REVIEW FOR ENGINEERING REGISTRATION

1. FUNDAMENTALS SECTION

By

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(2) To serve the industries, utilities, professional engineers, public departments, and engineering teachers by making investigations of interest to them.

(3) To publish and distribute by bulletins, circulars, and technical articles in periodicals the results of such studies, surveys, tests, investigations, and research as will be of greatest benefit to the people of Oregon, and particularly to the state's industries, utilities, and professional engineers.

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FOREWORD

Registration of professional engineers is now legally required throughout the United States and its territories, Canada, and many foreign countries. In most instances, a written examination is given in connection with registration procedure.

Most of the states grant registration simply as a "professional engineer" in recognition of the fact that the registrant has demonstrated his qualifications insofar as a knowledge of engineering sciences is concerned. In some states engineers are registered in such a manner to indicate the registrant is especially qualified in a particular branch by reason of a further investigation of his qualifications in a particular field of specialization, such as a civil, mechanical, electrical engineer, et cetera.

Even though the certificate of registration goes on to state the registrant is especially qualified in a particular branch, registration as a "professional engineer" in nearly all states permits that person legally to practice engineering in any branch he may wish. This gives the registered engineer maximum freedom in his choice of work and, at the same time, greatly simplifies administration of the registration laws.

Engineers in increasing numbers, for one reason or another, drift from the field of their original registration into some other branch. There are ample reasons, therefore, in support of the trend of various engineering registration boards throughout the country in placing increased emphasis upon the matter of examining the registration applicant in fundamentals of engineering.

In the State of Oregon, written examinations are divided into three sessions of substantially equal length; the first session being devoted to engineering fundamentals, with no reference text permitted. It has been observed by the Oregon Board that applicants making a passing grade in the section on fundamentals also invariably make a passing grade on those portions of the examination devoted to problems in the specific branch.

We believe Mr. Clayton has done an excellent job in the preparation of this circular and it should serve a definite need, particularly for engineers reviewing for state board registration examinations.

The problems presented are typical of those usually provided in such examinations and are of such nature that a reasonably competent engineer should be expected to work them out without the aid of handbooks or reference texts. They also involve the application of basic principles common to nearly all phases of specialized engineering. Long, drawn out computations are not required and, in general, one-half hour is ample for working out any one question. All the
various subjects usually included in an examination on fundamentals are covered in a single convenient review text which should, in itself, save much time for the engineer.

There is also a very definite need for similar books containing sample questions for review and devoted specifically to the fields of at least the major branches of engineering; i.e., civil, electrical, and mechanical engineering. Heretofore, applicants taking examinations have been required to turn in the copies of examination questions, and these were subsequently forwarded to certain libraries of the State of Oregon where they could be reviewed by prospective applicants. Because of the limited number of such copies available, the demand has far exceeded the supply.

Engineering registration examinations are conducted twice a year by the Oregon Board in Portland—in March and September. Anyone may apply for admission to the examination who has had at least six years of engineering experience, with at least one year in responsible charge of engineering work, either as principal or assistant. If the applicant is a graduate from an accredited engineering school, he is given four years' credit against the six-year requirement. Application blanks may be obtained by writing to the Board of Engineering Examiners, 717 Board of Trade Building, 310 SW Fourth Avenue, Portland, Oregon.

E. A. Buckhorn, Secretary
State Board of Engineering Examiners

ACKNOWLEDGMENTS

Appreciation is extended to engineers who have taken review courses and offered suggestions, criticism, and encouragement; Oregon State Board of Engineering Examiners and Mr. E. A. Buckhorn, secretary of the Board, for making examination questions available; Assistant Dean M. Popovich, Professor T. J. McClellan, Professor G. E. Thornburgh, and Professor L. N. Stone of Oregon State College for advice and assistance in preparing the text; and Mrs. Eloise Hout of the Engineering Experiment Station for preparation of the manuscript and sketches.
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I. INTRODUCTION

This circular is written for the engineering graduate who is preparing himself for the examination for registration as a professional engineer in the State of Oregon. This review is intended to cover only the fundamentals section (closed book) of the examination. Other circulars are planned to give reviews for the examination sessions (open book) on the various engineering branches—civil, mechanical, electrical, et cetera.

Example problems and solutions which form the body of this circular are drawn from eight examinations covering the period from March 1950 through September 1953. Nearly all of the problems from those eight examinations have been included. Approximately 75% of these problems have been used as illustrative problems in the body of the text. The remaining problems (some with answers) have been placed at the end of each section and may be referred to for additional review.

Analysis of the material covered in the fundamentals section of these examinations shows the relative weight given various topics, as follows:

<table>
<thead>
<tr>
<th>Topic</th>
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<tbody>
<tr>
<td>Mathematics</td>
<td>18%</td>
</tr>
<tr>
<td>Mechanics (statics, dynamics, energy relationships)</td>
<td>43%</td>
</tr>
<tr>
<td>Strength of Materials</td>
<td>12%</td>
</tr>
<tr>
<td>Electricity</td>
<td>15%</td>
</tr>
<tr>
<td>General Physics, Chemistry, Miscellaneous</td>
<td>12%</td>
</tr>
</tbody>
</table>

100%

This circular alone is not complete enough to give an adequate review for the examination, but it should give the candidate an indication of the areas in which further study is necessary. Supplementary review may be found in any number of good college level texts. A few of these are listed in the bibliography for those candidates who do not have such references at hand.
II. MATHEMATICS

Algebra. Substitutions of known quantities in many engineering formulas result in simple algebraic equations in one unknown.

A projectile is fired from a 75 mm gun at a velocity $V = 2400$ ft/sec. Neglecting such factors as air resistance and rotation of the earth, what is the maximum range (horizontal distance) in miles the projectile will travel when the equation describing the path of the projectile is:

$$ y = x \tan \theta - \frac{32 x^2}{2V^2 \cos^2 \theta} $$

For maximum range

$\theta = 45^\circ$

$V = 2400$ ft/sec

$y = 0$

$$ 0 = x(1) - \frac{32 x^2}{2(2400)^2(0.707)^2} $$

$$ 0 = 1 - \frac{16 x}{(1700)^2} $$

$$ x = 181,000 \text{ ft} = 34.2 \text{ miles} $$

It is required to select the diameter of a circular sedimentation basin which will settle to the bottom all suspended particles before particles contact the wall of the basin. The basin is to be 24 in. deep and will be fed from an overflow launder at the center. An inflow of 100 cu ft per minute is to be settled, and the smallest particle in the inflow has a sedimentation rate of 0.033 ft per minute. Assuming liquid velocities outward from the center are uniform at all depths and that no hindered settling is involved, what should be the diameter of the basin?

$$ \frac{2}{\text{time to settle}} = \frac{60.6 \text{ min}}{0.033} $$

$$ \text{vol basin} = \text{vol flow} $$

$$ (2)(\pi r^2) = (100)(60.6) $$

$$ r^2 = 966 $$

$$ r = 31.0 \text{ ft} $$

$$ D = 62.0 \text{ ft} $$
Quadratic equations may be solved by use of the quadratic formula, factoring, plotting, or trial and error. One convenient means of solution is completing the square; that is, solution by adjusting the equation so the unknown quantity appears in a term which has a square root that can be handled easily.

**Solution by Addition**

*A sheet of metal is 24 by 36 in. It is desired to remove a uniform border from the metal to obtain a smaller sheet with an area of 400 sq in. What must be the width of the border in inches?*

\[
400 = (36 - 2x)(24 - 2x)
\]

\[
100 = (18 - x)(12 - x)
\]

\[
100 = 216 - 30x + x^2
\]

\[
x^2 - 30x = -116
\]

\[
x^2 - 30x + (15)^2 = -116 + 225
\]

\[
x - 15 = \pm \sqrt{109} = \pm 10.45
\]

\[
x = \pm 25.45 \quad \text{(no)}
\]

\[= 4.55 \text{ in.} \]

Check:

\[(26.9)(14.9) = 401 \text{ in.}^2 \]

**Determine the values of x and y in the simultaneous equations**

\[
1.5 y^2 - 8x = 0
\]

\[
x - y - 1 = 0
\]

\[
1.5 y^2 - 8x = 0
\]

\[
8x - 8y - 8 = 0
\]

\[
1.5 y^2 - 8y = 8
\]

\[
y^2 - 5.34y = 5.34
\]

\[
y^2 - 5.34y + \left( \frac{5.34}{2} \right)^2 = 5.34 + \left( \frac{5.34}{2} \right)^2
\]

\[
y - 2.67 = \pm \sqrt{12.46} = \pm 3.54
\]

\[
y = \pm 6.21; -0.87
\]

\[
x = \pm 7.21; \pm 0.13
\]
The mathematical solution of a quadratic equation (see above) results in two possible values of the unknown quantity, each of which will satisfy the initial equation. In most engineering problems, however, one of the values will have no physical significance and can be discarded.

Algebraic expressions involving exponents greater than two, or fractional exponents, in general are solved more conveniently by plotting the curve to a sufficiently accurate scale and reading the desired values.

Simultaneous equations in two or more unknowns may be solved through a combination of equations, by addition, or by substitution until all but one of the unknowns is eliminated from the resulting equation. This equation can then be solved by methods previously outlined.

**Solution by Substitution**

*A man divides a tract of land into city lots. He sells all of the lots at the same price and realizes $4800. If the number of lots had been one greater, and the price $8 cheaper, he would have received the same amount of money. How many lots were there, and what was the price per lot?*

\[ \begin{align*}
  x &= \text{price of lots} \\
  y &= \text{number of lots} \\
  xy &= 4800; \quad y = \frac{4800}{x} \\
  (y + 1)(x - 8) &= 4800 \\
  xy + x - 8y - 8 &= 4800 \\
  4800 + x - 8\left(\frac{4800}{x}\right) - 8 &= 4800 \\
  x^2 - 8x &= (8)4800 \\
  x^2 - 8x + (4)^2 &= (8)4800 + 16 \\
  x - 4 &= \pm 196 \\
  x &= -192; + 200 \\
  x &= $200 \\
  y &= 24 \text{ lots}
\end{align*} \]

Check:
\[ \$200(24) = \$4800 \]
Calculus. The correct solutions of many engineering problems are found by establishing maximum or minimum values for certain combinations of variables. The minimum cost for a given design and the maximum efficiency of a turbine are typical examples.

If the problem can be stated in mathematical terms; for example, \[ y = x^3 - 3x, \]
the differential of this expression, \[ \frac{dy}{dx} = 3x^2 - 3, \]
is the slope of the curve at a given value of \( x \).

When the slope of the curve is zero, a maximum or minimum value of \( y \) has been reached, and

\[ \frac{dy}{dx} = 0 = 3x^2 - 3 \]
\[ x = \pm 1, \text{ for this case} \]

In many practical problems, the negative value of \( x \) would have no physical significance.

A box having a square base and containing 108 cu ft is to be constructed. The box is to have no cover. What would be the size of a box requiring the least amount of lumber to construct it?

Proof:

Box is \( \frac{1}{2} \) of cube

\[ y = \frac{x}{2} \]
\[ 108 = \frac{x^3}{2} \]
\[ x^3 = 216 \]
\[ x = 6 \text{ ft} \]

Amount of lumber (\( S \))

\[ S = x^2 + 4xy \]
\[ dS = 4x \frac{dy}{dx} + 4y \]
\[ 432 = 2x + (-1) \frac{432}{x^2} \]
\[ 2x = \frac{432}{x^2} \]
\[ x^3 = 216 \]
\[ x = 6 \text{ ft} \]
Areas are found conveniently by means of integration if the boundaries can be expressed in mathematical form.

Determine the area under the curve $y^2 = 64x$ between the limits $x = 5$ and $x = 50$.

A differential (very small) area under the curve is defined as the product of the ordinate $y$ and a differential distance along the $x$ axis, $dx$. If all of these differential areas are integrated (summed up), the result is the area under the curve between the limits desired.

$$\begin{align*}
y^2 &= 64x \\
y &= 8x^{1/2}
\end{align*}$$

$$\begin{align*}
A &= \int dy \\
&= \int_5^{50} 8x^{1/2} dx \\
&= 8 \left[ \frac{2^{3/2}}{3} x \right]_5^{50} \\
&= \frac{16}{3} (354 - 11) \\
&= \frac{1830}{3} \\
&= 1830
\end{align*}$$

This answer can be approximated by means of arithmetic integration if finite values are assigned to $dx$ and if the differential areas are treated as trapezoids; that is, if we approximate the curve by a series of short chords. The accuracy of the result will depend upon the value assigned to the differential distance $dx$.

The process of arithmetic integration can be applied to any area, however irregular, and is a valid and useful approach to engineering problems.
Assume \( \Delta x = 5 \text{ ft} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( y \text{ avg} )</th>
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<tbody>
<tr>
<td>5</td>
<td>17.9</td>
<td></td>
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<tr>
<td>10</td>
<td>25.3</td>
<td>21.6</td>
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<td>15</td>
<td>31.0</td>
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<td>20</td>
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<tr>
<td>40</td>
<td>50.7</td>
<td>49.1</td>
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<td>45</td>
<td>53.7</td>
<td>52.2</td>
</tr>
<tr>
<td>50</td>
<td>56.6</td>
<td>55.2</td>
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</table>

\[ \Sigma y \text{ avg} = 365 \]

\[ A = \sum \Delta A = \sum y \text{ avg} \Delta x = (365)(5) \]

\[ A = 1825 \]

Curves related mathematically, such that one is the derivative of the other, have certain characteristics that are dependent upon this relationship.

Assume some curve such that \( A = f(x) \) where \( A \) is equal to an expression in terms of \( x \).

Then the derivative of this curve is:

\[ \frac{dA}{dx} = B, \text{ where } B \text{ is still an expression in terms of } x \text{ but of a lower order; that is, if } A \text{ is a function of } x^3, \text{ then } B \text{ will be a function of } x^2, \text{ et cetera.} \]
From the relationship between the two curves, the following are true:

1. The ordinate to curve B at a given value of \( x \) is equal to the slope of curve A at the same value of \( x \) because \( \frac{dA}{dx} = B \) by the stated relationship.

2. Because \( dA = Bdx \), the area under curve B between \( x_1 \) and \( x_2 \) \( (Bdx) \) is equal to the change in value of A \( (dA) \) between the same two values of \( x \).

The observations made above will prove useful in establishing the relationship of displacement, velocity, and acceleration for motion problems, and load, shear, moment, slope, and deflection for beam problems.

PROBLEMS

1. What is the numerical value of the common logarithm of 100?
2. If the logarithm of 600 is 2.77815, what is the logarithm of the cube root of 600?
3. If the common logarithm of a number is 0.602060, what is the value of the co-logarithm of the number?
4. Find the value of \( x \) in the equation, \( 1.6^x = 1.0 \).
5. If the tangent of an angle is 1.00, what is the value of the secant of the angle?
6. Evaluate the following: \( \sin 270^\circ \), \( \csc \pi \), \( \log 1.0 \), \( \tan \pi/3 \).
7. A rectangular pyramid 16 ft high has a base 8 ft square. This pyramid is to be cut into two parts by passing a plane \( A-B \) through it, forming an angle of 45° with the horizontal, and passing through a point on the axis of the pyramid 10 ft above its base. What is the volume of the top part cut from the pyramid?

Answer: \( 20.5 \) cu ft

8. A doughnut has an inside diameter (the hole in the doughnut) of one inch, and its cross section is also one inch in diameter. What is the volume of the doughnut in cubic inches?

Answer: \( 4.95 \) cu in.
III. STATICS

This discussion of statics will be restricted to treatment of co-planar force systems and to mathematical analysis of the problems involved. Graphical analysis of statics problems can be equally accurate, and sometimes is more convenient.

It should be remembered that a force is a vector quantity and that it is necessary to give magnitude, direction, and point of application in order to fully define a given force. A couple or moment (force \( \times \) distance) is defined by magnitude and direction of rotation, but for purposes of definition need not be tied down to a specific location.

The resultant of a force system is defined as that single force or couple which replaces the force system and still produces the same net effect.

The most valuable tool in the solution of problems involving the action of forces is the free-body diagram. A free-body diagram is a sketch of the selected body showing all the external forces acting upon it. The body in question is thus isolated from contact with all other bodies and the effects of former contacts are shown as external forces acting on the free body. A carefully drawn free-body diagram is essential to the visualization and solution of most statics problems.

The conditions of static equilibrium require that the forces acting along two mutually perpendicular axes sum to zero, and that there be no tendency for the body to rotate. They are usually stated in the following manner:

\[
\begin{align*}
\sum F_x &= 0 \\
\sum F_y &= 0 \\
\sum M &= 0
\end{align*}
\]

The \( x \) and \( y \) axes are usually taken as being horizontal and vertical, respectively, but they may be rotated to any convenient position.

In the solution of problems in statics, the following procedure is recommended:

1. Draw a free-body diagram of the entire structure or machine. Include the effect of gravity and replace all contact surfaces with the forces or couples that could possibly act through them.

2. Write the equations of equilibrium. If three or fewer unknown quantities appear, solve for the desired forces.

3. If more than three unknowns appear, try a free-body diagram of some segment of the structure or machine, and repeat step 2.

4. If, after sketching and testing all possible free-body diagrams a solution is not possible, the problem is said to be statically indeterminate and other conditions in addition to those of static equilibrium must be considered in order to effect a solution.
A 6000-lb wheel with a radius of 3 ft is acted upon by a force $F$ as shown, which tends to pull the wheel over the obstruction $A$. At the instant the wheel is about to go over, the pressure between the wheel and the ground is zero. What is the magnitude of the force $F$ at this instant?

\[ F = 6000 \text{ lb} \]

In the figure shown, three smooth homogeneous cylinders whose radii are equal, each weighs 100 lb. What are the pressures on the cylinder $A$ at the surfaces $D$ and $E$?

\[ \Sigma F_x = 0 \]
\[ F = P \]
\[ \Sigma F_y = 0 \]
\[ 0.5 F + 0.5 P = 6000 \]
\[ F = 6000 \text{ lb} \]
The flywheel shown is acted upon by two belt pulls of 600 lb and 200 lb and the thrust $P$ from the connecting rod, as shown. Determine the amount of thrust $P$ necessary for constant rotative speed. Also determine the bearing reaction at $A$ when $P$ and the belt pulls are acting.
For constant speed, moments about A must balance

\[ \sum M_A = 0 \]

\[ 15P_x \cos 15^\circ + 15P_y \sin 15^\circ + 200(18) - 600(18) = 0 \]
\[ 15(0.866 \, P)(0.966) + 15(0.500 \, P)(0.259) + 3600 - 10,800 = 0 \]
\[ 12.6 \, P + 1.9 \, P = 7200 \]
\[ P = 496 \, lb \]

Balance horizontal forces: \( \sum F_x = 0 \)
\[ 0.866 \, P + (0.500)(200) - R_x = 0 \]
\[ R_x = 530 \, lb \]

Balance vertical forces: \( \sum F_y = 0 \)
\[ 0.500 \, P + R_y - (0.866)(200) - 600 = 0 \]
\[ R_y = 525 \, lb \]

\[ \tan \theta = \frac{525}{530} = 0.99 \]
\[ \theta = 44^\circ \, 40' \]
\[ R = \frac{525}{\sin \theta} = \frac{525}{0.703} = 746 \, lb \]
When one body slides or tends to slide over another, a friction force develops to oppose this motion. The friction force acts in the plane of contact, and its magnitude depends upon:

1. The magnitude of the normal force \( N \), which acts at right angles to the plane of contact and forces the two bodies together.
2. The coefficient of friction \( \mu \), which depends primarily on the roughness of the two surfaces in contact and also, to some extent, on the relative motion of the two bodies.

The friction force is expressed as \( F = \mu N \). It should be noted that friction is a passive force which does not act unless sliding or the tendency to slide exists.

A body weighing 200 lb rests on a plane inclined 30 degrees with respect to the horizontal. The body is acted on by a force of 120 lb. Determine the frictional force acting between the body and the plane. If the friction were not present, which way would the block move?

\[ P = 120 \text{ LB} \]

\[ \sum F_x = 0 \]

\[ F + 120(0.866) = \frac{200}{2} \]

\[ F = 100 - 104 \]

\[ F = -4 \text{ lb acting down the plane} \]

If \( F = 0 \), body moves up the plane.

A block of wood, \( W \), weighing 10 lb is placed on a plank 4 ft long. One end of the plank rests on a horizontal plane at \( A \), and the other end, \( B \), is to be raised to the point where the block \( W \) is at the incipient state of sliding down the plank. If the static coefficient of friction, \( f = 0.577 \), what would be the height, \( H \), if the end \( B \) should be raised so the block \( W \) would be at the incipient stage of sliding?
\[ f = \frac{F}{N} = 0.577 \]

but,
\[ \sum F_x = 0 \]
\[ F = W \sin \theta \]
\[ \sum F_y = 0 \]
\[ N = W \cos \theta \]

\[ 0.577 = f = \frac{W \sin \theta}{W \cos \theta} = \tan \theta \]
\[ \theta = 30° \]
\[ H = 4 \sin 30° = 2 \text{ ft} \]

Note that solution above is set up for a general case and that motion impends when \( \tan \theta \) is equal to the friction coefficient.

The forces exerted by an incompressible fluid against a confining surface are normal to that surface and proportional to the distance below the free fluid surface where atmospheric pressure exists. The intensity of pressure may be stated as \( p = wh \), where

\[ p = \text{pressure (lb/sq ft)} \]
\[ w = \text{specific weight of fluid (62.4 lb/cu ft for water)} \]
\[ h = \text{vertical distance from free fluid surface to point where pressure is desired (ft)} \]
Water stands 37 ft above the top edge of a 6-ft-square sluice gate which weighs 3000 lb. What vertical lift will be required to open the gate if the coefficient of friction between gate and guides is 0.30?

Total force normal to gate:
\[ F = P \cdot \text{avg area} \]
\[ F = (40) \cdot (62.4) \cdot (36) \]
\[ F = 90,000 \text{ lb} \]

Force to lift gate:
\[ T = \text{wt of gate} + \text{friction} \]
\[ T = 3000 + (0.3) \cdot 90,000 \]
\[ T = 3000 + 27,000 = 30,000 \text{ lb} \]

PROBLEMS

1. A body, \( \cdot \cdot \), which weighs 100 lb, rests upon a 30-degree plane, as shown. What is the coefficient of static friction if the body started to move when the angle was increased to slightly more than 30 degrees?

2. A circular cylinder, 4 in. in diameter and 1 ft high, rests with its base on a table. The table is tilted slowly until the cylinder either topples over or slides. Assuming the table has a surface friction factor of 0.4, which will it do?

3. A ladder 20 ft long weighs 40 lb and its center of gravity is at the center of the ladder. The ladder is placed against a vertical wall so that it makes an angle of 60 degrees with the horizontal ground surface. How far up the ladder can a 160-lb man climb before the ladder is on the verge of slipping? Assume the coefficient of friction at all contact surfaces at 0.20. Answer: 6.78 ft
IV. DYNAMICS

In studying the motion of a particle or rigid body without regard to the forces causing that motion, it is necessary to evaluate the displacement, velocity, and acceleration, which are defined as follows:

- **displacement (ft)** = \( s = \text{change in position} \)
  
- **velocity (ft/sec)** = \( V = \frac{ds}{dt} = \text{rate of change of displacement} \)
  
- **acceleration (ft/sec/sec)** = \( a = \frac{dV}{dt} = \text{rate of change of velocity} \)

Note that the velocity is the derivative of the displacement curve and that acceleration is the derivative of the velocity curve. These curves are related as outlined below.

Assume a constant acceleration (gravity) and relate \( a, V, \) and \( s \). Because the ordinate to the \( a \) curve is constant, the \( V \) curve must have a constant slope equal to \( a \).

The change in velocity from time zero to \( t \) is equal to the area under the \( a \) curve during that interval, and the velocity at any time is given as

\[ V = V_0 + at \]

Because the ordinate to the velocity curve has an initial value and increases with time, the slope of the displacement curve must follow a similar course, and the displacement curve will be a parabola. The change in displacement is given by the area under the velocity curve and the displacement at any time is

\[ s = s_o + V_o t + \frac{1}{2} at^2 \]
If the velocity and displacement are initially zero, these equations reduce to the familiar form

\[ V = at \]

\[ s = \frac{-at^2}{2} \]

The approach outlined above applies to the case where acceleration is not constant.

Assume that a baseball will fall 16 ft the first second, 48 ft the next, 80 the next, etc. A baseball was dropped from the top of Washington Monument, 550 ft high, and caught by an American League catcher. Approximately how fast was the ball falling (in fps) when caught?

\[ a = 32 \text{ ft/sec}^2 \]

\[ s = \frac{-at^2}{2} \]

\[ 550 = \frac{-32 t^2}{2} \]

\[ t = \sqrt{34.4} = 5.86 \text{ sec} \]

\[ V = at \]

\[ V = (32)(5.86) = 188 \text{ ft/sec} \]

A stone is dropped down a well 100 ft deep to the water surface. If the velocity of sound in the air is 1120 ft/sec, how many seconds should it take before the sound of the splash in the water can be heard at the top of the well?

For stone to drop

\[ s = \frac{-at_i^2}{2} \]

\[ 400 = \frac{-32.2 t_i^2}{2} \]

\[ t_i = \sqrt{24.8} = 4.98 \text{ sec} \]
For sound to come back

\[ s = V_t t \]

\[ 400 = 1120 t_2 \]

\[ t_2 = 0.357 \text{ sec} \]

total time = \[ t_1 + t_2 = 4.98 + 0.357 = 5.34 \text{ sec.} \]

A pitcher throws a ball so that it leaves his hand horizontally at a height of 5 ft. What must have been the velocity of delivery if the ball passed 2 ft above home plate 60 ft away, assuming no resistance due to the atmosphere? Ball drops 3 ft in 60-ft horizontal.

Time to drop 3 ft

\[ s_v = -a_v t^2 \]

\[ \frac{1}{2} \]

\[ 3 = -32.2 t^2 \]

\[ \frac{1}{2} \]

\[ t = \sqrt{0.186} \]

\[ t = 0.432 \text{ sec} \]

Horizontal component of velocity does not change

\[ s_h = V_h t \]

\[ 60 = V_h (0.432) \]

\[ \frac{60}{0.432} \]

\[ V_h = 139 \text{ ft/sec} \]

If the motion of a particle or rigid body is considered with respect to the forces which cause that motion, the relationship may be stated as

\[ F = Ma \]

where

\( F \) (lb) is the resultant force causing motion

\( M \) (slugs) is the mass of the body which may be taken as the weight (lb) divided by the acceleration of gravity (32.2 ft/sec/sec)

\( a \) (ft/sec/sec) is the acceleration which results and is in the same direction as \( F \)
Two weights, A and B, are suspended by a cord over a pulley, as shown. Assuming the pulley and cord as weightless and frictionless, and the weights A and B permitted to move from the position shown due to the force of gravity, when A is 1000 lb and B is 500 lb:

1. What would be the acceleration of the weights?
2. With what velocity would weight A hit the floor?
3. With what momentum would weight A hit the floor?
4. What would be the tension in the cord in pounds?

For entire system
\[ \sum F = Ma \]
\[ 1000 - 500 = \frac{1000 + 500}{32.2} = a \]
\[ a = \frac{500}{46.6} = 10.7 \text{ ft/sec}^2 \] \hfill (1)

Time for A to drop
\[ s = \frac{-at^2}{2} \]
\[ 1 = \frac{-10.7t^2}{2} \]
\[ t = 0.966 \text{ sec} \]

Velocity of A at floor
\[ v = at \]
\[ v = 10.7(0.966) = 10.4 \text{ ft/sec} \] \hfill (2)

Momentum of A
\[ mom = Mv \]
\[ 1000 \]
\[ = \frac{10.4}{32.2} = 323 \text{ lb/sec} \] \hfill (3)
The relationships developed above apply to linear motion, but the principles developed apply as well to rotation of a rigid body about an axis, where the following relationships hold:

- angular displacement (radians) = \( \theta \)
- angular velocity (radians/sec) = \( \omega = \frac{d\theta}{dt} \)
- angular acceleration (radians/sec/sec) = \( \alpha = \frac{d\omega}{dt} \)

The force-acceleration relationship may be written

\[ T = I\alpha \]

where

- \( T \) (ft-lb) is the resultant torque or unbalanced moment about the center of rotation
- \( I \) (slug-ft\(^2\)) is the mass moment of inertia about the center of rotation

It is frequently necessary to establish the absolute (relative to the earth) velocity of some body \( A \) when its motion relative to some other body \( B \) is known. This may be found by the vector addition of the absolute velocity of \( B \) and the velocity of \( A \) relative to \( B \). The same procedure applies to displacements and accelerations.
The wind drives a boat east with a force which would carry it 10 miles per hour, and the boat's propeller is driving it south with a force which would carry it 20 miles per hour. Find its direction and its actual speed in miles per hour.

\[
\begin{align*}
\tan \theta &= \frac{10}{20} = 0.500 \\
\theta &= 26^\circ 34' \\
V &= \frac{10}{\sin \theta} = \frac{10}{0.447} = 22.4 \text{ mph}
\end{align*}
\]

Bearing, S 26° 34' E

PROBLEMS

1. A 3-ton cage descending a mine shaft with a speed of 9 yards per second is brought to rest with a uniform deceleration in a distance of 18 feet. What is the tension in the cable while the cage is coming to rest? Answer: 9770 lb.

2. Assume an automobile moving 30 miles an hour on a horizontal road surface suddenly runs off the open end of a swing-span bridge. If the water surface in the stream was 30 feet below the roadway, how far horizontally from the end of the bridge would the car strike the water? Answer: 60 ft.

3. A man drives his car from Albany to Portland at an average speed of 60 miles per hour, and then returns by the same route at an average speed of 45 miles per hour. What is his average speed for the round trip? Answer: 50 mph.

4. In the figure, block A weighs 96.6 pounds and block B weighs 64.4 pounds. The coefficient of friction under A is 0.20 and under B 0.25. If \( P = 200 \) pounds, determine the acceleration of A and B, and the tension \( T \) in the connecting rope.

5. Find the value of a constant force which, acting on a weight of 100 pounds for 2 seconds, produces an increase in velocity of 32.2 feet per second.

6. An automobile traveling due north at 60 miles per hour is struck by a stone moving southwest at 50 feet per second (in a plane parallel to the roadbed). Find the velocity of the stone relative to the automobile.
V. ENERGY RELATIONSHIPS

Work (ft-lb) is defined in engineering terms as the product of a force and the distance through which it moves. The British thermal unit (Btu) is 778 ft-lb. Energy (ft-lb) is defined as the ability to do work, or work held in storage. Power (ft-lb/sec) is defined as the rate of doing work, or the rate of energy flow. One horsepower is $[550(ft-lb/sec)]$. The efficiency of a machine or process is the ratio of energy of work output to energy or work input, and is usually expressed as a percentage.

Assume the average drawbar pull per ton of a loaded train is 6 pounds, that the train itself weighs as much as its load, and that the locomotive develops one horsepower-hour from 4 pounds of coal used. The coal costs $8.00 per ton.

1. How many foot-pounds of work is done to haul one ton of freight 1000 miles?
2. How many foot-pounds of work is done for each pound of coal used by the locomotive?
3. How many pounds of coal are used to haul one ton of freight 1000 miles?
4. What is the cost of hauling one ton of freight 1000 miles?

1. Work/ton-mile = $2 \times 6 \times 1000 \times 5280 = 6.33 \times 10^7$ ft-lb.

2. Work/lb coal = $\frac{6.33 \times 10^7}{4} = 1.58 \times 10^6$ ft-lb.

3. W coal/ton-mile = $\frac{6.33 \times 10^7}{4.95 \times 10^6} = 128$ lb.

4. Cost/ton-1000 miles = $\frac{8}{2000} = 0.004$.

A man weighing 180 pounds runs up a flight of 26 steps, each 7 inches high, in 4 seconds. At what horsepower does he work?

$$\text{power} = \frac{180(26) \left( \frac{7}{12} \right)}{4 \times 550} = 1.24 \text{ hp}$$
If complete combustion of one pound of a certain grade of coal develops 13,000 Btu of heat, how much work in foot-pounds would it perform if used in a heat engine of 8% efficiency?

\[
\text{work} = 13,000 \times \frac{778}{805} = 805,000 \text{ ft-lb.}
\]

A spring in the recoil mechanism of a gun is compressed 5 inches by a force of 4 tons. What average horsepower does the recoiling gun develop if it recoils 8 inches in one second?

Spring constant = \( \frac{4 \times 2000}{5} = 1600 \text{ lb} \)

Average force acting = \( \frac{1600}{8} = 6400 \text{ lb} \)

Work = \( \frac{6400}{12} = 4270 \text{ ft-lb} \)

\[
\text{hp}_{\text{avg}} = \frac{4270}{550} = 7.76 \text{ hp}
\]

A fluid may possess energy in a variety of forms. A number of these forms pertinent to engineering practice are outlined below.

\[
Z \left( \frac{\text{ft-lb}}{\text{lb}} \right) = \text{potential energy due to position above a horizontal datum plane (elevation head)}
\]

\[
P \left( \frac{\text{lb}}{\text{ft}^2} \right) = \text{potential energy due to pressure (pressure head)}
\]

\[
\omega \left( \frac{\text{lb}}{\text{ft}^3} \right) = \frac{\text{ft}}{\text{sec}^2} = \text{kinetic energy due to motion (velocity head)}
\]

\[
V^2 \left( \frac{\text{ft}^2}{\text{sec}^2} \right) = \frac{V^2}{2g} \text{ ft-lb}
\]
ft-lb
\[ U = \text{internal energy due to motion of individual lb molecules in fluid mass—depends upon temperature of fluid} \]

Note that the units of each component are given as foot-pounds of energy per pound of fluid flowing—a convenient unit for analysis of problems involving a continuous fluid flow. These units also may be read simply as feet of fluid. Thus a pressure head of \( Z \) feet implies a pressure equal to that exerted at the bottom of a column of fluid \( Z \) feet high, or a kinetic energy equivalent to that attained by a pound of fluid falling freely through a vertical distance of \( Z \) feet.

According to the law of conservation of energy, energy may be neither created nor destroyed, but it may change form. The energy equation written between two points in a fluid body is actually a bookkeeping system used to account for additions, withdrawals, or changes in form of energy.

Referring to the sketch above, the general energy equation may be written

\[ E_1 + q_{1-2} = E_2 + W_{1-2} \]

where \( q_{1-2} \) (ft-lb) represents heat or thermal energy in transition added or removed as the fluid passes through the system.

\( W_{1-2} \) (ft-lb) represents mechanical energy or shaft work added or removed as the fluid passes through the system.
In more specific terms, the energy equation may be written

\[
\frac{V_1^2}{2g} + \frac{P_1}{\omega_1} + U_1 + Z_1 + q_{1-2} =
\]

\[
\frac{V_2^2}{2g} + \frac{P_2}{\omega_2} + U_2 + Z_2 + W_{1-2}
\]

In analysis of specific fluid systems, the general energy equation may be simplified by suitable choice of points 1 and 2 in relation to the datum plane and neglecting those terms which do not apply to the system under consideration.

In flow of incompressible fluids (liquids), the \( U_1 \) and \( U_2 \) terms are generally neglected, and any energy used to raise the temperature of the fluid (pipe friction) is classed as energy withdrawn or lost. For compressible fluids (gases) the change in potential energy (\( Z \)) is small and is usually neglected.

In thermodynamic study of vapors, internal energy and flow energy (\( P/\omega = p\nu \)) are combined and designated as the enthalpy (\( H \)) of the fluid. Enthalpy then becomes a property of the fluid and is a function of the temperature of the fluid. A change in enthalpy may be evaluated as the product of a change in temperature and the specific heat at constant pressure (\( C_p \)) of the fluid.

\[
(p_1\nu_1 + U_1) - (p_2\nu_2 + U_2) = C_p(T_1 - T_2)
\]

\[
H_1 - H_2 = C_p(T_1 - T_2)
\]

In the analysis of fluid flow problems, a convenient method of solution is:

1. Write the energy equation including all terms.
2. Equate to zero those terms which do not apply to the problem.
3. From the data given, evaluate all but one term and solve for the desired term.

_A steam turbine receives 3600 pounds of steam per hour at 110 ft/sec velocity and 1525 Btu/lb enthalpy. The steam leaves at 810 ft/sec and 1300 Btu/lb. What is the horsepower output?_

For each pound of steam

\[
\frac{V_1^2}{2g} + \frac{P_1}{\omega_1} + U_1 = \frac{V_2^2}{2g} + \frac{P_2}{\omega_2} + U_2 + W_{\text{turbine}}
\]

\[
190 + 1525(778) = 10,200 + 1300(778) + W_T
\]
\[ W_T = 778(1525 + 0.2 - 1300 - 13.1) \text{ ft-lb} \]
\[ W_T = 778(212) \frac{\text{ft-lb}}{\text{lb}} \]
\[ \text{lb/sec} \left( \frac{\text{ft-lb}}{\text{lb}} \right) \]
\[ \text{hp} = \frac{3600}{550} \frac{778(212)}{3600} = 300 \text{ hp} \]

Barometric pressure in one locality is 28.5 inches Hg absolute, and in another locality one inch of Hg greater; both measured at 60° F. If the wind generated by this potential pressure difference is half of that theoretically possible, what would be its velocity in miles per hour? (Assume air at 0.075 lb/cu ft.)

For each pound of air
\[ \frac{V_i^2}{2g} + \frac{P_i}{w} = \frac{V_f^2}{2g} + \frac{P_f}{w} \]
\[ \frac{V_i^2}{2g} + 28.5'' Hg = 0 + 29.5'' Hg \]
\[ \frac{V_i^2}{2g} = 1'' Hg = \frac{1}{12} \times 62.5 \times 13.6 \times 0.075 \]
\[ \frac{V_i^2}{2g} = 950 \text{ ft-lb/lb air} \]
\[ V_i = 247 \frac{\text{ft}}{\text{sec}} = 168 \text{ mph (theoretical)} \]
\[ V = 84 \text{ mph (actual)} \]
How many kilowatts of electrical power are represented by 3000 cfs of water falling a vertical distance of 100 feet?

\[
\text{hp} = \frac{3000 \times (62.4) \times (100)}{550} = 34,000
\]

\[1 \text{ hp} = 0.746 \text{ kw}\]

\[
\text{kw} = 34,000 \times 0.746 = 25,400 \text{ kw}
\]

The equation of continuity, as applied to fluid systems with steady flow, states that the weight rate of flow past any point in the system must remain constant.

\[
G = w_1A_1 V_1 = w_2A_2 V_2 = w_3A_3 V_3
\]

and for incompressible fluids it may be further simplified to state that the volume rate of flow remains constant.

\[
Q = A_1 V_1 = A_2 V_2 = A_3 V_3
\]

What is the horsepower of a jet of water 11 inches in diameter and moving with a velocity of 265 feet per second?

\[
Q = \frac{V^2}{2g} = \frac{1090 \times (265)^2}{2g} = H_v
\]

\[
Q = VA = (265) (0.785) \left(\frac{11}{12}\right) = 175 \text{ cfs}
\]

\[
\text{hp} = \frac{Q \times 110}{550} = \frac{175 \times (62.4) \times (1090)}{550} = 21,700 \text{ hp}
\]

A centrifugal pump is to be directly connected to an electric motor and large enough to lift 1,000,000 gallons of water a vertical distance of 100 feet in 24 hours. Assuming the friction head in the pipe to be 10 feet, the pump efficiency 75%, and the motor 90%, what size horsepower motor must be purchased?

\[
Q = \frac{1,000,000}{24 \times 3600 \times 7.5} = 1.54 \text{ cfs}
\]

\[
\text{hp} = \frac{1.54 \times 62.4 \times 110}{550 \times 0.75 \times 0.90} = 28.8 \text{ hp}
\]
In evaluating terms in the energy equation, it is sometimes necessary to use the equation of state for gases

\[ pv = RT \]

where

\[ p \left( \frac{lb}{ft^2} \right) \text{is the absolute pressure} \]

\[ v \left( \frac{ft^3}{lb} \right) \text{is the specific volume of the gas} \]

\[ R \left( \frac{ft}{\text{degree}} \right) \text{is the gas constant (53.3 for air)} \]

\[ T(\text{degrees}) \text{is the absolute temperature (degrees Rankine)} \]

\[ (460 \text{ degrees Rankine} = \text{zero degrees Fahrenheit}) \]

For systems which can be considered as being closed without flow, the mechanical or shaft work is evaluated as a pressure acting through a change in volume. Since the process is one in which there is no flow, the energy equation may be simplified to

\[ iq_s = (U_2 - U_1) + iW_2 \]

The solution of this type of problem involves consideration of the state of the system at successive intervals of time. Thus work may be represented graphically on a pressure and volume diagram.

In order to evaluate mechanical work as the system changes from state 1 to state 2, the work \((iW_2)\) is equal to \(\int_{V_1}^{V_2} PdV\). The relationship of \(V\) with respect to \(P\) must be known, and the following reversible changes in volume have become general in the thermodynamic systems for gases:

**Constant volume:** Because there is no change in volume, the work is zero and the heat added is equal to the change in internal energy.

**Constant pressure:** Represented as a horizontal line on a \(PV\) diagram the process is equivalent to a change in enthalpy and the work is the product of the pressure and the change in volume

\[ iW_2 = P(V_2 - V_1) \]

**Isothermal:** For this process the temperature is constant and \(P_1V_1 = P_2V_2\). There is no change of internal energy and

\[ iW_2 = P_1V_1 \ln \frac{P_2}{P_1} \]

**Isentropic process:** There is no heat transferred during this process and any work done is at the expense of internal energy,
\[ W_2 = \frac{P_1 V_1 - P_2 V_2}{K - 1}, \]
where \( K \) is ratio of specific heats at constant pressure and constant volume. \( (K = 1.4 \text{ for air.}) \)

**Polytropic**: Defined as any reversible process in which \( PV^n \) is a constant, where \( n \) can be any value between 0 and infinity. In evaluating the work for this process, it becomes

\[ W_2 = \frac{P_1 V_1 - P_2 V_2}{n - 1} \]

The temperature of a gas sample is increased from 80° F to 130° F, with a volume increase of 10%. If the pressure before heating was 2 atmospheres, calculate the new pressure.

\[
\frac{P_1 V_1}{T_1} = R = \frac{P_2 V_2}{T_2}
\]

\[
\frac{P_2}{P_1} = \frac{V_1 T_2}{V_2 T_1} = \frac{100}{110} \left( \frac{460}{460} \frac{135}{80} \right) = 100 \frac{595}{540} = 1.00
\]

No change in pressure.

A cylinder contains 3 cubic feet of nitrogen at 15 psia and 60° F. The gas is compressed reversibly and isothermally to a pressure of 100 psia. Compute the heat transferred and the work done. \( R \) for nitrogen is 55.1.

\[
Q_2 = (U_2 - U_1) + W_2
\]

\[
Q_2 = 0 + \int_1^2 PdV
\]

\[
PV = P_1 V_1 = P_2 V_2
\]

\[
Q_2 = P_1 V_1 \int_1^2 \frac{dV'}{V'} = P_2 \frac{3 \times 15 \times 144}{778} \frac{100}{15} = 15.78 \text{ Btu}
\]
PROBLEMS

1. Work is the product of torque and pressure and temperature and.

2. For steady work during a day of 8 hours, a draft equal to 1/10 the weight of the horse, exerted at 2.5 miles per hour, is regarded as full demand for the animal. Under these circumstances, at what horsepower does a horse work if he weighs 1000 pounds? Answer: 0.667 hp.

3. A box of freight weighing 300 pounds is pushed up a plank into a truck. The plank is 8 feet long and makes an angle of 30 degrees with the horizontal. If the friction coefficient between box and plank is 0.40, compute the total work done. What horsepower would be required to accomplish this work in 10 seconds? Answer: 0.37 hp.

4. A bomber whose engine develops 8000 horsepower with an efficiency of 25% makes a 6-hour trip. The gasoline has a heating value of 20,200 Btu per pound and weighs about 6.1 pounds per gallon. If it takes 778 ft-lb to make one Btu, how many gallons of gasoline are needed for the trip? Answer: 3980 gallons.

5. A jet of water at atmospheric pressure is 7 inches in diameter and has a velocity of 250 feet per second. What is the horsepower?

6. If a centrifugal pump delivers 3 cfs of water under a total delivery head of 100 feet at an efficiency of 65%, what horsepower is required to drive the pump? Answer: 52.5 hp.

7. A municipal pumping plant has a maximum capacity of 48,000 gallons of water per minute, pumping it against a total head of 122 feet. The pumps are 62.1% efficient and the electric motors are 92.3% efficient. The plant is operating at full load for 4 hours per day, and under 60% load the rest of the day. If the power cost is 0.8 cents per kw-hr, compute the monthly power bill.

If a part of the water flows through a 12-inch line and the flow is measured by an orifice meter with vena contracta taps and a diameter of 6 inches, what is the flow through the line in gallons per minute when the pressure differential across the orifice is 24 inches of water? The flow coefficient for the above conditions of flow is 0.625.
VI. STRENGTH OF MATERIALS

For purposes of design, engineering materials are commonly assumed to be homogenous and elastic. It is further assumed that materials follow Hooke's law

\[ \frac{f}{\varepsilon} = \text{a constant} = E \]

where

- \( f \) (lb/in.\(^2\)) is the unit stress which is equal to the applied load \( (P) \) divided by the cross-sectional area for axially-loaded members.
- \( \varepsilon \) (in./in.) is unit strain (change in length) due to load and is equal to the total change in length \( (\delta) \) divided by the length \( (L) \) of the member.
- \( E \) (lb/in.\(^2\)) is the modulus of elasticity and is taken as the slope of the stress-strain curve below the proportional limit of the material.

For members that are axially loaded, the total change in length is given as

\[ \delta = \frac{PL}{AE} \]

Each cable of the Golden Gate Bridge over San Francisco Harbor consists of 27,572 steel wires, each having a diameter of 0.192 inches. What is the in the cable when the unit stress in each wire has the value used in design, 82,000 lb/sq in.?

\[ F = \pi \left( \frac{0.192}{2} \right)^2 27,572 \]

\[ F = 65,500,000 \text{ lb} \]

When a material is subjected to a change in temperature it tends to expand or contract. The change in length of a given member can be determined by use of a coefficient of linear expansion, \( \alpha \), (in./in./\(^\circ\)F), which is determined experimentally

\[ \delta_T = \alpha L (T_2 - T_1) \]
An iron rail is 32 feet long at 32 degrees F. How long is it on a hot day when at 100 degrees F, assuming it is free to expand?

For steel, coefficient of expansion $\alpha = 0.0000065$ in./in./°F.

$$\delta_T = \alpha L \Delta T$$

$$= 0.0000065(32)(12)(100 - 32)$$

$$= 0.17 \text{ in.}$$

$L + \delta_T = 32 \text{ ft-3/16 in. approximately}$

If the member when subjected to a temperature change is restrained so it is not free to expand or contract, a thermal stress develops in the member.

A heavy steel structure 10 feet long is set between two identical masonry walls. If the compressive stress in the strut increases 5000 psi under a temperature rise of 40° F, how much did each wall yield?

Coefficient of expansion per degree is 0.0000065 in./in./°F, modulus of elasticity is 30,000,000 psi. If beam is not restrained, expansion is

$$\delta_T = 0.0000065(120)(40)$$

$$= 0.0312 \text{ in.}$$

Shortening due to 5000 psi

$$\delta = \frac{120 \times 5000}{30 \times 10^6} = 0.0200 \text{ in.}$$

each wall yields

$$\Delta = \frac{0.0312 - 0.0200}{2} = 0.0056 \text{ in.}$$

In analysis and design of structures, it is necessary to satisfy all of the following:

1. Conditions of static equilibrium.
2. Geometry of the deflected structure and of the assumed stress distribution.
3. Properties of the material.

A procedure for design may be summarized as follows:

1. Determine the external forces necessary to place the structure in static equilibrium.
2. Cut successive free-body diagrams and determine internal forces necessary to place the structure in static equilibrium.
3. Select a suitable material, properly proportioned, to resist internal forces.
Specifications of the American Water Works Association provide that a 36-inch diameter Class A (wall thickness 0.99 in.) cast-iron pipe must withstand a hydrostatic pressure of 150 lb/sq in. What circumferential unit stress does this pressure cause?

For 1 in. slice of pipe:

\[
2T = 150(36) \\
T = 150(18) \\
s = \frac{T}{A} = \frac{150(18)}{0.99} \\
s = 2730 \text{ psi}
\]

Construction of shear (\(V\)) and bending moment (\(M\)) diagrams for members subjected to bending may be classified as step 2 of the design procedure outlined previously. The \(V\) and \(M\) diagrams show internal forces acting at any cross section taken perpendicularly to the longitudinal axis of the member.

Consider a simple beam uniformly loaded with some load \(P\), pounds per lineal foot. The external forces necessary to place the beam in equilibrium are forces of \(PL/2\) at each reaction.

Cut a section some distance \(dx\), to the right of the left reaction

\[
V = \frac{PL}{2} - P \, dx
\]

In order to sum the vertical forces to zero (\(\sum F_y = 0\)), a vertical force (shear) must act at the cut section.
Note from the above sketch $PL/2 - V = dV = Pdx$, or the change in shear along the length of the beam is $dV/dx = P$, which shows the $P$ curve and the $V$ curve are related curves and the ordinate to the $P$ curve is equal to the slope of the $V$ curve for any given value of $x$. Because the ordinates to the load curve are constant, the slope of the shear diagram must be constant in this case.

A straight line beginning at $+PL/2$ at the left reaction and ending at $-PL/2$ at the right reaction, defines the shear in the beam for any given value of $x$. It is also true that the area under the load diagram between any two points on the beam is equal to the change in shear between those same two points.

If the free-body diagram of the cut section of the beam is examined closely, it will be seen that although the vertical forces are in balance, there is still a tendency for this segment of the beam to rotate in a clockwise direction; therefore, an internal couple, $M$ (bending moment), must be present to hold the free body in equilibrium.

$$M = \frac{PL}{2} d x$$

Assume that $(dx)^2$ is small enough to be neglected, and note that change in moment $= O + M = dM = V dx$, because $PL/2$ is the shear near the end of the beam. Then

$$\frac{dM}{dx} = V$$

and the same relationships must exist between the $V$ and $M$ diagrams as existed between the $P$ and $V$ diagrams. Because the ordinate of the $V$ diagram is large at the left reaction and decreases to zero at the center of the beam, the slope of the $M$ diagram must follow the same pattern. It follows that a point of zero shear is a point of maximum or minimum moment. The change in moment from the left end of the beam to the center is equal to the area under the shear diagram between those points.

$$M = \frac{1}{2} \left( \frac{PL}{2} \right) \left( \frac{L}{2} \right) = \frac{PL^2}{8}$$

The above method is most convenient for construction of shear and moment diagrams. Other methods that may be used are outlined below:

1. Formulate algebraic equations for $V$ and $M$ in terms of the loading $P$, and the horizontal distance $x$, and sketch curves by evaluating critical points.
2. Cut successive free-body diagrams along the beam, compute values of $V$ and $M$ for each section, and plot the resulting curves.

*A beam 30 feet long carries a uniform load of 400 pounds per linear foot. The beam is supported on a simple span of 20 feet, with an overhang of 10 feet at one end. Draw the shear diagram and find the location and values of the maximum bending moment.*

In proportioning members for given loads, the use of certain area relationships obtained mathematically is necessary. For some area as shown
the first moment of the area about the $x$-$x$ axis is

$$fy \, dA = A \, \bar{y}$$

where $\bar{y}$ is the distance to the centroid (center of gravity) of the area. The second moment (moment of inertia) of the area about the $x$-$x$ axis is

$$I_{x-x} = \int y^2 \, dA = Ak^2$$

where $k$ is defined as the radius of gyration.

If it is desired to determine properties with respect to the $g$-$g$ axis through the centroid, then the first moment is zero and the moment of inertia

$$I_{0-g} = I_{x-x} - A \, \bar{y}^2$$

Find the least radius of gyration of a timber column whose net cross section is 9 inches by 12 inches.

$$k = \text{radius of gyration}$$

$$k = \sqrt{I/A}$$

$$I = \frac{1}{12} \, bd^3$$

$$A = bd$$

$$\frac{1}{A} = \frac{1}{bd} \frac{bd^3}{12} = 81$$

$$6.75 \text{ lb/sq in.}$$

$$k = \sqrt{6.75} = 2.6 \text{ in.}$$
PROBLEMS

1. In engineering literature the term cement is understood to mean the finely pulverized product obtained by the burning of a suitable mixture of argillaceous and calcareous materials, or by artificial mixture of such materials after burning which will possess the property of hardening into a solid mass when mixed with water. Name the three essential ingredients in the manufacture of cement.

2. Within the limit of elasticity of a substance, “stress is proportional to strain.” This is known as __________.

3. Fill in the spaces with the proper words in each of the following:
   a) Brass is composed of __________ and __________.
   b) Bronze is composed of __________ and __________.
   c) Besides iron, stainless steel contains __________ and __________.
   d) Cement is a ternary mixture of __________, __________, and __________.
   e) Buna S rubber is made of __________ and __________.
   f) Concrete is usually a mixture of __________, __________, and __________. Made plastic with water.

4. Find the proper thickness of a wrought-iron steam pipe, 16 inches in diameter, to resist a pressure of 200 pounds per square inch, with a factor of safety of 10. Ultimate strength of wrought iron is 50,000 lb/sq in. Answer: 0.32 in.

5. In the discussion of beams, the “general flexure formula” and the formulas for deflection are based upon the assumption that the values of E for stress and deformation below the neutral axis are the same as for stress and deformation above the neutral axis. Write the “general flexure formula,” and define each term used.

6. Determine the reactions at A and B on the beam shown. Neglect the weight of the member.

Draw the shear diagram for the beam.

Draw the moment diagram for the beam.

7. A wood column is made from a 12-inch by 12-inch timber 16 feet long. What is the value of the least radius of gyration of that column?
VII. ELECTRICITY

Ohm's law is stated as

\[ E = IR \]

where

- \( E \) is the electrical potential difference across a circuit in volts
- \( I \) is the current flow in amperes
- \( R \) is the circuit resistance in ohms

Electrical power is given as

\[ \text{watts} = EI \times \text{power factor} \]

For direct-current circuits, the power factor may be taken as unity.

An electric stove with a resistance of 20 ohms when hot is connected to a 115-volt circuit. Find the power used (1) in kilowatts, and (2) horsepower.

\[ E = IR \]

\[ I = \frac{E}{R} = \frac{115}{20} = 5.75 \text{ amps} \]

Power = \( EI = (115)(5.75) = 661 \text{ watts} \) = 0.661 kw

Power = \( \frac{661}{0.746} = 0.886 \text{ hp} \)

An electric water heater is rated at 220 volts and has a resistance of 11 ohms. What is its rated load in kilowatts, and how much would it cost to run this unit for 10 hours when power costs 3 cents per kw-hr?

\[ I = \frac{220}{11} = 20 \text{ amps} \]

\[ \text{Power} = \frac{(220)(20)}{1000} = 4.4 \text{ kw} \]

\[ \text{Cost}/10 \text{ hr} = (4.4)(10)(0.03) = \$1.32 \]

The flow of electrical current through fundamental circuits is analogous to the flow of water through pipes. Thus resistances in parallel may be visualized as parallel pipes between two reser-
voirs with the voltage drop \( E \) corresponding to the head loss through the pipes. The head loss in each pipe is equal to the difference in water surface elevations of the two reservoirs, and the voltage drop across each of the resistances is the same.

Assume the resistors \( R_1, R_2, \) and \( R_3 \) are in parallel and connected across the source of potential difference \( V \), and that the resistance of the ammeters and connecting wires as shown is negligible. If \( R_1 = 50 \) ohms, \( R_2 = 100 \) ohms, and \( R_3 = 50 \) ohms when the voltage \( V = 100 \):

1. Calculate the current in each branch \( I_1, I_2, \) and \( I_3 \).
2. Calculate the total line current \( I_L \).
3. Calculate the equivalent resistance of the three resistors in parallel.

\[
\begin{align*}
I_1 &= \frac{100}{50} = 2 \\
I_2 &= \frac{100}{100} = 1 \\
I_3 &= \frac{100}{50} = 2 \\
I_L &= 5 \text{ amp} \\
R_{\text{equi}} &= \frac{100}{5} = 20 \text{ ohms}
\end{align*}
\]

 Resistances of different values connected in series are similar to pipes of different diameters in a line connecting two reservoirs. The voltage drop through the electrical system is equal to the sum of the voltage drops across the individual resistors, just as the head loss through the hydraulic system would be the sum of the head losses for the individual pipes.

An electric circuit is made up of resistances as shown. The voltage across \( R_2 \) is measured as 20 volts. Calculate the current in \( R_5 \).

\[
\begin{align*}
R_1 &= 300 \text{ ohms} \\
R_2 &= 4 \text{ ohms} \\
R_3 &= 20 \text{ ohms} \\
R_4 &= 5 \text{ ohms} \\
R_5 &= 25 \text{ ohms}
\end{align*}
\]
Assume $E_1 = 0$ \( \Rightarrow \)

then $E_2 = 20$

\[
\begin{align*}
I_2 &= \frac{20}{4} = 5 \\
I_3 &= \frac{20}{20} = 1 \\
I_4 &= \frac{20}{5} = 4 \\
I_5 &= 10 \text{ amps} \\
E_{2-2} &= (10)(300) = 3000 \\
E_3 &= 3020 \text{ volts} \\
I_5 &= \frac{3020}{25} = 120.8 \text{ amps}
\end{align*}
\]

PROBLEMS

1. An instrument for measuring an electric current by its magnetic effects is called a _______.

2. By diagram show the construction of a Wheatstone bridge. What is this device used for?

3. Draw a diagram showing the manner in which a SR-4 electric strain gage is connected with a Wheatstone bridge for measuring structural stresses, and briefly explain the theory involved.

4. When two different metals are joined together in a circuit and one junction is heated, an electric current flows through the circuit. This device, commonly used to measure temperatures, is called _______.

5. There are two general types of alternating-current electric motors. Name them.
6. An induction motor and a synchronous motor both use alternating current as a source of power. In what fundamental way do these two motors differ in their construction and operation?

7. An alternating-current generator may be one of two types: revolving armature type, or revolving field type. Which type is commonly used for large kilowatt and high voltage where the speed may be either of low or high rpm? Explain briefly.

8. An electric stove with a resistance of 20 ohms when hot is connected to a 115-volt circuit. What is the cost of operating this stove for 3 hours if electric power costs 3 cents per kw-hr? Answer: $0.06.

9. Twelve batteries having a voltage of 1.5 volts each, and a resistance of 0.1 ohm are connected in two parallel groups of 6 in series. They are connected to an external resistance of 4.2 ohms. What current will flow?

10. A single phase induction motor draws 6 amperes at 230 volts. If the motor has an efficiency of 90% and is operating at a power factor of 0.77, what is the horsepower output?
VIII. MISCELLANEOUS PROBLEMS

1. A walkway is to be lighted by a series of lamps mounted on 20-foot poles. The poles are to be spaced so illumination on the walk between poles is one-fourth of that immediately beneath the pole. It may be assumed the effect of one lamp is negligible at the base of the next. Determine the proper spacing. Answer: 106 feet.

2. Two light sources, A and B, have candle power of 16 and 97, respectively. If a screen is placed between the two light sources, how far from the smaller light should the screen be placed to be equally illuminated by the two? The distance between light sources is 33 feet. Answer: 9.55 feet.

3. What is the principal constituent of a solid, liquid, and gaseous fuel?

What is a mixture called which consists of 75 parts by weight of potassium nitrate, 15 parts of charcoal, and 10 parts of sulfur, made by grinding together the ingredients with enough water to moisten, then compressing into cakes and breaking into grains?

So far as now known, what are the two principal constituent parts of an atom?

What is a substance called which, by its mere presence, alters a chemical reaction, and which may be recovered unaltered in nature or amount at the end of the reaction?

4. If the molecular weight of hydrogen is 1.008, that of sulfur 32.06, and that of oxygen 16.00, how many pounds of sulfur would be required to make 100 pounds of sulfuric acid?

5. How much water must be added to 5 quarts of acid, which is 10% full strength, to make the mixture 8.3% full strength?

6. At atmospheric pressure, water freezes at 32° F, and benzine freezes at 41.9° F. Water ice floats on liquid water at the freezing point and benzine ice sinks in liquid benzine at its freezing point. What effect would an increase in pressure have upon the freezing points of each of these materials, and what general principle applicable to all action and reaction can be used to explain these effects?

7. Name three ways in which heat may be transferred from one body to another.

What element is the principal constituent of all fuels, whether liquid, solid, or gaseous?

8. There are four principal methods of manufacturing steel. Name three of the four processes.

9. In a study of shoveling it was found that the load per shovelful and the number of shovelfuls handled per minute vary with the condition of the shovel, as shown in the accompanying table.
Period of hours after sharpening: 0-8, 8-16, 16-24, 24-32, 32-40

Average shovel load in pounds over given period: 3.23, 2.87, 2.65, 2.51, 2.42

Average shovelfuls per minute over period: 6.72, 6.08, 5.66, 5.30, 5.04

If the cost of sharpening a shovel is $0.28 and the changes of both load and pace are due entirely to the dulling of the shovel, what reduction in cost in terms of dollars per 100 pounds of earth moved will result if shovels are sharpened at intervals of 16 hours instead of 40 hours? The shoveler's wage is $1.00 per hour.

What is the optimum interval of sharpening, assuming shovels can be sharpened only at the end of an 8-hour interval?

10. Fill in the spaces with the proper words in each of the following:
   Geodetic surveying differs from plane surveying in that it takes into account...
   At any point in a tube through which a liquid is flowing, the sum of the pressure energy, potential energy, and kinetic energy is...

11. A gasoline engine and a diesel engine both use petroleum products as a source of power. In what fundamental way do these two engines differ in their construction and operation?
12. A city water treatment plant and a city sewage treatment plant both are used for conditioning water. In what fundamental ways do these two plants compare, and in what ways do they differ in their construction and operation?
13. In general, all methods of measuring flowing water may be classed in one of two divisions, examples of which are the pitot tube and the venturi meter. In what fundamental way do these two meters differ in the method of measuring flowing water?
14. When an observer is in motion toward a source of sound, the pitch of the note heard is higher than when he is at rest. This phenomenon is known as...
15. The heat required to raise a pound of water one degree Fahrenheit is commonly called...
16. During a heavy storm period the mercury barometer read 28.9 inches. What was the approximate atmospheric pressure in pounds per square inch?
17. One inch of runoff from a watershed in 24 hours is equivalent to how many cfs per square mile?

18. Twenty-eight and eight-tenths horsepower are required to operate a pump and motor. The motor is rated at 30 hp. Find the monthly power bill for operating the pump continuously (30-day month) under the following system: $1.00 per month per horsepower of installation as a demand charge, plus 2.5¢ per kw-hr for the first 200 hours’ use of the active connected load, plus 1.5¢ per kw-hr for all energy over 200 hours’ use of the active connected load (full power of motor).

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