Robust underwater visibility parameter

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Abstract: We review theoretical models to show that contrast reduction at a specific wavelength in the horizontal direction depends directly on the beam attenuation coefficient at that wavelength. If a black target is used, the inherent contrast is always negative unity, so that the visibility of a black target in the horizontal direction depends on a single parameter only. That is not the case for any other target or viewing arrangement. We thus propose the horizontal visibility of a black target to be the standard for underwater visibility. We show that the appropriate attenuation coefficient can readily be measured with existing simple instrumentation. Diver visibility depends on the photopic beam attenuation coefficient, which is the attenuation of the natural light spectrum convolved with the spectral responsivity of the human eye (photopic response function). In practice, it is more common to measure the beam attenuation coefficient at one or more wavelength bands. We show that the relationship: visibility is equal to 4.8 divided by the photopic beam attenuation coefficient; originally derived by Davies-Colley [1], is accurate with an average error of less than 10% in a wide variety of coastal and inland waters and for a wide variety of viewing conditions. We also show that the beam attenuation coefficient measured at 532 nm, or attenuation measured by a WET Labs commercial 20 nm FWHM transmissometer with a peak at 528nm are adequate substitutes for the photopic beam attenuation coefficient, with minor adjustments.

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References and Links

1. R. J. Davies-Colley, "Measuring water clarity with a black disk," Limnol. Oceangr. **33**, 616-623 (1988).
2. R.W. Preisendorfer, *Hydrologic Optics Vol 1,* NOAA,*(*1976).
3. S.Q. Duntley, "Light in the Sea," J. Opt. Soc. Am. **53**, 214-233 (1963).
4. N.G. Jerlov, *Marine Optics,* (Elsevier, Amsterdam, 1976).
5. J.N. Lythgoe, "Vision," in *Underwater science*, J.D. Woods and J.N. Lythgoe (eds.), (Oxford University Press, London, 1971).
6. C.D. Mobley, *Light and Water*, (Academic Press, San Diego, CA 1994).
7. R.W. Preisendorfer, "Secchi disk science: Visual optics of natural waters," Limnol. Oceanogr. **31**, 909-926 (1986).
8. M. S. Twardowski, E. Boss, J. B. MacDonald, W. S. Pegau, A. H. Barnard, and J. R. V. Zaneveld, "A model for estimating bulk refractive index from the optical backscattering ratio and the implications for understanding particle composition in case I and case II waters," J. Geophys. Res. **106**, 14129-14142 (2001).
9. R.M. Pope and E.S. Fry, "Absorption spectrum (380- 700 nm) of pure water. II Integrating cavity measurements.," Appl. Opt. **36**, 8710- 8723 (1997).
10. H.R. Blackwell, "Contrast thresholds of the human eye," J. Opt. Soc. Am. **36**, 624-643 (1946).
11. A.H.Barnard, W.S.Pegau, and J.R.V. Zaneveld, "Global relationships of the inherent optical properties of the oceans". J.Geophys. Res. **103**(C11) 24,955- 24,968 (1998)
12. K.J. Voss, "A spectral model of the beam attenuation coefficient in the ocean and coastal areas," Limnol. Oceangr. **37**, 501-509 (1992).

13. T.E. Bowers, "Visbility for general viewing direction of objects submerged in vertically inhomogeneous waters," (Ocean Optics Conference 2002).

1. Introduction

Special Operations and Mine Warfare require the prediction of visibility for divers and cameras using ambient (natural) light. Visibility is a rather poorly defined concept that can mean many things. It ultimately predicts the ability of some observer (human or instrumental) to detect some object in a given environment. The resolution of the visibility problem can thus be very complex. In general it can be stated that if all the following are known: 1) all the characteristics of the target (size, shape, spectral reflectivity, markings, etc.) ; 2) the optics of the detector; 3) the inherent optical properties (spectral directional light scattering, absorption, and fluorescence characteristics) of the medium and the complete reflectance characteristics of the bottom; and 4) the external lighting conditions of the medium, one can, by use of classical radiative transfer, combined with Fourier optics, predict with great accuracy what a given object will look like in a given detector system at a given distance. The number of parameters that enter in such calculations however, are very large, far too large to be of use in operational diver situations. While, in theory, solutions to such problems can be obtained, the large number of parameters involved, of which a number must be guessed, guarantee that the solution will not likely be close to reality (see Table 1 in the discussion section). Such a complete solution is thus not appropriate for conflict situations, where quick deployment decisions must be made.

What is needed is a simple but accurate approach to visibility of objects that would include mines and divers. Visibility of these objects in typical situations is not limited by the angle they subtend. For such objects in ambient lighting conditions it is appropriate to look at the contrast reduction as a means to describe visibility. That is the approach taken in this paper. Once one bypasses Fourier optics (visibility of objects based on modulation transfer functions), however, one can no longer expect to be able to predict visibility of small features, such as letters, numbers, or other small scale details on objects. Only the detectibility of the objects themselves is analyzed. We will show below that the horizontal visibility of a black target meets all the requirements for a robust underwater visibility parameter.

2. Background

Starting in World War II and continuing until the mid 1970's the U.S.Navy extensively funded research in visibility. This effort laid the ground work for much of Ocean Optics as we know it today. The works of Preisendorfer [2] and Duntley [3] are legendary and are still frequently used today. Their visibility equations were derived directly from the equation of radiative transfer and are presented below. In the review below we often reference Preisendorfer [2], Duntley [3], and the review by Jerlov [4]. These references summarize much of the decades long theoretical and experimental work at the former Visibility Laboratory of the Scripps Institute of Oceanography. We use the IAPSO nomenclature of Jerlov [4] in this paper, which are now most commonly used. The reader should be aware that different conventions are used in some of the references leading to some sign differences.

A fundamental law of visibility as derived by Duntley [3], Jerlov [4], and Preisendorfer [2], is that the difference of the target and background radiances at a given wavelength attenuates as e^{-cr}, where c is the beam attenuation coefficient at that wavelength, and r is the range from the observer to the target. The derivation of this law follows directly from the equation of radiative transfer, and is presented below:

The equation of radiative transfer for a plane parallel medium without internal sources at a specific wavelength is given by:

$$\cos(\theta)\frac{dL(\theta,\phi,z)}{dz} = -c(z)\,L(\theta,\phi,z) + \int_0^{2\pi}\int_0^{\pi} \beta(\theta,\phi,\theta',\phi',z)L(\theta',\phi',z)\,\sin\theta'\,d\theta'\,d\phi' \quad (1)$$

we can replace the last term by $L*(\theta,\phi,z)$. This is the so-called path function. We then get:

$$\cos(\theta)\frac{dL(\theta,\phi,z)}{dz} = - c(z) L(\theta,\phi,z) + L*(\theta,\phi,z). \tag{2}$$

For the target radiance $L_T(\theta,\phi,z)$ and the adjacent background radiance $L_B(\theta,\phi,z)$ we can write the following equations, assuming c is a constant and the path functions for the adjacent target and background radiances are the same:

$$\cos(\theta)\frac{dL_T(\theta,\phi,z)}{dz} = - c L_T(\theta,\phi,z) + L*(\theta,\phi,z). \tag{3}$$

$$\cos(\theta)\frac{dL_B(\theta,\phi,z)}{dz} = - c L_B(\theta,\phi,z) + L*(\theta,\phi,z). \tag{4}$$

We take the difference of Eqs. (3) and (4):

$$\cos(\theta)\frac{d[L_T(\theta,\phi,z)- L_B(\theta,\phi,z)]}{dz} = - c [L_T(\theta,\phi,z) - L_B(\theta,\phi,z)] . \tag{5}$$

Integration of Eq. (5) along a line of sight from r'=0 at the target to r'=r at the observer gives (note that $r = z/\cos\theta$ and that $dr = dz/\cos\theta$):

$$[L_{Tr}(\theta,\phi,z) - L_{Br}(\theta,\phi,z)] = [L_{T0}(\theta,\phi,z_T) - L_{B0}(\theta,\phi,z_T)] \exp(- cr). \tag{6}$$

The equation of radiative transfer thus shows that the difference of the target and background radiances attenuates as e^{-cr}. This result was obtained by Duntley [3], Jerlov [4], and Preisendorfer [2].

3. The Visibility Laboratory contrast model

The contrast used by Preisendorfer, Duntley, and Jerlov is the visibility contrast defined by

$$C_v = \frac{L_T(\theta,\phi,z)- L_B(\theta,\phi,z)}{L_B(\theta,\phi,z)} . \tag{7}$$

A combination of Eqs. (7) and (6) shows that we may write [2,4]:

$$\frac{C_{vr}(\theta,\phi,z)}{C_{v0}(\theta,\phi,z_T)} = \exp(-cr) \frac{L_{B0}(\theta,\phi,z_T)}{L_{Br}(\theta,\phi,z)} . \tag{8}$$

The background radiance has an attenuation coefficient defined by:

$$K_B(\theta,\phi,z) = \frac{-1}{L_B(\theta,\phi,z)} \frac{dL_B(\theta,\phi,z)}{dz} , \tag{9}$$

so that

$$\frac{L_{B0}(\theta,\phi,z_T)}{L_{Br}(\theta,\phi,z)} = \exp [K_B(\theta,\phi,z)z] = \exp [K_B(\theta,\phi,z) r \cos\theta], \tag{10}$$

and

$$\frac{C_{vr}(\theta,\phi,z)}{C_{v0}(\theta,\phi,z_T)} = \exp[-cr + K_B(\theta,\phi,z) r \cos\theta] \tag{11}$$

Equation (11) was derived by Preisendorfer [2], Duntley [3], and Jerlov [4] and is the fundamental contrast model. It was extensively tested by Duntley and his group, and was found to be remarkably robust based on very extensive experimental work. Note that the

experimental work did not include use of a monochromatic beam attenuation measurement but rather of a relatively broad band green light source meter. The effects of the wavelength dependence of the attenuation measurement will be discussed in detail below. Suffice it to say for now that Eq. (11) was tested extensively and found to be applicable within reasonable bounds when using broad band green attenuation meters.

Duntley [3] sums up decades of underwater visibility experiments by stating:

" Along an underwater path of sight a remarkable proportion of the objects ordinarily encountered can be seen at limiting ranges between 4 and 5 times the distance $1/[c-K(\theta,\phi,z) \cos\theta]$, regardless of their size or the background against which they appear, providing ample daylight prevails."

For horizontal contrast reduction $\cos\theta=0$ in Eq. 11, so that

$$\frac{C_{vr}(\pi/2,\phi,z_T)}{C_{v0}(\pi/2,\phi,z_T)} = \exp[-cr] \qquad (12a)$$

Blackwell [10] found that there was a limiting contrast for human beings, that was reasonably constant. We will set this limit $= C_L$. Note that this limiting contrast can be either positive or negative depending on whether the target is brighter or darker than the background. In our case the target will always be darker than the background, so we take the limiting contrast to be a negative number. It follows from Eq. (7) that the inherent contrast of a black target is –1. For a black target the observed contrast will change from –1 to zero as one moves away from the target. When the contrast reaches C_L, the target can no longer be distinguished from the background by a human observer. Substitution of these parameters into Eq. (12a) gives the visibility range of a black target for monochromatic light:

$$\text{visibility range} = - (1/c)\ln |C_L| \qquad (12b)$$

The horizontal visibility range of a black target is thus predicted to be inversely proportional to the beam attenuation coefficient of the monochromatic light. Experimental results by Lythgoe [5] and Davies-Colley [1] showed an excellent relationship between horizontal sighting range of a black 200 mm diameter disk and c, where c was measured with a white light source transmissometer equipped with a Wratten #61 green filter, which approximates the photopic (human eye sensitivity) response function. The slope Ψ of the visibility range versus photopic c was found to be 4.8 with little curvature in the relationship. The small dependence of Ψ on c was also determined by Davies-Colley [1]. When the two linear relationships were taken into account it was found that c could be determined from the visibility range to an accuracy of 8%. Inverting this relationship would thus seem to indicate that the visibility range of a black target in the horizontal direction can be predicted from c to an accuracy of better than 10 %. This would seem to be more than sufficient for operational situations.

We should take the extensive theoretical and experimental work of Preisendorfer and Duntley, into account when designing a "simple" visibility parameter. All contrast reduction depends to first order on c, and only on c in the horizontal direction. When viewing in directions other than the horizontal, the $K(\theta,\phi,z)\cos\theta$ term plays a role. Near the surface $K(\theta,\phi,z)$ depends strongly on direction, but at greater depths can be approximated by asymptotic K, the irradiance attenuation coefficient. Note that for looking vertically down, we get the well known Secchi depth dependence on c + K.

There is thus ample historic evidence to suggest that the horizontal visibility of a black target, y, is governed by a simple law y = 4.8/c, where c is a green light attenuation measurement. The horizontal visibility of a black target would thus be an ideal underwater visibility standard as it depends on one parameter only. The remainder of the paper will examine this parameter further.

4. Photopic versus monochromatic attenuation coefficients

Equation (12) was strictly valid only for monochromatic light, as it was derived from Eq. (1), yet when used by experimentalists it was found to apply to broad band green light sources as well. This observation deserves further analysis. The eye perceives photopic parameters, that is, it observes light spectra convolved with the spectral sensitivity of the human eye. Photopic quantities are described in various tomes on Photometry (see for example Mobley [6] chapter 2). Suffice it to say here that we wish to determine if the photopic equivalent of Eq. (11) is valid.

Rewriting Eq. (11) for photopic quantities when viewing a black target with inherent contrast equal to -1, in the horizontal direction, yields:

$$C_{vr} = \frac{N_T(r) - N_B(r)}{N_B(r)} = - \exp[-\alpha\, r] \qquad (13a)$$

and hence, using similar arguments used to obtain Eq. 12b from 12a, we get:

$$\text{visibility range} = y = - (1/\alpha) \ln |C_L|. \qquad (13b)$$

We have simplified the notation, so that $N_T(r)$ is the photopic radiance (luminance) of the target a horizontal distance r from the target, and $N_B(r)$ is the photopic radiance of the background. α is the attenuation coefficient of the image forming light. This notation is the original Visibility Laboratory notation and seems appropriate here. All parameters in Eq. (13) refer to photopic quantities. What is the meaning of contrast for a photopic receiver such as the human eye? What is the meaning of a photopic beam attenuation coefficient?

Equation (13a) can be rewritten as:

$$N_T(r) = N_B\, [1 - \exp(-\alpha\, r)] \qquad (14)$$

The background radiance is not a function of r as we assumed it to be constant at a given depth in a given direction. (This is the fundamental plane parallel assumption of Eq. (1).)

Equation (14) is so far only postulated to be true for the image forming light, but it is true for monochromatic light as we saw in Eq. (12) :

$$L_T(\lambda, r) = L_B(\lambda)\, [1 - \exp(-c(\lambda)r)] \qquad (15)$$

We now need to reconcile Eqs. (14) and (15) by deriving the dependence of α on $c(\lambda)$ and the radiances.

From the definition of luminance [6] we find that:

$$N_T(r) = K_m \int_{400}^{700} L_T(r, \lambda) Y(\lambda)\, d\lambda \qquad (16)$$

and similarly for $N_B(r)$. $Y(\lambda)$ is the photopic luminosity function, which describes the relative sensation of brightness perceived by the human eye, when illuminated by light with the same radiance, but at different wavelengths. K_m is the maximum luminous efficacy. Substitution into Eq. (15) yields:

$$N_T(r) = K_m \int_{400}^{700} L_T(r, \lambda) Y(\lambda)\, d\lambda = K_m \int_{400}^{700} L_B(\lambda) Y(\lambda)\, [1 - \exp(-c(\lambda)r)] d\lambda \qquad (17)$$

A combination of Eqs. (17) and (12) shows that:

$$C_{vr} = \frac{N_T(r) - N_B(r)}{N_B(r)} = - \exp[-\alpha \, r] = \int_{400}^{700} L_B(\lambda) Y(\lambda) \, [-\exp(-c(\lambda)r)] d\lambda \; [\int_{400}^{700} L_B(\lambda) Y(\lambda) \, d\lambda \;]^{-1}$$

$$(18)$$

From Eq. (18) we see that the photopic beam attenuation coefficient, α, is a function of the spectral background (horizontal) radiance at a given depth, $L_B(\lambda)$, as well as the distance between the target and the observer, r. Provided α is defined as in Eq. (18), Eqs. (14) and (12) are compatible. It is clear that for a spectral coefficient such as α, Beer's law does not strictly apply. Nor does it for any non-monochromatic attenuation coefficient. It is therefore impossible to construct a photopic beam attenuation meter that precisely reproduces the effects of Eq. (18).

If one assumes that the background radiance is uniform with wavelength (Duntley [3] specified "ample daylight") and set its photopic luminosity equal to one (units of lumen), one can write the following equation for a uniform spectral light photopic attenuation coefficient, α_U :

$$\alpha_U = - (1/r) \ln \{ \int_{400}^{700} Y(\lambda) \, [-\exp(-c(\lambda)r)] d\lambda \; [\int_{400}^{700} Y(\lambda) \, d\lambda \;]^{-1} \}$$ (19)

Equation (19) shows how to construct a photopic α_U -meter: One puts a photopic filter in front of a spectrally flat white light source and measures the attenuation. It is for this reason that Duntley [3], Preisendorfer [2], and Davies-Colley [1] used white light source transmissometers with Wratten #61 filters, which approximate $Y(\lambda)$.

Preisendorfer [7] also derived a "photopic volume attenuation function" that is different from ours, yet he derived the same luminance difference law that for a black target results in Eq. (14). We started with the radiance difference law for black objects, Eq. (6), wrote the equivalent for photopic parameters, Eq. (14), and derived the definition of α, Eq. (18). Preisenforfer [7] started with the equation of radiative transfer, translated it into its photopic equivalent by multiplying all components by the photopic response function and integrationg over wavelength as in Eq. (16). This procedure results in a photopic beam attenuation coefficient that is different from ours in that it involves a spectral integration of the beam attenuation coefficient rather than an integration over the spectral transmission as in Eq. (18). In writing the photopic equivalents of Eq. (3) and (4), and taking the difference to obtain the equivalent of Eq. (6), the luminance difference equation, Preisendorfer [7] has to assume that the photopic beam attenuation coefficients for the target and background are the same. This implies that the radiance spectra of the perceived target and background are the same, which is most likely not correct. Our approach, starting with the radiance difference equation for a black target, Eq. (14) and then obtaining the photopic equivalent avoids that problem. More simply stated, the attenuation of a non-monochromatic light source is obtained by integrating the spectral beam transmittances, not by calculating the transmittance using the spectrally integrated beam attenuation coefficient. It would thus be impossible to produce a physical meter that would measure Preisendorfer's [7] photopic beam attenuation coefficient over a fixed distance. With the proper definition as in Eq. (19), one could construct such a meter.

The apparent complexity of Eq. (18), would seem to contradict the simple results of Davies-Colley [1]. We thus need to investigate the following questions: How well can a hypothetical α_U meter, a monochromatic beam attenuation meter, or an LED light source beam attenuation meter predict true α ?

5. Modeling photopic versus monochromatic beam attenuation.

In order to investigate the effects of Eq. (18), we need to model the spectral beam attenuation coefficient , $c(\lambda)$, as well as the spectral background lightfield, $L_B(\lambda)$.

In general $c(\lambda) = c_p(\lambda) + c_g(\lambda) + c_w(\lambda)$ i.e. the total attenuation coefficient at a wavelength is the sum of the attenuation coefficients of particles, yellow matter, and water at that wavelength. Twardowski et al. [8] have discussed the spectral shape of the particulate beam attenuation coefficient. They conclude that $c_p(\lambda)$ being proportional to $\lambda^{-\gamma}$ is a good model. Here we will use :

$$c_p(\lambda)/c_p(532) = (\lambda/532)^{-\gamma}. \tag{20}$$

The parameter γ has values that typically range from 0 to 2 . The absorption by yellow matter will be small as its absorption is weak where $Y_n(\lambda)$ is large. It is modeled as:

$$a_g(\lambda)/a_g(532) = \exp[-S(\lambda-532)]. \tag{21}$$

We have set S=0.012, although this parameter may vary by a small amount. The attenuation of water as determined by Pope and Fry [9] is used here.

Light spectra are modeled using Jerlov's [4] water types, which are given as spectral diffuse attenuation coefficients, $K(\lambda)$. These span the range from very clear ocean waters to extremely turbid coastal waters. The Jerlov diffuse irradiance attenuation parameters are used to determine the irradiance spectra as a function of depth. We then assign the background radiances the same spectrum as the irradiances. This is not strictly correct, but results in a large variety of reasonable spectra for modeling.

Equation (18) now contains six parameters, $c_p(532)$, γ , $a_g(532)$, Jerlov water type, depth, and r. A limit on the value of $a_g(532)$ is that it must be less than K(532), which follows directly from the condition that the total absorption coefficient must be less than the diffuse attenuation coefficient for optically deep waters (Gershun's equation in Jerlov [4]). The visibility range is determined from Eq. (18), by setting the contrast equal to the limit discernable by humans. This limit was set by Blackwell [10] at 0.02, but inversion of Davies-Colley's [1] data would show it to be 0.008. We determine the average α over the entire visibility range and compare it with other attenuation measures. It is this average α over the visibility range that has the simple relation with limiting contrast and visibility range in Eq. (18). The ratio of any attenuation measure and α is thus the error in the determination of the visibility range. We will therefore examine the relationship of a number of attenuation measures and α. First we need to examine the dependence of α on the light field spectrum.

We carried out numerical calculations with a large yellow matter load, since this will provide the largest gradient in the spectrum. We use $a_g(532) \leq 0.33c_{pg}(532)$, but with the condition that $a_g(532) \leq K(532)$. Therefore we take $a_g(532)$ to be the smaller of $0.33c_{pg}(532)$ and K(532). The cleanest ocean waters at great depths have nearly monochromatic light in the blue part of the spectrum, and so are likely to provide the largest difference between α and c(532). For the clearest ocean waters, Jerlov type I, we find that when comparing visibility range at the surface to that at 150m, the maximum difference encountered is 3%,. The reason for this small difference is that the small value of K(532) limits the value of $a_g(532)$. As mentioned before the particle plus water spectrum is not very steep, so that the influence of a shifted background light spectrum remains small. Similarly, at 20m depth in turbid coastal water type 5, a maximum difference of 4% was encountered. In the turbid water case, yellow matter absorption can be considerable in the blue part of the spectrum, but since the peak photopic transmission is near 550 nm, its effect is small.
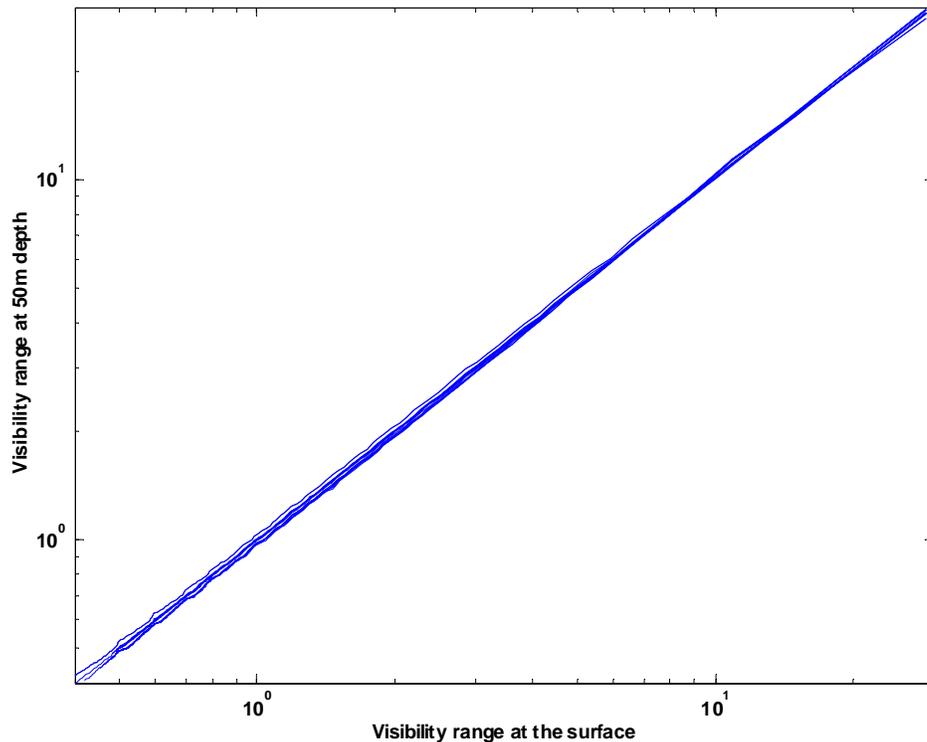
Fig. 1. Visibility range at 50m depth versus visibility range at the surface for Jerlov water types I, IA, IB, II, III, 1, 3, 5, 7, and 9. Calculations are for $a_g(532)=0.33c_{pg}(532)$, but $a_g(532)<K(532)$, $\gamma=1$ and $S=0.012$ (see text).

The figure shows that the effect of changing the background light spectrum is minimal. The primary reason for this is that the attenuation spectrum is relatively flat in the photopically important wavelength range (500 – 600 nm). The attenuation by pure water increases towards the red, whereas particulate attenuation usually decreases towards the red. These effects are somewhat offsetting. The steepest attenuation spectrum is provided by yellow matter, but its absolute value is limited by K, and its values are small in the 500 – 600 nm region. A spectrally flat beam attenuation coefficient will result in all measures of beam attenuation being the same, in which case any beam attenuation meter will correctly predict visibility. As we shall see, it is precisely because beam attenuation spectra are relatively flat in the range of 500 to 600 nm that various measures of beam attenuation other than α work quite well.

Combining the results of all calculations, we conclude that:

0.96 vis. range at surface < vis. range at depth < 1.04 vis. range at surface.

Since the product of α and the vis. range determines the limiting contrast, we obtain the same inequality for α . Going back to Eqs. (18) and (19) we have thus determined that the influence of the spectrum of the background light is small. We can thus use α_U in place of α, with an error of typically 1 to 2% and a maximum error of 4%.

We will next determine how well commonly used attenuation meters can predict the visibility range. Common measures of attenuation are narrow band attenuation measured with spectral attenuation meters such as the WET Labs ac-9, and red or green LED beam

attenuation meters. We will pay special attention to measurements at 532 nm, as this is the wavelength of doubled YAG lasers that are employed in a number of underwater applications. Hence a number of nominally 532 nm green LED tranmissometers are already in use. A large number of 650 nm LED source transmissometers are in use as well, so it is useful to look at how well these can be employed to predict visibility range.

First, we will compare the visibility range as determined by the photopic beam attenuation coefficient with that determined by a monochromatic beam attenuation measurement at 550 nm referenced to the beam attenuation coefficient of pure water. This comparison depends on the shape of the attenuation spectrum, which we have modeled using the various values for a_g/c_{pg} and γ mentioned earlier.

Modern attenuation meters are referenced to pure water, so that the water attenuation must be added back in. As Eq. (18) showed, α of pure water (α_W) is a function of visibility range. In order to use $c_{pg}(550)$ as a measure of α, we thus need to add α_W. α_W (r) can be calculated from Eq. (19) using the pure water attenuation values of [9]. We have added here α_W (12m) = 0.081. This range is chosen somewhat arbitrarily as representing very clear water. Our calculations show that monochromatic attenuation for dissolved plus particulate matter $c_{pg}(550) + 0.081$ is an excellent proxy for the prediction of visibility range. For the 534 cases calculated we obtain an r^2 value greater than 0.99. We note that if we had added the pure water attenuation value $c_W(550) = 0.067$, rather than the photopic attenuation value, there would have been a discernable effect at the larger visibility ranges. For $\gamma = 0$, all c-meters at all wavelengths, referenced against pure water should read the same, as the particulate attenuation spectrum is flat. We conclude that $c_{pg}(550) + \alpha_W(12m)$ is an excellent proxy for α.

We will next examine the visibility range as obtained from measures of the beam attenuation coefficient at 532 nm. The measures used are the monochromatic beam attenuation at 532 nm and the attenuation as obtained by a commercial 532 nm transmissometer, a WET Labs c-star with a peak wavelength of 528 nm and a FWHM of 20nm.

Figure 2 shows that the visibility range for different beam attenuation coefficient spectra as determined by three values of γ, and two values of a_g/c_{pg} , and for monochromatic measurements as well as for LED light source measurements (red and green dots in the figure), compared quite well with the true visibility range (blue dots in the figure) when a simple adjustment was made. We found that measures of attenuation at 532 nm can be reduced to nearly approximate α, by simple multiplication by 0.9 and addition of α for water at a range of 12m (α_W (12m) = 0.081). Note that we use the same water value as at 550 nm, since we are converting all attenuation measurements to the photopic attenuation. It was found that $c_{pg}(532)*0.9 = c_{pg}(550)$, approximately, hence the multiplier 0.9 was used. The 0.9 ratio is based on our models that have a larger range of γ than is typical for the ocean. Barnard et al. [11] found a ratio of $c_{pg}(550)/ c_{pg}(532)= 0.985$ for a large number of measured attenuation spectra and Voss [12] found a ratio of 0.96. This means that their observed γ was only slightly larger than 0. It also turns out that the commercial green LED attenuation meter modeled here provides a close proxy for $c_{pg}(532)$, so that treating it similarly to $c_{pg}(532)$ also provides an excellent proxy for α. Had we added $c_W(532) = 0.051$ m^{-1}, instead of $\alpha_W(12m)= 0.081$ m^{-1}, there would have been large changes at the longer visibility range, that would have made the log-log relationship of the attenuation measures at 532 nm and visibility range non-linear, which is not desirable for simple visibility algorithms.

Based on the model results presented in Fig. 3, we may conclude that an error of less than 10% is made for visibility predictions when using $c_{pg}(532)*0.9 +0.081$ either monochromatic, or from an LED transmissometer rather than photopic α.
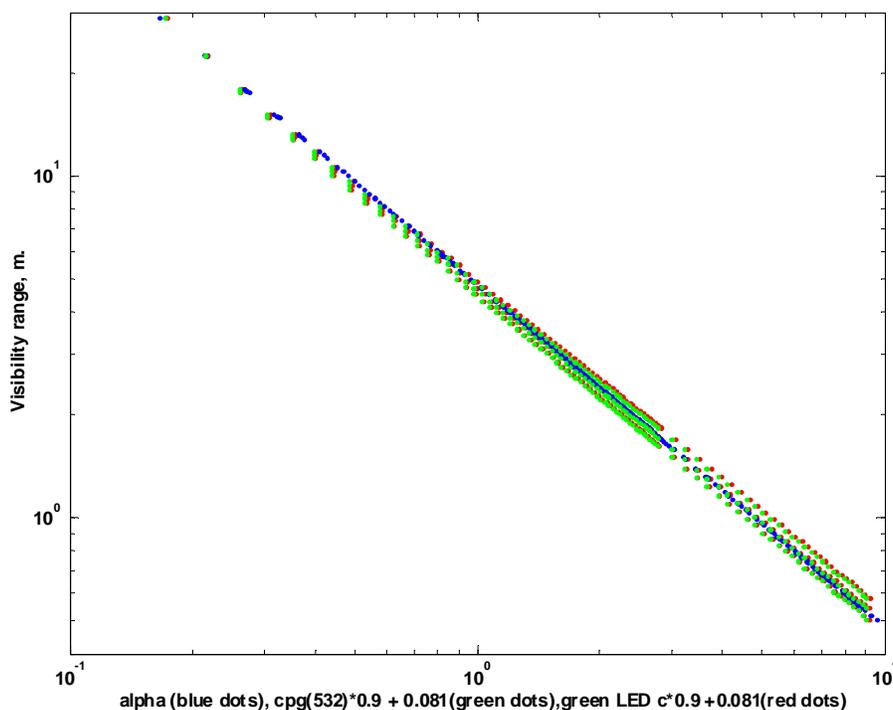
Fig. 2. Prediction of visibility range using α (blue dots); $c_{pg}(532)*0.9 + \alpha_W(12m)$ at 532 nm for $\gamma = 0$, 1, and 2 and for $a_g/c_{pg}=0$ and 0.2 (green dots), similarly for a green LED c *0.9 + $\alpha_W(12m)$ (red dots). Units are m^{-1} for attenuation measures and m for visibility range.

For 650 nm we found the relationship $c_{pg}(650)*1.18 + 0.081$ to be a reasonable proxy for α, with errors less than 20% for the three cases of γ and two cases of $a_g(532)/c_{pg}(532)$ examined. The reason is that $c_{pg}(650)/c_{pg}(550)$ is approximately equal to 1.18 (for $\gamma=1$ and $a_g(532)/c_{pg}(532) = 0.1$) and $\alpha_W(12m) = 0.081$. Barnard *et al.* [11] measured a ratio of 1.09 for $c_{pg}(550)/c_{pg}(650)$ and Voss [12] measured a ratio of 1.13. We have thus found that common measures of the beam attenuation coefficient can be converted into proxies for α, by means of simple multiplication and addition of α for pure water. The next section tests these theoretical relationships against observations.

6. Observations

Figure 3 shows a comparison between experimentally obtained visibility range and various measures of beam attenuatuion corrected to photopic attenuation by means of the expressions derived in section 5. Davies-Colley [1] showed an excellent relationship between horizontal sighting range of a black 200 mm diameter disk and Ψ/c (blue circles, Fig. 4), where c was measured with a white light source transmissometer equipped with a nearly photopic response filter. Ψ was found to be a weak function of c and averaged 4.8.

Our own beam attenuation data was taken with a WET Labs ac-9 spectral attenuation meter at 532 nm, and treated as described above ($c_{pg}(532)*0.9 +0.081$, red circles in Fig. 4). Visibility data was taken using a 200mm diameter black disk obtained from Dr. Davies-Colley, that matched the disk used for his observations. The disk is suspended in the water about 20 cm below the surface. The observer uses an inverted periscope to view the target at the depth of the target. The viewer moves away from the target until the edge of the target is no longer discernable relative to the background. The viewer must have normal vision; so-

called color blind people have different photopic response curves than the standard one, which will result in different visibility ranges. The radiance field must be nearly plane parallel, i.e. the radiance in the horizontal direction must be the same between the observer and the target and for a distance beyond the target. It is therefore important to have no objects behind the target for at least three optical depths. We have found no discernable effect due to the target being close to the surface. In addition, reflecting objects near the sight path, such as a white boat hull, can introduce errors by generating non-uniform light fields. These data contain a wide variety of locations, such as coastal ocean, estuaries, rivers, and lakes. Similarly, a wide range of illuminations (direct sun, 100% overcast, etc.) are included.

As shown in Fig. 2 the error of using $[c_{pg}(532)*0.9 + 0.081]$ rather than α should be less than 10%. This is well demonstrated by the experimental data in Fig. 3.
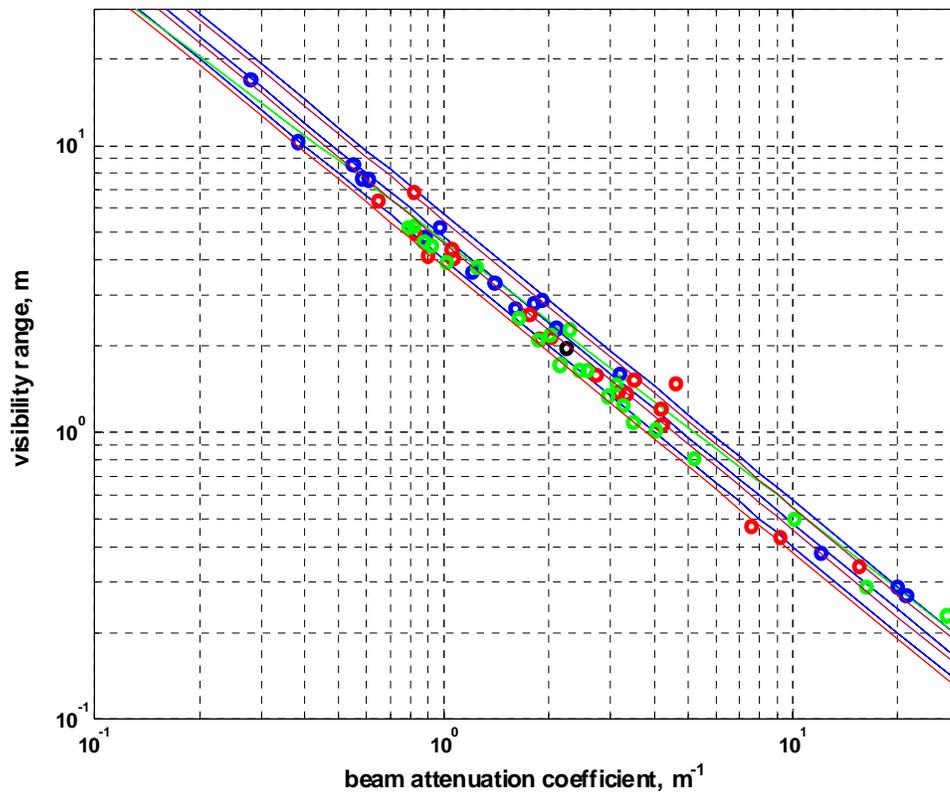

Fig. 3. Horizontal visibility of a 200 mm diameter black target. Blue points, Davies-Colley, "green" c-meter; red points, Zaneveld, c(532)*0.9+0.081; black point, Twardowski $c_{pg}(532)*0.9+0.081$; green points, Pegau, $c_{pg}(532)*0.9+0.081$;
blue lines vis. range = y = $4.8/\alpha$ and +/- 20% lines;
green line vis. range= y = $(5.207- 0.368\ lny)/\alpha$;
red lines vis. range = y = $4.55/\alpha$ and +/- 20% lines; r^2 = 0.985.

We see from Fig. 3 that our data nearly agrees with Davies-Colley's [1] conclusion. Our data tends to fall slightly below the 4.8 line. We found an average value of 4.55 for the product of α and vis. range. The central red line on figure 4 shows this relationship, and the upper and lower red lines are the +/- 20% lines. The difference is about 5% which is well within the experimental errors of the measurements.

We also examined experimentally the relationship between photopic visibility range and $c_{pg}(650)$. In this case the errors are slightly larger and the relationship was found to be:

Photopic visibility range y = 3.7 /[$c_{pg}(650)*1.18 +0.081$]. The correlation coefficient in this case was found to be $r^2 = 0.96$. Thus $c_{pg}(650)*1.18 +0.081$ is an adequate proxy for α, but it is not quite as good as using a green wavelength.

7. Discussion

Figure 3 shows that the observations of visibility at various wavelengths can be compared with theory. In Fig. 3 the central blue line is given by $4.8/\alpha$. We used the limiting contrast obtained from this relationship, 0.0082. Note that the $4.55/\alpha$ relationship that we found would lead to a limiting contrast of 0.010.

The theory and the data show that the horizontal visibility of a moderately sized black target (200 mm diameter) is quite well described by $4.8/\alpha$, where α is the photopic beam attenuation coefficient for the visibility range. When α_U (see Eq.(19)) is used in lieu of α, which as we have shown entails a very small error, the photopic beam attenuation coefficient becomes an inherent optical property as there is no dependence on the light field. An excellent proxy for the photopic beam attenuation coefficient is $c_{pg}(532)*0.90 +0.081$ or the attenuation obtained with a green LED source transmissometer, with small adjustments as described above. The reason for emphasizing the 532 nm wavelength is that 532 nm is the frequency of doubled YAG lasers that are used in the marine environment for mine detection, bathymetry, etc. Hence a number of attenuation meters are already in use that employ that wavelength. As we have seen use of such devices entails a very small error when making a slight correction for the wavelength offset and adding the pure water photopic attenuation at 550 nm. Even the attenuation in the red measured with a spectral attenuation meter (WET Labs ac-9), expressed by $c_{pg}(650)*1.18 +0.081$, gives an good correlation with visibility range. For practical purposes we thus have found experimentally that a single measured parameter, $c_{pg}(532)$, provides an excellent prediction of the visibility range of a black target in a plane parallel light field.

Other visibility arrangements, in terms of target properties, lighting and viewing arrangements require far more parameters and so do not qualify as a simple parameter that is useful to operational divers. Table 1 describes the parameters needed to predict visibility in other than the simple horizontal viewing arrangement of a black target described here.

Table 1. Parameters required to predict visibility range. All situations also require the inherent contrast of the target.

	Horizontal viewing angle	Vertical viewing angle	Arbitrary viewing angle
Large target; black target ; ample daylight	c	c, K	c, K , θ
Large target; arbitrary reflectance; ample daylight	c, ρ, TO(θ, ϕ)	c, K, ρ , TO(θ, ϕ)	c, K, ρ , θ, TD(D),TO(θ, ϕ)
arbitrary size target; black target; ample daylight	c, TD(D), C_L(D)	c, K, TD(D),C_L(D)	c, K, C_L(D), θ , TD(D),TO(θ, ϕ)
arbitrary size target; arbitrary reflectance; ample daylight	c, ρ, TD(D), C_L(D)	c, K, ρ , TD(D), C_L(D)	c, K, ρ , C_L(D), θ , TD(D), TO(θ, ϕ)
arbitrary size target; black target; arbitrary illumination	c, TD(D), C_L(D), C_L(E)	c, K, TD(D), C_L(D), C_L(E)	c, K, C_L(D), C_L(E) , θ, TD(D), TO(θ, ϕ)

arbitrary size target; arbitrary reflectance; arbitrary illumination	c, ρ, TD(D), C_L(D), C_L(E)	c, K, ρ , TD(D),C_L(D), C_L(E)	c, K, ρ, θ, C_L(D), C_L(E) , TD(D), TO(θ, φ)

c	spectral beam attenuation coefficient
K	spectral diffuse attenuation coefficient
ρ	spectral target reflectivity
θ	viewing angle
TD(D,S)	target description; diameter and shape
TO(θ, φ)	target orientation; zenith angle and azimuth
C_L(D)	threshold contrast as a function of target diameter
C_L(E)	threshold contrast as a function of light level

Bowers [13] describes calculations and observations of visibility for various sighting situations. If we take his calculated sighting range in the horizontal for a black target (90º and 270º in his figures 2 and 3, and if one were to use Davies-Colley's [1] experimentally determined 4.8/c visibility range rather than the 4/c used by Bowers (i.e. 20% larger) , we find good agreement between the calculated and predicted visibility range (about 10% error on average). This correlation was found independent of depth, confirming the calculations presented in Fig. 1, that showed very little influence of the daylight spectrum. Bowers' Fig. 4 refers to a white, spherical target, for which the 4.8/c rule does not hold as the target has to be black, or nearly so. A reflective spherical target has as its brightest value the reflected downwelling radiance. The 4.8/α rule is based on the contrast between the horizontal background radiance and a nearly zero inherent target radiance. In other words the contrast at the target needs to be nearly –1. For a reflective sphere, the contrast at the target is likely to be large as the downwelling radiance is usually much larger than the horizontal radiance. The data presented in Bowers [13] for the horizontal visibility range of the black target supports the Davies-Colley [1] rule.

We conclude that the simple visibility parameter, visibility range = 4.8/ photopic beam attenuation, investigated here is well grounded in theory, is readily measured by proxy apparatus, and is extremely useful to provide divers with a general sense of underwater visibility conditions.

Acknowledgments