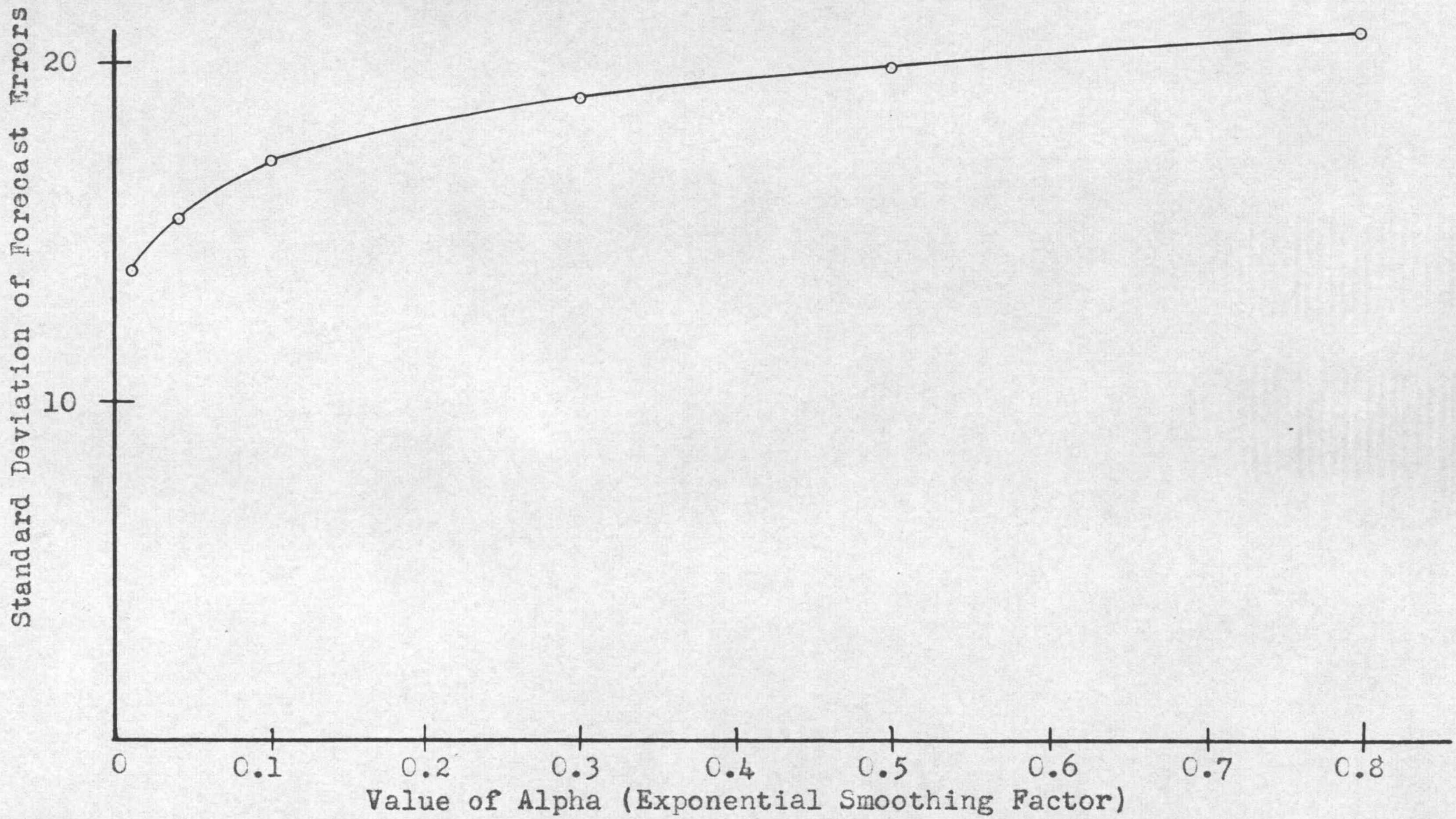


TABLE 8

FORECASTS FOR TRU-VUE CARD F-1

Obtained by Exponential Smoothing - Without Seasonal Pattern
Alpha = 0.1

Month	Forecasted Demand	Actual Demand	Error
November (1957)	55	63	-8
December	57	7	50
January (1958)	48	17	31
February	42	22	20
March	38	4	34
April	31	25	6
May	29	24	5
June	28	15	13
July	25	17	8
August	22	46	-24
September	26	12	14
October	23	23	0
November	22	26	-4
December	22	10	12
January(1959)	19	40	-21
February	22	17	4
March	20	5	15
April	17	17	0
May	16	6	10
June	13	5	8
July	11	11	0
August	10	23	-13

forecast errors is 17.2. These computations were made on the ALWAC III-E computer. The computer was programmed so that the standard deviation of forecast errors was included in the output data (see appendix 1). The results achieved by this method of forecasting are discussed further in the section which compares the various forecast methods.

Forecasting Tru-Vue Demand by Simplified Seasonal Pattern

A forecast prepared by the method designated simplified seasonal pattern requires knowledge of three factors: (1) Annual demand for the past two years; (2) The expected monthly change in demand; and (3) A forecast for next year's demand. The second factor, monthly changes, could be obtained from the 1954-1957 base series obtained for exponential smoothing in the previous presentation. This is presented in table 9.

In preparing a forecast for the calendar year 1959, it was noted that the average sales for 1958 were slightly over 60% of the 1954-1957 base. With the assumption that 1959 sales would be in the same vicinity as 1958 sales, use of the 1954-1957 base series produces disproportionately large changes. However, a new base series and monthly change is readily obtained by taking 60% of the old base. This leads to the data presented in table 10.

To demonstrate this method, forecasts for the

TABLE 9

EXPECTED MONTHLY DEMAND AND DEMAND CHANGE
FROM 1954-1957 BASE

Month	Expected Demand	Change
January	25.3	-7.4
February	23.7	-1.6
March	24.9	1.2
April	23.7	-1.2
May	21.9	-1.8
June	17.3	-4.6
July	24.1	6.8
August	29.8	5.7
September	46.3	16.5
October	48.3	2.0
November	45.9	-2.4
December	32.7	-13.2

TABLE 10

EXPECTED MONTHLY DEMAND AND DEMAND CHANGE FROM 1954-1957 BASE MODIFIED TO 1958 DEMAND

Month	Expected Demand	Change
January	16	-5
February	15	-1
March	16	1
April	15	-1
May	14	-1
June	11	-3
July	15	4
August	19	4
September	29	10
October	30	1
November	29	-1
December	21	-8

calendar year 1959 were prepared on six cards. These six are cards F-1, F-2, F-5, D-5, and D-6. The major problem encountered was the changing trend in demand. In every case the 1958 demand was significantly less than the 1957 figure. Since this downward trend started before 1957, it is rather hard to ignore. On the other hand, to extrapolate this trend in a linear fashion would enhance the risk of a low forecast, since there is a probability that the major decrease in demand has occurred in the period 1956 through 1958. The dilemma was resolved by choosing an arbitrary factor of ten percent. The change from 1958 to 1959 was taken as ten percent of the 1957-1958 change for each card.

1959 Forecast = F = (1958 demand) - (10%) (1957-1958 change)

1958 Demand = A

1957 Demand = Z

$$R = (-5)(12) + (-1)(11) + (1)(10) + (-1)(9) + (-1)(8) + (-3)(7) + (4)(6) + (4)(5) + (10)(4) + (1)(3) + (-1)(2) + (-8)(1) = -22$$

"Normalized demand for December 1958,

$$M = \frac{2A - R - Z}{12}$$

The constant factor added monthly,

$$k = \frac{F - 2A + Z}{78}$$

The forecast calculations are presented in appendix 3. Results of the forecasts are tabulated in table 11.

Use of the ALWAC III-E Computer

Forecasts prepared by the exponential smoothing method involve repetitive arithmetic operations. When several years' forecasts are being prepared on a month by month basis for several items, the calculations become quite tedious, and mistakes are probable. The digital computer's speed, accuracy, and ease of programming for a repetitive operation facilitated the forecasts in this paper. The computer used (ALWAC III-E) is part of the facilities of the Oregon State College Mathematics Department. It has a magnetic drum memory with 8,192 word capacity. A view of the control console, and the memory, logic and power cabinets is shown in figure 12.

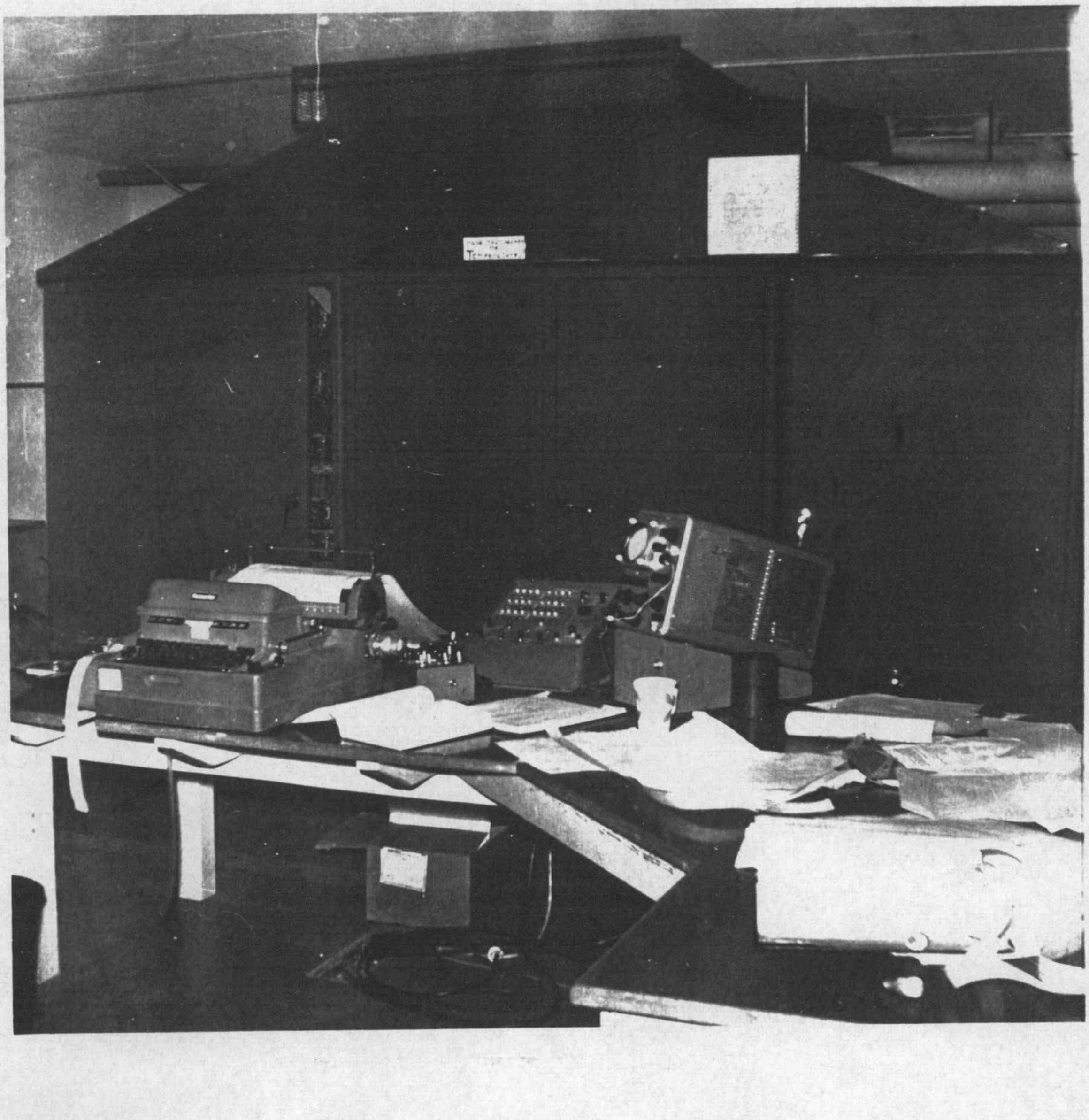
Programming was facilitated by use of the ALCOM (algebraic compiler) routine. This eliminates the necessity of placing information in machine language, and thereby greatly speeds the programming process for one with limited programming experience. Once the initial program is in the computer, a "machine language" program can be "read out" of the computer for future use. This eliminates the necessity of going through the ALCOM routine each time the program is used.

TABLE 11

1959 FORECASTS AND ERRORS FOR SIX TRU-VUE CARDS OBTAINED BY SIMPLIFIED SEASONAL PATTERN
(Error is difference between forecast and actual demand.)

Month	Card F-1		Card F-2		Card F-4		Card F-5		Card D-5		Card D-6	
	Fore-cast	Error	Fore-cast	Error	Fore-cast	Error	Fore-cast	Error	Fore-cast	Error	Fore-cast	Error
January	6	-34	11	-29	10	-8	2	-15	7	-62	2	-9
February	7	-10	11	11	10	4	3	-3	8	2	3	3
March	9	4	11	0	12	12	6	1	11	6	6	0
April	10	-7	11	2	12	1	7	1	12	-5	6	-6
May	11	5	10	-2	11	5	8	-3	13	1	7	1
June	10	5	8	-4	9	-2	6	0	12	7	6	-5
July	16	-5	12	6	14	8	12	0	18	6	11	11
August	21	-2	18	-4	18	1	18	7	24	1	17	-6
September	33	22	27	15	29	23	30	14	36	25	29	17
October	36	--	29	--	31	--	33	--	39	--	31	--
November	37	--	28	--	30	--	33	--	40	--	32	--
December	31	--	21	--	23	--	27	--	34	--	26	--

Figure 12. View of ALWAC III-E Computer



To program by ALCOM, a letter of the alphabet is chosen to represent each variable of input and output. The input instructions are typed into the machine as A in, B in, etc. The output instructions are similarly given as F out, G out, etc. The program is easier to understand if one reads the equal sign as "replaces". Thus, A . B = B means A times B replaces B. Similarly C / D = D means C divided by D replaces D. Once the information designated by a given letter is no longer needed, the letter may be used to designate a different item.

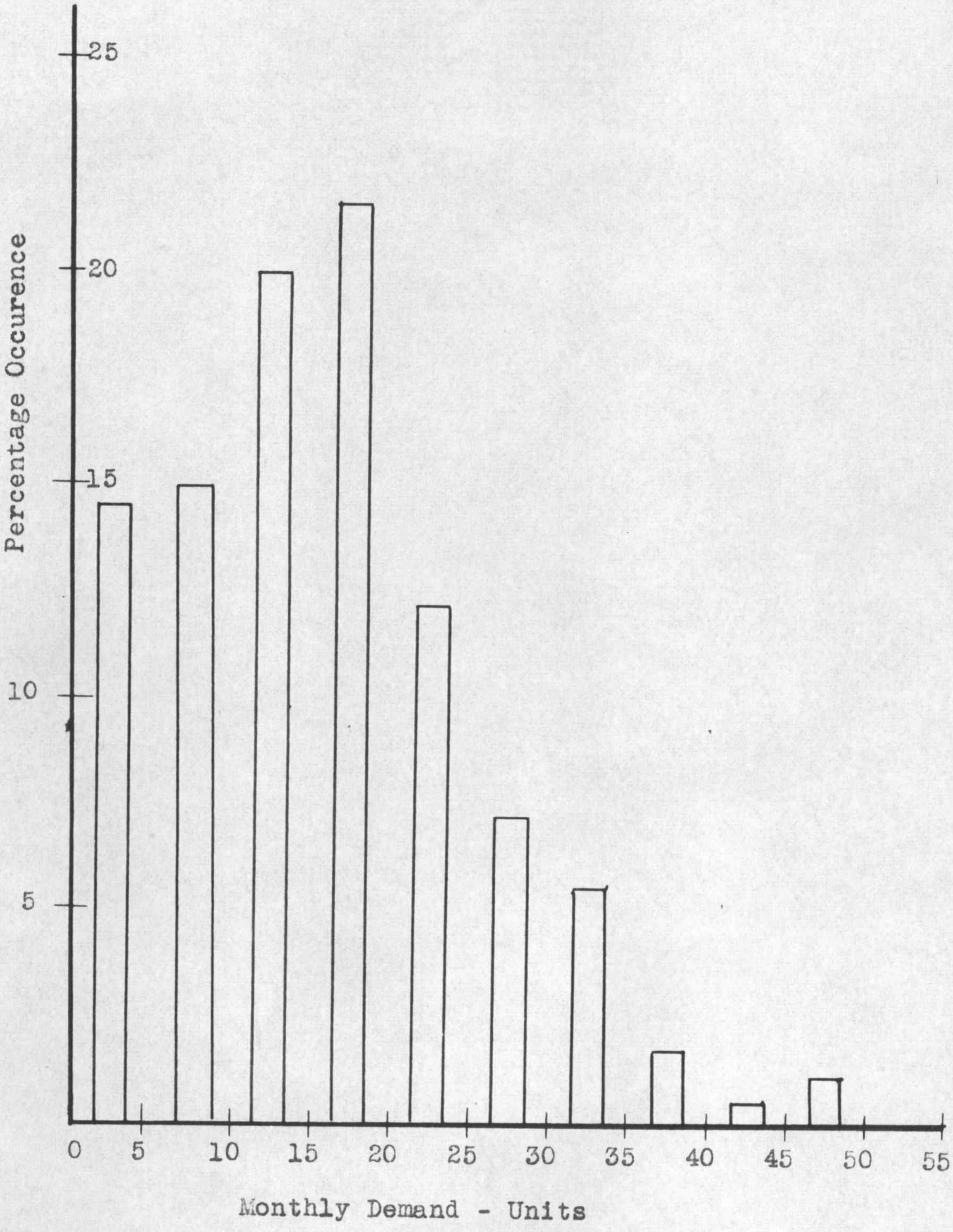
The ALCOM programs for forecasting by exponential smoothing with and without the seasonal pattern are presented in appendix 1. The corresponding machine language programs are presented in the same place. The format, or manner in which the information is typed out of the computer, is also presented for both exponential smoothing programs in appendix 1. The formats presented are exact replicas of portions of pages typed out by the computer.

The Demand Frequency Distributions

The demand pattern for nine cards[1] was studied for the 20 month period from January,1958 through August, 1959. The frequency distribution for the 180 demand values is presented in figure 13. The mean value for these data is

1. Cards D-5, D-6, F-1, F-2, F-4, F-5, D-23, T-20, T-23 were used.

Figure 13. Frequency Distribution - Tru-Vue Card Demand
January 1958 - August 1959
(Frequency that specific demand levels were experienced for nine cards)

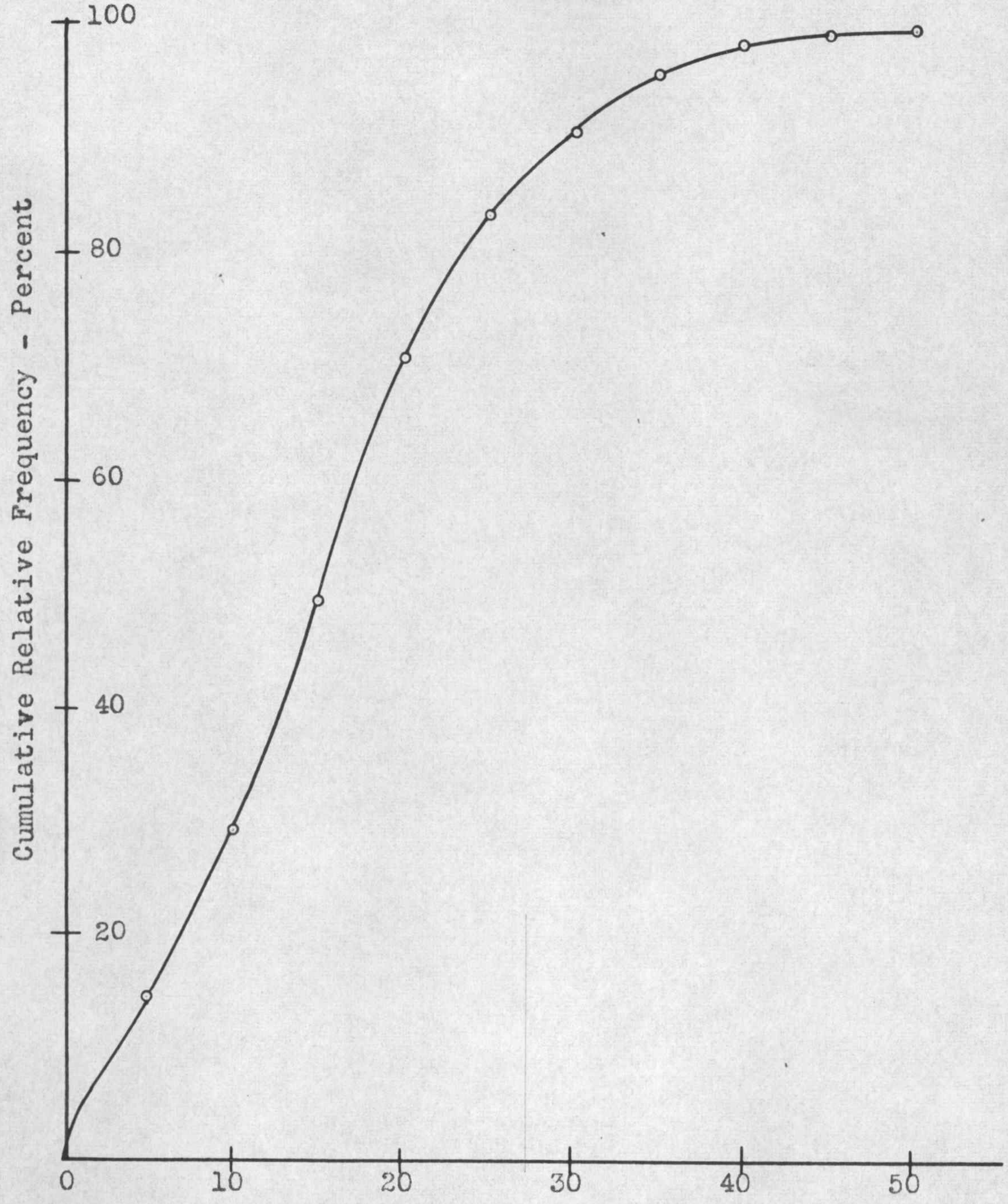


16.1, and the standard deviation is 10.96. The data and computations from which these values were obtained are presented in appendix 4. The cumulative frequency distribution for the same demand values is presented in figure 14. Magee, in his text on production planning and inventory control, describes the use of this curve for determining inventory levels (9, p. 72-76). The technique described in this reference is illustrated below by use of the Tru-Vue statistics. The information presented in figure 14 shows that 95% of the time, the average monthly order is for 40 cards or less. Thus, if 95% of all orders are to be filled without waiting, the beginning of the period inventory should be 40 cards.

This approach is presented for two purposes. (1) It is the method given by an authority in the field; and (2) It is an avenue to understanding the problem of forecasting with wide demand fluctuations. In Magee's text, the illustration given has a deviation from the mean to the 95% point of approximately one-third of the mean. The same deviation in the Tru-Vue values is more than seven times as large as the text example. The mean for Tru-Vue demand is approximately 16 and the 95% point is 40, which is $2\frac{1}{2}$ times the mean. One-third divides into $2\frac{1}{2}$, by a factor of $7\frac{1}{2}$ times.

If demand is to be satisfied with a minimum of

Figure 14. Cumulative Frequency Distribution
Tru-Vue Card Demand
January 1958 - August 1959
(cumulative frequency of specific
demand levels for nine cards)



shortages, a large safety stock is the inherent liability associated with such severe demand fluctuations. In the text on inventory control, Welch makes the following comments on this problem.

> "There should be no illusion that a set of formulas is a cure for poor or erratic data. The size of safety stocks to keep an inventory at a satisfactory performance level is a direct function of the unreliability of the data at hand (12, p. 135)."

The principal purpose of the forecast in such a situation is to set the monthly values for expected demand. A safety stock must be carried in addition to the forecasted amount in order to allow for the expected variations in demand. The amount of safety stock to carry is directly related to management's idea of what they consider a reasonable level of service.

To illustrate the manner in which these concepts would function, suppose that the decision is made that 95% of all orders are to be filled immediately. This is equivalent to the statement that the out of stock situation is reasonable up to 5% of the time. Based on the 1958 demand variation, a safety stock of 22 cards should be carried. The minimum inventory at the beginning of any month should be 22 plus the forecast for the month. The safety stock of 22 cards was obtained from the standard deviation of demand. This value was shown to be approximately 11. In a normal situation, one could expect a deviation of two standard deviation units or less 95% of the time. While the demand on Tru-Vue Cards is not exactly normally distributed, it is noted that two standard deviation units above the indicated mean of 16 is very near the 40 previously found as the 95% service level.

Comparison of Forecasting Systems

To compare the three methods of forecasting, individual forecasts for 1959 on six Tru-Vue Cards were prepared.[1] Actual demands for the first eight months of 1959 were used to obtain the monthly error for each forecast. The 144 errors obtained from the 144 monthly forecasts were analyzed graphically and statistically. The procedures and results are discussed below.

The forecasts prepared by exponential smoothing were obtained with an alpha value of 0.1. The modified seasonal pattern as presented in table 8 was used both for the exponential smoothing with seasonal pattern, and the simplified seasonal pattern. The forecasts prepared without the seasonal pattern were started in January, 1958 in order that the influence of the initial values would be negligible. The forecasts obtained in the previous section with the simplified seasonal pattern are used in this section. Table 12 presents the forecasts as well as the actual

---

1. The six selections used were cards F-1, F-2, F-4, F-5, D-5, and F-6.

TABLE 12

SUMMARY 1959 TRU-VUE FORECASTS

	F-1	F-2	F-4	F-5	D-5	D-6	Total
January							
Actual Demand	40	40	18	17	69	11	195
Exp Smoothing Seasonal Pattern	14	12	11	13	13	12	75
Exp Smoothing No Pattern (Rand)	20	18	15	16	22	16	107
Simplified Seasonal	6	11	10	2	7	2	38
Inventory	19	253	59	78	70	65	544
February							
Actual Demand	17	0	6	6	6	0	35
Exp Smoothing Seasonal Pattern	17	16	11	13	22	11	90
Exp Smoothing No Pattern (Rand)	24	22	15	16	31	15	123
Simplified Seasonal	7	11	10	3	8	3	42
Inventory	13	253	59	76	67	65	533
March							
Actual Demand	5	11	0	5	5	6	32
Exp Smoothing Seasonal Pattern	19	14	11	12	20	10	86
Exp Smoothing No Pattern (Rand)	23	18	13	14	26	12	106
Simplified Seasonal	9	11	12	6	11	6	46
Inventory	25	82	57	75	65	65	369

TABLE 12 Continued

	F-1	F-2	F-4	F-5	D-5	D-6	Total
April							
Actual Demand	17	9	11	6	17	12	72
Exp Smoothing Seasonal Pattern	15	12	8	10	17	8	70
Exp Smoothing No Pattern (Rand)	19	16	10	12	22	11	90
Simplified Seasonal	10	11	12	7	12	6	58
Inventory	19	79	53	73	59	61	344
May							
Actual Demand	6	8	6	11	12	6	49
Exp Smoothing Seasonal Pattern	14	11	8	8	16	8	65
Exp Smoothing No Pattern (Rand)	19	15	10	11	21	11	87
Simplified Seasonal	11	10	11	8	13	7	60
Inventory	17	76	51	69	55	59	327
June							
Actual Demand	5	12	11	6	5	11	50
Exp Smoothing Seasonal Pattern	10	8	6	7	12	6	49
Exp Smoothing No Pattern (Rand)	16	13	9	10	20	9	77
Simplified Seasonal	10	8	9	6	12	6	51
Inventory	15	72	48	65	54	55	309

TABLE 12 Continued

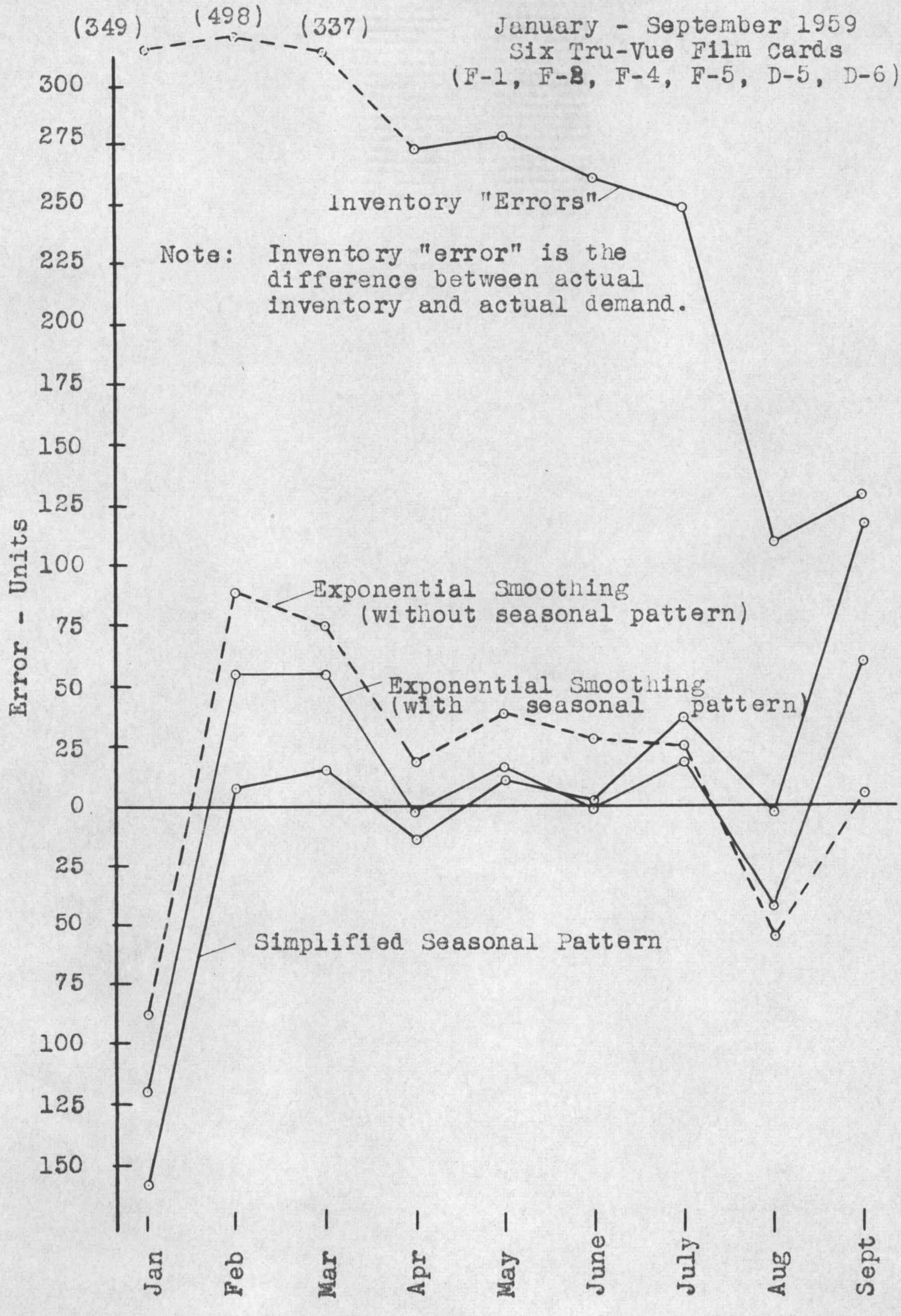
	F-1	F-2	F-4	F-5	D-5	D-6	Total
July							
Actual Demand	11	6	6	12	12	0	47
Exp Smoothing Seasonal Pattern	12	12	9	9	14	9	65
Exp Smoothing No Pattern (Rand)	14	13	9	9	17	9	71
Simplified Seasonal	16	12	14	12	18	11	83
Inventory	12	71	46	61	50	55	295
August							
Actual Demand	23	22	17	11	23	23	119
Exp Smoothing Seasonal Pattern	15	13	10	12	17	9	76
Exp Smoothing No Pattern (Rand)	13	11	8	9	15	7	63
Simplified Seasonal	21	18	18	18	24	17	116
Inventory	17	43	40	48	42	38	228
September							
Actual Demand	11	12	6	16	11	12	68
Exp Smoothing Seasonal Pattern	25	23	17	17	28	17	127
Exp Smoothing No Pattern (Rand)	15	13	9	9	17	10	73
Simplified Seasonal	33	27	29	30	36	29	184
Inventory	14	40	38	32	38	34	196

demands and inventories on each card. The end-of-the-month inventory on each card is presented also.

The graphical comparison was obtained by combining the six errors obtained on each card by each method. Thus, for the month of February, the total error obtained by exponential smoothing without the seasonal pattern was 88 cards. For the same month, the total forecast error was 55 cards for the method of exponential smoothing with the seasonal pattern. Similarly, the total February error for the simplified seasonal pattern forecast is seven cards. The total error obtained from each forecasting method is plotted in figure 15. The inventory "error" depicted in the same figure represents the monthly difference between the total demand for the six selections and their total inventory at the emd of the month. It serves mainly as an indication of potential improvement.

The use of the inventory "error" in the manner described above has not been described in any of the literature surveyed. While this factor is not presented as a purveyor of all information, it certainly is an historical record of the production planning. The large carry over at the beginning of the year is evident. Closer examination indicates that production continued into January and February even though several months' inventory was on hand. It serves as a partial answer to the question --why prepare

Figure 15. Total Monthly Errors of Forecasts and Inventory



a forecast?

The graph (figure 15) shows that the simplified seasonal pattern forecasts yielded the smallest error in four of the nine months. The exponential smoothing forecast was next, presenting the smallest error in three of the nine months. Thus, the simplified seasonal pattern wherein the entire forecast was prepared at the beginning of the year is at least as good as the other two methods wherein a new forecast is prepared each month. Moreover, a forecast prepared by the simplified seasonal pattern technique with September, 1959 as the first forecasted month yielded a total error of only nine for that month. (This particular forecast is presented in appendix 3.) There is also an indication that the seasonal pattern should be retained in the forecast. The forecast prepared without the seasonal pattern had the smallest error in only two months --January and September.

To determine statistically whether the errors obtained by the different forecasting techniques are significantly different, the technique of hypothesis testing is employed. This subject is explained in many statistical texts. Chapter 25 of Duncan (5, p. 443-463) is referenced. The same text describes the testing of non-independent data (5, p. 470-471). Since the forecasts tend to vary together as described in figure 15, the data dealt with in this paper cannot be considered as independent.

The procedure followed below is that two sets of forecasts are compared at one time. The monthly difference between each forecast is obtained, and the "t test" is applied to determine whether the mean difference is significantly different from zero. The hypothesis tested is therefore: The mean difference of the forecasts equals zero. If the hypothesis is accepted, no difference in the forecast is indicated. If the hypothesis is rejected, then it can be said that the forecasts obtained by the two systems are different.

The t statistic is obtained by the following expression.

$$t = \frac{\bar{d}}{\sqrt{\dfrac{\Sigma d^2 - \dfrac{(\Sigma d)^2}{n}}{n(n - 1)}}}$$

Where d represents the individual differences between forecasts; $\bar{d}$, the average difference; and n the number of months (therefore the number of differences) being compared. The forecasts compared are for nine months --January through September. The statistic computed as above will have the t distribution with nine minus one, or eight degrees of freedom. From the table of percentage points of t distribution (5, p. 870), the probability is 95% that t falls between -2.306 and 2.306 and 5% that t is outside these limits. If the computed value of the t function lies between these values, the hypothesis is accepted, and if outside the limits, it is rejected.

The forecasts and difference between forecasts for the six Tru-Vue Cards are presented in table 13. The differences between the forecast obtained by the simplified seasonal pattern method and the other two techniques are given. These differences are used in the t function given above. The value obtained from the differences between the simplified seasonal pattern forecast and the forecast made by exponential smoothing with seasonal pattern is 0.23. Since this is well within the limits of plus or minus 2.306, the hypothesis is accepted. The same conclusion is reached in comparing the simplified seasonal pattern forecast with the exponential smoothing without seasonal pattern. The t value obtained for the difference is 0.68. Thus, it has been shown that 95% of the time the variations found in the forecasts can be expected to stem from chance causes alone.

Discussion of Yield

In order to show the potential yield obtainable by use of a systematic forecast, consider the information presented in table 12. The average actual inventory for the six Tru-Vue Cards was 350 cards during the nine month period.

TABLE 13

FORECAST COMPARISON VALUES

Month	Simplified Seasonal Pattern Forecast	Exp. Smoothing With Seasonal Pattern		Exp. Smoothing Without Seasonal Pattern	
		Forecast	Diff.	Forecast	Diff.
January	38	75	37	107	69
February	42	90	48	123	81
March	46	86	40	106	60
April	58	70	12	90	32
May	60	65	5	87	27
June	51	49	-2	77	26
July	83	65	-18	71	-12
August	116	76	-40	63	-53
September	184	127	-57	73	-111

Exponential smoothing with seasonal pattern

$$d = 25$$

$$d^2 = 10{,}619$$

$$d = \frac{25}{9} = 2.778$$

$$t_1 = \frac{2.778}{\sqrt{\dfrac{10{,}619 - \dfrac{(25)^2}{9}}{(9)\ (8)}}}$$

$$t_1 = 0.23$$

Exponential smoothing without seasonal pattern

$$d = 119$$

$$d^2 = 32{,}625$$

$$d = \frac{119}{9} = 13.222$$

$$t_2 = \frac{13.222}{\sqrt{\dfrac{32{,}625 - \dfrac{(119)^2}{9}}{(9)\ (8)}}}$$

$$t_2 = 0.68$$

As a basis for the yield study, a comparable figure would be the average inventory that might have been obtained by use of the simplified seasonal forecast plus a safety stock.

A safety stock of 22 cards is indicated by use of two standard deviation units as discussed in the section on demand frequency distributions. While individual standard deviations could be prepared for each card, the accuracy of the forecasting system does not warrant the effort. A safety stock of 22 cards for this period is somewhat arbitrary, since it is the standard deviation of a composite lot of Tru-Vue Cards. However, it fits the theme of simplicity emphasized in the simplified seasonal pattern technique.

Using a safety stock of 22 cards and adding the individual simplified seasonal forecasts, the average monthly inventory for the six Tru-Vue Cards (table 12) would be 208 cards. This figure is 40 percent less than the 350 card average actual inventory. Considering that the annual carrying costs represent at least ten percent of the initial cost, this is significant. Since the annual or semi-annual forecasts for the entire Tru-Vue line can be prepared in less than one day by one person, the indicated yield is very significant.

The nine month forecast described above contains three shortages, even with a safety stock. However, all are in the month of January where abnormally high demands occurred on cards F-1, F-2, and D-5. By comparison, it is noted that the actual inventory is on the negative side five times, and these shortages occurred in January, February, April, July and August, wherein the month of January was the only abnormally high month. Card F-1 was low in all five instances.

One other factor noted is the high actual inventory in the first quarter when demand is low, and the drop in inventory during the last of the third quarter when demand is starting up. Had the inventory been based on the simplified seasonal pattern forecast, this situation would have been reversed. It is certainly more desirable to have a short forecast immediately after the peak season and a long forecast going into the peak season. Such would have been the case had this forecasting system been used as the basis for production.

CONCLUSIONS

Systematic preparation of demand forecasts can minimize inventory, and enhance the planning phases of production. Even with widely fluctuating demand pattern, demonstrated by the Tru-Vue Cards, an inventory based on systematic forecasting is shown to be considerably less than actually occurred. During the first nine months of 1959, an inventory based on the simplified seasonal pattern forecasting technique would have resulted in 40 percent less average inventory.

The forecasting systems applied to the Tru-Vue Cards were not statistically different as measured by their monthly differences. In evaluating the two techniques from the standpoint of ease of preparation, the simplified seasonal pattern forecast is most desirable. The more elegant exponential smoothing technique is tedious and time consuming. The use of a computer is warranted in preparing forecasts by this method. The simplified seasonal pattern forecasting technique is quite easily prepared on a desk calculator. Annual or semi-annual forecasts for the entire Tru-Vue line can be prepared in less than one day by one person with this technique.

The Tru-Vue Card demand follows a definite seasonal pattern, although the monthly fluctuations in 1958 did not follow the expected cycle. A forecast prepared without the seasonal pattern was not statistically different than the forecasts prepared by the other methods. However, retaining the monthly base series in the forecasting systems is justified by its regularity during the first four years of Tru-Vue history. Stated differently, to discard the monthly base series because one year did not conform is unwarranted.

The desirability of reducing inventory and the importance of the forecast in effecting this goal has been indicated. Other economic factors enter the picture in obtaining optimum inventory levels. Seasonal balancing of production loads and labor forces is one of the more important items. Production runs must also be based on economical lot sizes. But, always, the forecast is the basis for the planning activity. Intelligent scheduling of production can be accomplished only with assistance of a forecast. The formulas for economic lot size include a factor for future use. The forecast is the keystone, the basis for the entire planning process.

American Management Association's 1956 survey of forecasting in industry is pertinent to this discussion. Of the 297 companies surveyed, 244 (or five out of six) conducted a formal sales forecasting program (11, p. 141). General results of the survey are summarized by the following excerpts from the article.

> "Two out of three of the companies surveyed are planning some changes in their sales forecasting program. ----Better techniques are the goal of 63 companies. Of these the largest group hopes to eliminate or at least lessen the role of 'hunch estimates' by instituting a more formal and scientific approach to the basic problems (11, p. 156)."

With the use of forecasting techniques as prevalent in industry as this survey indicates, a systematic forecasting program is justified from a purely competitive aspect.

RECOMMENDATIONS

The use of the simplified seasonal pattern forecasting technique on a semi-annual basis is recommended with the following provisions: (1) Management should assist in predicting the next year's trend; (2) A safety stock equal to twice the standard deviation of the actual monthly demand for the past year should be used. This level of safety stock will give immediate service approximately 95 percent of the time.

Each time the forecast is prepared, the 1954-1957 demand base series should be ratioed to the most recent 12 months' actual demand. The monthly pattern of changes is then obtained from this ratioed base. With this new base and management's trend prediction, the forecast for each year becomes a routine operation.

It is pointed out that complete coverage of the subject of inventory control is beyond the scope of this paper. The student of the subject should investigate systems based on average usage and reserve stock. Other procedures which have not received attention in this paper are the aspects of the lead time, the reorder point, and the economic lot size. The text by Welch (12) is devoted to inventory control. Magee's text (9) contains several chapters on the subject. These references develope the specific techniques outlined above, and would be of value to anyone investigating the various aspects of inventory control.

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APPENDIX I

Computer Programs For Exponential Smoothing Forecasts

ALCOM PROGRAM

Forecasts by Exponential Smoothing With Seasonal Pattern

1.	X in	last month's demand
2.	R in	average ratio, initial value = 1
3.	T in	trend, initial value = 0
4.	M in	mean absolute deviation, initial value = 20
5.	E in	cumulative deviation, initial value = 0
6.	O in (letter)	initial value = alpha
7.	Y in	1st month's base
8.	Z in	next month's demand
9.	S in	next month's base
10.	U in	next two months' base
11.	V in	next four months' base
12.	W in	next six months' base
13.	A in	alpha
14.	CR out	
15.	O - A = F (letter)	
16.	If F neg to 67	
17.	A = O (letter)	
18.	X / Y = F	
19.	F - R = F	
20.	A . F = F	
21.	F + R = F	new average ratio
22.	F - R = C	change

23.	C - T = C	
24.	A . C = C	
25.	C + T = T	new trend
26.	F = F	new average ratio
27.	R - R = F	
28.	A . F = F	
29.	F + T = F	
30.	F / A = F	
31.	F out	expected ratio
32.	Char sp	
33.	Char sp	
34.	S . F = D	
35.	D out	next month's expected demand
36.	Char sp	
37.	Char sp	
38.	U . F = B	
39.	B out	next two months' expected demand
40.	Char sp	
41.	Char sp	
42.	V . F = B	next four months' expected demand
43.	B out	
44.	Char sp	
45.	Char sp	
46.	W . F = B	next six months' expected demand
47.	B out	

```
48.  Char sp
49.  Z - D = D          error deviation (one month's
                        lead time)
50.  D + E = E          cumulative deviation
51.  If D neg to 69
52.  D - M = D
53.  A . D = D
54.  D + M = M          mean absolute deviation
55.  M out
56.  Char sp
57.  Char sp
58.  E / M = D
59.  D out              normal cumulative deviation
60.  Char sp
61.  Char sp
62.  Z = X
63.  S = Y
64.  CR out
65.  CR out
66.  to 8
67.  0 = E (zero)
68.  to 17
69.  - D = D
70.  to 52
71.  end
```

MACHINE LANGUAGE PROGRAM FOR EXPONENTIAL SMOOTHING FORECASTS WITH SEASONAL PATTERN ON ALWAC III-E COMPUTER

```
4004
8529872e  872e9d60  9d60491b  4914411d
9d60491d  4914872e  872e9d60  5b1e9dc9
872e9d60  9d60491e  491c872e  490b410b
4917872e  872e9d60  83431121  5b179d47
9d604919  491f872e  df4d4114  490b4106
872e9d60  9d604918  5b069d47  83411120
4912872e  872e9d60  490b790b  83431124
9d60490a  491a872e  1d3b7906  00001b20
4104
5b0b9dc7  5b089dc7  490b410b  9dc74909
490b410b  49084108  5b199d41  4109872f
5b179d41  5b199d41  490b410b  9d60df20
490b410b  4919790b  5b069dc9  df20411a
5b179d47  49174117  490b410b  5b0b9dc7
49084108  5b199d47  972f9d60  49074107
5b199d47  490b4106  df20df20  872f9d60
49084106  5b0b9dc7  41185b0b  83421120
4204
df20df20  872f9d60  49094106  9dc94909
411b5b0b  df20411f  5b099dc7  4109872f
9dc74907  5b099d47  49094109  9d60df20
4107872f  49094109  5b129d41  df20791f
9d60df20  5b0a9d41  49124112  491d7918
df20411c  490a7909  872f9d60  491edf4d
5b0b9dc7  1d3f4109  df20df20  83431120
49074107  5b129d47  410a5b12  834311a8
4304
df4d113f  9d604906  00000000  00000000
793b490a  89440000  00000000  00000000
11377909  83401132  00000000  00000000
2e004909  00000000  00000000  00000000
11330000  00000000  00000000  834211b9
834011bf  00000000  00000000  834011be
00000000  81448529  00000000  00000000
00000000  83401b32  00000000  8340112d
3a00
40 d2db364f
41 63887f82
42 731b3441
43 645e5d57
```

COMPUTER FORMAT FOR EXPONENTIAL SMOOTHING WITH SEASONAL PATTERN

Data is for Tru-Vue Card F-1 From December 1958 to September 1959. Alpha = 0.1

1	2	3	4	5	6	7
4020 23 1 0 10 0 .1 30 26 29 50 81 112 .1 (input)						
.95566642	27.714324	47.783317	77.408973	107.03463	9.1714324	-.18692007
10 21 37 68 97 .1 (input)						
.94254457	19.793436	34.874149	64.093025	91.426818	9.2336320	-1.2462874
40 16 31 62 87 .1 (input)						
.85158729	13.625396	26.399206	52.798412	74.088088	10.947728	1.3579841
17 15 31 60 86 .1 (input)						
1.1586816	17.380224	35.919128	69.520896	99.646614	9.8909778	1.4446294
5 16 31 56 90 .1 (input)						
1.1611745	18.578792	35.996410	65.025772	104.50570	10.259758	.08848408
17 15 29 55 103 .1 (input)						
1.0069571	15.104356	29.201757	55.382640	103.71658	9.4233474	.29750239
6 14 25 59 118 .1 (input)						
1.0310546	14.434763	25.776365	60.832331	121.66444	9.3244886	-.60392534
5 11 26 74 133 .1 (input)						
.91769176	10.094609	23.859985	67.909187	122.05300	8.9015007	-1.2049546
11 15 34 93 143 .1 (input)						
.82591176	12.388675	28.080999	76.809791	118.10537	8.1502180	-1.4864116
23 19 48 107 144 .1 (input)						
.80082589	15.215691	38.439640	85.688369	115.31892	8.1136264	-.53370356
29 29 59 109 140 .1 (input)						
.87049812	25.244444	51.359386	94.884292	121.86973	9.7977075	-2.9889352

EXPLANATION OF COMPUTER FORMAT FOR EXPONENTIAL SMOOTHING WITH SEASONAL PATTERN

(see actual copy of format on preceding page)

Each complete row presents one month's forecast information as follows:

Column 1: Expected ratio of demand.
Column 2: Demand forecast for one month lead time.
Column 3: Demand forecast for two months' lead time.
Column 4: Demand forecast for four months' lead time.
Column 5: Demand forecast for six months' lead time.
Column 6: Mean absolute deviation of forecast error for one month lead time.
Column 7: Normal cumulative deviation of forecast error for one month lead time.

There are fourteen initial inputs, in the following order:

1. Informs computer where program is stored (4020).
2. Last month's demand.
3. Average ratio, initial value is one.
4. Trend, initial value is zero.
5. Mean absolute deviation, initial value is ten.
6. Cumulative deviation, initial value is zero.
7. Alpha value.
8. Base series demand value corresponding to month used in item 2, above.
9. This month's demand (month following input 2, above).
10. Base series demand value corresponding to month used in item 9, above.
11. Base series demand value for two months' lead time.
12. Base series demand value for four months' lead time.
13. Base series demand value for six months' lead time.
14. Alpha.

Each succeeding month's inputs are the monthly value for items 9 through 14 above.

ALCOM PROGRAM

Forecasts by Exponential Smoothing Without Seasonal Pattern

1.	B in	value of base series for last month
2.	T in	initial value of trend = 0
3.	N in	count of month's initial value = 1
4.	G in	cumulative error (initial value = 0)
5.	H in	sum of errors squared (initial value = 0)
6.	W in	1st month's demand
7.	X in	2nd month's demand
8.	A in	alpha
9.	Char sp	
10.	Char sp	
11.	CR out	
12.	W - B = F	
13.	A . F = C	change in average
14.	C + B = B	new average
15.	C - T = F	
16.	A . F = F	
17.	F + T = T	
18.	T out	new trend
19.	Char sp	
20.	Char sp	
21.	B - T = F	

22. A . F = F

23. F + T = F

24. F / A = D

25. D out — expected demand

26. Char sp

27. Char sp

28. D - X = E

29. E . E = S

30. G + E = G

31. S + H = H

32. N - 2 = K

33. If K neg to 44

34. N . H = F

35. G^2 = K — G . G = K

36. F - K = F

37. N - 1 = K

38. N . K = K

39. F / K = F

40. $\sqrt{F}$ = F — root F = F

41. F out — standard deviation of errors

42. Char sp

43. Char sp

44. X = W

45. N + 1 = X

46. to 7

47. end

MACHINE LANGUAGE PROGRAM FOR EXPONENTIAL SMOOTHING FORECASTS WITHOUT SEASONAL PATTERN ON ALWAC III-E COMPUTER

```
4004
8529872e  872e9d60  41065b0b  410b5b19
9d604907  491c872e  9dc74908  9d414919
872e9d60  9d60491d  41085b07  4119872f
4919872e  872e9d60  9a414907  9d60df20
9d604913  4906df20  41085b19  df204107
872e9d60  df20df4d  9d47490b  5b199d47
490c872e  411c5b07  41065b0b  83411120
9d60490d  9d47490b  9dc7490b  00001b20
4104
490b4106  df20df20  9d41490d  410b5b10
5b0b9dc7  41095b1d  41135b3f  9d47490b
490b410b  9d47490a  9d474910  41135b37
5b199d41  410a5b0a  79101d3b  9d474910
490b410b  9dc74918  41135b0d  83421120
5b069dc9  410c5b0a  9dc7490b  80000081
49094109  9d41490c  410c5b0c  834211a1
872f9d60  41185b0d  9dc74910  80000082
4204
41135b10  df64791d  00000000  00000000
9dc74910  491c4113  00000000  00000000
410b5b10  5b3f9d41  00000000  00000000
9dc9490b  4913113b  00000000  00000000
8735410b  834011bf  00000000  00000000
9d60490b  00000000  00000000  00000000
410b872f  00000000  00000000  834011a5
9d60df20  00000000  00000000  80000081
3a00
40 eda9be73
41 edbb5fe0
42 1404c538
```

COMPUTER FORMAT FOR EXPONENTIAL SMOOTHING FORECAST
WITHOUT SEASONAL PATTERN

Data is for Tru-Vue Card F-1 from October
1957 to August 1959. Alpha at 0.1

4020 46.3 0 1 0 0 95 63 .1

.48699989	55.552986		7 .1
.55659985	57.362384	40.877414	o17 .1
.04741001	48.244377	29.451789	o22 .1
- .26550787	42.346343	24.146511	o4 .1
- .46631628	38.265480	21.445934	o25 .1
- .80430787	31.377321	20.748983	o24 .1
- .86003798	29.514137	20.002611	o15 .1
- .90657901	27.769821	18.699022	o17 .1
-1.0252113	24.609222	17.911455	o46 .1
-1.0910514	22.333045	21.398626	o12 .1
- .84347146	25.946012	20.300615	o23 .1
- .97449684	22.613059	19.781801	o26 .1
- .96088248	21.897235	19.496486	o10 .1
- .91024601	21.898443	18.732313	o40 .1
-1.0201278	18.900438	19.899271	o17 .1
- .79893106	22.083049	19.251724	o5 .1
- .84177225	20.470134	18.707984	o17 .1
- .98805582	16.848972	18.285747	o6 .1
- .97666501	16.077342	17.773046	o5 .1
-1.0676717	13.371546	17.299341	o11 .1
-1.1407104	10.916138	16.972991	o23 .1
-1.1284646	10.008096	17.180355	o

Explanation of format is presented on the following page.

EXPLANATION OF COMPUTER FORMAT FOR EXPONENTIAL SMOOTHING WITHOUT SEASONAL PATTERN

(see actual copy of format on preceding page)

The first row contains the initial input instructions. Nine instructions are required in the following order:

1. Informs computer where program is stored (4020).
2. "Old" value of average demand (base series value for last month).
3. Trend, initial value is zero.
4. Informs computer that this is first month in forecast, initial value is one.
5. Cumulative error, initial value is zero.
6. Sum of errors squared, initial value is zero.
7. This month's demand.
8. Next month's demand.
9. Alpha.

Each succeeding month's inputs are the monthly values for items eight and nine.

Each row below the initial input presents one month's forecast information as follows:

Column 1: New trend.
Column 2: Expected demand next month.
Column 3: Standard deviation of forecast errors.
Column 4: Succeeding months' input instructions.

APPENDIX II

Procedures For Desk Calculator Forecast Computations By Exponential Smoothing

PROCEDURES FOR DESK CALCULATOR FORECAST COMPUTATION BY EXPONENTIAL SMOOTHING FACTOR METHOD WITHOUT DEMAND PATTERN

1. Record most current demand figure (D_o).

2. Calculate new average ratio in three steps. (Initial value for average ratio is the base series value for the month immediately prior to the starting month)

   a. Subtract the average ratio last month from the new demand (D_o).

   b. Multiply the difference obtained in step a by alpha. Record this value as the change in the average.

   c. Add the product in step b to the average ratio last month to obtain the new average ratio.

3. Compute the trend in the three steps. (Initial value for the trend can usually be taken as zero)

   a. Subtract the old trend from the value of the change calculated in 26.

   b. Multiply the difference in step 4-a by alpha.

   c. Add the product in step 4-b to the old trend to obtain the new trend.

4. Compute the expected demand next month in two steps.

   a. Evaluate $\frac{1-\alpha}{\alpha} \cdot T_n$.

      When T_n is the new value of trend 4-c.

   b. Add the results from step a to the new average (2-c) to obtain the expected demand.

5. To obtain the forecast over the lead time, multiply the forecast (4-b) by the number of months in the lead time. To extrapolate the trend, add the results of the

following three steps.

a. Multiply the number of months in the lead time by the same number plus one.

b. Divide the product obtained in a by two.

c. Multiply the quotient in step 5 times the trend (3-c) to obtain the trend extrapolating factor.

6. Calculate control points as in steps 11 through 14 of the procedure for cyclic demand, exponential smoothing forecasts.

PROCEDURE FOR DESK CALCULATOR FORECAST COMPUTATION BY EXPONENTIAL SMOOTHING FACTOR METHOD WITH SEASONAL DEMAND PATTERN

1. Record most current demand figure (D_o).

2. Record value of base series for corresponding month (D_b).

3. Calculate ratio: $D_o / D_b = R$

4. Calculate average ratio, R_a, in three steps:

   a. Subtract the average ratio last month (R_{a-1}) from the new ratio (R) obtained in step 3 above $R - R_{a-1}$ (initial value of R_{a-1} equals one)

   b. Multiply the difference obtained in step a by alpha. Record this value or the change in the average ratio.

   c. Add the product in step b to the average ratio last month to obtain the new average ratio (R_a).

5. Calculate the new trend (T_n) in three steps:

   a. Subtract the trend last month (T_{n-1}) from the new value of change obtained in step 4-b above. (Initial value of T_{n-1} equals zero).

   $$C - T_{n-1}$$

   b. Multiply the difference obtained in step a by alpha.

   c. Add the product in step b to the trend last month (T_{n-1}) to obtain the new trend (T_n).

6. Calculate the new expected ratio (R_e) in two steps:

   a. Evaluate $\frac{1 - \alpha}{\alpha}$ (T_n) (Where T_n is the new value of trend).

   b. Add the product from step 7-a to the new average ratio (R_a)obtained in step 4-c. Their sum is the new expected ratio (R_e).

7. Calculate expected demand next month by multiplying R_e times the value of the base series for next month.

8. Calculate expected demand for the next two by multiplying R_e times the sum of the base series for the next two months.

9. Calculate the expected demand for the next four months by multiplying the sum of the base series for the next four months by R_e.

10. Calculate expected demand for the next six months by multiplying the sum of the base series for the next six months by R_e.

11. Calculate the deviation (forecast error) by subtracting the current demand D_o from the one month forecast made <u>last</u> month.

12. Calculate the new mean absolute deviation d_n in three steps:

    a. Subtract the old mean absolute deviation from the <u>absolute</u> value of current deviation obtained in <u>step 11</u> above. (The initial value of the old mean absolute deviation is 20 for Tru-Vue Cards).

    b. Multiply the difference obtained in step a by alpha.

    c. Add the product in step b to the old mean absolute deviation to obtain the new mean absolute deviation.

13. Calculate the cumulative deviation (cumulative errors) by adding the deviation this month (step 11) to the cumulative deviation last month (add algebraically).

14. Calculate the control point by dividing the new cumulative deviation (step 13) by the new mean absolute deviation (step 12-c). As long as the quotient is within plus or minus four the forecast is within the three-sigma control limits.

APPENDIX III

Forecasts Prepared By Simplified

Seasonal Pattern Technique

TABLE 14

1959 FORECAST TRU-VUE F-1

SIMPLIFIED SEASONAL PATTERN

1957 Sales = Z = 395

1958 Sales = A = 241 change = - 154

1959 Forecast = F = 225 10% of change = - 15.4

R = - 22

$$M = \frac{2\ A - R - Z}{12} = \frac{(2)\ (241) + 22 - 395}{12} = 9.08$$

$$k = \frac{F - 2A + Z}{78} = \frac{225 - (2)\ (241) + 395}{78} = 1.77$$

	Previous Month	Normal Change	Trend	Forecast
January	9.1	-5	1.8	6
February	6	-1	1.8	7
March	7	+ 1	1.8	9
April	9	-1	1.8	10
May	10	-1	1.8	11
June	11	-3	1.8	10
July	10	+ 4	1.8	16
August	16	+ 4	1.8	21
September	21	+ 10	1.8	33
October	33	+ 1	1.8	36
November	36	-1	1.8	37
December	37	-8	1.8	31

TABLE 14 Continued

1959 FORECAST TRU-VUE F-2

SIMPLIFIED SEASONAL PATTERN

1957 Sales = Z = 298

1958 Sales = A = 232 change = - 66

1959 Forecast = F = 225 10% change = - 6.6

$$M = \frac{2\ A - R - Z}{12} = \frac{(2)\ (236) + 22 - 298}{12} = 15.7$$

$$k = \frac{F - 2\ A + Z}{78} = \frac{225 - (2)\ (232) + 298}{78} = 0.492$$

	Previous Month	Normal Change	Trend	Forecast
January	15.7	-5	.5	11
February	11	-1	.5	11
March	11	+ 1	.5	11
April	11	-1	.5	11
May	11	-1	.5	10
June	10	-3	.5	8
July	8	+ 4	.5	12
August	12	+ 4	.5	18
September	18	+ 10	.5	27
October	27	+ 1	.5	29
November	29	-1	.5	28
December	28	-8	.5	21

TABLE 14 Continued

1959 FORECAST TRU-VUE F-4

SIMPLIFIED SEASONAL PATTERN

1957 Sales = Z = 280

1958 Sales = A = 218 change = - 62

1959 Forecast = F = 212 10% change = - 6.2

$$M = \frac{2A - R - Z}{12} = \frac{(2)\ (218) + 22 - 280}{12} = 14.8$$

$$k = \frac{F - 2A + Z}{78} = \frac{212 - (2)\ (218) + 280}{78} = 0.72$$

	Previous Month	Normal Change	Trend	Forecast
January	14.8	-5	0.7	10
February	10	-1	0.7	10
March	10	+ 1	0.7	12
April	12	-1	0.7	12
May	12	-1	0.7	11
June	11	-3	0.7	9
July	9	+ 4	0.7	14
August	14	+ 4	0.7	18
September	18	+ 10	0.7	29
October	29	+ 1	0.7	31
November	31	-1	0.7	30
December	30	-8	0.7	23

TABLE 14 Continued

1959 FORECAST TRU-VUE F-5

SIMPLIFIED SEASONAL PATTERN

1957 Sales = Z = 377

1958 Sales = A = 209 change = - 168

1959 Forecast = F = 182 10% of change = - 16.8

$$M = \frac{2A - R - Z}{12} = \frac{(2)\ (209) + 22 - 377}{12} = 5.25$$

$$k = \frac{F - 2A + Z}{78} = \frac{182 - (2)\ (209) + 377}{78} = 1.81$$

	Previous Month	Normal Change	Trend	Forecast
January	5.25	-5	1.8	2
February	2	-1	1.8	3
March	3	+1	1.8	6
April	6	-1	1.8	7
May	7	-1	1.8	8
June	8	-3	1.8	6
July	6	+ 4	1.8	12
August	12	+ 4	1.8	18
September	18	+ 10	1.8	30
October	30	+ 1	1.8	33
November	33	-1	1.8	33
December	33	-8	1.8	27

TABLE 14 Continued

1959 FORECAST TRU-VUE D-5

SIMPLIFIED SEASONAL PATTERN

1957 Sales = Z = 453

1958 Sales = A = 274 change = - 179

1959 Forecast = F = 252 10% change = - 17.9

$$M = \frac{2A - R - Z}{12} = \frac{(2)\ (274) + 22 - 453}{12} = 9.75$$

$$k = \frac{F - 2A + Z}{78} = \frac{252 - (2)\ (274) + 453}{78} = 2.01$$

	Previous Month	Normal Change	Trend	Forecast
January	9.75	-5	2.0	7
February	7	-1	2.0	8
March	8	+ 1	2.0	11
April	11	-1	2.0	12
May	12	-1	2.0	13
June	13	-3	2.0	12
July	12	+ 4	2.0	18
August	18	+ 4	2.0	24
September	24	+ 10	2.0	36
October	36	+ 1	2.0	39
November	39	-1	2.0	40
December	40	-8	2.0	34

TABLE 14 Continued

1959 FORECAST TRU-VUE D-6

SIMPLIFIED SEASONAL PATTERN

1957 Sales = Z = 344

1958 Sales = A = 194 change = - 150

1959 Forecast = F = 179 10% change = - 15

$$M = \frac{2A - R - Z}{12} = \frac{(2)\ (194) + 22 - 344}{12} = 5.5$$

$$k = \frac{F - 2A + Z}{78} = \frac{179 - (2)\ (194) + 344}{78} = 1.7$$

	Previous Month	Normal Change	Trend	Forecast
January	5.5	-5	1.7	2
February	2	-1	1.7	3
March	3	+ 1	1.7	6
April	6	-1	1.7	6
May	6	-1	1.7	7
June	7	-3	1.7	6
July	6	+ 4	1.7	11
August	11	+ 4	1.7	17
September	17	+ 10	1.7	29
October	29	+ 1	1.7	31
November	31	-1	1.7	32
December	32	-8	1.7	26

TABLE 15

SEPTEMBER 1959 THROUGH AUGUST 1960 TRU-VUE FORECASTS

BY SIMPLIFIED SEASONAL PATTERN

Factor	F-1	F-2	F-4	F-5	D-5	D-6
	Tru-Vue Card					
A	200	160	148	149	227	130
Z	364	315	245	313	393	300
Change	-164	-155	-97	-164	-166	-170
F	184	145	138	133	210	113
M	2.8	0.25	4.1	-1.4	4.9	-3.5
k	1.9	1.8	1.1	1.9	1.9	1.96

$R = 2$

A = Actual demand for twelve month period, September 1958 through August 1959.

Z = Actual demand for twelve month period, September 1957 through August 1958.

Change = A - Z

F = A plus 10% of change

$$M = \frac{2A - R - Z}{12}$$

$$k = \frac{F - 2A + Z}{78}$$

TABLE 15 Continued

Tru-Vue Card

Month	F-1	F-2	F-4	F-5	D-5	D-6
September	15	12	15	10	17	9
October	18	15	17	13	20	11
November	18	16	17	14	21	13
December	12	9	11	8	14	7
January	9	6	7	5	11	4
February	10	7	7	6	12	4
March	13	10	9	9	15	8
April	14	11	9	10	16	8
May	15	11	9	11	17	10
June	14	10	7	10	16	8
July	20	16	12	15	22	12
August	26	22	17	21	28	17

APPENDIX IV

Actual Demands

January 1958 - August 1959

TABLE 16

ACTUAL DEMANDS - JANUARY 1958 - AUGUST 1959
Nine Selected Tru-Vue Cards

Card

Month	D-5	D-6	F-1	F-2	F-4	F-5	D-23	T-20	T-23
January	20	13	17	23	17	12	18	17	29
February	28	20	22	35	29	34	21	16	46
March	7	0	4	12	28	0	4	13	40
April	30	17	25	29	32	23	13	29	47
May	18	18	24	12	13	25	22	20	16
June	32	20	15	23	14	14	20	3	18
July	35	19	17	12	12	17	16	17	17
August	26	28	46	34	20	29	35	23	23
September	20	9	12	18	17	6	20	15	17
October	26	20	23	16	12	23	20	12	17
November	32	32	26	29	24	17	32	23	23
December	0	0	10	0	0	9	0	14	6
January	69	11	40	40	18	17	11	11	11
February	6	0	17	0	6	6	0	0	0
March	5	6	5	11	0	5	0	6	6
April	17	12	17	9	11	6	6	6	5
May	12	6	6	8	6	11	12	6	12
June	5	11	5	12	11	6	10	12	6
July	12	0	11	6	6	12	6	5	5
August	23	23	23	22	17	11	23	17	23

$$\text{Mean demand} = \frac{\text{Sum demands}}{n} = \frac{2896}{180} = \underline{16.1}$$

$$\text{Standard deviation} = \sqrt{\frac{n\ [\Sigma\ (\text{demands})^2] - [\Sigma\ \text{demands}]^2}{n\ (n-1)}}$$

$$= \sqrt{\frac{(180)(68{,}088) - (2896)^2}{(180)(179)}}$$

$$= \underline{10.96}$$