AN ABSTRACT OF THE THESIS OF

JAMES DOUGLAS ARNEY for the DOCTOR OF PHILOSOPHY
(Name) (Degree)

in FOREST MANAGEMENT presented on ______________
(Major) (Date)

Title: COMPUTER SIMULATION OF DOUGLAS-FIR TREE AND STAND GROWTH

Abstract approved: ______________
David P. Paine

A mathematical model has been developed to describe the growth of individual Douglas-fir trees under a variety of stand conditions and management schedules. The model was developed using empirical data from open-grown trees and reducing growth capacities of height, stem, and crown as functions of intertree competition indices. Any initial pattern of spacing for up to 30 trees may be assumed. The user must supply the stem coordinates, estimate of site, and estimate of the number of years to reach a height of 4.5 feet for each tree. Any intensity of thinning schedule may be requested up to an annual basis. The model simulates the form of the entire bole and crown for up to 80 years of growth.

All calculations were performed on a CDC 3300 electronic computer. Individual tree and stand summaries can be listed for any periodic interval of interest. A Cal-Comp plotter output is available

for a complete stem analysis output for one specified tree in the stand. All coefficients for regressions fitted in the model are listed in a separate file for easy access and modification.

The mathematical model developed here satisfactorily describes the growth of Douglas-fir stands on an individual tree basis for a wide range of stand conditions and management schedules. It was tested against seven permanent field plots for a range of conditions in both Oregon and Washington. Although predictions were not accurate there are no gross errors and the results demonstrate the sensitivity and stability of the model.

The model developed here can be a valuable tool to the forester in training students, analyzing basic growth relationships, testing management schedules, evaluating fertilization and irrigation responses, and planning allowable cuts, sustained yields, and economic returns of forest products.

# Computer Simulation of Douglas-fir Tree and Stand Growth

by

James Douglas Arney

A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Doctor of Philosophy

June 1972

APPROVED:

Associate Professor of Forestry

Head of Department of Forest Management

Dean of Graduate School

Date thesis is presented

Typed by Mary Jo Stratton for James Douglas Arney

# ACKNOWLEDGEMENTS

The author would like to express his appreciation to the many people who gave assistance during the course of this study. He is indebted to his major professor, Dr. David Paine, for assistance and review of the manuscript. He would also like to thank Dr. Scott Overton, Mr. Dave Bruce, and Dr. Robert Curtis for their critical reviews. Appreciation is also expressed to Dr. John Bell, Dr. Robert Strand, Dr. Jim Lin, and Dr. John Beuter for their encouragement and support throughout this study.

The financial assistance was supplied for all computer related expenses by the OSU Computer Center. Without this assistance the study could not have been completed.

# TABLE OF CONTENTS

# LIST OF TABLES

# LIST OF FIGURES

# COMPUTER SIMULATION OF DOUGLAS-FIR TREE AND STAND GROWTH

## INTRODUCTION

As demand on the Douglas-fir resource becomes more intense and diversified, foresters are striving to maximize growth and yield for each acre of commercial forest land. This requires that each stand be maintained at some near optimum growing condition throughout its rotation. To find and maintain these levels, more specific information is needed about growth characteristics of individual trees at various levels of competition, spacing, site, and age. There are numerous long-term studies in progress that will eventually give us the information needed; but that will require waiting at least one complete rotation on each study. Even if a field study had been started 20 years ago, it will be 40 or more years before the end results are known. More disappointing yet, is the realization that the particular treatments applied in these long-term studies may not result in finding the optimum growth and yield possible for the site and age of the stand. Because of this last problem, foresters are applying an array of treatments in hope of finding the optimum conditions they desire.

With the advent of the computer the forester has been able to consider alternative methods of determining optimum management schedules for growth and yield of forest stands. Through computer simulation techniques it is possible to duplicate the functions of growth documented in the literature from past and present field studies. The forester can then make inferences about the results of any number of years of growth under the constraints imposed. As is readily apparent, many more alternatives may be attempted and refined management schedules may be introduced as predicted growth is observed and appropriate adjustments can be made in the simulator.

## Background

Tree boles taper toward the tip of the tree, but the rate of decrease in diameter with increase in height above ground may be variable because of varying thickness of the annual sheath of new wood. Although the controlling physiological processes are not fully known, reviews by Farrar (1961) and Larson (1963) have concluded that the variation is correlated with crown size and crown position on the bole. Numerous studies of stem form have shown that trees with long, vigorous crowns produce strongly tapering stems with a rather high proportion of earlywood to latewood. As the crown recedes, either from stand closure or artificial pruning, the stem becomes more cylindrical and the proportion of earlywood to latewood decreases. Lateral branches nearest the top of the tree are the most vigorous and contribute the greatest quantity of auxin to the main stem. In the lower-most branches of stand-grown trees, the cambial stimulus fails to reach the branch base or it may be visibly expressed only in the form of latewood. Stem analyses have shown the maximum radial growth occurs in the general vicinity of the live crown base (Duff and Nolan, 1953; Reukema, 1961). Therefore, the maximum ring width may be found low on the bole of long-crowned trees and a gradual upward shift parallels crown recession. Within an individual tree the increment of the lower bole is relatively less stable and reflects seasonal changes in weather and environment far more readily than the bole of the active crown. Three general cases of stem form in the branch free bole have been recognized (Larson, 1963):

1) Trees with strongly developed crowns, such as those growing free or in small openings in the stand, show a downward increase in both area increment and radial increment.
2) Stand grown trees with side development hindered, but not overtopped, show approximately equal area growth but decreasing radial growth downward.
3) Stand grown trees overtopped and with small crowns relative to their bole length show a downward decrease in both measurements.

Stem form is a composite reflection of both stand density and crown class. These factors control stem form so consistently that the degree of taper can be regulated by stand management. However, quantitative expression of stand density in an accurate, useful fashion has long puzzled forest mensurationists. Better progress might have been achieved had it not been for the widespread acceptance and use of the normal stocking standard. It required the effort of many researchers over several years to demonstrate that complete area occupation and nearly full increment could occur over a wide range of stand densities (Staebler, 1958; Braathe, 1957).

Krajicek (1961) introduced a crown competition measure which he called Crown Competition Factor (CCF). Based on measurements of DOB at breast height throughout a stand, CCF is the accumulated, predicted open-grown crown areas for all stem diameters divided by 435.6. This yields a continuous index of stocking and crowding simultaneously from an open-grown tree condition to a dense stagnated stand (350-500). Hypothetically an acre fully stocked with open growing trees and crowns just touching would have an index equal to 100. Normally stocked stands tend to level off after 40 years near 340-370. Gerrard (1967) independently developed an individual tree competition index for white oak which he called Competition Quotient. He tried various ratios of competing areas to stem diameter in an attempt to predict response of diameter growth to release by thinning. His lack of success was most likely due to the small potential for response the 300 year old stand was capable of achieving. Although possibly not aware of the size of the most successful competing area, the area equal to the open-grown crown of trees appears to work well. Bella

(1969) developed an index based on open-grown crown width identical to Gerrard's with the exception that a weighting factor is applied subject to the size of the competitor.

Opie (1968) developed a competing basal area index for predicting growth of Eucalyptus in Australia which he called the zone count method. He tested various BAF's and found those corresponding to crown areas the best predictors for growth. The zone count (field method) was not as useful because additional competition estimates were made at a distance out from the subject tree which bring in unrelated competitors.

All of the above mentioned competition indices are based on a competitive area around the subject tree equal to the area of the open-grown crown for the particular stem diameter. An index of competition as applied above is the accumulated area overlap of all competitor crowns of neighboring trees expressed as a percentage of the subject tree crown area. This index appears to be consistent over all ages and sites which has contributed to its popularity for growth prediction. Gerrard stated that area overlap is a better predictor of periodic increment than Spurr's point density (1962), Staebler's linear overlap (1951), or Newnham's percent of circumference overlap (1966).

## Objective of the Study

This study attempts to go beyond the development of traditional yield tables which directly predict net growth in board or cubic feet.

Instead, the development of each component of volume growth (height growth, diameter growth, crown growth, mortality, stems per acre, site index, and age) is predicted based on previous conditions and treatments which may have altered the final component values. Volume increases are predicted at later stages from these primary components. The object is to build a basis whereby the effect of the history of development of each component may be evaluated in terms of stand volume growth distribution between trees and portions of trees.

The objectives of this study are to quantify basic components of tree and stand growth, to develop models of these components whereby the combined responses may be used to predict volume growth, and to build a basis for predicting stand volume growth response from one period to another depending on stand condition and treatment for previous periods.

## Methods

A number of successful stand models recently have been initiated (Clutter, 1963; Newnham, 1964; Lee, 1967; Leary, 1968; Myers, 1968; Lin, 1969). Of particular interest are individual tree models that incorporate measures of competition for each tree (Mitchell, 1969; Bella, 1969). Mitchell simulated the irregular crown expansion across a horizontal plane for each tree in the stand at five year intervals. This allowed neighboring trees to compete individually for available growing space. As discussed by Lee, these models do not lend themselves to statistical tests of precision and accuracy. The most frequently used approach has been one of comparing values for parameters such as volume over all ages simulated against a record of volume measurements on some field plot. The effort by Mitchell is an example of this for trees per acre, crown width, crown length, and bole diameter. The definitions of success as used above refers to this type of residual analysis. These stand models have predicted values which yield a horizontal band of homogeneous variance of residuals over the range of years simulated. A horizontal band of residuals indicates no abnormality in the model. More confidence can be assigned as the band becomes narrower through refined models. Mitchell was able to reduce this band for crown width to within 3% of actual values.

The procedure for this study is to model the growth of each individual tree in relation to its competitors. The growth of the bole and crown at each whorl down the entire length of the tree will be simulated for the model specified. The growth that results will be a function of age, site, and crown competition. It will be possible for the forester to observe the effect on the form, density, and quality of the wood laid down along the bole.

Only one tree species has been modeled in order to determine if the approach is feasible. At a later date other species can be incorporated. Site is an input since it is the most consistent indicator of dominant or open-grown height growth. Age provides a measure of time and potential growth rates. Tree size and spacing determines when competition is important in the growth of a tree on a given site. No attempt has been made to describe the variation in growth that occurs due to genetic variability, multiple species composition, or climatic fluctuation.

A new and logical extension of the crown width-DBH relationship discussed earlier will be tested in this study. The proposed hypothesis is that open-grown crown widths at any point on the open-grown stem may be expressed as a function of the diameter outside bark (DOB) at that point. If this is the case then simulation of open-grown trees should not be difficult.

The most formidable problem in simulating the growth of a stand-grown tree is finding a biologically sound function that predicts the death of a whorl and crown base recession up the bole. The second hypothesis that will be tested is that a limit for death of a whorl can be predicted from site, age, and an index of competition at that whorl.

As the crown width-DOB relationship was extended to include any point in the crown of open-grown trees, so too the competition indices will be applied to each whorl in the stand-grown trees. Crown Competition Factor measured on a horizontal plane high in the crowns of stand-grown trees will yield an index less than 100 (no competition). As the horizontal plane is lowered, the index increases until it reaches a maximum at DBH. It is reasonable to expect that measurements of DOB at crown base in stand-grown trees should yield an index of competition that represents the maximum competition a particular whorl can endure before dying.

The model simulates the growth of each tree in turn on an annual basis. Unless restricted by a marginal crown size the height is incremented at the beginning of each year. This provides a basis for predicting a maximum stem increment and crown width increment for the first whorl down from the top. Crown increment is reduced as a function of the proximity and size of competitor whorls equal to or slightly higher than the subject whorl. Stem increment is reduced as competition increases on the whorl. This procedure continues for each whorl down the bole until such crown competition occurs that the whorl can no longer survive. From this point the stem increment is approximated by a stem area increment equal to the diameter increment at the last live whorl. This procedure is followed on each tree in the model until all trees have completed one year's growth.

# APPROACH

## Description and Analysis of Components

Each component of the model is based on carefully chosen field measurements or accurate and complete presentations from available literature. A determined effort was made to see that each component of the model is biologically sound in relationship to the others and uniquely descriptive of some growth or death parameter acting on individual trees.

### Site

A primary consideration of a growth model is some index of the productivity potential for a given combination of available nutrients, light, and water. The only present index appropriate is the observed height growth of dominant or open-grown trees of similar species for a given period of years.

### Height

A very useful and precise predicting equation for the height growth of Douglas-fir was prepared by King (1966). It incorporates three simple linear equations to yield coefficients for height at any age given an index of site (see Figure 1). The equations are:

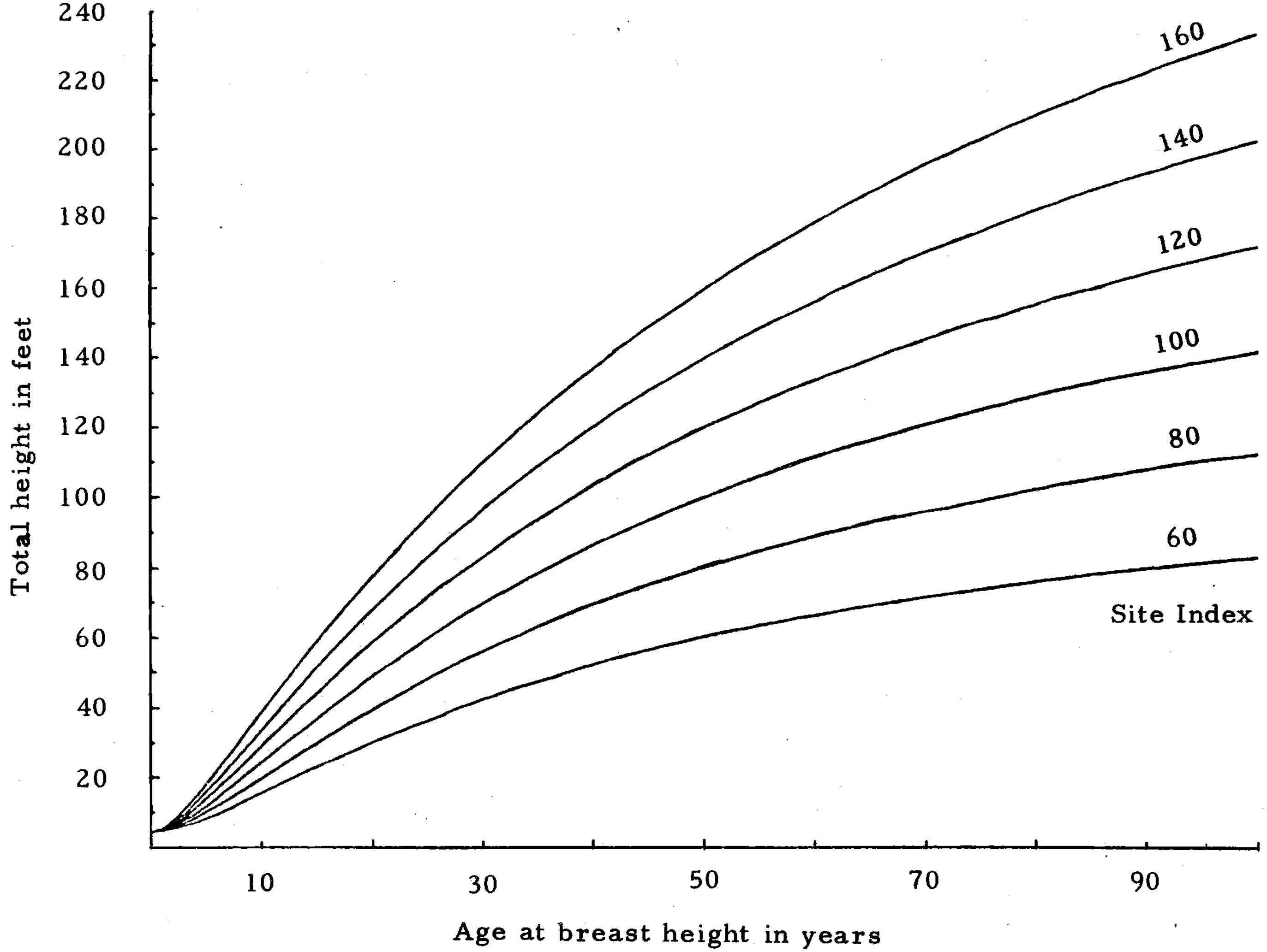


Figure 1. Height as a function of age and site. (from King, 1966)

$$B_1 = -0.954038 + 0.109757\,[2500/(\text{site} - 4.5)]$$

$$B_2 = 0.0558178 + 0.00792236\,[2500/(\text{site} - 4.5)]$$

$$B_3 = -0.000733819 + 0.000197693\,[2500/(\text{site} - 4.5)]$$

The predicting equation for height from age takes the form:

$$H = \frac{(\text{Age})^2}{B_1 + B_2\,(\text{Age}) + B_3\,(\text{Age})^2} + 4.5$$

This equation predicts total height for young-growth Douglas-fir with a standard error of approximately four feet at 50 years for a site II. The first derivative of this formula with respect to age is used in the model to predict annual height increment. It has the following form:

$$\frac{dH}{dA} = \frac{2\,B_1\,(\text{Age}) + B_2\,(\text{Age})^2}{[B_1 + B_2\,(\text{Age}) + B_3\,(\text{Age})^2]^2}$$

This allows a given tree to receive increment in height for a given age even though, as a result of other factors, the total height may be somewhat different for the accrued age.

### Stem Diameter

Since no comprehensive analysis of diameter growth at all points on the stem is available in the literature an independent study was initiated expanding on recent work by Paine.[1] Crown width, stem

---

[1] Paine, David P. 1968-1971. Personal contact and joint effort in part of analysis.

diameter at breast height, age, and total height had been recorded on 173 open-grown trees in western Oregon. The data was collected over sites and ages as described in Table 1. The equation for open-grown stem diameter is:

$$DOB = -2.5596 + 0.1963 (Age) + 0.2800 (L)$$

This equation had a standard error of 2.3 and correlation coefficient of 0.956 for 173 observations.

Table 1. Distribution of sample trees for open-grown crown diameter and stem diameter by site index and age.

| Age | Site | | | | | Total |
|---|---|---|---|---|---|---|
| | I | II | III | IV | V | |
| 1 - 10 | 6 | 3 | 14 | 6 | 16 | 45 |
| 11 - 20 | 4 | 15 | 30 | 23 | 7 | 78 |
| 21 - 30 | 2 | 3 | 17 | 7 | 2 | 32 |
| 31 - 40 | 1 | 2 | 7 | 4 | 2 | 16 |
| 41 - 50 | | | 1 | 1 | | 2 |
| Total | 13 | 23 | 69 | 41 | 27 | 173 |

To this data 64 additional observations of diameter outside bark (DOB) were made at higher points in open-grown trees. These additional measurements were used to prove that given the site, age, and length (L) from the top of the tree, diameter outside bark at any point on an open-grown tree can be predicted.

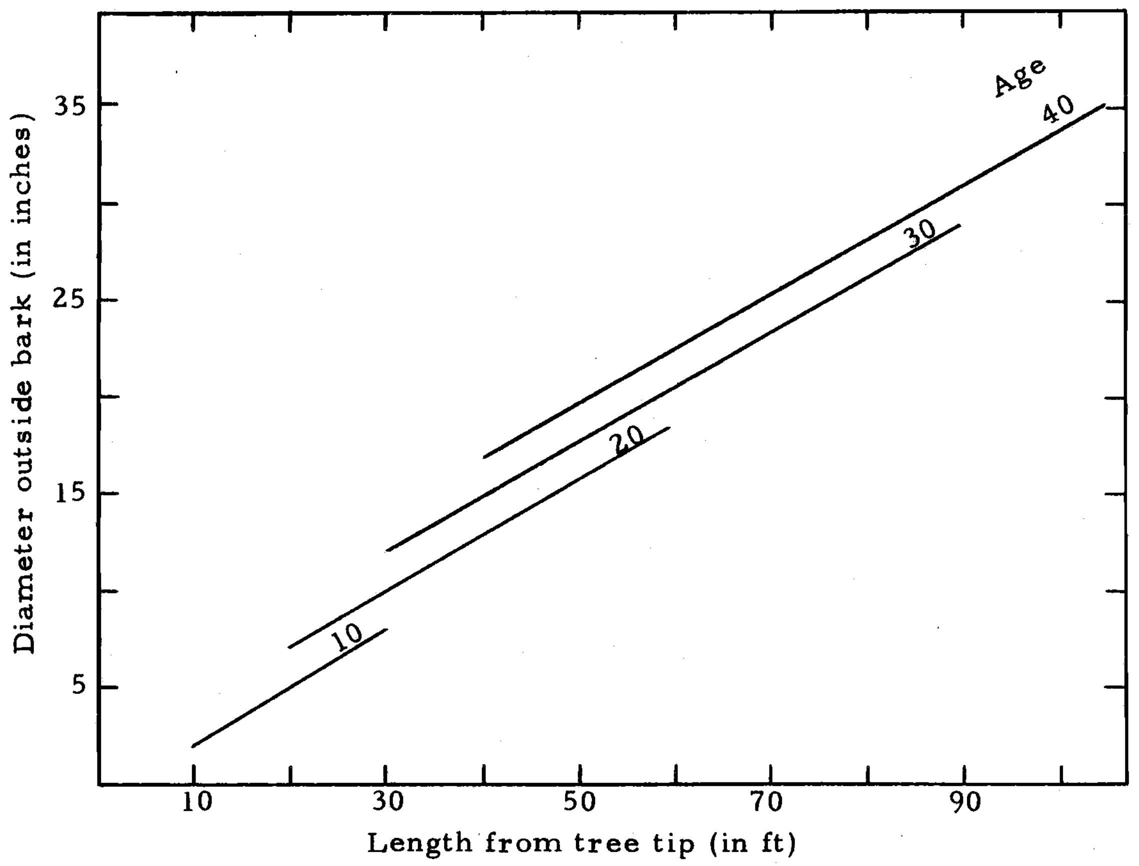


Figure 2. Diameter outside bark as a function of length to tip and age for open-grown trees.

Since it is also advantageous to predict diameter increment in the model without dependence on past diameter, the form used differentiates to:

$$dDOB = 0.1963\ (dAge) + 0.2800\ (dL)$$

The dAge will always be unity as long as the model makes annual increments and may therefore be handled as a constant.

## Crown Widths

Paine originally collected 173 observations of crown width and stem diameter on open-grown trees. This resulted in a predicting equation for open-grown crown widths which is independent of site or age. The form of the final equation is:

$$CW = 4.5685 + 2.0360\ (DOB) - 0.0191\ (DOB)^2$$

This equation has a standard error of 2.6, correlation coefficient of 0.97, and is based on 181 paired observations of crown and stem diameter. It is applied in the model both as above and for crown increment as follows:

$$dCW = 2.0360\ (dDOB) + 2\ (-0.0191)\ (DOB)\ (dDOB)$$

## Intertree Competition

Crown Competition Factor (CCF) is a consistent and adequate index of occupancy and competition for generally uniform, even-aged stands. It is not dependent on age or site and is biologically meaningful in its interpretation. Gerrard's Competition Quotient (CQ) is similar to CCF except that it is defined for individual trees and the area ratio is not based on open-grown crown widths (see Appendix IV). To apply an index of competition to each whorl in a tree a new index based on CCF and CQ is appropriate. Competition between whorls is expressed as:

$$CCQ = \frac{100\Sigma a_i}{A}$$

where

CCQ = Crown Competition Quotient

$a_i$ = area in square feet of open-grown crown overlap with the <u>ith</u> competitor whorl

A = area in square feet of an open-grown crown with specified DOB

Crown Competition Quotient yields index values similar to CCF. An index less than 100 describes a whorl on a tree in which no other tree has a crown overlap with that whorl. At the tip of the tree CCQ equals zero because there is no crown area. Proceeding downward whorl by whorl eventually a point is reached where crowns just touch. CCQ is near 100 at this point and by definition intertree competition begins. The index increases as the horizontal plane where the index is applied descends to DBH. Derivation of equations for determining crown area overlap may be found in Appendix I. Figure 3 helps to

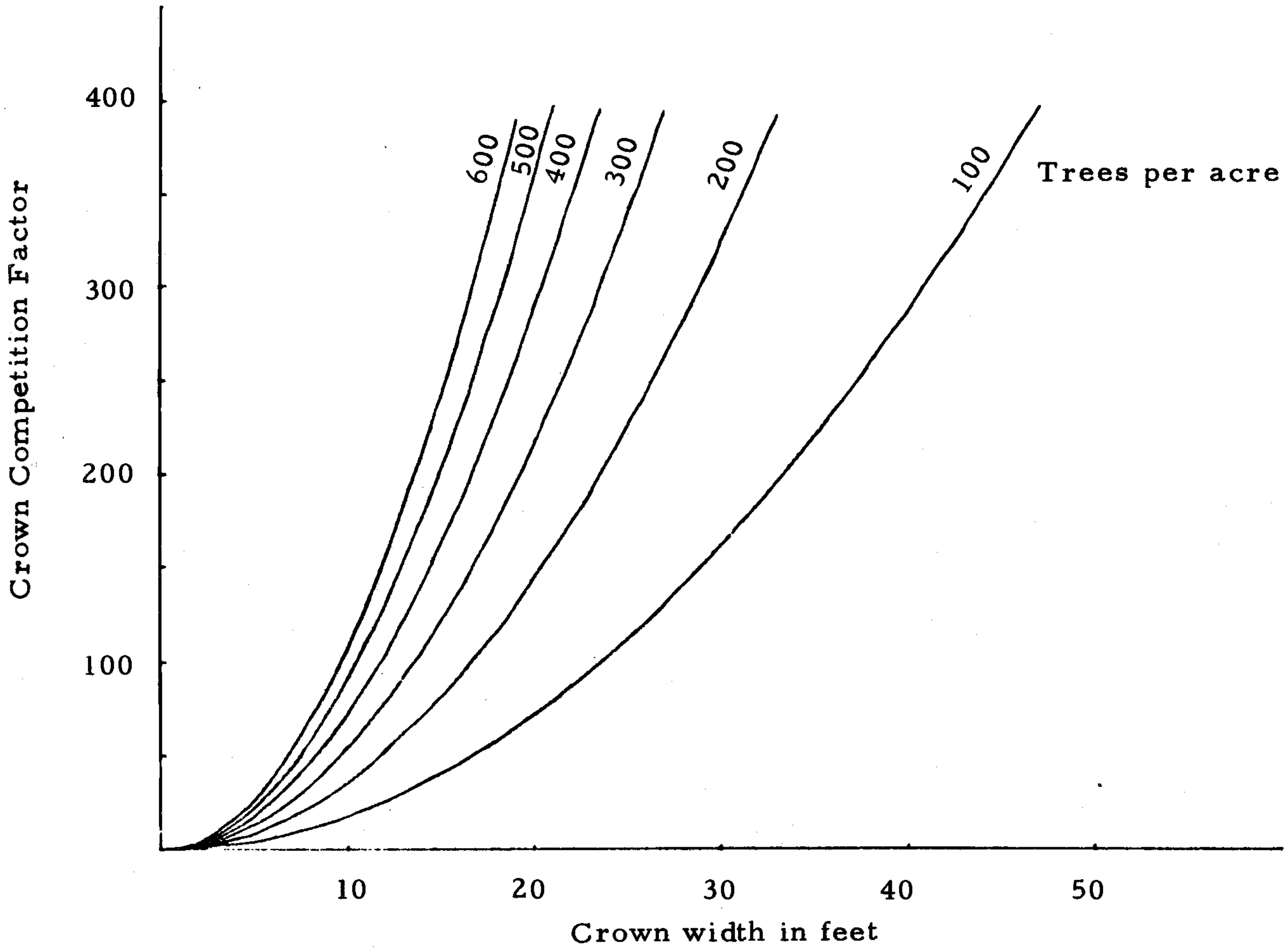


Figure 3. Crown Competition Factor as a function of crown width and number of trees per acre.

visualize the effect of crown width increment for a given number of trees per acre on the change in CCF index. In uniform stands with over 600 stems per acre a small increment in crown width produces a significant rise in the competition index. Such a description demonstrates the importance of mortality allocating more growing space to residual trees.

## Tolerance Limits

At some point immediately below where intertree competition begins in Douglas-fir the crown base is usually found. If not, then the stand has had some recent history of disturbance or the stand is not uniformly closed and sunlight is penetrating deep into the canopy through some large opening.

An independent study was initiated to characterize tolerance as some level of the Crown Competition Quotient described above. The derivation of the equations and the field procedure are explained in Appendix II.

Nineteen points were measured in uniform undisturbed Douglas-fir stands throughout western Oregon. At first it appeared that a given Crown Competition Quotient characterized the tolerance limit over all sites and ages. The index of tolerance (CCQ) actually decreases with height. This may be due to the whipping action between crowns that occurs as trees get taller (Mitchell, 1969). Small, young trees are known to intertwine their branches with little apparent reduction in growth. Intertwining is not possible as trees become taller and more limber, accounting for a reduced tolerance index level. The following prediction equation is based on data illustrated in Figure 4.

$$TOL = 133 + (2025 - 133)e^{-0.051\ HT}$$

## Reducing Growth by Suppression

Height growth generally is not affected by density until the live crown length becomes less than 30% of the total height. Little evidence is found in the literature describing the rate at which height growth declines as the live crown loses its status in the canopy. As a general model a negative exponential would characterize height increment drop off quite well. The equation chosen is:

$$dH = dH_m \left[1 - e^{-200(CL)^3}\right]$$

where

$dH$ = predicted increases in height

$dH_m$ = maximum increase in height for given site and age

$CL$ = proportion of total height in live crown

This relationship between height increment and live crown length is a key constraint on the maximum stem diameter increment and crown width increment that is predicted at every whorl in each current year's growth.

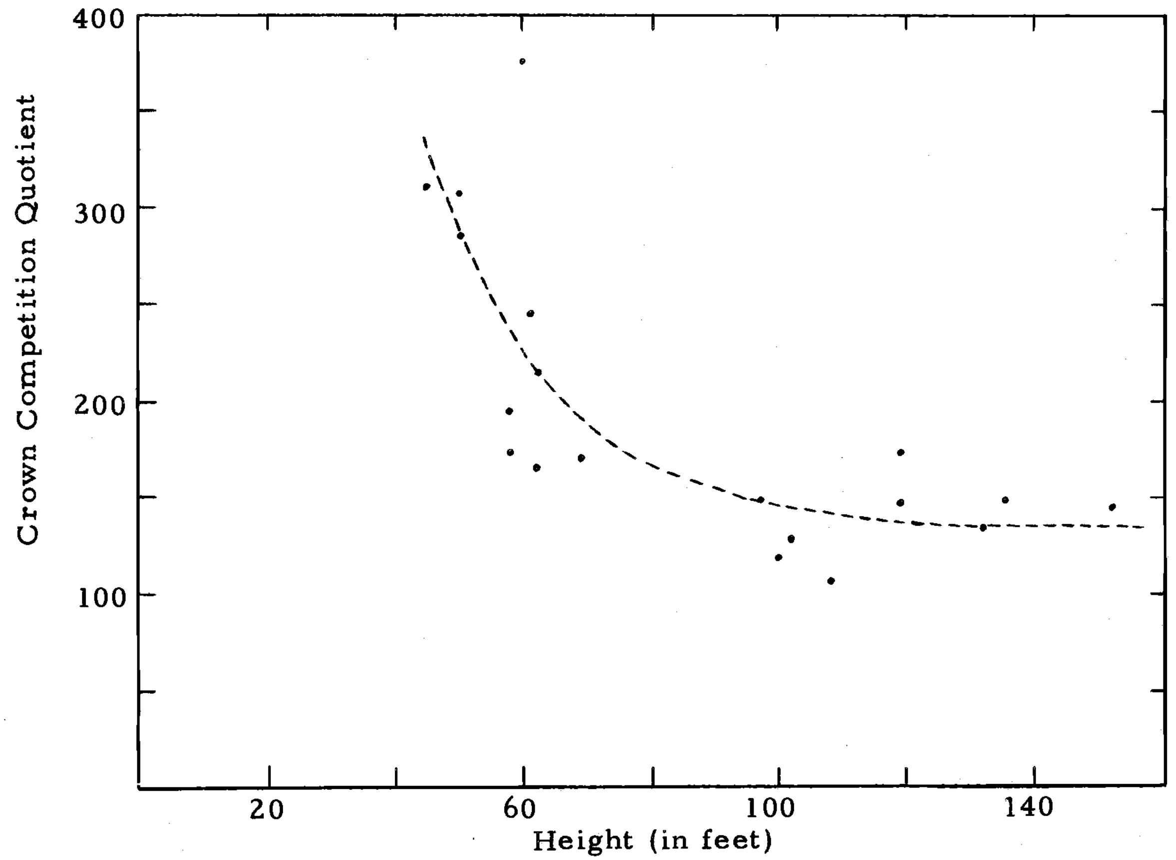


Figure 4. Tolerance as affected by total height.

## Volume Growth Response

Basal area increment has been one basis for judging success or failure of various thinning schedules. Other than total height growth, which is basically independent over the range of densities usually tested, basal area has been the most frequently measured point on a tree. With the increased use of dendrometers (both optical and recording) and intensive observation at additional points high on the stem, growth is being observed along the entire length of the stem. The entire bole of the tree responds to growing conditions. The maximum diameter growth occurs at or near crown base and in some trees under marginal growing conditions a year's increment measured at crown base may never appear at DBH (Larson, 1963). Other changes in form that may have occurred further up the bole of the tree have gone unrecorded. In some instances, thinning schedules that produced no significant response at DBH could have produced interesting responses in bole increment high in the tree.

In the model described here, simulated diameter increment is recorded at each whorl along the entire bole of the tree. It is a simple matter in the model to calculate the conic volume for each internode summed over the entire length. In this way the simulation should characterize the growth that occurs over the entire bole of each tree. This will, in turn, yield the most sensitive response to treatment, since it is the individual tree that responds.

# RESULTS

This model is unique in that increments occur annually and that they occur at every whorl on every tree. Each tree is incremented in total height, stem diameter, and crown width at the maximum expectation for the site and age and constraints of surrounding competition. Initial individual tree input consists of site index, number of years required to reach 4.5 feet in height, and two dimensional coordinates from a stem map. The coordinates may be from any source--field plot, random number generator, or predesigned spacing. The model simulates up to 30 individual trees on any size of square plot predesignated by an input parameter.

## Inferred Components of the Tree Simulator

This model is based on the hypothesis that maximum growth of height and stem diameter for Douglas-fir on a specified site results when no constraints on crown growth are imposed. Any constraint on the natural expansion of the crown causes a reduction in stem radial increment. Constraints equal to or greater than the tolerance level set for crown competition cause the whorl to die and crown base moves up the bole one whorl. Reductions resulting in live crown length less than 30% of the total height cause a reduction in height increment. To simulate reduction on radial increment downward from crown base as described by Larson (1963), area increments equal to increment at crown base are imposed. As area increment is applied to larger circumferences, radial increment becomes smaller. A three-dimensional matrix in the computer keeps data on height, crown width, and stem area for up to 80 continuous years for each of a maximum of 30 trees.

In order to derive a stable and reliable model for individual tree development, some schedule for crown and stem increment suppression from open grown conditions had to be generated. Based on seven plots described later in Table 3 essential equations and coefficients were generated by repeated iterations of the simulation. A particular equation was modified only after the best coefficients failed to yield good results. Consistent prediction of a tree component over time and stand development was favored over a close prediction at one or two points. Over 100 iterations and associated analyses were necessary to arrive at the functions and equations on the following pages.

## Suppressing Diameter Increment

Each year growth begins at the top of the tree with height and crown increment. Progressing downward, each whorl is incremented, competition observed, and appropriate reductions in stem and crown increment made. Since the lower most whorls contribute little if any stimulus to stem increment some reduction capacity may be appropriate as competition approaches the tolerance index limit. After considering published research on growth potentials near crown base, a simple linear reduction equation was applied as a first approximation. The inferred reduction equation was of the form:

$$dDOB = dDOB_m [0.8 + (1 - 0.8)(TOL - CCQ)/(TOL - 100)]$$

where

$dDOB$ = diameter increment under competition

$dDOB_m$ = maximum possible diameter increment for current year

$TOL$ = tolerance limit for whorl to remain alive

$CCQ$ = crown competition quotient of current crown interaction

As may be seen in Figure 6, diameter growth is reduced only a small proportion in addition to any reduction due to effect of a short live crown length. Little growth stimuli originates from the lowermost live branches and, in fact, some researchers have stated that dying branches may have a negative effect.

After repeated attempts at simulating volume increment the above equation was discarded because it consistently produced stems almost cylindrical in form. To attain a more realistic stem form the following equation was adopted:

$$dDOB = dDOB_m [.3 + (1 - .3)e^{-2.3(1 - GPC)}]$$

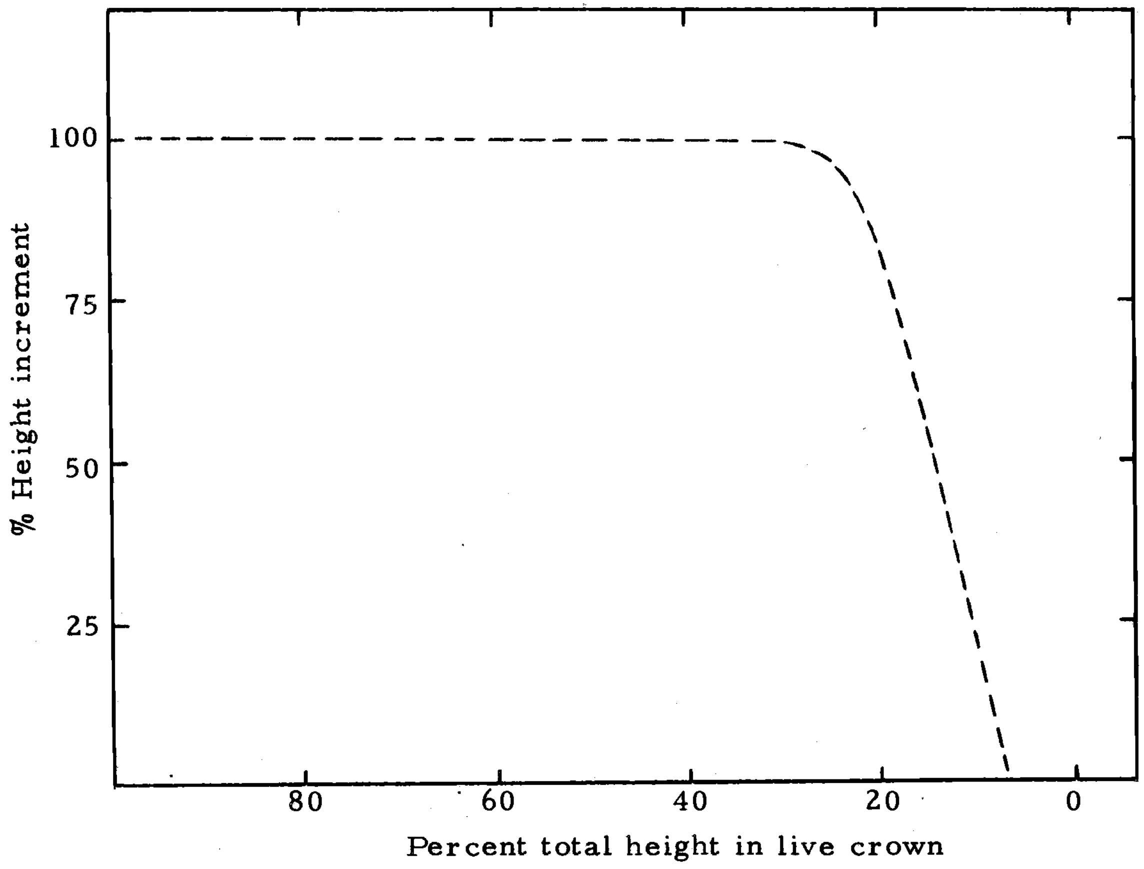


Figure 5. Height reduction rates for suppressed trees.

where

GPC = (TOL - CCQ)/(TOL - 100)

.3 = asymtote representing the minimum residual proportion of diameter increment when CCQ equals TOL

2.3 = reduction rate coefficient applied to relative crown competition

Since accrued diameters at given ages high in the tree are smaller, annual crown increment is reduced which in turn reduces intertree competition allowing a higher number of trees per acre to survive to any given stand age.

## Suppressing Crown Increment

As the tips of whorls come into competition with one another the amount and quality of light is severely reduced. Growth of the whorl drops off due to this competition and, in taller stands, due to physical abrasion of lateral buds on opposing trees. No previous work has quantitatively described the form of the relationship between competition and crown increment. Most researchers have tended to classify crown shapes as essentially parabolic when faced with this problem. A linear crown reduction factor similar to the diameter reduction equation was initially applied. It soon proved to be in error when compared to documented stand growth. A nonlinear form gave good results and was substituted resulting in crowns lengthening and assuming parabolic shapes similar to those described by previous workers. The nonlinear crown competition model assumed the form:

$$dCW = 2.0360\,(dDOB_c) + 2\,(-.0191)(DOB)(dDOB_c)$$

where

$$dDOB_c = (dDOB)e^{-5.6(1 - GPC)}$$

This equation predicts a rapid drop off in increment as competition begins and a slow approach to zero growth under extreme competition indices. The general form may be seen in Figure 7.

## Mortality

Criteria for death has been one of the major concerns in developing a realistic mathematical model of a dynamic forest stand. The longevity of suppressed trees has a direct effect on the potential diameter and volume of associated dominant and codominant trees in a stand. If mortality occurs too slowly stagnation becomes a problem because the growth of all trees in the stand is reduced; in some cases, growth responses in increased growing space exhibit subdued increments. Thus mortality schedules early in the life of a stand can have critical effects throughout the stand development.

Mortality at a high rate causes the number of trees to be reduced much too rapidly and residual trees maintain excessively long, overdeveloped crowns. Some minimum size of active crown is apparently necessary to maintain maximum height increment.

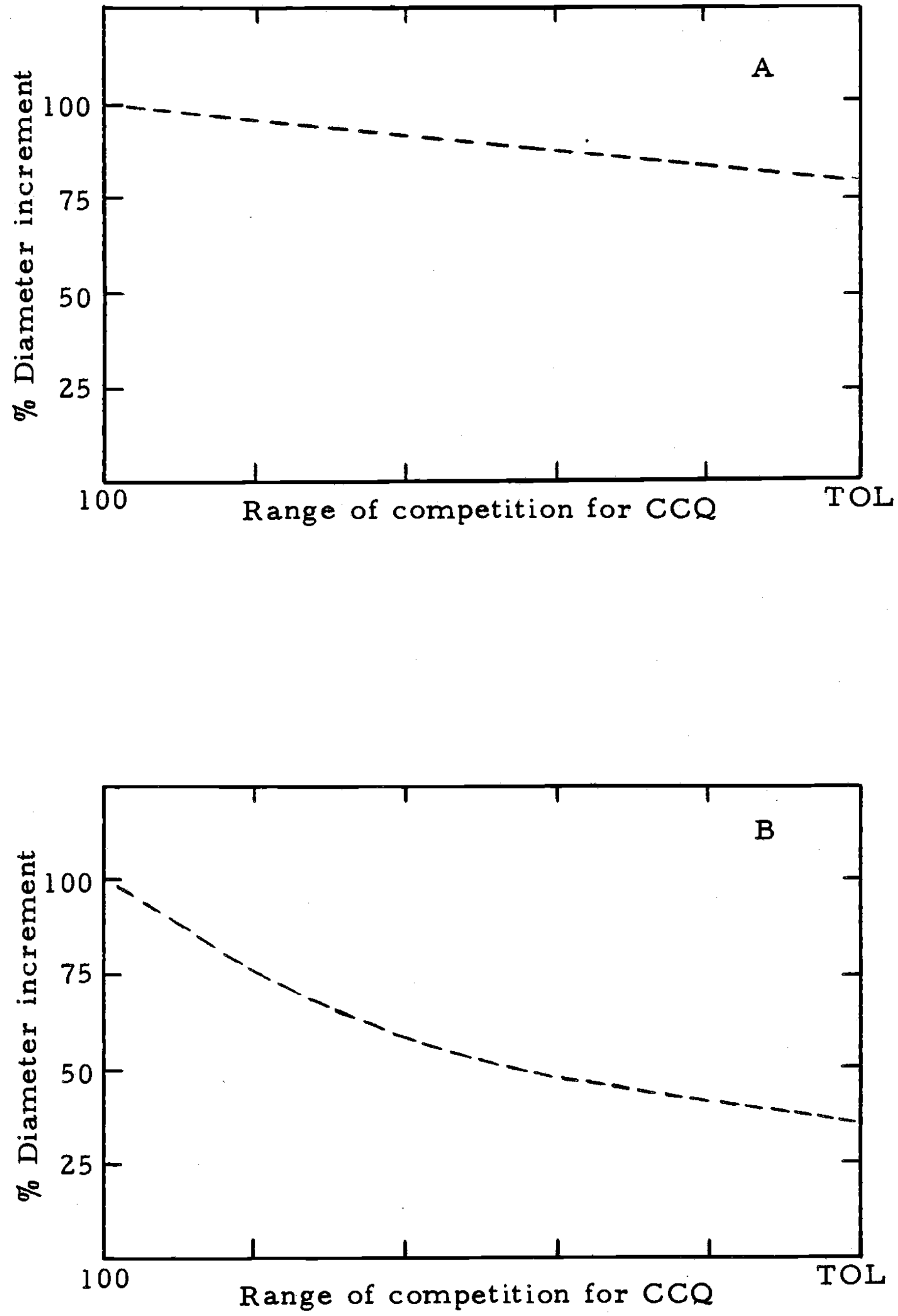


Figure 6. DOB increment reduction as a function of CCQ.
A: initial approximation; B: final approximation.

This minimum size changes with accrued height and age, but is generally 30% of the total height in live crown for Douglas-fir. As an arbitrary mortality level, 5% or less of the total height in live crown was deemed sufficient to carry on minimum plant functions of transpiration and respiration. This is an assumption of the model and does not necessarily occur in the field.

Some basis for mortality attributable to the individual tree is preferable. However, in lieu of adequately performing individual tree mortality schedules some stand schedule may suffice. A likely candidate is the maximum Crown Competition Factor that has been observed in normal stands. The simulation model could easily monitor development of CCF over time. The probability of death increases with density and poorer competitive position. Characteristics of deleted trees were then monitored in hope of detecting some stable individual tree characteristic useful in mortality prediction.

## Sequential Yearly Increments

All trees grow simultaneously each year. Since this is not possible in a simulator, each tree, in turn, receives increments of stem and crown based on the previous year's dimensions of surrounding trees. After all trees in the simulator have had an opportunity for growth the increments are accumulated on each tree.

The second attempt consisted of increasing dimensioned arrays to handle yearly height, stem diameter, and crown diameter for up to 30 trees for 80 years. A separate dimensioned array stores temporary yearly increments of stem diameter and crown diameter for 30 trees for as many as 80 yearly nodes. However, the core capacity of the CDC 3300 was exceeded by this approach and the 2 by 80 by 30 array was discarded in favor of incrementing each tree sequentially during the growth cycle.

The first trees in sequential incrementing always have a slight advantage over those incremented later. Adverse effects of this constraint are minimized when, as in this simulator, the order of sequential incrementing is randomized. The result is a variance in sizes of trees not unlike that which may be produced from genetic variation among individual trees in a stand.

## Average Tree Simulator

In order to assure stability of the model it was tested on a number of stand densities, sites, and ages. Verification of a complex model such as this becomes very difficult when basic parameters such as site index, mortality schedules, and initial live stems per acre are correlated with other 'independent' parameters (i. e., trees per acre), ill defined, or undocumented in past or current research.

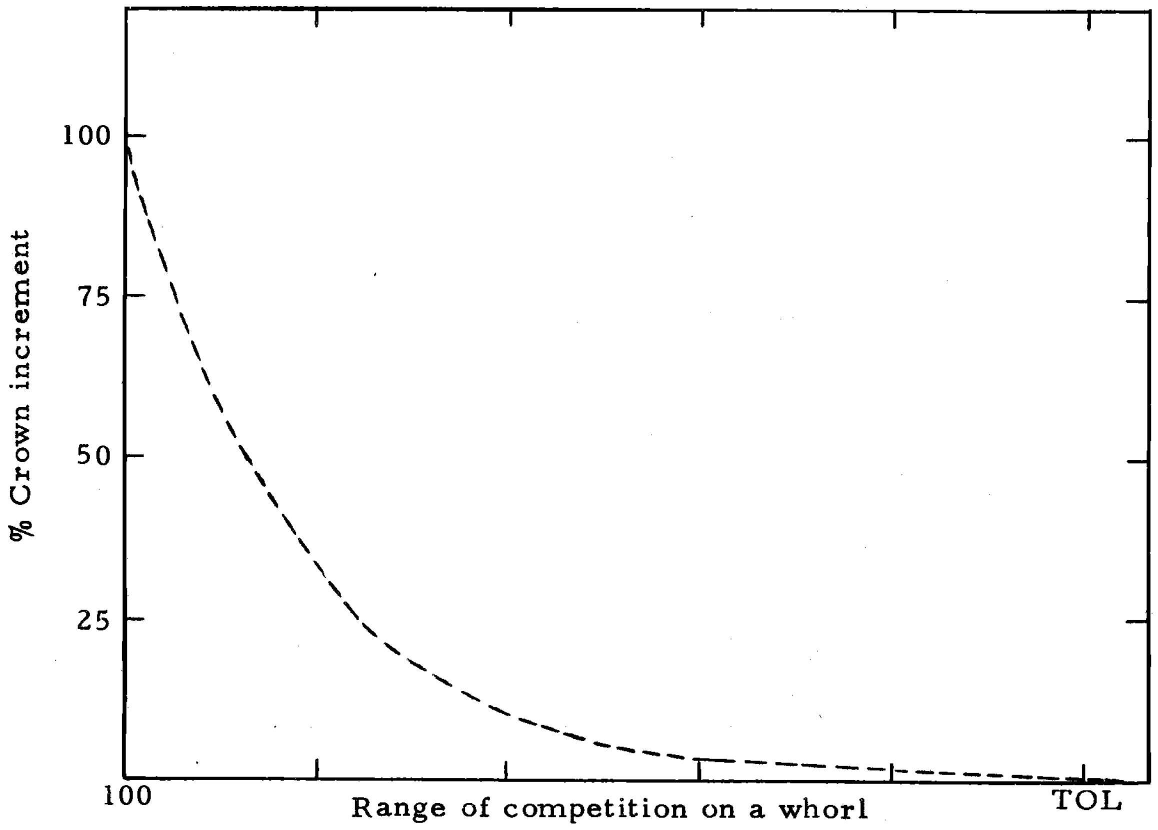


Figure 7. Crown increment reduction for suppressed whorls.

In spite of gaps in basic knowledge of the mechanisms of Douglas-fir growth under extremes of stand density, certain basic sensitivity tests were made. Most foresters are familiar with open-grown versus stand-grown tree forms as well as the typical variations in growth resulting from thinnings. The sensitivity and stability of this model were tested by observing profiles of simulated trees grown in the open, dense stands, and under intensive thinning.

Four average tree simulations were run for a site 135 (King) to a maximum age of 60 years. The first was essentially open-grown at 40 stems per acre. The following three were identical with 681 stems per acre except that the third and fourth were thinned to 300 stems at 20 years. The fourth received an additional thinning to 100 stems at 40 years. Profiles of a residual tree from each of these four simulated stands may be observed in Figures 8, 9, 10, and 11.

Even at 40 stems the open-grown simulation came into inter-tree competition finally forcing the crown base to recede. The plot of crown base position over time definitely demonstrates the type of reaction one would expect for these levels of site, age, and density. The thinning at 20 years was moderate and the short-term increase in radial increment until competing crowns again overlap is the kind of effect one would anticipate by such methods. Although few foresters would know exactly what to expect from the fourth simulation, researchers would agree with the general trend of stem form as compared to the unthinned dense stand.

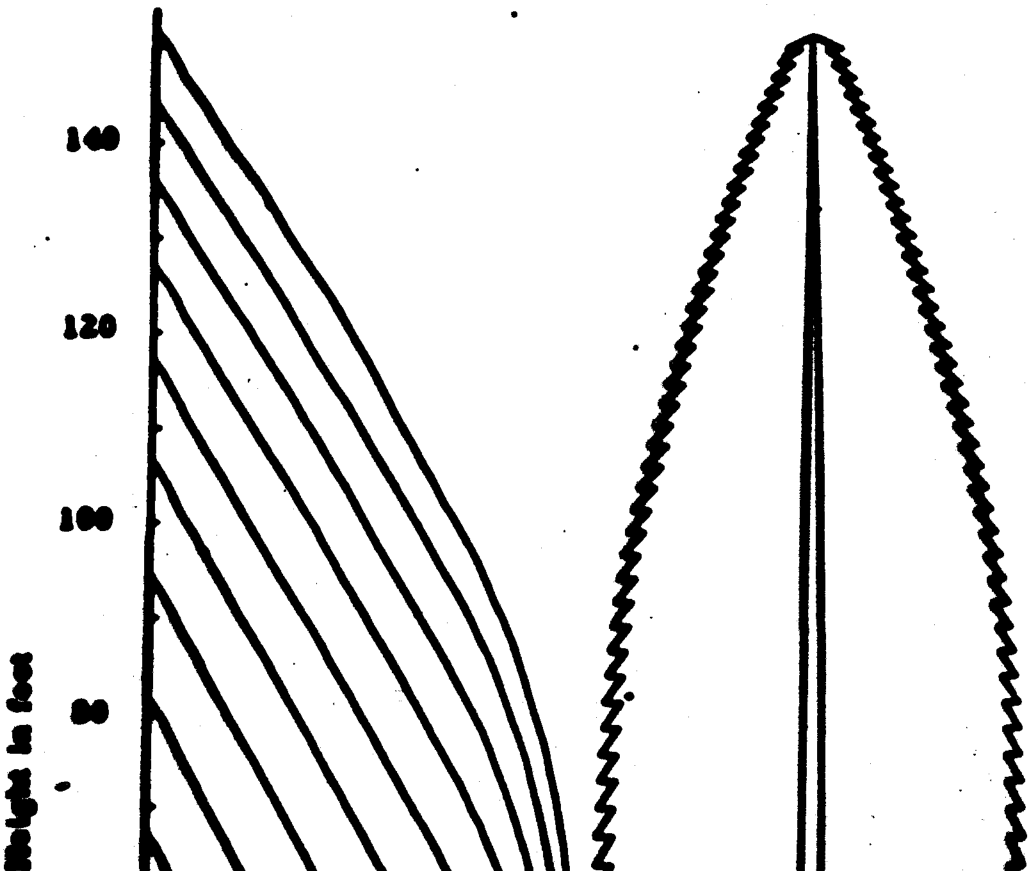


Figure 8. Simulated stem profile at 80 years showing 5 year increments from average tree simulation of 60 trees per acre.

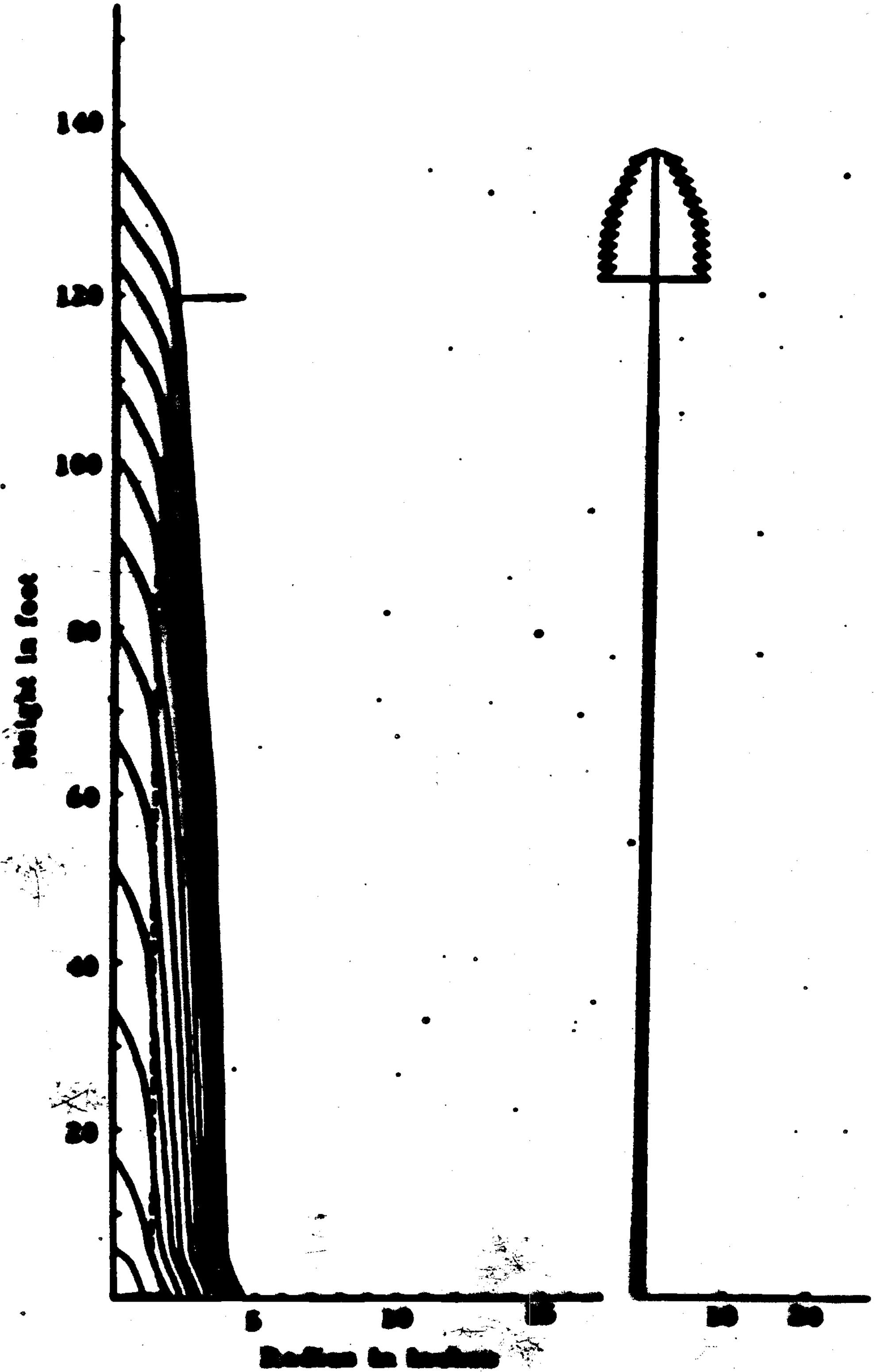


Figure 9. Simulated stem profile at 80 years showing 5 year increments. Initially 681 trees per acre.

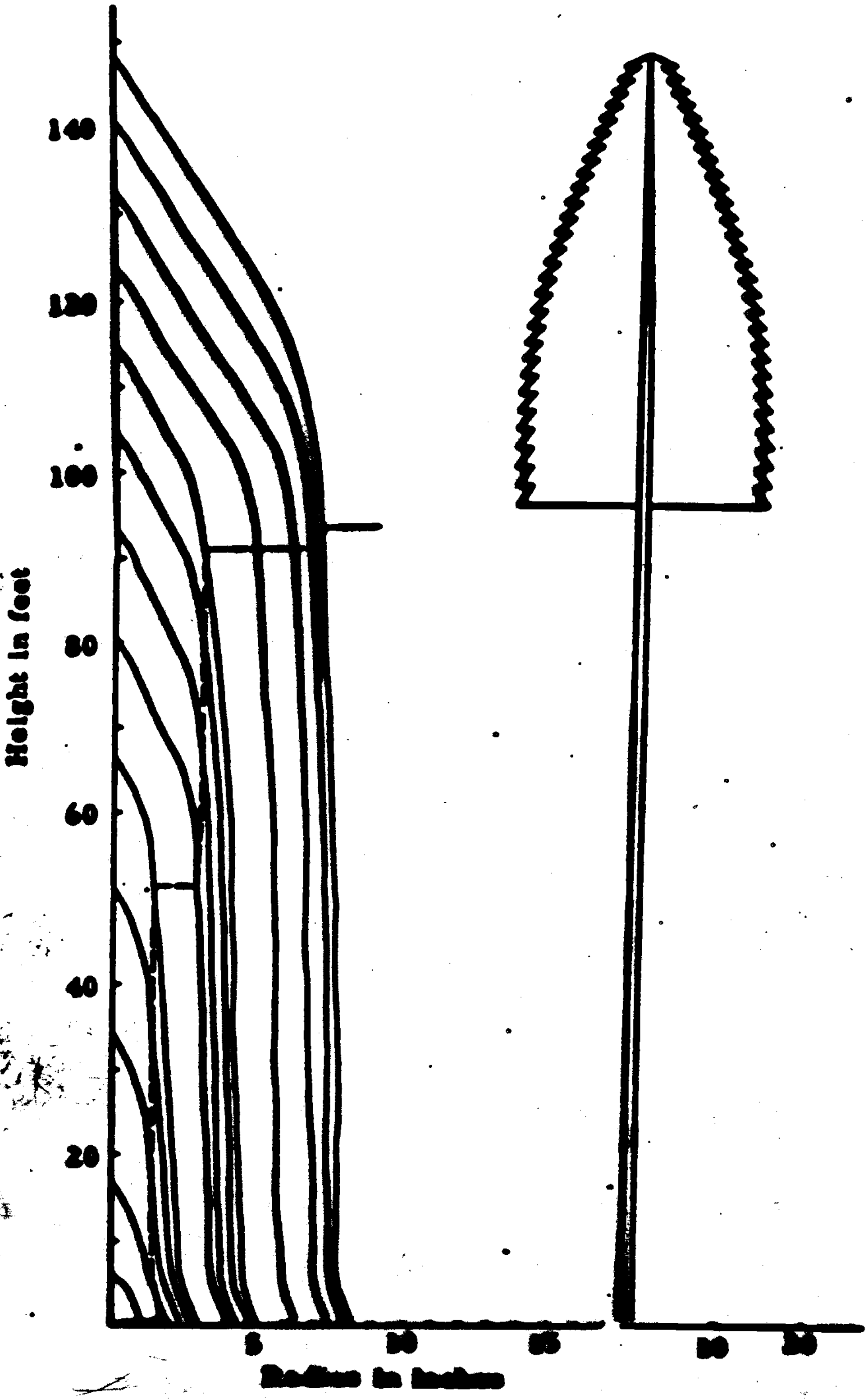


Figure 11. Simulated stem profile at 80 years showing 5 year increments. Initially 681 trees per acre, thinned at 30 and 60 years.

Other comparisons may be made as interest dictates by reading Table 2. The model is sensitive to environmental conditions imposed, yet stable enough to reflect these changes over a wide range of conditions.

## Stand Simulator

It is possible to simulate any stand condition, but coefficients acting on the various parameters have in many cases only been approximated. Only by chance would the model accurately describe a particular stand without further testing and parameter refinement. As individual relationships are understood, more exact and comprehensive field data can be collected to stabilize them. In this study seven plots from four localities were modeled. Five of the plots are from western Oregon and two are from Washington. None of the Oregon plots were established early enough in the stand history to obtain initial number of live stems per acre.

Plots A, B, and C (Table 3) represent part of a Levels of Growing Stock study established by the Oregon State Forest Research Laboratory in 1963. The stand was predominately Douglas-fir uniformly stocked with approximately 1, 700 stems per acre. Plot A, in this paper, is the average of three 1/5 acre plots, and B and C are averages of six replications each. Plots B and C (Figures 13 and 14) received calibration thinnings at initial plot installation leaving 345

Table 2. Data from average tree simulation, age 60 years, site 135 (King). (volume in cubic feet)

| | DBH | Height | TPA | Volume | Thinned | Mortality | Total |
|---|---|---|---|---|---|---|---|
| Open-grown (40 TPA) | 29.5 | 150 | 40 | 11,017 | -- | -- | 11,017 |
| Unmanaged stand (681 TPA) | 8.1 | 136 | 472 | 9,372 | -- | 3,000 | 12,372 |
| Thinned at 20 yrs. (681 TPA) | 10.8 | 145 | 300 | 12,594 | 1,285 | -- | 13,879 |
| Thinned at 20 and 40 yrs. (681 TPA) | 15.5 | 146 | 100 | 9,721 | 6,035 | -- | 15,756 |

Table 3. Description of sample plots.

| | Plot | | | | | | |
|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G |
| Location | Hoskins | Hoskins | Hoskins | Clackamas | Black Rock | Wind River | Wind River |
| Treatment | Control | Thinning | Thinning | Control | Control | 8 x 8 | 12 x 12 |
| Site (King) | 116 | 116 | 116 | 92 | 113 | 80 | 98 |
| Initial TPA | (2020)[1] | (2020) | (2020) | (2200) | (440) | 681 | 303 |
| Data span | 20-27 | 20-27 | 20-27 | 25-35 | 47-54 | 29-43 | 29-43 |
| Present TPA | 1272 | 327 | 207 | 1244 | 266 | 520 | 265 |
| Present DBH | 5.7 | 8.8 | 9.4 | 5.0 | 12.2 | 6.9 | 10.0 |
| Present age | 27 | 27 | 27 | 40 | 61 | 48 | 48 |
| Stand establishment | 1943 | 1943 | 1943 | 1930 | 1909 | 1922 | 1922 |
| Elevation | 1000 | 1000 | 1000 | 1450 | 1200 | 1350 | 1350 |

[1] Figures in parentheses are assumed initial number of trees per acre.

A, B, C: located approximately 22 miles west of Corvallis, Oregon near Hoskins, Oregon.

D: located 10 miles east of Mollala, north of the North Fork of the Mollala River.

E: located at Black Rock approximately 5 miles west of Falls City, Oregon.

F, G: located at Wind River, near Carson, Washington.

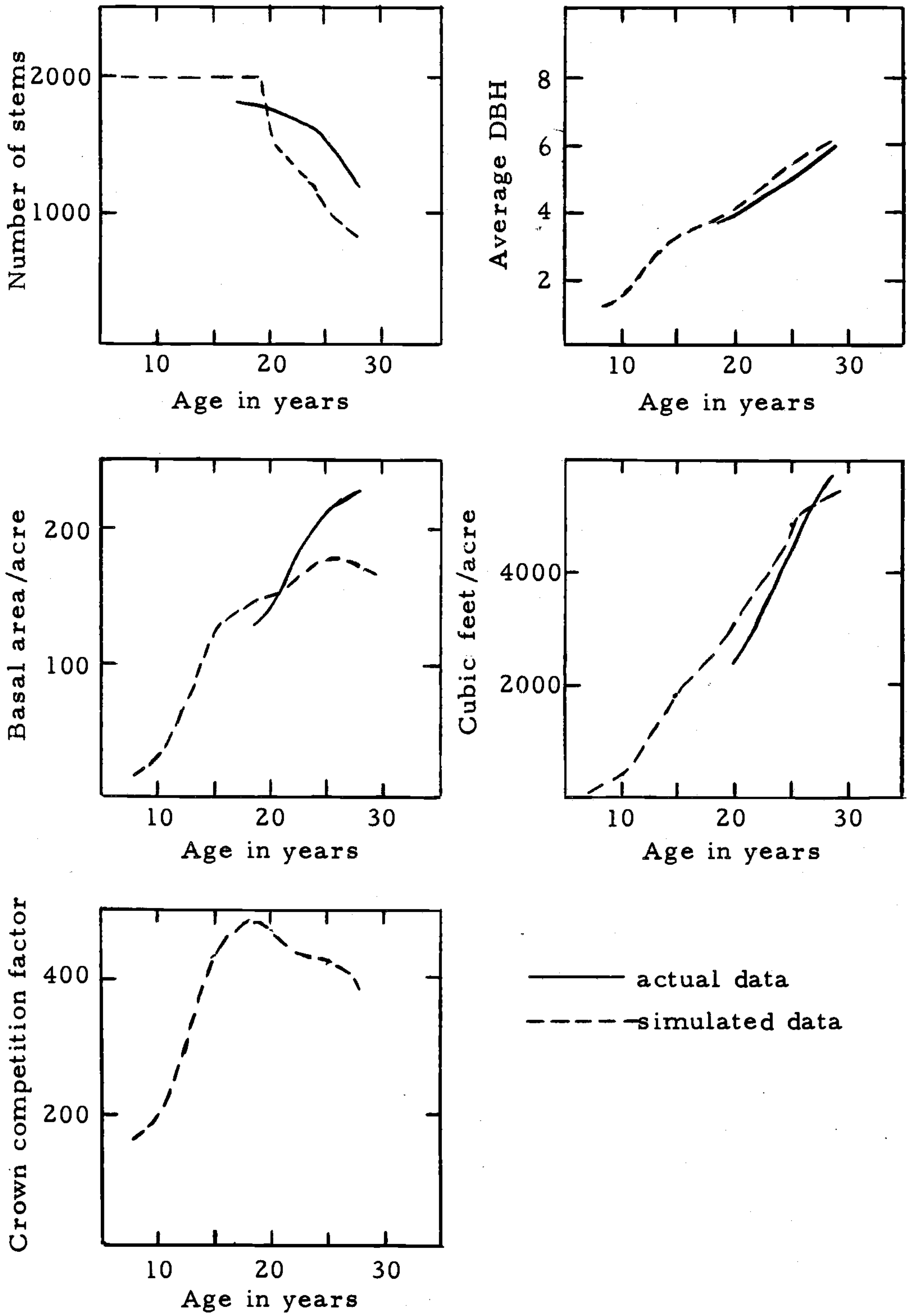


Figure 12. Plot A.

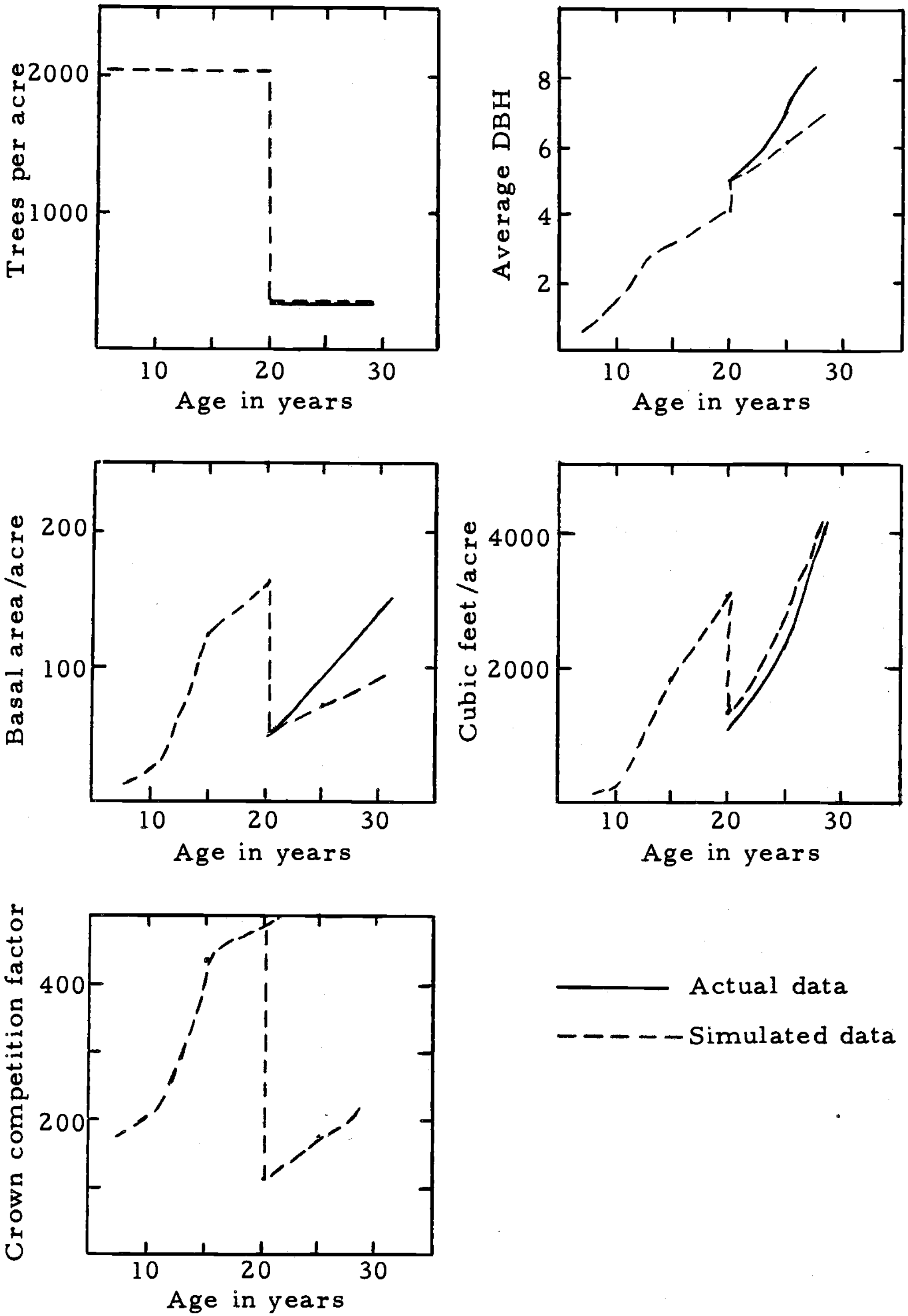


Figure 13. Plot B.

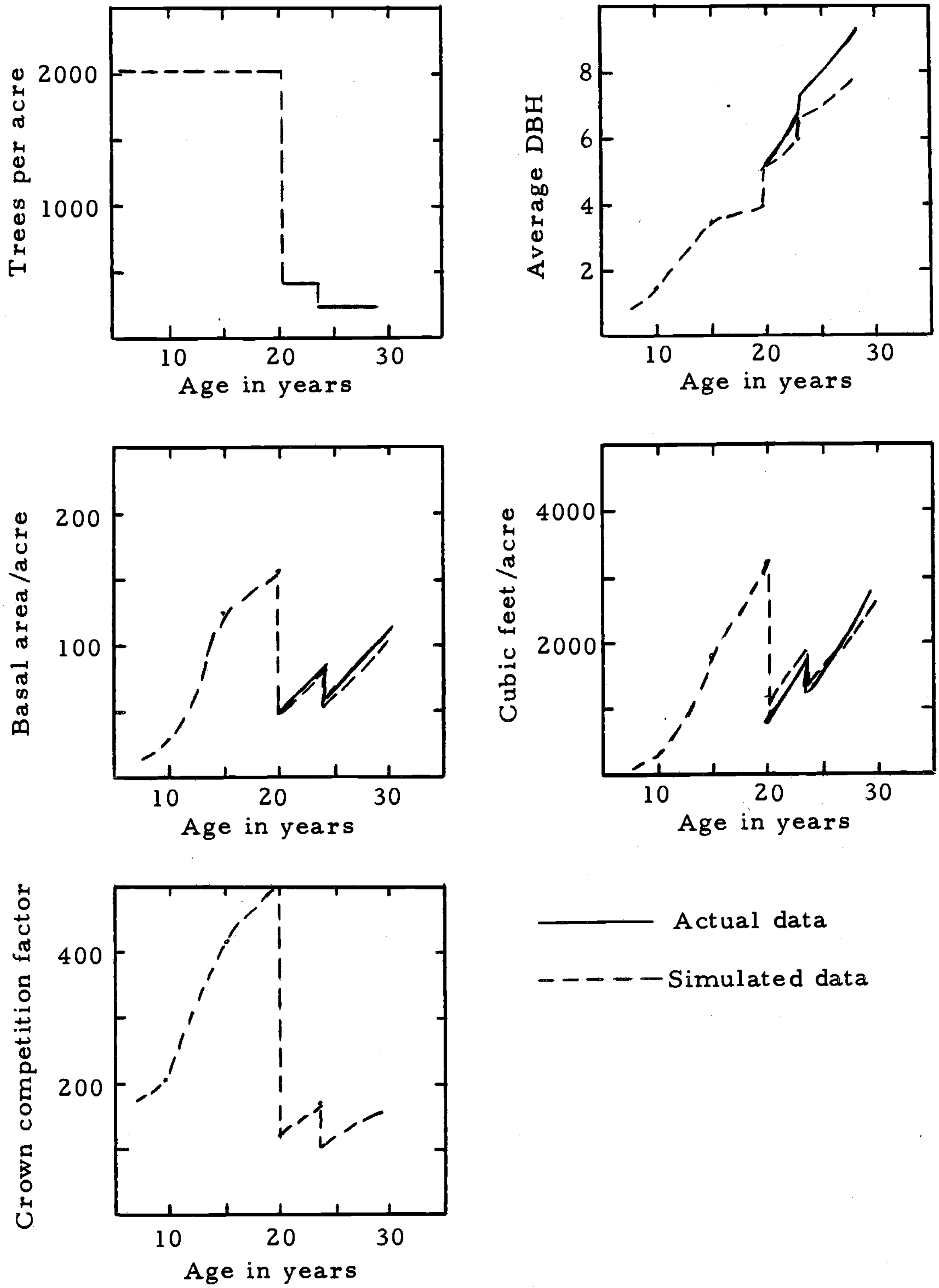


Figure 14. Plot C.

and 332 trees per acre respectively. Three years after the calibration thinning plot B was reduced to approximately 325 trees per acre and plot C to approximately 211 trees per acre.

Because of the limitations on the number of trees (30) that can be handled by the simulator, a square plot 25 feet on a side with 29 randomly distributed trees simulates 2,021 trees per acre. Based on 1,700 trees at 20 years, slightly over 2,000 trees would be required at 8 years when the stand averages 4.5 feet tall.

Since the simulator is being compared against the average of a number of replications a random distribution of stems offers the least difficulty for comparison tests. It also interjects an essentially wide range of growth potentials between trees in the model.

Various minimum limits of percent of total height in live crown were attempted as a criterion for mortality of individual trees in the model. Values over 5% produced excessive mortality early in the life of all stands simulated.

As may be seen in Figure 12, in which a mortality constraint of 4% of total height in live crown was applied, the stand is allowed to develop for 19 years before any mortality occurs. At this late date the crowns of many trees in the stand have been overly suppressed. The result is an overkill from the 19th to the 21st year before the stand again reaches some sort of stability.

Even though the average DBH of plot A maintains close agreement with actual data, mortality has reduced the residual stems per acre so severely that simulated basal area lags far behind by the 25th year.

Although the evidence is not sufficient to adjust coefficients, post thinning volume growth on plot B suggests radial increment on upper boles much accelerated over DBH growth. Without more complete upper stem radial increment data it is difficult to determine if this is an error in the model or evidence of field estimated volume not accounting for form change with thinning. Field volume estimates are based on a height-diameter access volume table used locally by the School of Forestry.

Figure 15 represents a comparison of simulated and actual data from a thinning and fertilization study by Crown Zellerbach near Mollala. Referred to as Plot D in this paper, the plot is one of the unthinned controls for their study. In order to determine a reasonable site index over the period of data collection, an index representative of actual heights recorded in the field was applied rather than the index listed by Crown Zellerbach. This method yields a site index of 92 based on 10 years to reach DBH height.

The simulation was based on 29 trees in a 24 foot square plot representing 2, 193 trees per acre. Mortality was modified to include a primary schedule limiting the maximum CCF at 400 and a secondary

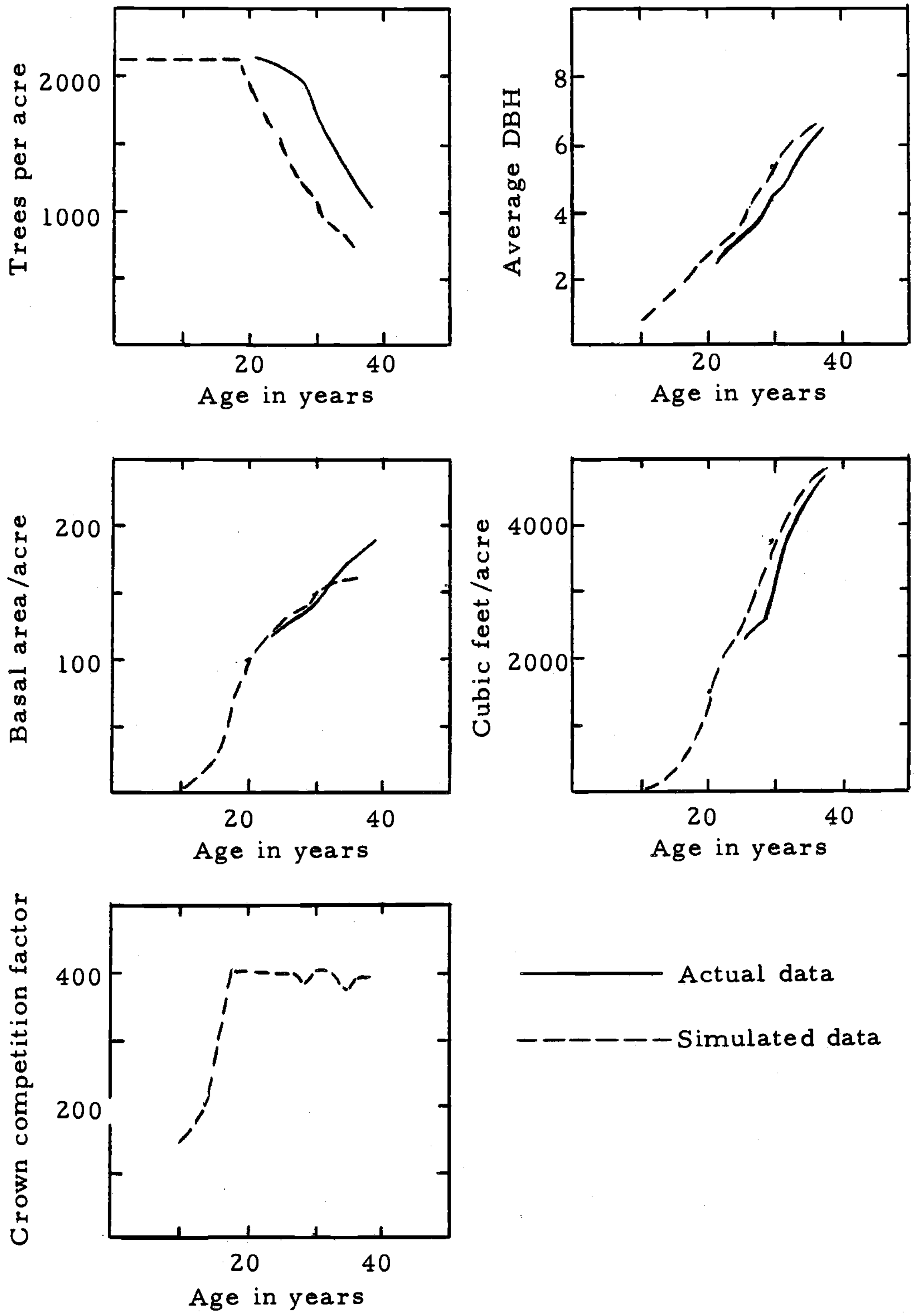


Figure 15. Plot D.

schedule limiting minimum percent live crown length at 2%. As stated previously, live crown length in this model has no relationship to field conditions.

The results are acceptable. Diameter breast high, total basal area, and total cubic foot volume all perform well. Mortality occurs at a similar rate to actual records. Trees in the model, however, are apparently too intolerant of one another accounting for the premature initiation of mortality at 19 years. This may be handled by adjusting the maximum CCF limit upward, thereby allowing all trees to remain alive until 20 or 22 years. However, there is no justifiable reason to expect a stand to develop CCF values to this excess. The difficulty, therefore, lies within the tolerance constraints of the individual trees in the model. The mortality schedule will have to be revised before modifications are made in functions of other components.

The Black Rock data represent a complex problem for individual tree simulation models. The data chosen for simulation comparison are from Black Rock Plot 12, a control plot one full acre in size. The stand was already 47 years of age when studies were initiated. There is no information about the initial number of stems per acre nor about intermediate stocking levels prior to 47 years. Only through iterative runs of the simulator adjusting for initial stems per acre is it possible to produce a stand 47 years old with the same characteristics of size, number, and potential for future growth. This is possible only if the simulator has been tested previously for accuracy throughout the younger age classes. Figure 15 demonstrates that 436 stems per acre was much too few. Diameter increment started at a high rate and reached comparable size to actual DBH's 11 years ahead of schedule. The problem of intolerance discussed on the previous stand simulation is also expressed here. Mortality is described by the 4% crown length minimum as before. However, the CCF levels out just under 340 while mortality continues. CCF calculated on the actual stand from 47-54 years showed a definite leveling effect near CCF equal to 360. The simulated stand was not able to reach higher CCF levels because of the intolerance problem.

The final two plots, F and G (Figures 17 and 18), are simulations of spacing tests where the initial number of stems are known exactly. These plots provide information not available in plots previously discussed. However, site index turns out to be highly dependent on stand density; therefore, the confounded site index estimate had to be adjusted for each plot by comparing height growth. Diameter increment predicted by the simulator is much too high for these low sites even in the presence of greater numbers of trees per acre. This overestimate is also depicted by the high levels of CCF that occur.

## Discussion

After more than 100 simulation runs on the computer some interesting stand relationships became apparent. Repeated

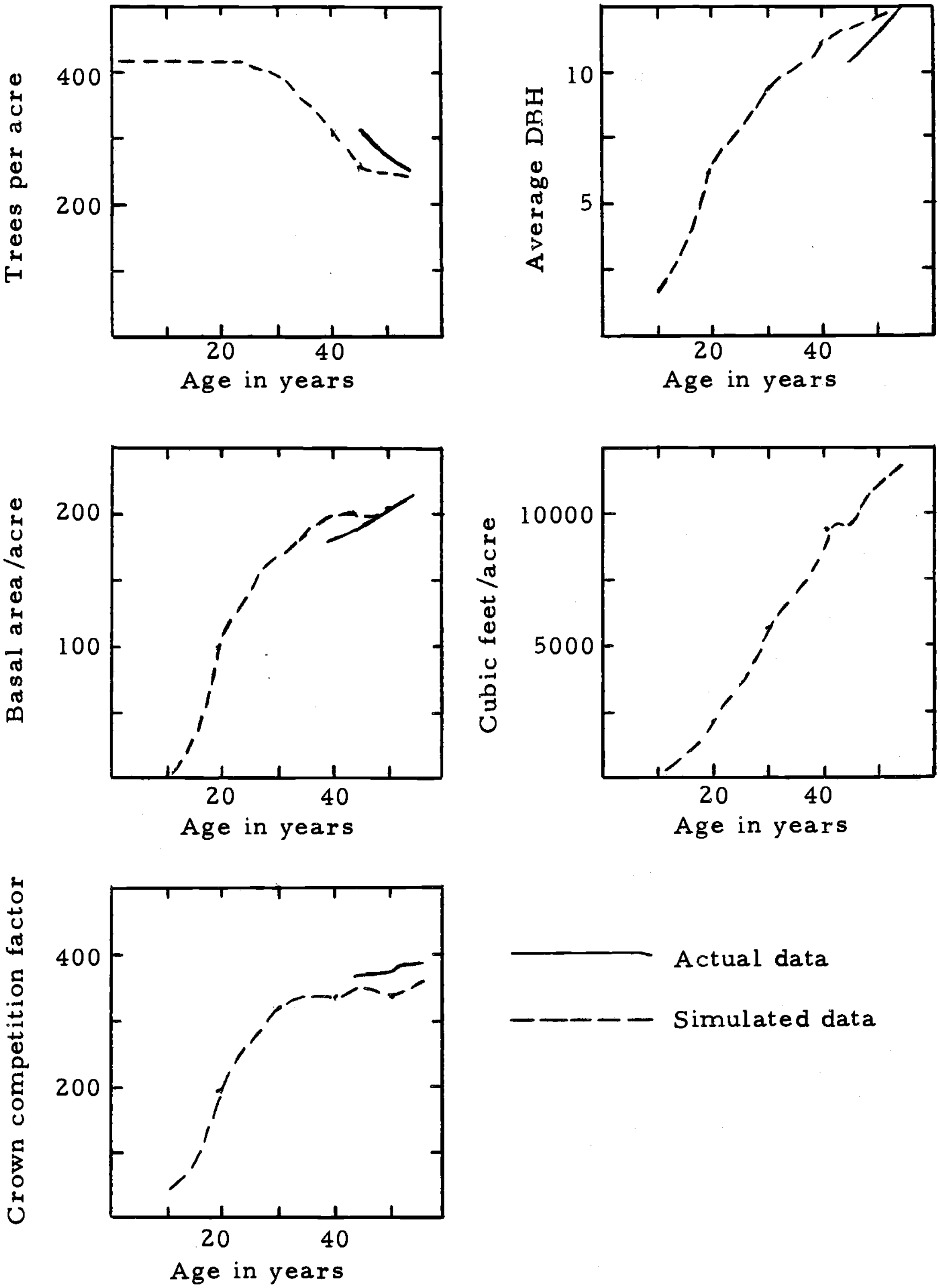


Figure 16. Plot E.

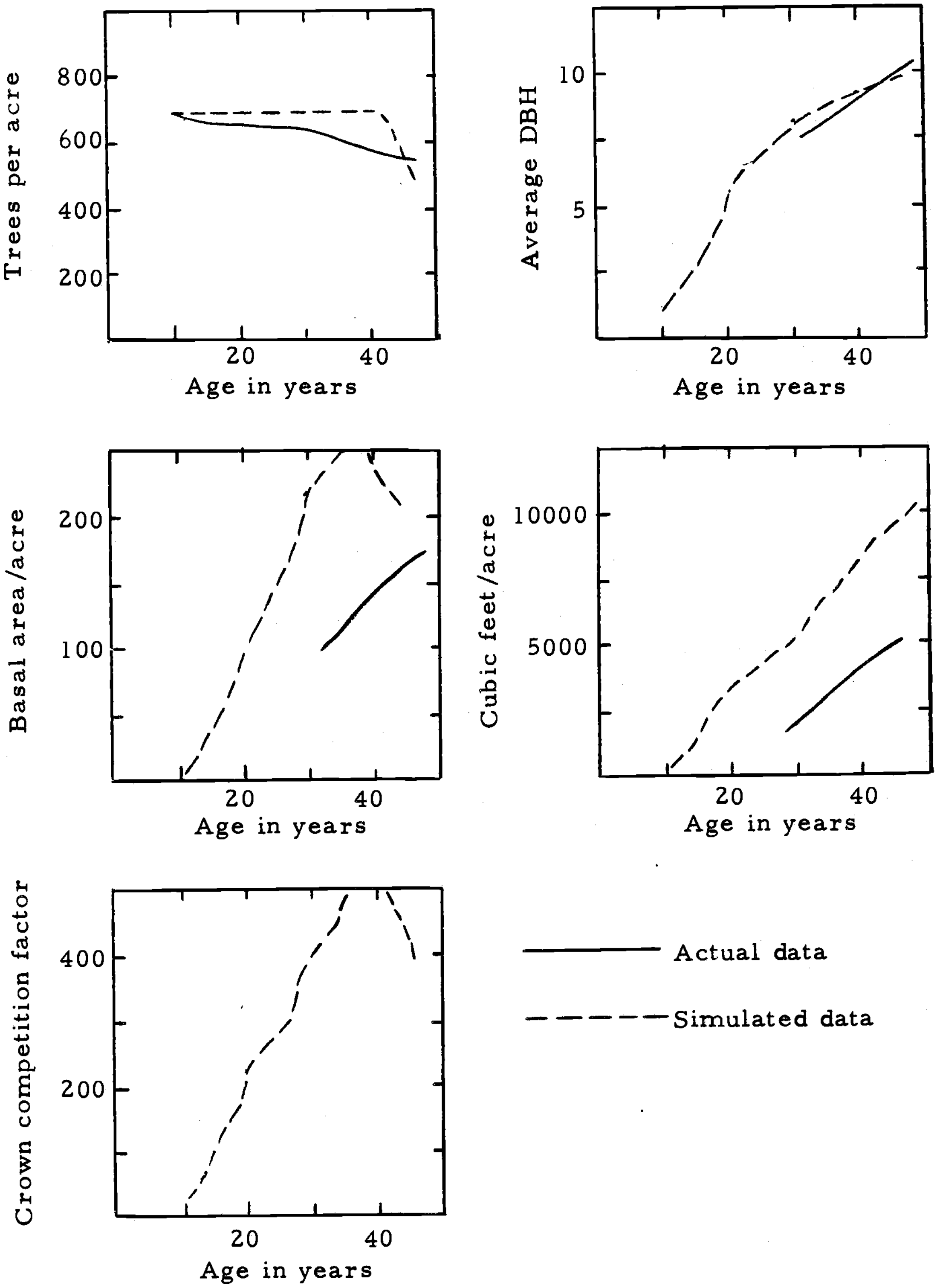


Figure 17. Plot F.

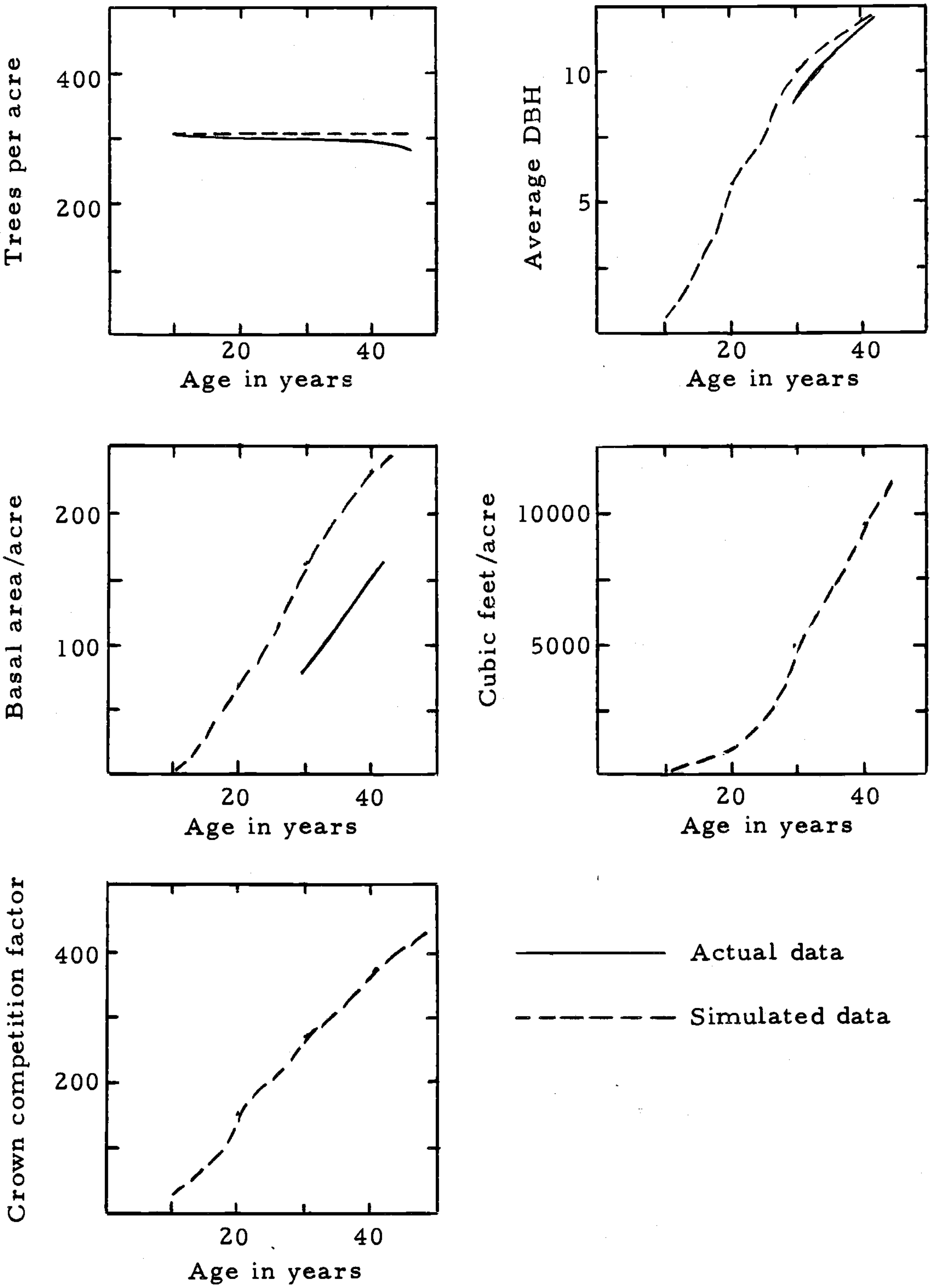


Figure 18. Plot G.

simulations of the Wind River spacing trials demonstrate that the effect of microsite variation and initial age to reach DBH between trees is similar to random spacing in their effect on tree growth. When all trees in the model had identical site index and number of years to reach DBH the stand tends toward stagnation. This is understandable when one realizes that the model assumes a perfectly flat forest floor with uniform light input. Each tree is no larger nor smaller than the next. All trees uniformly become suppressed and die within a two or three year period. Only sequential incrementing is responsible for some trees lasting an additional year or two.

Intertree variation on the Wind River spacing trials was supplied by distributing site index on approximately half of the trees above and below the mean. A deviation of two units of site index appeared to have similar results as a one year deviation in initial age for young stands. As the stands develop further the relationships change with characteristics somewhat unique to each condition.

Further study may reveal that uniformly spaced stands require more frequent thinnings to maintain a high level of growth.

Interesting effects on potential response of stand-grown trees have been observed when coefficients regulating crown expansion are altered. When crown increment is constrained competition does not build up as rapidly resulting in longer crowns. This changes the form of the tree, reduces mortality, and increases potential radial increment responses due to thinning.

Stem form is directly affected by competition on successive whorls down from the tip of the tree. This empirical relationship has a direct effect on volume and is easily manipulated by the user.

To simulate the growth of a stand such as those described in this paper for 60 years cost approximately $10, including all output and a plot of one tree profile.

## Limitations

The most frustrating limitation in the model is the instability of site index. Since site is the expressed height growth for dominant trees under generally normal stand conditions, any management scheme that significantly alters these "normal" conditions alters the basic potential height growth for the stand.

Since the accrued dimensions of each tree at the end of each year influence the processes during subsequent years it is not generally possible to initiate the simulation at intermediate points in the development of the stand. To initiate a simulation from some intermediate time would require extensive stem analysis data for stem form on each tree.

As the computer program is presently structured, mortality occurs through suppression only. A list of all live trees is updated annually in the simulation. A small modification would allow mortality to enter as a result of windthrow, insect or fungal attack, or some other limiting constraint.

The single-most controlling factor on stem radial increment is the estimate of competition endured by each whorl. The rate of increment damping with an approach of competition toward the tolerance limit was prescribed without prior knowledge. Further investigation may reveal more applicable functions. Until basic information is collected to prove otherwise, the simple relationship described in this paper yields good results.

## Applications

Although generally difficult to develop, estimating individual tree growth and yield and aggregating to stands is thought to be the best approach to projecting growth in mixed stands, composed of a number of species, ages, and quality classes with widely differing growth rates. This program was carefully written to ensure that each tree is entirely autonomous. Each tree has a file stating the number of years that tree will wait to reach 4.5 feet in height. A second list contains the exact site for each tree. Varying these two files makes it possible to study site variation and time or establishment on stand dynamics.

These characteristics give the tree physiologist and the silviculturalist a very powerful tool to study intertree dynamics in natural and managed stands. With a small modification it is possible to alter site by tree over time. Effects of fertilization may be tested using this modification. The average site may be increased for a short period and reduced back to its original level similar to fertilizer application.

Growth and yield for any spacing and management schedule may be tested. North and East coordinates may be input directly from field plots along with exact age and site for each tree.

Log grade and quality yields may be predicted. Every diameter at every node is recorded along with length between nodes. Plotter outputs or data outputs may be used to measure number of rings per inch along any portion of the stem. Position of crown base over time in conjunction with final stem diameters may be used to estimate volume in clear veneer produced by a management schedule of interest. A similar model can be structured from this basic approach to handle two or more species, uneven-aged stands, or sloping or uneven topography.

Successful individual tree models of this nature may be useful tools in many aspects of forest research, training, and planning. Management tables may be produced giving the forester a basis by which a stand may be managed to maintain maximum increment under a range of stand conditions. Models of this type not only predict yield of cubic feet, basal area, stems per acre, quality class, and form, but also point out which parameters of the stand are most important to measure and control. The next step in this model for applications would be growth predictions for various combinations of site, age, and CCF. The forester need only to apply the point sample technique described in Appendix II to determine the present competition level of a stand. With the stand site and age it would be possible to estimate growth rates over the next period. The forester may compare expected growth with predicted growth for various thinning, fertilization, and irrigation schedules in conjunction with anticipated net incomes. Inputs to decision-making of this nature fulfill many of the needs that "normal" yield tables, stand tables, and various classifications of growth, stocking, and stand condition have attempted to fill in the past.

The tree model may be used to simulate tree and stand dynamics to clarify and expand classroom training in silviculture, mensuration, timber management, and forest valuations. It would provide a needed input to timber supply studies, determination of allowable cut and sustained yield predictions.

# CONCLUSIONS

An individual tree model was designed and programmed for Douglas-fir on a CDC 3300 electronic computer. It is based on empirical measurements of height, stem diameter, and crown diameter of open-grown Doublas-fir over all sites and ages. Through iterative simulations of documented permanent field plots and intensive analysis of observed growth relations from cited literature, a number of inferences were successfully made which describe constraints on most aspects of tree growth under stand conditions.

The model provides a sound, simple approach for prediction of growth and yield in a highly complex dynamic forest stand. Much more effort is required to describe the functional relationships as they truly exist. Relationships developed and discussed in this paper such as measuring tolerance in the field have opened new avenues of investigation. It is hoped that they will provide insight for other researchers in ways yet undiscovered by this author.

# BIBLIOGRAPHY

Bella, I. E. 1969. Competitive influence-zone overlap: a competition model for individual trees. Bi-Monthly Research Notes. Vol. 25(3):23-24. Department of Fisheries and Forestry. Ottawa, Canada.

Braathe, P. 1957. Thinning in even-aged stands--a summary of European literature. Faculty of Forestry, University of New Brunswick, Fredericton, N. B., 92 p.

Clutter, Jerome L. 1963. Compatible growth and yield models for loblolly pine. Forest Science 9(3):354-371.

Duff, G. H. and N. J. Nolan. 1953. Growth and morphogenesis in the Canadian forest species. I. The controls of cambial and apical activity in red pine. Canadian Journal of Botany 31:471-513.

Farrar, J. L. 1961. Longitudinal variation in the thickness of the annual ring. Forest Chronical 37:323-349.

Gerrard, Douglas J. 1967. Competition quotient: an index of the competitive stress affecting individual forest trees. Ph. D. thesis, Michigan State University. 64 p.

King, James E. 1966. Site index curves for Douglas-fir in the Pacific Northwest. Weyerhaueser Forestry Paper No. 8. Forestry Research Center, Centralia, Washington. 49 p.

Krajicek, J. E., K. A. Brinkman and S. F. Gingrich. 1961. Crown competition--a measure of density. Forest Science 7(1):35-42.

Larson, P. R. 1963. Stem form development of forest trees. Forest Science Monograph 5. 42 p.

Leary, Rolfe Albert. 1968. A multi-dimensional model of even-aged forest growth. Ph. D. thesis. Purdue University.

Lee, Yam (Jim). 1967. Stand models for lodgepole pine and limits to their application. U. B. C. Faculty of Forestry. Ph. D. thesis. 332 p.

Lin, Jim. 1969. Growing space index and stand simulation of young western hemlock in Oregon. Ph. D. thesis, Duke University School of Forestry. 182 p.

Mitchell, K. J. 1969. Simulation of the growth of even-aged stands of white spruce. School of Forestry, Yale University. New Haven, Connecticut. Bulletin No. 75. 48 p.

Myers, Clifford A. 1968. Simulating the management of even-aged timber stands. USDA Forest Service Res. Paper RM-42, Rocky Mountain Forest and Range Expt. Sta. (Revised 1969)

Newnham, R. M. 1964. The development of a stand model for Douglas-fir. Ph. D. thesis, University of British Columbia. 201 p.

Paille, G. 1970. Description and prediction of mortality in some coastal Douglas-fir stands. Ph. D. thesis. UBC Faculty of Forestry. 300 p.

Opie, Jack E. 1968. Predictability of individual tree growth using various definitions of competing basal area. Forest Science 14(3):314-323.

Reukema, Donald L. 1961. Crown development and its effect on stem growth of six Douglas-firs. Journal of Forestry 59:370-371.

Spurr, Steven H. 1962. A measure of point density. Forest Science 8:85-96.

Staebler, G. R. 1951. Growth and spacing in an even-aged stand of Douglas-fir. Unpublished M. F. thesis, University of Michigan.

__________________ 1958. Some mensurational aspects of the level-of-growing stock problem in even-aged stands. Journal of Forestry 56:112-115.

# APPENDICES

# APPENDIX I

# CROWN AREA OVERLAP

Formulas for determining crown area overlap between two competing tree crowns where:

d = distance, in feet, between two competing tree centers

$r_1$ = radius, in feet, of the larger crown

$r_2$ = radius, in feet, of the smaller crown

$\theta$ = angle of incidence of the two crowns measured from the larger crown

$x_1 = (r_1^2 - r_2^2 + d^2) / 2d$

$x_2 = d - x_1$

The derivation of the overlap formula is as follows for conditions where d is greater than $x_1$:

$$\text{Area ABD} = x_i \sqrt{r_i^2 - x_i^2}$$

$$\text{Area ABCD} = r_i^2 \cos^{-1}(x_i/r_i)$$

$$\text{Area BCD} = r_i^2 \cos^{-1}(x_i/r_i) - x_i \sqrt{r_i^2 - x_i^2}$$

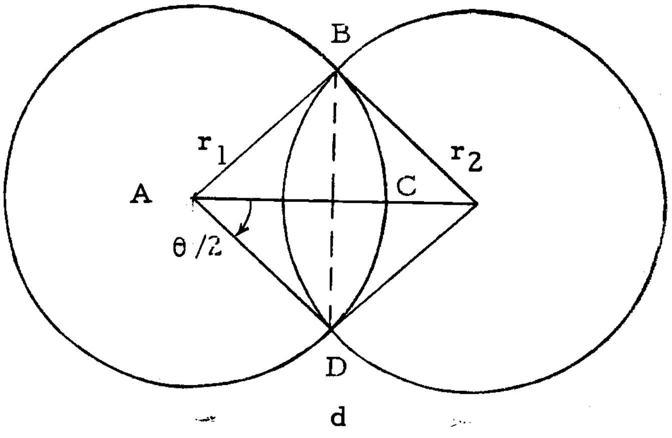
$$\text{Area of Overlap} = r_1^2 \cos^{-1}(x_1/r_1) + r_2^2 \cos^{-1}(x_2/r_2) - d \sqrt{r_1^2 - x_1^2}$$

Since the computer handles sine more efficiently than cosine, the following transformation is applied:

$$r_i^2 \cos^{-1}(x_i/r_i) = \pi r_i^2/2 - r_i^2 \sin^{-1}(x_i/r_i)$$

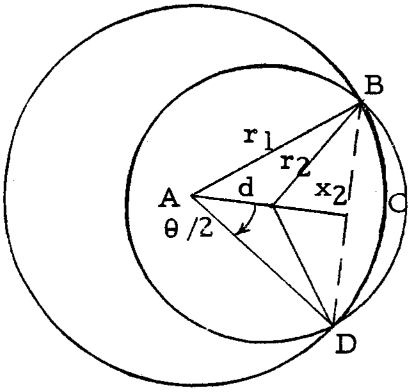
Therefore:

$$\text{Area of Overlap} = \frac{\pi}{2}(r_1^2 + r_2^2) - [d \sqrt{r_1^2 - x_1^2} + r_1^2 \sin^{-1}(x_1/r_1) + r_2^2 \sin^{-1}(x_2/r_2)]$$



In cases where $x_1$ is greater than d the equation changes to the form:

$$\text{Area of Overlap} = \frac{\pi}{2}(r_2^2 + r_1^2) + r_2^2 \sin^{-1}(x_2/r_2) - r_1^2 \sin^{-1}(x_1/r_1) + x_2\sqrt{r_2^2 - x_2^2} - x_1\sqrt{r_1^2 - x_1^2}$$



# APPENDIX II

## SAMPLING FOR AN INDEX OF TOLERANCE

Commercial tree species have long been characterized according to their relative tolerance to reduction in light quality and quantity. Various levels of tolerance, however, have never been related to an unbiased measurement scheme. Since it was necessary in this study to rely on a sound basis for tolerance a new measurement system was implemented.

In the description of intertree competition, Crown Competition Quotient (CCQ) is defined as being less than 100 when the crowns of neighboring trees do not touch. As crowns overlap, competition becomes more severe and eventually exceeds the tolerance limit for Douglas-fir. In uniform undisturbed stands it has been commonly observed that the base of the live crowns form a horizontal plane through the forest. Upon investigation, estimates of competition at crown base could be made by measuring the stem diameters at crown base of all trees on a large plot. Potential crown widths were calculated and an estimate of competition derived. Since the number of trees within a unit area is necessary to measure competition, the plot must be large to increase the number of trees relative to the number with crowns extending beyond the plot boundary.

Such a sampling system as just described is tedious and time consuming. To yield a single estimate of competition requires measuring 50-100 diameters on a plot at least 1/5 acre in size. An alternative sampling system was worked up to reduce the number of trees to be measured and to do away with crowns extending outside plot boundaries. A sample with probability proportional to stem area at crown base will accomplish these objectives. The estimator for Crown Competition Quotient is:

$$\hat{T}_y = \Sigma y_i / P_i$$

where

$$y_i = \frac{\pi D_i^2 (100)}{4(43560)},$$ maximum crown area of the ith tree as a percent of one acre

$D_i$ = crown diameter as a function of stem diameter on the ith tree at crown base

$$P_i = \frac{x_i \mathrm{CSC}^2 (\frac{\theta}{2})}{43560},$$ inclusion probability of the ith tree in the sample

$x_i$ = stem area, in square feet, of the ith tree

$\theta$ = horizontal angle of predetermined size which defines the size of the imaginary circle for a given sized tree

Therefore,

$$\hat{T}_y = \frac{\Sigma y_i 43560}{x_i \mathrm{CSC}^2 (\frac{\theta}{2})}$$

$$= F \, \Sigma y_i / x_i$$

where

$$F = \frac{43560}{\mathrm{CSC}^2 (\frac{\theta}{2})},$$ basal area factor

Given that n samples are taken in a stand the estimator takes the form

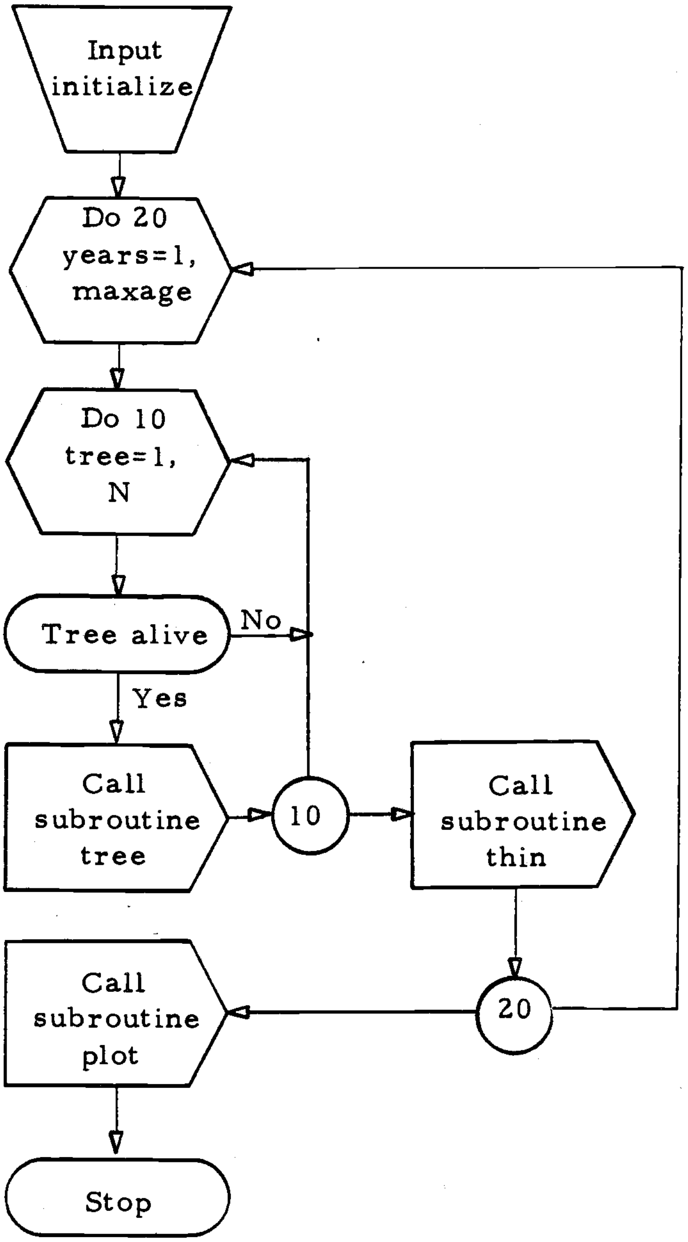
$$\hat{T}_y = \frac{F}{n} \sum_{j=1}^{n} \sum_{i=1} y_{ij}/x_{ij}$$

In applying the theory to measuring stands, a dominant or codominant tree was chosen in a homogeneous, undisturbed portion of the stand. This tree constituted the sample point and in effect competition was measured for this tree by the sample. A 10 basal area factor was used because it approximated an imaginary circle about equal to the maximum crown width for the sampled stem diameter. A McClure Mirror caliper was used to measure stem diameter at crown base for all possible competitors including the central tree. The McClure caliper was most efficient for this sample because most diameters are small and the horizontal range from the measurement point need not be known. Stem diameter was input into a table of boarder line distance to determine which trees were competitors for the sample. Horizontal distance between trees were then measured on the ground. Height and age were also measured on a dominant tree within or near the sample. Site index was subsequently determined for input into an analysis to test for interaction between Crown Competition Quotient and site, age, and height.

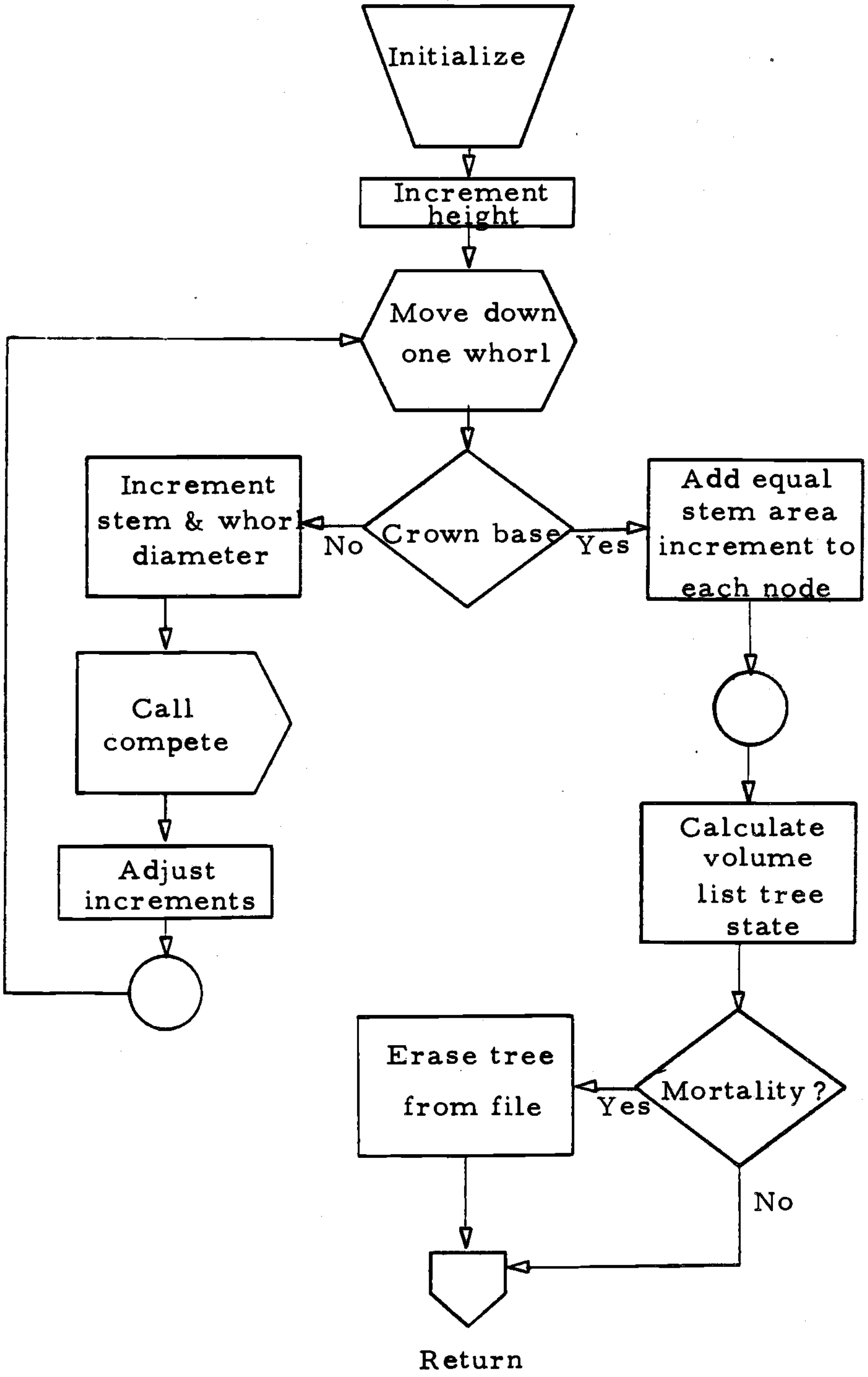
# APPENDIX III

## FLOW CHART OF PROGRAM

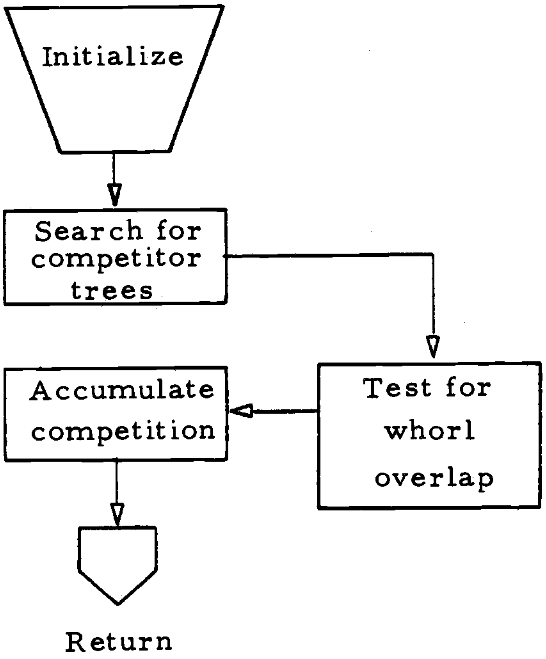
Program Stand



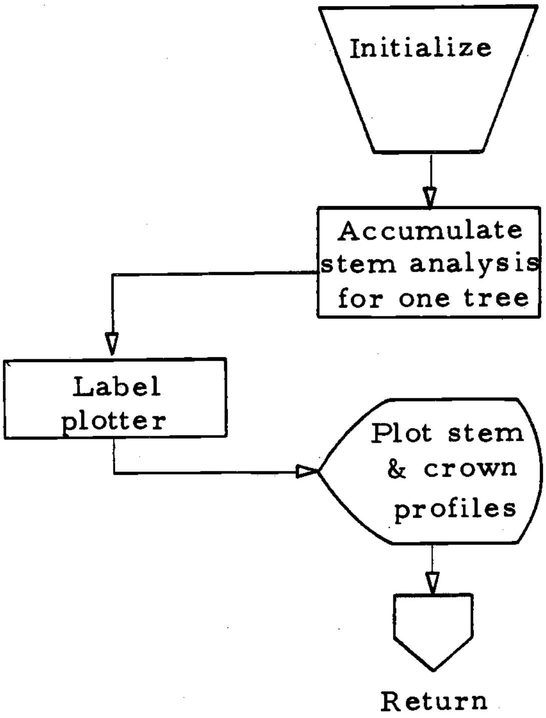
Subroutine Tree



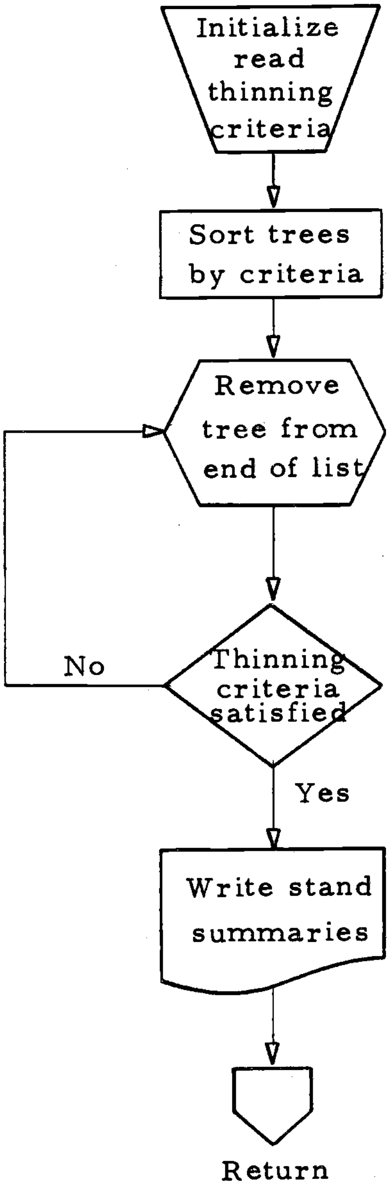
Subroutine Compete



Subroutine Plott



Subroutine Thin



# APPENDIX IV

## COMPARISON OF CCF TO COMPETITION QUOTIENT (GERRARD)

$$CCF = \frac{100 \sum_{j=1}^{n} A_j}{43560\ B}$$

where

$A_j$ = maximum crown area of jth tree

B = number of acres in the sample

$$CCF = \frac{100}{43560\ B} \sum_{j=1}^{n} (A_j - \sum_{i=1}^{k} a_{ij} + \sum_{i=1}^{k} a_{ij})$$

$$= \frac{100}{43560\ B} \sum_{j=1}^{n} (A_j - \sum_{i=1}^{k} a_{ij}) + \frac{100 \sum_{j=1}^{n} \sum_{i=1}^{k} a_{ij}}{43560\ B}$$

$$= \frac{100 \sum_{j=1}^{n} (A_j - \sum_{i=1}^{k} a_{ij})}{43560\ B} + 100 \left(\frac{\sum_{j=1}^{n} \sum_{i=1}^{k} a_{ij}}{43560\ B}\right)$$

where

$a_{ij}$ = area of overlap by the ith competitor on the jth tree

$\frac{\sum_{j=1}^{n} \sum_{i=1}^{k} a_{ij}}{43560\ B}$ = Competition Quotient on a per acre basis for maximum crown area of each tree

Therefore:

$$\hat{CCF} = 100 + 100(CQ) \text{ for } CCF > 100$$