AN ALGEBRA PLACEMENT AND DIAGNOSTIC EXAMINATION
FOR PORTLAND STATE EXTENSION CENTER

by

WILLIAM OWEN BUSCHMAN

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APPROVED:

Redacted for Privacy

Professor of School of Education

In Charge of Major

Redacted for Privacy

Head of Department of Education

Redacted for Privacy

Chairman of School Graduate Committee

Redacted for Privacy

Dean of Graduate School

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W. O. B.

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# AN ALGEBRA PLACEMENT AND DIAGNOSTIC EXAMINATION FOR PORTLAND STATE EXTENSION CENTER

## CHAPTER I

## INTRODUCTION

At Portland State Extension Center all students are admitted who present a high school diploma or its equivalent. The lack of specific high school course requirements in mathematics means that the mathematical preparations of the entering students will be varied. Many of them will have taken only one year of general mathematics while in high school; others will have taken as many as four years of the more traditional high school subjects, algebra, geometry and trigonometry. A single mathematics course offering, therefore, is not suitable for all new college students.

Provision must be made for the varied preparation students will have received in high school. Elementary algebra, intermediate algebra and geometry classes are provided for those who need them. College algebra classes are open to those students who come adequately prepared. The variety of preparations with which students come creates a difficult problem in placing them in the proper courses.

All students who intend to take mathematics are required to take a placement examination. This examination was designed to help the student select the proper course with which to start his mathematical training. The examination was prepared by several members of the staff of Portland State Extension Center and revised as needed. Students are placed in the elementary, intermediate, or college algebra classes on the basis of their scores on the placement examination. The examination scores which were required for entry into the intermediate or college algebra course were set arbitrarily by someone on the staff. However, no study has been made of the success or failure of students who were placed by the use of this method. The large number of withdrawals and failures in the intermediate and college algebra courses has indicated that this method of placement was not too satisfactory.

## School Situation

Portland State Extension Center, located in downtown Portland, Oregon, is a part of the Extension Division of the State System of Higher Education. It operates both a day and an evening program. In most fields of study, the course offerings of the day program are the same as the lower division offerings at the other schools of the state system. The evening program offers upper division and graduate work in certain fields of study. In general the students look upon the day program as that of a junior college, and most of them plan to continue their education at other colleges. For the benefit of those who do not plan to go on to college, a few terminal courses are offered.

The majority of the students live at home, and for a large portion of them it is necessary to work at least part time in order to go to school. The results of a recent survey indicated that most of the students felt part-time employment was necessary, and that approximately half of the students are working.

Most of the students are graduates of one of the ten large city high schools or of high schools in the Portland area. Generally these schools provide a dual mathematics program: (a) general mathematics for those whose interests lie outside of the field of science, and (b) the traditional high school mathematics courses for those who wish to take them. Some students who have taken as many as four years of the traditional high school mathematics courses are only prepared for beginning algebra when they start their college work.

Few of the entering students have had more than one or two years of high school mathematics to their credit.

Often these courses were taken in the first two years of high school, and much of the material learned in these courses will have been forgotten by the time the students enter college. The college, therefore, has found it necessary to provide courses at the high school level for those students whose background was deficient; these review courses are offered in elementary algebra, intermediate algebra and in plane geometry.

Though all of the instructors in the mathematics department teach some review courses, most of these courses have been taught by instructors from other departments, such as engineering, chemistry, physics, business and languages. Each instructor, of course, emphasizes different topics and uses different methods of instruction. These courses in general have emphasized the manipulative skills and rote processes, rather than generalization and transfer of processes.

## Reasons for Mathematical Placement Problems

The entering students' lack of competence in regular college mathematics work may be due to the fact that the high school offerings were too limited. This is particularly true of many of the smaller high schools, but may also be true of larger ones. Schorling states that this lack of competence "is due in part to the fact that too many administrators, guidance workers, teachers and others fail to realize the large number of persons who are now needed to do scientific, professional or other highly technical tasks of modern society" (171, p.26). High school students too frequently lack reliable information about the mathematics needed on jobs and in professional careers. A lack of knowledge of the applications of mathematics to engineering, agriculture, mining, medicine, and the like on the part of high school teachers is usually responsible for this situation (171, p.27).

According to Benz (14, p.341), "Many colleges and universities, especially in the Middle West and Far West, do not require evidence of the successful pursuit of algebra and plane geometry for entrance." High school students who plan to go on to college will be influenced to a considerable extent by the college entrance requirements. Many of those who plan to go on to college will not take algebra and geometry if these courses are not required for entrance by colleges in their community.

Among high school students who do not take a college preparatory course are many who plan to go directly to work. These students may omit mathematics from their programs. This group is usually of lower mental ability, but may also include others who cannot plan for college because of economic or other conditions (14, p.338). Some of these may later go to college. For them the choice of a major field which requires mathematics may necessitate a program which is longer than most students' because of their deficiency in mathematical preparation. Such students should be made aware of this situation at the earliest opportunity, as financial or other considerations may make an alternate choice more practical.

Entering college students reveal wide differences in their ability to do college mathematics. Tiegs states that the wide differences in ability found among those who present the same credentials constitute a difficult guidance problem at the college and university level. High school graduation does not at present guarantee any degree of competence (190, p.255). The same variations in background were noted in a study made by Held (87, p.39) even among students who presented evidence of the same courses.

Occasionally a student has so little mathematical ability that he should not attempt any mathematics course whatsoever while in college (171, p.25). Brown says that this does not occur as frequently as is generally believed. He cited Laura Guggenbull's conclusion that among the students who failed a college mathematics course only one in thirty ever made an <u>A</u> in any college course, while five out of six who failed in mathematics courses also failed at least one other course (20, p.26). The student who has no mathematical ability should be discovered as early as possible, so that he can choose a program which is more appropriate for his interests and aptitude.

## Nature of the Problem

Since entering students have a variety of backgrounds in mathematics, it is necessary to establish criteria for selecting the proper course with which to begin their mathematical training. Three courses are offered for beginning students: (a) elementary algebra, (b) intermediate algebra, and (c) college algebra. The student's lack of familiarity with these courses makes it impossible for him to choose properly among the three. Those who are familiar with the courses cannot help the student select the proper course unless they have some method of evaluating the student's ability to do work in college mathematics.

Proper placement is of importance to the student for two reasons: (a) to prevent his enrollment in a course which is too advanced and in which he would be unable to do satisfactory work, and (b) to prevent his enrollment in a course which is too elementary. The first is of importance because a student may become discouraged if the work is too difficult for him; further, if he fails the course, the grade becomes a part of his permanent record. The second situation may leave a student bored and disinterested, and consequently may cause the development of careless study habits. Either situation may hinder the proper development of the student in his studies and therefore should be avoided.

The use of high school records in the placing of freshman college students in mathematics classes is unreliable because mathematics is one of the subjects which is most readily forgotten if not used constantly (173, p.4). This problem has been of special significance since World War II because many students have not gone directly from high school to college. By the time they return to school after an interruption of several years, students have forgotten much of the mathematics which they had previously learned. Monroe notes (140, p.885), in reference to college work, that there was "a tendency for any record to lose prognostic value after a year or two and that the best single indicator of success in any given semester is the previous semester's record. These findings have been confirmed by numerous studies."

The mathematics staff of Portland State Extension Center has continually rejected the use of available standardized examinations because: (a) they do not follow the course content closely enough, and (b) the norms provided are not based on the group on which they are to be used.

The staff agrees with Ross that "For purposes of classification what is needed is a set of norms for the school itself. . . . the more specific the norm the more useful it becomes" (163, p.307).

The examination which has been used in the past is unsuitable for placement purposes because: (a) it is difficult to score, (b) no norms have been established and (c) it was not prepared from a careful study of the course content. The difficulty in scoring is mostly due to the fact that each answer must be read. The person scoring the examination must decide whether the answer is acceptable if given in an equivalent form.

## Purpose and Scope of the Study

This study was undertaken: (a) to develop algebra examinations to be used for placement purposes at Portland State Extension Center, and (b) to establish some meaningful norms for their use at Portland State Extension Center.

The contents of the elementary and intermediate algebra courses were studied, and examinations for each were prepared. These will be referred to as Part I and Part II respectively. The examinations, in preliminary form, were given at the end of the fall term of 1952 to all elementary and intermediate algebra classes at Portland State Extension Center to determine the scores that can be reasonably expected of students who had completed these courses. From a study of the preliminary forms of the examination and from the data obtained on the fall term groups, a shortened form of each test was prepared. Items for the short forms were selected on the basis of their validity and reliability, as well as on their relation to the aims and content of the courses.

After the selection of the items for the final forms of the examinations, the papers were rescored on the basis of these items. This was done to provide a score for each student on the new forms of the examinations. Correlation of these scores with the grades earned winter term gave an estimate of the usefulness of the test in forecasting grades for the following term. The shortened revised forms of the tests were then given during winter term to the elementary and intermediate algebra classes to determine the reliability of these forms.

In the fall term two hundred fifty-two students took Part I of the examination and one hundred six took Part II. The size of these groups was similar to other studies (22, pp. 272,278; 171, p.7; 7, p.241) and was sufficiently large to provide adequate data.

## Limitations of the Study

It is not the purpose of this study to develop algebra examinations for general use, nor to develop national or regional norms. To be most useful an examination must be standardized on the group on which it is to be used (37, pp.75-77). For this reason the examinations used in this study were not given at other schools in the area or in other regions. They were not given to high school students for the same reason.

To give the examinations in other localities would necessitate more general types of examinations. It would then be impossible to follow the content of the courses at Portland State as closely as was done in the preparation of these examinations. The development of standards based on students' scores at other schools would decrease the effectiveness of the examinations for the purpose for which they were intended.

Any set of criteria which may be established for the placement of students is not infallible. Changes of interest may cause a student whose examination indicates high capability to do unsatisfactory work in a course. Illness, outside work, and other factors may interfere with the achievement of a student who is otherwise able to do satisfactory work (193, p.293). On the other hand, students whose schooling has been interrupted may quickly recover and find themselves in a course which is too elementary. These personal factors often make placement tests seem unreliable.

Factors which prevent a capable student from doing satisfactory work should be located by the counseling program if possible. A good guidance program can help solve problems of this type as they arise. An examination which is given at the beginning of the school year is not capable of predicting circumstances which may arise later.

## Procedures Used in This Study

The development of the examinations used in this study included the following:

1. The aims of the courses were stated and used as a basis for the selection of test items.

2. An analysis of the course content was made for the elementary and intermediate algebra courses and used as a basis for the selection of test items.

3. Objective algebra examinations were prepared in preliminary form and used to determine the scores which can reasonably be expected of students who have completed the courses.

4. The validity and reliability of the examinations were studied.

5. An analysis was made of the test items for the purpose of selecting the better items for the final forms of the tests.

6. The tests were rescored on the basis of the items to be retained in the final forms of the examinations. These revised scores were compared with the grades earned in the following course to establish the validity of the tests for predicting grades. These scores and grades will provide a basis for use of the tests in placement.

7. The reliability of the revised forms of the examinations was studied.

8. Diagnostic charts were prepared for use with the revised forms of the examinations to aid in the location of areas of difficulty.

## Significance of the Study

An examination based on the content of the courses which are given at Portland State Extension Center will be more reliable for guidance purposes than one which has been prepared for national or regional use. Standardized examination scores are usually interpreted in the light of national norms; as a result they are often inflexible and too general in scope to meet fully the local requirements (163, p.184). Data based upon the group on which the test is to be used will be more significant than data from any other group. Thus, this examination can contribute to better counseling in helping the new student to prepare his program.

Fewer failures should result from the placement of students in courses commensurate with their abilities. Students who need to repeat elementary work, which they may have taken at an earlier date, can be shown this need in a manner which is understandable to them. This is essential to a good guidance program.

Diagnosis of difficulties at an early date will provide a better opportunity for the student to remedy his problems. Remedial work will be more useful to the student if his difficulties are discovered before greater obstacles are encountered in more advanced courses. If deficiencies are limited to certain specific areas, the student may be able to remedy them without repeating a whole course. This saves time by allowing the student to proceed to more advanced work without repeating material that he has already mastered.

Mathematical difficulties which are so acute as to make the student's success doubtful should be discovered at the earliest possible date. Proper counseling can then be used to make the student aware of the implications of his difficulties, including the possible extension of his training over a longer period of time or the assumption of a heavier academic load. Only if the student is aware of his difficulties can he cope with them. The existence of deficiencies in ability to do successful work in college mathematics can best be shown through a comparison of actual performances by students who have taken the courses. Then the basis on which conclusions are founded can be made understandable to the new student.

# CHAPTER II

# PREDICTION OF SUCCESS IN MATHEMATICS

## College Entrance Requirements

The secondary mathematics program has always been closely related to college entrance requirements. Due largely to the influence of the academy, algebra was recognized as a requirement for college entrance in 1820; geometry, in 1844 (104, p.420). Since then the prominence of mathematics in the secondary curriculum has fluctuated considerably. The most pronounced rises in popularity took place prior to 1920 and during the impact of World War II; the greatest recessions occurred during the decade immediately preceding the last world war (24, pp.12-13).

From 1890 until shortly before World War II, the trend in the United States was toward the requirement of one year each of algebra and geometry for college entrance. Fine (24, p.29) considered these courses typical requirements for college entrance in 1946, though he noted that there was an increasing number of colleges which did not insist on these courses. This decline in mathematical requirements has influenced the place of mathematics in the secondary school curriculum.

## Place of Mathematics in the Curriculum

During the latter part of the nineteenth century (24, p.28), a trend towards a new arrangement of subject matter in the high school mathematics program began to develop. This trend resulted from a demand for a more functional mathematical program. The demand arose from students who did not plan to go on to college and from those who found algebra too difficult. Courses which were developed as a result of this trend were designed to include topics from various fields of mathematics and to bring about a better integration of subject matter. "Previous to the last world war these courses were becoming quite common. . . . However, the prescribed military programs forced a quick return to the manipulative mathematics of a traditional type." (143, p.24)

The trend before World War II was to relegate mathematics to a secondary place in the school curriculum (132, p.291). This was due in part to the increased emphasis on the social sciences. Another cause may have been the outmoded subject matter and methods which were sometimes found in mathematics courses (179, p.76).

> In the schools of the armed forces there was considerable testimony to the effect that too many men with good native ability came to the

> technical jobs of the Army and Navy with no clear understanding of such important concepts as vector, tolerance, interpolation, representative fractions, scale drawing, tangent of an angle, micrometer, vernier, gage block, metric system, practical constructions, logarithms, and the slide rule. Yet these are among the very things that would have added vitality and meaning to the abstract symbolism and theory which their former mathematics teachers struggled to teach them (179, pp.76-77).

During World War II there was considerable criticism of the secondary mathematics program because of the great deficiencies in mathematics found among the members of the armed forces. Hedrick (86, p.253) termed the shortage of people trained in mathematics "the Pearl Harbor of education." This shortage was due to the fact that many students took only a general mathematics course while in high school. As college entrance requirements relaxed, many who planned to enter college often took only a single year of mathematics while in high school.

The changes in high school mathematics courses brought criticism from the colleges, as students often were not prepared to do college work after graduation from high school. Colleges then found it necessary to give courses which had formerly been taught in high school. As some beginning students were prepared to proceed to college mathematics while others lacked preparatory courses, a decision had to be reached concerning the proper course in which the entering student should be enrolled. Kossack classes this as a recent problem:

> A few years ago this problem was non-existent, since students were assigned to one standard beginning course, usually college algebra. In recent years the amount of high school mathematics has become so variable that colleges have been forced to introduce new freshman mathematics courses. With the advent of these new courses came the problem of placement (114, p.234).

## Methods of Predicting Mathematical Success

Various methods have been used for predicting achievement in mathematics courses with varying degrees of success. According to Monroe (140, p.722),

> The order of merit in accuracy of prediction of mathematical achievement at the secondary level is: (a) good prognostic tests, (b) mathematical marks for the previous year, (c) intelligence quotient, (d) mental age, (e) achievement tests in arithmetic and algebra, (f) average mark in previous years.

Prognostic tests. Prognostic tests in mathematics have been designed for use in predicting success in algebra and other courses. Such tests, however, are intended to be given to students before they begin the study of algebra. This makes them unsuitable at the college level because many of the students will have studied at least the elementary topics of algebra while in high school. Others will have considerable work in algebra to their credit.

Mathematical marks for the previous year. Many students will not enter college directly from high school.

Others will not have studied algebra for several years prior to college entrance. School records lose their value for prognosis after a year or two (140, p.890). Therefore, mathematical marks are not suitable criteria for placing students in algebra courses.

<u>Intelligence quotient and mental age</u>. Mental age has been shown to have some predictive value in mathematics for high school students (140, p.722). It is unsuitable for older adolescents and adults "since performance ceases to improve regularly as maturity is reached. There is little difference in test performance between 15-year-olds, 18-year-olds, and 21-year-olds" (37, p.119). Since the intelligence quotient is based upon mental age, it is likewise unsuitable.

<u>Achievement tests</u>. According to Lindquist, the prediction of success in a particular course from an appropriate achievement test may be better than the general prediction. He says, "this is particularly true in the physical sciences and mathematics. In these fields it is not unusual to obtain correlations in the neighborhood of .60" (122, p.92). Mathematical achievement tests are widely used to predict success in college mathematics and to place pupils. Keller and Jonah studied placement and concluded that "preliminary assignments should be made on the basis of a mathematics training test given prior to

the beginning of classes since this is the best single criterion available" (105, p.355). Kossack also designated this as the most important criterion for placing students (114, p.237).

Average mark in previous years. For entering students average marks are not suitable criteria for prediction of success in college mathematics. It is difficult to evaluate properly records of students who come from various high schools (*supra*, p.6). For placement purposes average marks are unsuitable for the same reasons as mathematics marks, which were discussed above. According to the listing of Monroe, this is the least reliable of his six criteria. Monroe, in summarizing other studies, found a median correlation of mathematics grades with previous school marks of .37 (140, p.881).

Several authors have reported limited results with various types of criteria for predicting grades in college mathematics. The methods are not as efficient as is desirable. Correlations of grades with mental test scores are typically in the range from .40 to .50 (37, pp.264-265). High school grades have been shown to make only fair predictions. Hart, in discussing collegiate mathematics, maintained that "the collegiate performance of a student is not significantly affected by the quantity of secondary mathematics which he presents at entrance to college"

(81, p.356).

Those studies which have made use of mathematics tests for predicting grades have usually indicated that this method is a more reliable procedure than others (140, p.881 and 83, p.434). The achievement test seems to furnish the only method of obtaining correlations with grades above .50 with any degree of frequency.

No previous study of the success of placement procedures has been made at Portland State. Most of the mathematics staff members believe that the placement procedures could produce more satisfactory results. For prediction of success in college mathematics it appears from the above that an achievement test offers the most probable success. For this reason this study was undertaken to develop an algebra achievement test.

# CHAPTER III

## OBJECTIVE EXAMINATIONS

Examinations are probably as old as formal education. China, for example, used written examinations as a part of the civil service system as early as 2200 B. C. (161, p.1). The early types of examinations were essay or oral. These forms of tests were accepted and used with little or no improvement being made until recent times, mainly since World War I. The historical development of testing is summarized by Ross (163, pp.27-64).

Essay examination scores may be influenced by factors such as the quality of spelling, handwriting, and English usage, as well as bluffing (163, p.157). Also, the limited sampling which is possible with an essay examination tends to keep the reliability low. By using more questions, the objective test reduces the chance of a student getting a "lucky break" in the selection of questions (37, p.74). "Essay tests, no matter what their merits may be, are commonly considered impractical if the number of subjects is at all sizable, because of the great difficulty in scoring them reliably and because of the time required to score them" (4, p.6). In spite of the criticisms of the essay type examination in recent years, it is still accepted as the best type of examination for showing the student's ability to express himself in the language.

One of the first persons to criticize the traditional forms of examinations and to point out the need for improvement of the validity and objectivity of measuring devices was Horace Mann (165, p.2). It was not until 1920, however, that McCall published an article suggesting the objective type of examination (163, p.30). During the next ten years, many books and articles appeared which urged the use of the "new-type" examinations. Much work was done in developing various types of objective test questions and in improving the results of these tests. In more recent years, as the new-type test became more widely used, there has been considerable emphasis on the development of statistical procedures for the scoring and improvement of tests.

Objective examinations are "those in which the answers are scored . . . in such a way that all judges would agree as to the score assigned . . . for each person" (37, p.16). The examination score must also be such that it will not fluctuate from one time to another when scored by the same individual. Ruch (166, pp.91-96), Starch and Elliot (37, p.74) and others (161, pp.2-6) have shown the low reliability of grading examinations of the essay-type, both when scored at various times by the same instructor and when scored by different instructors.

Objective tests provide a large number of small units for grading instead of a small number of large ones, and by the exclusion of extraneous and unrelated material from the answers, reduce the opportunity for bluffing. A student, even though he may know little or nothing about a question, can usually add a few points to his score on an essay examination by writing something which may be only remotely related, or entirely unrelated, to the question.

Types of Objective Examinations.

There are two methods of classifying objective examinations: according to purpose, or according to type of question. Ross classifies objective tests according to type of question into two main groups (163, p.127): (1) recall types, and (2) recognition types.

Recall types. Recall-type questions are those which ask a direct question which can be answered by a single word or phrase; or they may be a sentence with an important word omitted which is to be supplied by the person writing the test. These two types of questions are widely used in the classroom and often occur on standardized tests.

In long tests, however, recall-type questions become difficult to score as they require a reading of each answer, which is an unwise procedure (166, p.184). Answers to this type of question will not be uniform; the person correcting the paper must then make a decision as to whether or not the answer is acceptable. When this is necessary, objectivity is lost and extra time is required in scoring. Travers states, "It is almost impossible to develop a completion item for which the correct answer has only one possible form" (191, p.40). Because of the fact that mathematical expressions can often be expressed in alternative forms, this type of test was not considered satisfactory for the purposes of this study.

Recognition types. Several forms of the recognition types are: alternative-response (true-false), multiple-choice, matching, rearrangement, identification, analogy and incorrect statement. The first three are the most commonly used forms. Each of these types has its own advantages and disadvantages, with some questions more suitable to one form than to the others.

Alternative-response. Among the recognition types of examinations the alternative-response (true-false) has the advantage of requiring the least time per question. This form permits a greater number of questions than any of the others, which adds to the comprehensiveness and to the reliability of the examination. However, much of this advantage is lost in scoring because of the high proportion of correct responses which may be due to guessing.

Methods of correcting for guessing have been developed (155, p.163), but they add to the difficulty of evaluating and interpreting results. Questions of a mathematical nature, especially when computations are involved, are not particularly suited to this type of question.

*Matching*. Matching questions are useful but are limited by the number of questions which may be used. Only ten to fifteen items of this type should be used because further increase in the number decreases the reliability (91, p.7). Such questions can then be used in a long test only by having a short section of matching questions, while the rest of the test must be made up of other types of questions.

If more than one type of question is used on an examination, it is usually necessary to develop some procedure for assigning different weights to different types of questions; this contributes materially to the difficulty of scoring. It may also require the various parts of the test to be timed separately, which adds unnecessarily to the difficulty of administering the test. When a timed test is administered by more than one person, careful timing is necessary, or the results will not be comparable. For these reasons matching questions were not included in the examination used in this study.

*Multiple-choice*. The multiple-choice test offers most of the advantage of speed that is possible through the use of an alternative-response type of test (157, pp.367-372). The comprehensiveness which is possible with a multiple-choice test is nearly as great as for any of the other forms of objective tests. The multiple-choice question is easier than the recall, because it is possible to recognize more on the basis of previous experience (study, hearing, lectures, etc.) than can actually be recalled (148, p.13). Other advantages are similar to those for the recall-type question.

The number of choices to be offered on each multiple-choice question usually ranges from two to seven. If only two are used, this form has all of the disadvantages of the alternative-response question and no additional advantages. At the other extreme, if seven responses are used it may often be difficult to select a sufficient number of responses which are apparently logical or acceptable. If they are not, then the various responses do not attract equally and seven choices are not actually being offered. Ruch advocates the use of at least four or five responses in order to minimize the chances of guessing the correct response (166, p.275). Ordinarily, all items should have the same number of answers (70, p.194); for most practical purposes this limits the number of choices to four or five.

All of the objective-type examinations have one very distinct advantage over the essay-type; namely, that expertness in preparation of examination papers is substituted for expertness in correction of papers (148, p.13). Much of the time and effort expended in the preparation of objective questions can be used in the development and construction of future examinations while time and effort used in the scoring of papers are lost.

Another advantage of objective-type examinations is that the use of a large number of questions reduces the effect of coaching. Only a small portion of the questions can be passed on from one class to another. Conversely, on an essay examination in which only a few questions are used, it is easy for students to pass the whole examination on to members of the next group. If used only on an incoming class, coaching probably has little or no significance. Hopkins states that the results of coaching when an objective examination is used from one period to the next, or from one year to the next, are negligible (91, p.7). Later use of objective examinations has shown that Hopkins overestimated this advantage.

With regard to a test which is to be used for diagnostic purposes, Rinsland writes (161, pp.303-304), "The objective form of test lends itself to detailed diagnosis, and this is one of its characteristics that make it so greatly superior to the usual essay or subjective examination. . . . The very nature of the objective test permits the measurement of detail." One of the purposes of the test used in this study is to serve as a diagnostic test. This is one of the reasons that it was developed as an objective-type test.

Objective examinations may be divided according to purpose into achievement, placement and diagnostic examinations. These will be discussed below.

## Achievement Examinations

"The distinguishing characteristic of an achievement test . . . is that performance in a given 'field,' whatever the limits of the field, is expressed in terms of a single score" (85, p.24). The field may cover broad topics such as mathematics, history, or English, or it may cover more specific areas. Usually such tests are designed to cover a single school subject such as algebra or geometry or some area within such a subject. Tests of educational achievement are usually designed to measure the amount of learning that has taken place in specialized fields.

Since the achievement test expresses achievement in terms of a single score, it is best suited for ranking the pupils in a given group in the order of their total achievement within the given field (85, p.23). The purposes of achievement tests are set forth by Greene

(69, pp.48-58) as follows:

1. For comparison with other pupils.
2. For comparison with other classes.
3. For measuring the efficiency of instruction.
4. For class diagnosis.
5. For individual diagnosis.
6. For pupil gradation and guidance.
7. For obtaining advance information about a class.
8. To check progress from time to time.
9. For determining the placement of a pupil entering the system for the first time.
10. For the purpose of locating pupils of superior ability.

Cronbach maintains that an attempt to predict underlies every use of testing, and that prediction is the ultimate justification of the achievement test. Testing is used for prediction of the pupil's ability to carry on some future activity; this is true even when used to establish a grade for a course, even though the teacher and pupil may overlook this aspect of the test results (37, pp.9-10).

According to Cronbach, "achievement tests attempt to determine how much a person has learned from some educational experience" (37, p.270). This is one of the purposes for which the tests used in this study were prepared. It is important to determine how much of the material the student retains at the time he enters college. Only if the student retains a sufficient knowledge of the topics which are covered in elementary and intermediate algebra classes will he be ready to do more advanced work.

Traxler divides achievement tests into two types: survey tests and diagnostic tests (195, p.190). The survey test records the pupils' total scores; these can then be used to determine a student's overall relation to the group, or they can be used for the purpose of determining the level of the group. This type of test may be useful in locating individuals whose achievement may be too low for them to do successful work, or may be used to indicate those who need further scrutiny through the use of a diagnostic procedure. "The same test may, of course, be intended to serve both of these immediate objectives; that is, it may at the same time be considered as a test of general achievement in a given field and as an instrument for diagnosing achievement within that field" (69, p.214).

The use of an achievement test may hide the fact that in certain types of achievement within a field an individual may deviate significantly from his own general level. A student may make a high total score on a general achievement test and still be relatively weak in certain areas. It is not always necessary, however, to analyze each field into homogeneous skills, traits, or abilities to get a meaningful result. "Rather, we recognize the limitation and accept it in exchange for the greater convenience in administration and interpretation of a single general achievement test as compared to a diagnostic battery of tests of special abilities" (85, p.28).

## Placement Examinations

When used for the purpose of segregation, examinations are referred to as placement examinations. These examinations should not be used only to locate the students who are likely to have difficulty, though that is the use to which these tests are usually put. It is equally important to locate those of superior ability and unusual aptitude (24, p.217).

One of the most difficult tasks which the teacher has is that of determining the proper placement of the pupil (70, p.100). In the lower schools many things such as social and emotional development, chronological age and ability as well as achievement are taken into account in the placement of pupils. At the college level, however, achievement is customarily the main basis for placement. As indicated above (*supra*, p.8), high school records are not satisfactory for placement of beginning college students, particularly in the field of mathematics.

McCall notes that some educators believe that groups which are homogeneous in ability will make more satisfactory progress, due to the fact that the teacher can teach the group much as a single unit (130, p.156). The needs of all pupils are then similar and the work can better be adapted to all of the students in the group. This matter is of considerable importance where students have such a variety of training and ability as do those covered by this study. Without segregation into the three groups, elementary, intermediate and college algebra, it would be extremely difficult to meet the needs of the pupils in any course.

## Diagnostic Examinations

Any achievement examination can be used for diagnostic purposes if the time and trouble are taken to classify and tabulate the errors. "There is no sharp dividing line between achievement tests and diagnostic tests" (135, p.356). In a sense all tests are diagnostic (70, p.19) in that they yield information about the pupil taking the test. The main characteristic of the diagnostic test which makes it distinct from the usual achievement test is that it yields records or scores on various parts of the test.

> A diagnostic test . . . is one intended to discover specific deficiencies in learning or teaching. It is a test in which a single total or composite score is of little or no significance, but on which the part scores or the percentages of correct responses to individual items are the measures sought. Tests may be diagnostic in various degrees (85, p.25).

Greene (69, p.39) defines this type of test as "A test which yields separate measures of specific abilities . . . because it points out or diagnoses the specific weakness of pupils." These definitions place emphasis on the location of weaknesses.

Woody (207, p.371) describes diagnostic tests as those which give scores indicating the relative strengths and weaknesses in various phases of a subject or ability. Cronbach (37, p.284) similarly extends the description to include both strengths and weaknesses: "A diagnostic test attempts to determine, not the level attained, but the specific strengths and weaknesses of the student." The determination of areas of strength, though frequently overlooked, is just as much a function of diagnosis as is the location of areas of weakness.

The diagnostic tests serves an entirely different function in the analysis of the individual than does the achievement test, and both have their place in guidance (35, p.2). The former is designed to emphasize the differences among characteristics of the same individual, emphasizing his strengths so that he may capitalize on them or his particular weaknesses so that he may adapt to or correct them (37, p.10). It is not necessary to limit the use of diagnostic tests to individual analysis. They may be used either to identify particular strengths or weaknesses on the part of an individual student or of a class as a whole. The location of defects in individuals and in classes is one of the most important and helpful teaching procedures.

According to Lindquist (122, p.37), diagnostic tests are for the purpose of determining the status in a learning sequence. They are administered after a period of instruction to determine points of faulty or inadequate learning in a detailed and analytical manner with a view to correction. He lists the following as functions of diagnosis (122, p.36):

1. Provide a basis for predicting individual pupil achievement in each learning area.

2. Serve as a basis for the preliminary grouping of pupils in each learning area.

3. Discover special aptitudes and disabilities.

4. Determine the level of problem solving ability.

Cronbach (37, p.284) says, "The teacher cannot teach effectively unless he knows just what is wrong with the student."

The teacher must analyze pupil difficulties either with or without a diagnostic test. The diagnostic test is designed to call attention to the various aspects of the subject where the pupil might have difficulty or make errors. If the teacher who gives the tests uses a careful survey, then the instruction can be built around the needs of the pupils. Greene states (70, p.301), "the underlying purpose of all testing is the accurate determination of class and individual pupil difficulties to the end that remedial instruction may follow."

There is a need for a continuing process for determining the status of the pupil in any given area of achievement and need for continuous adjustment of the instruction to fit this status. From the use of diagnostic procedures may come a basis for preventive work based on the knowledge gained. Accurate and detailed diagnosis can provide the needed information for the development of such a program.

Standardized tests for diagnosis are superior to direct observation for the following reasons (122, p.38):

1. They are more thoroughly analytical than most teachers are able to prepare.

2. They make the teachers aware of the important elements, necessary sequences, and difficulties of the process.

3. They save the teacher's time and energy in diagnosis and leave more for individual remedial work.

4. They help the pupil recognize his learning needs by systematically emphasizing his errors.

5. Remedial procedures are usually suggested or provided which save the teacher's time and also aid in systematizing the process.

A diagnostic test may be useful in guidance to point out to the student particular areas which may be weak, even though his total achievement may be satisfactory. To be effective, "Diagnosis must be more exact than broad statements. . . . The more specific the diagnostic information revealed the more exactly the remedial material can be made to fit the need" (69, p.214).

Ross (163, p.269) indicates that when an achievement or total score is below what it should be then a need for more careful diagnosis is indicated. It would be very difficult and time consuming to apply the diagnostic procedures to the mathematics placement tests of all entering students at the time the tests are scored. Such procedures could be applied, however, by the students themselves in order to determine areas of difficulty. This procedure would also enable the student to become aware of the nature of his difficulties and he could take steps to remedy them. If this information were then accumulated by the instructors, it would furnish a basis for emphasizing needed topics in the class instruction.

# CHAPTER IV

## PROCEDURES USED IN DEVELOPING PRELIMINARY FORMS OF THE EXAMINATIONS

The preparation of the preliminary form of an examination usually begins with a general statement of the objectives or aims. The next step, or sometimes the first, is the outlining of the course content to be covered by the examination. From the statement of aims and the course outline the questions for the preliminary form of the examination are prepared. The preliminary form of the examination is then given to a group of students and the results are studied. In describing the development of the examinations used in this study, the first step will be to review the generally accepted statements of aims and objectives for college courses in mathematics.

### Statements of Course Aims and Objectives

"The most authoritative and widely accepted statement of the aims of mathematical instruction is contained in The Reorganization of Mathematics in Secondary Education, the 1923 report of the National Committee on Mathematical Requirements" (85, p.377). Regarding this report Butler writes (in 1951), "it has exerted so much influence that it has come to represent a sort of standard frame of reference for objectives in this field" (24, p.74). The eleven aims listed in this report are discussed under three main headings (109, pp.54-55).

Practical Aims

1. The immediate and undisputed utility of the fundamental processes of arithmetic.

2. An understanding of the language of algebra.

3. A study of the fundamental laws of algebra.

4. The ability to understand and interpret correctly graphic representations.

5. Familiarity with the geometric forms common in nature, industry and life; mensuration of these forms; development of space relations and perceptions; exercise of spatial imagination.

Disciplinary Aims

1. The acquisition in precise form of those ideas or concepts in terms of which the quantitative thinking of the world is done.

2. Development of ability to think clearly in terms of such ideas and concepts. This involves training in analysis of a complex situation, recognition of logical relations, and generalizations.

3. Acquisition of good mental habits and attitudes.

4. The idea of relationship or dependence.

Cultural Aims

1. Appreciation of beauty in the geometric forms of nature, art, and industry.

2. Ideals of perfection as to logical structure, precision of statement and of thought, logical reasoning, discrimination between the false and the true.

3. Appreciation of the power of mathematics.

In 1924 Young discussed the principal values of the study of mathematics under three general headings: (1) practical values of mathematics, (2) mathematics as a mode of thought, and (3) other functions of mathematics. Under this third and rather indefinite heading he mentions values which are in the nature of attitudes, habits, and ideals (24, p.74).

Reeve (152, pp.385-405), in 1925, distinguished between specific mathematical objectives and general objectives.

General Objectives

1. To give exercise in and appreciation of the fundamental mode of thought which mathematical thinking best represents.

2. To establish, organize and make cooperative certain important and specific habits: of action, of thinking, of moral conduct, of character.

Specific Mathematical Objectives

1. To increase and extend the power of understanding and of analyzing relations of quantity and space; of reading, interpreting, organizing, and expressing statistical and graphical data; and of recognizing, understanding, and using symbolic notation.

2. To extend and refine the power gained in applying mathematics so as to furnish a basis for an interest in future mathematical work.

3. To furnish and increase the incentive of study of mathematics for the love of the subject.

In 1930 Breslich classified the objectives of mathematics as (1) understandings, (2) skills, (3) problems and methods, (4) appreciations, (5) attitudes, and (6) habits (109, p.59).

A more extensive and more specific list of objectives for mathematical instruction was presented by Schorling in 1936 (109, pp.59-60):

1. To increase skill in computation.

2. To enlarge understanding of the basic concepts and principles of mathematics.

3. To provide a wider range of application of mathematics to other school subjects and to life needs.

4. To achieve more power in mathematical

reasoning or problem solving.

5. To secure a thorough understanding of measurement.

6. To familiarize the pupil with the properties of the common geometric forms (intuitive geometry).

7. To know and to be able to use methods of expressing mathematical relationships.

8. To learn to use and appreciate symbolism.

9. To learn to see the interrelationships of the various branches of mathematics.

10. To understand how mathematics is related to practically all branches of learning.

11. To enjoy mathematics for its own sake.

The Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics issued a report in 1940. This report differed from that of the National Committee which had long been accepted as a standard. The objectives listed by the Joint Commission are as follows (109, p.56):

1. Ability to think clearly.
    a. Gathering and organizing data.
    b. Representing data.
    c. Drawing conclusions.
    d. Establishing and judging claims of proof.
2. Ability to use information, concepts, and

general principles.

3. Ability to use fundamental skills.

4. Desirable attitudes.

   a. Respect for knowledge.

   b. Respect for good workmanship.

   c. Respect for understanding.

   d. Social-mindedness.

   e. Open-mindedness.

5. Interests and appreciations (The appreciation of the role of mathematics in our culture, and interest in mathematical thinking for pleasure).

6. Other objectives (health, citizenship, and worthy home membership).

The Joint Commission stated (109, p.56):

> . . . objectives may be regarded as having either a factual and impersonal aspect or a personal, psychological bearing. Thus, when we study a given domain in a purely scientific way, irrespective of the learner's personal reactions, we are mainly interested in facts, skills, organized knowledge, accurate concepts, and the like. If, on the other hand, we scrutinize the way in which the pupil behaves in a given situation, or his modes of reaction, we are led to such categories as habits of work or study, attitudes, interests, insight, modes of thinking, types of appreciation, creativeness, and the like.
>
> A clear recognition of these two essentially different yet complementary types of objectives is one of the achievements of recent educational theory. It is generally conceded that in the past the chief emphasis was on impersonal or factual objectives . . . .

The 1940 report of the Commission on Secondary School Curriculum, Progressive Education Association, differed from those above by emphasizing the role of mathematics in the aims of general education (32, p.48):

> In stating objectives for mathematics instruction it has not been customary to stress the development of such qualities of personality as tolerance, cooperativeness, self-direction, creativeness, social sensitivity, and esthetic appreciativeness. Yet teachers of mathematics can share with other teachers in this task, in some instances through the choice of problems on which students work, in others through the special resources of mathematics, and in still others through the way they guide the conduct of the class as a social group.

A more recent statement regarding the aims of college mathematics was made by Nowlan in 1950. He gives the aims, in order of importance, as (145, p.74): "(1) Training in precise thinking and a grasp of basic principles. (2) The acquisition of information and a mastery of certain technical skills." In this more recent statement emphasis is placed on principles and skills. No mention is made of the more general objectives which were emphasized in earlier lists.

The last statement of aims above was devoted to fundamental principles and specific skills. Hawkes states (85, p.340):

> The instructional materials now available and in general use are devoted almost exclusively to the development of specific skills and abilities, and current methodology is quite

> silent with reference to techniques for achieving disciplinary and cultural outcomes through the agencies of such materials. Courses of study may urge on the teacher the importance of ultimate objectives but they offer him little direction in the manner of their attainment. . . . He tries to develop in his pupils an understanding of and ability in the application of those mathematical principles and processes which are presented in the text or outlined in the course of study. He trusts that in the very nature of things many disciplinary and cultural values will accrue, but for the most part he is ignorant of techniques which can be counted on to insure their realization. He is disposed to judge the effectiveness of his teaching by the degree to which immediate objectives are attained.

An analysis of the content of the elementary and intermediate algebra courses shows that emphasis has been placed on the immediate aims, which are easily recognizable and measurable. The only statement of the objectives for the algebra courses at Portland State are those set forth by Peterson (149, p.vii), the author of the textbook. He lists two objectives: (1) "To serve as a terminal course in algebra for non-scientific studies," and (2) "To serve as a foundation course in algebra preparing for more advanced college mathematics." These are not objectives in the sense used by most authors. They simply state the type of course for which the book is suitable.

A list of more specific aims was needed for the preparation of a test covering the elementary and intermediate courses. For this purpose the following list of specific aims for each course was prepared.

<u>Aims for Elementary Course</u>

1. To recognize, understand, and use the language, definitions, symbols and notations of algebra.

2. To develop skill in evaluation of algebraic expressions and formulas.

3. To know and use common formulas.

4. To develop skill in the operations of addition and subtraction with signed expressions, including grouping operations.

5. To develop skill in the operations of multiplication and division with signed expressions, including grouping operations.

6. To develop skill in the solution of linear equations.

7. To learn to use equations and formulas in stated problems.

8. To develop skill in various types of factoring.

9. To develop skills in the various operations using fractions.

10. To develop techniques of handling fractions in equations and formulas.

<u>Aims for Intermediate Course</u>

1. To develop methods for solving simultaneous linear equations and their applications.

2. To learn the laws of exponents and to develop skill in their use.

3. To understand the use of roots and radicals, including imaginary numbers.

4. To learn the use of square root and the Pythagorean theorem.

5. To develop methods of solution of quadratic equations in one unknown and equations of higher order.

6. To develop ability to understand and interpret graphs of linear, quadratic and higher equations.

7. To develop graphic and algebraic methods for solving quadratic systems with two unknowns.

8. To learn to use ratio and proportion.

9. To develop the understanding of and use of variation.

10. To learn to use the binomial theorem.

## Analysis of Course Topics

According to Travers (191, p.22) the next step after setting forth the general objectives is to outline the course. Weitzman (198, pp.8-9) lists this as the first step. Ruch (166, p.150) refers to this procedure as "drawing up a table of specifications" and attaches considerable importance to it. The purpose of such a list is to prevent the omission of important topics and to prevent the overemphasis of minor topics.

The list of course topics follows the outline of the textbook, Intermediate Algebra for College Students by Thurmond S. Peterson, which is used in the courses. This list is shown below. The numbers in parentheses indicate the approximate number of assignments devoted to each topic.

Elementary Course Topics

- Introductory topics. (5)
  - Definitions and rules. ($\frac{1}{2}$)
  - Evaluations. (1)
  - Formulas, recall and solution. ($1\frac{1}{2}$)
  - Algebraic representation. (2)
- Signed expressions. (5)
  - Addition. (1)
  - Subtraction. (1)
  - Parenthesis. (1)
  - Multiplication. (1)
  - Division. (1)
- Equations and stated problems. (14)
  - Solution of equations. (5)
  - Various types of stated problems. (6)
  - Solution of literal equations. (2)
  - Solution of formulas. (1)

Factoring. (8)

Common monomial factors. (1)

Common binomial factors. (1)

Trinomials. (2)

Difference of two squares. (2)

Sum and difference of two cubes. (1)

Factor theorem. (1)

Fractions. (8)

Reduction. (1)

Multiplication. (1)

Division. (1)

Addition. (1)

Subtraction. (1)

Signs. (1)

Mixed expressions. (1)

Complex fractions. (1)

<u>Intermediate Course Topics</u>

First degree simultaneous equations. (6)

Indeterminate equations. ($\frac{1}{2}$)

Simultaneous equations. ($\frac{1}{2}$)

Solution by substitution. (1)

Solution by addition or subtraction. (1)

Equations with the reciprocals of the unknowns. (1)

Literal equations. (1)

Equations with three unknowns. (1)

Exponents, roots and radicals. (12)

Laws of exponents. (1)

Fractional exponents. (1)

Zero and negative exponents. (1)

Multiplication of radicals. ($1\frac{1}{2}$)

Division of radicals. ($1\frac{1}{2}$)

Addition and subtraction of radicals. (1)

Irrational expressions. (1)

Imaginary numbers. (1)

Equations involving radicals. (1)

Square root. (1)

Pythagorean theorem. (1)

Quadratic equations. (8)

Incomplete quadratic equations. (1)

Solution by factoring. (3)

Solution by completing the square. (1)

Solution by formula. (2)

Ratio and proportion. (1)

Variation. (2)

Direct variation. (1)

Inverse variation. (1)

Binomial theorem. (1)

Graphing. (7)

First degree functions. (2)

Second degree functions. (1)

Functions of higher order. (1)

Quadratic equations in two unknowns. (1)

Solution of pairs of linear equations. (1)

Solution of second degree and higher order equations. (1)

Quadratic systems in two unknowns. (2)

One linear and one quadratic equations. (1)

Two quadratic equations. (1)

Topics optional with instructor (frequently omitted).

Progressions. (0)

Logarithms. (0)

## Preliminary Forms of the Examinations

After the list of aims and the outline of the course had been prepared, the next step was to write the questions for the preliminary form of the examination. Hopkins (91, p.6) states that questions should cover the entire range of the unit selected for measurement; this is to give as wide a sampling of pupil knowledge as possible. In selecting the questions for the preliminary examination, an attempt was made to include as many of the topics listed in the course outlines as possible. Robinson (162, p.230) indicates that such material should be selected from the students' actual courses. Questions included on the examination were generally similar in form to those found in the text which is used in the courses.

Lindquist (122, p.152) indicates that "it should always be the fundamental goal of the achievement test constructor to make the elements of his test series as nearly equivalent to, or as much like, the elements of the criterion series as considerations of efficiency, comparability, economy, and expediency will permit." He further indicates, "achievement tests have been largely confined within the framework of the prevailing subject-matter organization of the curriculum and of the generally accepted immediate objectives of instruction" (122, p.124). Unless tests are designed in this way, they do not usually gain ready acceptance by the instructors who will use them. In general the instructors "are given certain subjects to teach and the content is prescribed for them in detail (the textbooks have been selected for them) and they are responsible for 'getting the content across' to their pupils" (122, p.125). Lindquist defends the formal type of question when the test is to be used for diagnostic purposes:

> Diagnostic tests frequently deal almost exclusively with the more mechanical aspects of a learning sequence and neglect the higher abilities requiring relational thinking and problem solving. This is defensible in that efficiency in learning the more mechanical aspects leaves

> more time for the development of the higher mental processes (122, p.38).

For the above reasons the questions for the examinations used in this study were prepared with emphasis on the basic skills.

The preliminary forms of the examinations, Forms P, were prepared with multiple-choice questions offering five responses. Cronbach (37, p.73) states, "Current practice in group testing favors the multiple-choice form." Buros (22, p.274) indicates that with this type of question the correct answer can sometimes be found by the process of elimination or substitution. Cronbach (37, p.73), however, says that though this form of test

> . . . measures recognition of correct answers rather than recall, it is satisfactory for many purposes. . . . in a mathematics achievement test at the college level multiple-choice questions had reliability coefficients and correlations with grades in later mathematics essentially the same as those for free answer questions.

Lindquist (122, p.173) notes that as few forms of questions as possible should be used and prefers only one form.

For a final form of the test which requires fifty minutes or an hour for each part, it was estimated that approximately forty questions would be needed for Part I and a similar number for Part II. According to Ruch (166, p.154), from twenty-five to fifty per cent more items should be prepared than will be used on the final form of the test. The preliminary examination was prepared with eighty questions on each part, double the number needed for the final form. This number was more than adequate to provide a wide selection of questions in preparing the final forms.

The preliminary forms of the test were administered with a time limit of one hour and forty minutes, the equivalent of two regular fifty-minute class periods. More than one half of the papers were turned in before time was called. Only a few students showed any significant shortage of time by leaving questions unanswered at the end of the test. According to Lindquist (122, p.338), "test authors usually set the time limits so that between eighty and ninety per cent of the pupils can consider or attempt all of the items."

Since teacher estimates of the time required by a student for taking an examination are often unreliable, the questions were not grouped according to topic for the preliminary examination. Questions were arranged so that if the estimate of time were too low, students would not omit a section on a topic occurring at the end of the examination. In this way enough questions on each major topic were answered to insure an adequate distribution of topics for the final form of the examination. Capron (25, p.695) studied three different arrangements of items in tests. As a result of this study she concluded that the arrangement of items was unimportant as it produced no significant difference in the test scores.

There is some disagreement as to the importance of speed in taking an examination. Hawkes and Lindquist state (85, p.377), "Some premium should be placed upon speed as well as upon power . . . by including a sufficient number of items so that most pupils will be given an opportunity to do the maximum amount of work of which they are capable in the time given." On the other hand, Travers (191, p.40) states, "Most tests of achievement should be power tests rather than speed tests, and enough time should be allowed for the examinee to do all he can do." When a test is used as a placement examination, its purpose is to determine the retained mathematical ability of the student; there is little need for emphasis upon speed. "All students in the class, however, should have the same length of time to prepare the answers" (136, p.7).

A test should include items of varying degrees of difficulty. The reason for this, according to Hawkes (85, p.377), is that "Items of different degrees of difficulty will discriminate between pupils at different levels of ability." The test items should be of appropriate difficulty so that no pupil will earn either a perfect score or a zero score (165, p.121).

Various methods have been developed for correcting a test score for chance to cpmpensate for the possibility of guessing correct answers. The need for correction is decreased as the number of possible responses is increased. Ruch recommends that the score on multiple-choice exercises be determined by counting the number of correct items and assuming that all individuals will profit more or less equally by guessing. He says (165, p.128), "guessing is probably sufficiently eliminated in the five or more response situations to neglect it entirely for practical purposes." Elsewhere, he writes (166, p.338) that "five and seven response tests are never corrected (for guessing) in actual practice." Lindquist questions the use of methods of correcting for chance as they imply that the same number of persons will select each of the incorrect alternatives. Rarely, however, are all of the incorrect responses equally attractive (122, p.269).

<u>The Preliminary Examination, Part I</u>

The preliminary examination appears in Appendix A, page 126. Table I, page 59, shows the scope of the course content and the sequence in which it is developed. The distribution of questions among the course topics for the preliminary forms (Forms P) of the examinations is also shown in Table I. Each topic in the list is represented by at least one question on Form P except "reduction" and "signs" of fractions. Both of these operations are necessary in the solution of other fraction problems. The distribution of questions for Form A (the revised form) of Part I of the examination is also shown in Table I; this form will be discussed in the next chapter.

TABLE I

DISTRIBUTION OF PRELIMINARY QUESTIONS FOR PART I AMONG COURSE TOPICS

| Topic | Number of Questions: Form P | Form A |
|---|---|---|
| Introductory topics | | |
| Definitions and rules | 5 | 4 |
| Evaluations | 8 | 5 |
| Formulas, recall and solution | 7 | 4 |
| Algebraic representation | 5 | 3 |
| Signed expressions | | |
| Addition | 1 | 1 |
| Subtraction | 3 | 1 |
| Parenthesis | 4 | 2 |
| Multiplication | 5 | 3 |
| Division | 7 | 2 |
| Equations and stated problems | | |
| Solution of equations | 5 | 4 |
| Various types of stated problems | 3 | 1 |
| Solution of literal equations | 5 | 2 |
| Solution of formulas | 4 | 1 |
| Factoring | | |
| Common monomial factors | 1 | 0 |
| Common binomial factors | 1 | 0 |
| Trinomials | 4 | 2 |

TABLE I (CONTINUED)

| Topic | Number of Questions Form P | Form A |
|---|---|---|
| Factoring (continued) | | |
| Difference of two squares | 3 | 1 |
| Sum and difference of two cubes | 1 | 1 |
| Factor theorem | 1 | 1 |
| Fractions | | |
| Reduction | 0 | 0 |
| Multiplication | 1 | 0 |
| Division | 1 | 0 |
| Addition | 1 | 0 |
| Subtraction | 1 | 1 |
| Signs | 0 | 0 |
| Mixed expressions and complex fractions | 3 | 1 |
| Totals | 80 | 40 |

The distribution of questions among the specific aims of the course is shown for the preliminary form (Form P) of the examination in Table II, page 61. The larger number of questions under the fifth and eighth aims is due to the variety of problems in these categories and does not imply greater importance. The distribution of questions for the revised forms (Forms A) of the examinations is also shown in Table II; this will be discussed in the next chapter.

TABLE II

DISTRIBUTION OF PRELIMINARY QUESTIONS FOR PART I

AMONG SPECIFIC AIMS

| Specific Aims | Number of Questions Form P | Form A |
|---|---|---|
| 1. Language, definitions, symbols and notation. | 7 | 4 |
| 2. Evaluation of algebraic expressions and formulas. | 7 | 5 |
| 3. Knowledge and use of formulas. | 7 | 4 |
| 4. Addition and subtraction of signed expressions. | 8 | 4 |
| 5. Multiplication and division of signed expressions. | 10 | 5 |
| 6. Solution of linear equations. | 9 | 4 |
| 7. Equations and formulas in stated problems. | 8 | 4 |
| 8. Factoring of various types. | 11 | 5 |
| 9. Operations with fractions. | 7 | 2 |
| 10. Fractions in equations, formulas, and other algebraic expressions. | 6 | 3 |
| Totals | 80 | 40 |

<u>The Preliminary Examination, Part II</u>

The preliminary examination, Part II, appears in Appendix A, page 140. Table III, page 63, shows the scope of the course content and the sequence in which it is developed. The distribution of questions among the course topics for the preliminary form (Form P) is also shown in Table III. No question on simultaneous literal equations is included because the methods are the same as used for other simultaneous equations. Questions on equations with three unknowns were omitted because the methods of solution are the same as for two unknowns and these problems are long and time consuming. Graphical solutions of pairs of equations are usually not emphasized in the courses and are frequently omitted. For this reason they were omitted from the examination.

At the suggestion and request of some of the instructors who gave the preliminary form of the examination, questions on the optional topics of logarithms and progressions were included. These were omitted from the revised form for reasons discussed in the next chapter.

The distribution of questions for the revised form (Form A) of Part II of the examination is also shown in Table III. This part of the table will be discussed in the next chapter.

TABLE III

DISTRIBUTION OF QUESTIONS FOR PART II AMONG COURSE TOPICS

| Topic | Number of Questions Form P | Form A |
|---|---|---|
| First degree simultaneous equations | | |
| Indeterminate equations | 1 | 0 |
| Simultaneous equations | 2 | 0 |
| Solution by substitution | 1 | 0 |
| Solution by addition or subtraction | 1 | 1 |
| Equations with the reciprocals of the unknowns | 1 | 1 |
| Literal equations | 0 | 0 |
| Equations with three unknowns | 0 | 0 |
| Exponents, roots and radicals | | |
| Laws of exponents | 2 | 2 |
| Fractional exponents | 2 | 1 |
| Zero and negative exponents | 1 | 1 |
| Multiplication of radicals | 2 | 2 |
| Division of radicals | 1 | 1 |
| Addition and subtraction of radicals | 1 | 0 |
| Irrational expressions | 1 | 1 |
| Imaginary numbers | 1 | 1 |
| Equations involving radicals | 2 | 2 |
| Square root | 2 | 1 |
| Pythagorean theorem | 3 | 1 |
| Quadratic equations | | |
| Incomplete quadratic equations | 2 | 0 |
| Solution by factoring | 3 | 1 |
| Solution by completing the square | 1 | 1 |
| Solution by formula | 3 | 3 |
| Higher order equations | 2 | 1 |
| Ratio and proportion | 6 | 4 |

TABLE III (CONTINUED)

| Topic | Number of Questions Form P | Form A |
| --- | --- | --- |
| Variation | | |
| Direct variation | 3 | 2 |
| Inverse variation | 2 | 2 |
| Binomial theorem | 6 | 2 |
| Graphing | | |
| First degree functions | 2 | 2 |
| Second degree functions | 2 | 1 |
| Functions of higher order | 1 | 1 |
| Quadratic equations in two unknowns | 4 | 2 |
| Solution of pairs of linear equations | 0 | 0 |
| Solution of second degree and higher order equations | 0 | 0 |
| Quadratic systems in two unknowns | | |
| One linear and one quadratic equation | 2 | 1 |
| Two quadratic equations | 4 | 2 |
| Topics optional with instructor | | |
| Progressions | | |
| Arithmetic | 3 | 0 |
| Geometric | 4 | 0 |
| Logarithms | | |
| Definition and laws | 3 | 0 |
| Computations | 3 | 0 |
| Totals | 80 | 40 |

The distribution of questions among the specific aims of the course is shown for the preliminary form (Form P) of the examination in Table IV below. The distribution of questions for the revised form (Form A) of the examination is also shown in Table IV; this will be discussed in the next chapter. Optional topics are omitted from this table.

TABLE IV

DISTRIBUTION OF QUESTIONS FOR PART II BY SPECIFIC AIMS

| Specific Aims | Number of Questions Form P | Form A |
|---|---|---|
| 1. Simultaneous linear equations. | 6 | 2 |
| 2. Exponents. | 5 | 4 |
| 3. Roots and radicals. | 8 | 7 |
| 4. Square root and the Pythagorean theorem. | 5 | 3 |
| 5. Quadratic and higher equations in one unknown. | 11 | 6 |
| 6. Graphing equations. | 5 | 4 |
| 7. Quadratic equations in two unknowns. | 9 | 5 |
| 8. Ratio and proportion. | 7 | 4 |
| 9. Variation. | 5 | 3 |
| 10. Binomial theorem. | 6 | 2 |
| Totals | 67 | 40 |

The questions for the preliminary forms (Forms P) of the examinations were prepared and selected on the basis of the course outlines and lists of aims given above. At the end of fall term the preliminary forms were given to all elementary and intermediate algebra classes at Portland State. Part I of the test was given to 254 students in eight elementary algebra classes which were taught by six different instructors. Part II was given to 106 students in four intermediate algebra classes taught by four different instructors. Two of the papers of students in the elementary class were discarded for the purposes of this study because the students had omitted one of the middle pages. The papers used for the following analysis were the remaining 252 papers from the elementary classes and 106 papers from the intermediate classes. The procedures used in the analysis of these scores are discussed in the next section.

## Analysis of Scores on Preliminary Examinations

An analysis of the preliminary examination scores was made for the purpose of determining the usefulness of the examinations. This analysis consisted of examining the distribution of the scores, and determining the validity and reliability of the examinations.

Distribution of Scores

The distribution of a set of examination scores usually is similar to that of a normal distribution. Two types of departure from the normal distribution can occur: (1) the distribution may be asymmetrical, the mean and the median being different; and (2) the distribution may be symmetrical but may have an excess or deficit near the center of the range (178, p.174). The first of these conditions may be tested by measuring the skewness. The second condition may be tested by measuring the kurtosis.

Neither the skewness nor the kurtosis of the distributions of scores for Part I or Part II of the examination differs significantly from the normal.

The individual scores on Part I, Form P, of the examination are shown in Table IX, Appendix B, page 154. The distribution is shown in Table V, page 68. The mean of the distribution of scores for Part I, Form P, was 37.40 and the standard deviation was 9.70.

TABLE V

DISTRIBUTION OF SCORES BY CLASSES ON PART I, FORM P

| | Class Number | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Scores | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
| 60-64 | | | 1 | | | | | | 1 |
| 55-59 | | 1 | 5 | | 4 | | | | 10 |
| 50-54 | 2 | 1 | 4 | 1 | 5 | 1 | 3 | 2 | 19 |
| 45-49 | 2 | 3 | 4 | 5 | 11 | 2 | 4 | 1 | 32 |
| 40-44 | 3 | 5 | 2 | 6 | 8 | 4 | 9 | 6 | 43 |
| 35-39 | 2 | 9 | 4 | 10 | 3 | 11 | 5 | 5 | 49 |
| 30-34 | | 10 | 1 | 6 | 2 | 5 | 11 | 5 | 40 |
| 25-29 | 2 | 4 | 5 | 5 | 3 | 4 | 3 | 5 | 31 |
| 20-24 | 2 | 2 | 5 | 2 | | 5 | 2 | 3 | 21 |
| 15-19 | | | 1 | 2 | | 3 | | | 6 |
| Totals | 13 | 35 | 32 | 37 | 36 | 35 | 37 | 27 | 252 |

Mean 37.40

Standard Deviation 9.70

The individual scores on Part II, Form P, of the examinations are shown in Table X, Appendix B, page The distribution of these scores by classes and the total distribution is shown in Table VI, page 69. The mean of the scores for Part II, Form P, was 42.38 and the standard deviation was 8.70.

TABLE VI

DISTRIBUTION OF SCORES BY CLASSES ON PART II, FORM P

| Scores | Class Number 1 | 2 | 3 | 4 | Total |
|---|---|---|---|---|---|
| 65-69 | 1 | | | | 1 |
| 60-64 | | | 1 | | 1 |
| 55-59 | 1 | 2 | 1 | 2 | 6 |
| 50-54 | 2 | 5 | 2 | 4 | 13 |
| 45-49 | 7 | 2 | 4 | 5 | 18 |
| 40-44 | 7 | 8 | 5 | 14 | 34 |
| 35-39 | 3 | 6 | 2 | 6 | 17 |
| 30-34 | 2 | | 1 | 4 | 7 |
| 25-29 | 2 | 1 | 3 | | 6 |
| 20-24 | | | 1 | | 1 |
| 15-19 | 1 | 1 | | | 2 |
| Totals | 26 | 25 | 20 | 35 | 106 |

Mean 42.37

Standard Deviation 8.70

An examination of the distributions of scores in Tables V and VI has shown that the sets of scores on both examinations were distributed in a pattern which did not differ significantly from that of a normal curve and can be treated as such.

Probable Error of a Pupil's Score

McCall states, "The best and most meaningful indication of the reliability is the probable error of a pupil's score" (130, p.288). The probable error for Part I of the test is 2.62 while that for Part II is 2.66.

Validity of the Examinations

Validity is defined as "the degree to which an evaluation device measures what it purports to measure" (155, p.195). It is a specific concept and must be used in reference to a specific purpose and a specific group of pupils. "It is meaningless to speak of any given test as being valid or invalid apart from any consideration of the purpose it is intended to serve or of the group to which it is to be given" (85, p.21).

Criteria which may be used to determine the validity of a test are of two types: (1) criteria which compare with course content, and (2) criteria which compare with test scores.

Curricular validity. Validity which is determined by comparison with course content is termed curricular validity. Some of the criteria by which the content of a course may be compared are: (1) analysis of courses of study, (2) statements of objectives, (3) analysis of textbooks, (4) judgments of competent persons, and (5) analysis of examination questions.

Validation of the test content by comparison with the textbook is useful because it assures that the pupils will be tested on material which they have actually studied or which they will be expected to know (155, p.198). Greene states (69, p.75),

> In the case of the achievement test . . . the validity of the test depends more largely upon the opportunity the pupil has had to master the information measured in the test. In such a situation the teacher himself is probably the best judge of the validity of the test, since he knows best what material he has taught the class.

Ebel notes (50, p.93), "There is need for emphasis on the fact that judgments . . . are inevitably involved in the validation process. If not applied to the test itself, then they must be applied in selecting criteria." The judgments made by different individuals as to which tests possess validity will vary greatly. This limits the usefulness of this criterion as a measure of the validity of a test (1, pp.320-328).

Statistical validity. Methods of determining validity which make use of test scores as a criterion yield what is termed statistical validity. Criteria with which test scores may be compared are: (1) school grades, (2) differences in scores obtained by two or more groups known to be widely separated in ability, and (3) correlations with other valid and reliable tests.

A statistical measure of the validity of a test is secured by computing a coefficient of correlation between scores on the test and an outside criterion. The difficulty encountered in using this procedure is in the selection of the outside criterion. Monroe writes (140, p.1296), "The use of school marks in determining the validity of achievement tests is a common procedure." However, some criticism has been made of the use of this procedure because of the variations and unreliability of the marks themselves. In spite of these criticisms this continues to be one of the common methods of establishing validity.

Validity correlations are not often obtained which are as high as those usually obtained for reliability. Cronbach takes note of this (37, p.256):

> Psychologists have now abandoned their insistence on validity coefficients of .70 or .80 for all tests. While we would be pleased to reach these or better levels, the experience of 30 years of practical testing shows that we very often cannot attain such standards. . . . It has been found repeatedly that coefficients as low as .30 are of definite practical value.

Lindquist notes that validity coefficients vary from zero to .70. "It is probably fair to say that the median correlation of tests . . . using freshman grades as the criterion, would fall somewhere near .45" (122, p.90). He also says (122, p.92),

> From a battery of achievement measures we get about as good a prediction of freshman average or other general measure of college success as we would get from rank in high school class; a validity coefficient of about .55 would be typical.

Validity of the tests used in this study. A comparison of the test questions with the course content can be made for Part I from Table I, page 59, and for Part II from Table III, page 63. A comparison of the test questions with the aims of the courses can be made for Part I from Table II, page 61, and for Part II from Table IV, page 65. The instructors who used the tests judged them as giving good doverage of the content of the courses. Instructors who had not included logarithms and progressions (optional topics) in their courses questioned the inclusion of these topics on Part II of the examination. The others, however, were quite insistent that these topics be included if the examination was to be used in their classes.

The scores on the preliminary forms of the tests were correlated with the fall term grades. This gave a measure of the validity of the tests for use with these courses. In determining these coefficients of correlation the scores for one of the elementary classes were omitted because they had been used in determining the course grades. In the remaining classes they had not been used for this purpose. One student in the intermediate class received a grade of incomplete. Thus these coefficients were computed on 204 cases for Part I and 105 cases for Part II.

The coefficients of correlation of test scores with fall term grades were .77 for Part I and .66 for Part II. These are as high as are usually obtained when grades are used as the outside criterion.

Reliability of the Examinations

Reliability cannot be determined by any method which is comparable to the face validity of a test. It is entirely a statistical concept. The validity of a test is a measure of its correlation with some outside criterion. Reliability, on the other hand, is a measure of the correlation of the test with itself. There are three common methods of determining the self-correlation of a test: (1) by repeating the same test at a later date, (2) by using two parallel forms of the test, and (3) by the split-half method of dividing a test into two comparable halves (75, p.421).

Repeated test. The first of these methods is not often used because it requires that the same test be applied to the group of pupils after an interval of time. The scores are then correlated. If too short a period of time elapses, the memory factor will tend to make the correlation higher than it should be. If too long an interval elapses, factors such as growth, intervening learning, and forgetting, tend to make the correlation too low (155, p.202).

Parallel forms. The second method uses two comparable forms of the test, and the correlation between the two sets of scores is determined. If the correlation is high, then both forms are said to be reliable. This method overcomes the disadvantages which are a result of giving the same test again at a later date. There are, however, certain difficulties involved in this method. Twice as much material must be prepared to have two forms of the test, even if one form is sufficient for the purpose for which the test is desired. Even more important is the difficulty in preparing two forms of the test which are truly equivalent, and yet have entirely different items.

Even when the two forms of the test are given one immediately following the other, such factors as fatigue, interest and motivation may be different at the time the student is taking the two parts of the test. Though such factors will probably be less variable than when the test is given at two different periods, the variation due to taking the test at different times can never be completely eliminated (130, p.56).

Split-half method of determining reliability. The split-half method of estimating the reliability avoids the difficulties encountered in the other two methods, and consequently is the most frequently used. This method divides the items of the test into two halves. The usual method of splitting the test is by taking the odd-numbered questions for one half and the even-numbered questions for the other half. The coefficient of correlation between the two halves is then determined. This is the reliability coefficient. Any one of several formulas (122, p.581) may be used to estimate the reliability of the whole test.

Jordan states (100, p.426), "The coefficient derived from odd-even items is probably a truer measure of reliability of a test than that obtained from corresponding duplicate forms because the odd-even method eliminates pupil variability." A study by Read (151, p.704) showed the results from various methods of selecting the chance halves to be small. However, the possibility is always present that the results may be atypical from any method used to split the test. If very high reliability coefficients are desired, methods of item analysis may be used to balance the two halves in difficulty.

<u>Reliability of the tests used in this study</u>. The split-half method was adopted to furnish an estimate of the reliability of the test used in this study to avoid the difficulties encountered in using the other two methods.

The odd-even split used here does not give an exact balance in difficulty as is shown by comparison of the means for the odd and even-numbered questions. This data is shown in Table IX, page 154, and Table X, page 162, for Part I and Part II respectively.

Correlations between the odd and even-numbered questions of .73 for Part I and .67 for Part II were found. The lower coefficient for Part II is attributed to the inclusion of questions on the optional topics of logarithms and progressions. When these questions are eliminated from the scores, the coefficient of reliability for Part II is found to be .73, the same as for Part I of the examination. The half-test scores are shown in Appendix B.

Lindquist states that if a test is split to yield two scores (122, p.617), "The procedure logically most defensible would appear to be to split it into two equivalent halves, balanced in terms of difficulty and content." Any split such as this necessarily involves some method of item analysis for determination of the difficulty of the individual items as well as an individual analysis of the content of the items. Careful balancing of the items in this way is usually necessary for obtaining the high reliability coefficients which are sometimes reported with tests. Item analysis could be used to balance the difficulty of the odd and even-numbered questions. The use of this procedure would yield still higher coefficients of reliability.

The Spearman-Brown prophecy formula was applied to the split-half correlation coefficients to estimate the reliability coefficients of the whole of Part I and Part II.

The estimate of the coefficient of correlation for the whole test was .84 for Part I and .80 for Part II. If the optional topics are excluded from Part II, this coefficient is increased to .84.

Index of reliability. The whole test is more reliable than the coefficient of correlation would indicate, because it "shows how close a test which is somewhat inaccurate corresponds to another test which is also somewhat inaccurate" (130, p.528). Tiegs states (190, p.386), "The index of reliability represents the correlation between the scores actually obtained by a test and the theoretically true scores." Lindquist (122, p.685) recommends the use of the index of reliability and states that the reliability for all tests should be reported in this way.

The index of reliability for Part I is .92; for Part II it is .89.

# CHAPTER V

## PROCEDURES USED IN DEVELOPING THE FINAL FORMS OF THE EXAMINATIONS

The preparation of the preliminary forms of the tests began with statements of aims for the courses. The content of the courses was then outlined. The preliminary forms of the examinations were prepared and the sets of scores on these forms were studied to establish the validity and reliability of the tests. The questions were then analyzed to determine which should be retained for the final forms of the examinations.

To develop the final forms of the examinations the preliminary forms were revised and shortened. To determine which questions were to be retained for the final forms, item analysis was used. The preliminary papers were then rescored and these scores were compared with the Ohio Psychological Examination scores and with grades earned the following term to determine the prognostic value of the tests. Reliability of the new forms was established by the odd-even split-half method and application of the Spearman-Brown formula.

## Item Analysis

Objective test construction usually involves trying out more items than will be needed for the final form of the test. Wood (206, p.16) has pointed out that "two objective test items may prove equally difficult and may hold the pupils responsible for equally valid content from the curriculum viewpoint, and yet the actual responses made to one may be much more highly related to general achievement than those made to the other." Item analysis provides a basis for selection or rejection of such items according to their contribution towards the measurement of whatever the whole test measures.

If the total score on a test is used as the criterion variable for judging the discriminating power of each item, the resulting indices reflect the extent to which the item measures the same functions as the total score. Some items are more useful than others in discriminating between students of different levels of ability; this means that for the group tested they are better measures of whatever the whole test actually measures. Wood comments on discrimination as follows (206, p.17): "An item is said to discriminate if the pupils who respond correctly to that item are, on the average, superior in general achievement to those who respond incorrectly."

The validity coefficient of a test is its correlation with some outside criterion as described above (supra, p.70). "Likewise, the validity coefficient of an item is a measure of its correlation with a criterion" (2, p.180). The validity of individual test items may be investigated by methods which are analagous to those used for determining the validity of the test.

Lindquist (122, p.286) comments on the difficulty of obtaining an adequate criterion for use in analysis of individual items, "A well constructed test may constitute the best available measure of the criterion; in a sense, the test itself defines the function it is to measure." The total scores derived from a test are often used as the immediate criterion with which the individual items in the test are correlated. These correlations may be estimated by any of a number of techniques. Items which have very low or negative correlations are then discarded or revised in preparing the final forms of the test. The methods of analyzing individual test items will be discussed below.

Item-Criterion Indices

Adkins states that about forty different indices have been devised to express the relation of a dichotomous variable and a criterion which is distributed over more than two categories (2, p.180). Most of these are approximations of correlation coefficients. Nineteen of the most important methods are summarized by Guilford (75, pp.429-456). Many of the others are variations of these methods or short-cut procedures based on them. The main difficulty of item analysis is that the methods are very intricate and time consuming. When working with large groups the use of item analysis may result in considerable expense.

Non-correlation methods. Several methods of item analysis which do not make use of correlation coefficients are based on a procedure which Guilford refers to as the precision method (75, p.429). In general the criterion subjects are classified in two or more groups that contain equal numbers. For each group the proportion passing the item is determined. The diagnostic value of the item is indicated by the steepness of the curve.

Hawkes describes another method which does not make use of correlation coefficients (85, p.42). For this method, the average score on the criterion test for the students who succeeded on the given item and for those who failed on the item is computed. If the average score on the criterion of those who succeeded on the given item was higher than that of those who failed on the item, then that item could be said to have discriminating power, the degree of this power depending upon the magnitude of the difference in the two averages.

Common correlation methods. The most common types of correlation coefficients for the purpose of item analysis are (2, p.180), (1) biserial r (2) tetrachoric r and (3) point-biserial r.

Biserial r. Biserial r assumes that the item variable is represented by a dichotomization of a continuous and normally distributed variable and that the criterion is a continuous but not necessarily normal variable.

Lindquist says that this index was found to be the best indication of the value of an item in contributing toward the ranking of the student (122, p.479). This procedure is laborious, which is probably one of the reasons that it has not been used more often. In order to reduce the labor of computing this coefficient, Flanagan devised a procedure using only the upper and lower portions of the group (55, pp.674-680). Dunlap (49, pp.51-58) and Chapanis (29, pp.297-304) have proposed methods for shortening the amount of computation in determining the biserial r index.

However, Richardson and Stalnaker (160, pp.463-464) indicate that the biserial r is not a suitable measure, for item analysis where the responses are dichotomized because it assumes: (1) that the distribution of the dichotomized variable is normal, and that (2) the dichotomized variable is continuous. They indicate that unless these conditions can be established and be shown to fit the case being studied that use of the biserial *r* is not warranted. These authors then proposed what is known as the point-biserial coefficient of correlation, which they consider more suitable.

*Tetrachoric r*. The tetrachoric *r* assumes that both the item variable and the criterion variable are represented by dichotomizations of continuous and normally distributed variables. This index was not suitable for the purposes of this study because the criterion variable was not dichotomized.

*Point-biserial r*. Point-biserial *r* assumes that the item is point-distributed, or that all the cases are concentrated at two points, and that the criterion is continuous but not necessarily normal. Guilford (73, pp.500-501) compares six of the most common indices of item analysis and says, "It is interesting to note that the point-biserial *r* gave results which correlated highest on the average with results from all other indices. This may or may not support the idea that this index is a kind of common denominator because it is the most realistic one."

*Other methods*.

*Analysis of variance*. A study by Lev (121, pp.623-630) which was concerned with the methods of the analysis of variance for the selection of test items concludes that this method is unnecessarily difficult.

*Factorial methods*. Regarding factorial methods of item analysis, Lindquist states, "The use of factorial methods for item analysis purposes awaits the development of electronic computers capable of extracting factors from large matrices of inter-correlation. . . ." (122, p.301). He indicates that when such instruments become widely available, it may be that items will be selected on the basis of their factor loadings.

*Chi-square*. On the use of Chi-square for the purpose of item analysis, Herfindahl (89, p.371) comments, "Direct calculation of Chi-square for each question would require a prohibitive amount of labor." He further states (89, p.377), "Chi-square provides a measure of only the probability of association and not the degree of association as would be provided by . . . the bi-serial *r*."

## Indices Used in This Study

Selection of a method of item analysis must be related to the method of test scoring which is used. In this study the responses to the test items were scored either correct or incorrect. Only item analysis methods which use this information are suitable. A form of the point-biserial coefficient of correlation has been selected for this study because of this method of scoring and for other reasons cited above. Derivation of the simplified formula which was used is shown in Appendix C, page 166.

In order to reduce the labor involved in the computation of biserial r Dunlap (49, pp.674-680), Chapanis (29, pp.297-304) and others have developed formulas which are easier to use and reduce the amount of computation which is necessary. Chapanis (29, pp.301-302) also developed what he terms a "reduced form" of the McCall-Long-Bliss validity coefficient, which was also used in the selection of items for the final forms of the tests used in this study.

The reduced point-biserial coefficient ($r_{rpb}$) and the McCall-Long-Bliss validity coefficient ($V_{MLB}$) were computed for each item on the tests. The two coefficients selected here are particularly suitable for use together because of the similarity in the calculations which are required to determine each of them. Table VII, page 87, shows these coefficients for Part I and Table VIII, page 88, shows them for Part II of the examinations. Items which were retained in the final form of the examinations are indicated by an asterisk (*).

TABLE VII

ITEM VALIDITY COEFFICIENTS FOR PART I OF THE EXAMINATION

| Item | $r_{rpb}$ | $V_{MLB}$ | Item | $r_{rpb}$ | $V_{MLB}$ |
|---|---|---|---|---|---|
| 1* | .15 | 52 | 31 | .19 | 49 |
| 2 | .11 | 34 | 32* | .54 | 64 |
| 3* | .34 | 48 | 33 | .55 | 58 |
| 4* | .21 | 27 | 34* | .52 | 69 |
| 5* | .17 | 35 | 35 | .42 | 54 |
| 6 | .08 | 42 | 36 | .32 | 31 |
| 7 | .06 | 20 | 37* | .34 | 85 |
| 8* | .24 | 90 | 38* | .35 | 85 |
| 9* | .30 | 75 | 39 | .17 | 40 |
| 10* | .29 | 48 | 40 | .16 | 26 |
| 11 | .05 | 34 | 41 | .13 | 30 |
| 12 | .06 | 22 | 42 | .02 | 4 |
| 13* | .34 | 82 | 43* | .25 | 35 |
| 14* | .24 | 65 | 44 | .25 | 20 |
| 15 | .13 | 11 | 45* | .35 | 60 |
| 16* | .25 | 66 | 46* | .64 | 87 |
| 17* | .24 | 32 | 47 | -.15 | -17 |
| 18* | .52 | 73 | 48* | .26 | 43 |
| 19* | .41 | 91 | 49 | .16 | 22 |
| 20 | .35 | 67 | 50* | .41 | 50 |
| 21* | .42 | 47 | 51 | .06 | 13 |
| 22* | .37 | 47 | 52 | .22 | 22 |
| 23 | .28 | 39 | 53 | .18 | 14 |
| 24 | .49 | 55 | 54* | .22 | 53 |
| 25 | .62 | 63 | 55* | .31 | 51 |
| 26 | .09 | 11 | 56 | .06 | 13 |
| 27* | .50 | 57 | 57* | .24 | 40 |
| 28* | .36 | 28 | 58 | .11 | 18 |
| 29* | .39 | 40 | 59 | .11 | 25 |
| 30* | .34 | 32 | 60* | .25 | 47 |

*Items selected for revised form, Form A, of examinations.

TABLE VII (CONTINUED)

| Item | $r_{rpb}$ | $V_{MLB}$ | Item | $r_{rpb}$ | $V_{MLB}$ |
|---|---|---|---|---|---|
| 61* | .62 | 52 | 71* | .32 | 51 |
| 62* | .29 | 34 | 72 | .13 | 22 |
| 63* | .46 | 69 | 73* | .21 | 46 |
| 64 | .17 | 17 | 74 | .16 | 28 |
| 65 | .44 | 27 | 75 | .19 | 33 |
| 66* | .48 | 86 | 76* | .33 | 53 |
| 67 | .35 | 88 | 77 | .15 | 22 |
| 68 | .41 | 77 | 78 | .13 | 22 |
| 69 | .13 | 40 | 79* | .26 | 43 |
| 70 | .14 | 37 | 80 | .16 | 26 |

TABLE VIII

ITEM VALIDITY COEFFICIENTS FOR PART II OF THE EXAMINATION

| Item | $r_{rpb}$ | $V_{MLB}$ | Item | $r_{rpb}$ | $V_{MLB}$ |
|---|---|---|---|---|---|
| 1 | .01 | 5 | 16* | .52 | 30 |
| 2 | .02 | 0 | 17* | .37 | 38 |
| 3 | .07 | 7 | 18* | .41 | 31 |
| 4 | .04 | 6 | 19* | .51 | 30 |
| 5* | .22 | 29 | 20* | .72 | 76 |
| 6* | .21 | 15 | 21* | .30 | 22 |
| 7* | .25 | 27 | 22 | .23 | 9 |
| 8* | .35 | 14 | 23 | .19 | 15 |
| 9* | .31 | 7 | 24 | .02 | 3 |
| 10* | .59 | 20 | 25 | .29 | 26 |
| 11 | .22 | 7 | 26 | -.09 | -12 |
| 12* | .32 | 14 | 27* | .20 | 11 |
| 13* | .45 | 34 | 28* | .50 | 26 |
| 14* | .54 | 47 | 29* | .55 | 26 |
| 15 | .28 | 15 | 30* | .37 | 15 |

*Items selected for revised form, Form A, of examinations.

TABLE VIII (CONTINUED)

| Item | $r_{rpb}$ | $V_{MLB}$ | Item | $r_{rpb}$ | $V_{MLB}$ |
|---|---|---|---|---|---|
| 31 | .19 | 49 | 56 | .06 | 13 |
| 32* | .54 | 64 | 57* | .24 | 40 |
| 33 | .55 | 58 | 58 | .11 | 18 |
| 34* | .52 | 69 | 59 | .11 | 25 |
| 35 | .42 | 54 | 60* | .25 | 47 |
| 36 | .32 | 31 | 61* | .62 | 52 |
| 37* | .34 | 85 | 62* | .29 | 34 |
| 38* | .35 | 85 | 63* | .46 | 69 |
| 39 | .17 | 40 | 64 | .17 | 17 |
| 40 | .16 | 26 | 65 | .44 | 27 |
| 41 | .13 | 30 | 66* | .48 | 86 |
| 42 | .02 | 4 | 67 | .35 | 88 |
| 43* | .25 | 35 | 68 | .41 | 77 |
| 44 | .25 | 20 | 69 | .13 | 40 |
| 45* | .35 | 60 | 70 | .14 | 37 |
| 46* | .64 | 87 | 71* | .32 | 51 |
| 47 | -.15 | -17 | 72 | .13 | 22 |
| 48* | .26 | 43 | 73* | .21 | 46 |
| 49 | .16 | 22 | 74 | .16 | 28 |
| 50* | .41 | 50 | 75 | .19 | 33 |
| 51 | .06 | 13 | 76* | .33 | 53 |
| 52 | .22 | 22 | 77 | .15 | 22 |
| 53 | .18 | 14 | 78 | .13 | 22 |
| 54* | .22 | 53 | 79* | .26 | 43 |
| 55* | .31 | 51 | 80 | .16 | 26 |

## Revised Forms of Tests, Forms A

### Selection of Items

After the determination of the discrimination indices, McCall-Long-Bliss validity coefficient, and the reduced point-biserial coefficient, the items showing the highest validity were selected for the revised form of the test. These were then sorted according to the outline of course topics and aims to determine whether a proper distribution among the various topics was provided. As was to be expected, this method of selection did not provide an entirely satisfactory distribution and some reselection was necessary. The best items available for each topic were chosen. In some cases very similar items show high indices, but all such items can not be used and yet provide sufficient variety to distribute the questions properly among the various topics. The questions selected for the final form are indicated with an asterisk (*) in Tables VII and VIII above.

Both Lindquist (122, p.314) and Gulliksen (78, p.365) emphasize the importance of fitting the questions to the course outlines. This is the most important aspect in the selection of questions. It may mean that some questions with lower item criterion relationships must be used rather than some with higher criterion relationships which are rejected. Within each topic or group of questions those with the highest discrimination indices should, of course, be selected.

Item analysis results may show that an item should be deleted, while a consideration of subject matter may show that essential material is being tested in the item. In this case Lindquist (122, p.314) advocates a study of the item to determine the flaw in it and revision so that it will satisfy both the item analysis and subject matter criteria. He indicates that such changes should be made even though a subsequent tryout will not be made to determine their efficiency.

Various authors have indicated that the best level of difficulty is about fifty percent (188, p.340; 126, p.676; and 122, p.314). Inasmuch as the McCall-Long-Bliss validity coefficient discriminates in favor of such items, no additional measure of difficulty was used in the selection of items for the tests. Use of an additional measure would tend to overemphasize this factor in the choice of questions.

<u>Part I of the revised examination</u>. The revised form, Form A, of Part I of the examination appears in Appendix E, page 178. The distribution of questions among the topics listed in the course outline is shown in Table I on page 59. The distribution of questions among the aims

is shown in Table II, page 61.

Part II of the revised examination. The revised form, Form A, of Part II of the examination appears in Appendix E, page 183. The questions were selected in the same manner as for Part I. The distribution of questions among the course topics is shown in Table III, page 63.

The questions on the optional topics all showed low item validity coefficients. This is probably due to the fact that only about one half of the students received instruction on these topics. For this reason these questions were omitted from the revised form of the examination.

The distribution of the questions among the aims for Form A of Part II of the examination is shown in Table IV, page 65.

## Diagnostic Charts

Diagnostic charts were prepared for use with the revised examinations, Forms A. These are shown on pages 189 and 190 for Part I and Part II respectively. These charts may be used to locate areas of difficulty for the individual student.

The diagnostic charts may also be used by the instructor to locate areas of difficulty which are common in a class. In this way the instructor can select review material to fit the needs of the particular class.

Revised Scores on Form P

Flanagan (57, pp.603-604) has proposed that in selecting the best possible combination of test items, use should be made of one of the short methods of determining the item-criterion correlation. On the basis of these correlations a group of questions should be selected to serve as the trial test. These scores should then be used to determine the correlation of the trial test with the criterion.

When the selections of the questions for the final forms of the tests had been made, the complete set of papers on the original forms of the tests was rescored. These scores were based on the questions which were to be retained on the revised form of the test. In order to determine how well the revised form of the test would predict grades for the following term, these scores were correlated with the grades earned the next term. The coefficients of correlation will furnish a measure of the validity of the tests for the purpose for which they were constructed.

Part I of the examination. The revised scores and grades earned the next term are shown in Table XI, Appendix D, page 171. The coefficient of correlation of the revised scores with the grades earned the following term for Part I of the examination was .67. As was expected, this correlation was not as high as that using the grades earned in the same term as the examination was given. The mean for this distribution of scores was 22.61. The standard deviation was 5.70.

Part II of the examination. Revised scores and grades earned the next term are shown in Table XII, Appendix D, page 174. The coefficient of correlation of the revised scores for Part II of the examination with the grades earned the following term was .56. The mean revised score was 22.40. The standard deviation was 6.20.

The lower coefficient of correlation for the revised scores on Part II of the examination with grades earned the following term may be due to several causes: (1) the size of the group for Part II is smaller, (2) students who took this examination were going on to a more theoretical type of algebra, and (3) the students of this group did not use the same textbook the following term as did those who took Part I of the examination. Though not as high as for Part I, the correlation coefficient for scores on Part II is well above the median for such correlations.

## Prediction of Grades

The mean revised scores for students who earned various letter grades the following term are shown in Table XIII, Appendix C, page 175. This table can be used to predict from his score on the test the grade that a student is likely to earn the following term.

Data on Forms A

The revised forms (Forms A) of the tests were given to the elementary and intermediate algebra classes at the end of winter term 1953 for the purpose of establishing the reliability of these forms. The individual scores for Part I are shown in Table XIV, Appendix F, page 192, and those for Part II in Table XV, Appendix F, page 193.

For Part I the mean was 22.60 and the standard deviation was 6.15. For Part II the mean was 18.20 and the standard deviation was 6.70. Variations in this data from that which was determined for revised scores on Forms P (*supra*, p.94) were caused by this data being determined on different groups of students. Another reason for the differences is that the length of time available for instruction is different for fall and winter terms.

To determine the reliability of revised forms of the tests each was again split into two halves on the basis of odd-numbered and even-numbered problems. These two halves were then correlated to determine the coefficient of reliability. For Part I the split-half reliability coefficient was .79. For Part II it was .80.

The Spearman-Brown prophecy formula was applied to the above coefficients of correlation to determine reliabilities of the whole tests. These were found to be .88 for Part I and .89 for Part II respectively.

The index of reliability for Part I of the test was .93; that of Part II was .94.

Use of Ohio Psychological Examination

The Ohio Psychological Examination is given to the entering classes at Portland State Extension Center. The purpose of using this examination is to provide "a measure of scholastic aptitude to predict ability to do college work." The scores on this examination and the grades that the students earned in mathematics courses are shown in Table XVI, Appendix G, page 197, for the students in the elementary group. The Ohio scores together with the grades earned are shown in Table XVII, Appendix G, page 200, for the intermediate group.

The coefficients of correlation of the Ohio Psychological Examination with the grades which the students earned the following term were: for the elementary course .35, for the intermediate course .21. These correlation coefficients are lower than those obtained by using the algebra examination scores with grades. Ohio Psychological Examination scores and the revised scores for Part I of the algebra examination are shown in Table XVIII, Appendix G, page 202. Table XIX, Appendix G, page 205, shows the Ohio scores and revised scores for Part II, Form A of the algebra examination. The correlation coefficient was obtained using the Ohio scores with the revised algebra examination scores and was .44 for Part I; for Part II it was .25.

<u>Multiple Correlation Coefficients</u>

To study the relation of the course grade to both the algebra examination score and the Ohio Psychological Examination score, the multiple correlation coefficient was computed. For Part I of the examination the multiple coefficient of correlation was .66 and for Part II it was .56. For both groups, the multiple coefficient of correlation with grade was lower than that of the algebra examination alone.

# CHAPTER VI

## SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The mathematical backgrounds of entering students at Portland State Extension Center are so varied that a single course offering does not meet their needs. Some method of placing the students in mathematics courses which are commensurate with their abilities is needed. An achievement examination is the best criterion for such placement in mathematics courses.

An examination which is used for placement should be developed for the particular group of students who are required to take the examination. This study was undertaken for the purpose of developing algebra placement examinations for entering students at Portland State Extension Center. The examinations were developed in two parts: Part I, covering topics of the elementary algebra course; and Part II, covering topics of the intermediate course.

The questions for the preliminary forms, Forms P, of both parts of the examinations were developed from the content of the courses and the aims of the courses as stated above (supra, pp.59-65). These questions were prepared in multiple-choice form offering five responses.

Forms P of the examinations were given to all students in the elementary and intermediate courses at the end of the fall term, 1952. The scores on these examinations were correlated with the grades earned in the respective courses. The coefficients of correlation yielded a measure of the validity of the tests. For Part I this validity coefficient was .77 and for Part II it was .66.

The reliability of Forms P was measured by using the odd-even split-half method. The coefficient of correlation between the halves of Part I was .73; for Part II it was .67. Elimination of the questions on the optional topics, logarithms and proportions, raised the correlation coefficient to .73 between the halves of Part II of the examination. The coefficients of reliability of the whole tests were estimated by the Spearman-Brown formula. This gave reliability coefficients of .84 for Part I and .80 for Part II. Removal of the optional topics from Part II increased this to .84. The index of reliability for Form P, Part I was .92 and for Part II was .89 (.92 with optional topics excluded).

Item analysis methods were used for the selection of questions for the revised forms of the test. A simplified formula for the point-biserial coefficient of correlation was developed and used. Reduced point-biserial coefficients of correlation and McCall-Long-Bliss validity coefficients were computed for each item on the examinations. Questions were selected for the revised forms on the basis of these indices and the material contained in the course outlines. The questions were compared with the topics in the course outlines to assure an adequate distribution.

All Form P papers were then rescored on the basis of the questions which were retained for the revised forms of the examinations. Scores on the revised forms were thus obtained for all students who took the original examinations. These revised scores were compared with the grades which the students earned in their next algebra courses. The coefficients of correlation between these scores and grades earned the following term were .67 for Part I and .56 for Part II.

The coefficients of correlation between the revised scores on the algebra tests and the scores on the Ohio Psychological Examination were also determined. These coefficients were .44 for Part I and .25 for Part II.

The coefficient of correlation between the Ohio Psychological Examination scores and the grades earned by students in the elementary mathematics course was .35. For students in the intermediate mathematics course this coefficient was .21.

Multiple correlation coefficients of grades with revised algebra examination scores and Ohio Psychological Examination scores were computed. These coefficients were .66 for the elementary group and .56 for the intermediate group. These multiple coefficients of correlation of the scores on the two tests with grades are no higher than the coefficients obtained by using only the algebra examinations. Therefore, the use of the two tests together does not predict grades with any more accuracy than do the algebra examinations alone.

The mean revised algebra examination scores for students earning each letter grade the following term were determined. These were shown in Table XIII, page 175. These tables can be used to predict, within limits, the grade a student will probably earn the following term. Predicted grades, of course, will be influenced by the instructor and his methods of instruction and grading as well as by changes in attitudes and interests on the part of the student.

The revised forms of the tests, Forms A, were given to the elementary and intermediate classes during the winter term, 1953, for the purpose of establishing the reliability of this form of the test. The odd-even split-half method was used. This yielded correlation coefficients of .79 for Part I and .80 for Part II. The estimated reliability coefficients of the whole tests were .88 and .89 for Parts I and II respectively. The index of reliability for Part I, Form A was .93. For Part II, Form A it was .94.

As a means of diagnosing difficulties within the elementary course, a chart listing the topics covered by each question on Part I was prepared. This is shown in Appendix E, page 189. These charts can be used for analyzing the individual student's difficulties.

## Conclusions

High school records are unsatisfactory for placement of students in college mathematics courses. This is due in part to the time lapse which frequently occurs between the last mathematics course the student has taken and his enrollment in a mathematics course at Portland State. The variability of high school records also tends to make them unsuitable for placement purposes.

The Ohio Psychological Examination which is given to the entering students at Portland State is also inadequate, either for predicting success in mathematics courses generally or for placing the students in specific mathematics courses.

Comparison of the entering students' test scores with those of students who have successfully completed the courses will provide a more satisfactory basis for placement than has been available in the past. Students who start their mathematics training with the intermediate or college algebra courses will be competing with many students who have taken the earlier courses at Portland State. It is important in placing such students to know how their backgrounds compare with those of other students who will be in the same classes.

A student is not considered to be doing satisfactory work in mathematics at Portland State unless he is earning a grade of C or better. Thus it would seem that a student should make a score on Part I of the placement test which indicates he is likely to earn a C in the intermediate course if he is to enter the intermediate course directly. Similarly, he should make a score on Part II of the examination that indicates he is likely to earn a C in the college algebra course if he is to enter this course directly.

From Table XIII, page 175, it appears that the most suitable score for determining whether a student should be permitted to enter the intermediate course would be midway between the mean C and D scores as this would be the approximate division point between these two groups of students.

The examination developed in this study closely parallels the content of the elementary and intermediate algebra courses given at Portland State Extension Center. Part I is intended for determining a student's mastery of the elementary course; Part II for determining a student's mastery of the intermediate course. Therefore, these examinations are more suitable for use at Portland State than are other examinations prepared elsewhere.

The curricular validity of the examinations for use with the courses at Portland State has been established. The statistical validity of the examinations for predicting a student's grade also has been established. The reliability of the examinations has been established and the reliability coefficient and the index of reliability have been determined for each part of the examinations. Reliability of the tests is sufficiently high to make the tests useful for individual diagnosis.

Individual diagnosis may be provided by use of the diagnostic charts which have been prepared for each test. These charts provide a basis for locating areas of weakness and for making the student aware of them. Use of the diagnostic charts will enable a student to locate his deficiencies at an early date before such deficiencies cause trouble in more advanced courses. Early location of extreme mathematical difficulties may prevent the loss of a student's time in a field of study for which he is not suited.

The diagnostic charts may also be used by the instructors for determining areas of general weakness which need review by their classes. Such analysis would enable the instructors to select review topics to suit the needs of their particular classes.

Use of the examinations developed here will provide an understandable and meaningful basis for placement of entering students. Entering students can be shown how their scores compare with those of students who have completed the courses. Provision should be made, however, for the student who, despite all precautions, finds himself misplaced. When such cases occur, either through incorrect placement or because of unforeseen circumstances, the student should be moved to either a more advanced or a less advanced course as needed.

The function of a mathematics placement examination is to segregate students according to their ability to do work in the field of mathematics. To be suitable for this purpose, an examination should be objective and should be easy to use and to score. It should possess a high degree of reliability and should also possess a high degree of validity for the purpose for which it is to be used. The tests developed in this study have been shown to possess these characteristics. They also possess the additional advantages of closely paralleling the content of the courses at Portland State and of providing a basis for diagnosis of difficulties in areas within these courses.

## Recommendations

In view of the findings of this study it appears that the algebra examinations developed here will adequately meet the mathematical placement needs at Portland State Extension Center. Therefore, the examinations are recommended for adoption and use in placement of entering mathematics students at Portland State.

Students who enter with one year or less of high school algebra need to take only Part I of the examination to determine whether they are ready for the intermediate course. Those entering with more than one year of high school algebra should take Part II as well to determine whether they are ready for college algebra.

Use of the diagnostic charts for students in the intermediate and advanced classes is recommended to aid the students in discovering areas of weakness. Use of these charts will enable the students to take positive steps to remedy such weaknesses as may be found. Examination of the charts by the instructors will furnish information for the selection of review material to meet the particular needs of a class.

Study of the results of future use of the examinations through diagnostic methods is needed to determine whether modification of the mathematics curriculum might provide a more suitable program for incoming mathematics students.

Continuous study of the examination scores of incoming classes should be made to keep the information on the tests up to date. Minimum scores to be required on the examinations for entry into the intermediate and college algebra classes should be reviewed by the staff from time to time in view of new information obtained on new groups of students.

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# APPENDICES

# APPENDIX A

## PRELIMINARY FORMS OF EXAMINATIONS, FORMS P

Directions to Students

Part I, Form P, Elementary Examination

Part II, Form P, Intermediate Examination

# ALGEBRA EXAMINATION
## FORM P

STUDENTS FILL IN THE FOLLOWING
Answer All Questions Below

SCORES: Part I ____
Part II ____
Total ____

Date ____________ Student's Name ______________________
(Last) (First)

Major Field __________________

**Previous Mathematical Training**

| High School | Years | College | Years |
|---|---|---|---|
| General Math. | ____ | Algebra | ____ |
| Algebra | ____ | Geometry | ____ |
| Geometry | ____ | Trigonometry | ____ |
| Trigonometry | ____ | Analysis | ____ |
| Senior Math. | ____ | | |
| Total | ____ | Total | ____ |

DIRECTIONS TO STUDENTS. READ CAREFULLY.

1. Do not open this test till told to begin.
2. Work rapidly and carefully. Answer as many questions as you can but do not make wild guesses.
3. Take the questions in any order that you prefer.
4. Stop promptly when time is called. You may not have time to answer all questions.
5. Use backs of pages and margins for your scratchwork.
6. Do not ask questions after the examination begins.

# ALGEBRA EXAMINATION

## PART I FORM P

## (Elementary)

Directions: Select the proper answer to the problem and check the appropriate answer in the space at the right.

Example: The sum of 6 and 2 is — Ex. a ( ) b ( ) c ( ) d (✓) e ( )

(a) 7 (b) 3 (c) 12 (d) 8 (e) none of these

Since 8 is the correct answer a check should be placed in the (d) space.

---

1. An exponent in the expression $12ax^3$ is — 1. a ( ) b ( ) c ( ) d ( ) e ( )

   (a) x (b) 12 (c) 12a (d) 2 (e) 3

2. The coefficient of x in the expression $12a^3b^2x$ is — 2. a ( ) b ( ) c ( ) d ( ) e ( )

   (a) 12 (b) $a^3b^2$ (c) $12a^3b^2$
   (d) 12ab (e) ab

3. An expression of the type $4x^2 - 2ax - ab^2$ is called a (an) — 3. a ( ) b ( ) c ( ) d ( ) e ( )

   (a) trinomial (d) quotient
   (b) binomial (e) product
   (c) equation

4. The distributive law of multiplication states — 4. ( ) ( ) ( ) ( ) ( )

   (a) $x(y + z) = xy + xz$
   (b) $x(y + z) = (y + z)x$
   (c) $(x + y) + z = x + (y + z)$
   (d) $x(yz) = xy \cdot xz$
   (e) none of these

5. The commutative law of addition states — 5. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $x + (y + z) = (x + y) + z$
(b) $x(y + z) = xy + xz$
(c) $x + y = y + x$
(d) $\frac{x + y}{z} = z(x + y)$
(e) $xyz = x + y + z$

6. The expression for the area of a circle is — 6. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $2\pi r$ (b) $\frac{1}{4}\pi d^2$ (c) $\pi r^2 h$ (d) $\frac{4\pi r^3}{3}$
(e) $\frac{1}{2}\pi d^2$

7. The formula for the total area of a closed box of dimensions a, b, and c, is — 7. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $2a + 2b + 2c$
(b) $2(ab + ac + bc)$
(c) $4ab + 2bc$
(d) $6abc$
(e) $(a + b + c)^2$

8. The circumference of a circle is — 8. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $\pi r^2$ (b) $\frac{\pi r}{2}$ (c) $\pi r$ (d) $\pi d$ (e) $2\pi d$

9. The units and tens digit of a number are x and y respectively. The number is — 9. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $xy$
(b) $10y + x$
(c) $10x + y$
(d) $10xy$
(e) $10(x + y)$

10. If the sum of two numbers is 17 and their difference is 5, then the equation needed to find the numbers is — 10. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $x + (x + 5) = 17$
(b) $x = (x - 5) + 17$
(c) $x + 5 = 17 + x$
(d) $5x = x + 17$
(e) $x = 17 + x$

| | a | b | c | d | e |
|---|---|---|---|---|---|

11. An expression for a number added to 30% of itself is

(a) n + .30%n (d) 1.30n
(b) (n + 30)n (e) n + .30
(c) 1.30%n

11. ( ) ( ) ( ) ( ) ( )

12. If the first 10 words of a telegram cost 50¢ and each additional word costs 5¢, the total cost <u>C</u> in cents of <u>n</u> words is

(a) 100 + 5n (d) 50 + 5n
(b) (50 + 5)n (e) none of these
(c) 50n + 5

12. ( ) ( ) ( ) ( ) ( )

13. A formula for <u>time</u> is

(a) $t = \frac{d}{r}$ (b) $d = rt$ (c) $r = \frac{d}{t}$

(d) $t = rd$ (e) $t = \frac{r}{d}$

13. ( ) ( ) ( ) ( ) ( )

14. Find the area of a circle of diameter 14($\pi$ = 22/7).

(a) 308 (b) 44 (c) 88 (d) 616
(e) 154

14. ( ) ( ) ( ) ( ) ( )

15. If pv = 6t, find <u>v</u> if p = 24 when t = 2.

(a) 8 (b) 4 (c) $\frac{1}{2}$ (d) 16 (e) 24

15. ( ) ( ) ( ) ( ) ( )

16. Find the area of a trapezoid of height 8 and bases 4 and 7.

(a) 88 (b) 112 (c) 84 (d) 46
(e) none of these

16. ( ) ( ) ( ) ( ) ( )

17. If an investment of $1000 earns 4% interest and another investment of $2000 earns 6%, then the expression for the annual earnings in dollars is

(a) 4(1000) + 6(2000) (d) .04(1000) + .06(2000)
(b) .04(2000) + .06(1000) (e) (.04%)(1000) + (.06%)(2000)
(c) (.10)(3000)

17. ( ) ( ) ( ) ( ) ( )

18. $(a + 2a^2 + 1) - 2(1 - a) =$

(a) $2a^2 - a + 1$ (d) $2a^2 - 2$
(b) $1 - a + 4a^2$ (e) $3a + 2a^2$
(c) $2a^2 + 3a - 1$

18. a ( ) b ( ) c ( ) d ( ) e ( )

19. Subtract $(4x - 7y + z)$ from $(3x - y)$.

(a) $x - 6y + z$ (d) $x - 8y + z$
(b) $-x + 6y - z$ (e) $-x + 8y + z$
(c) $x + 6y - z$

19. a ( ) b ( ) c ( ) d ( ) e ( )

20. $(x^3 - 3x - 2) - 3(1 - x) =$

(a) $x^3 + x - 5$ (d) $x^3 - 6x - 5$
(b) $x^3 + 1$ (e) $x^3 - 5x + 1$
(c) $x^3 - 5$

20. a ( ) b ( ) c ( ) d ( ) e ( )

21. Simplify: $7y + (2y - 5y) =$

(a) $2y$ (b) $14y$ (c) $7y - 3y$
(d) $4y$ (e) $4$

21. a ( ) b ( ) c ( ) d ( ) e ( )

22. $(26xy) \div (-2x) =$

(a) $13y$ (b) $-52x^2y$ (c) $24y$
(d) $-xy$ (e) $-13y$

22. a ( ) b ( ) c ( ) d ( ) e ( )

23. $(15a^2b^3c^4 - 20a^3b^2c^3) \div (-5a^2b^2c^2) =$

(a) $4bc - 3bc^2$ (d) $3bc + 4ac$
(b) $4ac - 3b^2c^2$ (e) $3b^2c^2 - 4ac$
(c) $4ac - 3bc^2$

23. a ( ) b ( ) c ( ) d ( ) e ( )

24. Simplify:
$(x^3 - x^2 + 4) - (5 + x^2 - x^3) =$

(a) $x^3 - x^2 - 1$ (d) $2x^3 - 2x^2 - 1$
(b) $x^3 - 1$ (e) $-2x^2 - 1$
(c) $2x^3 + 2x^2 + 9$

24. a ( ) b ( ) c ( ) d ( ) e ( )

25. $(3x + 2y) - (2x - y) =$

(a) $x - y$ (d) $5x - 3y$
(b) $5x + 3y$ (e) $x - 3y$
(c) $x + 3y$

25. a ( ) b ( ) c ( ) d ( ) e ( )

26. Simplify:
$5a - 2a[-2(1 - 2a) - 1] + 6 =$

(a) $7a + 3$ (b) $7a + 5$ (c) $a + 5$
(d) $7a + 6$ (e) none of these

26. a ( ) b ( ) c ( ) d ( ) e ( )

27. $(x - 3)(2x + 1) =$

(a) $2x^2 - 5x - 3$ (d) $3x - 2$
(b) $2x^2 - 3$ (e) $2x^2 - 5x + 3$
(c) $2x^2 - 7x - 3$

27. a ( ) b ( ) c ( ) d ( ) e ( )

28. Add: $2a^2 - 6ab + 3b^2$; $-a^2 + b^2$; and $3ab - 4b^2$

(a) $a^2 - 2ab + 4b^2$ (d) $a^2 - 3ab$
(b) $a^2 - 6ab$ (e) $2a^2 - 3ab - b^2$
(c) $a^2 - 6ab + b^2$

28. a ( ) b ( ) c ( ) d ( ) e ( )

29. Evaluate the expression $x - \frac{1}{2}x$ when $x = 0.8$.

(a) 1.6 (b) .12 (c) 0.8 (d) -1.2
(e) 0.4

29. a ( ) b ( ) c ( ) d ( ) e ( )

30. If $L = a + (n - 1)d$, find $\underline{L}$ when $d = 2$, $a = 6$, and $n = 11$.

(a) 60 (b) 26 (c) 28 (d) 78
(e) 24

30. a ( ) b ( ) c ( ) d ( ) e ( )

31. The circumference of a circle of radius 14 is ($\pi = 22/7$)

(a) 308 (b) 44 (c) 88
(d) 616 (e) 154

31. a ( ) b ( ) c ( ) d ( ) e ( )

32. $(-2xy)(2x^2y)^2 =$

(a) $16x^6y^4$ (b) $-8x^4y^3$ (c) $-16x^5y^3$
(d) $-8x^5y^3$ (e) $8x^4y^3$

32. a ( ) b ( ) c ( ) d ( ) e ( )

33. $(-xy)^3(xy)(x^2y) =$

(a) $-x^4y^3$ (b) $-x^6y^5$ (c) $x^6y^5$
(d) $x^5y^6$ (e) $-x^3y^3$

33. a ( ) b ( ) c ( ) d ( ) e ( )

34. $(x^2 - 2xy + y^2) + (x - y) =$ — 34. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $x - y$ (b) $x + y$ (c) $x - 2y$
(d) $x - 2$ (e) $x - xy + y$

35. $(x^3 - 2x^2 - 14x - 5) + (x - 5) =$ — 35. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $x^2 + 1$ (d) $x^2 - 2x + 1$
(b) $x^2 - 2x - 14$ (e) $x^2 + 3x + 1$
(c) $x^2 - 3x + 1$

36. $(3m^2n^3 - 9mn^2 - 6m^2n) + (-3mn) =$ — 36. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $mn + 3n + 2m$ (d) $mn + 3n^2 + 2m^2$
(b) $-mn^2 + 3n + 2m$ (e) $mn^2 - 5mn$
(c) $mn^2 - 3mn + 2$

In problems 37 to 39 evaluate using the values $a = 1$, $b = -1$, $c = 2$, and $d = -3$,

37. $(a + b)(c + d) =$ — 37. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 12 (b) -1/6 (c) -12 (d) 0
(e) none of these

38. $(b - c)^3 =$ — 38. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 8 (b) -27 (c) -8 (d) 27
(e) none of these

39. $\frac{a}{c} + \frac{d}{c-2} =$ — 39. a ( ) b ( ) c ( ) d ( ) e ( )

(a) -6 (b) 1/2 (c) -1 1/2 (d) -1
(e) none of these

40. If $x^5 - 3x^2 + 2x + 1$ is divided by $x - 1$ the remainder is — 40. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 7 (b) 1 (c) -5 (d) 3 (e) 5

41. A method of long division which makes use of only the coefficients is called — 41. a ( ) b ( ) c ( ) d ( ) e ( )

(a) short division (d) functional notation
(b) synthetic division (e) long division
(c) synthetic substitution

42. If $g(x) = 2x - 3$ find $[3g(-1)]^2$ — 42. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 81 (b) 9 (c) 225 (d) 64
(e) none of these

43. If $f(x) = x^3 - 3x + 2$ find $f(-2)$. — 43. a ( ) b ( ) c ( ) d ( ) e ( )

(a) -8 (b) 16 (c) 4 (d) -16 (e) 0

44. Solve for <u>x</u>: $6x - 2 = 3x + 10$ — 44. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 14 (b) 4 (c) 4/3 (d) -4/3
(e) -4

45. Solve for <u>x</u>: $x - (1 - x) = 7$ — 45. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 4 (b) 6 (c) -1 (d) 8
(e) none of these

46. Solve for <u>x</u>: $2 - 2x = 6$ — 46. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 2 (b) -4 (c) -2 (d) -1/2
(e) -3

47. Solve for <u>C</u>: $F = \frac{9}{5}C + 32$ — 47. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $\frac{5F - 32}{9}$ (d) $\frac{5F - 160}{9}$

(b) $9(5F - 32)$ (e) $\frac{9}{5}F - 32$

(c) $\frac{5}{9}F - 32$

48. Solve for <u>x</u>: $4ax - 9ab = 3ab$ — 48. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 3ab (b) 3b (c) 3(a + b)
(d) -6a + 3b (e) 6ab

49. Solve for <u>x</u>: — 49. a ( ) b ( ) c ( ) d ( ) e ( )
$(x + 3c)(x - 2c) = (x + 4c)(x - c)$

(a) 4c (b) 2c (c) -c (d) 2 (e) 0

50. Solve for <u>x</u>: $3x - (2x + 5) = 3$ — 50. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 2 (b) 8 (c) -3/4 (d) -2 (e) 0

51. Solve for h: $V = \frac{1}{4}\pi d^2 h$ — 51. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $\frac{V}{4\pi d^2}$ (b) $\frac{4V}{\pi d^2}$ (c) $\sqrt{\frac{4V}{\pi h}}$

(d) $4V\pi d^2$ (e) $V - \frac{1}{4}\pi d^2$

52. The sum of two numbers is 68 and half of their difference is 4. Find the numbers. — 52. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 30 and 36 (d) 32 and 36
(b) 28 and 40 (e) 24 and 34
(c) 30 and 38

53. Find a number such that one half of it added to one third of it is equal to ten. — 53. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 10 (b) 12 (c) 60 (d) 24 (e) 15

54. Two autos traveled at rates of 40 and 25 miles per hour respectively. If the faster car travels 3 hours longer and goes twice as far, what is the distance traveled by the faster car? — 54. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 600 (b) 400 (c) 1200 (d) 300
(e) 120

55. Solve for x: $\frac{x + 1}{4} - \frac{2x + 3}{11} = 1$ — 55. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 7 (b) -22/3 (c) 11 (d) 15
(e) 44

56. Solve for x: $\frac{c + d}{x + 1} = \frac{c - d}{x - 1}$ — 56. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 1 (b) c/d (c) cd (d) c + d
(e) c - d

57. Solve for C: $S = \frac{1}{2}(A + B + C)$ — 57. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 2(S - A - B) (d) 2A - 2S + B
(b) 2(S - A + B) (e) 2A + 2B + 2S
(c) 2S - A - B

58. Solve for W: $F = \frac{hW}{e}$

58. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $F - \frac{h}{e}$ (b) $\frac{eF}{h}$ (c) $eFh$
(d) $\frac{h}{eF}$ (e) $\frac{h}{e} - F$

59. Solve for x: $\frac{x}{a} - \frac{x}{b} = \frac{b}{a} - \frac{a}{b}$

59. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $a + b$ (b) 2 (c) 1
(d) $\frac{1}{a - b}$ (e) $b - a$

60. Solve for f: $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

60. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $\frac{fq + fp}{fpq}$ (b) $\frac{1}{pq}$ (c) $\frac{pq}{p + q}$

(d) $p + q$ (e) $\frac{1}{p + q}$

61. Factor $x^2 - 5x + 6 =$

61. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $(x - 6)(x - 1)$ (d) $(x - 4)(x - 1)$
(b) $(x - 3)(x - 2)$ (e) $(x + 6)(x + 1)$
(c) $(x + 3)(x + 2)$

62. Factor $4x^2 - 81y^2 =$

62. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $(2x - 9y)(2x + 9y)$
(b) $(2x - 9y)^2$
(c) $(4x + 9y)(x - 9y)$
(d) $(2x + 9y)^2$
(e) $(4x - 9y)(x + 9y)$

63. Factor $4xy^2 + 4xy + 8y =$

63. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $(2xv + 4y)(2y + 2)$ (d) $4y(x + 2y^2)$
(b) $y(4x + 4)(x + y)$ (e) none of these
(c) $4y(xy + x + 2)$

64. Factor $2\pi r^2h + 2\pi r^2 =$

64. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $2\pi r(r + 2)(h + 2)$ (d) $2\pi r^2(h + 1)$
(b) $2\pi(\pi rh + r)$ (e) $2(\pi rh + r)(r + 2)$
(c) $2\pi(rh + r)(r + 1)$

a b c d e

65. Factor $2y^2 - 5y - 18 =$ 65. ( ) ( ) ( ) ( ) ( )

(a) $(2y - 9)(y + 2)$
(b) $(2y - 3)(y + 6)$
(c) $(y - 18)(2y + 1)$
(d) $(2y + 9)(y^2 - 2)$
(e) $(2y + 3)(y - 6)$

a b c d e

66. Expand: $(2x - y)^2 =$ 66. ( ) ( ) ( ) ( ) ( )

(a) $4x^2 - y^2$ (d) $4x^2 + 4xy + y^2$
(b) $4x^2 - 2xy + y^2$ (e) $4x^2 - 4xy + y^2$
(c) $4x^2 + y^2$

a b c d e

67. Expand: $(x + y - 2)^2$ 67. ( ) ( ) ( ) ( ) ( )

(a) $x^2 + y^2 + 4$
(b) $x^2 + y^2 - 4$
(c) $x^2 + y^2 + 4 + 2xy - 4x - 4y$
(d) $x^2 + y^2 + 4 + xy - 2x - 2y$
(e) none of these

a b c d e

68. Factor $x^2 - 6xy + 9y^2$ 68. ( ) ( ) ( ) ( ) ( )

(a) $(x - 3y)(x + 3y)$ (d) $(x - 3y)^2$
(b) $(x - 9y)(x + y)$ (e) none of these
(c) $(x + 3y)^2$

a b c d e

69. A prime factor of $y^4 - 81$ is 69. ( ) ( ) ( ) ( ) ( )

(a) $y + 9$ (b) $y^2 - 9$ (c) $y^2 + 9$
(d) $y - 9$ (e) $y - 27$

a b c d e

70. Factor $4 - (x - y)^2$ 70. ( ) ( ) ( ) ( ) ( )

(a) $(2 + x - y)(2 - x - y)$
(b) $(2 - x + y)(2 + x - y)$
(c) $(2 - x - y)^2$
(d) $(2 - x + y)^2$
(e) none of these

a b c d e

71. Factor $8a^3 + 27y^3$ 71. ( ) ( ) ( ) ( ) ( )

(a) $(2a + 3y)^3$
(b) $(2a + 3y)(4a^2 + 9y^2)$
(c) $2a + 3y)(4a^2 - 6ay + 9y^2)$
(d) $(4a^2 + 6ay + 9y^2)(2a + 3y)$
(e) $(2a - 3y)(4a^2 - 12ay + 9y^2)$

72. Factor $2m^3 - 3m^2 - 4m + 6$

72. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $(m^2 - 2)(2m - 3)(2m + 3)$
(b) $(m^2 - 2)(2m - 3)$
(c) $(m^2 + 2)(2m - 3)(2m - 3)$
(d) $(m^2 - 2)(2m - 3)^2$
(e) $(2m + 3)(m^2 - 2)$

73. A factor of $x^3 + x + 10$ is

73. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $x + 2$ (b) $x - 2$ (c) $x + 5$
(d) $x - 5$ (e) $x^2 + 5$

74. Simplify: $\dfrac{3x + \frac{4y}{3}}{\frac{9x}{2} + 2y}$

74. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 1 (b) $3x - 2y$ (c) 3
(d) $\frac{2}{3}$ (e) $\frac{2y}{3x}$

75. Simplify: $\dfrac{1 - \frac{1}{x}}{x - 1} =$

75. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $x$ (b) $x - 1$ (c) $1/x$

(d) $\dfrac{x}{x - 1}$ (e) $\dfrac{x - 1}{x}$

76. Simplify: $\dfrac{2 + \frac{1}{x}}{\frac{1}{x} + 4} =$

76. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $\dfrac{2x + 1}{4(x + 4)}$ (b) $\dfrac{1 + 4x}{2x + 1}$ (c) $\dfrac{2x}{1 + 4x}$

(d) $\dfrac{2x + 1}{4x + 1}$ (e) none of these

77. $\dfrac{1}{1 - x} \div \dfrac{x}{1 - x^2} =$

77. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $1 + x$ (b) $1 - x$ (c) $\dfrac{1 + x}{x}$

(d) $\dfrac{x}{(1 - x)(1 - x^2)}$ (e) $\dfrac{1}{1 + x}$

78. $\left(4 - \frac{1}{x}\right)\left(\frac{x}{1 - 4x}\right) =$

78. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $\frac{4x - 1}{1 - 4x}$ (b) $\frac{1 - 4x}{4x - 1}$ (c) $-1$

(d) $\frac{4x + 1}{1 + 4x}$ (e) $\frac{3}{1 - 4x}$

79. Combine and simplify:

79. a ( ) b ( ) c ( ) d ( ) e ( )

$$\frac{a}{x - a} - \frac{a^2}{x^2 - a^2} =$$

(a) $\frac{ax}{x^2 - a^2}$ (d) $\frac{x^2 - a^2x}{(x - a)(x^2 - a^2)}$

(b) $\frac{ax}{(x - a)(x^2 - a^2)}$ (e) none of these

(c) $\frac{-2a^3}{x^2 - a^2}$

80. Combine and simplify: $\frac{1}{x} + \frac{1}{y} + \frac{2}{xy} =$

80. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $\frac{x + y + 2}{xy}$ (b) $4xy$ (c) $xy(x + y + 2)$

(d) $\frac{x + y + 2}{xy}$ (e) $\frac{x + y + 2}{x^2y^2}$

ALGEBRA EXAMINATION

PART II FORM P

(Intermediate)

Directions: Place a check indicating the correct answer in the appropriate space at the right.

Example: The sum of 6 and 2 is  Ex. a ( ) b ( ) c ( ) d (✓) e ( )

(a) 6 (b) 3 (c) 4 (d) 8 (e) 12

Since the correct sum is 8 a check should be placed in the (d) space.

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1. The equations $x - y = 8$ and $x + y = 8$ are  1. a ( ) b ( ) c ( ) d ( ) e ( )

   (a) dependent  (d) consistent
   (b) equivalent  (e) none of these
   (c) inconsistent

2. The equations $3m - 2n = 5$ and $2m + 3n = 12$ are  2. a ( ) b ( ) c ( ) d ( ) e ( )

   (a) consistent  (d) dependent
   (b) inconsistent  (e) none of these
   (c) equivalent

3. Solve: $7x + y = 15$ and $x = 2y$  3. a ( ) b ( ) c ( ) d ( ) e ( )

   (a) 1,2 (b) 1,1/2 (c) -1,2
   (d) 2,1 (e) 1/2,1

4. The equations: $3x - 4y = 6$ and $6x - 8y = 12$  4. a ( ) b ( ) c ( ) d ( ) e ( )

   (a) should be solved by addition
   (b) should be solved by subtraction
   (c) should be solved by completing the square
   (d) should be solved by substitution
   (e) can't be solved

5. The equation of the x-axis is

(a) $y = 0$ (b) $x = 0$ (c) $x - y = 0$
(d) $x = y = 0$ (e) $x + y = 0$

5. a ( ) b ( ) c ( ) d ( ) e ( )

6. The equation $2x - y = 3$ crosses the x-axis at

(a) 3 (b) 3/2 (c) -2/3 (d) -3
(e) 1

6. a ( ) b ( ) c ( ) d ( ) e ( )

7. The first step in the solution of the following equations should be

$$\frac{1}{x} - \frac{1}{y} = 6$$

$$\frac{1}{x} - \frac{2}{y} = 4$$

(a) substitution
(b) addition
(c) subtraction
(d) clear fractions
(e) none of these

7. a ( ) b ( ) c ( ) d ( ) e ( )

8. Solve: $3x - 4y = 13$
$2x - 3y = 12$

(a) $x = 9$, $y = 2$ (d) $x = -9$, $y = -10$
(b) $x = 7$, $y = 2$ (e) $x = -7$, $y = -2$
(c) $x = -9$, $y = 10$

8. a ( ) b ( ) c ( ) d ( ) e ( )

9. Simplify: $(-2x)^2 =$

(a) $-4x^2$ (b) $\frac{1}{4x^2}$ (c) $2x^2$

(d) $-2x^2$ (e) $4x^2$

9. a ( ) b ( ) c ( ) d ( ) e ( )

10. Simplify: $(x^{m-n})(x^{2n}) =$

(a) $x^{m+n}$ (b) $x^{m-2n}$ (c) $x^{m-n}$
(d) $x^{m-2}$ (e) $x^{n-2m}$

10. a ( ) b ( ) c ( ) d ( ) e ( )

11. Simplify: $(a^{1/2}b^{1/3})^6 =$

(a) $a^2b^3$ (d) $6a^{1/2}b^{1/2}$
(b) $a^3b^2$
(c) $a^{1/3}b^{1/2}$ (e) $a^{6\ 1/2}b^{1/3}$

11. a ( ) b ( ) c ( ) d ( ) e ( )

12. Simplify: $x^{5/6} + x^{1/3} =$ 12. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $x^{7/6}$ (b) $x^{2/3}$ (c) $x^{3/2}$
(d) $x^{1/2}$ (e) $x$

13. Simplify: $(1/4)^{-1/2} =$ 13. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 1/8 (b) -1/16 (c) -1/4
(d) 4 (e) 2

14. Multiply: $(\sqrt{a})(\sqrt[3]{a}) =$ 14. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $\sqrt[5]{a}$ (b) $\sqrt[5]{a^2}$ (c) $\sqrt[6]{a^2}$
(d) $\sqrt[6]{a^5}$ (e) $\sqrt[6]{a}$

15. Simplify: $2\sqrt{8} + \sqrt{50} =$ 15. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 40 (d) $2\sqrt{400}$
(b) $9\sqrt{2}$ (e) none of these
(c) $4\sqrt{2} + 2\sqrt{5}$

16. Multiply: $(2\sqrt{3} - \sqrt{5})(2\sqrt{3} + \sqrt{5}) =$ 16. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $4\sqrt{3} - \sqrt{5}$ (d) $7 - 4\sqrt{15}$
(b) $12 - 4\sqrt{15} + \sqrt{25}$ (e) 11
(c) 7

17. Simplify: $\dfrac{2}{\sqrt{3} + 1}$ 17. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $\sqrt{3} + 1$ (b) $1/2\ (\sqrt{3} + 1)$
(c) $\sqrt{3} - 1$ (d) $1/2\ (\sqrt{3} - 1)$
(e) none of these

18. Simplify: $i^{17} =$ 18. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 1 (b) $-i$ (c) $-1$ (d) $i$
(e) $1/i$

19. $\sqrt{-45}$ in simplest form is 19. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $-3\sqrt{5}$ (b) $3\sqrt{5i}$ (c) $5\sqrt{3}\ i$
(d) $3\sqrt{5}\ i$ (e) $-3\sqrt{5}\ i$

20. Solve: $\sqrt{9x^2 - 5} = 3x - 5$

(a) 1 (b) 3/2 (c) -2/3 (d) 3
(e) has no root

20. a ( ) b ( ) c ( ) d ( ) e ( )

21. Solve: $1 + \sqrt{x} = \sqrt{x + 3}$

(a) 2 (b) -3 (c) 1 (d) -1
(e) has no root

21. a ( ) b ( ) c ( ) d ( ) e ( )

22. Solve for <u>x</u>: $x^2 - x - 12 = 0$

(a) 4,-3 (b) -6,2 (c) 3,-4
(d) 1,12 (e) none of these

22. a ( ) b ( ) c ( ) d ( ) e ( )

23. What quantity, if any, must be added to $x^2 + 4$ to make it a perfect square?

(a) 4x (b) x (c) 4 (d) $3x^2$
(e) nothing need be added--already is a perfect square

23. a ( ) b ( ) c ( ) d ( ) e ( )

24. The quantity which needs to be added to make the expression $4x^2 - 6x$ a perfect square is

(a) 3/2 (b) 9 (c) 3/4
(d) -9 (e) 9/4

24. a ( ) b ( ) c ( ) d ( ) e ( )

25. Solve for <u>x</u>: $2x^2 - 5x = 3$

(a) 1,3/2 (b) 3,2 (c) 3,-1/2
(d) 3/2,2 (e) -3,-2

25. a ( ) b ( ) c ( ) d ( ) e ( )

26. If plotted, the equation $2x^2 - 5x - 3 = 0$ will intersect the y-axis

(a) at two positive points
(b) at one point
(c) at two points, one positive, one negative
(d) won't intersect
(e) at four points

26. a ( ) b ( ) c ( ) d ( ) e ( )

27. The function $y = x^2 - x - 12$ will cross the x-axis

(a) at one point which is negative
(b) at two points, both positive
(c) at two points, both negative
(d) at one point which is positive
(e) at two points, one positive and one negative

27. a ( ) b ( ) c ( ) d ( ) e ( )

28. Solve: $x^2 - x - 3 = 0$

(a) $3,-1$
(b) $-3,1$
(c) $\frac{-1 \pm \sqrt{13}}{2}$
(d) $\frac{+1 \pm \sqrt{13}}{2}$
(e) $\frac{-1 \pm \sqrt{13}}{2}$

28. a ( ) b ( ) c ( ) d ( ) e ( )

29. Solve: $2x^2 + x + 3 = 0$

(a) $\frac{-1 \pm \sqrt{25}}{2}$
(b) $\frac{-1 \pm \sqrt{23}\ i}{4}$
(c) $-1 \pm \frac{\sqrt{-23}}{2}$
(d) $-3,2$
(e) $+3,-3/2$

29. a ( ) b ( ) c ( ) d ( ) e ( )

30. Solve for <u>x</u>: $x^2 - 7abx + 6a^2b^2 = 0$

(a) $6a,b$ (b) $-6a^2,b^2$ (c) $-6ab,-ab$
(d) $3ab,2ab$ (e) $6ab,ab$

30. a ( ) b ( ) c ( ) d ( ) e ( )

31. A correct form of the quadratic formula is

(a) $-b \pm \frac{\sqrt{b^2-4ac}}{2a}$
(b) $b \pm \frac{\sqrt{b^2-4ac}}{2a}$
(c) $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$
(d) $\frac{b \pm \sqrt{b^2-4ac}}{2}$
(e) none of these

31. a ( ) b ( ) c ( ) d ( ) e ( )

32. The quadratic formula is derived from the general quadratic equation by

32. a ( ) b ( ) c ( ) d ( ) e ( )

(a) squaring both sides
(b) completing the square
(c) factoring
(d) addition or subtraction
(e) eliminating the constant

33. The equation $x^3 - x^2 - 12x = 0$ can be solved by

33. a ( ) b ( ) c ( ) d ( ) e ( )

(a) completing the square
(b) dividing out an x
(c) quadratic formula
(d) factoring
(e) no single one of these methods

34. The equation $x^3 + x^2 - 3x = 0$ can be solved by

34. a ( ) b ( ) c ( ) d ( ) e ( )

(a) completing the square
(b) factoring
(c) quadratic formula
(d) dividing out an x
(e) no single one of these methods

35. The graph of a cubic equation will always intersect the x-axis

35. a ( ) b ( ) c ( ) d ( ) e ( )

(a) three times
(b) once only
(c) not more than three times
(d) twice
(e) will never intersect

36. The number of solutions which the following pair of equations have is

36. a ( ) b ( ) c ( ) d ( ) e ( )

$x^2 + y^2 = 25$
$x - y + 1 = 0$

(a) 0
(b) 1
(c) 2
(d) 4
(e) 8

37. The following pair of equations can best be solved by

37. a ( ) b ( ) c ( ) d ( ) e ( )

$2x^2 + y^2 = 6$
$5x + y = 9$

(a) eliminating the constant
(b) addition or subtraction
(c) reducing to simpler systems
(d) substitution
(e) graphing

38. The number of distinct solutions for the following equations is

38. a ( ) b ( ) c ( ) d ( ) e ( )

$x^2 + y^2 = 13$
$4x^2 - y^2 = 7$

(a) 4 (b) 3 (c) 2 (d) 1 (e) 8

39. Solve for x and y: $x - y = 1$
$x^2 + y^2 = 13$

39. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $x = 3$, $y = 2$ and $x = -2$, $y = -3$
(b) $x = 3$, $y = -2$ and $x = -3$, $y = 2$
(c) $x = 3$, $y = 2$; double root
(d) $x = 3$, $y = -2$ and $x = 2$, $y = -3$
(e) $x = 3$, $y = -2$ and $x = -2$, $y = 3$

40. The number of solutions for two quadratic equations in two unknowns is

40. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 1 (b) 8 (c) 4 (d) 2
(e) at most 4

41. Ratio is a comparison of two numbers by

41. a ( ) b ( ) c ( ) d ( ) e ( )

(a) subtraction
(b) division
(c) multiplication
(d) addition
(e) progression

42. The expression $1/2 = 2/4$ is a (an)

42. a ( ) b ( ) c ( ) d ( ) e ( )

(a) equation
(b) ratio
(c) algebraic expression
(d) proportion
(e) formula

43. Which of the following is a proportion?

(a) $1/3 = 2/5$
(b) $3/12 = 6/18$
(c) $14/7 = 28/21$
(d) $5/3 = 15/12$
(e) $35/21 = 5/3$

43. a ( ) b ( ) c ( ) d ( ) e ( )

44. In a proportion the product of the first and fourth proportional is equal to the product of the

(a) first and third
(b) second and third
(c) first and fourth
(d) second and fourth
(e) third and fourth

44. a ( ) b ( ) c ( ) d ( ) e ( )

45. Solve the proportion for x:

5:x :: x:15

(a) $5/3$ (b) $3/5$ (c) $3\sqrt{5}$
(d) $5\sqrt{3}$ (e) 3

45. a ( ) b ( ) c ( ) d ( ) e ( )

46. The ratio of 2 feet to 5 yards is

(a) 2/5 (b) 5/2 (c) 2/15 (d) 15/2
(e) none of these

46. a ( ) b ( ) c ( ) d ( ) e ( )

47. A number y varies directly as x if their (?) is a constant.

(a) product
(b) quotient
(c) difference
(d) if x is a constant
(e) if y is a constant

47. a ( ) b ( ) c ( ) d ( ) e ( )

48. If y increases as x increases then y varies (?) as x varies.

(a) inversely
(b) conversely
(c) equally
(d) directly
(e) none of these

48. a ( ) b ( ) c ( ) d ( ) e ( )

49. If y varies directly as the square of x and y = 8 when x = 2, find the constant of variation.

(a) 2 (b) 1/2 (c) 4 (d) 16 (e) 6

49. a ( ) b ( ) c ( ) d ( ) e ( )

50. If y varies inversely as $x^2$, and $y = 9$ when $x = 2$, find y when $x = 3$.

(a) 9 (b) 5 (c) 4/9 (d) 94 (e) 4

a b c d e
50. ( ) ( ) ( ) ( ) ( )

51. If p varies directly as r and inversely at t, and $p = 10$ when $r = 10$ and $t = 12$, find p when $r = 7$ and $t = 6$.

(a) 10 (b) 12 (c) 14 (d) 18
(e) 21

a b c d e
51. ( ) ( ) ( ) ( ) ( )

52. The square root of 56 to the nearest hundredth is

(a) 7.49 (b) 2.04 (c) 7.48
(d) 7.483 (e) 2.045

a b c d e
52. ( ) ( ) ( ) ( ) ( )

53. To find the square root of 243 from a table of numbers from 1 to 100 the number 243 should be expressed as

(a) $27\sqrt{9}$ (b) $9\sqrt{3}$ (c) $3\sqrt{81}$ (d) $3\sqrt{9}$
(e) can't be found from such a table

a b c d e
53. ( ) ( ) ( ) ( ) ( )

54. If two legs of a right triangle are 10 and 24 then the third side is

(a) $\sqrt{676}$ (b) 26 (c) $\sqrt{476}$
(d) 240 (e) 34

a b c d e
54. ( ) ( ) ( ) ( ) ( )

55. Find the diagonal of a square to the nearest tenth of an inch whose area is 400 square inches.

(a) 800.0 (b) 20.0 (c) 28.1
(d) 282.8 (e) none of these

a b c d e
55. ( ) ( ) ( ) ( ) ( )

56. The number $\sqrt{73}$ is called

(a) transcendental (d) fractional
(b) rational (e) none of these
(c) irrational

a b c d e
56. ( ) ( ) ( ) ( ) ( )

57. The equation $4x^2 + 9y^2 = 36$ represents a (an) — 57. a ( ) b ( ) c ( ) d ( ) e ( )

(a) hyperbola (d) ellipse
(b) circle (e) two straight lines
(c) parabola

58. The graph of $4x^2 - 9y^2 = 36$ crosses the y-axis at — 58. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $\pm 2$ (b) $\pm 3$ (c) $+3, -2$ (d) $-3, +2$
(e) doesn't cross the y-axis

59. The equation $x = y^2 - 4y + 6$ represents a parabola which opens — 59. a ( ) b ( ) c ( ) d ( ) e ( )

(a) upwards (d) to the left
(b) downwards (e) none of these
(c) to the right

60. The equation $9x^2 - 4y^2 = 36$ represents a (an) — 60. a ( ) b ( ) c ( ) d ( ) e ( )

(a) two straight lines (d) ellipse
(b) circle (e) hyperbola
(c) parabola

61. Solve: $x^2 + y^2 = 25$, $xy = 12$ — 61. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $x = \pm 4,\ y = \pm 3$
(b) $x = 3, -3, 4, -4$; $y = 3, -3, 4, -4$
(c) $x = 3, -3, 4, -4$; $y = -4, 4, -3, 3$
(d) $x = 3, -3, 4, -4$; $y = 4, -4, 3, -3$
(e) $x = 3, -3, 4, -4$; $y = -3, 3, -4, 4$

62. A man who developed an arrangement for writing the coefficients of a binomial expansion was — 62. a ( ) b ( ) c ( ) d ( ) e ( )

(a) Pasteur (d) Pythagoras
(b) Euclid (e) unknown
(c) Pascal

63. If the binomial $(2x - a)^n$ is expanded the eighth term will be

(a) always positive
(b) positive only if n is even
(c) always negative
(d) negative only if n is odd
(e) can't tell unless n is known

a b c d e
63. ( ) ( ) ( ) ( ) ( )

64. The number of terms in the expansion of $(ax - 2b)^{17}$ is

(a) 17 (b) 16 (c) 34 (d) 18
(e) infinite

a b c d e
64. ( ) ( ) ( ) ( ) ( )

65. The 14th term in the expansion of of $(x^2 - y)^{19}$ will contain x to the power

(a) 6 (b) 10 (c) 5 (d) 12 (e) 8

a b c d e
65. ( ) ( ) ( ) ( ) ( )

66. Which of the following cannot be readily evaluated by the binomial theorem?

(a) $(.998)^7$ (b) $(103)^5$ (c) $(.788)^6$

(d) $(1 + \sqrt{2})^7$ (e) $(1.005)^4$

a b c d e
66. ( ) ( ) ( ) ( ) ( )

67. The fourth term of the expansion of $(x + \frac{y}{2})^{16}$ is

(a) $70x^{13}y^3$
(b) $35x^{12}y^4$
(c) $35x^{13}y^4$
(d) $70x^4y^{13}$
(e) none of these

a b c d e
67. ( ) ( ) ( ) ( ) ( )

68. The following sequence of numbers is a (an) 1, 7, 13, 19, . . .

(a) harmonic progression
(b) arithmetic progression
(c) geometric series
(d) arithmetic series
(e) geometric progression

a b c d e
68. ( ) ( ) ( ) ( ) ( )

69. In the progression 1/2, -1/2 . . . the tenth term is — 69. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 128 (b) 512 (c) 256
(d) -128 (e) -256

70. The geometric mean between 2 and 72 is — 70. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 38 (b) +36 (c) -35
(d) 37 (e) +12

71. If an infinite geometric series has a sum it must be — 71. ( ) ( ) ( ) ( ) ( )

(a) divergent (d) convergent
(b) concave (e) both convergent and divergent
(c) convex

72. Three arithmetic means between 7.5 and 21.5 are — 72. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 11, 14.5, 18 (d) 14.5, 14.5, 14.5
(b) 10, 14, 18 (e) -11, 14.5, -18
(c) -10, 14, -18

73. Find the sum of the first 10 terms of the progression 6, 12, 24, . . . — 73. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 3069 (b) 6138 (c) 6132
(d) 3063 (e) none of these

74. Find the sum of an infinite number of terms of 3/2, 9/4, 27/8, . . . — 74. ( ) ( ) ( ) ( ) ( )

(a) 3 (b) -3 (c) 3/2 (d) -2/3
(e) none of these

75. The logarithm of 27 to the base 3 is — 75. a ( ) b ( ) c ( ) d ( ) e ( )

(a) 3 (b) 9 (c) 1/9 (d) 24
(e) 1/3

76. The expression log (27)(13) equals — 76. ( ) ( ) ( ) ( ) ( )

(a) log 27 - log 13 (d) (log 27)(log 13)
(b) log (27 + 13) (e) $\frac{\log 27}{\log 13}$
(c) log 27 + log 13

77. The expression $\log_a (34)^3$ equals

77. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $(\log_a 34) + 3$
(b) $3a(\log 34)$
(c) $3(\log_a 34)$
(d) $\frac{3 \log 34}{a}$
(e) $34^3 \log a$

78. If the logarithm of 5 is .6990, then the logarithm of 2500 is

78. a ( ) b ( ) c ( ) d ( ) e ( )

(a) .3980 (b) 3.4950 (c) 4.4886
(d) 3.3980 (e) 0.4886

79. If the $\log_a N = b$ then which of the following is true?

79. a ( ) b ( ) c ( ) d ( ) e ( )

(a) $N = a^b$ (b) $N = ab$ (c) $N = b + a$
(d) $N = a/b$ (e) $N = b^a$

80. If the log of 3 is .3010 and the log of 4 is .4771, then log 48 is

80. a ( ) b ( ) c ( ) d ( ) e ( )

(a) .2552 (b) .7771 (c) 1.0791
(d) .1761 (e) 1.2552

# APPENDIX B

## INDIVIDUAL EXAMINATION SCORES ON FORMS P, AND GRADES EARNED THE SAME TERM

TABLE IX

INDIVIDUAL EXAMINATION SCORES ON PART I, FORM P AND GRADES EARNED THE SAME TERM

| | Scores | | | |
|---|---|---|---|---|
| Number | Odd | Even | Total | Grade |
| 1 | 17 | 14 | 31 | C |
| 2 | 26 | 29 | 55 | A |
| 3 | 22 | 20 | 42 | B |
| 4 | 14 | 6 | 20 | C |
| 5 | 21 | 21 | 42 | D |
| 6 | 19 | 25 | 44 | A |
| 7 | 26 | 26 | 52 | A |
| 8 | 23 | 27 | 50 | A |
| 9 | 24 | 26 | 50 | B |
| 10 | 25 | 23 | 48 | A |
| 11 | 15 | 22 | 37 | C |
| 12 | 8 | 8 | 16 | C |
| 13 | 23 | 29 | 52 | A |
| 14 | 25 | 25 | 50 | A |
| 15 | 21 | 26 | 47 | C |
| 16 | 16 | 15 | 31 | C |
| 17 | 27 | 23 | 50 | A |
| 18 | 15 | 7 | 22 | C |
| 19 | 22 | 25 | 47 | A |
| 20 | 26 | 29 | 55 | A |
| 21 | 18 | 19 | 37 | B |
| 22 | 24 | 20 | 44 | B |
| 23 | 22 | 23 | 45 | D |
| 24 | 15 | 18 | 33 | C |
| 25 | 16 | 23 | 39 | B |
| 26 | 22 | 22 | 44 | A |
| 27 | 21 | 22 | 43 | C |
| 28 | 26 | 27 | 53 | B |
| 29 | 8 | 12 | 20 | D |
| 30 | 21 | 17 | 38 | B |

TABLE IX (CONTINUED)

| Number | Scores Odd | Even | Total | Grade |
|---|---|---|---|---|
| 31 | 16 | 14 | 30 | C |
| 32 | 15 | 13 | 28 | F |
| 33 | 18 | 18 | 36 | B |
| 34 | 19 | 20 | 39 | C |
| 35 | 13 | 14 | 27 | D |
| 36 | 25 | 21 | 46 | B |
| 37 | 24 | 26 | 50 | A |
| 38 | 17 | 18 | 35 | C |
| 39 | 21 | 23 | 44 | B |
| 40 | 19 | 19 | 38 | C |
| 41 | 20 | 20 | 40 | C |
| 42 | 8 | 11 | 19 | D |
| 43 | 26 | 29 | 55 | A |
| 44 | 14 | 14 | 28 | C |
| 45 | 15 | 18 | 33 | C |
| 46 | 22 | 20 | 42 | A |
| 47 | 17 | 24 | 41 | B |
| 48 | 20 | 25 | 45 | A |
| 49 | 17 | 19 | 36 | A |
| 50 | 12 | 21 | 33 | B |
| 51 | 14 | 10 | 24 | D |
| 52 | 20 | 20 | 40 | C |
| 53 | 24 | 25 | 49 | A |
| 54 | 13 | 12 | 25 | C |
| 55 | 16 | 16 | 32 | B |
| 56 | 21 | 14 | 35 | A |
| 57 | 20 | 18 | 38 | D |
| 58 | 8 | 16 | 24 | C |
| 59 | 18 | 16 | 34 | D |
| 60 | 14 | 19 | 33 | C |
| 61 | 17 | 21 | 38 | C |
| 62 | 17 | 16 | 33 | C |
| 63 | 16 | 17 | 33 | D |
| 64 | 8 | 13 | 21 | D |
| 65 | 12 | 10 | 22 | F |

TABLE IX (CONTINUED)

| Number | Scores Odd | Even | Total | Grade |
|---|---|---|---|---|
| 66 | 13 | 14 | 27 | C |
| 67 | 16 | 20 | 36 | C |
| 68 | 17 | 19 | 36 | A |
| 69 | 15 | 21 | 36 | C |
| 70 | 22 | 17 | 39 | C |
| 71 | 10 | 15 | 25 | F |
| 72 | 21 | 20 | 41 | B |
| 73 | 23 | 21 | 44 | B |
| 74 | 17 | 21 | 38 | C |
| 75 | 16 | 21 | 37 | B |
| 76 | 25 | 22 | 47 | A |
| 77 | 22 | 20 | 42 | C |
| 78 | 25 | 25 | 50 | B |
| 79 | 18 | 20 | 38 | B |
| 80 | 18 | 22 | 40 | B |
| 81 | 16 | 21 | 37 | C |
| 82 | 23 | 16 | 39 | C |
| 83 | 14 | 15 | 29 | C |
| 84 | 26 | 23 | 49 | A |
| 85 | 22 | 24 | 46 | C |
| 86 | 17 | 21 | 38 | A |
| 87 | 16 | 22 | 38 | C |
| 88 | 19 | 20 | 39 | A |
| 89 | 17 | 20 | 37 | B |
| 90 | 10 | 13 | 23 | C |
| 91 | 14 | 5 | 19 | C |
| 92 | 19 | 18 | 37 | A |
| 93 | 19 | 18 | 37 | C |
| 94 | 19 | 18 | 37 | A |
| 95 | 30 | 28 | 58 | A |
| 96 | 20 | 21 | 41 | A |
| 97 | 13 | 10 | 23 | C |
| 98 | 22 | 20 | 42 | B |
| 99 | 12 | 13 | 25 | C |
| 100 | 10 | 9 | 19 | D |

TABLE IX (CONTINUED)

| Number | Scores | | | Grade |
|---|---|---|---|---|
| | Odd | Even | Total | |
| 101 | 20 | 22 | 42 | B |
| 102 | 10 | 10 | 20 | F |
| 103 | 17 | 15 | 32 | B |
| 104 | 21 | 25 | 46 | D |
| 105 | 14 | 17 | 31 | C |
| 106 | 17 | 17 | 34 | C |
| 107 | 18 | 19 | 37 | C |
| 108 | 20 | 21 | 41 | C |
| 109 | 13 | 16 | 29 | C |
| 110 | 11 | 18 | 29 | D |
| 111 | 19 | 14 | 33 | A |
| 112 | 17 | 21 | 38 | C |
| 113 | 15 | 17 | 32 | F |
| 114 | 26 | 27 | 53 | A |
| 115 | 14 | 17 | 31 | B |
| 116 | 21 | 24 | 45 | B |
| 117 | 22 | 24 | 46 | B |
| 118 | 24 | 26 | 50 | B |
| 119 | 12 | 14 | 26 | F |
| 120 | 24 | 24 | 48 | A |
| 121 | 11 | 14 | 25 | F |
| 122 | 24 | 22 | 46 | B |
| 123 | 18 | 19 | 37 | C |
| 124 | 24 | 28 | 52 | A |
| 125 | 23 | 25 | 48 | A |
| 126 | 19 | 18 | 37 | A |
| 127 | 22 | 23 | 45 | C |
| 128 | 11 | 9 | 20 | F |
| 129 | 16 | 17 | 33 | B |
| 130 | 17 | 17 | 34 | C |
| 131 | 17 | 20 | 37 | C |
| 132 | 15 | 18 | 33 | C |
| 133 | 20 | 27 | 47 | B |
| 134 | 12 | 6 | 18 | D |
| 135 | 19 | 25 | 44 | C |

TABLE IX (CONTINUED)

| Number | Scores | | | Grade |
|---|---|---|---|---|
| | Odd | Even | Total | |
| 136 | 17 | 17 | 34 | D |
| 137 | 23 | 21 | 44 | C |
| 138 | 26 | 29 | 55 | A |
| 139 | 17 | 20 | 37 | B |
| 140 | 23 | 20 | 43 | D |
| 141 | 20 | 23 | 43 | B |
| 142 | 10 | 9 | 19 | F |
| 143 | 12 | 12 | 24 | B |
| 144 | 15 | 17 | 32 | C |
| 145 | 8 | 14 | 22 | D |
| 146 | 13 | 14 | 27 | C |
| 147 | 13 | 12 | 25 | F |
| 148 | 10 | 11 | 21 | D |
| 149 | 13 | 17 | 30 | D |
| 150 | 21 | 23 | 44 | B |
| 151 | 19 | 19 | 38 | B |
| 152 | 21 | 18 | 39 | C |
| 153 | 12 | 8 | 20 | C |
| 154 | 18 | 19 | 37 | C |
| 155 | 13 | 13 | 26 | C |
| 156 | 18 | 25 | 43 | A |
| 157 | 19 | 26 | 45 | B |
| 158 | 11 | 12 | 23 | D |
| 159 | 29 | 30 | 59 | A |
| 160 | 28 | 29 | 57 | A |
| 161 | 12 | 13 | 25 | D |
| 162 | 18 | 17 | 35 | D |
| 163 | 17 | 16 | 33 | B |
| 164 | 19 | 18 | 37 | C |
| 165 | 20 | 23 | 43 | B |
| 166 | 12 | 13 | 25 | C |
| 167 | 23 | 31 | 54 | A |
| 168 | 23 | 28 | 51 | A |
| 169 | 18 | 15 | 33 | A |
| 170 | 15 | 13 | 28 | D |

TABLE IX (CONTINUED)

| Number | Scores Odd | Even | Total | Grade |
|---|---|---|---|---|
| 171 | 31 | 30 | 61 | A |
| 172 | 20 | 20 | 40 | B |
| 173 | 16 | 14 | 30 | C |
| 174 | 22 | 19 | 41 | C |
| 175 | 24 | 25 | 49 | B |
| 176 | 19 | 24 | 43 | B |
| 177 | 17 | 18 | 35 | C |
| 178 | 21 | 22 | 43 | B |
| 179 | 15 | 22 | 37 | D |
| 180 | 20 | 23 | 43 | B |
| 181 | 23 | 24 | 47 | A |
| 182 | 13 | 15 | 28 | C |
| 183 | 18 | 18 | 36 | B |
| 184 | 17 | 15 | 32 | D |
| 185 | 24 | 28 | 52 | B |
| 186 | 20 | 16 | 36 | B |
| 187 | 25 | 21 | 46 | A |
| 188 | 20 | 20 | 40 | A |
| 189 | 17 | 19 | 36 | D |
| 190 | 13 | 11 | 24 | D |
| 191 | 20 | 23 | 43 | B |
| 192 | 12 | 13 | 25 | F |
| 193 | 22 | 18 | 40 | D |
| 194 | 20 | 27 | 47 | D |
| 195 | 18 | 19 | 37 | A |
| 196 | 24 | 21 | 45 | C |
| 197 | 24 | 27 | 51 | A |
| 198 | 21 | 14 | 35 | B |
| 199 | 22 | 28 | 50 | A |
| 200 | 16 | 18 | 34 | B |
| 201 | 14 | 15 | 29 | C |
| 202 | 17 | 16 | 33 | B |
| 203 | 14 | 13 | 27 | C |
| 204 | 14 | 17 | 31 | C |
| 205 | 23 | 22 | 45 | D |

TABLE IX (CONTINUED)

| Number | Scores Odd | Even | Total | Grade |
|---|---|---|---|---|
| 206 | 16 | 11 | 27 | C |
| 207 | 13 | 11 | 24 | D |
| 208 | 30 | 28 | 58 | A |
| 209 | 22 | 26 | 48 | A |
| 210 | 13 | 13 | 26 | C |
| 211 | 15 | 11 | 26 | C |
| 212 | 17 | 22 | 39 | B |
| 213 | 16 | 12 | 28 | C |
| 214 | 13 | 8 | 21 | D |
| 215 | 19 | 13 | 32 | F |
| 216 | 14 | 16 | 30 | D |
| 217 | 14 | 20 | 34 | C |
| 218 | 16 | 17 | 33 | D |
| 219 | 23 | 23 | 46 | A |
| 220 | 21 | 21 | 42 | B |
| 221 | 30 | 29 | 59 | A |
| 222 | 18 | 25 | 43 | B |
| 223 | 13 | 13 | 26 | D |
| 224 | 9 | 14 | 23 | F |
| 225 | 20 | 29 | 49 | D |
| 226 | 14 | 20 | 34 | D |
| 227 | 19 | 15 | 34 | D |
| 228 | 19 | 22 | 41 | C |
| 229 | 20 | 23 | 43 | B |
| 230 | 25 | 26 | 51 | A |
| 231 | 21 | 24 | 45 | B |
| 232 | 13 | 14 | 27 | D |
| 233 | 16 | 17 | 33 | D |
| 234 | 25 | 24 | 49 | C |
| 235 | 22 | 25 | 47 | C |
| 236 | 18 | 18 | 36 | B |
| 237 | 17 | 17 | 34 | B |
| 238 | 23 | 19 | 42 | B |
| 239 | 18 | 15 | 33 | C |
| 240 | 14 | 15 | 29 | D |

TABLE IX (CONTINUED)

| Number | Scores | | | Grade |
|---|---|---|---|---|
| | Odd | Even | Total | |
| 241 | 18 | 15 | 33 | C |
| 242 | 20 | 20 | 40 | C |
| 243 | 14 | 9 | 23 | D |
| 244 | 21 | 20 | 41 | C |
| 245 | 20 | 29 | 49 | B |
| 246 | 26 | 29 | 55 | A |
| 247 | 18 | 18 | 36 | C |
| 248 | 13 | 12 | 25 | D |
| 249 | 24 | 26 | 50 | A |
| 250 | 15 | 12 | 27 | C |
| 251 | 20 | 24 | 44 | B |
| 252 | 21 | 23 | 44 | B |
| Mean | 17.73 | 19.10 | 37.40 | |
| Std. Dev. | 8.10 | 9.30 | 9.70 | |

TABLE X

INDIVIDUAL EXAMINATION SCORES ON PART II, FORM P AND GRADES EARNED THE SAME TERM

| Number | Scores: Odd | Even | Total | Grade |
|---|---|---|---|---|
| 1 | 18 | 22 | 40 | C |
| 2 | 19 | 22 | 42 | A |
| 3 | 17 | 24 | 41 | D |
| 4 | 27 | 25 | 52 | A |
| 5 | 19 | 18 | 37 | C |
| 6 | 22 | 18 | 40 | C |
| 7 | 17 | 21 | 38 | D |
| 8 | 19 | 22 | 41 | C |
| 9 | 21 | 18 | 39 | D |
| 10 | 11 | 12 | 23 | D |
| 11 | 23 | 26 | 49 | C |
| 12 | 27 | 24 | 51 | B |
| 13 | 20 | 21 | 41 | C |
| 14 | 22 | 27 | 49 | B |
| 15 | 21 | 26 | 47 | C |
| 16 | 12 | 14 | 26 | D |
| 17 | 18 | 24 | 42 | D |
| 18 | 16 | 19 | 35 | C |
| 19 | 23 | 22 | 45 | D |
| 20 | 12 | 22 | 34 | C |
| 21 | 22 | 19 | 41 | A |
| 22 | 25 | 26 | 51 | A |
| 23 | 16 | 15 | 31 | F |
| 24 | 5 | 12 | 17 | F |
| 25 | 15 | 18 | 33 | D |
| 26 | 22 | 22 | 44 | D |
| 27 | 30 | 24 | 54 | C |
| 28 | 22 | 27 | 49 | B |
| 29 | 24 | 26 | 50 | C |
| 30 | 28 | 31 | 59 | B |

TABLE X (CONTINUED)

| Number | Scores | | | Grade |
|---|---|---|---|---|
| | Odd | Even | Total | |
| 31 | 4 | 13 | 17 | F |
| 32 | 27 | 27 | 54 | B |
| 33 | 17 | 26 | 43 | C |
| 34 | 28 | 26 | 54 | B |
| 35 | 16 | 21 | 37 | D |
| 36 | 25 | 27 | 52 | B |
| 37 | 16 | 26 | 42 | D |
| 38 | 21 | 21 | 42 | B |
| 39 | 10 | 18 | 28 | D |
| 40 | 10 | 18 | 28 | C |
| 41 | 19 | 21 | 40 | C |
| 42 | 16 | 24 | 40 | C |
| 43 | 18 | 19 | 37 | D |
| 44 | 23 | 19 | 42 | A |
| 45 | 24 | 22 | 46 | B |
| 46 | 18 | 15 | 33 | D |
| 47 | 18 | 25 | 43 | D |
| 48 | 20 | 23 | 43 | C |
| 49 | 22 | 22 | 44 | C |
| 50 | 23 | 26 | 49 | C |
| 51 | 25 | 24 | 49 | B |
| 52 | 26 | 23 | 49 | A |
| 53 | 25 | 23 | 48 | B |
| 54 | 17 | 16 | 33 | F |
| 55 | 24 | 20 | 44 | C |
| 56 | 21 | 19 | 40 | C |
| 57 | 18 | 21 | 39 | C |
| 58 | 23 | 24 | 47 | A |
| 59 | 17 | 20 | 37 | B |
| 60 | 12 | 14 | 26 | C |
| 61 | 22 | 20 | 42 | C |
| 62 | 24 | 24 | 48 | C |
| 63 | 17 | 18 | 35 | D |
| 64 | 23 | 22 | 45 | A |
| 65 | 24 | 30 | 54 | B |

TABLE X (CONTINUED)

| Number | Scores Odd | Even | Total | Grade |
|---|---|---|---|---|
| 66 | 24 | 23 | 47 | C |
| 67 | 19 | 23 | 42 | D |
| 68 | 20 | 20 | 40 | B |
| 69 | 17 | 22 | 39 | D |
| 70 | 27 | 28 | 55 | A |
| 71 | 23 | 22 | 45 | C |
| 72 | 18 | 22 | 40 | B |
| 73 | 26 | 30 | 56 | B |
| 74 | 18 | 20 | 38 | D |
| 75 | 24 | 23 | 47 | A |
| 76 | 21 | 21 | 42 | C |
| 77 | 28 | 26 | 54 | B |
| 78 | 23 | 26 | 49 | B |
| 79 | 20 | 21 | 41 | C |
| 80 | 26 | 30 | 56 | A |
| 81 | 18 | 21 | 39 | C |
| 82 | 21 | 19 | 40 | C |
| 83 | 22 | 21 | 43 | C |
| 84 | 22 | 16 | 38 | D |
| 85 | 27 | 32 | 59 | A |
| 86 | 18 | 21 | 39 | D |
| 87 | 27 | 25 | 52 | B |
| 88 | 24 | 22 | 46 | C |
| 89 | 11 | 17 | 28 | D |
| 90 | 25 | 30 | 55 | A |
| 91 | 16 | 19 | 35 | C |
| 92 | 33 | 27 | 60 | A |
| 93 | 34 | 31 | 65 | A |
| 94 | 31 | 27 | 58 | B |
| 95 | 12 | 19 | 31 | F |
| 96 | 10 | 16 | 26 | F |
| 97 | 20 | 23 | 43 | C |
| 98 | 18 | 15 | 33 | C |
| 99 | 16 | 24 | 40 | B |
| 100 | 19 | 23 | 42 | C |

TABLE X (CONTINUED)

| Number | Scores Odd | Even | Total | Grade |
|---|---|---|---|---|
| 101 | 20 | 23 | 43 | C |
| 102 | 21 | 21 | 42 | A |
| 103 | 23 | 19 | 42 | I |
| 104 | 13 | 24 | 37 | B |
| 105 | 25 | 20 | 45 | C |
| 106 | 27 | 25 | 52 | B |
| Mean | 20.62 | 21.76 | 42.37 | |
| Std. Dev. | 5.55 | 4.29 | 8.70 | |

# APPENDIX C

## REDUCED FORMULA FOR POINT-BISERIAL COEFFICIENT OF CORRELATION

# APPENDIX C

## REDUCED FORMULA FOR POINT-BISERIAL COEFFICIENT OF CORRELATION

The point-biserial coefficient of correlation is given by the following formula (64, p.353):

$$r_{pb} = \frac{M_p - M_f}{\sigma_T} \sqrt{pq} \qquad (1)$$

in which:

$M_p$ = the mean score of individuals passing the item.

$M_f$ = the mean score of individuals failing the item.

$\sigma_T$ = the standard deviation of the total distribution of scores.

p = the proportion of individuals passing the item.

q = the proportion of individuals failing the item.

For purposes of comparing the validity of items on a single test and not for comparison with items on other tests, the reduction to standard form by division by $\sigma_T$ is unnecessary and may be omitted. This gives what will be termed the reduced point-biserial coefficient of correlation. The formula for this may then be written:

$$r_{rpb} = (M_p - M_f) \sqrt{pq} \qquad (2)$$

The difference $M_t - M_f$ is as adequate as $M_p - M_f$ (29, p.303). Thus the formula can be expressed in the alternative form:

$$r_{rpb} = (M_t - M_f)\sqrt{pq} \qquad (3)$$

in which $M_t$ is the mean of the total distribution of scores.

If deviations are taken from the same step in each case, then the means $M_t$ and $M_f$ may be replaced by the deviations of the true means from the guessed means as the difference will be cancelled in the subtraction $M_t - M_f$. The formula can then be expressed:

$$r_{rpb} = \left(\frac{\Sigma f_t d}{N_t} - \frac{\Sigma f_f d}{N_f}\right)\sqrt{pq} \qquad (4)$$

in which $N_t$ represents the total number of scores and $N_f$ the number failing the item. This is a more usable form than either (2) or (3) above. Further simplification of the computations are possible by taking the deviations (d) in terms of class intervals since the class interval is constant throughout any problem. If the zero deviation is taken at the lowest interval in which any frequencies occur, the negative deviations may be avoided.

The reduced formula will give values which are just as accurate and adequate as the formulas from which they are derived. The results can be applied only to the problem on which the values are computed; however, this is the usual way in which this coefficient is used. Later, if comparisons are sought, the data obtained by the reduced formula can be converted to the conventional formulas, but only in rare cases would this furnish useful information. In such instances it probably would be equally simple to apply the ordinary formula for point-biserial correlation in the first place.

A similar reduction is shown by Chapanis (29, pp.297-304) for the McCall-Long-Bliss validity coefficient in order to develop a less cumbersome form. This formula, which Chapanis calls the reduced McCall-Long-Bliss validity coefficient is (29, p.302):

$$r^{V}{}_{MLB} = N_f \left( \frac{\Sigma f_t d}{N_t} \right) - \Sigma f_t d \qquad (5)$$

This author points out that the reduced formulas are in every sense as accurate and adequate as those from which they were derived (29, p.304). He indicates that the work involved is considerably reduced. This formula favors items of fifty percent difficulty.

A table for $\sqrt{pq}$ is found in Guilford's Fundamental Statistics in Psychology and Education (73, pp.606-608). In using this table note should be taken of the fact that $\sqrt{pq}$ for "the small area" (C) = .015 is obviously in error. This item should read .1226.

# APPENDIX D

## REVISED EXAMINATION SCORES ON FORMS P AND GRADES EARNED THE FOLLOWING TERM

TABLE XI

REVISED EXAMINATION SCORES ON PART I, FORM P

AND GRADES EARNED THE FOLLOWING TERM

| Number | Score | Grade | Number | Score | Grade |
|---|---|---|---|---|---|
| 1 | 19 | B | 31 | 26 | B |
| 2 | 31 | A | 32 | 23 | D |
| 3 | 21 | C | 33 | 21 | B |
| 4 | 12 | C | 34 | 24 | C |
| 5 | 27 | C | 35 | 20 | C |
| 6 | 31 | B | 36 | 22 | D |
| 7 | 27 | B | 37 | 28 | B |
| 8 | 30 | C | 38 | 13 | F |
| 9 | 26 | A | 39 | 20 | D |
| 10 | 27 | B | 40 | 20 | D |
| 11 | 30 | A | 41 | 20 | C |
| 12 | 26 | B | 42 | 20 | F |
| 13 | 19 | C | 43 | 19 | F |
| 14 | 30 | A | 44 | 13 | F |
| 15 | 24 | C | 45 | 21 | F |
| 16 | 29 | B | 46 | 23 | C |
| 17 | 22 | D | 47 | 20 | B |
| 18 | 28 | B | 48 | 20 | C |
| 19 | 27 | C | 49 | 23 | C |
| 20 | 18 | D | 50 | 23 | B |
| 21 | 27 | F | 51 | 24 | D |
| 22 | 9 | D | 52 | 21 | F |
| 23 | 15 | C | 53 | 30 | C |
| 24 | 19 | F | 54 | 28 | C |
| 25 | 27 | B | 55 | 20 | C |
| 26 | 19 | C | 56 | 26 | C |
| 27 | 23 | B | 57 | 20 | D |
| 28 | 31 | A | 58 | 21 | F |
| 29 | 16 | D | 59 | 26 | C |
| 30 | 20 | C | 60 | 24 | B |

TABLE XI (CONTINUED)

| Number | Score | Grade | Number | Score | Grade |
|---|---|---|---|---|---|
| 61 | 22 | B | 96 | 12 | F |
| 62 | 20 | C | 97 | 24 | F |
| 63 | 12 | F | 98 | 14 | B |
| 64 | 11 | F | 99 | 23 | B |
| 65 | 22 | B | 100 | 25 | D |
| 66 | 21 | C | 101 | 33 | B |
| 67 | 33 | A | 102 | 32 | A |
| 68 | 13 | F | 103 | 16 | D |
| 69 | 24 | B | 104 | 16 | C |
| 70 | 22 | C | 105 | 26 | C |
| 71 | 21 | B | 106 | 18 | D |
| 72 | 16 | D | 107 | 32 | B |
| 73 | 21 | C | 108 | 20 | B |
| 74 | 14 | C | 109 | 34 | B |
| 75 | 14 | F | 110 | 22 | C |
| 76 | 18 | C | 111 | 21 | C |
| 77 | 29 | B | 112 | 22 | C |
| 78 | 25 | C | 113 | 27 | C |
| 79 | 29 | B | 114 | 25 | C |
| 80 | 26 | C | 115 | 29 | B |
| 81 | 29 | A | 116 | 19 | C |
| 82 | 26 | B | 117 | 30 | A |
| 83 | 23 | C | 118 | 23 | C |
| 84 | 18 | C | 119 | 26 | A |
| 85 | 18 | D | 120 | 23 | B |
| 86 | 23 | B | 121 | 20 | C |
| 87 | 20 | F | 122 | 12 | D |
| 88 | 31 | A | 123 | 26 | C |
| 89 | 22 | C | 124 | 23 | B |
| 90 | 21 | D | 125 | 21 | D |
| 91 | 26 | B | 126 | 27 | B |
| 92 | 14 | B | 127 | 28 | B |
| 93 | 20 | B | 128 | 18 | B |
| 94 | 25 | C | 129 | 26 | C |
| 95 | 26 | B | 130 | 22 | C |

TABLE XI (CONTINUED)

| Number | Score | Grade | Number | Score | Grade |
|---|---|---|---|---|---|
| 131 | 19 | D | 151 | 29 | C |
| 132 | 15 | C | 152 | 18 | F |
| 133 | 16 | F | 153 | 26 | C |
| 134 | 24 | F | 154 | 26 | D |
| 135 | 14 | C | 155 | 20 | C |
| 136 | 32 | A | 156 | 23 | C |
| 137 | 28 | B | 157 | 16 | C |
| 138 | 15 | D | 158 | 16 | C |
| 139 | 22 | C | 159 | 20 | C |
| 140 | 15 | C | 160 | 22 | C |
| 141 | 16 | C | 161 | 27 | C |
| 142 | 14 | F | 162 | 18 | D |
| 143 | 17 | D | 163 | 30 | B |
| 144 | 22 | D | 164 | 25 | D |
| 145 | 34 | A | 165 | 26 | C |
| 146 | 27 | B | | | |
| 147 | 12 | F | | | |
| 148 | 18 | F | | | |
| 149 | 18 | C | | | |
| 150 | 23 | C | | | |

TABLE XII

REVISED EXAMINATION SCORES ON PART II, FORM P

AND GRADES EARNED THE FOLLOWING TERM

| Number | Score | Grade | Number | Score | Grade |
|---|---|---|---|---|---|
| 1 | 25 | D | 31 | 31 | C |
| 2 | 22 | B | 32 | 31 | C |
| 3 | 22 | F | 33 | 26 | A |
| 4 | 28 | D | 34 | 15 | D |
| 5 | 25 | D | 35 | 12 | F |
| 6 | 25 | B | 36 | 23 | C |
| 7 | 23 | C | 37 | 24 | C |
| 8 | 20 | D | 38 | 22 | A |
| 9 | 22 | A | 39 | 30 | B |
| 10 | 6 | F | 40 | 19 | D |
| 11 | 30 | C | 41 | 26 | C |
| 12 | 29 | D | 42 | 28 | B |
| 13 | 29 | D | 43 | 30 | B |
| 14 | 28 | B | 44 | 28 | B |
| 15 | 26 | D | 45 | 28 | C |
| 16 | 28 | C | 46 | 24 | C |
| 17 | 19 | C | 47 | 30 | B |
| 18 | 22 | B | 48 | 32 | B |
| 19 | 29 | F | 49 | 27 | C |
| 20 | 18 | D | 50 | 18 | D |
| 21 | 15 | F | 51 | 23 | C |
| 22 | 30 | B | 52 | 23 | D |
| 23 | 29 | B | 53 | 32 | C |
| 24 | 28 | C | 54 | 23 | F |
| 25 | 28 | D | 55 | 29 | F |
| 26 | 34 | C | 56 | 33 | D |
| 27 | 33 | A | 57 | 33 | F |
| 28 | 24 | D | 58 | 30 | C |
| 29 | 30 | B | 59 | 32 | A |
| 30 | 21 | D | 60 | 15 | D |

TABLE XII (CONTINUED)

| Number | Score | Grade | Number | Score | Grade |
|---|---|---|---|---|---|
| 61 | 31 | A | 76 | 19 | D |
| 62 | 20 | C | 77 | 35 | A |
| 63 | 32 | C | 78 | 38 | B |
| 64 | 30 | C | 79 | 27 | D |
| 65 | 24 | D | 80 | 22 | B |
| 66 | 32 | C | 81 | 22 | D |
| 67 | 23 | D | 82 | 22 | C |
| 68 | 25 | D | 83 | 16 | F |
| 69 | 24 | C | 84 | 33 | C |
| 70 | 34 | A | 85 | 39 | B |
| 71 | 34 | B | 86 | 18 | F |
| 72 | 24 | D | 87 | 34 | B |
| 73 | 13 | F | 88 | 36 | A |
| 74 | 32 | C | 89 | 33 | D |
| 75 | 32 | B | | | |

TABLE XIII

MEAN REVISED EXAMINATION SCORES

FOR VARIOUS LETTER GRADES EARNED THE FOLLOWING TERM

| | F | D | C | B | A |
|---|---|---|---|---|---|
| Part I | 17.0 | 19.7 | 22.4 | 24.7 | 30.8 |
| Part II | 19.2 | 23.4 | 27.2 | 29.6 | 30.3 |

# APPENDIX E

## REVISED EXAMINATIONS, FORMS A

Directions to Students

Part I, Form A, Elementary Examination

Part II, Form A, Intermediate Examination

Answer Sheet

Diagnostic Charts

# ALGEBRA EXAMINATION

## FORM A

## DIRECTIONS TO STUDENTS

## READ CAREFULLY

1. The following examination is for the purpose of determining your preparation and readiness for college algebra. The first test, Part I, covers elementary topics. The second test, Part II, covers intermediate topics.

2. Each test is composed of forty multiple-choice questions. Each question offers five answers; among the five there is one answer which is best.

3. Do not open the test until told to begin.

4. Do not mark the test booklet. Do your work on scratch paper and indicate the answers that you select on the answer sheet.

5. Work rapidly but carefully. Answer as many questions as you can, but do not make wild guesses.

6. Do not ask questions after the examination begins.

ALGEBRA EXAMINATION

PART I FORM A

(Elementary)

Directions: Do not mark the test booklet. Use scratch paper for your work. When you have determined the answer for a problem place an x on the answer sheet to indicate the letter of the answer.

Example: The sum of 6 and 2 is

(a) 12 (b) 4 (c) 8 (d) 3 (e) none of these

Correct answer sheet marking:

| | a | b | c | d | e |
|---|---|---|---|---|---|
| Example: | ( ) | ( ) | (x) | ( ) | ( ) |

---

1. An expression of the type $4x^2 - 2ax - ab^2$ is called a (an)

   (a) trinomial (b) binomial (c) equation (d) proportion
   (e) product

2. The distributive law of multiplication states:

   (a) $x(y + z) = xy + xz$
   (b) $x(y + z) = (y + z)x$
   (c) $(x + y) + z = x + (y + z)$
   (d) $x(yz) = xy \cdot xz$
   (e) none of these

3. The commutative law of addition states

   (a) $x + (y + z) = (x + y) + z$
   (b) $x(y + z) = xy + xz$
   (c) $x + y = y + x$
   (d) $\frac{x + y}{z} = z(x + y)$
   (e) $xyz = x + y + z$

4. An exponent in the expression $12ax^3$ is

   (a) x (b) 12 (c) 12a (d) 2 (e) 3

5. If $f(x) = x^3 - 3x + 2$, find $f(-2)$.

   (a) -8 (b) 16 (c) 4 (d) -16 (e) 0

6. If $L = a + (n - 1)d$, find <u>L</u> when $d = 2$, $a = 6$, and $n = 11$.

(a) 60 (b) 26 (c) 28 (d) 78 (e) 24

7. Evaluate the expression $x - \frac{1}{2}x$ when $x = 0.8$.

(a) 1.6 (b) .12 (c) 0.8 (d) -1.2 (e) 0.4

8. If $a = 1$, $b = -1$, $c = 2$, and $d = -3$, find the value of $(a + b)(c + d)$

(a) 12 (b) -1/6 (c) -12 (d) 0 (e) none of these

9. If $b = -1$ and $c = 2$, find the value of $(b - c)^3$

(a) 8 (b) -27 (c) -8 (d) 27 (e) none of these

10. A formula for <u>time</u> (t) in terms of distance (d) and rate (r) is

(a) $t = \frac{d}{r}$ (b) $d = rt$ (c) $r = \frac{d}{t}$ (d) $t = rd$ (e) $t = \frac{r}{d}$

11. Find the area of a trapezoid of height 8 and bases 4 and 7.

(a) 88 (b) 112 (c) 84 (d) 46 (e) none of these

12. Find the area of a circle of diameter 14, ($\pi = 22/7$).

(a) 308 (b) 44 (c) 88 (d) 616 (e) 154

13. The circumference of a circle (r=radius, d=diameter) is

(a) $\pi r^2$ (b) $\frac{\pi r}{2}$ (c) $\pi r$ (d) $\pi d$ (e) $2\pi d$

14. Add: $2a^2 - 6ab + 3b^2$; $-a^2 + b^2$; $3ab - 4b^2$

(a) $a^2 - 2ab + 4b^2$ (b) $a^2 - 6ab$ (c) $a^2 - 6ab + b^2$
(d) $a^2 - 3ab$ (e) $2a^2 - 3ab - b^2$

15. Subtract $(4x - 7y + z)$ from $(3x - y)$.

(a) $x - 6y + z$ (b) $-x + 6y - z$ (c) $x + 6y - z$ (d) $x - 8y + z$
(e) $-x + 8y + z$

16. Simplify: $7y + (2y - 5y) =$

(a) 2y (b) 14y (c) 7y - 3y (d) 4y (e) 4

17. $(a + 2a^2 + 1) - 2(1 - a) =$

(a) $2a^2 - a + 1$ (b) $1 - a + 4a^2$ (c) $2a^2 + 3a - 1$
(d) $2a^2 - 2$ (e) $3a + 2a^2$

18. $(-2xy)(2x^2y)^2 =$

(a) $16x^6y^4$ (b) $-8x^4y^3$ (c) $-16x^5y^3$ (d) $-8x^5y^3$ (e) $8x^4y^3$

19. $(x - 3)(2x + 1) =$

(a) $2x^2 - 5x - 3$ (b) $2x^2 - 3$ (c) $2x^2 - 7x - 3$ (d) $3x - 2$
(e) $2x^2 - 5 + 3$

20. Expand: $(2x - y)^2 =$

(a) $4x^2 - y^2$ (b) $4x^2 - 2xy + y^2$ (c) $4x^2 + y^2$
(d) $4x^2 + 4xy + y^2$ (e) $4x^2 - 4xy + y^2$

21. $(26xy) \div (-2x) =$

(a) $13y$ (b) $-52x^2y$ (c) $24y$ (d) $-xy$ (e) $-13y$

22. $(x^2 - 2xy + y^2) \div (x - y) =$

(a) $x - y$ (b) $x + y$ (c) $x - 2y$ (d) $x - 2$ (e) $x - xy + y$

23. Solve for $\underline{x}$: $3x - (2x + 5) = 3$

(a) 2 (b) 8 (c) -3/4 (d) -2 (e) 0

24. Solve for $\underline{x}$: $2 - 2x = 6$

(a) 2 (b) -4 (c) -2 (d) -1/2 (e) -3

25. Solve for $\underline{x}$: $4ax - 9ab = 3ab$

(a) $3ab$ (b) $3b$ (c) $3(a + b)$ (d) $-6a + 3b$ (e) $6ab$

26. Solve for $\underline{x}$: $x - (1 - x) = 7$

(a) 4 (b) 6 (c) -1 (d) 8 (e) none of these

27. If an investment of \$1000 earns 4 per cent interest and another investment of \$2000 earns 6 per cent, then the expression for the annual earnings in dollars is

(a) 4(1000) + 6(2000)
(b) .04(2000) + .06(1000)
(c) (.10)(3000)
(d) .04(1000) + .06(2000)
(e) (.04%)(1000) + (.06%)(2000)

28. If the sum of two numbers is 17 and their difference is 5, then the equation needed to find the numbers is

(a) $x + (x + 5) = 17$ (b) $x = (x - 5) + 17$ (c) $x + 5 = 17 + x$
(d) $5x = x + 17$ (e) $x = 17 + x$

29. Two autos traveled at rates of 40 and 25 miles per hour respectively. If the faster car travels 3 hours longer and goes twice as far, what is the distance traveled by the faster car?

(a) 600 (b) 400 (c) 1200 (d) 300 (e) 120

30. The units and tens digit of a number are <u>x</u> and <u>y</u> respectively. The number is

(a) $xy$ (b) $10y + x$ (c) $10x + y$ (d) $10xy$ (e) $10(x + y)$

31. Factor $x^2 - 5x + 6 =$

(a) $(x - 6)(x - 1)$ (b) $(x - 3)(x - 2)$ (c) $(x + 3)(x + 2)$
(d) $(x - 4)(x - 1)$ (e) $(x + 6)(x + 1)$

32. Factor $4x^2 - 81y^2 =$

(a) $(2x - 9y)(2x + 9y)$ (b) $(2x - 9y)^2$ (c) $(4x - 9y)(x + 9y)$
(d) $(2x + 9y)^2$ (e) $(4x + 9y)(x - 9y)$

33. Factor $4xy^2 + 4xy + 8y =$

(a) $(2xy + 4y)(2y + 2)$
(b) $y(4x + 4)(x + y)$
(c) $4y(xy + x + 2)$
(d) $4y(x + 2y^2)$
(e) none of these

34. Factor $8a^3 + 27y^3 =$

(a) $(2a + 3y)^3$
(b) $(2a + 3y)(4a^2 + 9y^2)$
(c) $(2a + 3y)(4a^2 - 6ay + 9y^2)$
(d) $(4a^2 + 6ay + 9y^2)(2a + 3y)$
(e) $(2a - 3y)(4a^2 - 12ay + 9y^2)$

35. A factor of $x^3 + x + 10$ is

(a) $x - 5$ (b) $x - 2$ (c) $x + 5$ (d) $x + 2$ (e) $x^2 + 5$

36. Simplify: $\dfrac{2 + \frac{1}{x}}{\frac{1}{x} + 4} =$

(a) $\dfrac{2x + 1}{4(x + 4)}$ (b) $\dfrac{1 + 4x}{2x + 1}$ (c) $\dfrac{2x}{1 + 4x}$ (d) $\dfrac{2x + 1}{4x + 1}$ (e) none of these

37. Combine and simplify: $\dfrac{a}{x - a} - \dfrac{a^2}{x^2 - a^2} =$

(a) $\dfrac{ax}{x^2 - a^2}$ (b) $\dfrac{ax}{(x-a)(x^2-a^2)}$ (c) $\dfrac{-a^3}{x^2 - a^2}$ (d) $\dfrac{ax^2 - a^2x}{(x-a)(x^2-a^2)}$
(e) none of these

38. Solve for <u>x</u>: $\dfrac{x + 1}{4} - \dfrac{2x + 3}{11} = 1$

(a) 7 (b) -22/3 (c) 11 (d) 15 (e) 44

39. Solve for <u>C</u>: $S = 1/2(A + B + C)$

(a) $2(S - A - B)$ (b) $2(S - A + B)$ (c) $2S - A - B$
(d) $2A - 2S + B$ (e) $2A + 2B + 2S$

40. Solve for <u>f</u>: $\dfrac{1}{f} = \dfrac{1}{p} + \dfrac{1}{q}$

(a) $\dfrac{fq + fp}{fpq}$ (b) $\dfrac{1}{fpq}$ (c) $\dfrac{pq}{p + q}$ (d) $p + q$ (e) $\dfrac{1}{p + q}$

ALGEBRA EXAMINATION

PART II FORM A

(Intermediate)

Directions: Do not mark the test booklet. Use scratch paper for your work. When you have determined the answer for a problem place an x on the answer sheet to indicate the letter of the answer.

Example: The sum of 6 and 2 is

(a) 12 (b) 4 (c) 8 (d) 3 (e) none of these

Correct answer sheet marking:

| | a | b | c | d | e |
|---|---|---|---|---|---|
| Example: | ( ) | ( ) | (x) | ( ) | ( ) |

---

1. Solve: $\begin{cases} 3x - 4y = 13 \\ 2x - 3y = 12 \end{cases}$

(a) $x = 9$, $y = 2$
(b) $x = 7$, $y = 2$
(c) $x = -9$, $y = 10$
(d) $x = -9$, $y = -10$
(e) $x = -7$, $y = -2$

2. The first step in the solution of the following equations to give the most direct solution should be

$$\frac{1}{x} - \frac{1}{y} = 6$$

$$\frac{1}{x} - \frac{2}{y} = 4$$

(a) substitution
(b) addition
(c) subtraction
(d) clear of fractions
(e) none of these

3. Simplify: $(-2x)^2 =$

(a) $-4x^2$ (b) $\frac{1}{4x^2}$ (c) $2x^2$ (d) $-2x^2$ (e) $4x^2$

4. Simplify: $(x^{m-n})(x^{2n}) =$

(a) $x^{m+n}$ (b) $x^{m-2n}$ (c) $x^{m-n}$ (d) $x^{m-2}$ (e) $x^{n-2m}$

5. Simplify: $x^{5/6} \div x^{1/3} =$

(a) $x^{7/6}$ (b) $x^{2/3}$ (c) $x^{3/2}$ (d) $x^{1/2}$ (e) $x$

6. Simplify: $(1/4)^{-1/2} =$

(a) 1/8 (b) -1/16 (c) -1/4 (d) 4 (e) 2

7. Multiply: $(2\sqrt{3} - \sqrt{5})(2\sqrt{3} + \sqrt{5}) =$

(a) $4\sqrt{3} - \sqrt{5}$
(b) $12 - 4\sqrt{15} + \sqrt{25}$
(c) $7 + 4\sqrt{15}$
(d) $7 - 4\sqrt{15}$
(e) None of these

8. Simplify: $(\sqrt{a})(\sqrt[3]{a}) =$

(a) $\sqrt[5]{a}$ (b) $\sqrt[5]{a^2}$ (c) $\sqrt[6]{a^2}$ (d) $\sqrt[6]{a^5}$ (e) $\sqrt[6]{a}$

9. Simplify: $\dfrac{2}{\sqrt{3} + 1}$

(a) $\sqrt{3} + 1$
(b) $1/2\ (\sqrt{3} + 1)$
(c) $\sqrt{3} - 1$
(d) $1/2\ (\sqrt{3} - 1)$
(e) none of these

10. Solve: $1 + \sqrt{x} = \sqrt{x + 3}$

(a) 2 (b) -3 (c) 1 (d) -1 (e) has no root

11. Solve: $\sqrt{9x^2 - 5} = 3x - 5$

(a) 1 (b) 3/2 (c) -2/3 (d) 3 (e) has no root

12. When expressed in simplest form $\sqrt{-45} =$

(a) $-3\sqrt{5}$ (b) $3\sqrt{5i}$ (c) $5\sqrt{3}\ i$ (d) $3\sqrt{5}\ i$ (e) $-3\sqrt{5}\ i$

13. Simplify: $i^{17} =$

(a) 1 (b) $-i$ (c) -1 (d) $i$ (e) none of these

14. Find the diagonal of a square to the nearest tenth of an inch whose area is 400 square inches.

(a) 800.0 (b) 20.0 (c) 28.1 (d) 282.8 (e) None of these

15. The number $\sqrt{73}$ is called

(a) transcendental (b) rational (c) irrational
(d) imaginary (e) none of these

16. To find the square root of 243 from a table of numbers from one to 100 the number 243 should be expressed as

(a) $27\sqrt{9}$ (b) $9\sqrt{3}$ (c) $3\sqrt{81}$ (d) $3\sqrt{9}$
(e) can't be found from such a table

17. A correct form of the quadratic formula is

(a) $x = b \pm \frac{\sqrt{b^2-4ac}}{2a}$ (b) $x = -b \pm \frac{\sqrt{b^2-4ac}}{2a}$ (c) $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

(d) $x = \frac{b \pm \sqrt{b^2-4ac}}{2a}$ (e) none of these

18. Solve for <u>x</u>: $x^2 - 7abx + 6a^2b^2 = 0$

(a) 6a,b (b) $-6a^2,b^2$ (c) -6ab,-ab (d) 3ab,2ab (e) 6ab,ab

19. Solve: $2x^2 + x + 3 = 0$

(a) $\frac{-1 \pm \sqrt{23}}{2}$ (b) $\frac{-1 \pm \sqrt{23}\, i}{4}$ (c) $\frac{-1 \pm \sqrt{23}\, i}{2}$

(d) -3,2 (e) +3,-3/2

20. Solve: $x^2 - x - 3 = 0$

(a) 3,-1 (b) -3,1 (c) $\frac{-1 \pm \sqrt{13}}{2}$ (d) $\frac{+1 \pm \sqrt{13}}{2}$ (e) $\frac{1 \pm \sqrt{11}}{2}$

21. The quadratic formula is derived from the general quadratic equation by

(a) squaring both sides (d) completing the square
(b) factoring (e) eliminating the constant
(c) addition or subtraction

22. The equation $x^3 - x^2 - 12x = 0$ can be completely solved by which <u>one</u> of the following methods?
(a) completing the square (d) factoring
(b) dividing out an x (e) no single one of these methods
(c) quadratic formula

23. The equation $2x - y = 3$ crosses the x-axis at

(a) 3 (b) 3/2 (c) -2/3 (d) -3 (e) 6

24. The equation of the x-axis is

(a) $y = 0$ (b) $x = 0$ (c) $x - y = 0$ (d) $x = y = 0$
(e) $x + y = 0$

25. The function $y = x^2 - x - 12$ will cross the x-axis

(a) at one point which is negative
(b) at two points, both positive
(c) at two points, both negative
(d) at one point which is positive
(e) at two points, one positive and one negative

26. The graph of a cubic equation will cross the x-axis

(a) three times
(b) once only
(c) not more than three times
(d) twice
(e) will never cross

27. The equation $4x^2 + 9y^2 = 36$ represents a (an)

(a) hyperbola (b) circle (c) parabola (d) ellipse
(e) two straight lines

28. The equation $9x^2 - 4y^2 = 36$ represents a (an)

(a) pair of straight lines (b) circle (c) parabola
(d) ellipse (e) hyperbola

29. Solve for <u>x</u> and <u>y</u>:

$$\begin{cases} x - y = 1 \\ x^2 + y^2 = 13 \end{cases}$$

(a) $x = 3$, $y = 2$ and $x = -2$, $y = -3$
(b) $x = 3$, $y = -2$ and $x = -3$, $y = 2$
(c) $x = 3$, $y = 2$; double root
(d) $x = 3$, $y = -2$ and $x = 2$, $y = -3$
(e) $x = 3$, $y = -2$ and $x = -2$, $y = 3$

30. The following pair of equations can best be solved by

$2x^2 + y^2 = 6$
$5x + y = 9$

(a) eliminating the constant
(b) addition or subtraction
(c) reducing to simpler systems
(d) substitution
(e) graphing

31. The number of distinct solutions for the following equations is

$$\begin{cases} x^2 + y^2 = 13 \\ 4x^2 - y^2 = 7 \end{cases}$$

(a) 4 (b) 3 (c) 2 (d) 1 (e) 8

32. In a proportion the product of the first and fourth proportional is equal to the product of the

    (a) first and third (b) second and third (c) first and fourth
    (d) second and fourth (e) third and fourth

33. Solve the proportion for x: $5:x = x:15$

    (a) 5/3 (b) 3/5 (c) $3\sqrt{5}$ (d) $5\sqrt{3}$ (e) 3

34. Ratio is a comparison of two numbers by

    (a) subtraction (b) division (c) multiplication
    (d) addition (e) progression

35. The expression $1/2 = 2/4$ is a (an)

    (a) binomial (b) ratio (c) algebraic expression
    (d) proportion (e) formula

36. If y varies directly as the square of x and $y = 8$ when $x = 2$, find the constant of variation.

    (a) 2 (b) 1/2 (c) 4 (d) 16 (e) 6

37. If y varies inversely as $x^2$, and $y = 9$ when $x = 2$, find y when $x = 3$.

    (a) 9 (b) 5 (c) 4/9 (d) 94 (e) 4

38. A number y varies directly as x if their (?) is a constant.

    (a) product (b) quotient (c) difference
    (d) if x is a constant (e) if y is a constant

39. If the binomial $(2x - a)^n$ is expanded, the eighth term will be

    (a) always positive
    (b) positive only if n is even
    (c) always negative
    (d) negative only if n is odd
    (e) can't tell unless n is known

40. The number of terms in the expansion of $(ax - 2b)^{17}$ is

    (a) 17 (b) 16 (c) 34 (d) 18 (e) infinite

# ALGEBRA EXAMINATION

SCORE __________ NAME ______________________

PART __________ FORM ____ DATE ______________________
I or II

| | a | b | c | d | e |
|---|---|---|---|---|---|
| 1. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 2. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 3. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 4. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 5. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 6. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 7. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 8. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 9. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 10. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 11. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 12. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 13. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 14. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 15. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 16. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 17. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 18. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 19. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 20. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 21. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 22. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 23. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 24. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 25. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 26. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 27. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 28. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 29. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 30. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 31. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 32. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 33. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 34. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 35. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 36. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 37. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 38. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 39. | ( ) | ( ) | ( ) | ( ) | ( ) |
| 40. | ( ) | ( ) | ( ) | ( ) | ( ) |

DIAGNOSTIC CHART FOR ALGEBRA EXAMINATION
PART I, FORM A

Date__________ Age___ Name______________________________
Major Field____________ (Last) (First)

Previous Mathematical Training

Years of High School Math.

General___ Algebra___
Geometry___ Trigonometry___
Senior (Review)___

Years of College Math.

Algebra___ Analysis ______
Trigonometry___
Other (Specify)____________

Date last mathematics course was taken____________________________

Analysis of Algebraic Difficulties

If the student's total score is low and it is desired to locate specific areas of difficulty, the following chart may be used. The numbers following each sub-title indicate the question number. Areas of difficulty may be indicated by circling the number of each question which was answered incorrectly.

Introductory Topics
- Definitions. . . . . 1,4,5
- Rules. . . . . . . . . 2,3
- Evaluation . . . 5,6,7,8,9
- Formulas, recall . . 10,13
- Solution of formulas 11,12

Signed Expressions
- Addition . . . . . . . 14
- Subtraction. . . . . . 15
- Parenthesis. . . . . 16,17
- Multiplication . . 18,19,20
- Division . . . . . . 21,22

Equations and Stated Problems
- Solution of equations . . 23,24,25,26
- Stated problems 27,28,29,30

Factoring
- Common monomial . . . . 33
- Trinomials. . . . . . . 31
- Difference of two squares . . . . . . . 32
- Sum and difference of two cubes . . . . . . 34
- Factor theorem. . . . . 35

Fractions
- Addition. . . . . . . . . 40
- Subtraction . . . . . . 37,38
- Division. . . . . . . . . 36
- Fractional equations 38,39,40

# DIAGNOSTIC CHART FOR ALGEBRA EXAMINATION
## PART II, FORM A

Date__________ Age____ Name_______________________________
Major Field____________ (Last) (First)

### Previous Mathematical Training

**Years of High School Math.**

General____ Algebra____
Geometry____ Trigonometry____
Senior (Review)____

**Years of College Math.**

Algebra____ Analysis______
Trigonometry____
Other (Specify)__________

Date last mathematics course was taken____________________

### Analysis of Algebraic Difficulties

If the student's total score is low and it is desired to locate specific areas of difficulty, the following chart may be used. The numbers following each sub-title indicate the question number. Areas of difficulty may be indicated by circling the number of each question which was answered incorrectly.

Simultaneous Equations of the First Degree
- Simultaneous equations . 1
- Reciprocal equations . . 2

Exponents, Roots and Radicals
- Laws of exponents. . 3,4,5
- Fractional exponents . . 6
- Negative exponents . . . 6
- Multiplication of radicals . . . . . . . .7,8
- Division of radicals . . 9
- Equations with radicals . . . . . 10,11
- Imaginary numbers. . 12,13
- Pythagorean theorem. . . 14
- Irrational expressions . 15
- Square root. . . . . . . 16

Quadratic Equations and Higher Order
- Quadratic formula. . 17,21
- Solution by factoring. . 18
- Solution by formula. 19,20
- Higher order equations . 22

Graphing
- Linear functions. . 23,24
- Quadratic functions . . 25
- Cubic functions . . . . 26
- Quadratics in two . . unknowns. . . . . 27,28

Quadratic Systems
- One linear and one quadratic equation 29,30
- Two quadratic equations . . . . . . 31

Ratio and Proportion
- Ratio . . . . . . . . . . 34
- Proportion. . . . 32,33,35

Variation
- Direct. . . . . . . . . 38,39
- Inverse . . . . . . . . . 37

Binomial Theorem
- Expansion . . . . . . 39,40

# APPENDIX F

## INDIVIDUAL EXAMINATION SCORES ON FORMS A

TABLE XIV

INDIVIDUAL EXAMINATION SCORES ON PART I, FORM A

| | Score | | | | Score | | |
|---|---|---|---|---|---|---|---|
| Number | Odd | Even | Total | Number | Odd | Even | Total |
| 1 | 9 | 10 | 19 | 33 | 15 | 15 | 30 |
| 2 | 6 | 8 | 14 | 34 | 9 | 9 | 18 |
| 3 | 13 | 12 | 25 | 35 | 16 | 17 | 33 |
| 4 | 14 | 14 | 28 | 36 | 16 | 16 | 32 |
| 5 | 14 | 16 | 30 | 37 | 11 | 16 | 27 |
| 6 | 4 | 1 | 5 | 38 | 15 | 17 | 32 |
| 7 | 8 | 15 | 23 | 39 | 13 | 16 | 29 |
| 8 | 12 | 12 | 24 | 40 | 7 | 12 | 19 |
| 9 | 4 | 6 | 10 | 41 | 10 | 14 | 24 |
| 10 | 11 | 11 | 22 | 42 | 14 | 13 | 27 |
| 11 | 14 | 14 | 28 | 43 | 11 | 11 | 22 |
| 12 | 4 | 7 | 11 | 44 | 13 | 15 | 28 |
| 13 | 9 | 9 | 18 | 45 | 16 | 15 | 31 |
| 14 | 4 | 3 | 7 | 46 | 14 | 15 | 29 |
| 15 | 15 | 14 | 29 | 47 | 12 | 11 | 23 |
| 16 | 14 | 14 | 28 | 48 | 9 | 8 | 17 |
| 17 | 16 | 16 | 32 | 49 | 11 | 8 | 19 |
| 18 | 5 | 5 | 10 | 50 | 11 | 17 | 28 |
| 19 | 10 | 11 | 21 | 51 | 13 | 15 | 28 |
| 20 | 9 | 15 | 24 | 52 | 9 | 8 | 17 |
| 21 | 14 | 16 | 30 | 53 | 13 | 14 | 27 |
| 22 | 9 | 7 | 16 | 54 | 16 | 16 | 32 |
| 23 | 13 | 11 | 24 | 55 | 13 | 17 | 30 |
| 24 | 9 | 6 | 15 | 56 | 8 | 13 | 21 |
| 25 | 12 | 15 | 27 | 57 | 6 | 11 | 17 |
| 26 | 15 | 14 | 29 | 58 | 16 | 18 | 34 |
| 27 | 4 | 4 | 8 | 59 | 13 | 14 | 27 |
| 28 | 8 | 8 | 16 | 60 | 13 | 15 | 28 |
| 29 | 12 | 10 | 22 | 61 | 9 | 13 | 22 |
| 30 | 9 | 8 | 17 | 62 | 13 | 17 | 30 |
| 31 | 14 | 16 | 30 | 63 | 13 | 15 | 28 |
| 32 | 14 | 18 | 32 | 64 | 14 | 17 | 31 |

TABLE XV

INDIVIDUAL EXAMINATION SCORES ON PART II, FORM A

| | Score | | | | Score | | |
|---|---|---|---|---|---|---|---|
| Number | Odd | Even | Total | Number | Odd | Even | Total |
| 1 | 12 | 8 | 20 | 31 | 17 | 15 | 32 |
| 2 | 6 | 7 | 13 | 32 | 14 | 14 | 28 |
| 3 | 7 | 8 | 15 | 33 | 13 | 13 | 26 |
| 4 | 14 | 10 | 24 | 34 | 13 | 14 | 27 |
| 5 | 10 | 9 | 19 | 35 | 10 | 13 | 23 |
| 6 | 10 | 4 | 14 | 36 | 12 | 11 | 23 |
| 7 | 11 | 11 | 22 | 37 | 14 | 12 | 26 |
| 8 | 7 | 10 | 17 | 38 | 9 | 7 | 16 |
| 9 | 11 | 14 | 25 | 39 | 13 | 9 | 22 |
| 10 | 6 | 6 | 12 | 40 | 12 | 9 | 21 |
| 11 | 13 | 10 | 23 | 41 | 9 | 9 | 18 |
| 12 | 11 | 10 | 21 | 42 | 8 | 6 | 14 |
| 13 | 9 | 8 | 17 | 43 | 8 | 3 | 11 |
| 14 | 6 | 8 | 14 | 44 | 9 | 8 | 17 |
| 15 | 13 | 13 | 26 | 45 | 7 | 3 | 10 |
| 16 | 7 | 3 | 10 | 46 | 10 | 11 | 21 |
| 17 | 10 | 9 | 19 | 47 | 13 | 9 | 22 |
| 18 | 14 | 16 | 30 | 48 | 11 | 7 | 18 |
| 19 | 11 | 10 | 21 | 49 | 9 | 9 | 18 |
| 20 | 14 | 12 | 26 | 50 | 7 | 2 | 9 |
| 21 | 10 | 10 | 20 | 51 | 5 | 4 | 9 |
| 22 | 8 | 13 | 21 | 52 | 11 | 10 | 21 |
| 23 | 11 | 9 | 20 | 53 | 9 | 7 | 16 |
| 24 | 5 | 6 | 11 | 54 | 12 | 14 | 26 |
| 25 | 15 | 14 | 29 | 55 | 5 | 5 | 10 |
| 26 | 10 | 8 | 18 | 56 | 11 | 11 | 22 |
| 27 | 7 | 5 | 12 | 57 | 7 | 3 | 10 |
| 28 | 10 | 3 | 13 | 58 | 4 | 4 | 8 |
| 29 | 7 | 5 | 12 | 59 | 13 | 14 | 27 |
| 30 | 8 | 8 | 16 | 60 | 5 | 2 | 7 |

TABLE XV (CONTINUED)

| | Score | | | | Score | | |
|---|---|---|---|---|---|---|---|
| Number | Odd | Even | Total | Number | Odd | Even | Total |
| 61 | 7 | 10 | 17 | 96 | 9 | 8 | 17 |
| 62 | 10 | 6 | 16 | 97 | 5 | 7 | 12 |
| 63 | 13 | 12 | 25 | 98 | 12 | 17 | 29 |
| 64 | 11 | 3 | 14 | 99 | 8 | 10 | 18 |
| 65 | 15 | 16 | 31 | 100 | 10 | 10 | 20 |
| 66 | 12 | 7 | 19 | 101 | 7 | 8 | 15 |
| 67 | 8 | 8 | 16 | 102 | 8 | 11 | 19 |
| 68 | 15 | 13 | 28 | 103 | 12 | 14 | 26 |
| 69 | 13 | 12 | 25 | 104 | 17 | 16 | 33 |
| 70 | 8 | 8 | 16 | 105 | 3 | 2 | 5 |
| 71 | 7 | 3 | 10 | 106 | 14 | 7 | 21 |
| 72 | 7 | 4 | 11 | 107 | 9 | 9 | 18 |
| 73 | 15 | 16 | 31 | 108 | 9 | 8 | 17 |
| 74 | 4 | 2 | 6 | 109 | 15 | 13 | 28 |
| 75 | 9 | 2 | 11 | 110 | 10 | 12 | 22 |
| 76 | 8 | 11 | 19 | 111 | 6 | 6 | 12 |
| 77 | 10 | 12 | 22 | 112 | 12 | 5 | 17 |
| 78 | 6 | 5 | 11 | 113 | 5 | 7 | 12 |
| 79 | 8 | 9 | 17 | 114 | 9 | 10 | 19 |
| 80 | 10 | 10 | 20 | 115 | 13 | 12 | 25 |
| 81 | 11 | 11 | 22 | 116 | 11 | 15 | 26 |
| 82 | 7 | 3 | 10 | 117 | 6 | 4 | 10 |
| 83 | 4 | 1 | 5 | 118 | 6 | 8 | 14 |
| 84 | 6 | 1 | 7 | 119 | 14 | 15 | 29 |
| 85 | 9 | 7 | 16 | 120 | 12 | 15 | 27 |
| 86 | 8 | 2 | 10 | 121 | 5 | 5 | 10 |
| 87 | 6 | 3 | 9 | 122 | 7 | 9 | 16 |
| 88 | 7 | 6 | 13 | 123 | 4 | 5 | 9 |
| 89 | 7 | 8 | 15 | 124 | 8 | 7 | 15 |
| 90 | 10 | 15 | 25 | 125 | 10 | 6 | 16 |
| 91 | 8 | 7 | 15 | 126 | 8 | 8 | 16 |
| 92 | 12 | 13 | 25 | 127 | 8 | 10 | 18 |
| 93 | 4 | 1 | 5 | 128 | 8 | 8 | 16 |
| 94 | 10 | 8 | 18 | 129 | 12 | 12 | 24 |
| 95 | 9 | 6 | 15 | 130 | 9 | 10 | 19 |

TABLE XV (CONTINUED)

| | Score | | | | Score | | |
|---|---|---|---|---|---|---|---|
| Number | Odd | Even | Total | Number | Odd | Even | Total |
| 131 | 9 | 8 | 17 | 151 | 10 | 5 | 15 |
| 132 | 9 | 11 | 20 | 152 | 6 | 4 | 10 |
| 133 | 10 | 12 | 22 | 153 | 10 | 15 | 25 |
| 134 | 14 | 15 | 29 | 154 | 4 | 2 | 6 |
| 135 | 6 | 2 | 8 | 155 | 14 | 15 | 29 |
| 136 | 11 | 14 | 25 | 156 | 15 | 12 | 27 |
| 137 | 6 | 6 | 12 | 157 | 12 | 13 | 25 |
| 138 | 14 | 13 | 27 | 158 | 6 | 4 | 10 |
| 139 | 5 | 4 | 9 | 159 | 13 | 12 | 25 |
| 140 | 5 | 3 | 8 | 160 | 6 | 3 | 9 |
| 141 | 6 | 12 | 18 | 161 | 9 | 3 | 12 |
| 142 | 12 | 9 | 21 | 162 | 13 | 13 | 26 |
| 143 | 7 | 5 | 12 | 163 | 13 | 12 | 25 |
| 144 | 10 | 11 | 21 | 164 | 10 | 7 | 17 |
| 145 | 9 | 5 | 14 | 165 | 7 | 8 | 15 |
| 146 | 8 | 12 | 20 | 166 | 9 | 3 | 12 |
| 147 | 7 | 7 | 14 | 167 | 6 | 4 | 10 |
| 148 | 8 | 11 | 19 | 168 | 5 | 5 | 10 |
| 149 | 13 | 12 | 25 | 169 | 11 | 5 | 16 |
| 150 | 10 | 8 | 18 | | | | |

# APPENDIX G

## DATA FOR STUDY OF GRADES BY MULTIPLE CORRELATION

TABLE XVI

OHIO PSYCHOLOGICAL EXAMINATION SCORES AND ALGEBRA GRADES

EARNED THE FOLLOWING TERM BY ELEMENTARY GROUP

| Number | Ohio Score | Grade | Number | Ohio Score | Grade |
|---|---|---|---|---|---|
| 1 | 66 | B | 31 | 58 | A |
| 2 | 58 | C | 32 | 37 | C |
| 3 | 9 | C | 33 | 15 | B |
| 4 | 46 | B | 34 | 88 | C |
| 5 | 53 | A | 35 | 20 | B |
| 6 | 79 | A | 36 | 66 | B |
| 7 | 31 | C | 37 | 54 | D |
| 8 | 27 | B | 38 | 25 | C |
| 9 | 26 | D | 39 | 29 | F |
| 10 | 1 | D | 40 | 18 | C |
| 11 | 53 | B | 41 | 12 | C |
| 12 | 37 | B | 42 | 44 | A |
| 13 | 66 | D | 43 | 33 | D |
| 14 | 60 | B | 44 | 12 | C |
| 15 | 59 | C | 45 | 10 | C |
| 16 | 38 | D | 46 | 80 | B |
| 17 | 16 | F | 47 | 64 | D |
| 18 | 18 | C | 48 | 18 | F |
| 19 | 4 | F | 49 | 3 | F |
| 20 | 27 | C | 50 | 22 | B |
| 21 | 32 | C | 51 | 55 | C |
| 22 | 37 | B | 52 | 42 | D |
| 23 | 33 | F | 53 | 17 | C |
| 24 | 54 | C | 54 | 20 | C |
| 25 | 33 | D | 55 | 6 | F |
| 26 | 35 | C | 56 | 21 | B |
| 27 | 63 | B | 57 | 66 | C |
| 28 | 5 | F | 58 | 7 | F |
| 29 | 16 | B | 59 | 95 | A |
| 30 | 38 | F | 60 | 38 | B |

TABLE XVI (CONTINUED)

| Number | Ohio Score | Grade | Number | Ohio Score | Grade |
|---|---|---|---|---|---|
| 61 | 32 | C | 96 | 64 | B |
| 62 | 12 | D | 97 | 37 | C |
| 63 | 10 | C | 98 | 21 | F |
| 64 | 20 | B | 99 | 29 | C |
| 65 | 45 | B | 100 | 40 | C |
| 66 | 41 | C | 101 | 10 | C |
| 67 | 14 | B | 102 | 24 | F |
| 68 | 50 | A | 103 | 41 | D |
| 69 | 52 | B | 104 | 16 | B |
| 70 | 39 | B | 105 | 10 | C |
| 71 | 8 | B | 106 | 58 | F |
| 72 | 10 | F | 107 | 55 | B |
| 73 | 30 | D | 108 | 74 | B |
| 74 | 71 | A | 109 | 25 | D |
| 75 | 41 | C | 110 | 7 | D |
| 76 | 56 | B | 111 | 65 | B |
| 77 | 40 | B | 112 | 55 | C |
| 78 | 8 | C | 113 | 7 | C |
| 79 | 8 | C | 114 | 16 | C |
| 80 | 25 | B | 115 | 56 | C |
| 81 | 21 | A | 116 | 64 | C |
| 82 | 6 | A | 117 | 42 | B |
| 83 | 41 | C | 118 | 20 | D |
| 84 | 1 | C | 119 | 4 | B |
| 85 | 50 | B | 120 | 62 | B |
| 86 | 46 | B | 121 | 29 | C |
| 87 | 24 | F | 122 | 17 | F |
| 88 | 61 | A | 123 | 48 | B |
| 89 | 10 | D | 124 | 9 | C |
| 90 | 4 | C | 125 | 5 | C |
| 91 | 1 | F | 126 | 47 | D |
| 92 | 66 | D | 127 | 81 | A |
| 93 | 34 | F | 128 | 17 | F |
| 94 | 9 | C | 129 | 26 | C |
| 95 | 15 | C | 130 | 43 | F |

TABLE XVI (CONTINUED)

| Number | Ohio Score | Grade | Number | Ohio Score | Grade |
|---|---|---|---|---|---|
| 131 | 45 | C | 136 | 42 | C |
| 132 | 21 | C | 137 | 75 | C |
| 133 | 35 | F | 138 | 37 | C |
| 134 | 48 | C | 139 | 21 | D |
| 135 | 5 | B | 140 | 53 | D |
| | | | 141 | 27 | C |

TABLE XVII

OHIO PSYCHOLOGICAL EXAMINATION SCORES AND ALGEBRA GRADES EARNED THE FOLLOWING TERM BY INTERMEDIATE GROUP

| Number | Ohio Score | Grade | Number | Ohio Score | Grade |
|---|---|---|---|---|---|
| 1 | 21 | D | 31 | 51 | B |
| 2 | 63 | F | 32 | 13 | D |
| 3 | 42 | D | 33 | 42 | B |
| 4 | 52 | C | 34 | 63 | D |
| 5 | 8 | A | 35 | 50 | F |
| 6 | 48 | C | 36 | 31 | D |
| 7 | 31 | D | 37 | 42 | D |
| 8 | 38 | C | 38 | 1 | C |
| 9 | 27 | C | 39 | 86 | D |
| 10 | 32 | B | 40 | 27 | D |
| 11 | 95 | C | 41 | 49 | A |
| 12 | 21 | D | 42 | 15 | D |
| 13 | 65 | C | 43 | 65 | C |
| 14 | 41 | D | 44 | 42 | F |
| 15 | 18 | C | 45 | 68 | B |
| 16 | 23 | D | 46 | 52 | C |
| 17 | 29 | B | 47 | 33 | B |
| 18 | 57 | B | 48 | 57 | C |
| 19 | 48 | C | 49 | 33 | B |
| 20 | 50 | B | 50 | 8 | B |
| 21 | 8 | C | 51 | 38 | D |
| 22 | 63 | C | 52 | 4 | F |
| 23 | 13 | F | 53 | 61 | D |
| 24 | 61 | F | 54 | 54 | C |
| 25 | 51 | A | 55 | 55 | D |
| 26 | 42 | C | 56 | 67 | C |
| 27 | 60 | C | 57 | 26 | D |
| 28 | 54 | C | 58 | 34 | D |
| 29 | 28 | D | 59 | 16 | C |
| 30 | 85 | A | 60 | 85 | B |

TABLE XVII (CONTINUED)

| Number | Ohio Score | Grade | Number | Ohio Score | Grade |
|---|---|---|---|---|---|
| 61 | 33 | D | 66 | 24 | F |
| 62 | 47 | C | 67 | 47 | B |
| 63 | 52 | D | 68 | 48 | A |
| 64 | 65 | B | 69 | 13 | D |
| 65 | 87 | B | 70 | 11 | D |
| | | | 71 | 62 | C |
| | | | 72 | 26 | F |

TABLE XVIII

OHIO PSYCHOLOGICAL EXAMINATION SCORES AND REVISED SCORES ON PART I, FORM P OF THE EXAMINATION USED IN THIS STUDY

| Number | Ohio Score | Revised Score | Number | Ohio Score | Revised Score |
|---|---|---|---|---|---|
| 1 | 66 | 19 | 31 | 17 | 30 |
| 2 | 58 | 21 | 32 | 54 | 28 |
| 3 | 3 | 26 | 33 | 33 | 20 |
| 4 | 15 | 31 | 34 | 32 | 28 |
| 5 | 88 | 30 | 35 | 21 | 24 |
| 6 | 24 | 22 | 36 | 58 | 31 |
| 7 | 20 | 27 | 37 | 37 | 12 |
| 8 | 66 | 26 | 38 | 9 | 27 |
| 9 | 31 | 24 | 39 | 46 | 27 |
| 10 | 27 | 28 | 40 | 53 | 26 |
| 11 | 26 | 18 | 41 | 39 | 7 |
| 12 | 29 | 27 | 42 | 79 | 30 |
| 13 | 30 | 28 | 43 | 46 | 17 |
| 14 | 18 | 15 | 44 | 54 | 22 |
| 15 | 4 | 15 | 45 | 25 | 27 |
| 16 | 53 | 27 | 46 | 51 | 23 |
| 17 | 12 | 19 | 47 | 21 | 25 |
| 18 | 14 | 10 | 48 | 1 | 9 |
| 19 | 66 | 16 | 49 | 10 | 19 |
| 20 | 60 | 21 | 50 | 27 | 22 |
| 21 | 10 | 20 | 51 | 67 | 24 |
| 22 | 80 | 28 | 52 | 37 | 23 |
| 23 | 64 | 20 | 53 | 44 | 31 |
| 24 | 18 | 20 | 54 | 33 | 23 |
| 25 | 4 | 13 | 55 | 59 | 24 |
| 26 | 30 | 15 | 56 | 38 | 22 |
| 27 | 27 | 23 | 57 | 16 | 13 |
| 28 | 32 | 20 | 58 | 1 | 18 |
| 29 | 55 | 23 | 59 | 18 | 20 |
| 30 | 42 | 24 | 60 | 12 | 13 |

TABLE XVIII (CONTINUED)

| Number | Ohio Score | Revised Score | Number | Ohio Score | Revised Score |
|---|---|---|---|---|---|
| 61 | 3 | 21 | 96 | 58 | 12 |
| 62 | 22 | 20 | 97 | 55 | 23 |
| 63 | 19 | 15 | 98 | 74 | 11 |
| 64 | 37 | 23 | 99 | 71 | 32 |
| 65 | 33 | 21 | 100 | 25 | 16 |
| 66 | 27 | 23 | 101 | 7 | 18 |
| 67 | 20 | 26 | 102 | 65 | 20 |
| 68 | 6 | 21 | 103 | 40 | 34 |
| 69 | 35 | 26 | 104 | 22 | 17 |
| 70 | 21 | 21 | 105 | 57 | 26 |
| 71 | 63 | 22 | 106 | 66 | 20 |
| 72 | 5 | 12 | 107 | 7 | 11 |
| 73 | 36 | 21 | 108 | 16 | 22 |
| 74 | 95 | 33 | 109 | 59 | 26 |
| 75 | 38 | 13 | 110 | 38 | 24 |
| 76 | 8 | 16 | 111 | 13 | 10 |
| 77 | 32 | 22 | 112 | 64 | 21 |
| 78 | 32 | 25 | 113 | 12 | 16 |
| 79 | 37 | 21 | 114 | 66 | 21 |
| 80 | 26 | 23 | 115 | 10 | 14 |
| 81 | 21 | 14 | 116 | 30 | 21 |
| 82 | 20 | 29 | 117 | 1 | 19 |
| 83 | 29 | 25 | 118 | 45 | 29 |
| 84 | 1 | 13 | 119 | 42 | 28 |
| 85 | 10 | 15 | 120 | 40 | 26 |
| 86 | 36 | 20 | 121 | 20 | 9 |
| 87 | 41 | 18 | 122 | 84 | 22 |
| 88 | 10 | 20 | 123 | 14 | 23 |
| 89 | 22 | 11 | 124 | 24 | 20 |
| 90 | 50 | 31 | 125 | 41 | 21 |
| 91 | 52 | 26 | 126 | 15 | 9 |
| 92 | 27 | 10 | 127 | 24 | 16 |
| 93 | 16 | 14 | 128 | 52 | 14 |
| 94 | 10 | 11 | 129 | 39 | 20 |
| 95 | 10 | 25 | 130 | 8 | 26 |

TABLE XVIII (CONTINUED)

| Number | Ohio Score | Revised Score | Number | Ohio Score | Revised Score |
|---|---|---|---|---|---|
| 131 | 10 | 24 | 166 | 75 | 16 |
| 132 | 30 | 25 | 167 | 13 | 12 |
| 133 | 74 | 33 | 168 | 48 | 27 |
| 134 | 66 | 18 | 169 | 31 | 28 |
| 135 | 41 | 16 | 170 | 21 | 18 |
| 136 | 56 | 32 | 171 | 62 | 25 |
| 137 | 8 | 14 | 172 | 5 | 30 |
| 138 | 55 | 22 | 173 | 4 | 11 |
| 139 | 8 | 21 | 174 | 53 | 25 |
| 140 | 7 | 22 | 175 | 26 | 23 |
| 141 | 44 | 17 | 176 | 8 | 27 |
| 142 | 27 | 21 | 177 | 16 | 25 |
| 143 | 25 | 29 | 178 | 17 | 15 |
| 144 | 56 | 19 | 179 | 21 | 30 |
| 145 | 64 | 23 | 180 | 6 | 26 |
| 146 | 42 | 23 | 181 | 41 | 20 |
| 147 | 20 | 12 | 182 | 1 | 26 |
| 148 | 55 | 11 | 183 | 4 | 23 |
| 149 | 50 | 27 | 184 | 62 | 28 |
| 150 | 46 | 18 | 185 | 29 | 22 |
| 151 | 24 | 16 | 186 | 17 | 24 |
| 152 | 61 | 32 | 187 | 48 | 28 |
| 153 | 17 | 14 | 188 | 10 | 15 |
| 154 | 9 | 22 | 189 | 21 | 15 |
| 155 | 4 | 15 | 190 | 5 | 16 |
| 156 | 1 | 14 | 191 | 33 | 18 |
| 157 | 47 | 17 | 192 | 32 | 30 |
| 158 | 66 | 22 | 193 | 81 | 34 |
| 159 | 34 | 12 | 194 | 8 | 11 |
| 160 | 33 | 27 | 195 | 17 | 18 |
| 161 | 9 | 18 | 196 | 26 | 23 |
| 162 | 15 | 29 | 197 | 12 | 25 |
| 163 | 43 | 18 | 198 | 21 | 18 |
| 164 | 45 | 26 | 199 | 42 | 20 |
| 165 | 21 | 23 | 200 | 9 | 19 |
| | | | 201 | 51 | 22 |
| | | | 202 | 37 | 22 |
| | | | 203 | 27 | 26 |

TABLE XIX

OHIO PSYCHOLOGICAL EXAMINATION SCORES AND REVISED SCORES ON PART II, FORM P OF THE EXAMINATION USED IN THIS STUDY

| Number | Ohio Score | Revised Score | Number | Ohio Score | Revised Score |
|---|---|---|---|---|---|
| 1 | 1 | 19 | 31 | 21 | 25 |
| 2 | 51 | 22 | 32 | 63 | 22 |
| 3 | 13 | 28 | 33 | 42 | 25 |
| 4 | 52 | 23 | 34 | 63 | 20 |
| 5 | 47 | 21 | 35 | 8 | 22 |
| 6 | 50 | 6 | 36 | 48 | 30 |
| 7 | 31 | 29 | 37 | 29 | 23 |
| 8 | 42 | 26 | 38 | 74 | 24 |
| 9 | 11 | 22 | 39 | 38 | 28 |
| 10 | 50 | 16 | 40 | 27 | 8 |
| 11 | 86 | 18 | 41 | 30 | 26 |
| 12 | 32 | 30 | 42 | 27 | 28 |
| 13 | 95 | 34 | 43 | 47 | 7 |
| 14 | 49 | 33 | 44 | 21 | 24 |
| 15 | 15 | 21 | 45 | 65 | 31 |
| 16 | 41 | 15 | 46 | 42 | 12 |
| 17 | 18 | 24 | 47 | 68 | 30 |
| 18 | 23 | 19 | 48 | 52 | 26 |
| 19 | 29 | 28 | 49 | 33 | 30 |
| 20 | 57 | 28 | 50 | 48 | 24 |
| 21 | 33 | 30 | 51 | 23 | 17 |
| 22 | 26 | 16 | 52 | 50 | 32 |
| 23 | 63 | 28 | 53 | 71 | 23 |
| 24 | 8 | 27 | 54 | 38 | 18 |
| 25 | 12 | 12 | 55 | 63 | 32 |
| 26 | 4 | 23 | 56 | 13 | 29 |
| 27 | 61 | 33 | 57 | 54 | 30 |
| 28 | 76 | 18 | 58 | 51 | 32 |
| 29 | 30 | 31 | 59 | 55 | 15 |
| 30 | 53 | 25 | 60 | 42 | 20 |

TABLE XIX (CONTINUED)

| Number | Ohio Score | Revised Score | Number | Ohio Score | Revised Score |
|---|---|---|---|---|---|
| 61 | 67 | 32 | 76 | 60 | 30 |
| 62 | 26 | 24 | 77 | 54 | 32 |
| 63 | 34 | 23 | 78 | 25 | 25 |
| 64 | 28 | 25 | 79 | 16 | 24 |
| 65 | 85 | 34 | 80 | 33 | 24 |
| 66 | 73 | 35 | 81 | 57 | 32 |
| 67 | 24 | 13 | 82 | 47 | 32 |
| 68 | 52 | 19 | 83 | 48 | 35 |
| 69 | 65 | 38 | 84 | 68 | 37 |
| 70 | 13 | 27 | | | |
| 71 | 87 | 22 | | | |
| 72 | 11 | 22 | | | |
| 73 | 44 | 30 | | | |
| 74 | 27 | 24 | | | |
| 75 | 62 | 22 | | | |