Most general circulation models of earth's atmosphere use the sigma coordinate system which simplifies the modeling of flows where the orography is important, while observations are usually presented on constant pressure surfaces.

A method for dealing with the conversion of sigma-coordinate data to pressure coordinates is suggested for the comparison of the time averaged statistics of the circulation simulated by the OSU TWO-LEVEL...
AGCM with observations. Using the proposed method the mass balance and the total energy budget of the model for January and July are discussed and compared with the corresponding results from observations and from the original model data in sigma-coordinates.

The mass circulation in the vicinity of significant orographic features is more easily interpreted in the new diagnostic coordinate system. Near the Tibetan Plateau, most of the flow goes around the mountain with a stronger flow to the north of the mountain. A substantial fraction of the mass coming into the plateau region goes up to the upper layer on the upwind side and is compensated by the downward motion from the upper layer on the downwind side. Also, equatorial wave structures resembling mixed-Rossby gravity waves and Kelvin waves can be clearly seen in the Indian Ocean area in the new coordinate system.

The secondary circulations in the vicinity of the major jet streams are closely examined. Unlike the Namias and Clapp's hypothesis, the thermally indirect circulation does not appear in the jet exit region. In this region, the rotational component of ageostrophic flow is more important than the divergent component in decelerating the jet stream.

The new coordinate system is found to be useful in interpreting the total energy budget of the model atmosphere. The residuals of the budget which are due to
the numerical sink and the use of the six-hourly data instead of the ten-minute time step of the model integration are found to be comparable to those of the budgets in $\Theta$ coordinates.
The Energy Budget and Mass Balance of the OSU Atmospheric General Circulation Model Viewed in Terms of Constant Pressure Layers

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The Energy Budget and Mass Balance of the OSU Atmospheric General Circulation Model Viewed in Terms of Constant Pressure Layers

1. Introduction

During the past three decades, the development of numerical models of the global atmosphere has progressed significantly from the pioneering calculation of Phillips (1956). Calculations of the general circulation by Smagorinsky et al. (1965), Manabe et al. (1965, 1970), Mintz (1965) and others have reached the point where the time and space-averaged distribution of many climate elements have been successfully simulated.

As the general circulation models (GCMs) of the earth's atmosphere continue to improve, it is very important to interpret the numerical results in terms of the various dynamical and physical mechanisms responsible for the simulated circulation. Comparison of the GCM results with our limited observations of the general circulation can lead to new conceptual models and improvements in our ability to simulate the general circulation.
Most GCM's use a coordinate system in which the vertical coordinate is pressure divided by surface pressure as in Phillips' introduction of $\sigma (= P/P_s)$. This sigma-coordinate system simplifies the modeling of flows where the orography is important because the lower boundary is always a coordinate surface. Thus, it is relatively easy to guarantee conservation of important integral properties such as mass and energy.

However, the use of sigma coordinates causes interpretational difficulties. We would like to compare the time-averaged statistics of the circulation simulated by the model with the observed statistics of the corresponding atmospheric parameters. However, observations are usually presented on constant pressure surfaces. The time averages as defined on a sigma level are quite different from those defined on an isobaric level in regions where orography is prominent.

Also, as noted by Smagorinsky et al. (1965) and Kurihara (1968), the use of the $\sigma$-coordinate system may create some computational difficulties over steeply sloping terrain. Reducing the vertical increment of the grid mesh in the sigma-coordinate system does not reduce the slope of the mountain. It may generate fictitious influences of mountains all the way to the top of the atmosphere even in a multilayer model. However, it is not our purpose to discuss these errors of discretization.
The purpose of this thesis is to suggest methods for dealing with the conversion of $\sigma$-coordinate model data to pressure coordinates and to use these methods to examine the mass balance and the total energy budget of the OSU TWO-LEVEL ATMOSPHERIC GENERAL CIRCULATION MODEL (OSU TWO-LEVEL AGCM). Here the total energy means the sum of the enthalpy, latent heat, potential energy, and kinetic energy. Evaluation of sources, sinks, transports, transformations, and the storage of energy in its various forms provides valuable information on the dynamics of the simulated circulation. For example, the evaluation of these various terms makes it possible for us to explore quantitatively how the surplus energy of tropics is transported to the polar region, even though there is a significant convergence of the sensible and latent heat in tropics in the lower branch of the Hadley circulation.

In Chapter 2, it is suggested that the OSU-AGCM be divided into two layers for the purposes of dynamical interpretation and comparison with observation. The upper layer is bounded by the 200 mb and 600 mb surfaces and variables in this layer may be regarded as representing data on the 400 mb surface. The lower layer is bounded by the 600 mb surface and the earth's surface. Variables in this layer may be regarded as representing
data on the 800 mb surface over the ocean and at a variable height over land. Thus, the proposed vertical structure for the model diagnosis is similar to the structure proposed by Kasahara and Washington (1971) for prognostic purposes. In Section 2.4, the proposed finite difference form of the total energy and hydrostatic relation are compared to the continuous form of those equations in sigma coordinates. A method for removing inconsistency errors in the horizontal mass divergence (convergence) and the horizontal divergence (convergence) of mass-weighted quantities is introduced in Section 2.5. The vertical interpolation schemes which are used to produce the variables in the new coordinate system from the data of OSU TWO-LEVEL AGCM are provided in Chapter 3.

In Chapter 4 the mass balance is examined on the basis of three Januaries and Julys simulated by the OSU TWO-LEVEL AGCM. The reconstructed three-dimensional mass circulation is examined and compared with previous observational and numerical studies (Namias and Clapp, 1949; Blackmon et al., 1977; Lau, 1978; Kim and Grady, 1982). The mean meridional circulation is characterized by the steady response of a stratified atmosphere to an excess heating at low latitudes and a deficit at high latitudes while the mean zonal circulation is characterized by the strong westerly flow at mid-latitudes and easterlies in the tropics. The Walker circulation which
was used by Bjerknes (1969) to refer to an overturning of the troposphere in the quadrant of the equatorial plane spanning the Pacific ocean appears to be related to the sea surface temperature (SST) and the convergence of moisture in agreement with the studies of Cornejo-Garrido and Stone (1977) and Geisler (1981).

For better understanding of the dynamics of these circulations, the field of the mass flux in each layer is decomposed into four components: the geostrophic divergent (irrotational) component, the geostrophic rotational (non-divergent) component, the ageostrophic divergent component, and ageostrophic rotational component. This specific decomposition of mass flux is intended to provide a clue on the as yet unclear three-dimensional global structure of the ageostrophic motions in the atmosphere.

We examine the three-dimensional characteristics of the total energy budget of the OSU-AGCM in Chapter 5. It is well known that the excess incoming solar radiation at low latitudes compensates the deficit at high latitudes by the transporting the excess energy in its various forms to high latitudes. To understand this process, the total energy budget of the reduced data is studied on the basis of a three-year simulation with seasonally-varying incoming solar radiation and sea surface temperature. Included in the analysis are
the monthly balances of the shortwave and longwave fluxes of radiation, the surface sensible and latent heat fluxes, the net cumulus and large-scale condensational heating, the dissipation of energy by friction, the divergence (convergence) of each form of the total energy, and the energy conversions.

Chapter 6 presents the summaries of simulated results and some suggestions for further research.
2. Basic governing equations

2.1 Vertical coordinate

The isobaric pressure coordinate system has computational disadvantages in the vicinity of the mountains because the lower boundary of the atmosphere is not a coordinate surface. In particular, when coordinate surfaces intersect the earth's surface, arbitrary methods must be used to assign the "underground" portions at the reference pressure level. To circumvent this difficulty for prognostic applications, Kasahara and Washington (1971) developed a special vertical structure for the NCAR global circulation model. They defined the variables at half-integer levels and considered those variables to represent vertical averaged values of the variables within the layer bounded by the two nearest whole-integer levels. When the pressure of reference level was larger than the surface pressure, the half-integer level terminated at the surface of the mountain.

The OSU TWO-LEVEL ATMOSPHERIC GENERAL CIRCULATION MODEL (OSU TWO-LEVEL AGCM) uses the sigma coordinate in the vertical and has only two layers. Since it is difficult to interpret model results directly in the
sigma-coordinate system, it is desirable to transform
the output into a quasi-horizontal system such as the
isobaric coordinate system.

Here we propose a new coordinate system, suitable
for diagnostic studies of the OSU TWO-LEVEL AGCM. We
let $p_T$ be the pressure at the top of the model atmosphere
(200 mb), taken as a constant and $p_s$ be the pressure at
the earth's surface which varies with the horizontal co-
dinates and time. Defining $p_m$ as the pressure at the
middle level (approximately 600 mb), a new vertical co-
dinate $\sigma^*$ is then defined by

$$
\sigma^* = \frac{p - p_m}{\Pi_4}
$$

where

$$
\Pi_4 = \begin{cases} 
\Pi_u = p_{m} - p_{T} & \text{for } p_{T} \leq p < p_{m} \\
\Pi_l = p_{s} - p_{m} & \text{for } p_{m} < p \leq p_{s} 
\end{cases}
$$

Note that $\Pi_u$ is constant, except for the region where
the mountain penetrates the middle level, whereas $\Pi_l$ is
a function of the horizontal coordinates and time. It
follows from Eqs. (2.1) and (2.2) that

$$
\sigma^* = \begin{cases} 
-1 & \text{for } p = p_{T} \\
0 & \text{for } p = p_{m} \\
1 & \text{for } p = p_{s}
\end{cases}
$$

Fig. 2.1. shows surfaces of constant $\sigma^*$ in a verti-
cal cross section. The lower boundary, which follows
the earth's topography, is a coordinate surface and the
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Fig. 2.1. Definition of the layers in terms of vertical $\sigma^*$-coordinate.
Fig. 2.2. Monthly mean surface pressure for January. Unit is mb.
Fig. 2.3. As in Fig. 2.2. except for July.
As shown in the Fig. 2.2 and 2.3, there are a few grid points during January where the surface pressure is less than 600 mb level. The middle level pressure ($P_m$) of that area is the surface pressure. The mass-weighted variables in lower layer are therefore equal to zero.

2.2 Basic equations of the pressure coordinate system

Under the traditional approximation (Eckart, 1960; Phillips, 1966), the hydrostatic primitive equations for the atmosphere on the sphere in pressure coordinates are:

\[
\frac{\partial u}{\partial t} = -\nabla \cdot \mathbf{u} - \frac{\partial u}{\partial p} + (2 \omega + \frac{u}{a \cos \phi}) \nabla \cdot \mathbf{p} + \mathbf{F}_u
\]

\[
\frac{\partial v}{\partial t} = -\nabla \cdot \mathbf{v} - \frac{\partial v}{\partial p} + (2 \omega + \frac{u}{a \cos \phi}) \nabla \cdot \mathbf{p} + \mathbf{F}_v
\]

\[
\frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{u} - \frac{\partial T}{\partial p} - \frac{\omega}{a} \omega \frac{\partial T}{\partial p} + \omega \alpha + \mathbf{H}
\]

\[
\frac{\partial \mathbf{u}}{\partial t} = \nabla \cdot \mathbf{u} + \frac{\omega}{a}
\]
and the moisture balance equation is given by

\[ \frac{\partial \theta}{\partial t} = -\nabla \cdot \mathbf{q} - \omega \frac{\partial \theta}{\partial p} + \dot{Q} \]

where \( u \) and \( v \) are the zonal and meridional components of horizontal wind vector \( \mathbf{V} \), \( \nabla \) is the horizontal gradient operator, \( \mathbf{\Phi} \) is the specific geopotential, \( p \) is the pressure, \( \omega \) is the angular velocity of earth, \( a \) is the radius of earth, \( \omega \) is the individual time derivative of pressure, \( \dot{H} \) is the diabatic specific heating rate, \( \dot{Q} \) is the condensation rate, \( \varphi \) is the heat capacity at constant pressure, \( q \) is the water vapor mixing ratio, and \( \mathbf{F}_u \) and \( \mathbf{F}_v \) are the zonal and meridional frictional term.

Multiplying (2.4) by \( u \) and (2.5) by \( v \), and adding these equations, we obtain the kinetic energy equation,

\[ \frac{\partial K}{\partial t} = -\nabla \cdot \mathbf{q} - \omega \frac{\partial K}{\partial p} - \nabla \cdot \mathbf{F}_u + \nabla \cdot \mathbf{F}_v \]

where \( K \) is the kinetic energy \((1/2(u^2 + v^2))\), and \( \mathbf{F} \) is the frictional force vector. Combining (2.7) and (2.11), we obtain the flux form of conservation equation for the atmospheric kinetic energy,
\( (2.12) \quad \frac{\partial q}{\partial t} = - \nabla \cdot (\nabla p) - \frac{\partial}{\partial p} (\omega k) - \nabla \cdot (\nabla k) \\
\quad - \frac{\partial}{\partial p} (\omega k) - \omega \alpha + \nu \cdot \mathbf{E}.
\)

Similarly, we can obtain the flux form of the conservation equations for the atmospheric thermodynamic energy and water vapor,

\( (2.13) \quad \frac{\partial q}{\partial t} = - \nabla \cdot (\nabla p) - \frac{\partial}{\partial p} (\omega q + \omega H) \\
\quad + \dot{Q}, \)

and

\( (2.14) \quad \frac{\partial q}{\partial t} = - \nabla \cdot (\nabla q) - \frac{\partial}{\partial p} (\omega q) + \dot{Q}. \)

Multiplying (2.14) by the latent heat of evaporation \( (L) \), and adding the result to the sum of Eqs. (2.12) and (2.13), we obtain the flux form of the conservation equation for the total energy,

\( (2.15) \quad \frac{\partial}{\partial t} (K + q + L g) = - \nabla \cdot [(K + q + L g + \omega) \nabla] \\
\quad - \frac{\partial}{\partial p} [(K + q + L g + \omega) \nabla] \\
\quad + \nu \cdot \mathbf{E} + q + \dot{Q} + L \dot{Q}. \)

Next, we vertically integrate this equation to obtain

\( (2.16) \quad \frac{1}{\tau} \int_0^P \frac{\partial}{\partial t} (K + q + L g) \, dp \\
\quad = - \frac{1}{\tau} \int_0^P \nabla \cdot [(K + q + L g + \omega) \nabla] \, dp \\
\quad - \frac{1}{\tau} \int_0^P [(K + q + L g + \omega) \nabla] \, dp \\
\quad + \frac{1}{\tau} \int_0^P \nabla \cdot \mathbf{E} \, dp + \frac{1}{\tau} \int_0^P \nabla \cdot \mathbf{F} \, dp. \)
The terms \( \frac{1}{3} \int_{0}^{P_s} c_p H \, dp \), \( \frac{1}{3} \int_{0}^{P_s} L \dot{Q} \, dp \), and \( \frac{1}{3} \int_{0}^{P_s} (\nabla \cdot \mathbf{E}) \, dp \) represent, respectively, the vertically integrated heat added or lost by the atmosphere to space, the vertically integrated latent heat added or lost by the atmosphere to liquid water, and the vertically integrated frictional dissipation.

Next the differential operators \( \partial / \partial t \) and \( \nabla \cdot \) are moved outside of the integration with respect to \( p \), taking into account the variable upper limit \( p \) by the use of Leibniz rule for the differentiation of an integral quantity.

\[
(2.17) \quad \frac{1}{3} \left\{ \frac{\partial}{\partial t} \int_{0}^{P_s} (K + c_p T + L_g) \, dp - \frac{\partial P_s}{\partial t} (K + c_p T + L_g) \right\} \\
= -\frac{1}{3} \left\{ \nabla \cdot \int_{0}^{P_s} [(K + c_p T + L_g + \Theta) \, \omega] \, dp \\
- \nabla P_s \cdot [(K + c_p T + L_g + \Theta) \, \omega] \right\} + [(K + c_p T + L_g \\
+ \Theta) \, \omega] \left|_{s} \right. + \frac{1}{3} \int_{0}^{P_s} (c_p \dot{H} + L \dot{Q} + K \cdot \mathbf{E}) \, dp.
\]

The value of \( \omega \) at the surface is given by the kinematic boundary condition,

\[
(2.18) \quad \omega_s = \left( \frac{dP_s}{dt} \right)_s = \frac{\partial P_s}{\partial t} + \frac{V_s \cdot \nabla P_s}{s}.
\]

Substituting (2.18) into (2.17), we obtain
\[ (2.19) \quad \frac{\partial}{\partial \theta} \left\{ \frac{1}{\delta} \int_{0}^{\phi} \left( K + C_T + L + \frac{\partial P}{\partial \phi} \right) d\phi \right\} \]
\[ = -\frac{1}{\delta} \int_{0}^{\phi} \left[ \left( K + C_T + L + \frac{\partial P}{\partial \phi} \right) \right] d\phi \]
\[ + \frac{1}{\delta} \int_{0}^{\phi} \left( \varphi H + L \hat{\theta} + \nabla \cdot \mathbf{E} \right) d\phi \]

where \( \delta P \) is a two-dimensional quantity. The left side of (2.19) is the time rate of change of total energy while the first term of right side is the horizontal flux of total energy. The remaining terms represent the total sensible heat and latent heat added to the system by the surface flux of sensible heat and latent heat, the condensation, the pentrating convection and middle-level convection, and the frictional dissipation of kinetic energy.

2.3 Continuous equations in the \( \varphi^* \)-coordinate system

The OSU TWO-LEVEL AGCM, like most primitive equation models of the general circulation, uses the \( \varphi \) coordinate in the vertical. In this section, we will derive the analogues of the pressure coordinate equations obtained in Section 2.2 in the \( \varphi^* \)-coordinate system presented in Section 2.1. The development here follows Haltiner and Williams (1980).

The basic equations using the \( \varphi^* \) coordinate can be
written as follows:

\[(2.20) \frac{\partial u}{\partial t} = -v \cdot \nabla u - \dot{\phi} \frac{\partial u}{\partial \phi} + (2\omega + \frac{u}{a \cos \phi}) \sin \phi - \frac{1}{a \cos \phi} \left( \frac{\partial \tilde{\phi}}{\partial \phi} \right)_{\phi} - \frac{\sigma z}{a \cos \phi} \left( \frac{\partial \tilde{z}}{\partial \phi} \right)_{\phi} + \tilde{F}_\phi, \]

\[(2.21) \frac{\partial v}{\partial t} = -v \cdot \nabla v - \dot{\phi} \frac{\partial v}{\partial \phi} - (2\omega + \frac{u}{a \cos \phi}) u \sin \phi - \frac{1}{a} \left( \frac{\partial \tilde{\phi}}{\partial \phi} \right)_{\phi} - \frac{\sigma z}{a} \left( \frac{\partial \tilde{z}}{\partial \phi} \right)_{\phi} + \tilde{F}_\phi, \]

\[(2.22) \frac{\partial T}{\partial t} = -v \cdot \nabla T - \frac{\partial \tilde{\phi}}{\partial \phi} \frac{\partial T}{\partial \phi} + \omega \chi + \phi \dot{\phi}, \]

\[(2.23) \frac{\partial \chi}{\partial t} + \nabla \cdot \pi \chi + \pi \frac{\partial \tilde{\chi}}{\partial \phi} = 0, \]

\[(2.24) \pi \chi = RT, \]

\[(2.25) \frac{\partial \tilde{\phi}}{\partial \phi} = -\pi \chi, \]

\[(2.26) \frac{\partial \tilde{z}}{\partial \phi} = -v \cdot \nabla \tilde{z} - \dot{\phi} \frac{\partial \tilde{z}}{\partial \phi} + \dot{\phi}. \]

The change of pressure of air parcel with respect to time (\(\omega\)), is given by

\[\omega = \pi \dot{\phi} + \nu \left( \frac{\partial T}{\partial t} + v \cdot \nabla T \right). \]

Note that some of the terms in the equations involving
the derivatives of $\pi$ are zero in the upper layer where $\pi u$ is constant.

The following alternative forms of the hydrostatic equation can be derived from (2.25) and will be useful later in imposing constraints on the finite difference equations,

\begin{align}
(2.27a) \quad & \delta (\psi \pi^*) = (\psi - \psi^* \pi^*) \delta \pi^*, \\
(2.27b) \quad & \delta \psi = -RT \delta \ln P , \\
(2.27c) \quad & = -C_P \delta (P/P_0)^K , \\
(2.27d) \quad & = C_P \frac{d \ln \theta}{d (\psi^*)} \delta (P/P_0)^K , \\
(2.27e) \quad & \delta (C_P T + \psi) = (P/P_0)^K C_P \delta \theta
\end{align}

where $\theta = T (P_0/P)^K$ is the potential temperature, $K \in R/C_P$, and $\delta$ indicates a differential in the vertical.

The total derivative of an arbitrary scalar can be written

\begin{equation}
(2.28) \quad \frac{dA}{dt} = \frac{\partial A}{\partial t} + \nabla \cdot \nabla A + \hat{\psi} \frac{\partial A}{\partial \psi^*} \tag{2.28}
\end{equation}

which is the advective form of $dA/dt$. The flux form can be found by multiplying (2.28) by $\pi$ and (2.23) by $A$ and adding:

\begin{equation}
(2.29) \quad \pi \frac{dA}{dt} = \frac{\partial}{\partial t} (\pi A) + \nabla \cdot (\pi \nabla K) + \frac{2}{\partial \psi^*} (\pi \hat{\psi}^* A) .
\end{equation}
We now obtain the balance equation of total energy. First, adding the product of (2.20) and $\pi u$ to the product of (2.21) and $\pi v$ gives the kinetic energy equation as follows:

\[
(2.30) \pi \frac{\partial }{\partial t} \left( \frac{u^2}{2} + \frac{v^2}{2} \right) = - \pi \nu \cdot \nabla \left( \frac{u^2}{2} + \frac{v^2}{2} \right) - \pi \nu \cdot \frac{\partial }{\partial x} \left( \frac{u^2}{2} + \frac{v^2}{2} \right)
\]

\[
- \pi u \cdot \nabla \left( \frac{u^2}{2} \right) - \pi \nu \cdot \frac{\partial }{\partial y} \left( \frac{u^2}{2} \right) + \pi \nu \pi_y
\]

or

\[
(2.31) \pi \frac{\partial K}{\partial t} = - \pi \nu \cdot \nabla K - \pi \nu \cdot \frac{\partial K}{\partial t} - \pi \nu \cdot \left( \nabla \times \sigma_k \nabla \right) K + \pi \nu \cdot \bar{F}
\]

in vector form. Also (2.31) can be written in flux form by setting $A = K$ in (2.29) which allows the equation to be written as follows:

\[
(2.32) \frac{\partial }{\partial t} (\pi K) = - \nabla \cdot (\nu \nabla K) - \frac{\partial }{\partial t} \nabla \cdot \left( \pi \nabla \pi K \right) - \nu \cdot \left[ \pi \nabla \pi \right]
\]

\[
+ \sigma_k \nabla \times \pi + \pi \nu \cdot \bar{F}
\]

The rate of kinetic energy production by the pressure gradient force can be rewritten as follows with the use of (2.23), (2.25), and (2.27a):

\[
(2.33) \quad \nu \cdot \left[ \pi \nabla \pi + \sigma_k \nabla \times \pi \right] = - \nabla \cdot (\nabla \nu) + \nabla \times \pi - \nu \cdot \pi \nabla \times \pi.
\]
The thermodynamic energy equation (2.22) and the moisture balance equation (2.26) can be put in flux form using (2.29) which gives:

\[ 2.34 \frac{d}{dt} (\pi \phi T) = -\nabla \cdot (\pi \phi T \nabla T) - \frac{d}{d\sigma^*} (\pi \phi T \sigma^*) + \pi \omega \pi + \pi \rho \dot{H} \]

and

\[ 2.35 \frac{d}{dt} (\pi \phi) = -\nabla \cdot (\pi \phi \nabla \phi) - \frac{d}{d\sigma^*} (\pi \phi \sigma^*) + \pi \dot{\theta} \]

The equation for total energy is found by substituting (2.33) into (2.32), multiplying (2.35) by the latent heat of evaporation \(L\), and adding the results to (2.34). Integration of this equation from \(\sigma^*=1\) to \(\sigma^*=1\) yields:

\[ 2.36 \frac{d}{d\sigma^*} \left[ \frac{d}{dt} (\pi \phi T \sigma^*) + \frac{d}{d\sigma^*} (\pi \phi T \sigma^*) \right] \]

\[ = -\nabla \cdot \int_{-1}^{1} \pi \left( \nabla \phi T + L \phi \right) d\sigma^* \]

\[ + \int_{-1}^{1} \pi \left( \nabla \phi + \phi \dot{H} + L \dot{\theta} \right) d\sigma^* \]

with the use of the conditions, \(\dot{\sigma}^*=0, \frac{d\pi}{d\sigma^*} = \frac{d\rho}{d\sigma^*} \) at \(\sigma^*=1\), and \(\dot{\sigma}^*=0, \frac{d\pi}{d\sigma^*} = 0 \) at \(\sigma^*=-1\). Thus (2.36) corresponds
to (2.19), which was derived in isobaric pressure coordinates.

2.4 Vertical differencing of the total energy equation and hydrostatic relation

In this section, we obtain the explicit form of the vertical differencing to be used diagonostically in this study. The finite-differencing scheme used to solve the governing equations of the OSU AGCM was developed by Arakawa (see Arakawa, 1972, Arakawa and Lamb, 1977, and Ghan et al., 1982, for detailed description), who proposed the finite-difference schemes of the governing equations should satisfy important integral constraints of the continuous equations. In the OSU TWO-LEVEL AGCM, these constraints are the conservation of mass, kinetic energy, and, in the absence of sources and sinks, total energy and momentum. In addition, an integral property of vertically integrated pressure gradient force is maintained, and the global mean potential temperature and its square are conserved under adiabatic conditions.

For diagnostic purposes we divide the atmosphere into two layers as shown in Fig. 2.1. The upper and lower layers are identified with U and L respectively, and the level that divides the layers with M normally at
600 mb. Also, the top and bottom are identified with $T$ and $S$. The grid interval in $\sigma^*$ is given by $\Delta \sigma_u^* = \Delta \sigma_L^* = 1$.

The continuity equation (2.23) is finite differenced in the vertical as follows:

$$\frac{\partial \Pi}{\partial t} + \nabla \cdot (\Pi u \nu_u) + \frac{1}{\Delta \sigma^*} \left[ (\Pi \dot{\sigma}^*)_M - (\Pi \dot{\sigma}^*_T) \right] = 0,$$

and

$$\frac{\partial \Pi}{\partial t} + \nabla \cdot (\Pi u \nu_L) + \frac{1}{\Delta \sigma^*} \left[ (\Pi \dot{\sigma}^*)_M - (\Pi \dot{\sigma}^*)_L \right] = 0.$$

But $\dot{\sigma}^*_T = 0$, and $\dot{\sigma}^*_L = 0$ by the definition of vertical coordinate. Thus (2.37) and (2.38) become

(2.39) \( (\Pi \dot{\sigma}^*)_M = - \frac{\partial \Pi}{\partial t} - \nabla \cdot (\Pi u \nu_u) \),

(2.40) \( (\Pi \dot{\sigma}^*)_M = \frac{\partial \Pi}{\partial t} + \nabla \cdot (\Pi u \nu_L) \).

By the definition of $\omega$,

(2.41) \( \omega = \frac{d \sigma}{dt} = \frac{d}{dt} (\Pi \dot{\sigma}^*) = \dot{\Pi \dot{\sigma}^*} + \Pi \ddot{\sigma}^* \).

At the level of $\sigma^* = 0$, $\omega_M$ is therefore

(2.42) \( \omega_M = (\Pi \dot{\sigma}^*)_M \).

Substituting (2.42) into (2.39) and (2.40), we obtain
\[ (2.43) \quad \omega_u = -\frac{\partial u}{\partial x} - \nabla \cdot (\sigma_u V_u), \]

\[ (2.44a) \quad \frac{\partial}{\partial x} + \nabla \cdot (\sigma_u V_u), \]

and

\[ (2.44b) \quad \frac{\partial}{\partial x} (\pi_u + \pi_l) = -\nabla \cdot (\sigma_u V_u + \pi_l V_l). \]

For any variable A, which is carried at level U and L, the flux form of (2.29) can be written:

\[ (2.45) \quad (\pi \frac{dA}{dt})_u = \frac{\partial}{\partial x} (\pi_u A_u) + \nabla \cdot (\pi_u A_u V_u) + \frac{1}{\Delta \sigma_u^x} \left[ (\pi \hat{v}^x)_u \hat{A}_u - (\pi \hat{v}^x)_T \hat{A}_T \right], \]

\[ (2.46) \quad (\pi \frac{dA}{dt})_l = \frac{\partial}{\partial x} (\pi_l A_l) + \nabla \cdot (\pi_l A_l V_l) + \frac{1}{\Delta \sigma_l^x} \left[ (\pi \hat{v}^x)_l \hat{A}_l - (\pi \hat{v}^x)_M \hat{A}_M \right], \]

where \( \hat{A}_M \) is defined by an interpolation of A from the upper-level value \( A_u \) and the lower-level value \( A_l \).

(2.45) and (2.46) can be rewritten with the use of (2.37), (2.38), and the boundary conditions as follows:

\[ (2.47a) \quad (\pi \frac{dA}{dt})_u = \pi_u \left( \frac{\partial}{\partial x} + \nabla \cdot \hat{v}_u \right) A_u + \frac{1}{\Delta \sigma_u^x} \left[ (\pi \hat{v}^x)_M \right. \]

\[ \cdot (\hat{A}_M - A_u) + (\pi \hat{v}^x)_T (A_u - \hat{A}_T), \]

or
Since the choice of $\tilde{A}$ is arbitrary, as long as it is consistent, it is possible to satisfy an additional constraint. We now require that the global integral of $f(A)$ be conserved by advective effects. Let 

$$f_u \equiv f(A_u) \quad \text{and} \quad f'_u = \frac{df(A_u)}{dA_u},$$

$$f_L \equiv f(A_L) \quad \text{and} \quad f'_L = \frac{df(A_L)}{dA_L}.$$ 

Multiply (2.47a) by $f'$ and (2.48a) by $f'$ to obtain

$$\frac{d f}{d t} \Big|_u = \pi u \left( \frac{\partial}{\partial t} + u \cdot \nabla \right) f_u + \frac{1}{4 \sigma_c} \left[ \pi \nabla \cdot f_u (\tilde{A}_u - A_u) \right]$$

and

$$\frac{d f}{d t} \Big|_L = \left( \frac{\partial}{\partial t} + u_L \cdot \nabla \right) f_L + \frac{1}{4 \sigma_c} \left[ \pi \nabla \cdot f_L (A_L - \tilde{A}_L) \right].$$
\[ \pi_c \left( \frac{\partial \mu^*}{\partial x} + \mathbf{v} \cdot \nabla \right) f_L + \omega_n \left[ f_L \left( \mathbf{A}_c - \mathbf{A}_n \right) \right]. \]

With the use of (2.43), (2.44a), and the boundary conditions, (2.49) and (2.50) can be rewritten as follows:

\[ (2.51) \quad \left( \frac{\partial \mu}{\partial t} \right)_U = \frac{2}{\Delta t} (\pi_c f_L) + \mathbf{v} \cdot (\pi_c f_L \mathbf{v}_L) + \omega_n \left[ f_L \left( \mathbf{A}_n - \mathbf{A}_L \right) + f_L \right], \]

and

\[ (2.52) \quad \left( \frac{\partial \mu}{\partial t} \right)_L = \frac{2}{\Delta t} (\pi_c f_L) + \mathbf{v} \cdot (\pi_c f_L \mathbf{v}_L) - \omega_n \left[ f_L \left( \mathbf{A}_L - \mathbf{A}_n \right) + f_L \right]. \]

If \( \mathbf{A}_L = \mathbf{v} \) and \( f(A) = A \), the advective terms will conserve kinetic energy if

\[ (2.53) \quad \bar{\mathbf{v}}_n = \frac{1}{2} \left( \mathbf{v}_U + \mathbf{v}_L \right). \]

This method of determining the intermediate level constraint was first presented by Lorenz (1960). The advection terms of kinetic energy equation are then:

\[ (2.45a) \quad \left[ \pi \frac{\partial}{\partial t} \left( \frac{1}{2} \mathbf{v}^2 \right) \right]_U = \frac{2}{\Delta t} (\pi_c \frac{1}{2} \mathbf{v}^2) + \mathbf{v} \cdot (\pi_c \mathbf{v}_U \frac{1}{2} \mathbf{v}^2) + \omega_n \left[ \mathbf{v}_U \cdot (\mathbf{A}_n - \mathbf{A}_U) + \frac{1}{2} \mathbf{v}_U^2 \right], \]

\[ (2.54b) \quad = \frac{2}{\Delta t} (\pi_c \frac{1}{2} \mathbf{v}^2) + \mathbf{v} \cdot (\pi_c \mathbf{v}_U \frac{1}{2} \mathbf{v}^2) + \omega_n \left[ \mathbf{v}_U \cdot (\frac{1}{2} \mathbf{v}_U + \frac{1}{2} \mathbf{v}_L - \mathbf{v}_U) + \frac{1}{2} \mathbf{v}_U^2 \right]. \]
(2.54c) \[ \frac{\partial}{\partial x} (\frac{1}{2} u^2) + \frac{\partial}{\partial y} (\frac{1}{2} v^2) + \frac{1}{2} \omega \eta (\nu \cdot \nu) \]

(2.55a) \[ \frac{\partial}{\partial x} (\frac{1}{2} u^2) = \frac{\partial}{\partial x} (\frac{1}{2} u^2) + \frac{\partial}{\partial y} (\frac{1}{2} v^2) + \omega \eta (\nu \cdot \nu - \frac{1}{2} \nu \cdot \nu) \frac{1}{\nu} \]

(2.55b) \[ = \frac{\partial}{\partial x} (\frac{1}{2} u^2) + \frac{\partial}{\partial y} (\frac{1}{2} v^2) - \frac{1}{2} \omega \eta (\nu \cdot \nu) \]

The source and sink term of kinetic energy equation for the upper level are:

(2.56a) \[ S_u^k = -\nu (\nu \cdot \Delta u + \nu \nu \cdot \nu u \otimes \nu u) + \nu u \cdot \nu u \]

(2.56b) \[ = -\nu (\nu \nu \nu \nu) - \frac{1}{2} \frac{\partial}{\partial x} [\nu \nu \nu \nu - (\nu \nu \nu \nu)] + \frac{1}{2} \frac{\partial}{\partial x} \]

(2.56c) \[ - \nu (\nu \nu \nu \nu) - \frac{1}{2} \frac{\partial}{\partial x} \]

(2.56d) \[ = -\nu (\nu \nu \nu \nu) - \frac{1}{2} \frac{\partial}{\partial x} \]

Here $(\omega_k)_u$ is defined by

$$
(\omega_k)_u = (\nu^u)_u \left( \frac{\partial}{\partial x} + \nu \cdot \nabla \right) \pi_u - \frac{1}{\tau_{\omega u} \partial x} \left[ (\pi \nu^u)_u \right. \\
\left. \left( \tilde{\omega}_m - \tilde{\omega}_u \right) + (\pi \nu^u)_u \left( \tilde{\omega}_m - \tilde{\omega}_u \right) \right].
$$

The following expression was used:

$$
(\omega_k)_u = \frac{1}{\tau_{\omega u} \partial x} \left[ \tilde{\omega}_u - (\tilde{\omega}_m - \tilde{\omega}_u) \right]
$$

which can be obtained by (2.27a). Similarly we can obtain the source and sink term of kinetic energy equation for the lower level as follows:

$$
S_L^k = - \nu \cdot (\tau_L \delta \nu_L) - \frac{1}{\tau_{\omega l} \partial x} \left[ (\pi \nu^c)_l + \nu^c \cdot \nabla \right. \\
\left. \frac{\partial}{\partial x} \right] \tilde{\omega}_l - \left( \tilde{\omega}_m \right) - (\pi \nu^l)_l + \pi_L \nu \cdot \nabla
$$

where $S_U^k$ and $S_L^k$ are the source and sink of kinetic energy for the upper and lower level, and

$$
(\omega_k)_L = (\nu^l)_l \left( \frac{\partial}{\partial x} + \nu \cdot \nabla \right) \pi_L - \frac{1}{\tau_{\omega l} \partial x} \left[ (\pi \nu^l)_l \right. \\
\left. \left( \tilde{\omega}_m - \tilde{\omega}_L \right) + (\pi \nu^l)_l \left( \tilde{\omega}_m - \tilde{\omega}_L \right) \right].
$$
With the use of boundary conditions $\frac{\partial \phi}{\partial z} = 0$, $\frac{\partial \psi}{\partial z} = 0$, $\frac{\partial \omega}{\partial z} = 0$, $\Delta \phi = \Delta \psi = 1$, and (2.42), (2.54c), (2.55b), (2.56e), (2.57), (2.59), and (2.60), the finite difference form of the kinetic energy equation for the upper and lower level can be written as follows:

\begin{align*}
\frac{\partial}{\partial x} (\Pi L) + \frac{\partial}{\partial y} (\Pi L) + \frac{\partial}{\partial z} (\Pi L) &= -\frac{\partial}{\partial x} (\Pi U) + \frac{\partial}{\partial y} (\Pi V) + \frac{\partial}{\partial z} (\Pi W) - \omega m \Phi + \Pi L - \Pi V \cdot \nabla \right) \tag{2.61}
\end{align*}

where

\begin{align*}
(\omega m)_u &= -\frac{1}{\Pi L} \omega m (\Phi - \Phi_u), \\
(\omega m)_L &= (\Pi L) (\frac{\partial}{\partial y} + \nabla x) \Pi V - \frac{1}{\Pi L} \omega m (\Phi - \Phi_u) \\
\text{and}
(\Pi L (\nabla x))_L &= \frac{\nabla - \nabla^*}{\Delta \omega},
\end{align*}

Note that $\Phi_u$ has not yet been specified.

The thermodynamic energy equation will be developed in terms of the potential temperature ($\theta$) and then
rewritten in terms of temperature $T$. If there is no source and sink, the thermodynamic energy will be con-
served, so that

$$\frac{dE}{dt} = 0.$$ \hspace{1cm} (2.66)

With the use of (2.47a) and (2.48a), the finite differ-
ence form of (2.66) can be written:

$$\left( \pi \frac{\partial \theta}{\partial t} \right)_u = \frac{\partial}{\partial x} \left( \pi u \theta_u \right) + \frac{\partial}{\partial x} \left( \pi u \theta_v u \right) + \frac{1}{\delta x} \left[ (\pi \tilde{\theta}^u) \right] \delta_x$$

$$- (\pi \tilde{\theta}^u)_T \theta_T].$$ \hspace{1cm} (2.67)

$$\left( \pi \frac{\partial \theta}{\partial t} \right)_L = \frac{\partial}{\partial x} \left( \pi L \theta_L \right) + \frac{\partial}{\partial x} \left( \pi L \theta_v L \right) + \frac{1}{\delta x} \left[ (\pi \tilde{\theta}^L) \right] \delta_x$$

$$- (\pi \tilde{\theta}^L)_M \theta_M].$$ \hspace{1cm} (2.68)

Here

$$\theta_u \equiv T_u / (P_a / P_m)^x = T_u / P_a \text{ where } P_a = (P_c / P_m)^x, \hspace{1cm} (2.69)$$

$$\theta_L \equiv T_L / (P_c / P_m)^x = T_L / P_c \text{ where } P_c = (P_c / P_m)^x. \hspace{1cm} (2.70)$$

The OSU TWO-LEVEL AGCM conserves $\theta^*$ so that

$$\delta_M = \frac{1}{3} (\theta_u + \theta_L). \hspace{1cm} (2.71)$$

From (2.49) and (2.50) the advection form of (2.67) and

$$\left( \pi \frac{\partial \theta}{\partial t} \right)_u = \pi u \left( \frac{\partial}{\partial x} + \nu u \cdot \nabla \right) \theta_u + \frac{1}{\delta x} \left[ (\pi \tilde{\theta}^u) \right] \delta_x (\theta_4 - \theta_u)$$

$$+ (\pi \tilde{\theta}^u)_T (\theta_u - \theta_T)] \hspace{1cm} (2.72)$$
and

\[
(2.73) \left( \frac{d}{dt} \right)_L = \pi_L \left( \frac{2}{\alpha_L} + \nabla \cdot \mathbf{x} \right) + \frac{1}{\Delta \theta_{L, T}} \left[ (\pi \phi^{*, \alpha})_L (\partial_L - \partial_L) \right. \\
\left. + (\pi \phi^{*, \alpha})_M (\partial_L - \partial_M) \right].
\]

When \((2.69)\) and \((2.70)\) are introduced into \((2.72)\), it becomes:

\[
(2.74) \left( \frac{d}{dt} \right)_u = \pi_u \left( \frac{2}{\alpha_u} + \nabla \cdot \mathbf{x} \right) + \frac{1}{\Delta \theta_{u, T}} \left[ (\pi \phi^{*, \alpha})_M (\partial_L - \partial_M) \right. \\
\left. + (\pi \phi^{*, \alpha})_T (\partial_L - \partial_M) \right].
\]

This equation can be combined with \((2.37)\) to yield the flux form:

\[
(2.75) \left( \frac{d}{dt} \right)_u = \frac{2}{\alpha_u} \left( \mathbf{C}_L \mathbf{T}_u \mathbf{T}_u \right) + \nabla \cdot \left( \mathbf{C}_L \mathbf{T}_u \mathbf{T}_u \mathbf{x} \right)
\]

\[
+ \frac{\mathbf{G}_u}{\Delta \theta_{u, T}} \left[ (\pi \phi^{*, \alpha})_M (\mathbf{I}_u - (\pi \phi^{*, \alpha})_T) \right]
\]

\[
- \frac{\mathbf{G}_u}{\Delta \theta_{u, T}} \left[ (\pi \phi^{*, \alpha})_M (\mathbf{I}_u - (\pi \phi^{*, \alpha})_T) \right]
\]

\[
\left[ \nabla \cdot \left( \mathbf{C}_u \mathbf{T}_u \mathbf{T}_u \mathbf{x} \right) \right]
\]

With the source and sink, \((2.75)\) can be written as follows:
To be consistent with Eqs. (2.63) and (2.64), the first two terms on the right-hand sides of Eqs. (2.76) and (2.77) must be equal to \( \Pi_u (W_u)_u \) and \( \Pi_L (W_L)_L \). We first note that

\[
(2.78) \quad \Pi_L \frac{G_T L_x}{P_L} \frac{dP}{d\Pi_L} = \Pi_L \frac{G_T L_x}{P_L} \frac{1}{C_P / P_L} \sigma^L \frac{dP}{d\Pi_L} \left( -\frac{P}{P_L} \right)^k
\]

\[
= \frac{\Pi_L \sigma^L \sigma_L}{P_L} \sigma^L \frac{R T_L}{P_L},
\]

\[
= \Pi_L (\sigma^L)_L.
\]

The elimination of \( \Pi_L (\sigma^L)_L \) between (2.65) and (2.78) gives
On equating the other terms in \((\omega d)_{u.i.}\) the following condition is obtained:

\[ (2.80a) \quad C_p (\hat{T}_m - B \delta_m) = \hat{\omega}_x - \hat{\omega}_y, \]

\[ (2.80b) \quad C_p (B \delta_m - \hat{T}_m) = \hat{\omega}_m - \hat{\omega}_l. \]

When (2.70) is used, (2.80a) and (2.80b) can be rewritten as follows:

\[ (2.81a) \quad (C_p \hat{T}_m + \hat{\omega}_m) - (C_p \hat{T}_m + \hat{\omega}_m) = B \cdot C_p (\delta_m - \alpha_m), \]

\[ (2.81b) \quad (C_p \hat{T}_m + \hat{\omega}_m) - (C_p \hat{T}_m + \hat{\omega}_m) = B \cdot C_p (\alpha_m - \hat{\omega}_m) \]

where \(\delta_m\) is given by (2.71). These equations correspond to (2.27e).

Thus, the thermodynamic energy equation (2.76) and (2.77) can be rewritten as follows:

\[ (2.82) \quad \frac{\partial}{\partial t} (C_p \hat{T}_m \hat{V}_m) = - \vec{V} \cdot (C_p \hat{T}_m \hat{V}_m) - \omega \hat{T}_m + T_m (\omega d)_m \]

\[ + C_p T_m \hat{V}_m, \]

\[ (2.83) \quad \frac{\partial}{\partial t} (C_p \hat{T}_m \hat{V}_l) = - \vec{V} \cdot (C_p \hat{T}_m \hat{V}_l) + \omega \hat{T}_m + T_l (\omega d)_l \]

\[ + C_p T_m \hat{V}_l. \]

With the use of (2.47b) and (2.48b), the flux form of moisture balance equation (2.26) can be written as
follows:

\( \frac{\partial}{\partial x} (\Pi_L \delta_u) = -\nu \cdot (\Pi_L \delta_u \nu_L) - L \omega_n \delta_m + L \Pi_L \dot{\delta}_u \)

or

\( \frac{\partial}{\partial x} (\Pi_L \delta_L) = -\nu \cdot (\Pi_L \delta_L \nu_L) + L \omega_n \delta_m + L \Pi_L \dot{\delta}_L \).

Thus, we obtain the finite difference form of total energy equation for each layer by adding (2.61), (2.62), (2.82), (2.83), (2.86), and (2.87) as follows:

\[
(2.88a) \quad \frac{\partial}{\partial x} \left[ \Pi_L \left( K_L + C_p T_L + L \delta_L \right) \right] \\
= -\nu \cdot \left[ \Pi_L \left( K_L + C_p T_L + L \delta_L + \delta_m \right) \nu_L \right] - \omega_n \left( \frac{1}{2} \nu \cdot \nu_L \right) \\
+ C_p \dot{T}_m + L \dot{\delta}_m + \dot{\delta}_n \right] + \Pi_L \left( C_p \dot{\nu}_L + L \dot{\dot{\delta}}_L + \nu \cdot \nu_L \right)
\]

for the upper layer, and

\[
(2.88b) \quad \frac{\partial}{\partial x} \left[ \Pi_L \left( K_L + C_p T_L + L \delta_L \right) \right] \\
= -\nu \cdot \left[ \Pi_L \left( K_L + C_p T_L + L \delta_L + \delta_m \right) \nu_L \right] + \omega_n \left( \frac{1}{2} \nu \cdot \nu_L \right) \\
+ C_p \dot{T}_m + L \dot{\delta}_m + \dot{\delta}_n \right] + \Pi_L \left( C_p \dot{\nu}_L + L \dot{\dot{\delta}}_L + \nu \cdot \nu_L \right)
\]
for the lower layer. Thus the vertically integrated total energy budget equation can be written as follows:

\[
(2.89) \frac{2}{\sigma} \left[ \tilde{E}_p + \tilde{E}_u (K_u + G_T u + L_\theta u) + \tilde{E}_L (K_L + G_T L + L_\theta L) \right] = -\tilde{Q} \left[ \tilde{T}_u (K_u + G_T u + L_\theta u) V_u \right] + \tilde{Q} \left[ \tilde{T}_L (K_L + G_T L + L_\theta L) V_L \right] + \tilde{P}_u (G_T u + L \bar{\theta}_u + u \cdot \bar{\theta}_u) + \tilde{P}_L (G_T L + L \bar{\theta}_L + u \cdot \bar{\theta}_L)
\]

This is the finite-difference analogue of (2.36) which was a continuous total energy budget equation for \( \sigma \)-coordinates and (2.19), derived in pressure coordinates.

The geopotential of level \( \nu \) and \( L \), \( \Psi_u \) and \( \Psi_L \), can be obtained by the hydrostatic relations (2.79), (2.81a), and (2.81b). With the use of (2.69) and (2.70), the addition of (2.81b) to (2.81a), leads to

\[
(2.90) \quad \Psi_L - \Psi_u = -\frac{\tilde{Q}}{\sigma} \left( \frac{P_u - P_L}{\sigma} \right) \bar{\theta}_u
\]

This hydrostatic relation is a finite difference form of (2.27c). By the definition of \( \bar{\theta}_u \) in Eq. (2.71), (2.90) becomes

\[
(2.91) \quad \Psi_L - \Psi_u = -\frac{\tilde{Q}}{\sigma} \left( \frac{P_u - P_L}{\sigma} \right) (\theta_u + \theta_L)
\]

This corresponds to (2.27d).

Thus, the geopotential at the upper level, \( \Psi_u \), and the lower level \( \Psi_L \), obtained by (2.78), (2.79), and
(2.90) as follows:

\[
\begin{align*}
\text{(2.92)} \quad \mathbf{\omega} = \mathbf{\omega}_0 + \pi L (\mathbf{\omega} \cdot \mathbf{\omega})_L,
\end{align*}
\]

\[
\begin{align*}
\text{(2.93)} \quad \mathbf{\omega}_u = \mathbf{\omega}_0 + \pi L (\mathbf{\omega} \cdot \mathbf{\omega})_L + \mathbf{\omega} (\mathbf{\rho} - \mathbf{\rho}_0) \mathbf{\omega}_0,
\end{align*}
\]
or

\[
\begin{align*}
\text{(2.94)} \quad \mathbf{\omega}_u = \mathbf{\omega}_0 + \pi L (\mathbf{\omega} \cdot \mathbf{\omega})_L + \frac{\mathbf{\omega}}{L} (\mathbf{\rho} - \mathbf{\rho}_0) (\mathbf{\omega} + \mathbf{\omega}_0).
\end{align*}
\]

2.5 Correction of the diagnostic mass budget

For practical reasons the simulated data of the OSU AGCM is archived at six-hour intervals instead of at each ten-minute time step of the model integration. This procedure leads to inaccurate budgets primarily because the advection terms cannot be reconstructed from the six-hourly data, and, to a lesser extent, because of sampling errors. It is important to try to reduce these errors, particularly in the case of the heat budget calculations.

If we assume that there is an error in the estimates of the terms in the mass continuity equation given by (2.44b), then we may rewrite the equation as follows:

\[
\begin{align*}
\text{(2.95)} \quad \frac{\partial \pi}{\partial t} + \nabla \cdot (\pi \mathbf{\nu}_L + \pi L \mathbf{\nu}_u) = \mathbf{\varepsilon}.
\end{align*}
\]

1. The OSU AGCM uses a semi-implicit time scheme.
continuity equation without error can be obtained as follows:

\[(2.96) \quad (\nabla \cdot \pi u \nabla) \cdot \tau = (\nabla \cdot \pi u \nabla) - \varepsilon_u,\]

\[(2.97) \quad (\nabla \cdot \pi u \nabla) \cdot \tau = (\nabla \cdot \pi u \nabla) - \varepsilon_u\]

so that

\[(2.98) \quad \frac{\partial \pi}{\partial t} + (\nabla \cdot \pi u \nabla) \cdot \tau + (\nabla \cdot \pi u \nabla) \cdot \tau = 0,\]

or

\[(2.99) \quad \frac{\partial \pi_u}{\partial t} + (\nabla \cdot \pi_u \nabla) \cdot \tau + \omega_m = 0,\]

\[(2.100) \quad \frac{\partial \pi_l}{\partial t} + (\nabla \cdot \pi_l \nabla) \cdot \tau - \omega_m = 0\]

where

\[(2.101) \quad \varepsilon = \varepsilon_u + \varepsilon_l,\]

\[(2.102) \quad \varepsilon_u = \frac{\pi_u}{\pi_u + \pi_l} \varepsilon,\]

\[(2.103) \quad \varepsilon_l = \frac{\pi_l}{\pi_u + \pi_l} \varepsilon.\]

In Eqs. (2.102) and (2.103), we have made the assumption
that the error is proportional to the mass of each layer. Thus, with the use of (2.96), (2.97), (2.99), and (2.100), the total time derivative of conserved quantity (\( f \)) multiplied by the mass can be rewritten as follows:

\[
(2.104) \quad \left( \frac{\partial df}{\partial t} \right)_M = \frac{\partial}{\partial t} (\text{time term}) + \frac{\partial}{\partial t} (\text{space term}) - E_u f_n + \omega_n \left[ f_n (\text{mass term}) + f_n \right],
\]

and

\[
(2.105) \quad \left( \frac{\partial df}{\partial t} \right)_L = \frac{\partial}{\partial t} (\text{time term}) + \frac{\partial}{\partial t} (\text{space term}) - E_u f_L + \omega_L \left[ f_L (\text{mass term}) + f_L \right].
\]

We can now rewrite Eqs. (2.88a) and (2.88b) for the total energy budget equation in each layer as:

\[
(2.106a) \quad \frac{\partial}{\partial t} \left[ T_u (K_u + C_T u + L E_u + \omega_u) \right] = - \frac{\partial}{\partial x} \left[ T_u (K_u + C_T u + L E_u + \omega_u) \right] V_u + \omega_u \left[ \frac{1}{2} \left( V_u \cdot V_u \right) + C_T \tilde{T}_u + L \tilde{E}_u + \tilde{\omega}_u \right] + E_u (K_u + C_T u + L E_u + \omega_u) + T_u (C_p \tilde{H}_u + L \tilde{Q}_u + V_u \cdot \tilde{Q}_u),
\]

and

\[
(2.106b) \quad \frac{\partial}{\partial t} \left[ T_L (K_L + C_T L + L E_L + \omega_L) \right] = - \frac{\partial}{\partial x} \left[ T_L (K_L + C_T L + L E_L + \omega_L) \right] V_L + \omega_L \left[ \frac{1}{2} \left( V_L \cdot V_L \right) + C_T \tilde{T}_L + L \tilde{E}_L + \tilde{\omega}_L \right] + E_L (K_L + C_T L + L E_L + \omega_L) + T_L (C_p \tilde{H}_L + L \tilde{Q}_L + V_L \cdot \tilde{Q}_L).
\]
The vertically integrated total energy budget equation is then rewritten as:

\[
\frac{\partial}{\partial t} [\bar{E}_t] + \bar{T}_u (K_u + \bar{G}_T u + \bar{L}_G u) + \bar{T}_L (K_L + \bar{G}_T L + \bar{L}_G L) \]

\[= -\bar{v} \cdot \bar{\nabla} (K_u + \bar{G}_T u + \bar{L}_G u + \bar{e}_u) \bar{\nabla} u + \bar{v} \cdot \bar{\nabla} (K_L + \bar{G}_T L + \bar{L}_G L + \bar{e}_L) \bar{\nabla} L + \bar{v} \cdot \bar{\nabla} C \frac{\partial u}{\partial t} + \bar{v} \cdot \bar{\nabla} (C_p H_u + L \bar{\theta}_u + \bar{u} \cdot \bar{\nabla} \bar{\theta}_u).\]
3. Creation of variables

Because of the coarse vertical resolution in both the \( \varphi \)- and \( \varphi^* \)-coordinate systems, interpolation schemes must be carefully considered to avoid large errors in the energy budgets. Here, the interpolation schemes for the thermodynamic and kinematic quantities are developed using the constraint that the atmospheric energy must be conserved by the interpolation. To satisfy this constraint, the vertically integrated mass-weighted thermodynamic and kinematic quantities in \( \varphi^* \)-coordinates must be the same as those in \( \varphi \)-coordinates.

Standard interpolation schemes are used to obtain the variables \( U, V, T, q, \bar{z} \), and source and sink term at given pressure levels from the same variables of the upper and lower level of the OSU AGCM. The horizontal velocity \( \mathbf{v} \) is assumed to be linear in pressure \( p \), while \( T \) and \( \bar{z} \) are obtained from the assumption that the potential temperature is linear in \( p^* (1 = R/\rho) \). The details of the interpolation schemes are given below.

3.1. Horizontal wind velocity \( (\mathbf{v}) \) and geostrophic wind velocity \( (\mathbf{\nu}) \)

The zonal and meridional component of wind velocity
in the $\sigma^*$-coordinate system are interpolated linearly in $p$ from the velocities on level 1 and 3 of the OSU TWO LEVEL AGCM.

\begin{align}
(3.1) \quad \nu_u &= \nu_1 + \frac{\sigma_u - \sigma_1}{\sigma_3 - \sigma_1} (\nu_3 - \nu_1), \\
(3.2) \quad \nu_u &= \nu_1 + \frac{\sigma_u - \sigma_1}{\sigma_3 - \sigma_1} (\nu_3 - \nu_1)
\end{align}

where

\begin{align}
(3.3) \quad \sigma_u &= \frac{P_u - P_T}{P_3 - P_T}, \\
(3.4) \quad \sigma_l &= \frac{P_l - P_T}{P_3 - P_T}, \\
(3.5) \quad P_l &= \begin{cases} 
\frac{1}{3} (P_s - 600 \text{ mb}) & \text{for } P_s > 600 \text{mb}, \\
P_s & \text{for } P_s \leq 600 \text{mb},
\end{cases} \\
(3.6) \quad P_u &= 400 \text{ mb}
\end{align}

$\sigma_1$, $\sigma_3$, $\nu_1$, and $\nu_3$ are the sigma values and horizontal velocities defined at level 1 and 3; $P_s$ is the surface pressure. Here the variables which are subscripted by numbers are the variables of the OSU TWO-LEVEL AGCM.

When the surface pressure ($P_s$) is less than 600mb, the horizontal velocity on the lower level ($\nu_l$) is defined as the surface velocity rather than above definition, so that

\begin{align}
(3.7) \quad \nu_l &= \nu_s, \\
(3.8) \quad &= 0.7 (\frac{3}{2} \nu_s - \frac{1}{2} \nu_3).
\end{align}
The geostrophic wind velocity at level 1 and 3, \( V_g_1 \) and \( V_g_3 \), can be written as follows:

\[
V_{g_1} = -\hat{z} \times \frac{1}{f} (\nabla \Phi_1 + \sigma_1 \nabla \Phi),
\]

\[
V_{g_3} = -\hat{z} \times \frac{1}{f} (\nabla \Phi_3 + \sigma_3 \nabla \Phi),
\]

where \( \hat{z} \) is the unit vector of vertical direction, \( f \) is the Coriolis parameter, \( \Phi_1 \) and \( \Phi_3 \) are the geopotential at level 1 and 3, and \( \sigma \) is the horizontal differential operator.

The geostrophic wind velocity at the upper level, \( V_{g_u} \), and the lower level, \( V_{g_l} \), are interpolated in a manner similar to Eqs. (3.1) and (3.2) from the geostrophic wind velocity at level 1 and 3, \( V_{g_1} \) and \( V_{g_3} \). Thus

\[
V_{g_u} = V_{g_1} + \frac{\sigma_u - \sigma_1}{\sigma_3 - \sigma_1} (V_{g_3} - V_{g_1}),
\]

\[
V_{g_l} = V_{g_1} + \frac{\sigma_l - \sigma_1}{\sigma_3 - \sigma_1} (V_{g_3} - V_{g_1}).
\]

Using the above definitions, the total and geostrophic mass fluxes are defined as follows:

\[
\Pi_u V_u = \Pi_u \left\{ V_1 + \frac{\sigma_u - \sigma_1}{\sigma_3 - \sigma_1} (V_{g_3} - V_{g_1}) \right\},
\]

\[
\Pi_l V_l = \frac{1}{2} \Pi (V_1 + V_3) - \Pi_u V_u
\]

\[
\Pi_u V_{g_u} = \Pi_u \left\{ V_{g_1} + \frac{\sigma_u - \sigma_1}{\sigma_3 - \sigma_1} (V_{g_3} - V_{g_1}) \right\},
\]

\[
\Pi_l V_{g_l} = \Pi_l \left\{ V_{g_1} + \frac{\sigma_l - \sigma_1}{\sigma_3 - \sigma_1} (V_{g_3} - V_{g_1}) \right\}.
\]
\[(3.16) \quad M_{UL} = \frac{1}{2} \pi (V_{U1} + V_{U3}) - \pi_u V_{UL}\]

where \(M_{UL}\) and \(M_{UL}\) is a total mass flux on level U and L, \(\pi_u V_{UL}\) and \(\pi_u V_{UL}\) is a geostrophic mass flux on level U and L.

3.2 Potential temperature (\(\theta\)) and temperature (\(T\))

The potential temperature, \(\theta\) is assumed as linear in \(p^k\) so that

\[(3.17) \quad \theta_u = \theta_1 + \left( \frac{P_{u}^k}{P_{1}^k} - \frac{P_{l}^k}{P_{1}^k} \right) (\theta_2 - \theta_1)\]

where \(\theta_1\) and \(\theta_2\) are the potential temperatures of the OSU TWO-LEVEL AGCM. The potential temperature at the lower level, \(\theta_L\), is given by

\[(3.18) \quad \theta_L = \frac{1}{\pi_L} \left\{ \frac{\pi}{2} (\theta_1 + \theta_2) - \pi_u \theta_u^2 \right\} .\]

The temperature \(T\) is calculated from \(\theta\) by (2.70).

\[(3.19) \quad T_U = \theta_u \left( \frac{P_u}{P_{00}} \right)^k\]

\[(3.20) \quad T_L = \theta_L \left( \frac{P_l}{P_{00}} \right)^k\]

where \(P_{00} = 1000\text{ mb}\).
3.3 Geopotential ($\Phi$)

The geopotentials $\Phi_1$ and $\Phi_3$ are computed by a vertical integration of the hydrostatic equation upward from the surface geopotential $\Phi_s$, assuming that the potential temperature is linear in $p^\kappa$ space. Thus,

\begin{equation}
(3.21) \Phi_1 = \Phi_s + \frac{\kappa}{2} \left[ \frac{P_1}{P_{s0}} - \frac{P_1}{P_{s0}} \right] + \frac{\kappa}{2} (\alpha_s - \alpha_1),
\end{equation}

\begin{equation}
(3.22) \Phi_3 = \Phi_s - \frac{\kappa}{2} \left[ \frac{P_3}{P_{s0}} - \frac{P_3}{P_{s0}} \right] + \frac{\kappa}{2} (\alpha_s + \alpha_3).
\end{equation}

(See Ghan et al., 1982, for details)

The geopotential $\Phi_1$ and $\Phi_3$ are interpolated from the geopotential at level 1 and 3, $\Phi_1$ and $\Phi_3$. The geopotential at an arbitrary pressure $P$ is given by the expression

\begin{equation}
(3.23) \Phi = \Phi_s + \frac{R}{\kappa} \left[ T_1 \left( \frac{P_s - P_1}{P_1} + \frac{P_3 - P_1}{P_1} + \frac{P_3 - P_1}{P_1} \right) \right. \\
\left. + T_3 \left( \frac{P_3 - P_3}{P_3} + \frac{P_3 - P_3}{P_3} + \frac{P_3 - P_3}{P_3} \right) \right]
\end{equation}

where $\Phi_s = \Phi_s$ is the surface geopotential, $R$ is the dry-air gas constant, $P_T$ is the tropopause pressure, $\kappa$ is the thermodynamic ratio $R/\varphi$ ($=0.286$), and the potential temperature $\varphi$ has been assumed linear in $p^\kappa$ space.
3.4 Mixing ratio $q$

The relative humidity of the level higher than level 1 where $\sigma = 1/4$ is set equal to that of level 1, while the mixing ratio below $\sigma = 1/4$ is chosen by linear interpolation of relative humidity. The upper-level mixing ratio of $q$ is thus obtained from

\[
(3.24) \quad \delta u = \delta u^* \left\{ RH_1 + \frac{\delta u_1 - \delta u^*}{\delta u_2 - \delta u^*} (RH_3 - RH_1) \right\}
\]

where $\delta u^*$ is the saturation mixing ratio of the upper layer, and $RH_1$ ($= q_1/q_1^*$) and $RH_3$ ($= q_3/q_3^*$) are the relative humidity at level 1 and level 3. The quantities $q_1$, $q_1^*$, $q_3$, and $q_3^*$ are the mixing ratio and the saturation mixing ratio at the level 1 and level 3 of the OSU TWO-LEVEL AGCM, $q_u^*$ is the saturation mixing ratio at the upper level (calculated by the method of Lowe and Ficke, 1974). In the region where $P_u$ is less than $P_1$, $q_u$ is interpolated as follows:

\[
(3.25) \quad \delta u = \frac{\delta i}{\delta i^*} \times \delta u^*.
\]

The mixing ratio at the lower level $q_L$ is calculated from

\[
(3.26) \quad \delta L = \frac{1}{P_L} \left\{ \frac{P}{2} (\delta i_1 + \delta i_3) - P_L \delta u \right\}.
\]
3.5 Friction and the energy conversion term

Friction \( \mathbf{F} \) and the energy conversion term \( \omega \) in the upper layer are calculated by linear interpolation in pressure between values of level 1 and level 3 of the OSU TWO-LEVEL AGCM.

\[
(3.27) \quad \mathbf{A}_u = A_1 + \frac{\mathbf{F}_u - \mathbf{F}_i}{\mathbf{F}_3 - \mathbf{F}_1} (A_3 - A_1)
\]

where \( \mathbf{A}_u = \mathbf{F}_u \) and \( (\omega u)u ) \).

The terms at the lower level are given as

\[
(3.28) \quad \mathbf{A}_L = \frac{1}{L_a} \left( \frac{\mathbf{F}_1}{L_a} (A_1 + A_3) - \Omega u \mathbf{A}_u \right)
\]

where \( \mathbf{A}_L = \mathbf{F}_L \) and \( (\omega L)u_L ) \).

3.6 Diabatic and latent heating term

The diabatic heating term of each layer may be written as follows:

\[
(3.29) \quad \mathbb{H}_a = \frac{3}{4} \mathbb{N}R_a + L C_u + \mathcal{F}_a \left( \frac{\partial T}{\partial x} \right)_{MCL} + \mathcal{F}_a \left( \frac{\partial T}{\partial x} \right)_{PC},
\]

\[
(3.30) \quad \mathbb{H}_L = \frac{3}{4} \mathbb{N}R_L + H_2 + L (C_L - C_u) + \mathcal{F}_a \left( \frac{\partial T}{\partial x} \right)_{MCL} + \mathcal{F}_a \left( \frac{\partial T}{\partial x} \right)_{PC}
\]

where \( \mathbb{N}R_a \) is the area-mean net radiation of kth level,
LCₜₜ is the latent heat release at kth level,
\[ \zeta \left( \frac{\partial T}{\partial x} \right) _{MC} \] is the diabatic heating at kth level due to middle-level convection, and \[ \zeta \left( \frac{\partial T}{\partial x} \right) _{PC} \] is the diabatic heating at kth level due to penetrating convection. Here
\[ \zeta \alpha, \zeta \left( \frac{\partial T}{\partial x} \right) _{MC}, \text{ and } \zeta \left( \frac{\partial T}{\partial x} \right) _{PC} \] are assumed to be proportional to the mass of each layer so that

\[ (3.31) \quad S_u = S_i + \frac{T_u - T_i}{S_j - S_i} (S_3 - S_i) \]

where

\[ S_i = \frac{\alpha}{n} \sum \zeta \alpha, \quad \zeta \left( \frac{\partial T}{\partial x} \right) _{MC}, \text{ and } \zeta \left( \frac{\partial T}{\partial x} \right) _{PC} \]

and

\[ \alpha = 1, 3, 4, \text{ and } L \]

These terms at the lower level can be written in a form which is similar to Eq. (3.28); namely,

\[ (3.32) \quad S_L = \frac{1}{n_L} \left\{ \frac{n}{S_j} (S_i + S_3) - T_u S_u \right\} \]

The large-scale condensational heating \( LC_u \) and \( LC_L \) are interpolated as follows:

\[ (3.33) \quad LC_u = \begin{cases} \frac{1}{n_u} \left\{ \frac{n}{S_j} LC_i + (P_m - 0.5 P_3) LC_3 \right\} & \text{for } P_m \geq 0.5 P_3 \\ \frac{n_u}{n_i} LC_i & \text{for } P_m < 0.5 P_3 \end{cases} \]

and

\[ (3.34) \quad LC_L = \frac{1}{n_L} \left\{ \frac{n}{S_j} (LC_i + LC_3) - T_u LC_u \right\} \]
The moisture source term of each layer can be written in a manner similar to (3.29) and (3.30) as follows:

\[ (3.35) \quad \dot{q}_u = (\frac{2}{\partial t})_{MCL} + (\frac{2}{\partial t})_{PC} - C_u, \]

\[ (3.36) \quad \dot{q}_l = (\frac{2}{\partial t})_{MCL} + (\frac{2}{\partial t})_{PC} + C_u - C_l + \frac{2}{\pi} E_s \]

where \( E_s \) is surface flux of moisture, \( (\frac{2}{\partial t})_{MCL} \) is the rate of moisture change at level \( k (k = U, L) \) due to middle-level convection, and \( (\frac{2}{\partial t})_{PC} \) is the rate of moisture change at level \( k \) due to penetrating convection.
4. Mass balance

To better understand the dynamics of the planetary-scale flow, particularly the role of the ageostrophic flow, we decompose the mass flux in each layer into a geostrophic part and an ageostrophic part. These parts are further decomposed into rotational (non-divergent) and divergent (irrotational) parts. The separation of the rotational and divergent parts of the horizontal mass flux $\Pi_k$ is represented by the two-dimensional Helmholtz equation

\begin{equation}
\Pi_k = \mathbf{x} \times \mathbf{z} \mathbb{H} + \mathbf{x} \mathbb{X}
\end{equation}

where $\mathbb{H}$ is the stream function, $\mathbb{X}$ is the potential function, $\mathbf{z}$ is vertical unit vector and $\mathbf{x}$ is the horizontal gradient operator. Decomposing the circulation by layers, we have

\begin{equation}
\Pi_k u = \Pi_k \mathbb{H} u + \Pi_k \mathbb{X} u, \quad = \mathbb{G} u + \mathbb{A} u,
\end{equation}

\begin{equation}
= \mathbb{G} u + \mathbb{A} u + \mathbb{G} u + \mathbb{A} u,
\end{equation}

\begin{equation}
= \mathbb{G} u + \mathbb{A} L + \mathbb{G} u + \mathbb{A} L,
\end{equation}

\begin{equation}
= \mathbb{G} u + \mathbb{A} L + \mathbb{G} u + \mathbb{A} L,
\end{equation}

\begin{equation}
= \mathbb{G} u + \mathbb{A} L + \mathbb{G} u + \mathbb{A} L.
\end{equation}
where $\nu_u = \nu - \nu_0$ is the ageostrophic part of wind velocity of the upper level, $\nu_l = \nu - \nu_0$ is the ageostrophic part of wind velocity of the lower level, and

\begin{align}
(4.4) \quad g_u &= \pi \nu_l = g_{ru} + g_{lu}, \\
(4.5) \quad g_l &= \pi \nu_l = g_{rl} + g_{ll}, \\
(4.6) \quad a u &= \pi (\nu_u - \nu_0) = a_{ru} + a_{lu}, \\
(4.7) \quad a_l &= \pi (\nu_l - \nu_0) = a_{rl} + a_{ll}.
\end{align}

By the definition of $\nu$ and $\chi$,

\begin{align}
(4.8) \quad \pi \nu_u &= g_{ru} + a_{ru} = \times \chi_\nu, \\
(4.9) \quad \pi \nu_l &= g_{lu} + a_{lu} = \times \chi_\nu, \\
(4.10) \quad \pi \nu_l &= g_{rl} + a_{rl} = \times \chi_\nu, \\
(4.11) \quad \pi \nu_l &= g_{ll} + a_{ll} = \times \chi_\nu.
\end{align}

The divergent part of mass flux is usually an order of magnitude less than the rotational part of mass flux for the planetary-scale flows (Haltiner and Williams, 1980, p.56). Similarly
\[ (4.12) \quad (\phi_u, \phi_L) = \nabla \times \nabla \times (\psi_u, \psi_L), \]

\[ (4.13) \quad (\phi_u, \phi_L) = \nabla (X_u, X_L), \]

\[ (4.14) \quad (A_u, A_L) = \nabla \times (\psi_a, \psi_a), \]

\[ (4.15) \quad (A_u, A_L) = \nabla (X_a, X_a) \]

where \((\psi_u, \psi_L)\) and \((X_u, X_L)\) are the geostrophic part of stream function and potential function, \((\psi_a, \psi_a)\) and \((X_a, X_a)\) are the ageostrophic parts.

From (4.1) and the definition of \(\psi\) and \(X\), it may be shown that

\[ (4.16) \quad \nabla^2 \psi = \nabla \cdot \nabla \times (\nabla \times \omega), \]

\[ (4.17) \quad \nabla^2 X = \nabla \cdot (\nabla \times \omega). \]

Similarly

\[ (4.18) \quad \nabla^2 (\psi_u, \psi_L, \psi_a, \psi_a) = \nabla \cdot \nabla \times (\nabla \times (\nabla \times \omega)), \]

\[ (4.19) \quad \nabla^2 (X_u, X_L, X_a, X_a) = \nabla \cdot (\nabla \times (\nabla \times \omega)). \]

By means of a zonal harmonic analysis in the longitudinal direction and by using the method of Lindzen and Kuo (1969) in the latitudinal direction, the Poisson equations (4.16) - (4.19) can be solved for \(\psi_u, \psi_L, \psi_a, \psi_a\), \(X_u, X_L, X_a, X_a\).
Fields of stream function and potential function were obtained from the interpolated zonal and meridional wind components in the upper and lower layers in \( \theta^\ast \) coordinates for three Januaries and Julys of the OSU TWO-LEVEL AGCM.

4.1 Stream function

The resulting \( \psi_u, \psi_v, \psi_{gu}, \psi_{gv}, \psi_{au}, \psi_{av}, \) and \( \psi_{al} \) are shown in the Fig. 4.1 - Fig. 4.12 for the composite January and July. The rotational component of mass flux is parallel to the contours of the stream function with clockwise flow around a high. The mass flux is essentially rotational and zonal except in the tropics where as the strength of the divergent component of mass flux is comparable to that of rotational component in agreement with the observational results of Krishnamurti et al. (1973).

The rotational component of total mass flux of the upper layer shows that the locations of jet streams and blocking ridges for both January and July are in good agreement with observations (Newell et al., 1972; Blackmon et al., 1977; Lau, 1978; Blackmon and Lau, 1980).

During January, the ridges which are located in the
Fig. 4.1. Upper layer stream function of the monthly mean total mass flux for January. Contour interval is $5 \times 10^{10}$ kg/s.
Fig. 4.2. Lower layer stream function of the monthly mean total mass flux for January. Contour interval is $5 \times 10^{10}$ kg/s and contour sub-interval is $2.5 \times 10^{10}$ kg/s.
Fig. 4.3. As in Fig. 4.1. except for July.
Fig. 4.4. As in Fig. 4.2. except for July.
northeastern Pacific, the northeastern Atlantic, and the eastern European - western Asian region give the circulation in the Northern Hemisphere a three-wave structure. The eastern Pacific ridge is the most prominent. The waves in the Southern Hemisphere have much smaller amplitudes.

In the lower layer, the rotational component of mass flow is strongly affected by topography. The flow over upwind side of the Himalayas is anticyclonic and that over downwind side is cyclonic. Thus, most of the flow in mountain region goes around the mountain with stronger flow to the north of the mountain.

During July, the blocking ridges, which are located at the eastern Asia and western North America, are quite weak compared to those of January even though they still show the same three-wave structure. In the Southern Hemisphere, the weak amplitude ridge which is spread over the region eastward from the middle Pacific to southern Africa results in a one-wave structure.

The jet streams are located in the trough regions - the western North Pacific, the American-North Atlantic region, and the Mediterranean region in the Northern Hemisphere, and the southern Australian and South American region in the Southern Hemisphere during January. The jet streams of July are located approximately same region of the Northern Hemisphere compared to those of
January, although their strength is quite weak. In the Southern Hemisphere, the jet shifts equatorward in the broad region from the southern Indian Ocean to the central South Pacific.

Although there is an appreciable seasonal variation in the strength of jet stream in the Southern Hemisphere, it is much smaller than in the Northern Hemisphere. This is related to the smaller seasonal variation in intensity of the Hadley circulation (Fig. 4.23 - Fig. 4.26) and is probably due to the lack of major topographic features in the Southern Hemisphere.

The presence of some distinct anticyclonic systems in the upper layer in the region of the tropical West Pacific and Central America in the Northern Hemisphere, and southern Africa, Australia, and South America in the Southern Hemisphere during January, agrees with the observations presented by Newell et al. (1972). Similar systems found from Africa to the Pacific-Central American region in the Northern Hemisphere, and from the Indian Ocean area to the western Pacific in the Southern Hemisphere during July are also comparable to the observations of Newell et al. (1972).

As suggested by Ramage (1971), the upper-layer anticyclonic maxima appear to be directly caused by sensible heating and latent heat release in the upper troposphere over the continents and to the latent heat
realease in the convective activity in the ITCZ. Allan (1983) showed similar upper-layer anticyclonic maxima over the western Pacific and Australian regions during the summer season in his observational study of the monsoon and teleconnections over Australian region. In the lower latitudes, the rotational component is relatively weak and tends to be organized in closed circulations on smaller scales.

From Figs. 4.5 - 4.8, it is clear that the rotational component of the motion is almost entirely geostrophic. The jet streams are essentially characterized by this component of circulation.

Basically the ageostrophic stream function (Figs. 4.9 - 4.12) represents the strength and position of the rotational (horizontal) secondary circulation associated with jet stream (the divergent (vertical) secondary circulation associated with jet stream will be discussed at Section 4.2). The horizontal and divergent secondary circulation are relatively small compared to the zonal circulation at the midlatitudes, but are necessary to explain the dynamics of jet stream maxima which are important components of the midlatitude zonal circulation.

Namias and Clapp (1949) hypothesized the so-called "confluence mechanism" for the maintenance of middle-latitude jet stream maxima. They noted that the jet streams along the east coasts of Asia and North Am-
Fig. 4.5. Upper layer stream function of the monthly mean geostrophic mass flux for January. Contour interval is $5 \times 10^{10}$ kg/s.
Fig. 4.6. Lower layer stream function of the monthly mean geostrophic mass flux for January. Contour interval is $5 \times 10^{10}$ kg/s and contour sub-interval is $2.5 \times 10^{10}$ kg/s.
Fig. 4.7. As in Fig. 4.5. except for July.
Fig. 4.8. As in Fig. 4.6. except for July.
Fig. 4.9. Upper layer stream function of the monthly mean ageostrophic mass flux for January. Contour interval is $0.5 \times 10^{10}$ kg/s.
Fig. 4.10. Lower layer stream function of the monthly mean ageostrophic mass flux for January. Contour interval is $0.25 \times 10^{10}$ kg/s.
Fig. 4.11. As in Fig. 4.9. except for July.
Fig. 4.12. As in Fig. 4.10. except for July.
erica appear to be maintained by cross-isobaric flow. This is supported by the observational evidence of Blackmon et al. (1977) and Lau (1978). These recent papers noted that the vertical circulations in the vicinity of the jet stream tend to be thermally direct in the entrance regions of the jet stream maxima and thermally indirect in the exit regions of jet stream maxima.

Recently, the secondary circulation around jet stream maxima simulated by the OSU TWO-LEVEL AGCM was studied by Kim and Grady (1982). They noted that the divergent secondary circulation appears as the ageostrophic divergent component of mass flux, while the rotational secondary circulation appears as the ageostrophic rotational component of mass flux. The schematic structure of divergent and rotational secondary circulation is shown in Fig. 4.13.

The rotational secondary circulation is anticyclonic (cyclonic) around the westerly (easterly). The strength of the rotational secondary circulation of the upper layer is as large as or even larger than that of the upper parts of the divergent secondary circulation as shown in Figs. 4.9, 4.11, 4.22, and 4.24.

In the lower layer, the mass flux is strongly affected by the topography. In particular, the geostrophic as well as ageostrophic rotational mass flux over the Tibetan plateau is quite weak due to the small depth
Fig. 4.13. Schematic structure of the rotational and divergent secondary circulation associated with the jet stream.
of the lower layer in this region.

4.2 Potential function

Figs. 4.14 - 4.17 show that the divergent component of mass flux obtained from (4.17) and (4.19) (negative centers imply divergence and the positive centers imply convergence) is a combination of the flows associated with the Hadley circulation, the Walker circulation, and the east-west circulation over Northern Pacific. Observational estimates of this component of the circulation have been given by Krishnamurti (1971) and Krishnamurti et al. (1973).

In the upper layer, there is a strong divergence at the upwind side and a strong convergence at the downwind side of the Tibetan plateau in the divergent mass flux of the upper layer (Figs. 4.14 and 4.15). Since the height of 600 mb level is approximately the boundary between the upper layer and lower layer in this study as shown in Fig. 4.18, it is clear that a substantial fraction of the mass coming into the plateau region goes up to the upper layer on the upwind side and is compensated by the downward motion from the upper layer on the downwind side.

The convergence over Indonesia during January and
Fig. 4.14. Upper layer potential function of the monthly mean total mass flux for January. Contour interval is $0.25 \times 10^{10}$ kg/s.
Fig. 4.15. Lower layer potential function of the monthly mean total mass flux for January. Contour interval is $0.25 \times 10^{10}$ kg/s.
Fig. 4.16. As in Fig. 4.14 except for July.
Fig. 4.17. As in Fig. 4.15 except for July.
Fig. 4.18 Schematic diagram of the mass flow over the mountain area.
that over S.E. Asia during July is the center of the planetary-scale monsoon circulation and its seasonal change in the Indian and tropical West Pacific oceans. The local Hadley cell in eastern Asia is associated with the cold surface outflow from Siberia, the rising motion in the region of heaviest equatorial rain fall over Malaysia and Indonesia (Fig. 5.5), and the vigorous upper tropospheric return-flow during January.

The intensity of Hadley circulation is stronger during the winter season than the summer season in both hemispheres. This agrees well with the observational studies of Palmen and Vuorela, 1963; Kidson et al., 1969; Oort and Rasmusson, 1970, 1971; Newell et al., 1972. But, in comparison with the observed results of potential fields of Krishnamurti (1971) and Krishnamurti et al. (1973), the simulated northward mass flux in the South Asian - Indonesian region during January and the southward mass flux in the Indonesian - Australian region during July in the upper layer are quite strong. This characteristic of the OSU TWO-LEVEL AGCM was noted by Schlesinger and Gates (1980). In their results, the intensity of the simulated Hadley cell is $75 \times 10^5 \, g s^{-1}$ more intense than that observed in the Northern Hemisphere and $25 \times 10^5 \, g s^{-1}$ more intense in the Southern Hemisphere during January. The Southern Hemisphere Hadley circulation is about twice as intense
as observed during July.

The low-level convergence center over Indonesia during January and S.E. Asia during July, and the divergence center near the Peruvian coast, may be identified with the Walker circulation. As discussed by Bjerknes (1969), this circulation is associated with the westward flow near the ocean surface and the eastward flow in the upper troposphere in the Pacific. This Walker circulation is distinguished from other tropical circulations by an east-west exchange of air which taps the potential energy generated by the large-scale rise of warm-moist air and descent of cold-dry air (Bjerknes, 1969). The intensity of the simulated Walker circulation is stronger during July since the land-ocean heating and thermal contrasts, and zonal asymmetries during the Northern summer are much more pronounced than in other seasons.

The upper-level pattern in Figs. 4.14 - 4.16 compares well with Fig. 1 of Krishnamurti (1971) and Fig. 1 of Krishnamurti et al. (1973). The lower-layer convergence over Indonesia is associated with the convergence of sensible heat (Figs. 5.15 and 5.16) and water vapor (Figs. 5.17 and 5.18) which play an important role in the heat and water vapor balance of the tropics.

The east-west flow of tropical air accumulates latent and sensible heat, and carries this energy great
distances downstream where the latent heat is finally released in major convective systems. This heating of tropics is the basic source of the available potential energy for the Hadley circulation. The considerable east-west asymmetry in the heating near the equator is apparently the major energy source of the Walker circulation. The observational study by Cornejo-Garrido and Stone (1977) and the numerical study by Geisler (1981) showed that indeed there is a high correlation between condensational heating and the sea surface temperature (SST) and indicated that while SST may be important during the initial response of the Walker circulation it only plays a minor role in maintaining the circulation. They suggested that release of latent heat over Indonesia (Figs. 5.5 and 5.6) is the primary driving force of the Walker cell and the source of the heating is largely from dynamical moisture convergence.

An interesting difference from the observational results of Krishnamurti (1971) is that there is no significant east-west circulation in the Northern Pacific in the OSU TWO-LEVEL AGCM. Krishnamurti et al. (1973) showed that the east-west circulation in the Northern Pacific appeared to be the major circulation in zonal vertical planes; the Walker circulation is weaker in his velocity potential field at 200 mb (Fig. 1). As shown in Fig. 4.14 - Fig. 4.17, the east-west circulation over
Northern Pacific is weaker than the Walker circulation during January and it is difficult to find during July.

If we compare Fig. 4.19 and Fig. 4.20 with Fig. 4.23 and Fig. 4.24 for January or Fig. 4.21 and Fig. 4.22 with Fig. 4.25 and Fig. 4.26 for July we see that the geostrophic potential function, $\zeta_g$, depicts mostly east-west motions whereas the ageostrophic potential function, $\zeta_a$, depicts the north-south motions. This is most pronounced in the lower layer in the geostrophic fields, where the horizontal structures are characteristic of Kelvin waves whose maximum amplitudes appear at equator over the Peruvian coast and the west African coast. Over southern Asia and western Australia the two convergence (divergence) centers found in the upper (lower) layer, and over the central Atlantic and Brazil-South Atlantic the two divergence (convergence) centers found in the upper (lower) layer during January are more similar to Rossby-wave structures such as those appearing in the theoretical studies by Matsuno (1966) and Gill (1980).

The zonally asymmetric heating produces the Kelvin waves which carry the information rapidly eastward, thereby creating easterly trade winds in this region, with the trades providing inflow to the region of heating and resulting in a Walker-type circulation with rising over source region and sinking to the east. Accord-
Fig. 4.19. Upper layer potential function of the monthly mean geostrophic mass flux for January. Contour interval is 0.25 x 10^{10} kg/s.
Fig. 4.20. Lower layer potential function of the monthly mean geostrophic mass flux for January. Contour interval is $0.25 \times 10^{10}$ kg/s.
Fig. 4.21. As in Fig. 4.19 except for July.
Fig. 4.22. As in Fig. 4.20 except for July.
Fig. 4.23. Upper layer potential function of the monthly mean ageostrophic mass flux for January. Contour interval is $0.25 \times 10^{10}$ kg/s.
Fig. 4.24. Lower layer potential function of the monthly mean ageostrophic mass flux for January. Contour interval is $0.25 \times 10^{10}$ kg/s.
Fig. 4.25. As in Fig. 4.23 except for July.
Fig. 4.26. As in Fig. 4.24 except for July.
ing to Lettau (1974), the planetary wave response, which carries information westwards into Indian Ocean region, can explain the low-level westerlies over that region in the lower layer as being a result of the heating over the Malaysian-Indonesian region. The westerly flow in the lower layer over the Indian Ocean region can be seen in Figs. 4.2 and 4.15 for January, and 4.4 and 4.17 for July. But Gill (1980) reported that this planetary wave speed is much slower than that of Kelvin wave in his numerical study. The geostrophic motions (especially the rotational component) show a good agreement with the results of Lettau (1974). But there is strong convergence center over the tropical eastern African coast corresponding to the strong divergent geostrophic easterly flow over the Indian Ocean appearing in the geostrophic potential function field of the lower layer (Fig. 4.22). This geostrophic easterly flow represents the so-called Somali jet. Also during July, due to the ageostrophic flow there is a strong divergence center over the tropical east African coast so that there is strong northward mass flux as well as easterly flow over the Indian Ocean (Fig. 4.26) during July.

Thus, in the lower layer the moisture carried by the divergent geostrophic easterly flow is transported to India by the divergent ageostrophic flow. Apparently the divergent component of mass flux, whose strength is
comparable to that of rotational component in the tropics, plays an important role in the simulated Asian monsoon.

We can see that the mean meridional circulation of the simulated Hadley cell is mostly made up of the divergent component of ageostropic mass flux. Figures 4.23 - 4.26 show that the rising motion at the thermal equator and simultaneous sinking motion in the belt of subtropical highs are connected by the equatorward component of mass flow in the lower layer and by compensating flow away from the equator in the upper layer. As mentioned above, the strong meridional mass flow near the equator which is shown in the both upper and lower layers during July (Figs. 4.25 and 4.26) is a characteristic of the OSU TWO-LEVEL AGCM.

The upper branch of the divergent secondary circulation associated with the middle latitude jet streams in January are well-represented by the ageostrophic potential function field (Figs. 4.23 and 4.24). The poleward divergent mass flow in the upper layer and the equatorward divergent mass flow in the lower layer over the entrance regions of jet streams (eastern Asia and southern North America) and the equatorward divergent mass flow in the upper layer and the poleward divergent mass flow in the lower layer over the oceanic storm tracks are associated with the direct circulations of
entrance regions and the thermally indirect circulations of exit regions.

During July, the dipole patterns of the ageostrophic mass flow do not appear to be significant in comparison with those of January (Figs. 4.23 and 4.25). The magnitude of zonal fluctuation of the jet stream and its associated ageostrophic flow in the Southern Hemisphere is less than that in the Northern Hemisphere even during the winter of the Southern Hemisphere.

Secondary circulations are necessary to explain the acceleration and deceleration of the middle latitude jet streams. Lau (1978) showed that the observed time-mean ageostrophic flow plays a dominant role in producing the westerly acceleration at the entrance region of jet stream while the equatorward ageostrophic flow is important to the deceleration in the exit region.

However, the lower part of the simulated thermally indirect circulation in the jet exit region does not appear in the ageostrophic potential function fields of the lower layer (Figs. 4.24 and 4.26), although that of the direct circulation in the entrance region is quite evident. The strength of ageostrophic rotational flows are much stronger than those of divergent flows in the jet exit regions while those are comparable in the jet entrance regions. Thus, the jet streams are decelerated mostly by the ageostrophic rotational flows in the exit
regions. Lau's (1978) observational data also does not support an indirect circulation in the exit region.
5. Total energy budget

To understand the large-scale motion of atmosphere, it is necessary to understand the processes by which the energy is generated, transported, and converted from one to another. These processes are governed by two fundamental relationships - the equation of mechanical energy which can be derived directly from the equation of motion, and the first law of thermodynamics which governs the maintenance of temperature field.

For the annually averaged circulation the incoming solar energy, which is the ultimate driving force for the atmospheric and oceanic circulations, is more intense in low than in high latitudes. Although the atmosphere loses more energy by infrared radiation than it absorbs from the solar radiation, the sensible heat exchange with earth's surface, and the release of latent heat through the precipitation processes result in excess heating at low latitudes and a deficit at high latitudes. Thus the poleward transport of energy is needed to maintain the climatic energy balance.

There is a significant seasonal variation in this process due to the continuous change in solar declination. It is well known that there are large differences at all latitudes between the basic winter and summer circulation regimes.
In this chapter, the energy budget simulated by the OSU TWO-LEVEL AGCM is presented for composite January and July months. Equation (2.36) provides the framework for the presentation of the results. The total energy is divided into its four components - potential energy, enthalpy, latent energy, and kinetic energy. Equation (2.36) is the well-known energy equation which relates the time change of total energy in a fixed volume of the atmosphere to the total heat added and diminished by the flux of enthalpy, latent heat, potential energy, and kinetic energy across the boundary of the volume.

The schematic diagram of the globally averaged total energy balance (Eq. 2.106a, 2.106b, and 2.107) is expressed in Fig. 5.1. The first and second row in Fig. 5.1 represents the sensible heat budget and latent heat budget of the model atmosphere while the third and fourth row represents the potential energy and the kinetic energy budget of the model atmosphere. The last row is the sum of the terms in the other component budgets and represents the total energy budget of the model atmosphere.

The left-hand-side terms in Fig. 5.1 are the time change of each form of the energy. Thus, they represent the amount of the gain or loss of each form of the energy during the simulation period. The first terms on the
Fig. 5.1. Schematic diagram of total energy budget.
right-hand-side terms in Fig. 5.1 are the vertical fluxes of the various forms of energy at the middle level: the energy exchange between the upper layer and the lower layer. The second terms on the r.h.s. of Fig. 5.1 represent the sources or sinks of energy. The third term represents the energy conversion which plays the role of sink in the sensible heat budget and an energy source in the kinetic energy budget. This term does not affect the total energy since it is only a transformation of energy from one form to another. The fourth terms are due to sampling errors involving the mass continuity equation as discussed in Section 2.5.

Even though we include all physical source, sink and conversion terms, and the exchange of energy between layers, the true storage of energy computed on the left-hand-sides of the equations may not be balanced with the storage of energy implied by the four terms on the right-hand-sides due to computational sinks of energy, sampling errors, and the errors involved in interpolating from $\sigma$ to $\sigma^*$ coordinates. The last column in Fig. 5.1 represents the imbalance between the true storage and the implied storage for each form of the energy. As mentioned above, the mass compensation terms and the residuals are not physical and exist only in the energy budget of the model atmosphere.
5.1 Net radiation

The net radiative fluxes at the top of the model atmosphere and at the surface of earth are given by

\[ N_0 = S_i - AS_{st} - S_r - R_0, \]

\[ N_s = AS_s - R_s \]

where \( S_i \) is the incoming solar radiation at the top of the atmosphere, \( AS_{st} \) is the solar radiation absorbed by the atmosphere above top of model atmosphere (200 mb), \( S_r \) is the total solar radiation reflected or scattered to space, \( R_0 \) is the net outgoing long-wave radiation at 200 mb level, \( AS_s \) is the solar radiation absorbed by the surface, and \( R_s \) is the net long-wave radiation at the surface (See, Ghan et al., 1982, for a detailed description).

The latitudinal distributions of net radiation at top of the model atmosphere and the surface of earth are shown in Fig. 5.2. The maximum radiative energy gain \((S_i - R_0, S_i - R_s > 0)\) is located in the subtropics of the summer hemisphere and the maximum radiative energy loss \((S_i - R_0, S_i - R_s < 0)\) is located in the winter hemisphere at subpolar latitudes. These data clearly show that in both January and July the atmosphere is radiatively cooled at all latitudes while the earth is radiatively
Fig. 5.2. Net radiation at the top of atmosphere(....), at the surface of earth(----), and net radiative cooling(----). Unit is W/m².
heated everywhere except in a winter polar cap at latitudes higher than 60°. Thus, the balance requirements in the model are such that the heat transfer must occur from the earth to the atmosphere. The net radiative cooling of the atmosphere \((R_N = N_o - N_s)\) is remarkably uniform with latitude and has a global-mean value of -89.8 W/m² for January and -94.2 W/m² for July. This suggests that the net radiative heating within the atmosphere is not a major source of the energy required to drive the atmospheric circulation.

5.2 Surface sensible and latent heat flux

The turbulent fluxes of sensible heat \(H_s\) and latent heat \(LE_s\) at the earth's surface are parameterized by the bulk aerodynamic method:

\[
H_s = S_u C_P C_D V_s (T_s - T_k),
\]

\[
LE_s = S_u C_L V_s \beta (E_s^* - E_0),
\]

where \(S_u\) is the surface air density, \(C_D\), the surface drag coefficient, \(V_s\), the effective surface windspeed, \(E_s^*\), the surface saturation mixing ratio, and \(E_0\), the ratio of the actual evapotranspiration to the potential evapotranspiration (See, Ghan et al., 1982, for a
detailed description).

The zonally-averaged values of the surface sensible heat flux $H$ and the surface latent heat flux $LE$, for both January and July are given in Fig. 5.3. The surface flux of sensible heat is upward in the subtropics in January and in the subtropics and middle latitudes in July: the sensible heat flux poleward of approximately 60 degrees latitude is downward in both January and July. The major cause of the sensible heat flux in the model is the large radiative heating of the underlying surface in both January and July as evident by the solid curves in Fig. 5.2, and the consequent destabilization of the surface boundary layer with respect to convective turbulence (Kim and Wang, 1981). Schlesinger and Gates (1980) reported that the major differences between the simulated and observed surface sensible heat fluxes are that the model simulates a downward sensible heat flux in the high latitudes of the winter hemisphere while the observations show an upward flux.

As shown in Fig. 5.3, the surface latent heat flux is concentrated in the region within 20 degrees of latitude around thermal equator. Through the convective precipitation which will be discussed below, this surface latent heat flux plays a major role in generating the energy needed to drive the meridional circulation of atmosphere.
Fig. 5.3. Surface flux of sensible heat(—) and latent heat(....). Unit is W/m².
5.3 Condensational heating (precipitation)

The zonally-averaged simulated and observed condensational heating rates are shown in Fig. 5.4 for both January and July, along with that portion of the simulated total condensational heating rate due to large-scale condensation. The major difference when compared to observation (Jaeger, 1976) is that the simulated precipitation rates in the ITCZ and the mid-latitudes of the winter hemisphere (especially on the east coasts of the continents) are larger than observed. In Figs. 5.5 and 5.6 the excess precipitation in the ITCZ would tend to increase the model's Hadley circulation, and the excess in the mid-latitudes would tend to prevent the generation of a thermally indirect meridional (Ferrel) circulation.

The condensational heating near the thermal equator, which is about 160 W/m² for January and 230 W/m² for July, is essentially due to parameterized cumulus convection and exceeds the surface latent heat flux by about 100 - 140 W/m² as shown by Fig. 5.3. This means that an equatorward influx of moisture is necessary to maintain the release of latent heat which is in excess of latent heat supplied by the surface evaporation. This convergence of moisture is clearly shown in Fig. 5.17 and 5.18.
Fig. 5.4. Zonal average of simulated (full line) and observed (dashed line) condensational heating for January and July. The dotted line is the condensational heating associated with large scale (nonconvective) processes. Unit is W/m². The observed data are for January and July from Jaeger (1976).
Fig. 5.5. Condensational heating for January. Unit is W/m$^2$. 
Fig. 5.6. As in Fig. 5.5. except for July.
As shown in Fig. 5.6, there is excessive large precipitation over the western Indian Ocean and southern Africa. Fig. 5.7 demonstrates that this feature is due to the strong upward flux of moisture by the parameterized convective clouds. Also the strong mass convergence of lower layer over the western Indian Ocean plays an important role in this excessive precipitation.

5.4 Diabatic heating

The vertically integrated diabatic heating term in Eq. (2.107) may be written

\[ \varphi (\bar{\tau}_H \bar{H}_u + \bar{\tau}_L \bar{H}_s) = \varphi \left( \left[ (\bar{R}_u - \bar{R}_o) + (\bar{S}_o - \bar{S}_u) + H_s \right] + \varphi \right. \\
+ \varphi \left( \frac{\partial T}{\partial \varphi} \right)_{MCL} + \varphi \left( \frac{\partial T}{\partial \varphi} \right)_{PC} \]

where \( \bar{R}_o \) and \( \bar{R}_u \) are the area-mean net upward long-wave radiative flux at top of atmosphere and at the surface, \( \bar{S}_o \) and \( \bar{S}_u \), the area-mean net downward short-wave radiative flux at top of atmosphere and at the surface, \( H_s \), the surface flux of sensible heat, \( \varphi \), the latent heat release due to large scale condensation, \( \varphi \left( \frac{\partial T}{\partial \varphi} \right)_{MCL} \), the diabatic heating due to middle level convection, and \( \varphi \left( \frac{\partial T}{\partial \varphi} \right)_{PC} \), the diabatic heating due to penetrating convection.
Fig. 5.7. Latent cooling by the penetrating convection for July. Unit is W/m$^2$. 
The zonally-averaged diabatic heating for January and July is shown in Fig. 5.8. Schlesinger and Gates (1980) reported the most serious error in the simulated diabatic heating is the excessively large heating in the mid-latitudes of the winter hemisphere, particularly in January. These excessively large heating rates are due in large part to the excessively large precipitation off the east coasts of the continents in the winter hemisphere.

The vertically integrated diabatic heating is shown in Fig. 5.9 and 5.10 for January and July. Strong diabatic heating occurs in the ITCZ region and the storm tracks in middle latitudes due to the concentrated latent heat release, and the continents of the summer hemisphere due to the strong sensible heat flux at the earth's surface. The relatively strong heating over the Indonesia is probably important in generating the available potential energy needed to drive the divergent zonal (Walker) and the meridional (Hadley) circulations. The strong diabatic heating near the east coasts of the continents in the winter hemisphere which results from the excessive precipitation in those regions tends to prevent the development of the thermally indirect meridional (Ferrel) circulation (Schlesinger and Gates, 1980).

The global-mean value of diabatic heating of the
Fig. 5.8. Zonally-averaged diabatic heating for January and July. Unit is W/m$^2$. 
Fig. 5.9. Vertically integrated diabatic heating for January.
Unit is W/m².
Fig. 5.10. As in Fig. 5.8. except for July.
upper layer is negative (-12.95 W/m² for January and -11.68 W/m² for July) while that of the lower layer is positive (16.57 W/m² for January and 17.92 W/m² for July) due to the surface flux of sensible heat at the earth's surface.

The excessive heating over the western Indian Ocean and the southern Africa during July is associated with the abnormally strong condensational heating discussed in the previous section.

5.5 Latent heating

The vertically integrated latent heat source term in Eq. (2.106) may be written

\[ L(\theta_1 + \theta_2) = f L E_3 - L C + L \left( \frac{\partial \theta}{\partial z} \right)_{\text{mcl}} + L \left( \frac{\partial \theta}{\partial z} \right)_{\text{pc}} \]

where \( E_3 \) is the surface flux of moisture, \( \left( \frac{\partial \theta}{\partial z} \right)_{\text{mcl}} \), the rate of moisture change due to middle-level convection, and \( \left( \frac{\partial \theta}{\partial z} \right)_{\text{pc}} \), the rate of moisture change due to penetrating convection. Here the second and fourth terms of r.h.s. are exactly cancelled the fourth and sixth terms of r.h.s. of Eq. (5.5).

The vertically integrated latent heat source fields are shown in Figs. 5.11 - 5.12 for both January and July. In both months, the source regions occur over the
Fig. 5.11. Vertically integrated latent heating for January. The shaded area is cooling region. Unit is W/m².
Fig. 5.12. As in Fig. 5.11. except for July.
subtropics in both hemispheres due to the strong evaporation in these regions.

The global-mean latent heating of the upper layer is $-14.11 \text{ W/m}^2$ for January and $-13.57 \text{ W/m}^2$ for July while that of the lower layer is $15.00 \text{ W/m}^2$ for January and $12.10 \text{ W/m}^2$ for July. The sink in the upper layer is due to the excess condensation, while lower layer source is due to the evaporation at the earth's surface. This requires upward transport of total energy at the middle level when averaged over a month or longer. The high-latitude latent heat loss (shaded region) is due to the small surface fluxes associated with low surface temperatures; the cooling of the tropical region is due to excessive convective precipitation. In other words, the shaded region over ITCZ and most of higher latitudes is a moisture sink region while the unshaded region over subtropics is a moisture source region where the evaporation is larger than precipitation.

During July, the strong cooling over the western Indian Ocean and southern Africa shown in Fig. 5.12 is compensated by the strong diabatic heating over these regions (see Fig. 5.10).
5.6 Total energy dissipation by friction

The dissipation rate of kinetic energy and thus total energy of the upper layer for both January and July is shown in Figs. 5.13 - 5.14. The strong dissipation is concentrated in the mid-latitude jet-stream regions. Since the dissipation rate is proportional to the wind velocity, the local friction in the lower layer is not significant compared to that of the upper layer, even though the global-mean value of the dissipation in the lower layer (-1.19 W/m$^2$ for January and -1.03 W/m$^2$ for July) is comparable or exceeds that of the upper layer (-0.98 W/m$^2$ for January and -0.85 W/m$^2$ for July).

The seasonal variation of the dissipation rate in the Northern Hemisphere is much larger than that in the Southern Hemisphere. This is related to the seasonal variations of jet stream.

5.7 Divergence of total energy

The vertically integrated divergence of sensible heat for both January and July is shown in Fig. 5.15 and 5.16. Basically the divergence regions occur at subtropics while the convergence regions occur in the ITCZ and
Fig. 5.13. Friction of the upper layer for January. Unit is $W/m^2$. 
Fig. 5.14. As in Fig. 5.13. except for July.
Fig. 5.15. Vertically integrated divergence of sensible heat for January. The shaded area is convergence region. Unit is W/m².
Fig. 5.16. As in Fig. 5.15. except for July.
the polar regions due to the meridional circulation.

In the tropics, the convergence of sensible heat is due to the Hadley circulation. The tropical convergence of sensible heat is expected since the lower layer is warmer than the upper layer. But this does not mean a net convergence of total energy into the tropics. To understand the heat balance we must consider the fluxes of potential energy and latent heat since the each form of energy in itself is not conserved, and large energy conversions are taking place. This will be discussed in detail in the next section.

In the regions of rising motion, the latent heat is converted into internal energy and then into potential energy. Thus, it is possible for there to be divergence of total energy in the tropics even though there is a convergence of sensible heat. The opposite situation may occur in the regions of downward motion.

The simulated convergence of sensible heat in the tropics is located around the thermal equator (10°S during January, 12°N during July). The strong centers of divergence over central Asia and North America during January, and over South America and southern Africa during July are primarily due to the mass divergence in the lower layer since the temperature decreases with respect to height so that the influence of mass divergence (convergence) of the lower layer on the sensible
heat divergence (convergence) is much larger than that of the upper layer.

The vertically integrated divergence of latent heat is shown in Fig. 5.17 and 5.18. Divergences of latent heat occur in the subtropics and convergences of latent heat occur in the ITCZ and the polar regions for both January and July due to the meridional mass circulation.

As in the case of the sensible heat, the influence of mass divergence (convergence) of the lower layer on the latent heat divergence (convergence) is much larger than that of in the upper layer since the water vapor mixing ratio also decreases monotonically upwards at all latitudes below 200 mb. The most significant difference between the divergence of latent heat and that of sensible heat is there is no significant divergence of latent heat over land due to the lack of water vapor over land in comparison with that over the ocean in the lower layer, especially during January.

In Figs. 5.19 and 5.20, we can see that the vertically integrated transport of potential energy counterbalances that of sensible heat and latent heat. The sensible and latent heat which converges in the tropics is converted to potential energy during both January and July. This potential energy is transported to high latitudes by the meridional mass flux of the upper layer.

In contrast to the temperature or water vapor
Fig. 5.17. Vertically integrated divergence of latent heat for January. The shaded area is convergence region. Unit is W/m².
Fig. 5.18. As in Fig. 5.17. except for July.
Fig. 5.19. Vertically integrated divergence of potential energy for January. The shaded area is convergence region. Unit is W/m².
Fig. 5.20. As in Fig. 5.19. except for July.
field, the geopotential increases with height so that the influence of the mass flux of the upper layer (divergence in the tropics) is much larger than that of the lower layer (convergence in the tropics) in the flux of geopotential energy.

The zonally-averaged divergence of sensible heat, latent heat, and potential energy for both January and July are shown in Fig. 5.21. We can clearly see the compensation between the divergence of potential energy and the divergence of the sensible and latent heat flux. Since the effect of the divergence of potential energy is large, the transport of potential energy is in the same direction as the mean transport of total energy.

Starr (1951) and many others have noted that the poleward transfer of energy in its kinetic form is quite small compared to the sensible or latent heat transfers. Although it is not important in the budget of total energy, the kinetic energy transport can be very important in the budget of the kinetic energy itself.

The vertically integrated divergence of kinetic energy for January and July is shown in Fig. 5.22 and 5.23. The maximum divergence occurs in the entrance regions of the middle latitude jet-stream maxima and the maximum convergences occur at the exit region during January. During July, the divergence (convergence) of kinetic energy is significantly reduced in the Northern
Fig. 5.21. Zonally-averaged divergence of potential energy(——), sensible heat(....), and latent heat(----) for January and July. Unit is W/m².
Fig. 5.22. Vertically integrated divergence of kinetic energy for January. The shaded area is convergence region. Unit is W/m².
Fig. 5.23. As in Fig. 5.22. except for July.
Hemisphere due to the weakness of jet stream. In the Southern Hemisphere, there are small longitudinal differences in the divergence (convergence) patterns in both seasons.

5.8 Energy conversion

The energy conversion terms which appear in the thermodynamic energy equations ((2.82) and (2.83)) and the kinetic energy equations ((2.61) and (2.62)) are interpolated linearly with respect to pressure from those terms on the \( \varphi \)-levels of OSU TWO-LEVEL AGCM.

The vertically integrated energy conversion for both January and July is shown in Fig. 5.24 and 5.25. The region of energy conversion from the enthalpy to the potential energy is located in the tropics while the region of energy conversion from the potential energy to the enthalpy is located in the subtropics during both seasons. It shows that the energy conversion is highly correlated with the convergence of sensible heat and the divergence of potential energy in both strength and location during January and July.

The sensible and latent heat which converges in the tropics in the lower layer is converted to potential energy, and this potential energy is converted to kinetic
Fig. 5.24. Vertically integrated energy conversion for January. The unshaded area is inverse conversion (potential energy $\rightarrow$ enthalpy) region. Unit is W/m$^2$. 
Fig. 5.25. As in Fig. 5.24, except for July.
energy. Thus the energy conversion term plays a role as a sink of sensible heat and a generator of kinetic energy, even though it does not affect to the total energy of atmosphere.

5.9 Vertical flux of total energy

Figs. 5.26 - 5.27 show that the vertical flux of total energy at the middle level is closely related to the Hadley circulation. This vertical transport of energy to the upper layer compensates the diabatic cooling and latent heat loss in the upper layer.

Strong upward fluxes of sensible heat, latent heat, and potential energy occur in the ITCZ, while downward fluxes occur in the subtropical region during both January and July as shown in Figs. 5.26 - 5.31. As expected, the local sensible heat flux dominates the other forms of energy. But the global-mean value of latent heat flux (15.02 W/m² for January and 14.11 W/m² for July) is comparable to that of sensible heat flux (16.73 W/m² for January and 14.37 W/m² for July). This shows the important role of latent energy for the global energy balance.

Even though the global-mean value of vertical flux of kinetic energy and potential energy (-0.07 W/m² and
Fig. 5.26. Vertical flux of sensible heat at the midlevel for January. Unit is W/m². The shaded area is upward flux region.
Fig. 5.27. As in Fig. 5.26. except for July.
Fig. 5.28. Vertical flux of latent heat at the midlevel for January. Unit is W/m². The shaded area is upward flux region.
Fig. 5.29. As in Fig. 5.28. except for July.
Fig. 5.30. Vertical flux of potential energy at the midlevel for January. Unit is W/m². The shaded area is upward flux region.
Fig. 5.31. As in Fig. 5.30. except for July.
0.28 W/m² for January and -0.02 W/m² and 0.09 W/m² for July) are not significant, they are important in kinetic energy balance itself. Fig. 5.32 and 5.33 show that there are downward fluxes in the entrance regions of the mid-latitude jet-stream maxima while there are upward fluxes in the exit regions during both January and July due to the vertical secondary circulations associated with these jet streams.

5.10 True storage of total energy

The vertically integrated true storage of total energy in the OSU TWO-LEVEL AGCM is shown in Fig. 5.34 and 5.35 for both January and July. The regions of maximum heat gain are over the northern Pacific and the northern Atlantic in the Northern Hemisphere, and western Australia, the Pacific - South American region, and southern Africa during January. The maximum heat loss regions are the Pacific - North American region, the middle Atlantic - European region, and northeastern Asia in the Northern Hemisphere, and southwestern Australia, the southeastern Pacific and the southern Atlantic region in the Southern Hemisphere. During July, the maximum heat gain regions are the polar regions from 30° to 50° and southern Atlantic while the maximum heat loss regions
Fig. 5.32. Vertical flux of kinetic energy at the midlevel for January. Unit is W/m². The shaded area is upward flux region.
Fig. 5.33. As in Fig. 5.32. except for July.
Fig. 5.34. Vertically integrated true storage of total energy for January. The shaded area is energy loss region. Unit is W/m².
Fig. 5.35. As in Fig. 5.34. except for July.
are in tropics, southeastern Australia, and southeastern South America.

Generally total energy is stored in lower latitudes (20°N - 40°N) and released in higher latitudes during January. During July heat is stored in higher latitudes (20°N - 90°N, 50°S - 90°S) and released in lower latitudes (20°N - 50°S).

5.11 Global total energy balance

Disregarding the residual, the total energy of the model atmosphere is maintained essentially by the sensible and latent heat flux from the earth's surface. The net radiational cooling balances the surface fluxes while the energy dissipation by friction and the storage of energy constitute a small residual.

A schematic diagram of the simulated vertically integrated total energy budget is given in Fig. 5.36 and 5.37 for January and July. During January, the gain of the total energy of the model atmosphere is 0.7309 W/m² by which the gain by sensible heat is 0.2352 W/m², that by latent heat is 0.4892 W/m². The loss of potential energy is 0.0112 W/m², and the gain of kinetic energy is 0.1770 W/m². This is balanced by the net source of total energy (0.8917 W/m²) and the dissipation of ki-
Vertically integrated total energy balance for January.
Unit is W/m².

<table>
<thead>
<tr>
<th>Component</th>
<th>Storage</th>
<th>Source (sink)</th>
<th>Conversion</th>
<th>Mass compensation</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensible heat</td>
<td>0.2352</td>
<td>3.6220</td>
<td>-2.8647</td>
<td>0.1419</td>
<td>-0.9857</td>
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<tr>
<td>Latent heat</td>
<td>0.4892</td>
<td>0.8917</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Latent heat</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Latent heat</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Potential energy</td>
<td>-0.0112</td>
<td>-2.1718</td>
<td>2.8647</td>
<td>-0.1092</td>
<td>-0.5933</td>
</tr>
<tr>
<td>Kinetic energy</td>
<td>0.0177</td>
<td></td>
<td></td>
<td>0.0161</td>
<td></td>
</tr>
<tr>
<td>Total energy</td>
<td>0.7309</td>
<td>2.3417</td>
<td>0.0000</td>
<td>-0.0320</td>
<td>-1.5790</td>
</tr>
</tbody>
</table>

Fig. 5.36.
Fig. 5.37. As in Fig. 5.36. except for July.
netic energy (-2.1718 W/m²).

During July, the gain of the total energy of the model atmosphere is 1.2800 W/m² of which the gain by sensible heat is 0.9868 W/m² and that of latent heat is 0.2875 W/m². The loss of potential energy is 0.0126 W/m², and the gain by kinetic energy is -0.0069 W/m². This is balanced by the net source of total energy (2.8808 W/m²) and the dissipation of kinetic energy (-1.8832 W/m²).

Even when the mass compensation correction is included in the sensible heat balance, the net heating, which is the net diabatic heating minus adiabatic cooling, exceeds the increase of atmospheric temperature during both January and July. Since 1 W/m² net global heating corresponds to an increase of the global average temperature by about 0.32K per month, we suspect that there may be a significant computational sink of heat in the model. This computational sink appears as the residual in Figs. 5.36 - 5.41.

The kinetic energy is maintained basically by a balance between the energy conversion which plays the role of a source of kinetic energy and friction which plays the role of a sink of kinetic energy.

During January, the net generation of the kinetic energy is 0.694 W/m² while 0.626 W/m² are generated during July. This shows that there is also computational
**Fig. 5.38.** Total energy balance of the upper layer for January. Unit is W/m².
<table>
<thead>
<tr>
<th></th>
<th>Storage</th>
<th>Vertical flux</th>
<th>Source (sink)</th>
<th>Conversion</th>
<th>Mass compensation</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensible heat</td>
<td>-0.0409</td>
<td>-16.7323</td>
<td>+16.5737</td>
<td>-0.9839</td>
<td>+0.9991</td>
<td>0.6606</td>
</tr>
<tr>
<td>Latent heat</td>
<td>-0.4155</td>
<td>+15.0210</td>
<td>15.0007</td>
<td>+0.1223</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Potential energy</td>
<td>-0.0111</td>
<td>-0.2844</td>
<td>-1.1883</td>
<td>0.9839</td>
<td>+0.1055</td>
<td>0.3066</td>
</tr>
<tr>
<td>Kinetic energy</td>
<td>0.0038</td>
<td>+0.0686</td>
<td></td>
<td>+0.0008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total energy</td>
<td>0.3673</td>
<td>-31.9591</td>
<td>30.3861</td>
<td>0.9831</td>
<td>+0.9672</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 5.39. Total energy balance of the lower layer for January. Unit is W/m².
<table>
<thead>
<tr>
<th>Storage</th>
<th>Vertical flux</th>
<th>Source (sink)</th>
<th>Conversion</th>
<th>Mass compensation</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensible heat</td>
<td>0.4624</td>
<td>14.3667</td>
<td>-11.6838</td>
<td>-0.5152</td>
<td>-0.5504</td>
</tr>
<tr>
<td>Latent heat</td>
<td>0.0352</td>
<td>14.1134</td>
<td>-13.5747</td>
<td>0.0284</td>
<td>+</td>
</tr>
<tr>
<td>Potential energy</td>
<td>0.0000</td>
<td>0.0872</td>
<td>-0.9546</td>
<td>-0.1243</td>
<td>-0.7879</td>
</tr>
<tr>
<td>Kinetic energy</td>
<td>-0.0063</td>
<td>-0.0218</td>
<td>1.6868</td>
<td>0.0083</td>
<td>+</td>
</tr>
<tr>
<td>Total energy</td>
<td>0.4913</td>
<td>28.5455</td>
<td>-26.1131</td>
<td>0.0000</td>
<td>-0.6028</td>
</tr>
</tbody>
</table>

Fig. 5.40. As in Fig. 5.38. except for July.
Fig. 5.41. As in Fig. 5.39. except for July.
sink in the kinetic energy even though the magnitude is small compared to that of sensible heat.

These computational sinks are the result of the truncation error from the dissipative numerical integration scheme used in the OSU TWO-LEVEL AGCM, the sampling error due to the use of data every six hours versus the ten minute time step used by GCM itself, and the errors involved in interpolating from $\sigma$ to $\varphi$ coordinates.

Figs. 5.38 - 5.41 show the schematic diagrams of the total energy budget in the upper layer and the lower layer for both January and July.

During January the incoming vertical flux at the middle level (31.9691 W/m$^2$) is basically balanced with the net sink of total energy (-28.0442 W/m$^2$) in the upper layer. In the lower layer, the outgoing vertical flux of total energy at the middle level (-31.9691 W/m$^2$) is basically balanced with the net source of total energy (30.3861 W/m$^2$). The true storage of total energy in the upper layer is 0.3636 W/m$^2$ and that in the lower layer is 0.3637 W/m$^2$.

During July, the incoming vertical flux at the middle level (28.5455 W/m$^2$) is basically balanced with the net sink of total energy (-26.1131 W/m$^2$) in the upper layer. In the lower layer, the outgoing vertical flux of total energy at the middle level (-28.5455 W/m$^2$) is basically balanced with the net source of total energy
(28.9943 W/m$^2$). The true storage of total energy in the upper layer is 0.4913 W/m$^2$ and that in the lower layer is 0.7887 W/m$^2$.

Even though the mass compensation correction of total energy (-1.0151 W/m$^2$ in the upper layer and 0.9831 W/m$^2$ in the lower layer during January and -0.6028 W/m$^2$ in the upper layer and 0.2891 W/m$^2$ in the lower layer during July) is included, the residual is -2.5462 W/m$^2$ in the upper layer and 0.9672 W/m$^2$ in the lower layer during January and -1.3383 W/m$^2$ in the upper layer and 0.0508 W/m$^2$ in the lower layer during July. In spite of the numerical sink, the residual of the lower layer is positive for both January and July.
6. Summary and suggestions for further research

6.1 Summary

A new vertical coordinate system has been introduced for the diagnostic analysis of the OSU TWO-LEVEL AGCM. The model output is divided into two layers - surface to 600 mb and 600 mb to 200 mb. The model diagnostics in the 600 mb to 200 mb layer may be directly compared with global observations on constant pressure surfaces in the upper troposphere. The lower-layer diagnostics are roughly comparable to observations at 800 mb surfaces over the oceans for most variables.

The mass balance has been examined with the use of a stream function and a potential function which are further decomposed into geostrophic and ageostrophic terms.

The location of the mid-latitude jet-stream maxima in the Northern Hemisphere during January and in the Southern Hemisphere during July agree well with observations. There are two different secondary circulations associated with jet streams - one divergent and one rotational secondary circulation. The divergent secondary circulation, which appears as the ageostrophic divergent component of mass flux, shows a thermally direct circu-
lation over the entrance regions of mid-latitude jet streams. In contrast to Namias and Clapp's hypothesis the circulation in the exit region is not thermally indirect; there is an equatorward flow over the exit regions only in the upper layer. This agrees well with observations in both layers. The rotational secondary circulation, which appears as an ageostrophic rotational component of mass circulation, shows an anticyclonic flow around westerlies and cyclonic flow around easterlies in both layers. Its strength \( \mathbf{D} \times \mathbf{H} \) in the upper layer is comparable with or even larger than that of the upper part of divergent secondary circulation \( \mathbf{v} \times \mathbf{D} \).

The annual variation of Hadley circulation and Walker circulation are in qualitative agreement with observations. But the northward mass flux in the South Asian-Indonesian region during January and the southward flow in the Indonesian-Australian region during July are stronger than observed.

The ageostrophic divergent component of mass flux has horizontal structures which resemble the structure of quasi-geostrophic linear waves on an equatorial \( \beta \)-plane. In particular, this component of the flow shows Kelvin-wave-like structures near 130°E - 60°W and 30°W - 90°E which are associated with the Walker circulations, and Yanai-wave-like structures near 90°E - 130°E and 60°W - 30°W which are associated with wester-
lies in the lower layer over the Indian Ocean and the western Atlantic Ocean.

The total energy balance has been broken down into four components corresponding to the enthalpy, latent energy, potential energy, and kinetic energy balances. The atmosphere receives more energy in the low latitudes than in high latitudes due to the surface sensible and latent heat flux. The net effect of radiation on the model is to destroy energy uniformly from pole to pole. The excess heating and the convergence of sensible heat and latent heat in tropics is balanced by the conversion to potential energy. The divergence of potential energy exceeds the convergence of the sensible and latent heat so that total energy is transported toward the polar regions. This conversion to potential energy is the source of the kinetic energy which is then dissipated by friction.

The residual of the total energy budget appears to be due primarily to the computational energy sink of the OSU TWO-LEVEL AGCM. Table 6-1 lists the residual for each of the three simulated Januarys and Julys as computed by Kim and Wang (1981) and as computed here. The mean value of residual which is -1.5824 W/m² for January and -1.2890 W/m² for July is quite comparable with that of sigma layer energy budget which is -1.4476 W/m² and -1.9951 W/m².
Table 6.1. Residual (W/m²) for sigma layer energy budget (from Kim and Wang, 1981) and for pressure layer energy budget.

<table>
<thead>
<tr>
<th>Month</th>
<th>sigma</th>
<th>pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st January</td>
<td>-1.7663</td>
<td>-1.6025</td>
</tr>
<tr>
<td>2nd January</td>
<td>-1.2597</td>
<td>-1.0562</td>
</tr>
<tr>
<td>3rd January</td>
<td>-1.3169</td>
<td>-1.5824</td>
</tr>
<tr>
<td>Mean January</td>
<td>-1.4476</td>
<td>-1.5824</td>
</tr>
<tr>
<td>1st July</td>
<td>-2.4184</td>
<td>-1.8894</td>
</tr>
<tr>
<td>2nd July</td>
<td>-2.0498</td>
<td>-1.3397</td>
</tr>
<tr>
<td>3rd July</td>
<td>-1.0690</td>
<td>-0.6380</td>
</tr>
<tr>
<td>Mean July</td>
<td>-1.9951</td>
<td>-1.2890</td>
</tr>
</tbody>
</table>
6.2 Suggestions for further research

The present diagnostic analysis makes it possible to interpret the model behavior over much of the globe in terms of constant pressure-level data. According to the results, many observed geographic features of the mass balance and total energy balance are well simulated and the behavior of mass flow and the divergence (convergence) of each form of energy over regions with significant orographic features are closer to observations than those performed with sigma coordinates.

The present diagnostic method overcomes some of the disadvantages of sigma-coordinate analysis which are caused by the significant deviation of the sigma surfaces from quasi-horizontal surfaces over the regions where orography is important. A number of additional diagnostic studies and problems are suggested below:

1) Physical problems

A new definition of Available Potential Energy (APE) for the $\Theta$ diagnostic coordinate system.

The calculation of the energy cycle is one of the useful
checks on plausibility of the results from OSU TWO-LEVEL AGCM. Observational studies of the energy cycle of the atmosphere usually use the Lorenz (1955) approximation for APE defined on a constant pressure surface. Lorenz's definition of APE can be used directly to the data for the upper layer in the new diagnostic coordinate system but a special treatment is needed for the lower layer.

Recently, Ghan and Kim (1983) proposed a mass-consistent scheme and then used the Lorenz's approximate formula for APE for the simulated energy cycle of OSU model atmosphere. But since this scheme simply extrapolates values in sigma coordinates to 800 mb surface even in regions with significant orographic features the atmospheric energy cycle obtained by this scheme may not be consistent with the true energy cycle of the model.

Boer (1982) introduced an impulse function for the diagnostic equations and the Lorenz approximation for APE to alleviate the difficulty resulting from the lower boundary. Even though this method may be effective for observational data with many significant levels or multilevel model data, this method may have the same problem as Ghan and Kim's method since the OSU AGCM has only two levels.

The analysis of the model-simulated Walker circulation and equatorial westerlies.
In this study, the simulated walker circulation and equatorial westerlies in the lower layer agree well with observations. The analysis of these features using the new diagnostic coordinates and the comparison of the GCM results with those of simplified models may lead to better understanding of these circulations.

2. Diagnostic problems

The statistics of standing and transient eddies.

Even though there is no significant difference in the zonal-mean statistics of sigma coordinates compared to those in the new diagnostic coordinate system, the statistics of eddies in sigma coordinates will be quite different over regions with significantly orographic features. The use of the new diagnostic coordinates may overcome some of the problems in the interpretation of the simulated upper-tropospheric eddies. However, the fact that the lower layer is still a sigma-type layer leaves some unresolved problems.

The mass-consistent method of Ghan and Kim (1983) interpolates variables to a constant pressure level in the lower troposphere. However, even though the local behavior of the simulated eddies in the lower troposphere are more easily interpreted, this method can
be misleading when the variables extrapolated to near-surface and underground regions are used in statistical analysis. This may simply reflect the fact that there is not a coordinate system which is useful for all diagnostic purposes.
Bibliography


