AN ABSTRACT OF THE DISSERTATION OF

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Title: Essays on the Use of Distance Functions in Empirical Studies: Efficiency Measurement and Beyond.

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This dissertation explores the intricacies associated with the use of distance functions in empirical studies. It focuses on the measurement of advertising efficiency and investigates the properties of the models that seek to approximate the abatement costs of socially undesirable outputs. The first manuscript is devoted to the development of the algorithm that can be used to measure the efficiency with which firms market their brands in the presence of advertising by rivals. An empirical illustration is carried out using the data from the U.S. brewing industry. The second study analyzes the difficulties associated with the accurate approximation of the abatement costs of socially undesirable outputs. It contrasts the results from a variety of different shadow-pricing models, each of which relies on a different type of distance functions that are used to approximate the polluting

technology. The shadow prices of sulfur dioxide are computed using linear programming techniques and the data from the U.S. electric utility industry. The third manuscript shows how a generalized method of moments (GMM) algorithm can be used to estimate the parameters of certain types of distance functions; the empirical illustration is carried out using the dataset from the second study.

The first manuscript illustrates that advertising spillovers are important in brewing and shows that the estimates of marketing efficiency are inaccurate when spillover effects are present and ignored. The second study shows that the shadow price estimates of socially undesirable outputs are not invariant to the assumptions regarding the parametric form of production technology and can in fact be predetermined by selecting a specific model. Finally, the third study established the legitimacy of the GMM procedure as a choice of an algorithm for the shadow pricing of undesirable outputs.

Essays on the Use of Distance Functions in Empirical Studies: Efficiency Measurement and Beyond

by

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Mikayel Vardanyan, Author

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CONTRIBUTION OF AUTHORS

Dr. Victor J. Tremblay has assisted with the writing of Chapter 2. Dr. Dong-Woon Noh has furnished the data for Chapter 3 and Chapter 4. Dr. Rolf Färe and Dr. Shawna Grosskopf have provided the ideas and helped with the implementation of all aspects of this dissertation.

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Introduction

This dissertation addresses the details of empirical modeling of the production and marketing technologies using distance functions. Chapter 2 is devoted to the measurement of advertising efficiency in the presence of marketing spillovers using the directional input distance function. Chapter 3 contrasts the approximation properties of various distance functions in parametric shadow-pricing models. Chapter 4 shows how the parameters of certain types of distance functions can be estimated via a generalized method of moments (GMM) procedure.

In chapter 2, we develop a model of marketing efficiency based on a directional distance function that allows for marketing spillovers. A parametric model is used to test for spillovers from rival marketing and from a firm's marketing activity of its other related products. We then show how this information can be incorporated into a non-parametric model and used to estimate marketing inefficiency. We apply brand level data from the U.S. brewing industry to the non-parametric model to determine the effectiveness of television, radio, and print advertising.

In chapter 3, we use a panel of observations from the U.S. electric utility industry to analyze the estimates of the output set frontiers that are produced by approximating its polluting technology with several functional forms. Our specifications rely on a variety of parametric linear programming-based methodologies, and their outcomes are assessed against the general axiomatic framework of the production model. We report the shadow price estimates of socially undesirable outputs, or "bads," obtained using a number of different specifications. We also include a thorough benchmarking of our results by assessing the shadow price estimates, often interpreted as the opportunity cost of pollution reduction, against the market prices of pollution permits.

Finally, in chapter 4 we illustrate how the shadow pricing of bads can be alternatively implemented via a GMM procedure and provide an empirical illustration using the dataset from chapter 3. Our specification yields robust estimation results, which are assessed against the linear programming-based methodology and the market prices of SO_2 allowances that have evolved during the implementation of the Phase I of the Title IV of the 1990 Clean Air Act Amendments (CAAA) of the U.S. Acid Rain Program.

The three essays that comprise this dissertation are mainly empirical in nature. They are linked by their use of distance functions in models that can be of an interest to cost-minimizing firms and social welfare-maximizing policy makers—a common conceptual thread that ties this dissertation together.

Chapter 2

The Measurement of Marketing Efficiency in the Presence of Spillovers: Theory and Evidence

2.1 Introduction

Although there is a considerable body of work on production technology, efficiency analysis on the marketing side of the firm has just begun. Borrowing from production theory and Bresnahan's (1984) conceptual work on advertising, Färe *et al.* (2004) show how one can use data envelopment analysis (DEA) to characterize marketing technology and to estimate the effectiveness of a firm's marketing efforts. Such research is especially important in consumer goods industries where products are heavily advertised and marketing effectiveness is important to a firm's overall success.

In many industries, managers must diligently monitor both the production and marketing sides of the firm. That is, the production plant must use the fewest production inputs to produce output, and the marketing department must use the fewest marketing inputs to sell that output. Färe *et al.* (2004) identify several similarities between the models of technology in production and in marketing. An important distinction, however, is that the marketing efforts of one brand may impact the marketing effectiveness of other brands within the same market, a feature that is less common in production.[1] Such spillovers may be positive,

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[1]Important sources of production spillovers are pollution and economies or diseconomies of scope [Baumol, Panzar, and Willig (1982)].

negative, or zero, depending upon product substitutability and the nature of marketing.

In his survey of the economics of advertising, Bagwell (2005) discusses several reasons for advertising spillovers. For instance, advertising that expands the size of the market may increase demand for all brands, producing positive spillovers. In established markets where advertising affects market share alone, however, advertising that increases the demand for one brand will reduce the demand for other brands. This form of advertising is said to be business stealing or predatory and produces negative spillovers. For multi-brand producers, spillovers may also be internal to the firm. That is, the advertising of one brand may cannibalize or expand the sales of the firm's other brands within a particular market. The magnitude of these spillovers will depend on the extent to which competing products have similar characteristics and the degree to which advertising attracts new consumers to the overall product category. Spillovers are unlikely to be present in markets where brands have very different characteristics and where consumers have strong loyalties to particular brands. For example, persuasive forms of advertising have been shown to create subjective product differentiation and increase brand loyalty, which would tend to dampen spillovers in established markets [Tremblay and Polasky (2001) and Tremblay and Tremblay (2005)].

Empirical studies suggest that advertising spillovers are important. For example, Szegedy-Maszak (2004) finds that advertising for Strattera, a new drug to treat adult attention deficit disorder, increased demand for all drugs targeted to this disorder. Likewise, Tennant (1950) finds evidence of positive advertising spillovers in the early stages of the cigarette industry. As the cigarette industry matured, however, Farr *et al.* (2001) find that cigarette advertising became predatory, producing negative spillovers. This suggests that advertising may exhibit positive spillovers in emerging markets but negative spillovers in mature markets.[2]

Given the likelihood that marketing spillovers do exist in some markets, we extend the Färe *et al.* model of marketing technology to account for marketing spillovers.[3] This extension is especially important in markets where firms are multi-product producers and where non-cooperative behavior occurs. Marketing spillovers would not exist for a single product producer in a monopoly market or in a market like milk, where the product is homogeneous and producers cooperate in their marketing efforts [Liu and Forker (1988)]. Our work shows that the assessment of marketing efficiency is incorrect when these spillover effects are present but ignored. To the best of our knowledge, our work is the first to use

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[2] See McAfee (2002) for further discussion of the life cycle of a typical brand.

[3] Like Färe *et al.*, Luo and Donthu (2001) use data envelopment analysis to evaluate advertising effectiveness, with data at the firm level for the leading U.S. advertisers. Their method not only fails to account for spillovers, but it also pools data from firms of different industries. This leads to biased efficiency estimates if output prices are different and if the marginal effect of advertising differs from industry to industry. The authors find, for example, that fast food advertising is more efficient than cigarette advertising, an outcome that undoubtedly results from the fact that there is a ban on broadcast advertising in the cigarette industry.

brand level data from multi-product producers to analyze marketing efficiency in the presence of spillovers.

We develop the theoretical framework and then use it to estimate the degree of marketing efficiency in the U.S. brewing industry. Brewing is an ideal candidate because the mass-producing brewers use the same production technology and focus their marketing efforts on limited media: television, radio, and print advertising.[4] Brewers also market a multitude of brands in four main product categories: popular-priced beer, premium beer, super-premium beer, and light beer.[5] In 1995, these categories accounted for about 91 percent of all domestic malt beverage sales. Products in different categories have minor characteristic differences. Popular-priced brands are produced with a greater percent of cheaper adjuncts (i.e., corn or rice in place of more expensive malted barley) and receive little advertising promotion. Premium and super-premium brands are produced with more malted barley and receive greater advertising effort. Advertising of these more expensive brands is designed to promote a superior image. In contrast, light beer is lower in calories, carbohydrates, and alcohol than regular beer, and the advertisements for light beer tend to provide information about these characteristic differences. Another feature that makes

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[4] Although our application focuses on advertising, our methodology would apply to other marketing and sales activities.

[5] Data limitations make it impossible to evaluate excluded categories: malt liquor, dry beer, ice beer, and microbrewery beer. The domestic mass-producers produce traditional American lager beer and account for about 97 percent of total domestic beer production. See Tremblay and Tremblay (2005) for a detailed discussion of the U.S. brewing industry and these different beer categories.

brewing worthy of study is that brewing companies generally offer more brands than would be profitable under a monopoly setting [Schmalensee (1978), Tremblay and Tremblay (1996), and Tremblay and Tremblay (2005)]. When firm's brands proliferate in this way, the advertising of one brand is more likely to cannibalize sales of rival brands and the firm's own brands.

2.2 Theoretical Background

Consider for the moment a firm that produces and sells output without marketing. In this setting, the firm's technology can be described by its production set [Varian (1992) and Mas-Colell (1995)]. Let T_P denote the production set, which describes all input combinations that will generate a given level of output (y_P).

$$T_P = \{(x_P, y_P) : x_P \quad can \quad produce \quad y_P\}, \tag{2.1}$$

where x_P is a vector of production inputs. Another way of describing the technology of the firm is with an input requirement set, $L_P(y_P)$:

$$L_P(y_P) = \{x_P : (x_P, y_P) \in T_P\}, \quad y_P \in \Re_+ \tag{2.2}$$

Technical inefficiency can be defined over the input set by measuring the distance from a firm's input combination to the frontier of its input requirement set for a given output (i.e., its isoquant). This is illustrated in Figure 2.1 for two inputs, where the firm uses input combination A to produce output y_P. One issue involves

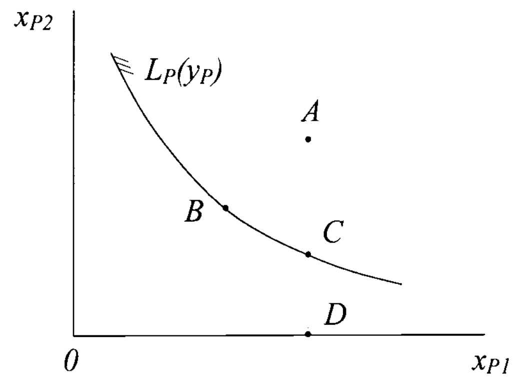

Figure 2.1 Input Requirement Set in Production

the direction of contraction. Following Shephard (1970), for example, an efficient input bundle is reached by making the maximum feasible contraction of all inputs along a ray from point A toward the origin (i.e., to point B). Greater deviation between points A and B implies greater technical inefficiency. Luenberger (1992) and Chambers *et al.* (1998) develop a more general method, one which allows movement in *any* direction from point A toward the isoquant. For example, if the direction of contraction is toward to the origin, as in Shephard, then the inefficiency indicator or score is AB/AO ≥ 0. Alternatively, we can contract in a southward direction by holding x_{P1} constant and moving from point A to point C, giving an inefficiency score is AC/AD. In both cases, a score of 0 implies technical efficiency, and a higher positive score implies greater inefficiency.

In a consumer goods industry like brewing, however, firm success depends on both production and marketing efficiency. To distinguish output that is produced from output that is marketed and sold to consumers, we specify them as production output (y_P) and sales output (y_M). Since firms generally use different inputs to produce and market their products and since production and marketing activities occur at different times, it is natural to follow Bresnahan (1984), Seldon *et al.* (2000), and Färe *et al.* (2004) by assuming that the production and marketing components of costs are separable. This produces the full cost function:

$$C(w_P, w_M, y_P, y_M) = C_P(w_P, y_P) + C_M(w_M, y_M), \tag{2.3}$$

where $C(w_P, w_M, y_P, y_M)$ denotes the full cost function of production and marketing, w_P is the price vector of inputs used to produce output y_P, and w_M is the price vector of inputs used to market and sell output y_M. $C_P(\cdot)$ represents the total production cost function, and $C_M(\cdot)$ represents the total marketing cost function.

Parallel to production, a firm's marketing technology can be described by a marketing possibility set. Let T_M denote the marketing set, which describes all possible marketing input combinations that will generate a given level of sales output.

$$T_M = \{(x, x', y) : (x, x', y) \ \ can \ \ sell \ \ y\}. \tag{2.4}$$

For notational convenience, we drop the subscript M on output and the input vector. In this case, x is a vector of the firm's marketing inputs for the brand in question. Unlike production, however, a firm's marketing efficiency may be affected by the marketing activities of other related brands (x'). For the moment, let us define other brands to include rival brands only.[6] The marketing of other brands that reduces the marketing efficiency of the brand in questions is predatory and produces negative spillovers. That which increases the brand's marketing efficiency produces positive spillovers.[7]

A firm's marketing technology can also be described by an input requirement set. Let $L_M(y)$ represent the marketing input requirement set, such that:

$$L_M(y) = \{(x,x') : (x,x',y) \in T_M\}, \quad y \in \Re_+. \tag{2.5}$$

Assuming just two marketing inputs, x and x', x' should be treated like a regular input when it produces positive spillovers as in Figure 2.2. Notice that an increase in x' (from A to B) when y is fixed causes the degree of marketing inefficiency to increase (from AC to BD when evaluated in a southern direction). With negative spillovers, the marketing isoquant will have a positive slope, since x' is a bad

[6] Later, we allow firms to be multi-product producers, and other brands will include the firm's other related brands.

[7] Note that when rival marketing produces positive spillovers, own and rival marketing are strategic substitutes. With negative spillovers, they are strategic complements.

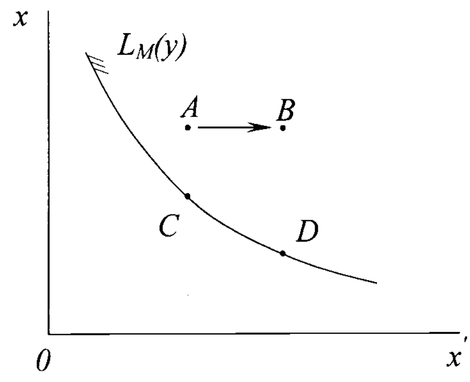

Figure 2.2 **Input Requirement Set in Advertising with Positive Spillovers**

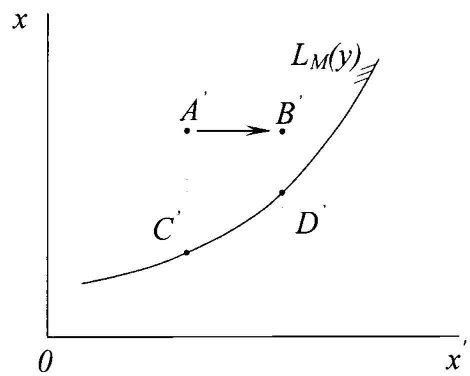

Figure 2.3 **Input Requirement Set in Advertising with Negative Spillovers**

(see Figure 2.3). In this case, an increase in x' (from A' to B') for a given y will lead to a decrease in marketing inefficiency (from A'C' to B'D').

Parallel to production, the marketing cost function derives from the marketing input set. This produces the total marketing cost function.

$$C_M(y,x',w_M) = \min_x \{w_M x : (x,x') \in L_M(y), x' = x'_0\}. \quad (2.6)$$

Because x' is external to the brand in question, it may generate positive or negative marketing spillovers. These characteristics of marketing technology can have important implications for the input disposability. It is natural to assume that the components of the marketing input vector x are *strongly* or *freely* disposable, such that:

$$(x,x') \geq (\hat{x},x') \in L_M(y) \Rightarrow (x,x') \in L_M(y). \quad (2.7)$$

This implies that an increase in a firm's marketing of a particular brand cannot reduce its sales, *ceteris paribus*. Given the discussion above, however, this need not be true for marketing efforts for other brands, x'. When the marketing of other brands generates positive spillovers, free disposability still applies. If the marketing of other brands pollutes, however, then x' will be *weakly* disposable or

$$(x,x') \in L_M(y), \lambda \geq 1 \Rightarrow (\lambda x, \lambda x') \in L_M(y). \quad (2.8)$$

This means that when rival brand marketing pollutes, own marketing of the brand in question must increase in order to offset these negative spillovers and keep sales of the brand constant.

A multi-brand producer that is sophisticated, however, will minimize the cost of marketing a particular brand and its other related brands, taking the marketing expenditures of rival brands as given.[8] To include these effects, we specify the following cost function.

$$C_M(y,x^r,w_M) = \min_{x,x^o}\left\{w_M x + w_M x^o : (x,x^o,x^r) \in L_M(y), x^r = x_0^r\right\} \quad (2.9)$$

where x is a vector of the firm's marketing inputs for the brand in question. For a multi-product firm, x' is divided into two parts: x^o is a vector of the marketing inputs for the firm's other related brands, and x^r is a vector of marketing inputs for rivals' related brands. This allows marketing spillovers from the firm's other brands to differ from marketing spillovers of rival brands.

We choose a non-parametric method to estimate marketing technology and inefficiency. Data envelopment analysis (DEA) is used to estimate the frontier of the marketing input set [Charnes *et al.* (1978)]. The main advantage of this approach is that it avoids imposing a specific function form on marketing technology. In addition, Kneip *et al.* (1998) show that the DEA algorithm produces

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[8] A sophisticated firm would take rival marketing as the only given, whereas a naïve firm would take as given the marketing efforts of both rival and its own other brands.

consistent estimates of relative inefficiency scores in large samples.[9] We use a directional input distance function to represent marketing technology and measure marketing inefficiency. Following Färe and Grosskopf (2004), this function is defined as follows:

$$\vec{D}(y, x, x^o, x^r; g_x, g_o, g_r) = \max\{\beta : (x - \beta g_x, x^o - \beta g_o, x^r - \beta g_r) \in L_M(y)\}, \quad (2.10)$$

where β is the marketing inefficiency indicator; g_x, g_o, and g_r are the sub-vectors that specify the direction of input contraction.[10] To illustrate the simple case of only one x and one x^r, if g_x equals x and g_r equals x^r, then the contraction is towards the origin. In this case, the directional distance function is similar to the Shephard distance function. In our application, however, we follow Chambers *et al.* (1998) by setting g_r equal to zero, since rival inputs are exogenous to the firm. That is, the contraction is such that x and x^o fall but x^r remains fixed. These methods allow us to measure the degree of marketing inefficiency for a particular brand. To illustrate, assume the two inputs in Figure 2.1 are marketing instead of

[9] Alternatively, one could estimate a stochastic marketing frontier [Aigner *et al.* (1977)], but a specific functional form must be assumed and efficiency estimates are sensitive to distributional assumptions regarding the composite error term.
[10] In our application $g_r = 0$, implying that rival advertising is taken as given. Similar to production theory, the cost function and directional distance function for marketing are dual, and the relationship between them is as follows:

$$C_M(y, x^r, w_M) = \min_{x, x^o}\left\{w_M x + w_M x^o : \vec{D}(y, x, x^o, x^r; g_x, g_o, 0) \ge 0\right\}$$

production inputs, where $x^o = 0$, $x_{P1} = x^r$, and $x_{P2} = x$. In this case, an estimate of the distance function provides a measure of marketing inefficiency, which equals AC/AD for a movement in a purely southern direction. Again, marketing is technically efficient when the inefficiency indicator equals 0, and a higher score indicates greater inefficiency.

In order to use the DEA approach, however, assumptions must be made regarding the disposability of marketing inputs. It seems sensible to assume that the marketing inputs of a brand cannot reduce its sales output and are strongly disposable. The marketing efforts of rival brands and of the firm's other related brands may have no effect or they may produce positive or negative spillovers, however. Recall that an input that generates positive spillovers is strongly disposable and one that produces negative spillovers is weakly disposable. To determine the signs of these spillovers, we first estimate a marketing function. The parameter estimates from this model allow us to conduct comparative static analysis and draw conclusions regarding the sign of marketing spillovers. The information regarding these spillovers is then used to impose appropriate constraints on the non-parametric DEA algorithm, and DEA methods are used to estimate the inefficiency scores.

2.3 Advertising Technology in the U.S. Brewing Industry

To illustrate this technique, we investigate marketing inefficiency in the U.S. brewing industry. The primary marketing tool in brewing is advertising, so the focus of our marketing application is on advertising. This and the fact that beer is heavily advertised make brewing a viable candidate for study. We focus our analysis on the primary categories of beer that are marketed by the mass-produced beer companies: popular-priced beer, premium beer, super-premium beer, and light beer.

We estimate advertising inefficiency at the brand level by collecting annual data on sales output and on television, radio, and print advertising for the leading 25 brands of beer.[11] Advertising is measured as the number of advertising messages, advertising expenditures divided by the price of an advertisement by media. To avoid bias, the pooled brands must have the same output prices. Tremblay and Tremblay (2005) find that prices are the same or are very similar within but not across categories. Thus, we estimate separately the marketing technologies of popular-priced, premium, super-premium, and light brands.[12] The period of analysis covers 1983-1992 and 1994-2000 (advertising data are

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[11] The use of annual data is appropriate, since Grabowski (1977-78) and Boyd and Seldon (1990) find that the demand effect of advertising dissipates within one year in the beer and cigarette markets, respectively. Print includes advertising in newspapers, in magazines, and on billboards.

[12] Even though we analyze these categories separately, the model controls for marketing spillovers from the firm's other related brands and from rival brands. Output prices are similar within categories but differ across categories, allowing pooling within but not across categories. This specification appears to be appropriate because marketing technologies differ across categories. For example, advertising expenditures per barrel of premium brands were generally twice that of popular-priced brands in 2000. In addition, the leading brewers typically use a different advertising agency to market their different brands.

unavailable for 1993), and the sample includes 298 total observations.[13] To control for the three marketing regimes identified by Sutton (1991) and Tremblay and Tremblay (2005), two dummy variables are added to the marketing function (1987-1995, 1996-2000). To control for differences in the relative size of the market, which change over time, total lagged output of the product category is added to the model. For example, the effectiveness of marketing may be greater in larger markets, *ceteris paribus*. The mean and standard deviations for the brand level variables are listed in Table 2.1.[14]

The first stage of estimation is designed to identify the disposability of rival advertising and the firm's own advertising on competing brands. Given the discussion above, we estimate the brand level marketing functions for each product category assuming that the sales output depends on controls for marketing regimes, the size of the market, and the firm's own as well as its rivals' marketing efforts.

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[13] Some brands were phased out during the sample period. Firm and brand level sales are obtained from *Beer Industry Update* (various issues), and brand advertising expenditures are obtained from *TNS Media Intelligence/Competitive Media Reporting*, as reported in *Beer Industry Update* (various issues). The price of advertising by media were graciously provided by Robert Coen of the marketing agency, Universal McCann. See Tremblay and Tremblay (2005) for a more complete description of the data.

[14] The Anheuser-Busch Brewing Company markets Bud Light, Michelob Light, Natural Light, Busch, Budweiser, Michelob, and Michelob Dark. The Coors Brewing Company markets Coors Light, Coors, Coors Extra Gold, and Killian's Red. The Heileman Brewing Company markets Lone Star, Rainier, Old Style, Henry Weinhard, and Special Export. The Miller Brewing Company markets Miller Lite, Miller Genuine Draft, Miller, and Lowenbrau. The Pabst Brewing Company markets Hamm's, and Pabst. The Stroh Brewing Company markets Stroh's Light, Stroh's, and Old Milwaukee.

$$y = f\left(x, x^{o}, x^{r}, Y, D_{87-95}, D_{96-00}\right) + \varepsilon \tag{2.11}$$

Recall that y is the sales output of a firm's particular brand of beer (e.g., Budweiser Light) and x is a vector that represents the firm's advertising of that particular brand. We define x^{o} to equal a vector of advertising for all of the firm's other brands (e.g., the sum, by medium, for Budweiser, Busch, Michelob, etc.), and x^{r} is a vector that represents rival advertising (e.g., the sum, by medium, for Coors Light, Miller Genuine Draft, Pabst, etc.). Y is the total output of the product type from the previous year (i.e, for all light beer brands), $D_{87\text{-}95}$ and $D_{96\text{-}00}$ are dummy variables that control for different marketing regimes, and ε is the disturbance term. Both x and x^{o} are three dimensional vectors, which include advertising from each of the three media (television, radio, and print). Given our interest in advertising spillovers but our relatively small sample, x^{r} is defined as a six dimensional vector, which includes the advertising of all rival brands within the product category (i.e., light beer) and the advertising of all rival brands in all other categories (i.e., non-light beer brands) for each of the three media.

Under certain regularity conditions, the parameter estimates in equation (2.11) can be calculated by specifying an appropriate econometric estimation technique for panel data and by assuming a particular parametric form.[15] Because the sample is small and includes data from only six firms, we choose a linear fixed-effects model.

With estimates of the parameters in equation (2.11), one can determine the sign of the spillovers from rival (own) advertising on the firm's advertising inefficiency for a particular brand and medium, $\partial y/\partial x^r$ and $\partial y/\partial x^o$. If a corresponding parameter estimate is positive and significant, then positive spillovers are present and strong disposability is assumed. That is, the variable is treated as a standard input in the non-parametric DEA model estimation to come. If, however, a derivative is negative and significant, the input generates negative spillovers, supporting weak disposability. Finally, if the derivative is insignificant, we conclude that a firm's advertising efficiency is unaffected by other advertising.

With the results from the first stage of estimation, we are able to calculate the directional distance function using a non-parametric DEA model. The non-parametric problem is constructed with respect to the advertising input requirement set, which is a piece-wise linear set that envelops all observations in the sample. For a brand $k = 1, 2, 3, \ldots$ K, the advertising input requirement set, along with its frontier, is defined as follows:

[15] To use this method, we must assume that firms face the same input prices and that output prices are the same within each product category. These maintained hypotheses are reasonable since Jung and Seldon (1995) find that firms face exogenously determined advertising prices. In addition, Tremblay and Tremblay (2005) find that output prices within a product category of beer.

$$
\begin{aligned}
L_M(y_k) = \{(Y, x, x^o, x^r) : \textstyle\sum_{k=1}^{K} z_k\, y_k \geq y_k, & \\
\textstyle\sum_{k=1}^{K} z_k\, x_{kn} \leq x_{kn}, & \quad n = 1,2,3, \\
\textstyle\sum_{k=1}^{K} z_k x_{kn}^o \leq x_{kn}^o, & \quad n = 1,2,3, \\
\textstyle\sum_{k=1}^{K} z_k x_{kn}^r \leq x_{kn}^r, & \quad n = 1,2,\ldots,6 \\
\textstyle\sum_{k=1}^{K} z_k Y_k \leq Y_k & \\
\textstyle\sum_{k=1}^{K} z_k = 1, z_k \geq 0, & \quad k = 1,\ldots,K\}
\end{aligned}
\tag{2.12}
$$

where n denotes a particular type of media, z_k parameters are the intensity variables that are to be calculated, and y_k in the right hand side of the inequality in line one represents the output of the particular brand. Recall that x_{kn} equals the quantity of advertising in media n for brand k. The inequalities in lines 2, 3, and 4 in equation (2.12) imply that all of the inputs exhibit strong disposability. If however, advertising from the firm's other brands and from rival brands generates negative spillovers, then the inequalities in lines 3 and 4 become equalities, implying weak disposability. Because returns to scale are unknown, a priori, we allow for variable returns to scale by constraining the non-negative intensities z_k to sum to unity in line 6. The directional input distance function for this technology is calculated for $g_x = x$, $g_o = x^o$, and $g_r = 0$ as follows:

$$\vec{D}\left(y_k, Y_k, x_k, x_k^o, x_k^r; x, x^o, 0\right) = \sup\{\beta$$

$$\begin{aligned}
\textit{subject to} \quad & \textstyle\sum_{k=1}^{K} z_k y_k \geq y_k, & & \\
& \textstyle\sum_{k=1}^{K} z_k x_{kn} \leq (1-\beta) x_{kn}, & n = 1,2,3, & \\
& \textstyle\sum_{k=1}^{K} z_k x_{kn}^o \leq (1-\beta) x_{kn}^o, & n = 1,2,3, & \qquad (2.13) \\
& \textstyle\sum_{k=1}^{K} z_k x_{kn}^r \leq x_{kn}^r, & n = 1,2,\ldots,6 & \\
& \textstyle\sum_{k=1}^{K} z_k Y_k \leq Y_k & & \\
& \textstyle\sum_{k=1}^{K} z_k = 1, z_k \geq 0, & k = 1,\ldots, K\} &
\end{aligned}$$

As before, the inequalities in lines 4 and 5 in equation (2.13) hold when own and rival advertising exhibit positive spillovers and are equalities when these types of advertising pollute. Whether they exhibit positive or negative spillovers will be determined in the first stage of estimation. Estimates of the distance function in (2.13) provide measures of advertising inefficiency for each brand.

2.4 Advertising Inefficiency Estimation

First, we use a fixed-effects regression model to determine if advertising from rival brands and from the firm's other brands exhibit positive, negative, or no spillovers. As discussed previously, the sign of these spillovers will depend on the degree to which competing products have similar characteristics and whether advertising is predatory or expands the size of the market. Because market demand is well established and total consumption has been stable during the sample period, one might expect rival advertising in brewing to be predatory.

Empirical results from the first stage of estimation are presented in Table 2.2. The model controls for firm effects and provides estimates of the marginal impact on sales from a brand's own advertising, rivals' advertising, and advertising from other brands marketed by a firm. To illustrate the notation, consider the advertising of Coors Light of the Coors Brewing Company. The marginal impact of the advertising of Coors Light is defined as Own-*A*, where *A* represents television (TV), radio, or print advertising. Own-Other-*A* indicates the marginal impact of the company's advertising in other beer categories (e.g., Coors' brands other than Coors Light). The marginal impact of the advertising of rival brands in the light beer category (e.g., Bud Light of the Anheuser-Busch Brewing Company) is defined as Rival-*A*. Finally, Rival-Other-*A* indicates the marginal impact of the advertising from rival brands outside the light beer category (e.g., Pabst and Lowenbrau).

The results indicate that firm effects and a brand's own and rival advertising influence sales output. In general, a firm's own advertising for a brand has a positive effect on sales. With about 90 percent of beer advertising devoted to television, the marginal impact of television advertising is of primary importance. As expected, television advertising always has a positive effect on a brand's own sales, and the television advertising of rival brands within the brand's category always has a negative effect. The effects of advertising from other product categories are mixed.

Statistically significant spillovers are present in the light and popular-priced categories. For light beer, rival advertising on television of light brands and own advertising on radio from other beer categories produce negative spillovers. For popular-priced beer, the firm's own advertising from other categories on television produces negative spillovers, while television advertising of rivals from other categories produces positive spillovers. In three of the four statistically significant cases, beer advertising is predatory, a result one might expect in a mature market.

Results from the first stage of estimation are used to identify the appropriate structure of the nonparametric DEA model in the second stage of estimation. When a marginal effect in Table 2.2 is positive (negative) and significant, the corresponding rival advertising input is modeled as being strongly (weakly) disposable. Regarding the directional vector, the frontier is reached by holding rival advertising fixed and contracting the brand's own advertising inputs and firm's advertising of other related brands toward the origin (i.e., g_x equals x, g_o equals x^o, and g_r equals 0).[16] Table 2.3 provides the average inefficiency estimates for each brand of beer. The inefficiency estimate of a particular brand (e.g., Bud Light) is calculated relative to other brands within its product category (light beer). In order to compare these with the results from the Färe *et al.* (2004) method, the

[16] For example, in the two input case in Figure 2, the contraction is from point A to point C if $x' = x^r$, $g_r = 0$ and $g_x = x$.

last column of the table provides inefficiency estimates assuming that other beer advertising produces no spillovers.[17]

Two results emerge from this analysis. First, when present, advertising spillovers have an important impact on the measurement of advertising inefficiency. For light and popular-priced beer, 69 percent of the brands experienced a change in their rank when we account for spillovers (see columns 4 and 6 in Table 2.3). This is especially pronounced for popular-priced beer, where there is a change in the efficiency ranking of all but the least efficient brand. This suggests that one must control for the effects of rival marketing efforts when estimating the marketing inefficiency of a brand or firm when spillovers are likely to be important.

Another implication of these and previous results is that advertising performance appears to be closely linked to overall firm success. Kerkvliet *et al.* (1998) demonstrate that overall efficiency in production is high in brewing, yet Tremblay and Tremblay (2005) find a large variance in brewing firm success. Table 2.4 lists the average inefficiency score, the growth rate in market share, and average profits per barrel (in 1982 dollars) for brewers with available data.[18] Inefficiency scores with (Column 3) and without (Column 4) spillovers indicate that the Anheuser-Busch Brewing Company is the most efficient advertiser. This is

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[17] Note that inefficiency estimates will generally fall with more variables in the model, which explains the relatively high scores or levels of inefficiency in column 5.

[18] Profits are obtained from company financial reports. For a more detailed discussion of the characteristics and relative firm success in brewing, see Tremblay and Tremblay (2005)].

consistent with its dominant-firm status, the dramatic increase in its market share during the sample period, and the fact that the company earned the highest profit rate among brewers sampled. The Miller Brewing Company is a distant second and is closely followed by the Coors Brewing Company. This efficiency ordering is certainly consistent with the average profit rates per barrel (Column 6), but other factors besides marketing efficiency are also at play. During the 1980s and early 1990s, for example, Coors was expanding into new geographic markets in an effort to become a national brewer, and this regional expansion explains the company's rapid growth in its national market share. An insufficient number of plants needed to take advantage of all multi-plant scale economies, along with a high degree of advertising inefficiency, caused Coors to have higher costs, which explains the company's relatively low profit rates. Both Anheuser-Busch and Miller were scale efficient. Thus, it appears that Miller's relatively low profit rates are due to poor advertising performance [Tremblay and Tremblay (2005)].

Although data limitations make more formal testing impossible, there is growing evidence to suggest that advertising success is an important contributor to overall firm success in brewing [Färe *et al.* (2004) and Tremblay and Tremblay (2005)]. In addition, beer industry experts claim that advertising efficiency is more important to overall firm success than production efficiency [Scherer *et al.* (1975)]. The relative importance of marketing efficiency to firm success may be common to other consumer goods industries where the production technology is well known but marketing campaigns are risky.

2.5 Concluding Remarks

Even though marketing plays a key role in many consumer goods industries, little research has been done on the nature of technology and efficiency measurement for the marketing side of the firm. We attempt to fill this gap by showing how a directional distance function can be used to estimate the degree of inefficiency in marketing. A distinguishing feature of marketing technology is that the marginal productivity of a firm's marketing may be affected by the marketing efforts of closely related brands. Advertising that expands the size of the market may generate positive spillovers, while marketing that is more predatory will impose negative spillovers.

We provide an empirical illustration of our technique by studying the advertising efficiency of the major brands in the U.S. brewing industry. Our results confirm that both positive and negative advertising spillovers are common in brewing. For light beer, rival advertising on television and the firm's own radio advertising of other brands produce negative spillovers. For popular-price brands, a firm's own television advertising in other categories produces negative spillovers, and television advertising of rival brands in other categories produces positive spillovers. Our results also show that inefficiency estimation will be incorrect when relevant spillovers are ignored.

Finally, the evidence is consistent with the hypothesis that advertising efficiency has a substantial effect on a firm's overall success. This may be true in other consumer goods industries where production methods are well established and marketing is important but risky, an important issue for future research.

Table 2.1
Means and Standard Deviations of Variables and Descriptions

Beer Category	Brand	Number of Observations	Sales Output	Quantity of Advertising TV	Radio	Print
Light	Bud Light	17	14,423	41,595	674	1,192
			(8,457)	(8,998)	(701)	(711)
	Coors Light	17	10,617	33,531	677	1,022
			(3,597)	(12,162)	(786)	(716)
	Michelob	14	2,440	11,614	342	750
	Light		(248)	(9,172)	(467)	(941)
	Miller Lite	17	17,288	50,443	1,184	2,293
			(1,545)	(11,904)	(1,934)	(1,350)
	Natural Light	10	2,414	2,010	27	44
			(1,369)	(1,305)	(56)	(94)
	Stroh's Light	6	748	4,327	24	28
			(117)	(3,508)	(59)	(31)
Popular	Busch	17	7,812	10,064	720	641
			(1,643)	(7,406)	(2,008)	(402)
	Hamm's	8	1,213	1,392	40	108
			(445)	(2,107)	(85)	(106)
	Lone Star	6	357	258	0	81
			(81)	(156)	(0)	(70)
	Old	13	5,840	7,878	348	165
	Milwaukee		(1,790)	(5,017)	(595)	(135)
	Pabst	13	3,522	2,304	343	386
			(1,675)	(4,332)	(616)	(458)
	Rainier	10	1,058	1,032	82	144
			(221)	(592)	(156)	(206)
	Stroh's	14	2,379)	6,852	285	651
			(1,848)	(8,779)	(352)	(458)
Premium	Budweiser	17	42,237	57,421	2,445	4,786
			(5,501)	(8,958)	(2,285)	(1,674)
	Coors	17	4,513	13,242	50	1,286
			(2,853)	(7,524)	(112)	(683)
	Coors Extra	7	624	9,801	302	154
	Gold		(269)	(6,702)	(340)	(132)
	Miller Gen.	14	5,068	27,813	896	1,901
	Draft		(1,529)	(10,969)	(1,546)	(954)
	Miller High	16	7,746	17,663	365	1,665
	Life		(3,943)	(18,762)	(823)	(1,568)
	Old Style	7	3,281	3,270	0	703
			(605)	(1,574)	(0)	(721)

Table 2.1 (Continued)

Beer Category	Brand	Number of Observations	Sales Output	Quantity of Advertising TV	Radio	Print
Super	Henry	11	365	1,219	37	181
Premium	Weinhard		(68)	(581)	(117)	(137)
	Killian's	7	637	2,098	98	173
	Red		(55)	(2,728)	(148)	(144)
	Lowenbrau	12	856	6,549	104	572
			(367)	(7,767)	(171)	(639)
	Michelob	17	3,411	13,402	1,314	1,045
			(1,668)	(14,611)	(1,839)	(960)
	Michelob	4	245	1,977	570	357
	Dark		(90)	(710)	(521)	(517)
	Special	6	277	884	0	116
	Export		(89)	(1,107)	(0)	(75)

Note: Sales are measured in thousands of barrels, and advertising is measured as nominal advertising expenditures in thousands of dollars divided by the appropriate media price index (1982 = 100).

Table 2.2
Spillover Effects from Own and Rival Advertising, Fixed Effects Heteroskedasticity-Consistent Model, 1983–2000

	Light	Popular	Premium	Super Premium
Intercept	1.292	13.936[a]	28.985[a]	2.008[a]
	(6.511)	(3.553)	(3.843)	(0.616)
Own-TV	0.108[b]	0.170[a]	0.026	0.048[b]
	(0.045)	(0.031)	(0.034)	(0.025)
Own-Radio	0.248	0.076	1.468[b]	0.149
	(0.511)	(0.181)	(0.672)	(0.191)
Own-Print	-1.104[b]	-0.450	0.251	0.063
	(0.456)	(0.315)	(0.220)	(0.187)
Own-Other-TV	0.037	-0.029[b]	-0.021	-0.005
	(0.032)	(0.016)	(0.021)	(0.007)
Own-Other-Radio	-0.337[c]	-0.043	-0.186	0.016
	(0.180)	(0.099)	(0.418)	(0.022)
Own-Other-Print	0.143	-0.166	-0.097	0.037
	(0.234)	(0.153)	(0.274)	(0.060)
Rival-TV	-0.465[c]	-0.112	-0.135	-0.053
	(0.263)	(0.191)	(0.290)	(0.167)
Rival-Radio	0.003	0.006	-0.472	0.010
	(0.439)	(0.059)	(0.331)	(0.084)
Rival-Print	0.306	-0.038	-0.010	-0.182
	(0.267)	(0.179)	(0.145)	(0.124)
Rival-Other-TV	0.012	0.012[b]	0.008	0.000
	(0.034)	(0.005)	(0.015)	(0.005)
Rival-Other-Radio	0.189	-0.006	0.093	0.004
	(0.137)	(0.030)	(0.161)	(0.014)
Rival-Other-Print	-0.142	-0.039	0.049	0.017
	(0.141)	(0.045)	(0.226)	(0.031)
$D_{87\text{-}95}$	0.224	0.068	-0.323	-0.299
	(1.350)	(0.381)	(1.009)	(0.341)
$D_{96\text{-}00}$	1.712	-1.491	-0.134	-0.692[c]
	(2.061)	(0.956)	(1.308)	(0.420)
Y	1.724[c]	-0.923	1.469[a]	1.995[b]
	(1.006)	(0.861)	(3.820)	(0.949)

Table 2.2 (Continued)

	Light	Popular	Premium	Super Premium
F_1^{AB}	-7.693[a] (1.781)	--	--	-2.357[a] (0.276)
F_2^{AB}	-4.738[a] (1.687)	--	--	--
F_1^{Coors}	-2.043 (1.832)		-32.639[a] (1.799)	-1.653[a] (0.296)
F_2^{Coors}	--	--	-35.897[a] (1.944)	--
F_1^{Miller}	2.903[c] (1.734)	--	-32.667[a] (1.241)	-2.159[a] (0.173)
F_2^{Miller}	--	--	-29.431[a] (1.774)	--
F_1^{Stroh}	-5.421[b] (2.167)	-10.955[a] (1.779)	--	--
F_2^{Stroh}	--	-7.939[a] (1.764)	--	--
F_1^{Pabst}	--	-11.701[a] (1.836)	--	--
F_2^{Pabst}	--	-9.211[a] (1.785)	--	--
$F_1^{Heileman}$	--	-12.103[a] (1.727)	-36.453[a] (2.482)	-2.704[a] (0.636)
$F_2^{Heileman}$	--	-11.464[a] (1.748)	--	-2.800[a] (0.636)
N	81	81	78	57
R^2	0.870	0.920	0.987	0.950

Notes: For light brands, "Intercept"=Bud Light, "F_1^{AB}"=Michelob Light, "F_2^{AB}"=Natural Light, "F_1^{Coors}"=Coors Light, "F_1^{Miller}"=Miller Light, "F_1^{Stroh}"=Stroh's Light.

For popular-priced brands, "Intercept"=Busch, "F_1^{Stroh}"=Stroh's, "F_2^{Stroh}"=Old Milwaukee, "F_1^{Pabst}"=Hamm's, "F_2^{Pabst}"=Pabst, "$F_1^{Heileman}$"=Lone Star, "$F_2^{Heileman}$"=Rainier.

For premium brands, "Intercept"=Budweiser, "F_1^{Coors}"=Coors, "F_2^{Coors}"=Coors Extra Gold, "F_1^{Miller}"=Miller Genuine Draft, "F_2^{Miller}"=Miller High Life, "$F_1^{Heileman}$"=Old Style.

For super premium brands, "Intercept"=Michelob, "F_1^{AB}"=Michelob Dark, "F_1^{Coors}"=Killian's, "F_1^{Miller}"=Lowenbrau, "$F_1^{Heileman}$"=Henry Weinhard, "$F_2^{Heileman}$"=Special Export.

[a] Significant at 1% in a two-tail test, [b] Significant at 5% in a two-tail test, [c] Significant at 10% in a two-tail test.

Table 2.3

Average Inefficiency Estimates by Brands of Beer

Beer Category	Brand	Average Efficiency (With Spillovers)		Average Efficiency (Ignoring Spillovers)	
		Level	Rank	Level	Rank
Light	Bud Light	0.174	3	0.469	4
	Coors Light	0.223	5	0.501	5
	Michelob Light	0.295	6	0.571	6
	Miller Lite	0.182	4	0.343	3
	Natural Light	0.000	1	0.037	1
	Stroh's Light	0.000	1	0.310	2
	MEAN	**0.146**		**0.402**	
Popular	Busch	0.111	5	0.259	1
	Hamm's	0.090	3	0.376	4
	Lone Star	0.052	2	0.354	3
	Old Milwaukee	0.028	1	0.327	2
	Pabst	0.110	4	0.396	5
	Rainier	0.111	5	0.485	6
	Stroh's	0.171	7	0.712	7
	MEAN	**0.096**		**0.417**	
Premium	Budweiser	0.067	1	0.067	1
	Coors	0.566	5	0.566	5
	Coors Extra Gold	0.514	4	0.514	4
	MGD	0.806	6	0.806	6
	Miller High Life	0.348	3	0.348	3
	Old Style	0.260	2	0.260	2
	MEAN	**0.423**		**0.423**	
Super Premium	Henry Weinhard	0.467	5	0.467	5
	Killian's Red	0.141	1	0.141	1
	Lowenbrau	0.427	4	0.427	4
	Michelob	0.180	2	0.180	2
	Michelob Dark	0.565	6	0.565	6
	Special Export	0.353	3	0.353	3
	MEAN	**0.328**		**0.328**	

Table 2.4

Firm Level Mean Advertising Inefficiency, Growth in Market Share, and Profits Per Barrel, 1983–2000

Company Name	Number of Observations	Average Efficiency (With Spillovers)	Average Efficiency (Ignoring Spillovers)	Percentage Change in Market Share	Average Profit Per Barrel (1982 dollars)
Anheuser-Busch, Inc.	96	0.0965	0.1572	60.8%	9.2
Miller Brewing Co.	60	0.3163	0.4069	6.9%	7.5
Coors Brewing Co.	48	0.3386	0.5413	65.9%	2.8

Note: The listing for the number of observations denotes the number of brands active during the subject period.

Chapter 3

On the Choice of the Approximation Procedure for Polluting Technologies in Parametric Shadow-Pricing Models

3.1 Introduction

Shadow pricing of undesirable outputs generated by polluting production processes has been analyzed in the environmental economics literature quite comprehensively during the past decade. Important recent parametric studies on pricing of socially undesirable outputs, or so-called "bads", using linear programming (LP) algorithms include Färe *et al.* (1993), Coggins and Swinton (1996), Swinton (1998, 2002, 2004), Färe, Grosskopf and Weber (2001), and Färe, Grosskopf, Noh and Weber (2005).[19] The key element of the shadow-pricing model is the duality between the distance function and a firm's revenue function, which enables us to retrieve the prices of bads, provided that the price of at least one socially desirable output is available.[20] The types of the distance functions that have been used in empirical studies are Shephard [Shephard (1970)] and directional [Chambers *et al.* (1998)] output distance functions, both of which can be used to recover the estimate of the reference technology (e.g. the output

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[19] Atkinson and Dorfman (forthcoming) show that the pricing of bads can be alternatively performed using a limited-information likelihood approach based on the moment conditions of the generalized method of moments estimator [Kim (2002)]. In contrast to the afore-mentioned studies, however, the authors treat the bads differently, assuming them to be an exogenous shifter of the technology.
[20] See, for example, Färe and Primont (1995) and Färe and Grosskopf (2004) for an in-depth analysis of duality.

possibilities set), thereby producing most of the information needed to compute the shadow prices of bads.

Parametric characteristics of algorithms for distance function estimation call for suitable assumptions regarding the functional form with which the true production technology is to be approximated. The transcendental logarithmic and the quadratic functional forms have both been used for this purpose. In addition to assuming a particular parametric structure for production technologies, one also needs to specify the regime according to which the model outputs are scaled for every observation in the sample, called the mapping rule. The mapping rule places each producer on the frontier of the reference technology, and the shadow prices are evaluated at these boundary points.

Modeling of the properties of distance functions under various parametric specifications is characterized by important caveats. The key is the choice of a suitable parametric approximation, as there exists no single functional form that can be used to parameterize both the directional and Shephard distance functions.[21] Therefore, the choice of one distance function *vis-à-vis* another to some extent also predetermines the selection of the functional approximation that will be used to parameterize it. In light of this limitation, the behavior of various algorithms in relation to one another, as well as with respect to the general assumptions defining the shadow-pricing model, becomes a matter of interest.

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[21] Färe and Lundberg (2004) prove that the suitable parameterizations of the directional distance functions can be achieved with only a limited number of functional forms.

The majority of the afore-mentioned essays on the pricing of bads make an attempt to estimate the shadow price of sulfur dioxide, a by-product produced in large quantities by coal-burning power plants together with electricity. Some adopt the Shephard/translog specification, and others use the directional/quadratic model, as we call them hereafter.[22] A comparison of their outcomes reveals that the shadow price estimates of SO_2 produced by the Shephard distance function are consistently lower than those generated in models that use the directional distance function. Although all of these studies rely on different datasets for empirical illustration, the gap between the reported shadow price estimates is so large it becomes apparent that it cannot be rationalized merely by the dissimilarity of data.

The studies that have discussed this phenomenon mention the difference in the mapping properties of various distance functions. For example, J.-D. Lee *et al.* (2002) cite varying estimate of the slope of the output possibilities set, but implicitly assume that the estimate of the frontier itself is invariant to the choice of a specific functional form of the distance function and/or the mapping rule. However, various parameterization methodologies would have to produce approximations of the technology that are sufficiently similar to one another in order to render adequate support for this presumption. Indeed, the dissimilarity of

[22] For instance, the directional/quadratic model is the one in which the duality is invoked between the revenue function and the quadratic directional distance function. Similarly, the Shephard/translog model is based on the Shephard distance function, which is parameterized via a translog functional form. The directional/translog model is not feasible, since the translation property of the directional distance functions cannot be modeled with the transcendental logarithmic function.

frontier estimates under various approaches can completely invalidate the comparison of any results between them.[23] Such divergence, if relevant, would consequently necessitate the evaluation of the relative strengths and weaknesses of each of the feasible parametric models.

This essay presents an empirical evaluation of output sets and shadow prices computed using various parametric methodologies and attempts to find a rational motivation for a choice of a specific shadow-pricing model. We assess the responsiveness of the shadow-pricing algorithm to the changes in the model assumptions concerning the choice of the mapping rule and the functional form of the distance function. We use a panel of observations on Phase I electricity-generating units in the U.S. electric utility industry for 1997, 1998, and 1999 for our illustration, and obtain the empirical analogues of the output possibilities set for each year separately using the Shephard/translog and directional/quadratic model for a number of mapping vectors. In addition, we use a variation of Shephard function, called the hyperbolic distance function, to compute shadow prices using a hyperbolic/translog algorithm. Consistent with most of the previous research regarding this matter, parametric LP algorithms are used throughout to calculate the coefficients of the distance functions.

[23] For example, Perroni and Rutherford (1998) show that some of the flexible functional forms are more appropriate in computable general equilibrium models than others due to their better global approximation properties. A similar conclusion is reached by McKitrick (1998), who illustrates that the choice of the functional form can strongly affect the performance of such models.

Our study is divided into two segments. In the first part we consider a variety of shadow-pricing models and show that estimates of the frontiers of the output set are extremely sensitive to both the choice of a functional form and a mapping rule. This essential result suggests that some parametric models may in fact be superior to others due to this quite pronounced difference in their approximation properties. The second part is devoted to the discussion of the relative relevance of these methodologies based on a comparison of the shadow price estimates to the market prices of pollution permits that have evolved during the implementation of the Phase I of the Title IV of the 1990 Clean Air Act Amendments (CAAA) of the U.S. Acid Rain Program.

The paper is organized as follows. The next section goes over the concept of the environmental output set with socially undesirable outputs, formally defines different types of the distance functions and shows how they can be used to elicit information on shadow prices of bads. Section 3.3 and Section 3.4 describe various calculation techniques and the datasets used for illustration, respectively. Section 3.5 is devoted to the comparison of outcomes produced by different models, testing and discussion. Section 3.6 concludes.

3.2 Theoretical Background

Below we discuss the axioms of the parametric environmental production model that characterizes the polluting technology and formally define the types of output distance functions that will be used in the analysis of shadow prices. See, for example, Färe, Grosskopf, Noh and Weber (2005) for a more in depth discussion of the theoretical underpinnings and the rationale behind each of the assumptions. The bads in the environmental production model are treated as outputs, so that the set of all possible input-output combinations is given by the production technology, which is defined in the following way:

$$T = \{(x, y, b) : x \text{ can produce } (y, b)\}, \tag{3.1}$$

where $x \in \Re_{+}^{N}$ is a vector of production inputs, $y \in \Re_{+}^{M}$ is a vector of socially desirable outputs, or "goods", such as kilowatt-hours of electricity, and $b \in \Re_{+}^{I}$ is a vector of bads, e.g. the emissions of sulfur dioxide. This polluting production technology can be equivalently described via the output possibilities set, given by

$$P(x) = \{(y, b) : (x, y, b) \in T\} \tag{3.2}$$

The concept of the environmental production model is characterized by an output set that also satisfies the following assumptions:

1. Null-Jointness, i.e. if $(y, b) \in P(x)$ and $b = 0$, then $y = 0$.
2. Weak disposability of an output vector, i.e. $(y, b) \in P(x)$ and $0 \leq \theta \leq 1$ imply $(\theta y, \theta b) \in P(x)$.

3. Free disposability of desirable outputs, i.e. $(y,b) \in P(x)$ and $(y^0,b) \leq (y,b)$ imply $(y^0,b) \in P(x)$.

This set of axioms, together with the assumptions of compactness of *P(x),* define an environmental output set, which is contained in the traditional production possibilities set.

Two types of distance functions that have been used in the shadow-pricing literature are the Shephard [Shephard (1970)] and the directional [Chambers, Chung, and Färe (1998)] output distance functions.[24] The Shephard function is defined as a measure that is based on maximal possible proportional expansion of all outputs onto the boundary of *P(x),* whereas the directional measure allows for a particular direction in which each output is to be expanded or contracted. The individual shadow prices are evaluated at these boundary points. The flexibility of the directional output distance function makes it particularly attractive in cases where theoretical specification may necessitate a particular mapping rule. For instance, some of the more recent shadow-pricing studies that use the directional distance function employ the rule that simultaneously increases the goods and reduces the bads, which is motivated by the negative externalities imposed on society by polluters. Correspondingly, the Shephard distance function and the directional output distance function in the direction $(g_y \in \Re^M_+, g_b \in \Re^I_+)$, called the mapping rule, are given formally by

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[24] The latter is the variation of the shortage function introduced by Luenberger (1992).

$$D(x, y, b) = \inf\{\theta : (y, b)/\theta \in P(x)\}, \tag{3.3}$$

$$\vec{D}(x, y, b; g_y, -g_b) = \sup\{\psi : (y + \psi g_y, b - \psi g_b) \in P(x)\} \tag{3.4}$$

In addition, we will also consider one of the variations of the Shephard distance function, referred to as the hyperbolic distance function, which resembles the directional output measure defined in (3.4) in that it enables us to map the observations to the frontier of the output set by simultaneously expanding the desirable outputs and reducing the bads, but is multiplicative in nature.[25] This contraction-based measure is given by

$$D_H(x, y, b) = \inf\{\theta : (y/\theta, \theta b) \in P(x)\} \tag{3.5}$$

The Shephard function and the hyperbolic function take values in the interval [0,1], whereas the directional function is bounded by zero from below and by positive infinity from above. Also, the closer a particular observation is to the frontier of environmental output set, the higher (lower) the value of the Shephard (directional) distance function. Table 3.1 provides a summary of some of the distance functions properties, more thorough discussion of which can be found in Färe and Grosskopf (2004). Like the traditional Shephard distance function, the

[25] For a detailed discussion of this and other variations of Shephard output distance function, as well as their empirical comparison, see Färe, Grosskopf, Lovell, and Pasurka (1989).

hyperbolic function possesses the representation and monotonicity properties, but is *almost* homogeneous [Aczél (1966)] in outputs, i.e. $D_H(x, \lambda y, \lambda^{-1} b) = \lambda D_H(x, y, b)$ for $\lambda > 0$. We shall see that the output homogeneity and the translation property will play a key role in choosing a specific functional form with which we parameterize each of these functions.

Let w, p, and r denote the price vectors of inputs, desirable outputs and the bads, respectively. Färe and Primont (1995), and Färe and Grosskopf (2004) demonstrate that the Shephard and the directional distance functions are dual to the firm's revenue function, defined as $R(x, p, r) = \max\{py - rb : (y, b) \in P(x)\}$, and apply this duality to find the shadow price estimates of the i^{th} output for every observation in the sample, provided that the price of the m^{th} desirable output is observed:

$$r_i^L = -p_m \left(\frac{\partial D(x, y, b)/\partial b_i}{\partial D(x, y, b)/\partial y_m} \right), \tag{3.6}$$

$$r_i^Q = -p_m \left(\frac{\partial \vec{D}(x, y, b; g_y, -g_b)/\partial b_i}{\partial \vec{D}(x, y, b; g_y, -g_b)/\partial y_m} \right) \tag{3.7}$$

where r_i^L is the shadow price estimate invoked by the duality between the Shephard distance function and the revenue function, and r_i^Q corresponds to the specification where $R(x, p, r)$ is dual to the directional distance function. In a similar way, the shadow prices can also be calculated using the appropriate gradient vectors of the hyperbolic function. Because the prices of the desirable outputs are commonly observed and all three distance functions can be suitably parameterized, the estimates of r_i^L and r_i^Q can be found without difficulty.

Parametric modeling of distance functions is characterized by the fact that there exists no single functional form that can be used to parameterize all of them. In particular, the homogeneity of the Shephard distance function and the almost-homogeneity of the hyperbolic function do not allow one to approximate them with the quadratic functional form and, likewise, the translation property of the directional distance function rules out translog specification as a choice for its parameterization. This important result is partly due to Färe and Sung (1986), and Färe and Lundberg (2004) who prove that there are but two functional forms that have the interpretation of the second-order Taylor's series approximation which can be used to parameterize the directional distance function, and the translog form is not one of them. Thus, one's choice of a particular distance function also predetermines the functional form with which it will be parameterized.

The mapping properties of both the Shephard and the directional distance function are rather encompassing. Figure 3.1 illustrates a hypothetical output set $P(x)$ with one undesirable and one desirable output given the above-mentioned axioms of the environmental production model and shows how various directional vectors and corresponding iso-revenue lines can result in different estimates of the

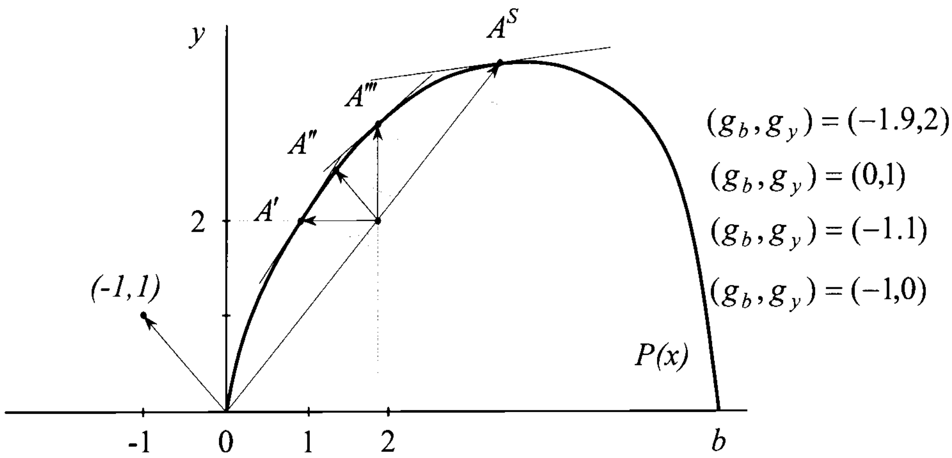

Figure 3.1 Environmental Output Set and Iso-Revenue Lines under Various Mapping Rules

shadow price of a bad for a particular observation in the sample, such as point A.[26] The iso-revenue lines are drawn tangent to the boundary counterparts of A, and their slope represents the estimate of r_i given an underlying mapping rule, which, except where the boundary counterparts of interior firms are located via simultaneous radial expansion of both the good and the bad, has been traditionally modeled with the directional distance function. This is because one needs to take $\underline{g}(A^S) \equiv (g_b^{A^S}, g_y^{A^S}) = (-b^A, y^A) \approx (-1.9,2)$ in order to map A to A^S with the directional distance function. Unfortunately, the parametric computation of the directional function given this directional vector is infeasible, since it does not

[26] Whereas the shadow price estimate of the boundary firms in this example is invariant to the mapping rule, the estimate of the bad for the interior firms will in contrast be significantly affected by the choice of the regime used to map these observations onto the frontier [recall that the shadow prices are assessed at the boundary of *P(x)*].

allow us to model the translation property. In contrast, simultaneous proportional expansion can be modeled without difficulty with the Shephard distance function given in (3.3). Either Shephard or the directional function can be used to map A to the boundary counterparts like A' and A''', but only the directional measure allows us to reach a particular point on the frontier in any northwestern direction.[27] Finally, the hyperbolic distance function scales the good and the bad simultaneously and in the northwestern direction, which is similar to the directional function with a mapping vector like $\underline{g}(A'') = (-1,1)$. In fact, these two distance measures are closely related, as the directional function can be retrieved from the hyperbolic function by taking its linear approximation [Chung (1996)].[28] Because all of the aforementioned aspects of modeling can potentially affect the estimate of the environmental output set frontier, a good deal of caution has to be exercised when comparing the shadow prices produced by different specifications. In particular, the difference in the average slope of the iso-revenue lines may be due to several reasons, such as the unequal approximation properties of various

---

[27] In case of the Shephard function this is done by using two of its other variations, which can be specified for a purely western and a purely northern move [Färe *et al.* (1989)]. Alternatively, the mapping can be performed with the directional function by taking $\underline{g}(A') = (-1,0)$ and $\underline{g}(A''') = (0,1)$, respectively. Finally, in order to map A to A'' one needs to take $\underline{g}(A'') = (-1,1)$.

[28] Since the output of the frontier (benchmark) firms does not require scaling, the difference in their shadow prices under various methodologies should help assess the sensitivity of the estimated boundaries to the changes in the model assumptions. For example, if the firms that are on the frontier in both the translog and the quadratic models also have similar shadow prices then the changes in assumptions regarding functional form would be unlikely to cause large divergence in the shadow price estimates for the remaining firms in the sample as well.

functional forms, or the different degree of responsiveness of the frontier estimates to the underlying mapping rule, or both. This intricacy has been overlooked on a number of occasions in the literature and so far no attempt has been made to look into the sensitivity of outcomes to the changes in some of the assumptions of the shadow-pricing model.

3.3 Estimation

This section is devoted to the description of computation techniques that we use in this study.[29] The goal programming method introduced by Aigner and Chu (1968) has proven to be particularly useful for parametric estimation of distance functions because it facilitates a straightforward modeling of all of the functions' properties, many of which, such as representation and monotonicity, have to be imposed through inequality constraints. We use the translog functional form to calculate the coefficients of the Shephard and hyperbolic distance functions defined in (3.3) and (3.5), respectively. The directional distance function, given by (3.4), will be parameterized with the quadratic form and computed for a variety of directions. Once the parameters of the distance functions have been computed, we will use them to plot the environmental output sets and to visually

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[29] We will calculate the shadow prices using parametric distance functions only. Boyd *et al.* (1996) and Ball *et al.* (1994) show that the shadow prices of bads can be alternatively found using nonparametric LP algorithms. These approaches allow dispensing with the functional form assumptions, but do not guarantee the differentiability of distance functions and are plagued with a number of other inaccuracies, such as the presence of outliers. Therefore, our shadow price estimates will be computed only parametrically.

assess their resemblance to the hypothesized set depicted in Figure 3.1. This will allow us to detect possible tendencies in the variation among the estimated boundaries that are due to the changes in the functional form and/or the mapping rule.

Suppose there are $k = 1,...,K$ observations in the sample. We first direct our attention to the Shephard/translog specification, which was introduced by Färe, Grosskopf, Lovell, and Yaisawarng (1993). If we assume that the polluting technology can be approximated with the translog functional form then the value of the Shephard output distance function for observation k will be equal to

$$\begin{aligned}\ln D(x_k, y_k, b_k) &= \alpha_0 + \sum_{n=1}^{N}\alpha_n \ln x_{nk} + \sum_{m=1}^{M}\beta_m \ln y_{mk} + \sum_{i=1}^{I}\gamma_i \ln b_{ik} \\ &+ \frac{1}{2}\sum_{n=1}^{N}\sum_{n'=1}^{N}\alpha_{nn'} \ln x_{nk} \ln x_{n'k} + \frac{1}{2}\sum_{m=1}^{M}\sum_{m'=1}^{M}\beta_{mm'} \ln y_{mk} \ln y_{m'k} + \frac{1}{2}\sum_{i=1}^{I}\sum_{i'=1}^{I}\gamma_{ii'} \ln b_{ik} \ln b_{i'k} \\ &+ \sum_{n=1}^{N}\sum_{m=1}^{M}\delta_{nm} \ln x_{nk} \ln y_{mk} + \sum_{n=1}^{N}\sum_{i=1}^{I}\eta_{ni} \ln x_{nk} \ln b_{ik} + \sum_{m=1}^{M}\sum_{i=1}^{I}\mu_{mi} \ln y_{mk} \ln b_{ik} \qquad (3.8)\end{aligned}$$

The parameter estimates can be obtained by solving a LP model that encloses all of the data points in the sample. Since the closer a particular observation is to the output set frontier, the larger the value of the Shephard distance function (i.e. the closer it is to $\ln 1 = 0$), this goal programming model entails the following minimization problem:

$$
\begin{aligned}
& Min \;\; \textstyle\sum_{k=1}^{K}(\ln 1 - \ln D(x_k, y_k, b_k)) \\
& s.t. \quad (i) \quad \ln D(x_k, y_k, b_k) \le 0; \qquad k = 1,\dots,K, \\
& \qquad (ii) \quad \partial D(x_k, y_k, b_k)/\partial y_{mk} \ge 0; \qquad m = 1,\dots,M, k = 1,\dots,K, \\
& \qquad (iii) \quad \partial D(x_k, y_k, b_k)/\partial b_{ik} \le 0; \qquad i = 1,\dots,I, k = 1,\dots,K, \\
& \qquad (iv) \quad \partial D(x_k, y_k, b_k)/\partial x_{nk} \le 0; \qquad n = 1,\dots,N, k = 1,\dots,K, \\
& \qquad (v) \quad \textstyle\sum_{m=1}^{M}\beta_m + \sum_{i=1}^{I}\gamma_i = 1, \\
& \qquad\qquad \textstyle\sum_{m'=1}^{M}\beta_{mm'} + \sum_{i=1}^{I}\mu_{mi} = 0; \qquad m = 1,\dots,M, \\
& \qquad\qquad \textstyle\sum_{i=1}^{I}\gamma_{ii'} + \sum_{m=1}^{M}\mu_{mi} = 0; \qquad i' = 1,\dots,I, \qquad\qquad (3.9) \\
& \qquad\qquad \textstyle\sum_{m=1}^{M}\delta_{nm} + \sum_{i=1}^{I}\eta_{ni} = 0; \qquad n = 1,\dots,N, \\
& \qquad\qquad \textstyle\sum_{m=1}^{M}\sum_{m'}^{M}\beta_{mm'} + \sum_{i=1}^{I}\sum_{i'}^{I}\gamma_{ii'} + \sum_{m=1}^{M}\sum_{i}^{I}\mu_{mi} = 0, \\
& \qquad (vi) \quad \alpha_{nn'} = \alpha_{n'n}; n \ne n', \;\; \beta_{mm'} = \beta_{m'm}; m \ne m', \;\; \gamma_{ii'} = \gamma_{i'i}; r \ne i'.
\end{aligned}
$$

The first set of constraints is used to model the representation property of the Shephard distance function. It assumes the present form because the logarithm of any number in (0,1] is non-positive. The second set of inequalities says that if some firm starts to produce more of its desirable output(s) then it should move closer to the frontier of $P(x)$, *ceteris paribus*. Similar logic is reflected in *(iii)* and *(iv)*, all of which model monotonicity. Homogeneity of degree one in the output vector is imposed in the fifth set of constraints and, finally, the symmetry of the parameters of the translog function is achieved by (v).[30] The vector of optimal solutions to this problem can be used to calculate the estimate of r_i^L for every undesirable output in the model.

[30] A more in-depth analysis of the Shephard distance function properties is given in Färe and Primont (1995).

We will be able to easily assess the sensitivity of the translog output set frontier to the changes in mapping rule, since both Shephard and hyperbolic function will be parameterized with this functional form. The coefficients of the hyperbolic distance function are calculated by specifying a linear programming algorithm very similar to the one given in (3.9). The difference, however, is in the way we model the almost-homogeneity of outputs [Aczél (1966)], which is achieved by modifying the fifth set of constraints in the following way:

$$\begin{aligned}
&\textstyle\sum_{m=1}^{M}\beta_m - \sum_{i=1}^{I}\gamma_i = 1, && \\
&-\textstyle\sum_{m'=1}^{M}\beta_{mm'} + \sum_{i=1}^{I}\mu_{mi} = 0; && m = 1,\dots,M, \\
&-\textstyle\sum_{i=1}^{I}\gamma_{ii'} + \sum_{m=1}^{M}\mu_{mi} = 0; && i' = 1,\dots,I, \\
&\textstyle\sum_{m=1}^{M}\delta_{nm} - \sum_{i=1}^{I}\eta_{ni} = 0; && n = 1,\dots,N, \\
&\textstyle\sum_{m=1}^{M}\sum_{m'}^{M}\beta_{mm'} + \sum_{i=1}^{I}\sum_{i'}^{I}\gamma_{ii'} - \sum_{m=1}^{M}\sum_{i}^{I}\mu_{mi} = 0. &&
\end{aligned} \tag{3.10}$$

Alternatively, one could calculate the coefficients of the directional distance function, which has an additive structure, and use the quadratic functional form in order to model its properties. This method is used in some of the later shadow-pricing models; see, for example, the study by Färe, Grosskopf and Weber (2001). In this case, the value of the directional output distance function for observation k and for any choice of the mapping vector $(-g_b, g_y)$ is assumed to be equal to

$$\vec{D}(x_k, y_k, b_k) = \alpha_0 + \sum_{n=1}^{N} \alpha_n x_{nk} + \sum_{m=1}^{M} \beta_m y_{mk} + \sum_{i=1}^{I} \gamma_i b_{ik}$$
$$+ \frac{1}{2} \sum_{n=1}^{N} \sum_{n'=1}^{N} \alpha_{nn'} x_{nk} x_{n'k} + \frac{1}{2} \sum_{m=1}^{M} \sum_{m'=1}^{M} \beta_{mm'} y_{mk} y_{m'k} + \frac{1}{2} \sum_{i=1}^{I} \sum_{i'=1}^{I} \gamma_{ii'} b_{ik} b_{i'k}$$
$$+ \sum_{n=1}^{N} \sum_{m=1}^{M} \delta_{nm} x_{nk} y_{mk} + \sum_{n=1}^{N} \sum_{i=1}^{I} \eta_{ni} x_{nk} b_{ik} + \sum_{m=1}^{M} \sum_{i=1}^{I} \mu_{mi} y_{mk} b_{ik} \tag{3.11}$$

The coefficients of this function can be computed for a wide variety of different directions of output contraction or expansion. For example, if we take $g_y = \sigma$ and $-g_b = v$ then the observations inside the output set will be mapped onto its frontier by simultaneously increasing the desirable output and decreasing the bad in the northwestern direction (σ, v). This resembles the mapping path that underlies the hyperbolic function. At the same time, the two distance measures are quite different, since the hyperbolic function has a multiplicative structure. As noted earlier, the directional function can also accommodate the simultaneous expansion of outputs toward the northeast, which corresponds to the mapping properties of the traditional Shephard function.

The parameter estimates of the quadratic function are found by minimizing the total distance between individual observations in the sample and the estimate of the output set frontier by solving the following LP model:

$$
\begin{aligned}
&Min \;\; \textstyle\sum_{k=1}^{K}(\vec{D}(x_k, y_k, b_k; \sigma, \nu) - 0) \\
&\quad s.t. \quad (i) \quad \vec{D}(x_k, y_k, b_k; \sigma, \nu) \geq 0; \qquad k = 1,...,K, \\
&\qquad\qquad (ii) \quad \partial\vec{D}(x_k, y_k, b_k; \sigma, \nu)/\partial y_{mk} \leq 0; \qquad m = 1,...,M, k = 1,...,K, \\
&\qquad\qquad (iii) \quad \partial\vec{D}(x_k, y_k, b_k; \sigma, \nu)/\partial b_{ik} \geq 0; \; i = 1,...,I, k = 1,...,K, \\
&\qquad\qquad (iv) \quad \partial\vec{D}(x_k, y_k, b_k; \sigma, \nu)/\partial x_{nk} \geq 0; \qquad n = 1,...,N, k = 1,...,K, \\
&\qquad\qquad (v) \quad \sigma\textstyle\sum_{m=1}^{M}\beta_m - \nu\textstyle\sum_{i=1}^{I}\gamma_i = -1, \\
&\qquad\qquad\qquad \sigma\textstyle\sum_{m'=1}^{M}\beta_{mm'} - \nu\textstyle\sum_{i=1}^{I}\mu_{mi} = 0; \quad m = 1,...,M, \qquad (3.12) \\
&\qquad\qquad\qquad \sigma\textstyle\sum_{m=1}^{M}\mu_{mi} - \nu\textstyle\sum_{i=1}^{I}\gamma_{ii'} = 0; \quad i' = 1,...,I, \\
&\qquad\qquad\qquad \sigma\textstyle\sum_{m=1}^{M}\delta_{nm} - \nu\textstyle\sum_{i=1}^{I}\eta_{ni} = 0; \quad n = 1,...,N, \\
&\qquad\qquad\qquad \sigma^2\textstyle\sum_{m=1}^{M}\sum_{m'}^{M}\beta_{mm'} + \nu^2\textstyle\sum_{i=1}^{I}\sum_{i'}^{I}\gamma_{ii'} - \sigma\nu\textstyle\sum_{m=1}^{M}\sum_{i}^{I}\mu_{mi} = 0, \\
&\qquad\qquad (vi) \quad \alpha_{nn'} = \alpha_{n'n}; n \neq n', \;\; \beta_{mm'} = \beta_{m'm}; m \neq m', \;\; \gamma_{ii'} = \gamma_{i'i}; r \neq i'.
\end{aligned}
$$

Again, the calculation of parameters in (3.12) has to be implemented in a way that satisfies all of the distance function properties. The representation property is imposed by the inequalities in the first set of constraints, the second through fourth sets model monotonicity, the translation property is imposed in *(v)*, and, finally, the symmetry of parameters of the quadratic functional form is modeled by *(vi)*.[31] Just as in the case of the Shephard/translog or the hyperbolic/translog model, the optimal solution can be used to find the estimate of r_i^Q for every *i*.

[31] Note that to model the problem in the northeastern direction such as $(g_y, g_b) = (\sigma, \nu)$ instead, one would need to modify the translation property constraints. This is achieved by changing all of the subtraction equalities to addition equalities in the fifth set of constraints, i.e. $\sigma\sum_{m=1}^{M}\beta_m + \nu\sum_{i=1}^{I}\gamma_i = -1$, $\sigma\sum_{m'=1}^{M}\beta_{mm'} + \nu\sum_{i=1}^{I}\mu_{mi} = 0$, and so on.

Hence, we will use the translog specification to calculate the coefficients of both the Shephard and the hyperbolic function. We then use problem (3.12) to obtain the parameter estimates of the quadratic directional function for several mapping vectors. Again, recall that our goal is to determine if the quadratic frontier estimates differ from the translog frontier estimates, as well as whether there are any particular tendencies in the variation of each type of these empirical boundaries as a consequence of changes in the mapping rule.

To visually assess the approximation properties of various functional forms with respect to the general assumptions of the environmental production model, we use the distance function coefficients from each of our specifications. They enable us to plot the estimates of the output set frontiers by simulating the optimal output values for every observation in the sample. We will denote them by $y_*^Q(\sigma,\nu)$ and $b_*^Q(\sigma,\nu)$ in the directional/quadratic specification and by y_*^L and b_*^L in the translog models. These optimal quantities place every producer on the output set boundary and thus generate the plots of empirical analogues of $P(x)$. In fact, assuming just one desirable output and one bad, only y_* will have to be simulated for every observation given $b \equiv b_*$ and $x_{nk} = \bar{x}_n, \forall n = 1,...N$. Alternatively, one could instead solve for b_* given $y \equiv y_*$. The quantities

$(\ln y_*^L, \ln b_*^L)$ and $(y_*^Q(\cdot), b_*^Q(\cdot))$ for the fixed values of inputs are found by solving K pairs of quadratic equations of the following form:[32]

$$\begin{aligned} 0 = \quad & \hat{\beta}_{11}(\ln y_*^L)^2 + \hat{\beta}_1 \ln y_*^L + \sum_{n=1}^{N} \hat{\delta}_n \ln \bar{x}_{nk} \ln y_*^L + \hat{\mu} \ln y_*^L \ln b_*^L \\ & + \hat{\alpha}_0 + \sum_{n=1}^{N} \hat{\alpha}_n \ln \bar{x}_n + \hat{\gamma}_1 \ln b_*^L + \frac{1}{2} \sum_{n=1}^{N} \sum_{n'=1}^{N} \hat{\alpha}_{nn'} \ln \bar{x}_{nk} \ln \bar{x}_{n'k} \\ & + \hat{\gamma}_{11}(\ln b_*^L)^2 + \sum_{n=1}^{N} \hat{\eta}_n \ln \bar{x}_{nk} \ln b_*^L \end{aligned} \tag{3.13}$$

$$\begin{aligned} 0 = \quad & \tilde{\beta}_{11}(y_*^Q)^2 + \tilde{\beta}_1 y_*^Q + \sum_{n=1}^{N} \tilde{\delta}_n \bar{x}_{nk} y_*^Q + \tilde{\mu} y_*^Q b_*^Q \\ & + \tilde{\alpha}_0 + \sum_{n=1}^{N} \tilde{\alpha}_n \bar{x}_n + \tilde{\gamma}_1 b_*^Q + \frac{1}{2} \sum_{n=1}^{N} \sum_{n'=1}^{N} \tilde{\alpha}_{nn'} \bar{x}_{nk} \bar{x}_{n'k} + \tilde{\gamma}_{11}(b_*^Q)^2 \\ & + \sum_{n=1}^{N} \tilde{\eta}_n \bar{x}_{nk} b_*^Q \end{aligned} \tag{3.14}$$

The plots of the estimates of the environmental output set should provide insight as to which of the various parametric specifications that we have defined earlier are particularly suitable for application in the shadow-pricing models and which are perhaps not appropriate at all—an outcome, which is quite plausible provided that some of these empirical analogues may bear only a distant resemblance to the set in Figure 3.1.

In light of this potential discrepancy, we need a benchmark against which one can assess the estimates of the output sets and the shadow prices. Shadow

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[32] Although the choice of x_n will affect the location of these estimates of the true frontier, their curvature will remain unaffected.

price estimates can be interpreted as the opportunity cost of reducing the emissions of sulfur dioxide; namely, they represent the proxies for the cost of pollution reduction. We will compare these proxies to the market prices of allowances, which have evolved during the implementation of the Phase I of the Title IV of the CAAA and which contain the real-world information regarding the actual cost of pollution reduction.[33] Since the shadow-pricing algorithms seek to approximate the costs associated with the pollution reduction, those models whose outcomes repeatedly resemble the real-world data more closely should be preferred to all other parametric specifications. Hence, we will conclude that the approximation properties achieved by the translog (quadratic) functional form are adequate for a given mapping regime provided the shadow prices from the Shephard/translog and hyperbolic/translog (directional/quadratic) specifications converge to the market price of allowances. On the contrary, if the opposite holds, then we will deduce that the estimates of the output set and shadow prices from the translog-based (quadratic-based) models are biased.

3.4 The Data

In the empirical example we use data from the U.S. electric utility industry for 1997, 1998, and 1999. Our model has one socially desirable output

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[33] Ellerman *et al.* (2000) argue that the emissions trading program established by Title IV resulted in transparent market prices of allowances, low transaction costs, and good participation on the part of buyers and sellers in later years of implementation of the Acid Rain Program, thereby resulting in the emergence of a "robust and efficient" allowance market.

(electricity), one bad (emissions of sulfur dioxide), and four production inputs, including generating capacity, fuel consumption, labor, and sulfur content of coal, i.e. $M=I=1$ and $N=4$.

We get the annual electricity production (GWh) and generating capacity (MW) of each generating unit in the sample from the Annual Steam Electric Unit Operation and Design Report (EIA767).[34] Since most of the 263 Phase I generating units (with the exception of two units) have a one-to-one relationship with the boiler, we assume that the boiler's data are the same as the generating unit's data.[35] However, in the case of multiple relationships between boiler and unit, we assume that the fuel (coal, oil, gas and other fuel) consumption and the SO_2 emission of generating units are proportional to the ratio that each boiler contributed to the total electricity generation of each unit. We exclude 52 units that do not produce any electricity, and instead receive it from other generating units, thereby reducing our sample size to 209 observations. Finally, to be able to use generating capacity as a proxy for capital, we exclude from consideration the units that had any scrubbing equipment during the period of consideration. There were 21 such units in 1997, 23 in 1998, and 22 in 1999.

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[34] The EIA767 Report compiled by the Energy Information Administration (EIA) of the Department of Energy (DOE) contains the unit- and boiler-level data. The EIA-767 data file is a steam-electric unit data file that includes annual data from organic- or nuclear-fueled steam-electric units with a generator nameplate rating of 10 or more megawatts. The data are derived from the Form EIA-767 "Steam-Electric Unit Operation and Design Report."

[35] In addition to 263 generating units that were legally required to participate in Phase I, some units, called the compensating units, have volunteered to become subject to Title IV. We exclude these units from the consideration because of the data limitations.

The SO_2 emissions data for each boiler are obtained from the Environmental Protection Agency's (EPA) Acid Rain Program database. These data are measured from Continuous Emissions Monitoring System (CEMS), which was installed with the implementation of the Acid Rain Program in 1994. We transformed the boiler's SO_2 emissions by multiplying them by the electricity production ratio to get the SO_2 emissions of each generating unit.

We can get the fuel consumption (million BTU) of each generating unit by multiplying the boiler's fuel consumption by the electricity production ratio of each generating unit, since the electricity production is assumed to be proportional to the heat input. We multiply the monthly quantity of fuel consumption by the heat content of each fuel. Then, we sum it to get the annual fuel consumption using the data from the EIA767 database, since it offers monthly fuel quantity and heat content for each boiler. To avoid biases in estimation that may arise from aggregating the consumption of various types of fuels into a single value of million BTU, we also include sulfur as one of the production input. Since various types of fuels differ in their sulfur content, such specification allows us to account for fuel switching in inputs as a source of pollution reduction.

We could not get the generating unit's labor data since they are available only at the power plant level. The labor data, which include management and sales staff, were obtained from the Federal Energy Regulatory Commission's Electric

Utility Annual Report (FERC-1).[36] We assume that the generating unit's labor is proportional to the electricity production ratio of each unit to the power plant's electricity production. We multiply the number of the plant's employees by the electricity generation ratio of each generating unit to get the labor of each generating unit. The labor of the half-time employee is assumed to be the half of the full-time employee.

Finally, to get the price of electricity we divide the sum of sales revenue by the sales of electricity from EIA861. The price data are available only at the utility level, so we assumed that the electricity price of units that belong to one utility is the same among the units. The summary statistics of the datasets along with the annual price of electricity are compiled in Table 3.2.

3.5 Results and Discussion

In the first stage of empirical investigation we compute the optimal solutions to the linear programming problems described in (3.9) and (3.10) for the Shephard/translog and hyperbolic/translog models, respectively, and in (3.12) for the directional/quadratic-type models. We then solve for pairs $(\ln y_*^L, \ln b_*^L)$ and $(y_*^Q(\cdot), b_*^Q(\cdot))$ by finding the roots of quadratic equations (3.13) and (3.14), which enable us to obtain the estimated boundaries of the environmental output set.

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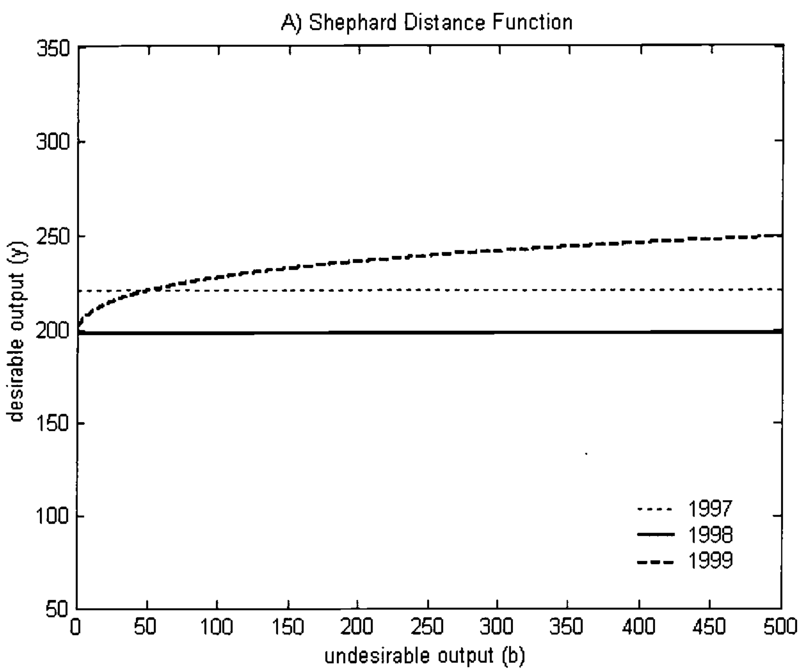
[36] The FERC-1 is a comprehensive and operating Report for Electric Rate regulation and financial audits. Major defined as (1) one million Megawatt hours or more; (2) 100 megawatt hours of annual sales for resale; (3) 500 megawatt hours of annual power exchange delivered or (4) 500 megawatt hours of annual wheeling for others (deliveries plus losses).

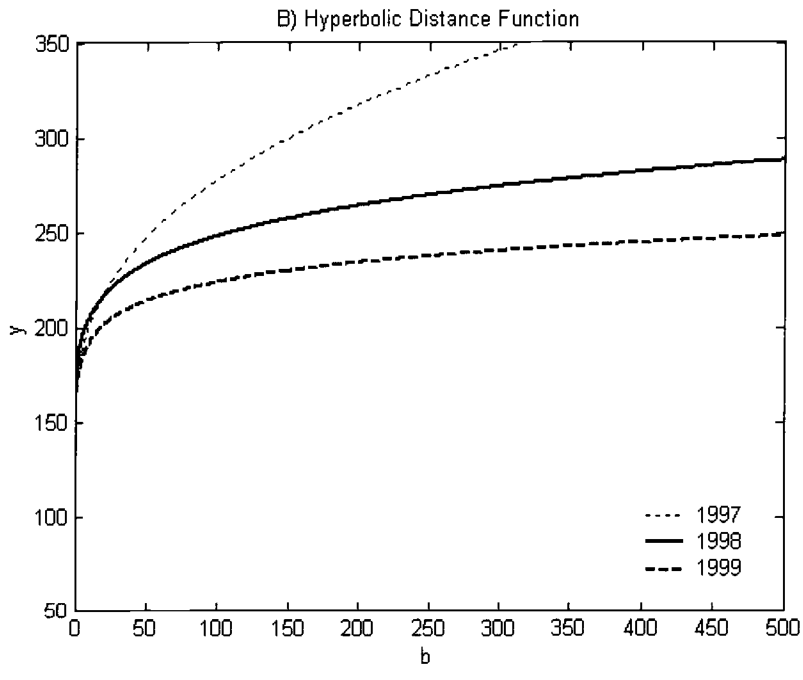
Hence, for every year in the sample we calculate two translog frontiers, one of which is retrieved from the traditional Shephard distance function and another one from the hyperbolic function, as well as five quadratic frontiers, computed using the coefficients of the directional distance function for the following mapping vectors (g_b, g_y): (1,1), (-1, 15), (-1, 9), (-1, 5), and (-1, 2). Such choice of the mapping regimes is sufficient in order to allow us to trace the dynamics of the change in the quadratic frontier estimates.

Figure 3.2 Environmental Output Sets Calculated Using the Directional and Shephard Output Distance Functions



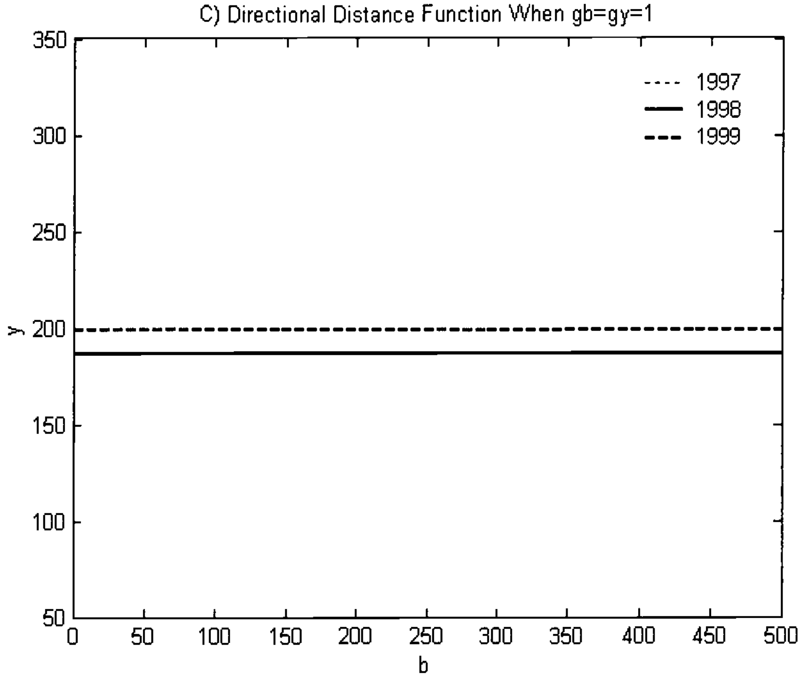
The plots of the estimated output sets point to rather interesting conclusions.[37] For instance, Figure 3.2 demonstrates four of the various types of frontiers—both of the translog estimates are depicted in panels A and B, whereas panels C and D illustrate two of the quadratic boundaries for the mapping vectors equal to (1,1) and (-1, 2), respectively.

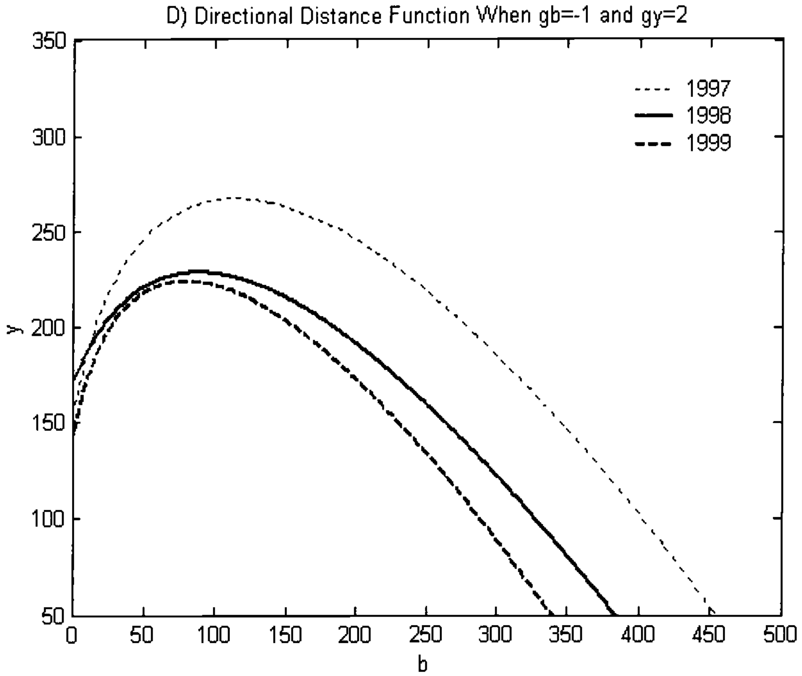
Figure 3.2 (Continued)



[37] We have rescaled our data to facilitate the convergence of the LP algorithm. Thus, for every observation in our samples, the electricity production is expressed in tens of GWh, the emissions of SO_2 and sulfur content are in thousands of tons, the generating capacity is in thousands of MW, labor is in hundreds of persons, and fuel consumption is in hundreds of billion BTU.

Figure 3.2 (Continued)





Both of the frontiers in panels A and C, which are based on the northeastern mapping regimes that simultaneously expand the good and the bad, are extremely "flat." In contrast, the contraction-based specifications yield far "steeper" boundaries for every year under consideration. Indeed, all but one of the estimated boundaries in panels A and C are horizontal, which corresponds to zero shadow price for every observation.[38] This finding attests to an extreme sensitivity of the frontier estimates to variation in the mapping rule. In addition, by comparing the plots in panels B and D, it becomes apparent that at least some of the cross-model divergence in shadow prices must be blamed upon the dissimilar approximation properties of the translog versus quadratic functional forms. For example, while the hyperbolic distance function-based translog frontiers in panel B exhibit rather poor global approximation properties, the quadratic boundaries in panel D resemble the hypothetical output set of Figure 3.1 much more closely. Note that the estimates in panels B and D do not necessarily violate the assumption of null-jointness. Recall that a downward shift of a desired magnitude can be achieved by choosing a specific value for input usage while solving equations in (3.14). Finally, both the translog estimates and the quadratic boundaries that are based on the northeastern mapping regimes appear to violate the assumption of compactness imposed on the environmental output set.

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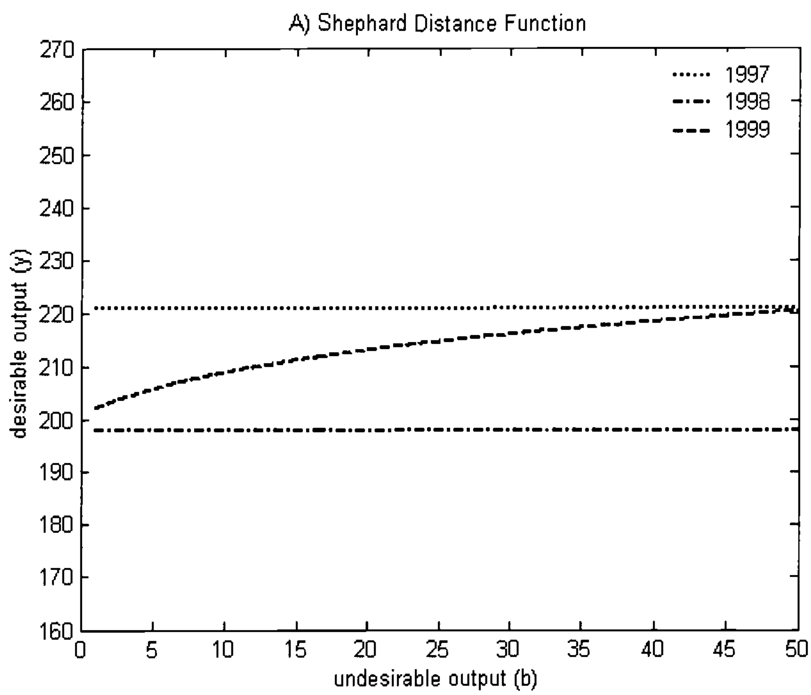
[38] Any of the quadratic models based on $g_b > 0$ has consistently produced $\gamma = \eta_1 = \eta_2 = \eta_3 = \mu = 0$, so that $\partial \vec{D}(x, y, b; g_y, -g_b) / \partial b = 0$ and the shadow price estimate is zero.

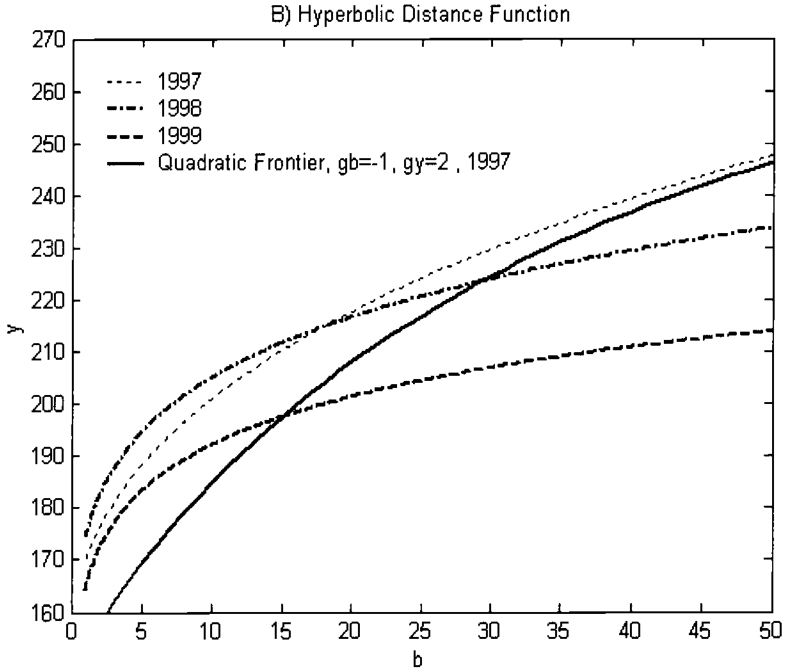
These tendencies are validated by the plots depicted in Figure 3.3, which are drawn in the vicinity of actual data on the output vector, as well as by the investigation of the observations that are among the benchmark firms in all of our models.[39] For example, the average shadow price estimate for such firms in 1999 is equal to \$229 in the Shephard/translog model, \$1,164 in the hyperbolic/translog model, and \$1,090 in the directional specification with $(g_b, g_y) = (-1,2)$. This trend has been confirmed for 1997 and 1998 data as well.

<u>Figure 3.3</u> Output Set Frontier Estimates; Some Cross-Model Comparisons



[39] On these diagrams the bad is bounded by 50 thousand tons and the good, which is expressed in tens of GWh, varies from 160 to 270.

Figure 3.3 (Continued)



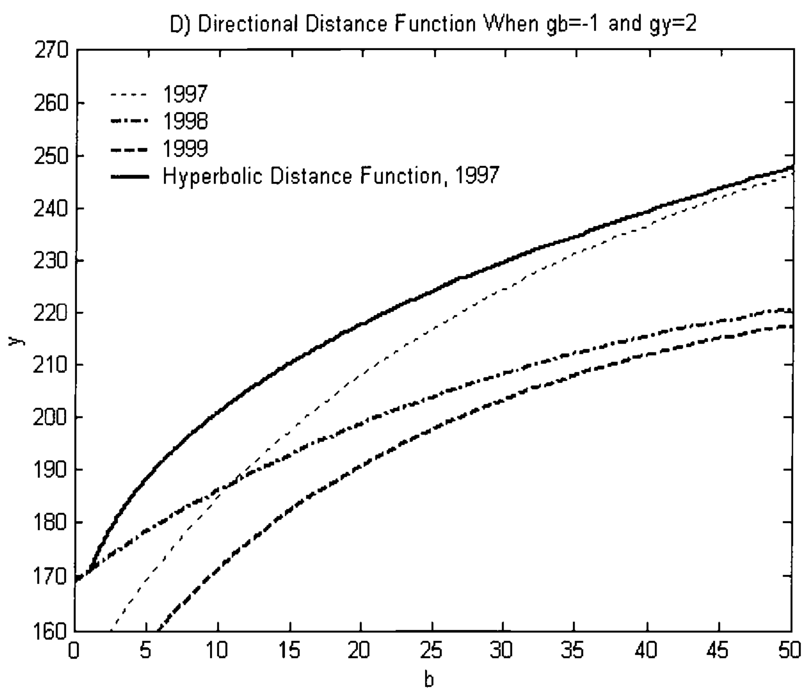
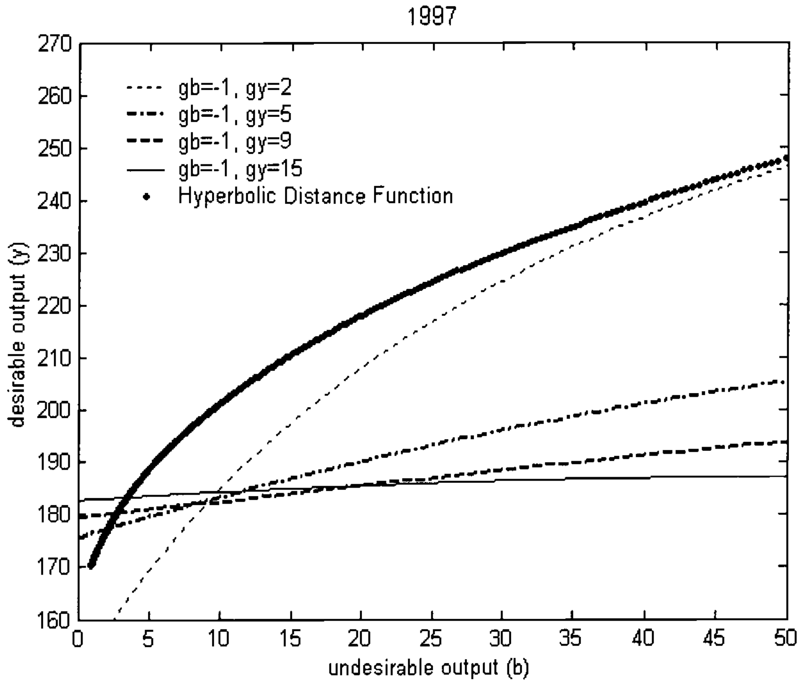


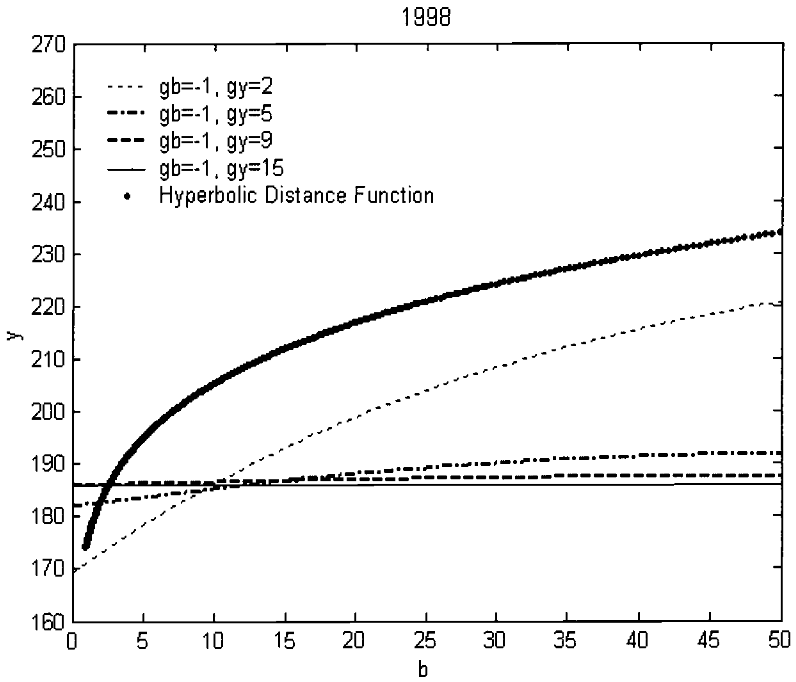
Figure 3.4 further investigates the responsiveness of the quadratic frontiers to the changes in the mapping rule for each of the three years; the relevant hyperbolic translog frontier estimates are depicted for comparison.

Figure 3.4 Output Sets Calculated Using the Parameters of the Directional Function; Various Mapping Rules



Although all of these models are based on a simultaneous expansion of the good and contraction of the bad, there is nonetheless a noticeable variation among the slopes of their estimated boundaries. In particular, as we start from a northernmost rule, i.e. $(g_b, g_y) = (-1,15)$, and rotate the mapping vector westward, the slope of the output set boundary increases dramatically, especially for 1997 and 1998 data.

Figure 3.4 (Continued)



Turning the vector even more toward the west yields the frontiers that have a "convex" rather than a "concave" contour, as in Figure 3.1, thereby violating some of the more important assumptions of the environmental production model.[40]

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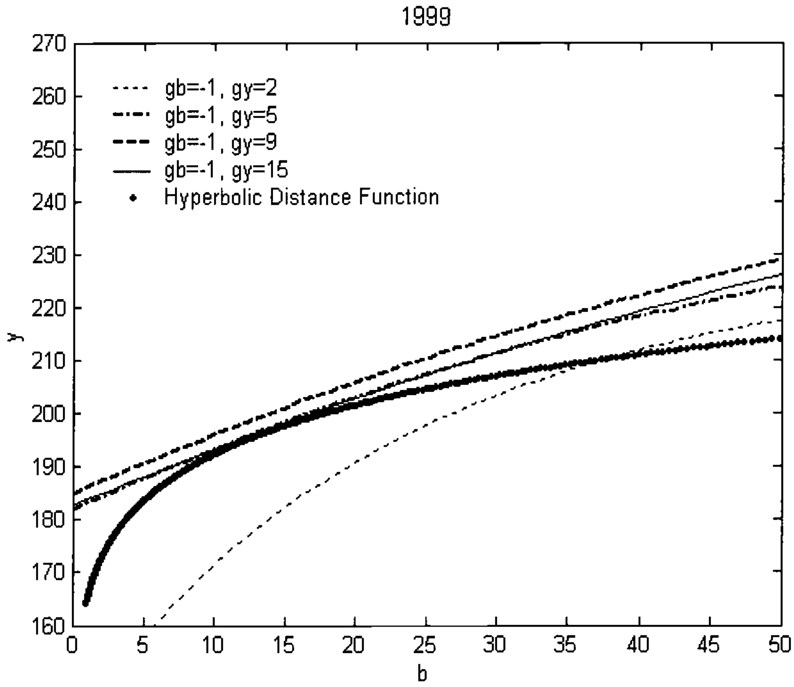
[40] The shape of the quadratic frontiers appears to be influenced by $\tilde{\beta}_{11}$, which denotes the estimate of β_{11} in models based on the directional distance function. For example, when $\tilde{\beta}_{11} < 0$ the frontier estimates are as in panel D of Figure 3.2; when $\tilde{\beta}_{11} = 0$ the frontiers are horizontal; when $\tilde{\beta}_{11} > 0$ the frontiers have a convex contour.

Figure 3.4 (Continued)



We believe that our results have very important policy implications. After all, it is obvious that one's choice of a specific mapping regime can greatly affect the estimates of the shadow prices of bads. Moreover, since the quadratic models yield extremely flat frontiers for all of the northeastern mapping vectors we tried, it becomes apparent that neither the translog nor the quadratic specification is ideal.

In Table 3.3 we list the average shadow prices generated by both types of the translog models and four of the directional/quadratic-type models, and contrast our results with the outcomes reported in a number of previous shadow-pricing studies based on parametric LP algorithms. Our findings are in line with the estimates reported in Coggins and Swinton (1996) and Swinton (1998, 2002, 2004), all of which rely on the translog distance functions, as well as Färe *et al.* (forthcoming), who utilize the quadratic methodology.[41] In particular, just as in previous studies, the Shephard/translog estimates are consistently lower than both the directional/quadratic and the hyperbolic/translog shadow prices. Hence, it can be concluded that the relatively low estimates reported in Coggins and Swinton (1996) and Swinton (1998, 2002, 2004) could be due not so much to the characteristics of the data as to quite specific approximation properties of the translog functional form that must be used to parameterize the Shephard function. A similar conclusion applies in case of the fairly high estimates reported in Färe *et al.* (2005).

In light of such dramatic variation in the estimates, as well as to provide recommendations regarding the choice of a particular parameterization technique and a specific mapping vector, one needs to assess the relative performance of these parametric models against some benchmark. As noted earlier, the market prices of SO_2 allowances represent one such benchmark for comparison, because they contain reliable information regarding the actual costs of pollution reduction. Reliable allowance-price data are reported by the following three private environmental brokerages: Cantor Fitzgerald, the Emission Exchange Corporation, and Fieldston Publications. In addition, EPA reports clearing prices of allowances

[41] Swinton (1998) groups the plants by the ones that were subject to Phase I and the ones that were not. Hence, consistent with our specification, the results reported in Table 3 correspond to the shadow prices of only those Phase I units that operated without any scrubbing equipment during the time period considered.

exchanged at its annual March spot auctions. Ellerman et al. (2000) show that by 1994 the prices published by the three market makers started to converge and were almost identical to the EPA's spot auction clearing prices a year later. Thus, in the second part of the analysis we gauge our first-stage results against the annual means of the allowance price indices published by these organizations.

Table 3.4 summarizes the results of such benchmarking. The average price of allowances was equal to approximately \$100 in 1997, \$151 in 1998, and \$195 in 1999. In addition to our original choice of mapping regimes, we also include the results from several other quadratic specifications. The standard errors of mean shadow price estimates reported in parentheses were drawn from 500 bootstrap iterations [Efron and Tibshirani (1993)]. They allow us to perform the test of the difference of the estimated mean shadow price from the average market price of allowances. Hence, Table 4 shows that the quadratic model based on $(g_b, g_y) = (-1,9)$ with the corresponding average shadow price of approximately \$97 produces a reliable estimate for 1997. Unfortunately, this model also yields rather inaccurate results using 1998 and 1999 data. Similarly, the quadratic specification that assumes $(g_b, g_y) = (-1,29.4)$ does a satisfactory job of approximating only the 1999 allowance price. We could not find a mapping vector that corresponds to an accurate estimate using 1998 data. For instance, the model based on $(g_b, g_y) = (-1,3.96)$ yields a mean shadow price estimate of roughly \$166, which then falls abruptly to \$136 as the mapping vector is changed to

$(g_b, g_y) = (-1, 3.97)$. Both of these estimates are significantly different from the 1998 allowance price of $151.

The approximation results produced by the translog models are even more disappointing. Although the 1999 Shephard/translog estimate of $208 is not significantly different from that year's allowance price, we suspect this result may be mainly due to a relatively large standard error, which is equal to nearly $60. None of the Shephard/hyperbolic models achieves the desirable degree of approximation, either. Moreover, the shape of all of our estimated translog frontiers appears to be in violation of the assumption of compactness imposed on the environmental output set. In contrast, most of the quadratic models based on the northwestern mapping vectors possess better global approximation properties. We believe that together with their superior mapping flexibility, this characteristic of the quadratic models makes them more appropriate for application in shadow-pricing studies than the translog-based specifications.

What are the conclusions that can be made from such an assessment of various shadow-pricing models? First, the products of model comparison reveal that different assumptions regarding the true production technology can lead to distinctly dissimilar degrees of its approximation. The estimates of the shadow prices of undesirable outputs depend crucially upon both the parameterization technique and the mapping rule. Second, although the specifications based on the directional distance function appear to be more flexible, no single parametric methodology produces the outcomes that are consistently close to the market prices of allowances. Finally, the translog models that have been used so extensively in the previous shadow-pricing studies may be inappropriate because the output sets they produce repeatedly violate some of the general assumptions imposed on the environmental production model.

3.6 Concluding Remarks

Many empirical studies on the pricing of so-called bads have implicitly assumed that all of the several available parametric methodologies provide a similar and an adequate approximation to the true production technology—a conjecture that is unsupported at best. Considerable differences in the shadow price estimates recorded in the literature have been attributed merely to the varying slope of the output set frontier, and no attempt has been made so far to look into potentially different degrees of its approximation achieved by various parametric models by performing some cross-model comparisons.

Using a panel of observations from the U.S. electricity industry we perform a detailed study of the empirical counterparts of an environmental output set produced by the parameterization of the distance functions via transcendental logarithmic *vis-à-vis* quadratic functional forms for a variety of mapping vectors. The shadow price estimates from each of our models are assessed against the prices of pollution permits. Since the market prices of SO_2 allowances represent decent proxies for the actual costs of pollution abatement—the costs that the shadow-price studies were developed to approximate—they can be used as a worthy benchmark for such a comparison. We illustrate that although no single parametric methodology is superior to all others, some specifications appear to attain much better degrees of approximation of the true production technology than the other shadow-pricing models do.

By demonstrating these distinctly different parameterization properties of various models our results provide a possible explanation for a sizeable divergence of the estimates of shadow prices of bads documented in the literature. The tendencies we were able to detect imply that these differences are due not so much to the varying slope of the output set frontier, but to an extreme sensitivity of its estimate to the changes in the parameterization methodology, especially to the variation in the mapping regime. Both the functional form assumptions and the underlying mapping rule used during the estimation can cause the entire boundary of the output set to change. As a consequence of this variation, the shadow price estimates can indeed be predetermined by a choice of a specific parametric technique—an outcome that may have very important policy implications.
Thus, our conclusions call for more extensive research of this subject. In light of our findings, the existence of alternative functional forms that can be used for the approximation of the true production technology becomes a matter of particular interest.

Table 3.1

Selected Properties of the Shephard and the Directional Output Distance Functions

	Shephard Distance Function	Directional Distance Function
Representation	$0 < D(x, y, b) \leq 1$	$\vec{D}(x, y, b; g_y, -g_b) \geq 0$
Monotonicity	$\partial D(x, y, b)/\partial x \leq 0$, $\partial D(x, y, b)/\partial y \geq 0$, $\partial D(x, y, b)/\partial b \leq 0$	$\partial \vec{D}(x, y, b; g_y, -g_b)/\partial x \geq 0$, $\partial \vec{D}(x, y, b; g_y, -g_b)/\partial y \leq 0$, $\partial \vec{D}(x, y, b; g_y, -g_b)/\partial b \geq 0$
Output Homogeneity of Degree +1	$D(x, \lambda y, \lambda b) = \lambda D(x, y, b)$, $\lambda > 0$	–
Translation	–	$\vec{D}(x, y + \rho g_y, b - \rho g_b; g_y - g_b) = \vec{D}(x, y, b; g_y - g_b) - \rho$, $\rho \in \Re$

Note: The proofs of these and other properties of output distance functions can be found in Färe and Primont (1995) and Färe and Grosskopf (2004).

Table 3.2

Dataset Descriptive Statistics

	Outputs		Inputs				
	Electricity (GWh)	SO_2 Emissions (1000 of tons)	Capacity (MW)	Fuel (Millions of BTUs)	Labor (Pers.)	Sulfur (tons)	Price of Electricity ($/MWh)
Arithmetic Mean							
1997	1,684.6	20.1	329	16,901,620	245	10,053	33.51
1998	1,719.9	20.4	331	17,260,723	231	9,942	31.40
1999	1,785.7	18.4	335	17,903,152	232	10,374	34.40
Standard Deviation							
1997	1,239.0	17.6	225	11,972,487	216	8,833	14.09
1998	1,240.0	17.6	226	11,997,303	209	8,381	10.54
1999	1,350.3	16.6	229	13,006,507	225	10,761	11.95
Minimum							
1997	4.4	3.00	19	47,659	2	0	10.50
1998	14.8	7.00	19	164,675	8	0	10.55
1999	3.7	23.00	19	40,792	2	0	10.99
Maximum							
1997	6,004.9	95.3	952	57,415,738	950	48,750	93.89
1998	6,570.7	120.3	952	62,523,743	1,051	55,378	63.06
1999	6,044.0	91.3	952	56,980,825	1,357	64,854	76.17

Note: The number of observations is equal to 188 in 1997, 186 in 1998, and 187 in 1999.

Table 3.3

ion Methodology and Its Impact on Average Shadow Price of Sulfur Dioxide; Selected Studies

Estimate of Average Shadow Price of SO_2 ($/ton)						
Shephard Distance Function	Hyperbolic Distance Function	Directional Distance Function				
		$g_b = -1$, $g_y = 1$	$g_b = -1$, $g_y = 2$	$g_b = -1$, $g_y = 5$	$g_b = -1$, $g_y = 9$	$g_b = -1$, $g_y = 15$
292						
7-127						
155-193						
143-269						
		1,117-1,974				
0	610		1,123	233	97	33
0	250		580	79	14	0
208	1,331		943	344	328	310

Table 3.4

Shadow Price Estimates and the Market Price of Allowances; Various Parameterization Methodologies

	1997	1998	1999
Shephard Distance Function	0.00 (0.00)	0.00 (0.00)	208.48 (59.62)
Hyperbolic Distance Function	609.77* (97.96)	249.50* (16.49)	1330.95* (148.76)
Directional Distance Function			
$g_b = -1, g_y = 2$	1127.81* (74.29)	579.77* (28.09)	943.25* (53.04)
$g_b = -1, g_y = 3.96$	428.46* (26.90)	165.65* (5.83)	412.63* (19.18)
$g_b = -1, g_y = 3.97$	428.37* (24.68)	135.60* (3.93)	412.22* (17.65)
$g_b = -1, g_y = 5$	233.00* (11.86)	78.79* (4.89)	343.85* (14.24)
$g_b = -1, g_y = 9$	97.27 (4.18)	13.78* (1.01)	328.00* (13.55)
$g_b = -1, g_y = 15$	33.05* (1.90)	0.00 (0.00)	309.82* (13.21)
$g_b = -1, g_y = 29.4$	31.91* (1.85)	0.00 (0.00)	189.11 (7.23)
Allowance Market Price	100.14	151.45	194.83

Notes: Market price of allowances is the average of the indices published by Cantor Fitzgerald Environmental Brokerage (1997, 1998), the Emissions Exchange Corp. (1997-1999), and Fieldston Publications (1997-1999).

Bootstrapped standard errors are in parentheses.

* Significant at 1% in a two-tailed test of difference from the average market price of allowances.

Chapter 4

Shadow Pricing of Undesirable Outputs Using Stochastic Distance Functions: Evidence from the U.S. Electric Utility Industry

4.1 Introduction

Shadow pricing of unwanted by-products generated by polluting technologies along with socially desirable outputs has been a popular topic of discussion in the relatively recent past. A plethora of studies has been dedicated to the investigation of various techniques for the pricing of these undesirable outputs, or the "bads," as they have also been called. The majority of empirical studies focus on the calculation of the shadow prices of sulfur dioxide, a socially undesirable by-product generated in the electric utility industry together with the electricity. These shadow-pricing algorithms are based on the duality between the producers' revenue function on the one hand and various types of distance functions on the other hand.[42] The duality between distance and revenue functions enables to retrieve the prices of bads provided that the price of at least one desirable output, or the "good," is known.[43]

Popular techniques used to compute the shadow prices of bads rely on linear programming algorithms, which can be parametric (goal programming) or

[42] In most simple terms, for every observation inside the technology set the distance function represents a proxy of the distance from that observation to the frontier of that set in a particular direction, called the mapping rule.
[43] The two types of distance functions that can be used for this purpose are Shephard [Shephard (1970)] and the directional [Chambers *et al.* (1998)] distance functions. An in-depth analysis of the duality is given in, for example, Färe and Primont (1995) and Färe and Grosskopf (2004).

non-parametric (data envelopment analysis, or DEA) in nature. Key shadow pricing studies based on goal programming methods include Färe *et al.* (1993), Coggins and Swinton (1996, 2002, 2004), Swinton (1998), Färe, Grosskopf and Weber (2001), and Färe, Grosskopf, Noh and Weber (forthcoming), whereas Ball et al. (1994), Boyd *et al.* (1996), and Lee *et al.* (2002) are among the papers that utilize the non-parametric methodology. Both techniques have their strengths but, at the same time, are plagued by important limitations. For example, the non-parametric character of the DEA-based models eliminates the need to impose certain potentially restrictive parameterization assumptions regarding the functional form of the underlying distance function. At the same time, this technique does not guarantee its differentiability, which, consequently, results in failure to yield a unique shadow price estimate for every firm in the sample. On the other hand, although the algorithms that rely on goal programming help to avoid this identification problem, their outcomes are dangerously susceptible to changes in the assumptions regarding a particular parameterization methodology.

Because of the duality between the revenue and the distance functions, the firm-level shadow price estimate will depend on the gradient vector of the latter. As mentioned above, this gradient has been traditionally computed using various linear programming (LP) techniques, so that many of the afore-mentioned studies are deterministic in nature. Although econometric estimation can also be used for this purpose, it has never served as a preferred choice for the shadow pricing methodology, which is due mainly to important caveats characteristic of the econometric estimation of distance functions in general. Atkinson *et al.* (2003a, 2003b) have recently demonstrated that the generalized method of moments (GMM) can be used to obtain consistent estimates of the parameters of a particular class of distance functions, called Shephard [Shephard (1970)] distance functions.[44] In this paper, we modify their algorithm and estimate the slope coefficients of the directional output distance function, which falls into another category of these distance-based measures. We then use the parameter estimates from the GMM procedure to find the stochastic shadow prices of undesirable outputs and assess our results against the outcomes of more familiar LP-based models.

Our analysis is innovative in two ways. First, the non-Bayesian estimation of the distance function parameters and the shadow prices of bads using a GMM procedure has never been implemented before. Since the shadow prices of undesirable outputs can serve as policy targets, it always helps to have several estimation methodologies in place. Second, unlike in parametric LP models, our algorithm does not map every firm in the sample onto the frontier of the production technology before the determination of shadow price estimates but, rather, allows for the production inefficiency to be taken into consideration. This is

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[44] Atkinson and Dorfman (forthcoming) show that the pricing of bads can be alternatively performed using a Bayesian framework. They use a limited-information likelihood approach based on the moment conditions of the generalized method of moments estimator [Kim (2002)]. In contrast to all of the previously mentioned studies, however, the authors treat the bads differently, assuming them to be an exogenous shifter of the technology.

important since, as argued by Lee *et al.* (2002), imposing the production efficiency constraint can result in an upward bias of the shadow price estimate.[45]

4.2 Theoretical Background

In what follows we summarize the theoretical foundation of the production model that described the polluting technology and define the distance function, which will have to be estimated in order to obtain our firm-level shadow prices. The bads in the environmental production model are treated as outputs, so that the set of all possible input-output combinations is given by the following production technology:

$$T = \{(x, y, b) : x \text{ can produce } (y, b)\}, \tag{4.1}$$

where $x \in \Re_+^N$ is a vector of inputs, $y \in \Re_+^M$ is a vector of desirable outputs, or "goods," such as kilowatt-hours of electricity, and $b \in \Re_+^I$ is a vector of bads, e.g. the emissions of sulfur dioxide. This polluting technology can be equivalently described via an output possibilities set, given in this case by

$$P(x) = \{(y, b) : (x, y, b) \in T\}. \tag{4.2}$$

[45] Lee et al. (2002) attempt to approach this problem of production inefficiency via the application of a deterministic nonparametric shadow-pricing algorithm. In contrast, our algorithm is parametric and stochastic in nature.

This set, sometimes referred to as the environmental output set, must also satisfy the following assumptions:

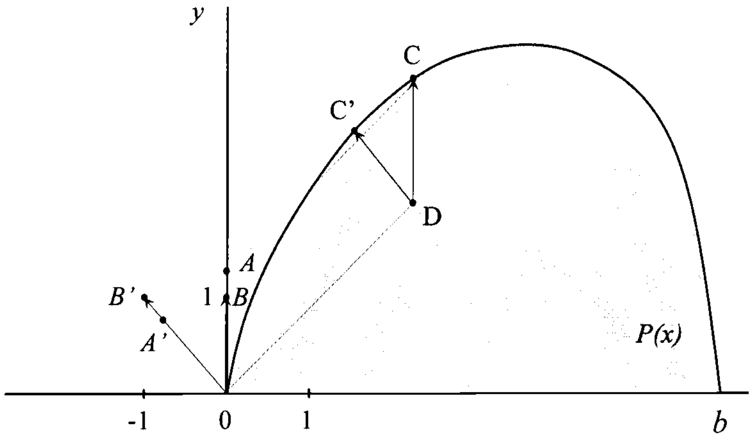
1. Null-Jointness, i.e. if $(y,b) \in P(x)$ and $b = 0$, then $y = 0$.
2. Weak disposability of an output vector, i.e. $(y,b) \in P(x)$ and $0 \leq \theta \leq 1$ imply $(\theta y, \theta b) \in P(x)$.
3. Free disposability of desirable outputs, i.e. $(y,b) \in P(x)$ and $(y^0, b) \leq (y,b)$ imply $(y^0, b) \in P(x)$.

Null-jointness implies that the production of a desirable output must always entail the production of a certain amount of the bad as well and, consequently, zero pollution has to be associated with no production of the good. The second assumption means that at the margin the joint decline in the production of outputs has to be proportional, i.e. a drop in a pollution level must be accompanied by a proportional decrease in the production of the good output as well for all of the observations that are on the frontier of the output set. Finally, free disposability of desirable outputs says that for a given level of x and b the reduction in y is always feasible. For a more in depth discussion of the theoretical underpinnings of the environmental production model and the rationale behind each of the assumptions see, for example, Färe *et al.* (2005) or Färe and Grosskopf (2004).

One of the distance measures that has been used in the shadow-pricing literature is the directional [Chambers, Chung, and Färe (1998)] output distance function, which has been used in later studies and represents a variation of the shortage function introduced by Luenberger (1992). It is characterized as a proxy of a distance from a particular observation in $\Re_{+}^{M+I}$ to the output set frontier and is defined for a given direction in which each output is to be expanded or contracted onto the boundary of *P(x)*, called a mapping rule. The shadow prices in the parametric linear programming models are evaluated at these boundary points, implying that this algorithm assumes production efficiency for every observation in the sample. The mapping flexibility of the directional output distance function makes it particularly attractive in cases where theoretical specification may necessitate a particular direction of output expansion or contraction. For instance, some of the later studies utilize the mapping rule that simultaneously increases desirable outputs and reduces the bads, which is motivated by negative externalities imposed on society by polluters. The directional output distance function with bads in the direction $(g_y \in \Re_{+}^{M}, g_b \in \Re_{+}^{I})$ is given formally by

$$\vec{D}(x, y, b; g_y, -g_b) = \sup\{\psi : (y + \psi g_y, b - \psi g_b) \in P(x)\}. \tag{4.3}$$

It takes values in the interval $[0, +\infty)$ and, as noted earlier, is related to the distance from a particular observation inside the output set *P(x)* to its frontier in the direction $g = (g_y, -g_b)$. The illustration of the hypothetical environmental output set with undesirable outputs together with the directional output distance function for two mapping regimes is presented in Figure 4.1. For example, the distance function associated with a purely northern mapping vector such as $g = (1,0)$ for observation D is equal to OA/OB. Alternatively, for the same observation a northwestern direction like g=(1,1) would yield a different value.



<u>Figure 4.1</u> Environmental Output Set with Bads and the Directional Distance Function

Both functions are nonzero since D is located inside the set, which, consequently, implies production inefficiency. Note also that due to the null-jointness and weak disposability of the output vector the environmental output set represents a subset of the traditional freely disposable output possibilities set. In addition to being non-decreasing in desirable outputs and non-increasing in the production inputs and the bads, the function in (4.3) also possesses a translation property, which will be introduced in the next section and which, as we shall see, will play a key role in achieving the practicability of econometric estimation of the directional distance functions.

Let w, p, and r denote the price vectors of production inputs, desirable outputs and the bads, respectively. Färe and Grosskopf (2004) demonstrate that the directional output distance function is dual to a firm's revenue function, defined as $R(x, p, r) = \max\{py - rb : (y, b) \in P(x)\}$, and apply this duality to find the shadow price estimates of the i^{th} undesirable output for every observation in the sample, provided that the price of the m^{th} good is observed, as follows:

$$r_i = -p_m \left(\frac{\nabla_b \vec{D}(x, y, b; g_y, -g_b)}{\nabla_y \vec{D}(x, y, b; g_y, -g_b)} \right) \tag{4.4}$$

Note that in Figure 4.1 the shadow price estimate for a particular mapping direction can be interpreted geometrically as the slope of the tangent to the appropriate boundary counterpart of D that can be reached in that direction. As the slope of the frontier changes, so does the shadow price, so that predominantly western directions are associated with relatively higher shadow prices, whereas the mapping rules based on mostly northern moves should produce lower estimates. Because the prices of the desirable outputs are commonly observed, the values of r_i are found without difficulty provided $\nabla_b \vec{D}(.)$ and $\nabla_y \vec{D}(.)$ can be estimated.[46]

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[46] The directional output distance function is not the only measure that can be used for this purpose. The two other types of distance functions that are also dual to a

In the majority of recent shadow-pricing studies this gradient is obtained by imposing a suitable parametric structure on the form of the distance function, where the parameters of the latter are calculated via a goal programming algorithm that derives from the method introduced by Aigner and Chu (1968). However, in the previous chapter we have demonstrated that the empirical counterparts of the output set frontier calculated using this LP methodology and, therefore, the respective shadow price estimates can vary widely and are extremely susceptible to the changes in the underlying mapping rule. In this regard, the existence of alternative estimation techniques becomes a matter of interest.

4.3 Estimation

Because of the duality between a firm's revenue and distance function, the calculation of shadow prices of bads will essentially come down to the estimation of the components of the gradient vector of the underlying distance function. Below we show how this task can be implemented using an algorithm that is based on stochastic estimation of directional output distance function using GMM.

The econometric estimation of the distance function parameters is complicated by the fact that the function values are not observed and instead have to be calculated based on the amounts of production inputs used and outputs produced. Here, we demonstrate how this idiosyncrasy can be resolved by suitably incorporating the properties of the directional distance function during estimation.

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firm's revenue function are Shephard [Shephard(1970)] and the hyperbolic [Färe, Grosskopf, Lovell, and Pasurka (1989)] output distance function.

Our analysis has been inspired by Coelli and Perelman (1996), Grosskopf *et al.* (1997), and Atkinson *et al.* (2003a, 2003b). In particular the last two of these studies illustrate how the practicability of econometric estimation can be achieved by taking advantage of the homogeneity property of Shephard [Shephard (1970)] distance functions.

Suppose $M=I=1$, so that the polluting production process generates just one socially desirable output with only one by-product. Since the directional distance function is bounded by zero from below and by positive infinity from above, we can write

$$0 = \vec{D}(x, y, b; g) - \varepsilon \tag{4.5}$$

where the random disturbance term ε is assumed to be nonnegative. The directional output distance function is characterized by a *translation* property, which can be summarized as follows:

$$\vec{D}(x, y + \lambda g_y, b - \lambda g_b; g_y, -g_b) = \vec{D}(x, y, b; g_y, -g_b) - \lambda, \quad \lambda \in \Re. \tag{4.6}$$

The translation property tells us that when the output vector (y,b) is translated into $(y+\lambda g_y, b-\lambda g_b)$ the value of the function will go down by a factor of λ.[47] Consequently, this allows us to rewrite (4.5) in the following way:

$$-\lambda = \vec{D}(x, y+\lambda g_y, b-\lambda g_b; g_y, -g_b) - \varepsilon. \qquad (4.7)$$

Suppose that $g_y = 1$, $-g_b = 0$, and $\lambda = -y$ so that (4.7) collapses to

$$y = \vec{D}(x,0,b) - \varepsilon. \qquad (4.8)$$

The parameters of the directional distance function in (4.8) can then be estimated by specifying a suitable parametric form for $\vec{D}(.)$ and choosing an appropriate goal programming or econometric technique. Note that because $\vec{D}(x,0,b) = \vec{D}(x,y,b) + y$, using our assumptions the shadow price estimate for every producer will equal $\hat{p}\nabla_b\vec{D}(x,0,b)$, where $\hat{p}$ is the observed price of the good.

Our choice for both the mapping vector g and the factor λ is not arbitrary. To estimate the above equation correctly one needs to specify either a purely northern, as in our case, or a purely western mapping direction, since for all other

[47] See Färe and Grosskopf (2004) for the proof of this and other properties of distance functions.

rules some of the regressors in (4.8) will enter $\vec{D}(.)$ as a function of the regressand. For example, if one were to take $g_y = -g_b = 1$ and $\lambda = -y$ instead, then one would have

$$-\lambda = \vec{D}(x, y+\lambda, b-\lambda) - \varepsilon, \tag{4.9}$$

so that the function to be estimated would take the following form:

$$y = \vec{D}(x, 0, (b+y)) - \varepsilon. \tag{4.10}$$

While the parameters of this distance function can still be found by utilizing a goal programming algorithm, the econometric estimation of equation (4.10) is not appropriate. This is because the dependent variable is being conditioned on a set of regressors, one of which is itself a function of the regressand. It is noteworthy that Atkinson *et al.* (2003a, 2003b) are unable to avoid this problem because they estimate the parameters of Shephard distance function, which is defined for a given mapping rule, which, therefore, does not allow them to choose a specific direction of output expansion or contraction.[48]

To parameterize the function in (4.8) we use the result of Färe and Lundberg (2004), who prove that there are but two functional forms that can be

[48] Note that to achieve the practicability of estimation assuming a purely western mapping direction instead one needs to take $g_y = 0$, $-g_b = 1$, and $\lambda = b$.

used for this purpose. We choose the quadratic function, since it has been used in parametric shadow pricing studies before and because it has the interpretation of the second-order Taylor's series approximation. Thus, the equation (4.8) takes the following form:

$$y=\alpha_0+\sum_{n=1}^{N}\alpha_n x_n+\gamma b+\frac{1}{2}\sum_{n=1}^{N}\sum_{n'=1}^{N}\alpha_{nn'}x_n x_{n'}+\frac{\gamma_{11}}{2}b^2+\sum_{n=1}^{N}\eta_n x_n b-\varepsilon. \quad (4.11)$$

A popular methodology for computing the parameters of the directional distance function involves a specification of a linear programming procedure, where the function coefficients are computed by minimizing the total sum of distance function values for all firms in the sample. Assuming that there are $k = 1,...K$ observations in the sample and for our choice of the mapping rule, this task requires finding the solution to the following problem:

$$\begin{aligned}
&Min \ \textstyle\sum_{k=1}^{K}\left(\vec{D}(x_k,b_k)-y_k\right)\\
&\quad s.t. \quad (i) \quad \vec{D}(x_k,b_k)-y_k\geq 0, \qquad k=1,...,K,\\
&\qquad\qquad (ii) \quad \nabla_{x_{nk}}\vec{D}(x_k,b_k)\geq 0, \qquad n=1,...,N,\ k=1,...,K,\\
&\qquad\qquad (iii) \quad \nabla_b\vec{D}(x_k,b_k)\geq 0, \qquad k=1,...,K,\\
&\qquad\qquad (iv) \quad \alpha_{nn'}=\alpha_{n'n},\ \ n\neq n'. \qquad (4.12)
\end{aligned}$$

The first set of constraints imposes the *representation* property of the distance function, which implies its non-negativity for every observation. The second and the third set of constraints impose *monotonicity* and reflect the fact that the directional distance function is non-decreasing in both the production inputs and undesirable outputs. Finally, the forth constraint models the symmetry of the parameters of the quadratic distance function.

Thus, the solution to problem (4.12) will be used to find the goal programming shadow prices, which will then be compared to the GMM estimates. The purpose of such assessment is to determine whether various estimation methodologies are likely to produce different results as well as to find out if any such difference can be attributed to the production efficiency assumption implicit in our LP model.

To satisfy the representation property of the distance function during econometric estimation one would have to impose a particular one-sided structure on the error term in equation (4.11) and proceed by estimating it via maximum likelihood procedure (MLE). This is the rationale behind the method used by Färe *et al.* (forthcoming), who take $g = (1,1)$ and apply the corrected ordinary least squares (OLS) algorithm. However, because of the monotonicity of the distance function, the production inputs and the bads are not independent of the error component, so that both the MLE and the OLS will produce inconsistent parameter estimates unless this dependence is properly accounted for. This is because the disturbance term ε, which equals the value of the directional distance function $\vec{D}(x, y, b; g)$, also enters equation (4.8). Yet, the monotonicity of the directional distance function requires that ε be non-decreasing in both x and b, creating an apparent problem of endogeneity. One can avoid this inconvenience by defining a set of instrumental variables and estimate the model using GMM instead. Although the instrumental variables procedure does not impose the one-sidedness of the error, it nevertheless yields consistent estimates of the slope parameters of the distance function, which is all that is needed to find its gradient. As was mentioned before, the estimation using GMM allows us to measure shadow prices without the required placing of every observation on the frontier of the output set $P(x)$ and thus takes account of production inefficiency.[49] Unfortunately, this algorithm entails certain costs as well, most important of which is the inability to impose monotonicity and the ensuing violation of this property for some observations in the sample, which, in turn, results in negative shadow price estimates.[50]

The chief motivation behind our study is the fact that the GMM estimation of shadow prices using a non-Bayesian framework has never been performed before. Hence, we apply our methodology to a sample of observations from the U.S. electric utility industry using the 1999 data from chapter 3.[51] As before, our model has one desirable output (electricity) and one bad (emissions of sulfur

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[49] Recall that the goal programming algorithm maps every observation onto the frontier of *P(x),* so that the firm-level shadow price represents the slope of the tangent to its boundary counterparts reached in a particular direction.

[50] This idiosyncrasy is the reason behind our assumption regarding the mapping rule. The models based on purely western mapping direction yield negative shadow prices for the majority of observations in our sample. In addition, they are associated with relatively low R^2 values.

[51] With minor modifications, the 1993 and 1997 quantities from this dataset have been used in Färe *et al.* (forthcoming).

dioxide), i.e. $M=I=1$. However, unlike in the previous chapter, we specify only three production inputs, including generating capacity, fuel consumption, and labor, so that $N=3$. Our choice of only three inputs is motivated by the fact that the sulfur values in out dataset are almost perfectly correlated with respective SO_2 quantities, which would have created a serious multi-collinerity problem if both of these variables were included as regressors.

The summary statistics of the dataset are presented in Table 3.1.

4.4 Results and Discussion

In the first step we estimate equation (4.8) using a GMM procedure and our 1999 data along with the respective 1997 input-output quantities as the only instrumental variables. The 1998 data, which we also observe, represent another natural choice for the set of model instruments. However, among all the models involving a variety of combinations of instrumental variables, our specification with the 1997 instruments yields the highest statistic in a test of joint significance of the distance function parameter estimates. These estimates are presented in the second column of Table 4.1 along with the asymptotic standard errors. Most of them are statistically significant and, as far as the distance function properties are concerned, have the expected sign. To account for the possible presence of heteroskedasticity, the inference regarding the statistical significance of our estimator is based on White's [White (1980)] consistent estimate of its covariance matrix. As was noted in the third section, we are unable to impose the monotonicity of the directional distance function with respect to our undesirable output and, as a result, it is violated for approximately 18% of firms in our sample of 184 observations. Consequently, the shadow price estimates for these firms are negative and are therefore excluded from the determination of the mean shadow price and the standard deviation, which are also reported in Table 4.1.

The shadow price estimates were also computed using a goal programming algorithm, which represents a more common methodology for this task and which for our choice of mapping direction involves solving problem (4.12). Note that unlike the GMM procedure, this model is deterministic in nature. Recall that this specification requires mapping of every observation onto the frontier of the output set prior to the determination of the shadow prices, whereas our GMM algorithm does not. Therefore, one would expect the goal programming shadow prices to be higher than their GMM counterparts, since by placing every firm of the boundary of $P(x)$ the LP model essentially assumes no production inefficiency. Also, because the LP algorithm allows for the imposition of inequality constraints, monotonicity can be imposed for all of the observations in the sample, so that the goal programming shadow price estimate is never negative. The LP parameters of the directional output distance function and the descriptive statistics of the shadow price estimates are reported in the third column of Table 4.1.

Our average goal programming shadow price is lower than its GMM counterpart, which is a little surprising. Firms that are productively inefficient will normally be associated with lower shadow prices than more efficient producers, since, as argued by Lee *et al.* (2002), the efficient producers of electricity cannot reduce their output of bads by means of the resource reallocation and, therefore, are likely to face higher marginal abatement costs of SO_2.[52] Instead, our GMM model, which allows for inefficiency during estimation, yields a higher average shadow price than the LP algorithm, which assumes no inefficiency. To some extent, this can be justified by an extreme sensitivity of shadow price estimates in goal programming models to variations in mapping rule shown in chapter 3. Recall that the average slope of the estimates of the environmental output set frontier and, therefore, the shadow prices tend to fall dramatically in models based on predominantly northern mapping rules compared to the specifications associated with northwestern mapping directions like $g_y = -g_b = 1$. Our algorithm assumes a purely northern move, so that the frontier estimate can be expected to be extremely 'flat' along the range of the undesirable output *b*. This is why the goal programming shadow prices are associated with relatively low estimated standard deviations as well.

Interesting conclusions can be drawn from the comparison of our average shadow price estimates to the market prices of sulfur dioxide allowances, which have evolved during the implementation of the Phase I of the Title IV of the 1990 Clean Air Act Amendments (CAAA) of the U.S. Acid Rain Program. Ellerman *et al.* (2000) argue that the emissions trading program established by Title IV resulted

[52] Shadow price estimates can be interpreted as the opportunity cost of reducing the emissions of sulfur dioxide; namely, they represent the proxies for the cost of pollution reduction.

in transparent market prices of allowances, low transaction costs, and good participation on the part of buyers and sellers in later years of implementation of the Acid Rain Program, thereby producing a "robust and efficient" allowance market. This implies that the market prices of allowances would contain reliable real-world information regarding the actual cost of pollution reduction. Since all shadow-pricing algorithms seek to approximate these actual costs, an ideal estimation methodology would resemble market prices of allowances as closely as possible. Thus, because the average market price of SO_2 allowances was equal to roughly $194 in 1999, we can conclude that our GMM methodology produces more accurate estimate of the average shadow price than does the goal programming specification.[53]

Our findings are further supported by the diagrams presented in Figure 4.2 and Figure 4.3. The former demonstrates the variation in the firm-level shadow prices from the two methodologies, whereas the latter contrasts their estimated kernel densities.[54] There are several observations that can be made with respect to

[53] Reliable allowance-price data are reported by the following three private environmental brokerages: Cantor Fitzgerald, the Emission Exchange Corporation, and Fieldston Publications.

[54] The Rosenblatt (1956) estimate of the shadow price density, $\hat{f}(d)$, at a point d for all $k=1,\ldots,K$ is found using $\hat{f}(d)=(Kh)^{-1}\sum_k \Psi\left(\left(\hat{r}-d\right)/h\right)$, where $\Psi(\cdot)$ is a standard normal kernel and h is a bandwidth calculated using a method proposed by Silverman (1986) from $h=0.79RK^{-1/5}$, where R is the interquartile range of the data.

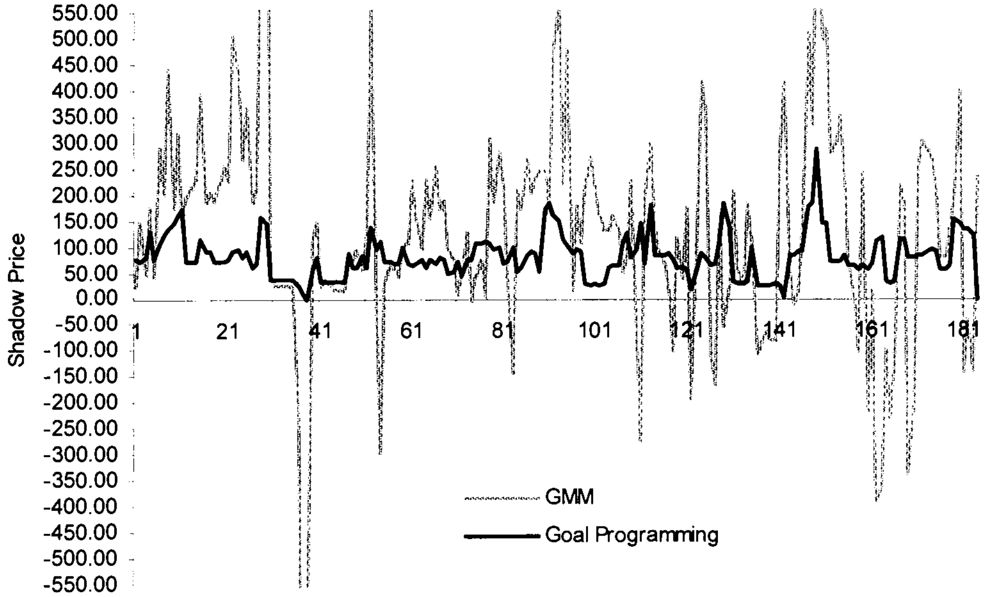

Figure 4.2 **Unit Level Shadow Prices; GMM versus Goal Programming**

the differences between the outcomes of our estimation algorithms. First, the shadow prices from the two models appear to be uncorrelated, since they are associated with the correlation coefficient of only about 0.42. Second, as illustrated in the first plot, our GMM estimates are characterized by a considerable amplitude of variation, which is reproduced via their fairly large variance in the second diagram. Finally, although the density estimate of the GMM shadow prices is not truncated at zero, the fraction of observations associated with negative shadow prices is nevertheless rather small. Along with the robustness of estimation results

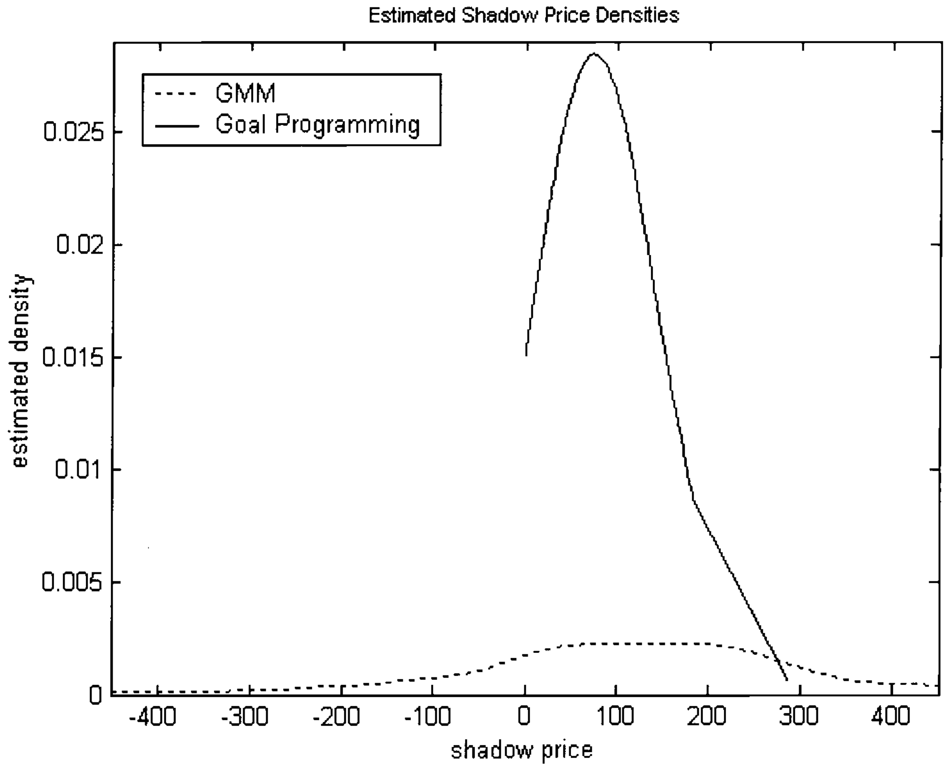

Figure 4.3 Kernel Density Estimates of Shadow Prices

and their closeness to actual costs of pollution reduction, this in effect establishes the legitimacy of the GMM procedure as a choice of an algorithm for the determination of shadow prices of undesirable outputs in parametric models.

4.5 Conclusions

Despite the fact that there are several theories regarding parametric estimation of shadow prices of undesirable outputs, or the "bads," the estimation techniques used in parametric shadow-pricing studies thus far have remained almost the same and, with very few exceptions, rely predominantly on the linear programming algorithm similar to the one that was originally introduced by Aigner and Chu (1968). In this paper we show how a generalized method of moments (GMM) procedure can be used as a suitable alternative to this methodology. The application of several estimation techniques may be useful in cases when various methods can result in potentially different outcomes even when the same data are used. Consequently, such differences may have important implications for making appropriate policy decisions.

We provide an empirical illustration using the 1999 data from the U.S. electricity industry and estimate the directional distance function parameters, which are subsequently used to calculate the firm-level stochastic shadow prices of sulfur dioxide. For comparison, we compute the deterministic goal programming shadow prices as well. Unlike in linear programming models, our specification does not assume that every firm in the sample is efficient and thus allows to account for a probable production inefficiency during the shadow price estimation. We demonstrate that the stochastic estimation of the directional distance functions can be performed for only a limited choice of mapping rules. Our model relies on an expansion of the desirable output that keeps the production of the bad unchanged. The cross-model comparison reveals that the GMM shadow prices are associated with a larger variance and a higher mean than the LP estimates. At the same time, the estimation results are robust, the distance function properties are satisfied for the majority of observations, and the shadow price estimate is very close to the average market price of SO_2 allowances. Hence, our analysis demonstrates that the GMM model represents an adequate and a legitimate choice of an algorithm that can be applied as an alternative to a more commonly used goal programming methodology in parametric shadow pricing models.

Table 4.1
Parameter Estimates of the Directional Distance Function; Various Estimation Methodologies

	GMM	Goal Programming
Intercept	-3.492 (3.295)	30.979
α_1	1.072[b] (0.618)	0.000
α_2	0.783[a] (0.090)	0.865
α_3	-0.023 (0.190)	0.269
γ	0.379[c] (0.242)	0.177
$\gamma_{11}/2$	-0.022[a] (0.001)	–0.003
$\alpha_{11}/2$	-0.122[b] (0.062)	0.000
α_{12}	0.025 (0.021)	0.000
α_{13}	0.069[a] (0.028)	0.000
$\alpha_{22}/2$	-0.001 (0.002)	0.000
α_{23}	-0.011[a] (0.004)	–0.001
$\alpha_{33}/2$	-0.004[c] (0.002)	–0.002
η_1	0.061[a] (0.020)	0.000
η_2	-0.008[b] (0.004)	0.001
η_3	0.007 (0.007)	–0.002
Adjusted R^2	0.993	-
Average Shadow Price Estimate	191.90 (161.86)	80.31 (42.09)

Notes: Asymptotic standard errors are in parentheses.
[a], [b], [c] Significant at 1%, 5%, and 10% in a one-sided test.

Chapter 5

Concluding Remarks

This dissertation addresses various methodological idiosyncrasies associated with the use of distance functions in empirical applications. Chapter 2 assesses the importance of advertising spillovers during the measurement of marketing inefficiency of the decision making units. Chapter 3 looks into the sensitivity of shadow-pricing models to the changes in assumptions regarding the parametric form of production technology. Chapter 4 shows how the generalized method of moments (GMM) can be used to estimate the parameters of the directional output distance functions with bads.

The results discussed in Chapter 2 reveal that advertising spillovers are important in brewing and that efficiency estimates are inaccurate when these spillover effects are ignored. The outcomes of our model suggest that marketing efficiency may be an important component to firm success in brewing, a result that may apply to other consumer goods industries.

Chapter 3 demonstrates that different parametric shadow pricing methodologies produce distinctly dissimilar empirical counterparts of the output set and, as a consequence, can generate rather different estimates of the shadow prices of bads as well. Our results suggest that the estimate of the output set boundary is not invariant to the choice of parameterization technique, which provides a possible explanation for the sizeable differences between the shadow prices of bads reported in the literature. Chapter 3 also demonstrates that among a variety of existing parametric methodologies no single technique is superior to all others.

Finally, the results presented in chapter 4 illustrate that the GMM algorithm can be used as an alternative to linear programming-based methodology in parametric shadow-pricing models. We show that in 1999 the mean GMM shadow price of sulfur dioxide was not significantly different from the average price of pollution permits traded in the market for SO_2 allowances.

Bibliography

Aczél, J', (1966). *Lectures on Functional Equations and Their Applications*, New York: Academic Press.

Aigner, D. and S.-F. Chu (1968). "On Estimating the Industry Production Function," *American Economic Review*, 58 (4), 826-39.

Aigner, D., C. A. Knox Lovell., and P. Schmidt, (1977). "Formulation and Estimation of Stochastic Frontier Production Models," *Journal of Econometrics*, 6, 21-37.

Atkinson, S. E., R. Färe, and D. Primont (2003a). "Stochastic Estimation of Firm Inefficiency Using Distance Functions", *Southern Economic Journal, 69* (3), 596-611.

Atkinson, S. E., C. Cornwell, and O. Honerkamp (2003b). "Measuring and Decomposing Productivity Change: Stochastic Distance Function Estimation Versus Data Envelopment Analysis", *Journal of Business and economic Statistics, 21*(3), 284-294.

Atkinson, S. and J. Dorfman (forthcoming). "Bayesian Measurement of Productivity and Efficiency in the Presence of Undesirable Outputs: Crediting Electric Utilities for Reducing Air Pollution," *Journal of Econometrics*.

Bagwell, K. (2005). "The Economics Analysis of Advertising," working paper, Department of Economics, Columbia University.

Ball, V., V. Eldon, C. A. Knox Lovell, R. Nehring, and A. Somwaru (1994). "Incorporating Undesirable Outputs into Models of Production: An Application to U.S. Agriculture," *Cahiers d'Economie et Sociologie Rurales*, 31, 60-74.

Baumol, W. J., J. C. Panzar, and R. D. Willig, (1982). *Contestable Markets and the Theory of Industry Structure*, New York: Harcourt Brace Javonovich, Inc.

Beer Industry Update: A Review of Recent Developments, Nanuet, NY: Beer Marketer's Insights (various issues).

Boyd, G., J. C. Molburg, and R. Prince (1996). "Alternative Methods of Marginal Abatement Cost Estimation: Nonparametric Distance Function," *Proceedings of the USAEE/IAEE 17th Conference*, 86-95.

Boyd, R., and B. J. Seldon, (1990). "The Fleeting Effect of Advertising: Empirical Evidence from a Case Study," *Economics Letters*, 34 (4), 375-379.

Bresnahan, T. F. (1984). "The Demand of Advertising by Medium: Implications for the Economies of Scale in Advertising," in Ippolito, Pauline M. and David T. Scheffman, (Eds.), *Empirical Approaches to Consumer Protection Economics*, Federal Trade Commission, Washington, DC, 1984, 135-163.

Cantor Fitzgerald Environmental Brokerage, *Allowance Price Indications*, various issues.

Chambers, R. G., Y. Chung, and R. Färe, (1998). "Profit, Distance Functions and Nerlovian Efficiency," *Journal of Optimization Theory and Applications*, 98 (2), 351-364.

Charnes A., W. W. Cooper, and E. Rhodes (1978). "Measuring The Efficiency of the Decision Making Units," *European Journal of Operational Research,* 2, 429-444.

Coelli, S. and S. Perelman (1996). "Efficiency Measurement, Multiple-Output Technologies and Distance Functions: With Application to European Railways," CREPP Working Paper 96/05, University of Liège, Belgium.

Coggins, J. S. and J. R. Swinton (1996). "The Price of Pollution: a Dual Approach to Valuing SO2 Allowances," *Journal of Environmental Economics and Management*, 30 (1), 58-72.

Company/Brand $, *TNS Media Intelligence/Competitive Media Reporting*, Multi-Media Service (various issues).

Chung, Y. (1996). *Directional Distance Functions and Undesirable Outputs*, PhD Dissertation, Southern Illinois University.

Efron, B. and R. J. Tibshirani (1993). *An Introduction to the Bootstrap, Monographs on Statistics and Applied Probability*, vol. 57. New York: Chapman and Hall.

Ellerman, D., P. Joskow, R. Schmalensee, J. P. Montero, and E. Bailey (2000). *Markets for Clean Air: The U.S. Acid Rain Program*, Cambridge: Cambridge University Press.

Emission Exchange Corp., *Exchange Value*, various issues.

Färe, R. & Sung, K. J. (1986). "On Second-Order Taylor's Series Approximation and Linear Homogeneity," *Aequationes Mathematicae*, 30, 180-86.

Färe, R., S. Grosskopf, C. A. Knox Lovell, and C. A. Pasurka (1989). "Multilateral Productivity Comparisons when Some Outputs are Undesirable: a Non-Parametric Approach," *The Review of Economics and Statistics*, 71 (1), 90-98.

Färe, R., S. Grosskopf, C.A. Knox Lovell, and S. Yaisawarng (1993). "Derivation of Shadow Prices for Undesirable Outputs: a Distance Functions Approach," *The Review of Economics and Statistics*, 75 (2), 374-80.

Färe, R. and D. Primont (1995). *Multi-Output Production and Duality: Theory and Applications*, Kluwer Academic Publishers, Boston/London/Dordrecht.

Färe, R., S. Grosskopf, and W. L. Weber (2001). "Shadow Prices of Missouri Public Conservation Land," *Public Finance Review*, 29 (6), 444-60.

Färe, R. and S. Grosskopf (2004). *New Directions: Efficiency and Productivity*, Kluwer Academic Publishers.

Färe, R., S. Grosskopf, B. J. Seldon, and V. J. Tremblay (2004). "Advertising Efficiency and Choice of Media Mix: A Case of Beer," *International Journal of Industrial Organization*, 22 (4), 503-522.

Färe, R. and A. Lundberg (2005). "Parameterizing the Shortage (Directional Distance) Function," Oregon State University Working Paper.

Färe, R., S. Grosskopf, D.-W. Noh, and W. L. Weber (2005). "Characteristics of a Polluting Technology: Theory and Practice," *Journal of Econometrics*, 126, 469-92.

Farr, S., V. J. Tremblay, and C. Horton Tremblay (2001). "The Welfare Effect of Advertising Restrictions in the U.S. Cigarette Industry," *Review of Industrial Organization*, 18 (2), 147-160.

Fieldston Publications, Inc., *Compliance Strategy Review*, various issues, Washington.

Grabowski, H. G. (1977-78). "The Effects of Advertising on the Interindustry Distribution of Demand," *Explorations in Economic Research*, 4 (5), 674-701.

Grosskopf, S., K. Hayes, L. Taylor, and W. Weber (1997). "Budget-Constrained Frontier Measures of Fiscal Equality and Efficiency in Schooling," *Review of Economics and Statistics*, 79, 116-124.

Jung, C. and B. J. Seldon (1995). "The Degree of Competition in the Advertising Industry," *Review of Industrial Organization*, 10 (1), 41-52.

Kerkvliet, J., W. Nebesky, C. Horton Tremblay, and V. J. Tremblay (1998). "Efficiency and Technological Change in the US Beer Industry," *Journal of Productivity Analysis*, 10 (3), 271-288.

Kim, J. Y. (2002). "Limited Information Likelihood and Bayesian Analysis," *Journal of Econometrics*, 107, 175-93.

Kneip, A., B. U. Park, and L. Simar (1998). "A Note on the Convergence of Nonparametric DEA Estimators for Production Efficiency Scores," *Econometric Theory,* 14 (6), 783-793.

Lee, J.-D., J.-B. Park, and T.-Y. Kim (2002). "Estimation of the Shadow Prices of Pollutants with Production/Environment Inefficiency Taken into Account: a Nonparametric Directional Distance Function Approach," *Journal of Environmental Management*, 64, 365-75.

Liu, D. J., and O. D. Forker (1988). "Generic Fluid Milk Advertising, Demand Expansion, and Supply Response: The Case of New York City," *American Journal of Agricultural Economics*, 70 (2), 229-236.

Luenberger, D. G. (1992). "Benefit Functions and Duality," *Journal of Mathematical Economics*, 21 (5), 461-481.

Luo, X., and N. Donthu (2001), "Benchmarking Advertising Efficiency," *Journal of Advertising Research*, 7-18.

Mas-Colell, A., M. D. Whinston, and J. R. Green (1995). *Microeconomic Theory*, New York: Oxford University Press.

McAfee, R. P. (2002). *Competitive Solutions: A Strategist's Toolkit*, Princeton, NJ: Princeton University Press.

McKitrick, R. R. (1998). "The Econometric Critique of Computable General Equilibrium Modeling: The Role of Functional Forms," *Economic Modelling*, 15, (4), 543-73.

Perroni, C. and T. F. Rutherford. (1998). "A Comparison of the Performance of Flexible Functional Forms for Use in Applied General Equilibrium Analysis," *Computational Economics*, 11, (3), 245-63.

Rosenblatt, M. (1956). "Remarks on Some Nonparametric Estimates of a Density Function", *Annals of Mathematical Statistics*, *27*, 642-69.

Scherer, F. M., A. Beckenstein, E. Kaufer, and R. D. Murphy (1975). *The Economics of Muli-Plant Operation: An International Comparisons Study*, Cambridge, MA: Harvard University Press.

Schmalensee, R. (1978). "Entry Deterrence in the Ready-to-Eat Breakfast Cereal Industry," *Bell Journal of Economics*, 9 (2), 305-327.

Shephard, R. W. (1970). *Theory of Cost and Production Functions*, Princeton: Princeton University Press.

Silverman, B. W. (1986). *Density Estimation for Statistics and Data Analysis*, New York: Chapman and Hall.

Sutton, J. (1991). *Sunk Costs and Market Structure: Price Competition, Advertising, and the Evolution of Concentration*, Cambridge, MA and London: MIT Press.

Swinton, J. R. (1998). "At what Cost do We Reduce Pollution? Shadow Prices of SO2 Emissioins," *The Energy Journal*, 19 (4), 63-83.

__________ (2002). "The Potential for Cost Saving in the Sulfur Dioxide Allowance Market: Empirical Evidence from Florida," *Land Economics*, 78 (3), 390-404.

__________ (2004). "Phase I Completed: An Empirical Assessment of the 1990 CAAA," *Environmental and Resource Economics*, 27, 227-46.

Szegedy-Maszak, M. Driven to Distraction, *U.S. News and World Report*, 52-64, April 26 2004.

Tennant, R. B. (1950). *The American Cigarette Industry; a Study in Economic Analysis and Public Policy*, New Haven: Yale University Press.

Tremblay, V. J., and S. Polasky (2001), "Advertising with Subjective Horizontal and Vertical Product Differentiation," *Review of Industrial Organization*, 22 (4), 503-522.

Tremblay V. J. and C. Horton Tremblay (1996). "Firm Success, National Status, and Product Line Diversification: An Empirical Examination," *Review of Industrial Organization*, 11 (2), 771-789.

__________ (2005). *The US Brewing Industry: Data and Economic Analysis*, The MIT Press.

Universal McCann Media Cost Indexes (August 2003). *Universal McCann*, 622 Third Avenue, New York, NY 10017.

Varian, H. R. (1992). *Microeconomic Analysis*, New York: W.W. Norton.