Material failures in heat exchangers are often closely tied to events associated with the conditions of service and operating parameters. These events can generally be attributed to adverse load application and higher than optimum operating temperatures that could lead to changes in the microstructure of the materials and fatigue failure of the component. However, fatigue failure in heat exchangers is usually associated with the presence of a biaxial stress condition. Two non-parallel forces create a two-dimensional stress field at the free surface of the structural element where the process and mechanism of fatigue failure normally initiate.

An experimental investigation was conducted to evaluate the biaxial fatigue behavior of commercially pure titanium Ti-50A (Grade 2) and low-alloy titanium
Ti-Code 12 (Grade 12) heat exchanger materials. The biaxial state of stress was composed of an axial stress and a superimposed torsional stress, applied in a thin-wall tubular specimen machined from titanium tubing. Torsional stress was applied independently using a torsion machine and a torque fixer assembly devised as part of this study. After applying the desired torsion, the torsionally stressed specimen was mounted on a closed-loop electrohydraulic machine for the application of axial cyclic loading. A minimum of four tests were conducted for each of three alternating stress levels at both high and low torsional stresses. The biaxial fatigue test under load control condition was done under fully reversed cycles equivalent to a biaxiality ratio of -1. These test parameters were determined from an analytical formulation based on Mohr's circle.

The results are presented in terms of the various measured or calculated quantities versus number of cycles to fracture. Biaxial fatigue curves were drawn through the experimental points corresponding to Weibull's mean life criterion. The four data points exhibit scatter that appears to be related to the applied stress amplitude. It was also found that a correlation exists between the magnitude of applied cyclic biaxial stress and fatigue life to failure. In addition, the results have been discussed taking existing failure criteria into account.
Biaxial Fatigue Behavior of Commercially Pure Titanium Ti-50A (Grade 2) and Low-Alloy Titanium Ti-Code 12 (Grade 12) Heat Exchanger Materials

by

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\( \sigma_{a_1}, \sigma_{a_2}, \sigma_{a_2} \)  
Alternating components of the principal stresses.

\( (\tau)_A, (\tau)_B \)  
Two torsional stresses designated as A and B as visualized in Mohr's circle with \((\tau)_A < (\tau)_B\).

\( (\tau_{wx}) = (\tau) \)  
Applied torsional stress

\( \tau \)  
Shear stress, ordinate in Mohr's circle

\( \tau_{\text{max}} \)  
Maximum shear stress

\( \tau_a \)  
Alternating shear stress

\( \tau_{xy} \)  
Shear stress in x-y

\( \alpha \)  
Alpha phase, solid solution in Ti-Mo phase.

\( \alpha_{\text{NMM}} \)  
Coefficient in Naomi-Masao-Masateru Theory

\( \theta \)  
Angular rotation of stressed element.

\( \theta_{\text{max}} \)  
Rotation of stressed element corresponding to axial load in tension.

\( \theta_{\text{min}} \)  
Rotation of stressed element corresponding to axial load in compression.

\( \overline{\theta}, \overline{\theta}_D, \overline{\theta}_t \)  
Angle of twist in torsion test, subscripts D and t correspond to disk and tube respectively.

\( \gamma \)  
Torsional strain

\( \mu \)  
Weibull's mean life

\( \mu_L \)  
Lod's parameter

\( \Gamma \)  
Gamma function obtained from mathematical table.
<table>
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<th>Symbol</th>
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<tr>
<td>$f(n)$</td>
<td>Weibull's probability density function</td>
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<td>$F(N)$</td>
<td>Weibull's probability cumulative distribution function</td>
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<td>$b$</td>
<td>Weibull's slope parameter</td>
</tr>
<tr>
<td>SD</td>
<td>Weibull's standard deviation</td>
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<tr>
<td>$N$</td>
<td>Specimen life</td>
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<tr>
<td>$Na$</td>
<td>Characteristic life parameter</td>
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<td>$No$</td>
<td>Minimum life parameter</td>
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<tr>
<td>$N_f$</td>
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<td>$N_f^{BF}$</td>
<td>Number of cycles to fracture for best fit.</td>
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<td>$\bar{S}_e$</td>
<td>Von-Mises Equivalent Stress</td>
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<tr>
<td>$\bar{S}_{eVM}$</td>
<td>Von-Mises equivalent stress made compatible with the applied biaxial stress.</td>
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<td>$\bar{S}_{eS}$</td>
<td>Sine's equivalent stress as applied corresponding to biaxial stress.</td>
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<td>$\Delta S_{NMM}$</td>
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<td>$\bar{S}_{eMVM}$</td>
<td>Modified Von-Mises Equivalent Stress</td>
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<td>$D$</td>
<td>Tube outside diameter</td>
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<tr>
<td>$d$</td>
<td>Mean linear intercept grain size</td>
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<tr>
<td>$L$</td>
<td>Effective tube length</td>
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<tr>
<td>LVDT</td>
<td>Linear Variable Differential Transformer, a transducer.</td>
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<tr>
<td>MTS</td>
<td>Trademark of Fatigue Testing Machine</td>
</tr>
</tbody>
</table>
Nomenclature -- Continued 3

\begin{align*}
\text{P} & \quad \text{Applied axial load} \\
\text{P}_{\text{max}} & \quad \text{Axial load in tension} \\
\text{P}_{\text{min}} & \quad \text{Axial load in compression} \\
\text{R} & \quad \text{Stress ratio} = \frac{\text{cmin}}{\text{cmax}} \\
\text{R}_t & \quad \text{Effective tube radius} \\
\text{R}_D & \quad \text{Disk radius} \\
\text{S}_D & \quad \text{Torsional displacement measured through LVDT} \\
\text{S}_{\text{ult}} & \quad \text{Ultimate tensile strength} \\
\text{S}_Y & \quad \text{Yield strength at 0.2\% offset} \\
\text{T} & \quad \text{Applied torsional load} \\
\text{t} & \quad \text{Wall thickness in gauge section}
\end{align*}
The design and construction of heat exchangers pose a challenge on the part of the material used in relation to its suitability and reliability under different service conditions. Metals and alloys that are used in heat exchanger applications have properties that can be utilized in certain situations but none is suitable for all design conditions. Moreover, while there is no general rule for the selection of heat exchanger materials, the possibility of fatigue failure is always an important consideration in the design process.

Failures in heat exchangers are found to occur either in the tube itself or in the tube-to-tube sheet connection. These failures could be attributed to inadequacies in the design and construction of the components and also to the behavior of the materials used when subjected to environmental and service conditions that often include biaxial states of stress.

The materials considered in this study are Ti-50A (Grade 2) and Ti-Code 12 (Grade 12) titanium supplied to ASTM B338-78 standards by the TIMET COMPANY, a titanium manufacturer in Irvine, California. These materials are
now commonly used in the industry; however, at this
time, there does not appear to be any information
published with regard to their behavior under biaxial
fatigue conditions.

This study has utilized a thin-wall tubular
specimen machined from the titanium tubing supplied by
TIMET. This specimen geometry is generally considered
appropriate for biaxial fatigue studies. In this ex-
perimental investigation, the biaxial state of stress
has been imposed by a combination of a predetermined
level of torsion and superimposed axial push-pull cycling.

A torsional moment is first applied to the
specimen with the use of a torsion machine. The applied
torsional stress is prevented from relaxing by mounting
a specially designed torque fixer assembly on to the
specimen. The torsionally stressed specimen is then
mounted on the MTS machine for axial push-pull cycling.
A constant maximum stress amplitude with sinusoidal
wave form has been used in this investigation.

A biaxiality ratio of -1, as modelled by the state
of stress is visualized and represented in Mohr's circle
has been used in this study. Moreover, from the biaxial
fatigue model used, equivalent expressions have been
formulated for the torsional stress as a function of the
applied test parameters. These tests were conducted
under fully reversed biaxial fatigue conditions at three
levels of constant maximum stress amplitude. Four tests were carried out at each level until complete fracture of the specimens occurred.

A statistical analysis of confidence limits requires a minimum test of four specimens at each level of stress amplitude. Weibull's statistical terms have been used in this study to define a qualitative \((\sigma - N_f)\) curve.

Two torsional stress levels have been used in this investigation in order to determine the effect of this variable on the biaxial fatigue behavior of the materials being tested.

The subject of biaxial stress failure criteria has recently received considerable attention. While there are several references in the literature that support these various theories, there is still no general agreement on their validity for general application. In light of this situation, an experimental approach has been adopted in this study for assessing the biaxial fatigue behavior of materials that are currently used in heat exchanger applications.

The purpose of this study was to obtain some information with regard to the biaxial fatigue properties of titanium materials and to determine whether the existing failure criteria are applicable under these conditions of loading. Correlation of the experimental data with the
failure criteria proposed in the literature shows the existence of some discrepancies between the two. It is felt that further research is necessary in order to better interpret biaxial fatigue data and to form the basis of a theory that would be more directly applicable in design.
II. BACKGROUND

A. Heat Exchanger Failure Attribution and Problem Areas

Heat exchanger tube materials in the power generation industry are always likely to encounter some operational problems which are bound to create unexpected difficulties. Even the most corrosion-resistant materials can experience leakage, either after unit startup or after years of operation. Most recently, David F. Goetcheus [1] while discussing some of the problems has indicated that a tube system designed to handle a circulating water flow rate of 6 ft/sec will encounter an inlet and erosion problem if improper inlet water box design gives rise to a localized turbulence. On the other hand, heat exchangers designed for 90°F cooling water and water velocities in the tube of 7 ft/sec may be throttled to 2 ft/sec when the cooling water temperature decreases to 45°F, resulting in tube leakage after three months of winter operation. In addition, commonly used in raw cooling water systems could result in blockage due to the small size tube becoming filled with debris [1]. In essence, the tube materials for the heat exchanger will perform as intended and tubing manufacturers will supply the tubes as specified. However, operational problems are always likely to occur
and ASME and ASTM specifications may not be entirely adequate to ensure trouble free operation on a long term basis.

In some cases failures in heat exchangers have been caused by vibration of the components, particularly the tubes, which led to fatigue failure [2]. On the other hand, in the case of heat exchangers with tubes in cross flow, the unsteady flow fluctuations were believed to give rise to tube vibrations [3].

Failure due to fatigue has been reported in shell and tube type heat exchangers [4]. The fatigue fracture was characterized by cracking due primarily to repeated vibration and other cyclic stressing of the tubes. This was induced by excessive vibration in adjacent rotating machinery, pulsing from a pump, and by fluid forces. The cracks were found to occur adjacent to the support plate and often midway in the span between support plates.

Figure 1 shows a typical shell and tube type of heat exchanger diagram indicating the relative position of its components. Failures have occurred in tube-to-tube sheet joints. Studies made by investigators in this field indicate that the actual stresses which the "U"-tube heat exchanger tube must withstand were due to tube deflection under pressure loading. The thermal stress was caused by thermal gradients and the axial
stress was caused by the pressure inside the tube [5]. However, in modern heat exchanger design and technology, it is now possible to control the thermal gradient effects by relieving the induced thermal stresses through longitudinal expansion of the tubes. Therefore, most of the stress is caused by tube sheet deflection under biaxial or triaxial stress, and in the absence of severe thermal cycling, by the tube joint subjected to mechanical cycling conditions. Moreover, when the tube sheet is pressurized, it flexes. Since the tubes are normally attached to the tube sheet face, there is a tendency to mechanically detach the tube from the tube hole and apply a high stress level on the joint. In cases where there is a large undetected root crack, or where there is lack of ductility in the tube joint, fatigue failure can be the cause of the problem [5]. In some instances, a tube joint may contain geometric and often microcracks induced by the service environment or by the metallurgical condition, leading to fatigue failure that could result leakage at a joint.

Hence, heat exchanger tube and joint failures are not only attributed to the vibration of components, thermal stress, and environmental effects but also to the state of applied stress which then influences the degradation of the desirable properties of the materials. This state of stress is often found to be biaxial in
character in the case of fatigue related failure.

B. Titanium for Heat Exchanger Materials

Titanium and its alloys, which are commercially available have been used in industry for more than 30 years. But is is only recently that the metal has been used in heat exchanger applications because of its relatively low thermal conductivity and low modulus of elasticity. However, in present heat exchanger technology, the influence of these two unwanted material properties of titanium have been controlled by using a thin-walled tube and providing supports along the length of the tube, thus making titanium competitive with other materials. The aerospace industry is using the metal extensively for its high strength-to-weight ratio. The primary reason that the metal is used in the process and other industries is due to its extremely good corrosion resistance. In fact, when used in surface condensers, the metal performs well in terms of corrosion effects. However, there is also a problem of tube-to-tube sheet joint leakage and tube variation [5].

Unalloyed and alloyed titanium exhibit desirable properties which make them ideal for different applications which utilize the superior strength-to-weight ratio, excellent elevated temperature performance, extreme corrosion resistance, good erosion resistance
and high overall coefficient of heat transfer. This provides high condensing and distillation rates [6]. However, there is no information at the present time concerning the influence of biaxial stress on the fatigue properties of commercial pure titanium, Ti-50A, and low alloy titanium, Ti-Code 12. The investigation of whether these two titanium heat exchanger materials respond favorably when subjected to different magnitudes of biaxial states of stress is the primary reason that these materials have been used in this study.

By far the most commonly used titanium is Ti-50A (Grade 2), while Ti-Code 12 is generally specified for severe corrosive environments. Moreover, the basic understanding of the behavioral response of these materials while undergoing biaxial fatigue requires some knowledge of microstructural interpretation from the standpoint of the material's prior history involving alloy content, working temperature and thermal treatment conditions.

Proper interpretation of titanium and titanium alloy microstructures requires some understanding of their phase relationships and constitutions. The materials are grouped in relation to the phases which are predominant in their microstructure. Grade 2 is commercially pure titanium that exhibits allotropic behavior and can exist in two crystal forms as illustrated in
Figure 2, an alpha phase at room temperature and a beta phase at elevated temperature. The alpha phase is characterized by a hexagonal close-packed crystal structure which is stable up to about 1620°F, while the beta phase is characterized by a body-centered cubic structure which is stable from 1620°F to the melting point of about 3040°F. Consequently, the addition of alloying elements will stabilize either the alpha or the beta phase by changing the temperature at which the phase transformation occurs, and the relative quantity of each phase, thereby producing a wide range of desirable mechanical and physical properties. Consider, for instance Grade 12 low alloy titanium: the addition of molybdenum (Mo) as an alloying element stabilizes the beta phase by lowering the temperature of transformation from alpha to beta. Figure 3 shows a Ti-Mo phase diagram which is a typical beta isomorphous system indicating that Grade 12 is a near alpha titanium alloy [7].

Most commercial alloys of titanium are either of the ternary or quarternary type. Production processes, however, do not usually approach equilibrium conditions so that for purposes of illustration, it is convenient to utilize the binary equilibrium phase diagrams in describing the microstructural changes. Consider for instance a beta eutectoid stabilized as shown in Figure 4. In this case, the addition of molybdenum will
stabilize the beta phase. However, under equilibrium conditions, the beta phase decomposes to form alpha and an intermetallic compound. The active beta eutectoid elements such as nickel (Ni), an element in Grade 12, cause the rapid decomposition of beta to a compound and alpha. On the other hand, a beta eutectoid element such as iron (Fe), an element common to both Grade 2 and Grade 12, results in sluggish eutectoid reactions. The addition of controlled amounts of elements will cause the beta phase to persist below the beta transition temperature down to room temperature resulting in an alpha plus beta two-phase structure. Thus, two-phase titanium alloys can undergo a significant improvement in mechanical properties when heat treatment is done in the $\alpha + \beta_g$ temperature range, followed by an aging cycle at a lower temperature. Moreover, adding a high percentage of beta stabilizing agents will result in a microstructure that is substantially beta which is susceptible to extensive strengthening by heat treatment [8, 9].

C. The Biaxial State of Stress and Fatigue Failure

As is generally known, stress is the term used to define the intensity and direction of the internal forces acting at a given point on a particular plane. It is a second order tensor quantity because not only
the magnitude and direction of the stress are involved in its description, but also the orientation of the plane on which it acts. A complete description of the magnitudes and directions of stresses on all possible planes through a point constitute the state of stress of the point [10]. However, when two non-parallel stresses act on a plane through a point then the state of stress is biaxial in character.

For most components in normal service, fatigue problems normally occur at locations of stress concentration that arise from structural discontinuities, metallurgical defects, or welding imperfections [11]. Actually, the fatigue analysis must often be concerned with stress states that are biaxial in character. For instance, pressure, thermal and rotational, when combined with axial induced stress fields are always biaxial at the free surfaces and are similar to those induced in structural discontinuities by uniaxial loading [12].

Predicting failure and establishing a geometry that will avert failure is a relatively simple matter if a uniaxial state of stress is acting in some structural elements. Application of the available uniaxial stress-strain curve obtained from either a tension or compression test of the material is relevant in this case. For instance, if yield is established as the governing
failure criteria for uniaxial loading, failure is predicted to occur when the maximum normal stress reaches the yield point as determined from the stress-strain curve.

However, if the structural element is subjected to a biaxial state of stress, failure prediction is far more difficult [10]. No longer is yielding predicted based on when the maximum normal stress reaches the yield point, because the other stress component can also influence yield criteria.

Despite the presence of such biaxial stresses, the incorporation of proven criteria in design procedures to adjust fatigue resistance for the effect of biaxial loading has been very limited. It appears that this is due to the difficulties in testing over a wide range of biaxial conditions. In reality, however, most of the criteria that are proposed or used are direct adaptations of those found successful in treating static deformation behavior such as the criteria based on the maximum shear stress and distortion energy [13].

Most of the information on yielding and fracture of materials under the action of biaxial stress, which is induced when two non-parallel forces act on the body, are obtained from experiments on thin-wall cylinders [14]. A typical arrangement for such an experiment for obtaining controlled ratios of principal stresses $\sigma_1$ and $\sigma_3$, is
accomplished by combining axial and torsional stresses as shown in Figure 5. Loading the thin-wall tubular specimen until yielding or fracture occurs will result in subjecting the elements of the wall to biaxial stress with constant principal stress ratio, $\sigma_1/\sigma_3$. In lieu of simple axial tension, a fluctuating axial stress of pre-determined amplitude is applied so that the elements in the wall are now subjected to biaxial stresses with a constant range of principal stress ratio $\Delta\sigma_1/\Delta\sigma_3$ under the imposed fatigue condition as illustrated in Figure 6.

A number of attempts have been made to determine the conditions governing biaxial fatigue failures in thin-wall tubular specimen. One of the earliest investigations used a specimen with 1.1 inches outside diameter and 1.0 inch bore. The test results agreed quite closely with those tests using a thick cylinder with a 1.0 inch bore and a wall thickness ranging from 1/10 to 1.0 inch [15]. Experimental studies have also been done on several materials with different specimen geometries suitable for their experimental set-ups [16, 17, 18, 19, 20]. On the other hand, extensive research and studies were also done on biaxial fatigue failure theories. In fact, Y.S. Garud [21] gives an in-depth review of the various theories and has in turn come up with his own theory. Hence, what this experimental and theoretical studies conclude is that there is still no general
consensus as to what governing rule is applicable in biaxial fatigue for design purposes.

The behavior of materials under biaxial state of stress is an important element in the prediction of fatigue life. The difficult problem of estimating the fatigue strength of materials under such loading conditions is of some concern in the design process. Although biaxially stressed specimens are used in laboratory tests to provide material property data, it is left to the design engineers to judiciously apply these results in real situations.

The study of the biaxial state of stress in simple static loading started at the turn of the century, but it was the biaxial fatigue situation which has recently attracted considerable attention.
III. THE EXPERIMENTAL STUDY

This experimental investigation has centered on the biaxial fatigue behavior of titanium materials using a thin-wall tubular specimen subjected to combined axial push-pull and independently applied torsion. An analytical model based on Mohr's circle was designed in order to address the problem of biaxial fatigue life at two different torsional stress levels and to investigate the fatigue behavioral response of the heat exchanger materials.

There is no doubt that the basic nature of biaxial fatigue failure in heat exchanger materials requires qualitative information with regard to their response when subjected to adverse load and operating conditions which can lead to a drastic deterioration of material properties in practice and a consequent increase in operating expenses.

Titanium and its alloys which are competitive materials in industry today deserve due consideration in heat exchanger application in terms of the conditions implemented in this study. It is hoped that the results of this study will help increase the knowledge of failure processes in such systems and provide useful information related to material performance. These results will hopefully lead to improvements in design and optimization.
of the properties of materials used in such applications. As knowledge of the science and technology of titanium as heat exchanger materials is enhanced, the use of titanium in industry could ultimately increase, for tomorrow's titanium heat exchanger materials are the products of today's research.

A. Material Characteristics

The tubular materials used in this experimental investigation have been supplied by the TIMET CO. as complimentary samples. The chemical compositions and mechanical properties are given in Table I [7, 9]. Grade 2 (Ti-50A) is commercially pure titanium. Grade 12 (Ti-Code 12) is low-alloy near alpha (α) titanium as shown in the Ti-Mo phase diagram, Figure 3. As a result of adding the alloying elements Mo and Ni, Grade 12 exhibits better mechanical and corrosion properties than Grade 2. Its characteristics in terms of phase and microstructure have been previously discussed and interpreted.

It is possible to determine the uniaxial cyclic changes of the materials from their monotonic mechanical properties utilizing the Manson, et. al. relations [22]. Accordingly, cyclic softening is expected to occur if the ratio of tensile strength to 0.2% offset yield stress (SUlt/Sy) is less than 1.2. On the other hand,
cyclic hardening should occur if this ratio is greater than 1.4. Calculation of Sult/Sy for both materials indicates that Grade 12 with Sult/Sy equal to 1.52 will cyclically harden, whereas in the case of Grade 2 with Sult/Sy equal to 1.36, which lies between 1.2 and 1.4, cyclic changes are difficult to predict.

This aspect of fatigue behavior was investigated in this study by generating hysteresis loops during the first ten cycles of the test.

B. Specimen and its Mounting Grips

1. Specimen Grips

The specimen and its grips were designed based on the following criteria: 1) The grip must provide good axial alignment when mounted in torsion and in the axial push-pull loading frame; 2) It must firmly grip the ends of the specimen and loading frame so that there is no backlash when an alternating stress is applied through zero stress; 3) It must have adequate provisions for internal heating of the specimen and, 4) It should be easy to assemble and disassemble.

The details of the grip design are shown in Figure 7. The button head end of the MTS grips, unerrated tube clamps, the Cajon hex coupling, and Swagelok tube fittings are adequate provisions for good alignment, firm grips and sturdy supports. The button head ends are
mounted on the MTS load frame and provide load transfer with zero backlash. On the other hand, the Cajon hex coupling is essentially used to align the pipe threaded button head end and the Swagelok tube fitting. Moreover, the Swagelok tube fitting has front and back ferrule components that serve as side supports and help tighten the specimen in position. Threaded holes for socket set screw were machined in the tube fitting as torsion supports. The Cajon hex coupling and Swagelok tube fitting are commercially available and these are well suited for use in biaxial fatigue testing under completely reversed cycles. The specimen is firmly supported during axial cycling by the unserrated tube clamps which are connected to a fixture at the ends of the coupling.

If desired, an internal heating element such as the Carborundum globar LL type made of silicon carbide can be internally installed in the grips assembly for the high temperature test. The grips can also be used for tension testing of tubes with 1 inch outside diameter.

2. Specimen Design

A tubular specimen loaded in torsional and axial push-pull for biaxial fatigue testing can be satisfactorily designed keeping in mind, failure due to
fatigue and premature buckling. The wall thickness is thin enough to minimize the influence of the stress gradient through the wall and yet thick enough to ensure the presence of an adequate number of grains across the cross-section and hence, bulk behavior of the material. The compressive principal stress generated during the fatigue test of a thin-walled tubular specimen may cause it to buckle. Buckling and stress gradients can be minimized if $D > 10t$. Similarly, alloys display bulk behavior if $t > 5$ to $10d$ [23]. Here, $D$ is the tube outside diameter, $d$ is the mean linear intercept grain size, and $t$ is the wall thickness. Generally, most engineering alloys have grain sizes ranging from 50 μm (0.00197 inch) to 100 μm (0.003937 inch), hence, the wall thickness is normally greater than 0.5 mm (0.0197 inch).

The thin-wall tubular specimen used in this investigation is shown in Photograph 1. This was loaded axi-symmetrically with a combination of push-pull and torsional moment so that a biaxial state of stress was imposed on the specimen. A gage section cross-sectional area of 0.0594 in$^2$ and a wall thickness of 0.0215 inch were used. The details of the design are shown in Figure 8. A gage length of 1.5 inches, wall thickness of 0.0215 inch, and six diametrically opposite holes were machined from the sample tubes of 1 inch outside diameter. These specimens were tested in the as-received condition.
IV. BIAXIAL TEST

A. Method

Several methods have been developed and used in obtaining a biaxial state of stress in laboratory specimens [24, 25, 26, 27, 28]. Moreover, there are biaxial stress systems adaptable for practical use such as the cruciform specimens loaded in orthogonal coordinates, tubes under combined tension and torsion loads [20, 25, 29, 30]. Each has its own limitations. For example, the complexity and inaccuracy of stress/strain measurements and stress concentration effects are disadvantages of the cruciform specimen. The influence of pressurized fluid on crack initiation and growth, and the effects of a stress/strain gradient in the specimen wall in axial loading and internal/external pressurization of tubes are common problems. However, these problems are overcome in thin-wall tubular specimens under tension-torsion loading. Although buckling and modification of crack development could introduce some problems, proper specimen design and test control could minimize its influence on the fatigue life of the material.

In this investigation the biaxial state of stress was obtained in a thin-wall tubular specimen under axial push-pull with superimposed torsion.
B. Analysis and Formulation of Stresses

The thin-wall tubular specimen shown in Figure 5 (a) minimizes the effects of stress gradients because the surface fibers are restrained from yielding by the less highly stressed inner fibers [13]. In this figure, the biaxial state of stress is simulated by the axial load, $P$, and torsional load, $T$. Consider an element A in the tube; the components of the biaxial state of stress as shown in Figure 5 (b). These components are the axial stress and the torsional shearing stress which can conveniently be visualized and defined by the principal stresses $\sigma_1$, $\sigma_2$, $\sigma_3$. $\sigma_1$ is the greatest principal stress, $\sigma_3$ the smallest principal stress and $\sigma_2$ the intermediate value is zero, as shown in Mohr's circle, Figure 5 (c). The Mohr's circle is especially useful in this study because it not only gives a geometrical representation of the equations used in the transformation of the stress components, but also is the best way to visualize the state of stress. Here, the normal stress, $\sigma_X$, is plotted along the X or $\sigma$ axis, while the torsional stress, $\tau_{WX}$, is plotted along the y or $\tau$ axis. These stresses have been plotted as points B and B₁ in the Mohr's circle in accordance with its proper sign convention. The intersection of the line BB₁ with the X axis, point C, locates the center of the circle, characteristic of Mohr's circle and...
resulting from stress equilibrium conditions with diameter $BB_1$. The maximum values of the principal stresses correspond to zero shear stress and are shown by points $A$ and $A_1$. The angle between the X axis and $\sigma_1$ is determined by the angle $A_1-C-B_1$ which is twice the angle between $\sigma_1$ and the X axis on the actual stressed body. From the Mohr's circle the values of the principal stresses and the maximum shear stress can be formulated as shown below:

\[
\sigma_1 = OC + CA_1
\]

\[
\sigma_2 = 0, \text{ for the biaxial case}
\]

\[
\sigma_3 = OC - CA_1
\]

but $OC = \sigma_x/2$

$CA_1 = CB_1$, radius of the Mohr's circle

\[
= [(\sigma_x/2)^2 + (\tau_{wx})^2]^{1/2}
\]

hence, the principal stresses are

\[
\sigma_1 = \sigma_x/2 + [(\sigma_x/2)^2 + (\tau_{wx})^2]^{1/2}
\]

(1)

\[
\sigma_3 = \sigma_x/2 - [(\sigma_x/2)^2 + (\tau_{wx})^2]^{1/2}
\]

(2)

and the maximum shear stress, $\tau_{max}$, is

\[
\tau_{max} = [(\sigma_x/2) + (\tau_{wx})]^{1/2}
\]

(3)

Equations 1, 2 and 3 were used in the formulation of the functional stress relations applicable in constant amplitude biaxial fatigue.
1. Formulation of Torsional Stress for Biaxial Fatigue Test: Analytical Model A - Generalized Formulation

The basic analysis of the Mohr's circle was expanded to include the influence of the alternating axial stress illustrated in Figure 6. Typical constant amplitude cyclic loading for a fatigue test is shown in Figure 9(B) which also defines the stress parameters for the test. Assume that the axial stress has a range from maximum stress, \( \sigma_{\text{max}} \) to minimum stress, \( \sigma_{\text{min}} \) with stress amplitude, \( \sigma_a \) and a non-zero mean stress, \( \sigma_m \). The minimum stress can be either be positive or negative. The equation for torsional shearing stress, \( (\tau) \) was formulated based on Figure 6 and Figure 9(B).

The state of stress acting on an element A of the tube has been plotted in Mohr's circle, showing the influence of constant amplitude cycling stress superimposed with torsional shearing stress. Here the principal stresses become principal stress ranges, \( \Delta \sigma_1 \) and \( \Delta \sigma_3 \) with a constant ratio \( \Delta \sigma_1 / \Delta \sigma_3 \). The radius of the circle when the axial stress is in tension is represented by line CB and by line C1D when it is in compression.

Deriving an expression for torsional shearing stress, \( (\tau) \) we have:

\[
\Delta \sigma_1 = \sigma_{1,\text{max}} - \sigma_{1,\text{min}}
\]

where: \( \sigma_{1,\text{max}} \) = maximum principal stress in positive axis, corresponding to axial stress in tension, \( \sigma_{\text{max}} \)
\[ \sigma_{max}/2 + CB \]  
(5)

\[ \sigma_{max}/2 + \left[ (\sigma_{max}/2)^2 + [\tau]^2 \right]^{1/2} \]  
(6)

\( \sigma_{min} = \text{minimum principal stress in positive axis, corresponding to axial stress in compression, } \sigma_{min}. \)

\[ \sigma_{min}/2 + C_1 D \]  
(7)

\[ \sigma_{min}/2 + \left[ (\sigma_{min}/2)^2 + [\tau]^2 \right]^{1/2} \]  
(8)

and,

\[ \Delta \sigma_3 = \sigma_{3min} - \sigma_{3max} \]  
(9)

where: \( \sigma_{3min} = \text{minimum principal stress in negative axis, corresponding to axial stress in compression, } \sigma_{min}. \)

\[ \sigma_{min}/2 - C_1 D \]  
(10)

\[ \sigma_{min}/2 - \left[ (\sigma_{min}/2)^2 + [\tau]^2 \right]^{1/2} \]  
(11)

\( \sigma_{max} = \text{maximum principal stress in negative axis, corresponding to axial stress in tension, } \sigma_{max}. \)

\[ \sigma_{max}/2 - CB \]  
(12)

\[ \sigma_{max}/2 - \left[ (\sigma_{max}/2)^2 + [\tau]^2 \right]^{1/2} \]  
(13)

However, from Figure 9, we have

\[ \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \]  
(14)

\[ \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \]  
(15)

\[ R = \frac{\sigma_{min}/\sigma_{max}}{} \]  
(16)

then, substituting Equations 6 and 8 into Equation 4, we have
\[ \Delta \sigma_1 = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} + \left[ \frac{(\sigma_{\text{max}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}} - \left[ \frac{(\sigma_{\text{min}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}} \]  

(17)

substituting Equation 14 into Equation 17,

\[ \Delta \sigma_1 = \sigma_a + \left[ \frac{(\sigma_{\text{max}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}} - \left[ \frac{(\sigma_{\text{min}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}} \]  

(18)

For \( \Delta \sigma_3 \), substituting Equations 11 and 13 into Equation 9, we have

\[ \Delta \sigma_3 = -\frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} + \left[ \frac{(\sigma_{\text{max}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}} - \left[ \frac{(\sigma_{\text{min}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}} \]  

(19)

\[ \Delta \sigma_3 = -\sigma_a + \left[ \frac{(\sigma_{\text{max}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}} - \left[ \frac{(\sigma_{\text{min}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}} \]  

(20)

Let the biaxiality ratio, \( \lambda \), be defined as the ratio of the principal stress range, that is

\[ \lambda = \frac{\Delta \sigma_1}{\Delta \sigma_3} \]  

(21)

substituting Equations 18 and 20 into Equation 21 and solving for the torsional shearing stress, \( \tau \), we get

\[ \lambda = \frac{\sigma_a + \left[ \frac{(\sigma_{\text{max}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}} - \left[ \frac{(\sigma_{\text{min}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}}}{-\sigma_a + \left[ \frac{(\sigma_{\text{max}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}} - \left[ \frac{(\sigma_{\text{min}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}}} \]  

simplifying and collecting like terms,

\[ (\lambda + 1) \sigma_a = (\lambda - 1) \left[ \frac{(\sigma_{\text{max}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}} - (\lambda - 1) \left[ \frac{(\sigma_{\text{min}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}} \]  

(22)

\[ \frac{(\lambda + 1)}{(\lambda - 1)} \sigma_a = \left[ \frac{(\sigma_{\text{max}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}} - \left[ \frac{(\sigma_{\text{min}}/2)^2 + \left[(\tau)\right]^2}{2} \right]^{\frac{1}{2}} \]  

(23)

let \( \beta = \frac{\lambda + 1}{\lambda - 1} \)  

(24)
hence,
\[
\beta \sigma_a + \left[ (\sigma_{\min}/2)^2 + [(\tau)]^2 \right]^{\frac{1}{2}} = \left[ (\sigma_{\max}/2)^2 + [(\tau)]^2 \right]^{\frac{1}{2}}
\] (25)
squaring both sides of the equation and simplifying, we get
\[
(\sigma_{\max}/2)^2 + [(\tau)]^2 = (\beta \sigma_a)^2 + 2\beta \sigma_a((\sigma_{\min}/2)^2 + [(\tau)]^2)^{\frac{1}{2}} +
(\sigma_{\min}/2)^2 + [(\tau)]^2
\] (26)
\[
(\sigma_{\max}/2)^2 - (\sigma_{\min}/2)^2 = (\beta \sigma_a)^2 + 2\beta \sigma_a((\sigma_{\min}/2)^2 + [(\tau)]^2)^{\frac{1}{2}}
\]
\[
(\sigma_{\max} + \sigma_{\min}) (\sigma_{\max} - \sigma_{\min}) = (\beta \sigma_a)^2 + 2\beta \sigma_a((\sigma_{\min}/2)^2 + [(\tau)]^2)^{\frac{1}{2}}
\] (27)
substituting Equations 14 and 15 into Equation 27,
\[
\sigma_m \sigma_a = (\beta \sigma_a)^2 + 2\beta \sigma_a((\sigma_{\min}/2)^2 + [(\tau)]^2)^{\frac{1}{2}}
\] (28)
\[
\frac{(\sigma_m \sigma_a - (\beta \sigma_a)^2)}{2\beta \sigma_a} = \left[ (\sigma_{\min}/2)^2 + [(\tau)]^2 \right]^{\frac{1}{2}}
\] (29)
again squaring both sides of the Equation and solving for \((\tau)\),
\[
(\tau)^2 = \frac{(\sigma_m \sigma_a - (\beta \sigma_a)^2)^2}{2\beta \sigma_a} - (\sigma_{\min}/2)^2
\] (30)
\[
(\tau)^2 = \frac{(\sigma_m \sigma_a)^2 - 2\sigma_m \sigma_a (\beta \sigma_a)^2 + (\beta \sigma_a)^4 - (\sigma_{\min}/2)^2}{(2\beta \sigma_a)^2}
\] (31)
\[
(\tau) = \left[ 1/4 \left( (\sigma_m/\beta)^2 - 2\sigma_m \sigma_a + (\beta \sigma_a)^2 - (\sigma_{\min}/2)^2 \right) \right]^{\frac{1}{2}}
\] (32)
\[
(\tau) = 1/2 \left[ (\sigma_m/\beta)^2 - 2\sigma_m \sigma_a + (\beta \sigma_a)^2 - (\sigma_{\min}/2)^2 \right]^{\frac{1}{2}}
\] (33)
for
\[
\beta \neq 0
\]
or
\[
\lambda \neq \pm 1
\]
Equation 33 gives an expression for the torsional shear stress that must be imposed on the specimen in order to achieve the necessary biaxial fatigue stress parameters. For instance, if $\lambda = -2$; together with

$\sigma_{\text{max}} = 4$ KSI; $\sigma_{\text{min}} = 1$ KSI for $R = 0.25$; $\sigma_a = 1.5$; and $\sigma_m = 2.5$, the required value of $(\tau)$ to be imposed is $(\tau) = 3.46$ KSI. However, Equation 33 is only valid when $\beta$ is not equal to zero (0) or infinity ($\infty$). But $\beta$ can be equal to zero if the biaxiality ratio, $\lambda$, is equal to -1 which is the case for fully reversed cycles, $R = -1$.

Figure 10 shows the relative magnitudes and comparisons of torsional stress ($\tau$) for different axial stress amplitudes. For a given $\sigma_{\text{max}}$, and stress ratio, $R$, $\sigma_{\text{min}}$ has been calculated and in turn $\sigma_a$, $\sigma_m$, and $(\tau)$, respectively. The higher the alternating stress, the greater is the torsional stress that needs to be superimposed on the tubular specimen.

2. Formulation of Stresses for Fully Reversed Biaxial Fatigue Test: Analytical Model B - Special Case of Generalized Formulation

This is a special case of Equation 33 when $\lambda$ is equal to -1, and consequently when stress ratio, $R$, is also equal to -1 for zero mean stress, $\sigma_m$. This statement can be verified as follows:

Figure 9(A) illustrates the above condition when
\( \sigma_{\text{min}} \) is just the negative value of \( \sigma_{\text{max}} \). A Mohr's circle shown in Figure 11 was constructed corresponding to a particular torsional shearing stress and alternating stress level from which the expressions for the range of principal stress \( \Delta \sigma_1 \) and \( \Delta \sigma_3 \) were formulated.

Here, we have

\[
\sigma_{1\text{max}} = \sigma_{\text{max}}/2 + CB \\
= \sigma_{\text{max}}/2 + [\left( \sigma_{\text{max}}/2 \right)^2 + \left( \tau \right)^2]^{1/2} \\
\sigma_{3\text{max}} = \sigma_{\text{max}}/2 - CB \\
= \sigma_{\text{max}}/2 - [\left( \sigma_{\text{max}}/2 \right)^2 + \left( \tau \right)^2]^{1/2} \\
\sigma_{1\text{min}} = \sigma_{\text{min}}/2 + C_1B \\
= \sigma_{\text{min}}/2 + [\left( \sigma_{\text{min}}/2 \right)^2 + \left( \tau \right)^2]^{1/2} \\
\sigma_{3\text{min}} = \sigma_{\text{min}}/2 - C_1B \\
= \sigma_{\text{min}}/2 - [\left( \sigma_{\text{min}}/2 \right)^2 + \left( \tau \right)^2]^{1/2}
\]

but from Equation 16, for \( R = -1 \)

\[
\sigma_{\text{min}} = -\sigma_{\text{max}}
\]

hence, Equations 39 and 41 become

\[
\sigma_{1\text{min}} = - \sigma_{\text{max}}/2 + [\left( -\sigma_{\text{max}}/2 \right)^2 + \left( \tau \right)^2]^{1/2} \\
\sigma_{3\text{min}} = - \sigma_{\text{max}}/2 - [\left( -\sigma_{\text{max}}/2 \right)^2 + \left( \tau \right)^2]^{1/2}
\]

The principal stress ranges, Equations 4 and 9, are now

\[
\Delta \sigma_1 = \sigma_{\text{max}}/2 + [\left( \sigma_{\text{max}}/2 \right)^2 + \left( \tau \right)^2]^{1/2} + \sigma_{\text{max}}/2 \\
- \left[ \sigma_{\text{max}}/2 + \left( \tau \right)^2 \right]^{1/2} \\
\Delta \sigma_1 = \sigma_{\text{max}}
\]

\[
\Delta \sigma_3 = - \sigma_{\text{max}}/2 - [\left( -\sigma_{\text{max}}/2 \right)^2 + \left( \tau \right)^2]^{1/2} - \sigma_{\text{max}}/2 \\
+ \left[ \left( -\sigma_{\text{max}}/2 \right)^2 + \left( \tau \right)^2 \right]^{1/2} \\
\Delta \sigma_3 = - \sigma_{\text{max}}
\]
therefore, the biaxiality ratio becomes

\[ \lambda = \frac{\Delta \sigma_1}{\Delta \sigma_3} \]

\[ = \frac{\sigma_{\text{max}}}{-\sigma_{\text{max}}} \]

\[ = -1 \]

It is important to note that in this case the torsional stress is not a function of \( \lambda \) and other stress parameters, as compared to Equation 33.

Indeed, for the fully reversed biaxial fatigue condition the value of \( \lambda \) is equal to -1. It was on this basis that this experimental investigation was done together with three stress levels of \( \sigma_{\text{max}} \) and two torsional stress levels. Most fatigue studies have been conducted under fully reversed conditions, although some studies have also looked at the influence of mean stresses. The influence of mean stresses can be considered by utilizing Equation 33, but this has one major problem, which is that the thin-wall tubular specimen has a maximum torsional stress capacity which must not be exceeded.

Figure 12 shows the dependence of the relative magnitude of the biaxial state of stress on the torsional stress. For the same \( \sigma_{\text{max}} \), \( (\sigma_{\text{max}}^A = \sigma_{\text{max}}^B) \), the higher the torsional stress \((\tau)^B\), as indicated in point E, the larger is the principal stress, \( \sigma_{1\text{max}}^B \), when compared to \( \sigma_{1\text{max}}^A \) as indicated in point D.

Appendix A1 is also a special case of the general-
ized formulation when the stress ratio, $R$ is equal to zero, or the minimum stress is equal to zero.

3. Biaxial Fatigue Failure Criteria and Data Correlation

Various theories in fatigue failure have been investigated such as the one proposed by Fuchs [32] for ductile metals. Most of these theories have been based on conventional static failure theories such as the Von-Mises, Tresca, etc. [31, 32, 33]. Although these conventional criteria have been formulated for use in triaxial or multiaxial states of stress, their application in fatigue failure under a biaxial state of stress could be justified since fatigue failures often initiate at the free surface where the stress field is two-dimensional. These criteria were then expanded to include the influence of alternating stresses and modified for use in systems under biaxial states of stress.

Some of these theories have been utilized in this study for correlation with the data obtained from the experimental investigation. These are as follows:

a. Von-Mises Equivalent Stress Criterion, $\sigma_{eVM}$

Here the conventional definition of equivalent stress, which is the uniaxial stress that is equidistant from the yield surface [32] is introduced and derived as
In terms of the applied stresses,

\[ \bar{S}_e = \frac{1}{\sqrt{2}} \left[ (\sigma_x - \sigma_y)^2 + \sigma_y^2 + (-\sigma_x)^2 + 6(\tau_{xy})^2 \right]^{\frac{1}{2}} \]  \hspace{1cm} (49)

for biaxial stress condition \( \sigma_y = 0 \), hence Equation 49 is reduced to

\[ \bar{S}_e = \frac{1}{\sqrt{2}} \left[ 2\sigma_x^2 + 6\tau_{xy}^2 \right]^{\frac{1}{2}} \]
\[ = \sqrt{2}/\sqrt{2} \left[ \sigma_x^2 + 3(\tau_{xy})^2 \right]^{\frac{1}{2}} \]
\[ = [\sigma_x^2 + 3(\tau_{xy})^2]^{\frac{1}{2}} \]  \hspace{1cm} (50)

Equation 49 predicts that yielding will occur when the quantity on the right side of the equation exceeds the yield stress in uniaxial tension, whereas Equation 50 is just an expression similar to the distortion energy theory of yielding \[31\].

In this study, the expression in Equation 50 has been modified to make it relevant to the applied biaxial stresses: the alternating stress, \( \sigma_a \) and the superimposed torsional stress, \( \tau \). In doing so, Equation 50 becomes

\[ \bar{S}_{eVM} = [(\sigma_a)^2 + 3(\tau)^2]^{\frac{1}{2}} \]  \hspace{1cm} (51)

An expression similar to Equation 51 has been used by Ellyin, et. al. \[17\] in their investigation.
b. Sine's Theory, $S_{eS}$

This again is a combined stress, fatigue failure criterion based on the distortion energy theory except that the equivalent alternating stress is expressed in terms of the alternating principal stresses and the mean stress component of the principal stresses. When applied to the stress conditions in this experimental investigation with no mean stress, the relationship becomes

$$S_{eS} = 1/\sqrt{2}[\sigma_{a1}^2 + (\sigma_{a2} - \sigma_{a3})^2 + (\sigma_{a3} - \sigma_{a1})^2]^\frac{1}{2}$$  \hspace{1cm} (52)

where:

$$a_i = \text{alternating component of the principal stresses in } "i" \text{ direction (}i = 1, 2, 3\text{) as described in the Mohr's circle, Figure 11, such that } \sigma_{a1} > \sigma_{a2} > \sigma_{a3}.$$  

For a biaxial state of stress $\sigma_{a2} = 0$, hence Equation 52 reduces to

$$S_{eS} = 1/\sqrt{2}[\sigma_{a1}^2 + (\sigma_{a3})^2 + (\sigma_{a3} - \sigma_{a1})^2]^\frac{1}{2}$$  \hspace{1cm} (53)

Expressing $S_{eS}$ as a function of the biaxiality ratio, $\lambda$ where $\Delta \sigma_3 = \sigma_{a3}$, and $\Delta \sigma_1 = \sigma_{a1}$ as delineated in the Mohr's circle, and recalling from Equation 21 that $\lambda = \Delta \sigma_1 / \Delta \sigma_3$ so that $\lambda = \sigma_{a1} / \sigma_{a3}$, 

$$S_{eS} = 1/\sqrt{2}[(\lambda \sigma_{a3})^2 + (\sigma_{a3})^2 + (\sigma_{a3} - \lambda^2 \sigma_{a3})^2]^\frac{1}{2}$$  \hspace{1cm} (54)

$$= 1/\sqrt{2}[(\lambda \sigma_{a3})^2 + (\sigma_{a3})^2 + \sigma_{a3}^2(1 - \lambda)^2]^\frac{1}{2}$$
= \frac{\sigma_{a3}}{\sqrt{2}} \left[ \lambda^2 + 1 + 1 - 2\lambda + \lambda^2 \right]^{1/2} \\
= \sqrt{2}/\sqrt{2} \left[ \sigma_{a3}^2 (\lambda^2 - \lambda + 1) \right]^{1/2} 
\text{(55)}

Again, for fully reversed biaxial conditions, \( \lambda = -1 \), and Equation 55 becomes

\[ \bar{S}_{eS} = \sigma_{a3} (3)^{1/2} \]

\[ = 1.7321 \sigma_{a3} \] \text{(56)}

but, \( \sigma_{a3} = \sigma_{\text{max}} = \sigma_a \) as deduced from Figure 9A and Figure 11, so that

\[ \bar{S}_{eS} = 1.7321 \sigma_a \] \text{(57)}

Fuchs [32] observed that the general criterion for crack initiation is governed by Sine's theory under the following conditions: 1) no gross yielding; 2) stress is above the crack propagation threshold; and 3) alternating stresses can be represented by principal alternating stresses along fixed principal axis.

c. Langer's Theory, \( \bar{S}_{eL} \)

The influence of the maximum alternating shear stress is considered in this theory, which states that if the alternating stresses are produced by a single alternating load, the maximum alternating shear stress can be determined from the maximum and minimum stress such that the equivalent stress is given by [32],
Langer's theory is similar to Tresca's yield criterion which assumes that yielding occurs when the maximum shear stress reaches the value of the shear stress in the uniaxial tension test. Here, the maximum shear stress is given by

$$\tau_{\text{max}} = \frac{\sigma_1 - \sigma_3}{2}$$  \hspace{1cm} (59)

where \(\sigma_1\) and \(\sigma_3\) are the algebraically largest and smallest principal stresses. In this experimental investigation and as observed from Figure 11, \(\tau_a\) in Equation 58 and \(\tau_{\text{max}}\) in Equation 59 can be related so the expression for \(\bar{S}_{eL}\) becomes,

$$\tau_a = \tau_{\text{max}} = \frac{\Delta \sigma_1 - \Delta \sigma_3}{2}, \text{ but } \Delta \sigma_3 = -\Delta \sigma_1 \text{ for } \lambda=-1$$

hence,

$$\tau_a = \frac{\Delta \sigma_1 + \Delta \sigma_1}{2} = \Delta \sigma_1 = \sigma_{\text{max}} = \sigma_a$$

substituting this into Equation 58 we have

$$\bar{S}_{eL} = 2(\sigma_a)$$  \hspace{1cm} (60)

It should be noted that Equation 60 is similar to Von-Mises if \((\tau)\) in Equation 51 is taken as \(\tau_a\).
d. The Naomi-Masao-Masateru Equivalent Stress Range, $\Delta S_{\text{NMM}}$

This criterion assumes an equivalent stress range based on the concept of crack opening displacement which includes the effect of the principal stress parallel to the fatigue crack [34]. This is expressed as a function of the biaxiality ratio, $\lambda$ such that

$$\Delta S_{\text{NMM}} = (\alpha_{\text{NMM}})\Delta \sigma_1 (2-\lambda)^m$$  \hspace{1cm} (61)

where the nondimensional coefficient, $\alpha_{\text{NMM}} = 1/\sqrt{2}$ obtained by satisfying the relation $\bar{S}_{\text{NMM}} = \Delta \sigma_1$ for the uniaxial test, and the exponent $m$ is equal to 0.5 only if $0 \leq \lambda \leq 1.$

If Equation 61 is applied to this experimental investigation where $\lambda$ is equal to -1, then it can be expressed as

$$\Delta S_{\text{NMM}} = (1/\sqrt{2})(2+1)^{0.5}(\Delta \sigma_1)$$ \hspace{1cm} (62)

$$= (\sqrt{3}/\sqrt{2})(\Delta \sigma_1)$$

$$= 1.2247 (\sigma_{\text{max}})$$

$$= 1.2247 (\sigma_a)$$  \hspace{1cm} (63)

It should be pointed out that the constant $\sqrt{3}/\sqrt{2}$ is similar to the criterion proposed by Sawert for long life fatigue [35], and for the octahedral shear stress yield theory [32].
e. **Modified Von-Mises Equivalent Stress Range**

The Von-Mises or the distortion energy yield criterion can be modified to include the effect of the intermediate principal stress by introducing Lod's stress parameter, $\mu_L$ [31].

Lod's stress parameter is defined by the equation

$$\mu_L = \frac{2(\sigma_2 - \sigma_3 - \sigma_1)}{(\sigma_1 - \sigma_3)}$$  \hspace{1cm} (64)

$\sigma_1$ and $\sigma_3$ are the algebraically largest and smallest principal stresses, while $\sigma_2$ is the intermediate principal stress as shown in the 3-dimensional Mohr's circle.

The intermediate principal stress, $\sigma_2$ can be expressed from Equation 64 as

$$\sigma_2 = \frac{[\mu_L(\sigma_1 - \sigma_3) + \sigma_3 + \sigma_1]}{2}$$  \hspace{1cm} (65)

substituting this expression of $\sigma_2$ into the Von-Mises or distortion energy equation below

$$\overline{S}_{eMVM} = \frac{1}{\sqrt{2}} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{\frac{1}{2}}$$  \hspace{1cm} (66)

will result in an expression for $\overline{S}_{eMVM}$ as a function of Lod's stress parameter $\mu_L$, that is

$$\overline{S}_{eMVM} = \left( \frac{\sigma_1 - \sigma_3}{2} \right) \left[ (3 + \mu_L^2) \right]^{\frac{1}{2}}$$  \hspace{1cm} (67)

If Equation 67 is expressed in terms of the biaxial
stress conditions used in this experimental investigation, then we have an equivalent stress range, such that

\[ S_{eMVM} = \left( \frac{\Delta \sigma_1 - \Delta \sigma_3}{2} \right) \left( \frac{3 + \mu L^2}{2} \right)^{\frac{1}{2}} \]  

(68)

where:

\[ \mu_L = \frac{2 \sigma_2 - \Delta \sigma_3 - \Delta \sigma_1}{\Delta \sigma_1 - \Delta \sigma_3} \]  

(69)

but \( \Delta \sigma_3 = - \Delta \sigma_1 \)

and the superimposed torsional stress is defined as

\[ \sigma_2 = (\tau) \]  

(70)

then Equation 64 becomes

\[ \mu_L = \frac{2 \sigma_2 + \Delta \sigma_1 - \Delta \sigma_1}{2 \Delta \sigma_1} \]

\[ = \frac{\sigma_2}{\Delta \sigma_1} \]

\[ = (\tau)/\sigma_{max} \text{ but } \sigma a = \sigma_{max} \]  

(71)

hence,

\[ S_{eMVM} = \left( \frac{\Delta \sigma + \Delta \sigma}{2} \right) \left( \frac{3 + \mu L^2}{2} \right)^{\frac{1}{2}} \]

\[ = (\Delta \sigma_1) \left( \frac{3 + \mu L^2}{2} \right)^{\frac{1}{2}} \]

\[ = (\sigma a) \left( \frac{3 + \mu L^2}{2} \right)^{\frac{1}{2}} \]  

(72)

The above theories which have been modified to suit this experimental investigation could be plotted together with the experimental data on stress against log number of cycles to failure coordinates for data correlations.
The equipment and instrumentation used in this study were designed to meet the needs and conditions necessary for biaxial fatigue testing in conjunction with the available Tinius Olsen torsion machine and the MTS servo-controlled electro-hydraulic system.

A. Torsion Machine

A photograph of the specimen mounted in the torsion machine is shown in Photograph 2. The block diagram of the torsion machine shown in Figure 13 illustrates the working components and instrumentation of the system. This electro-mechanical system provides a torsional moment in one rotating end at a constant rate through a torque converter while holding the other end fixed. The fixed end has a built-in torque measuring device by which the applied torque can be continuously monitored through the balance scale. A disk mounted in the specimen grip is connected to the moveable core of a linear variable differential transformer (LVDT) whose output voltage is fed into the voltage amplifier for appropriate amplification. This, in turn, is connected to a calibrated variable chart recorder to plot the torsional displacement. Some results are shown in Figures 14 and 15.
B. Torsion Fixer Facility

The photographs and details of design of the torsion fixer are shown in Photograph 3, Figure 16 and Figure 17 respectively. This was designed to facilitate constancy of applied torque in the specimen after being unmounted from the torsion loading frame. A helix of 1.5 hex was machined to fit the Swagelok tube fitting, while the brackets were designed to fix the torque level and for easier mounting and unmounting. Two guide slots were machined on the lower bracket to prevent relative side movement and to facilitate vertical motion of the specimen during the fatigue test. The machine bolts and the guide slots work similar to a journal bearing so that frictional forces were considered negligible relative to the applied alternating stresses. The lower edge of the right side bracket and the upper surface of the lower bracket were machined to finished smooth surfaces.

C. MTS Machine

The MTS machine shown in Photograph 4 is a servo-controlled electro-hydraulic system. Its block diagram shown in Figure 18 illustrates the relationships and functions of the system components. The push-pull action is provided by the hydraulic actuator whose vertical displacement is measured with an LVDT, and the
output signal is fed to the servo-controller. The counter-generator used electro-mechanical counters indicating the number of cycles which had taken place, and by setting it on oscillator function, a loading spectrum such as a sine wave could be generated. Voltage signals from the load cell and extensometer were fed back into the servo-controller with access for stress and strain measurements from which hysteresis loops could be generated and plotted on an X-Y recorder. The voltmeter and oscilloscope were used as monitoring devices to help with proper positioning of the actuator and with setting of applied alternating stress levels.
VI. EXPERIMENTATION

The experimental set up was based upon the availability of equipment, sample materials, generation of biaxial state of stress in a tubular specimen, and results desired. This study was designed to obtain maximum information at minimum cost and within the constraints of available test facilities.

Since fatigue test results are sensitive to slight variations in test parameters, such as the applied alternating stress and specimen geometry, a standard procedure was followed to recheck the specimen cross-sectional area before each test was performed.

Trial tests in torsion and axial push-pull were conducted in order to determine the optimum values of the test parameters, such as the torsional shearing stress and alternating stress, such that the specimen could withstand the stresses without premature buckling, either in torsion or during the fatigue test.

A. Experimental Procedure

The general procedure of the experimental work is presented in block diagram in Figure 19. Specimens were first machined from the sample tubing in the as-received condition. First, the torsional grips were assembled by mounting the Swagelok tube fittings, inner
tube support, and Cajon coupling on the specimen as shown in Figure 20. Rods were then inserted in both ends to provide axial alignment of the specimen when mounted in the torsion machine. The assembled unit was then mounted in the loading frame of the torsion machine. A disk for the LVDT connection was attached to the nut of the Swagelok tube fitting. Provisions for rigid support of the LVDT were connected to the frame of the torsion machine this mounting bracket is shown in Figure 21. Calibration of the torque scale was done after mounting the unit. A torsional moment was applied by running the torsion machine until the desired torsional stress level was attained on the scale. The applied torque was continually monitored through the LVDT and plotted on the chart recorder. After turning off the torsion machine, the LVDT connection was removed, the axial grips were installed on the specimen and tightly fitted. A Swagelok nut was then tightened against the axial grips.

In order to fix the applied torsional stress, a torque fixer facility, shown in Figure 16 was fitted on to the Swagelok hex nut. The attachments were properly installed so that the specimen could not relax after the unit was removed from the loading frame of the torsion machine. The top brackets were first mounted on the side brackets while the machine bolts were loosened from the guide slots. Alignment of the right side bracket and
bottom bracket were checked before tightening the bolts through the guide slots.

After unmounting the unit from the loading frame of the torsion machine, the lock nuts were first tightened against the Swagelok nuts. The MTS button head grips shown in Figure 22 and axial grip fixtures shown in Figure 23 were then assembled in the unit which was now ready for axial push-pull application, as shown in Figure 7 and Photograph 5.

The whole unit was then mounted in the loading frame of the MTS machine as shown in Photograph 6 by using the grips for the button head specimen with the right side bracket in the downward position. Standard mounting procedure as specified in the MTS manual was followed. The torque fixer bolts were then unloosened so that the ends of the specimen were permitted relative motion during the fatigue test.

Other components of the biaxial test unit are shown in Figure 24 and Figure 25.

B. Test Program

1. Torsion

Torque was applied to the specimen at a constant rate until reaching the desired torsional stress levels, which were 17.5 ksi and 8.75 ksi for Grade 2; 28.0 ksi and 14.0 ksi for Grade 1). These values were calculated
using Equation 77 as presented on Page 52. Linear torsional displacement was measured during the test and this was then converted into torsional strain using Equation 81, as presented on Page 53.

2. Axial Push-Pull

Fully reversed constant amplitude load controlled fatigue tests were conducted at two different levels of torsional stresses. These test conditions were set in accordance with the functional relationship of the biaxiality ratio, λ equal to -1 in which the mean stress, \( m \) was zero. The amplitude of the stress was the controlling parameter of the stress magnitude. A compromise between failure by premature buckling and fatigue failure set the maximum applied stress amplitude of 44 ksi which was about 75% and 79% of the yield strength of Grade 2 and Grade 12 respectively. Stress amplitudes of 37.5 ksi and 30 ksi were the other two stress amplitudes used in this investigation. Four tests were conducted at each stress amplitude. The number of cycles to failure was noted and periodic hysteresis loops and scope traces were recorded. A frequency of 10 Hz sine wave form was used for the constant amplitude cyclic loading and is shown in Photograph 7.
VII. EXPERIMENTAL RESULTS

The results of this investigation was summarized in Tables II, V, VIII and XI. Log number of cycles to failure was plotted against the constant stress amplitude as shown in Figures 26, 27, 28, 29, 30, and 31. \( \sigma - N_f \) curves were drawn through the mean values of the data distribution for each constant stress amplitude. These provide useful information with regard to the biaxial fatigue behavior of the materials investigated. In view of the scatter in the data obtained, a statistical analysis is presented below in order to obtain a meaningful \( \sigma - N_f \) curve.

A. Statistical Analysis of Biaxial Fatigue Data

Four tests were performed at each of the three constant stress amplitudes used in this research. This number of specimens were considered to be the minimum number needed to determine 95% confidence limits on the mean for a width of confidence interval that is twice the standard deviation and also to give some degree of statistical accuracy to the predicted behavior.

Statistical results show that the log normal of fatigue life at a constant stress amplitude level gives an adequate estimate of the statistical distribution function of fatigue life \([10, 13, 31, 32]\). Hence, if
the life distribution is taken to be log normal of the fatigue life, then the sample mean and standard deviation can be used to specify a probability of failure that is desired. This can be estimated from the form of the distribution. However, in order to verify the normality of biaxial fatigue life data, the following analysis is considered.

It must be recognized that because of the scatter of fatigue life data, the \( \sigma-N_f \) curve consequently consists of a family of \( \sigma-N_f \) curves with probability of failure as the parameter. Considering the data plotted on \( \sigma-\log N \) coordinates, a mean curve can be visually constructed through the data but substantial scattering about the mean negates the usefulness of such a curve.

In order to check whether the fatigue life is normally distributed, a normal probability graph was utilized. One such graph is the Weibull plot in which the Weibull distribution, expressed in Equation 73 as the Weibull probability density function, and Equation 74 for the cumulative Weibull distribution, are often used for representation of fatigue life data at a constant stress level [10]. If the data points are reasonably linear on the Weibull plot, and a best fit straight line is drawn through the data, then one can conclude that the fatigue life data are normally distributed.

\[
f(n) = \frac{b}{Na-No} \left[ \frac{N-No}{Na-No} \right]^{b-1} \exp\left( -\left( \frac{N-No}{Na-No} \right)^b \right)
\]  

(73)
\[ F(N) = 1 - \exp(-[N-No]/(Na-No))^b) \] (74)

where:  
\( N \) = specimen life, cycles
\( No \geq 0 \) = minimum life parameter, obtained from the Weibull plot
\( Na = \) characteristic life parameter occurring at a point where 63.2% have failed, obtained from the Weibull plot
\( b = \) Weibull slope obtained from Weibull plot by measuring the tangent of angle between the line and the \( N \) axis.

The Weibull probability density function represents a simple exponential distribution when the slope parameter \( b=1 \), Rayleigh distribution when \( b=2 \) and, a Gaussian distribution when \( b=3.57 \), that is, when the mean and median are the same.

If the data points plot as a straight line on the Weibull plot, the median - corresponding to 50% probability failure - and other probabilities are obtained and read directly from the Weibull plot; whereas, the mean and standard deviation are calculated from Equation 75 and Equation 76 respectively:

\[
\text{Mean, } \mu = No + (Na-No) \Gamma (1 + 1/b) \tag{75}
\]

\[
\text{Standard Deviation, SD}
\]

\[
SD = (Na-No)^2 [\Gamma (1 + 2/b) - \Gamma^2 (1 + 1/b)]^{1/2}
\]

\[
= (Na-No) \Gamma (1 + 2/b) - \Gamma^2 (1 + 1/b) \] \tag{76}

where:

\( \Gamma = \) Gamma function, Table of Gamma Function

It was desired to plot the experimental data on Weibull's probability graph so that appropriate statis-
tical parameters could be obtained and to check for normality of the data distribution at a constant stress level. Tables II, V, VIII and XI show biaxial fatigue test data arranged accordingly for Weibull's plot at three constant stress levels. It also shows the best fit straight line for the data in which the Y axis is taken as the probability axis and the X axis is taken as the number of cycles, N. The number of cycles for the best fit straight line, $N_{BF}$ corresponding to the plotting position was calculated, although a number of other data points were also considered.

To plot the data on the Weibull graph, consider for example the data for Grade 2 material at a torsional stress of 8.75 ksi and $\sigma_a$ of 44.0 ksi in Table II. First, the plotting positions were established using the equation, such that 20% is for Rank 1, 40% for Rank 2, etc. Linear regression was used for the best fit straight line to establish the linearity of the data. These data were then plotted on the Weibull graph as indicated in Figure 32, with a minimum life parameter, $N_0$, equal to zero, since the data initially plot as a straight line. The characteristic life, $N_a$ equal to $2.6 \times 10^4$ cycles was read directly from the plot at 63.2% probability. The median life of $2.23 \times 10^4$ was estimated by reading the intersection of 50% probability with the straight line. The slope parameter, 2.01, was
calculated by measuring the tangent of the angle made by the straight line with the X axis. Knowing the values of No, Na, and b, the Weibull mean and standard deviation were found using the expressions given in Equation 75 and Equation 76 respectively as illustrated below. We have, from Equation 75, the Weibull mean:

\[ \mu = No + (Na-No) \Gamma (1 + 1/b) \]

where: No = 0 (assumed)
Na = 2.6 x 10^4 cycles
b = tan 63.5°

\[ = 2.01 \]
\[ = 2.3042 \times 10^4 \text{ cycles} \]

and from Equation 76, the standard deviation:

\[ SD = (2.6 \times 10^4 - 0) \Gamma (1 + 2/2.01) - \Gamma^2 (1 + 1/2.01)]^{\frac{1}{2}} \]
\[ = (2.6 \times 10^4) [(1.0 - 0.7854)]^{\frac{1}{2}} \]
\[ = 1.20445 \times 10^4 \]

The mean locates the central location of the distribution. How the observations are spread out from the central region is described by the standard deviation; whereas the median represents the value above and below which half of the observations lie. It is important to note that the mean and the median are not usually equal due to the skewness of the distribution.

Since the data plotted as a straight line on the
Weibull plot, the distribution was regarded as approximately normal. In some cases the data curved downward, indicating that the minimum life, No, was not equal to zero. In this case, a trial and error procedure is used to obtain the best estimate of No until the transformed plotted as a straight line. A procedure is used in which an initial value of No is chosen corresponding to the life value which the curve approaches asymptotically. Then the quantity (N-No) is plotted on the Weibull graph for each data point against the initial probability values. This process is repeated until the data plots as a straight line, giving a best estimate of minimum life, No. The characteristic life, Na is obtained for a life value of (Na-No). From there the procedure for determining the Weibull statistical parameters is similar to the one previously discussed. At this point it is appropriate to note that the best fit plotted points deviate from the straight line at some extreme points but these deviation tails were normally disregarded [10].

The analysis thus presented was applied to the other experimental data, Tables V, VIII, and XI, and the results are summarized in Tables III, VI, IX and XII. Figure 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, and 43 show the data as plotted on the Weibull graph from which the Weibull statistical parameters were obtained.
to plot statistically acceptable $\sigma$-$N_f$ or fatigue life curves.

Since the Weibull statistical analysis showed that the distribution fatigue life data was approximately normal, the arithmetic mean and standard deviation in terms of logarithmic behavior of fatigue life could have been used to plot the $\sigma$-$N_f$ curve, but it was believed that this statistical approach was not appropriate in the present investigation.

B. Application of Pertinent Equations Relevant to the Experimental Set up

This investigation utilized some known equations in calculating the parameters necessary for analyzing the experimental data. On the other hand, governing equations were derived in conjunction with the experimentation. These are described below, taking for instance the experimental data obtained for Grade 2 material at a torsional stress of 17.5 ksi.

1. Torsional Stress

For a thin-walled tubular specimen the torsional stress is found by using the expression [36].

\[
(\tau) = \frac{T}{2\pi R_t t^2}
\]  

(77)
where:

\( (\tau) = \) torsional stress, psi

\( R_t = \) effective tube radius, inch

\( = 0.471 \) in.

\( T = \) applied torque, in-lb.

\( = 525 \) in-lb.

\( t = \) wall thickness in gage section, inch.

\( = 0.0215 \) in.

hence, substituting these values in Equation 77, we have

\[
(\tau) = \frac{525 \text{ in-lb}}{2\pi(0.471 \text{ in})^2(0.0215 \text{ in.})} = 17.510 \text{ ksi}
\]

2. The Torsional Strain

The torsional strain can be found using the LVDT experimental set up as illustrated in Figure 44. We have,

\[
\bar{\delta}_D = \frac{S_D}{R_D} \quad (78)
\]

\[
\gamma = \bar{\delta}_t \frac{(R_t/L)} \quad (79)
\]

but, \( \bar{\delta}_D = \bar{\delta}_t \) \quad (80)

substituting Equation 78 into Equation 79 we get

\[
\gamma = \frac{(S_D)(R_t)}{(R_D)(L)} \quad (81)
\]
where:

\[ S_D = \text{torsional displacement measured through LVDT, obtained in Figure 44 and converted using the calibration shown in Figure 45.} \]
\[ = 0.115 \text{ in.; (Arc AA)} \]
\[ L = \text{effective length of tube, 4.75 in.} \]
\[ R_D = \text{radius of disk, 2.375 in.} \]
\[ R_t = \text{effective radius of tube, 0.471 in.} \]
\[ \bar{\theta} = \text{angle of twist, radians. Subscripts D and t refer to disk and tube respectively} \]
\[ \gamma = \text{torsional strain, percent} \]

Substituting values and solving for the torsional strain

\[ \gamma = \left( \frac{0.115 \text{ in.}}{2.375 \text{ in.}} \right) \left( \frac{0.471 \text{ in}}{4.75 \text{ in.}} \right) (100) = 0.48\% \]

The above procedure was also used for the other data, and the results are shown in the biaxial fatigue data tables - Table III, VI, IX, and XII.

3. The Fatigue Test Parameters

The constant amplitude, completely reversed sinusoidal stress against time pattern shown in Figure 9(A) evaluating the biaxial fatigue behavior of the specimens. The relevant stress parameters were calculated as follows:

In this study the stress ratio, \( R \), is equal to -1, corresponding to the biaxiality ratio, \( \lambda \) of -1. Hence, solving for the mean stress, \( \sigma_m \) and alternating stress, \( \sigma_a \)
we have:

\[ R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} = -1 \]

therefore,

\[ \sigma_{\text{min}} = -\sigma_{\text{max}} \]

so that, the mean stress is found to be

\[ \sigma_{\text{m}} = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} \]

\[ = \frac{\sigma_{\text{max}} - \sigma_{\text{max}}}{2} \]

\[ = 0 \]

and the alternating stress is found to be

\[ \sigma_{\text{a}} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \]

\[ = \sigma_{\text{max}} \]

\[ = 30.0 \text{ ksi, for a constant stress level of} \]
\[ 30.0 \text{ ksi in particular.} \]

The above analysis was also used for the other stress levels and the results are shown in Tables II, V, VIII, and XI.

4. The Formulated Stresses

a. **Nonzero Mean Stress**

As previously derived, an expression for the torsional stress indicates the functional relationship
between the torsional stress, mean stress, alternating stress, minimum stress and biaxiality ratio. Equation 33 gives the required value of torsional stress needed to impose the desired biaxiality condition in the fatigue test.

Consider for instance a case when the biaxiality ratio, \( \lambda = -2 \), stress ratio, \( R = 0.25 \), and \( \sigma_{\text{max}} = 30.0 \) ksi. Using Equation 33, the torsional stress is calculated as follows:

\[
(\tau) = \frac{1}{2} \left[ \left( \frac{\sigma_{\text{m}}}{\beta} \right)^2 - 2(\sigma_a)(\sigma_m) + (\beta \sigma_a)^2 - (\sigma_{\text{min}})^2 \right]^\frac{1}{2}
\]

where:

\( \sigma_{\text{min}} = R(\sigma_{\text{max}}) \), from Equation 16

\[
= 0.25(30.0) \text{ ksi}
= 7.5 \text{ ksi}
\]

from Equation 15,

\[
\sigma_{\text{m}} = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2}
= \frac{30.0 + 7.5}{2}
= 18.75 \text{ ksi}
\]

from Equation 14,

\[
\sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2}
= \frac{30.0 - 7.5}{2}
= 11.25 \text{ ksi}
\]

and from Equation 24,
Substituting these values and solving for $(\tau)$, we have

$$(\tau) = \frac{\lambda+1}{\lambda-1}$$

$$= \frac{-2+1}{-2-1} = \frac{1}{3}$$

Hence, a torsional stress of 25.981 ksi is the value required to be superimposed for the biaxial fatigue test. Equation 33 is really an important tool in the evaluation of stress parameters for the biaxial fatigue test. However, the wall thickness of the thin-wall tubular specimen limits the maximum torsional stress it can withstand without failure by torsional buckling.

Torsional stresses for the other constant stress levels were calculated and the results are shown in Figure 10, which indicates the constant biaxiality ratio of $\lambda = -2$.

b. Zero Mean Stress, Fully Reversed Biaxial Fatigue Test

This is a case when $\lambda = -1$, which invalidates the use of Equation 33. As derived, the range of principal stresses are functions of the maximum stress or alternating stress for any torsional stress level, and are expressed as follows:

From Equation 45, we have
\[ \Delta \sigma_1 = \sigma_{\text{max}} \]
\[ = 30.0 \text{ ksi} \]

and from Equation 47, we have
\[ \Delta \sigma_3 = - \sigma_{\text{max}} \]
\[ = -30.0 \text{ ksi} \]

Consider the applied torsional stresses in Grade 2 material. These biaxial stresses were plotted in the Mohr's circle as shown in Figure 12. As delineated in the figure, the range of principal stresses is the same for both torsional stress levels which justified the biaxiality ratio, \( \lambda = -1 \). However, the magnitude of principal stress is greater at higher torsional stress even for the same alternating stress of 30.0 ksi. This illustrates the influence of the relative magnitude of biaxial state of stress on the fatigue behavior of the materials.

C. Application of Failure Theories

As previously presented, existing failure theories were formulated to make them relevant to the experimental investigation. Numerical calculations of the failure theories based on stress amplitude \( \sigma_a \) of 30.0 ksi, and torsional stress \( (\tau) \) of 17.5 ksi are presented below:

1. The Von-Mises Equivalent Stress Criterion

Using Equation 51, we have
\[ S_{\text{eVM}} = [(\sigma_a)^2 + 3(\tau)^2]^{\frac{1}{2}} \]
\[
\sigma = \left(30,000^2 + 3(17,510)^2\right)^{\frac{1}{2}}
\]
\[= 42.6 \text{ ksi}\]

2. Sine's Equivalent Alternating Stress Theory

Using Equation 57, we have
\[
\bar{\sigma}_{eS} = 1.7321(\sigma_a)
\]
\[= 1.7321(30.0)
\]
\[= 51.96 \text{ ksi}\]

3. Langer's Theory

Using Equation 60, we have
\[
\bar{\sigma}_{eL} = 2(\sigma_a)
\]
\[= 2(30.0)
\]
\[= 60.0 \text{ ksi}\]

4. The Naomi-Masao-Masateru Theory

Using Equation 63, we have
\[
\bar{\sigma}_{NMM} = 1.2247(\sigma_a)
\]
\[= 1.2247(30.0)
\]
\[= 36.7 \text{ ksi}\]

5. The Modified Von-Mises Criterion

Using Equation 72, we have
\[
\bar{\sigma}_{eMVM} = (\sigma_a)[3 + \left(\frac{\mu_L}{\sigma_a}\right)^2]^{\frac{1}{2}}
\]
where:
\[
\mu_L = \frac{\tau}{\sigma_a}
\]
\[= \frac{17.51}{30.0} \text{ ksi}
\]
hence,

\[ \bar{S}_{e\text{MVM}} = (30.0) \left[ 3 + \left( \frac{17.510}{30.0} \right)^2 \right]^{\frac{1}{2}} \]

\[ = 54.8 \text{ ksi} \]

The results for the other axial alternating stress levels and torsional stresses are summarized in Tables IV, VII, X and XIII. These values, together with the experimental results were plotted on \( \sigma \)-log \( N \) coordinates shown in Figures 46, 47, 48 and 49, based on Weibull's mean life.
VIII. DISCUSSION OF RESULTS

The maximum torsional stresses that could be applied without buckling the specimen were found from exploratory tests to be equivalent to a torque $T$, of 900 in-lb for Grade 12 and 600 in-lb for Grade 2. In axial push-pull, the value was found to be 45 ksi for both materials. Slightly lower values were used for the biaxial state of stress in the experimental work in order to avoid premature failure by buckling. The number of cycles to failure was taken through complete fracture of the specimen.

Test data for a biaxiality ratio of -1, torsional stress levels of 8.75 ksi and 17.5 ksi, at constant alternating stress levels of 44.0, 37.5 and 30.0 ksi for Grade 2 material are given in Tables II and V. Tables VIII and XI give the test data for Grade 12 material at torsional stresses of 14.0 ksi and 28.0 ksi. The tables also show the values of torsional strain as measured by an LVDT, while the fatigue test was done in the load control mode. These data are plotted on $\sigma$-log life coordinates as shown in Figures 26 and 27 for Grade 2 and Figures 29 and 30 for Grade 12. $\sigma$-$N_f$ curves based on Weibull's mean life were drawn through the data points.
A. Scattering of Biaxial Fatigue Life Data in Grade 2 and Grade 12

Based on the experimental results shown in Tables III, VI, IX, and XII which are plotted in Figures 26, 27, 29 and 30, we find some scatter of test data at a constant stress amplitude and for a particular torsional stress level. The scatter of data could be attributed to a variation in the preparation of the specimens, to testing variables and technique, to slight diversity in the manufacturing process of the materials, and to the influence of fatigue mechanisms.

As shown in the figures, the scatter is more pronounced at low than at high torsional stress levels in both materials, especially at a low level of axial stress amplitude. At higher applied torsional stress, scatter is not significant even at lower axial stress amplitudes, in both materials. However, in a comparison of the two materials, Grade 12 appears to show more scatter of data than Grade 2.

The influence of fatigue mechanisms on the scatter of data is known, and can be seen in their influence on the rates of fatigue crack initiation and propagation. Nevertheless, in the experimental data of this study, the scatter of the data could also be partly attributed to the magnitude of the biaxial state of stress. That is, a lesser degree of scatter was evident at higher
torsional stress levels for any given axial stress amplitude, indicating that a greater percentage of the life was spent in macrocrack propagation. On the other hand, scatter of data was more pronounced at lower torsional stress levels especially at lower axial stress amplitudes, again indicating that a greater percentage of the life was spent in microcrack initiation than at lower levels. This hypothesis is believed to be particularly appropriate for materials that cyclically harden, such as Grade 12 titanium.

The same conclusions could also be drawn from the oscilloscope traces shown in Photograph 7 which show the cycles being traced during the closing stages of one of the fatigue tests. Comparing the two photographs, it can be seen that a greater percentage of crack propagation tracing is evident at a higher magnitude of biaxial state of stress indicating that a greater percentage of biaxial fatigue life was spent in microcrack propagation, in turn resulting in reduced scatter of the data.

B. Biaxial Fatigue Life - the $\sigma$-$N_f$ Curves

The curves shown in Figures 26, 27, 28, 29, 30, and 31, show the $\sigma$-$N_f$ curves of the two materials under investigation and indicate that for a given axial stress, $\sigma_a$, the fatigue life $N_f$ increases with a decrease
in the level of superimposed torsion. This result satisfies the condition given in the Mohr's circle analysis of the biaxial state of stress namely, that a higher torsional stress results in a greater magnitude of principal stresses at a constant alternating stress level, even though the ratio of the range of principal stresses remains constant and equal to -1. Similar results have been reported for 2024-351 Al [17].

In comparing the two materials, it is seen that Grade 12 offers an enhanced level of biaxial fatigue behavior when compared with Grade 2, as shown by a higher and lower torsional stress levels. This superior performance of Grade 12 titanium could be attributed to the presence of its alloying elements.

Moreover, both the materials demonstrate higher fatigue life at 30.0 ksi alternating stress and lower torsional stresses, implying that crack initiation plays a larger part at lower torsional stress levels.

In essence, the $\sigma-N_f$ curves clearly indicate the nature of the fatigue response of the two materials in relation to the magnitude of the biaxial state of stress.

C. Hysteresis Loops

The response of the material in the form of elastic-homogeneous plastic flow was monitored on the X-Y recorder where hysteresis loops were generated during the first few cycles of the tests. These are
shown in Figure 51 and Figure 52 for Grade 2, Figure 53 and Figure 54 for Grade 12 material. As delineated in Figure 50, the stress controlled fatigue test was conducted with the stress range between $S$ and $R$. In all the cases investigated, the hysteresis loops stabilized and the material reached an equilibrium condition under the applied stress range, after approximately 10 cycles of loading. The influence of cyclic creep, a phenomenon normally associated with load controlled testing, was not evident during these first few cycles.

Measurements of the width, $PQ$, of the hysteresis loops revealed considerable contraction in the case of Grade 12 and a relatively small reduction for Grade 2 material. The contraction of the width, $PQ$, of the hysteresis loop is typical of materials that cyclically harden during fatigue, while expansion is typical of materials that cyclically soften. These results agree with those of Manson, et. al. [22], regarding the criterion for the prediction of cyclic changes. Nevertheless, hysteresis loops obtained for Grade 2 material do show some evidence of cyclic hardening, evident in a comparison of loop widths for Cycle 2 and Cycle 5. Moreover, cyclic hardening of both the materials is seen to occur at both torsional stress levels.
D. Data Correlation - the Failure Theories

Data computed for various theories were correlated with the experimental results as shown in Tables IV, VII, X and XIII. These correlations are illustrated in Figures 46, 47, 48 and 49 which show the relative positions of the $\sigma$-$N_f$ curves drawn for these theories in relation to the experimentally obtained curve.

An accurate failure theory should bring all fatigue data for different alternating and torsional stress levels into agreement. However, while the failure theories presented here do not correlate well with the biaxial fatigue data, the Naomi-Masao-Masateru failure criterion appears to show the least discrepancy at both the torsional stress levels used in the tests.

Results of the data correlation underscore the fact that these failure criteria are not entirely adequate for the prediction of failure under biaxial fatigue loading.
IX. CONCLUSIONS

The purpose of this experimental investigation was to evaluate the biaxial fatigue behavior of Grade 2 and Grade 12 titanium materials. The results show ample evidence that the two titanium materials performed differently under the biaxial fatigue loading conditions of this study.

In accordance with the statistical analysis used in evaluating the experimental results, the biaxial fatigue life data are found to be normally distributed. This conclusion is based on the fact that the four biaxial fatigue tests carried out at three different alternating stress levels plotted approximately as straight lines on Weibull's normal probability graph at both torsional stress levels.

Measurement of changes in material properties with cycling was carried out by analyzing the hysteresis curves obtained during the first ten cycles of the tests. The results show that both materials underwent cyclic hardening as evident from the contraction of the width of the curve along the strain axis. This result is in agreement with the prediction of cyclic behavior on the monotonic properties as defined by Manson's relation.

The analysis conducted with the aid of the Mohr's circle indicates the relative magnitude of the principal
stresses for the two different torsional stress levels. This allows a generalized formulation that helps define the parameters of the biaxial fatigue test. The experimental results relate this formulation to biaxial fatigue life. That is, under fully reversed loading conditions, the biaxiality ratio is equal to -1 and both materials behave in such a way that the life is dependent on the magnitude of the state of biaxial stress. As expected, a shorter life to failure is exhibited at elevated alternating stresses. The effect of torsional stress was such that a lower stress resulted in a higher fatigue life.

In accordance with the preceding, the $\sigma$-$N_f$ curves which were plotted corresponding to Weibull's mean life clearly show the comparison of biaxial fatigue lives at two different torsional stress levels. Life to failure is found to be longer at low values of torsional or principal stresses.

In comparing the two materials, it is found that the $\sigma$-$N_f$ curves of Grade 12 shift to the right, implying that low alloy titanium (Grade 12) exhibits a longer biaxial fatigue life than the commercially pure titanium (Grade 2). Thus, Grade 12 appears to possess better biaxial fatigue properties than Grade 2.

Referring to the failure theories utilized in this study, none appears to be adequately acceptable for
biaxial fatigue design purposes. However, it may be noted that the Naomi-Masao-Masateru failure criterion appears to exhibit the least discrepancy at both torsional stress levels utilized in this investigation. As such, this result reinforces the conclusion that relying on these failure criteria for prediction of fatigue life is not advisable or indeed, widely accepted in the context of biaxial fatigue. Hence, this study appears to indicate that an experimental investigation should normally be carried out for the evaluation of the biaxial fatigue behavior of titanium materials.

Finally, as a result of the design utilized for the torque fixer assembly, the axial and torsional stresses applied to the specimen may in actuality, have been somewhat lower than the values used in the calculations. However, examination of the stress-life curves does not reveal this effect, and it is assumed to be negligible in the context of this investigation.
X. FUTURE RESEARCH

This experimental investigation, together with the analytical formulation provides some basic biaxial fatigue data on titanium materials that can be utilized for advancing this research field. These could include the following:

1. Biaxial fatigue crack propagation studies in a thin-wall tubular specimen utilizing a monitoring device and a camera for photomicrography that can be moved around the tube as the crack propagates.

2. Biaxial fatigue test at elevated temperature by internal heating of tubular specimen using a Glo-bar heating element in order to investigate the behavior at high temperatures.

3. Perform the biaxial fatigue test at higher alternating stress levels to determine whether the $\sigma$-$N_f$ curves obtained at two different torsional stresses intersect, though failure by buckling of the tubes may pose a difficult problem.

4. Perform a biaxial fatigue test with non-zero mean stress in order to determine the influence of mean stress on the life.

5. Perform the biaxial fatigue test on tubular specimens with different wall thicknesses to determine the
influence of this parameter on the life.

6. Expand the experimental investigation at a single torsional stress level for both materials and check to see why the $\sigma$-$N_f$ curves of the two materials exhibit different angles tangent to the curve.

7. Conduct more tests at each alternating stress level in order to obtain a greater number of data points to help in the statistical analysis.
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XIII. APPENDICES
Appendix A

MODIFIED VON-MISES DERIVATION

\[ \sigma_2 = \frac{\mu_L (\sigma_1 - \sigma_3 + \sigma_3 + \sigma_1)}{2} \quad (a) \]

\[ \bar{S}_{eMVM} = \frac{1}{\sqrt{2}} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \quad (b) \]

Expanding and rearranging terms in Equation (b)

\[ 2 \sigma_2 [\sigma_2 - \sigma_1 - \sigma_3] = \left( \frac{\bar{S}_{eMVM} \sqrt{2}}{2} \right)^2 - \sigma_1^2 - \sigma_3^2 - (\sigma_3 - \sigma_1)^2 \]

\[ \sigma_2 = \frac{\left( \frac{\bar{S}_{eMVM} \sqrt{2}}{2} \right)^2 - \sigma_1^2 - \sigma_3^2 - (\sigma_3 - \sigma_1)^2}{2 [\sigma_2 - \sigma_1 - \sigma_3]} \quad (c) \]

Substituting Equation (a) to (c)

\[ \frac{1}{2} \left[ \mu_L (\sigma_1 - \sigma_3) + \sigma_3 + \sigma_1 \right] = \]

\[ \frac{\left( \frac{\bar{S}_{eMVM} \sqrt{2}}{2} \right)^2 - \sigma_1^2 - \sigma_3^2 - (\sigma_3 - \sigma_1)^2}{2 \left[ \frac{1}{2} (\mu (\sigma_1 - \sigma_3) + (\sigma_3 + \sigma_1) - \sigma_1 - \sigma_3 \right]} \]

Expanding, rearranging and combining terms,

\[ \mu_L^2 (\sigma_1 - \sigma_3)^2 - \sigma_1^2 - 2 \sigma_1 \sigma_3 - \sigma_3^2 = \]

\[ 2 \left( \frac{\bar{S}_{eMVM} \sqrt{2}}{2} \right)^2 - 4 \sigma_1^2 - 4 \sigma_3^2 + 4 \sigma_1 \sigma_3 \]

\[ \mu_L^2 \sigma_1^2 - 2 \mu_L^2 \sigma_1 \sigma_3 + \mu_L^2 \sigma_3^2 + 3 \sigma_1^2 - 6 \sigma_1 \sigma_3 + 3 \sigma_3^2 = \]

\[ 4 \frac{\bar{S}_{eMVM} \sqrt{2}}{2} \]

\[ \sigma_1^2 (\mu_L^2 + 3) + \sigma_3^2 (\mu_L^2 + 3) - 2 \sigma_1 \sigma_3 (\mu_L^2 + 3) = \]

\[ 4 \frac{\bar{S}_{eMVM} \sqrt{2}}{2} \]

\[ (\mu_L^2 + 3)(\sigma_1^2 - 2 \sigma_1 \sigma_3 + \sigma_3^2) = 4 \frac{\bar{S}_{eMVM} \sqrt{2}}{2} \]

therefore:
\[ \overline{S}_{\text{eMVM}}^2 = \frac{(\sigma_1 - \sigma_3)^2}{4} (\mu_L^2 + 3) \]

\[ \overline{S}_{\text{eMVM}} = \frac{\sigma_1 - \sigma_3}{2} (\mu_L^2 + 3)^{\frac{1}{2}} \]
Appendix Al

SPECIAL CASE OF GENERALIZED FORMULATION
WHEN R = 0

\[ \sigma_{1\text{max}} = \sigma_{\text{max}}/2 + ((\sigma_{\text{max}}/2)^2 + [(\tau)]^2)^{\frac{1}{2}} \]

\[ \sigma_{2\text{max}} = \sigma_{\text{max}}/2 - ((\sigma_{\text{max}}/2)^2 + [(\tau)]^2)^{\frac{1}{2}} \]

\[ \Delta \sigma_1 = \sigma_{1\text{max}} - \sigma_{1\text{min}} = \sigma_{\text{max}}/2 + ((\sigma_{\text{max}}/2)^2 + [(\tau)]^2)^{\frac{1}{2}} - (\tau) \]

\[ \Delta \sigma_2 = \sigma_{2\text{min}} - \sigma_{2\text{max}} = -(\tau) - \sigma_{\text{max}}/2 + ((\sigma_{\text{max}}/2)^2 + [(\tau)]^2)^{\frac{1}{2}} \]

\[ \lambda = \frac{\Delta \sigma_1}{\Delta \sigma_2} = \frac{\sigma_{\text{max}}/2 + ((\sigma_{\text{max}}/2)^2 + [(\tau)]^2)^{\frac{1}{2}} - (\tau)}{-(\tau) - \sigma_{\text{max}}/2 + ((\sigma_{\text{max}}/2)^2 + [(\tau)]^2)^{\frac{1}{2}}} \]

\[ = \frac{-\lambda(\tau) - \sigma_{\text{max}}/2 + \lambda((\sigma_{\text{max}}/2)^2 + [(\tau)]^2)^{\frac{1}{2}}}{\sigma_{\text{max}}/2 + ((\sigma_{\text{max}}/2)^2 + [(\tau)]^2)^{\frac{1}{2}} - (\tau)} \]

\[ = \frac{\sigma_{\text{max}}/2 (-1 - \lambda) + (\tau)((1 - \lambda))}{(1 - \lambda)((\sigma_{\text{max}}/2)^2 + [(\tau)]^2)^{\frac{1}{2}} - (1 - \lambda)((\tau))} \]

\[ = \frac{-\sigma_{\text{max}}/2 (1 + \lambda) = (1 - \lambda)((\sigma_{\text{max}}/2)^2 + [(\tau)]^2)^{\frac{1}{2}} - (1 - \lambda)((\tau))}{(1 - \lambda)((\sigma_{\text{max}}/2)^2 + [(\tau)]^2)^{\frac{1}{2}} - (\tau)} \]

\[ = \frac{\sigma_{\text{max}}/2 \left(\frac{1 + \lambda}{1 - \lambda}\right) = (\tau) - ((\sigma_{\text{max}}/2)^2 + [(\tau)]^2)^{\frac{1}{2}}}{(\sigma_{\text{a}})^2 + [(\tau)]^2)^{\frac{1}{2}} = (\tau) - \beta \sigma_{\text{a}}} \]

\[ \sigma_{\text{a}}^2 + [(\tau)]^2 = [(\tau)]^2 - 2\beta \sigma_{\text{a}}[(\tau)] + \beta^2 \sigma_{\text{a}}^2 \]

\[ 2\beta \sigma_{\text{a}}[(\tau)] = \beta^2 \sigma_{\text{a}}^2 - \sigma_{\text{a}}^2 - \sigma_{\text{a}}^2 (\beta^2 - 1) \]

\[ (\tau) = \frac{\sigma_{\text{a}}(\beta^2 - 1)}{2\beta} = \frac{\sigma_{\text{a}}(\beta^2 - 1)}{2\beta} \quad (a1) \]
Checking this with Equation 33

\[
(\tau) = \frac{1}{2} \left[ (\sigma_m/\beta)^2 - 2\sigma_a \sigma_m + (\beta \sigma_a)^2 - \sigma_{\text{min}}^2 \right]^{1/2}
\]

but \(\sigma_a = \sigma_m\), and \(\sigma_{\text{min}} = 0\) for \(R = 0\)

\[
= \frac{1}{2} \left[ \sigma_a^2/\beta^2 - 2 \sigma_a^2 + \beta \sigma_a^2 - 0 \right]^{1/2}
\]

\[
= \frac{1}{2} \left[ \frac{\sigma_a^2}{\beta^2} - 2 \beta^2 \sigma_a^2 + \beta^4 \sigma_a^2 \right]^{1/2}
\]

\[
= \sigma_a^2/(2\beta [1-2\beta^2 + \beta^4]^{1/2})
\]

\[
= \sigma_a^2/2[1-(\beta^2-1)^2]^{1/2}
\]

\[
= \sigma_a^2/2\beta (\beta^2-1)
\]

\[
= \frac{\sigma_a (\beta^2-1)}{2\beta}
\]  \hspace{1cm} (a2)

Equation (a2) verifies Equation 33 when \(R = 0\).
FIGURE 1: A Typical Schematic Diagram of A Shell and Tube Heat Exchanger
Melting temperature, 3040°F

\[ \beta \] (Beta) region

ECC (Body Centered Cubic) Structure

Beta transition temperature, 1620°F

\[ \alpha \] (Alpha) region

HCP (Hexagonal Close-Packed) Structure

Room temperature

FIGURE 2: Basic Crystal Structure of Unalloyed Titanium
FIGURE 3: A Titanium-Molybdenum Phase Diagram [37]
FIGURE 4: Beta Eutectoid Equilibrium Phase Diagram
FIGURE 5:

(a) Thin-wall tubular specimen subjected to torsion and axial load.

(b) Stressed element (A) in tube under biaxial state of stress of axial stress and torsional stress.

(c) Mohr's circle corresponding to stressed element shown in (b).
FIGURE 6: Analytical Model A: Mohr's circles of stressed element (A) under axial stress alternating from $\sigma_{\text{max}}$ to $\sigma_{\text{min}}$ with superimposed torsional stress.
FIGURE 7: Thin-wall tubular specimen grips and mounting supports used in biaxial stress fatigue test, schematic diagram
FIGURE 8: Detailed design of thin-wall tubular specimen

Original tube dimension:
outside diameter, 1
wall thickness, 0.049

Gage section:
length, 1 1/2; wall thickness, 0.018 in.
cross-sectional area, 0.0594 in.²

Scale: 1:1
Dimension: inches
Tolerances: ± 1/64, except as specified
FIGURE 9: Constant Amplitude Sinusoidal Stress-time Fatigue cycling.

(A) Completely reversed cycles, $R = -1$
(B) Nonzero mean stress, $\sigma_m \neq 0$
   \[ R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \]
(C) Zero minimum stress, $\sigma_{\text{min}} = 0$, $R = 0$
FIGURE 10: Mohr's circles of stressed element (A) using analytical Model A.
FIGURE 11: Analytical Model B, Mohr's circles of stressed element (A) under axial stress alternating from $\sigma_{\text{max}}$ to $-\sigma_{\text{max}}$ with superimposed torsional stress.
FIGURE 12: Mohr's circles of stressed element (A) indicating relative magnitude of biaxial state of stress.
Readout instrumentation (Torque—torsional stress)  

Torsion loading system

1--Drive, electric motor  
2--Torque converter  
3--Mounting frame  
4--Specimen grips  
5--Disk  
6--Torque measuring device  
7--Balance scale  
8--LVDT, transducer  
9--Schaevitz CAS Series  
10--Variable chart recorder  
11--Thin-walled tubular specimen

FIGURE 13: Torsion Schematic Diagram
FIGURE 14: Variable chart recording - strain data of torsion application for Grade 2.
FIGURE 15: Variable chart recording - strain data of torsion application for Grade 12.
Top removable hex plates

\(\frac{1}{4}\)"Drill (2 holes)

\(\frac{1}{4}\) hex

Fixed hex plate

Sliding hex plate

Threads for machine bolts (8)

Bottom plate support

"Journal bearing"

FIGURE 16: Torque Fixer Assembly Schematic Diagram
FIGURE 17: Detailed design of torque fixer assembly.
FIGURE 18: Schematic Diagram of MTS Fatigue Test Machine

1--Load Frame
2--Load Cell
3--MTS Bottom Head Mounting Grips
4--Biaxial Assembly with Mounted Tubular Specimen
5--Extensometer
6--Servo Valve
FIGURE 19: Flow Chart of Experimental General Procedure
FIGURE 20: Assembled Unit for Torsion Loading Frame Attachment
FIGURE 21: LVDT Mounting Bracket
FIGURE 22: Grip Fixtures

(a) Button head MTS grip

(b) Adapter

Material: Steel

Scale: 1:1

Tolerances: \( \frac{1}{64} \), except as specified
FIGURE 23: Detailed Design of Tubular Specimen Axial Grips

(A) Tube clamps
(B) Tube clamps support
FIGURE 24: Inner Tube Support
FIGURE 25: Threading #8 hexagonal socket set screw in Swagelok tube fitting
FIGURE 26: Biaxial Fatigue Life Curve ($\sigma-N_f$) of Grade 2 at torsional stress of 17.5 ksi indicating the data points. The curve corresponds to Weibull's mean life in Table VI.
FIGURE 27: Biaxial Fatigue Life Curve ($\sigma$-$N_f$) of Grade 2 at torsional stress of 8.75 ksi indicating the data points. The curve corresponds to Weibull's mean life in Table III.
FIGURE 28: Biaxial Fatigue Life curves ($\sigma$-N$_f$) of Grade 2 at torsional stresses of 8.75 ksi and 17.5 ksi. The curves correspond to Weibull's mean life.
FIGURE 29: Biaxial Fatigue Life Curve ($c$-$N_f$) of Grade 12 at torsional stress of 28.0 ksi indicating the data points. The curve corresponds to Weibull's mean life in Table XII.
FIGURE 30: Biaxial Fatigue Life Curve ($\sigma$-$N_f$) of Grade 12 at torsional stress of 14.0 ksi indicating the data points. The curve corresponds to Weibull's mean life in Table IX.
FIGURE 31: Biaxial Fatigue Life Curves ($\sigma$-$N_f$) of Grade 12 at torsional stresses of 14.0 ksi and 28.0 ksi. The curves correspond to Weibull’s mean life.
FIGURE 32: Weibull's probability plot for biaxial fatigue data in TABLE II at stress amplitude of 44 ksi.

\[ N_0 = 0 \text{ (assumed)} \]
\[ N_a = 2.6 \times 10^4 \]
\[ b = \tan 63.5^\circ = 2.01 \]

Weibull's mean life (\( \mu \)):
\[ = 0 + (2.6 \times 10^4 - 0) \left[ \Gamma(1 + \frac{1}{2.01}) \right] \]
\[ = (2.6 \times 10^4)(0.88623) \]
\[ = 2.3042 \times 10^4 \]

Weibull's standard deviation (SD),
\[ = (2.6 \times 10^4 - 0) \left[ \Gamma(1 + \frac{2}{2.01}) - \Gamma^2(1 + \frac{1}{2.01}) \right]^{\frac{1}{2}} \]
\[ = (2.6 \times 10^4) \left[ (1.0 - 0.7854) \right]^{\frac{1}{2}} \]
\[ = 1.20445 \times 10^4 \]

Weibull's median life
\[ = 2.23 \times 10^4 \]
FIGURE 33: Weibull's probability plot for biaxial fatigue data in TABLE II at stress amplitude of 37.5 ksi

No = (assumed)

Na = 8.2 x 10^4

b = tan 48.5° = 1.13

Weibull's mean life (M),

\[ M = 0 + (8.2 \times 10^4 - 0) \left[ \frac{1}{1.13} \right] \]

\[ = (8.2 \times 10^4) (0.95507) \]

\[ = 7.6406 \times 10^4 \]

Weibull's standard deviation (SD),

\[ = (8.2 \times 10^4 - 0) \left[ \frac{1}{1.13} + \frac{2}{1.13^2} \right] \]

\[ = (8.2 \times 10^4) \left[ (1.63 - 0.9121587) \right] \]

\[ = 5.74273 \times 10^4 \]

Weibull's median life

\[ = 6.6 \times 10^4 \]
FIGURE 34: Weibull's probability plot for biaxial fatigue data in TABLE II at stress amplitude of 30 ksi

\[ \text{No} = 0 \ (\text{assumed}) \]
\[ \text{Na} = 4.1 \times 10^5 \]
\[ b = \tan 31^\circ = 0.6 \]

Weibull's mean life \( (\mu) \),
\[ = 0 + (4.1 \times 10^5 - 0) \left[ \Gamma\left(1 + \frac{1}{b}\right) \right] \]
\[ = (4.1 \times 10^5) (1.5) \]
\[ = 6.15 \times 10^5 \]

Weibull's standard deviation (SD),
\[ = (4.1 \times 10^5 - 0) \left[ \Gamma\left(1 + \frac{2}{b}\right) - \left(\Gamma^2\left(1 + \frac{1}{b}\right) \right)^{\frac{1}{2}} \right] \]
\[ = (4.1 \times 10^5) \left[ (9.22 - 2.25) \right]^{\frac{1}{2}} \]
\[ = 1.082431 \times 10^6 \]

Weibull's median life
\[ = 2.5 \times 10^5 \]
Weibull's probability plot for biaxial fatigue data in Table V at stress amplitude of 44 ksi.

Weibull's mean life \( \mu = 0 \) (assumed).

\[ a = 1.37 \times 10^{10} \text{ ksi}, \]

\[ b = \tan 29.5^\circ = 5.4 \]

\[ R_a = 1.57 \times 10^{14} \]

\[ \sigma = (1.37 \times 10^{10}) R_a \]

Weibull's standard deviation (SD).

\[ \sigma = \sqrt{\left(1 + \frac{2}{b}\right) - \sqrt{2\left(1 + \frac{1}{b}\right)}} \]

Weibull's median life \( \mu = 0.92 \times 10^{10} \text{ ksi} \)
FIGURE 36: Weibull's probability plot for biaxial fatigue data in TABLE V at stress amplitude of 37.5 ksi

\(N_0 = 0\) (assumed)

\(\eta_a = 3.25 \times 10^4\)

\(b = \tan 62.5^\circ = 1.92\)

Weibull's mean life (\(\bar{L}\)),

\[= 0 + (3.25 \times 10^4 - 0) \left[ \Gamma(1 + \frac{1}{1.92}) \right]\]

\[= (3.25 \times 10^4)(0.88704)\]

\[= 2.8829 \times 10^4\]

Weibull's standard deviation (SD),

\[= 3.25 \times 10^4 \left[ \Gamma(1 + \frac{2}{1.92}) - \Gamma^2(1 + \frac{1}{1.92}) \right]^{\frac{1}{2}}\]

\[= (3.25 \times 10^4 \left[ (1.02 - 0.7868399) \right]^{\frac{1}{2}}\]

\[= 1.569316 \times 10^4\]

Weibull's median life

\[= 2.9 \times 10^4\]
FIGURE 37: Weibull's probability plot for biaxial fatigue data in TABLE V at stress amplitude of 30 ksi.

\[ N_0 = 0 \text{ (assumed)} \]
\[ N_a = 1.48 \times 10^5 \]
\[ b = \tan 71.0^\circ = 3.17 \]

Weibull's mean life (\( \bar{L} \)),
\[ = 0 + (1.48 \times 10^5 \text{ - 0}) \left[ \Gamma \left(1 + \frac{1}{3.17}\right) \right] \]
\[ = (1.48 \times 10^5)(0.89464) \]
\[ = 1.32407 \times 10^5 \]

Weibull's standard deviation
\[ = (1.48 \times 10^5 \text{ - 0}) \left[ \Gamma \left(1 + \frac{2}{3.17}\right) - \Gamma^2 \left(1 + \frac{1}{3.17}\right) \right]^{1/2} \]
\[ = (1.48 \times 10^5) \left[ (0.89724 - 0.800381) \right]^{1/2} \]
\[ = 4.60608 \times 10^4 \]

Weibull's median life
\[ = 1.52 \times 10^5 \]
FIGURE 38: Weibull's probability plot for biaxial fatigue data in TABLE VIII at stress amplitude of 44 ksi.

No = 0 (assumed)
Na = 1.05 x 10^5

b = tan 58° = 1.6

Weibull's mean life (t)

\[ t = 0 + (1.05 \times 10^5 - 0) \left[ \frac{\Gamma(1 + \frac{1}{1.6})}{\Gamma(1 + \frac{2}{1.6})} \right] \]

\[ = (1.05 \times 10^5)(0.89592) \]

\[ = 9.4072 \times 10^4 \]

Weibull's standard deviation (SD)

\[ = (1.05 \times 10^5 - 0) \left[ \frac{\Gamma(1 + \frac{2}{1.6})}{\Gamma(1 + \frac{1}{1.6})} - \frac{1}{2} \right] \]

\[ = (1.05 \times 10^5) \left[ (1.133 - 0.8027) \right]^{\frac{1}{2}} \]

\[ = 6.0345 \times 10^4 \]

Weibull's median life

\[ = 5.75 \times 10^4 \]
FIGURE 39: Weibull's probability plot for biaxial fatigue data in TABLE VIII at stress amplitude of 37.5 ksi

No = 0 (assumed)

Na = 2.4 \times 10^5

b = \tan 63^\circ = 1.963

Weibull's mean life (\mu),
\[
\mu = 0 + (2.4 \times 10^5 - 0) \left[ \frac{1}{1.963} \right]
\]
\[
= (2.4 \times 10^5)(0.5115)
\]
\[
= 1.2782 \times 10^5
\]

Weibull's standard deviation (SD),
\[
\sigma = (2.4 \times 10^5 - 0) \left[ \left( \frac{2}{1.963} \right)^2 - \left( \frac{1}{1.963} \right)^2 \right]^{\frac{1}{2}}
\]
\[
= (2.4 \times 10^5) \left[ (1.0075 - 0.786) \right]^{\frac{1}{2}}
\]
\[
= 1.29531 \times 10^5
\]

Weibull's median life
\[
= 2.0 \times 10^5
\]
FIGURE 40: Weibull's probability plot for biaxial fatigue data in TABLE VIII at stress amplitude of 30 ksi.

No = 0 (assumed)
Na = 8.25 x 10^5

b = tan 40° = 0.8391

Weibull's mean life (μ),

\[ μ = 0 + (8.25 \times 10^5 - 0) \left[ \Gamma(1 + \frac{1}{0.8391}) \right] \]

\[ = (8.25 \times 10^5)(1.1) \]

\[ = 9.08 \times 10^5 \]

Weibull's standard deviation (SD),

\[ \sigma = (8.25 \times 10^5 - 0) \left[ \Gamma(1 + \frac{2}{0.8392}) - \Gamma^2(1 + \frac{1}{0.8392}) \right]^{\frac{1}{2}} \]

\[ = (8.25 \times 10^5) \left[ (2.9302 - 1.201) \right]^{\frac{1}{2}} \]

\[ = 1.084 \times 10^5 \]

Weibull's median life

\[ = 5.8 \times 10^5 \]
FIGURE 41: Weibull's probability plot for biaxial fatigue data in TABLE XI at stress amplitude of 44 ksi.

No = 0 (assumed)
Na = $6.3 \times 10^4$

$b = \tan 72^\circ = 3.078$

Weibull's mean life ($\mu$),

$$= 0 + (6.3 \times 10^4 - 0) \left[ \Gamma(1 + \frac{1}{3.078}) \right]$$

$$= (6.3 \times 10^4)(0.89464)$$

$$= 5.6362 \times 10^4$$

Weibull's standard deviation (SD),

$$= (6.3 \times 10^4 - 0) \left[ \Gamma(1 + \frac{2}{3.078}) - \Gamma^2(1 + \frac{1}{3.078}) \right]^{\frac{1}{2}}$$

$$= (6.3 \times 10^4) \left[ (0.90012 - 0.8) \right]^{\frac{1}{2}}$$

$$= 1.9934 \times 10^4$$

Weibull's median life

$$= 5.7 \times 10^4$$
FIGURE 42: Weibull's probability plot for biaxial fatigue data in TABLE XI at stress amplitude of 37.5 ksi.

No = 0 (assumed)

Na = 1.15 x 10^5

b = tan 59° = 1.6643

Weibull's mean life (N):

\[ N = 0 + (1.15 \times 10^5 - 0) \left[ \frac{\Gamma(1 + \frac{2}{1.6643})}{\Gamma(1 + \frac{2}{1.5643})} \right] \]

\[ N = (1.15 \times 10^5) (0.89352) \]

\[ N = 1.02755 \times 10^5 \]

Weibull's standard deviation (SD),

\[ = (1.15 \times 10^5 - 0) \left[ \frac{\Gamma(1 + \frac{2}{1.5643})}{\Gamma(1 + \frac{2}{1.0043})} - \left( \frac{\Gamma^2(1 + \frac{2}{1.0043})}{\Gamma(1 + \frac{2}{1.0043})} \right)^{\frac{1}{2}} \right] \]

\[ = (1.15 \times 10^5) \left[ (1.178 - 0.7984) \right]^{\frac{1}{2}} \]

\[ = 7.0854 \times 10^4 \]

Weibull's median life

\[ = 9.5 \times 10^4 \]
FIGURE 43: Weibull's probability plot for biaxial fatigue data in TABLE XI at stress amplitude of 30 ksi.

\[ N_0 = 0 \text{ (assumed)} \]
\[ N_a = 3.26 \times 10^5 \]
\[ b = \tan 55.5^\circ = 1.455 \]

Weibull's mean life (\( \mu \)),
\[ \mu = N_0 + (3.2 \times 10^5 - 0) \left[ \Gamma \left(1 + \frac{1}{1.455}\right) \right] \]
\[ = (3.23 \times 10^5) (0.90678) \]
\[ = 2.97434 \times 10^5 \]

Weibull's standard deviation (SD),
\[ \sigma = (3.28 \times 10^5 - 0) \left[ \left(1 + \frac{2}{1.455}\right) - \left(1 + \frac{2}{1.455}\right)^2 \right]^{\frac{1}{2}} \]
\[ = (3.28 \times 10^5) \left[ (1.2224-0.8222) \right]^{\frac{1}{2}} \]
\[ = 2.0749 \times 10^5 \]

Weibull's median life
\[ = 2.7 \times 10^5 \]
FIGURE 44: Geometrical Representation of LVDT Connection for Torsional Strain Analysis.
FIGURE 45: Voltage to linear calibration of LVDT instrumentation.
FIGURE 46: Data Correlation of Failure theories with Experimental Results for Grade 2 at Torsional Stress (τ) = 8.75 ksi.
FIGURE 47: Data Correlation of Failure Theories with Experimental Results for Grade 2 at Torsional Stress ($\tau$) = 17.5 ksi.
FIGURE 48: Data Correlation of Failure Theories with Experimental Results for Grade 12 at Torsional Stress ($\tau$) = 14.0 ksi.
FIGURE 49: Data Correlation of Failure Theories with Experimental Results for Grade 12 at Torsional Stress ($\tau$) = 28.0 ksi.
FIGURE 50: A typical hysteresis loop showing the width PQ and stress limits RS.
FIGURE 51: Hysteresis loop of Grade 2 at low torsional stress.
FIGURE 52: Hysteresis loop of Grade 2 at high torsional stress.
FIGURE 53: Hysteresis loop of Grade 12 at low torsional stress.
FIGURE 54: Hysteresis loop of Grade 12 at high torsional stress with cycles being separately generated.
TABLE I: Titanium Chemical Compositions and Mechanical Properties

<table>
<thead>
<tr>
<th>Materials</th>
<th>Elements</th>
<th>Composition(%)</th>
<th>Mechanical Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Titanium, annealed</td>
<td>C</td>
<td>0.016</td>
<td>Yield stress, ksi 59</td>
</tr>
<tr>
<td>ASTM B-338 Specification (Grade 2)</td>
<td>Fe</td>
<td>0.12</td>
<td>Tensile strength, ksi 80</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>0.014</td>
<td>Elongation, % 31</td>
</tr>
<tr>
<td>Grade description</td>
<td>H</td>
<td>0.006</td>
<td>*Fatigue strength, ksi 46</td>
</tr>
<tr>
<td>(Ti-50A)</td>
<td>O</td>
<td>0.14</td>
<td>*Elastic Modulus, psi 14.6x10^6</td>
</tr>
<tr>
<td>Ti</td>
<td>Balance</td>
<td></td>
<td>*Shear Modulus, psi 5.6x10^6</td>
</tr>
</tbody>
</table>

| 2. Titanium, annealed | C | 0.013 | Yield stress, ksi 56 |
| ASTM B-338 Specification (Grade 12) | Fe | 0.08 | Tensile strength, ksi 85 |
| | N | 0.011 | Elongation, % 24 |
| Grade description | H | 0.007 | *Fatigue strength, ksi 50 |
| (Ti-Code 12) | O | 0.14 | *Elastic Modulus, psi 15.0x10^6 |
| Mo | 0.24 | | *Shear Modulus, psi 6.2x10^6 |
| Ni | 0.70 | | |
| Ti | Balance | | |

*Handbook values [37]
### TABLE II: Biaxial Fatigue Test Data Taken at Constant Stress Levels, $O_a$, Arranged According to Weibull's Plot

**Material: Grade 2**  
**Torsional Stress: 8.75 KSI**

<table>
<thead>
<tr>
<th>Rank (R)</th>
<th>Test No.</th>
<th>$N_f$</th>
<th>Plotting Position % ($100R/(n+1)$)</th>
<th>Best Fit Straight Line Linear Regression</th>
<th>$N_{bf}$</th>
<th>$O_a$ (KSI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12R2</td>
<td>$1.3736 \times 10^4$</td>
<td>20</td>
<td>$x=6349.5 + 320.56y$</td>
<td>$1.2761 \times 10^4$</td>
<td>44.0</td>
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<tr>
<td>2</td>
<td>12R0</td>
<td>$1.9825 \times 10^4$</td>
<td>40</td>
<td></td>
<td>$1.9172 \times 10^4$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>$2.1351 \times 10^4$</td>
<td>60</td>
<td></td>
<td>$2.5583 \times 10^4$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>12R</td>
<td>$3.4598 \times 10^4$</td>
<td>80</td>
<td></td>
<td>$3.1994 \times 10^4$</td>
<td></td>
</tr>
<tr>
<td>n=4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>13</td>
<td>$2.4068 \times 10^4$</td>
<td>20</td>
<td>$x=-28989 + 1961.2y$</td>
<td>$1.0236 \times 10^4$</td>
<td>37.5</td>
</tr>
<tr>
<td>2</td>
<td>13R</td>
<td>$4.1447 \times 10^4$</td>
<td>40</td>
<td></td>
<td>$4.9461 \times 10^4$</td>
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</tr>
<tr>
<td>3</td>
<td>13R1</td>
<td>$6.322 \times 10^4$</td>
<td>60</td>
<td></td>
<td>$8.8686 \times 10^4$</td>
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<tr>
<td>4</td>
<td>13S2</td>
<td>$1.4756 \times 10^5$</td>
<td>80</td>
<td></td>
<td>$1.2791 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td>n=4</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>15R2</td>
<td>$6.2012 \times 10^4$</td>
<td>20</td>
<td>$x=452710 + 17052y$</td>
<td>$-1.1167 \times 10^5$</td>
<td>30.0</td>
</tr>
<tr>
<td>2</td>
<td>15R3</td>
<td>$9.1284 \times 10^4$</td>
<td>40</td>
<td></td>
<td>$2.2937 \times 10^5$</td>
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<td>60</td>
<td></td>
<td>$5.7041 \times 10^5$</td>
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</tr>
<tr>
<td>4</td>
<td>15R0</td>
<td>$1.120730 \times 10^6$</td>
<td>80</td>
<td></td>
<td>$9.1145 \times 10^5$</td>
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## TABLE III: Biaxial Fatigue Test Data

Material: Grade 2  
Biaxiality ratio ($\lambda$): -1  
Torsional strain ($\gamma$): 0.23%  
Load Control Fatigue Test  
Applied Torque ($T$): 260 in-lb  
Frequency: 10 Hz  
Waveform: sinusoidal

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Axial Stress, ksi</th>
<th>Cycles to Failure, $N_f$</th>
<th>Weibull's Statistical Parameters, cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{max}$</td>
<td>$\sigma_{min}$</td>
<td>$\sigma_m$</td>
</tr>
<tr>
<td>12</td>
<td>44</td>
<td>-44</td>
<td>0</td>
</tr>
<tr>
<td>12R</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12R0</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>12R2</td>
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<td></td>
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<tr>
<td>13</td>
<td>37.5</td>
<td>-37.5</td>
<td>0</td>
</tr>
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<td>13R1</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>13R</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13R2</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>15R0</td>
<td>30</td>
<td>-30</td>
<td>0</td>
</tr>
<tr>
<td>15R1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15R2</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>15R3</td>
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</table>
TABLE IV: Failure Criteria--Biaxial Fatigue Data Correlation

<table>
<thead>
<tr>
<th>Material: Grade 2</th>
<th>Torsional Strain ($\gamma$): 0.23 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biaxiality Ratio ($\lambda$): -1</td>
<td>Load Control Fatigue Test</td>
</tr>
<tr>
<td>Applied Torque ($T$): 260 in-lb</td>
<td>Frequency: 10 Hz</td>
</tr>
<tr>
<td>Torsional Stress ($\tau$): 8.75 ksi</td>
<td>Waveform: sinusoidal</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Constant alternating stress amplitude ($\sigma_a$, ksi)</th>
<th>-Failure criteria-equivalent stress, ksi</th>
<th>Weibull's mean ($\mu$), cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>44</td>
<td>46.5</td>
<td>76.2</td>
</tr>
<tr>
<td>B</td>
<td>37.5</td>
<td>40.4</td>
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</tr>
<tr>
<td>C</td>
<td>30</td>
<td>33.6</td>
<td>52.0</td>
</tr>
</tbody>
</table>

Failure criteria no.

1 = Von-Mises Equivalent Stress, $S_{eVM}$
2 = Sine's Equivalent Alternating Stress, $S_{eS}$
3 = Langer's Theory, $S_{eL}$
4 = Naomi-Masao-Masateru Theory, $\Delta S_{NMM}$
5 = Modified Von-Mises, $S_{eMVM}$
TABLE V: Biaxial Fatigue Test Data Taken at Constant Stress Levels, $g_a$, Arranged According to Weibull's plot

Material: Grade 2  

<table>
<thead>
<tr>
<th>Rank (R)</th>
<th>Test No.</th>
<th>No. of Cycles to Failure $N_f$</th>
<th>Plotting Position % $(100)R/(n+1)$</th>
<th>Best Fit Straight Line Linear Regression</th>
<th>No. of Cycles for Best Fit $N_{fbf}$</th>
<th>$g_a$ (KSI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4R</td>
<td>$1.0515 \times 10^4$</td>
<td>20</td>
<td>$x=9426 + 66.995y$</td>
<td>$1.0766 \times 10^4$</td>
<td>44.0</td>
</tr>
<tr>
<td>2</td>
<td>9R2</td>
<td>$1.2522 \times 10^4$</td>
<td>40</td>
<td>$x=9416 + 66.995y$</td>
<td>$1.2106 \times 10^4$</td>
<td>44.0</td>
</tr>
<tr>
<td>3</td>
<td>1R0</td>
<td>$1.3366 \times 10^4$</td>
<td>60</td>
<td>$x=9426 + 66.995y$</td>
<td>$1.3446 \times 10^4$</td>
<td>44.0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>$1.4700 \times 10^4$</td>
<td>80</td>
<td>$x=9446 + 66.995y$</td>
<td>$1.4786 \times 10^4$</td>
<td>44.0</td>
</tr>
<tr>
<td>n=4</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3R2</td>
<td>$1.6200 \times 10^4$</td>
<td>20</td>
<td>$x=9343 + 366.73y$</td>
<td>$1.6700 \times 10^4$</td>
<td>37.5</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>$2.595 \times 10^4$</td>
<td>40</td>
<td>$x=9343 + 366.73y$</td>
<td>$2.4012 \times 10^4$</td>
<td>37.5</td>
</tr>
<tr>
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<td>3R0</td>
<td>$2.886 \times 10^4$</td>
<td>60</td>
<td>$x=9343 + 366.73y$</td>
<td>$3.1350 \times 10^4$</td>
<td>37.5</td>
</tr>
<tr>
<td>4</td>
<td>3R3</td>
<td>$3.9673 \times 10^4$</td>
<td>80</td>
<td>$x=9343 + 366.73y$</td>
<td>$3.8680 \times 10^4$</td>
<td>37.5</td>
</tr>
<tr>
<td>n=4</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5R</td>
<td>$8.2315 \times 10^4$</td>
<td>20</td>
<td>$x=51094.5 + 1519y$</td>
<td>$8.1480 \times 10^4$</td>
<td>30.0</td>
</tr>
<tr>
<td>2</td>
<td>10R</td>
<td>$1.06316 \times 10^5$</td>
<td>40</td>
<td>$x=51094.5 + 1519y$</td>
<td>$1.12000 \times 10^5$</td>
<td>30.0</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>$1.50827 \times 10^5$</td>
<td>60</td>
<td>$x=51094.5 + 1519y$</td>
<td>$1.42000 \times 10^5$</td>
<td>30.0</td>
</tr>
<tr>
<td>4</td>
<td>10R1</td>
<td>$1.68757 \times 10^5$</td>
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<td>$x=51094.5 + 1519y$</td>
<td>$1.72600 \times 10^5$</td>
<td>30.0</td>
</tr>
<tr>
<td>n=4</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE VI: Biaxial Fatigue Test Data

Material: Grade 2  
Biaxiality ratio (\(\lambda\)): -1  
Biaxiality ratio (\(\lambda\)): -1  
Applied Torque (\(T\)): 525 in-lb  
Torsional Stress (\(\tau\)): 17.50 ksi  
Torsional strain (\(\gamma\)): 0.48%  
Load Control Fatigue Test  
Frequency: 10 Hz  
Waveform: Sinusoidal

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Axial Stress, ksi</th>
<th>Cycles to Failure, (N_f)</th>
<th>Weibull's Statistical Parameters, cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\sigma_{max})</td>
<td>(\sigma_{min})</td>
<td>(\sigma_m)</td>
</tr>
<tr>
<td>1</td>
<td>44</td>
<td>-44</td>
<td>0</td>
</tr>
<tr>
<td>A</td>
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<tr>
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</tr>
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</tr>
<tr>
<td>3</td>
<td>37.5</td>
<td>-37.5</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
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</tr>
<tr>
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<td></td>
</tr>
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</tr>
<tr>
<td>5R</td>
<td>30</td>
<td>-30</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td></td>
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</tr>
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</tbody>
</table>
TABLE VII: Failure Criteria--Biaxial Fatigue Data Correlation

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Constant alternating stress amplitude ($\sigma_a$), ksi</th>
<th>-Failure criteria-equivalent stress, ksi</th>
<th>Weibull's mean ($\mu$), cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>44</td>
<td>53.4 76.2 88.0 53.9 78.2</td>
<td>$1.2618 \times 10^4$</td>
</tr>
<tr>
<td>B</td>
<td>37.5</td>
<td>48.2 65.0 75.0 45.9 67.3</td>
<td>$2.8829 \times 10^4$</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>42.6 52.0 60.0 36.7 54.8</td>
<td>$1.32407 \times 10^5$</td>
</tr>
</tbody>
</table>

Failure criteria no.

1 = Von-Mises Equivalent Stress, $\bar{S}_{eVM}$
2 = Sine's Equivalent Alternating Stress, $\bar{S}_{eS}$
3 = Langer's Theory, $\bar{S}_{eL}$
4 = Naomi-Masao-Masateru Theory, $\Delta \bar{S}_{NMM}$
5 = Modified Von-Mises, $\bar{S}_{eEMVM}$
<table>
<thead>
<tr>
<th>Rank (R)</th>
<th>Test No.</th>
<th>No. of Cycles to Failure $N_f$</th>
<th>Plotting Position % $(100)R/(n + 1)$</th>
<th>Best Fit Straight Line Linear Regression</th>
<th>No. of Cycles for Best Fit $N_fB_f$</th>
<th>$\sigma_a$ (KSI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TC12R3</td>
<td>$9.6751 \times 10^4$</td>
<td>20</td>
<td>$x = -56364 + 3009.8y$</td>
<td>$1.1656 \times 10^5$</td>
<td>37.5</td>
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<tr>
<td>2</td>
<td>TC12R1</td>
<td>$2.04634 \times 10^5$</td>
<td>40</td>
<td></td>
<td>$1.7675 \times 10^5$</td>
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</tr>
<tr>
<td>3</td>
<td>TC12</td>
<td>$2.40616 \times 10^5$</td>
<td>60</td>
<td></td>
<td>$2.3695 \times 10^5$</td>
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<tr>
<td>4</td>
<td>TC12R2</td>
<td>$2.85409 \times 10^5$</td>
<td>80</td>
<td></td>
<td>$2.9715 \times 10^5$</td>
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</tr>
<tr>
<td>n=4</td>
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<td></td>
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</tr>
<tr>
<td>1</td>
<td>TC13R1</td>
<td>$1.56081 \times 10^5$</td>
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<td>$x = -324220 + 20395y$</td>
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<td>30.0</td>
</tr>
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<td>TC13R4</td>
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<td>$4.9159 \times 10^5$</td>
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</tr>
<tr>
<td>3</td>
<td>TC13</td>
<td>$9.51747 \times 10^5$</td>
<td>60</td>
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<td>$8.9949 \times 10^5$</td>
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<td>TC13R2</td>
<td>$1.317460 \times 10^6$</td>
<td>80</td>
<td></td>
<td>$1.3074 \times 10^6$</td>
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</tbody>
</table>
TABLE IX: Biaxial Fatigue Data

Material: Grade 12
Biaxiality ratio ($\lambda$): -1
Applied Torque ($T$): 420 in-lb
Torsional Stress ($T$): 14.0 ksi
Torsional strain ($\gamma$): 0.21 %
Load Control Fatigue Test
Frequency: 10 Hz
Waveform: sinusiodal

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Axial Stress, ksi</th>
<th>Cycles to Failure, $N_f$</th>
<th>Weibull's Statistical Parameters, cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{max}$</td>
<td>$\sigma_{min}$</td>
<td>$\sigma_m$</td>
</tr>
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<td>-44</td>
<td>0</td>
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<tr>
<td>TCW11R2</td>
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</tr>
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<td>TCW1R4</td>
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<td>-37.5</td>
<td>0</td>
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<tr>
<td>TCW12R2</td>
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<tr>
<td>TCW12R3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TCW13</td>
<td>30</td>
<td>-30</td>
<td>0</td>
</tr>
<tr>
<td>TC13R1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TC13R2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TC13R4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE X: Failure Criteria--Biaxial Fatigue Data Correlation

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Constant alternating stress amplitude ($\sigma_a$), ksi</th>
<th>-Failure criteria-equivalent stress, ksi</th>
<th>Weibull's mean ($\mu$), cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>A</td>
<td>44</td>
<td>50.2</td>
<td>76.2</td>
</tr>
<tr>
<td>B</td>
<td>37.5</td>
<td>44.7</td>
<td>65.0</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>38.6</td>
<td>52.0</td>
</tr>
</tbody>
</table>

Failure criteria no.

1 = Von-Mises Equivalent Stress, $S_{eVM}$
2 = Sine's Equivalent Alternating Stress, $S_{eS}$
3 = Langer's Theory, $S_{eL}$
4 = Naomi-Masao-Masateru Theory, $\Delta S_{NMM}$
5 = Modified Von-Mises, $S_{eMVM}$
TABLE XI: Biaxial Fatigue Test Data Taken at Constant Stress Levels, $\sigma_a$, Arranged According to Weibull's Plot

Materials: Grade 12  
Torsional Stress: 28.0 ksi

<table>
<thead>
<tr>
<th>Rank (R)</th>
<th>Test No.</th>
<th>$x$ No. of Cycles to Failure $N_f$</th>
<th>$y$ Plotting Position % $(100)R_0$</th>
<th>Best Fit Straight Line Linear Regression</th>
<th>$y$ No. of Cycles for Best Fit $N_{bf}$</th>
<th>$\sigma_a$ (KSI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TC1R2</td>
<td>$3.8424 \times 10^4$</td>
<td>20</td>
<td>$x=26561 + 611.43y$</td>
<td>$3.879 \times 10^4$</td>
<td>44.0</td>
</tr>
<tr>
<td>2</td>
<td>TC1R</td>
<td>$4.8879 \times 10^4$</td>
<td>40</td>
<td>$x=13351 + 1719.9y$</td>
<td>$4.7748 \times 10^4$</td>
<td>37.5</td>
</tr>
<tr>
<td>3</td>
<td>TC1</td>
<td>$6.8622 \times 10^4$</td>
<td>60</td>
<td>$x=23279 + 5003.8y$</td>
<td>$1.2335 \times 10^5$</td>
<td>30.0</td>
</tr>
<tr>
<td>4</td>
<td>TC1R1</td>
<td>$7.2605 \times 10^4$</td>
<td>80</td>
<td>$x=23279 + 5003.8y$</td>
<td>$4.2358 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td>n=4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total

$\sigma_a = \frac{\sum N_{bf} \sigma_a}{n}$
TABLE XII: Biaxial Fatigue Test Data

Material: Grade 12  
Biaxiality ratio ($\lambda$): -1  
Applied Torque ($T$): 840 in-lb  
Torsional Stress ($\tau$): 28.0 ksi  
Torsional strain ($\gamma$): 0.58 %  
Load Control Fatigue Test  
Frequency: 10 Hz  
Waveform: sinusoidal

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Axial Stress, ksi</th>
<th>Cycles to Failure, $N_f$</th>
<th>Weibull's Statistical Parameters, cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{max}$</td>
<td>$\sigma_{min}$</td>
<td>$\sigma_{med}$</td>
</tr>
<tr>
<td>TC1</td>
<td>44</td>
<td>-44</td>
<td>0</td>
</tr>
<tr>
<td>TC1R</td>
<td>4.8879 x 10⁴</td>
<td>0</td>
<td>44</td>
</tr>
<tr>
<td>TC1R1</td>
<td>7.2605 x 10⁴</td>
<td>0</td>
<td>44</td>
</tr>
<tr>
<td>TC1R2</td>
<td>3.8424 x 10⁴</td>
<td>0</td>
<td>44</td>
</tr>
<tr>
<td>TC2R</td>
<td>37.5</td>
<td>-37.5</td>
<td>0</td>
</tr>
<tr>
<td>TC2R2</td>
<td>1.44375 x 10⁵</td>
<td>0</td>
<td>37.5</td>
</tr>
<tr>
<td>TC2R3</td>
<td>3.7465 x 10⁴</td>
<td>0</td>
<td>37.5</td>
</tr>
<tr>
<td>TC2R4</td>
<td>9.6146 x 10⁴</td>
<td>0</td>
<td>37.5</td>
</tr>
<tr>
<td>TC3R</td>
<td>30</td>
<td>-30</td>
<td>0</td>
</tr>
<tr>
<td>TC3R1</td>
<td>2.60435 x 10⁵</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>TC3R2</td>
<td>1.40414 x 10⁵</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>TC3R3</td>
<td>4.63647 x 10⁵</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>Test No.</td>
<td>Constant alternating stress amplitude ($\sigma_a$), ksi</td>
<td>Failure criteria-equivalent stress, ksi</td>
<td>Weibull's mean ($\mu$), cycles</td>
</tr>
<tr>
<td>---------</td>
<td>---------------------------------------------------</td>
<td>--------------------------------------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>A</td>
<td>44</td>
<td>65.5</td>
<td>76.2</td>
</tr>
<tr>
<td>B</td>
<td>37.5</td>
<td>61.3</td>
<td>65.0</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>57.0</td>
<td>52.0</td>
</tr>
</tbody>
</table>

Failure criteria no.

1 = Von-Mises Equivalent Stress, $\bar{\sigma}_{eVM}$

2 = Sine's Equivalent Alternating Stress, $\bar{\sigma}_{eS}$

3 = Langer's Theory, $\bar{\sigma}_{eL}$

4 = Naomi-Masao-Masateru Theory, $\Delta \bar{\sigma}_{NMM}$

5 = Modified Von-Mises, $\bar{\sigma}_{eMVM}$
Appendix D

PHOTOGRAPHS

PHOTOGRAPH 1: Unfractured and Fractured Thin-Wall Tubular Specimen
PHOTOGRAPH 2: Torsion Machine and Experimental Set-Up
PHOTOGRAPH 3: Torsion Fixer
PHOTOGRAPH 4: MTS Machine and Experimental Set-Up
PHOTOGRAPH 5: Assembled Unit
PHOTOGRAPH 6: Unit Mounted in MTS Machine
Low Torsional Stress

High Torsional Stress

PHOTOGRAPH 7: Oscilloscope Generated Cycles and Waveform