AN ABSTRACT OF THE THESIS OF

Amber D. Berger for the degree of Master of Science in Civil Engineering presented on June 5, 2015.

Title: Students' Conceptual Understanding of Mechanics and Materials for Axial and Bending Load Cases

Abstract approved:

____

Shane A. Brown

As people learn new concepts, they experience a process termed conceptual change. There are several theories that attempt to explain conceptual change, and this paper uses the framework theory to explain how students learn mechanics of materials. Semi-structured interviews with ninety students were qualitatively analyzed for this research in an attempt to find misconceptions students have regarding the concepts of mechanics of materials as they apply to axial loaded and bending members. Two common misconceptions were identified: 1) shear and normal stress depend on the direction of applied load; 2) shear and normal stress depend on the location of the applied load. This paper asserts that these misconceptions are part of a synthetic model of conceptual understanding that students create when assimilating presuppositions of mechanics of materials with newly acquired information of the scientific concepts. With an understanding of student misconceptions and an explanation of how these misconceptions form, it is possible to use instructional techniques that will help to prevent the formation of misconceptions.

©Copyright by Amber D. Berger
June 5, 2015
All Rights Reserved

Students' Conceptual Understanding of Mechanics and Materials for Axial and Bending Load Cases

by
Amber D. Berger

A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Master of Science

Presented June 5, 2015
Commencement June 2015

Master of Science thesis of Amber D. Berger presented on June 5, 2015.

APPROVED:

Major Professor, representing Civil Engineering

Head of the School of Civil and Construction Engineering

Dean of the Graduate School

I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

Amber D. Berger, Author

ACKNOWLEDGEMENTS

The author expresses sincere appreciation to Dr. Shane A. Brown for his guidance and expertise in helping to complete this research and write this thesis. His patience throughout the process has been very much appreciated, and his willingness to explain the abstract concepts of conceptual change theory has been invaluable. The author would also like to thank the faculty at Oregon State University for opportunities to learn and teach and the engineering education research group for their companionship in studying and critically analyzing concepts of engineering education.

This material is based upon work supported by the National Science Foundation under Grant Numbers 1361812- Building Theories That Inform Practice: Exploring Engineering Epistemologies Through Cross-Disciplinary Data Analysis and 1225036- Collaboratively Developing Research-Based Curricular Materials to Improve Conceptual Understanding in Engineering Education.

CONTRIBUTION OF AUTHORS

Amber D. Berger performed most of the qualitative analysis of the interviews used as input to this research, and wrote a majority of the contents contained within this thesis. Dr. Shane A. Brown provided guidance in performing the qualitative analysis and in applying the conceptual change theoretical framework. He also made edits to the texts of this thesis.

TABLE OF CONTENTS

Chapter Page

1. Introduction 1

2. Literature Review 2

3. Research Goals 6

4. Methodology 7

4.1. Interview Method 8

4.2. Interview Protocol 8

4.2.1. Axial Interview Protocol 9

4.2.2. Bending Interview Protocol 10

4.2.3. Axial-Bending-Torsion Interview Protocol 11

4.3. Analysis 11

5. Results 12

5.1. Axial Load Members 12

5.1.1. Review of Stresses in Axial Load Members 12

5.1.2. Normal Stress in Axial Loaded Members Results 13

5.1.3. Shear Stress in Axial Loaded Members Results 15

5.2. Bending Load Members 16

5.2.1. Review of Stresses in Bending Members 16

5.2.2. Normal Stress in Bending Members Results 17

5.2.3. Shear Stress in Bending Members Results 18

5.3. Axial and Bending Members Compared 19

5.3.1. Summary of Specific Load Case Results 19

5.3.2. Stress Dependency on Load Direction 20

5.3.3. Stress Dependency on Load Location 22
6. Discussion 23
6.1. The Framework Theory Summarized 23
6.2. Framework Theory of Mechanics of Materials 26
7. Recommendations for Instruction 30
8. Conclusion 34
9. Bibliography 35
10. Appendix A- Axial Interview Protocol 37
11. Appendix B- Bending Interview Protocol 39
12. Appendix C- ABT Interview Protocol 41

LIST OF FIGURES

Figure | Page

Fig. 1. Framework Theory Conceptual Change Process 5

Fig. 2. Axis Orientations 12

Fig. 3. Axial Loaded Member Cut Along a) Transverse Axis and b) Inclined Axis 13

Fig. 4. Angled Stress Element on Axially Loaded Member 14

Fig. 5. Normal Stress Distribution in Bending Member 16

Fig. 6. Axially Loaded Member with Sporadic Elements 23

Fig. 7. Example of a Child's Synthetic Model of Earth 26

Fig. 8. Example of Hypothesized Mechanics Naïve Theory 27

Fig. 9. Example of Hypothesized Mechanics Synthetic Model 28

Fig. 10. Axial Load on Rubber Sheet 29

Fig. 11. a) Bending Member b) Internal Loads on Bending Member 31

Fig. 12. Deformed Bending Member 31

Fig. 13. Finite Element Analysis Model of Cantilevered Bending Member 32

Fig. 14. Photoelastic Plastic Showing Stress Gradient 33

Fig. 15. Axially Loaded Member 41

Fig. 16. Bending Loaded Member 42

Fig. 17. Torsionally Loaded Member 42

Fig. 18. Concrete Cylinder Failed in Compression 43

Fig. 19. Wood Member Failed in Bending 43

Fig. 20. Chalk Member Failed in Torsion 43

LIST OF TABLES

Table Page

Table. 1. Example Protocol Questions 9

Table. 2. Summary of Results... 20

1. Introduction

The way people understand particular concepts is termed *conceptual understanding*, and conceptual understanding evolves and changes through the process of *conceptual change*. Conceptual change is a gradual process where an individual's understandings and the basis of these understandings change over time. Through everyday life, people create *naïve theories* to explain their understandings of the world as they experience it. Naïve theories are the explanations that people create to explain phenomena around them, and these theories are usually based on observations. For example, young children often believe the Earth is flat; the world looks and feels flat, therefore it must be flat (Vosniadou and Brewer 1992). The transition from a conceptual understanding of a flat to round Earth is an iterative process of developing and modifying a naïve theory to become a complete, correct, and coherent theory. Intermediate frameworks are called *synthetic models*. A synthetic model is created when a person integrates new information with presuppositions, naïve theories, and previously formed synthetic models as a way to conceptually understand a phenomenon before it is fully understood. This paper uses this theory of conceptual change developed by Stella Vosniadou, the *framework theory* (Vosniadou et al. 2008).

Most research in engineering education related to conceptual change has focused on the development of concept inventories (Hestenes et al. 1992; Martin et al. 2003; Midkiff et al. 2001; Richardson et al. 2001; Steif et al. 2005), with some work done on conceptual change theories (Streveler et al. 2014). A concept inventory was partially developed for mechanics of materials (Richardson et al. 2003), resulting in a set of questions that can be used and some data on students' correct and incorrect responses. However, little is known about students logic and approach to understanding mechanics of materials concepts, alternatively called strength of materials. Only one study has been completed using the framework theory to study student understanding of engineering knowledge (Brown 2013). The framework theory is relevant to mechanics of materials because it relates the observable and apparent behavior of phenomena (flat earth, members under load) to the abstract (round earth, stress and strain), and observable and abstract can be hard to integrate. This study investigates student understanding of axially loaded members and bending members using in-depth interview techniques and interpreting results in relation to the framework theory of conceptual change.

2. Literature Review

Conceptual understanding is an individual's fundamental understanding of a phenomenon; what they know without performing calculations. The ability to describe a phenomenon correctly and apply all underlying concepts is generally described to be correct conceptual understanding (Montfort et al. 2009). For example, conceptual understanding within engineering practice is more than performing calculations, memorizing analysis methodologies, or defining terms. A person can memorize the steps required to design a column or the steps to calculate the normal stress in a beam without understanding the concepts behind each step. For example, a person can use Eqn. 1 to calculate normal stress in a bending member (σ is normal stress, M is internal bending moment, c is the distance from the neutral axis to the top and bottom of the cross section, and I is the second moment of area), but still not be able to respond to questions such as, "Where is the biggest normal stress in this beam?"

Eqn. 1. $$\sigma = \frac{Mc}{I}$$

Conceptual understanding is more than memorizing or remembering, it is understanding the concept as truth (Montfort et al. 2009).

Theories of conceptual change attempt to describe how individuals categorize concepts and what rules are applied to these categories. There are three primary theories: *phenomenology primitives*, *ontological categorization*, and the *framework theory* of conceptual change. Conceptual change theories are critical to studies, because, like other disciplinary fields, they attempt to make sense of patterns in broad and generalizable terms.

One conceptual change theory developed by Andrea diSessa describes knowledge as a complicated structure of smaller parts called *phenomenology primitives* (p-prims). P-prims result from everyday interpretations of the physical world, and they use intuitive logic to explain physical experiences. One might think of p-prims as logic that explains the way things are without the need for further or underlying knowledge. People have the ability to choose or modify p-prims based on newly acquired information or experiences. They generally choose or cue p-prims based on reliability (does the idea seem credible) and priority (does the idea seem more feasible than all other ideas). Naïve theory is another term for intuitive logic or a person's conceptual understanding before they have achieved correct understanding of a concept. This level of understanding is mostly affected by observation and experiences from a person's everyday life. Conceptual change occurs when naïve knowledge transfers to expert knowledge. This happens when a person prioritizes p-prims with competing p-prims or replaces them by more advanced p-prims. As this happens, p-prims become less intuitive and must be based on more advanced justifications or physical law (DiSessa 1993).

Another conceptual change theory developed by Michelene Chi and colleagues asserts that when people learn, they place the new knowledge into ontological categories based on perceived properties. For example, a living creature that has fins and lives in water may be placed into a category called fish. Misconceptions occur when people place new knowledge into the wrong category, which would be the case if someone placed a whale into the fish category. To affect conceptual change, one must reassign or shift a concept from one category to another. This requires two conditions: people must be aware of their misconceptions and they must possess an alternate ontological category in which to place their misconception. When these two conditions do not exist, it is difficult to affect conceptual change, and misconceptions are said to be robust (Chi and Roscoe 2002).

Framework theory of conceptual change suggests that learning is a slow, gradual process. Throughout a person's life, their conceptual understanding evolves as new information is obtained, and during this evolution, synthetic models are created. Synthetic models are created when students try to assimilate new information with presuppositions as is the case when children learn that the Earth is not flat. "By forming these synthetic models children try to assimilate the information that the Earth is a sphere with their preexisting knowledge structures in a way that allows them to retain as many of their presuppositions as possible (Vosniadou and Brewer 1992)." Presuppositions may conflict with new information, so a student must either assimilate the conflicting information by creating an incorrect concept or modify the preconception to agree with the new information. Synthetic models are relatively intelligible to the person who possesses them even though misconceptions are prevalent. Due to the logical and intelligible nature of synthetic models, they can be difficult to change or said to be robust. However, synthetic models are required for a person to move from naïve understanding to full conceptual understanding. In other words, they create the building blocks that facilitate or hinder conceptual change (Vosniadou and Skopeliti 2014). A diagram of conceptual change based on the framework theory is shown in Fig. 1.

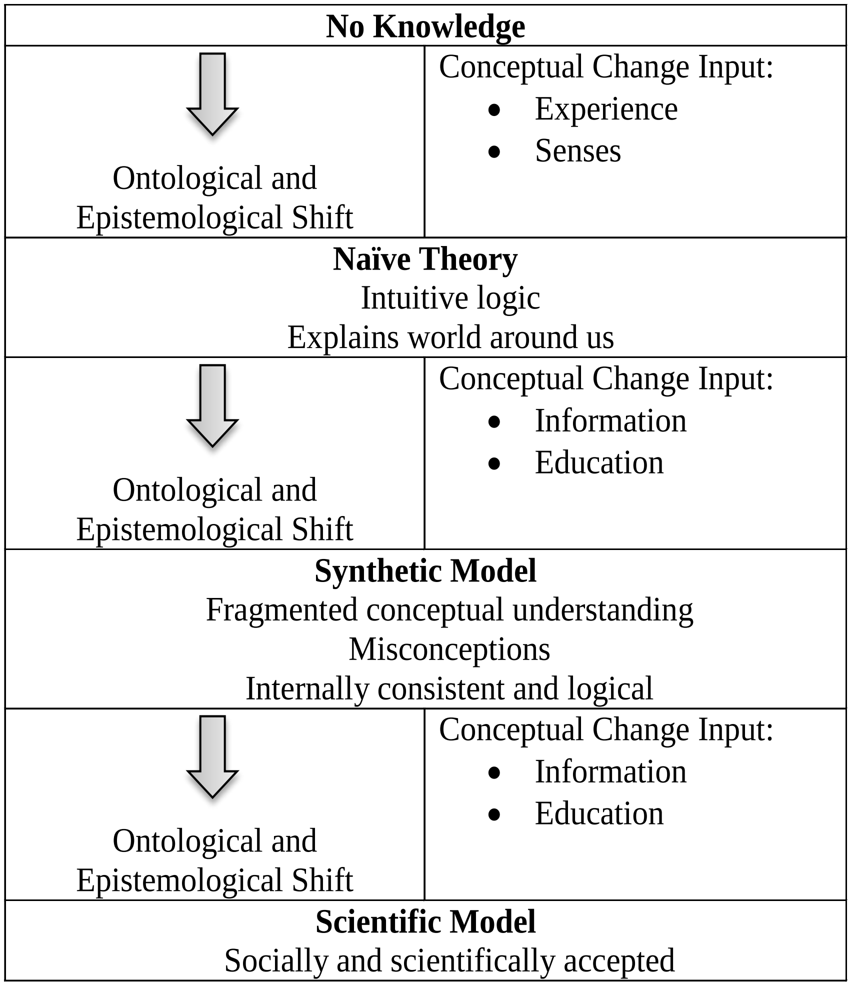


Fig. 1. Framework Theory Conceptual Change Process

The three conceptual change theories discussed in this section, p-prims, ontological categorization, and framework theory, have a few similarities. They each include some aspect of presupposition whether that is described as intuitive logic, naïve theories, or ontological commitments based on everyday experiences. The theories also include misconceptions that result from incomplete conceptual change, which might be termed competing or conflicting conceptual understandings, incorrectly categorized information, or synthetic models. They also theorize how a person moves from no conceptual knowledge to advanced conceptual understanding of a concept.

This paper will use the framework theory of conceptual change to analyze students' conceptual understanding of mechanics of materials (Vosniadou et al. 2001). The framework theory specifically explains conceptual change in science, and this theory relates the observable and unobservable and everyday phenomena. In mechanics of materials, students are given visible representations of structures; a simply supported beam with a point load at the center or a column with an axial load. These structures are tangible, and perhaps students can relate these visible representations to the real world. Stresses are internal to a structural member and invisible to the eye. A person cannot simply observe stress, and determining stresses in a member requires knowledge of internal force magnitudes, force types, and member geometry. Vosniadou often uses the shape of the Earth as an example to illustrate a person's understanding based on observable truths (Vosniadou and Brewer 1992). When people are children, they believe that the world is flat, because as they see the world, it is flat. The curvature of the Earth is too great to be visible from Earth's surface. It takes years for a person to develop a correct understanding that the world is round. The framework theory is discussed in further detail in the Discussion section to facilitate the explanation of how the framework theory can be used to describe conceptual change in mechanics of materials.

3. Research Goals

The goals of this research are to identify student misconceptions related to mechanics of materials and to interpret the results within the framework theory of conceptual change. Implications for instruction that are based on the framework theory of conceptual change are also considered.

4. Methodology

This research qualitatively analyzed dialogue from interviews with students who were taking or had recently taken mechanics of materials. Ninety interviews were conducted with students of sophomore standing taking mechanics of materials at Washington State University (WSU), Gonzaga University, Oregon State University (OSU), and Spokane Falls Community College. Ten of these interviews were conducted at the community college, seventeen at OSU, and the remaining interviews equally divided between WSU and Gonzaga. Forty students were interviewed regarding concepts of axial loading, and thirty-three students were interviewed regarding concepts of bending loads, referred to as *axial interviews* and *bending interviews* respectively in this paper. Seventeen students were interviewed with regard to mechanics of materials for axial, bending, and torsion loading conditions, referred to as *ABT interviews* in this paper. All students were either currently enrolled in or had completed a sophomore level course in mechanics of materials. Interviews were conducted after students had experienced lectures, completed homework problems, and been examined on the content areas of the interviews.

Although these interviews covered different content areas of axial, bending and torsion load cases, it is appropriate to compare the results of these interviews for several reasons. The concepts in question were the same across the different content areas; stress, strain, and deformation. Interview settings, formats, and question types were very similar. Interviews were conducted in private settings with only an interviewer and interviewee present. The students had sheets of paper with visual aids to guide them through the interview, and they were encouraged to write on the paper to assist in explaining their answers to the questions. All interviewers asked students to describe and explain various stresses and distributions of stresses. Interviewers asked follow-up questions to try to determine students’ reasoning or logic behind their understanding.

This research focuses on axial and bending load cases, both of which relate stresses to loading that is perpendicular or parallel to a member’s length and result in normal stresses in the direction of the length of the member. Axial and bending loading cases are common to civil engineering structural designs as most columns and beams are assumed to be subjected to axial loads, bending loads, or combined axial and bending loads.

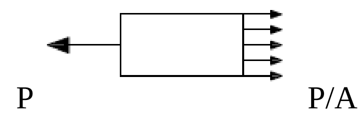
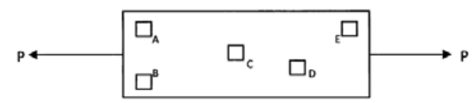
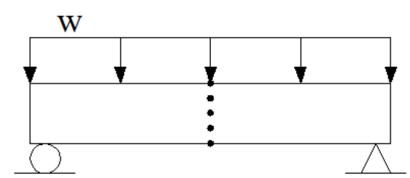
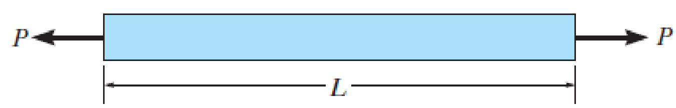
4.1. Interview Method

Students were interviewed following Piaget’s clinical interview methods (Piaget 1952). The interviewer asked students semi-structured sets of questions with the option to ask follow-up questions or clarifications to further probe student understanding of a concept. Clinical interview methods are used to explore students’ understanding of a particular concept, and these allow for researches to identify a student’s reasoning behind their understanding. This interview methodology has been used to study students’ understanding of engineering concepts (Montfort et al. 2009; Streveler et al. 2014).

4.2. Interview Protocol

Three different interview protocols were utilized in this research. As students answered the protocol questions, they were asked to explain the reasoning behind their answers and probed with follow up questions. They were also asked to draw sketches to show stress directions, stress distributions, stress elements, and deformed shapes. In most of the interviews, students were questioned on a particular concept at least two times to identify whether students truly understood the concept and could explain the concept from two different contexts. For example, during the bending interviews, students were ask to explain the stresses present in a simply supported beam with and single point load in the middle and alternatively with a uniformly distributed load along the length of the member. Some example protocol questions are included in Table. 1 and full protocols are included in the Appendices. The interview protocols for each are further discussed in the next three sections.

Table. 1. Example Protocol Questions

Interview Protocol	Questions asked of interviewees	Sketch shown to interviewees
Axial Interview	What is the distribution of the normal stress? Are there any other stresses present?	
	Rank the horizontally oriented stress elements based on their magnitude of shear stress.	
Bending Interview	Describe the distribution of normal stresses acting on the beam and where they will be maximum and minimum.	
	Rank points based on their magnitude of normal stress.	
ABT Interview	Tell me about the stresses the in the member.	
	Where are the largest shear stresses?	

4.2.1. Axial Interview Protocol

The axial interview protocol focused on questions related to stresses and strains in purely axial loaded members. Interviewers told the students at the beginning of the interviews that the internal loads were assumed to be uniform across the cross section and length of the member; not concentrated at the point of load application. Students were asked questions for several different scenarios of axially loaded members intended to examine conceptual understanding of stress at angles perpendicular and inclined to the applied loads and at different locations along the member. Three different contexts were presented as well, including a two dimensional drawing of an axially loaded member, a stretched rubber band with squares drawn on it, and a failed concrete cylinder.

4.2.2. Bending Interview Protocol

The bending interview protocol focused on concepts of bending moment and shear forces and normal and shear stresses in bending members. The first protocol questions focused on understanding of forces and stress, and how they are distributed along a bending member's length and cross section. Students were asked to provide their definitions of moment, normal stress, shear force and shear stress. Then the students were shown a simply support beam with a single point load placed at midspan, and they were asked to identify where the moment and shear would be maximum and minimum along the member length. Students were also asked to describe the distribution of normal and shear stresses on the cross section of the beam.

The subsequent protocol questions included a simply supported bending member where students were asked to rank points scattered along the member based on the magnitude of forces and stresses. Next a sketch was provided of a simply supported beam with a uniformly distributed load and five points placed vertically at the midspan, and students were asked to rank the points from highest to lowest in magnitude for moment, normal stress, shear force, and shear stress. Next the students were shown the same beam except with five points placed horizontally at the neutral axis of the beam, and they were asked the same set of questions about force and stress.

4.2.3. Axial-Bending-Torsion Interview Protocol

The axial-bending-torsion (ABT) protocol included three loading types; axial, bending and torsion loads. For this research, only the axial and bending load cases were analyzed. Students were first asked to explain their understanding of stresses, strains, and deformations for each load type given two-dimensional scenarios for an axially loaded member with tension loads on each end, a simply supported beam with a point load in the middle, and a circular shaft fixed at one end with a torsion load at the other end. Then students were asked to explain their understanding of stress for real-world, failed scenarios for all three loading types including a compression failed concrete cylinder, a piece of wood broken by bending, and a piece of chalk broken by torsion.

4.3. Analysis

Student interviews were recorded and saved as audio files, and the audio recordings were transcribed to text documents either via professional transcription company or by undergraduate and graduate researchers. The text documents were then uploaded into Dedoose, a web application for managing, analyzing, and presenting qualitative and mixed method research data ("Dedoose Version 5.0.11" 2014). The goal of coding was to identify the presence of and reasoning for student misconceptions. For example, a misconception was coded if a student believed that normal stress is not present in a bending member, and their dialogue was examined to identify why they believed this to be true.

Codes and code counts were reviewed to identify commonly occurring misconceptions. Particular attention was paid to whether the same or similar codes applied to both axial and bending load cases. Once all of the transcripts were coded, the codes were compared to identify commonly occurring codes especially with similarities between the two different load cases.

Codes that identified similar misconceptions in the interviews were combined to reduce the total number of codes. Then following the constant comparative method, the transcripts were recoded with the reduced number of codes (Patton 2001). Excerpts of dialogue highlighted by each code were examined to determine the logic that students used to justify their misconceptions. This would be used to identify naïve theories and synthetic models that students use or create in an effort to conceptually understand and explain the concepts in question.

5. Results

5.1. Axial Load Members

5.1.1. Review of Stresses in Axial Load Members

Pure axially loaded members are subjected to a compressive or tensile force that is assumed to be uniformly distributed over the transverse cross section. It's assumed that no bending of the cross section and no stress concentrations occur, which allows for the assumption that internal shear and normal stresses remain uniformly distributed across the entire length of the member. Normal stress is maximum on the transverse cross section. Normal stress can be computing using Eqn. 1, and the average shear stress on any axis can be computing using Eqn. 2; N is the internal force normal to the axis in questions, A is the cross sectional area of the axis, and V is the internal shear force on the axis.

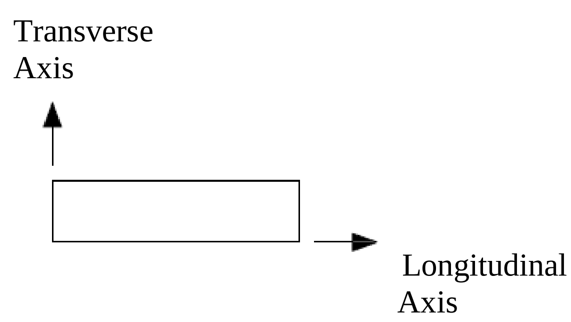


Fig. 2. Axis Orientations

Eqn. 1. $\sigma = \frac{N}{A}$

Eqn. 2. $\tau = \frac{V}{A}$

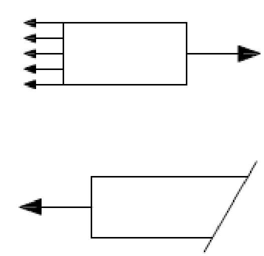
For axial loaded members, normal stress reduces to zero on the longitudinal cross section with axis orientations defined in Fig. 2. Shear stresses are not present on axes parallel to the transverse or longitudinal directions where normal stresses are maximum and minimum, and shear stresses are maximum on the axes at 45° angles from the longitudinal axis.

5.1.2. Normal Stress in Axial Loaded Members Results

Results from analyzing the axial and ABT interviews regarding normal stress in axial loaded members are recorded in this section. Including both sets of interviews, approximately ninety percent of students were able to identify the presence of normal stress in an axial loaded member. They were even familiar with the equation to calculate average normal stress on the transverse axis as shown in the interview excerpt.

> **Interviewer:** So looking at this top axially loaded member, what internal forces are present?
> **Student:** There is a normal stress P/A that would be the only internal force acting on it, I believe.

When shown a sketch of an axial loaded member with a cut along the transverse axis, see Fig. 3 (a), students initially identified normal stresses acting parallel to the applied load. Less than twenty percent of the time, they identified shear stress when given this sketch. However, when shown a sketch of an axial loaded member with a cut diagonal to the transverse axis, see Fig. 3 (b), students more often initially identified shear stress and were not as adept at correctly identifying normal stress. Students would represent normal stress acting on an inclined plane with an arrow parallel to the applied load rather than perpendicular to the inclined surface. When asked to draw normal stresses on an inclined stress element, many of the students, at least fifty percent of the



a)

b)

Fig. 3. Axial Loaded Member Cut Along a) Transverse Axis and b) Inclined Axis

students in the axial interviews, drew normal stress parallel to the longitudinal axis as shown in Fig. 4. These students did not indicate a conceptual understanding that normal stress is orthogonal to the axis in question, and instead took normal stress to act in the direction of or parallel to the applied load.

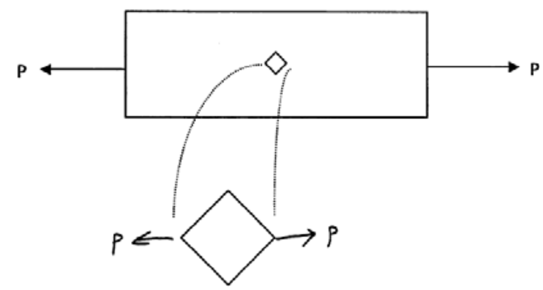


Fig. 4. Angled Stress Element on Axially Loaded Member

There were mixed results from students regarding the distribution of normal stresses in axial loaded members. Eighty-five percent of students accurately stated that stresses are uniformly distributed along an axial loaded member's length, reflecting the original assumptions. However, fifteen percent of the students who knew normal stress existed stated normal stresses were maximum at the location where the axial load was applied as the student quoted here.

> **Interviewer:** Where would the maximum normal stress be acting [in this axially loaded member with uniform cross section]?
> **Student:** It's going to happen at ends or near the ends because that's where the concentrated loads are being applied.
> **Interviewer:** How would [the magnitude of stress] change as you went to the middle?
> **Student:** It's going to decrease to a small extent cause we're farther away from the concentrated…I mean like ideally it would act uniformly across this beam, but as it plays out it'll decrease throughout the beam to be weakest shear forces acting at the center. Umm. So essentially, [the end] particles will be acted on before [the middle] particles…
> **Interviewer:** What about shear stresses? Where would the maximum shear stresses act or be acting?
> **Student:** Umm. In this case I think it's also at the ends.

The logic that this student used to justify their misconception is that loads concentrate at the location of the applied load and particles near the load get stressed first. Alternatively, a few students had the misconceptions that normal stresses were maximum at the midspan location as depicted in the interview quotation provided below.

> **Interviewer:** Okay. So then looking at this [axially loaded] member what internal forces are present?
> **Student:** Internal forces, that is going to be, well it's pointing out so there is going to be inching in that way [contracting along the transverse axis] but it's going to create kind of like a string going out so there is going to be more stress in the middle.
> **Interviewer:** Okay. And why is it going to be more stress in the middle?
> **Student:** Because it's further away, from the stress, from the load.

This student used logic that stresses increase at greater distances from the applied load to explain their misconception that the stresses are maximum at the midspan of the axial loaded member.

5.1.3. Shear Stress in Axial Loaded Members Results

Students' conceptual understanding of shear stress in axial loaded members was inconsistent throughout the axial and ABT interviews. Initially, sixty percent of students stated no shear stress occurred in a pure axial loaded member, which is only true when considering a axis parallel or perpendicular to the transverse axis. For the students who had the misconception that there was no shear present, all justified their answer by saying that there was no force parallel to the transverse axis as demonstrated by the interview quotation provided below.

> **Interviewer:** Do you think there would be any shear stress acting internally [to the axial loaded member]?
> **Student:** I wouldn't think so. You would need some vertical forces [parallel to the transverse axis] to cause that shear.

As mentioned in the results section regarding normal stress in axial loaded members, when students were presented with a sketch of a member with an inclined axis, they could identify the presence of shear stress. Also when shown a concrete cylinder that fractured from shear at a 45^{o} angle to the axial loading, eighty-five percent of the students identified the fracture as a shear failure and could explain the significance of the angle.

Students often did not demonstrate an understanding of concepts regarding shear stress distribution, and how stresses might change between different axes. They simplified shear stress to a singular axis that corresponds to the direction of loading as the student did from this interview excerpt.

> **Interviewer:** For this [axially loaded] member, are there shear forces acting in that member?
> **Student:** No.
> **Interviewer:** Are there shear stresses within this member?
> **Student:** I don’t remember shear I don’t believe so.
> **Interviewer:** Why do you say that?
> **Student:** Because again, the loads are in the horizontal…There are shear forces because if you cut it at diagonal, the next thing is you’ve got a plane and it’s going to have normal forces pulling out but it's also going to have shear forces going like that.

Rather than saying there is no shear stress present, it would be more accurate to say that shear stress is zero when considering the transverse axis, because this is the orientation with the minimum and maximum normal stress.

5.2. Bending Load Members

5.2.1. Review of Stresses in Bending Members

Bending members are subjected to loads perpendicular to the longitudinal axis. Normal stresses in a bending member depend on the internal moment force along the bending member’s length, and normal stresses are maximum at the highest internal moment along the member’s length and at the top and bottom fibers. Normal stress (σ) can be calculated using Eqn. 3; M is internal bending moment, c is distance from neutral axis to the location in question, and I is the second moment of area or moment of inertia. At the neutral axis, normal stresses are zero, see Fig. 5.

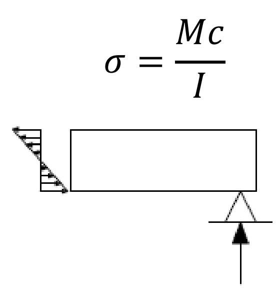


Fig. 5. Normal Stress Distribution in Bending Member

Eqn. 3. $\sigma = \frac{Mc}{I}$

Shear stresses depend on the shear force (V) internal to a bending member and the cross sectional properties of the member. Shear is not uniform about the cross section of a bending member. Shear stresses acting on the transverse cross section are zero at the top and bottom fibers the greatest distance away from the neutral axis, and shear stress is maximum at the neutral axis. Shear stress (τ) in a bending member can be calculated using Eqn. 4; V is internal shear, Q is the first moment of the portion of the cross-sectional area between the line where the stress is to be evaluated and the top or bottom fiber of the beam, I is moment of inertia, and t is the thickness of the cross section.

Eqn. 4. $$\tau = \frac{VQ}{It}$$

5.2.2. Normal Stress in Bending Members Results

For bending members, the most common normal stresses computed in mechanics of materials courses are the stresses that act orthogonal to the transverse face. These stresses are dependent on the value of the internal bending moment, the member section properties resisting the moment, and the distance in the transverse direction from the neutral axis to the top and bottom fibers. Students did not show a consistent understanding of the concept that normal stresses can occur in directions orthogonal to loads. For the bending cases, approximately half of students said no normal stress would be present, because there were no loads perpendicular to the transverse axis or in the direction parallel to the longitudinal axis. The quotations below demonstrate this.

> **Interviewer:** Where do you think the maximum normal stresses would be [in the bending member]?
> **Student:** Oh, normal stresses.
> **Interviewer:** Yeah.
> **Student:** Okay. It's going to be where [the load] is applied. Well, if you cut the beam here. The normal stresses in the beam is going to be, is going to be zero because we don't have a force in the [longitudinal] direction.

At first this student tried to say normal stress would be maximum at the location of the applied load. Then when they considered the beam cut at the location of the applied load, they changed their answer to say there was no normal stress, because there was no force parallel to the longitudinal direction.

> **Interviewer:** What type of stresses do you think caused [the wood bending member] to fail in the way that it did?
> **Student:** Ah, shear stress because, and that goes back to, umm, if you cut that member it going to have its maximum at either end, well this is going to be the minimum, so then in the middle is where you're going to have the least amount of shear.
> **Interviewer:** So these [arrows], these [forces] that you drew right here are shear, right?
> **Student:** Yeah.
> **Interviewer:** Okay. Why do you think they would be shear and not normal?
> **Student:** Umm, because if it's just in bending it's not going to have any normal or any forces in that [longitudinal] direction just in the [transverse] direction.

This student believed a bending member failed in shear, because there are only forces acting in the direction parallel to the transverse axis. Similarly, the following student thought that normal stresses in bending members would only be produced by forces acting in the direction parallel to the longitudinal axis.

> **Interviewer:** Do you think there would be any normal stresses in [the bending member]?
> **Student:** No, there aren't any normal stresses because there aren't any horizontal reactions [parallel to the longitudinal axis].

5.2.3. Shear Stress in Bending Members Results

Students' conceptual understanding of shear stress in bending members varied throughout the bending and ABT interviews. Twenty-five percent of students had a misconception that shear stresses were maximum at the locations of the member ends or supports. The student who provided the quotation below said the maximum shear stress would occur at the supports, because the supports are at a distance farthest away from the load at the midspan of the bending member.

> **Interviewer:** [Where would the] maximum shear stresses [occur in the bending member]?
> **Student:** I want to say back towards the ends just because it's the farthest from the force. But I'm not sure that's right. But I'll say back towards the supports on both sides.
> **Interviewer:** Okay. How do you think the shear stresses would change along the length of the [bending] member?
> **Student:** They'd increase back towards the supports.
> **Interviewer:** So…shear stresses would be at a minimum in the middle.
> **Student:** Yes.

A few students had a misconception that maximum shear stress occurred at the midspan location of the bending member where the load occurred. This student quoted below used the maximum deflection as the cause of maximum shear.

> **Interviewer:** How would the shear stress vary as you went from a support to the midspan [of the bending member]?
> **Student:** It's going to increase.
> **Interviewer:** Why do you think it's going to increase?
> **Student:** Is it going to increase though? Yeah because you have max deformation [at midspan] so our element is going to be deformed the most by the internal stresses.

A few students accurately stated that internal shear is uniform along the length of the bending member. Less than twenty percent of students were able to accurately explain how shear stress distribution varies along the transverse axis from maximum at the neutral axis to minimum at the top and bottom fibers the longest distance away from the neutral axis.

5.3. Axial and Bending Members Compared

5.3.1. Summary of Specific Load Case Results

A few misconceptions were identified in the previous sections for axial and bending load cases, and the approximate prevalence of these misconceptions are summarized in Table. 2. Some of the misconceptions share commonalities across the two load cases, which are termed *overlapping misconceptions*. The first overlapping misconception is that shear and normal stress depend on the direction of applied load. The second overlapping misconception is that shear and normal stress depend on the location of the applied load.

Table. 2. Summary of Results

	Load Case	Description of Misconception or Conceptual Understanding
90%	Axial	Identify presence of normal stress
Most	Axial	Did not acknowledge stresses on inclined axes, only initially identified stresses acting parallel to load
50%	Axial	Drew normal stress parallel to longitudinal axis on diagonal stress element
85%	Axial	Stresses uniformly distributed along length
15%	Axial	Normal stresses maximum at the point of applied load
Few	Axial	Normal stresses maximum at the middle of member’s length
60%	Axial	Initially stated no shear stress
Most	Axial	Changed mind about presence of shear stress when asked to consider inclined axis
85%	Axial	Stated shear failure for concrete cylinder
45%	Bending	No normal stresses, because no loads perpendicular to transverse axis
25%	Bending	Shear stress maximum at member ends
Few	Bending	Accurately described shear stress distribution

5.3.2. Stress Dependency on Load Direction

Students often used on the direction of load as logic to determine stresses, which is not always correct. For example, students had a misconception that there was only normal stress in axially loaded members, because the axial load was applied normal to the transverse cross section. Conversely for bending members, students had a misconception that that there was no normal stress, because there were no applied loads normal to the transverse cross section. This was the case with the student in the following quotation.

> **Interviewer:** Describe how the normal stresses are acting on the [simply supported bending member] where it can be maximum and minimum.
> **Student:** All right, let’s see here. So you cut it right there, you’re going to have P over 2. Actually, wait…There’s nothing acting axially on this guy.
> **Interviewer:** All right, so that would be [there is zero normal stress]?
> **Student:** Yeah, I think so.

Another commonality between the cases of normal stress in axially loaded members and no normal stress in bending members is that students had a misconception that normal stress acts on the transverse axis only. However, normal stresses act on an infinite number of axes or planes. With this conceptual understanding, in order to produce a normal stress, a perpendicular force is required on the transverse axis.

A similar relationship exists for shear stress. Students had a misconception that there was no shear stress in axially loaded members, because there was no load applied in the direction parallel to the transverse cross section. Conversely, they had a misconception that there was shear stress in bending members, because loads were applied in the direction parallel to the transverse cross section. This is demonstrated by the student in the first part of the interview excerpt below who didn't believe there was shear in the axially loaded member, because there wasn't a vertical applied load or a load perpendicular to the member's length. In the same way students attributed normal stress to forces acting perpendicular on the transverse axis, students attribute shear stress to shear force acting parallel to the transverse axis. Coincidentally, for an axially loaded member, the shear stress is zero on the transverses axis, so it's partially correct to say there is no shear stress in an axially loaded member when considering this axis. When interviewers asked students to consider an axis at an angle to the longitudinal or transverse axes, it was common for students to recognize the presence of shear in axially loaded members as demonstrated in the second part of the following quotation.

> **Interviewer:** Looking at that [axially loaded] member, are there any shear forces acting on this member?
> **Student:** I don't think so, no.
> **Interviewer:** And why can you say that?
> **Student:** I am, actually I don't really know.
> **Interviewer:** What would there need to be in order for there to be shear forces acting on that member? You can go ahead and draw what you want?
> **Student:** I think there need to be like a vertical [force].
> **Interviewer:** Force [perpendicular to the longitudinal axis] would…be shear?
> **Student:** Yeah.
> **Interviewer:** Then if we were to take a [diagonal] cut right here on this axial member, go ahead and draw the resulting forces acting on that [diagonal cross section]?

> **Student:** That would be the same axis the P is acting in. There would be a normal stress acting [perpendicular to the diagonal cross section]...And then there would be a shear acting along [the diagonal cross section].

5.3.3. Stress Dependency on Load Location

For both axial and bending cases, students used the logic of load location to explain magnitude of stresses. Students correlated the magnitude of stress with either the location of the applied load or the supports of the members. In some instances, such as a simply supported bending member with a single point load at midspan, the normal stress is maximum at the location of the applied load, because the internal bending moment is coincidentally maximum at this location. However, it is a misconception to say stress directly relates to the location of the applied load when stress actually relates to the magnitude of internal bending moment.

Similarly in axially loaded members, students had a misconception that the stress magnitude was dependent on the load location, despite the assumptions that loads distribute uniformly over the transverse cross section and stresses are uniform along the longitudinal length in axially loaded members. In the quotation provided below, the student believed that stress magnitude changed along the longitudinal length of the axial loaded member, because as you move farther away from the load on each end, then the stresses will increase.

> **Interviewer:** Alright and so then looking at this [axially loaded] member down here, we have elements A through E sporadically placed throughout the member [Fig. 6]. Go ahead and rank those elements from most to least based on their magnitude of stress.
> **Student:** Okay C>D, I hope this is right and I have it completely opposite C>D>E which is equal to A equal to B.
> **Interviewer:** Okay. Then what's your reason for that ranking?
> **Student:** Well it will be more in the middle than which I don't know if it is, but if that's the case then C will be the farthest away from both the stresses. So double check myself now but C will be farther away from both the stresses and then D second farthest away then the rest are all the equal distances away from the forces.

In the next quotation, the student believed that the normal stresses increased toward the middle of the axially loaded member, justifying their answer by saying the middle of the member experienced more force. These students demonstrated two different misconceptions regarding the stress distribution in axially loaded members, but they reach the same, incorrect conclusion.

> **Interviewer:** And then looking at these stress elements we have down below [Fig. 6], go ahead and rank them based on the magnitude of stress?
> **Student:** Okay. I am going to put down C is greater than D which is greater than E equals to A and equals to B.
> **Interviewer:** And what's your reasoning for that ranking?
> **Student:** Well I think A, B and E are going to all be equal because of distances because there is no actual distance but they all look like they are in basically the same area on the corner and if the P is going to be universal along that side then there wouldn't be any difference, there wouldn't be any difference on the height difference. And I thought C is the highest, so it is closest to the middle and has the most forces from both of them.

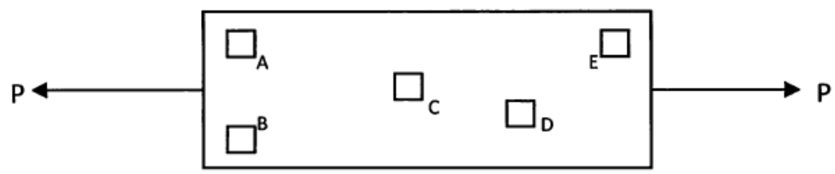


Fig. 6. Axially Loaded Member with Sporadic Elements

6. Discussion

6.1. The Framework Theory Summarized

The *framework theory* attempts to explain how conceptual change is achieved when a person learns science concepts. A framework is a coherent, whole explanatory system that comprises a person's conceptual understanding. It is not a singular concept or area of knowledge. Rather a framework is a complex network of embedded concepts that make up the whole system of a person's understanding of the world. Different frameworks are created for different concept areas called domains such as mathematics, physics, psychology, and language.

Before exposure to scientific theory, people develop initial frameworks called *naïve theories*. The physics framework of conceptual understanding starts as naïve physics that people use to explain the world around them. "Naïve physics are not fragmented observations but form coherent whole, a framework theory (Vosniadou et al. 2008 p. 4)." Naïve theories are not based on scientific concepts. They are based on and reinforced by everyday experiences and observations, and they create the basis of a person's conceptual framework. For the physics framework, naïve physics theories are supported by the notion that *the world is as we see it*. People see and feel their physical environments, and those observations shape the conceptual understanding or naïve theory of the physical world. As an example, Vosniadou studied how children believe that the Earth is flat. Children observe and feel throughout their daily lives that the Earth is flat, and they use those experiences to form a naïve theory regarding the shape of the Earth.

A naïve theory must change through the process of conceptual change in order for a person to learn a scientific theory. To change a naïve theory into a non-naïve theory, new information must be introduced. With the introduction of new information, a naïve theory will shift into a more sophisticated *framework theory* that is based on or includes the new information. A framework theory goes through this conceptual change process each time new information is added, moving through countless iteration of a framework theory until a person reaches a conceptual understanding of the scientific theory. Much of the conceptual understanding that comprises a naïve theory becomes the presuppositions that people have regarding a concept. Naïve theories can hinder the conceptual change process, because the presuppositions that make up a naïve theory are not always easy to change. Vosniadou, Vamvakoussi, & Skopeliti stated that, "The process of conceptual change appears to involve a gradual lifting of the presuppositions of the framework theory allowing the formation of more sophisticated models (2008 p. 9)."

Framework theories are comprised of countless concepts that make up the entire coherent framework. In some cases a concept may include information that is new or specific to that concept, and a *specific theory* within the framework must be created for that concept. For example the Earth might fit into a specific theory of astronomy that has its own set of concepts that apply. Specific theories may results from a set of observations, and from these observations, people create beliefs and mental models regarding those concepts. As a framework shifts into a more sophisticated theory, the framework will include more and more specific theories.

When new information is received, a person categorizes the concepts of the information and fits them within the most appropriate, existing framework or creates a new framework. To affect conceptual change, a person must add a category for the information, change an existing category to accommodate new concepts, or shift information from one category to another. These categories are sometimes referred to as ontological commitments; a person ontologically commits information to a certain category. Once a person ontologically commits information to a category, then that concept takes on the characteristics of that ontological category. For example, children will make an ontological commitment that the Earth is a physical object similar to all other physical objects. That means the Earth is supported probably at its base, stable unless pushed or pulled, and shaped as we observe it to be shaped. When children learn scientific theories regarding the shape and motion of the Earth, they must change or replace their previous ontological commitment of the Earth as a physical object. The new ontological commitment must accommodate the fact that the Earth is not supported, it moves without any apparent push or pull, and it is not flat as it appears.

Conceptual change based on the framework theory is a slow gradual process, because as people add new information through enrichment types of learning and modify their ontological commitments to concepts, they need to assimilate the new information with the existing framework and ontological commitments. Sometimes the new information is incompatible with

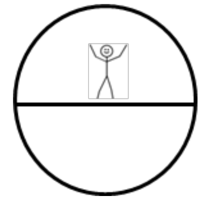


Fig. 7. Example of a Child's Synthetic Model of Earth

the existing, and assimilating incompatible information with existing leads to misconceptions, fragmentations, and internal inconsistencies, which are termed *synthetic models*. According to Vosniadou & Skopeliti (2014 p. 1430), "Synthetic [models] are produced when learners, in the search for coherence and internal consistency, incorporate the scientific information to their incompatible prior knowledge distorting it and creating an alternative conception or model which however has some internal consistency and explanatory value." Vosniadou and Brewer (1992) found that children create a few different synthetic models in an attempt to integrate their naïve theory that the Earth is flat with scientific theory that the Earth is a sphere. For example, some children construct a synthetic model that the Earth is a hollow sphere, and people live on a flat surface inside of the sphere, see Fig. 7.

6.2. Framework Theory of Mechanics of Materials

This paper uses the framework theory approach to explain the conceptual change process when students study and learn the concepts of mechanics of materials. Before studying the scientific theories of mechanics of materials, students develop a naïve theory that is based on their observations of the world; that the world is as they see it. A student can observe that objects move, bend, and break when they are pushed or pulled. If you push a child in a swing, the child moves. If you pull on silly putty, it stretches. If one child holds tightly to the ends of a wooden stick and another child hits the wood hard in the middle, the wood might break. These are examples of physical world observations, and this paper hypothesizes that similar observations contribute to a student's naïve theory of mechanics of materials.

The mechanics naïve theory has specific theories contained within it that are also based on observations of the world. As objects are pushed or pulled, they move or deform at the locations where they are pushed or pulled and in the direction where they are pushed or pulled. Objects may even break when they are pushed or pulled. From these observations of the world, a student may create the belief that pushing and pulling cause objects to move, deform, or break. Fig. 8 illustrates a mechanics of materials specific theory that fits within the mechanics naïve theory.

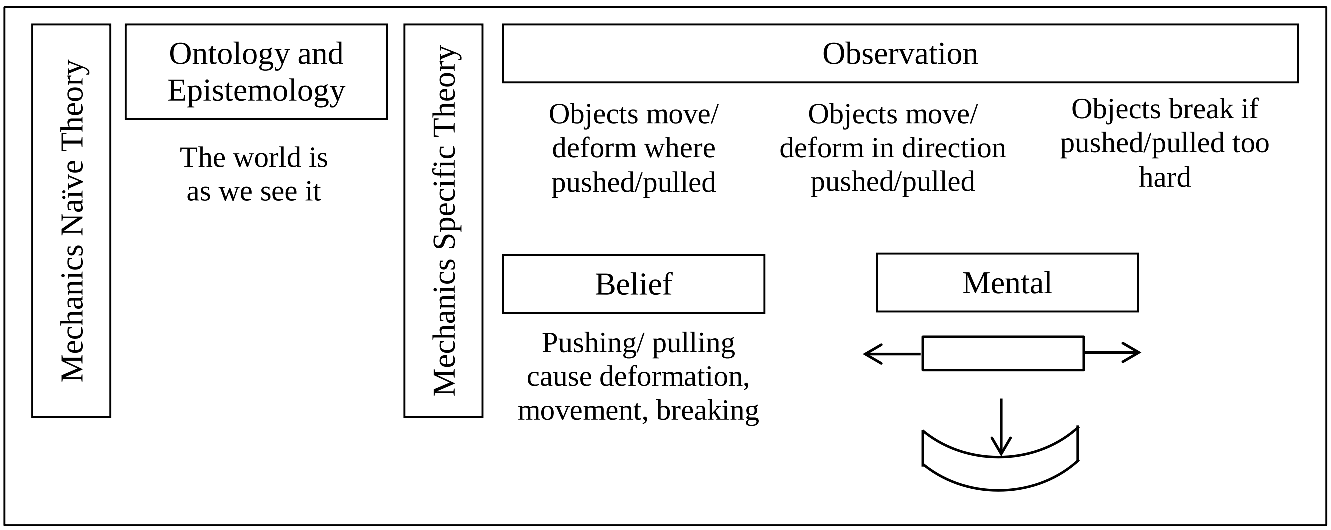


Fig. 8. Example of Hypothesized Mechanics Naïve Theory

When students start studying mechanics of materials, they must integrate the learned concepts with their naïve theory. This requires students to change or add to their ontological commitments of mechanics of materials concepts. Changing or adding ontological commitments is what allows a student to start to shift their naïve theory to a more sophisticated framework theory. Sometimes the mechanics of materials concepts do not agree with naïve theory and the ontological commitment that the world is as we see it, because the concepts cannot be observed. Internal forces and stresses are not visible to the eye. When there are conflicting ontological commitments within a framework theory, students create synthetic models in a way to synthesize the conflicting information. Synthetic models often contain misconceptions regarding scientific concepts. Common misconceptions that students have regarding mechanics of materials were identified in the results section of this paper: 1) shear and normal stress magnitude depend on the location of the applied load; 2) shear and normal stress depend on the direction of applied load.

A hypothesized synthetic model that students create may result from the introduction of equations, and it is depicted in Fig. 9. When students learn equations that can be used to calculate internal stresses, they ontologically commit to their internal interpretation of these equations, but they still hold onto the ontological commitment that the world is as they see it. This synthetic model allows a student to compute a numerical value for stress and believe their ontological commitment of the observable applied loads act in the direction of the invisible stresses. A misconception occurs when students don’t modify their ontological commitment of loads and stresses acting in the same direction. Internal stresses act in infinite number or directions, not only parallel to the applied load.

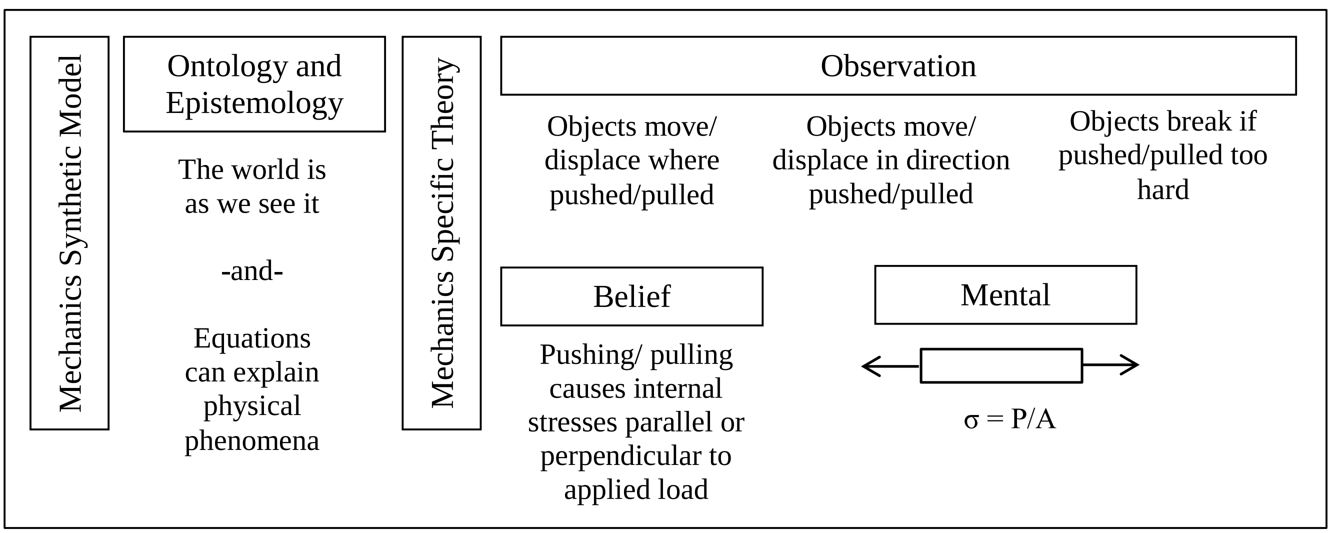


Fig. 9. Example of Hypothesized Mechanics Synthetic Model

Students’ synthetic models include misconceptions based on what they can see. In many mechanics of materials problems, students are shown a sketch or a picture of a structure with loads applied at certain locations. These sketches and pictures provide an observable object for students to base their conceptual understandings on. For example, students see a picture of a person pulling on a rubber sheet, see Fig. 10, and it looks like the person's fingers are going to poke a hole in the sheet

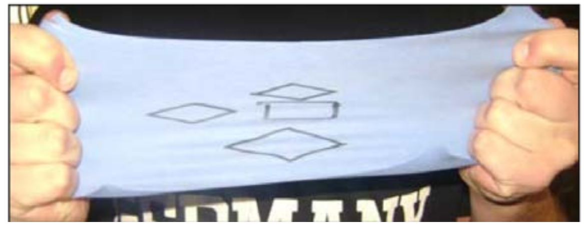


Fig. 10. Axial Load on Rubber Sheet

where the fingers are pulling. From this a student ontologically commits to the observation that stresses appear to be greatest where the person's fingers apply the load to the rubber sheet, i.e. at the location of the applied load. This is a misconception of the concept of stress distribution in an axial loaded member that is assumed to be uniform along the length of the member. Similarly for bending members, students observe that when a load is applied to the midspan of the member, it will displace in the direction of the applied load. Students observe this deflection and ontologically commit to the magnitude of internal force or stress is maximum at the location where the deflection is maximum as is the case in the following quotation.

> **Interviewer:** So where do you think the maximum moment would [be in the bending member with perpendicular point load applied at midspan]?
> **Student:** [Maximum moment is] going to happen at the center of the [bending member].
> **Interviewer:** Why do you think it would happen there?
> **Student:** Our strongest moment is going to be the maximum deflection.

Internal forces result from a combination of how a bending member is supported and how the loads are applied. They do not result from flexural deflection, so it is a misconception to say that the maximum moment will occur at the location of maximum deflection even though in some cases maximum moment coincidentally occurs at the same location as maximum deflection.

In summary, students start studying mechanics of materials with naïve theories that result from their observations of the world. These theories hinder a student's ability to conceptually understand mechanics of materials, and misconceptions arise in an attempt to integrate presuppositions with scientific theory. In moving from a naïve understanding to a conceptual understanding of the scientific theory of mechanics, students create synthetic models, which are coherent explanatory frameworks to the person who possesses them.

7. Recommendations for Instruction

The research presented in this paper identified misconceptions that students have or create regarding the mechanics of axially loaded and bending member. The misconceptions directly relate shear and normal stress magnitude with the location and direction of the applied loads, when shear and normal stress depend on internal loads and member geometry. Perhaps changes to traditional engineering educational instruction could help to correct or prevent these misconceptions.

For axially loaded members, instruction often uses visuals similar to those found on previous pages in Fig. 6 and Fig. 10. In either a sketch or a simulated real-life condition, students cannot see what happens internally to the axially loaded member. They cannot visually see stress. Students can see the arrows shown schematically on the ends of a two dimensional member and fingers prying a stretched rubber band. These visuals may relate to a student’s naïve theory that the world is as they see it, and therefore, they develop and conceptual understanding that stresses are maximum at the locations and in the direction of the applied, visible loads. If instructors use visuals that don’t allow students to connect the visual and tangible applied loads with their understanding of stress, then perhaps students will develop an understanding of stress that is not so closely related to their naïve theory. Instructors could also address this misconception head on. They can acknowledge that it is intuitive and correct to believe that stresses concentrate at the location of the applied load. Based on Saint-Venant’s principle, stress concentration at the location of applied load does exist. However these stress concentrations are negligible when compared to the uniformly distributed stresses throughout a member, and therefore stress concentrations are ignored in order to simplify computation of stresses.

For bending members, students often develop misconception that there is only shear stress and no normal stress. This may result from the misconception that stresses result from the direction of the applied load. Bending members are often represented as simply supported with an applied load shown as a single arrow at midspan, see Fig. 11. With this two dimensional representation, it is easy visually to see how the applied point load or arrow can transfer to an internal shear force or arrow. It is not so easy visually to see how the straight applied point load transfers to an internal moment or curved arrow. With this two dimensional representation, the unloaded condition and the condition with the internal forces do not change shape. Perhaps if instructors used a deformed bending member, see Fig. 12, that shows the member stretching horizontally at the bottom and compressing horizontally at the top, then students would use that visual que to develop a conceptual understanding that stresses act not only in the direction of the applied load but also perpendicular to the direction of the applied load.

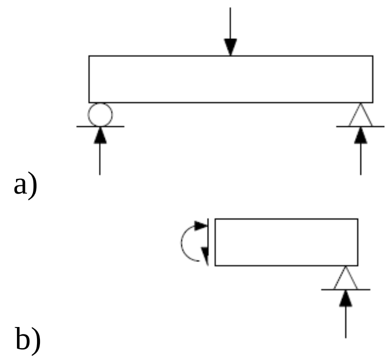


Fig. 11. a) Bending Member
b) Internal Loads on Bending Member

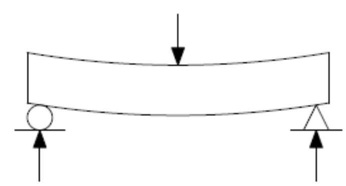


Fig. 12. Deformed Bending Member

The framework theory suggests that a student’s conceptual understanding is affected by their naïve or initial theory, and for mechanics of materials the naïve theory is based on everyday observations and the theory that the world is as we see it. As students study and learn, they struggle to assimilate the invisible, abstract concept of stress with the tangle and visual world that they can observe. What students can see are arrows as point loads, uniform loads, or moments that represent the magnitude and direction of applied forces. They can see member support types and locations. Using physical models, students can see members deform under applied loads. Instruction needs to be designed to bridge the gap between theses visible concepts and the invisible, abstract concepts of mechanics of materials. A couple potential tools that might help to bridge this gap between visible and abstract include graphics of finite element models or experiments using photoelastic plastic and polarized light that that can highlight stress concentrations.

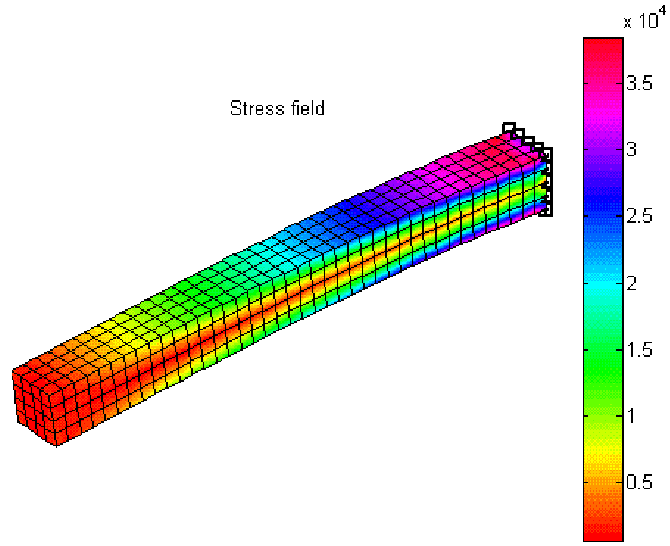


Fig. 13. Finite Element Analysis Model of

Finite element analysis computer programs create graphics using color gradients to represent stress magnitude in stressed structural elements such as the cantilevered bending member shown in Fig. 13. For this specific bending member, the color on the bottom of the vertical stress scale, which is orange, represents the lowest or zero internal stress. Looking at the graphic of the bending member in this figure, this orange color appears where the internal moment and internal stresses are zero on the left end opposite from the fixed support, and the color spreads through the length of the member near the center of its depth, which corresponds to the neutral axis also where stresses are zero. This finite element analysis graphic also shows the stress magnitudes are highest at the fixed support and the top and bottom of the bending member where the normal stresses are expected to be highest. The model shown in Fig. 13 may help students to conceptually understand that stresses exist internal to bending members even though stresses cannot be observed visually in real, everyday situations.

Photoelastic plastic and polarized light can illustrate how stresses distribute in structural elements as well. Fig. 14 shows an example of a bending photoelastic plastic plate. The colors of the plastic vary as the internal stress distribution changes and stresses increase or decrease. With these models, colors do not appear in locations along members where stresses are zero. Photoelastic plastic models may not be as exact as a finite element model at showing magnitudes of internal stress, but they can provide a visible representation that something happens internal to structural members when loads are applied, making abstract concepts of internal stress more tangible.

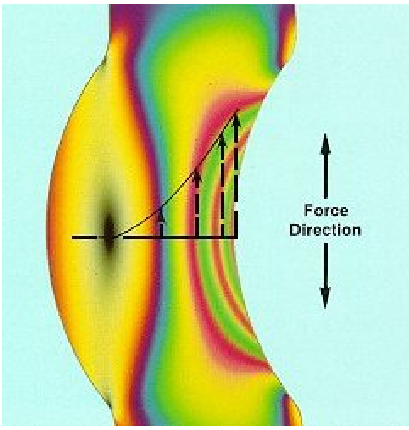


Fig. 14. Photoelastic Plastic Showing Stress Gradient (Missouri University of Science and Technology 2015)

In both the axial and bending cases, students developed the misconception that stresses are greatest at the location of the applied load. Given that stress concentration do in fact develop at the location of applied loads, one might argue that this is not actually a misconception at all. There is inherent truth to this logic. However, it seems that students focus on this intuitive logic that stresses occur at the location of applied loads and neglect the bigger picture that stresses occur throughout a member’s entire length and cross section, which is where the misconception occurs. Full conceptual understanding would require students to acknowledge stress concentration where loads are applied but also recognize that this phenomenon can be explained using specific theories such as Saint-Venant’s and overall internal stresses occur everywhere in a loaded member and can be computed using internal forces and geometry. Perhaps instruction can help students to develop this intuitive logic along with the lesser intuitive, more abstract logic that invisible internal forces result from externally applied loads. Finite element analysis models and photoelastic plastic can be used to show that student's intuition is correct about stress concentrations, but these tools also show that much more happens internal to members than just stress at the location of applied load. Instruction would need to better explain and make clear that despite the occurrence of stress concentrations, mechanic of materials concepts can follow the assumptions that stress concentrations can be ignored.

8. Conclusion

The results from this research show that students develop misconceptions from studying mechanics of materials. One common misconception is that students relate the location of applied load to the magnitude of stress. Another common misconception is students understand normal and shear stress to relate to load direction. In alignment with the framework theory of conceptual change, these misconceptions result from naïve theories of the correct conceptual understanding. Students start studying mechanics of materials with presuppositions of mechanics of materials from everyday experiences, and they must integrate new understandings with preconceived understandings, and sometimes the new and preconceived information conflict. When this integration occurs, students develop misconceptions or synthetic models to make sense of the conflicting information. Perhaps if instructional methods were designed to accommodate and address students' presuppositions and help students to realize their misconceptions, then misconceptions regarding mechanics of materials concepts could be prevented.

9. Bibliography

Brown, S. A. (2013). "Conceptual Change in Mechanics of Materials." *120th ASEE Annual Conference & Exposition*.

Chi, M. T. H., and Roscoe, R. D. (2002). "The Processes and Challenges of Conceptual Change." *Reconsidering Conceptual Change: Issues in Theory and Practice*, M. Limon and L. Mason, eds., Kluwer Academic, New York, NY, 3–27.

"Dedoose Version 5.0.11." (2014). SocioCultural Research Consultants, LLC, Los Angeles, CA.

DiSessa, A. A. (1993). "Toward an Epistemology of Physics." *Cognition and Instruction*, 10(2), 105–225.

Hestenes, D., Wells, M., and Swackhamer, G. (1992). "Force concept inventory." *The Physics Teacher*, 30(3), 141–158.

Martin, J., Mitchell, J., and Newell, T. (2003). "Development of a concept inventory for fluid mechanics." *33rd ASEE/IEEE Frontiers in Education Conference*, Boulder, CO.

Midkiff, K. C., Litzinger, T. A., and Evans, D. L. (2001). "Development of Engineering Thermodynamics Concept Inventory Instruments." *31st ASEE/IEEE Frontiers in Education Conference*, Reno, NV.

Missouri University of Science and Technology. (2015). "Photoelasticity."

Montfort, D., Brown, S., and Pollock, D. (2009). "An Investigation of Students ' Conceptual Understanding in Related Sophomore to Graduate-Level Engineering and." *Journal of Engineering Education*, 98(2), 111–129.

Patton, M. Q. (2001). *Qualitative Research & Evaluation Methods*. SAGE Publications, Inc, Thousand Oaks, CA.

Piaget, J. (1952). *The Origins of Intelligence in Children*. International Universities Press, New York.

Richardson, J., Morgan, J., and Evans, D. (2001). "Development of an Engineering Strength of Material Concept Inventory Assessment Instrument." *31st ASEE/IEEE Frontiers in Education Conference*, Reno, NV, F2A–4.

Richardson, J., Steif, P., Morgan, J., and Dantzler, J. (2003). "Development of a Concept Inventory for Strength of Materials." *33rd ASEE/IEEE Frontiers in Education Conference*, Boulder, CO, T3D–29.

Steif, P. S., Dollar, A., and Dantzler, J. A. (2005). “Results from a Statics Concept Inventory and their Relationship to other Measures of Performance in Statics.” *35th ASEE/IEEE Frontiers in Education Conference*, Indianapolis, IN, T3C–5.

Streveler, R. A., Montfort, D. B., Herman, G. L., Brown, S. A., and Matusovich, H. M. (2014). “Conceptual Change Across Engineering Disciplines.” *121st ASEE Annual Conference & Exposition*, Indianapolis, IN.

Vosniadou, S., and Brewer, W. F. (1992). “Mental Models of the Earth: A Study of Conceptual Change in Childhood.” *Cognitive Psychology*, 24(4), 535–585.

Vosniadou, S., Ioannides, C., Dimitrakopoulou, A., and Papademetriou, E. (2001). “Designing Learning Environments to Promote Conceptual Change in Science.” *Learning and Instruction*, 11(4), 381–419.

Vosniadou, S., and Skopeliti, I. (2014). “Conceptual Change from the Framework Theory Side of the Fence.” *Science & Education*, 23(7), 1427–1445.

Vosniadou, S., Vamvakoussi, X., and Skopeliti, I. (2008). “The Framework Theory Approach to the Problem of Conceptual Change.” *International Handbook of Research on Conceptual Change*, S. Vosniadou, ed., Routledge, New York, NY, 3–34.

10. Appendix A- Axial Interview Protocol

Question-Description	Description & Concepts Covered	Questions Asked
Figure of Axially Loaded member	An axially loaded member has a cut perpendicular to the applied load Normal stress	What is the distribution of the normal stress? Are there any other stresses present?
Figure of Axially Loaded member	Member has a horizontal stress element from the geometric center of the member Normal stress & shear stress	Draw and describe the stresses on the stress element. Are there any vertical normal stresses present? Are there any shear stresses present?
Ranking Task	An axially loaded member has 5 stress elements spaced randomly throughout its body Normal stress	Rank the stress elements based on their magnitude of normal stress.
Figure of Axially Loaded member	An axially loaded member with no cuts Normal strain & normal stress	Is there normal strain occurring on this member? Can you describe how the dimensions are changing? What are the stresses associated with the changes in dimension?
Ranking Task	An axially loaded member has 5 stress elements spaced randomly throughout its body Normal strain	Rank the stress elements based on their magnitude of normal strain.
Rubber Band	A rubber band with a horizontal stress element drawn on it Normal strain & normal stress	Pulling horizontally on the rubber band how is normal strain occurring on the stress element. Describe the normal stresses associated with the strain on the stress element.
Figure of Axially Loaded member	An axially loaded member with no cuts Shear stress	Are shear stresses acting within the axially loaded member? Why or why not? If so, how are they acting?
Figure of Axially Loaded member	An axially loaded member with a 45 degree cut at the center of the member Normal stress & shear stress	Draw the resultant forces acting on the cut. What stresses are present at the face of the cut? How are the stresses distributed? Is there an equation that you can recall?

Question-Description	Description & Concepts Covered	Questions Asked
Figure of Axially Loaded member	An axially loaded member with a stress element oriented at an angle Normal stress & shear stress	Draw and describe the stresses on the stress element. How would the stresses change if the stress element were rotated back to a horizontal orientation? Would their magnitudes change? Would their direction change?
Figure of Axially Loaded member	Member has a horizontal stress element from the geometric center of the member, along with Sigma & Tau axis for Mohr's circle Normal stress & shear stress	Draw the stresses that exist on the stress element. Draw Mohr's circle from the stresses indicated on the stress element you just drew. What points of significance did you use to draw Mohr's circle? How would Mohr's circle change if the stress element were oriented at an angle instead of being horizontal?
Ranking Task	An axially loaded member with 5 stress elements spaced randomly throughout its body Shear stress	Rank the horizontally oriented stress elements based on their magnitude of shear stress. Rank the diagonally oriented stress elements based on their magnitude of shear stress. Rank the mixed scenario of stress elements based on their magnitude of shear stress
Figure of Axially Loaded member	A diagonally oriented stress element in the center of the member Shear strain	Does the axially loaded member have shear strain? Why or why not? Is the stress element in the middle of the member experiencing shear strain? If so describe how.
Ranking Task	An axially loaded member with 5 stress elements spaced randomly throughout its body Shear strain	Rank the horizontally oriented stress elements based on their magnitude of shear strain. Rank the diagonally oriented stress elements based on their magnitude of shear strain. Rank the mixed scenario of stress elements based on their magnitude of shear strain.
Rubber Band	A rubber band with a horizontal stress element drawn on it Shear stress & shear strain	Pulling horizontally on the rubber band, how is shear strain occurring on the stress element? Describe the shear stresses associated with the strain on the stress element.
Failed Concrete Specimen	A concrete cylinder which has failed due to compression Normal stress & shear stress	What stresses caused this member to fail? Is there any significance to the angle at which the member failed?

11. Appendix B- Bending Interview Protocol

Question-Description	Description & Concepts Covered	Questions Asked
1. Definitions	I. moment II. normal stress III. shear force IV. shear stress	A. Without computations, verbally explain how you would define a bending moment. B. What units of measurement is a bending moment? C. How would you define normal stress? D. Do you know an equation for normal stress? E. How would you define shear force? F. How would you define shear stress? G. Do you know an equation for shear stress?
2. Simply Supported Beam Uniformly Loaded Beam with Points A through E varied throughout cross section of beam	I. Maximum, minimum moment II. Maximum, minimum shear III. Maximum, minimum normal stress IV. Maximum, minimum shear stress	A. Describe the distribution of bending moments and where moment will be maximum and minimum. B. Describe the distribution of shear force and where they will be maximum and minimum throughout the beam. C. Describe the distribution of normal stresses acting on the beam and where they will be maximum and minimum. D. Describe the distribution of shear stresses in the beam and where there will be maximum and minimum.

Question-Description	Description & Concepts Covered	Questions Asked
3. Uniformly Loaded Beam with Points A through E aligned horizontal through the center of the beam	I. Rank moment II. Rank shear III. Rank normal stress IV. Rank shear stress	A. Rank points based on their magnitude of bending moment. B. Rank points based on their magnitude of normal stress. C. Rank points based on magnitude of shear force. D. Rank points based on their magnitude of shear stress.
4. Cantilever beam with load P is directly on point D.		
5. A beam that's cut with moment about the X axis at cross-face.		
6. A beam that's cut with shear at cross-face		
7. A beam that's cut with shear and moment at cross-face		
8. A 3D and 2D view of a beam that's cut with shear, moment, and axial load at cross-face, Points A through E varied throughout cross section of beam		
9. A 3D and 2D view of a beam that's cut with shear, moment, and axial load at cross-face, Points A through E aligned horizontal through the center of the beam		
10. Uniformly loaded beam with a cut at point, blown up section with points A through E shown		
11. Confidence		A. How do you feel about your level of understanding? Which things do you think you did well, which things do you think you struggled with?

12. Appendix C- ABT Interview Protocol

Specifications

1. Deformation
2. Shear Stress
3. Shear Strain
4. Normal Stress
5. Normal Strain

Contexts

A. Axially Loaded Members
B. Bending Member
C. Torsional Loaded Member
D. Axially Loaded Concrete Column
E. Bending Failed Wood Member
F. Torsional Failed Chalk

Interview Prompts and Questions:

A. Axially Loaded Member
   1. Can you tell me what is going on here?
   2. Can you tell me of some instances where you would see this in the real world?
   3. Tell me about how this member would deform.
   4. How would the deformation change if you reversed the loading?
   5. Tell me about the strains/stresses the in the member.
   6. Tell me about any normal strains/stresses in the member
   7. Tell me about any shear strains/stresses in the member.
   8. Tell me about any shear strains/stresses in the member.
   9. Where are the largest normal stresses?
   10. Where are the largest shear stresses?

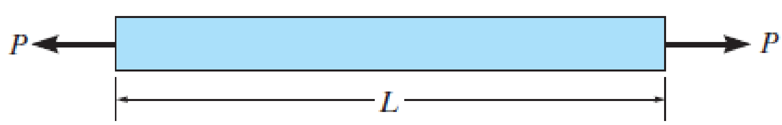


Fig. 15. Axially Loaded Member

B. Bending Member
   1. Can you tell me what is going on here?
   2. Can you tell me if some instances where you would see this in the real world?
   3. Tell me about how this member would deform.
   4. Tell me about the strains/stresses the in the member.

5. Tell me about any normal strains/stresses in the member
6. Tell me about any shear strains/stresses in the member.
7. Where are the largest normal stresses?
8. Where are the largest shear stresses?

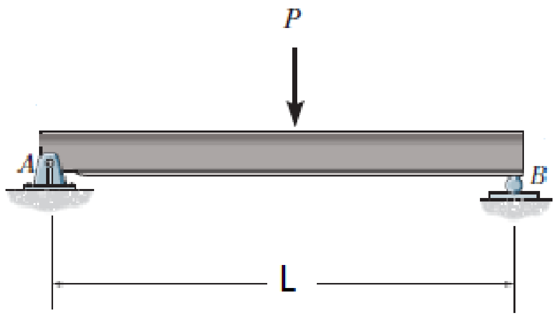


Fig. 16. Bending Loaded Member

C. Torsional Loaded Member
   1. Can you tell me what is going on here?
   2. Can you tell me if some instances where you would see this in the real world?
   3. Can you tell me about the stresses in the member?
   4. Can you tell me about the strains in the member?
   5. Can you tell me about the shear/normal stresses in the member?
   6. Can you tell me about the shear/normal strains in the member?
   7. Can you tell me about the torsion displacement in the member?
   8. Where are the largest normal stresses?
   9. Where are the largest shear stresses?

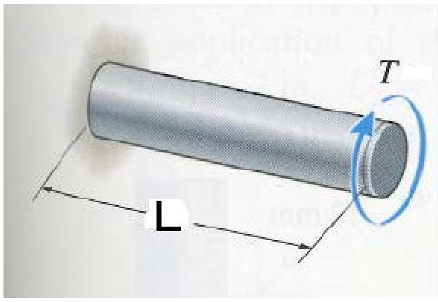


Fig. 17. Torsionally Loaded Member

D. Axially Loaded Member, Real World
E. Bending Member, Real World
F. Torsionally Loaded Member, Real World
   1. Can you tell me what happened here?

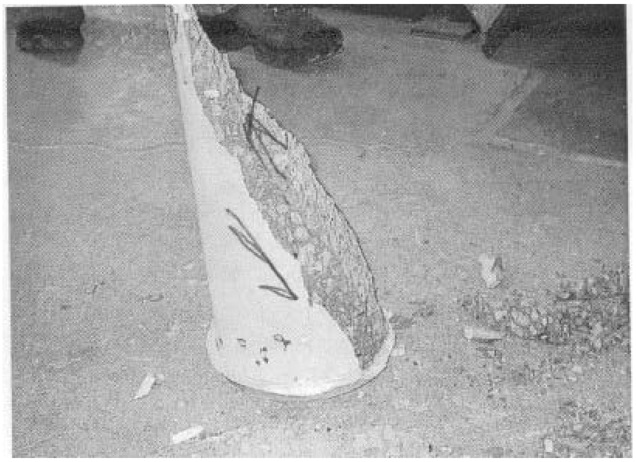


Fig. 18. Concrete Cylinder Failed in Compression

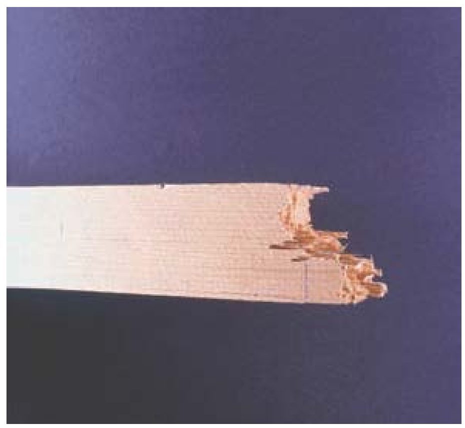


Fig. 19. Wood Member Failed in Bending

Fig. 20. Chalk Member Failed in Torsion