

Economics of Federal Range Use and Improvement For Livestock Production



Agricultural Experiment Station
Oregon State University
Corvallis



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AUTHORS: Darwin B. Nielsen, former graduate research assistant in agricultural economics at Oregon State University, is now assistant professor of agricultural economics, Utah State University; William G. Brown is professor of agricultural economics and Dillard H. Gates is Extension range management specialist, Oregon State University; Thomas R. Bunch, former graduate research assistant in range management at Oregon State University, is now Extension area range agent at Bend, Oregon.

ACKNOWLEDGMENTS: The authors appreciate the help of Dr. Russell D. Lloyd, Assistant Director, Rocky Mountain Forest and Range Experiment Station, U. S. Forest Service, in the conceptualization and planning of this study. Special thanks are also due to the cooperating ranchers of the East Cow Creek allotment for supplying the economic input-output data. Help from the personnel of the Vale office of the Bureau of Land Management is also appreciated. This study was initiated September 1, 1962, and supported by a grant from the Bureau of Land Management.

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DARWIN B. NIELSEN, WILLIAM G. BROWN,
DILLARD H. GATES, AND THOMAS R. BUNCH

SUMMARY AND CONCLUSIONS

The Bureau of Land Management (BLM) has the responsibility of managing millions of acres of federal land. One of the most important uses of this land is livestock grazing. Each year large investments of public capital are made to improve these federal rangelands. Seeding native ranges to crested wheatgrass and spraying sagebrush and rabbitbrush are common improvement practices undertaken by the BLM. Generally, there are also costs of fencing and water development associated with these improvements.

Administrators of these BLM rangelands are interested in analytical tools that would be useful to their range managers in making decisions relative to the use and improvement of the federal rangeland. This study was initiated and supported by the BLM to apply linear programming for the above purpose. A study was also set up with Oregon State University range management personnel to furnish the physical-biological coefficients needed for the economic study.

The three main objectives of this study were :

1. To determine rates of return from public investment in various range improvement practices on a given management unit of federal rangeland as measured by the effect upon costs and returns to individual ranches.
2. To estimate the marginal value product (MVP)¹ of an animal unit month (AUM) of grazing for various seasons and given range conditions of the management unit under study.
3. To evaluate the potential usefulness of programming models as an aid to decision-making by public land administrators.

¹ The MVP is the amount of money that would be added to total net income if one more unit of the resource were used. For example, suppose sprayable rangeland is one of the limiting resources and its MVP is \$12.50 per acre. This means that an additional acre of sprayable rangeland would add \$12.50 to total net income. If this acre of rangeland could be purchased for less than \$12.50, it would be profitable to purchase it.

Linear programming models were developed to reflect the physical-biological and economic situation of the East Cow Creek allotment. This allotment is located in the Vale grazing district of the BLM. MVP's of public capital at several levels of public investment were obtained from the solutions of these linear programming models. These MVP's were discounted over the life of the investment. The discount rate used was that rate which would equate the present value of the income stream over the life of the investment to the initial investment. Discount rates which perform the above function are known as "internal rates of return."

Internal rates of return for public investment in range improvements were computed for all relevant levels of public investment. These internal rates of return ranged from 31% to 3.25% for spraying and from 13% to 1% for seeding to crested wheatgrass. Whitmar beardless wheatgrass seeding was considered in one model, where its return ranged from about 16% to 2%. Objective number one was fulfilled by the computation of these internal rates of return.

As many as 23 different levels of public investment were considered in some of the models. At each level, a complete solution of the linear programming problem was obtained. MVP's per AUM for the various grazing seasons on the federal range were part of each solution. Weighted average MVP's for federal grazing were computed for the most relevant levels of public investment. At essentially zero public investment the weighted average MVP's were from \$7.91 to \$5.09 per AUM, depending on the assumptions of the model. These weighted average MVP's were from \$3.00 to \$3.76 per AUM at the optimum level of investment determined for each model. The optimum level of investment was considered to be where the internal rate of return was 3% (which was approximately the average rate of interest payable by the U. S. Treasury as recommended by Senate Document 97).

One of the advantages of using linear programming models to estimate these MVP's is that all measurable factors affecting them are considered simultaneously. Weighting each grazing season by the number of AUM's used by the individual ranchers further generalized these quantities. Thus, objective number two was accomplished.

With additional research and thought, these linear programming models could be used to get information relative to other public land management problems. The productive value of rangeland as measured through livestock can be estimated from present research results. With additional research, changes in management plans could be checked for feasibility in a model before funds had to be expended. If benefit-cost analysis is applied to range improvement projects, the rates of return and MVP's would provide valuable data for evaluating an additional AUM of range forage.

Several assumptions were built into these linear programming models, thus causing each one to be different from the others. Despite these differences in the models, there were certain consistencies in the results from which some general conclusions can be drawn:

- Returns, as measured through livestock production, are high enough to justify public investment in range improvement practices. However, at levels of public investment where the commensurate properties of the ranchers are being used near their capacity, these returns are soon pushed down to zero.

- A high degree of interdependence exists between private and public decision-making. Returns on public investment in range improvements are dependent on the investment of private funds to improve private properties. The amount of private investment required is indicated in the solutions of the linear programming models.

- Spraying federal rangeland for brush control under the assumptions used in this study returns more per dollar invested in range improvements than a dollar invested in seeding to crested wheatgrass.

It is concluded from the results of this study that the linear programming models have potential usefulness as an aid to decision-making by public land administrators.

INTRODUCTION

There are 31,969,038 acres of federally owned land in the state of Oregon, or about 52% of the total land area of the state.² The BLM holds title to about half of this, or 15,414,641 acres. Livestock are grazed on approximately 12.5 million acres of BLM lands in eastern Oregon and 500,000 acres in western Oregon.³ Grazing on BLM land supplements the production of privately owned ranches in these areas. In recent years over a million AUM's of grazing have been furnished by these federal lands in the five eastern Oregon grazing districts.

It is apparent from the above land statistics that the use of BLM rangelands in Oregon is very important to the economy of the state. The joint use of privately and federally owned range resources to produce livestock brings about a high degree of interdependence of one group upon the other. The BLM rangeland produces AUM's of grazing for certain seasons of the year; the particular season depends on the area. Ranchers using BLM lands provide feed for the livestock while they are off the federal lands. In most areas of eastern

² U. S. Department of the Interior, Bureau of Land Management, Public Land Statistics, 1963 (Washington, D. C., 1964).

³ *Ibid.*

Oregon, ranchers cannot maintain cattle numbers without BLM grazing. Grazing by livestock offers the best use at the present time for most of the federal lands. It is in the long-run interest of the public that the range resource be managed in such a way that it will make its maximum contribution to the nation and to local areas.

The BLM conducts other programs in addition to the administration of grazing districts. Programs for soil and moisture conservation to rehabilitate millions of acres of seriously eroded rangeland are conducted under the National Soil Conservation Act. A range improvement program authorized by the Taylor Grazing Act provides range use facilities to aid in range management and utilization. Many improvement practices such as seeding, brush control, water development, and fencing are undertaken. Each year large sums of public capital are invested in improvements on these federal rangelands. Of course, finances are not available now, and probably never will be, to improve all of the BLM rangeland that has the physical potential for improvement. The fact that not all of the BLM rangeland will be improved brings up the problem of deciding whether any rangeland should be improved, how much to improve, where to improve, and what improvements should be made.

To attain the necessary and sufficient conditions for optimum economic range improvement, present funds should be allocated to improvement practices and to range sites according to their marginal productivity.⁴ For example, if the marginal return to spraying is greater than the marginal return to seeding, then spraying should be undertaken first. Rangeland should be sprayed as long as its marginal return is greater than the marginal return for seeding. Given two range sites with different productive potentials, the marginal return would be the greatest on the one with the highest productivity potential; thus, this site should be improved first.

Problems and Purposes of the Study

This research was initiated for three main purposes. The first of these purposes was to determine the rate of return on public investment in range improvement practices as measured through domestic livestock use, the most profitable range improvement practice, and the optimum level of improvement. Seeding, spraying, and meadow fertilization were the improvements considered. The amount of public capital assumed available was varied, and a new solution was obtained

⁴ Marginal productivity is the ability of one additional unit of some variable input to increase the total product. In the above case, the variable input is dollars and they should be allocated according to the return on the last (marginal) dollar invested in a particular improvement practice.

for each level of available public capital. Purpose number two was to determine the MVP's for the different grazing seasons on the management unit under study as measured through domestic livestock use. The last purpose was to evaluate the potential usefulness of programming models as an aid to decision-making by public land administrators.

To accomplish the first purpose listed above brings up the problem of developing a method of analysis that will take into account the interdependence of public and private resources. The method of analysis should also reflect the economic environment of the ranchers, at a given point in time.

The problem encountered in trying to bring about the second purpose of this study is in many ways similar to the first problem, i.e., both public and private resources, as well as the economic situation, must all be considered at the same time. One might go so far as to say that the second problem has to be solved before or simultaneously with the first problem discussed. This is because the MVP of an AUM of grazing is an essential variable in determining the best range improvement practice and the amount of range improvement that should be undertaken. Linear programming is used in this study because it solves both of these problems simultaneously while considering the interdependence of the public and private resources.

Linear Programming

Linear programming originated largely during World War II as a method of finding minimum distance routes for the limited shipping facilities available. It was later applied to maximization and minimization problems in industry. Linear programming has been used to determine the optimum combination of crops and livestock enterprises on farms.⁵ It has also been used to determine the least-cost combination of feeds that will meet various nutritive requirements for livestock rations. In this study, linear programming is used to determine the optimum way to use a particular group of range and ranch resources.

The most profitable range improvement practices can be determined from the alternatives considered. Solutions also indicate the number of acres that can be profitably improved under the assumptions built into the linear programming model. Optimal seasonal use patterns for the rangeland are also determined by the solution of the models.

⁵ An extensive but mathematically simple treatment of linear programming is given by E. O. Heady and W. Candler, *Linear Programming Methods* (Ames, Iowa: Iowa State University Press, 1960).

The MVP's of limiting factors are determined simultaneously when linear programming is used, thus reflecting the value of the factor to the entire ranch operation. For example, the MVP of an AUM of April grazing takes into account all of the other input factors that are used in producing livestock. This method of estimating MVP's is more logical than simply using an animal turn-off value which only considers one grazing season and one type of rangeland and does not consider the other factors required to keep the animals through the entire year.

INPUT DATA FOR THE MODELS

Physical and Biological Coefficients

A management allotment, centered about a block of around 50,000 acres of federal rangeland grazed in common by nine cattle permittees, was selected. This selection was based on the number of ranch units in the allotment and the representative qualities of the ranches and range area.

The East Cow Creek allotment located just north and west of the town of Jordan Valley in Malheur County, Oregon, was selected. On this allotment the rangeland and types of cattle operations are quite typical of the high desert range country.

This area is essentially a plateau with some east- and southeast-oriented low ridges. The elevation varies from 4,000 to 4,800 feet above sea level. Some areas of the allotment are too steep, while others are too rocky to plow. Some areas are covered by comparatively recent lava flows which are practically void of vegetation.

The semiarid climate of the study area is characterized by warm, very dry summers and cold winters. Danner, located near the center of the study area, has a 20-year mean annual precipitation of 11.26 inches. Most of the moisture occurs as snow between the months of November and March. A secondary rainy period, however, usually occurs in May or June. The 20-year mean monthly precipitation shows that May is the wettest month of the year. Average annual runoff from the area is less than an inch.⁶

A joint study was set up between the Department of Agricultural Economics and the Division of Range Management at OSU. BLM personnel at the federal, state, and local levels and the ranchers involved agreed to cooperate in the study. Range management specialists

⁶R. C. Newcombe, *Ground Water in the Western Part of the Cow Creek and Soldier Creek Grazing Units, Malheur County, Oregon*, Washington, 1962, pp. 159-172. (U. S. Geological Survey Water Supply Paper No. 1475-E.)

furnished physical-biological yield coefficients to be used in the linear programming models.

These data are based on information collected during the summer of 1963 and the personal experience and judgment of the range management staff. Other research information was used to support these data where applicable.

Early in the spring of 1963, 34 plots (17 paired plots) were set up in and around the East Cow Creek allotment. One plot of each pair was on native unimproved rangeland and the other plot was on improved range. The plots in each pair were located quite close together on similar sites. Plots were fenced to prevent grazing by livestock.

Forage clippings were made to measure yield differences between improved and unimproved plots. Four 9.6 square-foot subplots were clipped in each of the 34 main plots. Holes were dug on each site and soil profiles were described. In addition, a vegetation classification was determined for each site. These data permitted correlation of site characteristics and potential production.

Range management personnel also estimated the amounts of rangeland, federal and private, which fell into the following categories: seedable, sprayable, and "other range." Other range was further broken down into two classes: (1) Other "good" (range too good to be seeded or sprayed); and (2) other "poor" (range too poor for seeding or spraying because of topography, soil, or lack of perennial grass understory).

To be classified as seedable, a block of land had to have all of the following characteristics: (1) Soil well enough developed to support a stand of crested wheatgrass; (2) topography and vegetative cover such that it could be physically prepared for seeding, i.e., not too steep, no brush species or trees that could not be plowed, and minimal rock outcroppings; (3) the perennial grass understory so depleted that spraying would not be feasible; and (4) in large enough blocks to be practical for seeding and management.

Sprayable range was land with a fair to good understory of perennial grasses with the potential to increase in growth and vigor when given a reduction in competition from brush species and a rest from grazing. Again, sprayable tracts had to be large enough for economical spraying and management.

Range types of all lands in the allotment, federal and private, were delineated on aerial photographs by range technicians. All delineations were field checked, and information concerning plant species, pertinent soils information, and estimated herbage production was recorded for each. Herbage production was broken into three categories: (a) less than 100 pounds per acre; (b) 100 to 200 pounds per acre; and (c) over 200 pounds per acre. Based on the ecological

factors indicated, a judgment was made as to the potential productivity of each delineation or range type. These categories were: (a) seedable, (b) sprayable, (c) other poor, and (d) other good. A mapping legend was developed to indicate type, productivity, and potential. This legend was placed directly on the aerial photograph for each delineation.

A square-inch grid system, with appropriate conversion factor, was used to determine acreages in each of the categories of seedable, sprayable, other poor, and other good. When acreages and estimated yields had been determined, carrying capacities expressed as AUM's were calculated. For these conversions, 800 pounds of air dry forage (10% moisture) were considered equivalent to one AUM. In making these conversions it was assumed that the following percentages of grasses could be utilized by grazing livestock: (1) cheatgrass, 75%; (2) bluebunch wheatgrass, 50%; (3) crested wheatgrass, 66%; and (4) the native range used only during the month of April would carry over 50% of the herbage produced to be used the following April.

All forage yield data were adjusted to the median year as determined by Sneva and Hyder.⁷ Several sources of weather data in the immediate vicinity of the allotment were used to give a more accurate index number.

For the purposes intended in this study the above outlined method of obtaining a resource inventory and yield estimate is sufficiently accurate. It should be kept in mind that these data were obtained by technicians trained in range resource management and were based on their experience and judgment. It should also be noted that these coefficients do not necessarily reflect management that is being used on this allotment, but they are based on what these professional range management specialists think management ought to be. It is important to keep this in mind when considering the results of the study.

Economic Input-Output Coefficients

Ranch budget data were collected for the calendar year 1962. A personal interview was made with each rancher in late December 1962 and early January 1963. These data were summarized and a net return per unit of breeding herd was calculated for each ranch.

Net return per unit of breeding herd was the gross return per unit of breeding herd minus the variable costs. The assumption was made that the variable inputs received a price equal to their MVP's (or marginal cost in most cases). Returns to the fixed factors are

⁷F. A. Sneva and D. N. Hyder, *Forecasting Range Herbage Production in Eastern Oregon*, *Oreg. Agric. Expt. Sta. Bull.* 588.

maximized by the procedures built into linear programming. An alternative assumption might be made, i.e., if there is any surplus it will be distributed to the fixed factors. In this study all costs were deducted except the costs of the fixed factors being considered in the model. These fixed factors were private rangeland, private meadow, federal rangeland, and public capital.

Four permittees had operations of such small size and diversity that they were omitted from the analysis. Also, because of complicated tenure arrangements on one ranch, only four of the five major ranch operations were considered. However, these four major ranch operations accounted for over 80% of the total use on the allotment.

The net return calculated for these ranches varied from ranch to ranch. Some variation was due to the type of operation, such as cow-calf-yearling versus cow-calf. Efficiency due to size also caused variation. Management was probably the most important factor causing variation. However, no attempt was made to adjust for differences in management, and each ranch was taken as it operated in 1962.⁸

Costs of seeding

Seeding cost per acre used in this study was based on projected seeding costs in the East Cow Creek allotment and adjacent areas.

These cost estimates were obtained from the BLM staff of the Vale grazing district in 1963 (Table 1).

It was assumed that the BLM staff had a better basis for estimating these costs than could have been obtained from secondary sources. Thousands of acres of BLM rangeland have been seeded in the Vale district, so they had many cases on which to base their estimates.

The initial investment for plowing and drilling (\$9.71) seems rather high compared to some studies.⁹ However, the assumption is being made that at this cost a 95 to 100% brush kill will be forthcoming and that proper care will be exercised to insure correct seeding

⁸ Each ranch budget is given by Darwin B. Nielsen, *Economics of Federal Range Use and Improvement*, Ph.D. thesis, Oregon State University, June 1965, pp. 159-162.

⁹ C. O. McCorkle and D. D. Caton, *Economic Analysis of Range Improvement*, Calif. Agric. Expt. Sta., Giannini Foundation of Agricultural Economics Bull. 235, p. 40, 1962; B. D. Gardner, *Costs and Returns from Sagebrush Range Improvements in Colorado*, Colo. Agric. Expt. Sta. Bull. 511-S, p. 2, 1961; D. D. Caton and C. Beringer, *Costs and Benefits of Reseeding Range Lands in Southern Idaho*, Idaho Agric. Expt. Sta. Bull. 326, p. 31, 1960; H. B. Pingrey and E. J. Dortignac, *Costs of Seeding Northern New Mexico Rangelands*, New Mexico Agric. Expt. Sta. Bull. 413, p. 43, 1957; and R. D. Lloyd and C. W. Cook, *Seeding Utah's Ranges*, Utah Agric. Expt. Sta. Bull. 423, p. 3, 1960.

Table 1. COST ESTIMATES FOR SEEDING CRESTED WHEATGRASS

<i>Initial costs:</i>	
Plowing and drilling	\$ 9.71 per acre
Fencing99 per acre
Water developments	2.20 per acre
Nonuse63 per acre
	\$13.53
<i>Annual costs:</i>	
Fence maintenance	\$.08
Water maintenance and use10
	\$.18
<i>20-year life of the seeding:</i>	
\$13.53	
———— = \$.68 + \$.18 = \$.86 per acre per year	
20	

rate and seed cover. It is further assumed that a seeding will last 20 years without additional investment. Length of seeding life is somewhat arbitrary at best. It depends on many factors, some of which can be influenced by management while others cannot.

Costs were computed for two years' nonuse on the seedings. The number of AUM's of grazing foregone for a two-year period (based on unimproved carrying capacity) were valued at \$3.00 per AUM. The \$3.00 per AUM is based on private grazing fees reported in Utah in 1961.¹⁰

Costs of spraying

As with seeding costs, the spraying costs used in this study are based on projected spraying costs for the East Cow Creek allotment and adjacent areas. The spraying costs are summarized in Table 2.

Nonuse in the case of spraying is for deferment until after the grasses mature for two years. This would amount to a loss of about one-half the use each year, or one year's nonuse. Perennial grasses are given a chance to increase in vigor during this two-year period.

The initial cost of spraying is somewhat higher than some of the more recent estimates from the Vale office of the BLM. Costs for fencing and water development were substantially less for spraying projects than for seeding in the East Cow Creek allotment to get the

¹⁰D. B. Nielsen, *Valuation of Public Rangeland for Grazing Purposes*, M.S. thesis, Utah State University, 1962.

Table 2. AERIAL SPRAYING COST ESTIMATES¹

<i>Initial cost:</i>	
Spraying (including materials and application)	\$3.42 per acre
Fencing28 per acre
Water developments67 per acre
Nonuse33 per acre
	\$4.70
<i>Annual costs:</i>	
Fence maintenance	\$.03
Water development maintenance and use02
	\$.05
<i>12-year life of the spraying:</i>	
\$4.70	
<hr style="width: 100%;"/>	
= \$.39 + .05 = \$.44 per acre	
12	

¹ Further details are given by Nielsen, *Economics of Federal Range Use and Improvement*, p. 164.

desired level of range management. Given the costs used, it was thought by the researchers involved that a spraying should last at least 12 years in the study area, i.e., a good enough kill of the brush species should be forthcoming from this much investment to last 12 years.

Costs of meadow fertilization

Time and research funds did not allow experiments to be conducted on the meadows of the cooperating ranches; therefore, estimates of the potential yields given some application of nitrogen on these meadows had to be gained from other sources. The most logical place to get these data was the Squaw Butte Experiment Station. This station is located about 130 miles from the study area.

C. S. Cooper, an agronomist at Squaw Butte, found the average increase in yield from the application of 60 pounds of nitrogen to be three-fourths of a ton per acre.¹¹ From these data it was decided to use an application of 60 pounds of nitrogen per acre with an expected increase in yield of .75 tons per acre. The ranchers in the study area do not use commercial fertilizer on their meadows. Unimproved meadow land is estimated to yield one ton per acre.

In order to get 60 pounds of nitrogen, it would require the application of about 180 pounds of ammonium nitrate or 300 pounds of

¹¹ C. S. Cooper, *More Mountain Meadow Hay with Fertilizer*, Ore. Agric. Expt. Sta. Bull. 550, 1955, p. 2.

ammonium sulfate per acre. The 1963 prices of ammonium nitrate and ammonium sulfate were around \$90 per ton and \$60 per ton, respectively. Thus, the cost of fertilizer would be between \$8.10 and \$9 per acre. The cost of application (\$.50 per acre) has to be added on to these figures. For this study, an annual cost of \$8.60 per acre for 60 pounds of applied nitrogen was used. It was assumed that this would increase the yield of meadow hay from 1.0 to 1.75 tons per acre.

Linear Programming Models

It was necessary to develop more than one model for this study so that different assumptions about the use of the range resources could be considered. The four ranchers considered in the models controlled 82% of the use on the allotment. Therefore, the acres of federal rangeland were reduced by 18%.

These linear programming models were set up in such a way that the yearly feed requirements for a breeding unit were specified for each rancher. To meet these feed requirements, both public and private resources were used. The public resources available, reduced by 18%, were: 3,874 acres of crested wheatgrass, 9,499 acres of seedable rangeland, 4,517 acres of sprayable rangeland, 1,034 acres of other "good" rangeland, and 22,117 acres of other "poor" rangeland. In addition to these resources, each rancher had some private rangeland and native meadow. The native meadow produces hay for winter feed plus aftermath grazing in late summer.

Range improvements enter the solution when profitable. If an acre of sprayable rangeland is sprayed at some annual cost, it is no longer available for use as sprayable range. Except for the last two models developed, range improvements are not considered on the private rangeland. However, meadow improvement was an alternative considered in all models.

Given all alternative seasonal uses of the various federal range resources, the alternative of improving or not improving these lands, the option of improving the private meadows, and the alternative use of the other private resources, the most profitable use plan for these resources is determined by the linear programming solution. Units of breeding herd for the different ranchers are added as long as it is profitable and as long as the resource requirements for these breeding units do not exceed the amount of resources available.

Several alternative ways of using these range and ranch resources are incorporated into the models. These alternative ways of using the resources are set up to reflect improved management practices and in some cases to more nearly simulate actual ranch management prac-

tices. Details of the programming models have been presented by Nielsen.¹²

Models I and II

Model I was set up so that 3,874 acres of crested wheatgrass were initially available. Also, it was assumed that any increased forage brought about by range improvements would be allocated in a fixed ratio according to the ranchers' relative proportion of use at the beginning of the study. If a particular rancher were getting 10% of the grazing on the allotment at the time of the study, he would continue to get 10% of the grazing, regardless of how much total grazing became available.

Model II was the same as Model I except that the initial 3,874 acres of crested wheatgrass seeding was omitted. The crested wheatgrass was omitted to see what the range improvement pattern would be if no improvements had been made on the allotment prior to this study. Both models gave essentially the same results at the higher levels of public investment. Differences at the lower levels of public investment resulted from the fact that in Model II all of the sprayable range was sprayed before any seeding came into the solution. Since the results lead to the same general investment decisions, only the results of Model II will be presented. All figures and data used in Model II are presented in Appendix Table 1.

Solution of Models I and II

Parametric programming was used to change the quantity of one of the resources.¹³ The parametric changer can be set up in such a way that it will increase the amount of a resource just enough to cause some change in the basic solution of the models. A change in the basic solution occurs when the variable that is altered causes a new variable to come into the solution.

Parametric programming was used to increase the amount of public capital. Eighteen parametric changes were needed to get the MVP of public capital below one dollar. Investment beyond this point was assumed irrational since cost of public capital would not be fully recovered. After each parametric change, a complete new solution was obtained so that the effects of increasing public capital could be traced out.

¹² Darwin B. Nielsen, *Economics of Federal Range Use and Improvement*.

¹³ A linear programming routine developed by James Boles was used to solve most of the problems on the IBM 1620 computer. Cf. Boles, LP 20 Linear Programming System, 1620 General Program Library, 10.1.009, University of California, Berkeley, n.d.

Results obtained from the solutions. As mentioned earlier, the results obtained from programming are a function of the assumptions and data, as reflected in the coefficients. Only a limited number of activities representing alternative ways of using the range resources can be considered. Each solution indicates the optimum way to use both public and private resources, given the assumptions, input-output coefficients, and alternatives explained above for the model. Finding the optimum use of these resources is important; however, additional information which is equally valuable is gained from the solution of a linear programming model.

MVP's of the limiting factors. The MVP's for all limiting factors of production are mathematically computed in the solution of a linear programming model (MVP's are usually called shadow prices in programming literature). With these MVP's several of the following questions can be answered: How would the total adjusted income to the allotment be affected if (1) another dollar of public capital was made available, (2) another AUM of grazing for some season was made available, or (3) another acre of some resource was made available? How many dollars of public capital would have to be made available before spraying or seeding would come into the solution? How much does the last dollar invested in some particular range improvement return to the system? The MVP's shed light on these and many other questions that are important in making public and private land policy decisions. These questions will be discussed in a later section on policy implications.

Some of the results obtained from the solutions of Model II are presented in Table 3. Nine of the eighteen solutions obtained are summarized in Table 3. At the \$10 (essentially zero) level of public investment, one more dollar of public capital would yield a gross return of \$3.77, or \$2.77 above cost. As more public capital is made available, the annual return becomes less. An annual investment of slightly under \$10,105 is the largest investment that will yield a positive return over the cost of capital.

The MVP of public capital at any given level of investment is applicable for each dollar invested up to the next higher investment level determined by the parametric routine of the linear program. For example, with an annual investment of \$5,122 the MVP is \$1.35 (Table 3). According to the parametric program, \$1.35 would be returned for each dollar invested up to an annual investment of \$8,719. The fact that the MVP computed for any level of public investment applies to each dollar up to the next higher investment level will be important later.

A weighted average MVP is used for the different grazing seasons at the various levels of public investment. An MVP for each

Table 3. GROSS RETURN PER DOLLAR FOR VARIOUS INCREMENTS OF PUBLIC INVESTMENT IN RANGE IMPROVEMENTS¹

Public levels of investment	\$10	\$654	\$1,989	\$3,210	\$4,688	\$5,122	\$8,719	\$9,602	\$10,105
MVP public capital	\$ 3.77	3.45	1.85	1.83	1.39	1.35	1.24	1.21	.85
MVP April grazing	\$ 8.16	8.43	5.56	5.48	4.81	4.72	4.21	4.23	4.46
MVP May grazing	\$11.78	11.77	9.02	8.88	7.80	7.64	6.83	6.86	7.23
MVP June grazing	\$.49	1.89	0	0	0	0	0	0	0
MVP July grazing	\$ 8.35	7.64	5.04	4.96	4.07	3.90	3.48	3.43	2.78
MVP Aug.-Sept. grazing	\$ 9.45	8.64	5.62	5.54	4.43	4.30	4.11	3.98	3.21
Average MVP public grazing	\$ 7.72	7.71	5.07	5.00	4.23	4.12	3.75	3.71	3.52
MVP Aug. 1-Oct. 1 private grazing	\$ 1.50	1.50	3.28	3.80	8.90	8.90	8.34	8.14	7.43
MVP Oct. 1-Nov. 20 private grazing	\$ 1.50	1.50	3.28	3.28	3.24	3.31	3.68	3.65	3.93
MVP hay-private	\$ 0	0	7.30	7.30	7.53	7.53	8.75	8.68	9.32
R I ² private investment	\$ 0	0	0	0	0	0	0	0	0
R II private investment	\$ 817	996	2,498	3,822	4,625	4,847	7,918	7,918	7,918
R III private investment	\$ 0	0	0	0	0	0	0	0	0
R IV private investment	\$ 27	51	96	130	171	182	582	748	841
Acres sprayed	23	1,488	4,517	4,517	4,517	4,517	4,517	4,517	4,517
Acres seeded	0	0	0	1,481	3,138	3,643	7,831	8,858	9,445

¹ It was assumed that the ranchers would utilize the increased forage in the same ratio as the present use. It was also assumed there was no seeded wheatgrass initially.

² R = rancher.

season is computed for each rancher. These MVP's are then averaged, using the number of AUM's allotted to each rancher as his respective weight. Therefore, the seasonal grazing MVP's shown in Table 3 represent an average of all the ranchers rather than for any one particular rancher.

Private investment required. Because of the proportionality assumption made in the model, Rancher II is required to make a large private investment at each level of public investment. In every case it is higher than the public investment. Rancher IV soon has to invest in meadow improvements because he has so few private resources available. From these figures in Table 3 it can be seen that private investment is essential to profitable use of increased forage brought about by public investment in range improvements on federal rangeland. Results from an earlier model gave indications that the price of private capital could be substantially higher before it would be unprofitable to invest. These results showed very little decrease in the amount of private investment when the price per dollar of private capital was raised from \$1.10 to \$1.50.

According to the bottom two lines of Table 3, all sprayable range is sprayed before any rangeland is seeded to crested wheatgrass. Given all of the factors and alternatives considered in the model, spraying yields a higher return than seeding. (Of course, for these high returns from spraying to hold, the rangeland classed as "sprayable" must have a fair to good understory of perennial grass with a potential of increasing in growth and vigor.)

Determination of land-use patterns. It was mentioned earlier that the linear programming solution gave the optimum seasonal use pattern for the rangeland, given the activities and constraints of the model. The seasonal use of each type of rangeland is changed as more improvements are made. These changes in seasonal use patterns are presented in Table 4.

At the lower levels of public investment, all 13,372 acres of unimproved seedable range are grazed from May 1 to July 30. As some of these acres are seeded to crested wheatgrass, the use of the remaining unimproved seedable range is shifted to May 1 to June 30.

Sprayable range is grazed August 1 to September 30. The sprayable range is soon sprayed as public capital is made available. After this land is sprayed, much of it is still grazed in the late summer season. However, as April grazing becomes a bottleneck, sprayed rangeland is used to supply this early season grazing.

Other "good" range which has a good stand of perennial grasses is used August 1 to September 30. One would expect this type of

Table 4. CHANGES IN SEASONAL USE PATTERNS FOR EACH TYPE OF FEDERAL RANGE WITH DIFFERENT LEVELS OF RANGE IMPROVEMENT FOR MODEL II

Public investment levels	\$10	\$654	\$1,989	\$3,210	\$4,688	\$5,122	\$8,719	\$9,602	\$10,106
<i>Season of use for each range type:</i>									
Seedable range (acres)									
May 1-Aug. 1	13,372	13,372	13,372	7,608	0	0	0	0	0
May 1-July 1	0	0	0	4,346	10,234	9,729	5,542	4,515	3,928
Sprayable range (acres)									
Aug. 1-Oct. 1	4,494	3,029	0	0	0	0	0	0	0
Other good range (acres)									
Aug. 1-Oct. 1	1,034	1,034	1,034	1,034	1,034	1,034	1,034	1,034	0
April 1-May 1	0	0	0	0	0	0	0	0	1,034
Other poor range (acres)									
April 1-May 1	9,491	10,066	11,146	11,961	11,412	10,476	3,496	1,308	73
May 1-Aug. 1	10,597	12,051	4,152	0	0	0	0	0	0
Aug. 1-Oct. 1	2,029	0	0	0	0	0	0	0	0
May 1-July 1	0	0	6,819	10,156	10,705	11,641	18,621	20,809	22,044
Crested wheatgrass (acres)									
July 1-Aug. 1	0	0	0	1,418	3,138	3,202	3,620	3,793	3,891
Aug. 1-Oct. 1	0	0	0	0	0	441	4,210	5,064	5,554
Spraying (acres)									
Aug. 1-Oct. 1	23	1,488	3,413	3,701	4,040	3,661	916	0	0
July 1-Aug. 1	0	0	1,104	816	0	0	0	0	0
April 1-May 1	0	0	0	0	477	856	3,601	4,517	4,517

rangeland to make its maximum contribution for summer grazing. At the highest level of investment other "good" range is grazed in April.¹⁴

Other "poor" range furnishes grazing from April 1 to September 30 at the first level of public investment. With additional investment its use is shifted to the earlier grazing (April 1 to June 30).

Crested wheatgrass comes in to furnish the grazing needed later in the season. The first crested wheatgrass seedings are used July 1 to August 31. However, as more acres are seeded, the use pattern shifts to furnish additional grazing August 1 to September 30.

After the above results were obtained, one of the later models was revised slightly to allow more flexibility in the use pattern for other "poor" and seedable range. In the original models, other "poor" and seedable range were allowed to furnish grazing for May 1 to July 1 but not for May 1 to June 1 and June 1 to June 30 separately. Consequently, it was not profitable for crested wheatgrass to provide June 1 to July 1 grazing since May 1 to June 1 grazing could not be efficiently supplied from any other rangeland.

The revision merely added two new activities. These two activities allowed other "poor" and seedable range to furnish grazing during the May 1 to June 1 season. In the revised model, seedable (unimproved) rangeland tended toward an earlier season of use—April 1 to May 1 and June 1 to July 1 at the higher levels of investment. Crested wheatgrass was used earlier in the season also. Some 3,600 acres were grazed in July and about 2,000 acres were grazed in June at an investment level of \$7,000. At investment levels greater than \$7,000, crested wheatgrass grazing shifted toward August 1 to October 1 use. Sprayed rangeland was grazed for the most part August 1 to October 1 as was other "good" rangeland. Other "poor" rangeland furnished most of the April grazing and all of the May grazing. These land-use patterns were more realistic in some cases than the original results. The revised land-use pattern is presented in Table 5.

Adding the more flexible land-use activities had very little effect on the MVP's. Therefore, for the purposes of this bulletin, the revised land-use pattern will be assumed to be applicable to all models except for a later model considering Whitmar beardless wheatgrass seeding.

¹⁴ Dry forage from the year before is generally the only forage available during the month of April. Land grazed in April is not grazed after May 1, and the current year's growth is left for use the next year. According to Robert J. Raleigh, Squaw Butte Experiment Station, this dry feed is just about as nutritious as meadow hay. Use of the range has the advantage of getting the cattle off the meadows during this wet month. Disease problems in the calves can be reduced by getting the cows out of the concentration on the meadows and scattered on the range.

Table 5. CHANGES IN SEASONAL USE PATTERNS AT DIFFERENT LEVELS OF RANGE IMPROVEMENT FOR EACH TYPE OF FEDERAL RANGE FOR REVISED GRAZING SEASONS

Levels of capital	\$10	\$1,989	\$4,368	\$6,051	\$6,815
<i>Range type</i>					
Seedable acres:					
May 1-Aug. 1	13,372	10,371	0	0	0
June 1-July 1	0	3,001	9,594	5,720	3,996
April 1-May 1	0	0	1,013	2,930	3,766
Crested wheatgrass acres:					
July 1-Aug. 1	0	0	2,765	3,381	3,545
June 1-July 1	0	0	0	1,341	1,935
Aug. 1-Oct. 1	0	0	0	0	130
Sprayable acres:					
Aug. 1-Oct. 1	4,494	0	0	0	0
Sprayed acres:					
Aug. 1-Oct. 1	23	3,446	4,081	4,464	4,517
July 1-Aug. 1	0	1,071	436	53	0
Other good acres:					
Aug. 1-Oct. 1	1,034	1,034	1,034	1,034	1,034
Other poor acres:					
April 1-May 1	9,491	11,241	12,025	11,188	10,823
May 1-June 1	0	3,156	10,092	10,929	11,294
May 1-Aug. 1	10,596	7,720	0	0	0
Aug. 1-Oct. 1	2,029	0	0	0	0

Federal range improvement decisions

The MVP's of public capital listed in Table 3 are directly applicable only if the decision-maker is willing to completely ignore time. Range improvements require an investment in time period t_0 , whereas the returns come in over the annual time periods t_1 to t_n . (For the case under discussion, n would equal 12 for spraying and 20 for seeding). If time is ignored, this implies that a dollar return at any time in the future is worth as much as a dollar today. Most decision-makers are not willing to ignore time, so a process called "discounting" is used to equate future returns to the present. The difference between the worth of a dollar today and a dollar in some time period t_i depends on the interest rate the decision-maker will accept.

Several interest rates could be assumed for the decision-maker. The level of investment could be determined for each rate of interest where costs of improvements are equal to the discounted annual returns from the improvements. The problem here is that one would never know the appropriate interest rate for any given decision-maker.

A better way of handling this problem would be to compute the rate of interest that would make the present value of costs and returns equal for each level of public investment. The interest rate that equates the present value of costs and returns is known as the internal rate of return.¹⁵

The assumption was made earlier that public funds are limited for range improvements, i.e., not all physically possible range improvements will be undertaken. The relative profitability of spraying versus seeding has already been determined by the linear programming solution, which eliminates many of the problems of ranking projects. Using the MVP's for the different levels of investment, an internal rate of return can be computed at each level.

One must be careful about making direct comparisons of the internal rate of return and the market rate of interest. For purposes of this study where public funds are being invested in range improvements, it is important to use the opportunity cost of public capital and not the market rate of interest as a standard to be compared with the internal rate of return. The internal rates of return are estimated in Table 6 for each level of public investment considered in Model II; the decision-maker can equate his own opportunity rate of interest with these rates.

The internal rates of return were computed using the following method: At each investment level the MVP of public capital represents an undiscounted rate of return per dollar invested. These MVP's must be discounted by that rate of interest which will make their present value over the life of the investment equal to the investment. Care must be taken to be sure to include each level of capital where a significant change in the MVP occurs. Every level of investment is not considered in Table 3; therefore, it is not complete enough for computing the internal rates of return. Table 6 includes each level of investment where there is a different MVP for public capital. The internal rate of return for each level of investment is computed in Table 6. Acres of sprayed and seeded range are also listed.

How could a public land manager use information, such as that in Table 6, to help decide on the level of investment to make in range improvements? First, he would need to have some idea of the opportunity rate of interest for public funds. There are alternative ways of estimating the opportunity rate of interest. If he had estimates of the rates of return of several projects, the opportunity rate of interest for any one project would be the highest rate of return on the other proj-

¹⁵ For more information see B. D. Gardner, *The Internal Rate of Return and Decisions to Improve the Range*, and a discussion by Allen LeBaron. In: Proceedings of the Committee on Economics of Range Use and Development, Western Agricultural Economics Research Council, Laramie, Wyoming, 1963.

Table 6. INTERNAL RATE OF RETURN FOR VARIOUS INCREMENTS OF PUBLIC INVESTMENT IN RANGE IMPROVEMENTS FOR MODEL II

Public levels of investment	\$10	\$654	\$89.82	\$890.93	\$1,989	\$3,043	\$3,209	\$3,213	\$4,146	\$5,688	\$5,122	\$5,448	\$8,719	\$9,602	\$10,106
MVP public capital..	\$ 3.77	3.45	2.93	2.24	1.85	1.85	1.83	1.52	1.41	1.39	1.35	1.28	1.24	1.21	.85
Internal rate of return		31%	27%	22%	15.5%	6.75%	6.75%	5%	4.5%	3.75%	3.25%	3%	2.5%	2.25%	2%
Acres sprayed	23	1,488	2,025	2,025	4,517	4,517	4,517	4,517	4,517	4,517	4,517	4,517	4,517	4,517	4,517
Acres seeded	0	0	0	0	0	0	1,481	1,484	2,506	3,138	3,643	4,023	7,831	8,858	9,445
Total adjusted ranch income	\$42,792	45,220	46,028	46,028	48,493	50,448	50,755	50,760	52,182	52,946	53,551	53,992	54,170	59,262	59,871
Animal units of federal grazing															
Rancher I	63	67	68	68	74	78	79	79	83	85	87	87	100	103	106
Rancher II	610	647	659	659	716	762	769	769 ¹	793	806	816	847	956	958	958
Rancher III	243	258	263	263	286	304	307	307	323	331	338	338	382	400	411
Rancher IV	82	87	89	89	96	102	103	103	109	111	114	114	130	135	139

¹ At investment levels above \$3,213 Rancher II comes in for use on the low price cow-calf-yearling activity which causes the proportionality assumption not to hold on an animal unit basis. However, this assumption still holds if figured on an AUM level. At the extremely high investment levels, it pays Rancher II to take nonuse so that the other ranchers can get additional use.

ects (assuming funds were to be invested in one or more of these projects).

Another way of identifying an opportunity rate of interest would be to use the method described in Senate Document 97. This procedure is described in the section on time considerations.¹⁶

For illustrative purposes the interest rate described in Senate Document 97 will be used as the opportunity interest rate. The interest rate used for discounting future returns by those doing benefit-cost analysis is 3% as of July 1964.

Equating the 3% interest rate with the internal rates of return in Table 6, the optimum annual level of investment is \$5,448. At this level of investment, 4,517 acres would be sprayed and 4,023 acres would be seeded to crested wheatgrass. The number of animal units allocated to each rancher is also listed in the lower portion of Table 6.

Model III

The assumption was made in Model I and Model II that the AUM's of grazing from the federal lands should be allocated to each rancher in a fixed ratio. With no hard and fast rules or regulations established by the BLM to cover the allocation of increased grazing, the fixed ratio assumption is believed to be a fair way to allocate the increased grazing on the East Cow Creek allotment. However, it could be argued that this is not a fair way to allocate the grazing. For instance, a rancher who does not have any grazing permits on the public range could argue that he should have the opportunity to graze these lands before those already grazing them are given more grazing privileges. In many areas a very good case could be made for this argument, but in the study area every ranch has a permit to graze the federal range.

A strong point against the fixed ratio assumption is that it may act as an obstacle to maximum economic efficiency in the use of available resources. For example, Rancher II gets such a large share of the increased grazing that he is forced to use his private land resources to the absolute maximum. At the highest levels of public investment it even pays him to take nonuse of federal grazing so that the other ranchers can increase further. On the other hand, Rancher III has resources going unused at most levels of investment because of the fixed ratio restriction. Rancher III does not have permits in other allotments, so all of his private resources can be used in connection with

¹⁶ United States Congress, Senate, The President's Water Resources Council, *Policies, Standards, and Procedures in the Formulation, Evaluation, and Review of Plans for Use and Development of Water and Related Land Resources* (Washington, D. C., May 29, 87th Congress, 2nd Session, Document No. 97, p. 12).

the study allotment. Rancher I has grazing permits in an allotment in Idaho, so nonuse of his resources is not serious. An alternative model was developed under the assumption that the forage from the public range would be allocated to these four ranchers according to their individual profitability.

Results obtained from Model III

Several levels of public investment determined by the parametric program are summarized in Table 7. The MVP of public capital and the average MVP of federal grazing are quite different than in Model II. At most levels of investment these MVP's are substantially higher in Table 7. However, at the highest levels of public investment these MVP's drop off much faster in Model III. The reason for this will be discussed later.

All sprayable federal rangeland is improved before any seeding takes place. One would expect this, since no changes were made in the respective costs or expected yield increases of these improvements. These improvements had a higher net return per dollar invested for most investment levels than was the case in Model II.

The amount of private investment at each level of public investment is much lower in this model, as one can see by comparing Table 7 with Table 3. Rancher II is required to invest far more than the other ranchers; nevertheless, his investment is much less than before. Private investment comes in first at the \$1,989 level of public investment with Rancher II having to invest \$179. It is profitable for Rancher I to start investing in meadow improvements at the \$5,259 level of public investment. Rancher I did not improve any meadow in the other models. Meadow improvement on Ranch III does not come in until public investment gets up to \$6,647. However, Rancher III would improve all of his meadow at the highest level of investment considered in Table 7.

The private investment pattern shown in Table 7 can be explained by the way the ranchers are allocated increased federal grazing. The number of animal units permitted to graze the federal range for each rancher at each level of public investment is shown in Table 8. Initially all of the grazing is allocated to three of the ranchers. Rancher IV has the high-cost operation and does not come into the solution until the other ranchers have used their private resources almost to the limit. As more federal grazing is made available at each level of investment in range improvements, the linear program determines which rancher can make the most profitable use of this forage and allocates it to him. Rancher I can make the most profitable use of the first forage brought about by range improvements on the federal range-

Table 7. GROSS RETURN PER DOLLAR FOR VARIOUS INCREMENTS OF PUBLIC INVESTMENT IN RANGE IMPROVEMENTS¹

Public levels of investment ..	\$0	\$653	\$1,553	\$1,989	\$2,869	\$4,606	\$5,259	\$5,930	\$6,647	\$7,448	\$9,331
MVP public capital	\$ 3.86	3.53	3.49	2.71	2.71	2.33	2.03	2.03	1.77	1.09	1.02
MVP April grazing AUM ..	\$ 8.35	8.63	8.66	8.15	8.15	7.81	6.90	6.90	6.04	3.72	3.58
MVP May grazing AUM	\$13.54	14.00	14.05	13.21	13.21	12.66	11.19	11.19	9.79	6.03	5.80
MVP June grazing AUM	\$ 0	0	0	0	0	0	0	0	0	0	0
MVP July grazing AUM	\$ 7.56	7.82	7.85	7.38	7.38	6.53	5.71	5.71	5.00	3.08	2.91
MVP Aug.-Sept. grazing AUM	\$ 9.67	8.84	8.74	8.23	8.23	7.28	6.44	4.44	5.63	3.47	3.28
Avg. MVP public grazing ...	\$ 8.13	8.02	8.00	7.53	7.53	6.93	6.11	6.11	5.35	3.30	3.14
R II private investment	\$ 0	0	0	179	479	1,058	1,058	1,058	1,058	1,058	3,693
R I private investment	\$ 0	0	0	0	0	0	204	204	204	242	242
R III private investment	\$ 0	0	0	0	0	0	0	0	218	218	688
R IV private investment	\$ 0	0	0	0	0	0	0	0	0	183	183
Total private investment	\$ 0	0	0	179	479	1,058	1,262	1,262	1,480	1,701	4,806
Acres sprayed	23	1,483	3,529	4,517	4,517	4,517	4,517	4,517	4,517	4,517	4,517
Acres seeded	0	0	0	0	1,023	3,043	3,804	4,584	5,419	6,352	8,543

¹ It was assumed that the forage would be utilized by the most profitable ranches. It was also assumed there was no reseeded crested wheatgrass initially.

Table 8. INTERNAL RATES OF RETURN FOR VARIOUS INCREMENTS OF PUBLIC INVESTMENT IN RANGE IMPROVEMENTS FOR MODEL III

Public levels of investment	\$10	\$654	\$1,553.78	\$1,553.81	\$1,989	\$2,870	\$4,146	\$4,606	\$4,688
MVP public capital	\$ 3.86	3.53	3.49	3.28	2.71	2.71	2.51	2.33	2.30
Internal rate of return		31%	27.5%	27%	26%	12.5%	12.5%	11%	10%
Acres sprayed	23	1,488	3,529	3,529	4,517	4,517	4,517	4,517	4,517
Acres seeded	0	0	0	0	0	1,023	2,506	3,043	3,138
Total adjusted ranch income	45,965	48,450	51,590	51,590	53,026	55,408	58,870	60,026	60,216
Allocation of public grazing (head)									
Rancher I	46	106	183	183	183	183	183	183	188
Rancher II	441	441	441	441	479	540	630	660	660
Rancher III	510	510	510	510	510	510	510	510	510
Rancher IV	0	0	0	0	0	0	0	0	0
Levels of investment	\$5,259	\$6,450	\$6,648	\$6,863	\$7,449	\$7,705	\$9,322	\$9,398	
MVP public capital	\$ 2.03	1.89	1.77	1.68	1.09	1.04	1.02	0.79	
Internal rate of return	9.75%	8%	7%	6.25%	5.5%	1%	0.5%	0.25%	
Acres sprayed	4,517	4,517	4,517	4,517	4,517	4,517	4,517	4,517	
Acres seeded	3,804	5,189	5,419	5,670	6,352	6,650	8,543	8,620	
Total adjusted ranch income	61,531	63,945	64,318	64,670	65,687	65,967	67,663	67,731	
Allocation of public grazing (head)									
Rancher I	225	225	225	225	225	237	237	237	
Rancher II	660	660	660	660	660	660	765	769	
Rancher III	510	510	523	523	523	540	540	540	
Rancher IV	0	77	77	77	114	114	114	114	

land. The forage allocated to him increases over the first four levels of investment, while the forage allocated to the other ranchers remains unchanged. As bottlenecks come about in Rancher I's feed program with this increased federal grazing, it becomes more profitable for Rancher II to get the increased forage. At about the \$5,259 level of investment the allocation is again made to Rancher I. Rancher IV is allocated forage for 77 head on the federal range at the \$6,450 level of investment. This shifting allocation pattern continues through the remaining levels of investment.

The method of allocating federal grazing described above explains the private investment pattern in Table 7. As long as the amount of federal grazing allocated to a particular rancher is unchanged, there is no need to change his private investment. Therefore, changes in private investment are tied directly to the way federal grazing is allocated.

Federal range improvement decisions

Internal rates of return were calculated for each level of public investment and summarized in Table 8. When reading Table 8, it should be remembered that the MVP at any particular level of investment holds up to the next investment level. Therefore, the internal rate of return corresponding to a particular MVP is shifted to the right by one column. The internal rate of return of 31% at the \$654 investment level is based on the \$3.86 MVP at the \$10 investment level.

Assuming again the 3% opportunity rate of interest based on Senate Document 97, the optimum level of investment is \$7,449. At this level of investment the internal rate of return is approximately 5.5%. Again the 4,517 acres of sprayable range would be sprayed and 6,352 acres of crested wheatgrass would be seeded. Some 1,329 more acres of crested wheatgrass would be seeded at the optimum level of investment in Model III than for Model II.

At the optimum level of investment, federal forage would be allocated in the following manner: Rancher I could graze 225 animal units, Rancher II could graze 660 animal units, Rancher III could graze 523 animal units, and Rancher IV could graze 114 animal units. As expected, the assumptions of Model III cause a reapportionment of the federal grazing to the four ranchers. Each rancher's relative share of the federal grazing was held constant in Models I and II. These percentages were 6.29, 61.12, 24.37, and 8.22, respectively, for the four ranchers in Model II. At the indicated optimum level of investment in Model III, they are 15%, 43%, 34%, and 8%. Ranchers I and III get a larger share, Rancher II gets a smaller share, and Rancher IV remains in about the same relative position. Some ques-

tion might arise as to the feasibility of allowing Rancher I to increase his relative share as much as indicated above. However, before more could be said about this, additional information would have to be gathered on the rancher's private resources located in Idaho.

In summary, dropping the fixed proportionality assumption has some advantages. The full potential of Rancher III's resources can come into the program. This is important since he has no permits in any other allotment. Another advantage is that the pressure which was put on Rancher II to expand because of the fixed proportionality is eliminated. Rancher I may be overextending his private resources in the study allotment, which is a disadvantage or limitation. This limitation could be remedied with more prior planning in getting private resource inventories. Model III causes a break with the institutional framework developed around federal rangeland use. That is, grazing is allocated on profitability and Rancher IV does not come in for any federal grazing until an annual investment of at least \$6,450 has been made in range improvements. At the optimum level of public investment determined for illustrative purposes by using a 3% interest rate, Rancher IV comes in for about the number of livestock that his private ranch resources can reasonably support.

Model IV

A grass that will furnish more acceptable summer forage would do much to round out seasonal grazing on this allotment. Whitmar beardless wheatgrass is such a grass. No experimental work has been done in the study area with beardless wheatgrass, but it appears to be a feasible alternative that should be considered for the allotment. Whitmar beardless wheatgrass is more difficult to get established and can be damaged more easily by improper grazing than crested wheatgrass. Because of these difficulties, it was assumed that only the best 5,000 acres of the 13,372 acres of seedable rangeland would be adaptable to Whitmar beardless wheatgrass seedings. It was further assumed that even on these best sites, Whitmar beardless wheatgrass would cost \$2.50 more per acre for seed and require one more year of nonuse than crested wheatgrass. The seeding cost for Whitmar beardless wheatgrass was \$1.00 per acre per year, as shown in Table 9. (Seeding cost for crested wheatgrass was \$0.86 per acre per year.)

Whitmar beardless wheatgrass would extend the higher quality grazing season on the federal land; thus, it would not be necessary to bring the salable livestock off the federal range in August to get the weight gain needed for the higher priced livestock activities. Some pressure would be taken off the private grazing as Whitmar beardless

Table 9. COST ESTIMATES FOR SEEDING WHITMAR WHEATGRASS

<i>Initial costs:</i>	
Plowing and drilling	\$12.21
Fencing99
Water development	2.20
Nonuse	1.00
	\$16.40
<i>Annual costs:</i>	
Fence maintenance	\$.08
Water maintenance and use10
	\$.18
<i>20-year life of the seeding:</i>	
\$16.40	
$\frac{\quad}{20} = \$0.82 + \$0.18 = \$1.00$ per acre per year	

wheatgrass replaced the aftermath grazing furnished by the private meadows. The aftermath grazing could then be used to provide forage for October 1 to November 20 grazing.¹⁷

Results obtained from Model IV

Some general trends in resource use are presented in Table 10, which is much like the seasonal use pattern tables discussed for the earlier models. Since some of the results have been discussed before, only the differences will be mentioned. The MVP's for public capital remain fairly high at large annual investments, such as \$1.23 at \$11,-100 annual investment. Private investment is the variable that shows the greatest change in this model. For Ranchers I and II it seems quite erratic—going up, then going back down as public investment increases. At the lower levels of public investment, Whitmar beardless wheatgrass comes in to furnish July grazing. Another interesting fact to note is that Rancher III is the only one who has to invest in meadow improvement for hay production.

Spraying, as in the other models, is the best alternative use of public capital. As soon as all of the sprayable rangeland is sprayed, Whitmar beardless wheatgrass seedings come in. After about 2,500 acres of Whitmar beardless wheatgrass are seeded, crested wheatgrass comes in, and they complement each other at the higher levels of investment.

¹⁷ For a detailed explanation of the livestock activities and grazing seasons, see Darwin B. Nielsen, *op. cit.*

Table 10. GROSS RETURN PER DOLLAR FOR VARIOUS INCREMENTS OF PUBLIC INVESTMENT IN RANGE IMPROVEMENTS¹

Levels of investment	\$10	\$1,989	\$4,020	\$4,951	\$6,113	\$6,878	\$11,110	\$11,340	\$11,348
MVP public capital	\$ 3.86	3.24	3.03	2.07	1.94	1.33	1.23	1.15	.93
MVP April grazing	\$ 8.35	8.15	8.43	7.07	6.60	4.54	4.30	4.02	3.72
MVP May grazing	\$13.54	13.21	13.67	11.47	10.70	7.35	6.97	6.52	7.95
MVP June grazing	\$ 0	0	0	0	0	0	0	0	0
MVP July grazing	\$ 7.56	7.38	7.06	5.08	4.74	3.25	3.03	3.26	3.06
MVP Aug.-Sept. grazing	\$ 9.67	8.22	7.86	6.60	6.15	4.23	3.94	3.68	3.45
Avg. MVP public grazing	\$ 7.89	7.42	7.42	6.06	5.65	3.89	3.66	3.50	3.64
R I private investment	\$ 0	0	0	242	242	242	0	0	0
R II private investment	\$ 0	180	1,005	1,058	1,058	1,058	270	270	270
R III private investment	\$ 0	0	0	0	0	0	218	390	396
R IV private investment	\$ 0	0	0	0	0	183	183	183	183
Acres sprayed	23	4,517	4,517	4,517	4,517	4,517	4,517	4,517	4,517
Acres seeded (crested)	0	0	0	477	1,670	2,455	4,884	5,017	5,078
Acres seeded (Whitmar)	0	0	2,033	2,557	2,695	2,786	4,921	5,000	5,000

¹ It was assumed that the forage would be utilized by the most profitable ranches, with Whitmar wheatgrass seeding considered. It was also assumed that initially there was no reseeded crested wheatgrass.

Federal range improvement decisions

Over 20 parametric changes were required to get the MVP of public capital below one dollar. Nineteen of these levels of public investment are presented in Table 11. The internal rates of return at the lower levels of investment follow a pattern very much like the pattern in Model III. One of the most significant points brought out in Table 11 is the fact that Whitmar beardless wheatgrass comes in at an internal rate of return of 16%. Crested wheatgrass does not come in until the internal rate of return gets down to about 13%. Whitmar beardless wheatgrass comes into the solution at a higher internal rate of return than crested wheatgrass in any of the previous models.

If the Senate Document 97 alternative rate of interest of 3% is again used, the optimum level of annual public investment is \$6,878. At the optimum level of investment, 2,786 acres of Whitmar beardless wheatgrass and 2,455 acres of crested wheatgrass would be seeded. This would indicate that the public land managers have already seeded too much crested wheatgrass on the allotment. It was assumed in Model IV that there was no crested wheatgrass already on the allotment, but 3,874 acres were seeded prior to this study.

Two additional models were developed to consider improvements on private rangeland simultaneously with the improvements discussed in the original models. However, because of the size of these models, a different computer had to be used. The larger computer did not have a parametric changer, so the information gained was limited. These models will not be presented in this bulletin.¹⁸

Implication and Additional Uses of Results of the Study

At various places throughout this bulletin the usefulness of the information gained from the different models has been mentioned. Some of the uses were discussed in considerable detail; for example, the use of the MVP's of public capital as the crucial factor in determining the internal rates of return. Internal rates of return were used as decision indicators to be compared with the appropriate rate of interest of the particular decision-maker.

The weighted average MVP of federal grazing is an important variable to know when the problem of setting grazing fees comes up. Even if the goal of the government land agency is something other than maximization of returns from these lands, these MVP's provide estimates of what the federal range resource is returning to society.

¹⁸ These models are presented and explained by Darwin B. Nielsen, *op. cit.*

Table 11. INTERNAL RATES OF RETURN FOR VARIOUS INCREMENTS OF PUBLIC INVESTMENT IN RANGE IMPROVEMENTS FOR MODEL IV

Levels of investment	\$10	\$653	\$1,553	\$1,989	\$2,828	\$4,019	\$4,159	\$4,463	\$4,739	\$4,951
MVP public capital	\$ 3,86	3.55	3.28	3.24	3.24	3.03	2.81	2.35	2.09	2.08
Internal rate of return		31.0%	28.0%	26.0%	16.0%	16.0%	14.0%	13.0%	10.0%	8.5%
Acres sprayed	23	1,488	3,529	4,517	4,517	4,517	4,517	4,517	4,517	4,517
Acres crested wheatgrass	0	0	0	0	0	0	0	282	477	477
Acres Whitmar seeded	0	0	0	0	839	2,033	2,173	2,479	2,536	2,557
Total adjusted ranch income	15,965	48,449	51,590	53,020	55,737	59,603	60,026	60,881	61,530	63,334
Animal units of federal grazing										
Rancher I	46	106	183	183	183	183	183	207	225	237
Rancher II	441	441	441	479	549	649	660	660	660	660
Rancher III	510	510	510	510	510	510	510	510	510	510
Rancher IV	0	0	0	0	0	0	0	0	0	0
Levels of investment	\$6,113	\$6,306	\$6,878	\$10,026	\$10,624	\$11,100.47	\$11,100.58	\$11,337	\$11,384	
MVP public capital ..	1.94	1.73	1.33	1.31	1.29	1.26	1.23	1.15	.93	
Internal rate of return	8.25%	7.75%	7.0%	2.8%	2.75%	2.5%	2.25%	2.0%	1.5%	
Acres sprayed	4,517	4,517	4,517	4,517	4,517	4,517	4,517	4,517	4,517	
Acres crested wheatgrass	1,670	1,868	2,455	4,248	4,600	4,884	4,884	5,068	5,078	
Acres Whitmar seeded	2,695	2,718	2,786	4,393	4,689	4,921	4,921	4,982	5,000	
Total adjusted ranch income	64,388	64,762	65,749	69,943	70,726	71,342	71,342	71,633	71,646	
Animal units of federal grazing										
Rancher I	237	237	237	237	252	264	264	264	264	
Rancher II	660	660	660	736	736	736	736	736	736	
Rancher III	510	523	523	523	523	523	523	533	534	
Rancher IV	77	77	114	114	114	114	114	114	114	

The point here is not to argue that grazing fees should be higher, but to show that knowing the MVP's of grazing is important. These MVP's give an indication of the value of federal range as measured through livestock use. If and when tools of analysis are applied that will yield comparable estimates of the value of the federal range for the competing uses, these values could then be used to help determine the allocation of the federal range between uses.

The hypothesis can be tested that the difference between the value of federal range to the rancher (MVP) and the grazing fee has been capitalized into the value of the commensurate property and/or the value of the grazing permits. The MVP's estimated by this study could be used to test this hypothesis.

For example, assume the weighted average MVP for federal grazing over a period of years is \$5.00, the grazing fee is \$.30, other associated costs of grazing are \$1.00, and the interest rate is 5.5%. The value of the grazing permit on an AUM basis would be as follows:

$$\begin{aligned} \$5.00 - \$.30 - \$1.00 &= \$3.70/\text{AUM}; \\ \$3.70 \text{ capitalized at } 5.5\% &= \$67.27/\text{AUM}. \end{aligned}$$

For a five-month grazing season, a permit would be worth about \$336. It is very doubtful that any grazing permits have sold for this price. Of course, the above situation is strictly hypothetical and would require much more thought and investigation than it was afforded here.

Many times public land administrators would like to have estimates of the productive value of the lands under their direction. The MVP's computed in the models of this study can be best used to estimate the productive value of these lands for grazing. For example, with an MVP of grazing of \$5.00 and 5.0 acres required per AUM, an estimate of the value of this range would be as follows:

$$\begin{aligned} \$5.00 \text{ capitalized at } 5.5\% &= \$90.90/\text{AUM}; \\ \$90.90 \div 5.0 &= \$18.18 \text{ per acre.} \end{aligned}$$

Using the MVP's to estimate productive values of federal rangeland can provide valuable information for use in land trades and/or land sales.

Changes in one or more of the coefficients could be traced through to see how they would affect the results obtained from the model. Information could be gained concerning the degree to which changes in the coefficients change the solution. This procedure would give insight to areas where more physical-biological research is needed. The ideal situation would be one where the results obtained from the solutions could be taken out and tested under actual range conditions. The land-use patterns could be tried to see if they were feasible under the gen-

eral open range conditions found in the West. It might be that the models where the seasonal use patterns are broken down into single months would indicate land-use patterns that would require excessive amounts of fencing for handling the livestock. If the plans developed from the models could be tried under actual range conditions, acreages per AUM for the various seasonal use of the different types of rangeland could be tested. All of these things would help in checking the validity of the assumptions of the models.

The feasibility of new range improvement practices could be checked by using models similar to Model IV, where the feasibility of Whitmar beardless wheatgrass seedings was investigated. Before public capital is invested in a new improvement practice, the proposed improvement practice could be worked out in one of the models. Knowledge could be gained concerning the relative profitability and the way the proposed improvement would fit into the overall grazing plan. By using this type of analysis, the decision-maker would have some idea of the effect of a proposed improvement practice without having to make a large investment in the practice.

The relative profitability of different improvement practices is brought out quite well in the models developed. Spraying consistently turned out to be the most profitable use of public funds, given the alternative improvement practices considered in the study. Also, a method of determining the optimum level of public investment has been described.

It is possible that a dynamic linear programming model or a dynamic programming model might be developed where the optimum time to spray a seeding for brush invasion could be determined. However, this approach was not successful in this study.

A few uses of linear programming models, such as the ones developed in this study, have been presented. Many of the uses would call for new research projects to get data needed to test the validity of the proposed uses. Nevertheless, it is believed that linear programming, properly used, is a tool that offers a great deal as a decision-making aid to the government land management agencies.

APPENDIX

Appendix Table 1. EQUATIONS OF LINEAR PROGRAMMING MODEL II¹

Equation number	Resources	Unit	B _i 's	
1	Reseeded range	Acres	1	$\cong 7.0X_{21} + 2.0X_{22} + 2.3X_{23} + 2.6X_{24}$
2	Reseeded range	Acres	13,372	$\cong 9.5X_1 + 7.7X_2 + 8.0X_3 + 7.0X_4 + 10X_5 + 11X_6 + 6.01X_{25} + 2.0X_{26} + 2.31X_{27} + 2.61X_{28}$
3	Sprayable range	Acres	4,517	$\cong 6.0X_7 + 10.6X_8 + 5.0X_9 + 5.0X_{10} + 3.0X_{16} + 5.3X_{20} + 2.5X_{21} + 2.8X_{22}$
4	Other (good) range	Acres	1,034	$\cong 6.02X_{11} + 10.61X_{12} + 5.01X_{13} + 5.51X_{14}$
5	Other (poor) range	Acres	22,117	$\cong 9.51X_{15} + 7.71X_{16} + 8.01X_{17} + 7.01X_{18} + 10.01X_{19} + 11.01X_{20}$
6	Public capital	Dollars	10	$\cong 5.17X_{25} + 1.73X_{26} + 1.99X_{27} + 2.24X_{28} + 1.32X_{29} + 2.33X_{30} + 1.11X_{31} + 1.23X_{32}$
7	Meadow R #1 ²	Acres	270	$\cong 1.0X_{33} + 1.0X_{34}$
8	Capital R #1	Dollars	0	$\cong -1.0X_{36}$
9	April 1-May 1 R #1	AUM's	0	$\cong -0.0629X_1 - 0.0629X_7 - 0.0629X_{11} - 0.0629X_{15} - 0.0629X_{20} + 1.9X_{46} + 1.9X_{50}$
10	May 1-June 1 R #1	AUM's	0	$\cong -0.03145X_2 - 0.021X_3 - 0.03145X_{10} - 0.021X_{17} - 0.03145X_{21} - 0.0629X_{25} + 1.9X_{46} + 1.9X_{50}$
11	June 1-July 1 R #1	AUM's	0	$\cong -0.03145X_2 - 0.021X_3 - 0.0629X_4 - 0.0629X_8 - 0.03145X_{16} - 0.021X_{17} - 0.0629X_{18} - 0.03145X_{21} - 0.0629X_{22} - 0.0629X_{26} - 0.0629X_{30} + 1.9X_{46} + 1.9X_{50}$
12	July 1-Aug. 1 R #1	AUM's	0	$\cong -0.021X_3 - 0.0629X_5 - 0.0629X_9 - 0.0629X_{13} - 0.021X_{17} - 0.0629X_{19}$

13	Aug. 1-Oct. 1 R #1	AUM's	0	$\cong -0.0629X_6 - 0.0629X_{10} - 0.0629X_{14} - 0.0629X_{20} - 0.0629X_{24} - 0.0629X_{28} - 0.0629X_{32} + 2.28X_{46} + 3.8X_{50}$
14	Oct. 1-Nov. 20 R #1	AUM's	270.5	$\cong -3.16X_{42} + 3.16X_{46} + 3.16X_{50}$
15	Hay R #1	Tons	0	$\cong -1.0X_{32} - 1.75X_{34} + 1.94X_{46} + 1.94X_{50}$
16	Meadow R # 2	Acres	923	$\cong +1.0X_{35} + 1.0X_{36} + 1.0X_{51}$
17	Capital R #2	Dollars	0	$\cong + 8.6X_{56} + 8.60X_{51} - 1.0X_{57}$
18	April 1-May 1 R #2	AUM's	0	$\cong -0.6112X_1 - 0.6112X_7 - 0.6112X_{11} - 0.6112X_{15} - 0.6112X_{29} + 1.65X_{47} + 1.65X_{51}$
19	May 1-June 1 R #2	AUM's	0	$\cong -0.3056X_2 - 0.2037X_9 - 0.3056X_{16} - 0.2037X_{17} - 0.3056X_{21} - 0.6112X_{25} + 1.65X_{47} + 1.65X_{51}$
20	June 1-July 1 R #2	AUM's	0	$\cong -0.3056X_2 - 0.2037X_3 - 0.6112X_4 - 0.6112X_8 - 0.6112X_{12} - 0.3056X_{16} - 0.2037X_{17} - 0.6112X_{18} - 0.3056X_{21} - 0.6112X_{22} - 0.6112X_{26} - 0.6112X_{30} + 1.65X_{47} + 1.65X_{51}$
21	July 1-Aug. 1 R #2	AUM's	0	$\cong 0.2037X_3 - 0.6112X_5 - 0.6112X_9 - 0.6112X_{13} - 0.2037X_{17} - 0.6112X_{19} - 0.6112X_{29} - 0.6112X_{27} - 0.6112X_{31} + 1.65X_{47} + 1.65X_{51}$
22	Aug. 1-Oct. 1 R #2	AUM's	0	$\cong -0.6112X_6 - 0.6112X_{10} - 0.6112X_{14} - 0.6112X_{20} - 0.6112X_{24} - 0.6112X_{28} - 0.6112X_{32} + 1.98X_{47} + 3.30X_{51}$
23	Oct. 1-Nov. 20 R #2	AUM's	474	$\cong -2.75X_{43} + 2.75X_{47} + 2.75X_{51} - 4.38X_{54}$
24	Hay R #2	Tons	0	$\cong -1.0X_{35} - 1.75X_{36} + 2.0X_{47} + 2.0X_{51}$
25	Meadow R #3	Acres	80	$\cong +1.0X_{37} + 1.0X_{38}$
26	Capital R #3	Dollars	0	$\cong + 8.60X_{28} - 1.0X_{55}$
27	April 1-May 1 R #3	AUM's	0	$\cong -2.437X_1 - 2.437X_7 - 2.437X_{11} - 2.437X_{15} - 2.437X_{29} + 1.2X_{48} + 1.2X_{52}$
28	May 1-June 1 R #3	AUM's	0	$\cong -1.2185X_2 - 0.812X_3 - 1.2185X_{16} - 0.812X_{17} - 1.2185X_{21} - 2.437X_{23} + 1.2X_{53} + 1.2X_{52}$

Appendix Table 1. EQUATIONS OF LINEAR PROGRAMMING MODEL II¹—continued

Equation number	Resources	Unit	B ₁ 's	
29	June 1-July 1 R #3	AUM's	0	$\cong -0.12185X_2 - 0.0812X_3 - 0.2437X_4 - 0.2437X_5 - 0.2437X_{12} - 0.12185X_{16} - 0.0812X_{17} - 0.2437X_{18} - 0.12185X_{21} - 0.2437X_{22} - 0.2437X_{26} - 0.2437X_{30} + 1.2X_{48} + 1.2X_{52}$
30	July 1-Aug. 1 R #3	AUM's	0	$\cong -0.0812X_3 - 0.2437X_5 - 0.2437X_9 - 0.2437X_{13} - 0.0812X_{17} - 0.2437X_{19} - 0.2437X_{23} - 0.2437X_{27} - 0.2437X_{31} + 1.2X_{48} + 1.2X_{52}$
31	Aug. 1-Oct. 1 R #3	AUM's	0	$\leq -0.2437X_6 - 0.2437X_{10} - 0.2437X_{14} - 0.2437X_{20} - 0.2437X_{24} - 0.2437X_{28} - 0.2437X_{32} + 1.44X_{48} + 2.4X_{52}$
32	Oct. 1-Nov. 20 R #3	AUM's	209	$\cong -2.00X_{34} + 2.00X_{48} + 2.00X_{52}$
33	Alfalfa R #3	Acres	343	$\cong + 1.0X_{29}$
34	Hay R #3	Tons	0	$\cong -1.0X_{37} - 1.75X_{38} - 2.0X_{39} + 1.8X_{48} + 1.8X_{52}$
35	Meadow R #4	Acres	175	$\cong -1.0X_{36} + 1.0X_{41} + 1.0X_{55}$
36	Capital R #4	Dollars	0	$\cong + 8.60X_{41} + 8.60X_{55} - 1.0X_{59}$
37	April 1-May 1 R #4	AUM's	0	$\cong -0.0822X_7 - 0.0822X_7 - 0.0822X_{11} - 0.0822X_{15} - 0.0822X_{20} + 1.3X_{49} + 1.3X_{53}$
38	May 1-June 1 R #4	AUM's	0	$\cong -0.0411X_2 - 0.0274X_3 - 0.0411X_{16} - 0.0274X_{17} - 0.0411X_{21} - 0.0811X_{25} + 1.3X_{49} + 1.3X_{53}$
39	June 1-July 1 R #4	AUM's	0	$\cong -0.0411X_2 - 0.0274X_3 - 0.0822X_4 - 0.0822X_8 - 0.0822X_{12} - 0.0411X_{16} - 0.0274X_{17} - 0.0822X_{18} - 0.0411X_{21} - 0.0822X_{22} - 0.0822X_{26} - 0.0822X_{30} + 1.3X_{49} + 1.3X_{53}$
40	July 1-Aug. 1 R #4	AUM's	0	$\cong -0.0274X_3 - 0.0822X_5 - 0.0822X_9 - 0.0822X_{13} - 0.0274X_{17} - 0.0822X_{19} - 0.0822X_{23} - 0.0822X_{27} - 0.0822X_{31} + 1.3X_{49} + 1.3X_{53}$
41	Aug. 1-Oct. 1 R #4	AUM's	0	$\cong -0.0822X_6 - 0.0822X_{10} - 0.0822X_{14} - 0.0822X_{20} - 0.0822X_{24} - 0.0822X_{28} - 0.0822X_{32} + 1.56X_{49} + 2.6X_{53}$

42	Oct. 1-Nov. 20 R #4	AUM's	72	$\cong -2.17X_{45} + 2.17X_{46} + 2.17X_{53} - 4.38X_{55}$
43	Hay R #4	Tons	0	$\cong 1.0X_{46} - 1.75X_{41} + 1.75X_{47} + 1.75X_{53}$
44	Rancher #1—aftermath	AUM's	180	$\cong + 3.16X_{42} + 1.52X_{46}$
45	Rancher #2—aftermath	AUM's	615	$\cong + 2.75X_{43} + 1.32X_{47}$
46	Rancher #3—aftermath	AUM's	1,082	$\cong + 2.00X_{44} + .96X_{45}$
47	Rancher #4—aftermath	AUM's	117	$\cong + 2.17X_{45} + 1.04X_{49}$
				Max. = $78X_{46} + 72.5X_{47} + 58X_{48} + 41X_{49} + 68X_{50} + 62.5X_{51}$ $+ 48X_{52} + 31X_{53} - 1.10X_{56} - 1.10X_{57} - 1.10X_{58} - 1.10X_{59}$

¹ Variables are defined as the following:
 X_1 is one AUM of grazing for the period April 1 to May 1.

X_2 —May 1-July 1
 X_3 —May 1-August 1
 X_4 —June 1-July 1
 X_5 —July 1-August 1
 X_6 —August 1-October 1
 X_7 —April 1-May 1
 X_8 —June 1-July 1
 X_9 —July 1-August 1
 X_{10} —August 1-October 1
 X_{11} —April 1-May 1
 X_{12} —June 1-July 1
 X_{13} —July 1-August 1
 X_{14} —August 1-October 1
 X_{15} —April 1-May 1
 X_{16} —May 1-July 1
 X_{17} —May 1-August 1
 X_{18} —June 1-July 1
 X_{19} —July 1-August 1
 X_{20} —August 1-October 1
 X_{21} —May 1-July 1

X_{22} —June 1-July 1
 X_{23} —July 1-August 1
 X_{24} —August 1-October 1
 X_{25} —May 1-June 1
 X_{26} —June 1-July 1
 X_{27} —July 1-August 1
 X_{28} —August 1-October 1
 X_{29} —April 1-May 1
 X_{30} —June 1-July 1
 X_{31} —July 1-August 1
 X_{32} —August 1-October 1
 X_{33} —Hay (1 ton meadow hay)
 X_{34} —60# N (1.75 tons meadow hay)
 X_{35} —Hay (1 ton meadow hay)
 X_{36} —60# N (1.75 tons meadow hay)
 X_{37} —Meadow hay (1 ton meadow hay)
 X_{38} —60# N (1.75 tons meadow hay)
 X_{39} —Alfalfa hay (2 tons)
 X_{40} —Hay (1 ton meadow hay)

X_{41} —60# N (1.75 tons meadow hay)
 X_{42} —R #1-Oct. 1-Nov. 20 (AUMs aftermath grazing)
 X_{43} —R #2-Oct. 1-Nov. 20 (AUMs aftermath grazing)
 X_{44} —R #3-Oct. 1-Nov. 20 (AUMs aftermath grazing)
 X_{45} —R #4-Oct. 1-Nov. 20 (AUMs aftermath grazing)
 X_{46} —R #1—Cow-calf-yearling
 X_{47} —R #2—Cow-calf-yearling
 X_{48} —R #3—Cow-calf
 X_{49} —R #4—Cow-calf
 X_{50} —R #1—Cow-calf-yearling
 X_{51} —R #2—Cow-calf-yearling
 X_{52} —R #3—Cow-calf
 X_{53} —R #4—Cow-calf
 X_{54} —R #2—Hay fed Oct.-Nov.
 X_{55} —R #4—Hay fed Oct.-Nov.
 X_{56} —R #1—Private capital at 10% interest
 X_{57} —R #2—Private capital at 10% interest
 X_{58} —R #3—Private capital at 10% interest
 X_{59} —R #4—Private capital at 10% interest

² R #1 refers to the operation of ranch 1; R #2 refers to the operation of ranch 2; R #3 refers to the operation of ranch 3; and R #4 refers to the operation of ranch 4.

Appendix Table 2. LIST OF COMMON AND TECHNICAL NAMES OF PLANTS MENTIONED IN THE TEXT

Grasses	
<i>Technical name</i>	<i>Common name</i>
1. <i>Agropyron desertorum</i> (Fisch.) Schult.	Desert wheatgrass or crested wheatgrass
2. <i>A. inerme</i> (Scribn. and Smith Rydb.)	Beardless wheatgrass
3. <i>A. spicatum</i> (Pursch.)	Bluebunch wheatgrass
4. <i>Bromus tectorum</i> (L.)	Cheatgrass (downy brome)
5. <i>Festuca idahoensis</i> (Elmer)	Idaho fescue
6. <i>Poa secunda</i> (Presl.)	Sandberg bluegrass
Shrubs	
1. <i>Artemesia arbuscula</i> (Nutt.)	Low sagebrush
2. <i>A. tridentata</i>	Big sagebrush
3. <i>Chrysothamnus nauseosus</i> (Pall. Britt.)	Grey rabbitbrush
4. <i>C. visidiflorus</i> (Hook.) (Nutt.)	Green rabbitbrush