CAPACITOR-EXCITED INDUCTION GENERATOR

by

JAMES WILLIAM LAUGHLIN

A THESIS

submitted to

OREGON STATE COLLEGE

in partial fulfillment of
the requirements for the
degree of

MASTER OF SCIENCE

June 1950

APPROVED:

[redacted]

Professor of Electrical Engineering

In Charge of Major

[redacted]

Head of Department of Electrical Engineering

[redacted]

Chairman of School Graduate Committee

[redacted]

Dean of Graduate School

Typed by: [redacted]

ACKNOWLEDGMENT

The author is grateful to Professor E. C. Starr for his help in the general supervision of the work and for his suggestions, particularly in connection with the calculations of magnetizing current; to Professor L. N. Stone for many helpful comments and suggestions particularly in connection with the incremental power tests.

The author also wishes to express his thanks to H. E. Todd, T. C. Dowerah, and Harold Haake for their assistance in conducting the laboratory tests, and to Professor H. B. Cockerline for his assistance in checking some of the more important calculations.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF FIGURES (CONT'D)

CAPACITOR-EXCITED INDUCTION GENERATOR

INTRODUCTION

The principle of capacitive excitation for induction generators is not new. The first known application of capacitive excitation for induction generators occurred in 1918 near Yakima, Washington on a power system operated by the Pacific Power and Light Company (7, p.964). The induction generator was connected to a high voltage distribution system, and it was discovered that when the synchronous generators were all disconnected from the system, the induction generator was still able to carry part of the load and retain its voltage. The distribution system transmission lines supplied the necessary magnetizing current due to the capacitance between lines. Power system loads at that time were largely resistive, and excess magnetizing kva was available, due to excess E^2Y.

In more recent times power system loads have changed and magnetizing current is no longer available, generally due to the increased use of induction motor loads, and increased loads with increased I^2X. This, combined with the high cost of static capacitors, has restricted the use of induction generators, until there are very few of them in use today.

In the past few years the cost of static capacitors has dropped until it might be practicable to use capacitor-excited induction generators, particularly where loads of near unity power factor or leading power factor are encountered. The criterion for this is that for the particular application considered the induction machine with capacitors and necessary control and regulator circuits, including the governor for the prime mover be cheaper than a synchronous machine of the same rating with its exciter and necessary regulator circuits. In some cases it might be desirable to use the induction machine even if it were more expensive due to its somewhat different characteristics which it would not be possible to obtain from the ordinary synchronous machine. The two main characteristics that are advantageous are excellent wave form and inability to back up or sustain a short circuit.

The scope of this thesis is to examine the induction generator and see what differences, if any, there are between capacitive excitation and operation in parallel with synchronous machines. Also, to measure and calculate load characteristics under constant-frequency constant-voltage conditions. These calculations will be made from the usual induction machine equivalent circuit, and it will be ascertained whether or not there are any differences in applying this circuit to a motor, a generator in parallel with synchronous machines, or a capacitor-excited induction generator. If necessary modifications in the methods of calculation will be made in order to secure agreement with measured characteristics to a sufficient degree of accuracy. If any modifications are necessary, an attempt will be made to give a theoretical justification, and to determine whether or not the method would be applicable to machines of all sizes. As a result of the tests and calculations it is hoped that it will be possible to specify the simplest laboratory test and calculations that would enable the determination of machine characteristics with sufficient accuracy to design control circuits for constant-frequency constant-voltage operation.

Due to availability of equipment and relative costs of wound-rotor and squirrel-cage machines, investigations have been limited to the cheapest most readily available machine, that is, the general purpose motor with a squirrel-cage rotor. The machine used had a cast aluminum rotor.

The induction generator has some advantages over synchronous machines. It also has some disadvantages (5, p. 561). The advantages are rugged construction, inability to back up a short circuit, excellent wave form, lack of hunting, and possibility of reduced cost. The disadvantages are variable speed necessary for constant-frequency operation, power factor fixed by the slip and voltage and not by the load, and necessity for providing a source of magnetizing current. It is believed that the advantages outweigh the disadvantages, and that most of the latter can be overcome by the use of capacitive excitation.

INDUCTION MACHINE THEORY

A polyphase induction motor consists of a distributed stator winding and a short-circuited rotor winding. The stator winding is similar to that of a polyphase alternator, and when supplied with the proper polyphase voltages sets up a magnetic field which rotates in space at synchronous speed. This flux cuts the rotor and induces a voltage in it. This induced voltage causes current to flow which lags behind the induced voltage by an angle depending upon the rotor impedance. In what follows the subscript 1 will refer to the stator or primary, and the subscript 2 will refer to the rotor or secondary, since the induction motor is actually a transformer with an air gap in the core and a short-circuited secondary which is free to move relative to the primary. If the rotor is free, the reaction between the flux produced by the stator and flux produced by the rotor current will cause the rotor to rotate. However, the rotor cannot rotate at the same speed as the rotating field of primary flux, for if it did there would be no relative motion between rotor and primary flux, and hence no voltage induced in the rotor and no rotor flux to react with stator flux to produce torque. The difference in speed between the rotor and rotating magnetic field is called slip, designated s, and usually expressed as a fraction of synchronous speed. Slip is considered to be positive when the rotor is below synchronous speed and negative when the rotor is above synchronous speed. The flux set up by the stator induces a voltage E_1 in the stator and a voltage E_2s in the rotor. Since the rotor reactance is proportional to the frequency of current flowing in the rotor, the actual reactance can be considered to be the reactance measured at stator frequency multiplied by the slip, and the rotor resistance will be the effective resistance, somewhat lower than the rotor resistance measured at stator frequency. Therefore, the equation for rotor current may be written as follows.

$$I_2 = \frac{E_2 s}{(r_2^2 + x_2^2 s^2)^{\frac{1}{2}}} \tag{1}$$

All quantities are referred to the primary (5, Eq.244, p.476). Equation (1) may be rewritten as

$$I_2 = \frac{E_2}{\left[\left(\frac{r_2}{s}\right)^2 + x_2^2\right]^{\frac{1}{2}}} \tag{2}$$

Thus, it is seen that the rotor acts as though it had a resistance equal to its actual resistance divided by the slip. The stator has resistance and reactance, and the whole machine has a certain magnetizing reactance. The

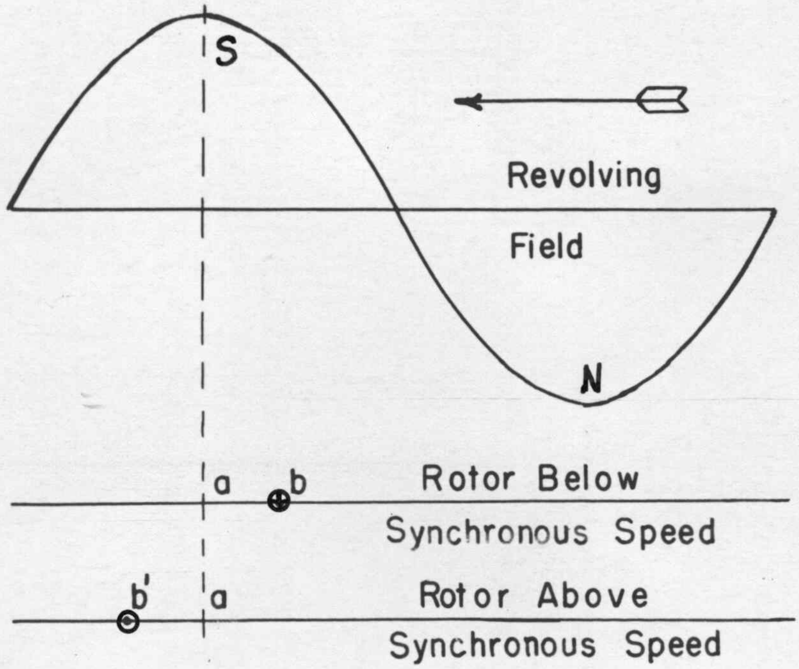


Fig. 1. Flux and Inductors of Induction Machine.

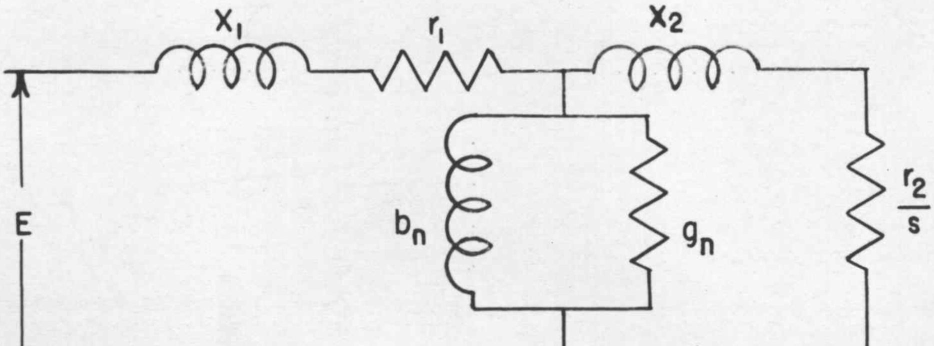


Fig. 2. Equivalent Circuit of Induction Machine.

performance of an induction machine is usually calculated from the equivalent circuit given in Fig. 2. It may be noted that this equivalent circuit is similar to that of a static transformer. X_1 and r_1 represent the primary leakage reactance and resistance, respectively, and x_2 and $\frac{r_2}{s}$ represent the rotor, referred to the stator. B_n and g_n are the magnetizing susceptance and conductance, respectively, referred to the stator. The primary and secondary induced voltages, E_1 and E_2, appear across this branch of the equivalent circuit and are less than the terminal voltage E by the primary impedance drop. Also, it should be noted that a source of error in using this equivalent circuit is the fact that the change in secondary resistance due to skin effect and eddy current loss is neglected. This introduces a nearly constant error, as the rotor resistance is usually measured at stator frequency, and the current flowing in the rotor under load conditions will be of varying frequency depending upon the slip, but this variation will be small as the slip rarely exceeds 5% over the useful range of the machine. Thus the difference due to differing slip will be small, and the difference between rotor resistance measured at stator frequency and at any actual frequency, though rather large, will be nearly constant.

It should be noted that the induction motor always operates at lagging power factor, since the line current must always have a magnetizing component.

If the rotor were rotating above synchronous speed the machine would act as a generator, providing the same rotating flux were present in the stator. This can be most readily seen by referring to Fig. 2. When the rotor is above synchronous speed the slip is negative, and the rotor acts as though it had a negative resistance, and thus the load component of current reverses while the magnetizing component remains the same.

It appears from the above that an induction generator must always operate at leading power factor. This can be explained by referring to Fig. 1 (5, fig.278, p.557). Fig. 1 shows a developed diagram of stator flux and one inductor on the rotor. When the rotor is operating below synchronous speed an inductor at position a has a maximum voltage induced in it. Due to rotor reactance, the inductor will reach some such position as b before the current in it reaches maximum value. Thus the rotor current will be reflected into the stator as lagging. However, when the rotor is above synchronous speed, the inductor will reach a position such as b' before the current flowing in it is maximum, and will be reflected into the stator as leading.

It is usual in measuring the constants in the equivalent circuit for motor applications to measure the magnetizing conductance and susceptance at a voltage corresponding to the terminal voltage E. This introduces a nearly constant error of 2 or 3 per cent in the induced voltages E_1 and E_2. Since the power and torque may be shown to vary as the square of these induced voltages, the error is often as high as 4 or 6 per cent (5, p.479). The effect of using this approximation for applications to generator action will be discussed in connection with calculating generator performance.

There are two ways in which the revolving flux may be produced. The first is the usual application, by operating the induction generator in parallel with synchronous machines, which control the terminal voltage and the frequency. The second is by capacitor excitation. The only difficulty encountered with capacitive excitation is the building up of terminal voltage. If sufficient capacitance is connected to the terminals, and the rotor rotated the voltage will be built up. This may be explained as follows. The rotor inductors cut the small flux that is present in the machine due to residual magnetism. This residual magnetism may be present in the stator or the rotor or both. A small voltage is induced in the rotor, which sets up a flux which in turn induces a voltage in the stator. Current flows in the stator charging the capacitors and setting up the revolving flux. The only difficulty arises in cases where there is no residual magnetism in the machine. It may be restored by discharging a charged capacitor through the windings, or passing a small direct current through them (1, p.543).

Thus, it appears that the performance of an induction generator can be predicted by the equivalent circuit, providing saturation is taken into account.

TEST PROCEDURE

Procedure for Determining Equivalent Circuit Constants

The two tests usually used to determine the main constants in the equivalent circuit are the blocked-rotor test and the running-light test. The blocked-rotor test is taken at normal frequency with the rotor blocked and reduced voltage applied to the terminals, using the arrangement shown in Fig. 3. Power, current, and voltage are read, usually only at rated current (4, p.711). Several readings at different voltages may be taken, but the reading at rated current should be used for determining the equivalent circuit constants. With the rotor blocked, the rotor currents are at stator frequency, and develop sufficient flux in opposition to the stator flux so that core loss is considered negligible compared to copper loss. Thus the impedance of the machine under blocked-rotor conditions consists of the primary and secondary resistances and leakage reactances. From the blocked-rotor test the sum of primary and secondary resistances, referred to the primary, and the sum of the primary and secondary leakage reactances referred to the primary are obtained. It is usual to assume that the leakage reactance divides equally between primary and secondary, since there is no satisfactory method of determining the actual division (5, p.540).

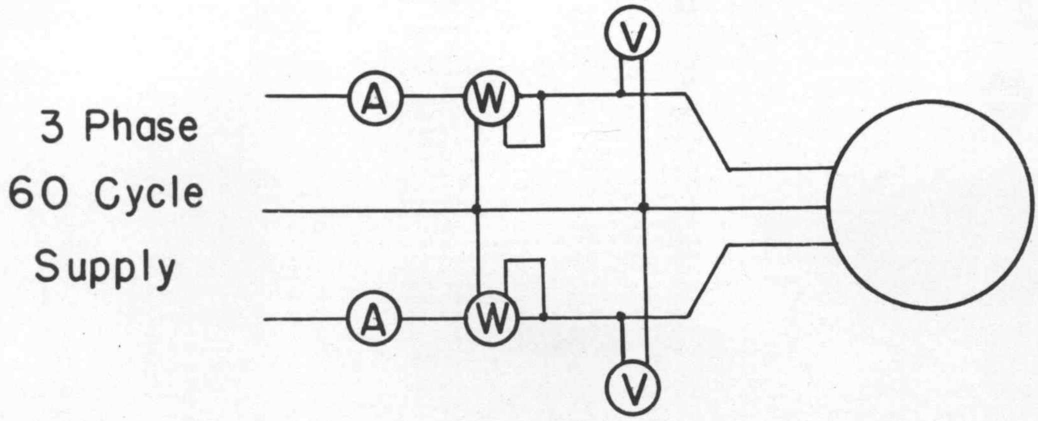


Fig. 3. Circuit Diagram, Blocked Rotor Test, Running Light Test.

The rotor resistance of a squirrel-cage machine is very difficult to determine. It is possible to calculate the rotor resistance from the rotor dimensions, but this is a complicated process and data are often not available (5, p.540). In some cases it is sufficiently accurate to measure the stator resistance with direct current and subtract this value from the total resistance measured with rotor blocked and consider the remainder to be rotor resistance (4, p.711). For more accurate results the effective stator resistance should be subtracted from the total resistance to get the rotor resistance (5, p.540). As previously pointed out, the total resistance measured in the blocked-rotor test is not the true total resistance of the motor under operating conditions due to the rotor resistance being higher due to skin effect and eddy currents in the inductors. Thus all the proposed methods are approximations. Whenever possible a slip test should be made (4, p.712). Knowing the slip as a function of load, even for one point, the value of rotor resistance to be used in the equivalent circuit may be calculated. A slip test was not taken, as the necessary equipment was not available. The rotor resistance was calculated from one reading of the generator load tests and will be described later.

The running-light test is taken at normal frequency, with the rotor free to rotate, using the arrangement shown in Fig. 3. Power, current, and voltage are read under varying voltage conditions. The voltage should be varied from about 125% to about 15% of rated voltage (4, p.710). The slip of the machine is very small except at low voltage, and the power input is made up of the friction and windage loss and the primary I^2R loss. The I^2R loss of the rotor is negligible, as the slip and hence the rotor current are very small. The power as a function of voltage may be extrapolated to zero voltage, and the value of power thus obtained is the sum of friction and windage losses at zero voltage. This value will change very little with increased voltage, as the change in speed of rotation is very small (4, p.710). The primary I^2R loss, calculated from the primary resistance obtained from the blocked-rotor test, and the friction and windage losses are then subtracted from the total power input, and the remainder considered to be core loss. The value of g_n is then calculated using this power with either the terminal voltage or the voltage obtained by subtracting the primary impedance drop from the terminal voltage. In most cases the former is sufficiently accurate (5, p.539). The value of b_n is calculated using the magnetizing current and either the terminal voltage or the voltage obtained by subtracting the primary impedance drop from the terminal voltage. In calculating the magnetizing current it is important to use the total power input, including friction and windage and primary I^2R loss, to calculate the power factor. From this power factor the reactive factor is obtained and used to calculate b_n. Normally the values of b_n and g_n at rated voltage and about 10% above and below rated voltage are all that are needed, but in order to calculate generator characteristics, these constants for a large number of terminal voltages are necessary. The running-light test as actually conducted in the laboratory included the friction and windage losses of a direct-current generator which was solidly coupled to the test machine. The friction and windage losses of this machine should remain constant during the test, and hence not affect the test results.

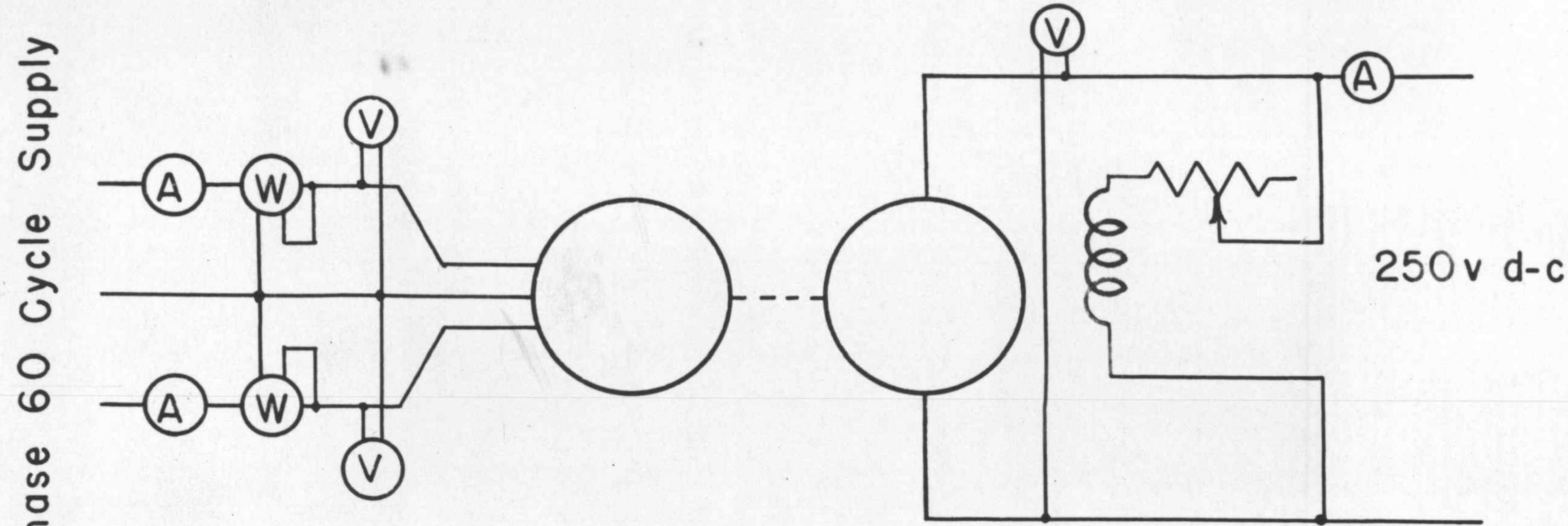


Fig. 4. Running Light Test, Synchronous Speed.

The uncertainty as to whether or not the friction and windage losses remain constant may be eliminated by taking the running-light test data with the machine driven at synchronous speed. This arrangement is shown in Fig. 4. The test procedure is the same. This test was conducted using the direct-current machine to drive the test machine, and a stroboscope to hold speed constant. It was noted that the power obtained using this method is not accurate. A slight increase in speed above synchronous speed will cause the test machine to act as a generator, and the power read will be too low. A decrease in speed below synchronous speed causes the power input to include a component due to part of the friction and windage losses being supplied from the supply lines to the test machine, and the power read will be too high. For this reason, this test should be conducted with a synchronous motor as the driving machine. This was impossible in this case, as the test machine was solidly coupled to the direct-current machine. It will be noted by examination of Table 3 in the Appendix that values of g_n are very irregular. These data were not used in the actual calculations. The values of b_n check very closely with values obtained from the normal running-light test.

The running-light data may also be obtained by operating the machine as a capacitor-excited generator. This

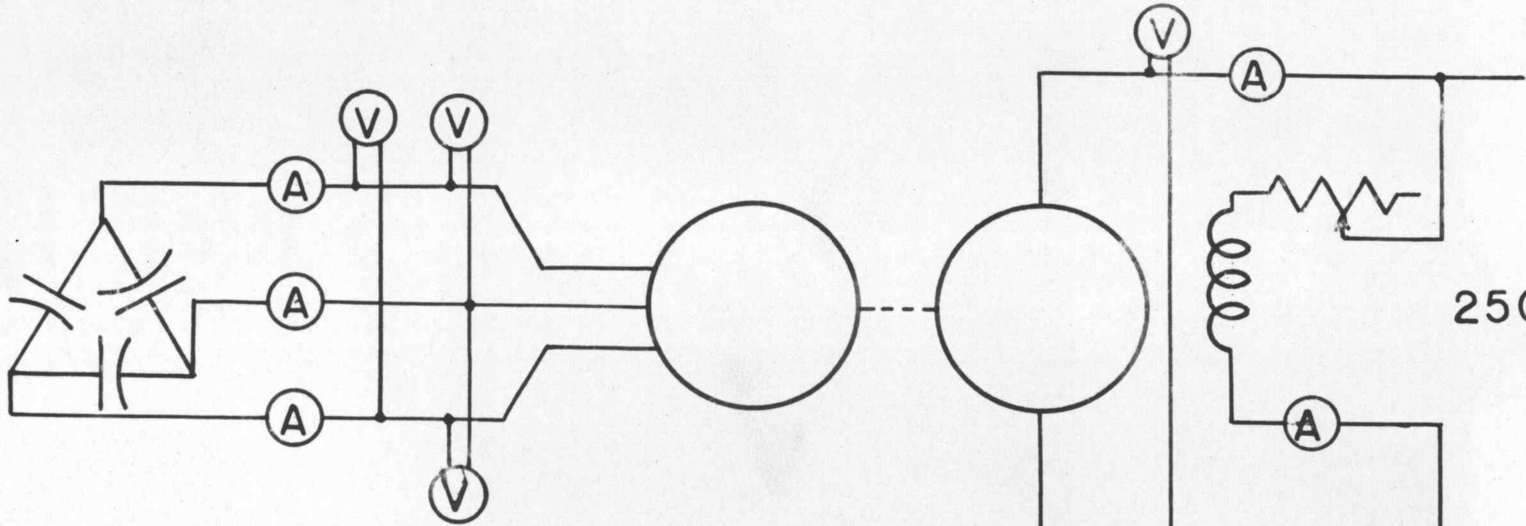


Fig. 5. Circuit Diagram, Incremental Power Test, d-c Drive.

arrangement is shown in Fig. 5. The machine is driven by the direct-current machine, and the terminal voltage and magnetizing current are measured directly. The magnetizing current flows through the primary resistance, and therefore the primary I^2R loss must be subtracted from the power input. The power input is measured by taking the incremental power input to the d-c machine. The conditions for conducting this test are as follows. The speed is held constant at synchronous speed by means of a stroboscope. The field current of the d-c machine is held constant, so that core loss in that machine will be constant. The power input to the armature is measured with the test machine turning at synchronous speed, but with the capacitors disconnected so that the terminal voltage is zero. Sufficient capacitance is then connected to the terminals to obtain the desired terminal voltage, and the terminal voltage and magnetizing current are read, keeping speed constant by adjusting the armature voltage of the d-c machine. The difference between this power and the power read with the test machine unexcited, corrected for d-c armature I^2R loss, is taken to be the sum of primary I^2R loss and core loss. By subtracting primary I^2R loss from the incremental power, the power corresponding to the core loss is obtained. G_n and b_n are calculated in a manner similar to that used in the running-light test. Table 4 in the Appendix shows that the values of g_n obtained in this test are very irregular. Better results could be obtained using a dynamometer or a d-c motor whose losses are accurately known for all load conditions. It will be noted that magnetizing current and values of b_n checked the running-light test.

Procedure for Load Tests

Load tests with shunt excitation were made using the arrangement shown in Fig. 6. The cathode-ray oscilloscope was used to hold frequency constant by the use of Lissajous figures. Slip was measured by the use of a stroboscope. A balanced three-phase liquid rheostat was used for a load. This type of load is pure resistance. The capacitors were connected in a delta to give maximum voltage across them, so that less capacitance was needed for a given magnetizing current, and to get maximum kva from a limited amount of capacitance. A wye connection would have worked equally as well, except that more capacitance would have been required. The capacitors were adjusted as the load was added to keep constant terminal voltage. The circuit was kept balanced by adjusting the capacitors, so that only one ammeter was necessary to read the machine current. The machine current was used to judge the per cent load in the machine. The machine was driven with a d-c shunt motor, and speed adjusted to hold frequency constant. Slight irregularities in the experimental data were caused by inability to hold the load and frequency absolutely constant while reading all the instruments and by the fact that capacitance was not continuously adjustable, so that the terminal voltage was not

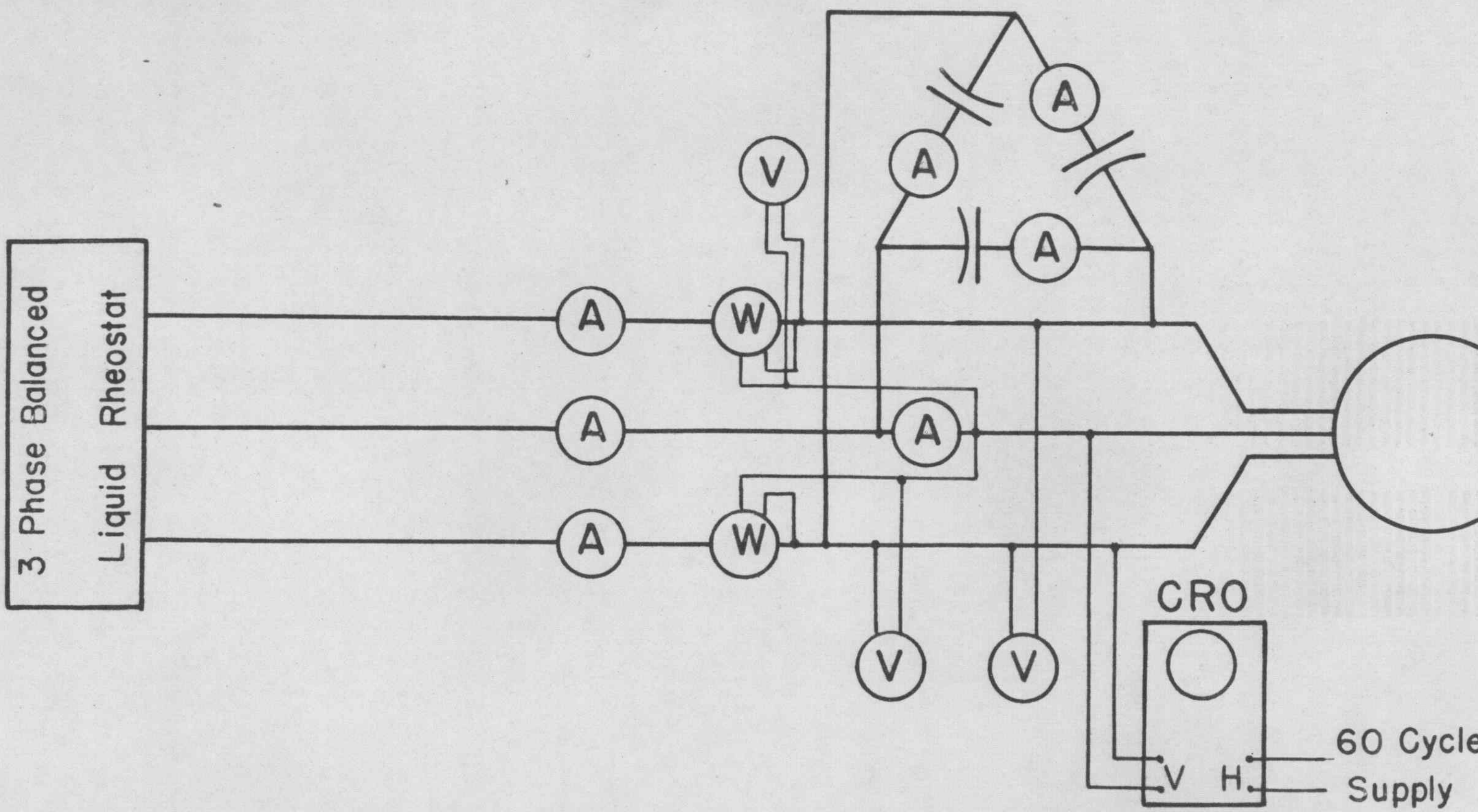


Fig. 6. Circuit Diagram, Generator Load Test, Shunt Excitation.

held absolutely constant. Readings of all instruments and slip were recorded. Wave shapes of phase voltage, line voltage, and line current were projected on the oscilloscope. It was noted that at lower voltages phase voltage was almost a perfect sine wave, and at higher voltages developed a third harmonic due to the magnetizing current. The line voltage was almost a perfect sine wave at all voltages, as the third harmonic was eliminated by the wye connection. Due to the uniform air gap, the machine acts somewhat as a cylindrical-rotor alternator, and has a good wave form. The line current showed some third harmonics, particularly at higher terminal voltages, due to the third harmonic in the magnetizing current. This agrees with other measurements (8, p.50).

It has been suggested (1, p.543) that the increase in magnetizing current necessary with increased load could be supplied by the use of series capacitors. Shunt capacitors would still be necessary, in order to maintain the terminal voltage at zero load. As load is increased the series capacitors change the phase angle and tend to compensate for drop in terminal voltage. The proper value of series capacitance to use (1, p.543) is a value that has a reactance approximately equal to that of the machine windings. An attempt was made at compounding, using the arrangement shown in Fig. 7. Sufficient

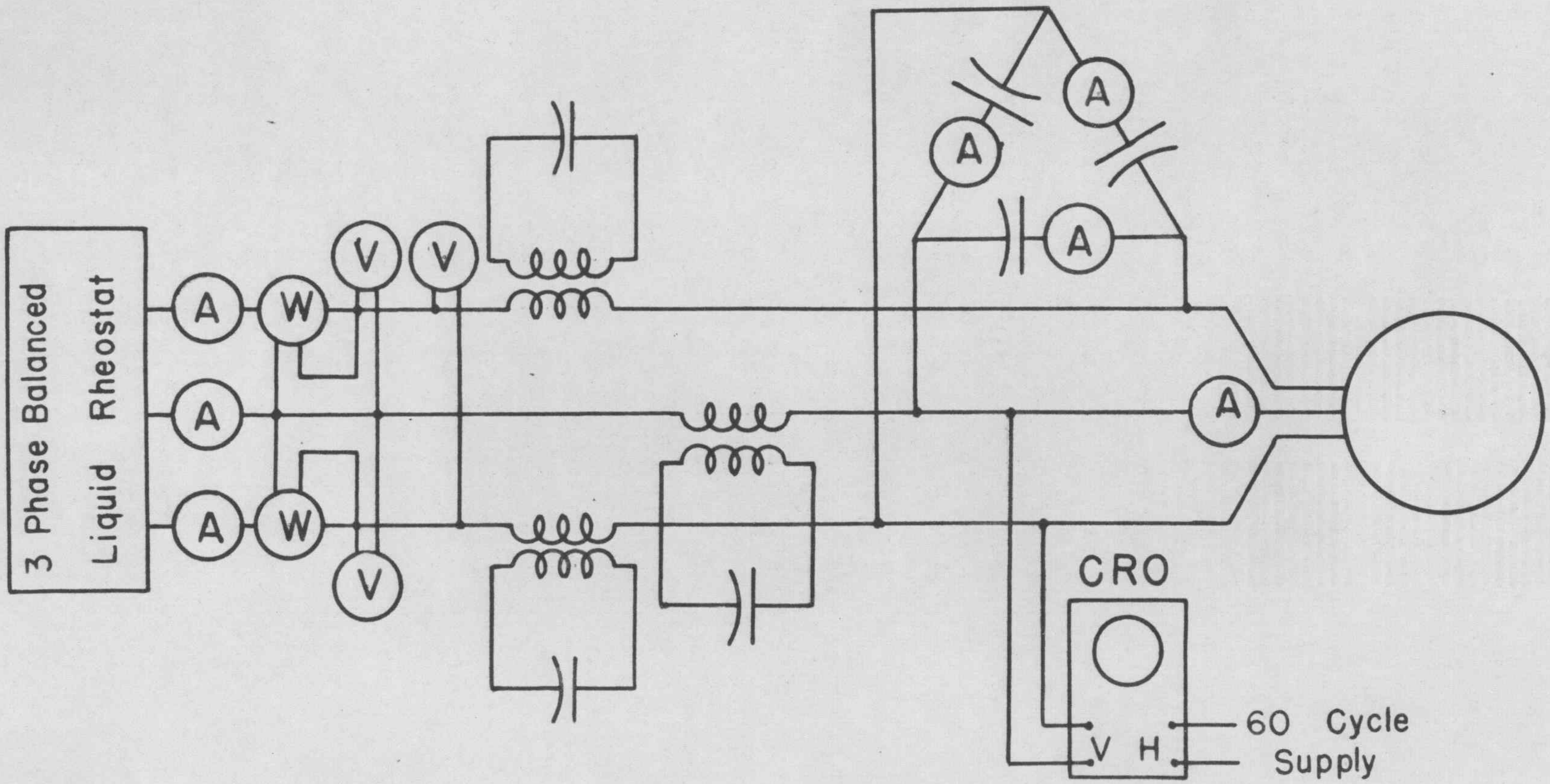


Fig. 7. Circuit Diagram, Generator Load Test, Compound Excitation.

capacitance was not available to insert directly, so series transformers were used. The boosting of voltage was observed, but the transformers didn't have sufficient capacity to go to more than 25% load. When the load was increased beyond this point, ferro-resonance resulted, with the resultant distortion of wave shape, and excessive vibration of the machine. The same results were obtained using auto-transformers of approximately three times the capacity, so no data were obtained. It is believed that this is due to lack of sufficient kva of capacitors. More capacitance is needed to get the same kva when series capacitors are used, due to the lower voltage across them. The use of series transformers would be feasible, except that they must have sufficient capacity so that they operate well below saturation.

Reports show (1, p.543) that a voltage regulation of about 15% can be obtained using compound excitation, but only for higher saturation in the generator. This voltage regulation is too large to be satisfactory in most cases, and it is felt that a control circuit for switching shunt capacitors would be necessary. For this reason, the tests and calculations were confined to shunt excitation.

RESULTS OF TESTS

Figures 8, 9, and 10 show the results of the various running-light tests. It will be noted that the shapes of the current curves are almost identical with the exception of the normal running-light test. The current curve turns upward at lower values of voltage. This is due to the fact that the friction and windage losses amount to an appreciable load at lower voltages, and the machine consequently has appreciable slip. The power curves of the incremental power test and the running-light test driven at synchronous speed have almost identical shape and values at any given voltage. The power curve of the normal running-light test has a different shape, tending to turn up more sharply at higher voltages. This is due to increased hysteresis and eddy current losses and some error. It will be shown later that this has little effect on the performance of the machine.

Table 1 of the Appendix shows the results of the blocked-rotor test. It will be noted that the impedance varies with varying applied voltage. This is due to changing conditions of saturation. The data taken at rated current were used in determining the equivalent circuit constants.

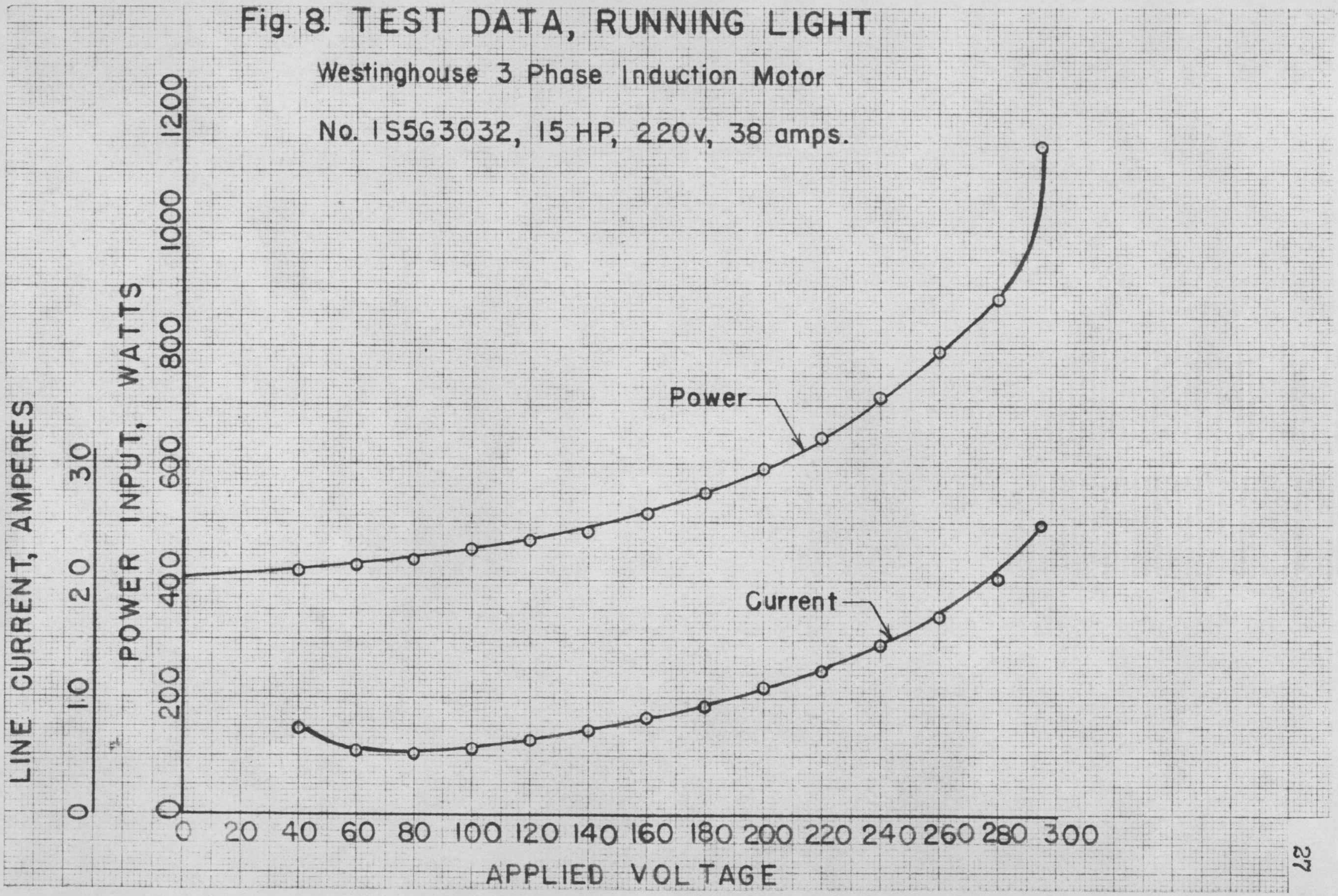


Fig. 8. TEST DATA, RUNNING LIGHT
Westinghouse 3 Phase Induction Motor
No. 1S5G3032, 15 HP, 220v, 38 amps.
LINE CURRENT, AMPERES
POWER INPUT, WATTS
APPLIED VOLTAGE
Power
Current
0
10
20
30
0
200
400
600
800
1000
1200
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300

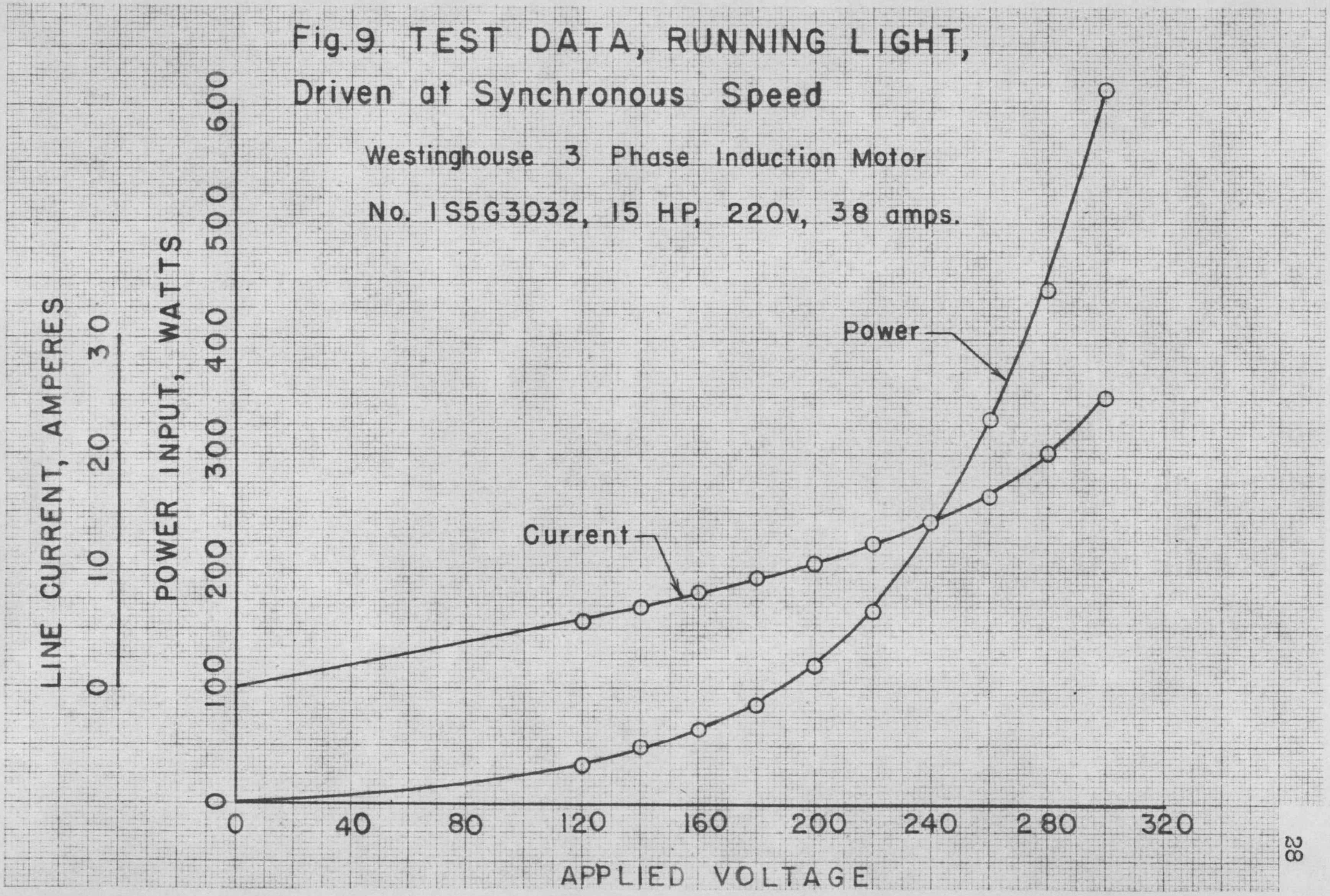


Fig. 9. TEST DATA, RUNNING LIGHT,
Driven at Synchronous Speed
Westinghouse 3 Phase Induction Motor
No. 1S5G3032, 15 HP, 220v, 38 amps.
LINE CURRENT, AMPERES
0
10
20
30
POWER INPUT, WATTS
0
100
200
300
400
500
600
Power
Current
0
40
80
120
160
200
240
280
320
APPLIED VOLTAGE

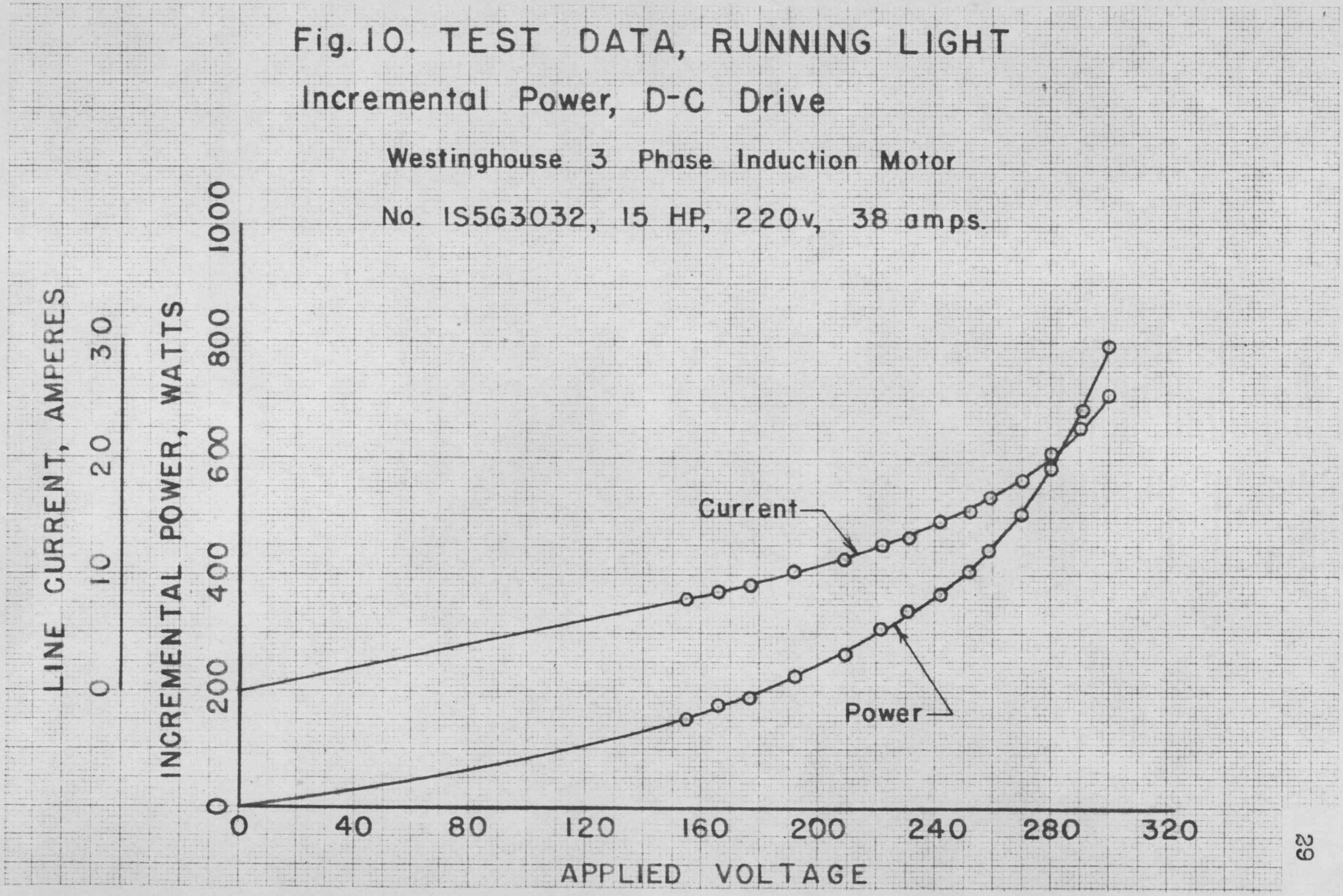


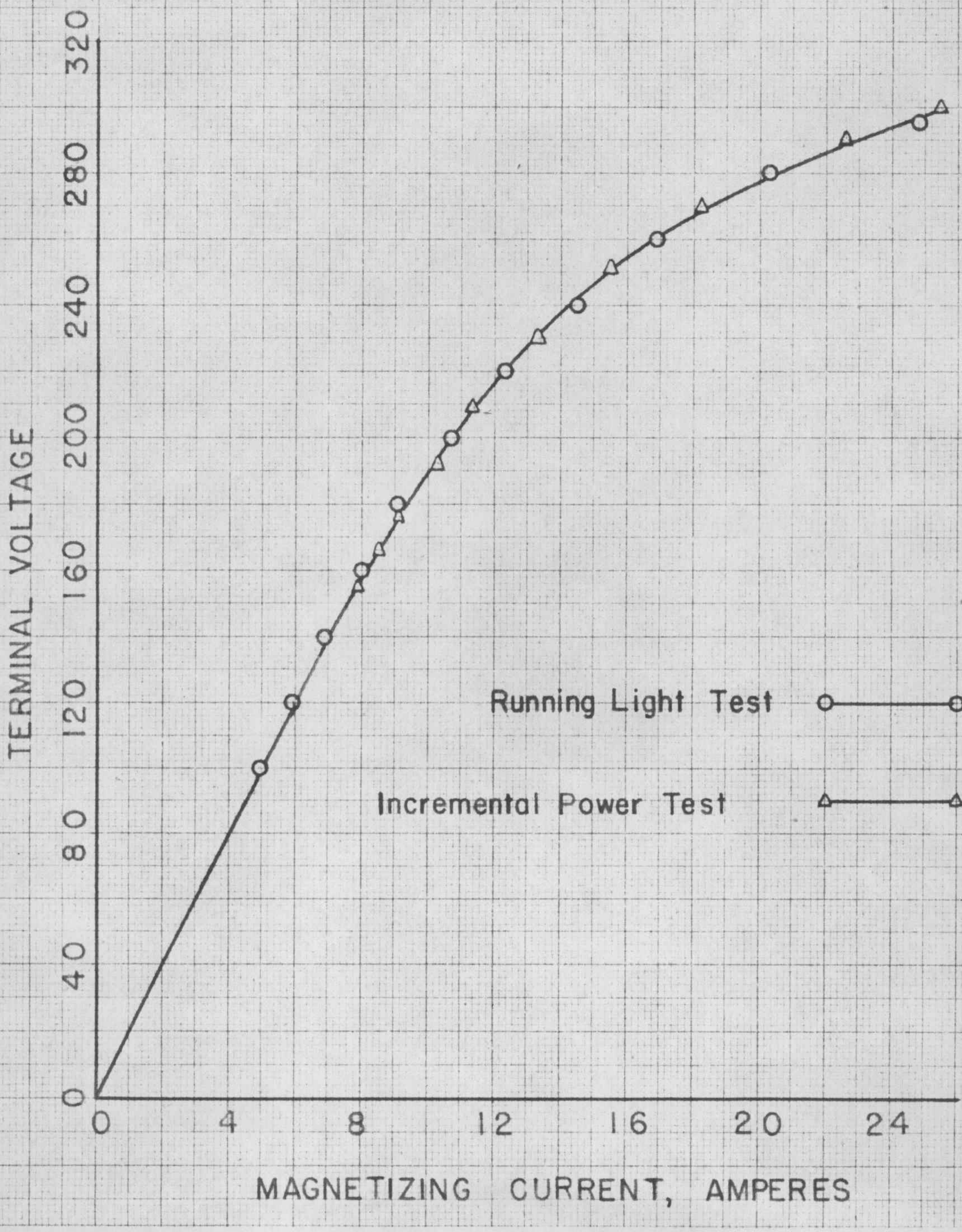
Fig. 10. TEST DATA, RUNNING LIGHT
Incremental Power, D-C Drive
Westinghouse 3 Phase Induction Motor
No. 1S5G3032, 15 HP, 220v, 38 amps.
LINE CURRENT, AMPERES
0
10
20
30
INCREMENTAL POWER, WATTS
0
200
400
600
800
1000
Current
Power
0
40
80
120
160
200
240
280
320
APPLIED VOLTAGE

Figure 11 shows the no load saturation curve with points calculated from the running-light test and the incremental power test. Results of the running-light test driven at synchronous speed are not shown, due to the fact that many of the points would coincide with the points plotted. If the voltage-current characteristics of any value of capacitance were plotted on the same scale, the result would be a straight line passing through zero, the slope of the line depending upon the value of capacitance. The value of capacitance necessary for any given terminal voltage may be found by plotting such a line on this figure. The intersection of this line with the no load saturation curve determines the no load voltage. It will be noted that if a value of capacitance is selected such that the line has exactly the same slope as the straight-line portion of the no load saturation curve, any number of terminal voltages are possible. Under such conditions the lowest stable terminal voltage would result, which would be zero. Any value of capacitance less than this would result in no intersection and hence no terminal voltage. Thus there is a critical value of capacitance which must be exceeded before the machine will build up a terminal voltage (1, p.541). It is recommended that the machine be operated well up on the saturation curve, so that any

Fig. 11. NO LOAD SATURATION CURVE

Westinghouse 3 Phase Induction Motor

No. 1S5G3032, 15 HP, 220v, 38 amps.



disturbance would not tend to bring the terminal voltage down to a point on the straight-line portion of the saturation curve, or the terminal voltage would drop to zero.

Figures 12 through 15 show the measured load characteristics of the machine, acting as a generator under capacitive excitation. Figures 12 and 13 show slip as a function of load. Figure 12 shows slip as a function of load component of current, not including the magnetizing current. At higher terminal voltages, and hence more saturation, the curves are closer together. This may be seen by the fact that only one line could be drawn for the 280 volt and 290 volt curves. It appears from this that at higher values of terminal voltage, a limit will be reached as to the amount of load current that can be drawn from the machine for a given value of slip. Figure 13 shows slip as a function of kw load on the machine. It will be noted that the curves at higher voltage are not so close together, the difference being that for any given value of load component of current the kw load will be different due to differing terminal voltages. Irregularities in the results are due to the inability to hold the conditions constant long enough to take accurate readings. It is believed, however, that the curves have the correct shape, that is, that slip is a straight line function of load over the range tested. The voltages used in

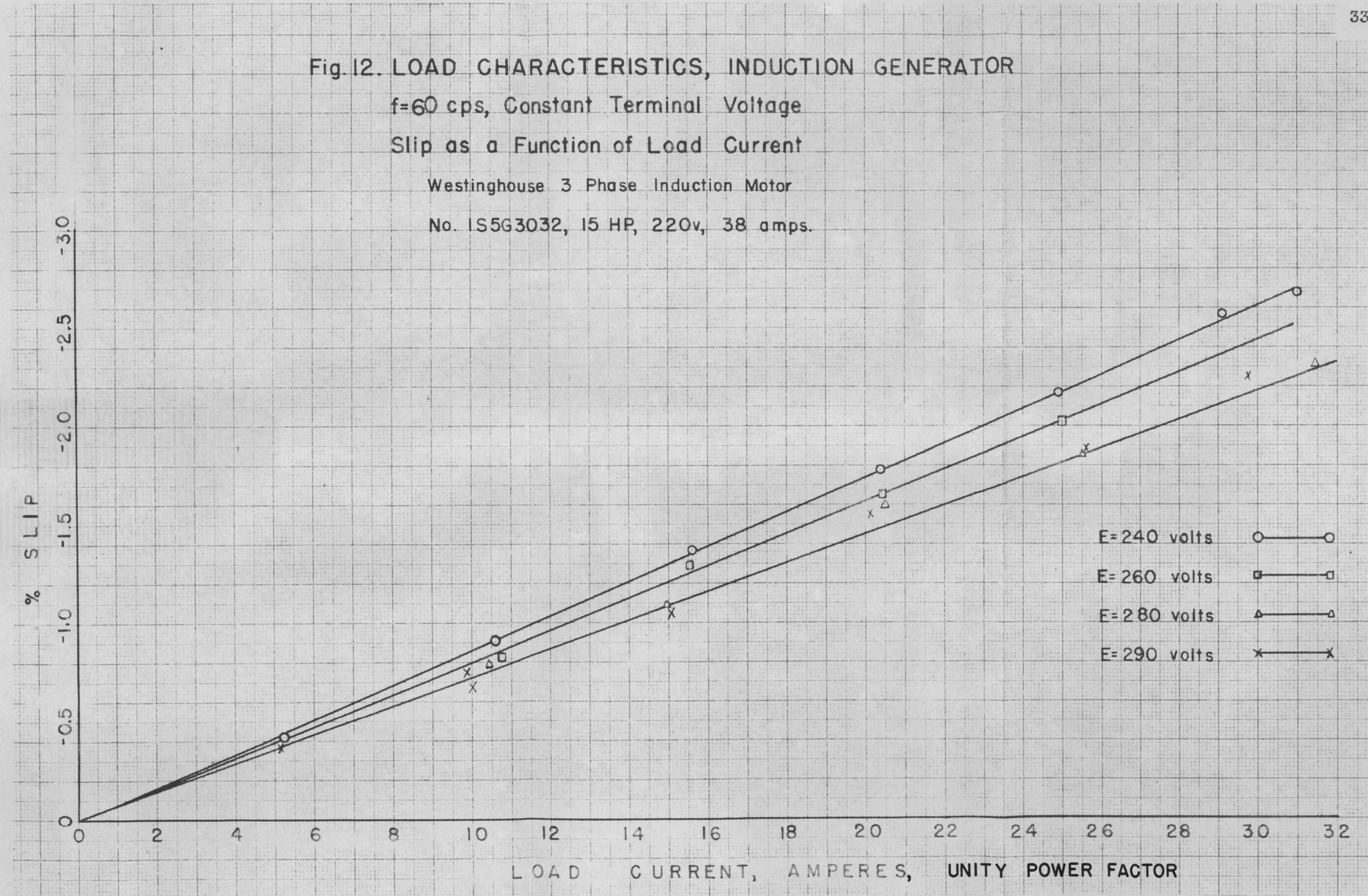


Fig. 12. LOAD CHARACTERISTICS, INDUCTION GENERATOR
f=60 cps, Constant Terminal Voltage
Slip as a Function of Load Current
Westinghouse 3 Phase Induction Motor
No. 1S5G3032, 15 HP, 220v, 38 amps.
% SLIP
0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
E=240 volts
E=260 volts
E=280 volts
E=290 volts
LOAD CURRENT, AMPERES, UNITY POWER FACTOR

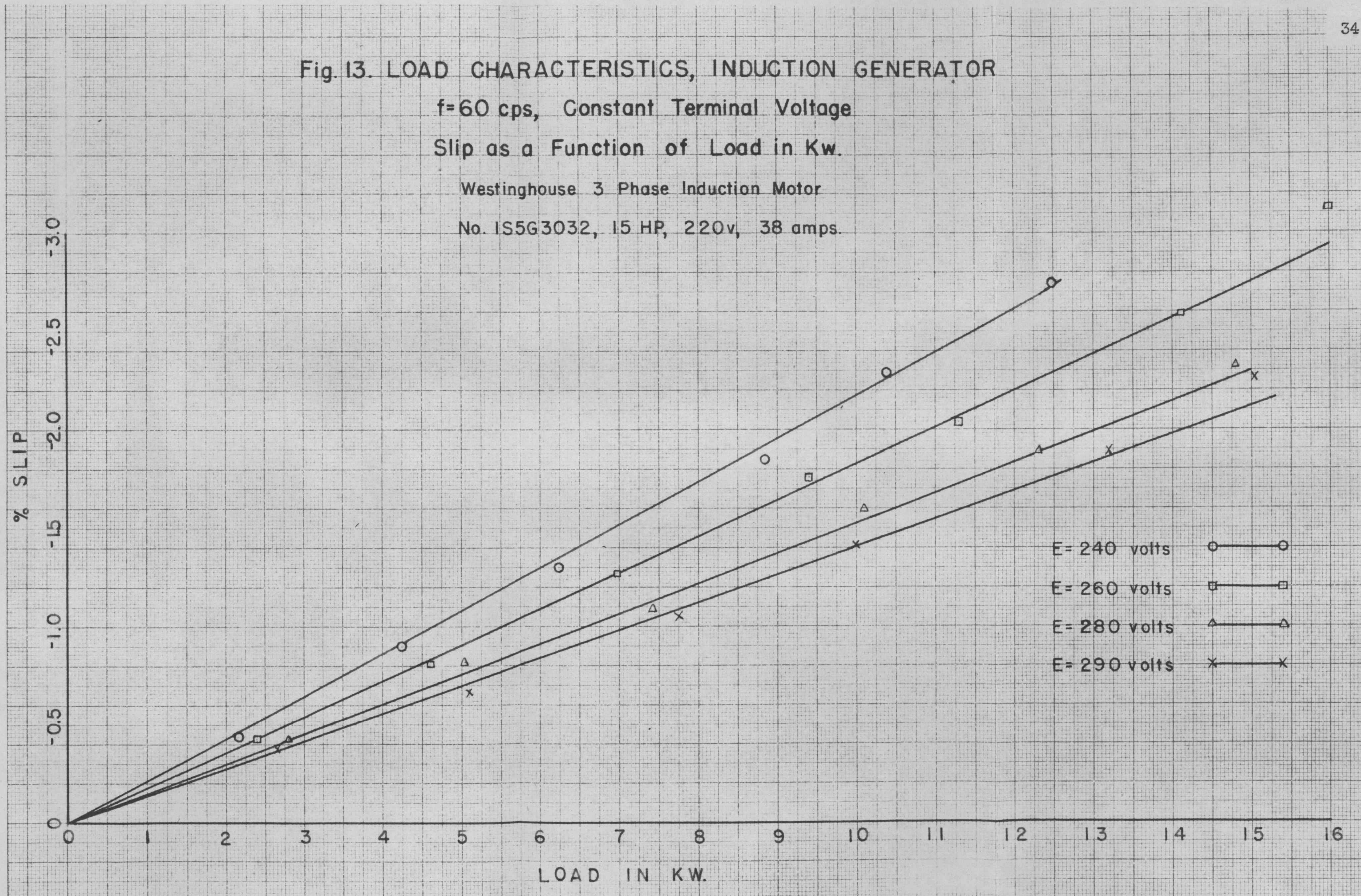


Fig. 13. LOAD CHARACTERISTICS, INDUCTION GENERATOR
f=60 cps, Constant Terminal Voltage
Slip as a Function of Load in Kw.
Westinghouse 3 Phase Induction Motor
No. 1S5G3032, 15 HP, 220v, 38 amps.
% SLIP
0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
LOAD IN KW.
E= 240 volts
E= 260 volts
E= 280 volts
E= 290 volts

conducting these tests were limited by two things. The lower limit is due to the tendency toward instability exhibited at the lower voltages, and the upper limit is due to lack of sufficient capacitance. It is believed that the load tests cover the practical voltage limits of the machine, with the possible exception that operation at 220 volts might be practicable.

Figures 14 and 15 show exciting current as a function of load current and load in kw, respectively. Results of this test were good. The exciting current as plotted is amperes per terminal. The actual magnetizing current measured was in the delta bank of capacitors as shown in Fig. 6. To obtain the values plotted it was necessary to multiply the values recorded in Table 5 of the Appendix by the square root of three. It will be noted that, for the lower voltages, when kw load is plotted against magnetizing current the excitation curves tend to approach each other at higher loads. This is due to the fact that when kw load is plotted, the same load may result from two different load currents at different terminal voltages.

It was noted that on short circuit the line current dropped rapidly to zero, due to loss of excitation. No short-circuit oscillograms were taken, as ample evidence has been presented (2, p.517) that the transient would be the same as for a cylindrical-rotor alternator with two exceptions. These exceptions are as follows. The initial surge of current would be large, due to the discharge of the capacitors in the first one-half cycle and the low subtransient reactance. The final value of short-circuit current would be zero.

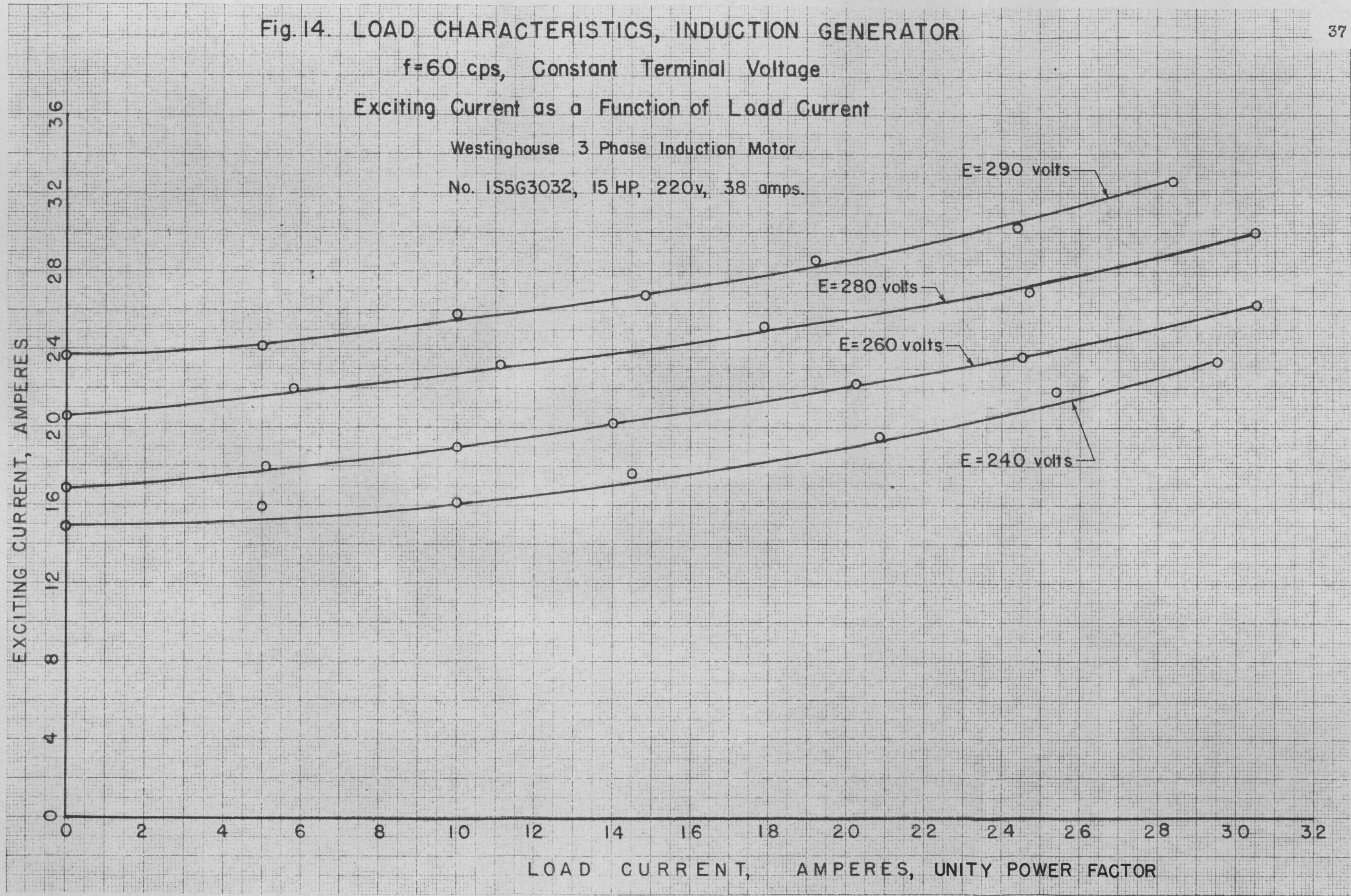


Fig. 14. LOAD CHARACTERISTICS, INDUCTION GENERATOR
f=60 cps, Constant Terminal Voltage
Exciting Current as a Function of Load Current
Westinghouse 3 Phase Induction Motor
No. 1S5G3032, 15 HP, 220v, 38 amps.
E=290 volts
E=280 volts
E=260 volts
E=240 volts
EXCITING CURRENT, AMPERES
LOAD CURRENT, AMPERES, UNITY POWER FACTOR
0
4
8
12
16
20
24
28
32
36
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32

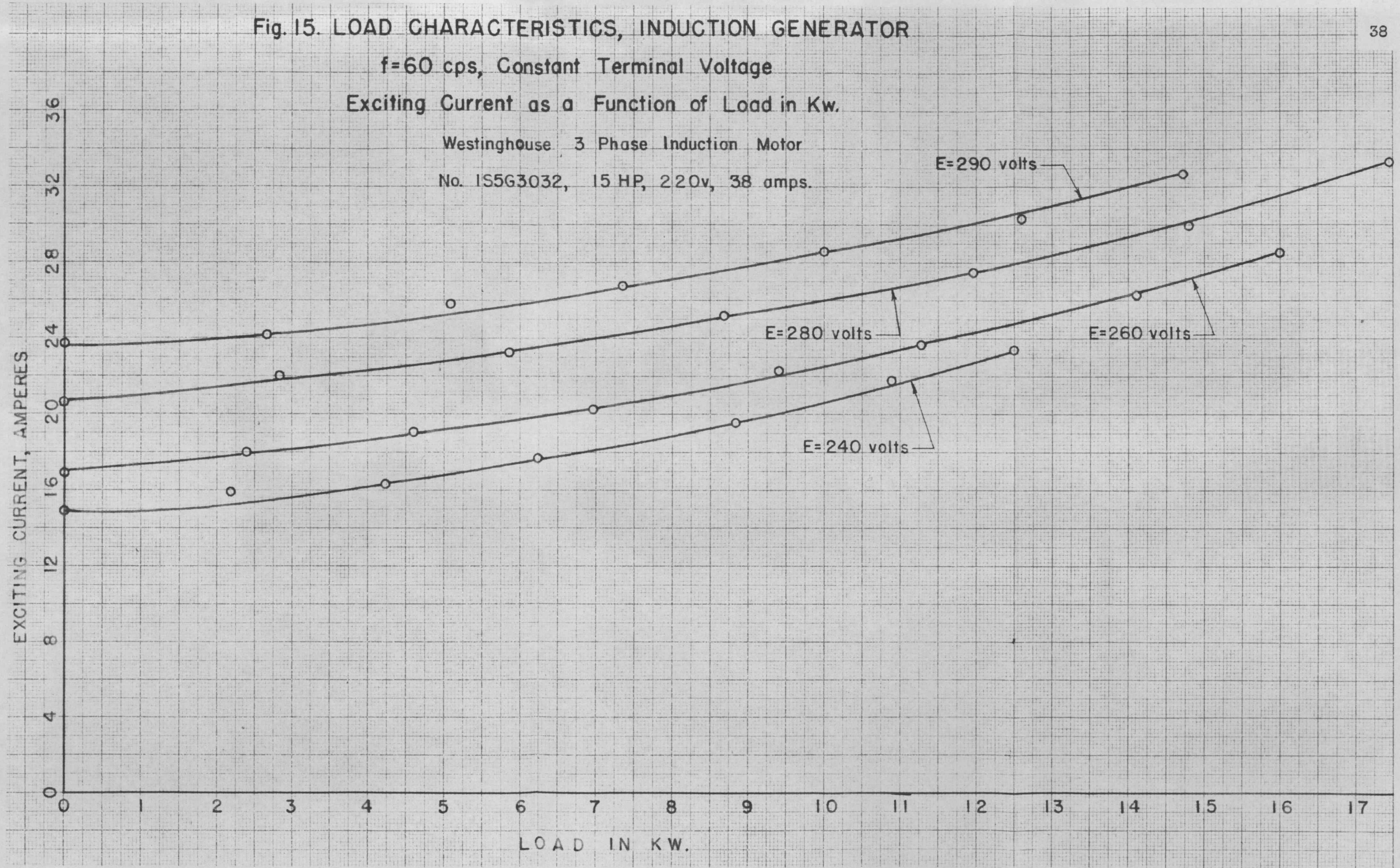


Fig. 15. LOAD CHARACTERISTICS, INDUCTION GENERATOR
f=60 cps, Constant Terminal Voltage
Exciting Current as a Function of Load in Kw.
Westinghouse 3 Phase Induction Motor
No. 1S5G3032, 15 HP, 220v, 38 amps.
E=290 volts
E=280 volts
E=260 volts
E=240 volts
EXCITING CURRENT, AMPERES
LOAD IN KW.
0
4
8
12
16
20
24
28
32
36
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

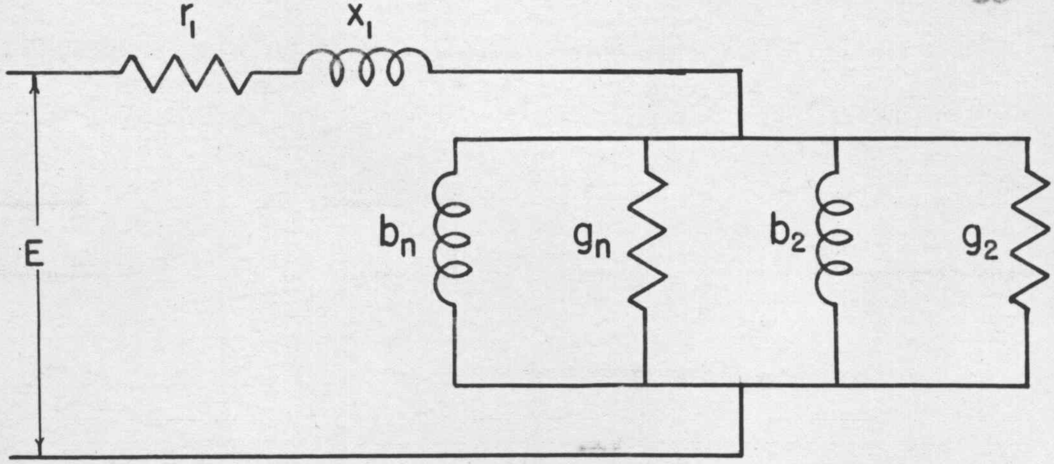


Fig. 16. Equivalent Circuit of Induction Machine Showing Secondary Admittance.

CALCULATION OF GENERATOR PERFORMANCE FROM EQUIVALENT CIRCUIT

In order to calculate the performance of the induction generator from the equivalent circuit, the equivalent circuit is arranged as shown in Fig. 16. The secondary part of the circuit is considered as being made up of an admittance, b_2 and g_2. The constants in the equivalent circuit are determined for the usual equivalent circuit shown in Fig. 2. The equivalent circuit must be treated as an impedance network, as it contains negative resistance, which makes it impossible to write enough independent equations for an analytical solution. A value of slip and terminal voltage are assumed, and the corresponding value of vector current is calculated. Since the network is treated as an impedance, and it is actually a source of power, the terminal voltage will be in opposition to that assumed, and the value of terminal voltage must be considered negative. The derivation of the equations for calculating performance follows.

$$Z_2 = \frac{r_2}{s} + jx_2$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{\frac{r_2}{s} + jx_2} = \frac{s}{r_2 + jsx_2} \cdot \frac{r_2 - jsx_2}{r_2 - jsx_2}$$

$$= \frac{r_2 s - js^2x_2}{r_2^2 + s^2x_2^2}$$

$$g_2 = \frac{r_2 s}{r_2^2 + s^2x_2^2}, \quad b_2 = \frac{-js^2x_2}{r_2^2 + s^2x_2^2}$$

$$-E = I\left\{r_1 + jx_1 + \frac{1}{g_n + \frac{r_2 s}{r_2^2 + s^2x_2^2} - j\left[b_n + \frac{s^2x_2}{r_2^2 + s^2x_2^2}\right]}\right\}$$

$$= I\left\{r_1 + jx_1 + \frac{r_2 + s^2x_2^2}{g_n(r_2^2 + s^2x_2^2) + r_2 s - j\left[b_n(r_2^2 + s^2x_2^2) + s^2x_2\right]}\right\}$$

$$\text{Let } g_n(r_2^2 + s^2x_2^2) + r_2 s = A \qquad (3)$$

$$\text{and } b_n(r_2^2 + s^2x_2^2) + s^2x_2 = B \qquad (4)$$

$$-E = I\left\{r_1 + jx_1 + \frac{r_2^2 + s^2x_2^2}{A - jB} \cdot \frac{A + jB}{A + jB}\right\}$$

$$= I\left\{\frac{(r_1 + jx_1)(A^2 + B^2)}{A^2 + B^2} + \frac{(r_2^2 + s^2x_2^2)(A + jB)}{A^2 + B^2}\right\}$$

$$I = \frac{-E(A^2 + B^2)}{(r_1 + jx_1)(A^2 + B^2) + (A + jB)(r_2^2 + s^2x_2^2)}$$

$$I = \frac{-E(A^2 + B^2)}{r_1(A^2 + B^2) + A(r_2^2 + s^2x_2^2) + j[x_1(A^2 + B^2) + B(r_2^2 + s^2x_2^2)]}$$

$$\text{let } r_1(A^2 + B^2) + A(r_2^2 + s^2x_2^2) = D \qquad (5)$$

$$\text{and } x_1(A^2 + B^2) + B(r_2^2 + s^2x_2^2) = F \qquad (6)$$

$$\text{then } I = \frac{-E(A^2 + B^2)(D - jF)}{(D + jF)(D - jF)}$$

$$I = \frac{-E(A^2 + B^2)}{D^2 + F^2} \cdot (D - jF) \qquad (7)$$

Using the constants A, B, C, and D as defined by Equations (3), (4), (5), and (6) with Equation (7), vector values of current may be calculated for varying values of slip and terminal voltage.

It would be possible to substitute a known value of slip, terminal voltage and vector current in the above equations and solve for r_2. The value of r_2 thus determined would be the value to use for calculation of performance. This was not done, as it would be very laborious to solve these equations for r_2. Rather, the proper value of r_2 to use was found by trial and error, substituting in the above equations. It was assumed that the value of $r_1 + r_2$ from the blocked-rotor test remained constant, and it was determined that the correct value of r_2 for very close agreement with the 240 volt measured data was 0.109 ohms. The primary resistance used was the difference between these values or 0.200 ohms. For accuracy g_n should be recalculated taking into account the difference in I^2R loss due to the change in primary resistance. This was not done, as the value of g_n had very little effect on the performance of the machine. In fact, some forms of the equivalent circuit (4, p.700) consider the exciting branch to be made up of only a magnetizing reactance. Using Equations (3), (4), (5), (6), and (7) the calculated generator characteristics shown in Figures 17 and 18 were determined. It will be noted that at zero load the slip was not zero. Actually, if zero slip is substituted into the equations, a negative load component of current results. The power represented by this current is supplied by the prime mover. In conducting the load tests, it was noted that a small slip was necessary at no load to maintain frequency and supply losses, but this slip was too small to be measured accurately, and hence was recorded as zero. As expected, saturation tends to make the curves of slip as a function of load current closer together, but seems to have little effect on the curves of slip as a function of load in kw. It will be noted that these calculated curves agree quite closely with the measured curves.

The calculation of exciting current was somewhat more difficult than the calculation of load current. Referring to Fig. 19, curve 1 shows the magnetizing current calculated.

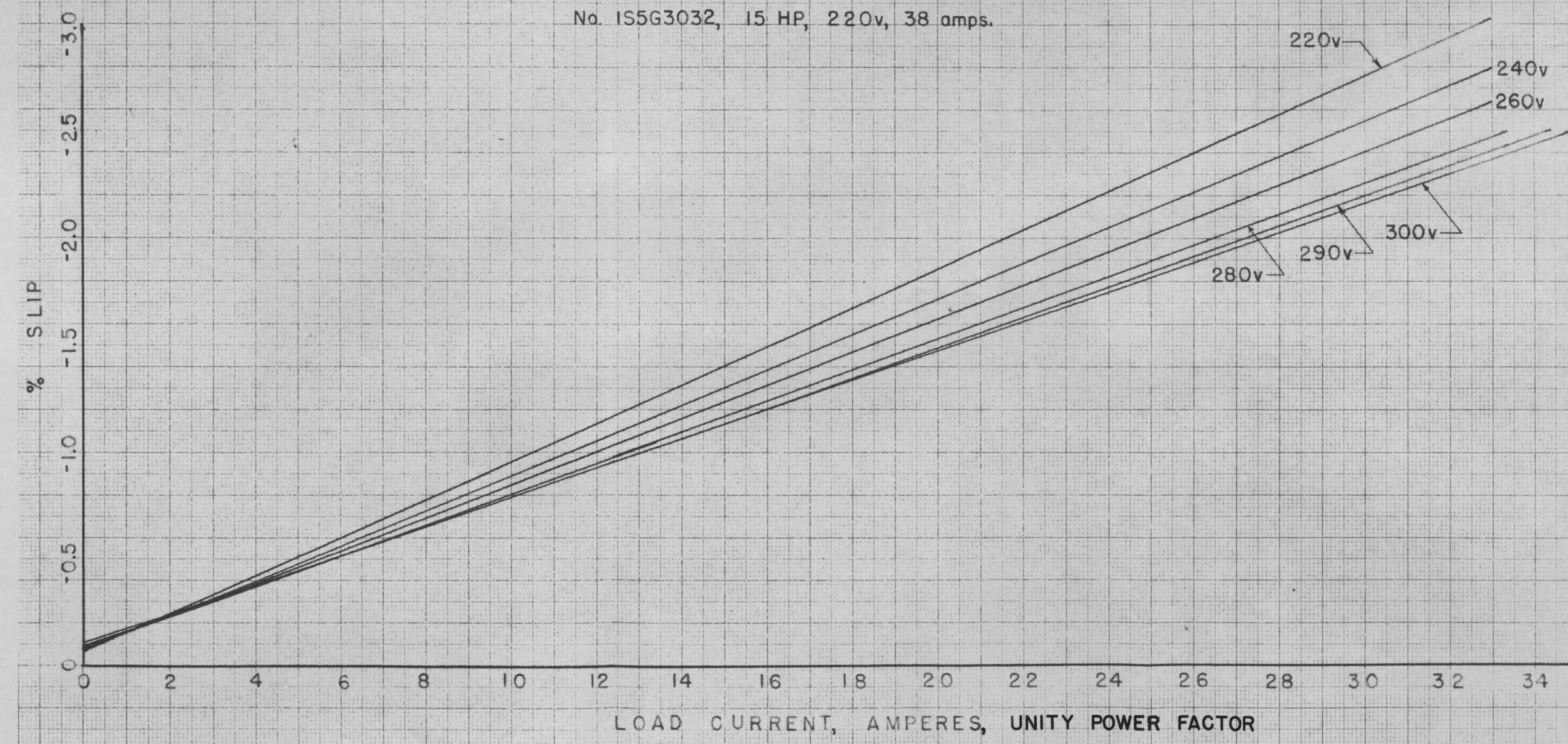
Fig. 17. CALCULATED GENERATOR CHARACTERISTICS

f = 60 cps, Constant Terminal Voltage

Slip as a Function of Load Current

Westinghouse 3 Phase Induction Motor

No. IS5G3032, 15 HP, 220v, 38 amps.



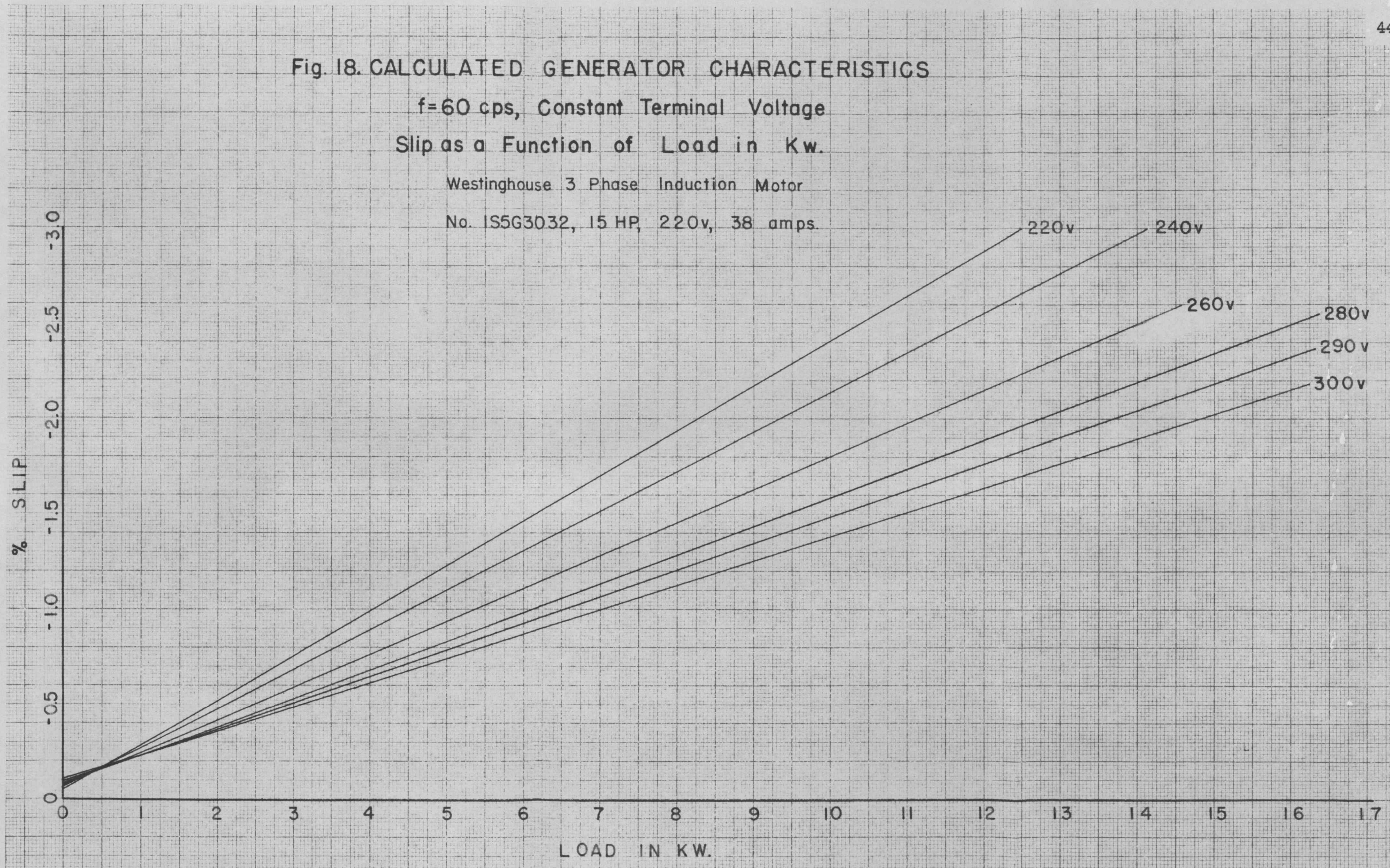


Fig. 18. CALCULATED GENERATOR CHARACTERISTICS
f=60 cps, Constant Terminal Voltage
Slip as a Function of Load in Kw.
Westinghouse 3 Phase Induction Motor
No. IS5G3032, 15 HP, 220v, 38 amps.
220v
240v
260v
280v
290v
300v
% SLIP
-3.0
-2.5
-2.0
-1.5
-1.0
-0.5
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
LOAD IN KW.

using the normal value of b_n measured at a terminal voltage of 280 volts. The correction of this value for primary impedance drop was made. It was found that, since the power factor on the running-light test was low, negligible error resulted if the voltage were corrected for reactance drop only. Curve 2, Fig. 19 shows the result of this calculation. It will be noted that this method of calculation still gives results that are about 13% too low for higher values of load. There was some uncertainty as to whether or not the correct values of x_1 and x_2 were being used in the calculations, as the blocked-rotor test showed that the value of $x_1 + x_2$ varied appreciably with applied voltage. Test calculations were made, using values of x_1 25% greater and 25% less, and the difference in load current and magnetizing current was negligible. The same results were observed when the value of x_2 was varied over the same range. Test calculations were also made with x_1 decreased and x_2 increased and with x_1 increased and x_2 decreased, and no appreciable difference in the calculated vector current was observed. When x_1 and x_2 were increased or decreased simultaneously, still no appreciable difference in the calculated vector current was observed. Hence, the only constant that could change with load to account for this increased magnetizing current was b_n. When the machine is loaded, the slip must be increased to hold constant frequency. When the slip is increased, a greater current of higher frequency flows in the rotor, and sets up a flux in opposition to the flux set up by the stator. The flux penetration of the rotor is decreased, and hence more of the flux is forced into the higher reluctance path offered by the air gap as the slip increases. This is enhanced by tooth saturation. This is analogous to armature reaction in synchronous machines. As the rotor flux increases in direct proportion to the slip, it is believed that, in order to obtain results that agree with measured values, the value of b_n used should increase in proportion to the slip. Until saturation in the rotor is reached the increase in b_n with slip will probably not be linear, but somewhat less. However, it seems reasonable to use values of b_n proportional to slip as a first approximation in calculating the magnetizing current. To make this clearer, an example of a specific calculation follows. For a terminal voltage of 280 volts, the adjusted value of b_n is 0.1325 mhos. This takes into account the primary reactance drop. Under blocked-rotor conditions the machine had a total reactance of 0.676 ohms. This is at 100% slip and it is assumed, for purposes of finding a method of calculation, that at 100% negative slip the reactance would be the same. This assumption, of course, is not true, but

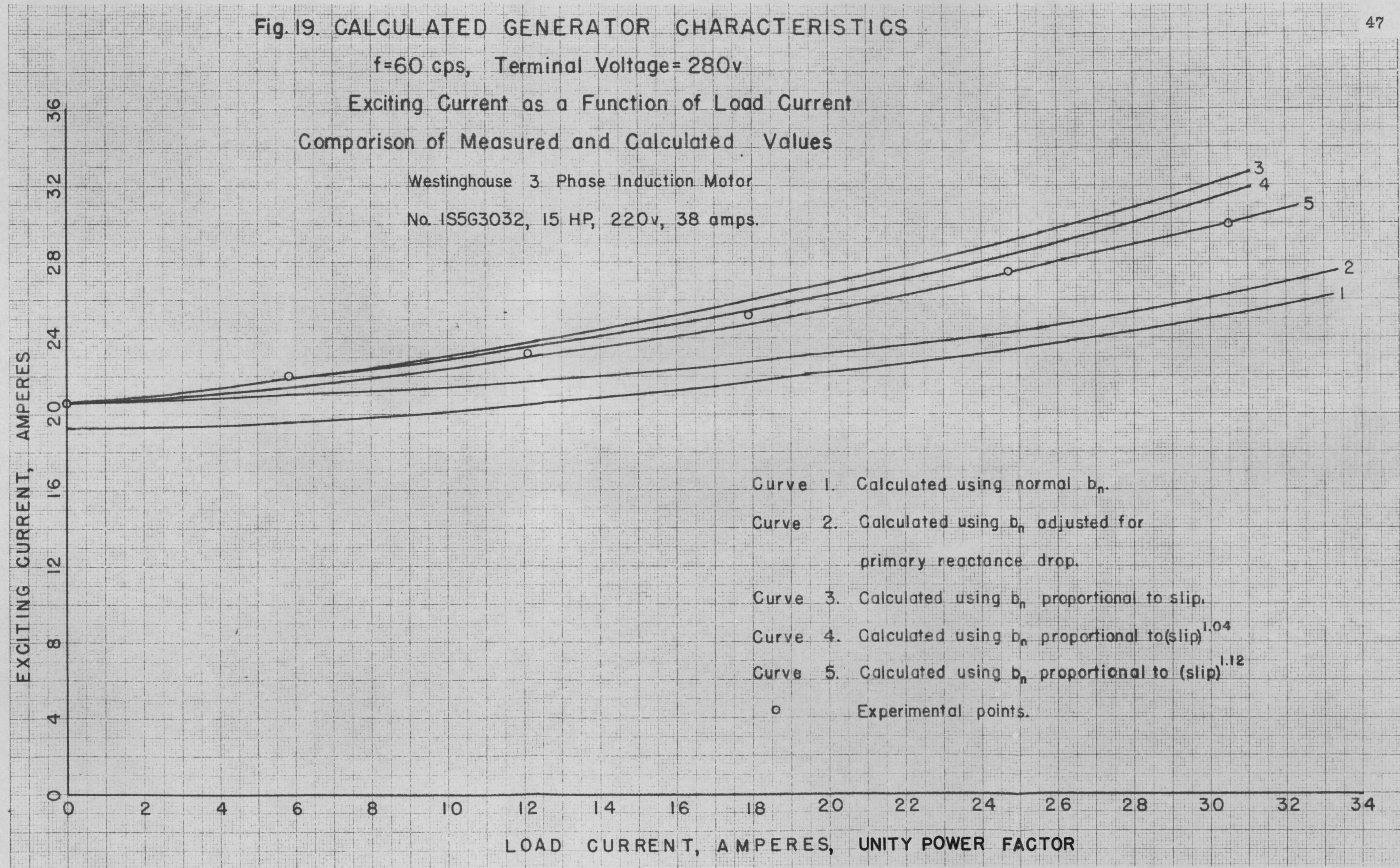


Fig. 19. CALCULATED GENERATOR CHARACTERISTICS
f=60 cps, Terminal Voltage= 280v
Exciting Current as a Function of Load Current
Comparison of Measured and Calculated Values
Westinghouse 3 Phase Induction Motor
No. IS5G3032, 15 HP, 220v, 38 amps.
Curve 1. Calculated using normal b_n.
Curve 2. Calculated using b_n adjusted for primary reactance drop.
Curve 3. Calculated using b_n proportional to slip.
Curve 4. Calculated using b_n proportional to $(slip)^{1.04}$
Curve 5. Calculated using b_n proportional to $(slip)^{1.12}$
o Experimental points.
EXCITING CURRENT, AMPERES
LOAD CURRENT, AMPERES, UNITY POWER FACTOR
0
4
8
12
16
20
24
28
32
36
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
1
2
3
4
5

the machine would never be operated at 100% negative slip. No tests were made at a slip of -100%, as that would require driving the machine at twice synchronous speed, and this would not be in the safe operating range. The value of susceptance corresponding to this value of reactance is its reciprocal, or 1.48 mhos. It is assumed that as the slip increases from zero towards -100%, the value of b_n increases linearly from 0.1325 mhos to 1.48 mhos. This gives the equation for b_n as a function of slip as follows.

$$b_n = 0.1325 + (1.347)(-\text{slip}) \qquad (8)$$

Using this equation, the magnetizing characteristic shown in Curve 3, Fig. 19 resulted. It is seen that this curve is about 8% too high at higher values of load.

As a second approximation, the magnetizing characteristic was calculated using b_n proportional to the 1.04 power of the absolute value of the slip. The equation relating b_n and slip in this case is shown in Equation (9).

$$b_n = 0.1325 + (1.347)(-\text{slip})^{1.04} \qquad (9)$$

The exponent 1.04 was chosen at random, since it was believed that this would give results closer to the measured values. The result of this calculation is shown in Curve 4, Fig. 19. The error at higher loads is still about 5%.

The last calculation was made using b_n proportional to the 1.12 power of the slip. This resulted in Equation (10).

$$b_n = 0.1325 + (1.347)(-\text{slip})^{1.12} \qquad (10)$$

This exponent was determined by finding a value of b_n that would give results agreeing very closely with the measured values at -2.5% slip. A general equation was then written, with its boundary conditions, Equation (11), and the value of b_n substituted, and the exponent of the slip determined. This is shown as follows.

$$b_n = K_1 + K_2(-s)^x \qquad (11)$$

When $s = 0$, $b_n = 0.1325$. When slip = -100%, $b_n = 1.48$. When slip = -2.5%, $b_n = 0.1540$.

When these values were substituted in Equation (11), the exponent 1.12 and Equation (10) resulted. Results of this calculation are shown in Curve 5, Fig. 19. It will be noted that very good agreement at higher values of slip results, but that at lower values the results are too low. It is believed that this error is partly due to inaccurate measurements at lower loads. Since it is intended that these curves be used to estimate the amount of capacitance necessary for excitation, it is believed that these results are of sufficient accuracy.

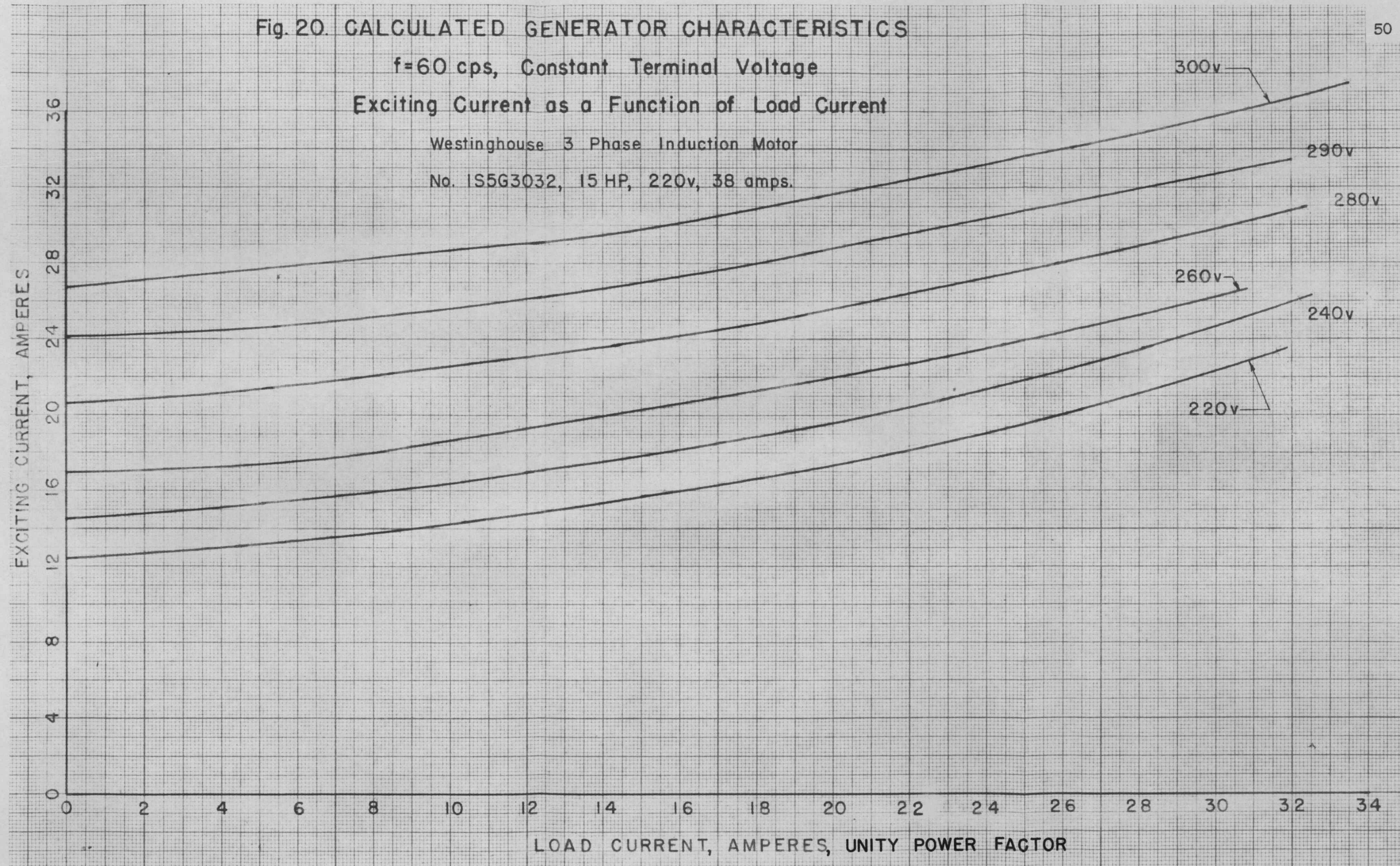


Fig. 20. CALCULATED GENERATOR CHARACTERISTICS
f=60 cps, Constant Terminal Voltage
Exciting Current as a Function of Load Current
Westinghouse 3 Phase Induction Motor
No. IS5G3032, 15 HP, 220v, 38 amps.
300v
290v
280v
260v
240v
220v
EXCITING CURRENT, AMPERES
0
4
8
12
16
20
24
28
32
36
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
LOAD CURRENT, AMPERES, UNITY POWER FACTOR

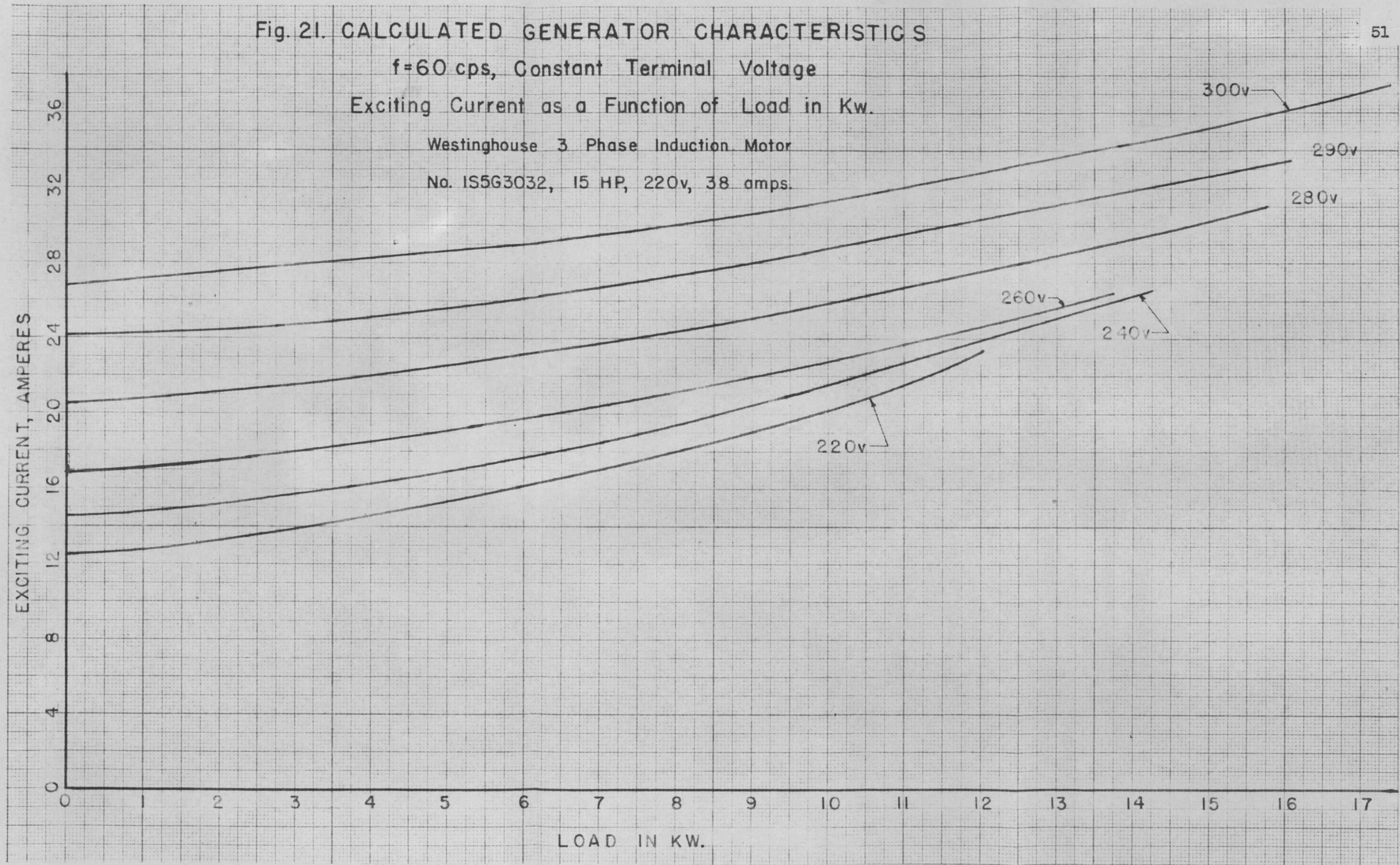


Fig. 21. CALCULATED GENERATOR CHARACTERISTICS
f=60 cps, Constant Terminal Voltage
Exciting Current as a Function of Load in Kw.
Westinghouse 3 Phase Induction Motor
No. IS5G3032, 15 HP, 220v, 38 amps.
300v
290v
280v
260v
240v
220v
EXCITING CURRENT, AMPERES
LOAD IN KW.
0
4
8
12
16
20
24
28
32
36
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

Using this method of calculation, the magnetizing characteristics shown in Figures 20 and 21 were calculated. It was assumed in all cases that the value of b_n at -100% slip would be 1.48, regardless of the value of terminal voltage. The value of b_n at zero slip was taken as the value resulting from the running-light test, corrected for primary reactance drop. It may be noted that the results of this calculation for 240, 260, and 290 volts at higher values of load agree within about 1 or 2% with the measured values.

It was noted in these calculations that slightly different values of power output resulted due to the changes in b_n and magnetizing current. This is due to two things. The change in I^2R loss and the very slight change in induced voltage, E_1 and E_2, due to the change in b_n. The change was of the order of 1 or 2%. Thus it can be seen that the most important constants in the equivalent circuit are b_n and r_2, and fairly close approximations on the other constants will result in little error. The magnetizing current is essentially dependent only upon b_n, and the load current upon r_2.

Power factor as a function of load is shown in Figures 22 and 23. These are calculated values based on the values of power shown in Figures 17 and 18, and upon the values of magnetizing current shown in Figures 20 and

21. These curves should agree accurately with measured values, as the values of in-phase and out-of-phase current agree with measured values. It will be noted that the effect of higher voltage or saturation is to lower the power factor. Apparently, there is a crowding of the flux to higher reluctance paths accompanied by tooth saturation, and the magnetizing current increases at a greater rate than the load current.

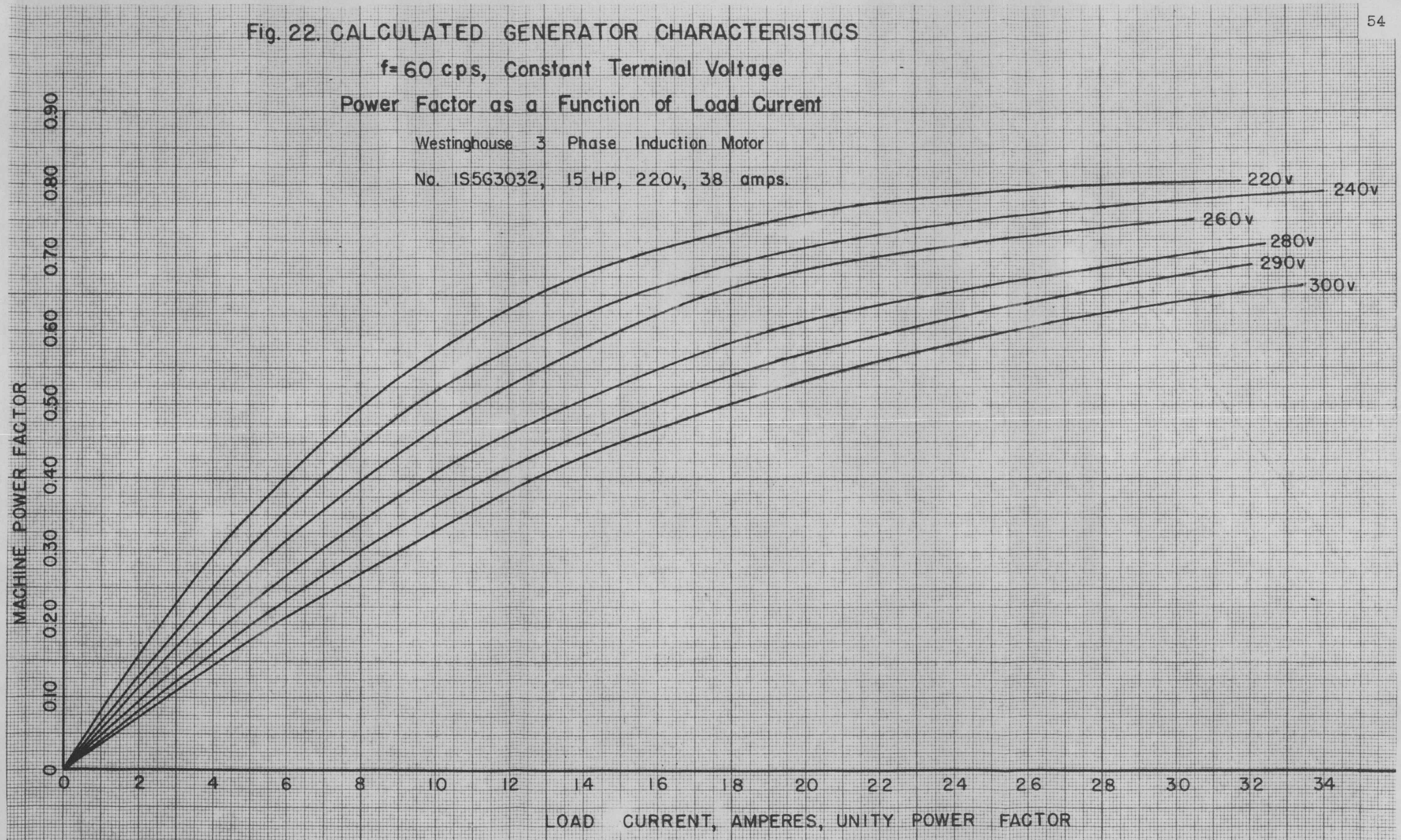


Fig. 22. CALCULATED GENERATOR CHARACTERISTICS
f=60 cps, Constant Terminal Voltage
Power Factor as a Function of Load Current
Westinghouse 3 Phase Induction Motor
No. IS5G3032, 15 HP, 220v, 38 amps.
220v
240v
260v
280v
290v
300v
MACHINE POWER FACTOR
0
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34
LOAD CURRENT, AMPERES, UNITY POWER FACTOR

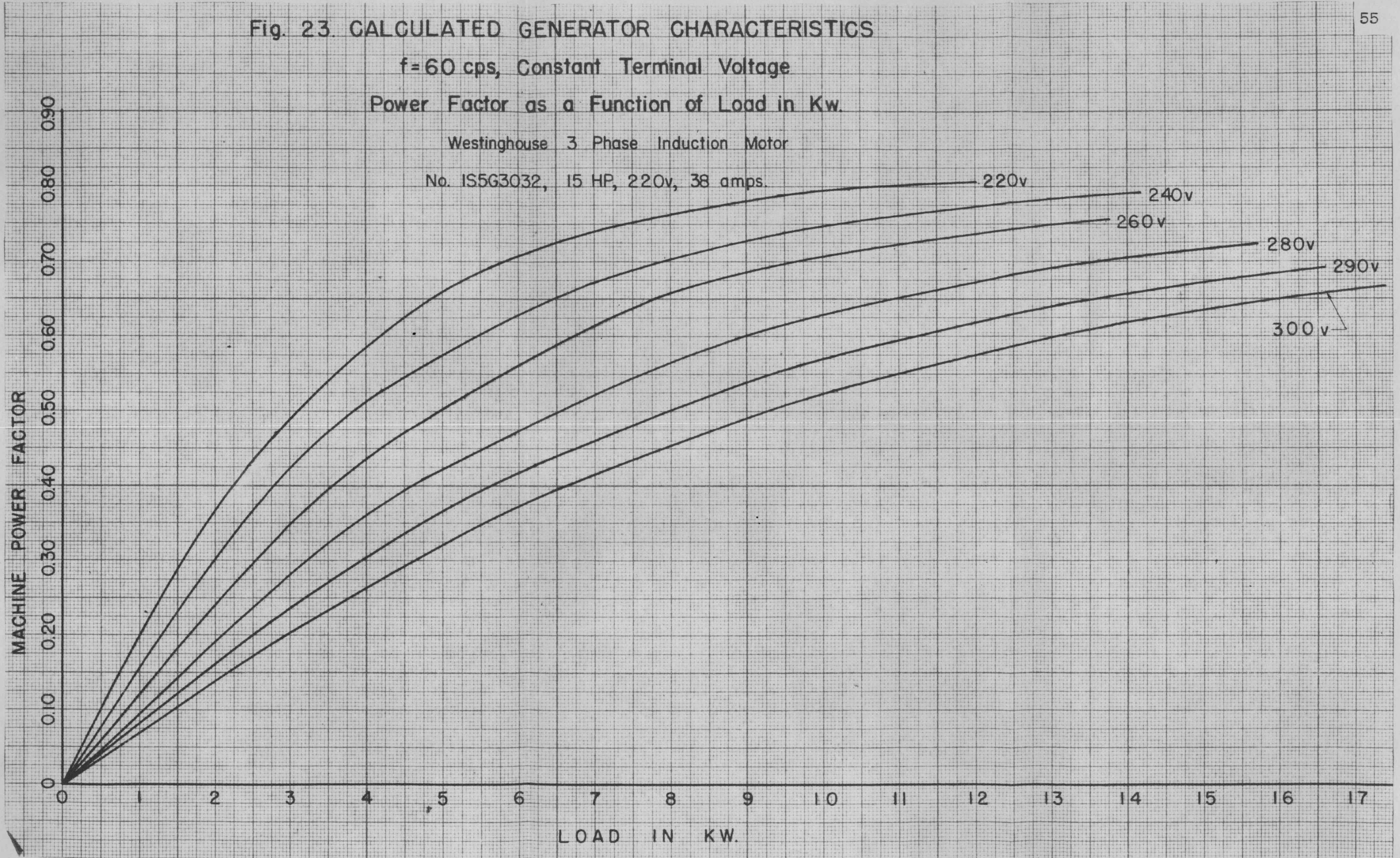


Fig. 23. CALCULATED GENERATOR CHARACTERISTICS
f=60 cps, Constant Terminal Voltage
Power Factor as a Function of Load in Kw.
Westinghouse 3 Phase Induction Motor
No. 15G3032, 15 HP, 220v, 38 amps.
220v
240v
260v
280v
290v
300v
MACHINE POWER FACTOR
0
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
LOAD IN KW.

CONCLUSIONS

It has been shown how the load characteristics of an induction generator may be calculated using an empirical method. Using a constant value of b_n for calculating the magnetizing current of a motor is accurate enough for practical purposes, but this does not apply to a capacitor-excited induction generator. It has been shown (6, Fig. 202, p.316) that the same machine acting as a motor and as a generator with synchronous excitation does not have the same exciting current. For the same absolute value of slip, the machine requires a greater magnetizing current when operating as a generator. It has been shown that the only difference between motor and generator action is that the rotor current reflects into the stator leading instead of lagging, due to the rotor speed being greater than the speed of revolution of the rotating field. The load current has a demagnetizing action which must be balanced by a larger magnetizing current. This is similar to armature reaction in synchronous machines.

It has also been shown (3, Fig. 14, p.1733) that increasing the air gap length of an induction machine decreases its power factor. It is believed that the effect on the equivalent circuit would be to increase the no load value of b_n, but that the value of the empirical exponent of slip in Equation (10) would not be changed a great deal by change in air gap length. It is believed that an empirical exponent of approximately the same magnitude could be used for all machines.

It has become evident from the various investigations that there is no difference in the performance of an induction generator with capacitor excitation and one with synchronous excitation, except that the wave form of a capacitor-excited generator is always very good, and the wave form of a synchronously-excited generator will be determined by the wave form of the synchronous apparatus in parallel. If there are any harmonics of appreciable magnitude present, they may cause some differences in characteristics, due to the harmonics present in the revolving flux, although the presence of the induction machine will improve the wave shape.

By comparison of the same machine operating as a motor and a synchronously-excited generator (6, Fig. 202, p.316), the following differences are observed, for the same absolute value of slip. For generator action, the magnetizing current is larger, the efficiency is greater, the load component of current is larger, but the power factor is less. All this, except for the increased magnetizing current, can be explained by the fact that the losses of the machine are supplied by the prime mover, and not from current flowing through the machine.

In cases where a load test is not feasible, the load characteristics for design purposes for generator applications may be determined using the methods contained herein, and the normal blocked-rotor test, and any one of the three running-light tests, taken under the proper conditions. It is also necessary to know the value of rotor resistance to use in the equivalent circuit. This may be determined from either a motor or generator load test, taken accurately at reduced voltage and load, or from the design data of the machine if that is available. If a motor test is used to determine the rotor resistance, it is necessary to know the friction and windage losses of the machine, so that the power due to them will not be used in determining the rotor resistance.

The only design change recommended for capacitor-excited generator applications is operation at higher saturation for standard voltages than is customary for motor operation. This would entail reducing the amount of iron in the machine, making it cheaper, except for production-line motors to be used as generators. In such cases, it might be cheaper to make changes in transformer ratios, or elsewhere on the system, depending upon the application. Reducing the iron would reduce the efficiency slightly, but this is of minor importance, since the change in efficiency is small.

It is recommended that capacitor-excited induction generators for constant-frequency constant-voltage operation be operated with shunt excitation. The use of compound excitation would improve the natural voltage regulation, but not to such an extent as to do away with the control circuits. There is no economic advantage in using shunt excitation, since the same kva of excitation is needed, regardless of the type of excitation. The use of series capacitors would require a spark gap or some other protection, as the machine voltage would rise to a high value on short circuit, due to the added excitation if not protected.

The effect of varying load power factor on a capacitor-excited generator is to cause variations in terminal voltage. A leading power factor load tends to raise the terminal voltage and a lagging power factor load tends to lower the terminal voltage. This is due to the fact that leading and lagging power factor loads increase and decrease the excitation, respectively. This disadvantage could be overcome by having sufficient capacity in the capacitor banks, and having an automatic voltage regulator. At present most power systems operate with loads of high power factor, so that excessive reactive kva in the capacitor banks would not be necessary.

It has been noted that for constant-frequency operation it is necessary to operate the prime mover at varying speed. This would present little difficulty, as the variation in speed would seldom exceed 4%. The only difficulty would be encountered in case of a short circuit. Since the terminal voltage and load drop to zero on short circuit, the prime mover with a naturally drooping speed characteristic would tend to speed up, but this would be counteracted by the governor. In case of governor failure, a centrifugal device to shut down the prime mover would offer adequate protection. Induction machines are inherently more rugged and able to stand overspeed without damage than are salient-pole alternators, especially in the large low-speed sizes.

It is believed that the largest application for capacitor-excited induction generators is in the smaller sizes, in isolated locations or on portable units.

It is also believed that under present conditions operation of capacitor-excited induction generators might be practicable, if the necessary work of designing and building of suitable control equipment can be done economically.

BIBLIOGRAPHY

1. Bassett, E. D. and Potter, F. M. Capacitive excitation for induction generators. Electrical engineering 54:540-545, May 1935.

2. Doherty, R. E. and Williamson, E. F. Short-circuit currents in induction motors and generators. Transactions, American institute of electrical engineers 40:509-523, 1921.

3. Hobart, H. M. and Knowlton, Archer E. The squirrel cage induction generator. Transactions, American institute of electrical engineers 31:1721-1749, 1912.

4. Knowlton, Archer E., Editor-in-Chief. Standard handbook for electrical engineers. 7th ed. New York, The McGraw-Hill book co., inc., 1941. 2303p.

5. Lawrence, Ralph R. Principles of alternating-current machinery. 3d ed. New York, The McGraw-Hill book co., inc., 1941. 678p.

6. Puchstein, Albert F. and Lloyd, Tom C. Alternating-current machines. 2d ed. New York, John Wiley and Sons, inc., 1942. 648p.

7. Ripley, C. M. Induction generator plants. General Electric review 22:963-967, Nov. 1919.

8. Wagner, C. F. Self-excitation of induction motors. Transactions, American institute of electrical engineers 58:47-51, Feb. 1939.

APPENDIX

MEASURED AND CALCULATED DATA
for machine described on p. 67

Table 1. Data, Blocked-Rotor Test.

Line Current, Amperes	Phase Voltage	Power per Phase, Watts	r_1+r_2, Ohms	x_1+x_2, Ohms
19	16.55v	123.2	0.338	0.798
38	28.3	449	0.309	0.676
48	34.5	720	0.322	0.648
57.3	41.3	1106.7	0.336	0.637

Primary resistance per phase, measured with direct current = 0.143 ohms.

Table 2. Data, Running-Light Test.

Line Voltage	Phase Voltage	Line Current	Power Input	Corrected Power Input	b_n, Mhos	g_n, Mhos
295v	170v	24.9a	1240w	1144w	0.1454	0.00550
280	162	20.4	960	882	0.1258	0.00388
260	150	17.0	860	793	0.1125	0.00398
240	138.5	14.6	770	713	0.1046	0.00385
220	127.0	12.4	690	642	0.0970	0.00366
200	115.5	10.8	632	592	0.0925	0.00355
180	104	9.3	600	550	0.0865	0.00348
160	92.4	8.25	570	513	0.0866	0.00328
140	80.8	7.2	513	483	0.0854	0.00310
120	69.3	6.3	495	466	0.0850	0.00340
100	57.7	5.6	446	432	0.0860	0.00300

Table 3. Data, Running-Light Test, Driven at Synchronous Speed.

Line Voltage	Phase Voltage	Line Current	Power Input	Corrected Power Input	b_n, Mhos	g_n, Mhos
300v	173.2v	25.0a	704w	616w	0.1440	0.00367
280	162	20.2	520	442	0.1246	0.00318
260	150	16.5	398	331	0.1097	0.00297
240	138.5	14.3	300	243	0.1030	0.00255
220	127.0	12.4	215	167	0.0974	0.00118
200	115.5	10.7	160	120	0.0926	0.00165
180	104.0	9.4	118	86	0.0905	0.00138
160	92.4	8.1	90	65	0.0877	0.00141
140	80.8	6.8	68	49	0.0842	0.00138
120	69.3	5.7	46	32	0.0824	0.00126

Table 4. Data, Running Light, Incremental Power, d-c Drive.

Line Voltage	Phase Voltage	Line Current	Power Input	Corrected Power Input	b_n, Mhos	g_n, Mhos
300v	173.2v	25.5a	848w	797w	0.1470	0.00520
291	168	22.6	734	686	0.1342	0.00530
280	162	20.6	630	585.6	0.1270	0.00496
270	156	18.2	546	505	0.1165	0.00482
259	149.6	16.68	485	447	0.1113	0.00473
252	145.5	15.55	441	405	0.1067	0.00461
242	140.0	14.55	402	369	0.1042	0.00462
231	133.4	13.34	369	339	0.1000	0.00480
222	128.2	12.65	338	310	0.0985	0.00477
209	120.8	11.42	289	264	0.0944	0.00467
192	111.0	10.30	248	227	0.0927	0.00484
177	102.3	9.18	209	191	0.0897	0.00487
166	95.9	8.55	194	178	0.0890	0.00525
155	89.5	7.95	166	153	0.0888	0.00515

Table 5. Data, Load Tests, Shunt Excitation.

Power Output	% Slip	Load Current	Exciting Current	Power Output	% Slip	Load Current	Exciting Current
	E = 240 volts				E = 260 volts		
0 kw	0	0a	8.3a	0 kw	0	0a	9.8a
2.17	-0.44	5.0	9.2	2.38	-0.42	5.08	10.4
4.24	-0.90	10.0	9.4	4.60	-0.81	10.0	11.0
6.24	-1.30	14.5	10.2	6.96	-1.27	15.0	11.7
8.84	-1.85	20.8	11.3	9.40	-1.76	20.25	12.9
10.88	-2.29	25.4	12.6	11.28	-2.04	24.5	13.7
12.48	-2.75	29.5	13.5	14.12	-2.59	30.5	15.2
				16.00	-3.13	34.7	16.5
	E = 280 volts				E = 290 volts		
0 kw	0	0a	11.9a	0 kw	0	0a	13.7a
2.83	-0.50	5.84	12.7	2.66	-0.375	5.0	14.1
5.87	-0.94	12.1	13.4	5.10	-0.667	10.0	14.9
8.68	-1.42	17.9	14.55	7.86	-0.866	14.85	15.5
11.97	-1.94	24.7	15.9	10.00	-1.515	19.20	16.5
14.8	-2.35	30.5	17.3	12.60	-1.94	24.4	17.5
17.4	-2.68	36.0	19.2	14.72	-2.26	29.4	18.9

Table 6. Data, No Load Saturation Curve.

Incremental Power Test		Running-Light Test	
Terminal Voltage	Magnetizing Current	Terminal Voltage	Magnetizing Current
300 v	25.45 amps.	295 v	24.8 amps.
291	22.56	280	20.3
280	20.57	260	16.9
270	18.18	240	14.5
259	16.65	220	12.3
252	15.52	200	10.67
242	14.53	180	9.1
231	13.31	160	8.0
222	12.62	140	6.9
209	11.38	120	5.9
192	10.28	100	4.97
177	9.16		
166	8.52		
155	7.95		

Table 7. Calculated Values of b_n, 280 Volts.

Slip	b_n Corrected For Primary X Drop	b_n Prop. to Slip	b_n Prop. to $(-slip)^{1.04}$	b_n Prop. to $(-slip)^{1.12}$
-0.000	0.1325	0.1325	0.1325	0.1325
-0.005	0.1325	0.1392	0.1379	0.1361
-0.010	0.1325	0.1460	0.1437	0.1402
-0.015	0.1325	0.1527	0.1496	0.1447
-0.020	0.1325	0.1594	0.1554	0.1496
-0.025	0.1325	0.1662	0.1604	0.1540

Table 8. Calculated Values of b_n Proportional to $(-slip)^{1.12}$

Slip	b_n for 220 v	b_n for 240 v	b_n for 260 v	b_n for 290 v	b_n for 300 v
-0.000	0.1010	0.1080	0.1166	0.1480	0.1607
-0.005	0.1047	0.1117	0.1203	0.1516	0.1643
-0.010	0.1089	0.1158	0.1244	0.1556	0.1682
-0.015	0.1136	0.1205	0.1290	0.1600	0.1727
-0.020	0.1187	0.1254	0.1339	0.1649	0.1773
-0.025	0.1231	0.1300	0.1384	0.1693	0.1818
-0.030	0.1280	0.1352	0.1352		

Table 9. Data, Calculated Load Characteristics, 280 Volts, Various Methods of Calculation.

Slip	Normal b_n	b_n Corrected For Primary X Drop	b_n Proportional To Slip
-0.000	-1.17 + j19.33	-1.20 + j20.62	-1.23 + j20.42
-0.005	5.74 + j19.60	5.86 + j21.10	5.57 + j21.85
-0.010	12.5 + j20.60	12.5 + j21.70	12.2 + j23.9
-0.015	19.4 + j22.10	19.42 + j23.3	18.96 + j26.45
-0.020	26.34 + j23.81	25.6 + j24.5	25.6 + j29.35
-0.025	33.25 + j26.22	33.4 + j27.5	32.1 + j32.7

Slip	b_n Proportional To $(-\text{slip})^{1.04}$	b_n Proportional To $(-\text{slip})^{1.12}$
-0.000	-1.23 + j20.42	-1.23 + j20.42
-0.005	5.53 + j21.85	5.46 + j21.4
-0.010	12.25 + j23.60	12.28 + j23.2
-0.015	18.90 + j25.90	19.05 + j25.2
-0.020	25.55 + j28.80	25.84 + j27.95
-0.025	32.1 + j31.9	32.4 + j30.95

Table 10. Data, Calculated Load Characteristics, Vector Current as a Function of Slip, using b_n Proportional to $(-\text{slip})^{1.12}$

Slip	VECTOR CURRENT		
	220 v	240 v	260 v
-0.000	-0.68 + j12.38	-0.85 + j14.35	-1.02 + j16.83
-0.005	4.48 + j13.15	5.03 + j15.28	5.35 + j17.75
-0.010	10.4 + j14.41	10.95 + j16.65	11.72 + j19.28
-0.015	15.6 + j16.05	16.9 + j18.52	18.02 + j21.2
-0.020	20.2 + j18.12	22.8 + j20.8	24.4 + j23.7
-0.025	27.2 + j20.6	28.1 + j23.45	30.5 + j26.4
-0.030	31.6 + j23.4	34.1 + j26.4	

Slip	VECTOR CURRENT	
	290 v	300 v
-0.000	-1.50 + j24.1	-1.65 + j26.4
-0.005	5.44 + j24.6	5.54 + j27.8
-0.010	12.4 + j26.3	12.55 + j29.2
-0.015	19.3 + j28.5	19.6 + j31.5
-0.020	26.1 + j31.3	26.6 + j34.3
-0.025	32.0 + j33.5	33.5 + j37.5

Table 11. Values of g_n Used in Calculations.

Terminal Voltage	220	240	260	280	290	300
g_n, Mhos	0.0037	0.0043	0.0046	0.0049	0.0053	0.0055

NAME PLATE DATA, TEST MACHINE

Westinghouse Induction **Motor**

Frame:360

Phases: 3, Cycles: 60

Volts: **128-220**

Amps. per Terminal: 66-38

RPM at Full Load: 1150

Type CS, Constant Speed

Continuous Rating at **Full** Load: 40° C Rise

Style: 5G3032

Serial Number: 1S5G3032

SAMPLE CALCULATIONS

Calculation of $x_1 + x_2$ and $r_1 + r_2$ **From** Blocked-Rotor Test.

Line voltage = 49 volts, Phase voltage = 28.3 volts
Line Current = 38.0 amperes, Corrected power input = 449 w

$$\text{Power Factor} = \frac{449}{(28.3)(38)} = 0.418$$

$$\text{Reactive Factor} = 0.908$$

$$Z = \frac{E}{I} = \frac{28.3}{38} = 0.744$$

$$r_1 + r_2 = (0.744)(0.418) = 0.309 \text{ ohms}$$

$$x_1 + x_2 = (0.744)(0.908) = 0.676 \text{ ohms}$$

$$x_1 = x_2 = 0.338 \text{ ohms}$$

Calculation of b_n and g_n From Running-Light Test.

Line voltage = 240 volts, Phase voltage = 138.5 volts,
Line current = 14.6 amps., Corrected power input = 713 w,
Friction and windage power = 400 watts

Primary copper loss = $(14.6)^2(0.143)$ = 30.5 watts per phase

$$\text{Power Factor} = \frac{713}{(1.732)(240)(14.6)} = 0.1173$$

Reactive Factor = 0.995

Magnetizing Current = (0.995)(14.6) = 14.5 amps.

$$\text{Core Loss Power, per phase} = \frac{713-400}{3} - 30.5 = 73.8 \text{ watts}$$

$$\text{Core Loss Current} = \frac{73.8}{138.5} = 0.553 \text{ amps.}$$

$$g_n = \frac{0.553}{138.5} = 0.00385 \text{ mho}, \quad b_n = \frac{14.5}{138.5} = 0.1046 \text{ mho}$$

Calculation of Vector Current From Equivalent Circuit.

E = 240 volts, Phase voltage = 138.5 volts, slip = -0.010,
$x_1 = x_2 = 0.338$ ohms, $r_1 = 0.200$ ohms, $r_2 = 0.109$ ohms

$$b_n = 0.108 + (1.372)(-s)^{1.12} = 0.108 + (1.372)(0.01)^{1.12}$$
$$= 0.108 + 0.0078 = 0.1158$$

$$g_n = 0.0043, \quad r_2^2 = 0.01189, \quad s^2x_2^2 = (0.001)(0.338)^2$$
$$= 1.14 \times 10^{-5}$$

$$r_2^2 + s^2x_2^2 = 0.01190, \quad A = g_n(r_2^2 + s^2x_2^2) + r_2s$$
$$= (0.0043)(0.01189) +$$
$$(0.109)(-0.010)$$
$$= -10.4 \times 10^{-4}$$

$$B = b_n(r_2^2 + s^2x_2^2) + s^2\, x_2 = (0.1158)(0.01190) + 0.338 \times 10^{-4}$$

$$= 14.13 \times 10^{-4}$$

$$A^2 + B^2 = (1.08 + 2.00) \times 10^{-6}$$

$$= 3.08 \times 10^{-6}$$

$$D = r_1(A^2 + B^2) + A(r_2^2 + s^2\, x_2^2) = (0.200)(3.08) \times 10^{-6} + (-10.4 \times 10^{-4})(0.01190)$$

$$= -11.76 \times 10^{-6}$$

$$F = x_1(A^2 + B^2) + B(r_2^2 + s^2\, x_2^2) = (0.338)(3.08) \times 10^{-6} + (14.13 \times 10^{-4})(0.01190)$$

$$= 17.86 \times 10^{-6}$$

$$D^2 + F^2 = (1.38 + 3.19) \times 10^{-10}$$

$$= 4.57 \times 10^{-10}$$

$$I = \frac{(-E)(A^2 + B^2)}{D^2 + F^2}(-D - jF)$$

$$= \frac{(-138.5)(3.08 \times 10^{-6})}{4.57 \times 10^{-10}}(-11.76 - j17.86) \times 10^{-6}$$

$$\underline{I = 10.95 + j16.65}$$