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The objective of this study is to develop an ordering policy which will minimize the total cost associated with maintaining an inventory under active constraints. Previous work in this area has concentrated on the application of either the Lagrangian multiplier technique or the exchange curve which allows non-linear trades-offs between cost elements. However, if the constraint depends on the total inventory level such as warehouse space limitation or maximum investment in inventory, we can obtain the optimum level for operating the system at its minimum cost.

The methodology presented herein is based on the composite

probability density function representing the total inventory level. This function results from the additive combination of demand rates for individual items in the inventory. Demand is assumed to be compound Poisson for each item and thus the total inventory level may be adequately represented by the normal distribution as the number of items increases in the system. The optimum ordering policy and the distribution of the total inventory level for the system are obtained using simulation after considering Schaack and Silver's algorithm.

In determining the optimum constraint level, the unit system cost, the penalty cost, and demand process are found to be important factors. The ratio of the unit system cost to the penalty cost determines the optimum constraint level for the given distribution of total inventory level. The distribution is also affected by the demand process. That is, the higher the demand the more variation which results. Thus increased savings result while stocking high-demand items.

Development and implementation of this methodology is illustrated through the use of several example problems.

Determination of Minimum Operating Cost
for a Multiple-Item Constrained Inventory

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To my parents, family members, and my wife, Sooyoun Kim, for their encouragements and sacrifice without which I would not have been able to finish my graduate work.

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DETERMINATION OF MINIMUM OPERATING COST FOR A MULTIPLE-ITEM CONSTRAINED INVENTORY

I. INTRODUCTION

1.1 Multi-Item Inventory systems

An inventory problem exists when it is necessary to stock physical goods or commodities for the purpose of satisfying demand over a specified period of time. Almost every business must carry stocks of goods in order to ensure smooth and efficient operation. Decisions regarding how much should be ordered for stocking and when it should be ordered are typical of every inventory situation. A high level of inventory (overstocking) requires higher invested capital per unit time but fewer shortages and placements of orders. A lower level of invenotry (understocking) decreases the invested capital per unit time but increases the frequency of ordering as well as the risk of running out of stock. Obviously, both cases are costly. Therefore, decisions regarding the quantity ordered and the time at which orders are placed should be based on the minimization of a cost function which balances the total costs resulting from overstocking and understocking.

In developing decision models, it is necessary to first explore the basic characteristics of an inventory system such as relevant costs and demand patterns which may affect the development of such models. As will be discussed later, inventory systems should be considered a function of the number of items involved in the system : that is, the single-item case or multi-item case. The problems of the single-item case are considerably simpler than those of the multi-item cases because we do not need to consider the interactions between items.

However, most organizations involved in the management of inventory are faced with making decisions for large number of individual items.

The control of such large numbers of items presents many problems that do not arise in considering just a single item. There is usually a diverse collection of factors (e.g., the demand pattern, the mode of shipment from supplier, and the methods of delivery to the customer, etc.) and constraints (e.g., budget limitations, warehouse space limitations, etc.). It is one thing to try to develop an optimal operating policy for just a single item, but it is something quite different to attempt to develop optimal operating policies for 1,000 or 10,000 items.

A number of studies have been performed on the topic of multi-item inventory control, where joint order of several items may save a part of the ordering cost. In an early work, Balintfy (1964) pointed out the interaction between items caused by the effect of certain combinations of order and proposed a random joint order policy as a reasonable strategy.

Some of the reasons why multi-item inventory control can reduce costs were discussed by Silver (1974):

1) Several items are produced on the same equipment, in which case coordination of run quantities may significantly reduce setup costs.
2) Several items are purchased from the same supplier, in which case coordination may allow use of group quantity discounts.
3) Several items share the same transportation facilities, in which case coordination may result in transportation economies (full car load).

Also, when we are dealing with many items, the total inventory level which affects the level of constraints, will not change constantly even though demand rate of each item is constant. Using this property of total inventory level, we can reduce the required constraint or reduce operating costs with the given constraint, which is a main idea of this thesis. We will discuss this property in more detail later.

Just as in the case of a single item, there are some basic decisions which must be made in a multi-item inventory situation. Basically, we need to know the following :

1) When should a replenishment order be placed ?
2) How large should the replenishment order be ?
3) How should the inventory status be determined and what are the review intervals ?

These variables affect the costs relevant to the inventory system and are subject to the control of the system design. Therefore the general inventory problem is to find the specific values of these variables that minimize the cost.

1.2 Relevant Inventory Cost

The costs incurred in operating an inventory system should play a major role in determining what the operating policy should be. The costs which influence the operating policy are clearly only those costs which vary as the operating policy is changed. Fundamentally, there are four types of costs which may be important in determining what the operating policy should be. These are

1) the costs associated with replenishing the units stocked (replenishment costs)
2) the costs of carrying the items in inventory (carrying costs)
3) the costs associated with demands occurring when the system is out of stock (shortage cost)
4) the costs of operating the data gathering and control procedures for the inventory system (system control costs)

These costs may be described in more detail as follows.

1.2.1 Replenishment costs

Replenishment costs are the costs incurred each time a replenishment action is taken. This cost can be expressed as the sum of two parts :

i) a fixed component often called the setup cost independent of the size of replenishment (AF)

ii) a component that depends on the size of replenishment(AJ).

In this paper, we deal with a multi-item dependent inventory system. The system is considered to be dependent because the replenishment cost varies with the number of items ordered. That is, the cost of placing an order is AF + AJ_i if item i is ordered. If there are n items altogether, the replenishment cost is AF + (AJ_1+ AJ_2+ ... + AJ_n). Thus we can save (n-1)*AF under joint replenishment condition which we have to pay if we order individually.

1.2.2 Carrying costs

Having materials in stock incurs a number of costs. One of them is the real cost. Real costs include insurance, taxes, breakage, pilferage at the storage site, warehouse rental or ownership expense, and the costs of operating the warehouse such as those for light, heat, and security.

Frequently, the most important cost is one which is not a direct expense but rather an opportunity cost which would never appear on an accounting statement. This is the cost incurred by having capital tied up in inventory rather than having it invested elsewhere, and it is equal to the largest rate of return which the system could obtain from alternative investments. By having funds invested in inventory, one forgoes this rate of return, and hence it represents a cost of carrying inventory.

We also need a kind of physical system cost to carry inventories. For example, we need a warehouse to keep inventories. The space requirement of a warehouse depends on the total inventory level. There will be a cost to build a warehouse of the proper size and a penalty cost for the number of items exceeding the warehouse space. That is, we have to lease or rent a warehouse thereby paying more than if we owned it. We call these costs physical system costs in carrying inventory.

1.2.3 Shortage costs

When inventory levels are not sufficient to satisfy customer demand, costs are incurred, whether or not they can be explicitly measured. Unsatisfied demand leads to immediate costs of backordering and/or lost sales. However, shortage cost are inherently extremely difficult to measure because they can include such factors as intangible losses of customers' good will.

Since poor service can have a long-range cost impact through loss of good will, in the short run, many companies will take almost any possible action to avoid shortages. In this paper, we do not attempt to quantify this cost. The reasons are, as we mentioned before. that (i) it is not easy to measure such a cost, and (ii) since most companies keep a high level of customer service. the actual shortage cost may be very small.

1.2.4 System control costs

In order to use any given operating policy, an inventory system must gather the information required for its use. The expense of obtaining the information necessary for decision making will depend on the type of operating policy used. This expense may include such things as the costs associated with having a computer continuously update the inventory records, the cost of making an actual inventory count, the cost of making demand predictions, and the cost of generally maintaining the system.

It is relatively easy to list the categories of costs as we have done above. However, their measurement in practice is a different story. Actually, it is quite difficult to represent mathematically all the cost components with complete accuracy. Consequently, it is desirable to make some approximations when representing these costs in the mathematical models to be developed. Sometimes surrogates or exchange curves can be used to portray aggregate tradeoffs between measures of interest as we vary a policy variable.

1.3 Classification of Inventory Systems

Two ways of classifying inventory models are presented in this section. In the first part, inventory models are divided according to their characteristics ; in the latter section, they are classified into two parts by the difference in implementation of ordering.

1.3.1 Classification by characteristics

Inventory control is quite broad and has a multitude of variations. A method of classification by Silver (1981) is discussed to show the variety of inventory problems. A number of the factors which may be used to classify inventory systems are shown below.

A) Single item case vs. multiple item case
Single item cases are classified into A1 while multi-item problems can take on a variety of forms according to item interactions, including

- A2 - Overall constraint on budget or space used by a group of items
- A3 - Coordinated control to save on replenishment costs
- A4 - Substitutable items
- A5 - Complementary demand

B) Deterministic vs. probabilistic demand

- B1 - Deterministic demand

  Where demand is not known with certainty, there are several versions of the probabilistic representation.

B2 - Known probability distribution

B3 - Special known distribution such as intermittent demands

B4 - Known form of the distribution in a unit time period but with parameters not assumed known

B5 - Unknown distribution of demand

C) Single (C1) vs. multiple period (C2)
In some situations, such as style goods and newspapers, there is a relatively short selling season or period, and remaining stock cannot be used to satisfy demand in the next season or period. This decoupling effect simplifies the analysis compared with the multiperiod case.

D) Stationary (D1) vs significantly time-varying (D2) parameters
We may find the common form of nonstationarity in the demand process. However, potentially as important are changes in other parameters such as costs, the effect of inflation or a one-time opportunity to purchase at a reduced unit cost.

E) Procurement cost structure
The unit value of an item may depend on the size of the replenishment. This replenishment may be a result of a supplier discount, or it can come about through freight consideration. In cases of multi-item inventory control with joint replenishment, the fixed cost of replenishment will be reduced. This is discussed in more detail in Chapter III.

F) Nature of supply process

A number of possibilities exist.

F1 - All of the material ordered is received after a known lead time; this is the case most commonly assumed in the literature.

F2 - All of the material ordered is received after a random lead time having known mean and variance

F3 - Only a random portion of the ordered material is received

G) Backorders (G1) vs. lost sales (G2)

Demand when an item is out of stock can be either backordered or lost; in fact, a combination of the two is likely in any specific context. From a mathematical standpoint, the case of complete backordering is usually the easiest to model. Moreover, because of the relatively low frequency of stockout occasions under any reasonable policy, the use of the simpler "complete backorders model" normally leads to a policy that produces a negligible cost increase over that found by a more exact "lost sales model" even when all demand in an out-of-stock situation are lost.

H) Shelf-life considerations

Most of the literature implicitly ignores the possibility of obsolescence or deterioration of stock. Obsolescence (H1) represents the situation where the stock is still in appropriate physical condition but can no longer be sold at anywhere near its original price(usually due to the appearance of a new competing product). Deterioration or perishability(H2) signifies that for legal and/or physical reasons the stock cannot be used for its original purpose after the passage of a certain time.

I) Single vs. multiple stocking point

In a significant number of companies, inventories are kept at more than one location (I1). In multi-echelon situations (I2), the orders generated by one location such as a branch warehouse become part or all of the demand on another location or central warehouse. In addition, one can have horizontal multiplicity (I3), that is, several locations at the same echelon level (e.g., several branch warehouse) with the possibility of transshipments. A situation analogous to the multi-echelon context exists in the production of an assembly where the assembly schedule dictates the needs through time of the various components.

1.3.2 Classification by ordering

There are two divisions in this classification.

A) Periodic ordering method
Orders are placed at fixed time intervals in this method. Order interval and order quantity of each item are determined to minimize total cost.

B) Continuous review method
This method records the stock levels of all the items in the system after each transaction, i.e., demand, receipt of items, order placed, etc., and whenever the stock reaches a certain predetermined level, an order is placed. There are two methods used to determine order quantity. They are

- i) to order the same quantity (Q_1) every time that item has to be ordered
- ii) to order a certain quantity in order to bring the inventory level of an item up to the predetermined level (S_i).

The values of Q_i or S_i are determined to minimize total cost. The coordinated ordering policy which will be discussed in Chapter III is a continuous review method.

1.4 Some Models of Inventory Systems

We will consider some inventory problems, according to the previous classification, to which mathematically developed decision rules have been widely applied or have substantial potential for application with minor modifications. Cross reference numbering to the classification scheme of the previous section is provided.

1.4.1 Single item with deterministic, stationary conditions (A1-B1-D1) - The economic ordering quantity (EOQ)

Deterministic stationary conditions lead to the classic EOQ, which has itself been directly used in practice but which, more importantly, represents a key building block of decision rules that cope with more complicated circumstances.

For example, if a quantity Q is ordered each time orders reolenishment stock, and D,H,A are the demand per unit period, average holding cost, and ordering cost, respectivelv, then after every Q demand, an order for Q units is placed. Thus, the time T between the placement of order is $T = Q/D$. Consequently, the inventory holding cost per unit time oeriod is average stock in unit periods times holding cost per unit.

The total cost(TC) per unit period is

$$TC = \frac{1}{T} A + \frac{Q}{2} H = \frac{D}{Q} A + \frac{Q}{2} H$$

Differentiating TC with respect to Q and setting the result to zero allows us to determinate the optimum ordering quantity Q^*.

$$\frac{\partial TC}{\partial Q} = -\frac{D}{Q^2} A + \frac{H}{2} = 0$$

Then,

$$Q^* = \sqrt{\frac{2DA}{H}}$$

This quantity Q^* is a well known economic ordering quantity(EOQ).

1.4.2 Multi-item with deterministic, stationary conditions under budget, space, or replenishment workload constraints (A2-B1-D1)

A Lagrangian multiplier approach leads to a decision rule where the scarce resource is properly allocated among the group of items involved. An exchange curve can be developed to show the benefit of relaxing the constraints. When demands are probabilistic instead of deterministic, decision rules have also been developed. These are based on a Lagrangian multiplier approach to allocate a given total safety stock among a group of items so as to minimize one of several possible measures of aggregate disservices. These are discussed in more detail in Chapter II.

1.4.3 Coordinated control of items under deterministic, stationary demand (A2-B1-C2-D1)

In general, a policy for ordering multiple interactive items will be called a coordinated replenishment policy. Several types of such a policy will be considered in this subsection.

Sometimes the term "joint ordering policy" is used in the literature to mean what is here called a coordinated ordering policy. In this paper, joint ordering is defined as follows. Suppose that whenever any item is ordered, every item in a group of family for which there has been any demand since the previous order is ordered. That is, every time an order is placed, every stock item is brought up to a specified inventory level. In this paper, the policy specifying the levels up to which each item is ordered as well as when to order is called "joint ordering policy".

In a sense, a joint ordering policy is at the end of a spectrum of coordinated replenishment policies. At the opposite end is independent ordering, under which each item is ordered according to its own single-item policy.

At this point, we need to explain briefly the coordinated ordering policy. When we are dealing with many items in the system, we are frequently faced with occasions where the coordination of replenishment orders for selected groups of items can lead to significant savings in the cost of replenishment. This is because there is dependency in the replenishment cost. In particular, if AI_i and AI_j are the replenishment costs for item i and j, respectively, under independent replenishment, then the cost involving both of the items at the same replenishment is less than $AI_i + AI_j$. This type of cost structure is particularly appropriate when a group of item is ordered from the same supplier and/or uses the same means of transportation.

The ordering policy considered in such a case is defined as (S, c, s) policy ; S stands for order-up-to level, c for coordinated joint-order point (or can-order point), and s for must-order point. This policy consists of bring up to its maximum inventory level S_i any item i below its coordinated joint-order point c_i whenever any item j (within the same family) hits its must-order point s_j. A number of authors have discussed methods of obtaining optimal control variables. This is briefly discussed in Chapter III.

And also, we can explain the independent ordering policy as follows : if the inventory level of item i falls to $x < s_i$, one orders up to level S_i ; i.e., a quantity $S_i - x$ is ordered.

Nevertheless, no real world situation encompasses multiple items having deterministic, stationary demand. However just as the EOQ is useful as a building block in single-item situations, useful models have been developed for determining replenishment sizes of a family of items that have interdependent costs for each family replenishment regardless of which items are included in the replenishment.

Of course we can consider a number of different problems for each specific case. However, it is not necessary to illustrate all kinds of problems. Our discussion involves only three simple cases which can provide some insight into developing real world problems which we will consider in the next chapter.

1.5 Statement of Problem

Let us consider a company which is going to build a new warehouse to store its raw material inventories. The company deals with many items, and the management would like to know the proper size of the warehouse while operating the system at minimum operating costs.

Since there are many items and their inventory levels vary independently with time, the total inventory level will also vary with time, and the distribution of the total inventory level may follow some probability distribution. If we know the distribution of the total inventory level, we can also find out the proper size of the warehouse and the operating policy by calculating the costs of building a warehouse and the penalty cost for the amount exceeding the given warehouse space and operating cost under given warehouse space constraint. The relationship between the total inventory level and cost is shown in Figure 1-1.

The most important difference between this situation and the model discussed previously is that the demand is uncertain and varies stochastically. In such a case, no direct analytical solution is available. However, we assume that the stochastic demand follows

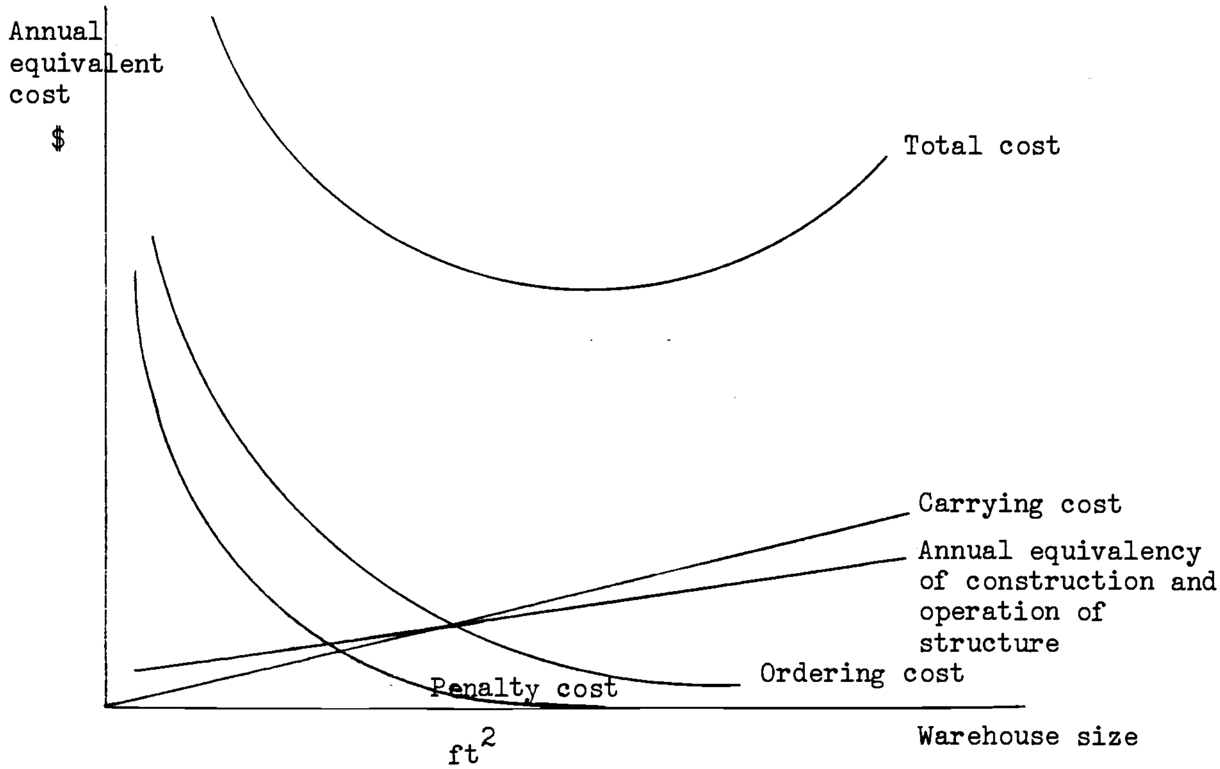

Figure 1-1 The relationship between total inventory level and cost

some probability property and that we can apply some probability distribution according to the collected data.

The ordering policy considered in muiti-item inventory control ia a coordinated ordering policy, as we mentioned in the previous section. The fundamental reason for adopting the coordinated ordering policy is that we can save costs in not only operating the system of carrying and ordering inventories but also system cost resulting from the interaction among items. In Chapter III, we will show the cost savings achieved by the coordinated ordering policy over the joint ordering policy and the independent ordering policy.

Briefly, the purpose of this study is to find the optimum control variables for the coordinated ordering policy which gives the minimum of operating cost and system cost.

1.6 Summary

We have discussed the general problems involved in controlling multi-item inventory systems. As was previously stated, when one deals with a large number of items, there will be many problems that do not arise in the single-item case. There will be interactions between items resulting from costs, resources, and demands which make the multi-item inventory control more difficult.

A number of authors have discussed methods of obtaining optimal policy by considering cost interaction caused by joint ordering of multiple items. But few have considered the system cost on the constrained case. If we consider such a system cost, the operating policy should be changed accordingly to represent the effect of such a system cost.

The deterministic model with a space constraint is the subject of Chapter II. We formulate a simple model of the multi-item case with deterministic demands. We show the optimum size of warehouse and operating policy which gives the minimum total cost, and we calculate the savings over the total cost of the system with the maximum size of the warehouse.

However, if the warehouse space is given as a constraint and if it is less than the optimum size of the warehouse, it is necessary to change the operating policy considering the given constraint. We use the Lagrangian multiplier method to get the optimum value of the control variable and update the value, considering the probability distribution of the total inventory level.

Chapter III considers a multi-item inventory control in more detail. A discussion of Schaack and Silver's work (1972) is followed by an introduction of characteristics and a review of previous research into obtaining optimum control variables for the multi-item inventory control.

The stochastic model with a space constraint is presented in Chapter IV. There determine the optimum size of the warehouse and test the operating policy using simulation. As in the case of deterministic model, we use the Lagrangian multiplier method for obtaining the optimum value of the control variables. FORTRAN is used for this simulation and its program is given in the Appendices.

Chapter V gives conclusions and offers recommendations for future areas of study.

II. CONSTANT DEMAND MODEL

2.1 Introduction

The model in this chapter is based upon the assumption that demands are constant and known. This is a significant simplification since demands are not usually constant in real world problems.

Since demands can rarely be predicted with certainty in practice, they must be described in probabilistic terms. However, the deterministic models discussed in this section are still of interest because they provide a simple framework for introducing the methods of analysis that will be used for more complicated situations. In addition, they are often useful in examining certain critical aspects of real world problems.

To provide uniformity throughout this paper, we use the following notation :

D_j : the demand of item j (units per period*)
H_j : the holding cost of item j ($ per unit per period*)
A_j : the ordering cost of item j ($ per order)
TCI : the total inventory operating cost ($ per period*)
* usually one year

2.2 Simple Model with a Space Constraint

We will first consider the case where there is an upper limit F to available warehouse space. Suppose that n items are being stocked and that one unit of item j takes up f_j square feet of floor space. If Q_j is the order quantity for item j, then the space constraint is not violated at any time, and it must be true that

$$\sum_{j=1}^{n} f_jQ_j = f_1Q_1 + f_2Q_2 + \ldots + f_nQ_n \leq F \qquad (1)$$

We assume that all demands should be met from on-hand inventory. So no backorders or lost sales are allowed. Then the average variable cost for all items per unit period is

$$TCI = \sum_{j=1}^{n} \left\{ \frac{D_j}{Q_j} A_j + \frac{H_jQ_j}{2} \right\} \qquad (2)$$

where n is the number of items being stocked

We want to find the absolute minimum of TCI in the region $0 < Q_j < \infty$, $j = 1, 2, \ldots, n$ subject to the active constraint (1).

The procedure is as follows;

First it is necessary to solve the problem ignoring the constraint. Thus we minimize over each Q_j separately. This yields

$$Q_j = \sqrt{\frac{2D_jA_j}{H_j}} \qquad j = 1, 2, \ldots, n \qquad (3)$$

which is just the Economic Ordering Quantity (EOQ). If the Q_j of (3) satisfies the constraint, then these Q_j are optimal. In such a case the constraint is not active, i.e., sufficient floor space is available.

On the other hand, if the Q_j of (3) exceed the given constraint, then the constraint is active and the Q_j of (3) are not optimal. To find the optimal Q_j, the Lagrangian multiplier technique is used. We form the function

$$L = \sum_{j=1}^{n} \left(\frac{D_j}{Q_j} A_j + \frac{H_j Q_j}{2} \right) + \theta \left(\sum_{j=1}^{n} f_j Q_j - F \right) \quad (4)$$

where the parameter θ is a Lagrangian multiplier.

Then the set of Q_j, $j = 1, 2, \ldots, n$ which yield the absolute minimum of TCI subject to (1) are solutions to the set of equations.

$$\frac{\partial L}{\partial Q_j} = 0 = -\frac{D_j}{Q_j^2} + \frac{H_j}{2} + \theta f_j \qquad j = 1, 2, \ldots, n$$

(5)

$$\frac{\partial L}{\partial \theta} = 0 = \sum_{j=1}^{n} f_j Q_j - F \quad (6)$$

These have the unique and hence optimal solution

$$Q_j^* = \sqrt{\frac{2 D_j A_j}{H_j + 2\theta^* f_j}} \qquad j = 1, 2, \ldots, n \quad (7)$$

where θ_j^* is the value of θ such that the Q_j^* of (7) satisfy (6)

The function

$$\sum_{j=1}^{n} f_j \left(2D_j A_j \left(H_j + 2\theta f_j \right)^{-1} \right)^{1/2} - F \quad (8)$$

is a monotone decreasing function of θ; consequently, there is a unique $\theta^{*} > 0$ such that (6) is satisfied.

Example 2-1

Consider a shop which produces and stocks three items. The maximum amount of storage space available equals 1,400 square feet. The items are produced in lots. The demand rate for each item is constant and can be assumed to be deterministic with no backorders allowed. Pertinent data for the items are provided in Table 2-1.

Table 2-1 Data for example

Item	1	2	3
Demand rate(D_j)	50	100	200
Space requirement(f_j)	50	50	50
Setup cost(A_j)	40	80	100
Holding cost(H_j)	40	160	100

The optimal lot sizes in the absence of the space constraint are $Q_1 = 10$, $Q_2 = 10$, $Q_3 = 20$ units, respectively. If these Q_j's were used, the maximum space requirement would be

$$F = (10 * 50) + (10 * 50) + (20 * 50) = 2{,}000 \text{ ft}^2$$

This is greater than the maximum allowable warehouse space in inventory. Hence, the constraint is active, and on introduction of a Lagrangian multiplier θ, we see by analogy with (7) that Q_j's are given by

$$Q_j^{*} = \sqrt{\frac{2 D_j A_j}{H_j + 2\theta^{*} f_j}} \qquad j = 1, 2, 3$$

where Q_j^{*} is the solution of the equation.

If we put these values of Q_j^* into equation (8), then

$$\sum_{j=1}^{3} \sqrt{\frac{2 D_j A_j}{H_j + 2\theta f_j}} * f_j - F$$

$$= \{\sqrt{\frac{2(50)(40)}{40 + 2\,\theta\,(50)}} * 50 + \sqrt{\frac{2(100)(80)}{160 + 2\,\theta\,(50)}} * 50$$

$$+ \sqrt{\frac{2(200)(100)}{100 + 2\,\theta\,(50)}} * 50 \} - 1400 = 0$$

Since this equation is a monotone decreasing function of θ, there is a unique solution of θ. This is shown in Figure 2-1.

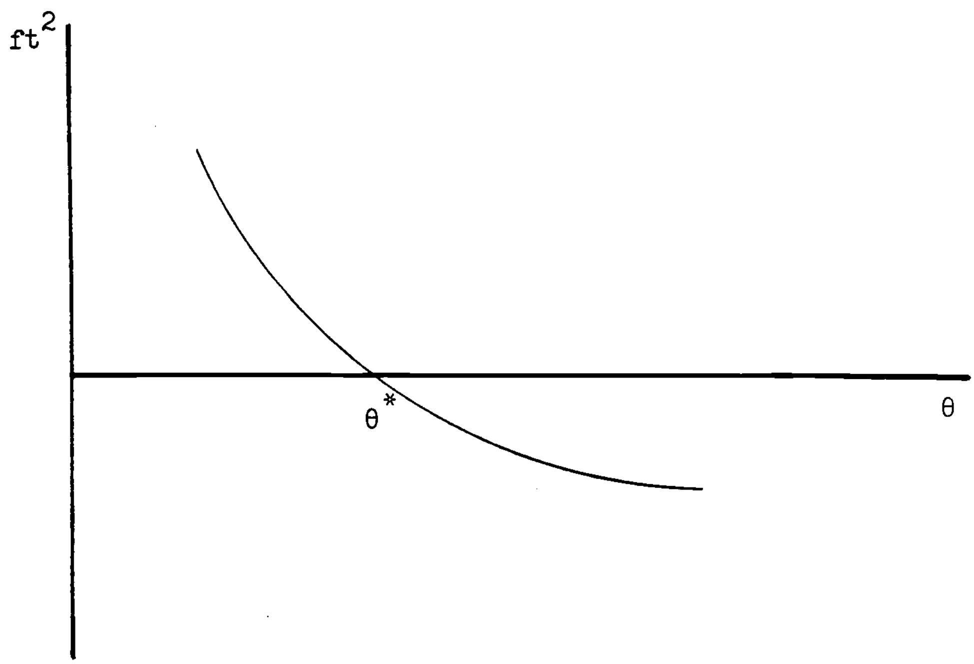

Figure 2-1 monotonic decreasing function

If we solve this equation numerically, we obtain the following solutions :

Solution ; $\theta^* = 0.9075$

$Q_1^* = 5.5310$

$Q_2^* = 7.9880$

$Q_3^* = 14.4810$

Substitution of these Q_j^* values into the constraint shows that it indeed hold as a strict equality. That is

$$\sum_{j=1}^{3} Q_j^* f_j - F$$

$$= (5.5310)(50) + (7.9880)(50) + (14.4810)(50) - 1400 = 0$$

Since the value of Q_j^* should be an integer, we round these Q_j^* off to the nearest integer even though it may cause slight loss of optimality. Then the solution will be

$Q_1^* = 6$ units $\quad Q_2^* = 8$ units $\quad Q_3^* = 14$ units

and

$$F = 6 * 50 + 8 * 50 + 14 * 50 = 1400 \text{ ft}^2$$

The minimum cost of setups and holding inventory for the three items in the absence of any constraint on warehouse space in inventory is as follows; if we put equation (3) into equation (2), total cost is expressed

√

$$TCI = \Sigma\sqrt{2 D_j A_j H_j} = \$4,000.00$$

$$TCI_{(\text{with constraint})} = \$4,221.90$$

The cost in the presence of the constraint is thus $221.90 per unit period higher than in the absence of such a constraint.

2.3 Optimum Level of Constraint

In the previous section, the system cost of building a warehouse was not considered. We only included the operating cost of holding and ordering inventories. We also assumed that the space constraint was not violated at any time.

However, since the inventory level of all items varies with time, it is necessary to see how this level changes with time. If we know this variation of total inventory level, we may save either system cost or operating cost, or both, in carrying inventories by relaxing the given constraint.

Since demands are constant and known, the inventory level of each item changes from zero inventory level assuming no safety stock to its maximum inventory level, i.e., the order quantity Q_j^*. The probability of each level of an item will be the same because demand is constant. This means the inventory level of each item is uniformly distributed ranging from zero to Q_j^*, with the probability of each level equal to $1/Q_j^*$. A graphical representation of this relationship is shown in Figure 2-2.

Therefore the mean and variance of each item will be

$$\mu_1 = \frac{Q_1^*}{2}, \quad \mu_2 = \frac{Q_2^*}{2}, \quad \ldots, \quad \mu_n = \frac{Q_n^*}{2} \tag{9}$$

$$\sigma_1^2 = \frac{Q_1^{*2}}{12}, \quad \sigma_2^2 = \frac{Q_2^{*2}}{12}, \quad \ldots, \quad \sigma_n^2 = \frac{Q_n^{*2}}{12} \tag{10}$$

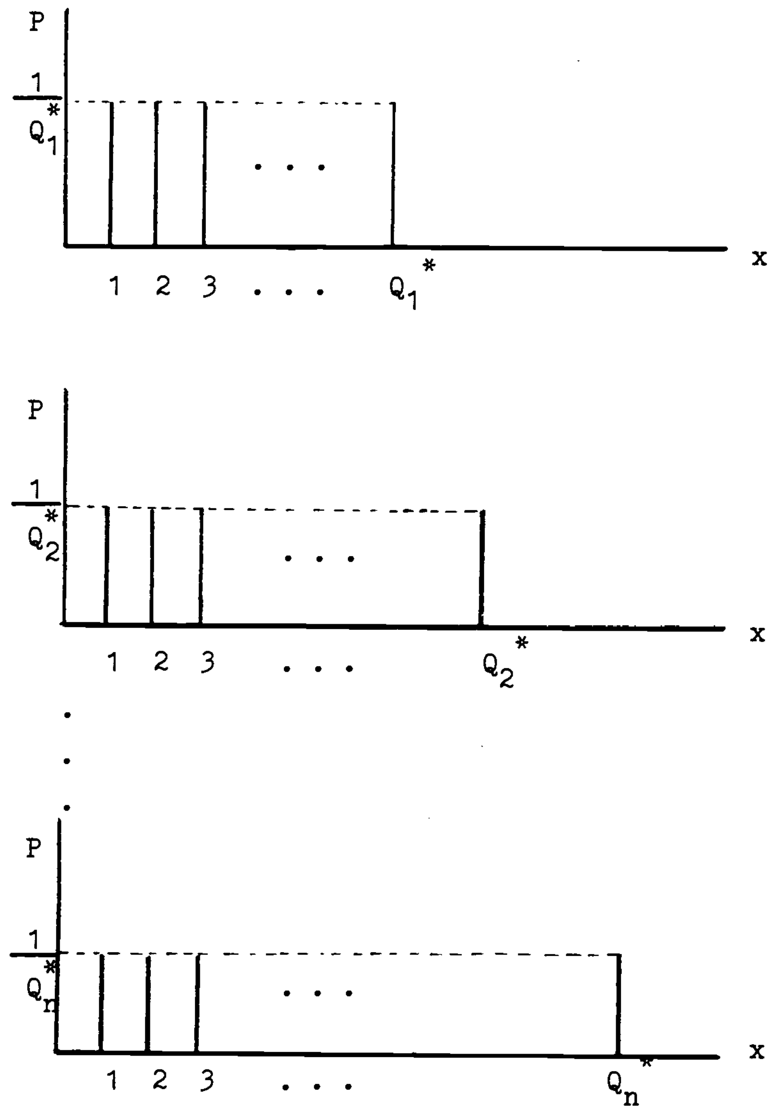

Figure 2-2 Distribution of each item level

Let us now consider the total inventory level. Since the inventory level of each item varies from zero to its maximum level Q_j^*, the level of all items will vary from zero to their sum of all items at its maximum level, i.e., $\Sigma_{j=1}^{n} Q_j^*$ (=MAX). Then the probability of each level of total inventory can be calculated as follows;

$$PT(0) = P_1(0) * P_2(0) * P_3(0) * \ldots * P_n(0)$$

$$PT(1) = P_1(1) * P_2(0) * P_3(0) * \ldots * P_n(0)$$
$$+ P_1(0) * P_2(1) * P_3(0) * \ldots * P_n(0)$$
$$\vdots$$
$$+ P_1(0) * P_2(0) * P_3(0) * \ldots * P_n(1)$$

$$\vdots$$

$$PT(MAX) = P_1(Q_1^*) * P_2(Q_2^*) * P_3(Q_3^*) * \ldots * P_n(Q_n^*)$$

where $P_i(N)$ = the probability of item i at the inventory level of N, and

$PT(M)$ = the probability of total inventory level at M, while $0 \leq M \leq \Sigma Q_j^*$ (=MAX)

Then the mean and variance of the total inventory level are

$$\text{Mean} = \Sigma M * PT(M) \qquad (= \mu \quad)$$

$$\text{Variance} = \Sigma \; (\text{Mean} - M)^2 * PT(M) \quad (= \sigma^2)$$

However, as the number of items increases, we can use the normal probability distribution to approximate the true probability distribution of the random variable because of the Central Limit Theorem. If we assume that there is no dependency among items in the inventory system, the distribution of the total inventory level will be approximate to normal distribution with mean and variance of (11) and (12), respectively. The distribution of the total inventory level is shown in Figure 2-3.

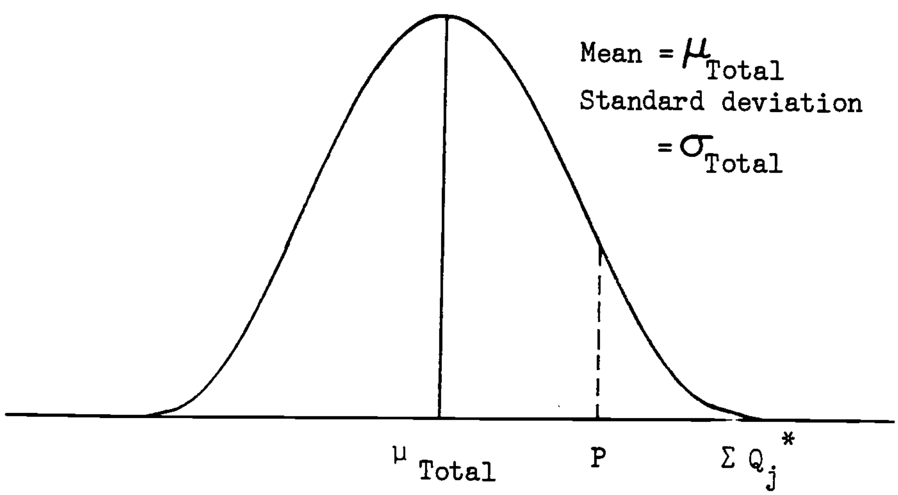

Figure 2-3 Distribution of total inventory level

$$\mu_{Total} = \mu_1 + \mu_2 + \mu_3 + \dots + \mu_n \tag{11}$$

$$\sigma_{Total}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \dots + \sigma_n^2 \tag{12}$$

We showed the relationship between the total inventory level and the cost in Figure 1-1. As shown in that figure, there are costs that depend on the total inventory level, i.e., cost for the construction and operation of the structure and a penalty cost.

Since the costs for the construction and operation of the structure increase while the penalty cost decreases as the total inventory level increases, there is a point that minimizes the total cost as shown in Figure 2-3 (point P). We define that point as the optimum level of constraint.

If we apply this property to the inventory system we may save the cost of building a warehouse and/or operating the inventory system at this optimum level of the constraint.

Let us now calculate the savings obtained from using this property of total inventory level. The following assumptions are made;

(i) The system cost of building a warehouse is proportional to the size of the warehouse. The relationship between system cost and the size of the warehouse can be linear, quadratic, or polynomial. It will be determined in each case according to the items to be stocked and how the warehouse would be constructed. However we assumed that the relationship is linear in this paper for the sake of simplicity. Therefore, the cost of building a warehouse can be expressed as follows;

$$COST(F) = U * F + V ,$$

where F is the size of the warehouse, U is the unit building cost and V is a constant which represent the fixed cost in building a warehouse regardless of the size of the warehouse

(ii) There is a penalty cost, PC, to the amount exceeding the size of warehouse. This cost may include transportation cost, extra handling cost, rent for additional warehouse space, etc.

(iii) We operate the system for a long period.

(iv) The probability exceeding the given constraint is small, which is often the case in practical application.

Then the total savings(Y) at the present time will be :

Savings = The cost for maintaining the system's maximum level
- The cost for maintaining its optimum level
- The penalty cost for the amount exceeding its optimal level

So,

$$Y = COST(F_{MAX}) - COST(F) - \{PC * (P/A,r,n) * \int_{F}^{F_{MAX}} (x - F)*P(x)\ dx\} \qquad (13)$$

where COST(F) = the cost of building a warehouse of size F

(P/A,r,n) = the present worth factor, while r is the interest rate and n is the number of the annual interest period

P(x) = the probability density function of the total inventory level

As we see in the above equation, the distribution of the total inventory is actually a truncated distribution. The range does not need to be greater than F_{MAX} of the maximum inventory level. Nevertheless, we can use the normal distribution because the probability that inventory level will be greater than F_{MAX} is negligible. And with this assumption we get the advantage of the simple calculations associated with the distribution.

To obtain the maximum savings, we differentiate equation (13) with respect to F and set the result equal to zero. Then,

$$\frac{\partial Y}{\partial F} = - U - PC * (P/A,r,n) * \frac{\partial}{\partial F} \int_{F}^{F_{MAX}} (x - F)P(x)\, dx$$

$$= - U - PC * (P/A,r,n) * \{ - H(F)\} = 0 \qquad (14)$$

where H(F) = the complementary cumulative distribution of P(F).

Therefore,

$$H(F) = \frac{U}{PC * (P/A,r,n)} \qquad (15)$$

The right hand side of equation (15) must be restricted to the interval $[0, 1]$, since H(F) is a probability. Therefore, equation (15) is only valid for suitable parameters of a particular problem. This restriction is necessary because of our previous assumption that the probability of exceeding the given constraint is small.

In general, it is not possible to solve equation(15) for F in closed form. For practical purposes, it can be solved iteratively for the desired value of F. However equation (15) is too complex to allow for generalized conclusions about the original problem. It will be necessary to further assume a particular type of density function in order to get specific results.

To obtain a manageable analysis, it is useful to further approximate the normal distribution tail with a simple exponential function. We will assume that:

$$H(x) = a \, \mathrm{Exp}\left(-b\left(\frac{x-\mu}{\sigma} \right) \right) \tag{16}$$

The constants a and b can be chosen to give a good fit for any particular problem. For example, if H(x), the probability of total inventory level exceeding the given constraint, should be relatively small,then the values of a and b can be chosen to give a good fit for the extreme right tail of the distribution. Figure 2-4 shows the standard normal distribution H(x) for $1 \leq x \leq 3$ and the corresponding fit from equation (16). In this case, the least square estimates for a and b were a = 2.88, b = 2.49. The fit achieved over this range was excellent,as shown in Figure 2-4. Then using this exponential approximation,P(x) can be obtained as follows:

$$P(x) = \frac{\sigma a}{b} \, \mathrm{Exp}\left(-\left(\frac{x-\mu}{\sigma} \right) b \right) \tag{17}$$

Now P(x) and H(x) both have a simple exponential form. From equation (15), the optimal level of the constraint(F_{opt}) can be calculated as follows:

$$F_{opt} = \mu + \sigma \left(- \frac{1}{b} * \ln\left(\frac{U}{a * PC * (P/A,r,n)} \right) \right) \tag{18}$$

Therefore, the optimum level of the constraint is determined by

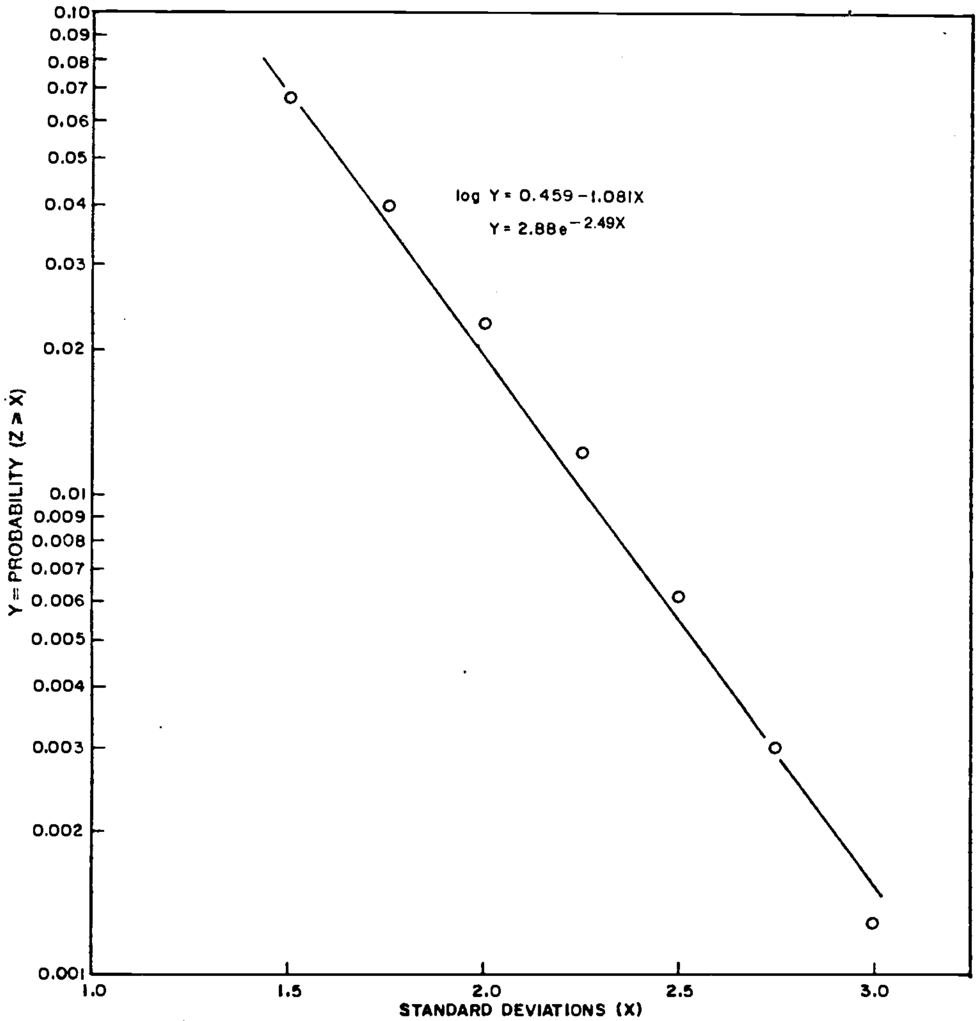

Figure 2-4 Approximation of normal distribution

the ratio of the unit system cost and the penalty cost. Using the above results, we can make a useful table for determining the optimum level of the space constraint according to the ratio of the unit system cost(U) and penalty cost(PC) under the given an interest rate and the number of years, as shown in Table 2-2.

Table 2-2. Optimum level of constraint

Ratio (U/PC)	x value in optimum level(= μ + x σ) (P/A,8,20) = 9.5826	(P/A,8,50) = 12.0026	(P/A,10,20) = 8.2215	(P/A,10,50) = 9.5079
0.1	2.2572	2.3476	2.1956	2.2540
0.25	1.8891	1.9796	1.8276	1.8860
0.5	1.6108	1.7012	1.5493	1.6088
1.0	1.3324	1.4228	1.2709	1.3293
1.5	1.1696	1.2600	1.1081	1.1664
2.0	1.0540	1.1445	.9925	1.0509
2.5	.9644	1.0549	-	.9613
3.0	-	.9816	-	-

Ratio	(P/A,12,20) = 7.1318	(P/A,12,50) =7.8239	(P/A,15,20) = 5.8715	(P/A,15,50) = 6.1768
0.1	2.1386	2.1757	2.0604	2.0807
0.25	1.7706	1.8077	1.6924	1.7127
0.5	1.4922	1.5294	1.4141	1.4343
1.0	1.2138	1.2510	1.1357	1.1560
1.5	1.0510	1.0862	.9729	.9931
2.0	.9355	.9726	-	-

Example 2-2

Let us consider the same example that was shown in section 2-2. The unit system cost to build one unit of warehouse is $450.00, and the penalty cost is $300.00. The optimum ordering quantities in the absence of the space constraint are

$Q_1^* = 10$ units $Q_2^* = 10$ units $Q_3^* = 20$ units

Then the mean and variance of each item will be

$$\mu_1 = \frac{10}{2} = 5, \ \mu_2 = \frac{10}{2} = 5, \ \ \mu_3 = \frac{20}{2} = 10 \text{ (units)}$$

$$\sigma_1^2 = \frac{10^2}{12} = 8.33, \ \ \sigma_2^2 = \frac{10^2}{12} = 8.33, \ \ \ \sigma_3^2 = \frac{20^2}{12} = 33.33$$

Thus, the mean and variance of total inventory are

$$\mu_{Total} = \mu_1 + \mu_2 + \mu_3 = 20 \text{ units}$$

$$\sigma_{Total}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 49.99$$

Since each item requires 50 square feet, the mean and standard deviation of total inventory distribution will be

$$\text{Mean} = 20 \text{ units} * 50 \text{ ft}^2/\text{unit} = 1{,}000 \text{ ft}^2$$

$$\text{Standard deviation} = 7.071 \text{ units} * 50 \text{ ft}^2/\text{unit} = 353.55 \text{ ft}^2$$

Then the optimum level of warehouse space constraint will be

$$F_{opt} = 1{,}000 \text{ ft}^2 + 353.55 \text{ ft}^2 * 1.0862 = 1{,}384.03 \text{ ft}^2 ,$$

where the interest rate is 12% and the building is assumed to last 50 years.

This optimum level is less than the given constraint. So we can operate the system without considering the constraint and save the operating cost $221.90 per year which we had to pay under the given constraint.

2.4 Summary

In this chapter, we have discussed the constant demand model with a warehouse space constraint. As we saw in section 2-2, we usually solve the problem of the constrainted case using the Lagrangian multiplier technique. In such a case we got the optimum policy which satisfied the given constraint all the time. But we had a higher total cost than without an active constraint. However, as shown in section 2-3, the probability that maximum or near maximum inventory level would happen was very small. Thus, if we consider the probability distribution for the variation of total inventory level, we could save either system cost or operating cost by relaxing the given constraint.

III. MULTI-ITEM INVENTORY CONTROL

3.1 Characteristics

The effective control of a multi-item inventory does not necessarily require a procedure different from those in the single-item case. All items in the system can be considered individually, assuming that demand for each item is unaffected by demand for the others. Each of the items can therefore be controlled separately. It is only true when there are no interactions among the stocked items. However, when organizations deal with many items, there are quite often interactions among items.

In this paper, we consider such interactions and a policy for controlling multiple items. Multiple-item ordering decision models ("multi-item model," for short) are necessary because they recognize interactions among the items involved.

As mentioned in the previous chapter, item interactions can be considered to be of three type : (1) interactions resulting from costs, (2) interactions resulting from resources, and (3) interactions resulting from demands. An example of a cost interaction is a reduction in ordering costs because of simultaneous ordering of multiple items. Another example is a material cost savings as a result of a quantity discount applied to the total dollar amount purchased in one order.

Resource interaction occurs when stock items compete for scarce resource. For example, the total size of an order may be limited by the capacity of a transport vehicle. A similar limitation may apply to the total quantities of materials stored (e.g., by warehouse capacity).

Demand interaction exists whenever the demand for one item can be affected by the demand for one or more other items being stocked.

In this paper, we consider cost interactions resulting from multiple ordering and resource interaction (constraint case). We will assume independence in demand from item to item.

In the following sections, we discuss the cost interaction between items and the controlling policy for such a problem. In general, we call a policy for ordering multiple interactive items a coordinated replenishment policy. Among several types of such policies, we discuss the (S, c, s) policy.

We talk briefly about the previous research in section 3-2. the algorithm of Schaack and Silver (1972) is presented in section 3-3. They developed a procedure for selecting the control variables of the (S, c, s) policy. The procedure, iterative in nature, is a combination of mathematical optimization and simulation.

3.2 Previous Studies

At this point we need to mention a few of the previous investigations in this area. A number of authors have developed methods of coordinating items for replenishment purposes.

Recognizing probabilistic demand, Balintfy (1964) was the first to advocate the use of an (S, c, s) system. However, he did not propose a practical means of obtaining the values of the control variables. Other references relevant to this type of policy include Silver (1965) and Curry (1970).

Ignall (1969) used Markov renewal programming to determine an optimal joint policy for the two-item inventory system. His results indicated that the (S, c, s) policy is not necessarily optimal. However, the loss of "optimality" may be more than justified by the simplicity of the (S, c, s) policy when compared with the supposed "optimal" policy.

Chern (1974) represented the multi-product inventory system by a Markov process. She determined the can-order point level by balancing the reduced cost of time-weighted backorders with the extra carrying cost. Her analysis involved certain approximations that allowed determination of the steady state probabilities of the associated Markov process. Furthermore, she used a fixed order quantity, Q_j, for item j rather than a prescribed order-up-to level S_j.

Silver (1974) presented an algorithm for selecting the control variables for an (S, c, s) policy for items under unit-sized Poisson demand and replenishment lead time greater than or equal to zero.

Thompstone and Silver (1975) demenstrated how control variables may be selected for the (S, c, s) policy when demand is compound Poisson, and not restricted to unit sized transactions with a lead time of negligible length.

Curry and Hartfiel (1975) proposed solving the problem of the joint setup cost inventory with constrained warehouse space availability. They assumed that the storage space could be segmented into fixed areas of equal size with the number of units which can be stored in each area varying by product and that different products could not share storage facility. Their solution method is an iterative optimization-simulation procedure. They used dynamic programming for optimizing the constraint on the allocation of storage space and simulated the system for obtaining control variables.

A number of other references are relevant to the general problem area of the multi-item inventory system. Johnson (1967) presented a reordering policy for a multi-item inventory system with periodic review. It consists of ordering an item i up to some level S_i if its stock level is within a reorder region. Veinott (1965) considered a multi-product, dynamic, nonstationary inventory problem in which the system is reviewed at the beginning of each of a sequence of periods of equal length. He chose an ordering policy that minimizes the expected discount costs over an infinite time horizon.

Ignall and Veinott (1969) studied certain inventory systems for which they obtained conditions under which a myopic ordering policy (i.e., a policy of minimizing the expected cost in the current period alone) is optimal for a sequence of periods.

As Ignall has shown, the (S, c, s) policy does not necessarily minimize the sum of replenishment, inventory carrying, and shortage costs. Therefore, we may not need to calculate the exact optimal control variables because the policy that would minimize these costs would be considerably more complex than the (S, c, s) policy. If one properly takes account of these costs, it is felt that the (S, c, s) policy achieves a solution to the coordinated replenishment problem which is close to the best attainable.

3.3 Schaack and Silver's Algorithm

In this section, we demonstrate the procedure for selecting the control variables of the (S, c, s) coordinated ordering policy as developed by Schaack and Silver (1972).

3.3.1 Assumption and notation

We consider the replenishment cost to consist of a fixed component AF and a variable component AJ_i depending on the item i. The independent replenishment cost of item i is $AI_i = AF + AJ_i$, whereas the joint replenishment cost of both item i and j is $AF + AJ_i + AJ_j$.

We assume compound Poisson demand; i.e., the arrivals of transactions are according to a Poisson process and the size of the transactions satisfy some prescribed probability distribution. It is also assumed that item i has a deterministic lead time L_i. The portion of the demand which is not satisfied directly out of stock is completely backordered.

The notation used in the analytical description of the problem is shown in Table 3-1.

Table 3-1. Notation

AF	:	fixed component of the setup cost
AJ_i	:	variable component of the setup cost for item i
AI_i	:	independent setup cost for item i ($AI_i = AF + AJ_i$)
D_i	:	expected demand for item i per period
m_i	:	mean transaction size for item i
σ_i	:	standard deviation of the transaction size for item i
H_i	:	carrying cost for item i per piece per period
NI_i	:	expected number of replenishment of item i where i is ordered alone
NJN_i		expected number of joint replenishment of item i not triggered by i
NJT_i		expected number of joint replenishments triggered by item i during the period
N_i		expected total number of replenishments of item i per period ($N_i = NI_i + NJT_i + NJN_i$)
R_i		average remnant stock of item i just before ordering, when item i is involved in a joint replenishment triggered by some other item
O_i		average remnant stock of item i when order is triggered by item i
EC_i		total expected cost for item i
TEC		total expected cost for the system
L_i		lead time of item i
π_i		desired service level of item i
S_i		order-up-to level for item i
c_i		can-order level for item i
s_i		must-order point for item i

3.3.2 Total inventory cost equation

Under these assumptions and with this notation, the total replenishment cost for the system for a unit time period is :

$$\sum_{i=1}^{n} ((NI_i + NJT_i) * (AF + AJ_i) + (NJN_i * AJ_i)) \qquad (19)$$

Since the transactions are not assumed to be unit-sized, each time item i triggers an order, its inventory level is at or below s_i and has an average value of O_i. Then the following equation must be satisfied:

$$D_i = (NI_i + NJT_i)(S_i - O_i) + NJN_i * (S_i - R_i) \qquad (20)$$

By setting $P_i = NJN_i/N_i$, i.e., the probability that replenishment involving item i is the result of another item hitting its reorder point, we can express NJN_i and NJT_i as follows :

$$NJN_i = P_i D_i \ / \ (S_i - O_i - P_i(R_i - O_i) \qquad (21)$$

$$NI_i + NJT_i = (1 - P_i) * D_i \ / \ (S_i - O_i - P_i(R_i - O_i)) \qquad (22)$$

Then the total replenishment cost is

$$\sum_{i=1}^{n} \frac{D_i}{S_i - O_i - P_i(R_i - O_i)} * ((P_i * AJ_i) + ((1 - P_i) * AI_i) \qquad (23)$$

If we make the approximation that the average duration of both independent and joint replenishment cycles are the same, then the carrying cost of the system for a unit time period is:

$$\sum_{i=1}^{n} \{ (P_i (\frac{S_i + R_i}{2}) H_i + (1 - P_i) (\frac{S_i + O_i}{2}) H_i) - E_i H_i \} \qquad (24)$$

where E_i is the expected demand of item i over the lead time period, i.e., $E_i = D_i L_i$

Thus, the total cost is as follows:

$$TEC = \sum_{i=1}^{n} EC_i = \sum_{i=1}^{n} \{ (1-P_i) (\frac{AI_i^* D_i}{S_i - O_i - P_i (R_i - O_i)} + \frac{S_i + O_i}{2} H_i) + P_i (\frac{AJ_i D_i}{S_i - O_i - P_i (R_i - O_i)} + \frac{S_i + R_i}{2} H_i) - D_i L_i H_i \} \qquad (25)$$

Setting,

$$\xi_i = S_i - O_i , \quad \rho_i = R_i - O_i \qquad (26)$$

the total cost equation above can be formulated as follows:

$$TEC = \sum_{i=1}^{n} EC_i = \sum_{i=1}^{n} \{ (1-P_i)*(\frac{AI_iD_i}{\xi_i - P_i\rho_i} + \frac{\xi_i H_i}{2}) + P_i*(\frac{AJ_iD_i}{\xi_i - P_i\rho_i} + \frac{(\xi_i + \rho_i) H_i}{2}) + O_iH_i - D_iL_iH_i \} \qquad (27)$$

However, backorder costs are not included in the above equation. Rather than explicitly costing backorders, we determine the safety stock by service level consideration of the maximum probability of running out of stock per unit time period.

Therefore, the problem is now to minimize the total cost with respect to S, c, and s, satisfying the service constraint.

However, it is impossible to get the optimal value of the control variables analytically because we do not know the functional relationships of the variables involved in the total cost equation, i.e., the ξ_i's, P_i's, ρ_i's, and O_i's.

The solution method is to update for each S, c, and s, using simulation. That is, for the given value of S, c, and s, we observe the behavior of the system through simulation and then make appropriate changes in the values of the control variables according to observations.

3.3.3 Updating the parameters

We assume that the system is in a given state of (S_i, c_i, s_i), $i = 1, 2, \ldots, n$, and that the P_i and R_i are known. The following section shows how separate optimizations on S_i, c_i, and s_i are realized.

i) Updating s_i

Using the normal approximation to the compound Poisson over the lead time demand, we come up with the following result.

An approximation of the probability of not running out of item i before the end of the lead time when item i has triggered the order is

$$\Phi\left(\frac{O_i - \mu_i}{\nu_i}\right)$$

where, $\mu_i = D_i L_i$, $\nu_i = \left(\frac{D_i L_i}{m_i} (\sigma_i^2 + m_i^2)\right)^{1/2}$

and,

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\overline{2\pi}} e^{-t^2/2} dt$$

This is an approximation since O_i is the average value of the available stock before ordering.

Similarily an approximation of the probability of not running out of item i when the order has been triggered by some other item is

$$\Phi\left(\frac{R_i - \mu_i}{\nu_i}\right)$$

Then the probability of not running out of stock per unit time period is

$$\{ \phi (\frac{O_i - \mu_i}{\nu_i}) \}^{(NI_i+NJT_i)} * \{ \phi (\frac{R_i - \mu_i}{\nu_i}) \}^{NJN_i} \quad (28)$$

If the allowed probability of running out of stock is π_i, the above equation must be greater than or equal to ($1 - \pi_i$).

$$\{ \phi (\frac{O_i-\mu_i}{\nu_i}) \}^{(NI_i+NJT_i)} * \{ \phi(\frac{R_i-\mu_i}{\nu_i}) \}^{NJN_i} \geq 1 - \pi_i$$

Using equation (21), (22), and (26), we can reformulate the last equation as follow :

$$\{ \phi(\frac{O_i-\mu_i}{\nu_i}) \}^{(1-P_i)} * \{ \phi (\frac{O_i+\rho_i-\mu_i}{\nu_i}) \}^{P_i}$$

$$\geq (1 - \pi_i)^{(\xi_i - P_i\rho_i)/ D_i} \quad (30)$$

Then we select the smallest O_i that satisfies the inequality. The value of s_i is given by

$$s_i = O_i + \beta_i , \quad \text{where } \beta_i = \frac{1}{2} (\frac{\sigma^2 + m_i^2}{m_i}) \quad (31)$$

if $S_i - s_i >> m_i$, under Karlin's assumption (1958)

ii) Updating S_i

Differenciating the equation (27) with respect to ξ_i and setting the result equal to zero gives

$$\frac{H_i}{2} - \frac{(\,(1-P_i)*AI_i + P_i*AJ_i\,)*D_i}{(\,\xi_i - P_i\rho_i\,)^2} = 0$$

Solving for ξ_i gives

$$\xi_i = P_i\rho_i + \sqrt{\frac{2D_i}{H_i} * (\,(\,1-P_i)*AI_i + P_i*AJ_i\,)} \qquad (32)$$

From Equation (26)

$$S_i = \xi_i + O_i \qquad (33)$$

iii) Updating c_i

Differentiating equation (27) with respect to c_i and setting the result equal to zero lead to

$$\frac{\delta TEC}{\delta c_i} = \{\, \frac{H_i}{2} + \frac{D_i}{(\,\xi_i - P_i\rho_i\,)^2} * (\,AI_i*(1-P_i) + AJ_i*P_i\,)\,\}$$

$$* \frac{\delta(P_i\rho_i)}{\delta c_i} - \frac{\delta P_i}{\delta c_i} * \frac{AF * D_i}{\xi_i - P_i\rho_i}$$

By using equation (32), we can reduce this equation to

$$\frac{\delta TEC}{\delta c_i} = H_i * \frac{\delta(P_i \rho_i)}{\delta c_i} - \frac{\delta P_i}{\delta c_i} * \frac{AF * D_i}{\xi_i - P_i \rho_i} \quad (34)$$

Then, the optimal value of c_i must satisfy

$$\frac{\delta TEC}{\delta c_i} = 0$$

Since we do not know the functional relationships between P_i, ρ_i, and c_i, we cannot obtain a closed-form solution for the optimal c_i. Instead we find it by making small changes in c_i until the derivative approaches zero. So, if $\delta TEC/\delta c_i > 0$, i.e., TEC increases with c_i, we decrease c_i and vice versa if $\delta TEC/\delta c_i < 0$.

3.3.4 Iteration method

We start the algorithm with the following initial values :

i) $s_i = O_{i0} + \beta_i$, where O_{i0} is the safety stock of item i in an independent system, in which case, using equation (27) with $P_i = 0$, it satisfies the inequality

$$\left\{ \phi \left(\frac{O_{i0} - \mu_i}{\nu_i} \right) \right\}^{D_i/EOQ_i} \geq 1 - \pi_i$$

ii) $S_i = O_{i0} + EOQ_i$

iii) $c_i = O_{i0} + (EOQ_i/10)$

After each complete updating of all S_i's, c_i's, and s_i's, a simulation is done in order to find the corresponding values of the P_i's and ρ_i's necessary for the next updating. The value of c_i is corrected at each iteration by an amount of $STEP_i$. If the value of $STEP_i$ is too large, the approximations in equation (34)

$$\frac{\delta P_i}{\delta c_i} \cong \frac{\Delta P_i}{\Delta c_i} \quad \text{and} \quad \frac{\delta(P_i \rho_i)}{\Delta c_i} \cong \frac{\Delta(P_i \rho_i)}{\Delta c_i}$$

are no longer valid. If $STEP_i$ is too small, the algorithm will require too many iterations to reach the optimal region. The starting value of EOQ/10 for the $STEP_i$ has proven a reasonable balance in several numerical examples. In order to obtain the convergence of the values of c_i, we cut the value of $STEP_i$ by two each time TEC increases after having first decreased. More details are given in the flowchart of Figure 3-1.

For the simulation language, we used FORTRAN, a general purpose language, rather than a simulation language such as GPSS or GASP. Some advantages of FORTRAN simulation are mentioned by Law and Kelton (1982). A flowchart and a simulation program listing are presented in Appendices II and III.

3.3.5 Cost savings

One important practical point to know before introducing the (S, c, s) policy in a particular context is whether or not the savings accomplished by the change will offset the cost of implementation. This section provides an approximation of the cost savings realized by a coordinated ordering over the continuous review independent ordering policy and the joint ordering policy.

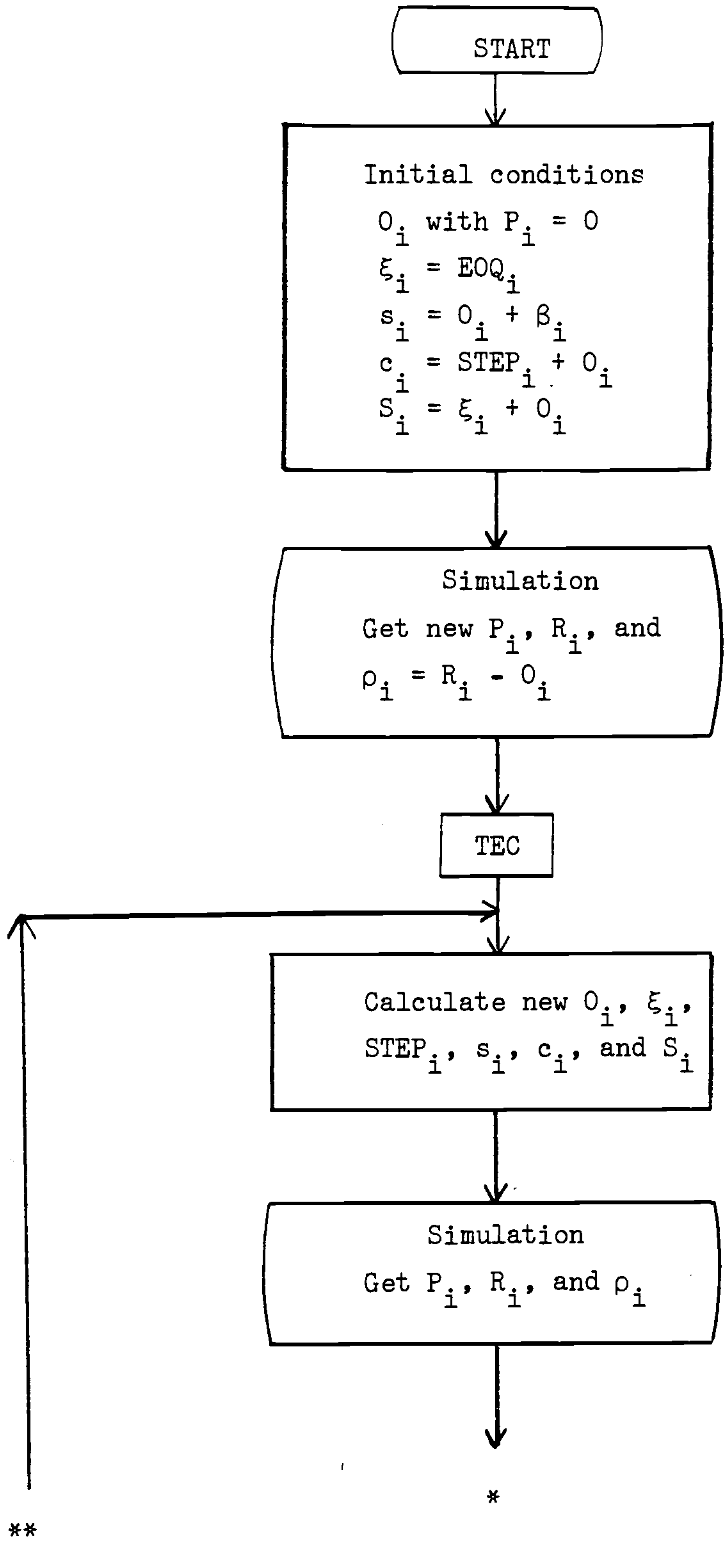

Figure 3-1 Flowchart

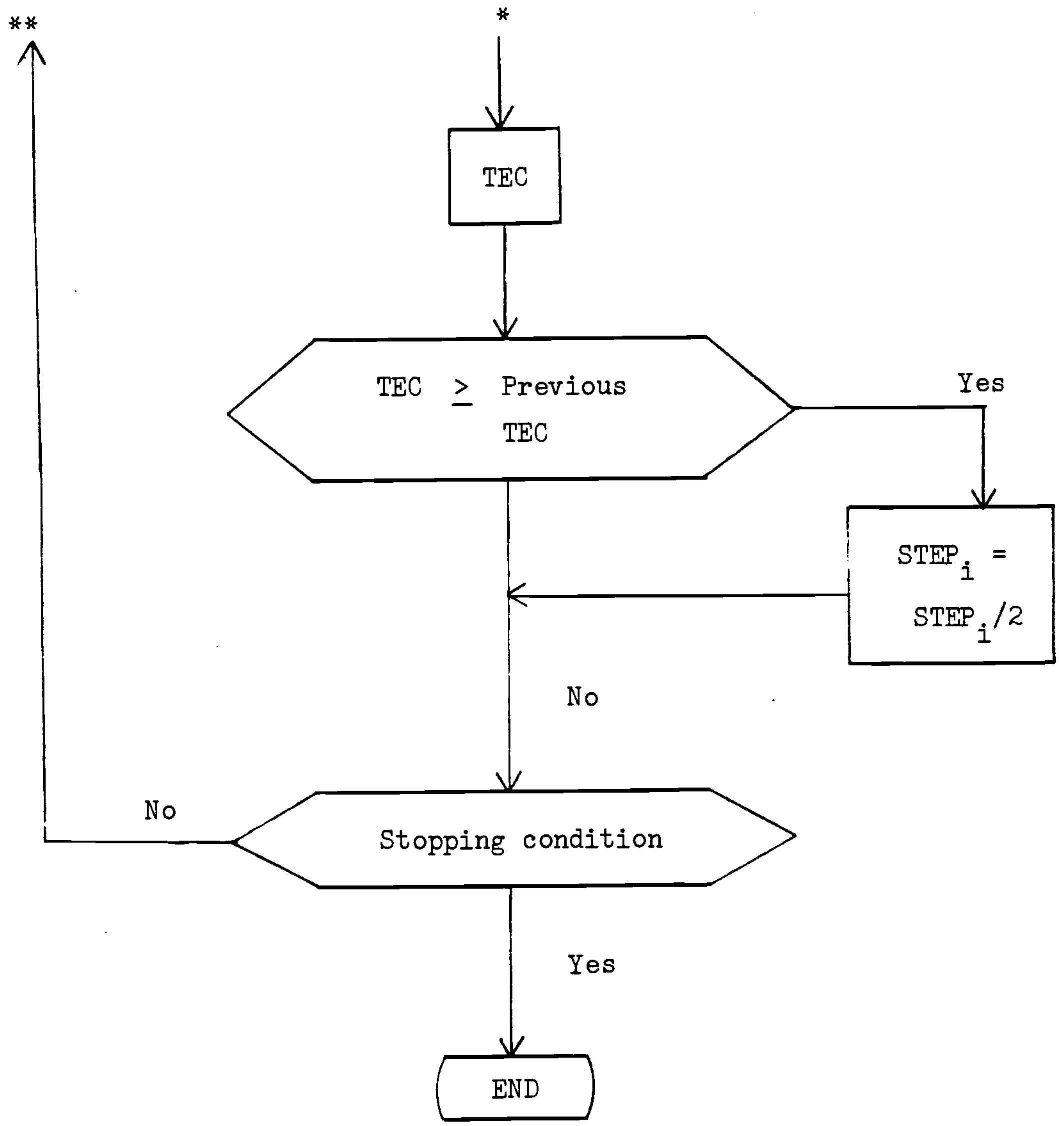

Figure 3-1 Flowchart
(continued)

As mentioned in Chapter I, under the independent ordering policy, each item is ordered according to its own single-item policy. Under the joint ordering policy, whenever any item is ordered, every item for which there has been any demand since the previous order is also ordered up to a specified inventory level. The relevant costs of each policy are expressed as TECI and TECJ, respectively. Then the savings are given by the expression (TECI - TEC) / TECI and (TECJ - TEC) / TECJ, where these values of total costs are calculated using simulation.

In the next example, we demonstrate the procedure for getting optimum control variables of the (S, c, s) policy and compare it with the joint ordering policy and the independent ordering policy.

Example 3-1

The example deals with a 30-item dependent system. The input data are given in Appendices 4.1 and 4.2. The input data consist of the mean interdemand time, required floor space, carrying cost, setup cost, lead time, and number of transactions. The data also show the transaction size and their probabilities.

The simulation results after each iteration are shown in Appendix 4.3, and the results of independent ordering and the joint ordering policy are shown in Appendix 4.4. The summary for the simulation are given in Table 3-2.

We can see the considerable cost savings of coordinated ordering policy over the independent ordering policy and the joint ordering policy. The simulation results for the independent ordering policy and the joint ordering policy show the total cost of $5,734.89 and $5,858.00, respectively. The cost obtained by the coordinated ordering policy after eight iterations is $5,133.88. This result shows a cost savings of 10.5 percent over the independent ordering policy and 12.4 percent over the joint ordering policy.

Table 3-2 Summary of simulation results

Iteration	Simulation cost		Calculated cost	
	Holding	Ordering	Holding	Ordering
1	5382.61	1120.35	5062.83	1239.60
		6502.51		6302.43
2	4712.35	899.25	4365.44	985.35
		5611.60		5350.79
3	4086.23	1195.90	3823.76	1259.59
		5282.13		5083.75
4	4071.79	1159.95	3835.25	1229.96
		5231.74		5065.21
5	4031.02	1162.45	3842.00	1188.32
		5193.47		5030.32
6	4002.68	1143.40	3766.64	1177.35
		5146.08		4943.99
7	3887.91	1250.80	3732.27	1296.38
		5138.71		5028.65
8	3849.28	1284.60	3668.18	1365.39
		5133.88		5033.57
Independent Ordering	3242.89	2492.00		
		5734.89		
Joint Ordering	4057.45	1800.55		
		5858.00		

IV. STOCHASTIC MODEL WITH CONSTRAINTS

4.1 Demand Characteristics

As discussed in the introduction, the model differs from actual application in that demand is not known with certainty. However, we assumed that the demand followed some identifiable probabilistic property and that we could determine the appropriate probability distribution based on collected data.

In developing realistic inventory control policies, one can use a probabilistic description of demand. A variety of statistical distributions is available to describe the demand distribution. For example, assumptions on the unit-sized transaction or normally distributed demand are used in most usable inventory control. The Poisson distribution also generally seems appropriate, but frequently provides a poor fit to the data, owing to having a coefficient of variation significantly greater than a Poisson distribution of appropriate mean.

When the demand pattern cannot be fit by the normal distribution or Poisson distribution, it is useful to view the demand pattern in a time period as having two components:

i) the number of transactions during the period and

ii) the magnitudes of the individual transactions.

Each component may have its own probability distribution. Empirical evidence suggests the pattern of arrival of transactions can be adequately represented by a Poisson process:

$$P_x(x_0) = \frac{(\lambda t)^{x_0} \exp(- \lambda t)}{x_0} \qquad t \geq 0,\ x_0 = 0, 1, 2, \ldots ,$$

where $P_x(x_0)$ is the probability that there are x_0 arrivals in the time t and λ is the expected number of arrivals per unit time period

If arrivals of transactions are thus described and the transaction sizes have some specified probability mass function(PMF), the total demand in a time period is described by a compound Poisson PMF. The probability mass function of transaction sizes for such an item is illustrated in Figure 4-1. We shall treat the case of a general distribution of transaction sizes, letting

$P_i(t_0)$ = the probability that the transaction is of magnitude t_0, $t_0 = 1, 2, \ldots, t_{max}$, where t_{max} is the largest transaction size of item i

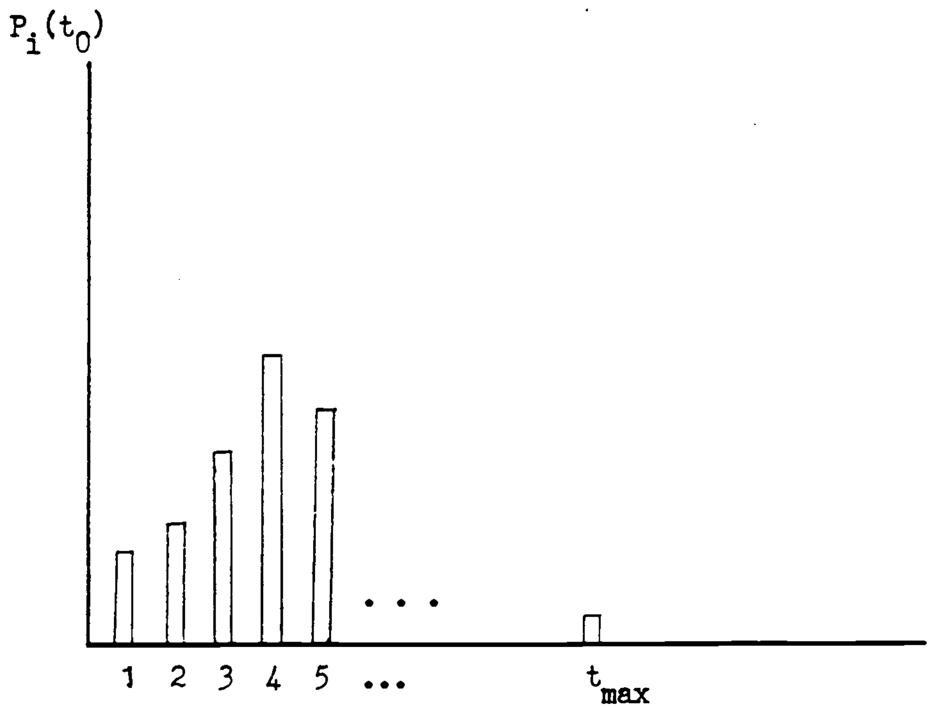

Figure 4-1 Probability mass function of transaction size

The variance of the compound Poisson distribution equals or exceeds its mean, a characteristics often exhibited by demand data. Empirically, this compound Poisson distribution has been shown to adequately fit many observed demand distributions as discussed by Silver (1970).

4.2 Inventory Level Distribution Assuming Compound Poisson Demand

In Chapter II, we showed the inventory distribution of each item under constant demand, assuming that there was no dependency between items in ordering. The inventory level of each item was a uniform distribution and in the multi-item case with constant demands, the total inventory level approximated the normal distribution as the number of items increased.

However, if there is dependency between items in ordering and if the demand is not constant, the inventory level distribution of each item will not be uniform, and the mean and variance of that distribution will differ from the constant demand case.

In this section we demonstrate how the mean and variance of the inventory level distribution may be calculated for the (S, c, s) policy when demand is compound Poisson i.e., not restricted to unit-sized transactions but the lead time is of negligible length.

Let us consider a single-item system. The problem consists of a single item faced with Poisson opportunities of rate μ to replenish at reduced cost. Opportunities may be viewed as being caused by another item triggering a replenishment and the rate μ is effectively the expected number of orders per year triggered by all other items in the family.

In the demand process, the time until the next transaction does not depend on the current level of the available stock. The probability distribution of the available stock immediately after a transaction is therefore equivalent to the probability distribution at a random point in time. The available inventory starts a cycle at the order-up-to level S, and as transactions occur, it moves in jumps downward until a replenishment is made. Then the available inventory instantaneously jumps back to S, starting a new cycle.

Since cpportunities for reduced replenishment cost and demand transaction occur according to Poisson process with rate μ and λ, respectively, the probability at any random point in time that the next event is a demand transaction is $\rho = \lambda / (\lambda + \mu)$, and the probability that the next event is a replenishment is $1 - \rho = \mu / (\lambda + \mu)$. When the available stock is in the range c+1, c+2, ..., S-1, S, all opportunities for reduced replenishment cost will be ignored. Thus the following equations govern probabilities of various inventory levels immediately after a transaction.

$$P_i(S-1) = P_i(S)P_i(1)$$

$$P_i(S-2) = P_i(S)P_i(2) + P_i(S-1)P_i(1)$$

$$P_i(S-3) = P_i(S)P_i(3) + P_i(S-1)P_i(2) + P_i(S-2)P_i(1)$$

$$\vdots$$

$$P_i(c+1) = P_i(S)P_i(S-c-1) + P_i(S-1)P_i(S-c-2) + \dots + P_i(c+2)P_i(1)$$

$$P_i(c) \quad = P_i(S)P_i(S-c) + P_i(S-1)P_i(S-c-1) + \dots + P_i(c+1)P_i(1)$$

$$P_i(c-1) = P_i(S)P_i(S-c+1) + P_i(S-1)P_i(S-c) + \dots + P_i(c=1)P_i(2)$$

$$+ (P_i(c)P\ (1))\ \rho$$

$$P_i(c-2) = P_i(S)P_i(S-c+2) + P_i(S-1)P_i(S-c+1) + \dots + P_i(c+1)*$$

$$P_i(3) + (P_i(c)P_i(2) + P_i(c-1)P_i(1))\ \rho$$

.

.

.

$$P_i(s+1) = P_i(S)P_i(S-s-1) + P_i(S-1)P_i(S-s-2) + \dots + P_i(c+1)*$$

$$P_i(c-s) + (P_i(c)P_i(c-s-1) + P_i(c-1)P_i(c-s-2) +$$

$$\dots + P_i(s+2)P_i(1))\ \rho$$

The probability distribution of the available stock immediately after a transaction is not equivalent to the probability distribution at a random point in time. For inventory levels c+1, c+2, ..., S, the expected time until the next event is the expected time until the next demand transaction; that is $1/\lambda$. For levels at or below c, the expected time until the next event is the expected time until either the next demand transaction or the next opportunity to replenish at a reduced setup cost, that is, $1/(\lambda + \mu)$. To calculate E(I), the mean of the available stock, we first weigh each level by the expected duration, add, and normalize:

$$P_i'(I_0) = \frac{P_i(I_0) * \frac{1}{\lambda}}{\sum_{I_0=c+1}^{S} P_i(I_0) \frac{1}{\lambda} + \sum_{I_0=s+1}^{S} P_i(I_0) \frac{1}{\lambda + \mu}}$$

where, I_0 = s+1, s+2, ...,

$$P_i'(I_0) = \frac{P_i(I_0) * \frac{1}{\lambda + \mu}}{\sum_{I_0=c+1}^{S} P_i(I_0) \frac{1}{\lambda} + \sum_{I_0=s+1}^{S} P_i(I_0) \frac{1}{\lambda + \mu}}$$

where, I_0 = s+1, s+2, ...,

Then,

$$E(I) = \sum_{I_0=s+1}^{S} P_i'(I_0) * I_0$$

$$Var(I) = \sum_{I_0=s+1}^{S} P_i'(I_0) * (I_0 - E(I))^2$$

Once we calculate the probability mass function of the available stock, we can calculate the expected mean inventory level and variance, Thus, it is possible to calculate the mean and variance of each item given (S, c, s) values and a known Poisson rate of opportunities to replenishment at a reduced cost. However, it is generally computationally intractable to calculate the mean and variance of all items for compound Poisson demand model.

4.3 Optimum Level of Constraint

In the previous sections, we showed the appropriateness of the compound Poisson distribution to the stochastic demand and considered the method to obtain the mean and variance of the inventory level under the compound Poisson demand distribution.

To obtain an optimal level of constraint, e.g., the optimum size of the warehouse, we must determine the distribution of all items. However, if we know the optimal control variables of (S, c, s) and the opportunity rate of replenishing at a reduced cost, we may easily compute this distribution.

However, since we are using the simulation method to obtain the optimal control variables and the opportunity rate, we may also determine the distribution of the total inventory level, the mean, and the variance of that distribution as by-products of simulation. The algorithm of the simulation is presented in Appendix II.

As in Chapter II, the distribution of the total inventory level of all items will be approximately normal with the mean and variance which are obtained by simulation as the number of items increases. With the given data of demands, lead times, holding and replenishment costs for each item, system cost to carry inventories, and penalty cost, we can determine the optimal control policy and the level of constraint. The procedure is as follows:

i) Solve the problem ignoring the constraint; i.e., calculate the optimal control variables of (S, c, s) which minimize the operating cost of holding and replenishing items.

ii) Under the obtained policy, calculate the optimum level of constraint using equations (14) and (15) in Chapter II.

iii) If the given constraint level is less than the above optimum level, reduce the mean total inventory level by the amount of difference between the optimum level of the previous distribution and the given constraint to get the maximum allowable level of constraint (F^1_{MAX} in Figure 4-2). Otherwise, the calculated level is optimal. In such a case, the constraint is inactive.

iv) Set this maximum allowable level of constraint (F^1_{MAX}) as the new constraint, and calculate new optimum control variables. The individual values of s and c will also be changed to keep the same customer service level as the previous one. Since the value of S should be smaller to satisfy the given constraint, the reorder cycle will be shorter than the unconstraint case. There will be more reorder cycle periods per year. Accordingly, the customer service level will be less with the same values as s's and c's of the unconstrained case. Therefore, we have to increase the levels of s and c to meet the same customer service level.

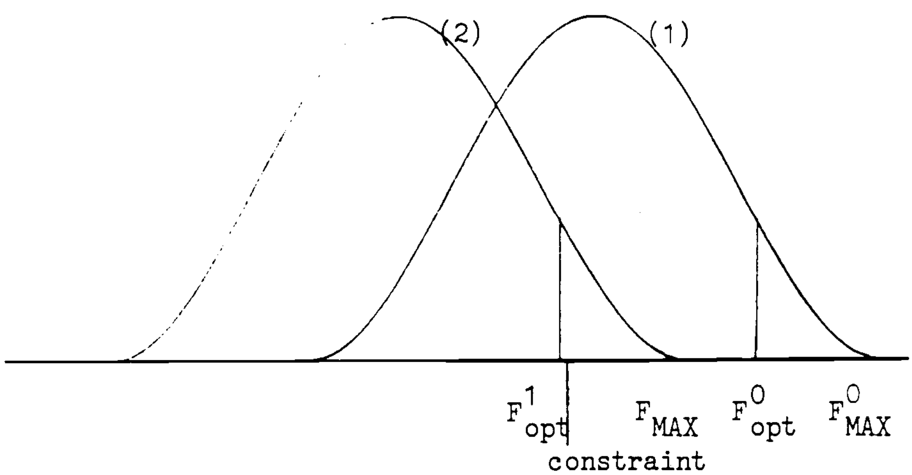

Figure 4-2 Distribution of total inventory level

graph (1) - distribution of total inventory level without constraint

graph (2) - moved distribution by the amount of (F^0_{opt} - $F_{constraint}$)

The superscripts, 0, 1, and ..., mean the order of simulation, and F^0_{opt}, F^1_{opt}, and ..., are the optimum levels of constraint after each simulation..

v) Simulate the system using new values of control variables, and determine the distribution of the total inventory level, mean and variance of this distribution.

vi) Calculate the optimum level of constraint. If this level is the same as the given constraint level, or if the difference between these two levels is small enough to be considered negligible, the obtained control variables and constraint level are optimal.

vii) If not, move the distribution to the left (if the given constraint is still smaller than the optimum level) or the right (if greater) and repeat from step (iii) to step (vi) until the distribution reaches the optimal conditions.

Example 4-1

Let us consider the same example as that in Chapter III. The input data are the same as in Example 3-1. We considered the simulation results in Table 3-2 after eight iterations as the optimum result. During the simulation, we reviewed the total inventory level every day for ten years and counted the number of days that how many days occupied each level of warehouse space. The results are shown in Table 4-1.

As shown in Table 4-1, the maximum required level is 137 units (one unit is 30 square feet). The mean and standard deviation of the total inventory level obtained by simulation are 92.025 and 6.392 units, respectively.

Table 4-1 Inventory level distribution

Level*	count	probability	Level	count	probability
71	2	.001	92	216	.059
72	2	.001	93	204	.056
73	3	.001	94	200	.055
74	3	.001	95	186	.051
75	9	.002	96	157	.043
76	9	.002	97	146	.040
77	17	.005	98	143	.039
78	17	.005	99	121	.033
79	26	.007	100	132	.036
80	26	.007	101	90	.025
81	58	.016	102	71	.019
82	77	.021	103	39	.011
83	77	.021	104	40	.011
84	83	.023	105	32	.009
85	100	.027	106	29	.008
86	155	.042	107	25	.007
87	190	.052	108	11	.003
88	203	.056	109	8	.002
89	236	.065	110	5	.001
90	238	.065	111	1	.000
91	262	.072			

$$\text{Maximum required level} = \sum_{i=1}^{30} S(i) * RFS(i) = 137 \text{ units}$$

where RFS(i) means the required floor space for item i and S(i) means order-up-to level for item i.

Mean inventory level = 92.025 units
Standard deviation = 6.392 units
* one unit = 30 square feet

We will show the cost savings achieved by operating the system at its optimum level when the system unit cost is \$150.00 and the penalty costs are \$50.00, \$75.00,and \$150.00.

The optimum levels of constraint will be

F_{opt}(PC=\$50.00) = 92.025 + 6.329 * (.9816) = 98.30 units

F_{opt}(PC=\$75.00) = 92.025 + 6.329 * (1.1445) = 99.34 units

F_{opt}(PC=\$150.00) = 92.025+ 6.329 * (1.4228) = 101.12 units ,

where the values of .9816, 1.1445, and 1.4228 are those of (P/A,i,n) in Table 2-1 when the ratio of unit system cost and penalty cost is 3.0,2.0 and 1.0,respectively.

Then the cost savings in system cost using same operating policy are given as follows .

First, if we operate the inventory system at its maximum level, the system cost will be

System cost(at maximum level of 137 units)
= 137 units * \$150.00 = \$20,550.00

The system costs at the optimum level when the ratios are 3.0, 2.0,and 1.0 are

System cost(at optimum level when ratio=3.0)
= 99 units * \$150.00 + 13.534 units * \$50.00 * (.9816)
= \$15,514.25

System cost(at optimum level when ratio=2.0)
= 100 units * \$150.00 + 9.96 units * \$75.00 * (1.1445)
= \$15,854.94

System cost (at optimum level when ratio = 1.0)

= 102 units * $150.00 + 7.44 units * $150.00 * (1.4228)

= $16,887.84

Therefore, the savings in the system csot are 24.5 percent, 22.8 percent, and 17.8 percent when the ratios of the unit system cost and the penalty cost are 3.0, 2.0, and 1.0, respectively. We can see from this example that the savings are directly dependent upon the size of the ratio.

In the above example, we showed the savings in the system cost when there are no constraints. However, if the constraints of the warehouse space are given and are less than the obtained optimum level of constraint, we have to change the operating policy of the control variables, i.e., S, c, and s. We will illustrate such a case in Example 4-2.

Example 4-2

We consider the same example in Example except that the demand rate is increased by a factor of four. The other input data remain the same. The ratio of unit system cost and penalty cost is 2.0, and the value of (P/A,r,n) is 1.1445 with a given constraint of warehouse space of 165 units. The simulation results after ten iterations are shown in Table 4-2. The maximum required level of warehouse space is 326 units, and the mean and standard deviation of the total inventory level are 151.379 and 20.024 units, respectively. The optimum level of the constraint is obtained as follows :

$$F_{opt} = 151.379 + 20.024 * (1.1445) = 174.296 \text{ units}$$

Since the given constraint is smaller than the optimum level, we have to change the operating policy of S, c, and s. The value of S can be obtained using the Lagrangian multiplier technique, as shown in Chapter II, to satisfy the given constraint. The values of

s_i and c_i are adjusted to keep the same customer service level. Then the simulation results with the constraint case are shown in Table 4-3.

Table 4-2 Simulation result without constraint

Item	s	c	S	Holding cost	Ordering cost	Item	s	c	S	Holding cost	Ordering cost
1	27	37	66	112.88	43.33	16	42	58	68	127.72	65.33
2	24	32	44	144.31	68.33	17	20	26	36	132.89	98.67
3	40	51	58	253.67	121.33	18	28	35	41	231.02	112.00
4	46	53	57	145.80	55.00	19	11	17	21	112.17	69.33
5	27	40	52	219.97	121.67	20	41	52	69	535.74	112.00
6	31	45	56	108.57	52.00	21	25	36	45	423.16	120.00
7	24	35	56	30.89	15.83	22	24	32	45	406.23	121.67
8	63	81	103	118.32	94.67	23	123	140	160	937.47	70.00
9	60	71	87	262.35	98.17	24	93	100	111	145.97	90.33
10	12	17	25	197.29	52.00	25	78	95	107	222.49	91.50
11	73	87	107	469.17	133.33	26	30	48	65	65.41	30.00
12	58	78	90	516.63	130.00	27	21	28	38	249.60	85.67
13	76	85	101	209.37	77.83	28	23	40	44	65.91	41.00
14	77	77	81	85.86	42.83	29	35	52	60	71.94	76.33
15	41	55	62	85.03	60.07	30	21	30	45	211.86	60.00
						Sub-total				6899.79	2410.84
						Total cost					9310.63

$$\text{Maximum required level} = \sum_{i=1}^{30} S(i) * RFS(i) = 326 \text{ units}$$

Mean inventory level = 151.379 units

Standard deviation = 20.024 units

* one unit = 30 square feet

Then the optimum level with constraint is obtained as follows ;

$$F^{*}_{opt} = 135.839 + 24.717 * (1.1445) = 164.128 \text{ units}$$

Table 4-3 Simulation results with constraint

Item	s	c	S	Holding	Ordering					Holding	Ordering
Item	s	c	S	cost	cost	Item	s	c	S	cost	cost
1	50	56	62	120.46	146.80	16	60	62	64	128.07	99.20
2	38	40	42	126.34	149.40	17	31	33	34	126.63	128.80
3	52	54	55	202.49	86.40	18	43	45	47	253.20	128.40
4	50	54	56	102.32	82.00	19	19	22	24	131.53	116.40
5	44	46	48	187.79	224.00	20	58	61	63	449.54	192.80
6	47	50	53	91.36	146.40	21	38	40	42	353.25	204.00
7	47	50	53	29.14	51.50	22	37	39	42	390.97	210.00
8	89	95	97	99.05	91.50	23	145	150	152	799.00	73.00
9	78	80	82	249.63	94.30	24	101	103	106	102.56	23.20
10	19	21	23	185.62	152.60	25	95	99	101	213.67	95.40
11	95	97	101	376.50	146.40	26	65	69	72	75.33	89.60
12	80	82	84	433.58	145.00	27	31	33	36	226.98	167.00
13	91	93	95	204.45	61.40	28	42	45	47	65.65	47.80
14	80	82	84	99.61	45.30	29	50	54	56	60.77	37.90
15	55	57	59	83.22	76.90	30	29	35	40	191.29	130.40
						Sub-total				6160.01	3443.80
						Total cost					9603.81

$$\text{Maximum required level} = \sum_{i=1}^{30} S(i) * RFS(i) = 312 \text{ units}$$

Mean inventory level = 135.830 units

Standard deviation = 24.717 units

Since this optimum value satisfies the given constraint, the F^*_{opt} is the optimum level of constraint. As we can see in Table 4-3, the cost in the presence of the constraint is thus $9603.81 per year higher than in the absence of such a constraint.

4.4 Cost Savings Under Different Demand Process

It is often convenient and economical to treat different items separately depending on the nature of the cost and stochastic processes involved. It is usually a poor policy to treat broad categories of items in the same way. However, it can become impossibly expensive if one attempts to develop and use sophisticated operating doctrines on each of thousands or more items. The answer to this problem lies in dividing the items up into a number of groups, with items in the different groups being treated differently. Since the distribution of the total inventory level which affects the optimum level of constraint depends on the demand processes, we may break down the items into several categories, usually three. Item in different categories are treated differently. In this paper, the items in the three different categories are referred to as high, medium, and low-demand items.

We will illustrate the cost savings achieved in each category in Example 4-3.

Example 4-3

We will consider the same example as that in Chapter IV. The input data given in Example 4-1 are categorized as low-demand items. The data given in Example 4-2, in which we increased the demand rates higher than those in Example 4-1, are categorized into high-demand items. And last, the data which has a demand rate twice that in Example 4-1 are categorized as medium-demand items.

The simulation results after eight iterations for the low-demand items and ten iterations for the medium and high-demand items are shown in Table 4-4.

If the unit system cost and the penalty cost are $150.00 and $75.00, respectively, and the value of (P/A,r,n) is 1.1445, then the cost savings achieved in the system can be calculated as follows.

First, if we operate the inventory system at its maximum level, the required system cost will be

System cost(low demanded item) = 137 units * $150.00 = $20,550.00

System cost(medium demanded item) = 211 units * $150.00
= $31,650.00

System cost(high demanded item) = 326 units * $150.00
= $48,900.00

And the optimum level of constraint at each category will be

F_{opt}(low-demand items) = 92.025 units + 6.329 units * 1.1445
= 99.34 units

F_{opt}(medium-demand items) = 119.593 units + 11.461 units * 1.1445
= 132.71 units

F_{opt}(high-demand items) = 151.379 units + 20.024 units * 1.1445
= 174.30 units

Table 4-4 Simulation results

Category	Total cost Holding	Total cost Ordering	Maximum required inventory level	Mean	Standard deviation
Low demanded item	3849.28	1284.60 5133.88	137	92.025	6.392
Medium demanded	5275.43	1691.60 6967.03	211	119.593	11.461
High demanded	6899.79	2410.63 9310.42	326	151.379	20.024

Then the system costs at the optimum level are

System cost_{opt}(low-demand items)
= 100 units * \$150.00 + 9.96 units * \$75.00 * 1.1445 = \$15,854.94

System cost_{opt}(medium-demand items)
= 133 units * \$150.00 + 14.35 units * \$75.00 * 1.1445 = \$21,181.77

System cost_{opt}(high-demand items)
= 175 units * \$150.00 + 15.96 units * \$75.00 * 1.1445 = \$27,619.97

Therefore, the cost savings achieved in each category are 22.85 percent, 33.07 percent, and 43.52 percent respectively. We can see in this example that we can get more savings as the demand increases.

4.5 Summary

In this chapter, we have developed an algorithm to determine the optimum level of constraint, using the probability distribution of the total inventory level. As shown in Table 4-1, the probability that maximum or near maximum inventory levels would occur was very small. Using this property, we found the optimum level of constraint and showed the savings in the system cost. The cost savings may vary in each case, depending on the unit system cost, the penalty cost, and the distribution of the total inventory level which varied according to the demand process.

As shown in Examples 4-1 and 4-3, it is evident that more cost savings are incurred in the system cost as the ratio of the unit system cost and the penalty cost and demand increase.

V. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The objective of this thesis is to develop a method to minimize the total inventory cost in a multi-item inventory system with constraint by determining the optimum level of constraint and operating policies.

Many authors have studied methods of obtaining the minimum operating cost of multi-item inventory systems. Among them were Schaack and Silver (1972), whose algorithm we use to obtain the minimum operating cost. However, if there is a constraint in the system, we generally apply either the Lagrangian multiplier technique, as in Chapter II, or the exchange curve to satisfy the given constraint.

If we use the Lagrangian multiplier technique, we can obtain the solution that always satisfies the given constraint, but we must pay higher operating costs. And with the exchange curve, we cannot get the exact solution. Instead we may get the relative comparative solution only enough to know that we may improve the present operating system if the present operating system is not optimal.

However, if the constraint-such as warehouse space limitation and maximum investment in inventory-depend on the total inventory level, we can get the exact optimum level of the constraint while operating the system at the minimum operating cost. That is, since the variation of the total inventory level follows the normal probability distribution as the number of items increases in the system, we can obtain the optimum level of constraint using a probabilistic property such as that shown in Chapter IV.

In Chapter II, we showed how to calculate the optimum level of constraint when demand was constant. In that case, we found that the unit system cost and the penalty cost were important factors. The optimum level was determined by the ratio of unit system cost and the penalty cost under the given interest rate and period of time. We have shown the relationship between the unit system cost and the penalty cost in Table 2-2.

In Chapter III, the coordinated ordering policy is used to study the multi-item dependent inventory system when demand is compound Poisson. Comparisions are given involving simulation results for the coordinated ordering policy, the joint ordering policy, and the independent ordering policy. The cost savings of the coordinated ordering policy with respect to the joint ordering policy and the independent ordering policy were 12.4 and 10.5 percent, respectively.

In Chapter IV, the extension of the multi-item dependent inventory system to a constrained warehouse space problem is handled using the iterative optimization-simulation technique.

In the mathematical model of examples 4-1 and 4-3, we show that we can get more savings in the system cost as the ratio of the unit system cost and the penalty cost increases.

Also, since the demand process affects the total inventory level, we consider the inventory system according to the demand process. We break down the items into three categories ; i.e., low-, medium-, and high-demand items, and calculate the cost savings achieved in each category. The brief result is shown in Table 5-1.

Table 5-1. Cost savings

ratio	1.0	2.0	3.0
cost saving	17.8%	22.8%	24.5%

demand rate	low	medium	high
cost saving	22.85%	33.07%	43.52%

Therefore, we can see that the cost savings advantage is increasingly better as the ratio of the unit system cost and the penalty cost, and demand increase.

5.2 Recommendations

Three specific fields for future research are

(1) Implementing a continuous review method is more expensive than a periodic ordering one because every transaction must be recorded. Even though such a system control cost is not considered in this paper, obtaining the optimal control variables using periodic ordering policy when inventory problem is constrained are desired. Because of the complexity, little work has been published in this area.

(2) In Chapter II, we assumed that the system cost of building a warehouse is linear to the size of warehouse for the sake of simplicity. However it is desired to use more exact system cost function for that relationship between the system cost and the size of a warehouse.

(3) In this paper, we considered the operating cost and the system cost of inventory system separately. That is, we first obtained the optimal control variables which gave the minimum operating cost to the inventory system. And next, we calculated the optimum level of the constraint. However it is also desired to consider an ordering policy in which an order is triggered for a group of items as soon as the total inventory level falls below a certain level. In such a case, we may obtain the optimal control variables in the inventory system and the optimum level of the constraint simultaneously.

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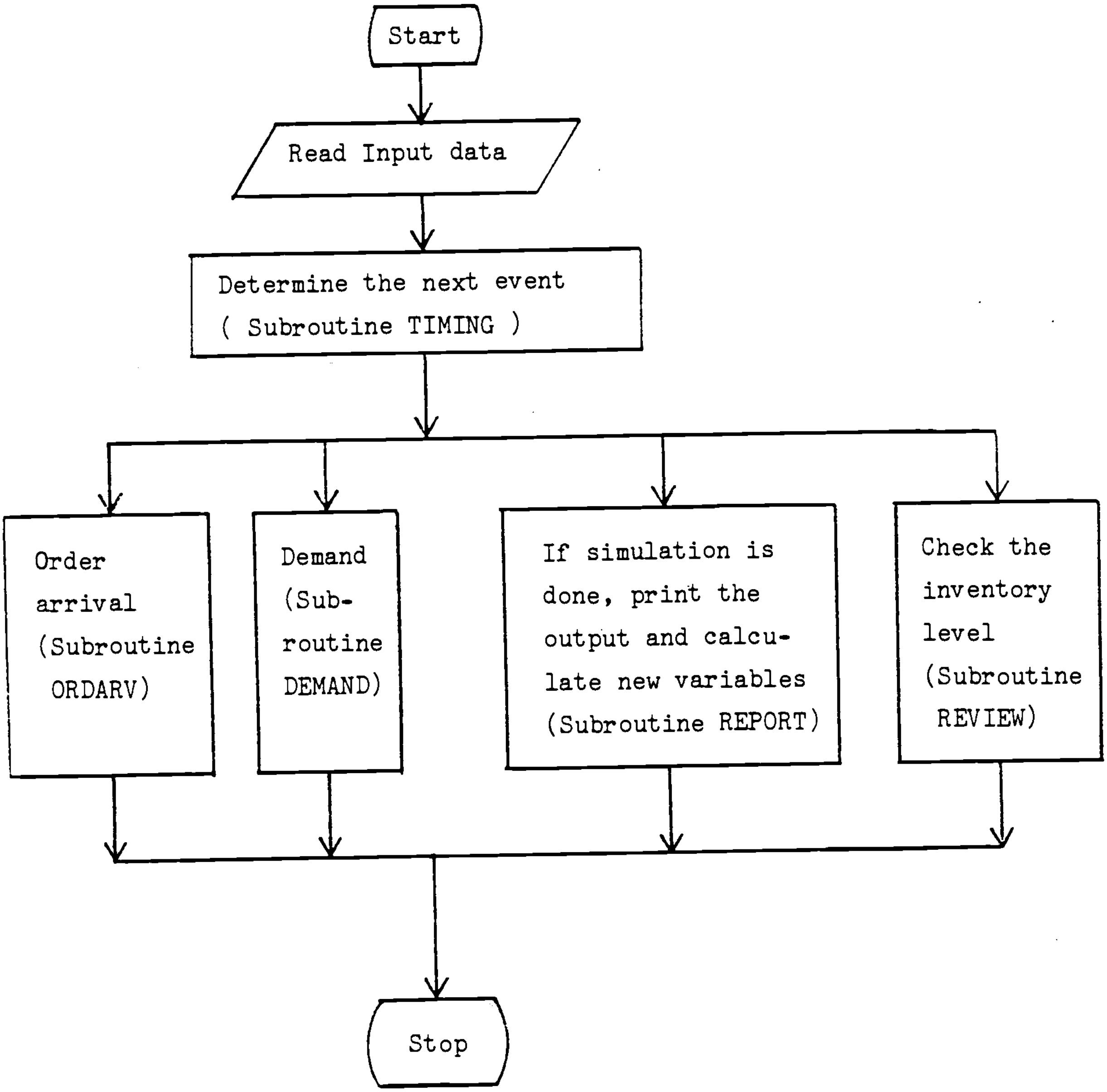
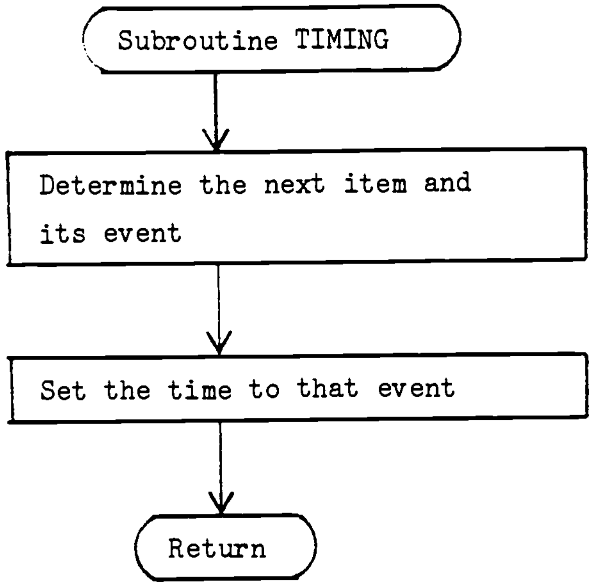
APPENDICES

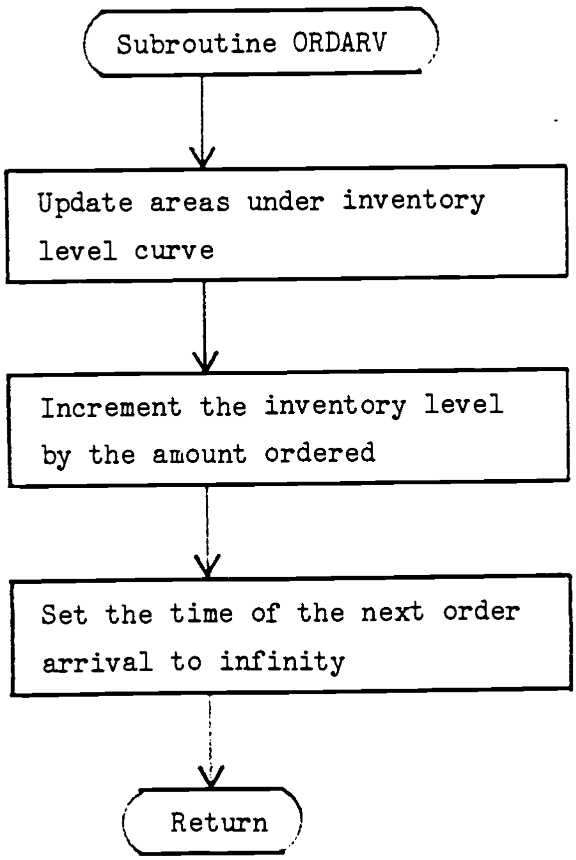
Appendix I. Glossary of symbols used in simulation program

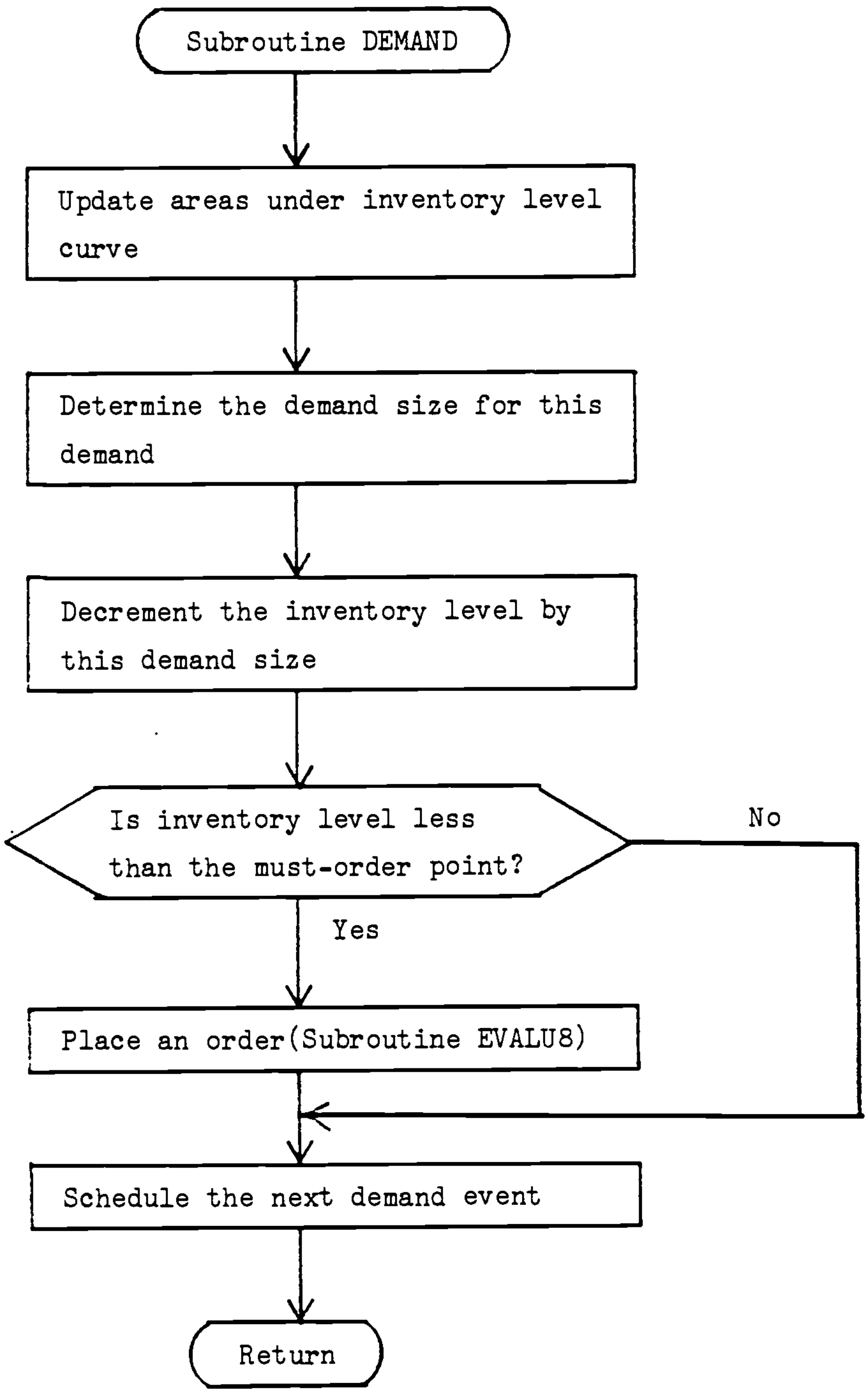
AMOUNT = Ordering quantity
BIGS = Order-up-to level
INITIL = Initial inventory level
INVLEV = Inventory level
NEVNTS = Number of event types for model
NEXT = Event type of next event to occur
NYEARS = Number of years for simulation
SMALLS = Must-order point
NITEMS = Number of items in the inventory system
NVALUE = Number of demand sizes
CAN = Coordinated joint order point
AMINUS = Area under inventory level when inventory level is less than zero
APLUS = Area under inventory level when inventory level is greater than zero
H = Average holding cost
RFS = Required floor space
PI = Penalty cost
TIME = Simulation clock
TLEVNT = Time of last event which changed the inventory level
MDEMDT = Mean interdemanded time
TNE(I) = Time of occurrence of event type I
TSLE = Time ~ince last event
TOTHLD = Total inventory holding cost
TOTORD = Total ordering cost
TOTCOST= Total operating cost
FIXSET = Setup cost of placing an order
VARSET = Variable ordering cost depending on the number of item
LEADT = Lead time

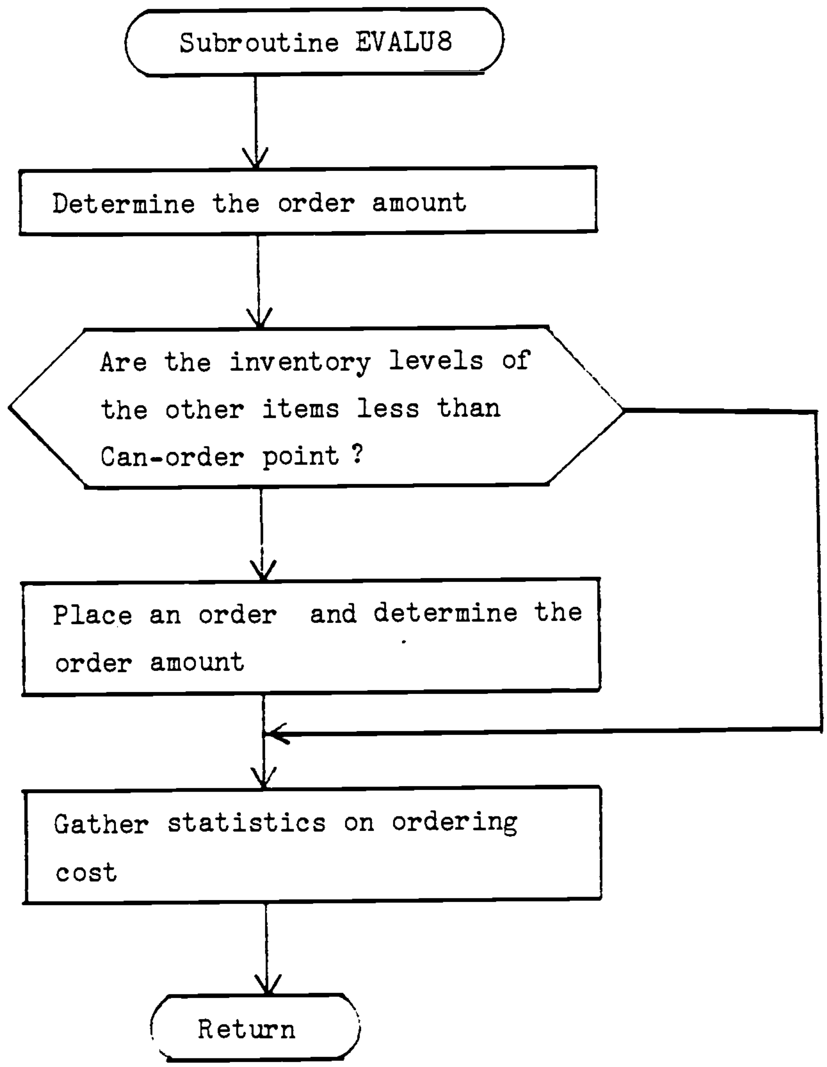
Appendix II. Flow chart for the simulation program

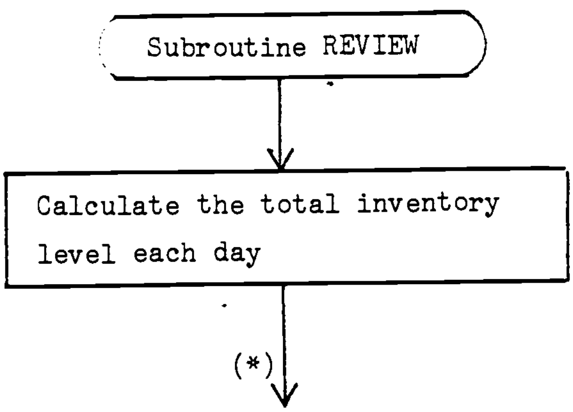


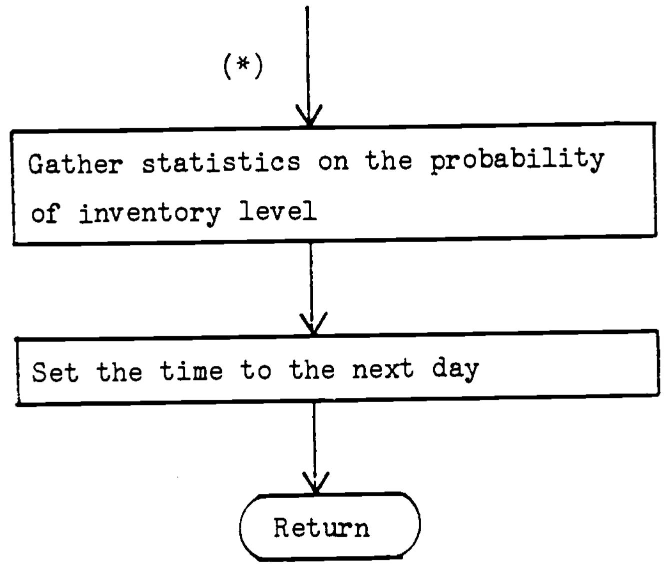

Subroutine TIMING
Determine the next item and its event
Set the time to that event
Return

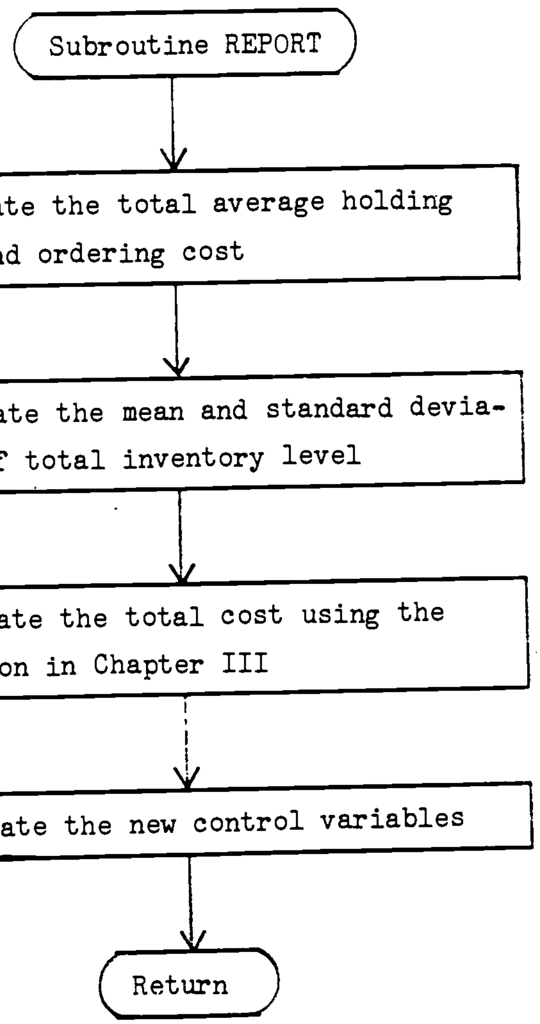

Subroutine ORDARV
Update areas under inventory level curve
Increment the inventory level by the amount ordered
Set the time of the next order arrival to infinity
Return


Subroutine DEMAND
Update areas under inventory level curve
Determine the demand size for this demand
Decrement the inventory level by this demand size
Is inventory level less than the must-order point?
No
Yes
Place an order(Subroutine EVALU8)
Schedule the next demand event
Return


Subroutine EVALU8
Determine the order amount
Are the inventory levels of the other items less than Can-order point ?
Place an order and determine the order amount
Gather statistics on ordering cost
Return


Subroutine REVIEW
Calculate the total inventory level each day
(*)


(*)
Gather statistics on the probability of inventory level
Set the time to the next day
Return


Subroutine REPORT
Calculate the total average holding cost and ordering cost
Calculate the mean and standard deviation of total inventory level
Calculate the total cost using the equation in Chapter III
Calculate the new control variables
Return

Appendix III. Simulaiton Program

```
      PROGRAM INVENT(INPUT,OUTPUT)
      PARAMETER (NUM=30)
      INTEGER AMOUNT(NUM),BIGS(NUM),INITIL(NUM),INVLEV(NUM),NEVNTS(NUM)
     &        ,NEXT(NUM),NYEARS(NUM),SMALLS(NUM),I,J,NITEMS,NVALUE(NUM)
     &        ,BACKLOG(NUM),ITEM,CAN(NUM)
      REAL AMINUS(NUM),APLUS(NUM),H(NUM),RFS(NUM),MDEMDT(NUM),PI(NUM)
     &     , TIME(NUM),TLEVNT(NUM),TNE(NUM,4),TORDC(NUM)
     &     ,TSLE(NUM),TCOST,TOTCOST,FIXSET,VARSET(NUM)
     &     ,LEADT(NUM)
      COMMON/MODEL/AMINUS,AMOUNT,APLUS,BIGS,H,RFS,INITIL
     &            ,INVLEV,MDEMDT,NEVNTS,NEXT,NYEARS,PI,LEADT,
     &            SMALLS,TIME,TLEVNT,TNE,TORDC,NITEMS,TSLE,CAN,TOTCOST
      COMMON/RANDOM/NVALUE,PROBD(NUM,25),NVAL(NUM,25),U
      COMMON/ITEMS/ITEM,BACKLOG
      COMMON/COST/FIXSET,VARSET
      NITEMS=30
      DO 10 I=1,NITEMS
      NEVNTS(I)=4
      NYEARS(I)=3
   10 CONTINUE
C
C*****READ INPUT PARAMETERS
      DO 20 I=1,NITEMS
      READ*,NVALUE(I),MDEMDT(I),RFS(I),H(I),VARSET(I),LEADT(I)
      PI(I)=2.0*H(I)
   20 CONTINUE
      DO 30 I=1,NITEMS
      READ*,SMALLS(I),CAN(I),BIGS(I)
      INITIL(I)=CAN(I)+0.2*(BIGS(I)-CAN(I))
   30 CONTINUE
      DO 40 I=1,NITEMS
      NV=NVALUE(I)
      READ*,(NVAL(I,J),J=1,NV)
      READ*,(PROBD(I,J),J=1,NV)
   40 CONTINUE
      READ*,SEED,FIXSET
      CALL RANSET(SEED)
C
C*****INITIALIZE THE SIMULATION
      CALL INIT
C*****DETERMINE THE NEXT EVENT
   50 CALL TIMING
      GO TO (60,70,80,90),NEXT(ITEM)
   60 CALL ORDARV
      GO TO 50
   70 CALL DEMAND
      GO TO 50
```

```
 80 CALL REPORT
    GO TO 100
 90 CALL REVIEW
    GO TO 50
100 STOP
    END
```

```
    SUBROUTINE INIT
    PARAMETER (NUM=30)
    INTEGER AMOUNT(NUM),BIGS(NUM),INITIL(NUM),INVLEV(NUM),NEVNTS(NUM)
   &        ,NEXT(NUM),NYEARS(NUM),SMALLS(NUM),I,J,NITEMS
   &        ,BACKLOG(NUM),ITEM,CAN(NUM)
   &        ,N(NUM),NJN(NUM),NJT(NUM),OMEGA(NUM),STOCK(NUM)
    REAL AMINUS(NUM),APLUS(NUM),H(NUM),RFS(NUM),MDEMDT(NUM),PI(NUM)
   &     , TIME(NUM),TLEVNT(NUM),TNE(NUM,4),TORDC(NUM)
   &     ,TSLE(NUM),FIXSET,VARSET(NUM),TCOST,TOTCOST
   &     ,LEADT(NUM)
    COMMON/MODEL/AMINUS,AMOUNT,APLUS,BIGS,H,RFS,INITIL
   &             ,INVLEV,MDEMDT,NEVNTS,NEXT,NYEARS,PI,LEADT,
   &             SMALLS,TIME,TLEVNT,TNE,TORDC,NITEMS,TSLE,CAN,TOTCOST
    COMMON/ITEMS/ITEM,BACKLOG
    LOGICAL ORDER(NUM)
    COMMON/LOG/ORDER
    COMMON/STAT/N,NJN,NJT,OMEGA,STOCK
    INTEGER IGAP(1000),COUNT(1000)
    COMMON/ST/IGAP,COUNT
    TOTCOST=0.
    DO 10 I=1,NITEMS
    TIME(I)=0.
    INVLEV(I)=INITIL(I)
    TLEVNT(I)=0.
    TSLE(I)=0.

    TORDC(I)=0.
    APLUS(I)=0.
    AMINUS(I)=0.
    AMOUNT(I)=0
    BACKLOG(I)=0
    ORDER(I)=.FALSE.
```

```
      N(I)=0
      NJN(I)=0
      NJT(I)=0
      OMEGA(I)=0
      STOCK(I)=0
      TNE(I,1)=1.E+20
      TNE(I,2)=EXPON(MDEMDT(I))
      TNE(I,3)=NYEARS(I)
      TNE(I,4)=0.002740
   10 CONTINUE
      DO 20 M=1,1000
      IGAP(M)=0
      COUNT(M)=0
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TIMING
      PARAMETER (NUM=30)
      INTEGER AMOUNT(NUM),BIGS(NUM),INITIL(NUM),INVLEV(NUM),NEVNTS(NUM)
     &        ,NEXT(NUM),NYEARS(NUM),SMALLS(NUM),I,J,NITEMS
     &        ,BACKLOG(NUM),ITEM,CAN(NUM)
      REAL AMINUS(NUM),APLUS(NUM),H(NUM),RFS(NUM),MDEMDT(NUM),PI(NUM)
     &     , TIME(NUM),TLEVNT(NUM),TNE(NUM,4),TORDC(NUM),RMIN
     &     ,TSLE(NUM),FIXSET,VARSET(NUM),TCOST,TOTCOST
     &     ,LEADT(NUM)
      COMMON/MODEL/AMINUS,AMOUNT,APLUS,BIGS,H,RFS,INITIL
     &            ,INVLEV,MDEMDT,NEVNTS,NEXT,NYEARS,PI,LEADT,
     &           SMALLS,TIME,TLEVNT,TNE,TORDC,NITEMS,TSLE,CAN,TOTCOST
      COMMON/ITEMS/ITEM,BACKLOG
      REAL TOTINV
      DO 10 I=1,NITEMS
      RMIN=1.E+29
      NEXT(I)=0
   10 CONTINUE
******DETERMINE THE NEXT EVENT******
      DO 30 I=1,NITEMS
      DO 20 J=1,NEVNTS(I)
      IF (TNE(I,J).GE.RMIN) GO TO 20
      RMIN=TNE(I,J)
      ITEM=I
```

```
      NEXT(I)=J
   20 CONTINUE
   30 CONTINUE
      IF (J.EQ.4) THEN
        DO 40 I=1,NITEMS
        TNE(I,4)=RMIN
   40   CONTINUE
      ENDIF
      IF (NEXT(ITEM).GT.0) GO TO 50
      PRINT*," EVENT LIST EMPTY"
      RETURN
   50 TIME(ITEM)=TNE(ITEM,NEXT(ITEM))
      RETURN
      END
```

```
      SUBROUTINE ORDARV
      PARAMETER (NUM=30)
      INTEGER AMOUNT(NUM),BIGS(NUM),INITIL(NUM),INVLEV(NUM),NEVNTS(NUM)
     &        ,NEXT(NUM),NYEARS(NUM),SMALLS(NUM),I,J,NITEMS
     &        ,BACKLOG(NUM),ITEM,CAN(NUM)
      REAL AMINUS(NUM),APLUS(NUM),H(NUM),RFS(NUM),MDEMDT(NUM),PI(NUM)
     &     , TIME(NUM),TLEVNT(NUM),TNE(NUM,4),TORDC(NUM)
     &     ,TSLE(NUM),FIXSET,VARSET(NUM),TCOST,TOTCOST
     &     ,LEADT(NUM)
      COMMON/MODEL/AMINUS,AMOUNT,APLUS,BIGS,H,RFS,INITIL
     &            ,INVLEV,MDEMDT,NEVNTS,NEXT,NYEARS,PI,LEADT,
     &            SMALLS,TIME,TLEVNT,TNE,TORDC,NITEMS,TSLE,CAN,TOTCOST
      COMMON/ITEMS/ITEM,BACKLOG
      LOGICAL ORDER(NUM)
      COMMON/LOG/ORDER
      INTEGER N(NUM),NJN(NUM),NJT(NUM),OMEGA(NUM),STOCK(NUM)
      COMMON/STAT/N,NJN,NJT,OMEGA,STOCK
      CALL UPDATE
      INVLEV(ITEM)=INVLEV(ITEM)+AMOUNT(ITEM)
      N(ITEM)=N(ITEM)+1
      TNE(ITEM,1)=1.E+20
      BACKLOG(ITEM)=0
      ORDER(ITEM)=.FALSE.
      RETURN
      END
```

```
      SUBROUTINE DEMAND
      PARAMETER (NUM=30)
      INTEGER AMOUNT(NUM),BIGS(NUM),INITIL(NUM),INVLEV(NUM),NEVNTS(NUM)
     &        ,NEXT(NUM),NYEARS(NUM),SMALLS(NUM),DSIZE(NUM),I,J,NITEMS
     &        ,BACKLOG(NUM),ITEM,CAN(NUM)
      REAL AMINUS(NUM),APLUS(NUM),H(NUM),RFS(NUM),MDEMDT(NUM),PI(NUM)
     &     , TIME(NUM),TLEVNT(NUM),TNE(NUM,4),TORDC(NUM)
     &     ,TSLE(NUM),FIXSET,VARSET(NUM),TCOST,TOTCOST
     &     ,LEADT(NUM)
      COMMON/MODEL/AMINUS,AMOUNT,APLUS,BIGS,H,RFS,INITIL
     &            ,INVLEV,MDEMDT,NEVNTS,NEXT,NYEARS,PI,LEADT,
     &            SMALLS,TIME,TLEVNT,TNE,TORDC,NITEMS,TSLE,CAN,TOTCOST
      COMMON/ITEMS/ITEM,BACKLOG
      LOGICAL ORDER(NUM)
      COMMON/LOG/ORDER
      CALL UPDATE
      DSIZE(ITEM)=RANDI(ITEM)
      INVLEV(ITEM)=INVLEV(ITEM)-DSIZE(ITEM)
      IF (INVLEV(ITEM).LT.0) THEN
      BACKLOG(ITEM)=BACKLOG(ITEM)-DSIZE(ITEM)
      ENDIF
******IF INVENTORY LEVEL IS LESS THAN SMALLS, PLACE AN ORDER******
      JITEM=ITEM
      IF(INVLEV(ITEM).LT.SMALLS(ITEM)) THEN
      IF(.NOT.ORDER(JITEM)) THEN
      CALL EVALU8(JITEM)
      ENDIF
      ENDIF
      TNE(ITEM,2)=TIME(ITEM)+EXPON(MDEMDT(ITEM))
      RETURN
      END
```

```
      SUBROUTINE EVALU8(JITEM)
      PARAMETER (NUM=30)
      INTEGER AMOUNT(NUM),BIGS(NUM),INITIL(NUM),INVLEV(NUM),NEVNTS(NUM)
     &        ,NEXT(NUM),NYEARS(NUM),SMALLS(NUM),I,J,NITEMS
     &        ,BACKLOG(NUM),ITEM,CAN(NUM)
      REAL AMINUS(NUM),APLUS(NUM),H(NUM),RFS(NUM),MDEMDT(NUM),PI(NUM)
     &     , TIME(NUM),TLEVNT(NUM),TNE(NUM,4),TORDC(NUM)
     &     ,TSLE(NUM),FIXSET,VARSET(NUM),TCOST,TOTCOST
     &     ,LEADT(NUM)
```

```
      COMMON/MODEL/AMINUS,AMOUNT,APLUS,BIGS,H,RFS,INITIL
     &           ,INVLEV,MDEMDT,NEVNTS,NEXT,NYEARS,PI,LEADT,
     &            SMALLS,TIME,TLEVNT,TNE,TORDC,NITEMS,TSLE,CAN,TOTCOST
      COMMON/COST/FIXSET,VARSET
      REAL MCOST(NUM),TMCOST
      COMMON/ITEMS/ITEM,BACKLOG
      LOGICAL ORDER(NUM)
      COMMON/LOG/ORDER
      INTEGER N(NUM),NJN(NUM),NJT(NUM),OMEGA(NUM),STOCK(NUM)
      COMMON/STAT/N,NJN,NJT,OMEGA,STOCK
      TCOST=0.
      AMOUNT(JITEM)=BIGS(JITEM)-INVLEV(JITEM)
      TNE(JITEM,1)=TIME(JITEM)+LEADT(JITEM)
      ORDER(JITEM)=.TRUE.
      OMEGA(JITEM)=OMEGA(JITEM)+INVLEV(JITEM)
      NJT(JITEM)=NJT(JITEM)+1
      TEMP=JITEM
******IF THE INVENTORY LEVEL IS LESS THAM CAN-ORDER POINT, PLACE AN ORDER*****
      DO 20 I=1,NITEMS
      IF(.NOT. ORDER(I)) THEN
      IF(I.NE.TEMP) THEN
      IF(INVLEV(I).LE.CAN(I)) THEN
      AMOUNT(I)=BIGS(I)-INVLEV(I)
      TNE(I,1)=TIME(I)+LEADT(I)
      ORDER(I)=.TRUE.
      STOCK(I)=STOCK(I)+INVLEV(I)
      NJN(I)=NJN(I)+1
      ENDIF
      ENDIF
      ENDIF
   20 CONTINUE
      DO 30 I=1,NITEMS
      IF (ITEMC(I).GT.0.0) THEN
      TCOST=TCOST+VARSET(I)
      ENDIF
   30 CONTINUE
      IF (TCOST.GT.0.0) THEN
      TOTCOST=TOTCOST+FIXSET+TCOST
      ENDIF
      RETURN
      END




      SUBROUTINE REVIEW
      PARAMETER(NUM=30)
```

```
      INTEGER AMOUNT(NUM),BIGS(NUM),INITIL(NUM),INVLEV(NUM),NEVNTS(NUM)
     &        ,NEXT(NUM),NYEARS(NUM),SMALLS(NUM),I,J,NITEMS
     &        ,BACKLOG(NUM),ITEM,CAN(NUM)
      REAL AMINUS(NUM),APLUS(NUM),H(NUM),RFS(NUM),MDEMDT(NUM),PI(NUM)
     &     , TIME(NUM),TLEVNT(NUM),TNE(NUM,4),TORDC(NUM)
     &     ,AORDC(NUM),AHLDC(NUM),ASHRC(NUM),ACOST(NUM),TSLE(NUM)
     &     ,FIXSET,VARSET(NUM),TCOST,TOTCOST,AVGORDC
     &     ,LEADT(NUM)
      COMMON/MODEL/AMINUS,AMOUNT,APLUS,BIGS,H,RFS,INITIL
     &           ,INVLEV,MDEMDT,NEVNTS,NEXT,NYEARS,PI,LEADT,
     &           SMALLS,TIME,TLEVNT,TNE,TORDC,NITEMS,TSLE,CAN,TOTCOST
      REAL TOTINV
      INTEGER IGAP(1000),COUNT(1000),JGAP
      COMMON/ST/IGAP,COUNT
      COMMON/ITEMS/ITEM,BACKLOG
      TOTINV=0.0
      DO 10 L=1,NITEMS
      TOTINV=TOTINV+RFS(L)*INVLEV(L)
   10 CONTINUE
      JGAP=INT(TOTINV/30.0)+1
      DO 20 K=1,JGAP
      IF ( JGAP.GE.K .AND. JGAP.LT.K+1 ) THEN
      COUNT(K)=COUNT(K)+1
      ENDIF
   20 CONTINUE
      DO 30 I=1,NITEMS
      TNE(I,4)=TNE(I,4)+0.002740
   30 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE REPORT
      PARAMETER (NUM=30)
      INTEGER AMOUNT(NUM),BIGS(NUM),INITIL(NUM),INVLEV(NUM),NEVNTS(NUM)
     &        ,NEXT(NUM),NYEARS(NUM),SMALLS(NUM),I,J,NITEMS
     &        ,BACKLOG(NUM),ITEM,NVALUE(NUM),CAN(NUM)
      REAL AMINUS(NUM),APLUS(NUM),H(NUM),RFS(NUM),MDEMDT(NUM),PI(NUM)
     &     , TIME(NUM),TLEVNT(NUM),TNE(NUM,4),TORDC(NUM)
     &     ,AORDC(NUM),AHLDC(NUM),ASHRC(NUM),ACOST(NUM),TSLE(NUM)
     &     ,FIXSET,VARSET(NUM),TCOST,TOTCOST,AVGORDC
     &     ,TOTORC(NUM),AVGORC(NUM),LEADT(NUM)
     &     ,TOTCN,MINVL,STDINV,PCOUNT(1000)
```

```
      COMMON/MODEL/AMINUS,AMOUNT,APLUS,BIGS,H,RFS,INITIL
     &             ,INVLEV,MDEMDT,NEVNTS,NEXT,NYEARS,PI,LEADT,
     &             SMALLS,TIME,TLEVNT,TNE,TORDC,NITEMS,TSLE,CAN,TOTCOST
      COMMON/COST/FIXSET,VARSET
      INTEGER N(NUM),NJN(NUM),NJT(NUM),OMEGA(NUM),STOCK(NUM)
      REAL P(NUM),CALOR(NUM),CALHL(NUM),TCALOR,TCALHL,PROX(NUM)
      COMMON/STAT/N,NJN,NJT,OMEGA,STOCK
      INTEGER IGAP(1000),COUNT(1000),MAX
      REAL TOTHLD,TOTSHR,TOTORD,U,FLAG(NUM,2)
      COMMON/ST/IGAP,COUNT
      COMMON/RANDOM/NVALUE,PROBD(NUM,25),NVAL(NUM,25),U
      REAL MSIZE(NUM),DPYR(NUM),TAU(NUM),CAL(NUM),TCAL,
     &     VSIZE(NUM),SSIZE(NUM),XI(NUM),BIGA(NUM)
      REAL CDF(400,2),X1(NUM),X2(NUM),ZX1(NUM),ZX2(NUM),MU(NUM),
     &     NU(NUM),Q(NUM),R(NUM),PS(NUM),YA(NUM),YB(NUM)
      REAL XA(NUM),XB(NUM),ZXA(NUM),ZXB(NUM),PX(NUM),PY(NUM),PZ(NUM)
      CALL UPDATE
      TOTHLD=0.0
      TOTSHR=0.0
      TOTORD=0.0
******CALCULATE THE AVERAGE HOLDING AND ORDERING COST******
      DO 10 I=1,NITEMS
      AHLDC(I)=H(I)*(APLUS(I)/NYEARS(I))
      ASHRC(I)=PI(I)*(AMINUS(I)/NYEARS(I))
      PROX(I)=AMINUS(I)/(APLUS(I)+AMINUS(I))
      TOTHLD=TOTHLD+AHLDC(I)
      TOTSHR=TOTSHR+ASHRC(I)
      TMCOST=TMCOST+MCOST(I)
   10 CONTINUE
      AVGORDC=TOTCOST/NYEARS(1)
      PRINT*," I      SS  C    BS    AHLD     ASHR  PROX "
      DO 20 I=1,NITEMS
      PRINT"(1X,I3,3I4,2X,2F7.2,F8.4)",I,SMALLS(I)
     &      ,CAN(I),BIGS(I),AHLDC(I),ASHRC(I),PROX(I)
   20 CONTINUE
      DO 30 I=1,NUM
      TOTORC(I)=NJN(I)*VARSET(I)+NJT(I)*FIXSET
      AVGORC(I)=TOTORC(I)/NYEARS(I)
      TOTORD=TOTORD+AVGORC(I)
   30 CONTINUE
      DO 40 I=1,NUM
      OMEGA(I)=OMEGA(I)/NJT(I)
      STOCK(I)=STOCK(I)/NJN(I)
      P(I)=(FLOAT(NJN(I))/FLOAT(N(I)))
   40 CONTINUE
      PRINT*,"  I   O   R     P     TORD   AORD   N  NJN  NJT "
      DO 50 I=1,NITEMS
      PRINT"(1X,3I4,F6.2,2F9.2,3I4)",I,OMEGA(I),STOCK(I),P(I),
     &      TOTORC(I),AVGORC(I),N(I),NJN(I),NJT(I)
   50 CONTINUE
```

```
******CALCULATE THE MEAN AND STANDARD DEVIATION OF TOTAL INVENTORY LEVEL******
      MAX=0
        DO 60 I=1,NITEMS
      MAX=MAX+INT(RFS(I)*BIGS(I)+0.5)
   60   CONTINUE
      MAX=MAX/30
      PRINT*,"  MAX=",MAX
      TOTCN=0.
      MINVL=0.
      STDINV=0.
      DO  70 J=1,MAX
      TOTCN=TOTCN+COUNT(J)
   70 CONTINUE
      DO  80 J=1,MAX
      PCOUNT(J)=COUNT(J)/TOTCN
      MINVL=MINVL+J*PCOUNT(J)
   80 CONTINUE
      DO  90 J=1,MAX
      STDINV=STDINV+((J-MINVL)**2.0)*PCOUNT(J)
   90 CONTINUE
      STDINV=SQRT(STDINV)
      PRINT"(1X,A,F10.3,A,F10.3)","MEAN INV. LEVEL=",MINVL,
     &    "      STD DEV OF INV LEVEL = ",STDINV
      DO 100 I=1,NUM
      MSIZE(I)=NVAL(I,1)*PROBD(I,1)
      VSIZE(I)=0.0
      SSIZE(I)=0.0
      NV=NVALUE(I)
      DO 110 J=2,NV
      MSIZE(I)=MSIZE(I)+NVAL(I,J)*(PROBD(I,J)-PROBD(I,J-1))
      VSIZE(I)=VSIZE(I)+( MSIZE(I)-NVAL(I,J) )**2.0*
     &         ( PROBD(I,J)-PROBD(I,J-1) )
      SSIZE(I)=SQRT(VSIZE(I))
  110 CONTINUE
      DPYR(I)=MSIZE(I)/MDEMDT(I)
      TAU(I)=BIGS(I)-OMEGA(I)-P(I)*(STOCK(I)-OMEGA(I))
  100 CONTINUE
******CALCULATE THE TOTAL COST USING THE EQUATION IN CHAPTER III******
      TCAL=0.0
      TCALOR=0.0
      TCALHL=0.0
      COMOR=0.0
      COMHL=0.0
      PRINT*,"           CALCULATED COST    "
      PRINT*," I   TOT     ORD     HLD      ORA    HLA   ORD     MSIZE
     &SSIZE"
```

```
      DO 120 I=1,NUM
      IF (P(I).GT.1.0) THEN
      P(I)=1.0
      ENDIF
      CALOR(I)=DPYR(I)/TAU(I)*( P(I)*VARSET(I) + (1.0-P(I))*(FIXSET
     &         +VARSET(I)) )
      CALHL(I)=( P(I)*(BIGS(I)+STOCK(I))/2.0 + (1.0-P(I))*(BIGS(I)+
     &          OMEGA(I))/2.0 - DPYR(I)*LEADT(I) ) * H(I)
      CAL(I)=CALOR(I)+CALHL(I)
      PRINT"(1X,I2,3F8.2,2F5.2)",I,CAL(I),CALOR(I),CALHL(I),MSIZE(I),
     &      SSIZE
  120 CONTINUE
      DO 130 I=1,NUM
      TCAL=TCAL+CAL(I)
      TCALOR=TCALOR+CALOR(I)
      TCALHL=TCALHL+CALHL(I)
      XI(I)=P(I)*( STOCK(I)-OMEGA(I) )+SQRT( 2.0*DPYR(I)/H(I)*
     &      ( (1.0-P(I) )*(FIXSET+VARSET(I)) + P(I)*VARSET(I)) )
      BIGA(I)=XI(I)+OMEGA(I)
      PRINT"(1X,I2,3F7.2,F6.4,2F7.2,3I4,2F7.2)",I,P(I),DPYR(I),TAU(I),
     &   LEADT(I),H(I)  ,VARSET(I),BIGS(I),OMEGA(I),STOCK(I)
     &      ,BIGA(I),XI(I)
  130 CONTINUE
      PRINT"(1X,A,F10.5)"," RANDOM NUMBER = ",U
******CLACULATE THE TOTAL OPERATING COST******
      PRINT*," "
      PRINT*,"                          SIMULATED COST     CALCULATED
     &COST"
      PRINT*," ITEM   SS  CAN  BIGS   HOLDING  ORDERING  HOLDING
     &ORDERING"
      PRINT*," "
      DO 140 I=1,NUM
      PRINT"(1X,4I5,4F10.2)",I,SMALLS(I),CAN(I),BIGS(I),AHLDC(I),
     &      AVGORC(I),CALHLA(I),CALOR(I)
  140 CONTINUE
      PRINT*," "
      PRINT"(1X,A,4F10.2)","        SUB TOTAL  ",TOTHLD,TOTORD,
     &      TCALHL,TCALOR
      TX=TOTHLD+TOTORD
      TY=TCALOR+TCALHL
      PRINT"(1X,A,10X,F10.2,10X,F10.2)","        TOTAL COST  ",TX,TY
******PRINT THE DISTRIBUTION OF TOTAL INVENTORY LEVEL******
      DO 150 I=1,MAX
      IF (COUNT(I).GT.0) THEN
      PRINT"(1X,I5,I10,F10.4)",I,COUNT(I),PCOUNT(I)
      ENDIF
  150 CONTINUE
******CALCULATE NEW CONTROL VARIABLES******
      DO 160 I=1,NUM
```

```
      MU(I)=DPYR(I)*LEADT(I)
      NU(I)=SQRT( MU(I)/MSIZE(I)*(VSIZE(I)+MSIZE(I)**2.0) )
  160 CONTINUE
      DO 170 I=1,NUM
      IF (OMEGA(I).LE.0.0) THEN
      OMEGA(I)=MU(I)
      ENDIF
      X1(I)=( OMEGA(I)-MU(I) )/NU(I)
      X2(I)=( STOCK(I)-MU(I) )/NU(I)
  170 CONTINUE
      DO 180 I=1,328
      READ*,(CDF(I,J),J=1,2)
  180 CONTINUE
      DO 190 I=1,NUM
      DO 200 K=1,328
      IF (X1(I).GE. 0.0) THEN
         IF(X1(I).GT.CDF(K,1) .AND. X1(I).LE.CDF(K+1,1) ) THEN
         ZX1(I)=CDF(K,2)
         ENDIF
      ELSE
         X1(I)=ABS(X1(I))
         IF (X1(I).GT.CDF(K,1) .AND. X1(I).LE.CDF(K+1,1) ) THEN
         ZX1(I)=1.0-CDF(K,2)
         ENDIF
      ENDIF
      IF (X2(I).GE. 0.0) THEN
         IF (X2(I).GT.CDF(K,1) .AND. X2(I).LE.CDF(K+1,1) ) THEN
         ZX2(I)=CDF(K,2)
         ENDIF
      ELSE
         X2(I)=ABS(X2(I))
         IF (X2(I).GT.CDF(K,1) .AND. X2(I).LE.CDF(K+1,1) ) THEN
         ZX2(I)=1.0-CDF(K,2)
         ENDIF
      ENDIF
  200 CONTINUE
  190 CONTINUE
      DO 210 I=1,NUM
      Q(I)=(FLOAT(NJT(I))/FLOAT(NYEARS(I)))
      R(I)=(FLOAT(NJN(I))/FLOAT(NYEARS(I)))
      PS(I)=(ZX1(I)**Q(I)) * (ZX2(I)**R(I))
  210 CONTINUE
      DO 220 I=1,NUM
      PRINT"(1X,4F8.3,2F5.2,3F7.3)",MU(I),NU(I),X1(I),X2(I),Q(I),R(I),
     &       ZX1(I),ZX2(I),PS(I)
  220 CONTINUE
      PRINT*," XA(I) XB(I)  ZXA(I)  ZXB(I)  PX(I)  PY(I)  Q   R   SS"
      DO 230 I=1,NUM
      DO 240 J=1,2
```

```
      FLAG(I,J)=0.0
240   CONTINUE
      PY(I)=(1.0-.20)**(((BIGS(I)-OMEGA(I))-P(I)*(STOCK(I)-OMEGA(I)))
     &       /DPYR(I))
250   XA(I)=(OMEGA(I)-MU(I))/NU(I)
      XB(I)=(STOCK(I)-MU(I))/NU(I)
      IF (XA(I).LT.0.0) THEN
      YA(I)=ABS(XA(I))
      ENDIF
      IF (XB(I).LT.0.0) THEN
      YB(I)=ABS(XB(I))
      ENDIF
      DO 260 IX=1,328
      IF(XA(I).GE.0.0) THEN
      IF(XA(I).GT.CDF(IX,1) .AND. XA(I).LE.CDF(IX+1,1)) THEN
      ZXA(I)=CDF(IX,2)
      ENDIF
      ELSE
      IF(YA(I).GT.CDF(IX,1) .AND. YA(I).LE.CDF(IX+1,1)) THEN
      ZXA(I)=1.0-CDF(IX,2)
      ENDIF
      ENDIF
      IF(XB(I).GE.0.0) THEN
      IF(XB(I).GT.CDF(IX,1) .AND. XB(I).LE.CDF(IX+1,1)) THEN
      ZXB(I)=CDF(IX,2)
      ENDIF
      ELSE
      IF(YB(I).GT.CDF(IX,1) .AND. YB(I).LE.CDF(IX+1,1)) THEN
      ZXB(I)=1.0-CDF(IX,2)
      ENDIF
      ENDIF
260   CONTINUE
      PX(I)=( ZXA(I)**(1-P(I)) ) * ( ZXB(I)**P(I) )
      PZ(I)=PX(I)-PY(I)
      IF(FLAG(I,1).EQ.1.0 .AND. FLAG(I,2).EQ.1.0) GO TO 270
      IF (PZ(I).GT.0.0) THEN
      OMEGA(I)=OMEGA(I)-1.0
      STOCK(I)=STOCK(I)-1.0
      FLAG(I,1)=1.0
      ELSE
      OMEGA(I)=OMEGA(I)+1.0
      STOCK(I)=STOCK(I)+1.0
      FLAG(I,2)=1.0
      ENDIF
      GO TO 250
270   SMALLS(I)=OMEGA(I)+0.5*( SSIZE(I)**2.0 + MSIZE(I)**2.0 )/MSIZE(I)
      CAN(I)=SMALLS(I)+(STOCK(I)-OMEGA(I))
      PRINT"(1X,7F7.3,4I4,I6)",XA(I),XB(I),ZXA(I),ZXB(I),PX(I),PY(I),
     &       PZ(I),OMEGA(I),STOCK(I),SMALLS(I),CAN(I),BIGA(I)
```

```
230 CONTINUE
    RETURN
    END



    SUBROUTINE UPDATE
    PARAMETER (NUM=30)
    INTEGER AMOUNT(NUM),BIGS(NUM),INITIL(NUM),INVLEV(NUM),NEVNTS(NUM)
   &        ,NEXT(NUM),NYEARS(NUM),SMALLS(NUM),I,J,NITEMS
   &        ,BACKLOG(NUM),ITEM,CAN(NUM)
    REAL AMINUS(NUM),APLUS(NUM),H(NUM),RFS(NUM),MDEMDT(NUM),PI(NUM)
   &     , TIME(NUM),TLEVNT(NUM),TNE(NUM,4),TORDC(NUM)
   &     ,TSLE(NUM),FIXSET,VARSET(NUM),TCOST,TOTCOST
   &     ,LEADT(NUM)
    COMMON/MODEL/AMINUS,AMOUNT,APLUS,BIGS,H,RFS,INITIL
   &             ,INVLEV,MDEMDT,NEVNTS,NEXT,NYEARS,PI,LEADT,
   &             SMALLS,TIME,TLEVNT,TNE,TORDC,NITEMS,TSLE,CAN,TOTCOST
    COMMON/ITEMS/ITEM,BACKLOG
    TSLE(ITEM)=TIME(ITEM)-TLEVNT(ITEM)
    TLEVNT(ITEM)=TIME(ITEM)
    IF(INVLEV(ITEM)) 10,20,30
 10 AMINUS(ITEM)=AMINUS(ITEM)+(-INVLEV(ITEM)*TSLE(ITEM))
 20 RETURN
 30 APLUS(ITEM)=APLUS(ITEM)+(INVLEV(ITEM)*TSLE(ITEM))
    RETURN
    END



    FUNCTION EXPON(RMEAN)
    REAL RMEAN,U
    U=RANF( )
    EXPON=-RMEAN*ALOG(U)
    RETURN
    END



    FUNCTION RANDI(ITEM)
    PARAMETER(NUM=30)
    INTEGER I,NVALUE(NUM)
    REAL U
    COMMON/RANDOM/NVALUE,PROBD(NUM,25),NVAL(NUM,25),U
```

```
      U=RANF()
      DO 10 J=1,NVALUE(ITEM)
      IF (U.LE.PROBD(ITEM,J)) GO TO 30
   10 CONTINUE
      PRINT*," PROBD ARRAY IS NOT COMPLETE"
      STOP
   30 RANDI=NVAL(ITEM,J)
      RETURN
      END
```

```
      FUNCTION UNIFRM(A,B)
      REAL A,B,U
      U=RANF()
      UNIFRM=A+(U*(B-A))
      RETURN
      END
```

Appendix 4.1 Input data

Item	Mean inter-demanded time	Required floor space	Carrying cost	Variable setup cost	Lead time	Number of transaction
1	.0417	2.0	3.0	2.0	.0417	8
2	.0458	3.0	6.0	2.0	.0625	5
3	.0500	4.0	9.0	4.0	.0833	7
4	.0583	2.5	7.2	5.0	.1250	7
5	.0458	5.0	7.2	5.0	.0417	8
6	.0542	6.0	3.6	2.0	.0625	7
7	.0667	4.0	1.2	2.5	.1250	5
8	.0542	3.5	2.4	3.5	.0833	13
9	.0750	8.0	6.0	4.5	.0833	15
10	.0708	6.0	12.0	3.0	.0417	4
11	.0375	1.0	9.0	4.0	.0625	10
12	.0400	5.0	12.0	5.0	.0625	11
13	.0567	5.5	6.0	5.5	.1250	10
14	.0600	6.0	4.2	1.5	.1250	11
15	.0700	4.0	3.0	1.5	.0833	11
16	.0650	4.0	4.2	2.0	.0833	10
17	.0550	3.0	7.8	2.0	.0625	5
18	.0625	2.5	9.6	3.0	.0625	8
19	.0708	6.5	10.2	3.0	.0625	3
20	.0642	7.5	12.0	4.0	.0417	14
21	.0525	8.0	15.0	4.0	.0417	7
22	.0608	8.0	15.0	5.0	.0417	9
23	.0433	7.0	15.6	5.0	.1250	13
24	.0483	7.5	3.6	1.0	.1250	10
25	.0450	6.0	4.8	1.5	.0833	13
26	.0558	5.0	1.8	2.0	.0625	10
27	.0608	3.0	10.2	3.0	.0417	8
28	.0692	4.0	4.2	3.0	.1250	7
29	.0733	2.0	3.0	3.5	.1250	6
30	.0633	2.0	7.2	4.0	.0417	9

Appendix 4.2 **Transaction sizes and their probabilities**

Item	Transaction sizes														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	.167	.183	.150	.200	.133	.067	.050	.050							
2	.150	.350	.250	.150	.100										
3	.200	.200	.200	.100	.100	.100	.100								
4	.150	.250	.200	.150	.100	.100	.050								
5	.100	.200	.200	.150	.100	.100	.100	.050							
6	.100	.200	.200	.150	.150	.100	.100								
7	.200	.300	.250	.150	.050										
8	.075	.075	.100	.100	.150	.100	.075	.075	.075	.050	.050	.050	.025		
9	.050	.050	.100	.150	.100	.100	.075	.075	.050	.050	.050	.050	.050	.025	.025
10	.200	.300	.300	.200											
11	.050	.050	.100	.100	.150	.150	.150	.100	.100	.050					
12	.100	.100	.150	.150	.100	.100	.075	.075	.050	.050	.050				
13	.100	.100	.150	.150	.100	.100	.075	.075	.075	.075					
14	.100	.150	.150	.150	.075	.075	.075	.075	.050	.050	.050				
15	.100	.150	.200	.150	.100	.050	.050	.050	.050	.050	.050				
16	.100	.100	.200	.150	.100	.100	.075	.075	.050	.050					
17	.200	.300	.300	.100	.100										
18	.150	.150	.200	.250	.100	.050	.050	.050							
19	.300	.400	.300												
20	.050	.050	.100	.150	.150	.100	.075	.075	.050	.050	.050	.050	.025	.025	

Item	Transaction sizes														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
21	.100	.150	.250	.200	.100	.100	.100								
22	.100	.200	.200	.150	.100	.100	.075	.075	.025						
23	.075	.075	.100	.150	.150	.075	.075	.050	.050	.050	.050	.050	.050		
24	.100	.120	.140	.120	.120	.100	.100	.100	.050	.050					
25	.085	.085	.080	.100	.150	.150	.075	.075	.050	.050	.050	.025	.025		
26	.100	.150	.150	.150	.100	.100	.100	.050	.050	.050					
27	.150	.200	.250	.150	.100	.075	.050	.025							
28	.200	.300	.200	.100	.100	.050	.050								
29	.100	.150	.250	.200	.200	.100									
30	.150	.150	.200	.150	.150	.100	.050	.050							

Appendix 4.3.1 Simulation 1

Item	s	c	S	Simulated cost Holding	Simulated cost Ordering	Calculated cost Holding	Calculated cost Ordering
1	13	18	45	83.15	14.00	82.20	17.15
2	12	15	30	119.07	27.80	110.39	28.28
3	18	21	33	208.42	47.60	186.85	47.80
4	25	28	41	205.94	32.50	196.94	38.98
5	15	19	35	177.35	42.50	164.60	47.08
6	15	19	40	94.19	25.40	87.80	27.32
7	14	19	47	35.04	5.00	33.41	5.14
8	23	35	68	107.34	20.40	99.60	22.27
9	27	32	47	192.91	42.85	187.78	42.98
10	8	10	16	147.56	33.60	135.49	35.39
11	31	36	53	324.34	83.20	302.67	92.36
12	27	31	44	376.11	88.50	341.39	112.75
13	30	35	51	200.23	36.00	189.68	37.50
14	28	33	51	133.54	24.35	130.83	23.32
15	18	22	44	82.47	23.15	80.46	20.37
16	20	24	42	114.19	33.80	109.73	34.37
17	11	13	24	129.40	28.00	119.16	27.19
18	14	17	27	190.09	33.90	172.80	37.50
19	7	9	16	115.00	23.00	107.56	22.37
20	22	26	35	351.02	59.20	339.02	82.00
21	14	16	24	268.11	72.40	249.23	31.47
22	13	15	24	270.17	74.50	243.56	85.48
23	47	53	62	594.48	70.50	609.51	83.37
24	32	39	61	142.06	14.80	132.53	14.99
25	32	38	59	182.31	32.45	178.17	29.33
26	17	23	57	62.61	13.60	61.74	12.14
27	11	13	23	174.61	39.50	155.83	48.10
28	14	17	32	86.19	19.90	78.50	25.40
29	17	21	40	74.49	15.35	72.38	14.99
30	12	15	28	145.23	37.60	133.03	41.82
Sub-total				5382.61	1120.35	5062.83	1239.60
Total cost					6502.51		6302.43

Appendix 4.3.2 Simulation 2

Item	s	c	S	Simulated cost Holding	Simulated cost Ordering	Calculated cost Holding	Calculated cost Ordering
1	8	14	33	59.31	18.00	55.70	16.74
2	8	11	25	95.50	21.30	84.64	20.92
3	14	19	30	184.55	31.60	164.81	37.00
4	15	20	38	162.72	18.50	154.98	22.31
5	10	15	32	158.67	38.50	138.20	45.45
6	11	16	35	82.69	16.60	74.63	17.61
7	8	11	32	20.79	11.25	19.50	10.91
8	21	29	57	87.33	22.75	72.78	25.88
9	21	29	43	187.56	26.00	169.34	22.70
10	5	8	15	134.15	15.00	118.69	18.35
11	26	33	51	303.51	54.40	282.89	61.57
12	23	27	44	340.36	61.50	322.44	70.00
13	24	30	47	164.38	35.80	161.59	36.77
14	21	30	44	110.74	21.70	109.58	18.60
15	14	20	37	73.46	23.30	66.09	23.53
16	16	22	39	114.37	17.80	99.54	23.77
17	7	10	21	104.99	23.00	94.76	25.00
18	11	14	24	164.30	36.40	143.04	41.76
19	4	7	14	97.98	10.20	89.09	9.42
20	17	22	36	326.15	49.20	288.31	53.75
21	10	14	24	263.52	40.00	230.99	55.13
22	10	14	24	260.91	60.50	233.84	56.76
23	40	47	56	487.06	68.50	515.26	75.36
24	24	32	49	101.69	25.40	96.72	25.43
25	25	33	51	164.17	24.60	146.85	26.99
26	11	18	41	45.14	23.80	41.65	25.50
27	8	12	21	153.23	32.60	140.89	32.01
28	12	16	29	73.71	19.20	70.82	18.33
29	12	16	33	59.30	18.65	53.90	19.90
30	9	12	26	129.85	32.80	115.20	36.88
Sub-total				4712.35	899.25	4365.44	985.35
Total cost					5611.60		5350.79

Appendix 4.3.3 Simulation 3

				Simulated cost		Calculated cost	
Item	s	c	S	Holding	Ordering	Holding	Ordering
1	9	15	26	51.06	25.20	49.70	24.67
2	9	12	23	89.12	29.40	81.17	29.66
3	14	18	25	148.63	47.60	139.03	48.85
4	12	20	27	121.79	29.00	117.69	27.36
5	10	15	25	128.77	42.50	115.91	47.84
6	10	15	24	56.25	35.60	52.83	31.93
7	9	12	23	16.00	19.75	14.67	19.36
8	20	30	48	77.39	18.70	70.74	20.94
9	23	29	41	160.67	56.65	158.70	51.42
10	4	8	11	103.18	42.20	93.73	33.59
11	24	32	43	256.74	65.20	244.13	69.38
12	22	27	37	302.15	75.00	280.33	86.28
13	26	31	42	155.22	53.25	147.96	61.40
14	23	29	37	84.31	35.50	93.12	30.16
15	13	20	28	61.66	18.60	54.14	21.66
16	16	21	33	92.60	31.60	85.88	28.55
17	8	11	17	91.61	29.80	84.17	29.95
18	10	15	20	146.33	42.70	128.41	46.88
19	2	7	9	66.80	32.10	63.02	23.69
20	17	23	30	285.65	66.80	256.73	81.05
21	10	14	20	233.80	56.40	204.40	63.17
22	9	15	19	234.51	55.50	207.16	66.93
23	40	44	56	503.64	68.50	497.52	80.40
24	25	35	40	74.12	41.10	84.75	36.28
25	25	35	45	137.18	42.85	135.39	41.44
26	9	19	31	38.79	16.00	43.91	18.72
27	8	13	16	131.58	48.00	120.79	47.15
28	12	15	23	64.03	20.40	56.69	25.56
29	14	17	28	57.43	18.40	48.56	24.90
30	8	13	21	114.49	31.60	101.55	40.42
Sub-total				4086.23	1195.90	3823.76	1259.59
Total cost					5282.13		5083.74

Appendix 4.3.4 Simulation 4

Item	s	c	S	Simulated cost Holding	Simulated cost ordering	Calculated cost Holding	Calculated cost Ordering
1	9	15	28	54.14	27.20	52.23	26.81
2	9	12	21	82.23	33.70	74.91	37.41
3	12	18	25	140.35	40.80	139.78	37.92
4	12	20	24	95.81	47.50	104.02	42.01
5	10	16	27	133.20	51.00	125.63	46.92
6	10	16	27	69.60	15.60	60.80	18.66
7	8	12	26	18.12	16.00	16.19	18.49
8	21	30	48	73.58	31.65	69.26	33.90
9	20	27	37	152.51	47.80	142.23	48.47
10	4	7	10	92.37	50.90	80.68	51.76
11	24	32	42	257.39	66.40	239.15	75.54
12	21	29	37	314.79	79.50	285.84	96.36
13	23	30	41	143.50	44.00	142.66	46.03
14	24	29	40	103.01	38.80	98.38	40.35
15	13	21	31	64.72	20.00	59.75	20.00
16	14	21	29	85.20	21.40	78.87	20.67
17	7	11	26	130.98	7.20	120.68	7.57
18	10	16	21	155.26	30.80	139.84	34.27
19	3	7	10	76.41	21.20	68.38	18.52
20	17	23	31	306.12	52.80	265.33	72.56
21	9	15	20	230.73	53.20	212.26	52.88
22	9	14	19	223.88	66.00	198.21	68.24
23	37	46	50	430.76	67.00	460.32	76.92
24	28	33	40	72.96	54.10	82.43	48.15
25	24	34	44	140.87	44.10	129.71	47.83
26	14	22	39	48.57	18.80	44.55	18.69
27	6	13	17	137.18	33.30	126.43	33.78
28	13	15	23	62.25	30.10	56.58	36.26
29	12	19	31	59.09	14.50	56.18	14.84
30	8	14	21	116.19	34.00	104.95	38.13
Sub-total				4071.79	1159.95	3835.25	1229.96
Total cost					5231.74		5065.21

Appendix 4.3.5 Simulation 5

Item	s	c	S	Simulated cost Holding	Simulated cost Ordering	Calculated cost Holding	Calculated cost Ordering
1	7	14	27	52.02	19.40	49.69	20.43
2	9	11	19	75.75	46.50	65.86	57.62
3	13	18	26	144.51	48.00	142.99	40.54
4	14	19	29	127.80	30.00	119.29	31.77
5	10	15	28	136.81	51.00	124.33	50.03
6	10	15	28	65.09	29.60	59.81	27.47
7	8	12	30	20.17	6.75	19.12	6.80
8	21	30	50	80.89	20.00	72.93	22.25
9	22	29	38	159.15	59.80	149.68	57.42
10	4	7	11	99.92	34.80	97.84	36.68
11	25	32	45	280.18	60.80	253.47	68.01
12	22	27	38	298.44	91.50	282.25	97.04
13	24	29	41	139.73	48.20	140.65	46.29
14	20	28	37	94.44	35.70	89.10	35.40
15	14	20	34	69.54	22.05	62.15	23.34
16	16	21	33	94.22	34.40	85.04	35.20
17	7	10	17	85.42	39.80	78.85	37.44
18	10	14	21	143.30	43.80	128.96	43.44
19	1	6	9	65.26	23.00	58.13	18.18
20	17	24	31	303.73	66.80	266.70	77.05
21	10	14	21	232.24	60.00	210.12	60.58
22	9	14	20	230.71	61.50	206.36	61.10
23	36	49	49	379.91	61.50	474.69	66.26
24	24	34	44	98.62	17.20	92.98	18.35
25	25	33	44	138.72	44.55	128.70	46.63
26	13	21	41	47.56	15.00	45.96	11.94
27	7	12	18	136.96	29.40	126.82	28.79
28	11	16	25	60.94	16.80	63.29	15.45
29	12	17	29	49.00	20.60	50.26	15.94
30	8	13	22	119.99	24.00	105.96	30.81
Sub-total				4031.02	1162.45	3842.00	1188.32
Total cost					5193.47		5030.32

Appendix 4.3.6 Simulation 6

Item	s	c	S	Simulated cost Holding	Ordering	Calculated cost Holding	Ordering
1	7	14	27	52.06	26.40	48.85	27.98
2	9	11	19	77.18	42.00	66.26	50.76
3	13	18	26	145.64	52.80	141.98	47.57
4	13	18	29	120.23	30.50	115.11	33.82
5	10	15	28	138.89	55.50	123.39	55.17
6	10	15	28	65.23	29.60	59.81	27.10
7	8	12	30	20.53	8.25	18.99	8.50
8	21	30	50	80.33	22.95	72.40	26.18
9	20	27	38	160.76	49.15	145.47	48.13
10	4	7	11	100.15	32.00	88.45	33.17
11	24	31	45	272.72	61.60	247.24	72.50
12	22	27	38	299.70	90.00	282.62	97.91
13	24	29	41	139.93	51.10	140.11	50.05
14	20	28	37	94.09	34.00	89.50	32.77
15	14	20	34	69.54	12.95	63.43	13.78
16	15	20	33	89.81	25.20	84.04	25.38
17	7	10	17	84.95	39.80	78.85	37.44
18	10	14	21	141.96	43.80	128.96	40.39
19	1	6	9	65.73	23.00	58.13	18.18
20	15	22	31	294.21	62.00	253.56	74.33
21	10	14	21	229.18	65.60	208.24	70.02
22	9	14	20	230.65	64.50	205.75	61.42
23	31	44	49	379.62	55.00	438.75	57.27
24	22	32	44	96.12	16.70	89.51	16.39
25	25	33	44	139.39	39.15	129.52	41.33
26	11	19	41	45.90	19.60	43.29	18.46
27	7	12	18	136.41	29.40	126.82	28.79
28	10	15	25	60.81	16.20	61.15	15.61
29	12	17	29	50.27	22.25	50.07	18.27
30	8	13	22	120.70	22.40	106.41	28.67
Sub-total				4002.68	1143.40	3766.64	1177.35
Total cost					5146.80		4943.99

Appendix 4.3.7 Simulation 7

				Simulated cost		Calculated cost	
Item	s	c	S	Holding	Ordering	Holding	Ordering
1	8	14	26	50.16	28.60	47.50	30.90
2	9	12	20	77.32	39.90	72.23	37.38
3	13	19	26	150.45	38.40	149.62	35.29
4	11	19	25	103.26	33.50	105.97	32.80
5	10	15	27	140.62	44.50	120.79	56.18
6	11	16	27	68.58	19.60	60.61	23.67
7	8	12	29	18,07	11.25	18.43	8.70
8	20	30	47	71.28	30.45	68.53	29.36
9	20	29	39	161.51	40.05	154.83	40.05
10	3	7	11	101.36	23.10	89.13	22.97
11	23	31	41	241.85	82.80	226.28	90.64
12	21	28	36	295.85	86.00	274.51	101.84
13	24	31	42	148.30	47.65	148.58	48.45
14	24	29	40	106.02	31.85	99.55	33.39
15	13	21	30	63.18	20.30	58.19	23.02
16	14	21	30	89.98	22.00	79.75	25.50
17	7	11	17	92.64	30.60	83.68	28.33
18	10	15	31	155.60	34.90	133.35	44.18
19	2	7	9	66.68	31.10	62.33	26.84
20	17	23	30	282.37	83.30	254.50	87.88
21	9	14	19	216.47	68.80	194.67	71.80
22	9	15	17	202.14	86.50	191.40	80.58
23	39	45	50	377.88	73.00	455.61	76.09
24	25	33	40	65.86	43.00	82.03	34.78
25	24	35	44	140.53	35.30	133.53	38.22
26	13	19	30	36.48	34.40	33.55	34.90
27	7	13	17	135.79	43.40	125.74	37.82
28	12	15	22	58.88	26.50	54.65	25.69
29	14	18	32	57.25	23.35	55.18	24.36
30	8	13	20	110.56	36.80	97.55	45.78
Sub-total				3887.91	1250.80	3732.27	1296.38
Total cost					5138.71		5028.66

Appendix 4.3.8 Simulation 8

				Simulated cost		Calculated cost	
Item	s	c	S	Holding	Ordering	Holding	Ordering
1	8	15	27	53.61	22.60	51.29	20.72
2	8	11	21	80.17	34.00	72.09	32.75
3	13	18	25	152.99	48.00	137.18	57.05
4	11	19	25	112.55	33.50	106.45	34.00
5	10	15	25	130.16	49.00	114.79	58.62
6	11	16	26	64.37	31.20	58.30	29.03
7	10	13	30	21.16	10.75	19.60	10.63
8	18	29	45	73.25	26.00	64.76	30.57
9	21	30	39	171.38	40.25	157.11	46.83
10	4	8	11	106.94	30.70	94.71	29.15
11	22	30	42	236.11	72.00	228.65	73.01
12	21	27	36	290.76	88.50	270.70	97.01
13	25	32	44	172.97	34.55	159.58	40.37
14	23	29	37	92.32	42.00	91.81	43.25
15	13	21	29	64.90	18.25	56.40	24.08
16	15	21	32	94.06	25.60	83.95	25.89
17	7	10	17	91.39	34.00	79.55	35.05
18	10	14	20	141.44	47.20	123.29	52.57
19	2	7	9	66.80	27.80	63.59	21.19
20	17	23	30	285.98	62.80	260.45	68.74
21	9	14	20	225.90	55.20	204.65	57.29
22	8	11	11	115.99	169.50	138.96	181.10
23	40	43	49	355.04	90.50	436.02	97.53
24	26	35	41	93.00	19.80	89.72	21.17
25	25	34	45	145.29	38.25	133.48	40.59
26	11	20	33	41.28	10.00	38.43	9.70
27	7	12	17	131.63	41.20	120.67	40.83
28	12	16	24	67.02	17.40	61.38	16.67
29	14	17	29	53.81	29.35	49.39	31.52
30	8	13	21	117.00	33.60	101.24	38.46
Sub-total				3849.28	1284.60	3668.18	1365.39
Total cost					5133.88		5033.57

Appendix 4.4 Simulation results for independent ordering policy and joint ordering policy

Item	s	S	Independent ordering Holding	Independent ordering Ordering	Joint ordering Holding	Joint ordering Ordering
1	8	27	43.80	80.00	41.70	86.33
2	8	21	63.88	80.00	64.89	84.74
3	13	25	134.19	82.00	120.02	102.00
4	11	25	82.68	66.00	78.66	83.24
5	10	25	109.74	88.00	100.27	119.03
6	11	26	51.02	84.00	51.03	80.11
7	10	30	18.41	38.00	18.04	40.63
8	18	45	59.87	62.00	53.28	81.97
9	21	39	151.06	70.00	135.02	91.87
10	4	11	83.05	74.00	72.33	90.24
11	22	42	206.83	130.00	210.75	141.54
12	21	36	264.38	128.00	246.84	158.59
13	25	44	151.82	66.00	140.86	93.69
14	23	37	80.68	86.00	83.13	87.79
15	13	29	54.92	54.00	46.40	68.01
16	15	32	75.97	68.00	73.54	75.35
17	7	17	75.73	78.00	70.55	86.67
18	10	20	116.70	84.00	109.44	101.91
19	2	9	42.55	68.00	38.09	72.19
20	17	30	234.84	104.00	232.90	123.95
21	9	20	177.00	102.00	172,86	122.45
22	8	11	95.34	158.00	101.86	100.46
23	40	49	333.65	122.00	423.99	128.57
24	26	41	80.47	88.00	75.04	101.24
25	25	45	117.30	100.00	116.51	106.49
26	11	33	30.88	64.00	30.33	69.75
27	7	17	106.09	88.00	98.61	91.87
28	12	24	64.82	58.00	53.60	64.26
29	14	29	48.88	54.00	46.34	63.23
30	8	21	96.33	68.00	86.61	88.86
Sub-total			3242.89	2492.00	4057.45	1800.55
Total cost				5734.89		5858.00