

AN ABSTRACT OF THE THESIS OF

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Title: Channel Estimation in TDD MU-MIMO Systems using Sub-Linear Sparse Signal Recovery Algorithms

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Massive Multiple-User Multiple-Input Multiple-Output (MU-MIMO) wireless communication systems incorporate promising advanced strong technologies for upcoming 5G communications. To obtain some of the high spectrum and energy efficiencies bonuses brought by MU-MIMO systems, the ability to obtain Channel State information, especially on the receiver side (CSI), is important. To minimize the amount of overhead and the complexity caused by the more common Frequency Division Duplexing standard, a Time Division Duplex scenario will be considered. The scenario will exploit the sparsity of the CSI in the angular domain to leverage compressive sensing techniques for channel estimation. The characteristics of the DFT matrix will be exploited to recover major channel components with sub-linear computational complexity. Simulation results will demonstrate that the algorithm can, with relatively low error, estimate the major components within a channel matrix while reducing the complexity of calculations.

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Channel Estimation in TDD MU-MIMO Systems using Sub-Linear
Sparse Signal Recovery Algorithms

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David Barry, Author

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Chapter 1: Introduction

1.1 Challenges

Over the past twenty years, the simple multiple-input multiple-output technologies were studied. These technologies improved the system capacity of wireless communicating systems. They also improved their reliability. While initial MIMO work focused on point-to-point links, multi-user multiple-input multiple-output (MU-MIMO) is becoming a widely discussed topic. This move has allowed the study of a single base station with hundreds of antennas to be connected with many individual and simple users. This leads to two important results. One, the expensive multi-antenna equipment is only required at the base station, so the users only need cheap single-antenna devices. Two, Because of the system's user diversity, the system generally suffers less from the propagation environment the the point-to-point system does [1].

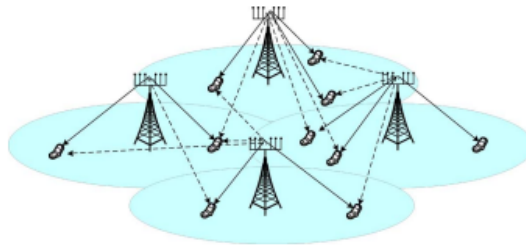


Figure 1.1: MU-MIMO System Illustration [1]

In general, the MU-MIMO format has become important in standards such as 802.11,

802.16, and LTE [1]. Most implementations only use a few antennas, but papers have proposed systems with well over 100 antennas. The 3 key reasons for this suggestion are that effects due to noise and small-scale fading are reduced/eliminated, the amount of users is independent from the size of a base station cell, and the required transmission energy trends toward a minuscule number. Of course these are idealistic in nature, but most of the effects can still be felt. This change in hardware structure and duplexing require development of new system algorithms to keep complexities low.

The goal of this thesis is to develop a method to acquire channel information at the base station side of the system while also keeping computational complexities below the linear scale. The common approach currently utilizes a Least Squares method [4],[5] which requires linear growth in both the computational and measurement complexity as the number of antennas increases. Another common approach is the Minimum Mean Square Error approach which has similar conditions [5],[6]. In this thesis we will discuss the sparsity of the channel propagation coefficient matrix and how we can use compressive sensing techniques to estimate the channel in a sub-linear time frame.

1.2 State of the Art

To cover these challenges, it is important to review similar discussions in other papers. In [7], Xiongbiao Rao, Vincent Lau, and Xiangming Kong employ a modified version of the Orthogonal Matching Pursuit (OMP) algorithm to the sparsity of a massive MIMO channel. This paper first identifies the common support within the assumed joint sparsity channel matrices. After the common support is identified, the individual remaining supports are identified to recover the whole support. The main theory of this method is again revised for the creation of a modified subspace pursuit algorithm in [8]. In

this documents, Xionbin Rao and Vincent Lau exploit the information quality and the support of the system to recover the channel matrix. In [9], Yang Nan, Li Zhang, and Xin Sun exploit the block sparsity of the channel matrix in TDD mode to recover the channel matrix. With an auxiliary block subspace pursuit algorithm and the path delay information acquired from the up-link signal, the authors propose a method to reduce the potential pilot overhead by near one-third.

Recent reports have made modifications to the standard Least Squared methods, Minimum Mean Squared Error Methods, or l_1 minimizations. [5] proposes an iterative search using a subgradient method. Similarly, [6] first compares the LS and MMSE methods before implementing a H-infinity approach for suppression of pilot contamination. [10] employs Multiple Shrinkage Factors to improve the LS approach.

Another common aspect for study is within methods to design pilot symbols in such a fashion as to reduce the amount needed. For example, in [11], compressive sensing theory is utilized to design the pilot symbols and their sequences in order to reduce the measurement and calculation complexities. In [12], You, Gao, Sindlehurst, and Zhong produce a plan to increase the amount of pilots by making the phase shifts adjustable. Also, Gao, Zhange, Dai, and Han present a spectrum-effecient channel estimation technique to reduce pilot overhead in [13].

Finally, this thesis would be remiss to not recognize the effort put forward by my colleagues in [14] and [15]. In Yichuan's thesis, the SPRIGHT framework presented in [16] was adapted and tested in terms of a TDD MU-MIMO system for channel estimation. This work is building directly off of that work's efforts. Meanwhile, [15] presented a Kalman filter approach to estimate the channel.

1.3 Proposed Method

The goal is to estimate the channel via uplink signals from mobile users in a sub-linear time frame. Functioning in TDD operation mode, the following work will take advantage of the intrinsic sparsity of the channel matrix and estimate the channel. The algorithm used will take advantage of the properties of the Discrete Fourier Transform matrix and Bipartite Graphs to separate the results into single, zero, and multi-ton bins based on how many relevant paths are within the channel matrix and affect that output. The algorithm will then use a peeling method based upon the known bipartite graph to estimate the single-ton bins and remove the critical pieces from multi-ton bins to create new single-ton bins. This methodology can be completed all within a sub-linear measurement philosophy and a sub-linear computational time.

1.4 Notation and Organization

Throughout this thesis, we use bold lowercase letters to represent vectors containing complex samples such as $\mathbf{x}$, with elements x_{ij} . Upper case letters represent matrices, X .

The thesis is organized as follows. Section 2 describes background information covering Telecommunication Systems, Wireless Channel Modeling, and Compressive Sensing. Section 3 continues by developing the system model in the angular domain. Finally, section 4 presents the channel estimation framework used and simulates the system providing detailed discussion on the results and their meanings.

Chapter 2: Background

2.1 Telecommunication Systems

Wireless communication has existed for nearly 120 years. In 1897, Marconi demonstrated wireless telegraphy. Following this demonstration, in 1901, radio reception reached across the Atlantic Ocean. Since this time frame, wireless systems have rapidly developed with some being iterated upon while others discarded for more efficient methods. In transmission systems, such as with the old-style television, the older wireless systems have been replaced for stronger wired systems. Meanwhile, telephones are leaving the wire-based home models to become easily transported wireless cellphones [2].

Cellular networks have developed to become a very unique and strong system, especially over the past twenty years. In general, cellular networks are pictured as hexagonal systems with a base station situated in the middle. This can be seen in Figure 1 below.

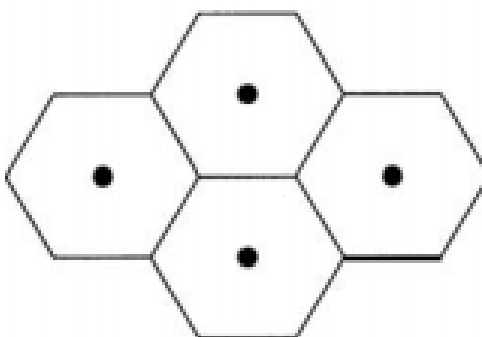


Figure 2.1: High Level Configuration of Cell Networks [2]

Cellular users are within the radius of each base station and connect to the base station when making a call. Because this is an ideal representation, this representation is not accurate. Due to costs, terrain changes such as mountains and other physical obstructions, base stations are placed irregularly throughout the landscape. The cell phone user would then be connected to the base station with the least interference, not based on a pre-defined radius as in the ideal model [2].

The communication between users and their base stations is a complicated transaction. There are downlink signals, from the base station to the cell phone user, and uplink signals, from the cell phone user to the base station. The downlink signals are sent out as a single signal: relying on cell phones to filter out or discriminate the signals meant for other users and obtain their own information. In the uplink process, each user sends a signal to the base station which is received as a combination of all the signals and any added noise [2].

To make matters more complicated, the mode of operation appears to be changing as well. Frequency division duplexing has been the standard and contains two frequency channels, one for uplink and one for downlink. In this thesis, we are assuming time division duplexing which only requires a single frequency band where the signals can be sampled regularly.

2.2 Wireless Channel Physical Modeling

2.2.1 Channel Fading

In reality, when discussing and modeling the characteristics of a cellphone network, there are specific issues and ideas that must be taken into account. It is key to understand

channel fading, grouped into large-scale and small-scale components, when discussing wireless communication. Large-scale fading occurs due to shadowing from large objects and other forms of path loss over scales the size of a single base station cell. It is usually only worried about when constructing base stations, but will still be added into the simulations here. Small-scale fading on the other hand is important for techniques which will be used in the future model. Small-scale fading takes into account constructive and destructive interference and other problems that occur at the scale of the carrier wavelength. In figure 2, the low frequency movement represents large-scale fading while the high-frequency effects represent the small-scale fading [2].

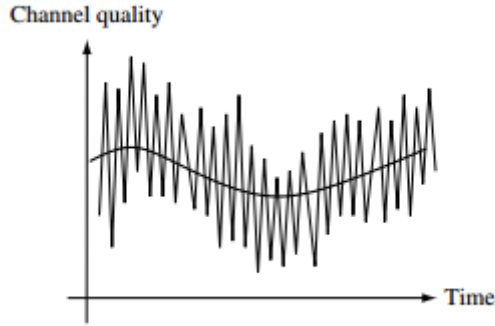


Figure 2.2: Graphical Representation of Fading [2]

2.2.2 Multipath Fading

A large amount of small-scale fading is caused due to multipath fading. In the real world, the electromagnetic signals are dispersed by the mobile user and reflected off objects in the environment. These signals can be reflected off of walls, buildings, etc. As the signal is reflected, part of the signal might reflect onto the proper path toward the receiver.

This reflection adds onto part of the direct waveform, creating a sum of the direct and the reflected waveform. Although the two signals are added, they have distinct phases. This is important because the two waves can add constructively, increasing the signal strength, or they could add destructively, decreasing the signal strength. How the waves add is purely based on the phase difference between the two wave forms.

Based on the difference between propagation delays, the delay spread, T_d , is important for determining the effects of the reflected wave onto the transmitted wave. If the frequency changes less than the coherence bandwidth, given by $\frac{1}{T_d}$, the constructive/destructive interference does not affect the signal noticeably.

2.2.3 Baseband Equivalent Model

Since the reflected paths superimpose onto the transmitted signal, the receiving antenna will receive summed replicas of the original signals. Furthermore, the variety of paths taken present a large assortment of attenuation factors and propagation factors. Usually the attenuation factor is due to the antenna patterns, reflections, and the distance the signal traveled. To mathematically describe this model, we call the attenuation factors a_i , the propagation delays τ_i , the signal x , and the received signal y . The following equation can be used to represent the baseband equivalent received signal without locally generated noise

$$y(t) = \sum_i a_i(t)x(t - \tau_i(t)) \quad (2.1)$$

The frequency independence is assumed because transmission is generally over a narrow frequency band when compared to the carrier frequency. As it turns out the

baseband equivalent signal is composed of not only real components, but also imaginary components. This involves a simple change as seen in the channel equation below

$$y(t) = \sum_i a_i(t)x(t - \tau_i(t))e^{-j2\pi f_c \tau_i(t)} \quad (2.2)$$

Furthermore, a Doppler shift can be considered by incorporating mobility by the user. This is seen by subtracting a component of

$$f_d(t - \tau_i(t)), \text{ where } f_d = \frac{v}{c}f_c \quad (2.3)$$

from the carrier frequency. This incorporates the slight change in frequency known as the Doppler shift. In the calculation, v is the movement speed of the mobile user, c is the speed of light, and f_c is the carrier frequency.

2.2.4 Rayleigh Fading

The Rayleigh fading model will be used in this thesis to discuss the compressive sensing application of channel recovery. The Rayleigh model is useful for its simplicity in normal cellular situations. It states that the gains of the signal will be circularly symmetric complex Gaussian random variables. This model is predicated on a large number of independent paths due to reflection and scattering as well as random amplitudes. Since the distance between the user and the reflectors is assumed to be much larger than the carrier wavelength, $\lambda = \frac{c}{f_c} \ll d$, it can be assumed that each path phase is uniformly distributed between zero and 2π . It is also safe to assume that the phase between each path is independent. This means we can represent the model as a circular symmetric complex random variable. Using the Central Limit Theorem, the system can be modeled

as a zero-mean Gaussian random variable.

2.3 Compressive Sensing

2.3.1 The Problem

In the future algorithm discussion, we will be making use of the potential sparsity inherent in the channel propagation coefficient matrix. Before the sparsity algorithm is used, the terms and techniques of sparsity must be addressed. In this case, we formulate a system defining equation in the form

$$Y = AX \tag{2.4}$$

with $Y \in R^{M \times 1}$ containing the received information, $X \in R^{N \times 1}$ containing the sparse signal, and $A \in R^{M \times N}$ containing the measurement matrix. X is considered κ -sparse if it contains κ nonzero values while the rest are zero. If the rest of the values are, instead, close to zero, X is considered compressible. The goal of compressive sensing is to use the inherent compressibility or sparsity of X and the ability to control A to manipulate and recover X in a below-linear time space, with $M < N$.

A simple solution to finding X is to use the N-by-N identity matrix for A to measure each value individually. As the amount of measurements in X increase, A also increases in size at a 1-to-1 rate. In the real world, this means more money is spent in constructing the system to add more measurement sensors or antennae. It is preferable to design the system so that the amount of measurements (M) required is less than the signal size (N).

2.3.2 Stable Measurements

Because there are fewer measurements than signal length, the initial problem seems poorly conditioned in its ability to recover X . To ensure our ability to recover the signal, the system must be proven to be stable and properly conditioned. The main property for showing this proof is the restricted isometry property, often labeled as RIP. As long as the system meets the RIP, introduced in [17], the system is well conditioned. This means that, for some $\epsilon \in (0, 1)$,

$$1 - \epsilon \leq \frac{\|Az\|_2}{\|z\|_2} \leq 1 + \epsilon \quad (2.5)$$

must hold true for some arbitrary κ -sparse vector, z . As it turns out, RIP is achieved, with high probability if A incorporates a probability density function with independent and identically distributed Gaussian variables.

2.3.3 Reconstruction Algorithms

The most basic reconstruction algorithm uses convex optimization. This means the use of l_p norms defined as

$$\|x\|_p^p \equiv \sum_{i=1}^N |x_i|^p \quad (2.6)$$

To solve for Y , a search over the vector norms needs to be completed. The conditions are given by

$$x_{est} = \operatorname{argmin} \| \hat{x} \|_p \quad (2.7)$$

$$\text{such that } A\hat{x} = y \quad (2.8)$$

Using an l_2 norm presents a simple solution to the previous problem, but rarely returns a sparse solution. The l_0 norm usually recovers the exact κ -sparse solution, but requires a search over all $\binom{n}{\kappa}$ locations of nonzero entities in X . Finally, using the l_1 norm will recover the approximate signal with high probability, but requires computational complexity of $O(N^3)$ [18].

Chapter 3: System Modeling

Before sparse signal techniques can be employed, it is important to prove that the channel matrix is sparse. To begin, a simple line of sight single-input multiple-output and line of sight multiple-input single-output systems will be analyzed. The system will then change to be discussed with physically separated antennas, angular resolvability, and the addition of a reflected path. The channel propagation coefficient matrix will be modeled into the angular domain where its sparsity is proven before finally discussing the intricacies of the model used.

3.1 Angular Domain Derivation

The first place to begin when attempting to prove the sparsity of the MU-MIMO system in the angular domain is to start with a Line of Sight single-input, multiple-output (SIMO) system. At this early stage, some of the initial terms used further in the discussion will be defined and some important properties will be discussed.

In this first model, there are no scatterers. As its name implies, the only path between the transmitting antennas and the receiving antennas is the straight line between the two nodes. Also, the receiving antenna is made up of multiple separate antennas, evenly spaced within a linear array. The distance, d , between the receiving and transmitting antennas is much larger than the separation between the antennas within the antenna array. The separation within the array is given by $\Delta_r \lambda_c$. Δ_r is the separation normalized to the carrier wavelength, λ_r .

Using the assumption that the attenuation path, a , remains constant and that d is significantly less than the speed of light divided by the transmission bandwidth, $d \ll \frac{c}{W}$, the baseband channel gain can be given by [2]

$$g_i = a \exp\left(-\frac{j2\pi d_i}{\lambda_c}\right) \quad (3.1)$$

Using this channel gain, the line of sight SIMO channel baseband equivalent received signal can be written as

$$\mathbf{y} = \mathbf{g}x + \mathbf{w} \quad (3.2)$$

In this case x is the transmission signal, $\mathbf{y}$ is the received vector, $\mathbf{g}$ is the vector of baseband channel gains, and $\mathbf{w}$ is circular Gaussian noise vector. Assuming the distance between transmit and receive antennas is much larger than the antenna array, $d \gg \Delta_r \lambda_c n_r$, the paths from the transmitting antenna to the receiving antenna can be treated as first-order parallel.

$$d_i = d + (i - 1)\Delta_r \lambda_c \cos\phi, \quad i = 1, \dots, n_r \quad (3.3)$$

d can be considered the distance between the first receiving antenna and the transmitting antenna while ϕ is the angle of incidence. To make things simple, $\cos\phi$ shall be called Ω . The spatial signature, $\mathbf{g}$ can be given by

$$\mathbf{g} = a \exp\left(-\frac{j2\pi d}{\lambda_c}\right) \begin{bmatrix} 1 \\ \exp(-j2\pi\Delta_r\Omega) \\ \vdots \\ \exp(-j2\pi(n_r - 1)\Delta_r\Omega) \end{bmatrix}, \quad (3.4)$$

where n_r is the number of receive antennas. For simplicity, the unit spatial signature vector can be given by

$$\mathbf{e}_r = \frac{1}{\sqrt{n_r}} \begin{bmatrix} 1 \\ \exp(-j2\pi\Delta_r\Omega) \\ \vdots \\ \exp(-j2\pi(n_r - 1)\Delta_r\Omega) \end{bmatrix} \quad (3.5)$$

A similar proof can be shown to obtain the same results for Line of Sight multiple-input, single-output (MISO) channels. The only difference would be to change Δ_r with Δ_t and n_r with n_t , where n_t is the number of transmit antennas.

If the problem is then combined to form a Line of Sight MIMO channel with similar arrays for the receiving and transmitting antennas, it can then be shown that the channel matrix, G can be given by

$$G = a \exp\left(-\frac{j2\pi d}{\lambda_c}\right) \begin{bmatrix} 1 \\ \exp(-j2\pi\Delta_r\Omega) \\ \vdots \\ \exp(-j2\pi(n_r - 1)\Delta_r\Omega) \end{bmatrix} \begin{bmatrix} 1 \\ \exp(-j2\pi\Delta_t\Omega) \\ \vdots \\ \exp(-j2\pi(n_t - 1)\Delta_t\Omega) \end{bmatrix}^* \quad (3.6)$$

$$= a\sqrt{n_r n_t} \exp\left(-\frac{j2\pi d}{\lambda_c}\right) \mathbf{e}_r \mathbf{e}_t^*, \quad (3.7)$$

where $\mathbf{e}^*$ is the complex conjugate transpose of $\mathbf{e}$ and a unique non-zero singular value given by $a\sqrt{n_r n_t}$. Since there is only a single-dimension space, there is only one spatial degree of freedom. The columns of the channel coefficient matrix are all along the direction as defined by $\mathbf{e}_r$. Although the power gain increases, the most important aspect to be dealt with is the degrees of freedom. To increase the amount of degrees of freedom, we will now consider a system where the transmitting antennas are placed significantly farther apart. Now, assuming there are two transmitting antennas, the spatial signature can be given by

$$\mathbf{g} = a_k \sqrt{n_r} \exp(-\frac{j2\pi d_k}{\lambda_c}) \mathbf{e}_r(\Omega_{rk}), \quad k = 1, 2 \quad (3.8)$$

The spatial signature $\mathbf{e}_r$ is a periodic function of Ω with period $\frac{1}{\Delta_r}$. This means that G has linearly independent columns as long as the difference in directional cosines is not a multiple of $\frac{1}{\Delta_r}$, or

$$\Omega_{r2} - \Omega_{r1} \neq \frac{c}{\Delta_r} \quad (3.9)$$

Because the directional cosines exist between $[-1, 1]$ and the difference cannot be greater than 2, equation (3.9) can be simply reduced to the antenna spacing $\Delta_r \leq \frac{1}{2}$ when the directional cosines are not equal. In terms of resolvability and conditioning, the channel matrix, G , is heavily dependent on how aligned the spatial signatures are.

If we denote the angle between the two spatial signatures as

$$|\cos\theta| := |\mathbf{e}_{r1}^* \mathbf{e}_{r2}| \quad (3.10)$$

and realize that the right hand side of the equation only relies on $\Omega_r := \Omega_{r1} - \Omega_{r2}$, then

we can define

$$f_r(\Omega_r) = \mathbf{e}_{r1}^* \mathbf{e}_{r2} = \frac{1}{n_r} \exp(j\pi \Delta_t (n_r - 1)) \frac{\sin(\pi L_r \Omega_r)}{\sin(\pi L_r \Omega_r / n_r)} \quad (3.11)$$

In this case, L_r is defined as the normalized length of the antenna array, $n_r \Delta_r$. Similar to $\mathbf{e}_r$, f_r is periodic based around $2/L_r$ and multiples of $1/\Delta_r$. To determine if a matrix is ill conditioned the following is true.

$$|\Omega_r - \frac{c}{\Delta_r}| \ll \frac{1}{L_r} \quad (3.12)$$

Knowing that Ω_r exists between $[-2, 2]$, the condition can reduce to

$$|\Omega_r| \ll \frac{1}{L_r} \quad (3.13)$$

when the antenna separation, Δ_r , is less than $\frac{1}{2}$. Given that any received signal within $\frac{1}{L_r}$ contributes to the same direction, adding more transmitting antennas within the same amount of space can only contribute to the resolvability of the system to a certain limit. This limit is inherently linked to the size of the array.

This problem of an ill conditioned channel matrix and a low number of degrees of freedom can be resolved via the simple inclusion of the reflected path. If we sample the angular domain at fixed angular spacings of $1/L_t$ and $1/L_r$ at the transmitter and receiver, respectively.

Assuming there are i paths, the angles made with the transmitting and receiving arrays will be denoted by Ω_{ti} and Ω_{ri} , respectively. The channel matrix can then be given by

$$G = \sum_i a_i^b \mathbf{e}_r(\Omega_{ri}) \mathbf{e}_t^*(\Omega_{ti}) \quad (3.14)$$

where a_i^b is defined by

$$a_i^b \equiv a_i \sqrt{n_r n_t} \exp\left(-\frac{j2\pi d_i}{\lambda_c}\right) \quad (3.15)$$

Using f_r from earlier, we can determine the orthonormal basis for the received and transmitted signal in the angular domain.

$$\begin{aligned} \delta_r &:= \left[\mathbf{e}_r(0), \quad \mathbf{e}_r\left(\frac{1}{L_r}\right), \quad \dots, \quad \mathbf{e}_r\left(\frac{n_r-1}{L_r}\right) \right] \\ \delta_t &:= \left[\mathbf{e}_t(0), \quad \mathbf{e}_t\left(\frac{1}{L_t}\right), \quad \dots, \quad \mathbf{e}_t\left(\frac{n_t-1}{L_t}\right) \right] \end{aligned} \quad (3.16)$$

To finally represent everything in the angular domain, a subscript 'a' will be used to denote the angular vectors of each appropriate term. U will be used to represent the $n \times n$ unitary matrices of which the columns are the vectors in δ . The $(k, l)^{th}$ entry of U is

$$\frac{1}{\sqrt{n}} \exp\left(\frac{-j2\pi kl}{n}\right) \quad (3.17)$$

Using the standard matrix setup, $\mathbf{y} = G\mathbf{x} + \mathbf{w}$, we can craft the angular representation of the channel matrix.

$$\begin{aligned}
\mathbf{x}_a &:= U_t^T \mathbf{x} \\
\mathbf{y}_a &:= U_r^T \mathbf{y} \\
&= U_r^T (G\mathbf{x} + \mathbf{w}) = U_r^T G U_t \mathbf{x}_a + U_r^T \mathbf{w} \\
&= G_a \mathbf{x}_a + \mathbf{w}_a
\end{aligned} \tag{3.18}$$

With the assumption that there are only a select few local scatterers, only a few angular bins contain active paths. This allows the approximation that G_a is sparse.

3.2 Assumptions

Now that the channel matrix has been expressed in the angular coordinates, let us discuss the channel model. The first assumption being made is that we assume TDD operations and perfect synchronization between the base station of interest and the other base stations.

For a multi-cell scenario, the $M \times K$ channel propagation coefficient matrix of the l^{th} cell as affected by the j^{th} cell is given by

$$\sum_{j=1}^L G_{lj} = \sum_{j=1}^L H_{lj} B_{lj}^{1/2}, \quad l = 1, \dots, L \tag{3.19}$$

where H_{lj} describes the small-scale fading coefficient matrix while B_{lj} is the large-scale fading coefficient matrix, including path loss and shadowing. B_{lj} can also be described as $L_{lj} D_{lj}$ for the path loss and shadowing aspects. D_{lj} is made up of diagonal elements that have a lognormal distribution and pdf of

$$f_D = \frac{\exp(-\frac{(\ln(d)-\mu)^2}{2\sigma^2})}{\sqrt{2\pi}\sigma d}, \quad d > 0 \quad (3.20)$$

where μ and σ are in dB. This means that B_{lj} is a diagonal matrix. The received vector at the base station of the l^{th} cell, $y_l(n)$, can be rewritten as

$$\mathbf{y}_l(n) = \sqrt{P_u} \sum_{j=1}^L G_{lj}(n) \mathbf{x}_j(n) + \mathbf{w}_l(n) \quad (3.21)$$

where P_u is the average mobile user (MU) transmitted power, $\mathbf{x}_j$ is the MU signal vector, and $\mathbf{w}_j$ is zero-mean white complex Gaussian noise vector. Using this knowledge, we need to estimate the channel matrix within a coherence time interval. Assuming a 7-cell scenario, with the cell of interest in the middle and a constant pilot sequence used by all cells, $x_j(n) = x_p(n)$ for $j = 1, 2 \dots 7$ and $n = 0, 1, \dots, N_s - 1$, where N_s is the length of the pilot sequence, we can write $\mathbf{y}_l(n)$ as

$$\mathbf{y}_l(n) = \sqrt{P_u} \left[\sum_{j=1}^L G_{lj}(n) \right] \mathbf{x}_p(n) + \mathbf{w}_l(n) \quad (3.22)$$

During the coherence time period, the channel matrix is assumed to be constant, i.e. $G_{lj}(n) \cong G_{lj}$ for $n = 0, 1, \dots, N_s - 1$. We can write $\mathbf{y}_l(n)$ as

$$\mathbf{y}_l(n) \cong \sqrt{P_u} \left[\sum_{j=1}^L G_{lj} \right] \mathbf{x}_p(n) + \mathbf{w}_l(n) \quad (3.23)$$

Furthermore, during the coherence time period, we can write the received signal block as

$$\begin{bmatrix} \mathbf{y}_l(0) & \mathbf{y}_l(1) & \dots & \mathbf{y}_l(N_s - 1) \end{bmatrix} \quad (3.24)$$

$$\begin{aligned}
&= \sqrt{P_u} \begin{bmatrix} \sum_{j=1}^L G_{lj} \mathbf{x}_p(0) & \sum_{j=1}^L G_{lj} \mathbf{x}_p(1) & \dots & \sum_{j=1}^L G_{lj} \mathbf{x}_p(N_s - 1) \end{bmatrix} \\
&+ \begin{bmatrix} \mathbf{w}_l(0) & \mathbf{w}_l(1) & \dots & \mathbf{w}_l(N_s - 1) \end{bmatrix}
\end{aligned} \tag{3.25}$$

Let

$$\begin{aligned}
Y_l &= \begin{bmatrix} \mathbf{y}_l(0) & \mathbf{y}_l(1) & \dots & \mathbf{y}_l(N_s - 1) \end{bmatrix} \\
X_p &= \begin{bmatrix} \mathbf{x}_p(0) & \mathbf{x}_p(1) & \dots & \mathbf{x}_p(N_s - 1) \end{bmatrix} \\
W_l &= \begin{bmatrix} \mathbf{w}_l(0) & \mathbf{w}_l(1) & \dots & \mathbf{w}_l(N_s - 1) \end{bmatrix},
\end{aligned} \tag{3.26}$$

then

$$Y_l \cong \sqrt{P_u} G_l X_p + W_l \tag{3.27}$$

where $G_l \equiv \sum_{j=1}^7 G_{lj}$ and the matrix X_p is given by

$$X_p = \begin{bmatrix} x_{p,1}(0) & x_{p,1}(1) & \dots & x_{p,1}(N_s - 1) \\ x_{p,2}(0) & x_{p,2}(1) & \dots & x_{p,2}(N_s - 1) \\ \vdots & \vdots & \ddots & \vdots \\ x_{p,K}(0) & x_{p,K}(1) & \dots & x_{p,K}(N_s - 1) \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{p,1}^T \\ \mathbf{x}_{p,2}^T \\ \vdots \\ \mathbf{x}_{p,K}^T \end{bmatrix} \tag{3.28}$$

where $\mathbf{x}_{p,k}$ is the pilot assigned to the mobile user $k, k = 1, 2, \dots, K$.

In this case, the assumed perfect synchronization actually presents the worst case scenario for interference from the surrounding cells. To show that this scenario does not contain a large interference from the surrounding cells, even when all pilot sequences are being broadcasted at the same time, the singular values from the generated matrices were calculated using $B_{lj}^{1/2} = I_K$ for the cell of interest and $B_{lj}^{1/2} = .1I_K, .08I_K, .05I_K, .02I_K$ for the surrounding cells where I_K is the K -dimensional identity matrix. Those values are

contained in the table below and serve to show the lack of interference from surrounding cells.

Table 3.1:

G_u	$G_{lj}, B_{lj}^{1/2} = .1I_K$	$G_{lj}, B_{lj}^{1/2} = .08I_K$	$G_{lj}, B_{lj}^{1/2} = .05I_K$	$G_{lj}, B_{lj}^{1/2} = .02I_K$
7.95	2.90	2.32	1.45	0.58
6.55	1.83	1.46	0.91	0.37
5.68	1.41	1.13	0.70	0.28
5.13	1.20	0.96	0.60	0.24
4.53	1.00	0.80	0.50	0.20
4.08	0.79	0.63	0.39	0.16
3.48	0.62	0.49	0.31	0.12
2.57	0.46	0.37	0.23	0.09

Finally, we will present an algorithm which is based in sparse signal processing techniques and will aim to estimate the small scale fading coefficient matrix H_u in one cell despite pilot contamination and circular Gaussian noise.

Chapter 4: Proposed Channel Estimation

The proposed algorithm we will discuss utilizes Sparse-Bipartite Graphs to separate and highlight the sparse locations within the channel. Assuming the problem being solved is in the form of $\mathbf{Y} = \mathbf{A}\mathbf{X} + \mathbf{W}$, our goal will be to create a matrix $\mathbf{A}$ such that the sparse components in $\mathbf{X}$ can be located and identified in a sub-linear time frame.

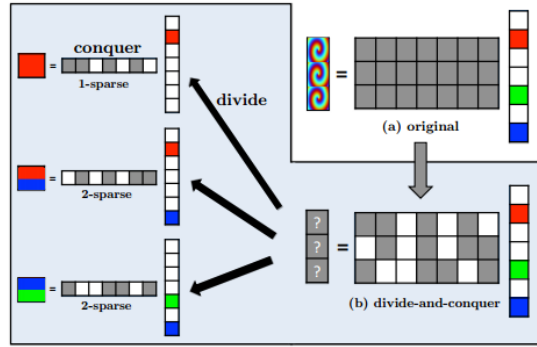


Figure 4.1: Divide and Conquer Strategy [3]

To perform this feat, a "divide and conquer" strategy will be performed. The basics of this strategy are shown in Figure 4.1. As shown, the red, green, and blue components are the non-zero values in $\mathbf{X}$. The grey components in $\mathbf{A}$ are used to represent any design. In part (a), the resulting measurements of $\mathbf{Y}$ are combinations of the components in $\mathbf{X}$.

The key to the proposed design is to introduce zeros into the measurement matrix as shown in part (b). Using the created design and specific rule sets, the resulting measurements can be inspected to find "one-ton" measurements, or measurements with only a single value reflected in the output. Using the calculated location, calculated value,

and designed measurement matrix, each value can be removed from other measurement results, forming new "one-ton" measurements.

In Figure 4.1, it is determined that the first output bin only has the red value of $\mathbf{X}$ within it. It is also known from the construction of $\mathbf{A}$ that the second measurement contains the red value, so the red value is removed from the second measurement output leaving only the blue value. This process is repeated with the blue component of $\mathbf{X}$ to isolate the green value.

Before continuing, it is important to discuss how these matrices relate to the MU-MIMO systems discussed and derived earlier. In affect, the designed $\mathbf{A}$ matrix will be replaced by the pilot sequence produced from the users. The $\mathbf{X}$ matrix is replaced by the channel matrix.

4.1 Ideal Algorithm Variation

4.1.1 Measurement Design

To understand the algorithm more completely, our goal is to construct the measurement, or pilot sequence, for the noiseless scenario. To perform this, two matrices will be needed: a sparse binary matrix with a known pattern and an n -sized DFT matrix. For the ideal case, only the first two rows will be needed from the DFT matrix. The two matrices might look something like the following:

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ w^0 & w^1 & w^2 & w^3 \end{bmatrix} \quad (4.1)$$

with $\mathbf{w} = e^{i\frac{2*\pi}{n}}$. In this example, we are discussing 4 users in the system. If we let the sparse binary matrix be generated randomly with a regular graph ensemble of degree two, the redundancy parameter as calculated in [3] implies that the minimum amount of rows needed to accurately recover the channel matrix is 2 multiplied by the number of nonzero components.

$$A^T = (H \odot S)^T = \begin{bmatrix} 1 & w^0 & 0 & 0 & 1 & w^0 & 0 & 0 \\ 0 & 0 & 1 & w^1 & 1 & w^1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & w^2 & 1 & w^2 \\ 1 & w^3 & 1 & w^3 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.2)$$

To create the pilot sequence signal, the next step is to use a row-tensor product. As explained in [3], this means multiplying the matrix $\mathbf{S}$ component-by-component with each row of the $\mathbf{H}$ matrix. The resulting matrix is displayed in equation 4.2.

4.1.2 Bin Definition

After the users send the pilot sequence to the base station, it is important that the base station has the capability of determining which MU sent a signal and the strength of the channel the signal traversed. To recover this information, the output measurement

matrix, $\mathbf{Y}$, should be separated into 'Bins.' These bins will be separated by each row of the binary matrix $\mathbf{H}$. For the ideal scenario, each bin will contain two outputs.

The bins can then be classified into zero-ton, single-ton, or multi-ton bins which contain no channel path, one channel path, or multiple channel paths. As we continue to step through the example, the definition of the bins and their uses will be revealed. First, let us create a $m = 1, n = 4$ channel matrix (1 antenna and 4 users).

$$G^T = \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} \quad (4.3)$$

If we continue by running the pilot sequence through the channel matrix, the output appears as shown in equation 4.3.

$$Y = G^T * A^T = \begin{bmatrix} 1+1 & w^0 + w^3 & 1 & w^3 & 1 & w^0 & 0 & 0 \end{bmatrix} \quad (4.4)$$

In this case, every two columns in $\mathbf{Y}$ make up a bin. The first bin contains contributions from both nonzero components of the channel matrix; therefore, it is considered a multi-ton bin. Both the second and third bin contain only one contribution from the channel matrix, so they are only single-ton bins. Finally, the last bin contains no contributions, so it is a zero-ton bin.

4.1.3 Bin Detection and Peeling

The most important part in this process is to be able to identify the type of bin, recovery the location of single-ton bins, and recover the value contained within a single-ton bin. Since the system is noiseless right now, it will be easy to classify the output bins.

- **Zero-ton Test:** $\|y_r\|^2 = 0$.

- **Single-ton Test:** There are two conditions that must be met. The first is the magnitudes must be equal or $|y_r(1)| = |y_r(0)|$. The second condition is that the angle division between the two bin components must be a factor of $\frac{2\pi}{n}$ or $\angle \frac{y_r(1)}{y_r(0)} = 0 \bmod \frac{2\pi}{n}$.
- **Multi-ton Test:** Obviously, if the bin does not meet any of the previous conditions, the bin can be considered multi-ton.

These tests are heavily reliant on vector addition. Because the second component of all bins will be the addition of vectors (magnitudes at differing angles), the resulting vector will not maintain the same magnitude as the first component. This is similarly true for the angle tests which acts as a redundancy condition in case a rare exception occurs.

Now that the conditions have been officially laid out, the example can be continued showing the peeling process of the algorithm. The first step is to inspect the first bin $[2 \ 1 - 1j]$. Obviously, this bin does not have zero energy and it can be shown that it will not meet the single-ton tests, so the bin is marked as a multi-ton bin and left for later.

The next bin is calculated as $[1 \ -1j]$ which meets both the angle and magnitude tests. To calculate the location and the magnitude of the value in $\mathbf{G}$, we can use the following equations.

$$\hat{k} = \frac{n}{2\pi} \angle \frac{y_r(1)}{y_r(0)}, \quad x(\hat{k}) = y_r(0) \quad (4.5)$$

Using this previous information and the knowledge of the binomial matrix used to construct $\mathbf{A}$, $\mathbf{H}$, we know one of the significant locations in $\mathbf{G}$ and can remove this factor from both the first and second output bins. It follows that both the first and third bin are the only single-ton bins remaining. They will also both recover the same location and

value, recreating the channel matrix. Once all multi-ton bins have been eliminated and all single-ton bins have been used to obtain channel estimates, the peeling and decoding of the system is finished.

4.2 Noisy Recovery Variation

Now that the general process and steps have been laid out in the non-complicated noiseless scenario, the sub-linear scenario will be dealt with. Because the same outline is followed as in the noiseless scenario, only noticeable changes need to be discussed in the coming algorithm discussion.

4.2.1 Measurement Design

The first major change is in the formation of the $\mathbf{S}$ matrix. The $\mathbf{S}$ matrix will still pull from the DFT matrix, but it will pull more rows in separate variations. $\mathbf{S}$ will now be made up of smaller sub-matrices pulled from the DFT: these will be called $\mathbf{S}_r$.

$$S = [S_0^T S_1^T S_2^T \dots S_{r-1}^T]^T \quad (4.6)$$

Each of the sub-matrices will contain $Q = O(\log^{0.3}(n))$ rows from the DFT matrix as defined in Definition 5 from [3]. Each of the submatrices will start at a random row in the DFT matrix and pull out every 3^r row until Q rows have been pulled out.

$$S_r = \begin{bmatrix} 1 & w^{l_r} & \dots & w^{n^*l_r} \\ 1 & w^{l_r+3^r} & \dots & w^{n^*(l_r+3^r)} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & w^{l_r+(Q-1)3^r} & \dots & w^{n^*(l_r+(Q-1)3^r)} \end{bmatrix} \quad (4.7)$$

4.2.2 Bin Detection

There are changes in the bin detection tests as well. The first change is that a single-ton search is included in the bin detection. This will be covered in section 4.2.3.

- **Zero-ton Test:** $\frac{1}{P} \| y_r \|^2 \leq (1 + \gamma)\sigma^2$.

The zero-ton search now has to account for noise. To do this, the average energy level in the bin is measured and compared against the expected noise variance multiplied by some constant where $\gamma \in (0, 1)$.

- **Single-ton Search:** Estimates the pair $(\hat{k}, x(\hat{k}))$ on the assumption the bin is single-ton.
- **Single-ton Verification:** Verifies the pair recovered in the search is single-ton or not. $\frac{1}{P} \| y_r - x(\hat{k})s_{\hat{k}} \|^2 \leq (1 + \gamma)\sigma^2$
- **Multi-ton Test:** If the bin does not meet the single-ton verification or zero-ton verification conditions, the bin can be considered multi-ton.

Since the system is now noisy, there is not a clean way to determine whether a bin is single or multi-ton. The solution now discussed is to treat all non-zero-ton bins as single-ton bins. After solving for the theoretical location and value, the output is checked

to determine if the recovered pair is indeed the only values contributing to the output. If it is not, the bin is disregarded and returned to later after more peeling has occurred.

4.2.3 Single-ton Search

The reason why the $\mathbf{S}$ matrices were changed is revealed in the Single-ton Search. In particular, $\mathbf{S}$ presents a Fourier structure that can be leveraged to estimate k . In general, estimating k can be treated as estimating the frequency of a sinusoid. Each measurement set associated with each sub-matrix $\mathbf{S}_r$ can be associated with some frequency $w_r = \frac{2\pi k}{n} * 3^r$ and amplitude $X(k)$.

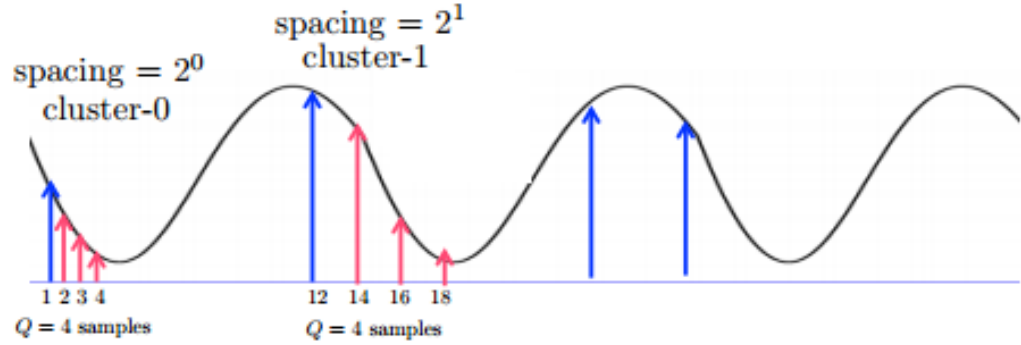


Figure 4.2: Frequency Visualization [3]

Figure 4.2 helps visualize a single-ton output bin as a sinusoid. Each cluster is defined by the sub-matrix $\mathbf{S}_r$ and the spacing is defined by 3^r (or 2^r as shown in Figure 4.2).

To estimate this frequency, it is important to take advantage of an efficient linear estimator. This estimator can be seen in Lemma 4 of [3].

To reach the final estimate of the location k , the linear estimation process must occur. First, the frequency obtained from the linear estimator must have its modulus

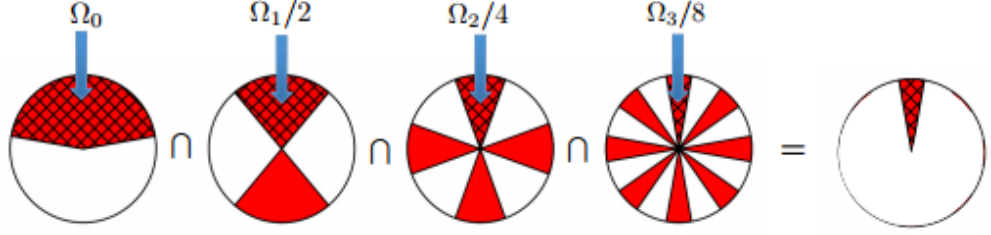


Figure 4.3: Frequency Combination [3]

taken by 2π . Then, depending on the the sub-matrix, the frequency must be unwrapped into multiple smaller slices as visualized in Figure 4.3. Each unwrapped frequency has "certainty regions" that add up to about π , but when combined with higher frequency estimations, a thin certainty region can emerge.

After finding the location $\hat{k}$, the corresponding channel can be calculated through a search over a predetermined set, Π . This is done by performing the below minimization

$$\hat{x}(\hat{k}) = \min_{x \in \Pi} \|a_{\hat{k}} - x\|^2 \quad (4.8)$$

$$\text{where } a_k = \frac{s_k^* y}{\|s_k\|^2} \quad (4.9)$$

A discussion for a future time would be on how to form the library Π . In general, a base station can be tuned by expected user counts and expected estimated ranges to form this library. As the user count drops, the library would need to lower the amount of values it searches over and compensate by using more power to broadcast its response over a wider area.

4.2.4 Adjustments

The previous changes can be made quite successfully to the algorithms, but a problem is faced when the channel estimate is made with a low amount of users, 'n'. The major problem is caused by the probability that the location of a single value falls outside the estimated zone: this probability is of at least $1 - O(1/n)$. This sort of problem can appear in significantly low-population density zones or in small size simulations such as ones performed in this paper. To compensate for this, a few methods can be attempted.

- **Expanding the Certainty Regions:** The first method tried involved adding extra room in the certainty regions. By increasing the certainty region, more room for estimating the correct location was allowed at the cost of computation complexity. This solution helped the result to a certain extent, but still suffered from the estimation problems.
- **Switch Methods at Low User Densities:** The second method tried was to switch algorithms at low densities. This is the method actually used in this paper. At low user densities, switching to a modified version of the noiseless recovery served to create a bump in correct recoveries while causing a drop in error.

The second method allows for a return to the ideal measurement design formation. The main changes made to modify the noiseless algorithm were to change the zero-ton test to the sub-linear version and then factor error in the magnitude and angle comparison for the single-ton test. The tests become $|y_r(1)| - |y_r(0)| \leq n\phi$ where ϕ is some error threshold. A similar measurement is used for the angle test. Because of how the noise adds on top of itself, these compensation techniques are not as accurate at larger amounts of users.

$$\hat{x}(\hat{k}) = \min_{k \in [\hat{k}-1, \hat{k}+1]} \|s_k a_k - y_r\|^2 \quad (4.10)$$

Because of this return $\hat{k}$, is calculated similar to the ideal case as in equation (4.5). The resulting calculation of $\mathbf{x}$ is shown in equation (4.10). The resulting value is then verified to determine if it single-ton or not.

4.3 Other Algorithms

4.3.1 Least Squares

For comparison, two other algorithms were implemented into the system. The least squares method is a fairly common method for sparse signal recovery, but requires a linear computational complexity of $O(nc^2)$. This method can be performed via a calculation of $G = (A^T A)^{-1} A^T Y$. While this may be a good comparison for methods, it must be noted that due to its linearity, it cannot meet the goals of sub-linearly determining a channel estimate.

4.3.2 SPRIGHT

The second method used for comparison uses the SPRIGHT framework presented in [16] and modified in [14]. The goal is to use the Walsh-Hadamard Transform and estimate the channel using a near-linear time scheme.

4.3.3 Computational Complexity Comparison

Before continuing to the simulation results, it is important to discuss computation complexity of the algorithms as a whole. The hope with the whole system is to obtain an accurate estimation in a sub-linear time frame, so it is important to confirm the computational complexity of all the algorithms. The following complexities are reported on a per antenna basis.

First, the computational complexity of the ideal version presented in this thesis is given by $O(k)$. This is simply because there is only one calculation performed for each non-zero entry in the channel matrix. The unmodified sub-linear algorithm functions has three separate contributions. The first is a contribution from the non-zero entries, or $O(k)$. The second consideration comes from the number of clusters needed which is given as $O(\log(n))$ and the third contribution comes from the amount of measurements within each cluster given by $O(\log^{1/3}(n))$. Together, this becomes a computational complexity of $O(k \log^{1.3}(n))$. The modified version is closer to the ideal version, but only functions at a low number of users. It requires three calculations for each single-ton search, so the computational complexity is given by $O(3k)$.

The near-linear algorithm reported in [16] and [14] reports a computational complexity of $O(n \log(n))$. The Least Squares method uses a standard linear algorithm, so the computational complexity is $O(n)$.

Chapter 5: Algorithm Performance Evaluation

5.1 Simulation Setup

There are two simulated measurement designs that were used to test the algorithms recovery performance: Gaussian generated and angular domain generation. To test the measurement design in each of these scenarios, mean-squared error and individual channel component tracking will be used to evaluate the results. The following tests will be run with 8 mobile users/64 base station antennas as well as 16 mobile users/128 base station antennas.

The first scenario the system was tested under is the circular Gaussian generated channel matrix scenario. In this case, the channel matrix was generated from a complex Gaussian distribution, $\mathbb{CG}(0, \sigma)$, with each column generated independently using distinct random generators to make sure no statistical dependence existed between each mobile user's channels. Six other matrices would then be generated and added as coming from the surrounding cells using $B_{lj}^{1/2} = .1I_K$ for the large-scale fading where I_K is the K -dimensional identity matrix, K is the number of users, and $l \neq j$. Afterward, every location under a certain threshold, 1.7σ , was ignored to create a sparse channel matrix. After the requisite mobile user uplink pattern was created, the system added circular Gaussian noise dependent on the SNR to create the measured signal at the base station.

The first measurements will inspect the mean-squared error (MSE) of the recovered signal compared to the sparsified channel matrix. The Sub-linear recovery framework, SPRIGHT framework, and Least Squares framework will have their MSE compared over

SNR levels to determine their quality. The MSE is defined by

$$MSE = \frac{1}{KM} \sum_{m=1}^M \sum_{k=1}^K \|\hat{h}_{mk} - h_{mk}\|_2^2 \quad (5.1)$$

Next, channel components will be monitored over individual channel estimations within the SNR level of 15dB. Each iteration has the channel matrix being generated independently; therefore, they can serve as individual coherence time periods.

Because the threshold of 1.7σ was chosen semi-arbitrarily, we are interested to see how the system performs with changing thresholds and how the sparsity of the system changes with different thresholds. In the following measurements, the threshold will be set to five separate values where the MSE and the computational complexity of the resulting system will be calculated. These five values are given at 1σ , 1.3σ , 1.5σ , 1.7σ , and 2σ .

Another interesting decision to look at is how the change in the large-scale fading factor affects the MSE of the system. $B_{lj}^{1/2}$ was tested at the values of $.1I_K$, $.08I_K$, $.05I_K$, $.02I_K$ to determine the error's change over varying degrees of interference.

The second method of generating the channel matrix is the sparsity proof discussed in [14]. In this case, eight users were generated within seven cells with random distances and angles from their local base stations. Each user was generated ten channel paths of fast-fading coefficients from a circular Gaussian distribution. The paths were then multiplied through the steering matrix to create the channel matrix for a single cell. After the noise from the surround cells channels were added, using $B_{lj}^{1/2} = .1I_K$, the system was then transformed into the angular domain by multiplying it with the unitary matrix U_r . This generates an inherently sparse matrix where only the MSE and individual channel tracking measurements are necessary.

5.2 Gaussian Channel Estimation Quality

The first simulations are the Mean-Square Error (MSE) over the signal-to-noise ration (SNR) in dB. These calculations were averaged over 500 separate tests per SNR level.

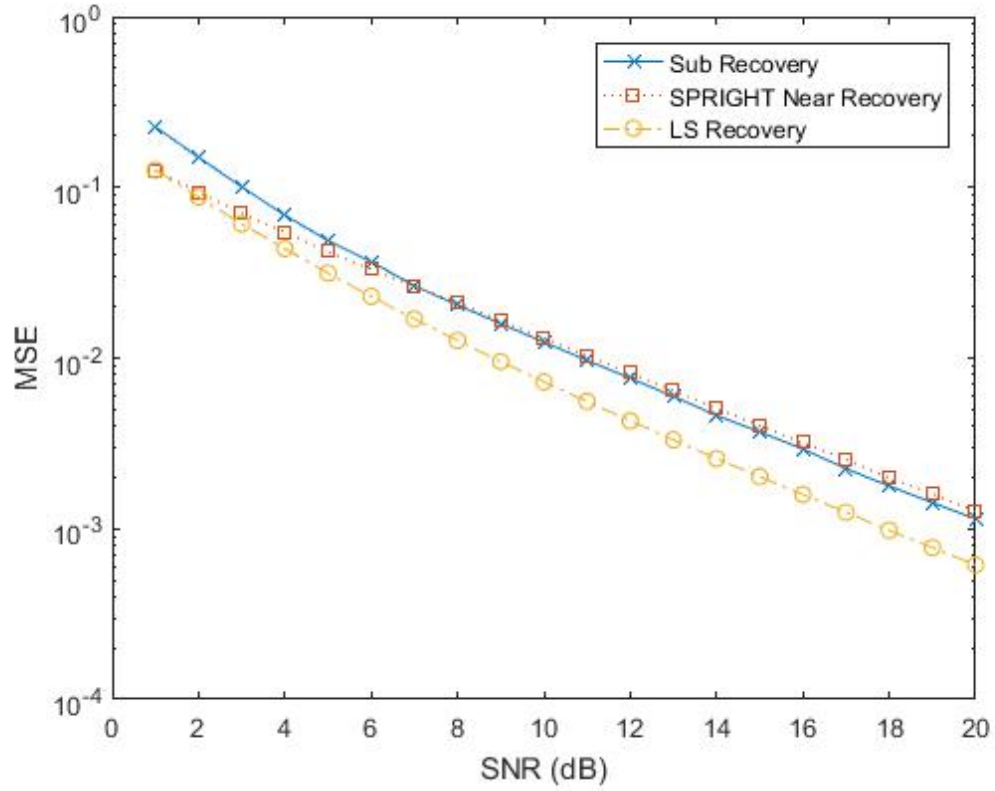


Figure 5.1: MSE calculated over SNR (dB) for 8 MU and 64 base station antennas.

As seen, the presented framework, named "Sub Recovery" matches a similar slope to the Least Squares recovery and, at lower SNR, slightly outperforms the SPRIGHT recovery. Figure 5.1 showcases the simulations utilizing 8 mobile users and 64 base station antennas.

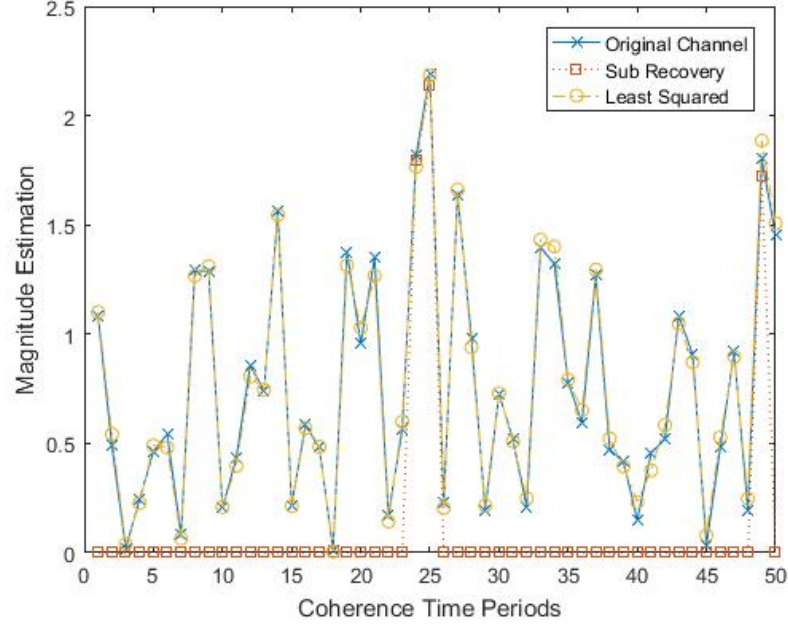


Figure 5.2: Random channel coefficient tracking for 8 MU and 64 base station antennas

During the 15dB SNR tests, two random channel coefficients, for mobile users 6 and 1 and their connection to the antennas 53 and 3, respectively, were picked and tracked over the multiple tests. The magnitude of the generated channel coefficients and the magnitude of the recovered channel coefficients were tracked.

Because of the cutoff point at 1.7σ , the only estimated channel coefficients have magnitudes above that threshold. As expected, the majority of channel coefficients are not recovered because they are considered noise.

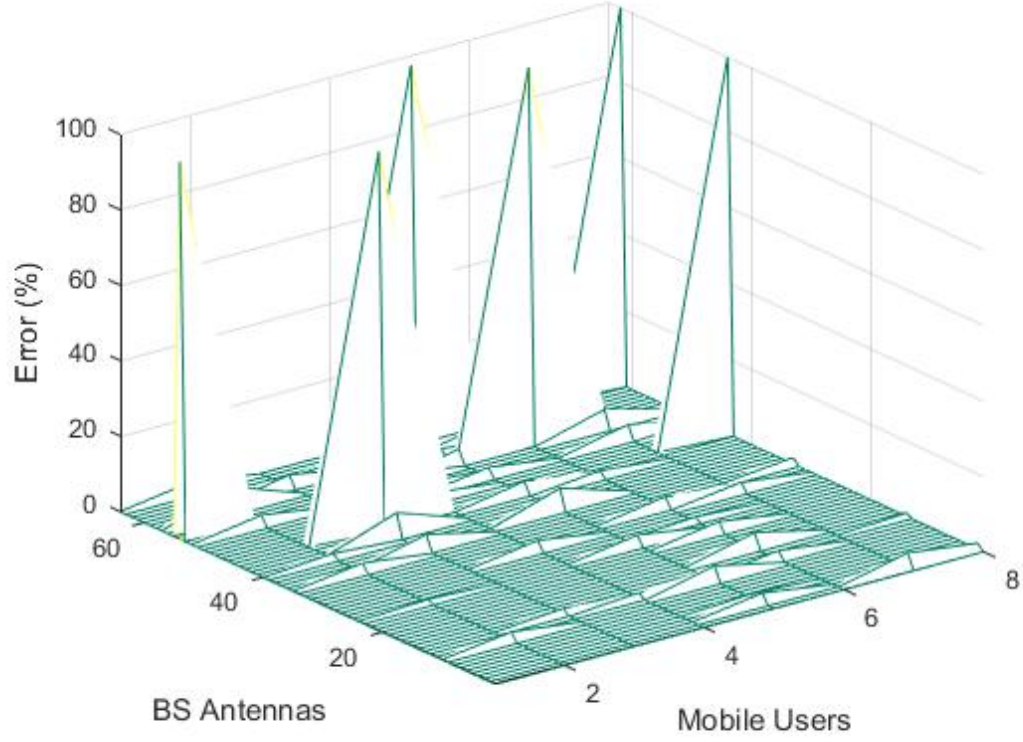


Figure 5.4: 15dB SNR channel estimation error for 8 MU and 64 base station antennas.

At 15dB SNR, a random snapshot of the channel matrix was taken. The recovered matrix was then compared with the noisy sparse channel to determine the error in the channel matrix. One thing to notice, which will be discussed later, is the six non recovered channel constants. In this case, there are three separate BS antennas with two non-recoveries each.

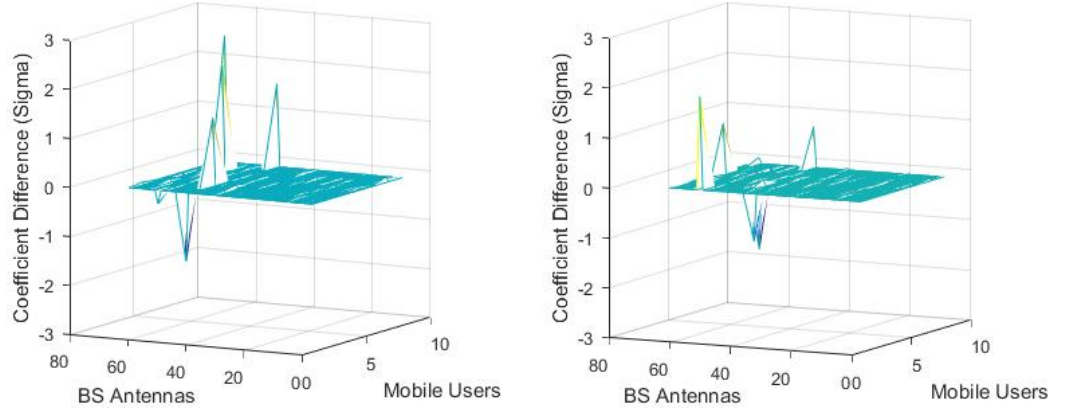


Figure 5.5: 15dB SNR channel estimation error for 8 MU and 64 base station antennas. (A) The magnitude difference between real components. (B) The magnitude difference between imaginary components.

An interesting breakdown into the system is to look at more than just the error of the channel matrix magnitudes by looking at the difference between the imaginary and real components of the system. Here we can see the same six coefficients that were completely missed by the algorithm in their various components. We can also see the minor errors in the estimations.

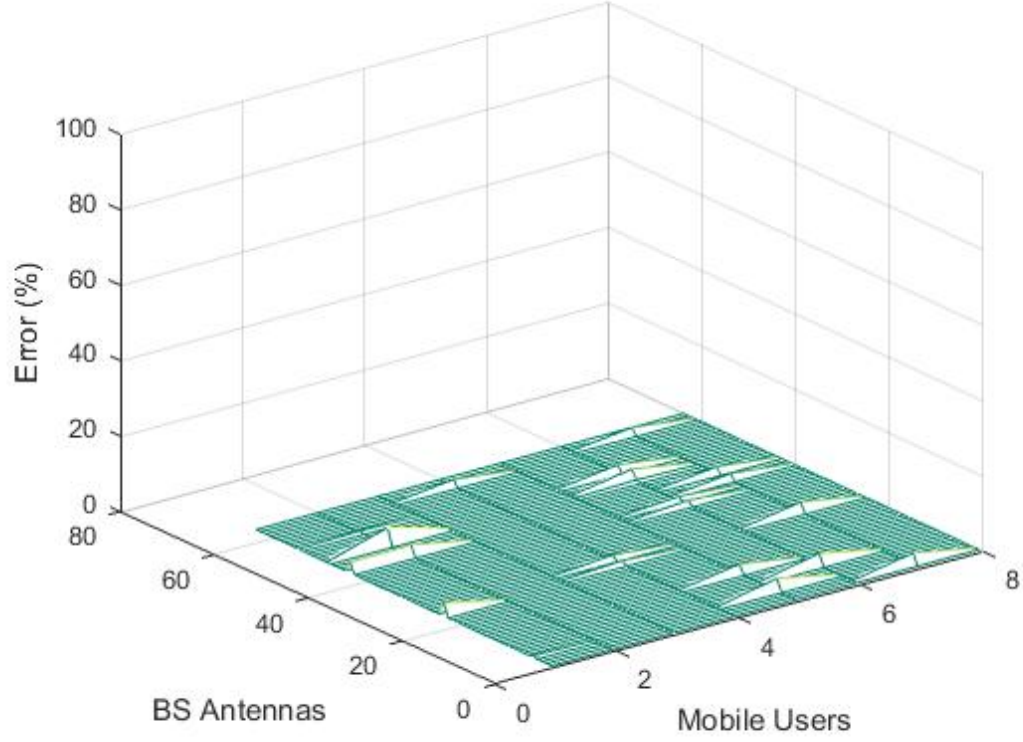


Figure 5.6: 2σ cutoff 15dB SNR channel estimation error for 8 MU and 64 base station antennas.

As a comparison, the 2σ threshold channel matrix error was also measured. Notice that there is no error in the channel matrix recovered above 5%. One of the main reasons for this difference is the nature of sparsity. This will be explained in section 5.6.

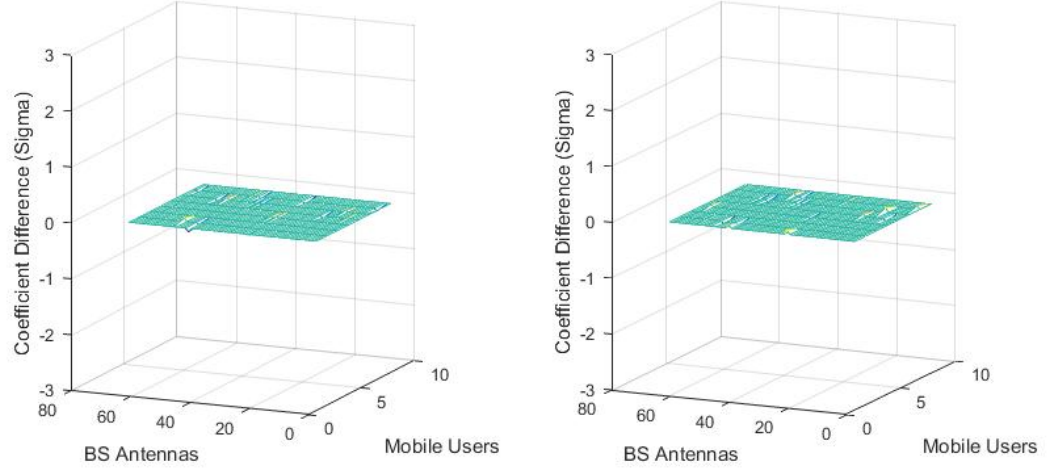


Figure 5.7: 2σ cutoff 15dB SNR channel estimation error for 8 MU and 64 base station antennas. (A) The magnitude difference between real components. (B) The magnitude difference between imaginary components.

Again, we can see the minute differences in the recovered matrices. There are very few differences in the real and imagined components of the recovered system.

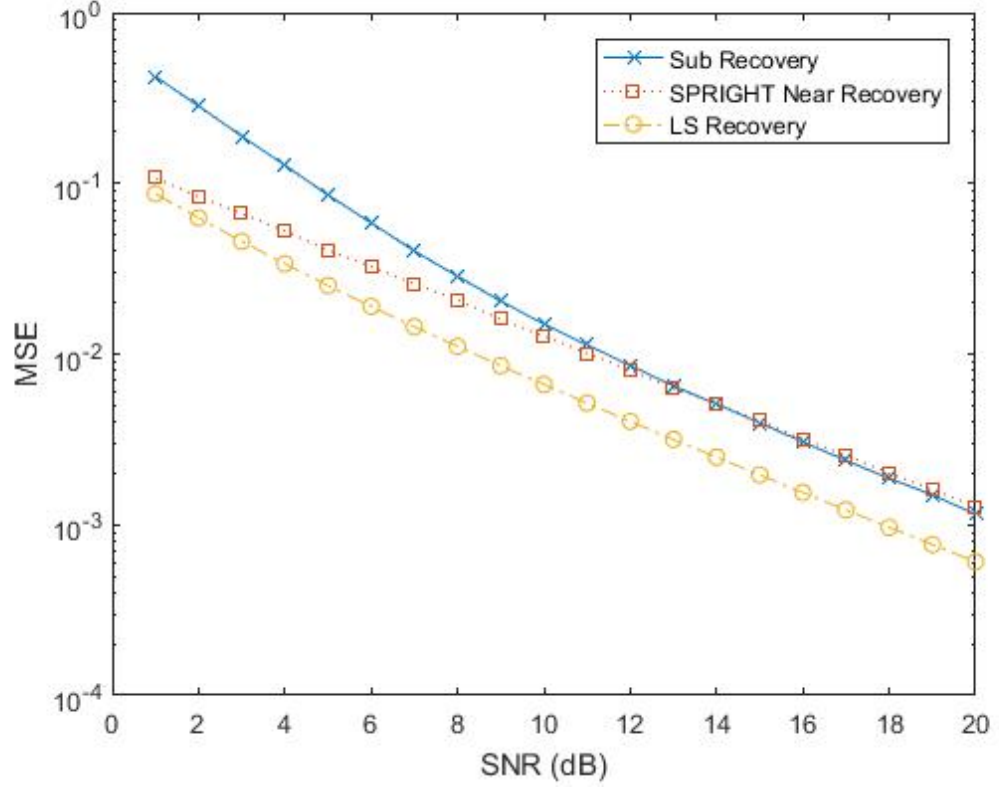


Figure 5.8: MSE calculated over SNR (dB) for 16 MU and 128 base station antennas.

Figure 5.8 shows the system performance with 16 mobile users and 128 base station antennas. The modified Sub Recovery framework is beginning to not perform as well, especially in noisier situations. The limitations that cause some of these differences between the sub-linear and SPRIGHT frameworks will be further discussed in section 5.6.

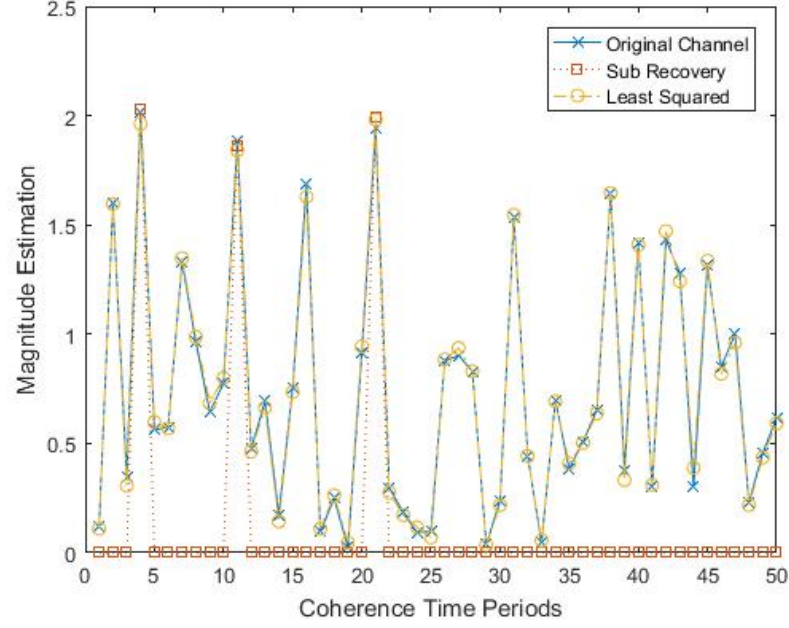


Figure 5.9: Random channel tracking for 16 MU and 128 base station antennas

Again, during the 15dB SNR tests, two random channel coefficients, mobile users 13 and 15 as they connect to antennas 123 and 33 respectively, were picked and tracked over the multiple coherence time periods. This time the channel coefficients were measured during the test involving 16 mobile users and 128 antennas at the base station.

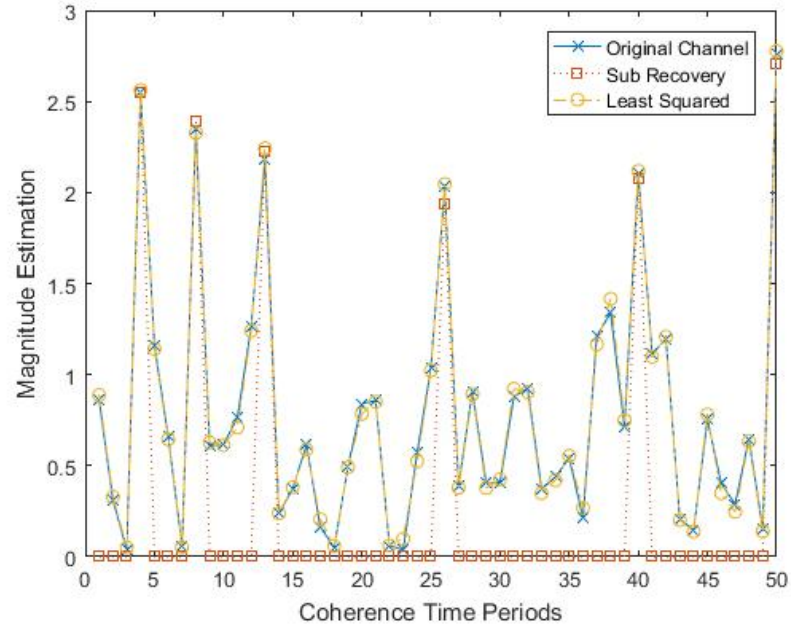


Figure 5.10: Random channel tracking for 16 MU and 128 base station antennas.

In these figures, the Least-Squared method estimates closely to the whole of the signal, while the sub-linear method discussed only finds the values above the given threshold.

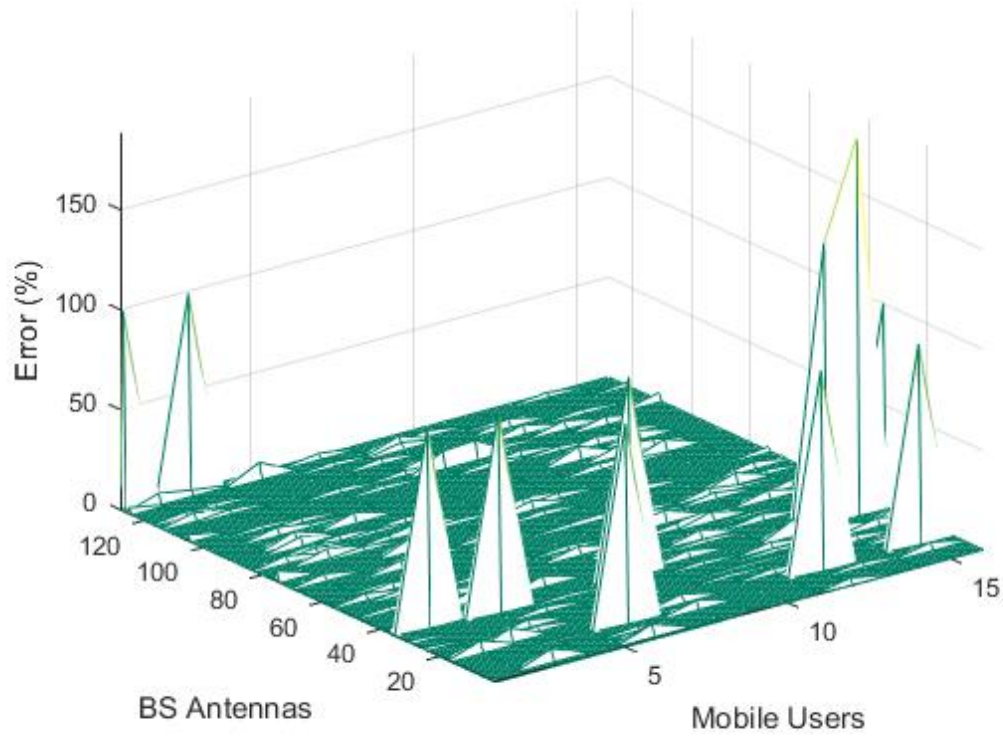


Figure 5.11: 15dB SNR channel estimation error for 16 MU and 128 base station antennas.

Again, a random snapshot of the channel matrix was compared with the noisy, sparse channel matrix. This snapshot shows similar amounts of failed recoveries. It also shows a new method of failure, an incorrect recovery. As it turns out, both of these forms of failures have the same root which will be discussed in section 5.6.

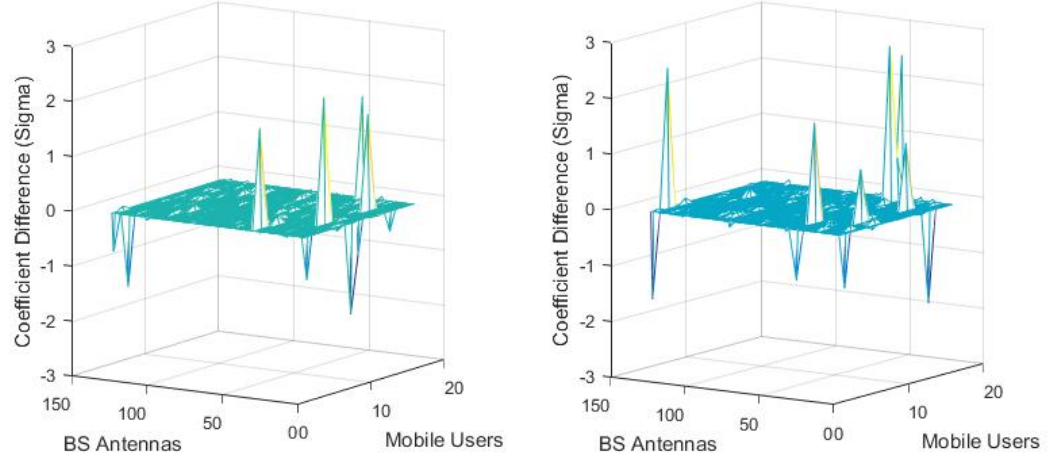


Figure 5.12: 15dB SNR channel estimation error for 16 MU and 128 base station antennas. (A) The magnitude difference between real components. (B) The magnitude difference between imaginary components.

The components in the system are displayed in real and imaginary component differences. From here, it's easy to spot the major error contributions that show up in Figure 5.11.

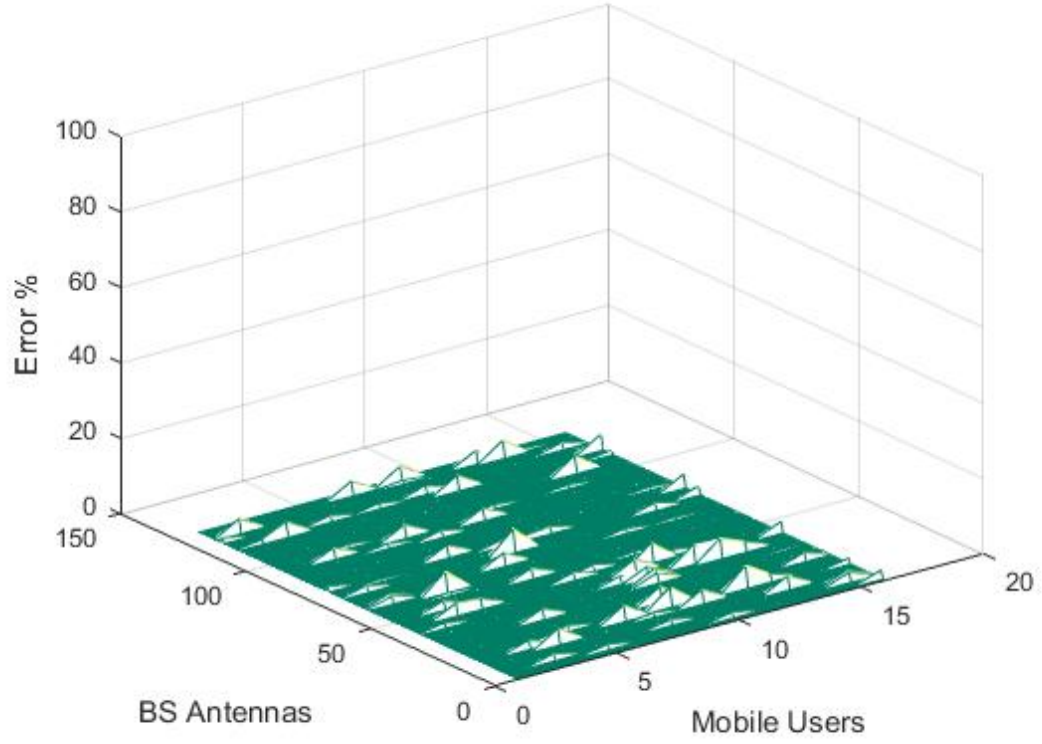


Figure 5.13: 2σ cutoff 15dB SNR channel estimation error for 16 MU and 128 base station antennas.

The recovered matrix with a 2σ threshold recovers most of the important channels in the given system. This is very clearly why section 5.3 will show recoveries near the LS MSE.

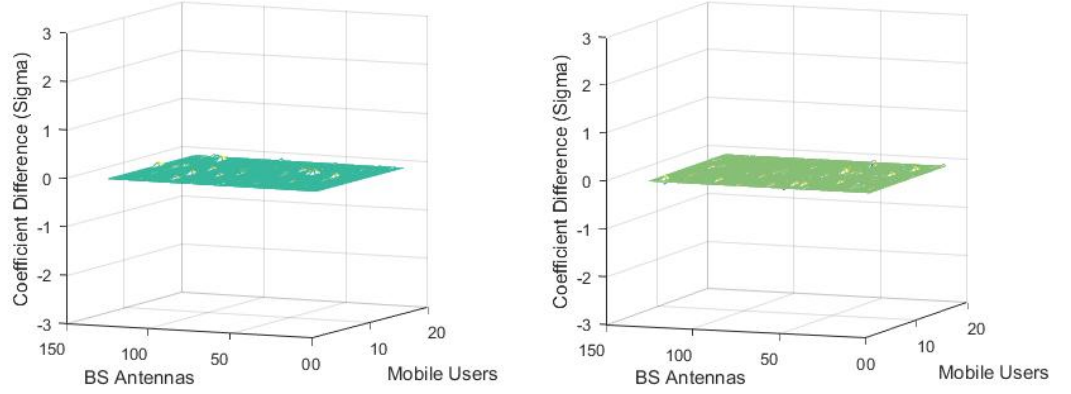


Figure 5.14: 2σ cutoff 15dB SNR channel estimation error for 16 MU and 128 base station antennas. (A) The magnitude difference between real components. (B) The magnitude difference between imaginary components.

Finally, the difference in real and imaginary components are again graphed. Like the calculated error, there is minimal error in the recovery.

5.3 Threshold Comparisons

Because we are inclined to test the system over various scenarios, we will want to identify how its performance changes with the level of the threshold, as well as the sparsity of the system.

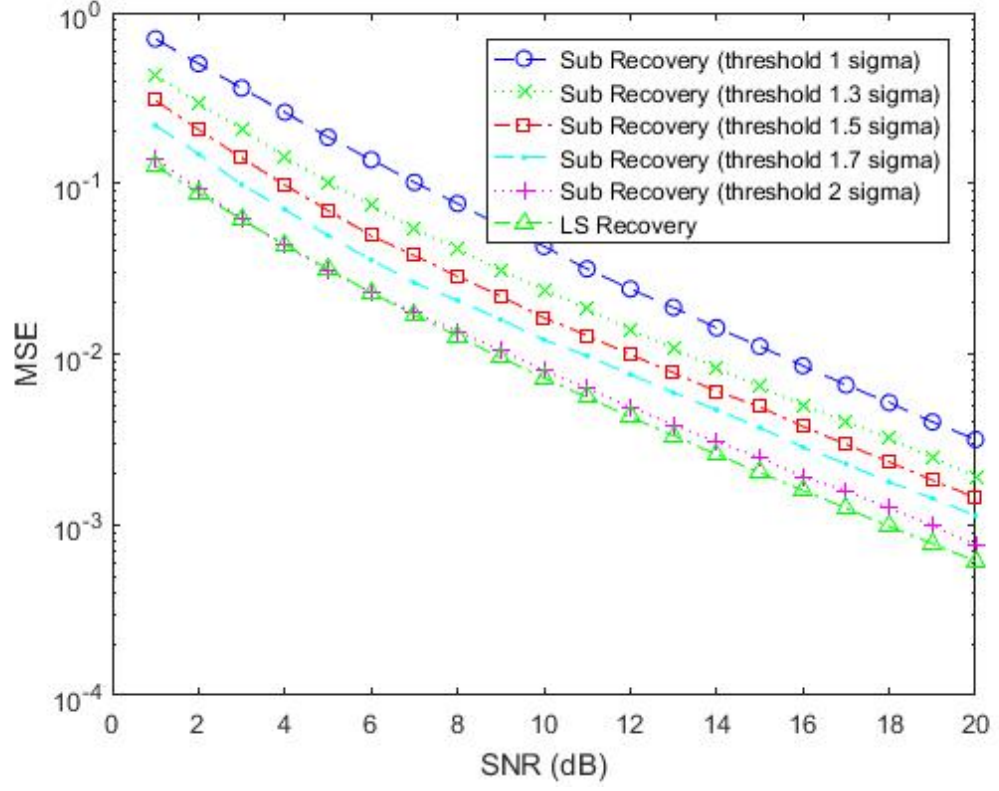


Figure 5.15: Threshold tracking for 8 MU and 64 base station antennas.

The first test to run is the same test as in Figure 5.1 and 5.8, but changing the threshold between 1σ , 1.3σ , 1.5σ , 1.7σ , and 2σ . For now, it is enough to know that the Sub Recovery framework performs better at higher thresholds.

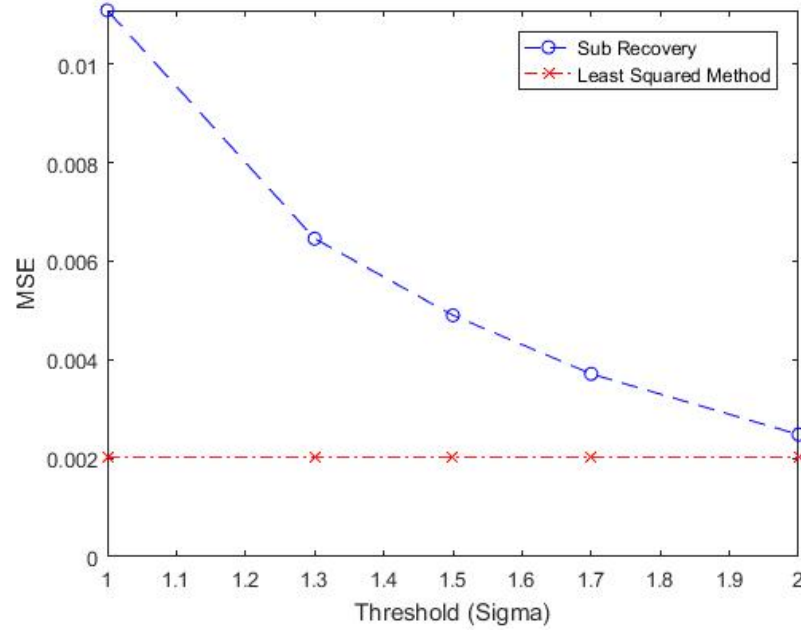


Figure 5.16: 15dB SNR performance for 8 MU and 64 base station antennas.

To take a closer look at this change, we can examine the MSE as a function of the threshold for 15db SNR. Again, we notice that the error diminishes as the threshold increases, but we also notice a slightly asymptotic curve to the measurements.

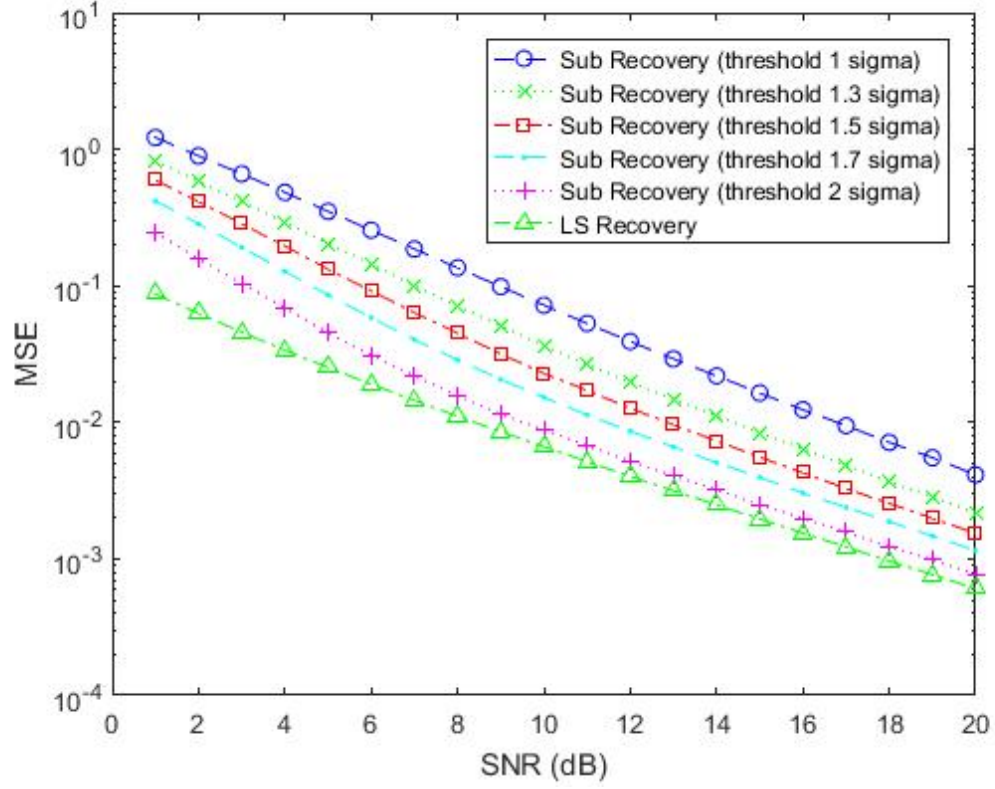


Figure 5.17: Threshold tracking for 16 MU and 128 base station antennas.

The same tests were run again for the 16 mobile user and 128 base station antenna scenario. Similar to Figure 5.8, we notice some of the odd behaviors caused by the modifications.

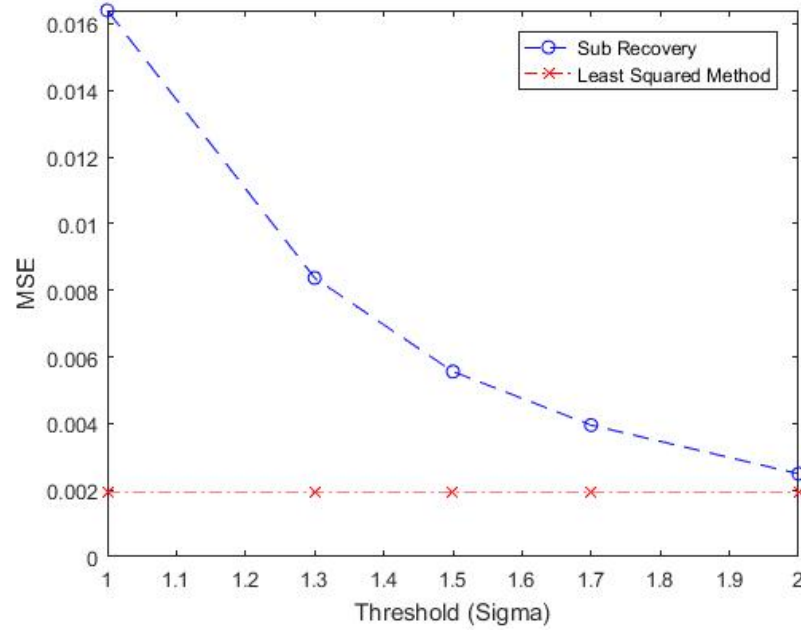


Figure 5.18: Random channel tracking for 16 MU and 128 base station antennas.

The 15dB SNR measurement was used to track the behavior of the Sub Recovery framework over the threshold changes. Again, there appears to be diminishing returns as the threshold increases.

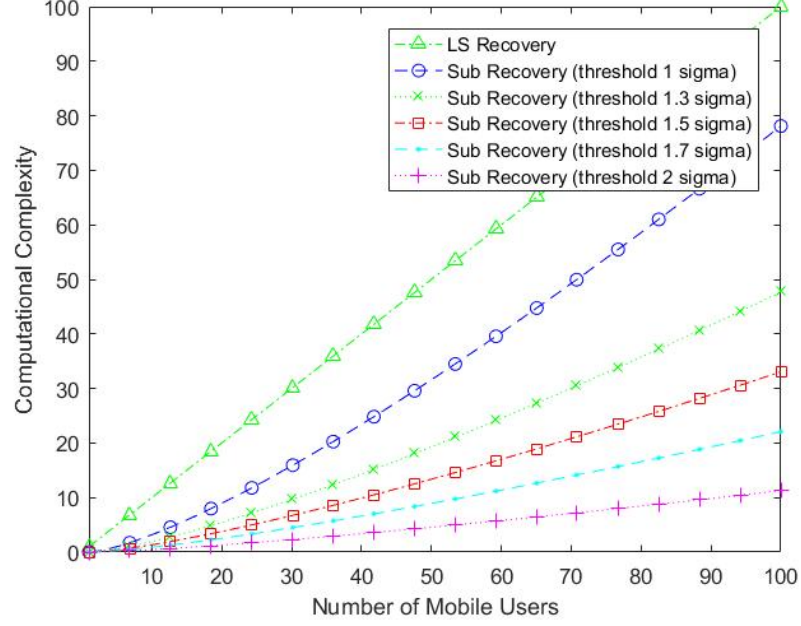


Figure 5.19: The computational complexity of the system calculated over the number of mobile users on a per antenna basis

The most important factor that can explain the previous decreases in error is the sparsity. Throughout the previous simulations, the sparsity, $\frac{\kappa}{KM}$ of the system was tracked, where κ is the number of important channel coefficients. The sparsity averaged out over the 500 iterations per scenario to be 31.73%, 19.36%, 13.42%, 8.97%, and 4.57%. This means that as the threshold increases, the amount of factors to recover decreases. As expected, the system can recover, with higher accuracy, matrices with higher sparsity.

Finally, the goal of this system is to solve the provided system in a sub-linear time frame. To do that, the measured sparsity of the threshold cutoff was used to calculate the system's computational complexity over the total number of mobile users, K . The computational complexity is calculated per base station antenna.

In general, all threshold complexities lie below the linear Least-Squared method for this amount of users. The 1σ cutoff exists fairly close to the linear line while the remaining four thresholds exist under the linear measurements.

5.4 Large-Scale Fading Comparisons

Another intriguing scenario is the effect of the large-scale fading on the MSE of the sub-linear recovery algorithm. The large-scale fading will be tested over the same scenarios as in section 3.2. This is another method of determining the total affects of the interference on the system.

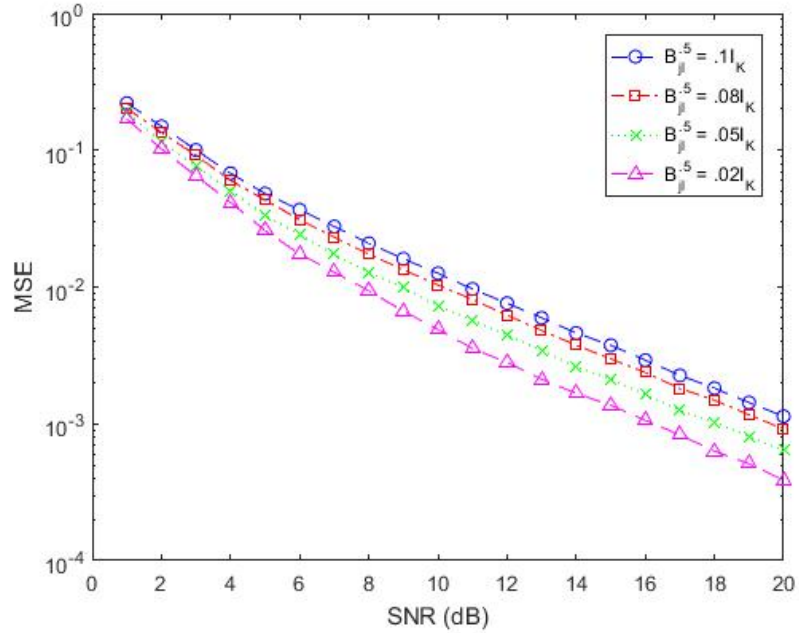


Figure 5.20: Affects of $B_{lj}^{1/2}$ on MSE for 8 MU and 64 BSA

A very noticeable pattern in the 8 Mobile User and 64 antenna simulation is how

closely grouped the mean-squared errors are. When there is high SNR, the measured errors are tightly grouped and grow farther apart as the SNR decreases.

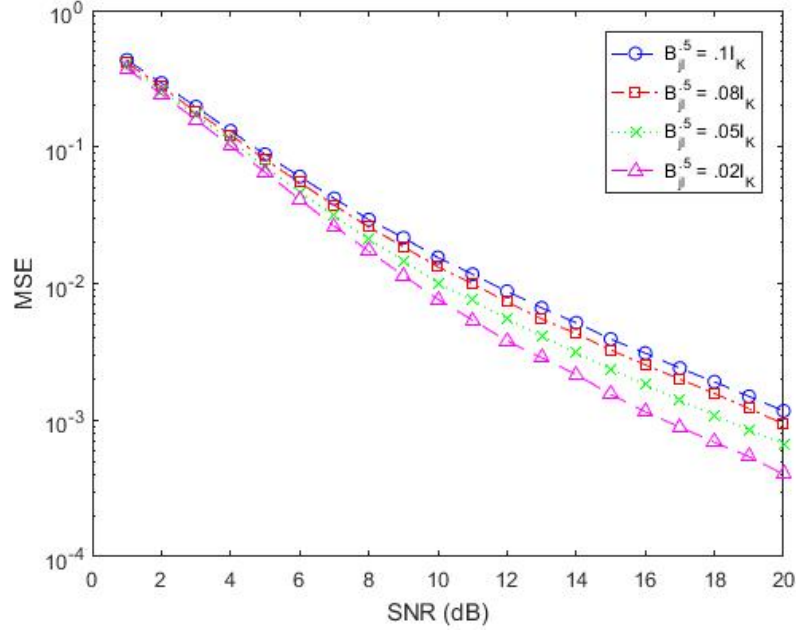


Figure 5.21: Affects of $B_{ij}^{1/2}$ on MSE for 16 MU and 128 BSA

A similar, but to a more noticeable degree, effect happens in the 16 users and 128 antennas scenario. This means that the SNR informs much of the error closer to 1 dB, but as the SNR increases, the error becomes more defined by other aspects such as large-scale fading.

5.5 Angular Domain Estimation Quality

For the angular domain models, the variance of the channel matrices is larger, so the MSE will also be larger in scale than the previous calculations.

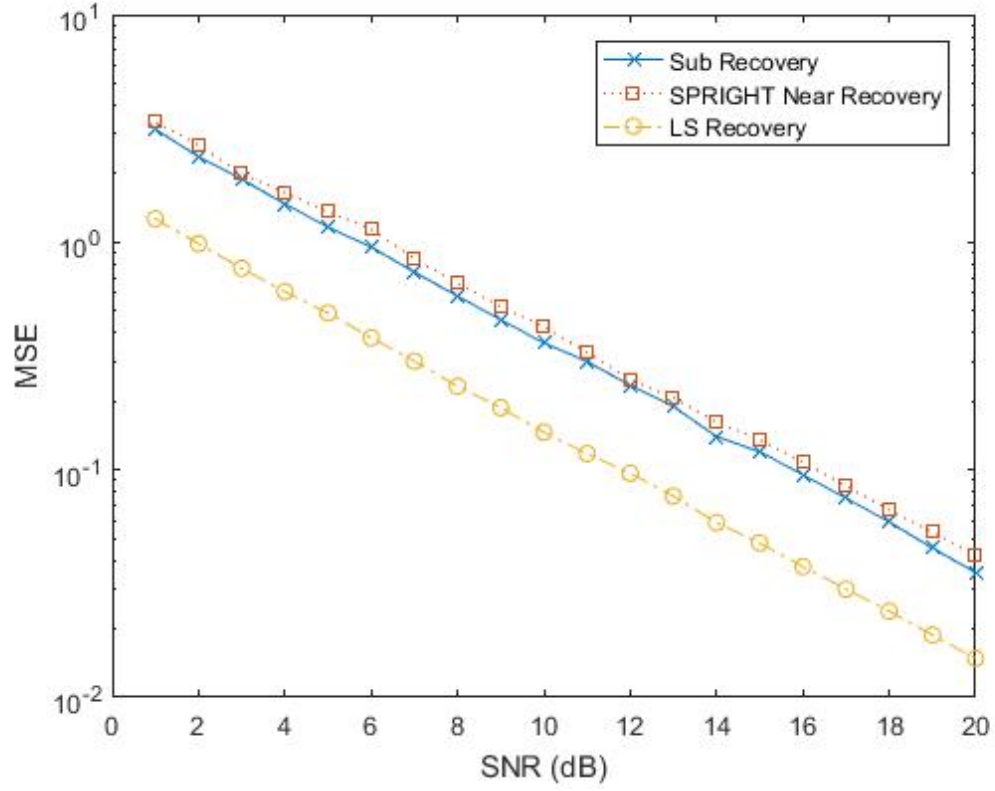


Figure 5.22: MSE calculated over SNR (dB) for 8 MU and 64 base station antennas.

Similar to section 5.2, the MSE of the system was graphed over the SNR (dB). Each SNR level had 500 channel matrices generated over which the algorithm attempted to recover all channel constants above a certain threshold. These simulations were performed with 8 mobile users and 64 base station antennas.

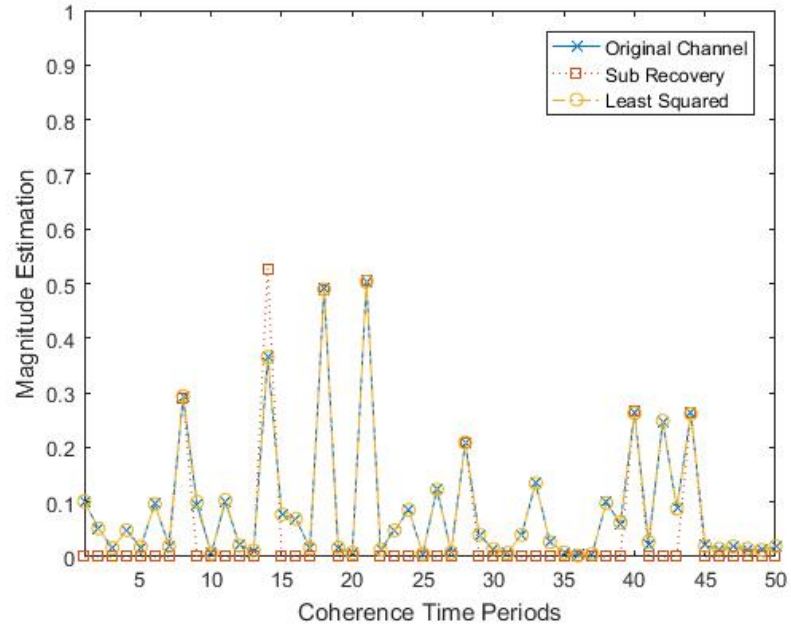


Figure 5.23: Random channel coefficient tracking for 8 MU and 64 base station antennas

As before, two individual channels were observed over various measurements. In this case, the magnitude was scaled to unity to show the data as a whole. In this first graph, one can notice that the algorithm does not attempt to estimate small magnitudes.

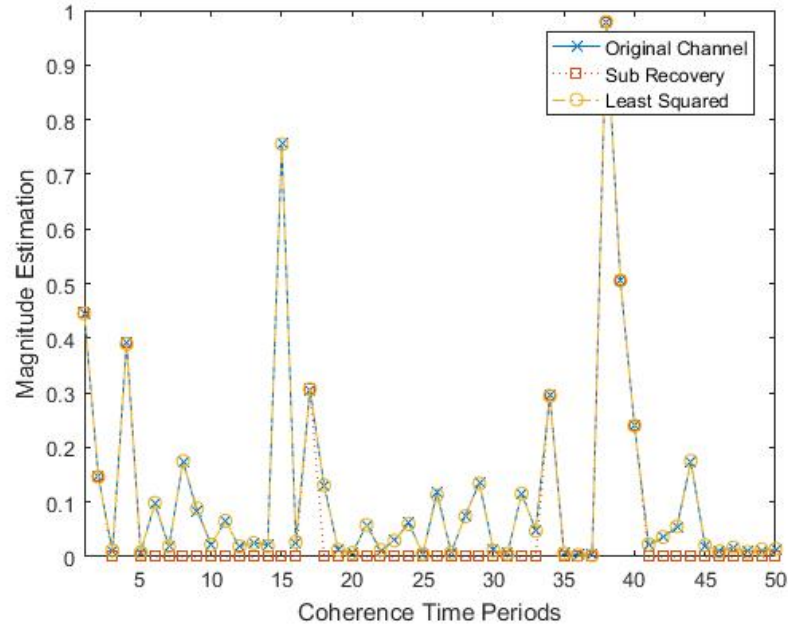


Figure 5.24: Random channel coefficient tracking for 8 MU and 64 base station antennas

The second channel tracked is useful because it displays even more so, the sparsity and compressability of the system. The significant channel constant magnitudes tend to be significantly larger than the noise of the system.

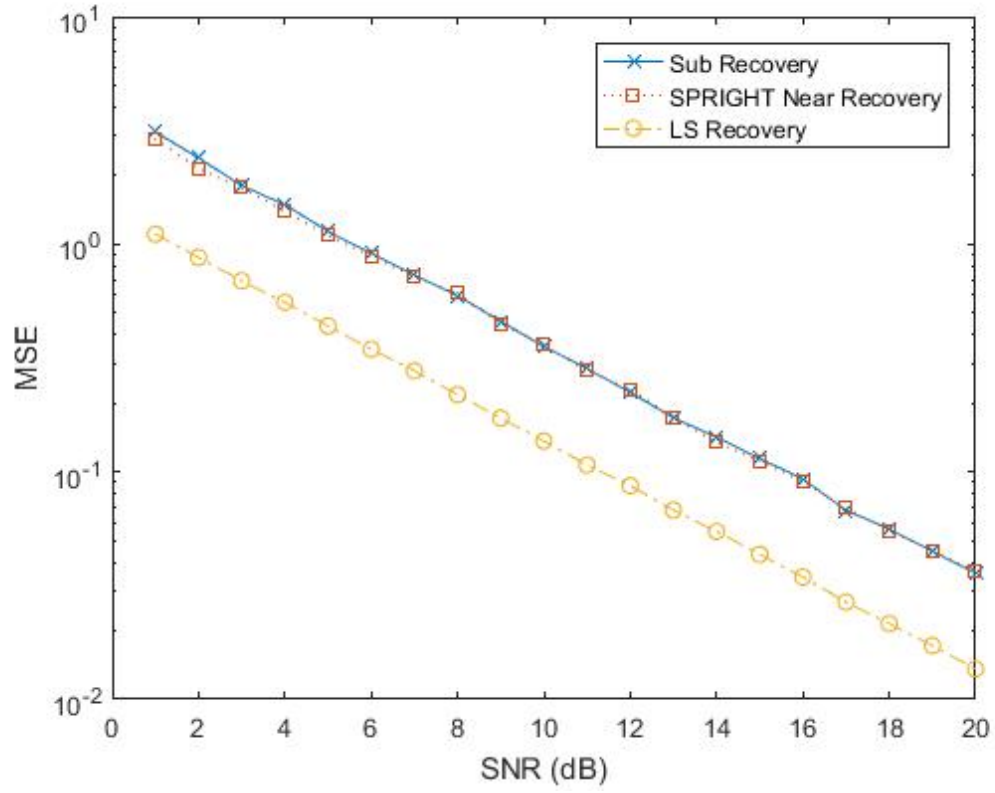


Figure 5.25: MSE calculated over SNR (dB) for 16 MU and 128 base station antennas.

Moving on to the measurements for the 16 mobile user and 128 base station antennas scenario, the errors remain about the same.

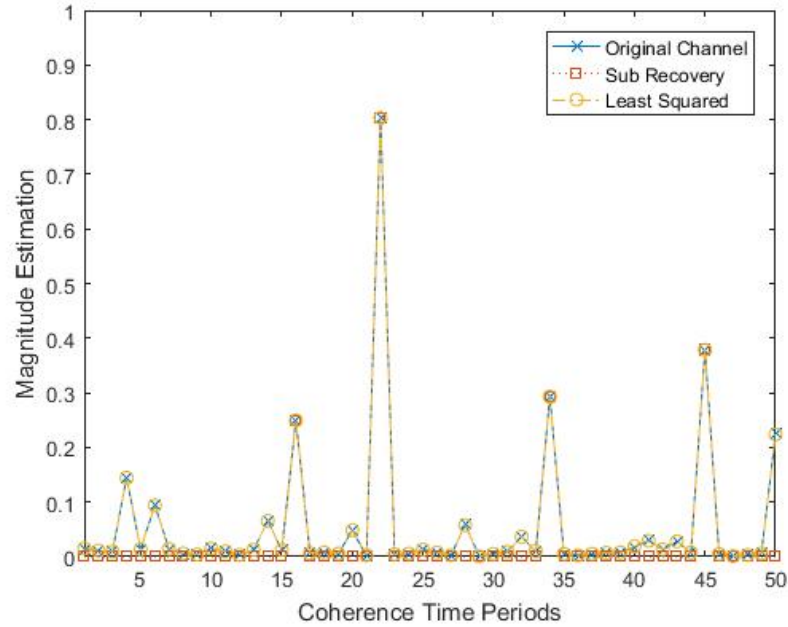


Figure 5.26: Random channel coefficient tracking for 16 MU and 128 base station antennas

As opposed to Figures 5.23 and 5.24, the sparsity of the system is better observed in the next few graphs. Notice how the Least Squared method follows along with the original channel constants, but the Sub Recovery method only obtains the major contributors.

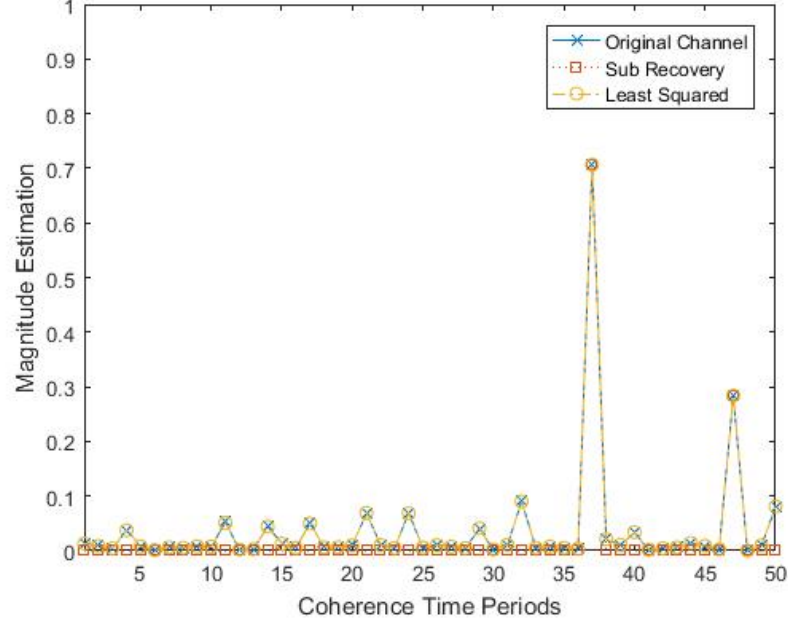


Figure 5.27: Random channel coefficient tracking for 16 MU and 128 base station antennas

Once again, the magnitude estimation only recovers the significant portions of the channel matrix.

5.6 Discussion

Within the measurement methods, there are a few observations to note. The first one is that sparse signals rely on a large data pools of mostly non-values to work at their maximum capability. In the test presented in this thesis, the size of the matrices, as defined by K , limits the accuracy and potential recovery possibilities of the system. Despite this drawback, we have shown through simulation that the system still works in

a sub-linear computational framework with only a small drop in accuracy caused by the limitations of the scenario sizes.

The second observation has to do with the measurement cost. Although not discussed thoroughly in this thesis, the measurement cost can be considered the length of the pilot sequence. Ideally, the measurement cost should operate in less than a linear framework as well. This limits the time that the channel matrix can change over and allow for an estimation of the system sooner. At such a low matrix size, it can be difficult to create the sub-linear design patterns necessary within the pilot sequence constraints, but some limit needed to be set on the pilot length. For this case, the approximate size of K was used as that limit. This forced some limitations on the system and caused another minor amount of error.

To return to the discussions around the channel matrix figures (Figure 5.4 and 5.11), the errors noticed are common in low dimension scenarios. The main issue is the same as discussed just prior, the limited pilot length. Because the H matrix, see equation (4.1), is generated randomly and it is limited by the pilot sequence length, there is a non-insignificant likelihood that both channel constants will affect the same output bins. This will either prevent the recovery of the channel components as shown in Figure 5.4 or it might cause a false recovery as seen in the large percent error in Figure 5.11.

The reason the 2σ figures do not have any false recoveries is because it has increased sparsity. The channel is less likely to have significant channel propagation coefficients that overlap, as they did in the 1.7σ scenario.

Chapter 6: Conclusion

Over the course of this thesis, a massive MU-MIMO system was modeled in the angular domain. The uplink channel matrix was shown to hold the sparsity conditions in the angular domain. The sub-linear algorithm proposed in [3] can be used to recover the sparse channel matrix. The algorithm was compared to a linear implementation of the least squared algorithm and the SPRIGHT framework presented in [16] and implemented in [14]. The below-linear methods yield a slightly large MSE in exchange for their quicker recovery frameworks. The sub-linear algorithm was also tracked to show the recovery over a sizable amount of consecutive coherence time periods.

There are a couple opportunities to expand this research framework that should be explored. The first opportunity is to expand upon the SPRIGHT framework, using the so-called SO-SPRIGHT and NSO-SPRIGHT algorithms presented in [16]. Another option would be to simulate the system over a much larger amount of users. This would allow the confirmation of the algorithm's recovery over a truly MU-MIMO system.

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