AN ABSTRACT OF THE THESIS OF

Sairam V Pamulapati for the degree of Master of Science in Industrial Engineering presented on February 25, 1997. Title: Computer Aided Tolerance Analysis and Process Selection for AutoCAD.

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The fundamental objective of a design engineer in performing tolerance technology is to transform functional requirements into tolerances on individual parts based on existing data and algorithms for design tolerance analysis and synthesis. The transformation of functional requirements into tolerances must also consider the existing process capabilities and manufacturing costs to determine the optimal tolerances and processes.

The main objective of this research is to present an integrated but modular system for Computer Aided Tolerance Allocation, Tolerance Synthesis and Process Selection. The module is implemented in AutoCAD using the ARX 1.1 (AutoCAD Runtime Extension Libraries), MFC 4.2, Visual C++ 4.2, Access 7.0, AutoCAD Development System, AutoLISP, and Other AutoCAD Customization tools.

The integrated module has two functions:

a. Tolerance analysis and allocation: This module uses several statistical and optimization techniques to aggregate component tolerances. Random number generators are used to simulate historical data used by most of the optimization techniques to

perform tolerance analysis. Various component tolerance distributions are considered (Beta, Normal, and Uniform). The proposed analysis technique takes into consideration the distribution of each fabrication of the component, this provides designers . The proposed tolerance analysis method takes into consideration the distribution of each fabrication process of the assembly. For assemblies with non-normal natural process tolerance distributions, this method allows designers to assign assembly tolerances that are closer to actual assembly tolerances when compared to other statistical methods. This is verified by comparing the proposed tolerance analysis method to the results of Monte Carlo simulations. The method results in assembly tolerances similar to those provided by Monte Carlo simulation yet is significantly less computationally-intensive.

b. Process Selection: This thesis introduces a methodology for concurrent design that considers the allocation of tolerances and manufacturing processes for minimum cost. This methodology brings manufacturing concerns into the design process. A simulated annealing technique is used to solve the optimization problem. Independent, unordered, manufacturing processes are assumed for each assembly. The optimization technique uses Monte Carlo simulation. A simulated annealing technique is used to control the Monte Carlo analysis. In this optimization technique, tolerances are allocated using the cost-tolerance curves for each of the individual components. A cost-tolerance curve is defined for each component part in the assembly. The optimization algorithm varies the tolerance for each component and searches systematically for the combination of tolerances that minimizes the cost. The proposed tolerance allocation/process selection method was found to be superior to other tolerance allocation methods based on manufacturing costs.

Computer Aided Tolerance Analysis and Process Selection for AutoCAD

by

Sairam V Pamulapati

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1.0 INTRODUCTION

1.1 Introduction

Tolerance analysis is receiving renewed emphasis as industry realizes that tolerance management is a key element in their programs for improving quality, reducing overall costs, and retaining market share. The specification of tolerances is being elevated from a menial task to a legitimate engineering design function. The quest for quality has focused attention on the effects of variation on cost and performance of manufactured products. Excess cost or poor performance will eventually show up as a loss of market share. Therefore, the specification of tolerance limits on each dimension and feature of engineering drawings is considered by many to be a vital design function. Tolerance analysis allows one to study the effect that component tolerances have on the output variability of a mechanism or system.

Tolerance stackups or accumulation in assemblies control the critical clearances or interferences (e.g. lubrication paths, bearing mounts) and thus affect the performance and functionality of the assembly. During assembly, parts are selected randomly from the individual populations and put together. The resulting assembly therefore gives a design function which varies depending upon the parts selected and the distribution of the individual parts.

1.2 Importance of Tolerance Technology

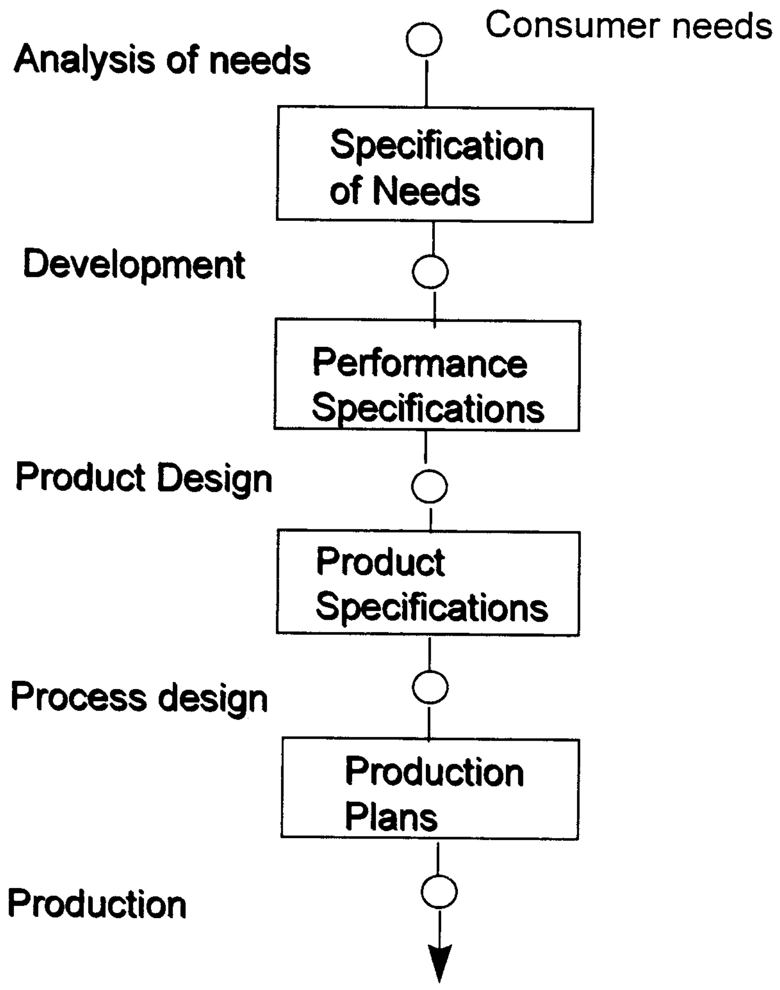


Fig 1.1 Importance of Tolerance Technology

In Fig 1.1 the first phase is the analysis of the consumer needs, leading to the specification of the needs. Then the development follows, resulting in the specifications of the performance of the product. Product specifications are the outcome of product design. Finally, process design results in production plans. Tolerances are determined at the product specifications stage. The product specifications made by the designer consist mainly of form, dimension, material and surface requirements each of which has a basic size and a tolerance.

The fundamental objective of tolerance technology is to transform functional requirements developed during product design into tolerances on individual parts based on systematic utilization of existing and/or priori knowledge of process capabilities, manufacturing costs, experience, handbooks or standard information. In real life situations this information does not apply for all kinds of manufacturing environments and therefore is seldom accurate. Many benefits would extend from the use of an interactive computerized procedure to aid designers in transforming functional requirements into tolerances such as time savings, improved quality of design, and, ultimately, reduced product cost.

1.3 Present computer applications in tolerance analysis

AnvilTOL is a tolerance analysis software application which utilizes an Anvil 5000 CAD database to perform interactive, computer aided linear tolerance analysis. AnvilTOL does not implement advanced methods of tolerance analysis(e.g. Monte Carlo simulation and Method of Moments), tolerance allocation or process selection and AnvilTOL does not consider non-normal distributions. AnvilTOL is implemented in GRAPL-IV programming language, which limits the application to only ANVIL CAD systems and if the GRAPL-IV language is changed in subsequent versions of ANVIL 5000, AnvilTOL may be rendered incompatible.

Mechanical Advantage, Analytix, DesignView and Mechanical Engineering Workbench follow the dimension-driven approach. All these packages are similar in that they are 2-D CAD systems. Mechanical Advantage and Analytix both perform linearized worst-case and statistical analysis. Both packages support only normal distributions, however actual manufacturing processes are rarely normal in their behavior. An assumption that each of the produced dimensions will be normally distributed is likely to give results that are highly optimistic.

Main disadvantages of the above mentioned approaches are

1. do not consider non-normal distributions
2. do not consider models other than worst case and root sum square model
3. tolerance allocation is not at all considered in any of the above mentioned software's
4. manufacturing cost considerations are ignored too.

1.4 Software approaches to tolerance analysis

Their are two main approaches to Computer aided tolerance analysis:

- Declarative modeling
- Procedural modeling

1.4.1 Declarative modeling

In declarative modeling the modeling system builds up a declarative representation of each of the geometric elements of the model (face, edges, and vertices). Typical declarative model is represented as a collection of geometric elements. And 2-D Declarative model is composed of just edges and vertices. The edges are defined relative to the vertices (a line segment connecting two vertices). The Variational coverage of the model does not depend on the way the model is defined. The user creates a sketch of the model using point. line and curve primitives. After the model is created the user adds dimensions. The model variables are the coordinates of the vertices and other defining points. The dimensions define constraint equations on the model variables.

In declarative modeling strategy the model does not retain any information about the sequence of operations used in its initial construction and therefore is less dependent on the choices made by the user.

1.4.2 Procedural modeling

A CAD model is an idealization which represents certain geometric properties at an ideal instance. However a variational model represents a collection of different instances of a part or an assembly. In procedural modeling the modeling system builds up a step-by-step procedure for constructing each of the geometric elements of the model.

Procedural approach to tolerance analysis:

1. the user defines a procedural model.
2. the user specifies a procedure for computing a particular design function of interest from the procedural model.
3. Finally, the software uses the procedural model to help analyze the design function.

Feature modeling is characterized by the parameters of location and shape. Feature based model offers similar characteristics to a CSG model. The part model is defined by performing a number of feature-forming operations in a well defined sequence. The Variational coverage of a feature based model is determined by the choice of features and by the sequence of in which they are applied. The user defines a model, specifies a procedure for computing a particular design function and finally tolerance analysis is performed.

1.5 Overview of the thesis

The three components considered in this thesis are

1. Tolerance analysis
2. Tolerance allocation.
3. Process selection

1.5.1 Tolerance analysis

Tolerance analysis is performed when the component natural process tolerances are known and the design tolerance of the assembly component needs to be calculated. Tolerance analysis should not only determine if the given tolerance specifications are adequate to meet the functionality of the product, but also should give guidance as to where the tolerance specifications can be tightened and where the specifications can be relaxed. Tolerance analysis allows one to study the effect component tolerances have on the output variability of a mechanism or system.

Advanced statistical tolerance methods can give much better estimates of the number of rejects than simple statistical tolerance analysis, when the component distributions are well known non-normal distributions. Non-symmetric and non-normal distributions are important to consider as naturally occurring shifts in a process can produce biased distributions, which result in increased assembly problems and a greater percentage of rejects than anticipated. This section discusses Monte Carlo and Method of Moments tolerance analysis model and proposes a tolerance analysis model using method of moments in conjunction with Monte Carlo model to overcome some of the disadvantages of Monte Carlo simulation and Method of Moments.

1.5.2 Tolerance allocation

One of the issues that design engineers commonly face is the problem of tolerance allocation rather than tolerance analysis. In tolerance allocation the assembly design tolerance is known and the component natural process tolerance are to be determined. In addition to the tolerance allocation models found in literature, this paper proposes a tolerance allocation model which considers non-normal distributions and natural process tolerance of the individual components.

1.5.3 Process selection

Components can be manufactured with different processes and different costs. Each process is optimal only at certain tolerance range. Therefore tolerances must be allocated along with the manufacturing process if costs are to be minimized. In this thesis simulated annealing optimization technique is implemented for process selection.

1.6 Terminology and definitions

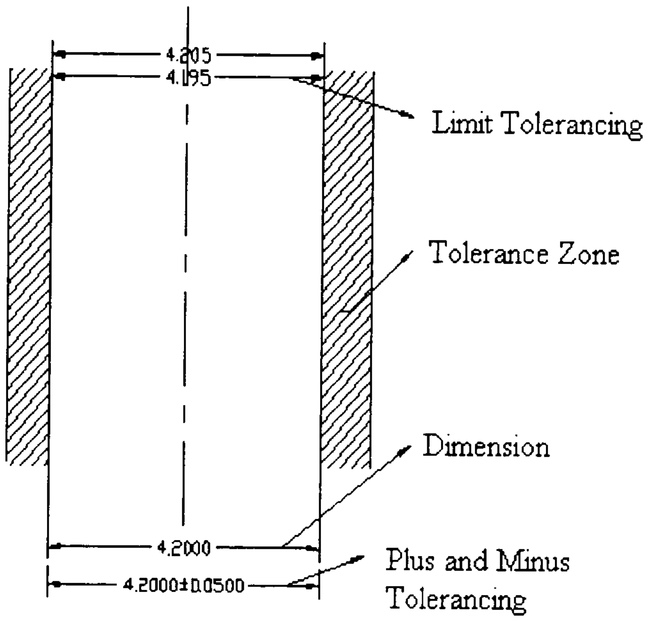


Fig 1.2 Terminology of tolerance analysis and allocation

Dimension: The nominal value of each component

Tolerance: Tolerance is the total amount by which a specific dimension is allowed to vary. Geometric tolerance is a general term applied to the category of tolerances used to control form, profile, orientation, location, runout, and so on. Tolerance of size and tolerance of form covers the location of geometric features and geometric properties like concentricity runout and straightness

Design function: A mathematical relationship which defines the assembly variable in terms of component variables.

Tolerance of size can be stated in two different ways:

Plus or minus tolerancing: Plus or minus tolerancing can be represented in two different ways bilateral and unilateral tolerancing.

Limit tolerancing: This type of the tolerancing is a variation of plus or minus system. It states actual size boundaries for the specific dimension. This eliminates any calculation on the part of the manufacturer.

Natural process tolerance: The natural process tolerance is defined as the maximum range of variation permissible for the size of a dimension in a particular process

Assembly design tolerance: The design tolerance requirement for proper functionality of the assembly component.

2.0 PROBLEM STATEMENT

2.1 Problem Statement

Even when all manufactured parts for an assembly are produced within limits, some parts may not function properly due to inadequate or erroneous tolerance analysis. Design engineers often assign tolerances arbitrarily mainly due to insufficient data, time consuming or incompatible tolerance analysis models. All tolerance analysis models may not be applicable for all assemblies in all situations due to the variations (mean shift) and uniqueness (process distributions) of manufacturing processes.

Design tolerances are often interrelated, and contribute to a given assembly tolerance of the design. These design tolerances specify various mechanical features, and the features are manufactured using different production processes. Production processes, however, have different production cost-tolerance relations due to the uniqueness and variations of the manufacturing conditions. The sensitivity of total production cost with respect to each tolerance depends on the tolerance and the production process used for forming the feature. Simply put, the problem is to identify the best combination of the interrelated design tolerances that satisfies the stack-up constraint and design requirements of the assembly leading to the least production costs.

Main reasons for computer aided tolerance analysis are:

1. Insufficient data or incomplete models
2. Arbitrary assignment of tolerances.
3. Tedious and time consuming calculations
4. Tolerances are largely concerned with the geometry of parts
5. CAD systems lacking tolerance representations cannot support many design and manufacturing activities that require tolerance representations.

The fundamental objective of tolerance technology is to transform functional requirements into tolerances on individual parts based on a systematic utilization of existing a priori knowledge of process capabilities and manufacturing costs or experience, handbooks and standard information.

In real life situations this information does not apply for all kinds of manufacturing environments and, therefore, is seldom accurate and use of statistical procedures in determining tolerances are time consuming. The transformation of functional requirements into tolerances should be done by an interactive computerized procedure by which the computer calculates and the designer makes the decisions.

2.2 Objectives

During the design of mechanical components and assemblies, mechanical tolerances are specified in conjunction with part geometry, material type and other technical specifications. These tolerances are used to ensure the expected assembly design function, and are used to provide guidelines for manufacturing the parts. However assigning proper tolerances requires that the following major objectives be met.

1. the design tolerances must satisfy a given set of design requirement
2. satisfy the stackup constraint of its assembly and
3. meet the design requirements and assembly constraints while minimizing production costs.

The primary objective of this thesis to develop an interactive, computerized software to aid designers in transforming the design requirements into tolerances which will result in:

- Improved tolerancing with respect to both product performance and cost.
- Designer time savings

The proposed approach gives the user options to perform various methods of tolerance analysis/allocation on existing AutoCAD drawings. This allows the user to choose attributes (e.g. process distribution, natural process tolerance etc.) and tolerance analysis models appropriate to the manufacturing conditions for each assembly component.

The two main modules considered are

- Tolerance analysis
- Tolerance allocation

Tolerance analysis is the applied when the natural component tolerance's are known and the assembly design tolerance needs to be calculated and on the other hand tolerance allocation is performed on assemblies when assembly design tolerance is known and the component natural process tolerance needs to be calculated.

Several models for tolerance analysis and allocation models are reviewed in the following chapters. Improvements for tolerance analysis and tolerance allocations models are proposed and implemented in the software.

3.0 TOLERANCE ANALYSIS

3.1 Introduction to tolerance analysis

Tolerance analysis is performed when the natural process tolerances of the components parts are known and the design tolerance of the assembly component needs to be calculated. A good tolerance model should predict assembly tolerance close to actual assembly tolerance limits, minimizing rejects and/or scrap.

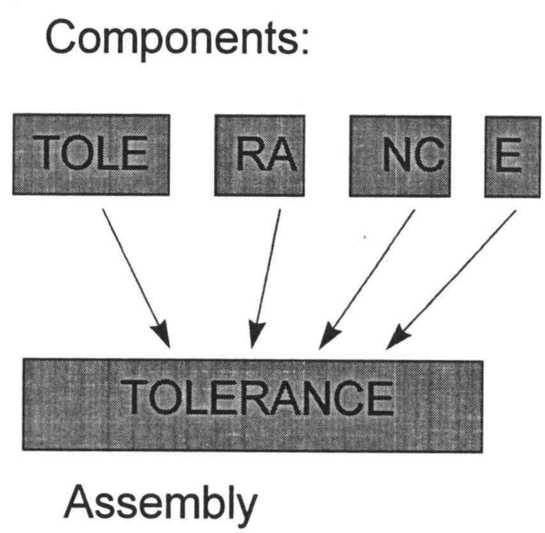


Fig 3.1 Tolerance Analysis

Tolerance analysis should not only determine if the given tolerance specifications are adequate to meet the functionality of the product, but also provide guidance as to where the tolerance specifications must be tighter and where the specifications can be relaxed. Tolerance analysis allow one to study the effect the component tolerances have on the output variability or the assembly tolerance of a mechanism or system.

A number of tolerance models exist with different levels of sophistication. The most common models for predicting the sum of component tolerances in an assembly are Worst case and root sum squares models.

3.2 Literature review

The following tolerance analysis models have been cited in literature and each model has some advantages and disadvantages

1. Worst case model
2. Statistical model (root sum square model)
3. Mean shift model
   a. Chase and Greenwood model
4. Advanced tolerance analysis models
   a. Monte Carlo model
   b. Method of moments

3.2.1 Worst case model

The assembly tolerance for worst case is

$$T_{asm} = \Sigma t_i \text{ Where i = 1,2, ... i components} \quad (1)$$

where T_{asm} is the assembly tolerance and t_i are component tolerances. The worst case tolerance analysis guarantees satisfaction of the specified assembly tolerance with 100% probability, for any distribution. The worst case model makes no assumption about the parts falling outside the tolerance range This results in large calculated assembly tolerance. Therefore to meet the functionality of the assembled component, the component are allocated tighter tolerances.

3.2.2 Root sum square model

The statistical model calculates the assembly tolerance by taking the root sum square of the component tolerances.

$$T_{asm} = \sqrt{\sum (t_i)^2} \quad \text{Where i = 1,2, ... i components} \qquad (2)$$

where T_{asm} is the assembly tolerance and t_i are component tolerances. Tolerances are commonly assumed to correspond to $\pm 3\sigma$ (where σ denotes standard deviation). When the tolerance limits are $\pm 3\sigma$, there are 2.7 components for one thousand components which do not conform to the specifications. Root sum square model assumes the components natural process tolerance follow normal distribution. For symmetric distributions the fraction of rejects is small but for asymmetric component distributions the fraction of rejects may be very high due to the mean shift.

3.2.3 Mean shift

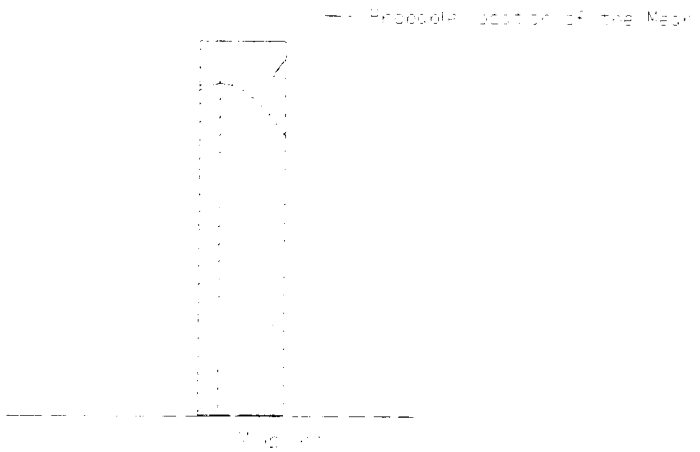


Fig 3.2 The location of the mean is not known precisely

In the real processes the mean of the distribution may be shifted away from the nominal dimension due to various reasons. The mean shift can occur from tooling or fixture errors, setup errors or tool wear or it may be deliberately introduced during setup to compensate to tool wear or to allow for rework. At early design stages the mean shift of the component distribution is difficult to determine because detailed data about mean shifts or distributions of the component is not available.

Mean shift tolerance model by Chase and Greenwood:

This mean shift model calculates the assembly tolerance

$$T_{asm} = \sum m_i t_i + \left(\frac{Z}{3}\right)\sqrt{\sum (1-m_i)^2 t_i^2} \qquad (5)$$

Z is the number of standard deviations desired for the specified assembly tolerance. And $m_i = \frac{\text{mean shift}}{t_i}$ = mean shift factor. The mean shift factor is expressed by Chase and Greenwood as a fraction of the specified tolerance range for the part dimension. Factors ranging between 0 and 1.0 have been suggested. It assumes a 3σ statistical variation in process tolerance from the specification limit. The mean shift factor is expressed as a fraction of the specified tolerance range for the part dimension (between 0 to 1.0). Mean shift factor for a tightly controlled process is assigned 0.1 to 0.2 and for less well known processes (e.g. supplied or contracted parts) a large factor of 0.7 to 0.8 is assigned. When the mean shift factor is 1, the assembly tolerance calculated is same as the value obtained by Worst Case model and on the other hand when the mean shift factor is 0 then the assembly tolerance calculated is same as the value obtained by Root sum square model

3.2.4 Advanced statistical analysis model

Advanced statistical tolerance methods can give much better estimates of the assembly tolerance range than simple statistical analysis, when the component distributions are well known non-normal distributions. Non-symmetric and non-normal distributions are important to consider as naturally occurring shifts in a process can produce biased distributions, which result in increased assembly problems and a greater percentage of rejects than anticipated.

3.2.4.1 Monte Carlo model

Monte Carlo simulation uses pseudo-random number generators to describe a wide variety of distribution shapes. A random dimension for each component is input into the assembly function. The value of the resultant assembly variable is determined. The procedure is described below:

a. Generate a random value for each of the assembly components' according to its user supplied distribution.
b. Evaluate the assembly corresponding to these values.
c. Compute the design function.

Design function is the mathematical relationship which defines the assembly tolerance in terms of the component tolerances. In tolerance analysis, the permissible rejection fraction is usually quite small and large samples on the order of 10,000 or 100,000 are required for accurate prediction of assembly range.

$T = X_{max} - X_{min}$

where T = tolerance of the assembly.

X_{max} = upper limit of the tolerance range

X_{min} = lower limit of the tolerance range

Where X_{max} and X_{min} are the upper and lower limits of the assembly tolerance range resulting from the Monte Carlo simulation. This model is particularly good at handling skewed distributions. However, before the Monte Carlo simulation can be performed, complete information about component tolerances distributions must be known. The computer time used for simulation is extremely long if an accurate result is desired. For special applications, Monte Carlo simulation is a very useful tool for modeling complex situations such as tolerance analysis in actual assembly operations where the product as well as process accuracy are very important (e.g. robot assembly). Monte Carlo requires advance knowledge of the component distributions. And Monte Carlo simulation produces assembly tolerance distributions very close to the actual.

3.2.4.2 Method of moments

The method of moments uses the statistical moments of the component distributions and the first and second derivatives of the assembly function to find the first four moments of the assembly distribution. These four moments are used to find the parameters of a general distribution such as the Pearson system, the Johnson system. With the parameters of a distribution determined, the fraction outside of the assembly limits can be found from statistical tables, numerical integration, or in some cases by algebraic equations. Tolerance analysis by Method of moments will be quite long and complex due to the need for numerical derivatives in most cases and the many series summations to get the assembly moments.

$$X_{max} = M + 3D$$

$$X_{min} = M - 3D$$

$$M = \Sigma m_i$$

$$D = \sqrt{(\Sigma \sigma_i^2)}$$

$$T = X_{max} - X_{min}$$

Where M = assembly mean tolerance

m_i = the ith component mean tolerance

σ_i = standard deviation of the ith component tolerance

X_{max} = component maximum dimension

X_{min} = component minimum dimension

In this model the first two moments are used, and the assembly tolerance is assumed to be normally distributed. For the moment model to be used, the mean and the standard deviations of each component tolerances distribution must be known beforehand. Like the Monte-Carlo model, the reject problem for non-normal and skewed distributions has been greatly improved by using the moment model.

3.3 Calculations for existing tolerance analysis models

Fig 3.3 is used for illustrating the different tolerance analysis models algorithms. The example is a step shaft. The assembly tolerance (Tasm) is the resultant of the stack of the three features.

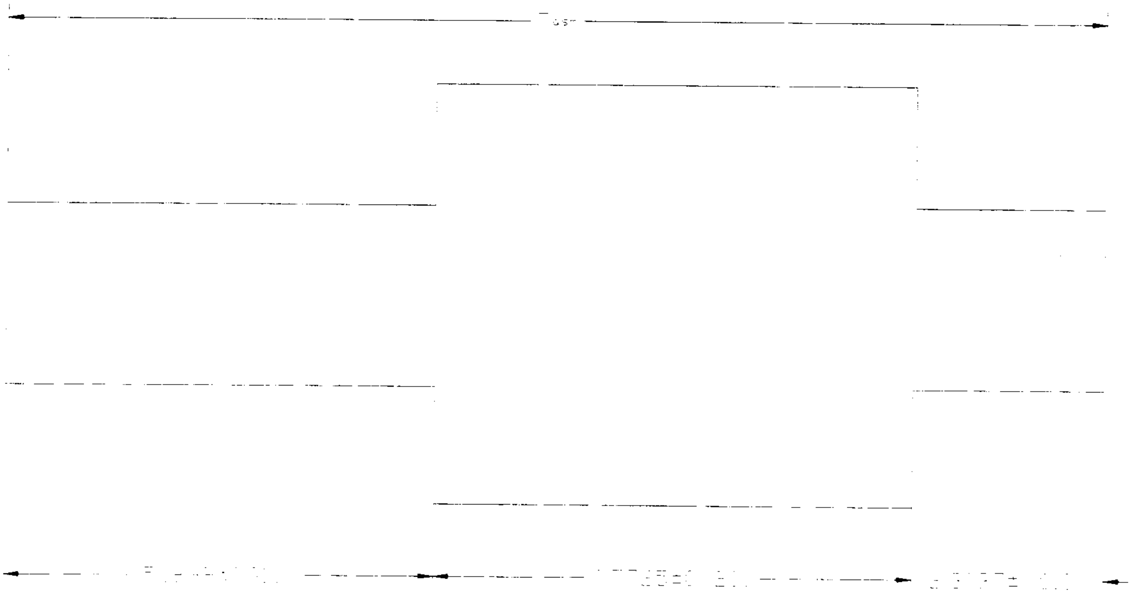


Fig 3.3 Example for Tolerance Analysis

<u>Worst case model:</u>

Step 1: Select the component features which effect the assembly design tolerance

Step 2: Apply the Worst case analysis model to the selected component features

Formula: $T_{asm} = \Sigma t_i$

Tasm = 0.02 + 0.04 + 0.02

= 0.08

Root Sum Square model:

Step 1: Select the component features which effect the assembly design tolerance

Step 2: Apply the root sum square analysis model to the selected component features

Formula: $T_{asm} = \sqrt{\sum (t_i)^2}$

$T_{asm} = Sqrt[(0.02)^2 + (0.04)^2 + (0.02)^2]$

$T_{asm} = 0.049$

Mean shift model by Chase and Greenwood:

Step 1: Select the component features which effect the assembly design tolerance

Step 2: Determine the mean shift factors for each of the features

Step 3: Apply the Mean shift model

Formula: $T_{asm} = \sum m_i t_i + \left(\frac{Z}{3}\right)\sqrt{\sum (1 - m_i)^2 t_i^2}$

If mean shift factors for the assembly $m_1 = 0.2$, $m_2 = 0.8$, and $m_3 = 0.5$

Tasm = $[m_1 * t_1 + m_2 * t_2 + m_3 * t_3] +$

$(3/3) * Sqrt \{[(1- m_1)^2 * t_1^2 + [(1- m_1)^2 * t_1^2 + [(1- m_1)^2 * t_1^2\}$

$= (0.2 * 0.02 + 0.8 * 0.04 + 0.5 * 0.02) +$

$Sqrt\{(1-0.2)^2*0.02^2 + (1-0.8)^2*0.04^2 + (1-0.5)^2*0.02^2\}$

= 0.046 + 0.021

= 0.067

Method of moments:

Step 1: Select the component features which effect the assembly design tolerance

Step 2: Find out the moments for each of the components. This can be done by finding the natural process tolerance distribution for each component.

Step 3: Apply the Method of moments model

$$X_{max} = M + 3D$$

$$X_{min} = M - 3D$$

Monte Carlo Simulation:

Step 1:The design function for the assembly is defined by the user.

Step 2: Each component feature is assigned a distribution (Default: normal distribution)

Step 3: A random value for each of the assembly components' is generated using unit random number generator (Mean = 0, Range = 1) according to its user supplied distribution.

Step 4: Evaluate the assembly corresponding to these values.

Step 5: Compute the design function.

Step 6: The above procedure is iterated for 10000 times

Step 7: The range of the assembly tolerance is found out by finding out the range of the assembly tolerance values obtained during the simulation.

3.4 Disadvantages of existing tolerance analysis models

While the Worst Case model is too conservative, the Root Sum Squares model generally predicts too few rejects compared to real assembly processes. This is due to the fact that the Root Sum Square model uses normal distribution and normal distribution is only an approximation of the true distribution which may be flatter or may be skewed than the classic bell shape of the normal curve. The mean of the distribution may also be shifted from the midpoint of the tolerance range.

The common models for assembly tolerance accumulation have distinct limitations when applied to tolerance analysis:

- The Worst Case model results in component tolerances which are tight and costly to produce. Statistical models allow looser tolerances, but often predict higher assembly yields than actually occur in production.
- Statistical models assume manufacturing variations follow a normal or classic bell-shaped distribution, symmetrically positioned at the midpoint of the tolerance limits.
- They do not take into account possible skewness or bias which is common in manufacturing processes.
- Time taken for performing Monte Carlo simulations is very large for assemblies with more than 10 components

Bias results in a shift in the nominal dimension. It is particularly harmful, since it can accumulate in an assemblies and cause unexpectedly high rejection rates. Bias can occur from tooling or fixture errors, setup errors, or tool wear. Mean shift model address some of the issues of bias but it does not take care of skewness of the component process distributions. Advanced statistical methods are used for tolerance analysis because they permit non-normal distributions. These methods can give much better estimates of the number of rejects than simple statistical analysis, when the component distributions are well-known non-normal functions.

3.5 Objectives to improve tolerance analysis models

Main objectives for an optimal tolerance analysis model are

- Consider non-normal distributions
- Perform tolerance analysis in less time than Monte Carlo simulation.

<u>Perform tolerance analysis in less time than Monte Carlo simulation:</u>

Monte Carlo simulation requires sample sizes in order of 10,000 or 100,000 are required to perform accurate calculations of assembly range and therefore requires a lot computation time and if Monte Carlo analysis is not run with enough samples, the results may be inaccurate. The time taken to perform tolerance analysis using Monte Carlo simulation depend on the number of components in the assembly and if there are more than 10 components in the assembly then the time taken to perform Monte Carlo simulation is very large.

Non-normal distributions:

Non-normal distributions are important to consider in tolerance analysis because of the random deviations inherent in the process. The dimensions on a part, resulting from machining, are dependent on the probability distribution of the process. The probability distribution of a process is a mathematical model that approximately represents the empirical distribution we would observe if we plot a large number of dimensions machined. The probability distribution of a process is influenced by random deviations in the process. Some of the important factors influencing the process are

- Tool life
- Machine tool reconditioning period
- Operator
- Machine tool life

Tool wear changes the process, both with respect to positioning and with respect to scatter. The change in temperature of the machine tool may change the workpiece dimensions. A lower frequency variation is caused by wear of the machine tool itself. The process distribution might also influenced by the skill level of the operator. Finally the machine tool gradually deteriorates during its lifetime, and the deviations are caused by this deterioration. Due to above reasons the mean of the process may shift or skew changing the process distribution from normal distribution. Therefore it is important to consider distributions other than normal to truly represent the natural process distributions of components.

Distribution	Advantages	Disadvantages
Normal	Ease of use	• The model has infinite range. • Cannot cover asymmetric sum dimensions • Cannot cover confidence levels in the neighborhood of 100%
Truncated Normal	Covers 100% confidence limits	Cannot cover asymmetrical cases
Beta	• Covers the actual distribution from normal to rectangular • Finite range. • Covers asymmetrical cases	More computation time.

Table 3.1 Advantages and disadvantages of various distributions

These distributions have been used in the simulation because they represent most of the typical distributions of mechanical components tolerance. The truncated normal distributions is non-normal and symmetric and the Beta distribution is non-normal skewed (Bjorke ,1989) . The probability distributions of processes are not limited to above distributions.

3.6 Improved Monte Carlo model

The two advanced statistical methods have advantages and limitations. The Monte Carlo method predicts an assembly distribution close to the actual assembly tolerances distribution. However, Monte Carlo simulation requires sample sizes in order of 10,000 or 100,000 to calculate assembly tolerance range and therefore requires a lot computation time. Monte Carlo analysis will result in inaccurate assembly tolerance range if the simulation is not run with enough samples. The Method of moments requires prior knowledge of the moments of every component in the assembly and is complex and is quite computationally intensive due to the need for numerical derivatives to calculate the third and fourth moments (i.e. skewness and kurtosis) in most cases.

3.6.1 Improved Monte Carlo model

The above mentioned shortcoming can be over-comed by blending Monte Carlo simulation and Method of Moments. This paper proposes a model which requires a moderately complicated program with moderate computation time. In this model Monte Carlo simulation is used to generate a smaller number of assembly values. The Monte Carlo simulation is used to create 1000 sample assembly tolerance values to calculate T_{asm}, which are then used to calculate the moments (mean and standard deviation) of the assembly tolerance distribution:

$$T_{asm} = \sum_{i=0}^{n} f(T_i * R_i)$$

$$M = 0.001 \sum_{i=0}^{1000} (T_{asm})_i$$

$$D = (0.001 \sum_{i=0}^{1000} (T_{asm})^2{}_i - M^2)^{0.5}$$

$X_{max} = M + 3D,$

$X_{min} = M - 3D,$

$T_{asm} = X_{max} - X_{min}$

Where T_{asm} = assembly tolerance

D = Deviation or 1 sigma limit

T_i = Component tolerance

f = design function

n = number of components in the design function

R_i = random number with a mean of 0 and range of 1

X_{max} = upper limit of the tolerance range

X_{min} = lower limit of the tolerance range

Most of the complexity of the Method of Moments is eliminated since the moments of the assembly tolerance are calculated from the Monte Carlo simulation data. And since the sample size can be on the order of 1000 to 5000, the computation is greatly reduced from the simple Monte Carlo simulation. The unit random number generators result in the random numbers depend on the user defined distribution and the probability density function of the distribution and do not generate uniform random numbers. The computation time is decreased due to smaller sample size and complexity is also reduced due to calculation of moments by values obtained by Monte Carlo simulation

3.6.2 Software modeling

The tolerance analysis by modified tolerance analysis is performed by the following steps.

- The assembly component drawing is opened in AutoCAD and the tolerance data for each feature is taken from the drawing.
- The design function for the assembly is defined by the user.
- Each component feature is assigned a distribution (Default: normal distribution)
- A random value for each of the assembly components' is generated using unit random number generator (Mean = 0, Range = 1) according to its user supplied distribution. The unit random number generators are not uniformly distributed.
- Evaluate the assembly corresponding to these values. And compute the design function.
- The above procedure is iterated for 1000 times
- Finally moments of the assembly are calculated and the design tolerance range of the assembly is found out using the moments of the assembly.

3.6.3 Model Verification

For the Fig. 3.3 modified Monte Carlo simulation is performed.

The tolerance data: $t_1 = 0.02$, $t_2 = 0.04$, $t_3 = 0.02$

Design function: $t_1 + t_2 + t_3$

Process distribution: Normal, Normal, Normal

Random values from random number generators:

Iterations	Component 1	Component 2	Component 3
1	0.8	0.8	0.8
2	0.3	0.3	0.3
3	-0.2	-0.2	-0.2
4	-0.21	-0.21	-0.21
5	-0.8	0.8	-0.8

Table 3.2 Sample values generated by the random number generator

Evaluate design function: The design function for the assembly in Fig. 3.3: $t_1 + t_2 + t_3$

$T_{asm} = t_1 * r_1 + t_2 * r_2 + t_3 * r_3 = 0.02 * 0.8 + 0.04 * 0.8 + 0.02 * 0.8$

T_{asm} = 0.064 for Iteration 1

T_{asm} = 0.018 for Iteration 2

T_{asm} = 0.052 for Iteration 3

T_{asm} = 0.0546 for Iteration 4

T_{asm} = 0.064 for Iteration 5

Mean (M) = [0.064 + 0.018 + 0.052 + 0.0546 + 0.064] / 5 = 0.0502 and S D: 0.02

Tolerance range:

Upper limit = M + 3D = 0.0502 + 0.06 = 0.1102

Lower limit = M - 3D = 0.0502 - 0.06 = - 0.0098

T_{asm} = 0.12 (The assembly tolerance is inaccurate because of the small number of iterations.). The above calculations are performed only to illustrate the procedure for performing Modified Monte Carlo simulation.

Results from computer software:

The tolerance data: $t_1 = 0.02$, $t_2 = 0.04$, $t_3 = 0.02$ and Design function: $t_1 + t_2 + t_3$

Number of Iterations: 5000, T_{asm} = 0.049

3.7 Comparison of Tolerance Analysis

Tolerance analysis comparison is based on the shaft and bearing assembly shown in Fig 3.4. The shaft and bearing assembly consists of

a. Retaining ring: The retaining ring holds the ball bearings and shaft in place
b. Housing: The housing encompasses the shaft and bearing assembly
c. Ball bearing: Ball bearing aid in the free movement of the shaft
d. Shaft: The shaft
e. Bearing Sleeve: Bearing sleeve holds the housing and bearing together
f. Clearance: The clearance between retaining ring and ball bearing. And the clearance is the assembly design tolerance because the proper functioning of the shaft and bearing assembly depends the clearance. If the clearance is negative, their is interference between retaining ring and shaft and the assembly component does not function properly on the other hand if their is too much clearance then their is radial runout and assembly component is subjected to more wear and tear.

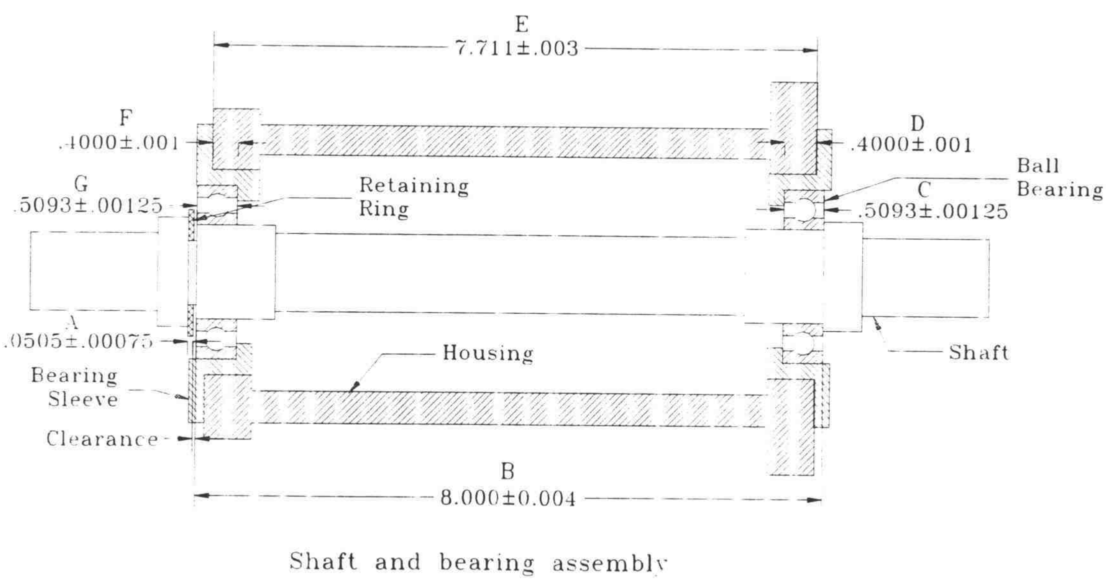


Fig 3.4 Shaft and bearing assembly

Dimension and tolerance information of the shaft and bearing assembly:

Dimension	A	B	C	D	E	F	G
Average	0.505	8.0	0.509	0.40	7.711	0.40	0.509
Tolerances (Design)		0.008		0.002	0.006	0.002	
Tolerances (Fixed)	.0015		0.0025				0.0025

Table 3.3 Dimension and tolerance data for the shaft and bearing assembly

Tolerance analysis using Normal distribution:

Comparison of tolerance analysis using the shaft and bearing assembly shown in Fig 3.4. Five tolerance analysis models are applied and the results are tabulated in Table 3.3. All the component's natural process tolerance follow normal distribution. The assembly tolerance range calculated by Hybrid model is equal to that of Statistical model because Root Sum Square model assumes normal distribution.

	Mean	Lower limit	Upper limit	**Tolerance range**	**Time (CPU) ms**
Actual (Monte-Carlo)	0	-0.0135	0.0137	**0.0258**	**428,502**
Worst-case	0	-0.035	0.035	**0.07**	**0.25**
Statistical	0	-0.0133	0.0132	**0.0265**	**0.59**
Mean shift	0	-0.0133	0.0132	**0.0265**	**0.6**
Hybrid	0	-0.0133	0.0132	**0.0259**	**9,028**

Table 3.4 Tolerance analysis for the shaft and bearing assembly (All components' natural process tolerance follow normal distribution)

The Monte-Carlo simulation is done by generating 50,000 tolerance values and the range of the distribution gives the upper and lower limits. Hybrid values are obtained by generating only 1000 iterations and the lower limit is -3σ value and upper limit is 3σ value of the resultant tolerance values. The design tolerance range calculated by Worst case model is very large. The assembly tolerance values obtained by Monte-Carlo simulation, Statistical, Mean shift and Hybrid model are approximately equal because all the assembly components follow normal distribution.

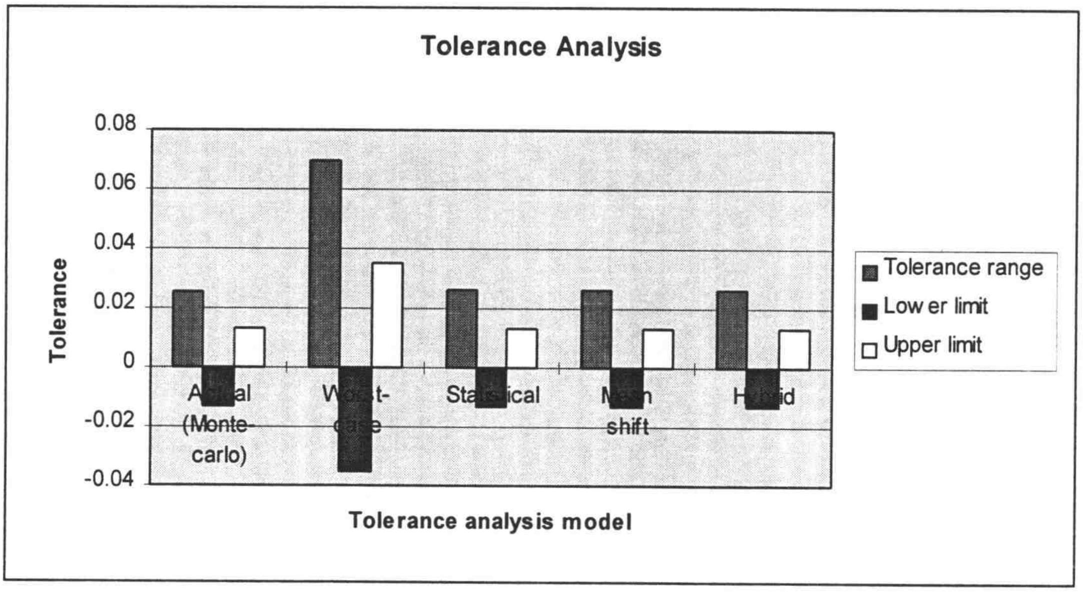


Fig 3.5 Chart showing the results of assembly tolerance applied to the shaft and bearing assembly (All components' natural process tolerance follow normal distribution)

Tolerance analysis using Beta distribution:

Comparison of tolerance analysis using the shaft and bearing assembly is shown in Fig 3.6. Five tolerance analysis models are applied and the results are tabulated in Table 3.5. The unit random number generators result in the random numbers depend on the user defined distribution and the probability density function of the distribution and do not generate uniform random numbers. All the component's natural process tolerance are assumed to follow beta distribution. The mean shift tolerance analysis model considers the mean shift but does not consider the skewness of the distribution. The tolerance range calculated by Monte-Carlo simulation is different than that of Statistical model because Statistical model does not consider mean shift or the skewness and therefore results in erroneous assembly tolerance values. Monte Carlo simulation considers skewness and mean shift but at the expense of lot of computational power. Whereas hybrid method gives approximately same results with less computational effort. The hybrid model calculates the assembly tolerance approximately equal to Monte-Carlo simulation but at less than **43** times the time taken by Monte Carlo simulation. Although the range of statistical model and Modified Monte Carlo model are same, the statistical model does not consider the mean shift of the assembly.

Statistical model assumes the mean and median of the assembly design tolerance will coincide but when the components' natural distribution do not follow normal distribution, the mean and median of the assembly may not coincide, resulting in mean shift.. In the Fig 3.4 the mean of the assembly design tolerance is shifted by **0.0143** which is predicted by both Monte Carlo simulation and hybrid model. Statistical model does not consider mean shift. Mean shift model incorporates the mean shift by increasing the range of the assembly design tolerance by increasing the range, which may result in improper functionality of the assembly because of too much clearance for the assembly component. Monte Carlo simulation and hybrid model shift the range of the assembly design tolerance instead of increasing the assembly design tolerance range. The beta factors for generating unit beta random numbers for assembly components are graphed in Appendix E.

	Mean	Lower limit	Upper limit	**Tolerance range**	**Time (CPU) ms**
Actual (Monte-Carlo)	-0.0143	-0.025	-0.0009	**0.0241**	**427,072**
Worst-case	0	-0.035	0.035	**0.07**	**0.25**
Statistical	0	-0.0133	0.0132	**0.0265**	**0.59**
Mean shift	0	-0.0221	0.0221	**0.0443**	**0.6**
Hybrid	-0.0143	-0.0266	-0.0021	**0.0245**	**8,568**

Table 3.5 Tolerance analysis for the Shaft and bearing assembly (All components' natural process tolerance follow Beta distribution)

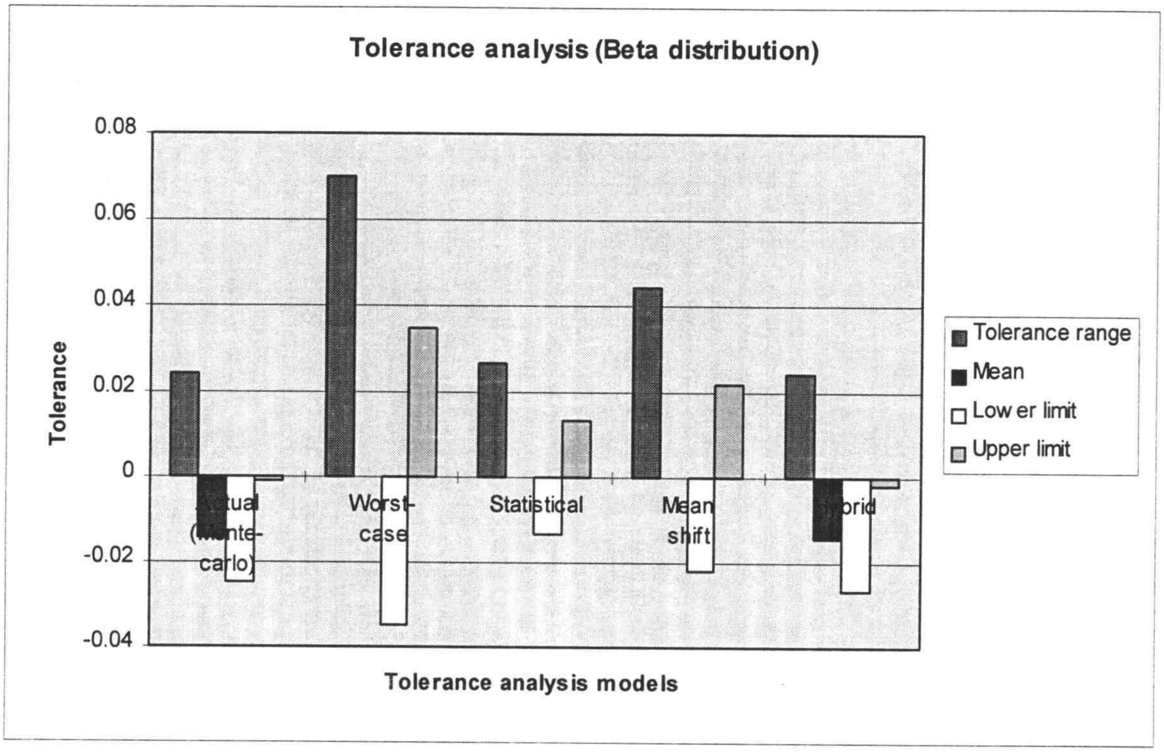
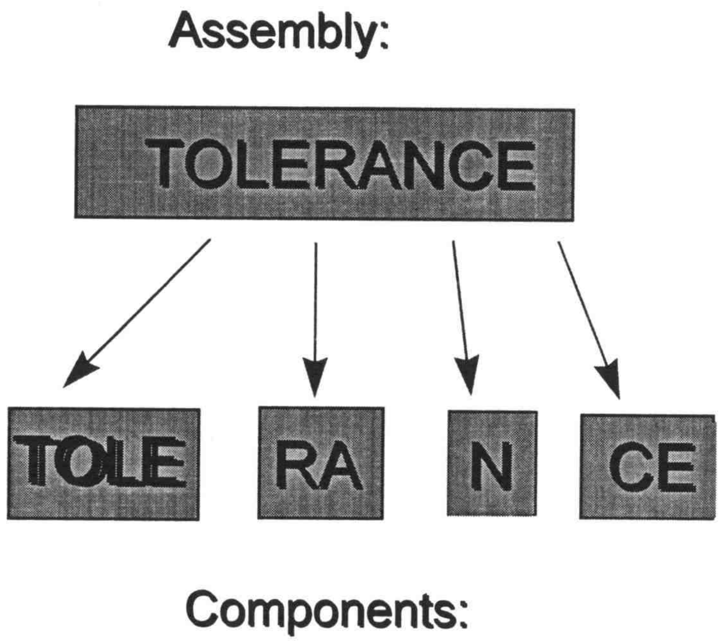


Fig 3.6 Chart showing the results of assembly tolerance applied to the shaft and bearing assembly (All components' natural process tolerance follow beta distribution)

4.0 TOLERANCE ALLOCATION

4.1 Introduction to tolerance allocation

In tolerance allocation the assembly design tolerance is known and the components' natural process tolerances are to be determined. The objective of Tolerance Allocation Models is to determine the tolerances of the individual dimensions based on the assembly design tolerance.



4.1 Tolerance Allocation

There are three approaches to tolerance allocation:

1. Tolerance allocation based on the dimensions', standard deviations and proportional scaling.

2. Tolerance allocation based on optimization techniques. Optimization techniques like Lagranges' multiplier and Linear Programming use Cost tolerance curves to minimize manufacturing cost.
3. Tolerance allocation considering alternative manufacturing processes: In this method component tolerances' are allocated by considering the manufacturing process costs. The algorithm varies tolerances for each of the component considering alternative manufacturing processes, and recommends processes and tolerances to minimize overall assembly cost.

4.2 Literature review

The following tolerance analysis models have been cited in literature and each model has some advantages and disadvantages

1. Tolerance Allocation by proportional scaling
   a. Tolerance Allocation using root sum square
   b. Tolerance Allocation using worst-case analysis
2. Tolerance Allocation by constant precision factor
   a. Tolerance allocation using root sum square
   b. Tolerance allocation using worst-case analysis

4.2.1 Tolerance allocation using proportional factor

Tolerance allocation by proportional scaling is performed by allocating reasonable tolerances (using historical data) on the components. The assembly tolerance is calculated by using Worst-case or Root sum square models to check if the calculated assembly tolerance meets the assembly design tolerance. If the assembly design tolerance constraint is not met then the tolerance on each component is scaled by a proportional constant, such that the assembly tolerance calculated using the proportionally scaled tolerances matches with the assembly design tolerance. The resultant tolerances on the components depend on the initial tolerances assigned by the designer.

Proportional scaling Worst Case Tolerance allocation model:

$$T_{asm}^{*} = \sum S_i$$

$$P = \frac{T_{asm}}{T_{asm}^{*}}$$

$$T_i = P \times S_i$$

Where

Si = initial tolerance allocated by the designer for ith component

T_i = Final component tolerances.

P = Proportionality constant

T_{asm} = Assembly design tolerance

T_{asm}^{*} = Initial assembly tolerance

Proportional scaling Root Sum Square Tolerance allocation model:

$$T_{asm}^{*} = \sqrt{\sum (S_i)^2}$$

$$P = \frac{T_{asm}}{T_{asm}^{*}}$$

$$T_i = P \times S_i$$

Where

Si = initial tolerance allocated by the designer for ith component

T_i = Final component tolerances.

P = Proportionality constant

T_{asm} = Assembly design tolerance

T_{asm}^{*} = Initial assembly tolerance

The allocation of initial tolerances affects the assembly tolerance. The resultant tolerances on the components depend on the initial tolerances assigned by the designer.

The designer allocates initial tolerances' based on process, historical data or design guidelines.

4.2.2 Tolerance allocation using precision factor

The constant precision factor is similar to proportional scaling, both models use a proportional scaling factor to calculate component tolerances. Constant precision factor model does not need the designer to allocate the initial tolerances, instead the initial tolerances are allocated using the nominal dimension of the component and the initial tolerances are scaled to meet the assembly design tolerance.

Worst-case tolerance allocation using precision factor:

$$P = \frac{T_{asm}}{\sum \sqrt[3]{D_i}}$$
$$\therefore T_i = P \times \sqrt[3]{D_i}$$

Root sum square tolerance allocation using precision factor:

$$P = \frac{T_{asm}}{\sqrt{\sum D_i^{\frac{2}{3}}}}$$
$$\therefore T_i = P \times \sqrt{D_i^{\frac{2}{3}}}$$

Where

D_i is the dimension of ith component

T_i = Final component tolerances.

P = Precision factor

T_{asm} = Assembly design tolerance

4.3 Algorithms for existing tolerance allocation model

The shaft and bearing assembly shown in Fig 4.2 consists of

a. Retaining ring: The retaining ring holds the ball bearings and shaft in place
b. Housing: The housing encompasses the shaft and bearing assembly
c. Ball bearing: Ball bearing aid in the free movement of the shaft
d. Shaft: The shaft
e. Bearing Sleeve: Bearing sleeve holds the housing and bearing together
f. Clearance: The clearance between retaining ring and ball bearing. And the clearance is the assembly design tolerance because the proper functioning of the shaft and bearing assembly depends the clearance. If the clearance is negative, their is interference between retaining ring and shaft and the assembly component does not function properly on the other hand if their is too much clearance then their is radial runout and assembly component is subjected to more wear and tear.

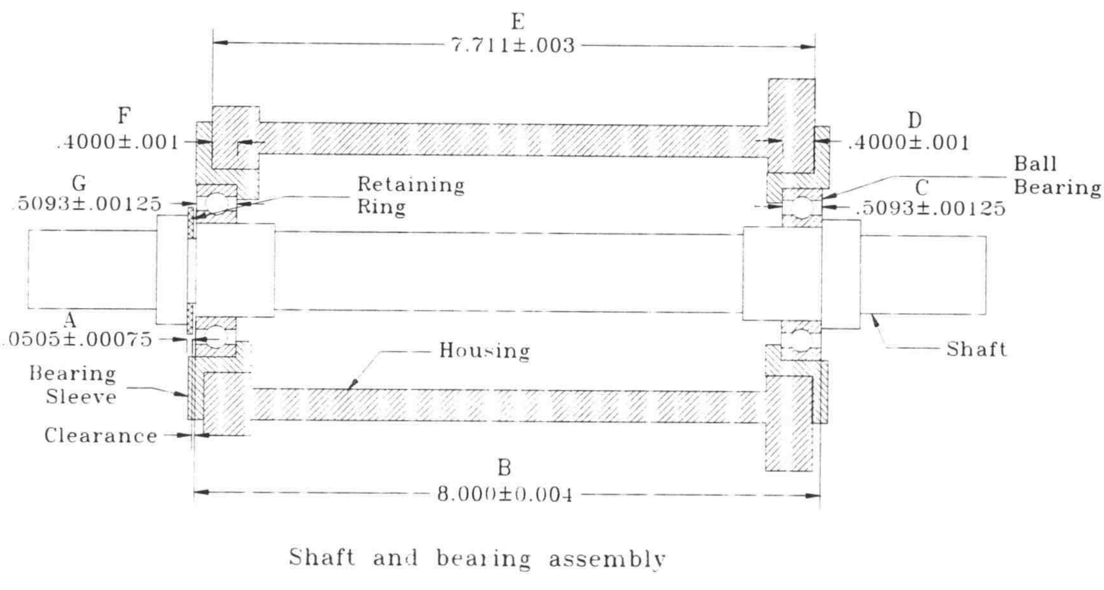


Fig 4.2 Shaft and bearing assembly

Dimension and tolerance information of the shaft and bearing assembly:

Dimension	A	B	C	D	E	F	G
Average	0.505	8.0	0.509	0.40	7.711	0.40	0.509
Tolerances (Design)		0.008		0.002	0.006	0.002	
Tolerances (Fixed)	.0015		0.0025				0.0025

Table 4.1 Dimension and tolerance data for the shaft and bearing assembly

The following calculations are based on the shaft and bearing assembly shown in the Fig 4.2. The retaining ring (A) and the two bearings (C and G) supporting the shaft are vendor supplied, hence their tolerances are fixed and must not be altered by the allocation process. The critical clearance is the shaft end-play, which is determined by tolerance accumulation in the assembly.

Initial tolerance specifications:

Required clearance = 0.020 ± 0.015

Average clearance = $A+B-C+D-E+F-G$

$= 0.505+8.00-0.5093+0.400-7.711+0.400-0.5093$

$= 0.020$

The clearance tolerance is obtained by computing the assembly tolerance sum by worst limits. Hence

$$T_{asm} = T_a + T_b + T_c + T_d + T_e + T_f + T_g$$

$$= 0.0015+0.008+0.0025+0.002+0.006+0.002+0.0025$$

$$= 0.0245 \text{ (too large)}$$

Solving for the proportionality factor:

$$T_{asm} = 0.015$$

$$= 0.0015+0.0025+0.0025 + P(0.008+0.002+0.006+0.002)$$

$$P = 0.4722$$

$T_b = 0.4722*0.008 = 0.00378$

$T_d = 0.4722*0.002 = 0.00094$

$T_e = 0.4722*0.006 = 0.00283$

$T_f = 0.4722*0.002 = 0.00094$

Each of the design tolerances has been scaled down to meet assembly requirements. If the same data is applied to get the tolerance allocation using Root sum square proportional scaling method then

$$T_{asm} = \sqrt{T_a^2 + T_b^2 + T_c^2 + T_d^2 + T_e^2 + T_f^2 + T_g^2}$$
$$T_{asm} = \sqrt{0.0015^2 + 0.008^2 + 0.0025^2 + 0.002^2 + 0.006^2 + 0.002^2 + 0.0025^2} =$$

0.011 (too small)

The tolerances for the components which do not have the tolerances not fixed have to be scaled up:

$$0.015^2 = 0.0015^2 + 0.0025^2 + 0.0025^2 + P^2(0.008^2 + 0.002^2 + 0.006^2 + 0.002^2)$$
$$P = 1.39$$
$$T_b = 1.39 * 0.008 = 0.01116$$
$$T_d = 1.39 * 0.002 = 0.00279$$
$$T_e = 1.39 * 0.006 = 0.00837$$
$$T_f = 1.39 * 0.002 = 0.00279$$

Tolerance allocation using Worst-case Precision factor:

$$\mathrm{T_{asm} = T_a + T_b + T_c + T_d + T_e + T_f + T_g}$$
$$0.015 = 0.0015 + 0.0025 + 0.0025 + P(8.0^{\frac{1}{3}} + 0.40^{\frac{1}{3}} + 7.7111^{\frac{1}{3}} + 0.40^{\frac{1}{3}})$$
$$P = 0.001568$$

$$T_b = P * 8.0^{\frac{1}{3}} = 0.00312$$

$$T_c = P * 0.40^{\frac{1}{3}} = 0.00115$$

$$T_e = P * 7.7111^{\frac{1}{3}} = 0.00955$$

$$T_f = P * 0.40^{\frac{1}{3}} = 0.00115$$

Tolerance allocation using RSS precision factor:

$$T_{asm} = \sqrt{T_a^2 + T_b^2 + T_c^2 + T_d^2 + T_e^2 + T_f^2 + T_g^2}$$

$$0.015^2 = 0.0015^2 + 0.0025^2 + 0.0025^2 + P^2(8.0^{\frac{2}{3}} + 0.40^{\frac{2}{3}} + 7.7111^{\frac{2}{3}} + 0.40^{\frac{2}{3}})$$

$$P = 0.004836$$

$$T_b = P * 8.0^{\frac{1}{3}} = 0.00976$$

$$T_c = P * 0.40^{\frac{1}{3}} = 0.00356$$

$$T_e = P * 7.7111^{\frac{1}{3}} = 0.00955$$

$$T_f = P * 0.40^{\frac{1}{3}} = 0.00356$$

Comparison of Proportional and Precision Factor tolerance allocation models

Table 4.2 shows the component tolerance values for Fig 4.2. The values obtained by Proportional scaling model are dependent on the initial tolerance allocation by the designer and therefore highly subjective. The resultant tolerance values do not depend on the process or the dimension of the component. Initial tolerances for Precision factor tolerance allocation model are based on the dimension of the component and the resultant tolerances are proportional to the dimension of the component. In Precision factor tolerance allocation model components which have larger dimensions are assigned a larger proportion of the assembly design and tolerance and components with smaller dimensions are assigned smaller proportion of the assembly design tolerance. The Precision factor tolerance allocation model assumes that the components' process follow normal distribution. In Worst case Proportional scaling and Precision factor allocation model, as the case with Worst case tolerance analysis model, the component tolerances are very tight due to the assumption of worst case scenario. Root sum square assigns relatively looser tolerances and assumes the component dimensions follow normal distribution.

Components	Proportional WC	Proportional RSS	Precision WC	Precision RSS
1	0.000918367	0.002030822	0.000757	0.000651
2	0.004897959	0.010831048	0.004026	0.018431
3	0.001530612	0.003384703	0.001622	0.002993
4	0.00122449	0.002707762	0.001498	0.002552
5	0.003673469	0.008123286	0.003977	0.017988
6	0.00122449	0.002707762	0.001498	0.002552
7	0.001530612	0.003384703	0.001622	0.002993
Assembly Tolerance	0.015	0.015	0.015	0.015

Table 4.2 Results of shaft bearing assembly using proportional and precision tolerance allocation models

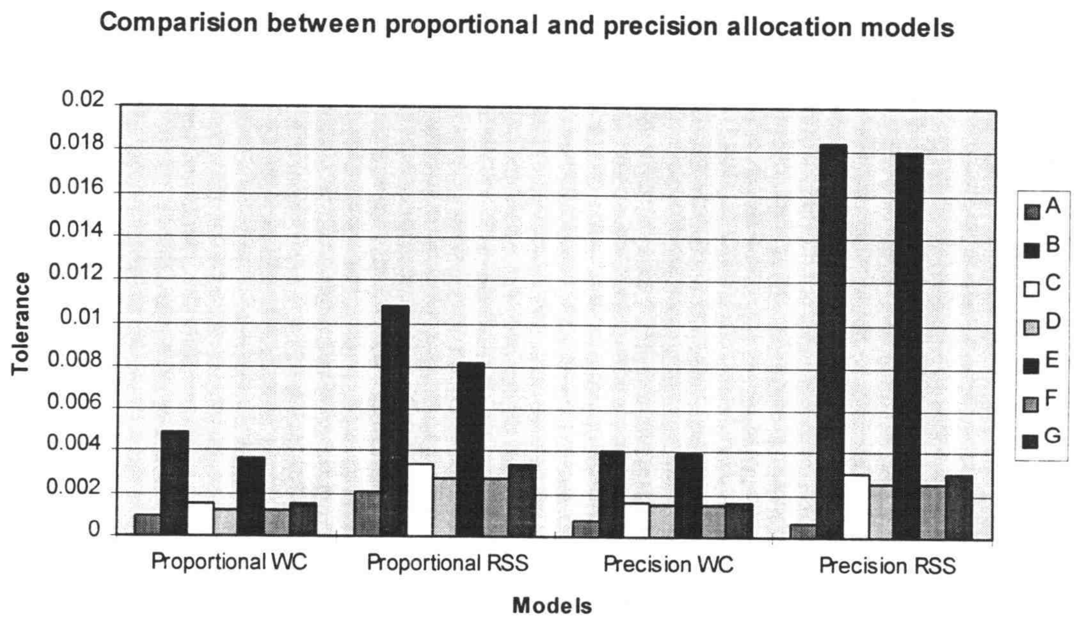


Fig 4.3 Chart of shaft bearing assembly using proportional and precision tolerance allocation models.

4.4 Disadvantages of existing models

The tolerance allocation models described above have the following limitations:

- The proportional scaling factor tolerance allocation model allocates tolerances by considering historical data which may or may not be appropriate for the present assembly and/or parts design and if the initial tolerances are allocated by designer, the allocated initial tolerances are highly subjective and depend on the designers' choice whether the initial tolerances are based on respective standard deviations, or magnitudes of the dimensions of the components.
- The precision factor allocates tolerances uses the dimension and not the process distribution of the manufacturing process thereby resulting in inaccurate allocation of the tolerances.
- And both of the models do not consider non-normal process distributions while calculating tolerances.

The tolerances allocated are not optimum due to the above reasons.

4.5 Objectives to improve tolerance allocation models

Objectives for improved tolerance allocation model:

a. initial tolerance assignment of component tolerance to components based on the dimension of the component and not based on subjective allocation the designer
b. consider normal and non-normal process distribution of components' natural process distribution to improve assembly functionality.
c. consider skewness and mean shift of components' process.
d. the model should use worst-case and root sum square method.
e. neither models consider the process cost or manufacturing processes while allocating tolerances.

4.5.1 Tolerance allocation using Monte Carlo simulation

The proposed tolerance allocation method using Monte Carlo simulation and Method of moments to take into consideration the distribution of the components and uses the both of the above discussed tolerance allocation models to allocate tolerance to individual components.

The tolerance allocation for the proposed method is done by following steps:

Step 1. The initial tolerances are found out by Precision factor method so that the initial tolerances allocated does not depend on the designer but is proportional to the dimensions. For Worst-case tolerance allocation:

$$P = \frac{T_{asm}}{\sum \sqrt[3]{D_i}}$$

$$\therefore T_i = P \times \sqrt[3]{D_i}$$

And for Root sum square tolerance allocation:

$$P = \frac{T_{asm}}{\sqrt{\sum D_i^{\frac{2}{3}}}}$$

$$\therefore T_i = P \times \sqrt{D_i^{\frac{2}{3}}}$$

T_{asm} is the assigned assembly tolerance

P is the precision scaling factor

D_i is the dimension of ith component

Step 2. The resultant tolerances are then used to find out the natural process tolerance (6σ limits) are found out by using Monte Carlo simulation and Method of moments. Monte Carlo simulation is incorporated into the model to consider skewness and mean shift for each component process.

$$T_i(MC) = \sum_{n=0}^{1000} T_i * R_n$$

$T_i(MC)$ is the tolerance value for component T_i

R_n is the random number with a mean of 0 and standard deviation of 1

T_i tolerances values from step 1

where i are the number of the components.

$T_i(MC)$ values are used to calculate the 6σ for each component.

Where T_{cal} *is* $= \Sigma T_i(MC)$ for worst case analysis

$T_{cal} = \sqrt{\sum (T_i(MC))^2}$ for root sum square model

Step 3. If T_{cal} is greater or lesser than the assembly tolerance assigned then proportional scaling method is used to scale the tolerances for the individual components to meet the assembly tolerance. The values we got from step 2 are used as the initial values of the proportional scaling method.

If $T_{cal} \neq T_{asm}$

$$P = \frac{T_{asm}}{\sum T_i(MC)}$$

$$T_i = P \times T_i(MC)$$

Where T_{cal} is $= \Sigma T_i(MC)$ for worst case analysis

$T_{cal} = \sqrt{\sum (T_i(MC))^2}$ for root sum square model

$t_i = 1, 2, \ldots\ i$ components

P is the proportionality constant

T_{asm} is the specified assembly tolerance

T_i are allocated component tolerances

4.5.2 Software modeling

The tolerance analysis by modified tolerance analysis is performed by the following steps.

- The assembly component drawing is opened in AutoCAD and the tolerance data for each feature is taken from the drawing.
- The design function for the assembly is defined by the user.

- Each component feature is assigned a distribution (Default: normal distribution)
- Initial component tolerances are calculated by Precision factor tolerance allocation model
- A random value for each of the assembly components' is generated using unit random number generator (Mean = 0, Range = 1) according to its user supplied distribution. The resultant initial component tolerances are used to perform Monte Carlo simulation.
- The above procedure is iterated for 1000 times
- The moments for each component are calculated and for 99.97% acceptance rate $\pm 3\sigma$ limits is the tolerance range.
- If assembly design tolerance constraint is not equal to assembly tolerance calculated by Monte Carlo simulation then the component tolerances are scaled using Proportional Scaling allocation model.

4.5.3 Model Verification

Tolerance allocation using modified Monte Carlo simulation is applied to Fig 4.2 and the dimension data is tabulated in Table 4.1.

Step 1:

Initial tolerances by Precision factor tolerance allocation model

$$T_{asm} = T_a + T_b + T_c + T_d + T_e + T_f + T_g$$

$$0.015 = 0.0015 + 0.0025 + 0.0025 + P(8.0^{\frac{1}{3}} + 0.40^{\frac{1}{3}} + 7.7111^{\frac{1}{3}} + 0.40^{\frac{1}{3}})$$

$$P = 0.001568$$

$$T_b = P * 8.0^{\frac{1}{3}} = 0.00312$$

$$T_c = P * 0.40^{\frac{1}{3}} = 0.00115$$

$$T_e = P * 7.7111^{\frac{1}{3}} = 0.00955$$

$$T_f = P * 0.40^{\frac{1}{3}} = 0.00115$$

Step 2:

Calculate moments by Modified Monte Carlo simulation

	Comp 1	Comp 2	Comp 3	Comp 4	Comp 5	Comp 6	Comp 7
1	0.8	0.8	0.8	0.8	0.8	0.8	0.8
2	0.3	0.3	0.3	0.3	0.3	0.3	0.3
3	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2
4	-0.21	-0.21	-0.21	-0.21	-0.21	-0.21	-0.21
5	-0.8	0.8	-0.8	-0.8	-0.8	-0.8	-0.8

Table 4.3 Sample values generated by the random number generator

For Component T_b:

= 0.003112 * 0.8 + 0.00312 * 0.3 + 0.00312 * -0.2

+ 0.00312 * -0.21 + 0.00312*-0.8 = 0.00144144

Standard deviation = 0.004516

Step 3:

Similarly component tolerance are found for all the components and proportionally scaled to meet the assembly constraint.

Above calculations are performed only to illustrate the procedure for implementing Modified Monte Carlo simulation. The values obtained by above calculations are inaccurate because of the small number of iterations.

Results from computer software

Component tolerance using the Modified Monte Carlo simulation for Fig 4.2:

1	2	3	4	5	6	7
0.000701	**0.018054**	**0.003093**	**0.002952**	**0.021988**	**0.002002**	**0.003593**

4.6 Tolerance allocation comparison

Table 4.4 shows the component tolerance values for Fig 4.2. The values obtained by Proportional scaling model are dependent on the initial tolerance allocation by the designer and therefore highly subjective. The resultant tolerance values do not depend on the process or the dimension of the component. Initial tolerances for Precision factor tolerance allocation model are based on the dimension of the component and the resultant tolerances are proportional to the dimension of the component. In Precision factor tolerance allocation model components which have larger dimensions are assigned a larger proportion of the assembly design and tolerance and components with smaller dimensions are assigned smaller proportion of the assembly design tolerance. In Worst case Proportional scaling and Precision factor allocation model, as the case with Worst case tolerance analysis model, the component tolerances are very tight due to the assumption of worst case scenario. Root sum square assigns relatively looser tolerances and assumes the component dimensions follow normal distribution.

The component tolerance values obtained for Fig 4.2 using Monte Carlo simulation are approximately similar to component tolerance values obtained using Precision RSS only when all the components' natural process distribution follow normal distribution and Precision RSS fails to assign optimal tolerance values when the process distribution of the component has mean shifts and skewed distributions. The Precision factor tolerance allocation model assumes that the components' process follow normal distribution. But the component tolerances depend on the variation occurring in the process rather than on the nominal dimension of the component. Modified Monte Carlo simulation tolerance allocation model assign component tolerances considering natural process distribution of similar processes. The resultant component tolerances consider normal and non-normal distributions and therefore can account for the mean and skewness of component distributions. (All the components follow normal distribution)

Components	Proportional WC	Proportional RSS	Precision WC	Precision RSS	Monte Carlo
1	0.000918	0.002030	0.000757	**0.000651**	**0.000701**
2	0.004897	0.010831	0.004026	**0.018431**	**0.018054**
3	0.001530	0.003384	0.001622	**0.002993**	**0.003093**
4	0.001224	0.002707	0.001498	**0.002552**	**0.002952**
5	0.003673	0.008123	0.003977	**0.017988**	**0.021988**
6	0.001224	0.002707	0.001498	**0.002552**	**0.002002**
7	0.001530	0.003384	0.001622	**0.002993**	**0.003593**
Assembly Tolerance	0.015	0.015	0.015	0.015	0.015

Table 4.4 Results of shaft bearing assembly using proportional, precision and Monte Carlo simulation tolerance allocation models

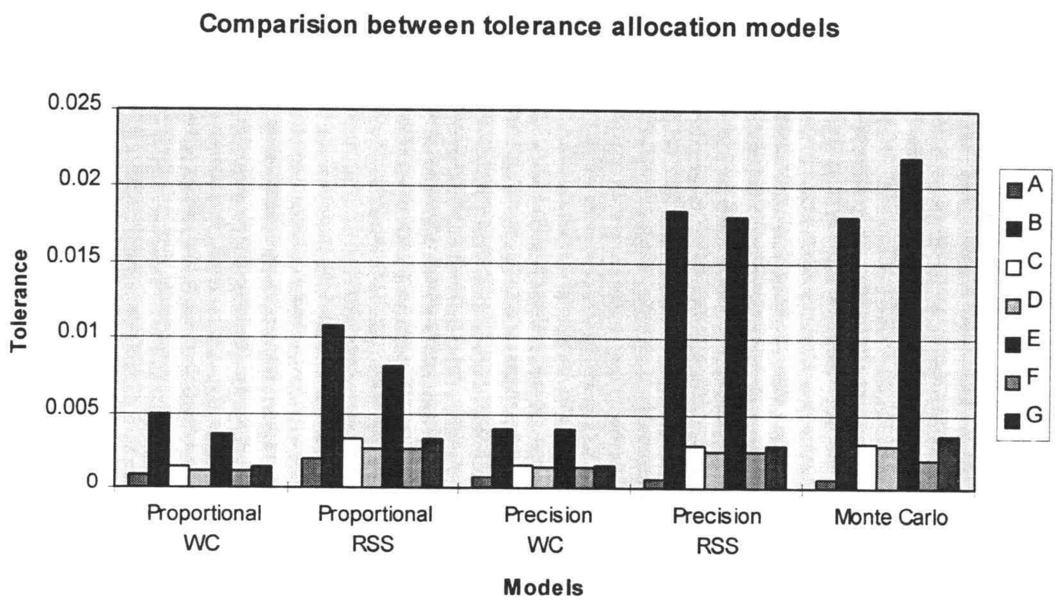


Fig 4.4 Chart of shaft bearing assembly using proportional, precision tolerance and Monte-Carlo simulation allocation models

5.0 PROCESS SELECTION

5.1 Introduction

The intent of concurrent design of mechanical systems is to break the barrier between current computer automated design software and manufacturing modules, and incorporate manufacturing considerations into the design phase, thereby generating designs which need fewer re-designs and have lower productions costs. The parts must be designed and manufactured such that no constraints are violated and the cost is kept to a minimum.

Often design conceptualization, detailed design, and manufacturing design are done independently causing inferior product quality and excessive cost. Allocation of tolerances alone is not enough, the tolerances must be selected along with the manufacturing process if costs are to be minimized. Manufacturing a part to tight tolerances can be an expensive process; thus parts are usually designed for as large a tolerance range as possible but large tolerances may result in defective assembly components. Components can be manufactured with different processes at different costs, and each process is best or optimally suited to hold different tolerance costs.

5.2 Objectives

In design of any assembly or mechanism it is necessary to assign tolerances to dimensions. The tolerances should be assigned such that the manufacturing cost should be minimum and should guarantee assembly functionality. At present tolerance analysis and tolerance allocation is largely performed without considering the production costs. An assembly has many mechanical features and each of these different features are manufactured using different production processes and at different production cost-tolerance relationships. One of the objective of this thesis is to implement a method to allocate tolerances optimally subject to minimum cost and assembly tolerance constraints.

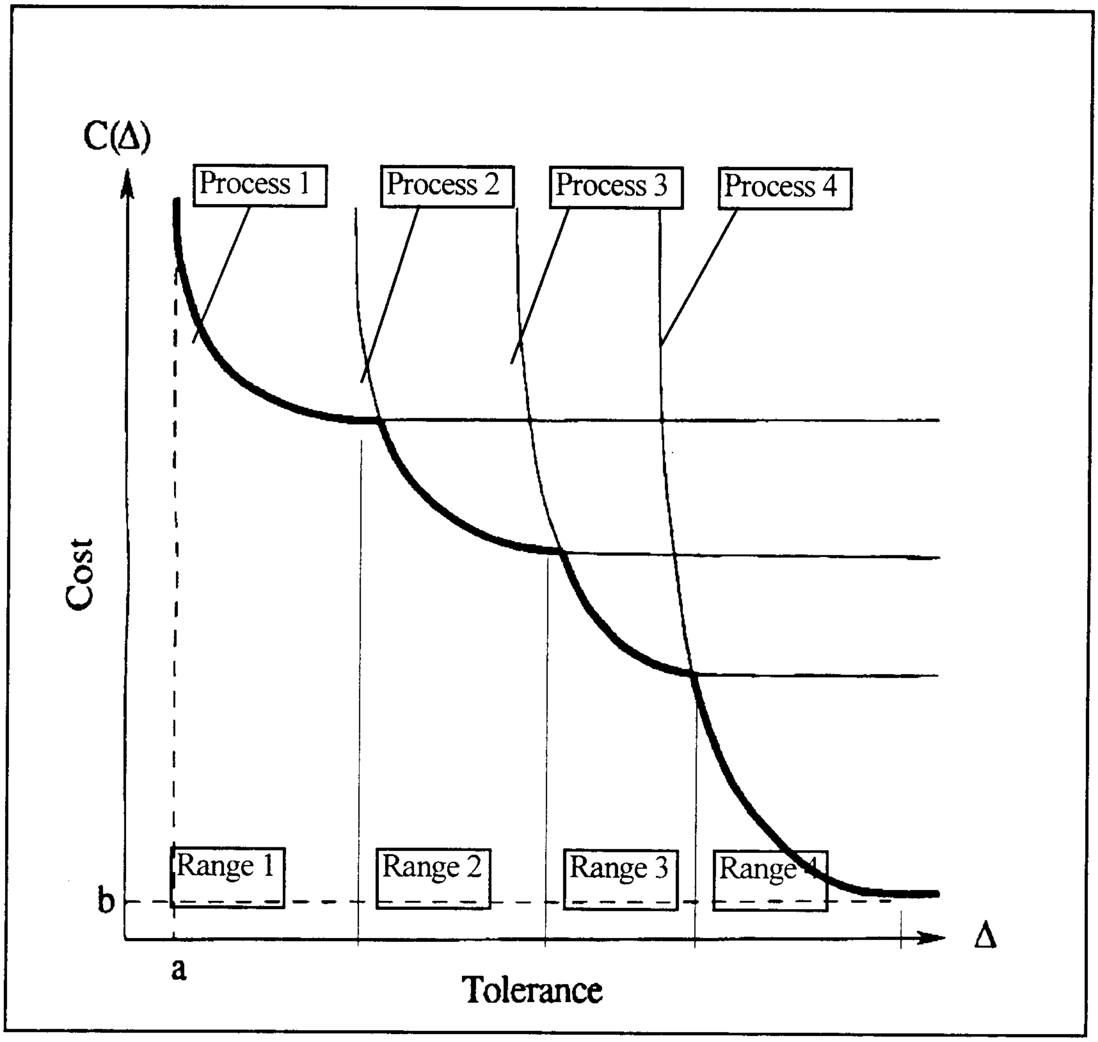


Fig 5.1 Cost tolerance curves for different processes

Fig 5.1 shows that a given tolerance range can be met by more than one different process at different costs. The cost-tolerance curves are non-linear in nature because the cost to meet tighter tolerances increases exponentially, tighter the tolerances higher the production cost. Currently tolerance allocation is performed mainly by trial and error and the tolerance assignment depends upon the experience and process knowledge of the designer. A certain cost is associated with a selection of particular tolerance on a component. Increasing the tolerance reduces the cost of production but such an increase is constrained by the assembly tolerance requirement or functionality of the assembly. Therefore cost of manufacturing a component will decrease with a widening of tolerances but on the other hand components need to meet the assembly design tolerance constraint. To minimize production costs, generally tolerances on parts that are expensive to machine are allocated maximum possible tolerances and tolerances on parts that are relatively less costlier to meet are reduced. The objective is to not only choose the process which has the minimum production cost but also to allocate maximum tolerances on processes which are expensive to manufacture and allocated minimum tolerances on process which are relatively cheap to manufacture.

The mathematical problem of finding the one set of tolerances that will minimize costs and still meet the performance criteria can be simply characterized as a problem of minimizing a non-linear function (cost function) subject to linear or non-linear constraints (assembly constraints).

5.3 Simulated annealing

Simulated annealing is a stochastic optimization technique which has been shown able to solve both ordered combinatorial problems and non-linear continuous problems even with objectives of discontinuous slope. The method of simulated annealing is a technique suitable for optimization problems where the desired global extremum is hidden among many local extrema. Simulated annealing method has been effectively used to solve traveling salesman problem of finding the shortest cyclical itinerary for a traveling salesman who must visit each of N cities in turn. A more detailed explanation of simulated annealing, Boltzmann constant and Metropolis algorithm is provided in Appendix C.

5.4 Simulated annealing algorithm and software modeling

- *Begin Simulated_Anneal*
  - *Determine cost function;*
  - *T = l;*
  - *Generate set of manufacturing processes;*
  - *Evaluate set of manufacturing processes;*
  - *While T > 0 do*
    - *Generate temp_set of manufacturing processes by mutation where range is function of T;*
    - *If (verify constraints of temp_set of manufacturing processes)*
    - *Then Begin*
      - *Evaluate temp_set of manufacturing processes;*
      - *Test temp_set of manufacturing processes with Metropolis;*
      - *If (accept)*
      - *Then*
        - *Set of manufacturing processes*
        - *= temp_set of manufacturing processes;*
      - *End*
    - *End*
    - *T = T * reduction_factor;*
  - *End*
  - *End*

Table 5.1 Algorithm of simulated annealing

Table 5.1 shows the simulated annealing algorithm (Cagan, 1992) used to allocate tolerances and toe assign process for each component. The approach to simulated annealing is to randomly pick a feasible set of manufacturing processes, S_1, and evaluate the cost of the assembly at that state, E_1. A different feasible set of manufacturing processes S_2, is then selected by randomly picking a new state within the given range of the available design space (which is called the mutation space in the algorithm). State S_2 is the evaluated to E_2. If $E_2 < E_1$, then S_2 becomes the new solution set of manufacturing processes. If $E_2 >= E_1$, then there is a probability P_r based on the tolerance that the new state of manufacturing processes will be accepted anyhow. A random number, r, uniformly distributed between 0 and 1 is generated and compared with the probability $P(E_2)$. If $r < P(E_2)$ then the new set of manufacturing processes is accepted anyhow other wise the old state of manufacturing processes is retained. The tolerance is reduced and the process continues until convergence is reached or the tolerance reaches 0.

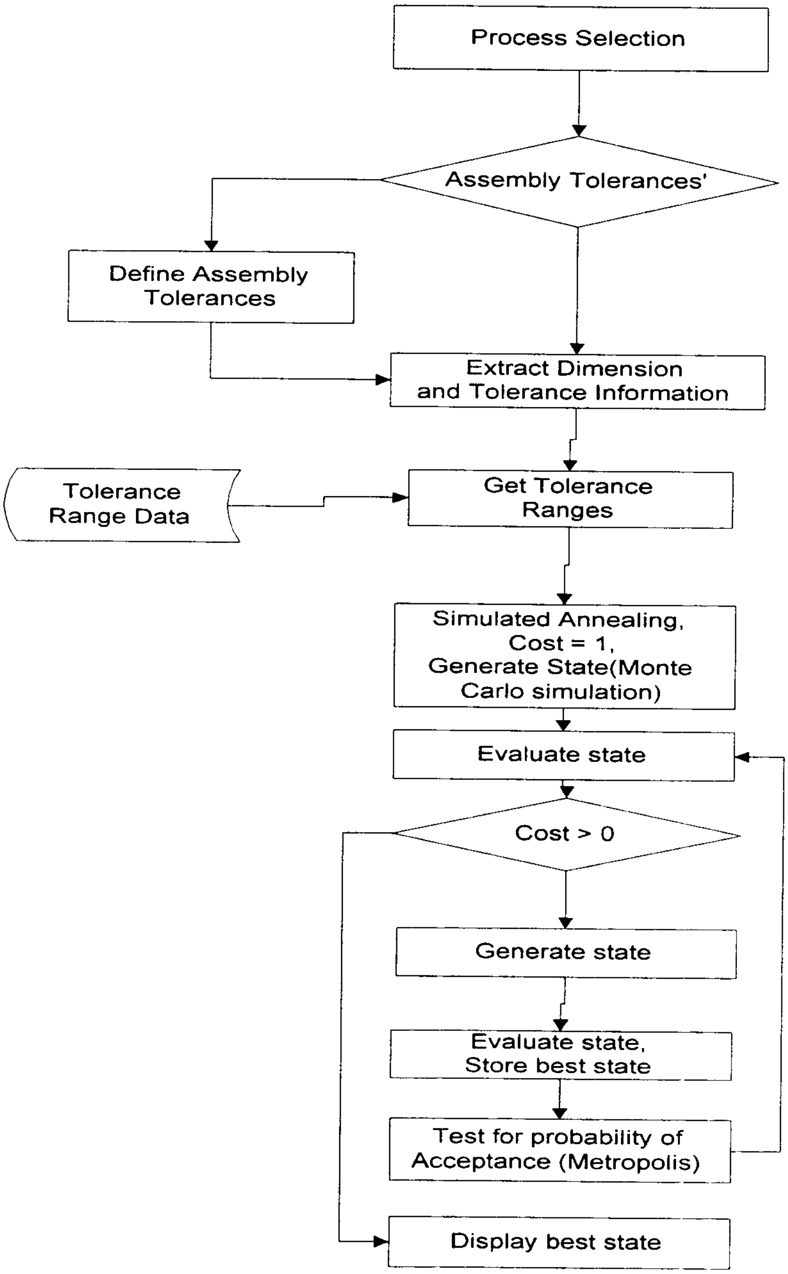


Fig 5.2 Flowchart of the process selection module

First the cost-tolerance curves for each component is chosen and then the initial starting tolerances are selected, making sure tolerances sum up to specified output tolerance. This can be done by either proportional scaling or by allocating the maximum allowable tolerance on the first of n components and continuing this process for n-1 parts. The total cost is evaluated. A detailed explanation of Boltzmann constant and Metropolis algorithm in Appendix C. The algorithm is then run by randomly generating new tolerances in a neighborhood (ε) about the tolerance of each of the first part and the final part is assigned the remaining tolerance. The new set of tolerances are then evaluated and the Metropolis algorithm determines whether it is accepted. As the tolerance is reduced, So is the range (ε) of the mutation space. The algorithm terminated when the cost converges or the tolerance reaches 0.

Software Procedure

- Select the components in the critical path.
- Select the dimension data for the components.
- Define the Design Function.
- Enter/Select the tolerance data for any components whose tolerances are fixed.
- Choose the cost-function for each component (Default Reciprocal Squared, if no cost function is specified for a component Reciprocal Squared is assumed).
- Choose the distributions for each component (Default Normal distribution, if no distribution is specified for a component normal distribution is assumed)
- Dimension each component from the results of the tolerance allocation.

5.5 Case study

Simulated annealing technique is applied to friction wheels in Fig 5.3. Friction wheels are good example to demonstrate process selection because the wheels can be manufactured to different process tolerance ranges. Friction wheels are used when the low levels of energy need to be transferred. (e.g. of friction wheels: friction wheels are used in watch mechanism to transfer energy from 'hour hand' to 'minute hand' and to 'second hand', friction wheels are also used in VCRs' and other mechanisms with low energy transfer requirements). Highly precise friction wheels can be produced by sequentially hot rolling, turning and grinding. A tradeoff exists between the level of precision needed and the production cost for each component. The problem is choose the tolerances levels such that assembly tolerance criteria is met while minimizing the total cost.

Problem statement
min: C_T
Subject to:

$$h1: C_T = \sum_{i=1}^{n} C_i$$

$$h2: C_i = \frac{K_i}{\Delta_i - a_i} + b_i$$ Cost function

$$g1: \sum_{i=1}^{n} \Delta_i \leq \Delta_{out}$$ Worst Case Analysis Assembly Contraint

Where:

C_i = cost of machining the component to Δ_i tolerance

Δ_i = tolerance of components, Δ_{out} = assembly tolerance

a_i, b_i, K_i = process constants.

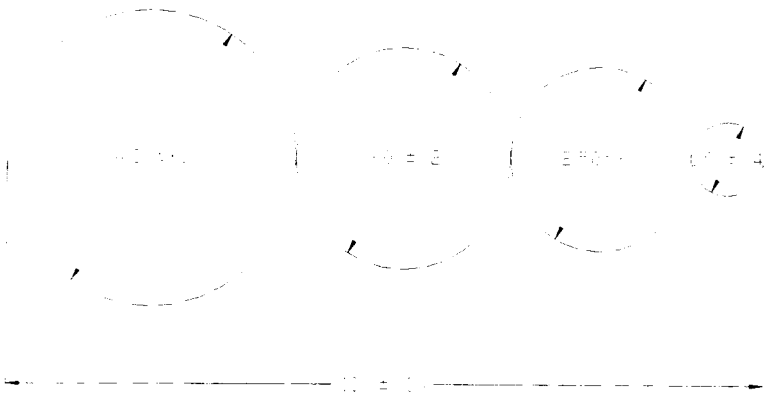


Fig 5.3 Friction Wheels

Process	Parameters	4.0 inch	3.0 inch	2.5 inch	1.0 inch
Hot Rolling	K	0.03	0.03	0.03	0.03
Hot Rolling	a	2.15	1.17	0.83	0.15
Hot Rolling	b	0.0279	0.0208	0.0172	0.073
Turning	K	0.005	0.005	0.005	0.005
Turning	a	2.67	1.63	1.2	0.3
Turning	b	0.0057	0.0052	0.00495	0.0042
Grinding	K	0.0005	0.0005	0.0005	0.0005
Grinding	a	2.95	1.89	1.45	0.52
Grinding	b	0.00031	0.00029	0.00028	0.00027

Table 5.2 Parameter values used for Fig 5.3

The above data is based on values obtained from Manufactures and Machinability Data Center(1980). Initial tolerance ranges for the simulated annealing are obtained from Appendix B.

If the friction wheel problem Δ_{out} is specified at 0.08 then the optimal configuration for parts 1-4 allocated tolerances of 0.0654, 0.0049, 0.0049, 0.0048, and the processes of hot rolling, turning, turning, and turning respectively and at a total cost of $6.714 compared to $12.24 obtained using precision factor worst case model. Initial tolerances for Precision factor tolerance allocation model are based on the dimension of the component and the resultant tolerances are proportional to the dimension of the component and therefore in precision factor tolerance allocation model components which have larger dimensions are assigned a larger proportion of the assembly design and tolerance and components with smaller dimensions are assigned smaller proportion of the assembly design tolerance. And precision factor model does not consider the cost of meeting the tolerance whereas simulated annealing allocates tolerances' considering the process and cost. Table 5.2 shows that the assembly cost obtained by simulated annealing is less the cost obtained by Modified Monte Carlo tolerance allocation.

	Allocation using Precision factor Worst Case	Cost Using Monte Carlo	Allocation using Process Selection	Cost Using Process Selection
	0.0041	4.06	0.0654	2.74
	0.02	1.98	0.0049	1.91
	0.015	1.69	0.0049	1.47
	0.004	4.5	0.0048	0.59
Total Cost	0.08	**$12.24**	0.08	**$6.714**

Table 5.3 Cost comparison of tolerance allocation using modified Monte Carlo tolerance allocation and simulated annealing

Cost comparison using precision factor tolerance allocation:

Cost analysis is done using precision factor tolerance allocation for various combinations of processes. Table 5.4 is the resultant table with all the possible combinations of the processes. Parameters in Table 5.2 are used to calculate the assembly costs. Component tolerances' are allocated using precision factor model. If precision factor allocation model is followed, the optimal processes are obtained by performing cost calculations on all the possible combinations of processes and selecting the combination with minimum cost. For Fig 5.3 the minimum cost is **7.17** with processes turning, turning, grinding and grinding for components 1 ,2,3, and 4 respectively with tolerances of 0.041, 0.02, 0.015, 0.004 and assembly design tolerance of 0.08. The assembly cost obtained by precision factor tolerance allocation model may not be optimal because precision factor tolerance allocation model assigns tolerance is direct proportion to the dimension and does not consider the cost of meeting the tolerance or the processes to meet the tolerances. The case study is an example of an assembly with just 4 components and three processes. If the assembly has more than 10 components and more than three processes, the time needed to obtain an optimal cost will be very large and the resultant cost may not optimal as was the case with Fig 3.3

Component	Process	Total Cost	Assembly Cost
4.0 inch	Hot Rolling	4.0590909	
3.0 inch	Hot Rolling	3.25	
2.5 inch	Hot Rolling	1.9766667	
1.0 inch	Hot Rolling	0.4307692	**9.716526807**
4.0 inch	Hot Rolling	4.0590909	
3.0 inch	Hot Rolling	3.25	
2.5 inch	Hot Rolling	1.9766667	
1.0 inch	Turning	4.5	**13.78575761**
4.0 inch	Hot Rolling	4.0590909	
3.0 inch	Hot Rolling	3.25	
2.5 inch	Turning	1.695	
1.0 inch	Turning	4.5	**13.50409091**
4.0 inch	Hot Rolling	4.0590909	
3.0 inch	Turning	1.9766667	
2.5 inch	Turning	1.695	
1.0 inch	Turning	4.5	**12.23075761**
4.0 inch	Turning	3.1283333	
3.0 inch	Turning	1.9766667	
2.5 inch	Turning	1.695	
1.0 inch	Turning	4.5	**11.3**
4.0 inch	Turning	3.1283333	
3.0 inch	Turning	1.9766667	
2.5 inch	Turning	1.695	
1.0 inch	Grinding	0.5971429	**7.397142857**
4.0 inch	Turning	3.1283333	
3.0 inch	Turning	1.9766667	
2.5 inch	Grinding	1.4693103	
1.0 inch	Grinding	0.5971429	**7.171453202**
4.0 inch	Turning	3.1283333	
3.0 inch	Grinding	1.9048718	
2.5 inch	Grinding	1.4693103	
1.0 inch	Grinding	0.5971429	**7.19965833**
4.0 inch	Grinding	3.6576543	
3.0 inch	Grinding	1.9048718	
2.5 inch	Grinding	1.4693103	
1.0 inch	Grinding	0.5971429	**7.68979318**

Table 5.4 Assembly cost for various process combinations for tolerance allocation using precision factor worst case model

6.0 CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The proposed tolerance analysis method takes into consideration the distribution of each fabrication process of the assembly. For assemblies with non-normal natural process tolerance distributions, this new method allows designers to assign assembly tolerances that are closer to actual assembly tolerances when compared to other statistical methods. This is verified by comparing the new method to the results of Monte Carlo simulations. The method results in assembly tolerances similar to those provided by Monte Carlo simulation yet is significantly less computationally-intensive.

Advantages of Hybrid method of tolerance analysis:

- Most of the complexity of the Method of Moments is eliminated since the moments of the assembly tolerance are calculated from the Monte Carlo simulation data.
- Decreased sample size in the order of 1000 to 5000, the computation time is greatly reduced from simple Monte Carlo simulation.

Non-symmetric and non-normal distributions are important to consider as naturally occurring shifts in a process can produce biased distributions, which result in increased assembly problems and a greater percentage of reject than anticipated. The Monte-Carlo, and Hybrid Monte-Carlo methods consider non-symmetric and non-normal distributions and also predict assembly tolerance close to the actual one, agrees with the statistical model for normal distributions. In conclusion, for tolerance analysis, if there is sufficient information available about the component distributions, then modified Monte-Carlo simulation is well-suited.

The proposed tolerance allocation/process selection method was found to be superior to other tolerance allocation methods based on manufacturing costs. The tolerance allocation/process selection technique is used to determine optimal tolerance allocation of tolerances and manufacturing processes to a system of components for minimum cost. Simulated annealing technique is applied to the design of a system of friction wheels considering the manufacturing processes of grinding, turning and sawing. The hyperbolic cost function is applied , however if the manufacturing process is modeled with a different cost function, simulated technique can still be applied by incorporating the new cost function in the simulated annealing model. Simulated annealing has the following advantages

- results in lesser cost than precision factor tolerance allocation
- takes lesser time compared to precision factor tolerance allocation model when manufacturing costs are considered.

The software developed has the following advantages over other software's like ANVILTOL, Mechanical Advantage, Analytix, DesignView and Mechanical Engineering Workbench

- Implemented on a popular CAD software: The software is implemented in AutoCAD R13, AutoCAD R13 has 1,600,000 Customer base.
- Does not assume normal distributions: The software is not limited to normal distributions, if sufficient information is available about the component distributions, then the component distributions are used to calculate the tolerances.
- Allows the designer to use as many or as few tolerances as functionally required: The designer can is not limited to the number of tolerances that can be calculated.

- User interactive interface allowing the user to make decisions: The software allows the designer to perform various tolerance analysis and tolerance allocation calculations interactively
- Works directly with CAD system geometry: The user does not have to input the dimension data, the software system gets the data from the drawings of the components, so there is less chance of erroneous data input.

6.2 Limitations

Monte-Carlo simulation, and the Hybrid method of tolerance analysis require advance knowledge of the distribution of the components, but, in the early stages of design, little information is available on distribution type.

The proposed tolerance allocation has same of limitation of requiring the knowledge of component distribution before hand to apply the model but the proposed tolerance allocation can be still be applied for assemblies assuming normal distribution if information about the component distribution is unavailable. Most of the Quality control methods are based on normal distributions and do not utilize information on the third and fourth moments because of the large sample size required, thereby the quality control techniques may not predict out-of-control conditions if only higher moments are changing. Simulated annealing technique, Linear programming, and Lagranges Multiplier method can only be applied if there is enough information to graph a cost tolerance curve.

In addition, to the worst-case and the simple statistical tolerance methods, other methods used for assembly tolerance analysis include: mean-shift tolerance model, Monte-Carlo simulation and the Hybrid model. Each of the models has some advantages and limitations when considering the different possible distributions of the components in an assembly.

6.3 Recommendations

Future tolerance analysis and allocation should consider:

1. Software must handle geometric tolerances
2. Capable to handle both 2- Dimensional and 3-Dimensional cad systems
3. Should also consider non-linear design functions
4. Incorporate manufacturing considerations into the design phase.

BIBLIOGRAPHY

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Cagan J. and Kurfess T. R., Optimal Tolerance Allocation Over Multiple Manufacturing Alternatives, Advances in Design Automation, ASME, Vol 2, 1992 pp 165-172

Chase, K. W. and Greenwood, W. H., Design issues in mechanical tolerance analysis, Manufacturing Review, Vol 1, No 1 (1988) pp 50-59

Dimensioning and tolerancing, ANSI Y14.5 M-1982, American Society of Mechanical Engineers, USA (1983)

Evans. D. H., Statistical tolerancing: the state of the art. Part II: Method of estimating moments, Journal of Quality Technology, Vol 7, No 1 (1975) pp 1-12

Evans, D. H., An application of numerical integration techniques to statistical tolerancing. Part III: General distributions, Technometrics, Vol 14, No 1 (1972) pp 23-35

Evans, D. H., An application of numerical integration techniques to statistical tolerancing, Technometrics, Vol 9, No 3 (1967) pp 441-456

Greenwood, W. H. and Chase, K. W., A new tolerance analysis method for designers and manufacturers, Trans ASME Journal of Eng. Indust., Vol 109, No 2 (1987) pp 112-116

Law A. M. and Kelton W. D., Simulation Modeling And Analysis, McGraw-Hill, 1982

Lee, W. J. and Woo, T. C., Optimum Selection of Discrete Tolerances, Journal of Mechanisms, Transmissions, and Automation in Design, (June 1989) pp 243-251

Lee, W. J. and Woo, T. C., Tolerances: their analysis and synthesis, Transactions ASME Journal of Engineering for Industry, Vol 112, No 2 (1990) pp113-121

Mansoor, E. M., The application of probability to tolerances used in engineering designs, Proceeding of Institute of Mechanical Engineering, Vol 178, No1 (1963) pp 82-88

Nigam S. D. and Turner J. U., Review of statistical approaches to tolerance analysis, Computer-Aided Design, Vol 25, No 1 1995 pp 6-15

Numerical Recipies In C: The art of scientific computing, Cambridge University Press, 1988

Oyvind Bjorke, Computer-Aided Tolerancing, ASME Press, 1989

Richard H. M., Statistical Methods For Engineers, Prentice Hall, 1985

Speckhart, F. H., Calculation of tolerances based on a minimum cost approach, Journal of Engineering for Industry, (May 1972) pp 447-453

Spotts, M. F., An application of statistics to the dimensioning of machine parts, Journal of Engineering for Industry, (Nov 1959) pp 317-322

Wu W. H., ElMaraghy H. A., Evaluation of Cost-Tolerance Algorithms for Design Tolerance Analysis and Synthesis, Manufacturing Review (Oct 1988) pp 168-178

Trucks H. E., Designing For Economical Production, Society of Manufacturing Engineers, 1974

APPENDICES

Appendix A-Manual for the software

Components in the critical path: The components in the critical path are the components which can influence the final assembly tolerance of the component

Design function: A mathematical relationship which defines the assembly variable in terms of component variables. The function specifies the influence of the component on the assembly tolerance.

Tolerance Analysis:

Step1: Choose the appropriate tolerance analysis model to apply the assembly component

Step 2: Select the components in the critical path

Step 3: Select the dimension data for the components

Step 4: Define the Design Function

Step 4a: Choose the distributions for each component (Default Normal distribution, if no distribution is specified for a component normal distribution is assumed)

Step 4b: Define the range for the assembly tolerance ($\pm 3\sigma$ or greater).

Step 4c: Choose the number of iterations (5000 for Modified Monte Carlo Simulation, 100,000 for Monte Carlo Simulation)

Step 4d: For Mean shift model the mean shift factor has to be defined.

Step 5: Dimension the assembly tolerance.

Steps 4a,4b and 4c for Monte Carlo Simulation and Modified Monte Carlo Simulation only.

Tolerance Allocation: (Proportional Scaling and Precision factor):

Step1: Choose the appropriate tolerance allocation model to apply the assembly component.

Step 2: Select the components in the critical path.

Step 3: Define the Design Function

Step 4: Select the dimension data for the components.

Step 4a: For Proportional Scaling initial tolerances should be allocated by the designer. (Historical data or subjective decision of the designer)

Step 5: Enter/Select the tolerance data for any components whose tolerances are fixed

Step 6: Dimension each component from the results of the tolerance allocation

Tolerance Allocation (Monte Carlo Simulation Tolerance Allocation Model):

Step 1: Select the components in the critical path.

Step 2: Select the dimension data for the components.

Step 3: Define the Design Function

Step 4: Enter/Select the tolerance data for any components whose tolerances are fixed

Step 5: Choose the distributions for each component (Default Normal distribution, if no distribution is specified for a component normal distribution is assumed)

Step 6: Choose the number of iterations (Default 5000 for Monte Carlo Simulation)

Step 7: Dimension each component from the results of the tolerance allocation

Process Selection:

Step 1: Select the components in the critical path.

Step 2: Select the dimension data for the components.

Step 3: Define the Design Function.

Step 4: Enter/Select the tolerance data for any components whose tolerances are fixed.

Step 5: Choose the cost-function for each component (Default Reciprocal Squared, if no cost function is specified for a component Reciprocal Squared is assumed).

Step 5: Choose the distributions for each component (Default Normal distribution, if no distribution is specified for a component normal distribution is assumed)

Step 6: Choose the number of iterations (Default: 5000 for Modified Monte Carlo Simulation)

Step 7: Dimension each component from the results of the tolerance allocation.

Appendix B - Tables

Range of sizes					TOLERANCES					
From	Through									
0.000	0.599	0.00015	0.0002	0.0003	0.0005	0.0008	0.0012	0.002	0.003	0.005
0.600	0.999	0.00015	0.00025	0.0004	0.0006	0.001	0.0015	0.0025	0.004	0.006
1.000	1.499	0.0002	0.0003	0.0005	0.0008	0.0012	0.002	0.003	0.005	0.008
1.500	2.799	0.00025	0.0004	0.0006	0.001	0.0015	0.0025	0.004	0.006	0.010
2.800	4.499	0.0003	0.0005	0.0008	0.0012	0.002	0.003	0.005	0.008	0.012
4.500	7.799	0.0004	0.0006	0.001	0.0015	0.0025	0.004	0.006	0.010	0.015
7.800	13.599	0.0005	0.0008	0.0012	0.002	0.003	0.005	0.008	0.012	0.020
13.600	20.999	0.0006	0.001	0.0015	0.0025	0.004	0.006	0.010	0.015	0.025
Laping & Honing		■	■	■						
Diamond Turning & Grinding		■	■	■	■					
Broaching			■	■	■	■				
Reaming				■	■	■	■	■		
Turnin, Boring, Planing, & Shaping						■	■	■	■	■
Millig							■	■	■	■
Drilling								■	■	■

Tolerance ranges for different process (Trucks, 1974)

Appendix C – Simulated Annealing

Simulated Annealing

Simulated annealing is a stochastic optimization technique which has been shown able to solve both ordered combinatorial problems and non-linear continuous problems even with objectives of discontinuous slope. The method of simulated annealing is a technique suitable for optimization problems where the desired global extremum is hidden among many local extrema. Simulated annealing method has been effectively used to solve traveling salesman problem of finding the shortest cyclical itinerary for a traveling salesman who must visit each of N cities in turn.

Simulated annealing can be described by analogy of annealing from thermodynamics. At high temperatures, the molecules of a liquid freeze and crystallize, or metals cool and anneal. At high temperatures, the molecules of a liquid move freely with respect to one another. If the liquid is cooled slowly, thermal mobility is lost. The atoms are often able to line themselves up and form a pure crystal that is completely ordered over a distance up to billions of times the size of an individual atom in all directions. The crystal is the state of minimum energy for this system. The amazing fact is that, for slowly cooled systems, nature is able to find this minimum energy state. In fact, if a liquid metal is cooled quickly or "quenched," it does not reach this state but rather ends up in polycrystalline or amorphous state having somewhat higher energy.

So the essence of the process is slow cooling, allowing ample time for distribution of the atoms as they lose mobility. This is the technical definition of annealing, and it is essential for ensuring that a low energy state will be achieved. Although the analogy is not perfect, there is a sense in which most of the minimization algorithms(i.e. integer programming) correspond to rapid cooling or quenching.

The simulated annealing technique can be adapted to choose optimal tolerance allocation and manufacturing processes of minimum cost.

Three important aspects of simulated annealing are

- Boltzmann Probability distribution
- Metropolis algorithm
- Monte Carlo Simulation

Boltzmann probability distribution:

$$P(E) \propto e^{\frac{-E}{kT}}$$

The Boltzmann probability distribution expresses the idea that a system in thermal equilibrium at temperature T has its energy probabilistically distributed among all different states E. Even at low temperature, there is a chance, albeit very small, of a system being in a high energy state. Therefore, there is a corresponding chance for the system to get out of a local energy minimum in favor of finding a better, more global, one. The quantity k(Boltzmann's constant) is a constant of nature that relates temperature to energy. In other words, the system sometimes goes uphill as well as downhill; but the lower the temperature, the less likely is any chance of the algorithm going uphill.

Metropolis algorithm:

A simulated thermodynamic systems was assumed to change its configuration from energy E_1 to energy E_2 with probability $p = \exp\frac{(E_2 - E_1)}{kT}$. If $E_2 < E_1$, this probability is greater than unity; in such cases the change is arbitrarily assigned a probability $p = 1$, i.e., the system always took such an option. This general scheme, of always taking a downhill step while sometimes taking an uphill step, has come to be known as the Metropolis algorithm.

The following elements are required to make use of the Metropolis algorithm

- A description of possible system configurations
- A generator of random changes in the configurations; these changes are the "options" presented to the system.

- An objective function E (analogy of energy) whose minimization is the goal of the procedure.

A control parameter T (analog of temperature) and an annealing schedule which tells how its is lowered from high to low values, e.g., after how many random changes in configuration is each downward step in T taken, and how large is that step. The meaning of "high" and "low" in this context.

A random number, r, uniformly distributed between 0 and 1 is generated and compared with $P(E_2)$. If $r < P(E_2)$ then the new state is accepted anyhow; otherwise the old state is retained. The tolerance is reduced. The tolerance is reduced by choosing a sub-range which is generally 5-10% of the range in simulated annealing problems. In this paper the sub-range is taken to be 5% of the tolerance of individual components and the process continues until convergence is reached or the tolerance reaches zero.

Metropolis algorithm uses Boltzmann's probability distribution to test for the acceptance of the state and Monte Carlo simulation is used to generate different states.

Appendix D - Software development in AutoCAD

AutoCAD as Software Tool:

The development of the tolerance analysis module for AutoCAD is done on AutoCAD (release 13) using ARX, AutoCAD development system, AutoLISP and AutoCAD customization tools.

AutoCAD is chosen to automate the tolerance analysis and allocation mainly due to the following reasons:

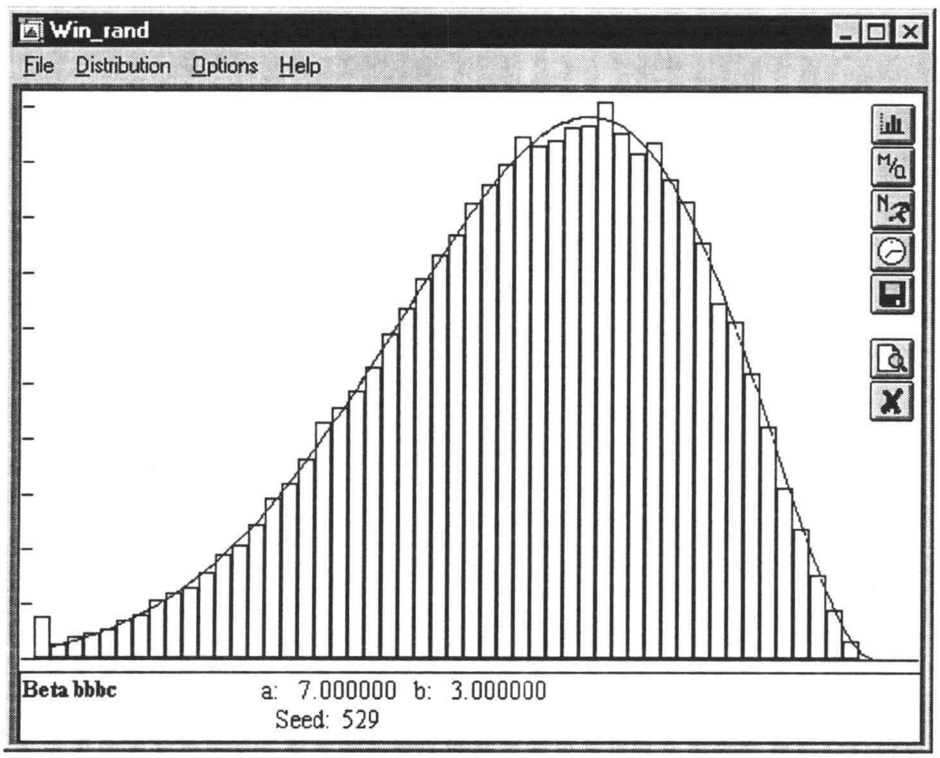
a. AutoLISP, a specialized implementation of the LISP programming language, is an integral part of AutoCAD. AutoLISP is very useful in manipulating the information of the entities stored in the AutoCAD database because the data is stored in the form of lists and AutoLISP is very effective to getting useful information from lists.

b. The AutoCAD development system programming interface lets the user to use high-level programming languages like C to develop customized applications. Therefore complicated and lengthy calculations like Monte Carlo calculations can performed using C/C++ language. We can design and implement dialogue boxes, similar to the ones employed by AutoCAD itself making the user interaction easy. Menu Customization can be used to tailor the AutoCAD interface to specific application.

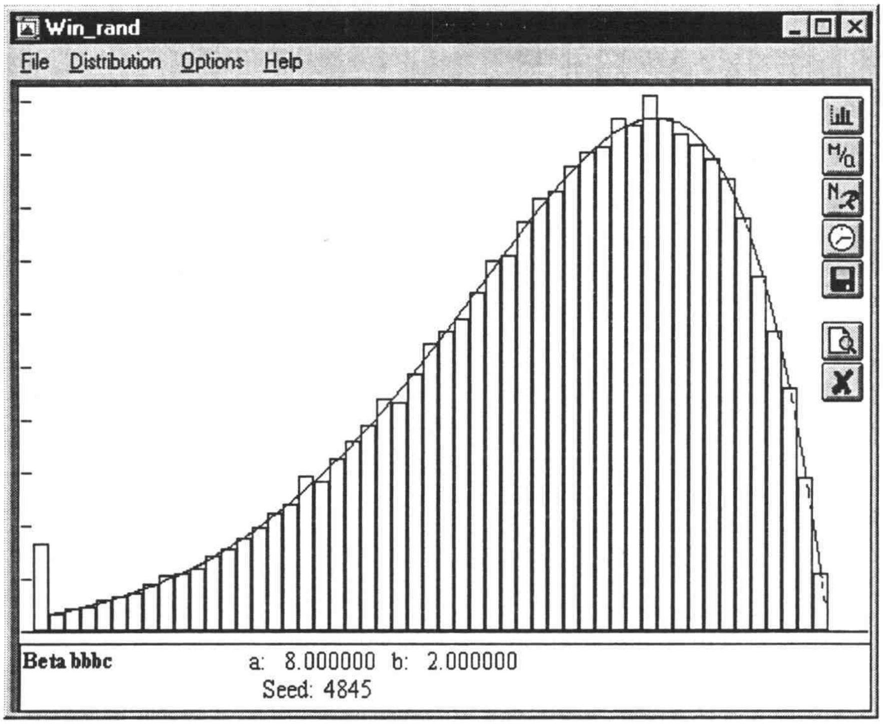
Appendix E - Beta Factors for components used for Tolerance Analysis

Beta factors and graphs:

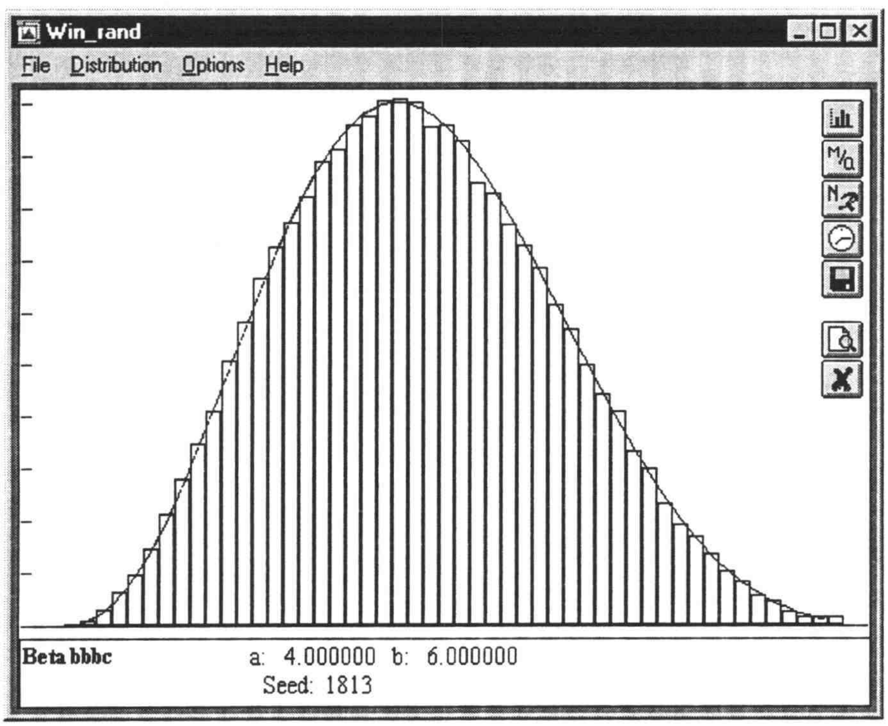
The following graphs show the random number generators used in the tolerance analysis for beta distribution generated by WinRand program. The beta distribution factors for components A, B, and C are a = 7.0 and b = 3.0. Fig 3.7 is the graph for unit beta random number generator for components A, B, and C. The beta distribution factors for components D and E are a = 8.0 and b = 2.0. Fig 3.8 is the graph for unit beta random number generator for components D and E. The beta distribution factors for components F and G are a = 4.0 and b = 6.0. Fig 3.7 is the graph for unit beta random number generator for components F and G.



Graph for unit random number distributor for Components A, B and C



Graph for unit random number distributor for Components D and E



Graph for unit random number distributor for Components F and G