

AN ABSTRACT OF THE THESIS OF

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Title: An Analysis of the Viability of Ranked Set Sample Methodologies Based on the Evaluation of Dyess AFB B-47 Crash Site Radiological Contamination Survey Data and Theoretical Modifications

Abstract approved: _____

Kathryn A. Higley

Since Ranked Set Sampling (RSS) was proposed in 1952 by G.A. McIntyre, it has been the subject of many statistical investigations and a large number of field studies. Throughout these studies RSS has been proven to provide a better estimate of population the mean and variance than Simple Random Sampling (SRS). Generally stated, RSS is the application of a screening methodology to evaluate a larger quantity of locations and then selectively choose those locations that would have more expensive and accurate sample analysis measurements. Specifically, its most commonly cited advantage is a more precise estimate of the mean if the same number of measurements is obtained using SRS. The most recent and publicly available methods for RSS use in radiological contamination evaluations have been accomplished by Oak Ridge Associated Universities (ORAU). Overall, RSS does not have extensive published literature regarding use or application for solely radiation clean up surveys. However, many literature articles and the US Environmental Protection Agency's guidance manuals on RSS often use a radiation contamination scenario as example where RSS could be useful compared to

SRS. The focus of this study is to utilize the Dyess Air Force Base B-47 crash site data as a framework for a retrospective analysis of RSS. Specifically, the considerations to be assessed are the following: 1) What are the effects on the precision and accuracy of RSS mean estimate compared to that of SRS, when considering the Dyess survey and theoretical modifications, 2) Is there an optimal means to apply RSS for this survey or other radiological contamination surveys and lastly, 3) Given the procedures for RSS, could the field survey techniques used in this survey be utilized as a screening tool for determining what soil samples should be used in this survey, or future surveys of a similar nature?

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An Analysis of the Viability of Ranked Set Sample Methodologies Based on the Evaluation of
Dyess AFB B-47 Crash Site Radiological Contamination Survey Data and Theoretical
Modifications

by

Vivien J. Miller

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Chapter 1: Introduction

1.1 Objectives

Ranked set sampling (RSS) is a sampling methodology proposed originally in 1952 by G.A. McIntyre, which since the 1960s has been primarily utilized in agricultural fields of study. Since then, a multitude of theoretical studies have evaluated statistical aspects of RSS. The primary benefit of RSS is a more precise estimation of the population mean. RSS is an unbiased estimator of the mean as is SRS, so both should be similarly accurate in estimating the mean for the same number of samples [14]. In some circumstances, RSS also is a more precise estimate of the population variance [19]. A review of the literature indicates that it was not until the 1990s when RSS became widely considered for environmental sampling applications, to include radiological contamination clean-up surveys [2]. For the past several years, RSS procedures have been utilized by Oak Ridge Associated Universities, and the incorporation of the procedure has been proposed in 2012 for the update to the Multi-Agency Radiation Survey and Site Investigation Manual (MARSSIM) [12]. MARSSIM details the primary methodologies for radiological environmental contamination surveys. ORAU's initial use of RSS was for independent verification and characterization surveys involving gamma emitting radionuclides of concern in soils as well as for characterization surveys for alpha, beta, and gamma emitters in structural surfaces when volumetric sampling was required. More recently, ORAU has developed a protocol for applying RSS for hard to detect alpha and beta emitters in soils [2], [21].

The basic premise behind RSS is the use of a low cost screening method that either qualitatively or quantitatively ranks observations for use in selection of the decision driving

sample measurements. For an environmental radiological survey, a quantitative screen methodology would be used (e.g. a handheld detector) to obtain the set(s) of data to be ranked. This ranking is then used to select specific points for the actual measurement to estimate certain aspects of the population, such as the mean. In the case of an environmental radiological survey, as discussed in this paper, the actual measurements are soil sample measurements. Since RSS was originally proposed, statisticians have published the results of a substantial amount of theoretical evaluations which indicate that this method will result in a more accurate estimate of the true mean and additional parameters of a distribution such as variance (and hence standard deviation), than the traditional method of simple random sampling (SRS). An additional advantage of RSS is that requires fewer samples with more rigorous and costly laboratory analysis than SRS to achieve the same level of accuracy [2].

This study considers the application of RSS retrospectively to a uranium contamination clean-up survey conducted by the US Air Force near Dyess Air Force Base, Texas. The contamination is not large or widespread over the area considered for clean-up, but the characterization survey did result in the need for remediation. The characterization survey consisted of three different techniques of field screening the area for radiological contamination and soil sample measurements [3]. In particular, the primary objectives of this study are to evaluate:

- 1) What are the effects on the precision and accuracy of RSS mean estimate compared to that of SRS, when considering the Dyess survey and theoretical modifications?

- 2) Is there an optimal means to apply RSS for this survey or other radiological contamination surveys?

3) Given the procedures for RSS, could the field survey techniques used in this survey be utilized as a screening tool for determining what soil samples should be used in this survey, or future surveys of a similar nature?

1.2 Background

The Dyess B-47 crash site radiological survey data were selected as a framework to assess whether RSS could be applied beneficially to a comparable future survey. The survey area consisted of a 36.5 acre area where a B-47 crashed on 4 November 1958, about 4.5 miles southwest of Dyess AFB, Texas, which is located near Abilene Texas as seen in Figure 1.1 below [3].

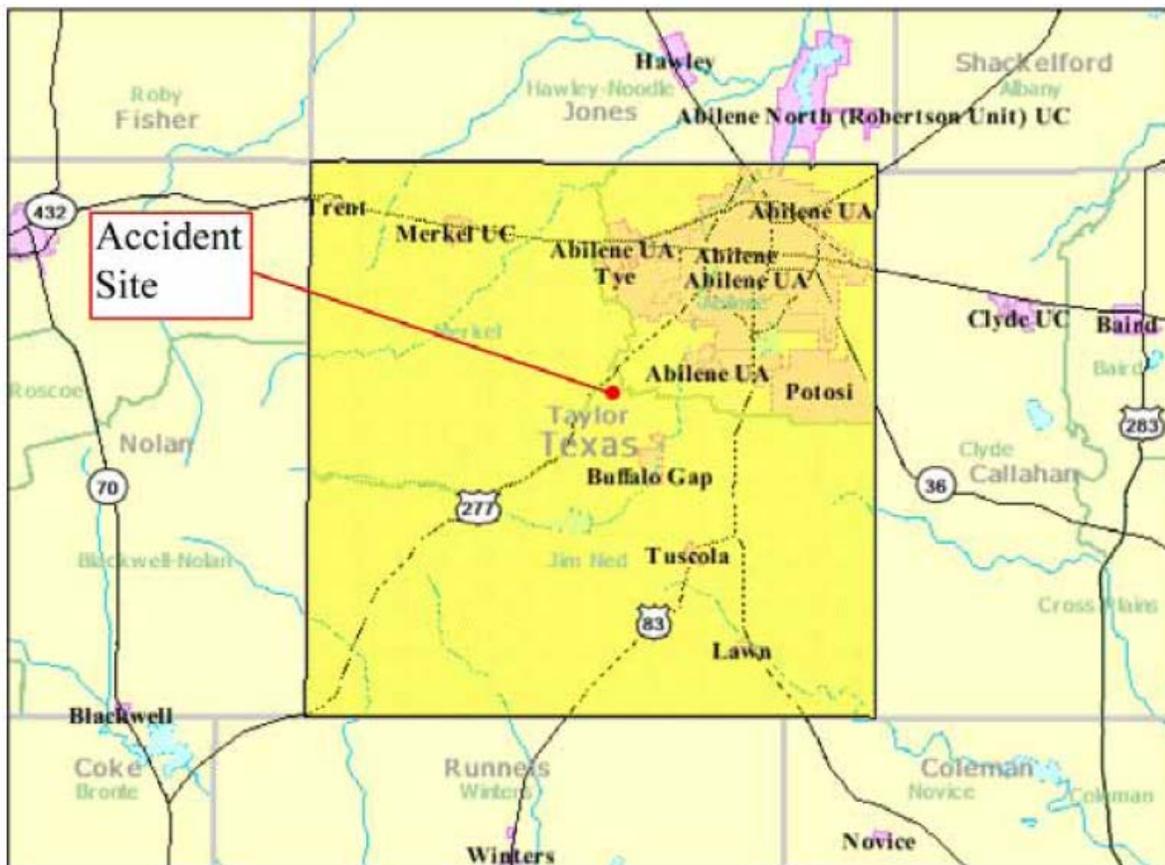


Figure 1.1 Accident Site Location, provided by USAF [3]

The aircraft crashed upon take off, carrying a full load of fuel and one nuclear weapon. The nuclear weapon fissile material was uranium. The conventional high explosives in the nuclear weapon detonated along with the explosion of fuel. The detonation scattered weapon materials and parts of the aircraft up to 800 feet from the impact crater, which was 35 feet in diameter and 6 feet deep. Survey teams identified alpha radiation on this debris and on the ground in the proximity of the crater. Contamination levels assessed during the initial analysis at this point were not documented, though the highest readings were within 100 feet of the crater. Responders did report a smoke plume from the incident, but survey records do not support a large contamination spread. Much of the debris was removed from the site in the initial clean-up effort shortly after the crash. Since the accident, the land has primarily supported cattle grazing operations. In 2010 the Air Force performed a characterization survey of the site, which drove a small remediation project in 2012 [3].

For the characterization survey, the site impact zone and surrounding area extending out 800 feet radially from the impact were considered the boundaries for investigation. This area was divided into 12 survey units, where units 1-5 were assumed to be MARSSIM Class 1 survey units for survey purposes [3]. MARSSIM Class 1 surveys are those with the highest potential for contamination and are expected to require a class 1 final status survey, and as such, meet the following criteria: “(1) impacted; (2) potential for delivering a dose above the release criterion; (3) potential for small areas of elevated activity; and (4) insufficient evidence to support reclassification as Class 2 or Class 3” [12].

Each survey unit had less than 2000 m² of land area. Figure 2.2 below is an aerial view of the impact/detonation location, with the survey units identified. Survey unit 1 included the crater and was anticipated to have the highest levels of contamination [3]. Survey unit 5 was

included as a Class 1 area because it is in the area covered by the plume based on the predominant wind direction recorded for the incident. Survey units 1-5 were used for the RSS assessment objective of this study.

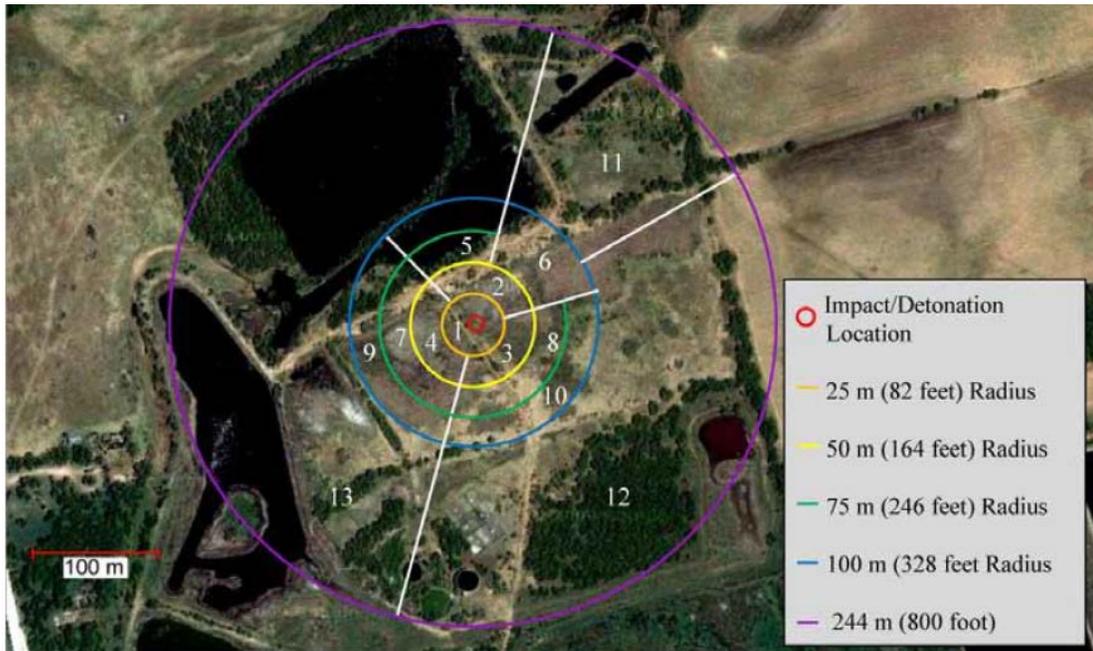


Figure 1.2 Aerial View of crash site survey units, provided by USAF [3]

Chapter 2: Literature Review

2.1 RSS Theory

The concept of ranked set sampling (RSS) was first presented by McIntyre in 1952 as a sampling method where a ranking technique is used to evaluate a set of screening observations and indicate which of these data points should have analysis measurements. These analysis measurements are those that are used to estimate the true population distribution's parameters, such as the mean, which can then be used to support the sampling event's ultimate decision. RSS is as an alternate sampling approach to simple random sampling (SRS). While both use the same analysis measurement technique in itself, the difference is in the method of selecting the data points for analysis measurement. Given the example of a radiological survey, the data points are the location where the soil sample analysis measurement is taken. For SRS, these points are randomly selected, and for RSS they are judgmentally selected based on the screening observations. Over the past 6 decades, many statistical investigations and some field studies have been performed comparing RSS and SRS. Given the correct application, the benefit of RSS is an improved estimate of the mean and standard deviation when compared to SRS. A second perspective on this benefit is that RSS can provide the same precision in the estimate as SRS with a lower number of analysis measurements [14].

In this paper, the data used for ranking will be referred to as the screening observations to differentiate them from the analysis measurements. Screening observations can be obtained by either qualitative or quantitative methods, as long as they provide an accurate relative indication

of the magnitude of the analysis measurement. The use of RSS is most practical when the screening observation technique is simple and quick to use and is considerably less expensive than the analysis measurement [21].

The procedure for RSS consists of selecting m random sample sets, each with m locations with values (screening observations) obtained by applying the screening observation technique. The m values (screening observations) of each of the m sets are ranked numerically within the set, smallest to largest. From each set the m judgment order statistic is sequentially selected [19]. For example, if a value of three was selected for m , then nine screening observations would be obtained (three values for each of the three sets, or m^2). From the first set the largest screening value would be selected for sample measurement. From the second set the middle value would be selected, and from the third set the lowest value would be selected. This same process is repeated for n cycles, as determined by the surveyor. This results in mn^2 screening observations and mn analysis measurements. The value of mn is referred to as the ranked set sample [14]. The use of cycles allows for an estimate of the variance of the estimated mean [21].

Figure 2.1 below illustrates a simple example how ranked set sampling would be used to characterize the population distribution of people's heights. In this example, the height of m^2 individuals is visually evaluated and ranked; specifically, since m is equal to 3, three individual's heights are evaluated for each of three sets resulting in a total of 9 screening observations and 3 resulting analysis measurements per cycle. This process is repeated n times, giving the subset distributions of 1, 2, and 3 seen in Figure 2.1. These mn analysis measurements are then used to estimate the mean of the population. The value of mn is selected prior to the start of the evaluation, keeping in mind the total number of screening observations will be m^2n [14].

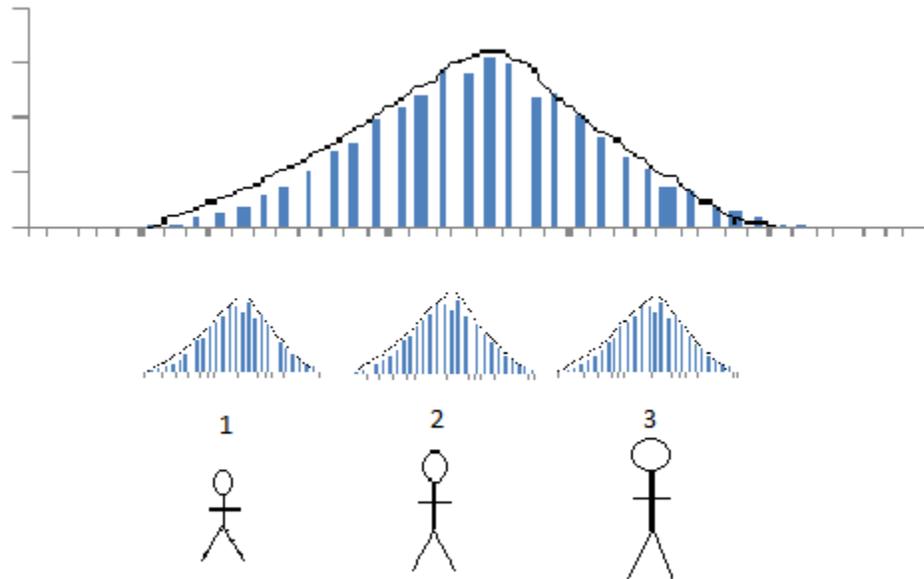


Figure 2.1. Illustration of Ranked Set Sampling adapted from Patil (1995)

A practical limit of m is imposed based on the screening observation methodology, particularly for qualitative methods. The above example of ranking individuals heights visually is a qualitative method of ranking. Based off of the limits of professional judgment (i.e. the human eye) to differentiate rankings, the EPA guidance on RSS recommends that m should be between two and five. If a quantitative method of ranking is applied than the value of m is limited by cost of the screening technique. For this type of ranking, Stokes (1977) suggests that m should be chosen as large as practical [17]. Once m is selected, the desired ranked set sample size is obtained by increasing n [18].

As mentioned, the primary advantage of RSS is an improved estimation of the mean and variance when compared to a random sample of size nm for parametric distributions. An important aspect of this is that the mean estimated by parametric RSS will always be as precise or better than that of SRS, given the analysis measurements (mn) of RSS is the same number as

that in SRS, even if the ranking process has error and is not a perfect judgment. An example of ranking error in the previous example of RSS of heights would be if the surveyor evaluated someone as the tallest person of a set when in actuality they should have been the middle height. The term perfect judgment indicates that there were no errors in ranking. If the ranking process is completely in error, then the efficiency of the sampling, when considered as relative precision (RP), will be the same as that of SRS for parametric distributions (such as a normal distribution) [5]. This relationship, originally proposed by McIntyre and given considerable further investigation in subsequent research, is demonstrated by the following equation:

$$RP = \frac{\text{variance of sample mean with SRS}}{\text{variance of sample mean with RSS}} \quad (\text{equation 1})$$

It is important to note that in this relationship the sample mean for both SRS and RSS is based on the same number of analysis measurements (nm) [5].

Dell and Clutter (1972) showed that the bounds of the RP given in equation 1 are expressed by equation 2 below.

$$1 < RP < \frac{m+1}{2} \quad (\text{equation 2})$$

This expression illustrates mathematically that the RP is not less than one. This indicates that the variance of the sample mean of SRS is expected to always be the same or larger the variance of the sample mean of RSS, for parametric distributions. As such, the equation also illustrates that the advantages of RSS remain regardless of errors in ranking as stated earlier [5].

Stokes (1980) displayed that a sufficiently large value of nm is needed to effectively estimate the variance of the distribution when utilizing the RSS observations. Unlike the estimate of the mean, RSS does not always estimate variance of small sample sizes more efficiently than SRS. The RP of estimating the variance is expressed by the following equation,

where the numerator is the sample variance of a SRS data and the denominator is the variance of RSS data :

$$RP_{var} = \frac{\text{var}(s^2)}{\text{MSE}(\hat{\sigma})} \quad (\text{equation 3})$$

As can be seen in equation 3 above, a RP_{var} value of 1 indicates that the variance of RSS is equivalent to SRS. A value greater than 1 indicates that the RP_{var} of RSS is better than that of SRS. In table 2.1 below, Stokes (1980) provides values of RP_{var} for a variety of m and n values applied to different distributions. His results indicate that even with large sample sizes, the benefit gained in estimating variance by use of RSS is not as considerable as that gained by using RSS to estimate the mean. One can note how the benefit is almost negligible for lognormal distributions (best RP_{var} is 1.04) and not very noteworthy for normal distributions until m equals 5 with a large n . These conclusions are applicable to radiological contamination or clean-up scenarios since they typically result in distributions that are comparable to normal or lognormal distributions [15].

Distribution	m	RP ($n=1$)	lim RP $n \rightarrow \infty$
normal	2	0.68	1
	3	0.81	1.11
	4	0.93	1.25
	5	1.03	1.4
lognormal	2	0.93	1
	3	0.95	1.01
	4	0.96	1.01
	5	0.97	1.02
	6	0.98	1.02
	7	0.99	1.03
	8	1	1.03
	9	1.01	1.04

Table 2.1. RP for estimating the variance (equation 3) for normal and lognormal distributions, adapted from Stokes (1980) [18]

Another cited advantage of RSS is that it remains a better technique to statistically characterize the population regardless of the distribution of the population [14]. This advantage is seen in Table 2.1 in the application of RSS to both normal and lognormal distributions for the theoretical situation where n approaches infinity or for higher values of m . However, the greater the distribution is skewed, the less efficient is the use of RSS compared to SRS and the closer the RP is to 1 (as seen in the lognormal distribution values of Table 2.1). This was acknowledged by McIntyre when he originally suggested the use of RSS.

Several application methods have been suggested to increase the efficiency of RSS for skewed distributions. In his article about RSS, McIntyre suggested using an unequal allocation of the screening observations. For example, consider a distribution skewed towards higher values (as would be seen in radiation clean-up surveys) and that the surveyor has decided that m equals 3. Instead of selecting the lowest, middle, and highest values across the three sets of screening measurements, the surveyor may select the lowest and two highest values. Figure 2.2 below adapted from Patil provides a detailed illustration of unequal allocation compared to equal allocation:

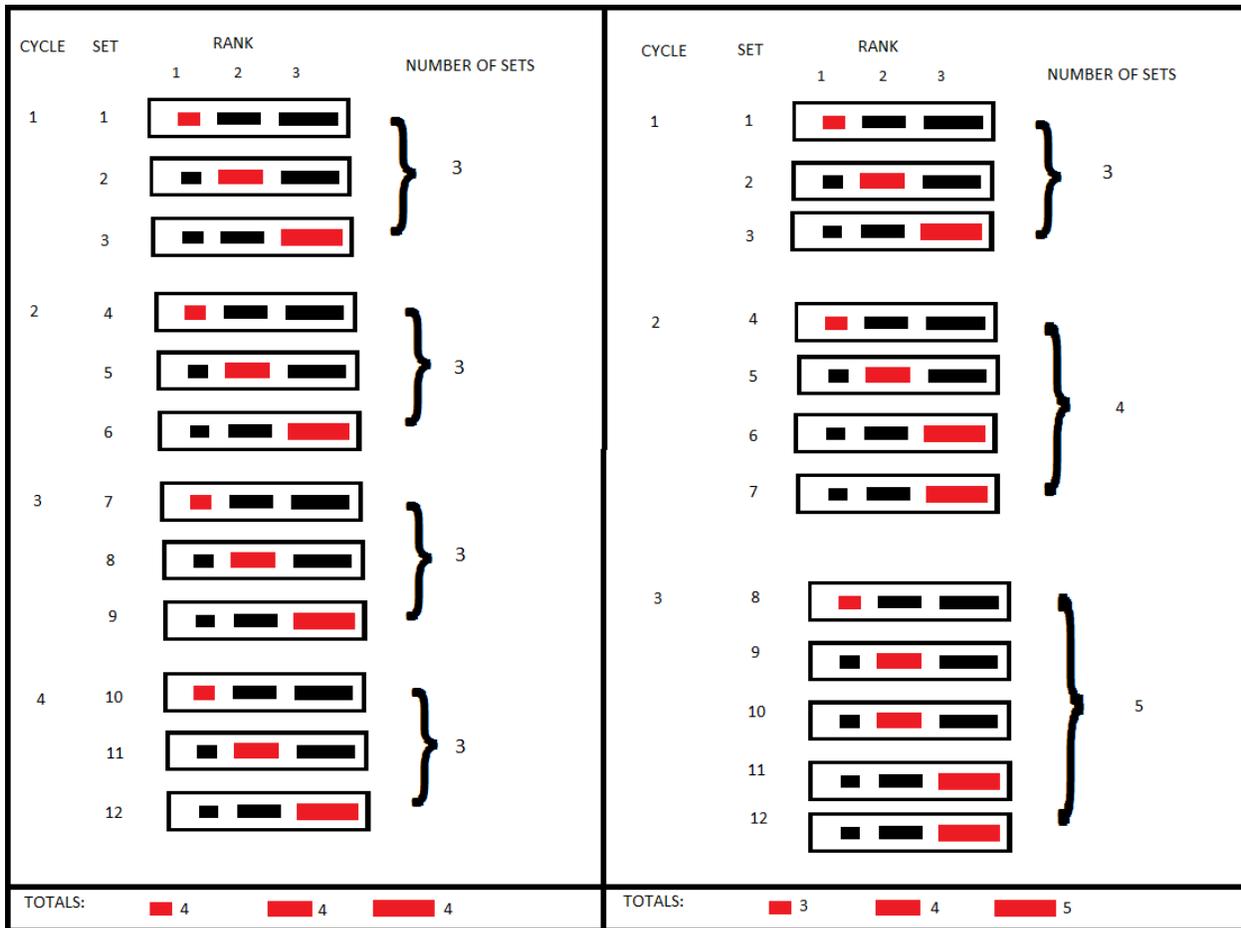


Figure 2.2. Comparison of equal (left side) and unequal (right side) allocation of RSS, adapted from Patil (2002)

Figure 2.2. illustrates the adjustment needed to create an unequal allocation. Instead of one selection of smallest, middle, and largest value respectively in each of the three sets of screening measurements per cycle, the scheme is modified to give more weight to the larger values. In Figure 2.2 the red highlighted boxes indicate whether the smallest, middle, or largest values are selected. For unequal allocation, Figure 2.2 shows that the smallest value is selected 3 of 12 times, the middle ranking has one additional value (4 of 12 selections) and the highest ranking has two additional values (5 of 12 selections). To determine how the sets will be unequally allocated, McIntyre suggested allocating the number of sets for each rank in proportion to the standard deviation of that rank; this process is termed Neyman's Allocation [14]. This method is

challenging to apply since it is difficult to determine reliable estimates of rank standard deviation prior to any sampling. Instead, Kaur (1997) proposed that if the surveyor has some knowledge of the degree of skewness expected, then two rules-of-thumb options for screening observation allocation can provides results very near to those of Neyman Allocation [9].

The first rule of thumb option is applied by selecting the largest screening value for sample measurement t times more than the rest of the screening values, where t is always greater than one; this option is referred to as the t -model. Specifically, the optimal value of t is proven by Kaur to be equal to the following equation,

$$t = \sqrt{\frac{b'(m-1)}{a'}} \quad (\text{equation 4})$$

where $a' = \sum_{i=1}^{m-1} \sigma^2_{(i:m)}$ and $b' = \sigma^2_{(m:m)}$

Kaur developed the below graphs of t in Figure 2.3 based on 225 positively skewed distributions that were founded on six different parametric distributions, to provide a large range of skewness, kurtosis, and coefficient of variation (CV). Given a surveyor selected value of m and knowledge of one for these statistical attributes, these graphs can be used to select a value of t versus needing to apply equation 4 to calculate t . As can be seen, the CV graph results have the least variability from the line of regression, and hence this graph is suggested for use in selecting t [9].

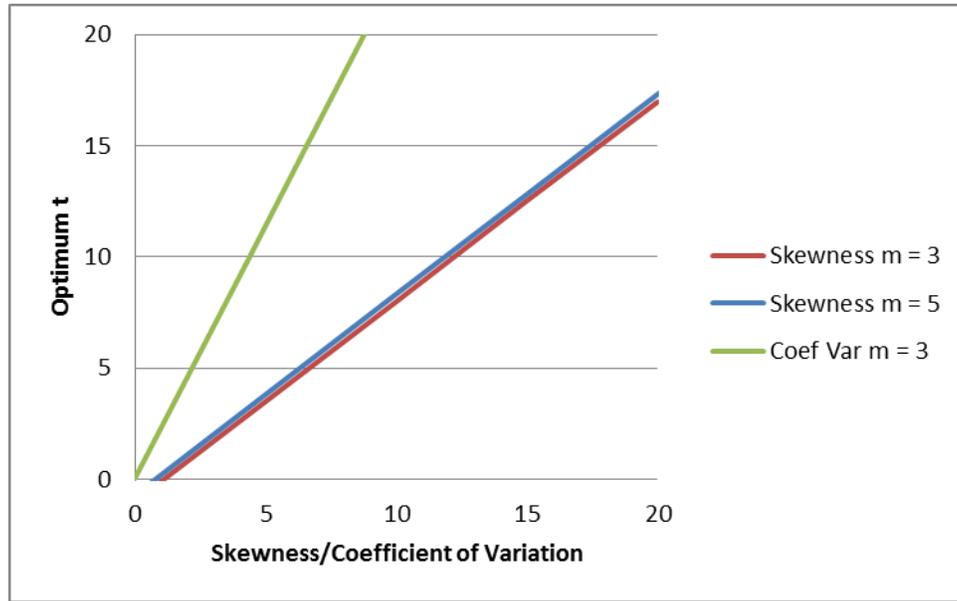


Figure 2.3. Rule of thumb plots of t_{opt} for a known skewness or CV; adapted from results given in Kaur (1997)

The second method, termed the (s, t) -model, is applied by selecting the two largest screening values for quantification more than the remaining screening values by the factors of s and t respectively, where $1 \leq s \leq t$. Kaur et al displays how s and t are calculated by the following equations:

$$t^* = \sqrt{\frac{(m-2)c}{a}} \quad \text{and} \quad s^* = \sqrt{\frac{(m-2)b}{a}} \quad (\text{equation 5})$$

where

$$a = \sum_{i=1}^{m-2} \sigma^2_{(i:m)} \quad \text{and} \quad b = \sigma^2_{(m-1:m)} \quad \text{and} \quad c = \sigma^2_{(m:m)}$$

Graphically s and t are shown in Figure 2.3 for a variety of values of skewness and CV. If these values can be estimated for the distribution, then values of s and t can be determined by use of the below plots instead of equation 5. Again, using the CV plot is recommended given that it has less variation in the data from the line of regression. When unequal allocation is applied correctly, the precision obtained will be greater than using the standard parametric RSS methodology [9]. The bounds of equation 2 are redefined by Kaur as:

$$1 < RP < m$$

(equation 6)

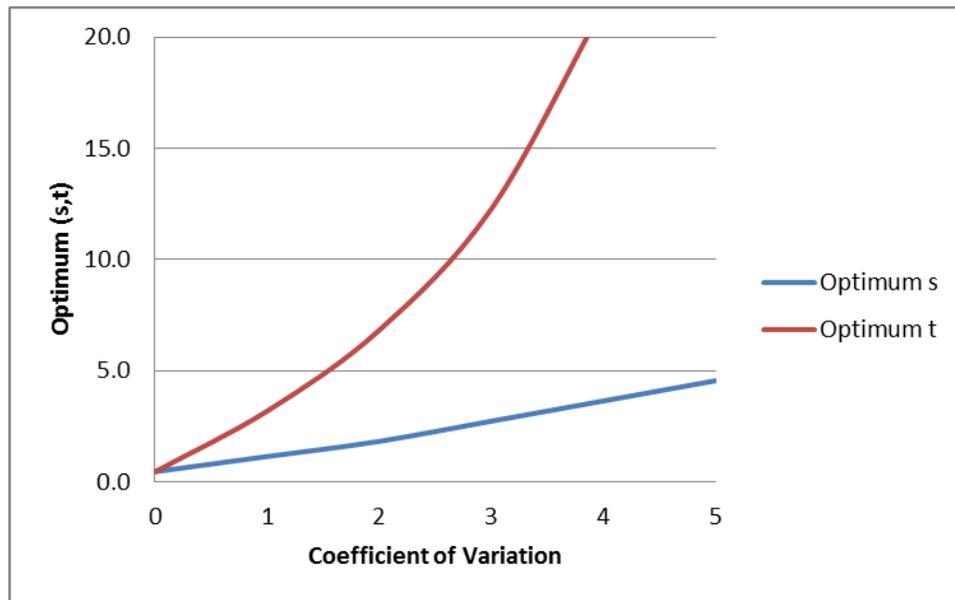


Figure 2.4. Rule of thumb plots of optimum s,t values for skewness and CV; adapted from Kaur (1997)

The use of an unbalanced RSS warrants consideration to environmental clean-up sampling scenarios where the contaminant is naturally present in background, due to the likelihood that the contaminant distribution is skewed right towards higher values [22]. By using this method of applying RSS, there is the potential that even more accuracy can be obtained for such distributions than using unbalanced RSS. However, there is also the possibility that the estimation of the mean can be worse if the population's underlying distribution is not estimated correctly [14]. For example, if it is assumed that the population is highly skewed but in reality it is not, then one can visualize how the unbalanced RSS sample measurements will result in too great an emphasis on the upper end of the distribution. Given this possibility, it is important to conduct a scoping survey or consider prior surveys that are representative of the proposed survey [20].

As mentioned previously, another benefit of utilizing RSS comes from the fact that less analysis measurements are needed to estimate the mean at the same level of accuracy compared to SRS, given balanced RSS is utilized. As stated by Patil (1990): “With SRS, the only way to increase the prospect of covering the full range of possible values is to increase the sample size. With RSS, however, we increase the representativeness with a fixed number of sample units, thus saving considerably on quantification costs.” Table 2.2 derived from the Environmental Protection Agency’s (EPA) section on RSS within their *Guidance on Choosing a Sampling Design for Environmental Data Collection* illustrates the benefit of reduced sample size when using RSS compared to SRS in obtaining the same degree of precision in estimating the mean [21]. The original table was produced by Mode et al (1999), but modified slightly by the EPA authors to indicate coefficient of variation, which is the standard deviation divided by the mean, versus Mode who presented the data for values of variance. The standard deviation is used in environmental clean-up procedures, to include radiological MARSSIM procedures, more prevalently than the variance. The data are based on the assumption of no errors in ranking the field locations [11].

Coefficient of Variation (CV)	m/SRS	specific precision of estimated mean with 95% confidence		
		10%	15%	25%
0.5	SRS	97	43	16
	2	66	30	12
	3	51	24	9
	5	35	20	10
0.707	SRS	193	86	31
	2	132	60	22
	3	102	45	18
	5	70	35	15
1	SRS	385	171	62
	2	262	118	42
	3	201	90	33
	5	140	65	25

Table 2.2. Comparison of Number of Samples for Laboratory Analysis Using RSS adapted from Mode et al (1999) and EPA's RSS guidance[21]

2.2 RSS Applied

14 years after McIntyre's original proposal of RSS, RSS was applied for the first time by Halls and Dell to estimate forest yields [20]. Since then, RSS has been utilized widely in agricultural fields and expanded to other fields, to include environmental clean-up sampling. One of the earliest uses of RSS for an environmental study was the application of the method in estimating PCB concentrations in soil. In this study, Patil, Gore, and Sinha (1994) evaluated a skewed contaminant distribution with balanced and unbalanced RSS. They evaluated the RP, and their results indicated that RSS performed better than SRS [20]. Since then, the use of RSS has been incorporated into state environmental remediation guidance, such as Indiana's Risk Integrated System of Closure (RISC) guidance. The use of RSS for PCB evaluations by use of a field test kit is a specifically listed example in this guidance [8].

The EPA's *Guidance on Choosing a Sampling Design for Environmental Data Collection*, 2002, has a designated chapter on RSS that provides further suggestions for

environmental applications and highlights how handheld detectors can provide a quantitative screening technique for RSS. One of these examples is an evaluation of lead contamination by use of a x-ray fluorescence handheld detector to provide the screening observation technique for ranking the locations to take soil samples for laboratory analysis of lead or other metals in soil. Another example is the use of a hand held radiation detector as the screening observation method to obtain analysis samples for estimating the mean plutonium concentration in surface soils of Nevada weapons testing sites [20]. Each of these examples uses a relatively inexpensive handheld detector to obtain the screening observations to selectively choose those locations with more expensive soil analysis measurements.

This EPA example of using a handheld radiation detector was derived from one of the only detailed scholarly articles available regarding use of RSS within the health physics field. This article was written by Richard Gilbert in the Data Quality Objectives (DQO) Statistics Bulletin in 1995. In particular, he discussed how RSS could be applied to estimate the mean plutonium concentration in soil at a Nevada Test Site test area to prepare for possible remedial actions at the site by use of the FIDLER (Field Instrument for Detection of Low Energy Radiation) handheld radiation detector [6]. Several locations at the test site were contaminated with plutonium as a result of nuclear weapons testing. Americium is a decay product of Pu-241 and is present in the environment in amounts that strongly correlate with the degree of plutonium [10]. Gilbert suggested using this correlation at nuclear weapons test sites, where Pu-241 is a primary plutonium isotope present (given a plutonium containing weapon was tested at the site). The FIDLER is very effective at detecting the low energy gamma rays of Am-241. Even though it is not effective at detecting Pu-241, which is a beta emitter [13], it provides an estimate for Pu-241 through the correlation that is known to exist between the two radionuclides. Given this, the

FIDLER measurements of americium can be used as a field surrogate for the relative amounts of plutonium. As such, they can be used as the ranking technique for selection of those sites that will have soil samples analyzed for the plutonium concentrations [6].

Gilbert did not apply his proposal to an actual survey, but acknowledged key points and considerations that should be taken into account when selecting an RSS methodology in a radiation survey. He did not consider that the increased precision of RSS could suggest that less soil samples are needed than SRS. Instead he cited that the number of soils samples should match the value given from the statistical process used in SRS. He also discussed how these soil samples could be selected in one of two ways. The first approach is that one cycle is used, $n=1$, so that m^2 locations are randomly selected over the survey area for the screening samples. In the example he gives of 12 soil samples required by the DQO process. So in the case of $n=1$, m will equal 12, and a total of 144 locations must be ranked. These locations are randomly allocated into 12 sets, and the locations of 1st through 12th largest value from each subsequent set would be the selected for the soil sample (resulting in 12 samples).

The alternative approach given by Gilbert is to have m equal to 3, and hence select 4 as the value of n cycles. For each cycle, the three sets would each have three screening locations. From the first set the highest screening value will indicate the soil sample location, the middle value will be selected from the second set, and the lowest from the third set. In this option, a total of 36 locations of screening observation will be ranked, and result again in 12 soil samples used as analysis measurements. This latter option will be less costly because there are less screening observation samples needed. However, the first option has the noteworthy advantage that it is expected to have a smaller variance in the estimate of the mean. It is possible to use such a large value of m given this is use of a quantitative method of ranking [6].

While the use of RSS in evaluating plutonium contamination is one of the examples highlighted in the EPA sampling guidance and often cited as a practical example of RSS in the literature, the literature does not illustrate wide application of RSS to radiation clean-up surveys. Recently it has been used by Oakridge Associates University (ORAU) within a MARSSIM framework, and in 2012 it was proposed for incorporation into the rewrite of the MARSSIM guidance by Timothy Vitkus of ORAU [22]. The ability of RSS to provide a more precise estimate of the mean has evident benefits given the high level of accuracy often desired from the results of final status surveys (FSS), and it can also be useful in a characterization survey. FSS are defined as “Measurements and sampling to describe the radiological conditions of a site, following completion of decontamination activities (if any) in preparation for release” [12]. Characterization surveys are utilized to determine the nature and extent of the contamination, and determine what approach is needed for the FSS [2].

Vitkus’s MARSSIM proposal is a RSS based method for augmenting FSSs that evaluate hard to detect radionuclides (specifically alpha and beta emitters). While this was the primary example given in the proposal, he also mentioned the concept of using RSS for detecting cesium-137 by use of a NaI gamma scintillation detector as the screening technique [22]. One of the benefits he cites of using of RSS in the FSS for hard to detect radionuclides is that RSS increases the probability of detecting areas of residual contamination within Class 1 FSS survey units that otherwise could have gone undetected because of the increased number of screening observations. The RSS procedure for these radionuclides suggested by Vitkus is to set m equal to three and adjust the number of cycles, n , to obtain the desired number of samples. The desired number of scanning observation locations in his procedure is determined by the MARSSIM procedure of selecting enough samples for hotspot consideration. A pre-defined amount of

surface soil (the study uses 100 grams) is collected for each screening measurement. It is shaken in a container for size reduction and placed in a container that allows for consistent geometry with the surface the same size as the detector (preferably one with an area of at least 100 cm²). This sample is measured by an alpha or beta field detector, for a one minute count.

Per Vitkus, this methodology is effective for alpha emitters and beta emitters where the maximum beta energy is greater than 250 keV. He defines minimum ranking capability as “the lowest activity level, that will be consistently greater than the instrument/background soil count rate, and therefore result in the ability to use professional judgment to rank a given count as low, medium, or high when varying HTD activity levels are truly present” [22]. The minimum ranking capability limitation for his method is 5 to 10 pCi/g alpha activity and 100 to 200 pCi/g for lower energy beta energies (such as Technetium-99). The concentrations would be lower for higher energy beta emitters [22].

RSS has also been included in the most recent version of MARSSIM Final Survey Design and Strategies, by Eric Albequist. Within his text he includes the suggestion to use the RSS applications previously discussed in this paper, and he also describes the suggested application of RSS for use within independent verification surveys of FSS for validating the licensee’s reported mean concentration in selected survey units. Independent verification surveys are those accomplished by an independent third party to evaluate the final radiological conditions at a decommissioned site and validate that the site release criteria were met. Two aspects of the independent verification survey are “to substantiate the credibility of the radiological survey procedures and to validate the accuracy of field measurements and laboratory analytical techniques” [2].

ORAU has utilized RSS during verification surveys for the past several years. A similar procedure as the one proposed for MARSSIM has been applied. The number of samples needed for verification surveys can be significantly smaller than that needed for the FSS. Figure 2.5 illustrates an example of a verification survey sample plan using RSS with m equal to three and the use of n equal to two, resulting in a total of 18 screening values and 6 soil measurements. A sample size of six in this case is assumed to be that needed statistically for independent verification of a selected survey unit. The survey points were randomly determined using visual sampling plan (VSP), with some judgment applied to ensure that there was not too many clusters of points [2]. ORAU has found that use of RSS in this application an effective means to increase the confidence in validating the conclusions of the FSS.

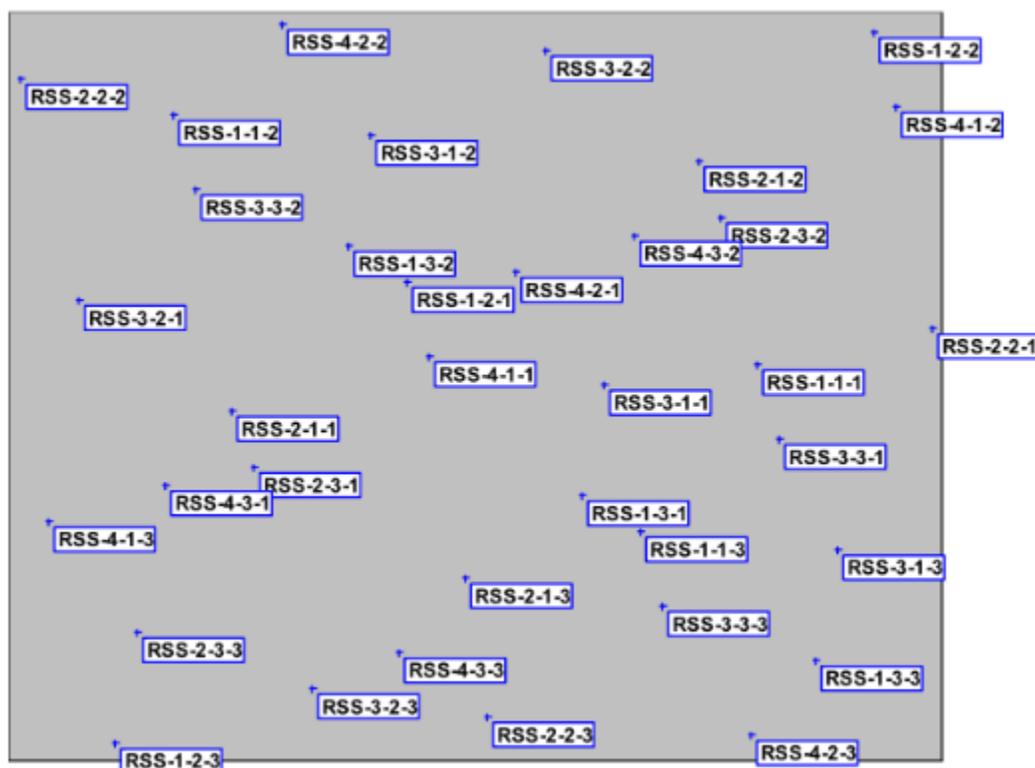


Figure 2.5. Example of RSS locations applied to a radiation survey unit, provided by Vitkus from his MARSSIM proposal [22]

Another valuable benefit of RSS mentioned in the EPA's guidance document is in providing an increased ability to detect differences in the mean of two populations. For many environmental contaminant scenarios (to include radiological contamination), these two populations are the background and contaminant [21]. Bohn and Wolfe (1994) validated theoretically that use of RSS obtained data in the Mann-Whitney-Wilcoxon test (used to evaluate differences in the mean of two populations) will perform better than traditional application of this test for both perfect and imperfect judgment [4]. Wilcoxon Rank Sum (WRS) tests are used in MARSSIM procedures when the contaminant is present in the background [2]. For RSS obtained data to be utilized for WRS tests, the EPA guidance suggests the following:

“If ranked set sampling data are used to test hypotheses, the data computations may differ from the standard computations that would be performed if the data were obtained using simple random sampling. For example, suppose the Wilcoxon Rank Sum test will be used to test for differences in the medians of two populations and that the data are obtained using ranked set sampling. Then the data computations for the Wilcoxon Rank Sum test described in Bohn and Wolfe (1992, 1994) should be used rather than the standard computations [for example, see Section 18.2 of Gilbert (1987)] that would be used if the data had been obtained using simple random sampling. If ranked set sampling data will be used to conduct tests of hypotheses or to compute confidence intervals on means or other statistical parameters, guidance from a statistician familiar with ranked set sampling should be sought.”

If applied correctly, another beneficial application of RSS obtained data given by the EPA RSS guidance is in conducting sign tests to check for compliance with a fixed remediation concentration limit [21]. The sign test is utilized in the MARSSIM procedure when contaminants are not present in the background [2]. Again, it is suggested by the EPA that a statistician should be consulted when a sign test is utilized with RSS data [21].

2.3. Sampling Strategy Considerations

The EPA guidance document provides a section with instructions on the practical application of RSS. Specifically, the EPA's guidance provides details on how to decide the number of samples for laboratory analysis and how to decide where in the field to collect these samples. One suggestion that McIntyre (1952) gives is to divide the area to be evaluated into different portions of equal size that have no well-defined distinguishing factors [21]. This fits within a MARRSIM structure where multiple survey units are defined for a radiation contamination impacted area that is classified as a Class 1 survey unit [12].

Another important consideration emphasized in the literature is the importance of evaluating the cost benefit of RSS compared to SRS. RSS's ranking method should not be so high as to cause the survey to be cost prohibitive. The cost evaluation varies depending on the application of RSS. Figure 2.6 illustrates the cost as a function of level of radiation dose per year for four different radiation measurement techniques. The unit cost of A is equivalent to one US dollar. The graphic indicates the dose in mrem/y that could be evaluated by each type of instrumentation. Figure 2.6. is helpful in a general cost consideration for RSS applied to radiation contamination scenarios because it provides a perspective of the cost for samples and laboratory analysis (analysis measurements) compared to in-situ spectrometry and survey meters (screening observation techniques). From this graph, laboratory analysis of samples costs at least approximately 50 times more than a survey meter measurement and 5 times greater than in-situ spectrometry.

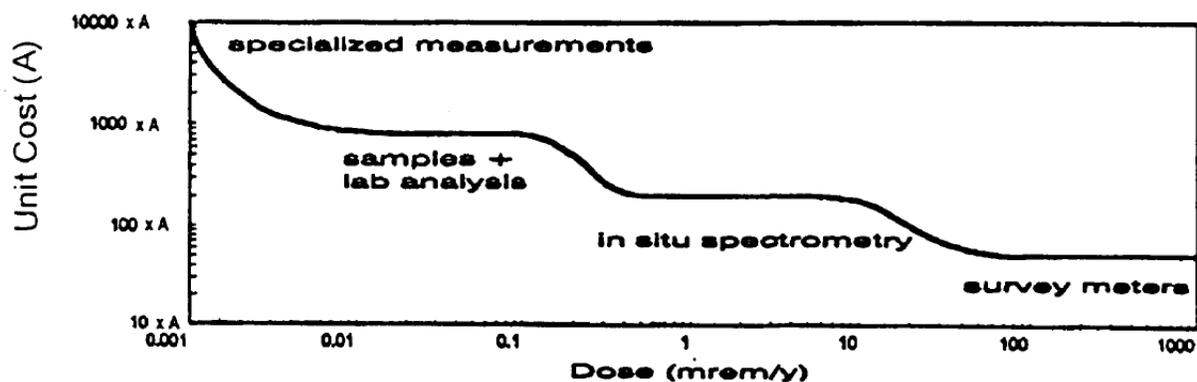


Figure 2.6. Cost for radiation sample methods, illustrated in IAEA radiological remediation guidance [23]

2.4. Considerations on Radiological Contamination Distributions

Radiological contamination distributions are typically considered lognormal or approximately normal with some frequency of hot spots that results in a right-skewed distribution. As an example, in his doctoral thesis dissertation on Dose Modeling and Statistical Assessment of Hot Spots for Decommissioning Applications, Albequist utilized both a normal and lognormal distribution for evaluation of hotspots [1]. The estimated mean and standard deviation of the distribution are both parameters utilized in the MARSSIM site evaluation process. The estimated standard deviation of the contaminant distribution is one of the parameters needed by the MARSSIM process to determine the number of random soil samples necessary for the FSS. The estimated mean of the results of these random samples is compared to a specified release criterion for the site, that indicates it has been remediated to the appropriate level. The MARSSIM method evaluates the biased samples that are taken separately from the random samples, and compares them to a separate additional release criterion level [2].

Per Albequist, the “FSS data has a substantial impact on the development of the posterior distribution.” The MARSSIM method assumes that the data are representatively proportional to the true contaminant concentration distribution at the site, so that if hot spots exist they are

proportionally reflected in the sample. If this is the case the posterior distribution will accurately reflect the estimate and its uncertainty. Given this, Albequist suggests the concept of evaluating the hot spot and random samples together as a distribution against one criterion versus separately [2]. If this approach was taken, RSS could be advantageous in providing more accurate descriptors of the distribution than SRS.

2.5 Properties of Radioactive Contaminants at the Dyess Survey

U-235, U-238, and U-234 all decay by emission of an alpha particle. Though none have a substantial level of photons emitted, U-235 and decay products of U-238 do emit detectable low energy photons. Th-234 (thorium) is a daughter product of depleted uranium (U-238), and emits a low-level photon of 92.6 keV and 63.3 keV. U-235 emits a 185.7 KeV gamma ray that can be useful for field detection through methods to include ex-situ gamma spectroscopy, FIDLER measurements, and scanning by NaI detectors. All three of these methods were used in the characterization survey of the B-47 crash site. U-234 itself does not have any significant photon emissions, and has to be evaluated by alpha spectroscopy unless the mixture ratio of it to either U-235 or U-238 is known. Alpha spectroscopy was used for the B-47 crash site survey sample analysis [3].

Chapter 3. Materials and Methods

The methodology used in this paper was designed to enable an evaluation of the viability of RSS through an assessment of the Dyess survey. The results of this evaluation would provide data on the feasibility of RSS for the Dyess survey and similar surveys in order to answer this study's three objectives, restated here:

1) What are the effects on the precision and accuracy of RSS mean estimate compared to that of SRS, when considering the Dyess survey and theoretical modifications?

2) Is there an optimal means to apply RSS for this survey or other radiological contamination surveys?

3) Given the procedures for RSS, could the field survey techniques used in this survey be utilized as a screening tool for determining what soil samples should be used in this survey, or future surveys of a similar nature?

To accomplish this, the Dyess survey data were used as a framework for investigating objective 1 and 2. Specifically to consider objective 2, different values of m and n were evaluated for the Dyess survey data and modified versions of this data. The field instrumentation results available from the survey were evaluated for objective 3.

3.1 Dyess Survey

The procedure and materials of the original Dyess survey will be discussed to the extent that it pertains to investigating the objectives of this paper. The Dyess survey utilized a MARRSIM approach to conduct the survey. In the retrospective evaluation of RSS accomplished by this paper, the data from survey units 1-5 were evaluated together because all were classified as Class 1 survey units. The radiological field survey portion of the survey data utilized three different survey instruments: the RS-700, a FIDLER detector, and an ORTEC ex-situ handheld gamma spectroscopy detector. Additionally, the Dyess surveys included taking soil samples at 99 random locations along with 12 biased soil samples for units 1-5. The soil samples were evaluated in the laboratory by gamma spectroscopy for U-235 and Th-234, and alpha spectroscopy for U-234.

The RS-700, manufactured by Radiation Solutions, is a rugged detector consisting of two 4 liter NaI(Tl) crystals that measure total counts per second (cps). It provides cps in up to nine different regions of interest (ROI), which can be set for a specific range of energies. According to the manufacturer:

“The RS-700 utilizes advanced DSP / FPGA (digital signal processing/field programmable gate array) technology and software techniques that provide laboratory levels of spectral performance that were previously unachievable on mobile platforms. Despite it's state-of-the-art technology, the RS-700 is extremely operator friendly and can be rapidly deployed. The system is also capable of unattended operation if required. The system has a built-in GPS receiver, which enables it to accurately locate each measurement taken” [16].

RS-700 background measurements were obtained in a similar area near the survey unit each day prior to surveying. The average of these values was subtracted from the ROI. The area surveyed of each unit was dependent on the MARRSIM category. Since units 1 through 5 are all Class 1, the survey team accomplished close to 100% scanning coverage for these units. The scan data for these units was obtained over two days. Areas that indicated higher radiation were evaluated further using FIDLER data. Consideration was given to whether there were geographical impacts on the data, such as scanning considerably non-even terrain. As warranted, biased soil samples were taken from areas of interest (AOI). Additionally, a few areas of interest were evaluated using an OrtecR Trans-SPEC-DX-100 HpGe ex-situ gamma spectroscopy handheld detector [3].

The Class 1 survey unit soil samples taken for the Dyess survey included surface measurements (first six inches of topsoil), subsurface (depth of six to twelve inches), and duplicate surface measurements for quality control. The subsurface measurements and duplicate measurements were removed from those considered as the soil sample data set in this paper. The 111 soil samples considered in this paper were taken from different locations and all were

measurements taken from the top six inches of soil. The specific soil data set values used in this paper's study were obtained by summing the soil results that could be measured by a gamma scanning technique such as the FIDLER or RS-700. These radionuclides were Th-234 and U-235 (obtained by laboratory gamma spectroscopy analysis). The U-234 results (obtained by alpha spectroscopy analysis) were not used since U-234 is an alpha emitter and cannot be detected by the FIDLER and RS-700 [3]. Including U-234 measurements would not allow for a comparable evaluation of the soil analysis measurements and screening observation techniques.

3.2 Evaluation of Data and Application of RSS Retrospectively

3.2.1. Assumptions

Since the original Dyess characterization survey plan did not include an RSS approach, the data available did not intentionally include screening observation values (handheld detector field measurements) that were purposefully linked to the analysis measurements (individual soil samples). However, in this paper all three field measurements techniques were evaluated to determine whether they could have been used as screening techniques for this survey or in future surveys. For this study, the RS-700 scan and soil data distributions were normalized and then characterized and compared by a set of statistical descriptors such as mean, standard deviation, and skewness. RSS was applied retrospectively to the RS-700 scan data set with the assumption that it adequately reflected the true radiological contaminant distribution. This assumption is slightly erroneous as further evaluation by the Dyess survey team of one of the hotspot areas identified from the scan data concluded that these scan data values were artificially high readings. However, this difference was not considered significant enough to effect the

conclusions in regard to the applications and viability of RSS. RSS was considered for application to the soil data set as well.

In a prospective field evaluation of RSS, both screening observations and analysis measurements would be taken for a large number of locations. This would allow for different selections of m and n to be compared in regard to a set of samples selected randomly (such as by SRS methods). It would also allow for an evaluation of the correlation of the screening observations to analysis measurements. In this study's retrospective evaluation of RSS, such a large data set was not available. To account for this, a key assumption was made. This assumption was that the screening technique magnitudes are perfect judgment indicators to the soil sample results. To illustrate this, in a set of $m=3$ it would be assumed that the lower, middle, and high scan data values selected from the existing scan data set would truly match with the low, middle, and high values of analysis measurements, respectively, if analysis measurements had been taken for the selected scanning observation locations. While this assumption of perfect judgment is not very likely to occur for the RS-700 scan data set, this data set is the largest available from Dysess survey. As such, it was most suitable to be used for evaluation of the application of RSS in this study.

3.2.2. General Procedure

The RSS techniques (based on the previously discussed assumptions) utilized for this study were applied only to the RS-700 scanning set. The soil data set was not evaluated in the same manner because it was determined during the initial data review along with use of some RSS application pre-trials that this data set was too small. Instead the soil data set distribution was only utilized in the aforementioned comparison to the scanning data set distribution. The RS-700 scanning data set resulted from the drive-over survey of units 1-5 and consisted of 7,500

values, each from a discrete location. For the purpose of evaluating RSS versus SRS, this data set was considered comparable to the contaminant population distribution. The mean and standard deviation of the entire data set was utilized for comparison of those estimated by RSS and SRS trials. The standard deviation estimate consideration was included in this paper for an additional perspective, but was not as thoroughly evaluated as it is not a key part of the primary objectives. The RSS and SRS data sets were also evaluated in comparison to each other. The conclusions of this comparison supported objective 1 of this paper.

Objective 2 was evaluated by the following tests applied to the RS-700 scanning data:

1. Applying different m and n values to the dataset
2. Modifying the scan data set to be a normal distribution with the original standard deviation and a larger standard deviation
3. Modifying the skewness of the scan data set to increase the right side skew

The details of these tests are outlined in section 3.2.4 of this paper. For each of these tests, four aspects were considered to evaluate if RSS provided an improved estimation of the mean and standard deviation when compared to SRS for 10,000 iterations. These aspects are: 1) Whether the RSS iteration means are statistically different in accuracy for estimating the mean than SRS, and if each was statically different from the true mean; 2) Whether there was a statistical difference in estimating the standard deviation between RSS and SRS, and if so was the RSS or SRS estimate of the standard deviation closer to the true standard deviation; 3) Whether more individual iterations indicate that RSS estimated the mean and standard deviation with a lower percent difference than SRS did, and 4) whether the RP is greater than 1, and if so, to what degree. The RP is a comparison of the variability of the mean for RSS and

SRS over the 10,000 trials. The RP of the standard deviation estimates were also provided for comparison.

3.2.3. Development of RSS Matlab Tool

RSS was applied by creating a Matlab code that is given in Appendix A. The code queries the user for m and n values, along with the number of iterations of the trial. The Matlab code randomly selects mn^2 points from the user inputted data set, without repeating a value, and separates these values into n number of sorted m by m matrices. Each column is a set of observations which is sorted from smallest to largest. As a result, the diagonal vector of each of these matrices results in the appropriate judgment statistic vector for RSS, with a quantity of m , as highlighted in the figure below:

$$\begin{pmatrix} 0.0898 & 0.0920 & 0.0920 \\ 0.1351 & 0.1112 & 0.1179 \\ 0.2954 & 0.1788 & 0.2146 \end{pmatrix}$$

Figure 3.1. Example of one of the n matrices produced by the Matlab code for $m=3$

The diagonal vectors from each n cycle result in the mn quantity of RSS selected values, which are representative of the analysis measurements for this paper. The code also randomly selects mn values from the user inputted data set, without repeating a value, as representative of the SRS values, for comparison to the RSS values. The Matlab code for one iteration generates a chart output to allow the surveyor to view the random locations selected for screening observations and analysis measurements. Figure 3.2 provides an example of this output for m equal to 3 and n equal to 4. To provide a more robust analysis between RSS and SRS, the Matlab code was optimized to perform a user specified number of iterations. The location figures are not produced in this version of the code.

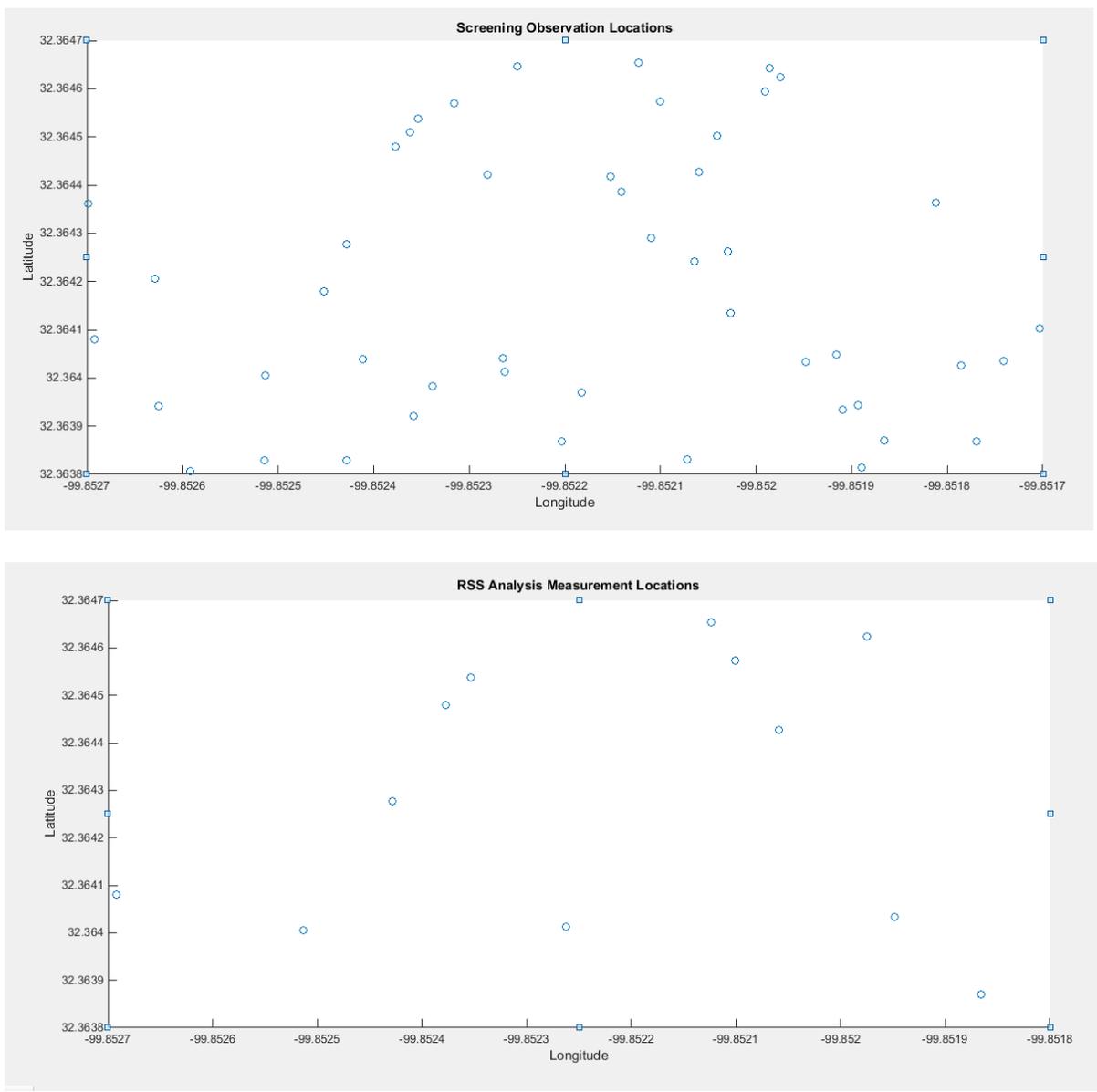


Figure 3.2 Matlab produced figures of RSS locations

3.2.4. Details on RSS Trial Procedures

For the first set of trials, applied to the RS-700 screening data, a value of $mn=100$ (or as close as possible for the combination of m and n) was selected to result in a total number of ‘analysis measurements’ that match the number of soil samples taken in the original survey. A total of 10,000 iterations of each trial were accomplished to provide a higher statistical

confidence in comparing RSS versus SRS. The following table gives the m and n values that were used for each trial where $mn=100$ or as close possible. The m^2n is also given indicating the number of screening observations that would hypothetically have been taken, though in this case of this approach this is the number of times a random number was selected from the entire data set.

m	n	mn	$m^2 n$
2	50	100	400
3	33	99	297
4	25	100	400
5	20	100	500
6	16	96	576
7	14	98	686

Table 3.1 Values of m and n applied to the scanning data set for mn approximately equal to 100

The Matlab code provides the mean and standard deviation of each iteration for the RSS and SRS data sets, and an overall average of each for comparison to the true values. In this analysis, the true values were considered to be the mean and standard deviation of the RS-700 scan data set, calculated from the entire set of 7,500 values (considered to be the population distribution). The RSS and SRS values for the estimated mean and standard deviation were averaged over all 10,000 iterations to provide a robust comparison of the percent accuracy of RSS and SRS in reference to the population distribution. To evaluate accuracy, the mean values were compared to see if they were statistically different from each other and from the population distribution using t tests. A t test was also used to evaluate whether the average of the iteration RSS estimates of standard deviation were different than the SRS estimates of standard deviation.

If this t test did indicate that they were different, the question was posed whether the average standard deviation was smaller in magnitude for RSS than SRS. This would indicate that RSS is a more precise method for estimating the mean than SRS for the individual trials.

The relative precision (RP) of the mean and standard deviation was also calculated over the entire set of iterations using equation 1 for the mean and the following equivalent of the equation 1 for the standard deviation, given below:

$$RP_{std} = \frac{\text{variance of sample standard deviations with SRS}}{\text{variance of sample standard deviations with RSS}} \quad (\text{equation 8})$$

The Matlab code also provides the percentile difference of both the mean and standard deviation from the true value for each iteration using the following equation:

$$\text{Percent Difference} = 100 * \left(1 - \frac{\text{abs}|\text{experimental value} - \text{true value}|}{\text{true value}} \right) \quad (\text{Equation 7})$$

Equation 7 was only used in evaluation of individual trials to illustrate the percent differences between the estimated mean and standard deviation and true mean and standard deviation.

Another set of trials was run at smaller values of mn for the scan data distribution, for the same considerations as mentioned for the mn equal 100 values. The following table gives the m and n values selected for this set of trials. These values were selected based on values interpolated from Table 2.1 of this paper for the scan data distribution coefficient of variation.

m	n	mn	$m^2 n$
3	22	66	198
2	23	46	92
3	12	36	108
5	6	30	150
2	12	24	48
3	6	18	54

Table 3.2. Values of m and n applied to the scanning data set for lower values of mn

The same n and m were applied to modified scan data results to see how changing the distribution affected the RSS to SRS comparison. Using the RS-700 scan data set as the framework, distribution properties were modified as follows: 1) changed the distribution to make it a normal distribution with the same standard deviation as the original RS-700 scan data distribution; 2) increased the standard deviation of this normal distribution three fold; and 3) increased the right side skew of the data set. The number of data points was kept at 7,500 as in the original scan data set to provide a similar comparison. The normal distribution was generated by using a normal distribution function based on random numbers generated by excel. The right skewed data set was generated by modifying the original normalized RS-700 data set directly. The number of high values over 0.3 in this data set was multiplied approximately 20 times. Some of the lower values were removed from the data set to keep the number of data points at 7500. The Matlab code was utilized for all these data sets with comparable m and n values to the previous trials where $mn \approx 100$. Additionally, the Matlab code was modified to apply the t_{opt} rule of thumb proposed by Kaur et al (1997) discussed in Chapter 2 of this paper,

and $mn \approx 100$ values were used along with some smaller values of m and n comparable to previous trials. This modified Matlab code is available in Appendix A.

Chapter 4. Results

4.1 Evaluation of B-47 crash site data

The B-47 crash site data were evaluated for the three objectives given in Chapter 3. The results in this section are provided first, for the evaluation of the field data techniques and their plausibility as an RSS scanning observation method and second, for the evaluation of the application of RSS to the Dyess survey RS-700 scan data and three other distributions that are applicable to radiation remediation projects.

4.1.1. Evaluation of Field Data Techniques

As mentioned, objective 3 of the assessment of the survey data from the B-47 crash site is to evaluate whether the field measurement methods utilized could be used as screening techniques for the purpose of RSS ranking. The data sets evaluated were the scan survey data and FIDLER data. There were not enough ORTEC gamma spectroscopy measurements to evaluate numerically, which is discussed in Chapter 5 of this paper along with discussion on

considerations of using this instrument for RSS. For a field screening technique to be an effective ranking method it needs to be a valid indicator of the soil sample relative magnitude. It does not need to be an exact correlation, though that would guarantee perfect judgment in ranking.

The scanning drive over data were evaluated to see if it could be a valid ranking method. The key concerns regarding the use of the scanning data were: 1) that it would not be accurate enough of a measurement given the sample time is essentially just one second per measurement; 2) geospatially, the scan measurements will not match with the soil sample coordinates; and 3) the minimum ranking capability (concept explained in Chapter 2 of this paper) will be too high to allow for the degree of ranking that would be needed. The first and third concerns will be discussed in Chapter 5 of this paper. The second concern was evaluated by comparing magnitudes of the scan data measurement and the soil sample measurement with the closest soil sample coordinates. Table 4.1. below provides the results of this evaluation.

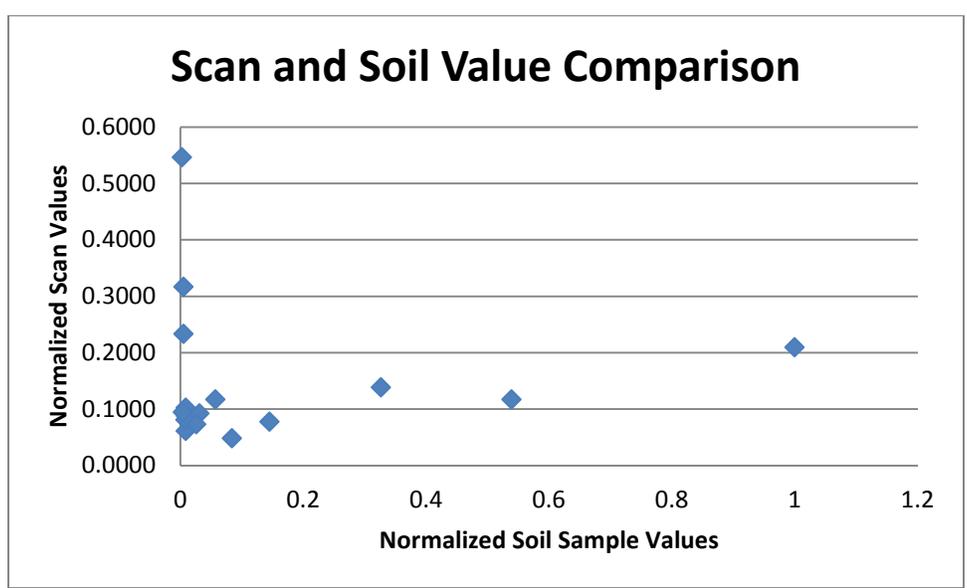


Figure 4.1. Scan data values compared to soil values with the most similar locations

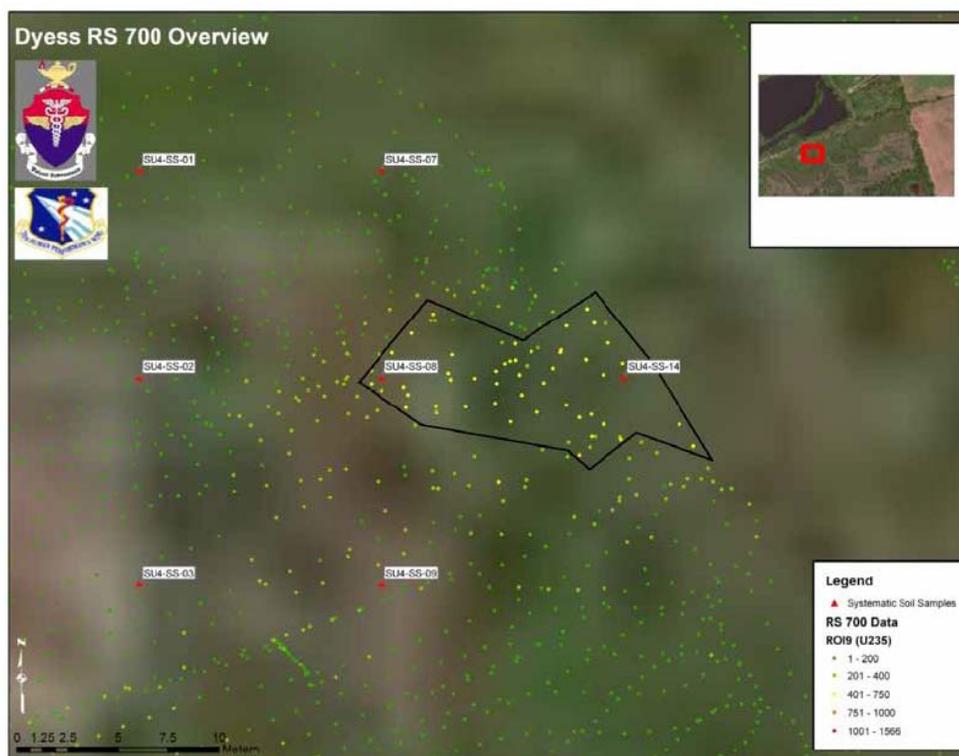


Figure 4.2. Drive over data hot spot with corresponding sample locations, provided by USAF [3]

Figure 4.1 above indicates that two of the systematic soil samples are in one of the drive over data identified areas of interest. Neither of these two random sample locations' soil sample results were elevated as would be anticipated if the data were correlated with a high degree of confidence and the RS-700 accurately captured this hot spot.

The FIDLER was anticipated to be a better ranking method than the RS-700 based off the fact that it has been a suggested technique in the literature and EPA guidance manual and because it is able to provide more accurate measurements. The Dyess survey FIDLER measurements included walkover measurements and static thirty second measurements in AOIs. However, the walk over data were for only a small area (Figure 4.2. below illustrates the area covered by the walk-over data). Additionally, due to an error of the blue tooth connection the data did not download correctly for a large majority of the walk-over data. The large yellow area

of data seen in Figure 4.2. resulted from instrument connection errors, and were artificially high [3]. Given these issues the walkover data were not evaluated as a screening technique for RSS. If the data could be evaluated, a possible issue would be that walkover FIDLER data measurements would not be accurate enough given the short measurement time of each point.



Figure 4.1. FIDLER Data from B-47 Crash Site Survey [3]

The biased sample location FIDLER measurements were taken for thirty seconds at each location. Given the increased sample time, more confidence exists in the accuracy of these results. Figure 4.3. illustrates the correlation of these measurements and the soil sample results for 7 points of comparison. While majority points are near the regression trend line, the highest and second highest FIDLER counts' soil sample measurements are not. While the R square value is only 0.388 for the data set, removal of the highest magnitude soil sample measurement

would give a R square value of 0.9045. The lower R square value indicates there is potential for error in ranking if the FIDLER was used to obtain screening observation measurements in the same manner it was employed in the Dyess survey.

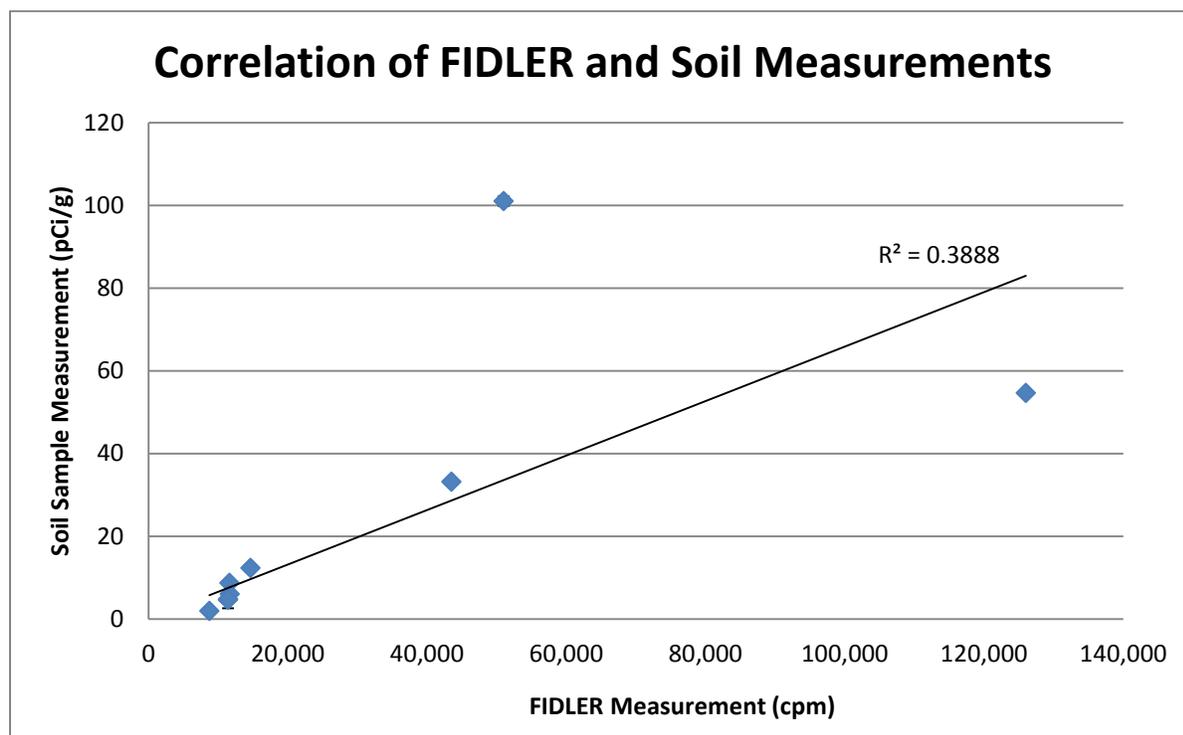


Figure 4.3. Correlation of FIDLER and Soil Measurements

4.1.2. Evaluation of scan data and soil distribution data set values and distributions

The following figure illustrates the area scanned for the systematic soil samples and the locations of the soil samples. Additionally, the figure indicates the few points where mid scanning values and high scanning values were noted. These locations are two of the primary AOIs identified for further evaluation by the survey team. The vast majority of values were considered comparable to background levels.

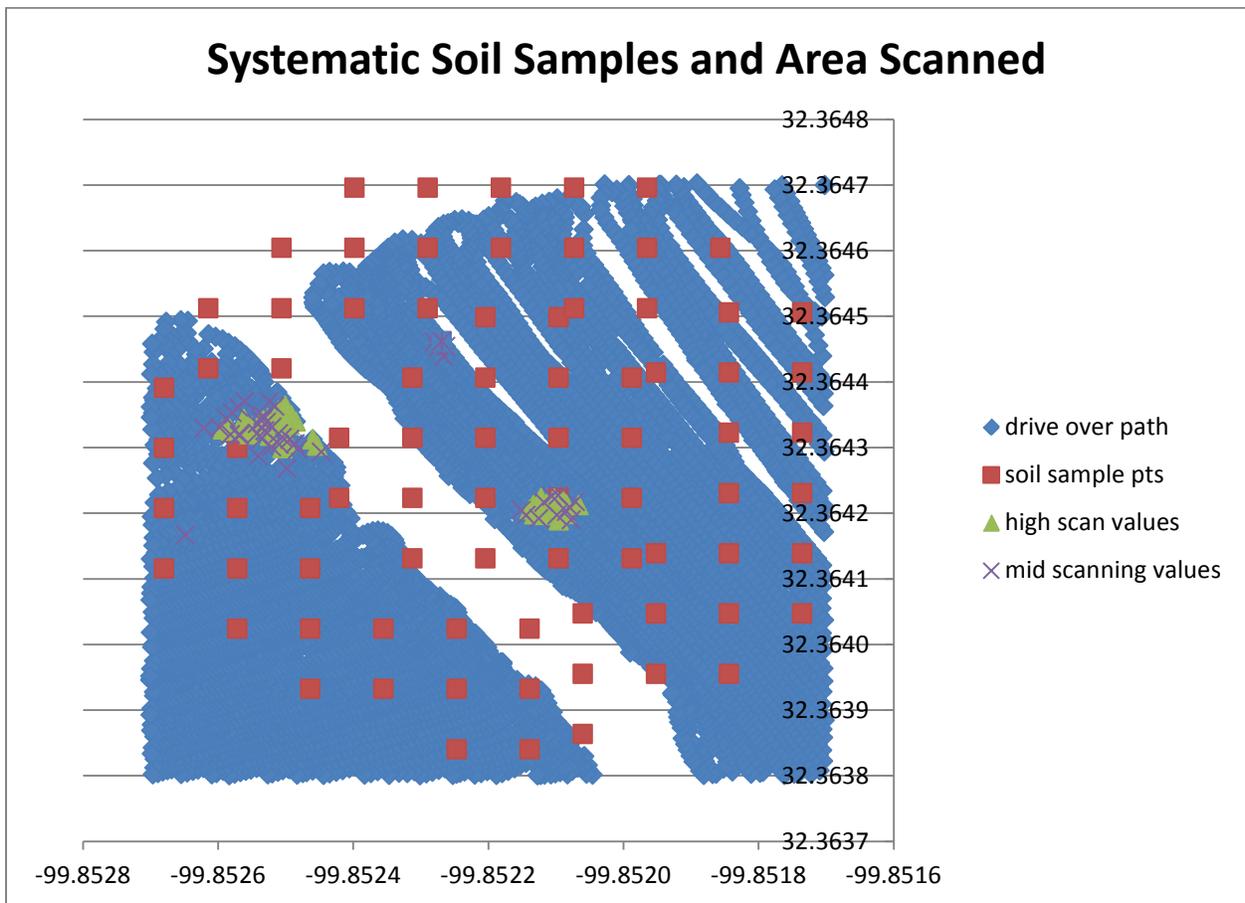


Figure 4.2. below illustrates the normalized scanning data from a three dimensional perspective. Again, the two areas with high values are clearly visible. The majority of the data is below a normalized value of 0.2.

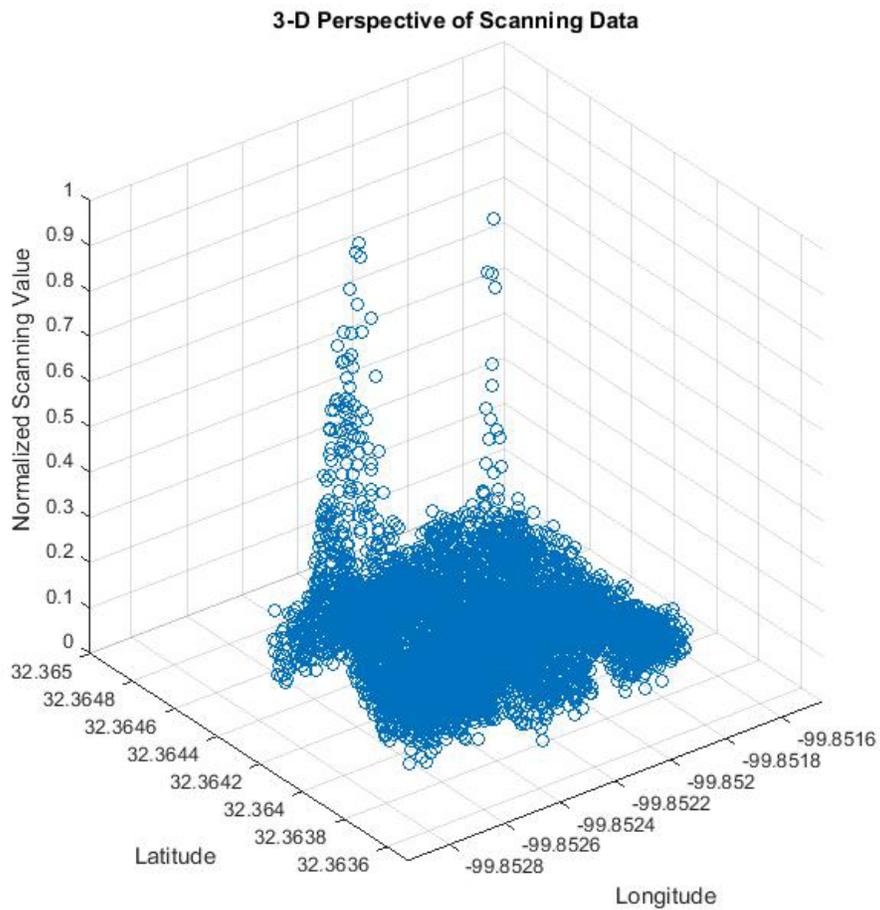


Figure 4.2. Three dimensional perspective of normalized scanning data

The distribution of the drive over data can be visualized by use of a histogram in Figure 4.3. below. Table 4.1 provides the statistical descriptors of this distribution. The specific frequency values for each bin are available in Appendix C.

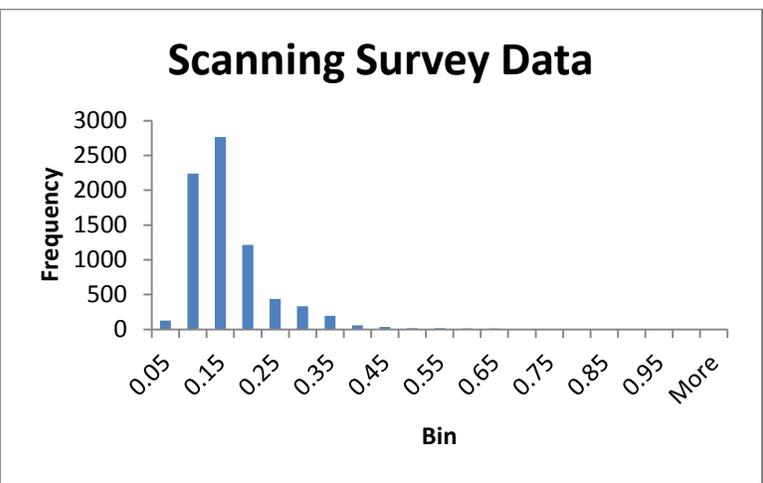


Figure 4.3. Normalized scanning data distribution

<i>Normalized Scanning Data Statistics</i>			
Mean	0.1438	Sample Variance	0.0076
Standard Error	0.0010	Kurtosis	16.9577
Median	0.1205	Skewness	3.1570
Mode	0.0888	Range	1
Standard Deviation	0.0871	Number of Data Pts	7500

Table 4.1. Normalized scanning data distribution statistics

The normalized soil sample data distribution is given in Figure 4.5. below. Table 4.2. provides statistical descriptors for this distribution. Figure 4.6. and Table 4.3. illustrate the distribution with the biased samples removed in comparison to Figure 4.5. and Table 4.2. Removing the biased samples reduces the number of soil samples in the data set to 99. The vast majority of values in both scenarios are below 0.1.

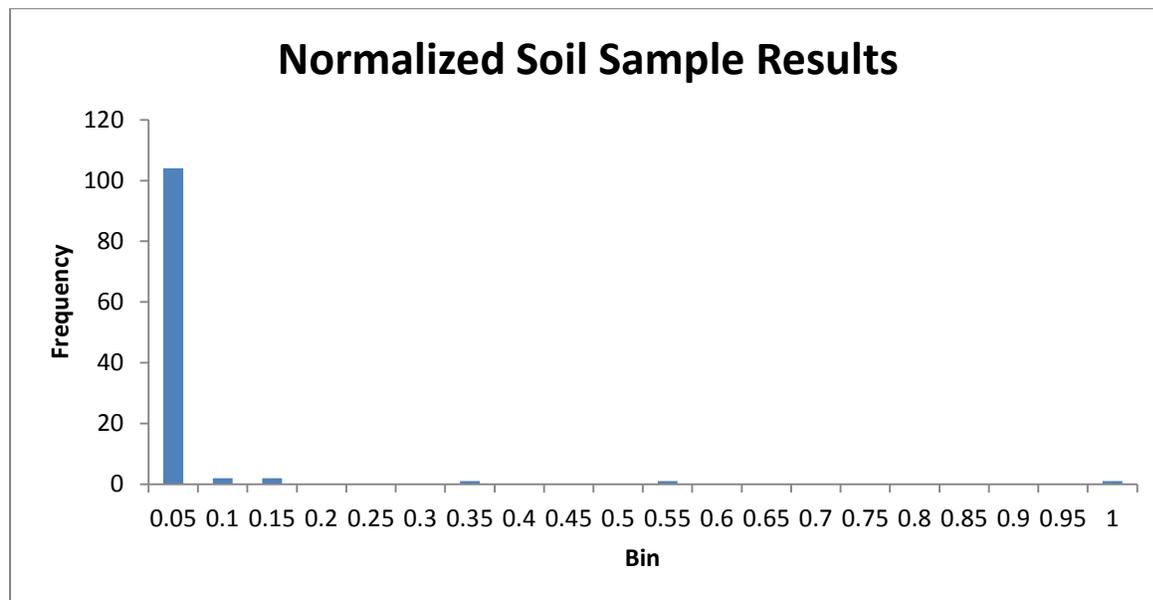


Figure 4.5. Normalized soil sample data distribution

<i>Normalized Soil Sample Data Statistics</i>			
Mean	0.0314	Sample Variance	0.0124
Standard Error	0.0105	Kurtosis	56.9848
Median	0.0093	Skewness	7.1921
Mode	0.0149	Range	1
Standard Deviation	0.1111	Number of Data Pts	111

Table 4.2. Normalized soil sample data distribution statistics

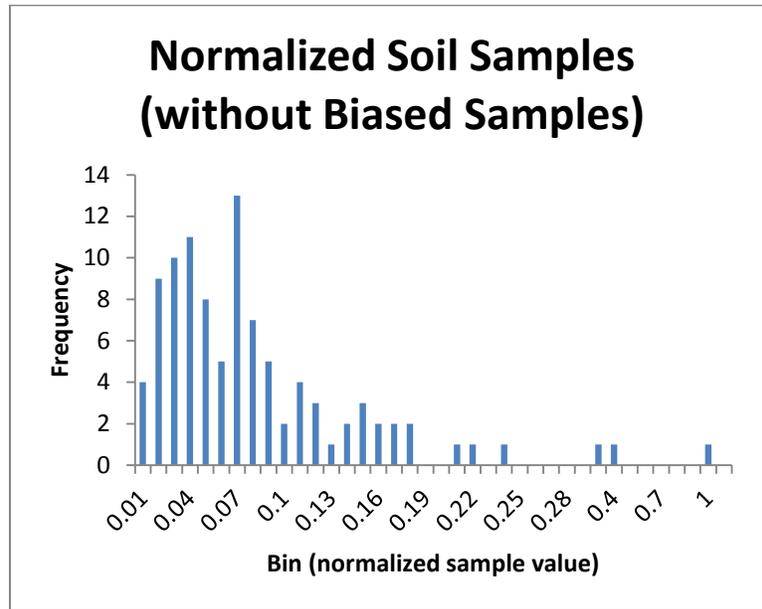


Figure 4.6. Normalized Soil Sample Data Distribution, without biased samples

<i>Normalized Soil Sample Data Statistics- Biased Samples Removed</i>			
Mean	0.08351	Sample Variance	0.01241
Standard Error	0.01120	Kurtosis	47.15397
Median	0.06037	Skewness	6.01803
Mode	0.10297	Range	1
Standard Deviation	0.11141	Number of Data Pts	99

Table 4.3. Statistics of normalized soil sample data without biased samples

4.2 RSS Trial Results

The results of objective 1 and 2 were obtained by using the scan data set as a framework to evaluate RSS applied to four different distributions. The RS-700 scan data set and three modifications were evaluated for $mn \approx 100$ and lower mn values using the Matlab code discussed

in Chapter 3 for 10,000 iterations. As discussed previously, mn is equivalent to the number of analysis measurements (soil samples for this survey). The normalized scan data set was utilized to provide a distribution framework that is representative of a radiological contamination scenario. The purpose of the application of RSS trials to the scan data set was to evaluate the overall effectiveness of RSS compared to SRS in estimating the mean accuracy and precision. This evaluation is more theoretical in nature, and is not dependent on the actual utility of the scan data as a RSS field scanning method as discussed in section 4.1. RSS trials were not applied to the FIDLER data set since it was not robust and did not provide an even coverage of the entire class 1 survey area.

Results are given in this chapter for each set of RSS and SRS trials for the following distribution and mn scenarios:

- 1) Normalized scan data set, $mn=100$
- 2) Scan data set, mn =lower values than 100
- 3) Normal distribution data set, $mn=100$
- 4) Normal distribution data set with larger standard deviation, $mn=100$
- 5) Skewed data set, $mn=100$ and mn =lower values than 100

T tests, with an alpha value of 0.05, were used to evaluate the null hypothesis that the averages of the iteration means were the same. Specifically to assess differences in the means, a t test was performed between the RSS estimated average and true average, SRS estimated average and true average, and RSS and SRS estimated average. The RSS and SRS average of the 10,000 iterations standard deviation results were also compared using a t test with alpha of 0.05. If the t test indicated that these means were different and the iteration average RSS standard deviation was less than that of SRS, then the RSS estimate of sample standard deviation is

improved compared to SRS. For all of the following results, T probability values must be less than 0.05 for the result to be statistically significant.

A RP over 1 indicates that there is less variability in the estimate of the mean over 10,000 iterations for RSS compared to SRS. Less variability in the mean between iterations indicates that RSS is a more efficient method than SRS. If the standard deviation is the value of interest to the surveyor, then the same is true for variability in the estimate of the standard deviation. A lower variability in the standard deviation RSS estimates increases the confidence in the RSS efficiency of estimating the standard deviation.

As discussed in Chapter 3, each iteration was also individually considered using the percent difference between the true and estimated RSS and SRS values of the mean and standard deviation, as if it had been a standalone application. For simplicity in display of the results, they are expressed by the following A through D category designators:

A – Number of individual iterations where RSS estimated the mean with a lower percent difference than SRS

B – Number of individual iterations where RSS estimated the standard deviation with a lower percent difference than SRS

C – Number of individual iterations where RSS estimated both the mean and the standard deviation with a lower percent difference than SRS

D - Number of individual iterations where SRS estimated both the mean and the standard deviation with a lower percent difference than RSS

Overall the results can be evaluated by four conditions as give in Chapter 3, and restated here:

1) Whether the RSS iteration means are statistically different in accuracy for estimating the mean than SRS, and if each was statically different from the true mean; 2) Whether there was a

statistical difference in estimating the standard deviation between RSS and SRS, and if so was the RSS or SRS estimate of the standard deviation closer to the true standard deviation;

3) Whether more individual iterations indicate that RSS estimated the mean and standard deviation with a lower percent difference than SRS did, and 4) Whether the RP is greater than 1, and if so, to what degree.

4.2.1. Trial Results for Scan Data Set, $mn=100$

The Matlab code described in the Chapter 3 was applied to multiple m and n values using the RS-700 scan data set. The results of the first set of trials, where m and n were chosen to give a value of mn as close to 100 as possible, are given in Table 4.4. and Table 4.5 below. Overall, it can be seen in Table 4.4. that RSS is statically the same as SRS for the estimate of the mean.

The lower error in the RSS mean estimator, listed as plus or minus the standard deviation, and the RP values over 1 indicate that RSS is a more efficient method of sampling then SRS.

Scan Data Set Distribution Mean = 0.14377							
m,n	mn	RSS_avg (+/- std)	SRS_avg (+/- std)	T prob RSS_avg to true	T prob SRS_avg to true	T prob RSS_avg to SRS_avg	RP
2,50	100	0.14376 (0.00771)	0.14381 (0.00864)	0.99856	0.996810	0.66658	1.25683
3,33	99	0.14371 (0.00712)	0.14374 (0.00868)	0.99386	0.997368	0.73443	1.48643
4,25	100	0.14382 (0.00673)	0.14370 (0.00863)	0.99531	0.992964	0.24774	1.64333
5,20	100	0.14382 (0.00629)	0.14378 (0.00846)	0.99561	0.998910	0.73314	1.80563
6,16	96	0.14374 (0.00604)	0.14379 (0.00886)	0.99727	0.998301	0.64941	2.15003
7,14	98	0.14388 (0.00584)	0.14377 (0.00873)	0.99035	0.999534	0.28969	2.23543

Table 4.4. Comparison of RSS and SRS in estimating the mean (avg) of the RS-700 scan data set for 10,000 iterations, where $mn \approx 100$

As can be seen in table 4.5 below, m,n of 5,20 and 7,14 t test results for the RSS standard deviation estimator and SRS standard deviation estimator were the only m and n values where the RSS and SRS standard deviation estimates were significantly different. The average standard deviation over 10,000 trials is closer to the true standard deviation for RSS than SRS for both of these combinations. The rest of the RSS and SRS estimators of standard deviation were not statistically different from each other. These results suggest that when considering individual trials, RSS is not always more precise than SRS.

Scan Data Set Distribution Standard Deviation= 0.08712					
m,n	mn	RSS_std (+/- std)	SRS_std (+/- std)	T prob RSS_std to SRS_std	RP_std
2,50	100	0.08536 (0.01781)	0.08522 (0.01801)	0.56920	1.02286
3,33	99	0.08536 (0.01758)	0.08530 (0.01796)	0.83049	1.04368
4,25	100	0.08555 (0.01728)	0.08510 (0.01795)	0.06908	1.07853
5,20	100	0.08585 (0.01719)	0.08521 (0.01768)	0.00887	1.05723
6,16	96	0.08539 (0.01705)	0.08508 (0.01835)	0.21904	1.15846
7,14	98	0.08608 (0.01711)	0.08511 (0.01811)	0.00010	1.11999

Table 4.5. Comparison of RSS and SRS in estimating the standard deviation (std) of the RS-700 scan data set for 10,000 iterations, where $mn \approx 100$

Table 4.6. below displays the results of A-D, as previously defined, for the normalized RS-700 scan data set trials where mn is equal to approximately 100.

m,n	A	B	C	D
2,50	5452	5080	3478	2946
3,33	5653	5051	3546	2842
4,25	5879	5140	3788	2769
5,20	5949	5142	3798	2707
6,16	6339	5330	4107	2438
7,14	6276	5242	4003	2485

Table 4.6. Results of RSS compared to SRS for individual iterations applied to the scan data set, where $mn \approx 100$

4.2.2. Trial Results for Scan Data Set, mn =lower values

The m and n values were lowered for the next set of trials to evaluate the effect this would have on the effectiveness of RSS in estimating the mean, when compared to SRS (with the same number of analysis measurements) and when compared to the RSS trials with mn equal to 100. The number of iterations was again set at 10,000 and the normalized RS-700 scan data set was evaluated. Tables 4.7 through 4.9 display the results of this set of trials. As can be seen in these trials, again there is no statistical difference in the accuracy of the estimate of the mean between RSS and SRS, and when comparing each to the true values. RSS consistently displays a RP over one for the mean, though marginally so for the standard deviation. The magnitude of RP decreases for decreasing values of mn . Trial 5,6 has the highest value of RP.

Scan Data Set Distribution Mean = 0.14377							
m,n	mn	RSS_avg (+/- std)	SRS_avg (+/- std)	T prob RSS_avg to true	T prob SRS_avg to true	T prob RSS_avg to SRS_avg	RP
3, 22	66	0.14382 (0.00866)	0.14368 (0.01075)	0.996495	0.993097	0.322722	1.54196
2, 23	46	0.14369 (0.01134)	0.14401 (0.01293)	0.994959	0.984700	0.065011	1.30019
3, 12	36	0.14350 (0.01188)	0.14355 (0.014618)	0.984219	0.987284	0.605306	1.51401
5, 6	30	0.14362 (0.01129)	0.14389 (0.01608)	0.992180	0.994098	0.180806	2.02743
2, 12	24	0.14373 (0.01570)	0.14379 (0.01757)	0.997764	0.998950	0.769940	1.25242
3, 6	18	0.14393 (0.01655)	0.14381 (0.02033)	0.993537	0.998424	0.254744	1.50948
3, 4	12	0.14340 (0.02036)	0.14374 (0.025228)	0.998833	0.998805	0.292569	1.53529

Table 4.7. Comparison of RSS and SRS in estimating the mean (avg) of the RS-700 scan data set for 10,000 iterations, where $mn \approx$ lower values than 100

Scan Data Set Distribution Standard Deviation= 0.08712					
m,n	mn	RSS_std (+/- std)	SRS_std (+/- std)	T prob RSS_std to SRS_std	RP_std
3, 22	66	0.08490 (0.02088)	0.08397 (0.02115)	0.00171	1.02637
2, 23	46	0.08360 (0.02463)	0.08374 (0.02505)	0.67459	1.03393
3, 12	36	0.08279 (0.026849)	0.08251 (0.027663)	0.465705977	1.00917
5, 6	30	0.08285 (0.02758)	0.08198 (0.02944)	0.03054	1.13958
2, 12	24	0.08096 (0.03132)	0.08082 (0.03147)	0.75736	1.00917
3, 6	18	0.08108 (0.03443)	0.07959 (0.03488)	0.00240	1.02658
3, 4	12	0.07883 (0.039213)	0.07698 (0.040252)	0.00100	1.05366

Table 4.8. Comparison of RSS and SRS in estimating the standard deviation (std) of the RS-700 scan data set for 10,000 iterations, where $mn \approx$ lower values than 100

In table 4.8. above, m,n combinations of (3,22), (5,6), (3,6), and (3,4) have standard deviation estimates that are statistically significant. The average standard deviation over 10,000 trials is closer to the true standard deviation for RSS than SRS for each of these combinations.

m,n	A	B	C	D
3, 22	5678	5095	3632	2859
2, 23	5372	5062	3460	3026
3, 12	5689	5184	3766	2893
5, 6	6073	5242	3999	2684
2, 12	5359	5003	3510	3148
3, 6	5640	5192	3730	2898
3, 4	5746	5205	3783	2832

Table 4.9. Evaluation of RSS compared to SRS for individual iterations for the RS-700 scan data set using 10,000 iterations, where $mn \approx$ lower values than 100

4.2.3. Trial Results for Normal Distribution Data Set #1, $mn=100$

The scan data set was adjusted to evaluate the effect the distribution had on RSS and SRS's estimate of the mean. As mentioned in Chapter 3, the total number of 7,500 points remained consistent. The first adjustment was to change the distribution to a normal distribution that had a standard deviation approximately equal to the RS-700 scan data standard deviation and had a mean of 0.5. This distribution is referred to as normal distribution data set #1. The distribution and corresponding statistics are in Appendix C. The number of analysis measurements, mn , was set at 100 for comparison to the previous results. Tables 4.10 through 4.12 give the results of this set of trials. The trends in the data are very similar to that seen for

the unaltered RS-700 scan data, where the accuracy estimates are statistically the same. RSS consistently displays a RP over one, though again marginally so for the standard deviation.

Normal Distribution #1 Mean = 0.50054							
<i>m,n</i>	<i>mn</i>	RSS_avg (+/- std)	SRS_avg (+/- std)	T prob RSS_avg to true	T prob SRS_avg to true	T prob RSS_avg to SRS_avg	RP
2,50	100	0.50053 (0.00729)	0.50061 (0.00883)	0.99919	0.999996	0.449836	1.46769
3,33	99	0.50050 (0.00635)	0.50056 (0.00875)	0.99680	0.999999	0.619326	1.90114
4,25	100	0.50063 (0.00575)	0.50053 (0.00877)	0.99210	0.999999	0.359580	2.33059
5,20	100	0.50056 (0.00523)	0.50058 (0.00881)	0.99795	0.999997	0.845418	2.83902
6,16	96	0.50063 (0.00500)	0.50069 (0.00900)	0.99232	0.999989	0.567937	3.23307
7,14	98	0.50046 (0.00471)	0.50059 (0.00904)	0.99297	0.999996	0.192125	3.68698

Table 4.10. Comparison of RSS and SRS in estimating the mean (avg) over 10000 iterations for normal distribution data set #1, where $mn \approx 100$

Normal Distribution #1 Standard Deviation= 0.08712						
<i>m,n</i>	RSS_std	(+/- error, 1 std)	SRS_std	(+/- error, 1 std)	T prob RSS_std to SRS_std	RP_std
2,50	0.08894	0.00636	0.08853	0.00638	0.000006	1.004754
3,33	0.08872	0.00620	0.08862	0.00641	0.255050	1.070721
4,25	0.08897	0.00589	0.08862	0.00638	0.000077	1.172863
5,20	0.08890	0.00569	0.08862	0.00642	0.001251	1.270957
6,16	0.08907	0.00556	0.08865	0.00648	0.000001	1.357673
7,14	0.08894	0.00530	0.08862	0.00636	0.000104	1.438937

Table 4.11. Comparison of RSS and SRS in estimating the standard deviation (std) over 10000 iterations for normal distribution data set #1, where $mn \approx 100$

In table 4.11. above, all m,n combinations except for 3,33 have standard deviation estimates that are statistically significant. The average standard deviation over 10,000 trials is not closer to the true standard deviation for RSS than SRS for any of these combinations.

m,n	A	B	C	D
2,50	5575	4949	2805	2281
3,33	6046	5080	3082	1956
4,25	6294	5256	3321	1771
5,20	6585	5394	3557	1578
6,16	6725	5547	3709	1437
7,14	6948	5646	3888	1294

Table 4.12. Evaluation of RSS compared to SRS for individual iterations for normal distribution data set #1, where $mn \approx 100$

4.2.4. Trial Results for Normal Distribution Data Set #2, $mn=100$

The normal distribution data set used for the previous trials was modified to see if having a larger standard deviation would impact the effectiveness of RSS over SRS. The results of the trials where the standard deviation was increased to approximately three times greater than that of the RS-700 scan data set is given in Table 4.13 through Table 4.15 below. General trends in this data are comparable to those previously noted. One deviation was that the t test of 3,33 indicated that the RSS estimate and SRS estimate of the mean are statistically different. Discussion on the comparison of these results to those of normal distribution data set #1 is given in Chapter 5.

Normal Distribution #2 Distribution Mean = 1.00336						
m,n	RSS_avg (+/- std)	SRS_avg (+/- std)	T prob RSS_avg to true	T prob SRS_avg to true	T prob RSS_avg to SRS_avg	RP
2,50	1.00370 (0.02165)	1.00369 (0.02637)	0.98997	0.99997	0.963696	1.48372
3,33	1.00356 (0.01930)	1.00290 (0.02649)	0.99413	0.98618	2.8727E-10	1.88474
4,25	1.00365 (0.01701)	1.00355 (0.02648)	0.99145	0.99998	0.749676	2.42502
5,20	1.00313 (0.01550)	1.00335 (0.02627)	0.99322	1.00000	0.457402	2.87330
6,16	1.00318 (0.01482)	1.00321 (0.02685)	0.99481	0.99999	0.930488	3.28444
7,14	1.00332 (0.01389)	1.00345 (0.02681)	0.99896	0.99999	0.678339	3.72592

Table 4.13. Comparison of RSS and SRS in estimating the mean (avg) over 10000 iterations for normal distribution data set #2, where $mn \approx 100$

Normal Distribution #2 Standard Deviation= 0.26530						
m,n	RSS_std	(+/- error, 1 std)	SRS_std	(+/- error, 1 std)	T prob RSS_std to SRS_std	RP_std
2,50	0.26523	0.01867	0.26469	0.01861	4.19E-02	0.99422
3,33	0.26536	0.01801	0.26458	0.01893	3.07E-03	1.10501
4,25	0.26574	0.01700	0.26456	0.01859	3.05E-06	1.19679
5,20	0.26549	0.01653	0.26475	0.01862	2.83E-03	1.26867
6,16	0.26562	0.01615	0.26475	0.01913	5.02E-04	1.40336
7,14	0.26592	0.01551	0.26466	0.01909	3.17E-07	1.51514

Table 4.14. Comparison of RSS and SRS in estimating the standard deviation (std) over 10000 iterations for normal distribution data set #2, where $mn \approx 100$

In table 4.14. above, all m,n combinations have standard deviation estimates that are statistically significant. Unlike normal distribution #1, the average standard deviation over 10,000 trials is closer to the true standard deviation for RSS than SRS for any of these combinations.

m,n	A	B	C	D
2,50	5680	4978	2853	2195
3,33	5963	5166	3023	1894
4,25	6353	5293	3349	1703
5,20	6560	5451	3565	1554
6,16	6745	5564	3776	1467
7,14	6974	5627	3919	1318

Table 4.15. Evaluation of RSS compared to SRS for individual iterations; applied to normal distribution data set #2, where $mn \approx 100$

4.2.5. Trial Results for Skewed Data Set, $mn=100$ and mn =lower values

The last theoretical modification to the original RS-700 scan data set was to skew the data stronger to the right. Such a distribution would have been plausible if there had been more areas of contamination. The skewed data histogram and statistical descriptors are presented in Appendix C. Table 4.16 through Table 4.18 illustrate the results of RSS applied to this distribution for various m and n values, and 10,000 iterations. The results given in these tables are comparable to the previous results. Again, RSS estimators have statically the same accuracy as SRS estimators and the RP is greater than 1 with 5,6 having the highest magnitude of RP.

Skewed Data Set Distribution Mean =0.205213							
m,n	mn	RSS_avg (+/- std)	SRS_avg (+/- std)	T prob RSS_avg to true	T prob SRS_avg to true	T prob RSS_avg to SRS_avg	RP
2,50	100	0.205255 (0.01489)	0.205342 (0.01691)	0.998086	0.994000	0.697312	1.290805

3,33	99	0.205078 (0.01377)	0.205303 (0.01700)	0.993728	0.995827	0.303466	1.523849
4,25	100	0.205317 (0.01283)	0.205349 (0.01695)	0.995171	0.993699	0.883011	1.74582
5,20	100	0.205188 (0.01217)	0.204974 (0.01717)	0.998833	0.988823	0.307874	1.990813
3,12	36	0.205244 (0.02309)	0.20514 (0.02836)	0.999142	0.997895	0.776627	1.508456
5,6	30	0.143623 (0.082849)	0.143886 (0.03098)	0.998304	0.996012	0.819830	2.027429
4,5	20	0.205388 (0.02887)	0.205387 (0.03866)	0.996308	0.996236	0.999589	1.792996
3,3	9	0.204679 (0.04010)	0.204914 (0.04871)	0.991009	0.994836	0.710179	1.476027

Table 4.16. Comparison of RSS and SRS in estimating the mean (avg) using 10,000 iterations applied to the skewed data set with various m,n values

Skewed Data Set Distribution Standard Deviation = 0.171219							
m,n	mn	RSS_std	(+/- error, 1 std)	SRS_std	(+/- error, 1 std)	T prob RSS_std to SRS_std	RP_std
2,50	100	0.169998	0.02315	0.169701	0.02360	0.367830739	1.03899
3,33	99	0.169515	0.02273	0.169697	0.02367	0.580108937	1.08428
4,25	100	0.17025	0.02181	0.169731	0.02368	0.107108059	1.17807
5,20	100	0.170296	0.02160	0.169587	0.02416	0.028735471	1.25057
3,12	36	0.167806	0.03809	0.166401	0.03997	0.010978503	1.10149
5,6	30	0.082849	0.03943	0.081976	0.04377	5.28773E-07	1.23184
4,5	20	0.165933	0.04953	0.162539	0.05369	3.40682E-06	1.17497
3,3	9	0.160353	0.06551	0.15662	0.06783	7.58442E-05	1.07218

Table 4.17. Comparison of RSS and SRS in estimating the standard deviation (std) using 10000 iterations applied to the skewed data set with various m,n values

In table 4.17. above, all m,n combinations have standard deviation estimates that are statistically significant except for 2,50, 3,33, and 4,25. Unlike normal distribution #1, the average standard deviation over 10,000 trials is closer to the true standard deviation for RSS than SRS for any of these combinations.

m,n	A	B	C	D
2,50	5388	5026	3672	3258
3,33	5663	5128	3885	3094
4,25	5930	5223	4002	2849
5,20	6130	5323	4244	2791
3,12	5616	5156	3859	3087
5,6	6073	5242	3999	2684
4,5	5915	5303	4098	2880
3,3	5635	5179	3922	3108

Table 4.18. Evaluation of RSS compared to SRS for individual iterations, applied to the skewed distribution data set and various m,n values

The modified Matlab code was used to apply one of Kaur (1997)'s rule of thumb for unequal allocation (concept explained in Chapter 2 of this document) to the skewed data set. Specifically, the method of quantifying the highest rank t_{opt} times more than equal allocation was utilized. A value of $t_{opt} = 2$ was selected based on Figure 2.3 for a CV of 0.8 for the skewed distribution data set. The same m and n values were used as listed above for the equal allocation skew data trials, with 10,000 iterations. The results are provided in Table 4.19. through 4.21. below. Overall, the marked difference of these results compared to Table 4.16. through 4.17. is that SRS is a better estimator of both the mean and standard deviation than RSS. The only deviation from this observation is for $m,n=5,6$ and $4,5$.

Skewed Data Set Distribution Mean = 0.2052130							
m,n	mn	RSS_avg (+/- std)	SRS_avg (+/- std)	T prob RSS_avg to true	T prob SRS_avg to true	T prob RSS_avg to SRS_avg	RP
2,50	100	0.23175 (0.01357)	0.20523 (0.01369)	0.157046231	0.99932	0.00000	1.01805
3,33	99	0.23942 (0.01396)	0.20521 (0.01491)	0.081561992	0.99999	0.00000	1.14068
4,25	100	0.24151 (0.01344)	0.20529 (0.01525)	0.068762353	0.99664	0.00000	1.28713
5,20	100	0.24177 (0.01295)	0.20526 (0.01552)	0.53782	0.99928	0.00000	1.43441
3,12	36	0.23921 (0.02312)	0.20513 (0.02446)	0.29343	0.99756	0.00000	1.11886
5,6	30	0.24142 (0.02382)	0.20499 (0.02841)	0.32398	0.99426	0.00000	1.42304
4,5	20	0.24216 (0.03074)	0.20535 (0.03461)	0.40762	0.99699	0.00000	1.26783
3,3	9	0.23917 (0.04681)	0.20482 (0.04920)	0.59589	0.00000	0.00000	1.10457

Table 4.19. Comparison of RSS and SRS in estimating the mean (avg) by use of Kaur (1997) rule of thumb for unequal allocations using 10000 iterations, applied to the skewed data set with $t_{opt}=2$, and various m,n values

Skewed Data Set Distribution Standard Deviation = 0.171219							
m,n	mn	RSS_std	(+/- error, 1 std)	SRS_std	(+/- error, 1 std)	T prob RSS_std to SRS_std	RP_std
2,50	100	0.18509	0.018103	0.17015	0.019105	0.00000	1.11383
3,33	99	0.19244	0.018962	0.16991	0.020836	0.00000	1.20750
4,25	100	0.19631	0.018922	0.16990	0.021112	0.00000	1.24482
5,20	100	0.19902	0.01899	0.16987	0.02184	0.00000	1.32302
3,12	36	0.19089	0.03155	0.16759	0.03453	0.00000	1.19801
5,6	30	0.19737	0.03518	0.16641	0.03976	0.00000	1.27751
4,5	20	0.19489	0.04374	0.16454	0.04831	0.00000	1.21976
3,3	9	0.18436	0.06495	0.15680	0.06823	0.00000	1.10359

Table 4.20. Comparison of RSS and SRS in estimating the standard deviation (std) by use of Kaur (1997) rule of thumb for unequal allocations using 10000 iterations, applied to the skewed data set with $t_{opt}=2$, and various m,n values

m,n	A	B	C	D
2,50	1529	4251	1138	5358
3,33	865	3655	707	6187
4,25	673	3176	555	6706
5,20	644	2998	544	6902
3,12	2825	4753	2082	4504
5,6	2982	4653	2233	4598
4,5	3719	5030	2727	3978
3,3	4657	5371	3366	3338

Table 4.21. Evaluation of RSS compared to SRS for individual iterations, applied to the skew data with use of Kaur rule of thumb

Chapter 5 – Discussion

5.1 Discussion of B-47 Crash Site Data

The Dyess B-47 crash site data proved to be useful for the consideration of RSS to a radiological contamination scenario with a low energy gamma emitter as the contaminant and a limited amount of elevated measurements. For this study the gamma emitter was U-235 and U-238, and the study indicated that there were two primary area of concern with elevated scan data measurements in survey units 1-5. The following discussion first provides a consideration on objective 3, the application of RSS using the field techniques from the Dyess survey. The second section discusses the comparison of the scan data set and the soil sample data set to evaluate whether the scan data set is representative of the radiological contamination of the site. This consideration is important to be able to extend the conclusions of RSS applied to the scan data set to the soil sample data set, since the latter was not used in the RSS trials.

5.1.1. Discussion of field data techniques

The RS-700 manufacturer's promotion that the RS-700 "*provide[s] laboratory levels of spectral performance*" and that "*the system has a built-in GPS receiver, which enables it to accurately locate each measurement taken*" (excerpts from quote cited in Chapter 2 of this paper) may lead a surveyor to consider the RS-700 as a potential screening method for RSS [16]. Based on the evaluation of the Dyess data, the RS-700 did not provide results that displayed any clear relative correlation to the soil data measurements (per Figure 4.1.). As a result, it would not be an appropriate screening technique for RSS in a field survey with conditions similar to those expressed in this study.

Even if the relative correlation had been notable, there are additional concerns with using the RS-700 scanning data as a screening method. One such concern is that the RS-700 can result in artificially high or low measurements due to changes in geometry of the detector in relation to the surface being scanned. An additional evaluation of the consistency of the surface prior to use of the RS-700 would be required to mitigate this concern. For example, a smooth dirt surface (unlikely to occur) or a surface such as concrete would help eliminate this concern.

As seen in Figure 4.3., the static FIDLER results showed significantly more promising correlation data to soil sample results that indicate this detector could be used as an improved judgment technique, albeit not a perfect one. Two considerations that could impact FIDLER data correlation are the degree of homogeneity in the area of contamination and the degree of disparity between areas of contamination. These considerations are dependent on the location of the FIDLER measurement in respect to the contamination. Since the FIDLER may detect nearby contamination that is not in the soil sample, different FIDLER measurements could result from the same level of soil contamination depending on the degree of contamination surrounding the soil sample (prior to taking the sample). Figure 5.1. below illustrates these considerations visually for a contamination distribution where there is one larger area of contamination and one small hot spot in the survey unit. Dependent on contaminant levels, FIDLER measurement 1 could result in being erroneously higher than FIDLER measurement 2 and FIDLER measurement 3 could be higher than FIDLER measurement 4, when in reality the soil measurement indicates they should not be higher. This issue is less likely to happen for measurements 5 and 6 that are in a more homogenous area. This example may be indicative of Figure 4.3., where the higher FIDLER measurements showed less correlation with the lower measurements.

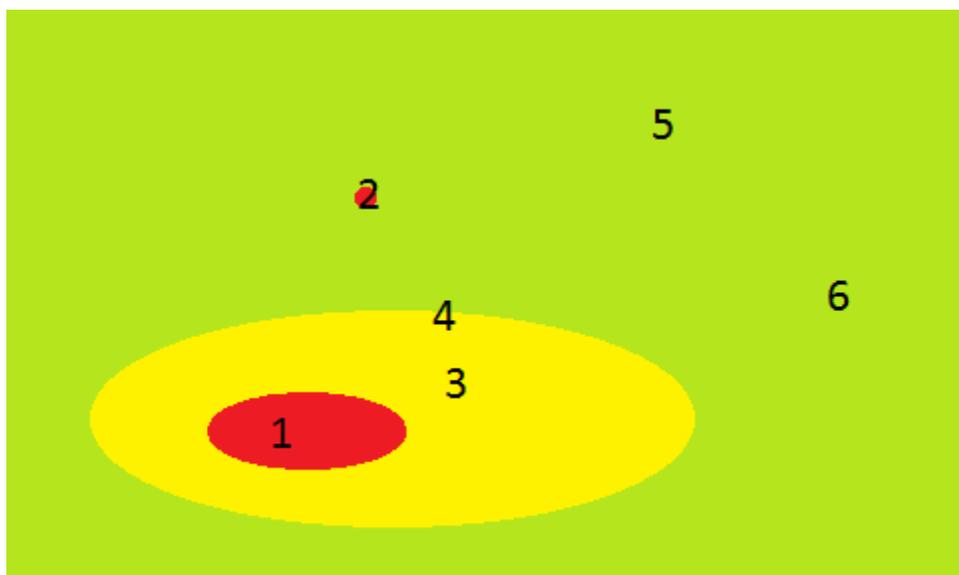


Figure 5.1. Illustration of scenarios that could result in relative correlation errors for FIDLER measurements compared to respective soil measurements

To prevent this FIDLER measurement disparity, a simple shield device could be created that would surround the soil sample area (possibly even insert into soil) to help ensure that the soil being measured is only that intended to be collected for the soil sample. Alternatively, the soil could be measured by the FIDLER after it is removed in a manner similar to that proposed by Vitkus (2012) and discussed in Chapter 2 of this paper.

A few ex-situ gamma spectroscopy field measurements were also obtained for the Dyess survey. Though not enough samples were obtained to do a thorough evaluation of correlation, this method is also a promising technique for use as a screening observation method. Figure 5.1 below illustrates the excellent resolution in the spectrum obtained by an Ortec high purity germanium (HpGe) handheld detector. Multiple photo energy emissions from U-235 are clearly indicated in this spectrum. Potentially something as simple as the height of the 187.5 keV gamma ray, emitted at a frequency of 54% for U-235, could be used as the judgment value. A drawback with this method is that these detectors are very expensive. Additionally, the impact

of surrounding contamination being detected as part of the sample could also affect the overall judgment ability of this technique. As discussed for the FIDLER, this latter concern can be mitigated by appropriate technique.

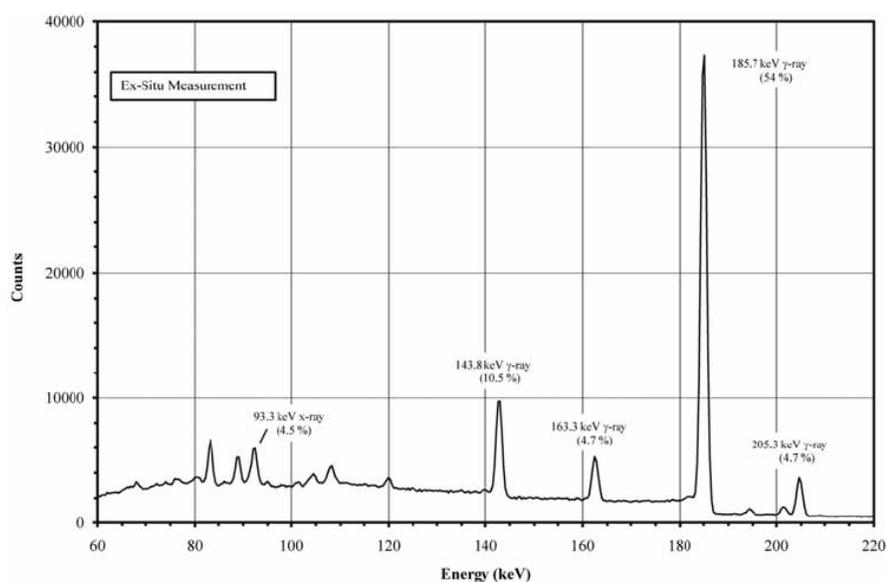


Figure 5.2. OreteR Tran-SPEC-DX-100 HpGe ex-situ gamma spectroscopy measurement of biased soil sample number SU1-BS-04, provided in USAF report [3]

5.1.2. Discussion of scan data and soil distribution data set values and distributions

Both the scan data and soil distribution data sets had a relatively low coefficient of variation and have a notable skew compared to a normal distribution. Additionally, since both distributions (scan 84%, soil 96%, soil no bias samples 93%), (lognormal scan 71%, soil 80%) had more than 65% of the data fall within plus or minus one standard deviation, they are not approximated well by a normal distribution. Specifically, the RS-700 scan data set and the soil data set had 84% and 96% of their data, respectively, within plus or minus one standard deviation. The soil data set with the biased samples removed had 93% of its data within one standard deviation. The reason for this is that there were outliers in all these data sets. When

converted to lognormal distributions the scan data set had 71% of its data within one standard deviation and the soil data had 80% of its data within one standard deviation. Though these lognormal data sets were not used in this evaluation of RSS, it is possible that screening observation data sets could be converted to lognormal form before applying the ranking and selecting the analysis measurement locations. Illustrations of the lognormal data sets are provided in Figure 5.3. below.

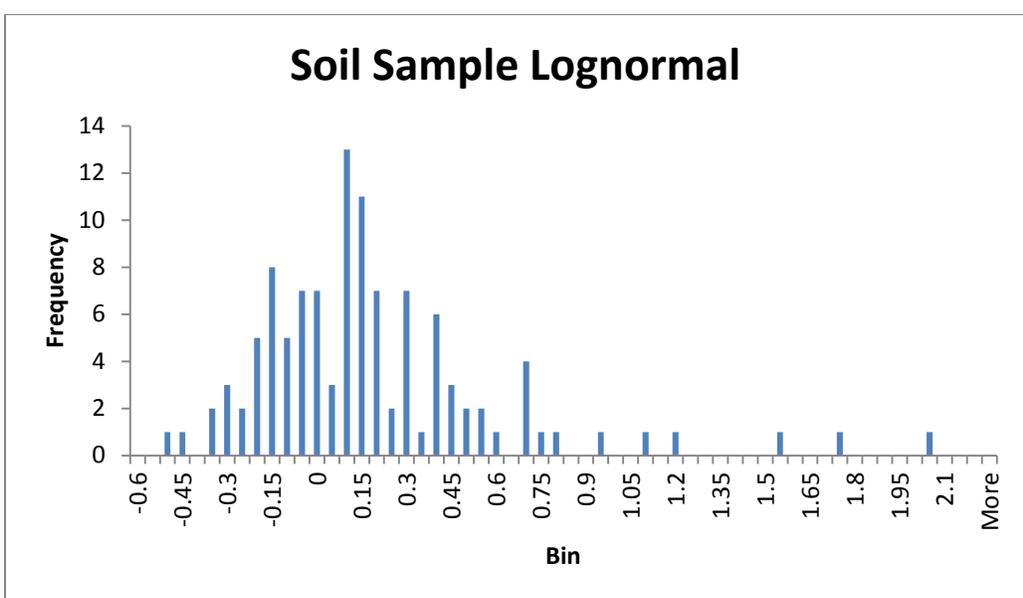
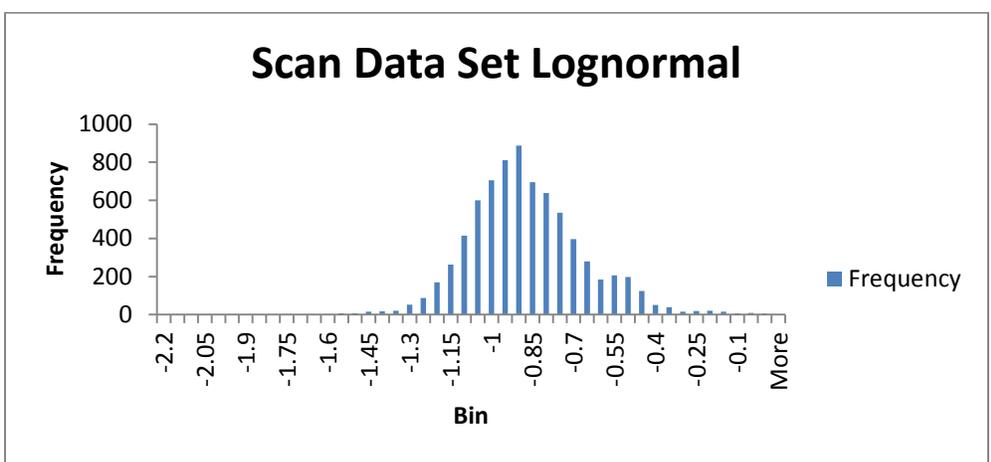


Figure 5.3. Illustration of RS-700 Scan Data Set and Soil Sample Set displayed as a lognormal distribution

Compared to a normal distribution skewness of 0, the scan data and soil data set's skewness were 3.16 and 7.19, respectively. When compared to each other, the standard deviation of the soil sample data were marginally greater than that of the scan data at 0.111 and 0.0871. However, as these two data sets were comparable, the assumption was made that the scan data distribution approximates the soil distribution, which is representative of the true distribution.

5.2 Discussion of RSS Trial Data

To evaluate objectives 1 and 2 of this paper, the RSS trial performance was compared to that of SRS and the results given in Chapter 4 were obtained. For RSS to most evidently be a better technique than SRS in estimating the mean it should display an increased precision and equivalent accuracy. As initially explain in Chapter 3, and with results given in Chapter 4, four aspects were considered to evaluate if RSS provided an improved estimation of the mean and standard deviation when compared to SRS for 10,000 iterations. These aspects are: 1) Whether the RSS iteration means are statistically different in accuracy for estimating the mean than SRS, and if each was statically different from the true mean; 2) Whether there was a statistical difference in estimating the standard deviation between RSS and SRS, and if so was the RSS or SRS estimate of the standard deviation closer to the true standard deviation; 3) Whether more individual iterations indicate that RSS estimated the mean and standard deviation with a lower percent difference than SRS did, and 4) Whether the RP is greater than 1, and if so, to what degree. The RP is a comparison of the variability of the mean for RSS and SRS over the 10,000 trials. The RP of the standard deviation estimates were also provided for comparison.

Of these conditions, the third provides a qualitative assessment of iteration performance. The second is provided as a perspective for comparison of RSS and SRS to the standard deviation estimate. The first and the last carry the most weight in consideration of the overall performance in RSS in estimating the mean. If RSS is not as accurate as SRS than RSS would not be a preferred technique. The variation between the 10,000 iteration trial means is an indicator of the precision in the RSS mean estimate. If this value is less than that of SRS (RP greater than 1), then RSS is a more precise estimator of the mean than SRS. While the RP of the standard deviation is given, the complete evaluation of the accuracy and precision of the standard deviation estimate are not the primary objectives of this paper. The RP of standard deviation were provided as a comparison to the SRS values, per the literature it was anticipated they would be lower than the SRS values and the results of this paper support that aspect. The RP values (of the mean) were also evaluated to see if they were comparable between lower and higher values of mn .

Overall, the relative precision (RP) results consistently indicated that RSS performed better than SRS per condition 3 above. The RP was over 1 for the vast majority of trials, indicating there was less variability in estimating the average by RSS than SRS. Evaluation of conditions 1 and 2 also supported RSS as a more accurate technique than SRS for balanced RSS overall, but not as evidently as seen for condition 3. In general, the percent accuracy was only improved by about 1% for RSS than SRS (for the balanced RSS). Given that the RP results had more variability between the different trials, the RP was a better comparison tool.

5.2.1. Discussion of Trial Results for Scan Data

RSS had equivalent accuracy to SRS in estimating the mean of the scan data set for the first set of trials (normalized scan data set unmodified, $mn \approx 100$). For all scan data set trials, over

10,000 iterations, the RSS and SRS mean estimator were not statistically different from the true population mean (mean of the 7500 data points) or statistically different from each other. This observation held true for lower m,n values. This validated RSS as an unbiased estimator of the mean, as is SRS.

Since in the field 10,000 iterations would not be applied, the evaluation of the performance of each iteration individually is an additional evaluator of the success of RSS to SRS. From Table 4.6., it can be seen that RSS estimated the mean and standard deviation more accurately than SRS for more than half of the iterations for each trial (column A and column B, respectively). However, the degree above 50% is not high, particularly for the standard deviation. The largest percentage of iterations where RSS estimated the mean more accurately than SRS is 63.39% ($m,n=6,16$) and the largest percentage of iterations with the standard deviation estimated more accurately for RSS than SRS is 53.30% ($m,n=6,16$). For $m,n=6,16$, there were 4,107 iterations out of 10,000 where both the mean and standard deviation were estimated more accurately by RSS than SRS (column C). The trial with the lowest number of iterations where both the mean and standard deviation were estimated more accurately by RSS than SRS (column D) was m,n of 2,50 with 3,478 iterations. Over the various trial sets, SRS estimated both the mean and standard deviation more accurately than RSS between 2,438 trials out of 10,000 ($m,n=6,16$) to the maximum of 2,946 trials out of 10,000 ($m,n=6,16$). When considering the results of the individual trials (condition 2) it is evident that RSS overall performed better than SRS. However, this success is still marginal until higher values of m are used. The increase of m is limited in real applications since the total number of scanning observations increases by m^2 , thereby increasing the cost and time of screening, which may not be practical in comparison to accepting the marginally less precise SRS technique.

The relative precision (RP), was also calculated for the second set of trials by use of equation 1. In line with the expectation referenced by the literature, the RP was greater than one for all combinations of m and n indicating that the variation in the iteration mean is less than that of SRS. However, the RP values were also compared to each other with consideration of their upper bounds as given theoretically in the literature. The RP values in Table 5.1. below are listed with their upper bound for their m value given by equation 2, and the ratio is provided for comparison to each other. When considering RP values as they are initially given, increasing the value of m results in a higher RP (for a consistent mn). However, as shown in Table 5.1. below, the higher values of n with lower values of m have greater ratios of RP to the upper bound of the RP.

m,n	RP	RP upper bound	Ratio of RP to upper bound
2,50	1.25683	1.5	0.837887691
3,33	1.48643	2	0.743214801
4,25	1.64333	2.5	0.657330309
5,20	1.80563	3	0.601878005
6,16	2.15003	3.5	0.61429408
7,14	2.23543	4	0.558858197

Table 5.1. RP comparison for m,n combinations where $mn \approx 100$ for the scan data set trials

As mentioned, for the second set of trials (where lower mn values were applied to the scan data set), again RSS was equivalent in accuracy when compared to SRS. In addition, the RSS percent difference results of lower mn values were compared to the RSS and SRS results of the higher mn values (first set of trials). Under this consideration, the results of the second set of

trials indicate that RSS was a less accurate estimator of the mean for lower values of m and n by approximately 3-6 percent, when compared to RSS results with higher mn results of the first set of trials.

The RP for the mean for the lower iterations are mostly about 1.5. For a FSS validation survey, low values of mn similar to these would be used. An additional benefit the surveyor should consider, though not specifically analyzed in this study, is that by having more screening observation locations the probability of detecting a hot spot that was not remediated is increased.

The t-tests performed between the mean RSS and SRS iteration standard deviation indicated that there was a statistical difference between the majority of the trials. Overall, values of the iteration average standard deviation percent difference are much lower than values of mean percent difference for both RSS and SRS, indicating that that RSS standard deviation estimate does not perform as well as the RSS mean estimate. The lowest of these values are in the 60% range for the lowest m,n values and the highest is about 80% for $m,n=3,22$. While the RP_{std} is greater than one, it is only greater by 0.13 at most for $m,n=5,6$. Additionally, the number of individual iterations (condition 2) that estimated the standard deviation more accurately by RSS than by SRS was only greater than 50% of the iterations (5,000) by a couple hundred or less. The observation that the standard deviation is not estimated as well by RSS as the mean is supported by the literature for the variance, which is comparable to the standard deviation if the distribution is estimated as a normal distribution. Given this, if the surveyor is most concerned with estimating the standard deviation, as in the scenario of a characterization survey, RSS may not be a very beneficial technique to apply.

When considering the individual iterations of the second trial (lower mn), a value of m,n equal to 5,6 had the best performance for RSS compared to SRS. This combination had the

highest number of iterations, for RSS compared to SRS, that had better estimators of the mean (6073), standard deviation (5242), and had the highest number of iterations (3,999) where the RSS estimators of both the mean and standard deviation were better than SRS. For condition 2, when $m,n=5,6$ is compared to the best m,n combination of the first trial ($m,n=6,16$), the number of iterations with better estimators of the mean is only less by 266 iterations. The standard deviation (for the same comparison) is only 88 iterations less. Additionally, $m,n=5,6$ did comparably well to $m,n=6,16$ for the number of iterations where both the estimated mean and standard deviation were better than SRS (the latter was better by 108 iterations). Based on this consideration, a m,n of 5,6 could be used as almost as good an estimator as 6,16 but with substantially less screening measurements (150 compared to 576) and sample measurements (30 compared to 96).

The relative precision (RP) of the mean, was also calculated for the second set of trials by use of equation 1. The RP ranged from 1.25 to ($m,n=2,12$) to 2.03 ($m,n=5,6$). The RP values in table 5.2. below are again listed with their upper bound for their m value given by equation 2, and the ratio is provide for comparison to each other. From this table, the $m=2$ values provide the best RP performance (condition 3). For condition 1, $m,n=2,23$ had a higher RSS percent accuracy of the mean than $m,n=2,12$. However, $m,n=5,6$ was comparable to $m,n=2,23$. Additionally, the $m=2$ combinations resulted in the lowest number of iterations where RSS estimated the mean better than SRS. This suggests that the best RP does not necessarily correspond to the best accuracy of the RSS estimation of the mean by conditions 1 and 2, making the evaluation of the optimal m,n combination more challenging.

m,n	RP	RP upper bound	Ratio of RP to upper bound
3, 22	1.54196	2	0.77098
2, 23	1.30019	1.5	0.866793
3, 12	1.51401	2	0.757005
5, 6	2.02743	3	0.67581
2, 12	1.25242	1.5	0.834947
3, 6	1.50948	2	0.75474
3, 4	1.53529	2	0.767645

Table 5.2. RP and comparison to upper bounds for selected values of m and n for the scan data set trials

One more noteworthy aspect from the comparison between the first and second set of trials is the observation that the RSS estimate of the mean by use of lower values of mn can provide as accurate an estimate as the SRS estimate of the mean by use of higher values of mn . To illustrate this, consider the example of $m,n=3,22$ and $m,n=3,33$. From the previous set of trials it was noted that $3,33$ had a RP of about 1.5. The respective values for $3,22$ are also 1.5 for the RP. This means that the surveyor can take 66 analysis measurements instead of 99, for about a 33% reduction in requirements, and still obtain the same level of RP. Perhaps the number of scanning observations to achieve this (198) is cost prohibitive for the surveyor or his requirement is a smaller number of analysis measurements. If this is the situation, the same comparison holds true between smaller m,n values. The RP for 30 analysis measurements ($m,n=5,6$) is comparable to that of 18 analysis measurements ($m,n=3,6$). This provides the surveyor a reduction in requirements of 40%.

5.2.2. Discussion of Trial Results for Modified Scan Data

The scan data set distribution was modified to further evaluate the comparison of RSS and SRS against different data distributions; specifically for normal distributions and skewed distributions. As before, both normal distributions had equivalent accuracy between RSS and SRS.

The estimate of the mean by RSS was improved over SRS for 200 to 500 iterations more than the same m,n combinations in the first scan data trial. The performance of the individual iteration estimates of the standard deviation were not overall significantly different than the previous trials. As can be seen by comparing values in column B of Table 4.11 with Table 4.6., m values of 6 and 7 are the only ones that display a notable increase. $m,n=2,50$ actually indicates that SRS performs better or as good as RSS. Column C results are notably lower in Table 4.11 than Table 4.6 for all m,n combinations assessed, particularly for the three lowest m values.

Results show benefit to RSS compared to SRS in estimating the mean when considering the RP values. The RP values are all above 1 and Table 5.3. below shows how they are all very close to their upper bounds. These RP results are the highest seen yet in this study.

m,n	RP	RP upper bound	Ratio of RP to upper bound
2,50	1.25683	1.5	0.98915
3,33	1.48643	2	0.94237
4,25	1.64333	2.5	0.97001
5,20	1.80563	3	0.95777
6,16	2.15003	3.5	0.93841

7,14	2.23543	4	0.93148
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Table 5.3. RP and comparison to upper bounds for selected values of m and n for normal distribution #1 trials

The same m,n values where $mn \approx 100$ were applied to a normal distribution with a standard deviation of 0.265 (compared to the previous 0.0888) to see if a wider distribution range would indicate differences to the observations made thus far. Interestingly, the results of Table 4.13 through Table 4.15 were very comparable to the previous set of trials for normal distribution #1, and hence lead to similar conclusions. As an example, the same observation can be made that for $m,n = 2,50$ RSS does not perform better than SRS in estimating the standard deviation when individual iterations are considered since its column C value is less than 5,000 (4,978). This is supported by the fact that the RP_{std} for this m,n combination is less than 1.

The last distribution modification considered was a data set that is more strongly skewed to the right than the original scan data set. Based off of previous literature, it was anticipated that a balanced RSS would not perform as well for this data set. However, the data given in Tables 4.16 and 4.17 do not support this prediction. The RSS estimated mean percent difference values range from about 88% to 95%. These values are 5-19% higher than their SRS counterparts. RP values are all above one.

Regarding, the evaluation of the individual trials for the skewed data, Table 4.18 indicates marginal improvement in performance for column A when compared to Table 4.6 for m,n values of 3,33, 4,25, and 5,20. This was not expected, given the anticipation for RSS to not perform as well for skewed data. The lower m,n column A values in Table 4.18 were very similar to those of Table 4.6, and again did not show lower performance that was expected of

skewed data. This may be because even after modifications to the RS-700 normalized scan data set, just a small amount of data proportionally were in the upper right tail of the distribution.

The last set of trials applied the rule of thumb of t_{opt} proposed by Kaur (1997). All of the indicators of RSS performance were notably poor compared to SRS and compared to RSS for previous trials. Though RP was still over 1, all were lower for the same m, n values in the previous trial. A reason for this could be that the data is not skewed enough for this rule of thumb to apply. However, the RSS surveyor may think the skewness of their data is high enough to warrant using this rule of thumb. Values of t_{opt} , the primary value used in the rule of thumb, are provided for a CV range of 0-10 and for a skewness range of 0-20 (as can be seen in Figure 2.3. of this paper). However, the results of this paper support that there is a minimum level of skewness where equal allocation RSS is a better option than applying this rule of thumb for unequal allocation. More research on unequal allocation RSS for various skewed distributions by use of this rule of thumb is needed to determine this level.

Chapter 6 – Conclusion

6.1. Overview of Study and Objective Results

This study aimed to provide a comprehensive assessment of how RSS could be applied within the framework of an actual radiological contamination scenario, the Dyess AF B-47 crash site, and for related radiological contamination scenarios. The distributions evaluated included the limited contamination distribution that was representative of the Dyess crash site, normal distributions that are more representative of what may result from background contamination after remediation, and lastly a more skewed data set that could have resulted if there had been more contamination. RSS is promoted as providing a more precise estimate of the mean than SRS. Additionally, with high mn values, RSS can provide a more precise estimate of the variance (and hence standard deviation) than SRS. While this study focused on the estimate of the mean, some consideration was given to the estimate of the standard deviation. Though not as comprehensively evaluated, the standard deviation results do indicate that SRS can provide statistically different estimates of the standard deviation than SRS that are more precise over multiple iterations for high mn .

The summary evaluation of each of the objectives given in this study is as follows:

1) What are the effects on the precision and accuracy of RSS mean estimate compared to that of SRS, when considering the Dyess survey and theoretical modifications?

RSS will provide as accurate estimation of the distribution mean than SRS for a radiological contamination scenario similar to the Dyess survey and for data sets that are closer to a normal distribution or similar to the skewed data set assessed in this study. This supports the

statement that RSS is an unbiased estimator of the mean. The results of this study support RSS as being a more precise estimate of the mean over a large number of iterations. If individual iterations are considered, RSS was not always a more precise estimate of the mean for the scenarios considered in this study. But with the high statistical confidence from 10,000 iterations RSS can be considered a more precise estimator of the mean.

2) Is there an optimal means to apply RSS for this survey or other radiological contamination surveys?

The optimal means depends on the surveyor's goals, cost comparison, and the expected radiological contamination distribution. While higher values had increased magnitude of RP, they did not have the highest values when the theoretical upper bound was considered. This variation is suggested for further evaluation. This study supported that a smaller number of analysis measurements can be used by RSS to obtain the same levels of accuracy and precision as gained by SRS with a larger number of analysis measurements.

3) Given the procedures for RSS, could the field survey techniques used in this survey be utilized as a screening tool for determining what soil samples should be used in this survey, or future surveys of a similar nature?

It is possible that the FIDLER and ex-situ gamma spectroscopy field survey instruments could be used to obtain screening observation measurements. The RS-700 data did not prove to have the correlation in quantity or location that would be needed to be a useful screening tool.

6.2. Further Considerations

Since the unequal allocation RSS procedure applied to the skewed data did not result in a better performance than the equal allocation procedure, it is suggested that for data comparable to this survey a user of RSS continues to use equal allocation or uses a different method of

unequal allocation than the one given by Kaur (1997). Further evaluation of distributions with higher values of skewness should be assessed to determine more specifically when Kaur (1997)'s rule of thumb's may become applicable.

As mentioned, of the three field survey techniques used in this survey, the FIDLER and the ORTEC instruments show potential to be effectively utilized as a RSS screening tool for determining what soil samples should be used in this survey, or future surveys of a similar nature. Further study in this area would be useful to concretely build procedures for using these instruments as a ranking potential and evaluate their performance within an actual RSS survey to include consideration of each method's minimum ranking capability.

Another follow on consideration for furthering the work done in this study would be to conduct a more in depth comparison of the hot spots detected by RSS compared to SRS. As suggested by Vitkus, the screening observations can all be used to identify hot spots. If a high value is identified that would not have normally been selected as a ranking judgment, a biased sample would still be taken [22]. If a scanning method is not already performing 100% coverage with high enough confidence that hot spots will be detected, the use of RSS will increase the probability that hot spots will be detected. This analysis could be done by overlaying the Matlab code that produces a figure of RSS locations with the actual or theoretical data set values that are being used to represent the population distribution.

In conclusion, though the concept of RSS has been proposed for over 60 years, RSS has not been widely applied in radiological contamination remediation sampling strategy. Current use and education provided on RSS methods by ORAU and the possible future inclusion of RSS in the MARSSIM strategy could change this paradigm. RSS has the potential to provide cost saving measures, while still obtaining accurate results. RSS can be useful for each survey aspect

of a remediation project. This paper supports the further consideration and use of RSS as a sampling technique for incorporation into radiation survey procedures.

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APPENDICES

Appendix A – Matlab Code

RSSmaster.m

```
%This RSS analysis program allows the user to import a data set of format
%(lat,long,value) and specify a value of m, n and number of iterations
%desired ("trial repeat value"). The input file should be selected to be a
%numeric matrix. The output is a excel file with one sheet containing the
%mean and standard deviation of the entire input data set, the RSS and
%SRS mean and standard deviation,and the percent error of each of these
%compared to the data set's mean and standard deviation for each
%iteration. The second sheet contains the average of these values over all
%of the iterations.
```

```
clear;
uiimport
prompt1='Enter a value for m:';
prompt2='Enter a value for n:';
m=input(prompt1)
n=input(prompt2)
prompt='Enter trial repeat value';
B=input(prompt);

trueA=mean(scandata(:,3));
true_sd=std(scandata(:,3));

for ii=1:B
    RSStrialL
    avgN(ii,:)=mean(VV);
    sdN(ii,:)=std(VV);
    varN(ii,:)=var(VV);
end

RSSpres_avgN=100*(1-(abs(trueA-avgN))/trueA);
RSSpres_sdN=100*(1-(abs(true_sd-sdN))/true_sd);

avgA=mean(avgN);
sdA=mean(sdN);
varA=var(avgN);
```

```

RSSpresA=100*(1-(abs(trueA-avgA))/trueA);
RSSpres_sdA=100*(1-(abs(true_sd-sdA))/true_sd);

BB=m*n;
for ii=1:B
SRS(ii,:)=datasample(scandata(:,3),BB,'Replace',false);
end

SRS1=mean(SRS,2);
SRS2=std(SRS,0,2);
SRSvar=var(SRS1);
compare=SRSvar/varA;
SRSavg=mean(SRS1);
SRSsd=mean(SRS2);
RSSpres_SRS1=100*(1-(abs(trueA-SRS1))/trueA);
RSSpres_SRS2=100*(1-(abs(true_sd-SRS2))/true_sd);
RSSpres_avg=100*(1-(abs(trueA-SRSavg))/trueA);
RSSpres_sd=100*(1-(abs(true_sd-SRSsd))/true_sd);

T1=table(avgN,sdN,SRS1,SRS2,RSSpres_avgN,RSSpres_SRS1,RSSpres_sdN,RSSpres_SRS2,'
VariableNames',{'RSS_avg','RSS_std','SRS_avg','SRS_std','RSS_avg_acur','SRS_avg_acur','RSS
_std_acur','SRS_std_acur'});
T2=table(trueA,true_sd,avgA,sdA,SRSavg,SRSsd,RSSpresA,RSSpres_avg,RSSpres_sdA,SRSpr
es_sd,compare,'VariableNames',{'true_avg','true_sd','RSS_avg','RSS_std','SRS_avg','SRS_std','R
SS_avg_acur','SRS_avg_acur','RSS_std_acur','SRS_std_acur','compare'});

writetable(T1,'RSStrial.xlsx','sheet',1);
writetable(T2,'RSStrial.xlsx','sheet',2);

```

RSStrialL.m

```

m2=m*m;
i=m2*n;
[R,idx]=datasample(scandata(:,3),i,'Replace',false);
idx=idx.';

i1=reshape(idx,[m2,n]);
for i=1:n
i2=reshape(i1(:,i),[m,m]);
i2=sort(i2);
i3(i,:)=diag(i2);
end

```

```
i4=reshape(i3,[numel(i3),1]);
```

```
S=reshape(R,[m2,n]);
for i=1:n
V=reshape(S(:,i),[m,m]);
V=sort(V);
L(i,:)=diag(V);
end
```

```
VV=reshape(L,[numel(L),1]);
```

RSSplot.m

```
% This program executes one RSS trial and gives the mean, standard deviation
% and a figure for both the screening observation locations and the analysis
% measurement locations.
```

```
clear;
uiimport
prompt1='Enter a value for m: ';
prompt2='Enter a value for n: ';
m=input(prompt1)
n=input(prompt2)
m2=m*m;
i=m2*n;
[R,idx]=datasample(scandata(:,3),i,'Replace',false);
idx=idx.';
```

```
i1=reshape(idx,[m2,n]);
for i=1:n
i2=reshape(i1(:,i),[m,m]);
i2=sort(i2);
i3(i,:)=diag(i2);
end
```

```
i4=reshape(i3,[numel(i3),1]);
```

```
N=[scandata(idx(:,1),1),scandata(idx(:,1),2)];
Nsel=[scandata(i4(:,1),1),scandata(i4(:,1),2)];
```

```
figure; scatter(N(:,1),N(:,2))
figure; scatter(Nsel(:,1),Nsel(:,2))
```

```
S=reshape(R,[m2,n]);
for i=1:n
V=reshape(S(:,i),[m,m]);
V=sort(V);
L(i,:)=diag(V);
end
```

```
VV=reshape(L,[numel(L),1]);
avg=mean(VV)
std=std(VV)
Matlab code for t opt for skewed distributions
```

RSSmaster2.m

```
%RSS analysis for skewed data, where t opt is selected from rule of thumb
%explained in Kaur et al(1997)
%This RSS analysis program allows the user to import a data set of format
%(lat,long,value) and specify a value of m, n, t opt value, and number of
%iterations desired ("trial repeat value"). The input file should be named
%"scandata.xlsx" and after choosing file as the source the user should select
%numeric matrix. The output is a excel file with one sheet containing the
%mean and standard deviation of the entire input data set, the RSS and
%SRS mean and standard deviation,and the percent error of each of these
%compared to the data set's mean and standard deviation for each
%iteration. The second sheet contains the average of these values over all
%of the iterations.
```

```
clear;
uiimport
prompt1='Enter a value for m: ';
prompt2='Enter a value for n: ';
m=input(prompt1)
n=input(prompt2)
prompt3='Enter t opt value: ';
topt=input(prompt3)
prompt4='Enter trial repeat value: ';
B=input(prompt4)
```

```
trueA=mean(scandata(:,3));
```

```

true_sd=std(scandata(:,3));

for ii=1:B
    RSStrial2
    avgN(ii,:)=mean(VV);
    sdN(ii,:)=std(VV);
    varN(ii,:)=var(VV);
end

RSSpres_avgN=100*(1-(abs(trueA-avgN))/trueA);
RSSpres_sdN=100*(1-(abs(true_sd-sdN))/true_sd);

avgA=mean(avgN);
sdA=mean(sdN);
varA=var(avgN);
RSSpresA=100*(1-(abs(trueA-avgA))/trueA);
RSSpres_sdA=100*(1-(abs(true_sd-sdA))/true_sd);

BB=numel(VV);
for ii=1:B
    SRS(ii,:)=datasample(scandata(:,3),BB,'Replace',false);
end

SRS1=mean(SRS,2);
SRS2=std(SRS,0,2);
SRSvar=var(SRS1);
compare=SRSvar/varA;
SRSavg=mean(SRS1);
SRSsd=mean(SRS2);
SRSpres_SRS1=100*(1-(abs(trueA-SRS1))/trueA);
SRSpres_SRS2=100*(1-(abs(true_sd-SRS2))/true_sd);
SRSpres_avg=100*(1-(abs(trueA-SRSavg))/trueA);
SRSpres_sd=100*(1-(abs(true_sd-SRSsd))/true_sd);

T1=table(avgN,sdN,SRS1,SRS2,RSSpres_avgN,RSSpres_SRS1,RSSpres_sdN,RSSpres_SRS2,'
VariableNames',{'RSS_avg','RSS_std','SRS_avg','SRS_std','RSS_avg_acur','SRS_avg_acur','RSS
_std_acur','SRS_std_acur'});
T2=table(trueA,true_sd,avgA,sdA,SRSavg,SRSsd,RSSpresA,SRSpres_avg,RSSpres_sdA,SRSpr
es_sd,compare,'VariableNames',{'true_avg','true_sd','RSS_avg','RSS_std','SRS_avg','SRS_std','R
SS_avg_acur','SRS_avg_acur','RSS_std_acur','SRS_std_acur','compare'});

writetable(T1,'RSStrial.xlsx','sheet',1);
writetable(T2,'RSStrial.xlsx','sheet',2);

```

RSStrial2.m

```
t=topt-1;
m2=m*(m+t);
Num=m*(m+t)*n; %modfies n to include one more set to be evaluated for its max value
[R,idx]=datasample(scandata(:,3),Num,'Replace',false);
idx=idx.';
```

```
kk=m+t;
i1=reshape(idx,[m2,n]);
for i=1:n
i2=reshape(i1(:,i),[m,kk]);
i2=sort(i2);
i3(i,1:m)=diag(i2);
i3(i,kk)=i2(m,kk);
    if t-1>0
        for j=1:t
            i3(i,kk-j)=i2(m,kk-j);
        end
    end
end
```

```
i4=reshape(i3,[numel(i3),1]);
```

```
S=reshape(R,[m2,n]);
for i=1:n
V=reshape(S(:,i),[m,kk]);
V=sort(V);
L(i,1:m)=diag(V);
L(i,kk)=V(m,kk);
    if t-1>0
        for j=1:t
            L(i,kk-j)=V(m,kk-j);
        end
    end
end
```

```
VV=reshape(L,[numel(L),1]);
```

Appendix B - Soil Sample Data

Survey Unit	Sample Number	Latitude	Longitude	Sum* (pCi/g)	Normalized Value	uncertainty
1	SU1-SS-01	32.364407	99.851988	14.93	0.1452	0.55
1	SU1-SS-02	32.364315	99.851988	3.77	0.0344	0.28
1	SU1-SS-03	32.364223	99.851988	1.81	0.0149	0.27
1	SU1-SS-04	32.364131	99.851988	1.29	0.0098	0.19
1	SU1-SS-05	32.364498	99.852096	1.97	0.0165	0.24
1	SU1-SS-06	32.364407	99.852096	2.45	0.0213	0.29
1	SU1-SS-07	32.364315	99.852096	0.82	0.0051	0.30
1	SU1-SS-08	32.64223	99.852096	2.47	0.0215	0.81
1	SU1-SS-09	32.364131	99.852096	1.24	0.0093	0.50
1	SU1-SS-10	32.364498	99.852204	2.50	0.0218	0.25
1	SU1-SS-11	32.364407	99.852204	2.64	0.0232	0.29
1	SU1-SS-12	32.364315	99.852204	1.81	0.0149	0.22
1	SU1-SS-13	32.364223	99.852204	0.46	0.0016	0.74
1	SU1-SS-14	32.364131	99.852204	1.39	0.0108	0.20
1	SU1-SS-15	32.364407	99.852312	2.86	0.0254	0.31
1	SU1-SS-16	32.364315	99.852312	2.74	0.0242	0.22
1	SU1-SS-17	32.364223	99.852312	1.71	0.0139	0.21
1	SU1-SS-18	32.364131	99.852312	1.79	0.0148	0.26
1	SU1-SS-19	32.364315	99.852421	0.59	0.0028	0.17
1	SU1-SS-20	32.364223	99.852421	0.97	0.0066	0.19
2	SU2-SS-01	32.364604	99.851857	0.58	0.0027	0.18
2	SU2-SS-02	32.364696	99.851965	0.87	0.0056	0.35
2	SU2-SS-03	32.364604	99.851965	0.98	0.0067	0.23
2	SU2-SS-04	32.364512	99.851965	0.79	0.0048	0.19
2	SU2-SS-05	32.364696	99.852073	0.55	0.0024	0.17
2	SU2-SS-06	32.364604	99.852073	1.14	0.0083	0.20
2	SU2-SS-07	32.364512	99.852073	2.25	0.0193	0.24
2	SU2-SS-08	32.364696	99.852181	0.78	0.0048	0.20
2	SU2-SS-09	32.364604	99.852181	1.19	0.0088	0.23
2	SU2-SS-10	32.364696	99.852290	1.02	0.0071	0.21
2	SU2-SS-11	32.364604	99.852290	1.57	0.0126	0.20
2	SU2-SS-12	32.364512	99.852290	1.62	0.0131	0.16
2	SU2-SS-13	32.364696	99.852398	1.58	0.0127	0.17
2	SU2-SS-14	32.364604	99.852398	1.31	0.0100	0.16
2	SU2-SS-15	32.364512	99.852398	1.48	0.0116	0.17
2	SU2-SS-16	32.364604	99.852506	1.19	0.0088	0.15

2	SU2-SS-17	32.364512	99.852506	1.32	0.0101	0.24
2	SU2-SS-18	32.364420	99.852506	1.22	0.0091	0.21
2	SU2-SS-19	32.364512	99.852614	1.12	0.0081	0.16
2	SU2-SS-20	32.364420	99.852614	1.42	0.0111	0.17
3	SU3-SS-01	32.364024	99.852139	1.55	0.0124	0.23
3	SU3-SS-02	32.363932	99.852139	1.78	0.0147	0.22
3	SU3-SS-03	32.36384	99.852139	0.90	0.0060	0.56
3	SU3-SS-04	32.364024	99.852247	2.76	0.0244	0.28
3	SU3-SS-05	32.363932	99.852247	1.15	0.0084	0.19
3	SU3-SS-06	32.363840	99.852247	0.43	0.0013	0.16
3	SU3-SS-07	32.364024	99.852355	0.59	0.0029	0.18
3	SU3-SS-08	32.363932	99.852355	1.31	0.0100	0.18
3	SU3-SS-09	32.364207	99.852463	0.81	0.0050	0.23
3	SU3-SS-10	32.364116	99.852463	1.22	0.0091	0.20
3	SU3-SS-11	32.364024	99.852463	1.48	0.0116	0.22
3	SU3-SS-12	32.363932	99.852463	0.83	0.0052	0.22
3	SU3-SS-13	32.364299	99.852572	0.78	0.0047	0.19
3	SU3-SS-14	32.364207	99.852572	0.90	0.0059	0.17
3	SU3-SS-15	32.364116	99.852572	0.98	0.0067	0.19
3	SU3-SS-16	32.364024	99.852572	1.35	0.0104	0.18
3	SU3-SS-17	32.364391	99.852680	0.89	0.0058	0.21
3	SU3-SS-18	32.364299	99.852680	0.95	0.0064	0.14
3	SU3-SS-19	32.364207	99.852680	0.85	0.0054	0.21
3	SU3-SS-20	32.364116	99.852680	0.94	0.0063	0.14
4	SU4-SS-01	32.364506	99.851735	1.35	0.0103	0.15
4	SU4-SS-02	32.364414	99.851735	1.17	0.0086	0.18
4	SU4-SS-03	32.364322	99.851735	1.94	0.0162	0.19
4	SU4-SS-04	32.364231	99.851735	5.22	0.0488	0.30
4	SU4-SS-05	32.364139	99.851735	1.94	0.0162	0.17
4	SU4-SS-06	32.364047	99.851735	2.16	0.0184	0.20
4	SU4-SS-07	32.364506	99.851844	1.18	0.0087	0.28
4	SU4-SS-08	32.364414	99.851844	3.32	0.0299	0.32
4	SU4-SS-09	32.364322	99.851844	3.46	0.0313	0.26
4	SU4-SS-10	32.364231	99.851844	2.46	0.0214	0.21
4	SU4-SS-11	32.364139	99.851844	2.30	0.0198	0.23
4	SU4-SS-12	32.364047	99.851844	1.16	0.0084	0.37
4	SU4-SS-13	32.363955	99.851844	4.59	0.0425	0.32
4	SU4-SS-14	32.364414	99.851952	1.37	0.0106	0.30
4	SU4-SS-15	32.364139	99.851952	1.37	0.0106	0.39
4	SU4-SS-16	32.364047	99.851952	1.33	0.0102	0.17

4	SU4-SS-17	32.363955	99.851952	1.23	0.0092	0.20
4	SU4-SS-18	32.364047	99.852060	1.26	0.0094	0.20
4	SU4-SS-19	32.363955	99.852060	1.19	0.0088	0.20
5	SU5-SS-01	32.364765	99.851690	0.54	0.0024	0.21
5	SU5-SS-02	32.364849	99.851789	0.67	0.0036	0.18
5	SU5-SS-03	32.364765	99.851789	0.75	0.0044	0.22
5	SU5-SS-04	32.364681	99.851789	0.74	0.0043	0.21
5	SU5-SS-05	32.364849	99.851888	0.66	0.0035	0.18
5	SU5-SS-06	32.364765	99.851888	0.62	0.0031	0.20
5	SU5-SS-07	32.364681	99.851888	0.66	0.0036	0.19
5	SU5-SS-08	32.364932	99.851987	0.88	0.0057	0.24
5	SU5-SS-09	32.364849	99.851987	0.65	0.0035	0.23
5	SU5-SS-10	32.364765	99.851987	0.70	0.0039	0.18
5	SU5-SS-11	32.364932	99.852085	0.67	0.0036	0.18
5	SU5-SS-12	32.364849	99.852085	0.35	0.0004	0.21
5	SU5-SS-13	32.364765	99.852085	0.64	0.0033	0.22
5	SU5-SS-14	32.364932	99.852184	0.65	0.0034	0.23
5	SU5-SS-15	32.364849	99.852184	0.45	0.0014	0.23
5	SU5-SS-16	32.364765	99.852184	0.57	0.0026	0.24
5	SU5-SS-17	32.364932	99.852283	0.49	0.0019	0.20
5	SU5-SS-18	32.364849	99.852283	0.48	0.0017	0.22
5	SU5-SS-19	32.364765	99.852283	2.93	0.0261	0.31
5	SU5-SS-20	32.364932	99.852382	0.30	0.0000	0.21
1	SU1-BS-1A	32.3642808	99.852321	8.76	0.0839	0.41
1	SU1-BS-2A	32.3642811	99.852346	33.19	0.3264	0.75
1	SU1-BS-3A	32.364298	99.852349	6.04	0.0569	0.34
1	SU1-BS-4A	32.364294	99.852342	54.63	0.5392	1.12
1	SU1-BS-05	32.363972	99.852503	101.05	1.0000	2.01
4	SU4-BS-02	32.364423	99.851886	4.83	0.0449	0.36
4	SU4-BS-01	32.364394	99.851876	4.66	0.0432	0.38
4	SU4-BS-03	32.364404	99.851934	12.37	0.1198	0.56
1	SU1-BS-1B	32.3642808	99.852321	1.44	0.0112	0.20
1	SU1-BS-1C	32.3642808	99.852321	1.01	0.0070	0.15
1	SU1-BS-2B	32.3642811	99.852346	4.60	0.0426	0.31
1	SU1-BS-3B	32.364298	99.852349	1.29	0.0098	0.16

Table B.1 Dyess AFB B-47 Crash Site Characterization Survey Soil Sample Results with Th-234 and U-235 values summed, and indicating normalized values[3]

Sample Number	Th-234				U-235			
	Value	Uncertainty (U)	Value-U	MDC	Value	Uncertainty (U)	Value-U	MDC
SU1-SS-01	2.23	0.29	1.94	0.67	12.70	0.26	12.44	0.03
SU1-SS-02	1.50	0.20	1.30	0.37	2.27	0.08	2.19	0.03
SU1-SS-03	1.21	0.21	1.00	0.32	0.60	0.06	0.54	0.03
SU1-SS-04	0.81	0.16	0.65	0.31	0.48	0.03	0.45	0.02
SU1-SS-05	1.37	0.18	1.19	0.38	0.60	0.06	0.54	0.02
SU1-SS-06	0.72	0.20	0.52	0.39	1.73	0.09	1.64	0.02
SU1-SS-07	0.30	0.25	0.05	0.35	0.52	0.05	0.47	0.03
SU1-SS-08	1.50	0.18	1.32	0.37	0.97	0.63	0.34	0.02
SU1-SS-09	0.75	0.44	0.31	0.36	0.49	0.06	0.43	0.02
SU1-SS-10	1.26	0.20	1.06	0.39	1.24	0.05	1.19	0.02
SU1-SS-11	1.20	0.20	1.00	0.41	1.44	0.09	1.35	0.02
SU1-SS-12	0.66	0.17	0.49	0.38	1.15	0.05	1.10	0.02
SU1-SS-13	0.42	0.12	0.30	0.30	0.04	0.62	-0.58	0.02
SU1-SS-14	0.95	0.17	0.78	0.36	0.44	0.03	0.41	0.02
SU1-SS-15	1.50	0.26	1.24	0.34	1.36	0.05	1.31	0.03
SU1-SS-16	1.72	0.18	1.54	0.35	1.02	0.04	0.98	0.03
SU1-SS-17	1.23	0.15	1.08	0.31	0.48	0.06	0.42	0.03
SU1-SS-18	1.34	0.23	1.11	0.34	0.45	0.03	0.43	0.03
SU1-SS-19	0.56	0.09	0.46	0.23	0.03	0.07	-0.04	0.10
SU1-SS-20	0.79	0.17	0.62	0.28	0.18	0.02	0.16	0.02
SU2-SS-01	0.55	0.10	0.45	0.28	0.03	0.08	-0.05	0.02
SU2-SS-02	0.80	0.25	0.55	0.31	0.07	0.10	-0.03	0.14
SU2-SS-03	0.86	0.13	0.73	0.30	0.12	0.10	0.02	0.14

SU2-SS-04	0.69	0.12	0.57	0.28	0.10	0.07	0.03	0.02
SU2-SS-05	0.41	0.15	0.26	0.38	0.14	0.02	0.12	0.02
SU2-SS-06	0.79	0.15	0.64	0.35	0.35	0.05	0.30	0.02
SU2-SS-07	1.34	0.20	1.14	0.44	0.91	0.04	0.87	0.03
SU2-SS-08	0.68	0.15	0.53	0.35	0.10	0.05	0.05	0.02
SU2-SS-09	0.99	0.18	0.81	0.39	0.20	0.05	0.15	0.03
SU2-SS-10	0.93	0.16	0.77	0.36	0.09	0.05	0.03	0.02
SU2-SS-11	1.35	0.15	1.20	0.32	0.22	0.05	0.17	0.03
SU2-SS-12	1.36	0.14	1.22	0.30	0.26	0.02	0.24	0.02
SU2-SS-13	1.42	0.15	1.27	0.31	0.16	0.02	0.15	0.02
SU2-SS-14	1.17	0.14	1.03	0.32	0.14	0.02	0.13	0.02
SU2-SS-15	1.29	0.15	1.14	0.34	0.19	0.02	0.17	0.03
SU2-SS-16	1.04	0.13	0.91	0.31	0.15	0.02	0.13	0.03
SU2-SS-17	1.17	0.14	1.03	0.32	0.15	0.10	0.05	0.14
SU2-SS-18	1.15	0.14	1.01	0.34	0.07	0.07	0.00	0.03
SU2-SS-19	0.96	0.14	0.82	0.36	0.16	0.02	0.14	0.03
SU2-SS-20	1.18	0.15	1.03	0.33	0.24	0.02	0.22	0.03
SU3-SS-01	0.90	0.17	0.73	0.36	0.65	0.06	0.59	0.02
SU3-SS-02	1.13	0.17	0.96	0.34	0.65	0.05	0.60	0.02
SU3-SS-03	0.65	0.13	0.52	0.34	0.25	0.43	-0.18	0.02
SU3-SS-04	0.95	0.21	0.74	0.41	1.81	0.07	1.74	0.02
SU3-SS-05	0.68	0.14	0.54	0.32	0.47	0.05	0.42	0.02
SU3-SS-06	0.08	0.14	-0.06	0.35	0.36	0.02	0.33	0.02
SU3-SS-07	0.55	0.13	0.42	0.32	0.04	0.05	-0.01	0.02
SU3-SS-08	0.77	0.15	0.62	0.34	0.54	0.03	0.51	0.02
SU3-SS-09	0.67	0.18	0.49	0.34	0.14	0.05	0.09	0.02

SU3-SS-10	0.91	0.15	0.76	0.37	0.31	0.05	0.26	0.02
SU3-SS-11	1.23	0.17	1.06	0.37	0.25	0.05	0.20	0.02
SU3-SS-12	0.78	0.15	0.63	0.35	0.05	0.07	-0.02	0.11
SU3-SS-13	0.69	0.14	0.55	0.32	0.09	0.05	0.04	0.02
SU3-SS-14	0.72	0.15	0.57	0.37	0.18	0.02	0.16	0.02
SU3-SS-15	0.77	0.14	0.63	0.36	0.21	0.05	0.16	0.02
SU3-SS-16	0.96	0.13	0.83	0.29	0.39	0.05	0.34	0.02
SU3-SS-17	0.87	0.12	0.75	0.29	0.02	0.09	-0.07	0.13
SU3-SS-18	0.84	0.12	0.72	0.30	0.11	0.02	0.10	0.02
SU3-SS-19	0.76	0.12	0.64	0.29	0.09	0.09	0.00	0.13
SU3-SS-20	0.79	0.12	0.67	0.27	0.15	0.02	0.14	0.02
SU4-SS-01	0.99	0.13	0.86	0.32	0.36	0.02	0.33	0.03
SU4-SS-02	0.88	0.13	0.75	0.30	0.29	0.05	0.25	0.02
SU4-SS-03	0.93	0.15	0.78	0.30	1.01	0.04	0.97	0.02
SU4-SS-04	2.72	0.22	2.50	0.36	2.50	0.08	2.42	0.03
SU4-SS-05	0.87	0.13	0.74	0.30	1.07	0.04	1.03	0.02
SU4-SS-06	1.02	0.15	0.87	0.31	1.14	0.05	1.09	0.02
SU4-SS-07	0.84	0.22	0.62	0.28	0.34	0.06	0.29	0.02
SU4-SS-08	1.23	0.25	0.98	0.37	2.09	0.07	2.02	0.03
SU4-SS-09	1.28	0.19	1.09	0.33	2.18	0.07	2.11	0.02
SU4-SS-10	1.12	0.16	0.96	0.32	1.34	0.05	1.29	0.02
SU4-SS-11	1.08	0.18	0.90	0.40	1.22	0.05	1.17	0.02
SU4-SS-12	0.78	0.32	0.46	0.33	0.38	0.05	0.33	0.02
SU4-SS-13	1.31	0.31	1.00	0.44	3.28	0.01	3.27	0.02
SU4-SS-14	0.93	0.25	0.68	0.34	0.44	0.05	0.39	0.02
SU4-SS-15	0.86	0.34	0.52	0.34	0.51	0.05	0.46	0.02

SU4-SS-16	0.66	0.14	0.52	0.33	0.67	0.03	0.64	0.02
SU4-SS-17	0.83	0.15	0.68	0.35	0.40	0.05	0.35	0.02
SU4-SS-18	0.72	0.15	0.57	0.35	0.54	0.05	0.49	0.02
SU4-SS-19	0.85	0.15	0.70	0.34	0.34	0.05	0.29	0.02
SU5-SS-01	0.50	0.14	0.36	0.35	0.04	0.07	-0.02	0.02
SU5-SS-02	0.64	0.10	0.54	0.25	0.03	0.08	-0.05	0.11
SU5-SS-03	0.67	0.13	0.54	0.30	0.08	0.09	-0.01	0.13
SU5-SS-04	0.73	0.12	0.61	0.30	0.01	0.09	-0.09	0.13
SU5-SS-05	0.61	0.10	0.51	0.26	0.05	0.08	-0.03	0.02
SU5-SS-06	0.54	0.11	0.43	0.29	0.08	0.09	-0.01	0.12
SU5-SS-07	0.62	0.11	0.51	0.27	0.04	0.08	-0.04	0.12
SU5-SS-08	0.83	0.14	0.69	0.34	0.05	0.10	-0.05	0.15
SU5-SS-09	0.58	0.14	0.44	0.28	0.07	0.09	-0.02	0.13
SU5-SS-10	0.61	0.10	0.51	0.26	0.09	0.08	0.01	0.12
SU5-SS-11	0.62	0.13	0.49	0.30	0.05	0.05	0.00	0.03
SU5-SS-12	0.33	0.14	0.19	0.34	0.02	0.07	-0.05	0.11
SU5-SS-13	0.58	0.15	0.43	0.35	0.06	0.07	-0.02	0.02
SU5-SS-14	0.64	0.15	0.49	0.40	0.01	0.08	-0.06	0.02
SU5-SS-15	0.34	0.15	0.19	0.38	0.11	0.08	0.03	0.02
SU5-SS-16	0.45	0.16	0.29	0.37	0.12	0.08	0.04	0.02
SU5-SS-17	0.46	0.13	0.33	0.32	0.03	0.07	-0.03	0.02
SU5-SS-18	0.46	0.15	0.31	0.39	0.02	0.07	-0.06	0.02
SU5-SS-19	1.57	0.26	1.31	0.34	1.36	0.05	1.31	0.03
SU5-SS-20	0.24	0.14	0.10	0.36	0.06	0.07	-0.01	0.12
SU1-BS-1A	2.48	0.26	2.22	0.56	6.28	0.15	6.13	0.03
SU1-BS-2A	2.99	0.23	2.76	0.99	30.20	0.52	29.68	0.04

SU1-BS-3A	2.54	0.25	2.29	0.46	3.50	0.09	3.41	0.03
SU1-BS-4A	2.13	0.25	1.88	1.29	52.50	0.87	51.63	0.05
SU1-BS-05	6.65	0.41	6.24	1.65	94.40	1.60	92.80	0.06
SU4-BS-02	1.45	0.27	1.18	0.45	3.38	0.09	3.29	0.03
SU4-BS-01	1.80	0.29	1.51	0.40	2.86	0.09	2.78	0.03
SU4-BS-03	2.27	0.32	1.95	0.62	10.10	0.24	9.86	0.03
SU1-BS-1B	0.88	0.14	0.74	0.32	0.56	0.06	0.50	0.03
SU1-BS-1C	0.72	0.13	0.59	0.32	0.29	0.02	0.27	0.03
SU1-BS-2B	1.30	0.21	1.09	0.44	3.30	0.10	3.20	0.03
SU1-BS-3B	0.72	0.13	0.59	0.30	0.57	0.03	0.55	0.02

Table B.2. Dyess AFB B-47 Crash Site Characterization Survey Soil Sample Results for U-235 and Th-234 Results [3]

Appendix C- Additional Distribution Data

1) Normal data distribution with standard deviation equivalent to original scan data set:

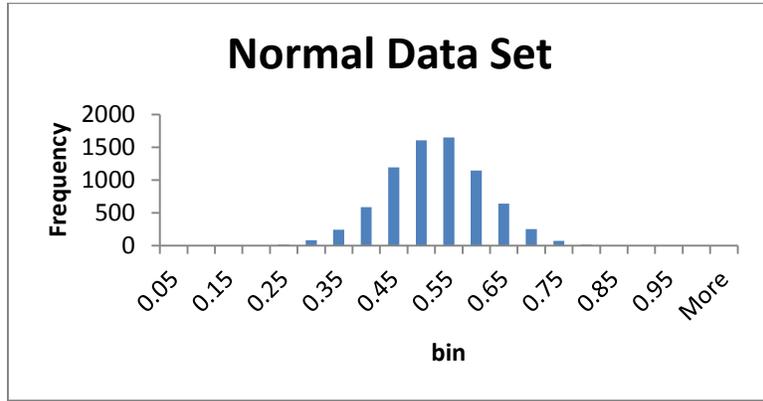


Figure C.1. Histogram Chart of Normal Data Set

<i>Distribution Statistics</i>	
Mean	0.500539062
Standard Error	0.001025779
Median	0.500592069
Standard Deviation	0.088835028
Sample Variance	0.007891662
Kurtosis	0.071905341
Skewness	0.000403509
Range	0.672420527
Minimum	0.176954813
Maximum	0.84937534
Count	7500

Table C.1. Normal Data Set Statistical Descriptors

<i>bin</i>	<i>Frequency</i>
0.05	0
0.1	0
0.15	0
0.2	2
0.25	16
0.3	82
0.35	244
0.4	587
0.45	1191
0.5	1605
0.55	1647
0.6	1143
0.65	640
0.7	251
0.75	71
0.8	16
0.85	5
0.9	0
0.95	0
1	0

Table C.2 Normal Data Set Histogram Values

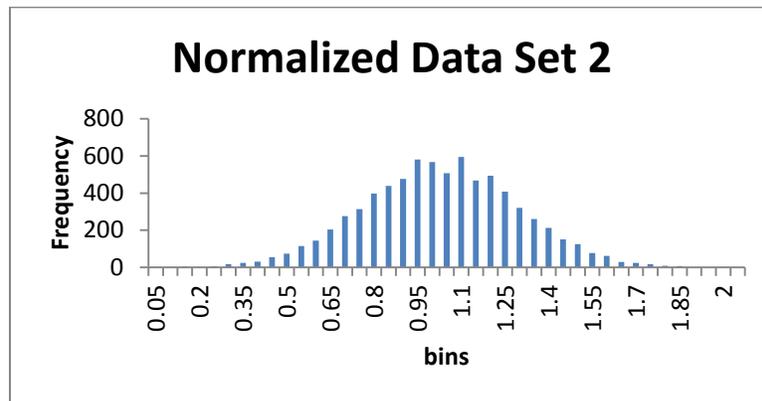


Figure C.2. Normalized Data Set #2 (increased standard deviation) Histogram Chart

<i>Distribution Statistics</i>	
Mean	1.00336
Standard Error	0.003063
Median	1.002172
Standard Deviation	0.265302
Sample Variance	0.070385
Kurtosis	-0.00664
Skewness	-0.01246
Range	1.96457
Minimum	0.011119
Maximum	1.97569
Count	7500

Table C.3 Normalized Data Set #2 (increased standard deviation) statistical descriptors

<i>bins</i>	<i>Frequency</i>
0.05	1
0.1	1
0.15	4
0.2	3
0.25	6
0.3	18
0.35	24
0.4	32
0.45	56
0.5	75
0.55	115
0.6	144
0.65	205
0.7	276
0.75	313
0.8	397
0.85	438
0.9	476
0.95	580
1	567
1.05	507

1.1	594
1.15	468
1.2	493
1.25	408
1.3	320
1.35	260
1.4	213
1.45	152
1.5	125
1.55	77
1.6	62
1.65	29
1.7	25
1.75	18
1.8	10
1.85	7
1.9	0
1.95	0
2	1

Table C.4. Normalized Data Set #2 (increased standard deviation) histogram values

Skew data:

<i>Skew Data Statistics</i>			
Mean	0.205213	Sample Variance	0.029316
Standard Error	0.001977	Kurtosis	5.670003
Median	0.144371	Skewness	2.282173
Mode	0.08877	Range	0.993377
Standard Deviation	0.171219	Count	7500

Table C.5. Skew Data Statistics

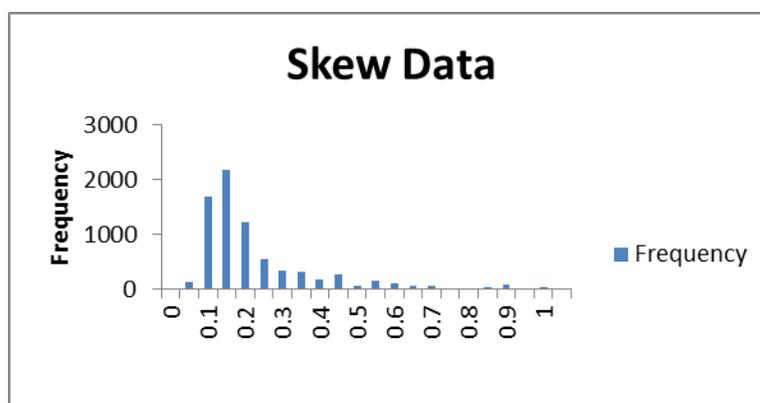


Figure C.3. Skew Data Histogram Chart

<i>Bin</i>	<i>Frequency</i>
0	0
0.05	126
0.1	1701
0.15	2178

0.2	1217
0.25	545
0.3	331
0.35	330
0.4	174
0.45	264
0.5	68
0.55	146
0.6	102
0.65	66
0.7	58
0.75	8
0.8	10
0.85	46
0.9	82
0.95	2
1	46

Table 1. Skew Data Histogram Values

scan data histogram values

Bin	Frequency	Cumulative %
0.05	128	1.71%
0.1	2240	31.57%
0.15	2763	68.41%
0.2	1217	84.64%
0.25	440	90.51%
0.3	331	94.92%
0.35	195	97.52%
0.4	60	98.32%
0.45	37	98.81%
0.5	15	99.01%
0.55	16	99.23%
0.6	13	99.40%
0.65	14	99.59%
0.7	10	99.72%
0.75	4	99.77%
0.8	5	99.84%
0.85	4	99.89%
0.9	3	99.93%
0.95	1	99.95%

1	4	100.00%
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Table C.7. Scan Data Histogram Values

<i>Bin</i>	<i>Frequency</i>	<i>Cumulative</i> %
0.05	104	93.69%
0.1	2	95.50%
0.15	2	97.30%
0.2	0	97.30%
0.25	0	97.30%
0.3	0	97.30%
0.35	1	98.20%
0.4	0	98.20%
0.45	0	98.20%
0.5	0	98.20%
0.55	1	99.10%
0.6	0	99.10%
0.65	0	99.10%
0.7	0	99.10%
0.75	0	99.10%
0.8	0	99.10%
0.85	0	99.10%

0.9	0	99.10%
0.95	0	99.10%
1	1	100.00%
More	0	100.00%

Table C.8. Soil sample results histogram values

<i>Bin</i>	<i>Frequency</i>	<i>Cumulative</i> %
0.01	4	4.04%
0.02	9	13.13%
0.03	10	23.23%
0.04	11	34.34%
0.05	8	42.42%
0.06	5	47.47%
0.07	13	60.61%
0.08	7	67.68%
0.09	5	72.73%
0.1	2	74.75%
0.11	4	78.79%
0.12	3	81.82%
0.13	1	82.83%
0.14	2	84.85%

0.15	3	87.88%
0.16	2	89.90%
0.17	2	91.92%
0.18	2	93.94%
0.19	0	93.94%
0.2	0	93.94%
0.21	1	94.95%
0.22	1	95.96%
0.23	0	95.96%
0.24	1	96.97%
0.25	0	96.97%
0.26	0	96.97%
0.27	0	96.97%
0.28	0	96.97%
0.29	0	96.97%
0.3	1	97.98%
0.4	1	98.99%
0.5	0	98.99%
0.6	0	98.99%
0.7	0	98.99%
0.8	0	98.99%
0.9	0	98.99%

1	1	100.00%
More	0	100.00%

Table C.9. Soil Sample Results w/out biased sample histogram values