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Brian Jerald Weaver for the degree of Master of Science in Civil Engineering presented on January 19, 2017.

Title: Hybrid Survey Networks: Combining Real-time and Static GNSS Observations for Optimizing Height Modernization.

Abstract approved:

______________________________________________________________
Daniel T. Gillins

To derive ellipsoid heights on passive marks with cm-level accuracy, many current specifications require the collection and adjustment of long-duration, static Global Navigation Satellite System (GNSS) sessions. To increase efficiency, a survey procedure that includes real-time kinematic vectors from a real-time GNSS network (RTN) was evaluated. Thirty different “hybrid” networks involving three to nine Network RTK (NRTK) vectors per mark and some static GNSS vectors were developed from surveys completed in Oregon and South Carolina. The variance-covariance matrices of the static and kinematic vectors were scaled by variance component estimation procedures to produce realistic error estimates for stochastic modeling. After least squares adjustment and formal random error propagation of the networks, the resulting ellipsoid heights on
the passive marks had network accuracies ranging from 0.6 to 3.6 cm (95% confidence). These network accuracies reduced to 2 cm when using six or more NRTK observations per mark. Further, the use of NRTK vectors obtained from both GPS and GLONASS observables were, on average, 19.2% more accurate vertically than vectors obtained solely from GPS observables.

Author keywords: GPS leveling; height modernization; GPS-derived heights; Accuracy of Real-time Networks; RTK; GNSS
Hybrid Survey Networks: Combining Real-time and Static GNSS Observations for Optimizing Height Modernization

by

Brian Jerald Weaver

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APPROVED:

Major Professor, representing Civil Engineering

Head of the School of Civil and Construction Engineering

Dean of the Graduate School

I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

________________________________________
Brian Jerald Weaver, Author
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CONTRIBUTIONS OF AUTHORS

Dr. Daniel Gillins assisted with creating the manuscript outline and spent countless hours revising the writing to help create a technical and concise paper. He also assisted with fieldwork and provided direction during analysis and presentation of results. Michael Dennis reviewed the text, provided edits and technical advice, and assisted with reducing the South Carolina GNSS data.
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1 INTRODUCTION

Differential geodetic leveling provides precise height differences between marks; however, it is a tedious process that requires line-of-sight optical measurements every 90 meters or less, and significant errors accumulate when leveling a long distance. Many researchers and practitioners have investigated methods for using advances in Global Navigation Satellite Systems (GNSS) technology to more efficiently derive orthometric heights on marks (e.g., Henning et al. 1998; Martin 1998; Ollikainen 1997; Baryla et al. 2013; Gillins and Eddy 2016). GNSS is an attractive and cost-effective alternative because it enables the simultaneous determination of both vertical and horizontal positions without the need for maintaining line-of-sight between marks on the ground. A report (NGS et al. 1998) to the United States Congress concluded that static GNSS surveys are more cost-effective than differential leveling surveys for projects with distances greater than 4 km in length. The report also noted 88% cost savings when using static GNSS surveys rather than differential leveling when measuring orthometric height differences between four marks spaced only 10 km apart.

Unfortunately, GNSS does not directly measure orthometric heights; rather, GNSS measures heights relative to the ellipsoid, a simple, geometric shape aimed to approximate the shape of the geoid (nominally global mean sea level). However, ellipsoid heights can be converted to orthometric heights using Eq. 1:

\[ H = h - N \]  

(1)
where $H$ is the orthometric height measured along a plumb line from the geoid to the mark on the ground surface; $h$ is the ellipsoid height measured along the ellipsoid normal (a line perpendicular from the ellipsoid to the mark on the ground surface), and $N$ is the geoid height measured along the ellipsoid normal from the ellipsoid to the geoid.

Technically, Eq. 1 is an approximation because the plumb line is not coincident with the ellipsoid normal and slightly curved; however, the error is considered insignificant, likely less than 1 mm in even the most extreme settings on Earth (Jekeli 2000).

Minimizing the error in $h$ will ultimately reduce error in estimating $H$ per Eq. 1. Accordingly, the United States National Geodetic Survey (NGS) developed detailed static GNSS surveying guidelines for deriving $h$ on marks, referred to herein as NGS-58 (Zilkoski et al. 1997). These guidelines are followed for performing GNSS “height modernization” surveys in the United States. The guidelines state that the ellipsoid heights measured on project marks are expected to have local and network accuracies less than 2 cm and 5 cm at 95% confidence, respectively. NGS-58 recommends observing multiple baselines at least twice between project marks assigned to four levels of hierarchy: control, primary, secondary, and local (Zilkoski et al. 1997). The guidelines also recommend long-duration static GNSS sessions depending on baseline length: baselines between 15 and 40 km in length should be observed for 5 hours on three consecutive days; baselines between 10 and 15 km in length should be observed twice for at least 1 hour, and baselines less than 10 km in length should be observed twice for at least 30 to 45 minutes (Zilkoski et al. 1997). Afterwards, the precision of the repetitive baseline observations should be verified to be less than 2 cm vertically, and NGS-58 then
recommends performing minimally and fully constrained least squares adjustments of the survey network in order to derive most probable ellipsoid heights at each mark.

More recently, in 2013, NGS released *OPUS-Projects*, which is free, web-based software that allows users to upload, manage, post-process, and adjust multiple static GNSS observations on numerous marks in a survey project. *OPUS-Projects* is becoming popular in the U.S., and many practitioners desire to use it for height modernization surveys. Gillins and Eddy (2016) showed in a case study in Oregon that ellipsoid heights derived by post-processing in *OPUS-Projects* following NGS recommendations (i.e., Armstrong et al. 2015) generally matched to within ± 1 cm with ellipsoid heights derived following the NGS-58 guidelines.

Although *OPUS-Projects* is a valuable tool, its baseline processing engine, *PAGES*, requires at least a 2-h static GNSS session in order to produce reliable results (Soler et al. 2006). Such long-duration sessions are less attractive to surveyors accustomed to deriving geodetic coordinates on marks in less than a few minutes by obtaining real-time kinematic (RTK) GNSS observations utilizing a real-time network (RTN). An RTN consists of a set of continuously operating GNSS reference stations that are combined and used to generate network RTK (NRTK) solutions. RTN technology has matured over the last several years and is widely used by surveyors and other geospatial professionals for high-accuracy applications. As discussed later in this paper, a number of studies (Allahyari 2016, Smith et al. 2014, and Janssen and Haasdyk 2012) have been completed showing cm-level accurate geodetic positions can be obtained in minutes or less using an RTN.
There are considerable indications that NRTK baseline observations would greatly optimize the derivation of cm-level accurate ellipsoid heights on marks--possibly reducing the required duration of the GNSS observation per mark from hours to only minutes. However, RTNs do have some limitations worth consideration. First, any error between the position of the RTN reference stations as provided by the network manager and the National Spatial Reference System (NSRS) is propagated to the user. NGS defines its network of continuously operating GNSS reference stations as the backbone of the NSRS. Only the reference stations in this NGS network will be referred to herein as “CORS.” Ideally, the network manager should ensure that the positions of the RTN reference stations are accurately referenced to the NSRS; however, this may not necessarily always occur. Second, to mitigate the first issue and check the alignment with the NSRS, it is best practice to tie the control survey project to multiple CORS. Because the spacing of the CORS often exceeds the spacing of the RTN references stations, tying a survey project network to multiple CORS will likely still require some static GNSS observations and post-processing.

This paper develops an efficient, campaign-style surveying approach involving the inclusion of NRTK GNSS vectors with some static GNSS observations referenced to multiple CORS for deriving high-accuracy ellipsoid heights on marks. The intent is to evaluate the accuracy of combining numerous NRTK and static baseline observations on multiple marks into a survey network that is subsequently adjusted by least squares. If sufficiently accurate, the surveying approach could greatly optimize the efficiency of height modernization surveys. To accomplish this objective, static and NRTK data were evaluated from two GNSS surveys in South Carolina and Oregon. The static data were
first post-processed in \textit{OPUS-Projects} to derive coordinates on each mark. Afterwards, additional survey networks known as “hybrid networks” were developed using only the static baseline observations at the active stations and varying sets of NRTK GNSS vectors. Each hybrid network was adjusted by least squares, and the resulting coordinates were compared with the coordinates of the static-only network from \textit{OPUS-Projects}. In addition, error estimates for the coordinates were computed by formal random error propagation, and plots were drawn depicting horizontal and vertical errors as a function of the number of NRTK baseline observations on each mark in the survey project. These plots are helpful for showing how many NRTK baseline observations on each mark are typically required to achieve a particular level of positioning accuracy.
2 BACKGROUND

2.1 Overview of OPUS-Projects

Fig. 1 presents a flowchart for processing static GNSS data in OPUS-Projects. This process was discussed in more detail in Gillins and Eddy (2016), and it is based on NGS recommendations in the OPUS-Projects User Manual (Armstrong et al. 2015). A user who has completed NGS OPUS-Projects Manager’s Training can upload each static GNSS data file for a survey to OPUS-Projects via the NGS online portal for OPUS-Static. OPUS-Static requires at least a 2-h-duration static observation on a mark, decimates all observations to a logging rate of 30 s, and only uses GPS observables (Soler et al. 2006). The user can specify desired CORS, and OPUS-Projects will upload 24-h-duration static files for each day (in GPS time) of a GNSS session.

After uploading the static data, the next step is to process the sessions in OPUS-Projects using its baseline processing engine, PAGES (Schenewerk and Hilla 1999). An advantage of PAGES is that research has demonstrated it is capable of accurately processing long-distance baseline observations. For example, Eckl et al. (2001) conducted a study investigating the accuracy of more than 500 GPS baseline observations developed from CORS data and processed in PAGES. The study could not find a correlation between accuracy and baseline length for baselines up to 300 km long; instead, Eckl et al. (2001) found the following relationship for estimating the vertical root-mean-square error ($VRMSE$) of a baseline processed in PAGES versus observation duration:
$VRMSE = \frac{3.7}{\sqrt{T}}$  \hspace{1cm} (2)

where $VRMSE$ is in terms of ellipsoid height and expressed in cm, and $T$ is the duration of the static session, in hours. Eckl et al. (2001) only recommended Eq. 2 for $T > 4$ h; however, Soler et al. (2006) found it was reasonable to extrapolate Eq. 2 to sessions as short as 3 h and occasionally found larger errors when $T$ was only 2 h. Soler et al. (2006) state that the accuracy of PAGES is dependent on observation duration due to the difficulty of fixing integer ambiguities during stagnant atmospheric conditions and satellite geometries associated with short-duration static observations.
Fig. 1. Flowchart used for developing, processing, and adjusting a survey network using *OPUS-Projects* and *ADJUST*.

During session processing, Armstrong et al. (2015) recommends developing a hub survey network design (Fig. 2). Per Armstrong et al. (2015), the user should do the following:
• Designate a single station within roughly 100 km of observed passive marks in
the project area as the hub. It is best practice to designate a CORS, temporary
CORS, or other type of active station as the hub (so that the hub has a minimum
of 24 h of static data for each session). The user may designate different stations
as the session hub when processing multiple sessions; for such projects, ensure
that a baseline observation connects all hubs in the survey network for every
session.

• Add and use data from nearby and distant CORS for every session. Use of nearby
CORS helps reference the network to the geometric reference frame of the NSRS,
and the use of data from at least one long-distance (i.e., ~1,000 km) CORS helps
solve for the wet component of its tropospheric delay modeling
corrections (Ugur 2013).

• Solve for baselines such that every observed mark in the session is directly
connected to the hub (per Fig. 2). Designating an active station as the hub is ideal
because its 24-h-duration data file is needed for accurately determining
tropospheric delay for the session, and it helps ensure there is sufficient mutual
satellite visibility between the hub and the distant CORS or other active stations
for double differencing. Observations shorter than 24 h in duration are typically
adequate for the shorter (i.e., < 100 km) baselines between the hub and the
passive marks observed in the session.

• Hold the coordinates of the CORS as control during session processing, but
unconstrain the hub (even if it is a CORS). In this manner, the position of the hub
is found by constraining the session to the CORS. The user can apply "tight,"
normal, or loose constraint weights which are meant to restrict the session solution to within 0.1 mm, 1 cm, and 1 m of the published geodetic coordinates of the selected CORS, respectively. Normal constraint weights are preferred because they allow slight shifts on the same rough order as the typical accuracy of the published coordinates of the CORS, thus developing a reasonable stochastic model for the constrained coordinates in the adjustment.

![Hub network design recommended for session baseline processing in OPUS-Projects](image)

**Fig. 2.** Hub network design recommended for session baseline processing in *OPUS-Projects*

Once satisfied with the results of the session baseline processing, the next step is to perform a network adjustment of the combined session solutions using another tool within *OPUS-Projects, GPSCOM.*
Upon completion of a network adjustment, *OPUS-Projects* outputs the multiple, unadjusted session solutions in a file known as a “G-file,” and it outputs the adjusted coordinates in another file known as a “B-file.” The G-file and B-file are required inputs for running NGS software, *ADJUST* (Milbert and Kass, 1993), and their format is described in the NGS “Bluebook” (NGS 2016a)). (It is worth noting that NGS is in the process of replacing *GPSCOM* with *ADJUST* so that steps 7-9 in Fig. 1 can be completed within *OPUS-Projects* in the near future.) *ADJUST* estimates the a-priori geodetic coordinates of the marks in the network using the B-file; the G-file contains earth-centered, earth-fixed components of the GNSS baseline observations and variance-covariance (VCV) matrices of the session solutions.

*OPUS-Projects* outputs the data in the G-file in the International GNSS Service of 2008 (IGS08) reference system at the weighted mean epoch of the measurement. NGS (2015) recommends transforming baseline observations to a common epoch in the western United States that experiences significant crustal motion using an NGS utility, Horizontal Time-Dependent Positioning (*HTDP*). *HTDP* transforms the baseline observations made at various epochs in one geodetic datum into observations made at a common epoch in another geodetic datum by accounting for time-dependent horizontal velocities at the ends of the baseline and the transformation parameters between datums (Snay and Pearson 2012). NGS (2015) recommends to use *HTDP* to transform all baseline observations in the G-file to the current geodetic datum and standard epoch of the NSRS (currently NAD 83(2011) Epoch 2010.00) prior to running *ADJUST*. 
The final steps shown in Fig. 1 are to input the B-file and transformed G-file in *ADJUST* and perform the desired network least squares adjustment. See Milbert and Kass (1993), NGS (2015), and NGS (2016b) for instructions on running *ADJUST*.

### 2.2 Overview of Real-time Networks

Over the past 15 years, GNSS surveying with RTK technology has become extremely popular for numerous fields, such as in surveying engineering, mobile mapping, machine control, precision agriculture, mining, and construction. Conventional RTK utilizes a stationary, single reference station, or “base” station, which transmits its precise coordinates and GNSS observables to a moving, “rover” receiver, enabling real-time derivation of GNSS baselines. However, such a configuration usually limits baseline lengths to 10 - 20 km due to the fact that at greater baseline lengths, broadcast satellite orbits and atmospheric delay errors do not sufficiently cancel by differencing GNSS observables collected at the base and rover, resulting in greater difficulty resolving integer ambiguities.

To overcome this limitation, a RTN uses a network of continuously operating reference stations to interpolate atmospheric delay and broadcasts ultra-rapid ephemerides to reduce satellite orbit error (Zhang et al. 2006; Janssen 2009). Current practice is to space the reference stations every 70 km or less, enabling derivation of sufficiently accurate baselines up to approximately 40 km in length. In an RTN, the set of continuously operating GNSS reference stations send data to a centralized network processing server. The rover receiver also transmits and receives data from this server using wireless communication, such as via a cellular data plan. RTN software uses the
data from the reference stations to generate NRTK corrections by fixing integer ambiguities of double-differenced GNSS phase observables. Using the correction messages and published geodetic coordinates of the RTN base stations, the rover can compute its precise coordinates quickly.

RTN implementation typically involves one of two popular methods for providing solutions: the Virtual Reference Station (VRS) method or the Master-Auxiliary Concept (MAC) method. These two methods have important differences worth noting. For the VRS method, the rover first transmits an uncorrected point position to the service. To reduce distance-dependent errors associated with relative positioning, the service then assigns this position as the location of an imaginary base station, and interpolates network corrections at this virtual location. These interpolated corrections are used to generate corrected pseudo-observables that are “transmitted” from the virtual base to the rover to be processed using conventional single-base RTK algorithms to solve for precise geodetic coordinates at the rover (Wang et al. 2010). Thus, it appears to the user as if a very short baseline (i.e., ~1 m) was obtained from the imaginary base station to the rover. The position of the VRS may change often, such as each time the rover is powered on or moved a certain amount of distance requiring a new uncorrected rover point position to be transmitted to the service to maintain quality network solutions (Leica 2005, Janssen 2009).

For the purpose of developing a height modernization survey network that typically consists of numerous repeat baseline observations on multiple marks, the VRS method is problematic because the GNSS baselines originate from a virtual position that is not repeatable or referenced to published geodetic control. Unlike the VRS method, the
MAC method produces GNSS baselines between the physical location of a reference station in the RTN, known as a “master station,” and the rover.

In the MAC method, the rover transmits an uncorrected point position to the central server, and then the server searches and selects a “cell” of reference stations for generating corrections. Typically, the nearest reference station is assigned as the master station, and several additional auxiliary stations within the appropriate cell are chosen. The phase ranges from all selected stations are reduced to a common ambiguity level, and dispersive and nondispersive errors for each frequency and satellite-receiver pair are computed relative to the master station. Afterwards, the residual corrections between the auxiliary stations and the master station, as well as the full corrections and published coordinates at the master station are transmitted to the rover. The rover then interpolates the received corrections and network information to derive its precise position (Janssen 2009).

Although the VRS and MAC method have important differences, Edwards et al. (2010), Wang et al. (2010), and Martin and McGovern (2012) found that both methods produce coordinates with similar accuracies. There is no standard method for evaluating the accuracy of NRTK solutions. Table 1 summarizes several recent studies where investigators evaluated the accuracy of NRTK data by comparing with coordinates derived from previous static GNSS surveys. The results summarized in Table 1 include tests with at least 50 NRTK baseline observations on multiple marks using an RTN with a recommended average reference station spacing (i.e., interstation spacing) approximately equal to 70 km or less. A small portion of the NRTK data in these studies were filtered prior to evaluation using various criteria, such as rejection of data with poor
coordinate quality from GNSS software reports, high position dilution of precision (PDOP), or significant differences from published coordinates at a mark. Some studies (Wang et al. 2010; Janssen and Haasdyk 2012) also used RTNs with greater reference station spacing and found the resulting NRTK data were less accurate; hence, these statistics are not shown in Table 1. Although each study was done with different RTNs using varying solution methods and observation durations, it is remarkable that nearly every study found the horizontal RMSE (HRMSE) equal to roughly 1 - 2 cm and the VRMSE equal to roughly 2 - 3 cm. It is also noteworthy that several of the studies found that the RMSE hardly reduced when increasing the duration of the observation session from just 6 s to as long as 600 s (e.g., Janssen and Haasdyk 2012; Smith et al. 2014; Allahyari 2016).
Table 1. Summary of empirical studies on the accuracy of NRTK baseline observations.

<table>
<thead>
<tr>
<th>Source</th>
<th>Location</th>
<th>HRMSE (cm)</th>
<th>VRMSE (cm)</th>
<th>Session Dur. (sec)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edwards et al. (2010)</td>
<td>Great Britain</td>
<td>1.0 - 2.0</td>
<td>1.5 - 3.0</td>
<td>1</td>
<td>Tested both MAC and VRS; found 1 - 4 mm reduction in RMSE when averaging 300 s of data</td>
</tr>
<tr>
<td>Wang et al. (2010)</td>
<td>Brisbane, Australia</td>
<td>2.0 - 3.0</td>
<td>4.0 - 5.0</td>
<td>1</td>
<td>Tested both MAC and VRS. Also tested longer than recommended interstation distances but results not shown here</td>
</tr>
<tr>
<td>Janssen and Haasdyk (2011)</td>
<td>New South Wales, Australia</td>
<td>0.5 - 1.2</td>
<td>0.9 - 2.1</td>
<td>1 - 600</td>
<td>VRS; collected NRTK data for 3 full days at each test mark</td>
</tr>
<tr>
<td>Martin and McGovern (2012)</td>
<td>Ireland</td>
<td>2.2</td>
<td>2.9</td>
<td>5</td>
<td>Tested both VRS and MAC using three independent RTNs; investigated effects of including GLONASS observables</td>
</tr>
<tr>
<td>Smith et al. (2014)</td>
<td>Texas, U.S.A.</td>
<td>1.5</td>
<td>2.7 - 2.8</td>
<td>6 or 180</td>
<td>VRS; data compared with 48-h static GNSS baseline observations processed in OPUS-Projects</td>
</tr>
<tr>
<td>Allahyari (2016)</td>
<td>Oregon, U.S.A.</td>
<td>1.1 - 1.6</td>
<td>2.0 - 2.7</td>
<td>5 - 900</td>
<td>MAC, GPS-only observables; compared with &gt;40-h static GNSS baseline observations processed in OPUS-Projects</td>
</tr>
<tr>
<td>Allahyari* (2016)</td>
<td>South Carolina, U.S.A.</td>
<td>1.1 - 1.6</td>
<td>2.1 - 2.8</td>
<td>5 - 600</td>
<td>MAC; collected both GPS-only and GPS+GLONASS observations; compared with &gt;30-h static GNSS baseline observations processed in OPUS-Projects</td>
</tr>
</tbody>
</table>

* = The South Carolina NRTK data evaluated in Allahyari (2016) were also used in this study

It is clear that ellipsoid heights with VRMSE less than 3 cm could be obtained in mere seconds using an RTN. However, typical height modernization survey guidelines
require development and least squares adjustment of a campaign-style survey network consisting of numerous repeat baseline observations on several marks in a project. Repeat observations and adjustments are beneficial for identifying blunders, outliers, or poor session solutions. Prior work (Table 1) does not evaluate the accuracy of coordinates obtained from adjustment of such a survey network.

Table 2 summarizes the key differences between GNSS baseline observations derived from an RTN versus OPUS-Projects.

**Table 2. Summary comparison of NRTK GNSS data versus static GNSS data processed in OPUS-Projects**

<table>
<thead>
<tr>
<th>Property</th>
<th>NRTK</th>
<th>OPUS-Projects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session Duration</td>
<td>~1 to 600 s</td>
<td>2 to 48 h</td>
</tr>
<tr>
<td>Recommended Baseline Length</td>
<td>&lt; 40 km</td>
<td>&lt; ~300 km&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Estimated Vertical Accuracy (VRMSE)</td>
<td>2 to 3 cm&lt;sup&gt;b&lt;/sup&gt;</td>
<td>See Eq. 2&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<sup>a</sup> While evaluating the baseline processing engine in OPUS-Projects (i.e., PAGES), Eckl et al. (2001) tested baseline observations up to 300 km in length and could not find a correlation between accuracy and baseline length. However, it is probable that OPUS-Projects can reliably process even longer baseline observations. For examples, Kerr (2015) and Gillins and Eddy (2016) successfully processed baseline observations longer than 1,000 km in length in OPUS-Projects.

<sup>b</sup> Based on empirical testing results presented in Table 1

<sup>c</sup> Although OPUS-Projects will process static sessions as short as 2 h in duration, Eq. 2 is only recommended for sessions longer than 3 h in duration (Soler et al. 2006).
3  SOURCE OF SURVEY DATA FOR ANALYSIS

3.1  South Carolina

To develop the hybrid networks for evaluation, this study used static GNSS data collected by Gustin, Cothern & Tucker, Inc. (GCT) Engineering and NGS from December 11 to December 17, 2013 (days of year 345 to 351) on 20 passive marks in Aiken, Lexington, and Richland counties, South Carolina (Dennis 2014 and Stinner 2014). For the 2013 survey, passive marks were selected in order to evaluate several factors that affect the accuracy of GNSS such as: multipath, overhead obstructions, occupation time, and number of repeat baseline observations. To test these factors, the surveyors attempted to collect GNSS baseline observations at several marks with unfavorable conditions, such as at marks set within two meters of wooden power poles or under tree canopies that obstruct one-fourth to one-half of the sky. The idea was to collect data at these challenging locations so that resulting recommendations would be conservative in case surveyors needed to collect GNSS data at similar places. Six of the 20 marks (3201, LEX, PELI, SURV, AIKP, D138) had minimal obstructions 15 deg. above the horizontal of the antenna, 12 marks were located near power poles or under tree canopies that obstructed up to 25% of the view of the satellites, and two marks (L186, BUTL) were under canopies obstructing up to 50% of the view of the satellites.

A minimum of 30 h of static GNSS data were collected at each passive mark through several occupations. Two 5-h-duration sessions were conducted on each day of the survey. Ten marks were simultaneously observed for both 5-h sessions on days of year 345-347, and the other ten marks were simultaneously observed for both 5-h
sessions on days of year 348, 350, and 351. The survey used three different types of
Trimble antennas (official IGS antenna names are given in parentheses): R8-Model 2
GNSS/SPS88x Internal (TRMR8_GNSS NONE), Zephyr Geodetic 2 (TRM55971.00
NONE), and Zephyr Geodetic 2 RoHS (TRM57971.00 NONE). The antennas were
attached to 2-m fixed-height tripods that had been calibrated prior to the campaign. Using
digital calipers, corrections to the antenna heights were also made to account for the
recess depths on the monuments of the passive marks. During each session, both GPS and
GLONASS observables were logged at a 1-Hz rate, later converted to receiver
independent exchange (RINEX) format, version 2.

The authors submitted the RINEX files for the observations on the passive marks
to OPUS-Projects. In addition, 12-h-duration static GPS data files for each day of the
survey (overlapping in time with the sessions on the passive marks) at five reference
stations in the South Carolina Real-Time Network (SCRTN) were downloaded and
uploaded to OPUS-Projects. Lastly, 24-h static GPS data files collected at 14 CORS for
each day of the survey were added in OPUS-Projects.

In addition to the static data, NRTK data utilizing the SCRTN collected at the
same 20 passive marks by the GCT and NGS from December 4 to 10, 2013 (Geoghegan
2014 and Stinner 2014) were evaluated. It is worth noting that these same NRTK data
were studied in Bae et al. (2015) and Allahyari (2016). The SCRTN is managed by the
South Carolina Geodetic Survey, has base stations spaced less than 70 km, and used
Trimble VRSNet3 software at the time of the NRTK survey. The aim of the 2013 survey
was to investigate the accuracy of NRTK vectors versus the duration of the observational
session, as well as to compare the NRTK vectors collected using differing RTN settings.
For three days at each mark, the surveyors collected a “series” of six baseline observations for the following durations at a 1-s epoch rate using a MAC method: 5, 30, 60, 180, 300, and 600 s. Afterwards, they would invert the antenna, rotate the tripod 120 degrees, re-set the antenna, and repeat the observation series. When using the full network of reference stations in the RTN, the surveyors repeatedly cycled through using only GPS and using both GPS and GLONASS observables. Five of the series of baseline observations were completed using GPS-only and five of the series of observations were completed using GPS+GLONASS per day, resulting in 12 distinct observational samples, and 15 independent observations (i.e., 5 per day, each one collected roughly every 2 hours) in each of these distinct samples.

Unfortunately, as explained in Allahyari (2016), it was discovered that four of the Trimble R8-Model 2 GNSS receivers utilized in South Carolina had out-of-date firmware (v. 4.12) which caused the rover to not correctly recognize the antenna model of the RTN base stations. After correspondence with Trimble engineers, the NRTK baseline observations produced from outdated firmware were likely biased in ellipsoid height by +8.546 cm. This positive bias is equal to the nominal vertical antenna phase center offset of the Trimble Zephyr Geodetic 2 (TRM55971.00 NONE) base antennas in the RTN as defined by Trimble. To correct this problem, the bias was subtracted from the ellipsoid heights of all NRTK vectors to seven stations G176, L186, W186, W53_, Q176, HUNT and PELI (Allahyari 2016). It is likely that subtracting the 8.546 cm resolved the vertical bias problem, but it is important to point out that this is a complex problem with many possible permutations. The observed height is affected not just by the rover firmware, but also by the network software and its settings. Even if all software versions and settings at
that time were known, it would still be necessary for Trimble engineers to analyze the
exact version of the firmware used at that time to determine specifically what occurred
(Briggs, 2016). Because of such uncertainties, some small ellipsoid height bias may
remain. Although this was an unfortunate occurrence for this research, it serves as a
valuable example of the complexity of real-time solutions and the importance of using
current software and firmware.

3.2 Oregon

This study also developed hybrid networks using static GNSS data collected from
Oct. 7, 2014 to Nov. 8, 2014 on 18 passive marks (Fig. 3) in the Willamette Valley,
Oregon, during a previous research study documented in Gillins and Eddy (2015, 2016).
The 18 marks selected for the 2014 survey were located at sites that were considered
suitable for obtaining satellite observations. Fifteen of the marks had no to only a few
minor overhead obstacles (e.g., distant tree canopies) more than 15 degrees above the
horizontal of the GNSS antenna. However, two marks (i.e., point names LBCC and
GLAS) were located next to traffic signs and had nearby tree canopies as tall as 45
degrees above the horizontal, and one mark (B726) was next to a wooden telephone pole.
The three marks with the less-ideal overhead obstacles and nearby features that could
cause some multipathing were included in the survey study to simulate some typical
types of field challenges surveyors encounter when attempting to make GNSS baseline
observations on existing passive marks. However, these field challenges were less severe
than at some of the marks in the aforementioned South Carolina survey.
During the 2014 static GNSS survey, 17 of the 18 marks were observed for a minimum of four sessions lasting 10 h in duration. One mark (D728) was observed for only three 10-h sessions. For each of the sessions, five to six Leica GS14 (IGS name “LEIGS14 NONE”) integrated receiver/antennas were attached to calibrated, 2-m fixed-height tripods set up over marks per the schedule in Gillins and Eddy (2016). During each session, both GPS and GLONASS observables were collected, stored both in a raw Leica proprietary format as well as in RINEX format. After completion of the survey, the RINEX files were submitted to *OPUS-Projects*.

In addition to the static data collected at the passive marks, 24-h static GPS data files collected at 11 active stations for every day of each of the survey sessions were uploaded to *OPUS-Projects*. Seven of the 11 active stations are part of the CORS Network, and four are base stations in the Oregon Real-Time GNSS Network (ORGN).

In addition to the static data, NRTK vectors utilizing the ORGN were collected on the same 18 passive marks from July 6 to July 8, 2016, in support of this study. The ORGN is managed by the Oregon Department of Transportation (ODOT), has base stations spaced at 70 km or less, uses *Leica GNSS Spider* RTN software, and only used GPS observables at the time of the survey to provide NRTK corrections. Three field crews were deployed for three consecutive days, each using a Leica GS14 integrated receiver/antenna that was receiving the NRTK corrections from the ORGN utilizing a MAC method. Each field crew was responsible for conducting nine 180-s-duration NRTK sessions per day at an epoch rate of 1 s, on six passive marks. In order to collect data using different overhead satellite geometries, each crew was assigned to drive a “loop” between six marks (Fig. 3a). At each mark, the crew would set up the receiver on
a 2-m fixed-height tripod and then would wait for the receiver to initialize and fix integer ambiguities (Fig. 3b). A 180-s duration NRTK session was then performed, and then the antenna was inverted such that it lost initialization. Afterwards, the antenna was set back up and the procedure repeated until three 180-s-duration, fixed NRTK vectors were obtained. The crew would then drive to the next mark in the loop and repeat the process. It took approximately 2-3 h for the crew to complete one revolution around the loop.

Three revolutions were completed per day so that each mark was observed for nine 180-s NRTK sessions per day (i.e., three sessions in the morning, three in the afternoon, and three in the evening per day). Each day, the crews rotated equipment to see if any receiver noise could be detected in the data. After three days, twenty-seven 180-s-duration NRTK vectors with fixed integer ambiguities were acquired on each of the 18 passive marks.

Fig. 3. (a) Loop assignments for the 2016 NRTK survey in Oregon; (b) typical setup on a passive mark, involving a 2-m fixed-height tripod, Leica GS14 receiver/antenna, and Leica CS15 data collector (image by Daniel T. Gillins).
4 DEVELOPMENT OF THE HYBRID NETWORKS

Using the static and NRTK GNSS data collected in South Carolina and Oregon, numerous hybrid networks were created per the simple schematic in Fig. 4. Each of the hybrid survey networks were developed by including: (1) a differing number of repeat NRTK vectors to each mark with fixed integer ambiguities; and (2) the same set of baseline observations post-processed in *OPUS-Projects* using the static GNSS observations collected at only the active stations in the survey network.

The following describes the procedure for preparing and combining the data from these two sources into the hybrid test networks. Fig. 5 provides a summary flow chart.

**Fig. 4.** Hybrid network conceptual design where (a) NRTK vectors are added with (b) vectors derived from post-processing static GNSS observations at the active stations to produce (c) a final combined or “hybrid” survey network for adjustment.
Fig. 5. Flowchart for developing and adjusting a hybrid survey network that consists of NRTK GNSS vectors and static GNSS vectors processed in *OPUS-Projects*
4.1 Preparation of the NRTK Data

Thirty hybrid networks were developed using a varying number of NRTK vectors per mark. Again, all of the NRTK vectors in this experiment were collected utilizing network corrections from an RTN, and the RTN server software was configured so that every baseline began at the physical location of a reference station (as illustrated in Fig. 4a).

For Oregon, two different sets of 3, 4, 5, 6, and 9 NRTK vectors to each mark were created by strategically selecting from the total number of 180-s-duration, GPS-only, fixed-integer NRTK vectors. The vectors in each of the ten sets were selected from different times of the day (e.g., one in the morning, afternoon, and evening for one set of 3 NRTK vectors per mark). This strategy simulates typical surveying guidelines (e.g., NGS-58) which recommend that surveyors collect repeat GNSS baseline observations on a mark at different times of the day so that independent measurements are made with significantly different satellite geometries. Such a strategy is especially important since dilution of precision and multipathing varies with time, and short-duration baseline observations are more vulnerable to multipath. It also ensured that each vector in a set was acquired with an independent setup of the antenna (in order to simulate instrument setup errors) as well as with an independent NRTK initialization.

Significantly more data are available from the South Carolina NRTK survey. However, for this study, only the 180-s-duration NRTK vectors were used in developing the hybrid networks. The reasons are: (1) previous research has found that the RMSE of NRTK vectors hardly improves after averaging single-epoch (1-s) solutions into 60 to 300-s duration, multi-epoch solutions (Edwards et al. 2010; Janssen and Haasdyk 2012; Smith et al. 2014; Allahyari 2016); (2) this duration is consistent with the available
NRTK vectors in Oregon; and (3) although a subjective reason, it seemed unnecessary to shorten the observation duration any further given the amount of time required to write some notes and/or collect photos during the survey.

In a similar method to the Oregon data, two samples different sets of 3, 4, 5, 6, and 9 vectors to each passive mark were made using the fixed, 180-s-duration, GPS-only, network NRTK solutions in South Carolina. In addition, in order to examine the influence of using GLONASS, two more sets of 3, 4, 5, 6, and 9 NRTK vectors to each mark were made using the fixed, 180-s, GPS plus GLONASS NRTK solutions.

Each of the 30 sets of unadjusted NRTK vectors were exported to 30 separate G-files (Step 1B, Fig. 5). The baseline components and VCV matrix in each of the G-files were then transformed from the IGS08 reference frame at the mean epoch of the measurement to NAD 83(2011) Epoch 2010.00 using HTDP (Step 1C, Fig. 5). Afterwards, each G-file was loaded in ADJUST, and a minimally constrained least squares adjustment of the vectors was performed (Step 1D). This preliminary adjustment has two important purposes.

First, the preliminary adjustment assists with the determination of whether or not a poor NRTK vector or blunder is present in the network. For this study, the 3-D residual for each adjusted vector was computed (3D-resi) as well as the 3-D RMSE for all residuals of all adjusted vectors in the network (RMSEi). If $3D{-}\text{resi} > 3x\text{RMSE}_i$, then the vector was considered an outlier and rejected, and a new vector was selected as replacement. This iterative process (Step 1E) was repeated until $3D{-}\text{resi} \leq 3x\text{RMSE}_i$ for all vectors in the network. This criterion resulted in only a small number of rejections. Only 0.2% (2/972) of the NRTK vectors in Oregon were rejected and replaced, and only
0.9% (10/1080) and 0.4% (4/1080) of the NRTK GPS-only and GPS+GLONASS vectors in South Carolina were rejected and replaced, respectively.

The second reason for the preliminary minimally constrained least squares adjustment of the GNSS baselines is to mitigate inconsistencies and other problems with the solution VCV matrices generated by the baseline processor. Han and Rizos (1995) showed that the VCV matrix resulting from different GNSS surveying modes (e.g., static versus kinematic) are incompatible. Furthermore, it is also a well-known problem that the VCV matrix output by GNSS baseline processing software is usually overly-optimistic, and that differing software programs have varying levels of over-optimism (e.g., El-Rabbany and Kleusberg 2003; Craymer et al. 1990; Kashani et al. 2004). For example, Kashani et al. (2004) found that the VCV matrix output by static baseline processing in Bernese GNSS Software were over-optimistic by a factor of 23.0, whereas the VCV matrices output by processing the same static baseline observations in GAMIT were over-optimistic by a factor of 1.9. The reason for the over-optimism is believed to be because software typically underestimates and handles systematic errors differently, and a-priori measurement errors are often arbitrary, thereby affecting the stochastic model used during baseline processing (El-Rabbany and Kleusberg 2003; Han and Rizos 1995).

One common remedy to this problem is to iterate a scale factor (also known as a variance component) to apply to the VCV matrix until the standard deviation of unit weight of a minimally constrained least squares adjustment equals 1 (Kashani et al. 2004; Craymer et al. 1990). The standard deviation of unit weight (σ) is computed as
\[ \sigma_0 = \sqrt{\frac{V^T W V}{r}} \]  

where \( V \) is a vector of length \( 3n \) containing the adjusted residuals of the baseline observation components, \( n \) is the total number of baseline observations in the survey network for adjustment, \( W \) is the weight matrix (the inverse of the VCV matrix), and \( r \) is the degrees of freedom (redundancy) of the survey network. If \( \sigma_0 \approx 1 \) after an adjustment, then it could be stated that the overall estimated error of the baseline components is consistent with the residuals of the adjusted baseline components.

The relative weights derived from the VCV matrices for an individual baseline processing software program are internally consistent (Han and Rizos 1995). Thus, the VCV matrix for only the NRTK vectors were first scaled by the preliminary minimally constrained least squares adjustment (Fig. 5, Step 1D). Later, as discussed below, other scalars were found for the VCV matrix of only the static GNSS baseline observations. As discussed in two NGS reports (Pursell and Potterfield 2008; Milbert 2009) and in NGS 2016b, \( ADJUST \) can be set to iterate and solve for both a horizontal and vertical factor for scaling the VCV matrix until \( \sigma_0 = 1 \). Table 3 presents the scale factors found by \( ADJUST \) from the preliminary minimally constrained adjustment of each of the 30 sets of NRTK vectors. Interestingly, the horizontal and vertical factors were, on average, both equal to approximately 2. These factors were applied to scale the VCV matrix in each of the 30 G-files in preparation for creating the hybrid survey networks for adjustment.
Table 3. Scale factors for the VCV matrix of the NRTK vectors in each hybrid network; scaling the VCV matrix with these factors and performing a minimally constrained least squares adjustment of the NRTK vectors resulted in $\sigma_0$ equal to 1.

<table>
<thead>
<tr>
<th>Hybrid Network Designation</th>
<th>No. of 180-s NRTK vectors per mark</th>
<th>Oregon (GPS-only)</th>
<th>South Carolina (GPS-only)</th>
<th>South Carolina (GPS+GLONASS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Scale Factor Horiz</td>
<td>Scale Factor Vert</td>
<td>Scale Factor Horiz</td>
</tr>
<tr>
<td>3A</td>
<td>3</td>
<td>2.16</td>
<td>2.13</td>
<td>1.51</td>
</tr>
<tr>
<td>3B</td>
<td>3</td>
<td>1.74</td>
<td>2.41</td>
<td>1.53</td>
</tr>
<tr>
<td>4A</td>
<td>4</td>
<td>2.08</td>
<td>1.83</td>
<td>1.64</td>
</tr>
<tr>
<td>4B</td>
<td>4</td>
<td>1.97</td>
<td>2.40</td>
<td>1.52</td>
</tr>
<tr>
<td>5A</td>
<td>5</td>
<td>1.99</td>
<td>1.95</td>
<td>1.62</td>
</tr>
<tr>
<td>5B</td>
<td>5</td>
<td>2.07</td>
<td>2.40</td>
<td>1.52</td>
</tr>
<tr>
<td>6A</td>
<td>6</td>
<td>2.05</td>
<td>1.94</td>
<td>1.52</td>
</tr>
<tr>
<td>6B</td>
<td>6</td>
<td>1.98</td>
<td>2.31</td>
<td>1.54</td>
</tr>
<tr>
<td>9A</td>
<td>9</td>
<td>2.03</td>
<td>2.16</td>
<td>1.68</td>
</tr>
<tr>
<td>9B</td>
<td>9</td>
<td>2.09</td>
<td>2.29</td>
<td>1.50</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>2.01</td>
<td>2.18</td>
<td>1.56</td>
</tr>
</tbody>
</table>

4.2 Preparation of the Static Data

In order to tie the set of NRTK vectors to the CORS, static GPS data continuously collected at each of the RTN base stations in the project were compiled for each day of the NRTK surveys (Step 2A, Fig. 5). This static data is advantageous because it does not require any additional fieldwork by the surveyor and enables careful referencing of the survey to multiple CORS.

Static GPS data files at each of the RTN base stations in the NRTK surveys and from multiple near and distant CORS were uploaded to OPUS-Projects. One active station was identified as the hub for each network in Oregon (CORV) and South Carolina (COLA), and the first 8 steps in Fig. 1 were completed for post-processing the static sessions in OPUS-Projects, version 2.6, following previously summarized NGS recommendations (Step 2B, Fig. 5). Two G-files containing the vectors derived in OPUS-
Projects were created, one from the static data of the active stations in Oregon and one from the static data of the active stations in South Carolina.

Similar to the NRTK data, each G-file was then input to ADJUST and a preliminary minimally constrained least squares adjustment was performed (Step 2C). The same outlier criterion (3D-Res; exceeding 3xRMSE) was applied for detecting possible blunders (Step 2D); fortunately, all of the vectors in both networks satisfied this criterion. The lack of rejections was not surprising, as all of the baseline observations were derived from long-duration, 12 to 48-h static observations at active stations with unobstructed view of the satellites.

For both sites, ADJUST was also set to solve for horizontal and vertical scalars that were necessary to scale up the horizontal and vertical components of the VCV matrix of baseline observations output from OPUS-Projects so that σ₀ of a minimally constrained adjustment equals 1. For Oregon, the horizontal and vertical scalars equaled 20.33 and 4.83, respectively; for South Carolina, the scalars were 13.75 and 6.24.

Based on the scalars found in ADJUST, it can be concluded that the VCV matrices of the vectors output by OPUS-Projects were 2.5 to 12 times more optimistic than the VCV matrices of the NRTK vectors. This finding underscores the importance of correctly scaling VCV matrices prior to combining vectors from different sources in a survey network for adjustment. The workflow (Fig 5) followed in this paper is meant to account for these differences by scaling the VCV matrices of the NRTK vectors and OPUS-Projects vectors separately using the preliminary minimally constrained adjustment. Otherwise, the vectors from OPUS-Projects would appear overly accurate relative to the NRTK vectors, and they would be weighted too high in an adjustment and
would skew the results. In addition, the relative scaling within the different baseline processors are not the same. The horizontal and vertical scalars were nearly equal for the NRTK vectors, whereas the horizontal scalars were about 2 to 4 times greater than vertical for *OPUS-Projects*.

### 4.3 Combination of the Data and Hybrid Network Adjustment

Satisfied that the poor baseline observations had been replaced and that the VCV matrix in the G-files were scaled appropriately, the next steps (Steps 3A - 3B, Fig. 5) were to simply merge each NRTK G-file with either the Oregon or South Carolina *OPUS-Projects* G-file and perform the final hybrid (static+NRTK) survey network adjustments in *ADJUST*. Figs. 6 and 7 illustrate a final hybrid survey network in Oregon and South Carolina, respectively.

Holding the published coordinates of the hub station fixed, a minimally constrained least squares adjustment of all 30 hybrid networks was first completed. Afterwards, fully constrained adjustments were completed by holding the published coordinates of the CORS in each network as control and by using the published standard deviations of these coordinates for developing weights on the constraints. If the a-priori VCV matrices for the NRTK vectors, static baseline observations, and coordinates for the control were appropriately scaled, then $\sigma_0$ for the least squares adjustments should remain approximately equal to 1. Table 4 presents $\sigma_0$ values for the minimally and fully constrained adjustments of each of the 30 hybrid networks. For all cases, $\sigma_0 \approx 1$. 
Table 4. Standard deviations of unit weight ($\sigma_0$) for the test hybrid networks from either a minimally constrained (MC) or fully constrained (FC) least squares adjustment

<table>
<thead>
<tr>
<th>Hybrid Network Designation</th>
<th>No. of 180-s NRTK vectors per mark</th>
<th>Oregon (GPS-only) $\sigma_0$</th>
<th>South Carolina (GPS-only) $\sigma_0$</th>
<th>South Carolina (GPS+GLONASS) $\sigma_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MC</td>
<td>FC</td>
<td>MC</td>
<td>FC</td>
</tr>
<tr>
<td>3A</td>
<td>3</td>
<td>1.00</td>
<td>1.04</td>
<td>1.00</td>
</tr>
<tr>
<td>3B</td>
<td>3</td>
<td>1.00</td>
<td>1.01</td>
<td>1.00</td>
</tr>
<tr>
<td>4A</td>
<td>4</td>
<td>1.01</td>
<td>1.02</td>
<td>1.00</td>
</tr>
<tr>
<td>4B</td>
<td>4</td>
<td>1.01</td>
<td>1.02</td>
<td>1.00</td>
</tr>
<tr>
<td>5A</td>
<td>5</td>
<td>1.01</td>
<td>1.03</td>
<td>1.00</td>
</tr>
<tr>
<td>5B</td>
<td>5</td>
<td>1.00</td>
<td>1.02</td>
<td>1.00</td>
</tr>
<tr>
<td>6A</td>
<td>6</td>
<td>1.00</td>
<td>1.03</td>
<td>1.00</td>
</tr>
<tr>
<td>6B</td>
<td>6</td>
<td>1.00</td>
<td>1.01</td>
<td>1.00</td>
</tr>
<tr>
<td>9A</td>
<td>9</td>
<td>1.00</td>
<td>1.01</td>
<td>1.00</td>
</tr>
<tr>
<td>9B</td>
<td>9</td>
<td>1.00</td>
<td>1.02</td>
<td>1.00</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>1.00</td>
<td>1.02</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Fig. 6. Diagram of hybrid network 6A in Oregon: (a) six distant CORS and the hub (CORV); (b) detail of project area, passive marks, and active stations in the ORGN

Fig. 7. Diagram of hybrid network 6A in South Carolina: (a) CORS and the hub (COLA); (b) detail of project area, passive marks, and active stations in the SCRTN
5 RESULTS AND DISCUSSION

There are two common methods to estimate the accuracy of the unknowns (i.e., coordinates at each mark) determined by a least squares adjustment: (1) using formal random error propagation theory; and (2) by comparison with coordinates of higher accuracy. The accuracy of the coordinates derived from the final constrained hybrid network adjustments were evaluated according to both of these methods.

5.1 Accuracy of Results per Formal Error Propagation

After a least squares adjustment, the VCV matrix of the unknowns (i.e., coordinates at each station) may be computed by multiplying the variance of unit weight ($\sigma_0^2$) by the inverse of the normal equation coefficient matrix. Using such a procedure to obtain realistic accuracy estimates requires critical assumptions that only random errors were present in the baseline observations and that the observations were properly weighted. ADJUST computes the VCV matrix of the unknowns after each adjustment and uses the resulting variances and covariances of the derived coordinates at each mark to compute standard deviations in north, east, and up components, as well as the horizontal correlation coefficient. Using these values, ADJUST then computes horizontal and vertical (ellipsoid height) accuracies at the 95% confidence level in order to meet U.S. federal standards (FGDC 1998) for reporting the “network accuracy” of each mark in a geodetic survey. Thus, unless explicitly stated, accuracies are reported at the 95% confidence level in this report. Further, vertical accuracies presented in this report are in terms of ellipsoid height.
Figs. 8 through 10 show the horizontal and vertical network accuracies at each mark and for each of the 30 hybrid test networks. The network accuracies generally become smaller as the number of NRTK vectors to each mark increases. As expected with GNSS data, the horizontal network accuracies are 2 to 3 times smaller than the vertical network accuracies. It is important to note that the expected vertical network accuracy for each mark in a static GNSS survey campaign following NGS-58 guidelines is 5 cm. Per Figs. 8a through 10a, vertical network accuracies at every mark in each hybrid network test were less than 3.6 cm, which shows that the hybrid network survey approach can yield results that satisfy height modernization standards in terms of accuracy. For the test networks consisting of at least six NRTK vectors to each mark, the vertical network accuracies were less than 2 cm and the horizontal network accuracies were less than 1 cm at every mark.
Fig 8. Resulting (a) vertical (in terms of ellipsoid height) and (b) horizontal network accuracy at each mark versus number of 180-s, GPS-only NRTK vectors to each passive mark; Oregon hybrid networks
**Fig. 9.** Resulting (a) vertical (in terms of ellipsoid height) and (b) horizontal network accuracy at each mark versus number of 180-s, GPS-only NRTK vectors to each passive mark; South Carolina hybrid networks
For comparison purposes, Fig. 11 presents the average of the network accuracies grouped by number of NRTK vectors to each mark as well as error bars at one standard deviation. As shown, the network accuracies for the marks in the Oregon networks are, on average, smaller than the accuracies for the South Carolina networks. This is likely
because more marks in South Carolina were under moderate tree canopies or next to wooden power poles.

Another interesting pattern observed in Fig. 11 is that for the hybrid networks in South Carolina, the average network accuracies were consistently smaller when using GPS+GLONASS NRTK vectors than when using GPS-only NRTK vectors. The inclusion of GLONASS was found to reduce the vertical network accuracy by an average value of 19% (4 mm) and the horizontal network accuracy by an average value of 7% (0.7 mm). In another effort to highlight the benefit of using GPS+GLONASS instead of only GPS, Fig. 12 depicts the number of NRTK vectors required to a mark to yield an ellipsoid height with an average vertical network accuracy less than 2.0 cm at 95% confidence. The vertical network accuracy was less than 2.0 cm at 15 of 20 marks versus 7 of 20 marks when using three NRTK GPS+GLONASS baseline observations versus three GPS-only observations per mark. Further, this level of accuracy was achieved at every mark rather than only at 10 of the 20 marks when using five NRTK GPS+GLONASS observations per mark versus five GPS-only observations per mark.
Fig. 11. Mean (a) vertical (in terms of ellipsoid height) and (b) horizontal network accuracy (at 95% confidence) for the coordinates derived at the passive marks versus number of 180-s NRTK vectors to each passive mark; Oregon and South Carolina hybrid networks (error bars are 1 standard deviation)

Fig 12. Number of 180-s NRTK observations required to produce an adjusted ellipsoid height with an average vertical network accuracy less than 2.0 cm; comparison of GPS-only and GPS+GLONASS South Carolina hybrid networks
Per Fig. 12, average vertical network accuracies were generally less than 2.0 cm after adjustment of three 180-s NRTK observations at each mark; however, at some marks, additional NRTK observations were required. Multiple factors affect the accuracy of GNSS baseline observations but a portion of the error is modestly correlated with baseline length (Fig. 13). To create Fig. 13, the average vertical network accuracy at each mark from the constrained adjustments of the 10 hybrid networks involving GPS-only NRTK vectors, and the average vertical network accuracy at each mark from the constrained adjustments of the 10 hybrid networks involving GPS+GLONASS NRTK vectors were computed. Afterwards, linear regression lines were fitted to these average vertical network accuracy values at each mark versus baseline length. As the baseline length or distance from the RTN base station increases, the network accuracy tends to worsen. Of course, there is considerable scatter in Fig. 13, likely due to local conditions such as if the mark was in close proximity to a metal sign or power pole which may have induced additional multipathing errors. Nonetheless, the modest correlation shows that minimizing the distance from the rover to a reference station in the RTN will improve the accuracy of an NRTK solution.
Comparison with Coordinates of Higher Accuracy

Ideally, exact or known coordinates at a mark could be used as a basis for evaluating the accuracy of the coordinates derived at a mark from a survey adjustment. Unfortunately, there is no method for deriving exact coordinates at a mark because all measurements contain some amount of error. This problem is often mitigated in practice by using coordinates of a higher-order of accuracy (rather than exact coordinates) as a basis for evaluating the accuracy of coordinates of a lower-order of accuracy. However, according to formal error propagation, the coordinates of the marks derived by adjustment of the hybrid survey networks had network accuracies ranging from only 0.6 to 3.6 cm in ellipsoid height and 0.3 to 1.7 cm horizontally at 95% confidence. Deriving coordinates at each mark of a higher-order of accuracy (i.e., 0.03 to 0.36 cm at 95% confidence) is practically impossible.

Fig 13. Average vertical network accuracy at each mark from the constrained adjustments of the 10 hybrid networks involving GPS-only NRTK vectors or GPS+GLONASS NRTK vectors versus baseline length; South Carolina

5.2 Comparison with Coordinates of Higher Accuracy

Ideally, exact or known coordinates at a mark could be used as a basis for evaluating the accuracy of the coordinates derived at a mark from a survey adjustment. Unfortunately, there is no method for deriving exact coordinates at a mark because all measurements contain some amount of error. This problem is often mitigated in practice by using coordinates of a higher-order of accuracy (rather than exact coordinates) as a basis for evaluating the accuracy of coordinates of a lower-order of accuracy. However, according to formal error propagation, the coordinates of the marks derived by adjustment of the hybrid survey networks had network accuracies ranging from only 0.6 to 3.6 cm in ellipsoid height and 0.3 to 1.7 cm horizontally at 95% confidence. Deriving coordinates at each mark of a higher-order of accuracy (i.e., 0.03 to 0.36 cm at 95% confidence) is practically impossible.
Hence, the best available option was to post-process all of the aforementioned 30 to 100 h of static GNSS observations collected at each passive mark along with the six to 15 days’ worth of static observations at the active stations and then adjust the network of resulting baseline observations by least squares. This was completed with all of the available static GNSS data using OPUS-Projects and ADJUST following the process given in Fig. 1. ADJUST output coordinates at each passive mark with accuracy estimates ranging from 0.5 to 1.7 cm in ellipsoid height and 0.2 to 0.6 cm horizontally at 95% confidence. These error estimates are only 1.2 to 3.1 times smaller than the error estimates for the coordinates derived by the hybrid survey network approach--not an order of magnitude smaller.

For each hybrid network, \( HRMSE_j \) and \( VRMSE_j \) of the coordinates of the passive marks were computed according to Eq. 4 - 5.

\[
VRMS_j = \sqrt{\frac{\sum_{i=1}^{n}(h_{\text{hybrid},i}-h_{\text{static},i})^2}{n}} \\
HRMS_j = \sqrt{\frac{\sum_{i=1}^{n}(n_{\text{hybrid},i}-n_{\text{static},i})^2 + \sum_{i=1}^{n}(e_{\text{hybrid},i}-e_{\text{static},i})^2}{n}}
\]

where \( h_{\text{hybrid},i}, n_{\text{hybrid},i}, e_{\text{hybrid},i} \) is the ellipsoid height, northing, and easting, respectively, at passive mark \( i \) from the constrained adjustment of hybrid network \( j \); \( h_{\text{static},i}, n_{\text{static},i}, e_{\text{static},i} \) is the ellipsoid height, northing, and easting at passive mark \( i \) from the constrained adjustment of the static-only network, and \( n \) is the number of passive marks in hybrid network \( j \).
Figure 14 presents $HRMSE_j$ and $VRMSE_j$ for each hybrid network versus the number of 180-s NRTK observations per passive mark. Differences between RMSE values for hybrid networks with the same number of NRTK observations per mark (i.e., between hybrid network designations “A” and “B”) were between 0-4 mm, with the largest differences occurring in the vertical component. The VRMSE values, which ranged from 1.3--2.2 cm for all hybrid networks, are nearly 1 cm smaller than the values reported in Table 1, which are based solely on evaluations of the accuracy of baseline observations obtained in real-time without adjustment. This finding highlights how much vertical error can be reduced by developing and adjusting a network of repeat baseline observations. Horizontal RMSE values ranged from only 0.6 to 1.0 cm with slight (sub-millimeter) reduction as the number of NRTK observation per mark increased.

![Diagram](image)

**Fig 14.** (a) $VRMSE_j$ and (b) $HRMSE_j$ of the coordinates of the passive marks derived from constrained adjustment of the hybrid network based upon coordinates derived from constrained adjustment of a network of > 30 h of static GNSS baseline observations
5.3 Comparison of Methods for Estimating the Accuracy of the Results

In order to compare the two methods presented in this paper for estimating the accuracy of the adjusted coordinates, the network accuracies obtained from formal error propagation were reduced from the conventional 95% confidence level to the 68% confidence level. This is because if systematic errors were mostly removed, then the values of RMSE should approximate accuracies at the 68% confidence level.

Dividing the formal vertical and horizontal network accuracies by 1.96 and 2.45, respectively, results in formal vertical network accuracies ranging from 0.3 to 1.8 cm and horizontal network accuracies ranging from 0.1 to 0.7 cm (68% confidence). Thus, the formal accuracy estimates are somewhat smaller than the empirical accuracy estimates found by computing $VRMSE_j$ and $HRMSE_j$ (Table 5). On one hand, it is reasonable based on these results to argue that the formal accuracy estimates are somewhat optimistic, up to as much as 1 cm vertically and 0.5 cm horizontally. On the other hand, it is also possible that some systematic error remained, thus causing the values of $VRMSE_j$ and $HRMSE_j$ to be slightly greater than the formal accuracies at 68% confidence. The coordinates derived from least squares adjustment of only the static GNSS baseline observations were treated as if they were errorless when used as a basis for computing $VRMSE_j$ and $HRMSE_j$. However, these coordinates have formal errors near the same order of magnitude as the formal errors of the coordinates derived from the hybrid survey networks. Errors in the static survey coordinates could have introduced bias in RMSE.
Table 5. Summary of the estimates of the accuracy of the adjusted coordinates from the 30 hybrid GNSS survey networks using formal error propagation versus empirical computations of error by comparing with coordinates from a static GNSS survey network.

<table>
<thead>
<tr>
<th></th>
<th>Range of Formal Accuracies (68% confidence)</th>
<th>Range of RMSE values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>0.3 - 1.8</td>
<td>1.3 – 2.2</td>
</tr>
<tr>
<td>Horizontal</td>
<td>0.1 - 0.7</td>
<td>0.6 – 1.0</td>
</tr>
</tbody>
</table>

6 CONCLUSIONS AND RECOMMENDATIONS

Existing NGS height modernization surveying guidelines require long-duration static GNSS baseline observations (e.g., NGS-58) in order to derive ellipsoid heights with network accuracies less than 5 cm at 95% confidence. In an effort to develop a more efficient approach, this paper explored the idea of creating “hybrid networks” by combining NRTK baseline observations from an RTN with static post-processed baseline observations and performing simultaneous least squares adjustments. A total of 30 hybrid test networks were developed using static GNSS data from active stations along with varying amounts of repeat NRTK baseline observations on 18 passive marks in Oregon and 20 passive marks in South Carolina. The results of the constrained least squares adjustments of the hybrid survey networks show great promise for significantly reducing the fieldwork required to derive ellipsoid heights with accuracies less than 5 cm at 95% confidence. Fig. 5 shows the recommended workflow; critical steps in this workflow are to scale the VCV matrix output from the static baseline processing software (Step 2C) as well as the VCV matrix of the NRTK vectors (Step 1D). Such scaling is crucial in order to estimate realistic and compatible variances and covariances for development of an appropriate stochastic model for the hybrid network adjustment. Another important step
in the hybrid network development is to ensure that the RTN is configured so as to provide NRTK vectors that are referenced to a physical base station in the RTN.

According to formal error propagation, network accuracies (at 95% confidence) on the passive marks in the hybrid networks ranged from 0.6 to 3.6 cm in ellipsoid height and 0.3 to 1.7 cm horizontally. These network accuracies are only 1.2 to 3.1 times larger than network accuracies achieved when post-processing and adjusting at least 30 h of static GNSS observations on the same passive marks in the test networks. Further, when comparing coordinates from the constrained adjustments of the hybrid networks with the coordinates from the constrained adjustment of a network of > 30 h of static GNSS observations, VRMSE ranged from 1.3 to 2.2 cm, and HRMSE ranged from 0.6 to 1.0 cm as the number of NRTK baseline observations per mark decreased from 9 to 3.

Generally, three to six 180-s-duration NRTK vectors to each passive mark were required to reduce the estimated vertical network accuracy to less than 2 cm (95% confidence). If survey specifications or guidelines were to be developed using the hybrid network approach, then for conservatism, the authors recommend six 180-s-duration network NRTK vectors to each mark. This conservatism is meant to account for the fact that site conditions at passive marks can be highly variable due to trees or other overhead obstacles, presence of power lines, etc., and that short occupations are more vulnerable to multipath. Since multipathing varies throughout the day and in an effort to make independent observations, the authors also recommend that the NRTK baseline observations on a mark are taken at different times of the day with independent antenna setups and NRTK initializations.
In addition, when available, it is recommended to use both GPS and GLONASS observables. The use of GPS and GLONASS NRTK vectors improved the network accuracy of the coordinates, on average, by 19% (4 mm) in ellipsoid height and by 7% (0.7 mm) horizontally versus the use of GPS-only observables. The use of GLONASS increases the number of available satellites on different orbital planes for deriving NRTK solutions which is especially beneficial at marks with overhead obstructions.

The hybrid networks used in this study only involved data collected from RTNs in Oregon and South Carolina. Future research should include testing the accuracy of hybrid networks constructed using NRTK vectors from other RTNs in differing geographic, topographic, and climatic settings. In addition, more testing is required to find the recommended separation of time between repeat NRTK observations on a passive mark. All of the hybrid networks described in this paper had a minimum time separation between repeat NRTK observations at a mark of approximately 2 hours. Further research could also analyze NRTK solutions using additional types and combinations of other viable GNSS (e.g., Galileo, BeiDou).
7 REFERENCES

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