AN ABSTRACT OF THE THESIS OF

Mohammad Assiri for the degree of Master of Science in Electrical and Computer Engineering presented on December 9, 2016.

Title: Massive MIMO Channel Characterization

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Mario E. Magaña

It has been estimated that data traffic from the different mobile devices that range from smartphones to machine-to-machine (M2M) devices will exceed 15.9 exabyte per month by the year 2018. With this immense data growth, the current wireless communication systems suffer from the scarcity of the radio spectrum and eventually cannot cope with such high demands. Massive MIMO technology is one of the most promising technologies that has the potential to be the standard technology for the next generation of wireless communication systems. This thesis aims to characterize massive MIMO channels in every possible scenario for a typical seven-cell hexagonal configuration. Each cell can be described as follows:

- Either 8 or 16 single-antenna mobile users.
- Either 64 or 128 basestation antennas.
- Either critically spaced or sparsely spaced antennas.
• Either rich or clustered scattering.

• Either fast fading or slow fading.

• Either flat or frequency selective fading.

• Including the attenuation due to distance and shadowing.

The characterization is considered in both spatial and angular domains.
Massive MIMO Channel Characterization

by

Mohammad Assiri

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____________________________________
Mohammad Assiri, Author
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To the memory of my aspiring father and to the loving and endearing mother, I dedicate this work. It is with their love, kindness and encouragement I prosper and pursue my dreams. I would like to express my gratitude to my supervisor professor Mario Magaña for his continuous support and guidance toward finishing my thesis and degree. I also would to thank my friends and graduate fellows who shared with me their advice and motivations.
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Chapter 1: Introduction

1.1 A Brief History and Introduction to Wireless Communications

Wireless communication systems emerged a century ago. However, the rise and the need of other types or media of communication fluctuated with the passage of time causing the popularity of wireless communication to be limited for a few applications. For instance, the early era of TV broadcasting used radio transmission only, but it was almost completely replaced by wired or cable transmissions due to the better quality they delivered for this type of application. There are different types of wireless communications including broadcasting systems e.g. AM, FM radio and TV and wireless LAN with its different varieties. The interest here is in the cellular type of communication systems. A cellular system consists of multiple cells or areas of coverage where each cell contains a base station that serves many cellular subscribers within or close to that cell. Typically, a cell is represented as a hexagonal shape that indicates the coverage area by the serving basestation located in the center of the cell. However, in a real life situations the shape of the cell is not regular but rather random and is dictated by the surrounding environment. Figure 1.1 illustrates that. The general working principle of a cellular network is as follows. A mobile user generates a signal e.g., text message or a call which is then transmitted through an uplink (reverse) channel to its serving base station. All
Figure 1.1: Cellular coverage examples (a) hexagonal cell shape (nonrealistic) (b) realistic cell shape

Base stations are either connected with wires or wirelessly to the mobile switching center (MSC) which handles channel assignments and handoffs. Once the placed call is sent to the MSC, it forwards the call to the public wired telephone network and the call will be received at its desired destination. Similarly for the opposite direction (downlink), a call made from any destination would have to pass through the MSC to the nearest base station or the one that has the best connection via a downlink or the forward channel to the receiving mobile user.

The early implementation of cellular communication systems was analog with the launch of the first generation in 1981 mmwave[25]. They were constructed by the advanced mobile phone service (AMPS) in which a voice signal was modulated on a carrier wave then transmitted to be received by the user with the same modulation frequency. Thus, AMPS used different sets of frequencies for each cell in order to prevent interference between adjacent cells. Then in 1992, the digital communication systems emerged with second generation of communication systems such as the global system for mobile communication (GSM), time division
multiple access (TDMA) and code division multiple access (CDMA). Nevertheless, the second generation communication systems suffered from low data rates since they were mainly designed to handle voice data. The third generation of mobile communication systems was implemented in 2001 for the purpose of processing higher data rates than its predecessors. In 2011, the fourth generation was released with even higher data rates that reached a peak of 100 Mbps. According to Cooper’s law, the demand and the growth of data rates will continue to increase exponentially and hence forcing the next generation to bring higher data rates. With the increasing demand for higher data rates and better quality, the next generations of mobile communication systems will continue to bring rather higher data rates and better quality.

1.2 Classification of Wireless Channels

A wireless signal is basically the propagation of an electromagnetic wave through the surrounding media. Thus, wireless communications are governed by their environment. The physical nature of the propagation of radio signal is the result of diffraction, reflection and scattering. That is, a radio wave can reflect from a large surface that it impinges upon, or diffracted by an obstructing object with irregular surface or sharp edges, or deviated to many paths by small objects that have dimensions smaller than its wavelength. This dependency on the surrounding environment makes wireless communications more susceptible to high fluctuations in their performance and uncertainty.
A Wireless signal is prone to one fundamental phenomenon on which signal characteristic analysis is mostly based on. This phenomenon is called channel fading which is essentially the channel variations over time and frequency. A general classification of these variations states that there are two major types of channel variations: a large-scale fading of which the fading is due to distance such as path loss and shadowing by large blockage of obstacles between the basestation and the mobile user. The second type is the small-scale fading where the variation of the signal strength is due to multi path propagation of signal and the relative movement of the mobile user. Both types are related, as their names indicate, on how they participate in the overall variations of the signal’s strength. The large-scale fading depends on the distance between the transmitter and the receiver, hence it is independent of the frequency. On the other hand, the small-scale fading is frequency dependent since its constructive/destructive pattern on the multiple signal paths is of the order of the wavelength of the carrier frequency.

Furthermore, the relationship between those two major types of fading is de-
picted in figure 1.2. It shows that the large-scale fading is dependent on the mean of the signal’s path loss which decreases as distance increases and the shadowing across that mean value. In addition to that, the signal due to the local scatterers would vary rapidly causing the small-scale fading to take place [3]. Our interest in this paper is in the small-scale fading type hence we will not mention more details on the large-scale fading.

1.3 The Next Generation of Wireless Communications

It is no secret that the demand for more reliable high data rate transmission systems is greatly needed than before. With the advance toward the implementation of internet of things (IoT), the exponential increase in data consumption and production makes such demands as a necessity [22, 28] which complies with the well-known Cooper’s law for the wireless spectral efficiency. Furthermore, it has been estimated that the multimedia data traffic from the different mobile devices that range from smartphones to machine-to-machine (M2M) devices will exceed 15.9 exabyte per month by the year of 2018 [27, 28]. With this immense data growth, the current wireless communication systems suffer from the scarcity of the radio spectrum. The current spectrum allocation is limited between 700 MHz and 5.2 GHz for carrier frequency leaving the wireless operators with a limited bandwidth of 200 MHz for all bands of frequencies [22]. The combination of the aforementioned issues of high data traffic growth and insufficient spectrum turns the attention to the large-scale multiple input multiple output (LS-MIMO) or equivalently called
massive MIMO technology. Evidently, the multiple input multiple output technology has been around for decade and is currently deployed in 4G LTE and its advanced version. It is known as the conventional or classical point-to-point MIMO technique where there are multiple antennas in the both ends of the communication link making every single antenna element in the receiver to be under the combined transmission from all elements in the transmitter. Massive MIMO, unlike conventional MIMO, employs large number of antennas at the basestation side on the order of more than 100 antennas that simultaneously serve a few single antenna users, e.g., tens of users, using the same time-frequency resources. That is also known in the literature as multiuser multiple input multiple output abbreviated by MU-MIMO. This configuration of very large MU-MIMO system implies that the single-antenna terminals do not cooperate neither for transmission nor for reception. Furthermore, massive MIMO technology provides both high data rates and spectrum efficiency. It also yields an even higher power efficiency than in conventional MIMO with less interuser interference as well. All of these features alongside millimeter wave deployment guarantee reliable and enabling technology for the next generation of wireless communication systems. Chapter 3 will provide more details on massive MIMO technology.

1.4 Thesis contribution

This work is aimed to exhaustively study the massive MIMO channel characteristics under different possible scenarios that take place in realistic channel propaga-
tion. Specifically, we are interested in characterizing the massive MIMO channels for a multicell (seven hexagonal cells) cellular systems where each cell is described as:

- Either 8 or 16 single-antenna mobile users.
- Either 64 or 128 basestation antennas.
- Either critically spaced or sparsely spaced antennas.
- Either rich or clustered scattering.
- Either fast fading or slow fading.
- Either flat or frequency selective fading.
- Including the attenuation due to distance and shadowing.

This characterization is crucial for further channel processing and for developing more sophisticated channel estimation and detection schemes. It is with solid channel characterizations, that massive MIMO systems can be modeled realistically and that eventually will pave the way for more profound transceiver designs. That is because acquiring accurate channel state information (CSI) is important for the performance of massive MIMO channels\cite{23}, at least for the coherent transceivers, e.g., for high accuracy beamforming and detection and in mitigating the directed interference effect in multicell configurations \cite{11,18}.
Chapter 2: Wireless Channel Modeling

2.1 Wireless Channel Modeling

In this chapter, we describe two fundamental modeling techniques. One is based on the physical attributes of the electromagnetic waves and the other is based on stochastic models.

2.1.1 Physical Modeling of Wireless Channels

Since wireless channels use electromagnetic waves for signal propagation, it is suitable to study the channel using the so called ray tracing technique in which the received signal is approximated as the sum of direct and reflected rays of the transmitted signal. In ray tracing techniques, a propagating signal would have different scenarios each of which would describe the environment of the wireless channel for fixed transmitter antennas with fixed or moving receiving antennas in the free space or in the presence of reflectors. These scenarios also help in understanding how the signal strength is going to be affected.\footnote{In this context, we assume noise free environment}

We start with scenario one which describes an ideal situation of fixed antennas with free space medium of communication. Because of the proportionality between the electric field and the magnetic field, we can use the electric field to describe a
propagating signal in form of the electric field at time $t$ and point $u$. That is,

$$E(f,t, (r, \theta, \psi)) = \frac{\alpha_s(\theta, \psi, f) \cos 2\pi f(t - \frac{r}{c})}{r}$$ \hspace{1cm} (2.1)$$

where the point $u = (r, \theta, \psi)$, $\theta$ and $\psi$ are vertical and horizontal angles respectively, $r$ is the distance between the transmitter and point $u$, and the term $\alpha_s(\theta, \phi, f)$ is the sending antenna’s radiation pattern.

It follows that when the receiving antenna is at point $u$ and time $t$, then the electric field of the received signal is

$$E_r(f,t, (r, \theta, \psi)) = \frac{\alpha(\theta, \psi, f) \cos 2\pi f(t - \frac{r}{c})}{r}$$ \hspace{1cm} (2.2)$$

Hence from (2.2), we can see that in a free space propagation environment as the distance increases the $E_r$ decreases and thus the power per unit area decreases as of order $r^{-2}$.

Now, the mobility of the receiving antenna with velocity $v$ presents another scenario in which the electric field of the received signal at point $u(t)$ appears as

$$E_r(f,t, (r_0 + vt, \theta, \psi)) = \frac{\alpha(\theta, \psi, f) \cos 2\pi f(t(1 - \frac{v}{c}) - \frac{r_0}{c})}{r_0 + vt}$$ \hspace{1cm} (2.3)$$

Equation (2.3) introduces the so called Doppler shift effect. An effect that is mainly due to the movement of the receiver i.e., the change of the frequency by $(-fv/c)$
which plays a crucial part of our analysis as we proceed further. Another situation to be considered is when there are obstacles between the fixed transmitter and receiver. Here, there will be two copies—ideally speaking—of the transmitted signal at the receiving end, one from the direct path and the other from the reflected path. Thus, the electric field at the received signal would be a superposition of these two rays. That is,

\[ E_r(f, t) = \frac{\alpha \cos 2\pi f \left( t - \frac{r}{c} \right)}{r} - \frac{\alpha \cos 2\pi f \left( t - \frac{2d - r}{c} \right)}{2d - r} \]  

(2.4)

We notice in (2.4) that the phase change \( \Delta \theta = \frac{2\pi f (2d - r)}{c} + \pi - \frac{2\pi fr}{c} \) causes a constructive/destructive interference pattern of which the signal variations is noticed. Such pattern is described by the coherence bandwidth or alternatively coherence distance. Both dictate when or where the constructive/destructive interference of the transmitted wave takes place. That is, the coming waveforms from the transmitter would not experience a significant changes in their envelopes as long as their frequencies or the traveling distance do not exceed these two parameters.

Taking this scenario a step further and assuming the movement of the receiving antenna brings about the effect of what is called multipath fading. In this case, the received signal strength fluctuates according to the constructive/destructive interference patterns which are dictated by the Doppler spread parameter \( D_s = 2f_c v/c \) i.e., the maximum Doppler shift among the multipath components. Therefore, the
electric field of the received signal will be

$$E_r(f, t) = \frac{\alpha \cos 2\pi f \left( t \left( 1 - \frac{v}{c} \right) - \frac{r_0}{c} \right) }{r_0 + vt} - \frac{\alpha \cos 2\pi f \left( t \left( 1 + \frac{v}{c} \right) + \frac{r_0 - 2d}{c} \right) }{2d - r_0 - vt} \quad (2.5)$$

Generally, when we want to describe the time requirement of fading in radio waveforms, we use the coherence time which determines the time interval of a change from the peak of the waveform to its valley and it is typically as $c/4fv$.

In a different scenario, the ground is considered to be a reflecting object, but when the receiving mobile is far enough from the transmitter, the difference between the line of sight path and the one reflected from the ground is small compared to the wavelength $\lambda$ so it becomes negligible. That follows similarly to what is described before in scenario one, but with greater energy loss of the order of $r^{-4}$.

In general, the power of the received signal decays exponentially with distance at proportional rate to the density of the surrounding obstacles. Adding this power degradation to the number of users in a cell determines cell size. A more realistic case is when the mobile receiver is moving in multiple reflecting object environment. Then, the received waveform can be aggregated from the reflecting paths as a sum or as an integral of the scattered paths. If the distance between transmitter and receiver is very large, the received power will decay proportionally with $(d(d - r(t)))^{-2}$. 
2.1.1.1 The physical model based on the input/output relationship

Nonetheless, the use of ray tracing method is not preferable since it depends on the electromagnetic field equations which in turn are complex and require accuracy that cannot be accomplished in a quick fashion that suits the situation in cellular networks. Instead, the wireless channel is modeled in terms of its input-output behavior. This model is achieved in four steps as follows. The first step is to model the wireless channel as a linear time variant (LTV) system. As we saw in the ray tracing method, the received signal is the weighted sum of the different delayed versions of the transmitted signal at frequency $f$ i.e., $\sum_i a_i(f,t)\phi(t-\tau_i(f,t))$ where $a_i(f,t)$ is overall attenuation and $\tau_i(f,t)$ is the propagation delay of the $i^{th}$ path.

Assuming independence for each individual path from the frequency $f$ allows us to represent the received signal for any transmitted signal $x(t)$ as

$$y(t) = \sum_i a_i(t)x(t-\tau_i(t)) \tag{2.6}$$

The linearity of this representation permits representing the received signal $y(t)$ in terms of the LTV impulse response $h(\tau,t)$ of an impulse that is transmitted at time $t-\tau$ and observed at $t$ as,

$$y(t) = \int_{-\infty}^{+\infty} h(\tau,t)x(t-\tau(t))d\tau \tag{2.7}$$

where the impulse response is $h(\tau,t) = \sum_i a_i(t)\delta(\tau-\tau_i(t))$. Clearly, this representation of the multipath fading channels as LTV system reduces the complexity and
the required accuracy of the ray tracing method into a simple input/output relation between the transmit and receive antennas. Analyzing the impulse response of the multipath fading channel in both time and frequency domains provides rich details of the influential physical parameters such as the delay spread of channel paths and their doppler spread which in turn leads to categorizing and modeling the channel accordingly.

The second step is to obtain an equivalent baseband model for the time-varying channel obtained at the first step. This is because the communication over the channel uses a passband bandwidth (around a center frequency \([f_c - w/2, f_c + w/2]\) whereas the processing in both ends of communication requires a baseband bandwidth (around the origin). By considering a real passband transmitted signal \(x(t)\) as the output of the transmitter and the input of the time-varying multipath channel, one can obtain its complex baseband equivalent \(x_b(t)\) in the frequency domain as

\[
X_b(f) = \begin{cases} 
\sqrt{2}X(f + f_c), & f + f_c > 0, \\
0, & f + f_c \leq 0
\end{cases}
\]

which carries the same information as \(x(t)\). Or in the time domain, the relation between \(x(t)\) and \(x_b(t)\) is

\[
x(t) = \sqrt{2} \Re [x_b(t) e^{j2\pi f_c t}]
\]

The conversion process at transmitter and receiver, respectively, from baseband sig-
nal $x_b(t)$ to passband signal $x(t)$ and then back to baseband is implemented through the QAM modulation/demodulation process. That is the complex baseband signal $x_b(t)$ in the transmitter is modulated by sinusoid carriers $\sqrt{2}\cos 2\pi f_c t \Re\{x_b(t)\}$ and $-\sqrt{2}\sin 2\pi f_c t \Im\{x_b(t)\}$ and their sum results in a passband signal $x(t)$ that gets propagated over the channel. The reverse occurs at the receiver side as a first step after receiving the passband signal $y(t)$ as the output of the wireless channel in (2.6) in a baseband form as

$$y_b(t) = \sum_i a_i^b x_b(t - \tau_i(t))$$  \hspace{1cm} (2.9)

where $a_i^b$ is

$$a_i^b \triangleq a_i(t) e^{-j2\pi f_c \tau_i(t)}$$  \hspace{1cm} (2.10)

then passing it through a low pass filter to get back the equivalent complex baseband $x_b(t)$. This baseband representation shows that the the equivalent baseband received signal $y_b(t)$ is the sum of the delayed versions of the baseband representation of the transmitted signal $x_b(t)$ over all paths. It also shows that a path’s magnitude changes significantly when a change in its phase by $\frac{\pi}{2}$ since this change requires a change in the length of the path by quarter of the wavelength i.e., $\frac{1}{4l}$. Equivalently for a moving receiver, the required time for a remarkable phase change (change in the path amplitude ) is of order of $\frac{1}{4D_s} = \frac{c}{4f_c}$ seconds which is the amount of coherence time $T_c$ of the channel.

The third step for modeling the wireless channel based on its input/output behavior is to obtain the discrete time representation of the continuous time baseband
model in previous step by utilizing the sampling theorem. The continuous-time baseband transmitted signal can be represented in \( n \) samples of multiples of \( \frac{1}{W} \) as

\[
x_b(t) = \sum_n x[n] \text{sinc}(Wt - n)
\]  

where \( x[n] = x_b\left(\frac{n}{W}\right) \) and \( \text{sinc}(Wt - n) \) represents the orthogonal basis. The corresponding sampling of the received baseband signal \( y_b(t) \) results in

\[
y[m] \triangleq y_b\left(\frac{m}{W}\right) = \sum_l h_l[m] x[m - l]
\]  

The term \( h_l[m] \) represents the \( l \)th channel tap at time \( m \) which is explicitly described as,

\[
h_l[m] = \sum_i a^b_i\left(\frac{m}{W}\right) \text{sinc}[l - \tau_i\left(\frac{m}{W}\right)]
\]  

where \( a^b_i\left(\frac{m}{W}\right) \) is

\[
a^b_i\left(\frac{m}{W}\right) \triangleq a_i\left(\frac{m}{W}\right) e^{-2\pi f_c \tau_i(m/W)}
\]  

Thus, the \( l \)th channel tap \( h_l \) is the convolution of the baseband channel responses of the contributing paths with a pulse shape \( \text{sinc}(W\tau) \). In fact this sampling process is a modulation/demodulation method of the complex valued symbol \( x[m] \) by using pulse shaping such as sinc pulse or the raised cosine. That is, the complex symbol \( x[m] \) can be modulated using sinc pulse (or raised cosine) before the up conversion step in the transmitter takes place and at the receiver the demodulation occurs after the down conversion step where the receiver baseband signal \( y_b(t) \) gets
sampled at $m/W$ times at the output of the low-pass filter. It is important to note that the sampling rate complies with Nyquist rate which makes the use of sinc pulse impracticable because of its slow decaying time compared to the more practical raised cosine pulse shape. Here, the mobility of the receiving antenna poses the doppler spread which affects the sampling process by causing the bandwidth of the received baseband signal to be bigger than in the transmitted signal. However, it can be ignored since it is smaller than the bandwidth.

Then to complete the model, we include the additive white noise so that it can model a wireless channel in a realistic way. Adding white Gaussian noise $w(t)$ or AWGN to the model means that what happens to the transmitted signal $s(t)$ in the receiver applies to AWGN $w(t)$. Therefore, the wireless channel can be represented as

$$y(t) = \sum_i a_i(t)x(t - \tau_i(t)) + w(t) \quad (2.15)$$

where $w(t)$ is a white gaussian random process i.e., its power spectral density PSD $S_{WW}(\omega) = N_0/2 \quad \forall f$. The discrete-time baseband equivalence of (2.15) is

$$y[m] = \sum_l h_l[m]x[m - l] + w[m] \quad (2.16)$$

where $w[m]$ is the complex sampled noise after the conversion down to a discrete-time baseband noise random sequence. Since this noise sequence is jointly gaussian and white, it is uncorrelated and that means its samples are independent random variables with variance $\frac{N_0}{2}$. In other words, these complex random variables are i.i.d gaussian and thus their random sequence $\{w[n]\}$ is a circular symmetric complex
gaussian i.e., \( \{w[n]\} \sim CN(0, \sigma^2) \). This choice of modeling the noise as AWGN implies that the source of the noise is independent from the received signal's propagation paths.

From the previous modeling steps and examples, we notice important physical parameters that are fundamentally used to characterize wireless channels. Since a channel filter tap \( h_l[m] \) in (2.13) is composed of different paths contributions each with different doppler shift, the change of the phase in each propagation path contributes in impacting the magnitude of that \( l \)th tap. That is, these doppler shifts collectively can be taken in terms of their maximum difference which is called Doppler spread i.e., \( D_s = \max_{i,j} \left| f_{c_i} \tau_i'(t) - \tau_j'(t) \right| \). In the time domain, we say that the required time for a significant change in the channel is confined to the so called coherence time of the channel i.e., \( T_c \approx \frac{1}{4D_s} \). Meaning, that as long as these changes are smaller than the amount of \( T_c \) interval, the channel magnitude remains roughly the same. Furthermore, the inversely proportional relationship between \( T_c \) and \( D_s \) states the role of Doppler spread of the channel. Based on the coherence time, a multipath fading channel can be classified as a fast fading channel if its \( T_c \) is smaller than its delay requirement, e.g., symbol period, or a slow fading channel if its \( T_c \) is much greater than its delay requirement. On the other hand, the analogous representation in the frequency domain is dictated by the so called coherence bandwidth \( B_c \) which determines how fast a multipath fading channel changes with respect to frequency. Coherence bandwidth is primarily determined through the delay spread \( T_d \) of the multipath fading channel. That is \( B_c \approx \frac{1}{2T_d} \) where \( T_d \) is considered to be the maximum difference between the propagation
path times i.e., \( T_d = \max_{i,j} |\tau_i(t) - \tau_j(t)| \). Typically, a multipath fading channel is classified as an underspread channel because of the fact that the multipath spread \( T_d \) is much shorter than the coherence time \( T_c \). Then, based on the coherence bandwidth, a multipath channel can be categorized as flat or frequency-selective fading channel. That is if the coherence bandwidth is much bigger than the the transmitted signal bandwidth then the channel is flat fading channel, and if its coherence bandwidth is much smaller than the bandwidth of the transmitted signal then it is a frequency-selective fading. These types imply that in the flat fading one tap can represent the channel since it has very small delay spread \( T_d \) making all copies of transmitted signal appear overlapped i.e., unresolvable multipath components. Whereas in frequency-selective fading channel, multiple taps are needed to represent the channel due to its larger multipath delay spread \( T_d \) which makes the transmitted signal appears as multiple delayed replicas i.e., resolvable multipath components.

2.1.2 Statistical Modeling

Here, we introduce the second approach for approximating wireless channels based on probabilistic models. The way to that is by modeling channel as filter taps because they represent channel variations over time/frequency. One of the popular stochastic models of the taps \( h[m] \) is Rayleigh fading model. In this model and based on the assumption that there is a large number of reflected/scattered paths which are independent and identically distributed, the tap gains can be modeled
as circular symmetric complex gaussian random variables i.e., $h_l[m] \sim \mathcal{CN}(0, \sigma_l^2)$, where both real and imaginary parts of $h_l[m]$ are i.i.d gaussian random variables.

It follows that the magnitude of the tap $|h_l[m]|$ is a Rayleigh random variable that has a probability density function that is given by,

$$f(x) = \frac{x}{\sigma_l^2} \exp\left(\frac{-x^2}{2\sigma_l^2}\right), \text{ for } x \geq 0$$  \hspace{1cm} (2.17)$$

Then, the tap power (squared magnitude) will have an exponentially distributed density function given by

$$f(x) = \frac{1}{\sigma_l^2} \exp\left(\frac{-x}{\sigma_l^2}\right).$$ \hspace{1cm} (2.18)$$

Therefore Rayleigh model considers no dominant line of sight where as mentioned above it assumes that the environment is rich scattering one $^{24}$.

On the other hand, when there is a dominant line of sight path, the tap gain is modeled as

$$h_l[m] = \sqrt{\frac{k}{k+1}} \sigma_l e^{i\theta} + \sqrt{\frac{1}{k+1}} \mathcal{CN}(0, \sigma_l^2)$$ \hspace{1cm} (2.19)$$

where $k$ is the ratio of the energy in the line of sight path to that in the scattered/ reflected paths which dictates the behavior of the channel. The magnitude of the tap $|h_l[m]|$ in this representation has a Rician distribution.

Nonetheless, modeling channel filter taps as random variables does not satisfy the need for studying the channel variations over time. Hence, the channel filter taps are treated as random sequences $\{h_l[m]\}$ for a given $l$ path at time $m$. This enables
us to study the channel tap variations using its autocorrelation function given by

\[ R_l[n] = E[h_l[m + n]h_l^*[m]] \]  \hspace{1cm} (2.20)

This autocorrelation function assumes that \( \{h[m]\} \) is a wide sense stationary random sequence (WSS). Hence, the average power of this random sequence is

\[ R_l[0] = E\{|h_l[n]|^2\} \]  \hspace{1cm} (2.21)

In this perspective, the delay spread is defined to be the normalized weighted sum of the power over the contributing paths in the \( l^{th} \) tap i.e., \( T_d = \frac{1}{W} \sum_0^\infty R_l[0] \) and the coherence time \( T_c \) is defined as the smallest time \( (n > 0) \) in which \( R_l[n] \) is considerably different than \( R_l[0] \) where this difference depends on the choice of the bandwidth.
Chapter 3: Multiple Input Multiple Output and Its Scaled Up Version

In this chapter, first we describe the spatial multiplexing capability of multiple input multiple output (MIMO) communication system and what physical attributes the channel should have to harvest both power and degree of freedom gains. Then, the scaled up version of MIMO i.e., the so called massive MIMO will be discussed with some details including what makes it promising technology and and what are the major challenges that hinder achieving its full potential as well as a brief literature review of the major detection and precoding techniques that have been reported well in this area.

3.1 Multiple Input Multiple Output Channel (MIMO)

3.1.1 A deterministic MIMO channel case

A multiple input multiple output or MIMO channel would have \( n_t \) transmit antennas and \( n_r \) receive antennas. one can analyze this channel through the channel matrix \( \mathbf{H} \). For simplicity, we start with the deterministic MIMO channel where the channel matrix \( \mathbf{H} \) is a time-invariant \( n_t \times n_r \) matrix. One way to analyze the channel is by investigating its capacity. The MIMO channel can be represented as

\[
\mathbf{y} = \mathbf{Hx} + \mathbf{w} \tag{3.1}
\]
where $\mathbf{y}$ is the received signal $\in \mathbb{C}^{n_r}$, $\mathbf{x}$ is the transmitted signal $\in \mathbb{C}^{n_t}$, $\mathbf{H}$ is the channel matrix $\in \mathbb{C}^{n_t \times n_r}$, and $\mathbf{w} \sim \mathcal{CN}(0, N_0 \mathbf{I}_{n_r})$ is the circular symmetric white Gaussian noise $\in \mathbb{C}^{n_r}$. In this MIMO channel representation, the channel matrix $\mathbf{H}$ contains channel gains.

### 3.1.2 The capacity of deterministic MIMO channel using SVD

In order to evaluate this channel matrix, it needs to be decomposed into independent and parallel sub-channels using singular value decomposition technique. This results in

$$\mathbf{H} = \mathbf{U} \Lambda \mathbf{V}^*$$  \hspace{1cm} (3.2)

where $\mathbf{U} \in \mathbb{C}^{n_r \times n_r}$ and $\mathbf{V} \in \mathbb{C}^{n_t \times n_t}$ are unitary matrices and $\Lambda \in \mathbb{R}^{n_r \times n_r}$ is a diagonal matrix with real non-zero elements which are the singular values $\lambda_i$’s such that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{n_{\min}}$. Hence, we can represent the MIMO channel accordingly as:

$$\tilde{\mathbf{y}} = \Lambda \tilde{\mathbf{x}} + \tilde{\mathbf{w}}$$  \hspace{1cm} (3.3)

where $\tilde{\mathbf{y}} = \mathbf{U}^* \mathbf{y}$, $\tilde{\mathbf{x}} = \mathbf{V}^* \mathbf{x}$, and $\tilde{\mathbf{w}} = \mathbf{U}^* \mathbf{w} \sim \mathcal{CN}(0, N_0 \mathbf{I}_{n_r})$. The capacity of the channel would be

$$C = \sum_{i=1}^{n_{\min}} \log(1 + \frac{P_i \lambda_i^2}{N_0}) \text{ bits/sec/Hz}$$  \hspace{1cm} (3.4)

\cite{1} Bold upper case letter denotes matrix and bold lower case denotes vectors for the rest of the paper.
where $\lambda_i^2$ is the eigenvalue of $H^*H$ (or $HH^*$) and $P_i^* = (\mu - \frac{N_0}{\lambda_i^2})$ is the power allocation for $\lambda_i$'s. From (3.3), each singular value $\lambda_i$ corresponds to an eigenchannel that carries data stream. Thus, MIMO channel induces $n_{min}$ data streams and that shows it can provide spatial multiplexing for those $n_{min}$ data streams [25].

### 3.1.3 Rank and condition number of channel matrix

The above decomposition of $H$ gives a clear insight on how the non-zero singular values of $H$ dictates the performance of the MIMO channel. It is known that these non-zero singular values are exactly the rank of $H$ which in turns determines eigenmodes of the MIMO channel. When analyzing the capacity of MIMO channels, one needs to consider two different regimes. One is the high “signal-to-noise ratio” SNR regime and the second is low SNR. In high SNR regime, the rank of $H$ plays an important role for having high capacity. The key here is allocating equal amount of power on the eigenmodes of the channel. The closer they have equal power, the higher the capacity of the channel. That can be shown as

$$C \approx \sum_{i=1}^{n_{min}} \log(1 + \frac{P_i^* \lambda_i^2}{k N_0}) \approx k \log SNR + \sum_{i=1}^{k} \log\left(\frac{\lambda_i^2}{k}\right) \text{bits/sec/Hz} \quad (3.5)$$

where $SNR = \frac{P}{N_0}$ and $k$ is the number of non-zero eigenchannels (rank of $H$) which is also called the spatial degrees of freedom. Hence, a full rank $H$ would have $n_{min}$ spatial degrees of freedom. However, the rank of $H$ is not the only measure here. The ratio $\max_i \lambda_i / \min_i \lambda_i$ is the condition number of $H$ which is crucial for
determining the channels performance. That is, a MIMO channel is said to be well conditioned if its condition number is close to 1 which in turn implies higher channel capacity. Whereas in the case of low SNR regime, the strategy is to assign power to the strongest eigenmode only. Thus the capacity of the channel will be \( C \approx SNR(\max_i \lambda_i^2) \log_2 e \) bits/sec/Hz. Clearly, MIMO channel in high SNR regime provides both spatial degree of freedom gain and power gain but in the low SNR regime it only provides power gain.

3.1.4 Physical modeling of MIMO channels

Next using some ideal scenarios, we investigate the physical impact on the multiplexing capability of MIMO channels with the assumption of uniform linear antenna arrays layout. The first scenario resembles a single transmit antenna and an array of receiving antennas that are separated by \( \Delta_r \lambda_c \) where \( \lambda_c \) is the carrier wavelength and \( \Delta_r \) is the normalized separation between the the antennas. The assumption here is there are no reflecting/scattering objects i.e., line sight path (LOS) between the transmitter antenna and the receive antenna array plus that the receive antenna array size is much smaller than the distance between them and transmitter. This SIMO channel would be represented as

\[
y = h x + w
\] (3.6)
where \( y \) is the received vector, \( x \) is the transmitted symbol, \( w \) is the noise vector distributed as \( \sim \mathcal{CN}(0, N_0 \mathbf{I}_{n_r}) \), and \( h \) is the channel vector or the spatial signature. Since there are \( n_r \) paths from the transmitter to the receive antenna array, the gain of the \( i^{th} \) baseband signal is

\[
h_i = a \exp\{-j2\pi \frac{d_i}{\lambda_c}\} \quad (3.7)
\]

where \( d_i \) is the distance between the transmit antenna and the \( i^{th} \) receive antenna. It follows that the spatial signature is \( h = [h_1 \cdots h_{nr}]^T \). But because the paths from the transmitter to every receiving antenna is almost in parallel i.e., \( d_i \approx d + (i - 1)\Delta_r \lambda_c \cos \phi \) where \( d \) is the distance between first receive antenna and transmitter and \( \phi \) is the incidence angle of the line of sight path onto the receive antenna array, there will be a phase difference of \( 2\pi \Delta_r \lambda_c \cos \phi \) between the received signals to each antenna in the receive array [25]. Thus, the unit spatial signature
can be represented in the directional cosine $\Omega = \cos \phi$ as

$$e_r(\Omega) \triangleq \frac{1}{\sqrt{n_r}} \begin{bmatrix} 1 \\ \exp(-j2\pi \Delta_r \Omega) \\ \exp(-j2\pi 2 \Delta_r \Omega) \\ \vdots \\ \exp(-j2\pi (n_r - 1) \Delta_r \Omega) \end{bmatrix}$$

(3.8)

The capacity of SIMO channel is

$$C = \log(1 + \frac{P a^2 n_r}{N_0}) \text{ bits/sec/Hz}$$

(3.9)

This capacity shows that SIMO channel gives only a power gain of order of $n_r$. The opposite configuration on the SIMO would yield MISO channel where there are $n_t$ transmit antennas and a single receive antenna and a reciprocal vector channel. That is shown as

$$y = h^* x + w$$

(3.10)
where $\mathbf{h}$ is represented as the unit transmit spatial signature in the direction of $\Omega$ as

$$
\mathbf{e}_t(\Omega) \triangleq \frac{1}{\sqrt{n_t}} \begin{bmatrix}
1 \\
\exp(-j2\pi\Delta_t\Omega) \\
\exp(-j2\pi2\Delta_t\Omega) \\
\vdots \\
\exp(-j2\pi(n_t - 1)\Delta_t\Omega)
\end{bmatrix}
$$

(3.11)

This MISO channel has similar capacity as the SIMO channel and thus it provides power gain only. Taking the previous example to the next level with a MIMO configuration where there are $n_t$ transmit antennas separated by $\Delta_t$ and $n_r$ receive antennas separated by $\Delta_r$. A single path gain between the $k^{th}$ transmit antenna and the $i^{th}$ receive antenna is defined as

$$
h_{ik} = a \exp(-j2\pi d/\lambda_c) \exp(j2\pi(k-1)\Delta_t\Omega_t) \exp(-j2\pi(i-1)\Delta_r\Omega_r)
$$

where $\Omega_t = \cos \phi_t$ and $\Omega_r = \cos \phi_r$. So the whole channel matrix is to be represented as

$$
\mathbf{H} = a\sqrt{n_t n_r} \exp(-j2\pi d/\lambda_c) \mathbf{e}_r(\Omega_r) \mathbf{e}_t(\Omega_t)^*
$$

(3.12)

where $\mathbf{e}_r(\Omega_r)$ and $\mathbf{e}_t(\Omega_t)$ are as in (3.8) and (3.11) respectively. This channel matrix $\mathbf{H}$ has only one non-zero singular value i.e., its rank is 1. This MIMO channel has
The capacity of
\[ C = \log(1 + \frac{P a^2 n_t n_r}{N_0}) \text{ bits/sec/Hz} \quad (3.13) \]

Therefore, it has a power gain of \( n_t n_r \) fold but lacks the spatial degrees of freedom gain due to rank-one channel matrix \( \mathbf{H} \) which can be geometrically interpreted as all the transmit signals are projected onto a single non-zero eigenchannel.

The setup in the previous scenarios may contrast with the concept that MIMO channels provide both power gain as well as degree of freedom gain. However, that as aforementioned is not always achievable. That implies different configuration of the MIMO channels in order to achieve both gains. Such goal can be achieved by either placing transmit \(^2\) antennas apart from each other or by utilizing the multi-path environment. In this way the receiving antennas will receive the transmitted signal form different angles which will ensure a spatial degree of freedom gain. In a scenario where two transmit antenna are geographically separated by distance of order much bigger than carrier wavelength, that can be accomplished when the difference or separation between the directional cosines \( \Omega_r \) meets this condition:

\[ \Omega_r \neq 0 \mod \left( \frac{1}{\Delta_r} \right) \quad (3.14) \]

where \( \Omega_r = \Omega_{r2} - \Omega_{r1} \) i.e., the angular difference between the two LOS paths. This angular separation ensures that the spatial signatures of the MIMO channel are being linearly independent and distinct which means that the channel matrix \( \mathbf{H} \) is going to be full rank thus having \( n_{\text{min}} \) spatial degrees of freedom. Yet,

\(^2\)or receive antennas
having full rank on the channel matrix does not mean at all that the spatial signatures of $H$ can be effectively resolved at the receiver antennas. The key to have complete effectiveness on the spatial signatures is having a well-conditioned $H$ matrix. This conditioning of $H$ depends on the angular separation condition above in (3.14). In the scenario where the two transmit antennas are geographically separated, the channel matrix $H$ has two spatial signatures i.e., $H = [h_1 \; h_2]$ and its conditioning requires these two spatial signatures to be less aligned in order to be well-conditioned matrix. Geometrically, these two spatial signatures are separated by an angle $\theta$ such that

$$|\cos \theta| = |e_r(\Omega_{r1})^* e_r(\Omega_{r2})|$$  \hspace{1cm} (3.15)

where the term $e_r(\Omega_{r1})^* e_r(\Omega_{r2}) = f_r(\Omega_r) = \frac{1}{n_r} \exp(j\pi \Delta_r \Omega_r (n_r - 1)) \frac{\sin(\pi L_r \Omega_r)}{\sin(\pi L_r \Omega_r / n_r)}$, 
and $L_r \triangleq n_r \Delta_r$ is the normalized length of the receive antenna array. Thus, the conditioning of $H$ relies on the parameter $|\cos \theta|$ and hence rewriting (3.15) as

$$|\cos \theta| = \left| \frac{\sin(\pi L_r \Omega_r)}{\sin(\pi L_r \Omega_r / n_r)} \right|$$  \hspace{1cm} (3.16)

From (3.16), we notice that the parameter $|\cos \theta|$ is periodic with period $\frac{1}{\Delta_r}$ making its angular representation to have main lobes at multiples of $\frac{1}{\Delta_r}$ which results $|\cos \theta| \approx 1$. This angular representation declares that $H$ matrix is going to be ill-conditioned if the angular separation $\Omega_r \ll \frac{1}{L_r}$ where $L_r$ is the resolvability parameter. This resolvability parameter measures the effectiveness of the number of degrees of freedom i.e., if the condition above is met then the receive antennas
cannot resolve the transmitted signals. It also implies that for a fixed antenna array size $L_r$ the increase in the number of antennas will not help in increasing the resolvability. Furthermore, this angular resolvability can be represented by the beamforming patterns in which the angular resolvability can be depicted by the beam width of the main lobe of the receive beam forming pattern. The larger the size of the array $L_r$, the the narrower the beam width and the higher its resolvability. On the other hand, if we take the geographic separation to the receive antenna array, we would have a reciprocal channel. That is the channel matrix $H$ is going to be $H = [h_1 \ h_2]^*$ where the spatial signature $h_i = a_i \exp(j2\pi d_{i1}/\lambda_c)e_t(\Omega_{ti})$ for $i = 1, 2$ and $\Omega_{ti} = \cos \phi_{ti}$ is the directional cosine of the departure angle of path from transmitters to the $i^{th}$ receiver and $d_{i1}$ is the distance between the first transmit antenna and the $i^{th}$ receive antenna. Since this geographic separation is similar to the one in the previous scenarios, then the conditioning of $H$ holds the same angular separation condition. That is for $H$ to be well-conditioned the angular separation $\Omega_t = \Omega_{t2} - \Omega_{t1} \geq \frac{1}{L_t}$ where $L_t$ is the normalized length of transmit antenna array. In terms of transmit beamforming, the same effect of antenna array size $L_t$ applies such that the larger the size $L_t$, the sharper the beam width in the intended direction and the more effective the spatial signatures are.

In another scenario suppose that the antennas in the transmit array or receive array are not geographically separated and the channel here has two different paths one from the LOS and the other from the reflecting object. In this case, both paths have angles with transmit antenna array i.e., $\Omega_{ti} = \cos \phi_{ti}$ and with receive antenna array i.e., $\Omega_{ri} = \cos \phi_{ri}$. Thus the channel matrix $H$ is a superposition of
the channels matrices of the two paths. This multipath affect makes the matrix of each path act as a virtual relay between the transmit and receive antenna arrays. The MIMO channel matrix $H$ here can be represented as

$$H = \hat{H} \bar{H}$$

(3.17)

where $\hat{H} = \begin{bmatrix} a_1^b e_r(\Omega_{r1}) & a_2^b e_r(\Omega_{r2}) \end{bmatrix}$ which is the channel from the relays to the receive antenna array and $\bar{H} = \begin{bmatrix} e_t(\Omega_{t1})^* \\ e_t(\Omega_{t2})^* \end{bmatrix}$ which is the channel from the transmit antenna array to the relays. Hence the overall channel matrix $H$ is

$$H = a_1^b e_r(\Omega_{r1})e_t(\Omega_{t1})^* + a_2^b e_r(\Omega_{r2})e_t(\Omega_{t2})^*$$

(3.18)

where $a_i^b = a_i \sqrt{n_t n_r} \exp(-j2\pi d^{(i)}/\lambda_c)$ and $d^{(i)}$ is the distance between the first transmit and first receive antennas. What we notice from these scenarios is that the configuration of the MIMO channel could not provide degree of freedom gain unless the angular separation conditions for both paths in the transmit and receive antenna arrays are met i.e., $\Omega_{t1} \neq \Omega_{t2} \mod \frac{1}{\Delta_t}$ and $\Omega_{r1} \neq \Omega_{r2} \mod \frac{1}{\Delta_r}$. Furthermore, the conditioning of $H$ is directly dependent on the resolvability at both ends. That is, if the angular separation at the transmitter array as $|\Omega_t| \geq \frac{1}{\Delta_t}$ and the angular separation at the receive array as $|\Omega_r| \geq \frac{1}{\Delta_r}$ then $H$ will be well-conditioned. In a real world situation, the channel matrix $H$ is not going to be well-conditioned because of the angular separation at the transmitter or Bs in cellular networks is small due to its height and the proximity of the reflectors from the receive antennas.
but still can be enhanced through manipulating the size of the transmit array.

3.1.5 Modeling of multipath MIMO channels

Modeling multipath MIMO channel in terms of the individual paths is not efficient way when it comes to reality where the channel is nondeterministic. Therefore, MIMO channel needs to be modeled by stochastic methods. The best approach to do so is by modeling the channel taps since they assemble the contributions of the resolvable bins (paths) and hence reliably model the channel [25]. Assuming \( n_t \) and \( n_r \) uniform linear arrays of transmit and receive antennas with sizes respectively \( L_t \) and \( L_r \), the MIMO channel can be written as in (3.1). In this multipath channel, each path has its own attenuation \( a \) and makes angles with both transmit array \( (\Omega_{ti} = \cos \phi_{ti}) \) and receive array \( (\Omega_{ri} = \cos \phi_{ri}) \). Additionally, the channel matrix \( H \) can be represented as time-invariant, i.e.

\[
H = \sum_i a_i^h e_t(\Omega_{ri})e_t(\Omega_{ti})^* \tag{3.19}
\]

3.1.6 Angular domain for MIMO channels

The angular representation for the MIMO channel is used to get the clearest and best representation for the relationship between the transmit/receive antenna arrays and the surrounding environment. It represents both transmitted and received signals as orthonormal basis that span the transmitted and received signal spaces respectively \( (\mathbb{C}^{n_t},\mathbb{C}^{n_r}) \). That is, the transmitted signals and the received ones are
respectively transmitted/ received at direction $\Omega$ along the unit spatial signatures $\mathbf{e}_t(\Omega)$ and $\mathbf{e}_r(\Omega)$ and due to the periodicity of the spatial signatures, they span the transmitted/ received signal spaces thus forming orthonormal basis respectively as:

$$
S_t = \left\{ \mathbf{e}_t(0), \mathbf{e}_t\left(\frac{1}{L_t}\right), \ldots, \mathbf{e}_t\left(\frac{n_t-1}{L_t}\right) \right\}
$$

(3.20)

and

$$
S_r = \left\{ \mathbf{e}_r(0), \mathbf{e}_r\left(\frac{1}{L_r}\right), \ldots, \mathbf{e}_r\left(\frac{n_r-1}{L_r}\right) \right\}
$$

(3.21)

The beamforming patterns for these angular basis show that the transmitted/received signal over any path concentrate most of their energy along one of these angular basis i.e., $\mathbf{e}_t\left(\frac{k}{L_t}\right) / \mathbf{e}_r\left(\frac{k}{L_r}\right)$. For fixed antenna arrays’ length $L_t/L_r$, the beam width the main lobes of the angular basis is the same but the number of main lobes depends on how far the antennas from each other in the same array i.e, $\Delta_t/\Delta_r$. If the antennas are critically spaced ($\Delta_r = \frac{1}{2}$ wavelength) then there will be one main lobe pair of the angular basis ($\mathbf{e}_r\left(\frac{k}{L_r}\right)$). If the antennas are densely spaced ($\Delta_r < \frac{1}{2}$ wavelength), then there will be no main lobes for some of the angular basis and if the antennas are sparsely spaced ($\Delta_r > \frac{1}{2}$ wavelength), then there will be multiple main lobe pairs for different angular basis. In fact, this angular representation is merely an inverse Fourier transformation of the signal $\mathbf{x}$ such that $\mathbf{x} = \mathbf{U}_t \mathbf{x}^a$ and $\mathbf{x}^a = \mathbf{U}_t^* \mathbf{x}$ where $\mathbf{U}_t$ is an $n_t \times n_t$ unitary matrix whose columns are the angular basis of $\mathbf{x}$. Therefore, the angular domain representation for MIMO

3the same applies for the transmit array
channel is of the form

\[ y^a = H^a x^a + w^a \]  

(3.22)

where \( y^a = U_r^* y \), \( x^a = U_t^* x \), \( H^a = U_r^* H U_t \), \( w^a = U_r^* w \sim \mathcal{C}\mathcal{N}(0, N_0 I_n) \) and \( U_t \), \( U_r \) are unitary matrices whose columns are the angular basis of \( S_t \) and \( S_r \), respectively. This angular representation of the channel matrix \( H \) shows that the \((k,l)\) entry of \( H^a \) is

\[ h^a_{kl} = \sum_i a^b_i \left[ e_r(\frac{k}{L_r})^* e_r(\Omega_{ri}) \right] \left[ e_t(\Omega_{ti})^* e_t(\frac{l}{L_t}) \right] \]  

(3.23)

in which the term \( e_r(\frac{k}{L_r})^* e_r(\Omega_{ri}) \) has direct influence in the window width of \(|\Omega_{ri} - \frac{k}{L_r}| < \frac{1}{L_r}\). Hence, there is a set \( R_k \) of all paths whose receive directional cosine falls in \( \frac{k}{L_r} \) window around the receive angular basis \( e_r(\frac{k}{L_r}) \). A similar set \( J_l \) represents all paths whose transmit directional cosine within \( \frac{1}{L_t} \) window around \( e_t(\frac{l}{L_t}) \). Clearly, the size of the antenna arrays \( L_t \), \( L_r \) dictates paths resolvability such that different unresolvable paths contribute to form one resolvable path that will have a gain of \( h^a_{kl} \) in the angular domain matrix \( H^a \).

### 3.1.7 Statical modeling in the angular domain

For statistical modeling of MIMO fading channels, the time varying channel matrix \( H[m] \) and its angular representation \( H^a[m] \) are considered for a reliable modeling such that at time \( m \) the \( i^{th} \) path would have attenuation \( a_i[m] \) and makes angles \( \phi_{ti}[m] \) and \( \phi_{ri}[m] \) with the transmitters and receivers, respectively. It utilizes the
gains \((h^a_{kl}[m])\) of the resolvable angular bins. The gains at time \(m\) are modeled as independent random processes \(\{h^a_{kl}[m]\}_m\) since the angles \(\{\phi_{ti}[m]\}_m\) and \(\{\phi_{ri}[m]\}_m\) change in slower pace than the attenuations \(\{a^a_i[m]\}_m\). Since the gains \(\{h^a_{kl}[m]\}_m\) are the aggregation of many physical paths, the central limit theorem yields a complex circular symmetric Gaussian process. Therefore, the entries of \(h^a\) depend on the angular bins \((k,l)\) that form its entries \(h^a_{kl}[m]\) such that if an angular bin across \(k,l\) has no path, then the entry \(h^a_{kl}[m]\) is approximated zero.

### 3.1.8 Degree of freedom and diversity

The spatial multiplexing in MIMO channels depends mainly on the degrees of freedom of the channel. The angular domain representation of the MIMO fading channel provides clear way for identifying the degrees of freedom. That is, number of degrees of freedom = \(\text{rank}(H^a) = \min(\text{number of non-zero rows, number of non-zero columns})\).

There are two physical factors controlling the number of non-zero rows/ columns of \(H^a\). The first is the surrounding environment represented by the amount of the scattering and reflection, the more scatterers/reflectors the more non-zero entries in \(H^a\) and thus the higher the degrees of freedom. The second factor is the antenna array lengths \(L_t\) and \(L_r\). The bigger the size the more resolvability of paths and more non-zero elements of \(H^a\) and eventually the more the number of degrees of freedom. It is common approach to group the scatters/reflectors into clusters and model them as group instead individually. However, diversity amount plays impor-
tant role in the slow fading MIMO channels. Using the angular representation $\mathbf{H}^a$ of the matrix channel, the diversity amount in the channel is said to be the number of non-zero entries in $\mathbf{H}^a$. In general, channels with multiple-bounced paths (more scatters/reflectors) have high amount of diversity than the single-bounced ones due to increased number of angles between transmit and receive antennas.

3.1.9 The impact of antenna spacing

For fixed antenna arrays sizes i.e, fixed $L_t$ and $L_r$, antenna spacing $\Delta_t$ and $\Delta_r$ would have equivalent impact on the resolvability of paths. That is the maximum achievable angular resolution can be achieved by critically spacing the antennas i.e., $\Delta_r = 1/2$. Making the antennas sparsely spaced i.e., $\Delta_r > 1/2$ will make the paths from different angular windows accumulate into single bin which in turn reduces the antenna resolution and thus reduces degrees of freedom. On the other hand, spacing antenna densely i.e., $\Delta_r =< 1/2$ causes some of the angular basis to have no main lobes which means that they do not have physical contribution to the taps and hence they do not add any value in terms of antenna resolution. Nonetheless, if increasing the antenna separation is not accomplished by reducing the number of antennas (hardware limitation) then, the separation can increase degrees of freedom only when of the scattering is clustered onto one direction since that allows the scattered signal to be received in more angular bins. That can be interpreted in terms of the spatial and angular representations of the channel.

\[^4\text{ or equivalently changing the number of antennas at the arrays}\]

\[^5\text{ the same applies for } \Delta_t\]
matrices $H$ and $H^o$, respectively, as the more degrees of freedom the less correlated entries in $H$ or the less zero row/columns in $H^o$.

3.1.10 Rayleigh fading model

Multipath fading MIMO channels are commonly modeled as i.i.d Rayleigh fading. It models the entries of the matrix channel $h_{kl}^m$ as i.i.d circular symmetric complex Gaussian random variables. The angular domain representation of $H$ is related to the spatial form by this linear transformation

$$H^o[m] = U_r^* H[m] U_t$$ (3.24)

Hence, $H^o[m]$ will have the same i.i.d circular symmetric Gaussian distribution for its entries. The important aspect of the Rayleigh fading model is that it assumes richly scattered environment i.e., the resolvable bins have to have multiple paths and they have approximately equal energy. Therefore, it suggests that the antenna separation has to be critical or sparse depending on the direction of scattered paths. That is, if scattering from particular direction occurs, then the antennas should be sparsely spaced, otherwise they should be critically spaced.
3.2 State of The Art: Massive MIMO

3.2.1 The potentials and the challenges

The intuitive notion that the more antenna elements in transmitter or receiver side the more degrees of freedom and so the better the performance, has led to the implementation of multiple input multiple output (MIMO) technology in wireless communication systems. It provides a channel gain of $n_tn_r$ fold when implemented in multiuser systems. Increasing the number of antenna elements in MIMO systems i.e., massive MIMOs will entail a disparity between the huge number of antennas within the serving array and the number of terminals (users) that are being served simultaneously. However, such disparity is considered practical and necessary since the number of terminals is limited by the lack of methods for estimating the channel state information (CSI) if we opt for a large number of terminals. One ultimate goal for massive MIMO is to reach minimum power consumption per antenna with maintaining constant total power consumption i.e., the power per antenna element will be inversely proportional to the number of antennas. This power saving strategy is yet to be fully achieved due to factors that hinder it such as errors in CSI, interference and the need for multiuser multiplexing gains [19, 23]. There are several consequences for scaling up MIMO arrays. First, as the number of antenna elements increases, the randomness of the channel matrix becomes deterministic [23, 14]. That appears in the distribution of the singular values of the channel matrix and the quickly obtained operations on the channel matrix as well as being well conditioned. Secondly, increasing array sizes helps in leveling the
amount of thermal noise of the system so that it becomes mostly constrained by the interference from neighboring transmit arrays. That is, the massive MIMO performance is restricted mostly because of interference imposed by the reuse of pilot signals in the neighboring cells which is inevitable since the channel’s coherence time is finite. Third consequence is the increased resolution i.e., scattered paths have sharper beamwidth and so they are more likely to be resolved by the arrays. Nonetheless, the scaling up here is not infinite due to physical limitations on the feasible dimensions and the technical difficulties for constructing unlimited or extremely high dimensions. Thus, the size of “Massive ” MIMO, here, would take between 100 to 1000 antennas at a basestation (BS).

3.2.2 Massive MIMO channels modeling approaches

Massive MIMO channels are commonly modeled by two main methods. First is the correlation-based stochastic models (CBSM). The CBSM models tend to be less complex and are used more for theoretical analysis of massive MIMO channels. However, they lack the accuracy for modeling realistic propagation phenomena such as nonstationarity phenomenon and the near-field effect [28]. CBSMs include: Rayleigh channel model where the fast fading matrix is composed of i.i.d circular symmetric gaussian random variables, the mutual coupling model in which the antenna and mutual and load impedances are considered, the correlation channel model where the correlation between transmit or receive antennas is considered.

\(^6\)In literature, the small-scale matrix is referred to as fast fading matrix and the large-scale fading matrix is referred to as slow fading matrix
The second method is the geometry-based stochastic models (GBSM) which tend to have higher complexity than CBSMs but accurately capture massive MIMO channel propagation characteristics. GBSMs include: 2D channel models where the linear antenna arrays are used, the 3D channel models where the spherical, cylindrical or rectangular arrays are used to model the propagation beam in 3D plane.

3.2.3 Point-to-point MIMO

Shannon theorem of noisy channel coding states that for any communication link, there is an achievable rate such that for any transmission rate less than that achievable rate, there is a coding scheme that yields an arbitrary small error probability.

Staring with the point-to-point MIMO link, there are $n_t$ transmit antennas and $n_r$ receive ones and each receiving antenna is exposed to a combined transmission from all antennas at the transmit array. The narrow band channel can be represented as

$$x = \sqrt{\rho} G s + w$$

(3.25)

where $x$ represents received $(n_r \times 1)$ message vector, $\rho$ is an SNR measure, $s$ is the $(n_t \times 1)$ transmitted message vector, $w$ is the $(n_r \times 1)$ noise vector distributed as $\mathcal{CN}(0, 1)$ and $G$ is the complex-valued $n_r \times n_t$ channel matrix. Whereas in the wideband channel, the channel matrix $G$ has to be decomposed into independent and parallel narrow bands that have the form in (3.25).
3.2.4 Achievable rate on point-to-point link

The best performance for this point-to-point MIMO channel is its capacity or the maximum achievable rate. With assumption of receiver knowledge of the channel matrix $G$ as an i.i.d complex Gaussian inputs, the channel capacity is

$$C = I(x; s) = \log_2 \det(I_{n_r} + \frac{\rho}{n_t} G G^H)$$  \hspace{1cm} (3.26)$$

where $I(x; s)$ is the mutual entropy between the transmitted and received vectors, $I_{n_r}$ is $n_r \times n_r$ identity matrix and $H$ in the exponent is the Hermitian transpose. In the case of one antenna at both ends, the capacity becomes $C = \log_2(1 + \rho|G|^2)$.

A vivid representation of the achievable rate of the channel is given by the singular values of the channel matrix $G$. That is

$$C = \frac{\min(n_r, n_r)}{\sum_{i=1}^{\min(n_t, n_r)} \log_2(1 + \frac{\rho v_i^2}{n_t})}$$  \hspace{1cm} (3.27)$$

where $v_i$’s are the singular values of $G$. This singular value representation yields that the achievable rate ($C$) is directly influenced by the distribution of the singular values of $G$. That is, the decomposition $^{7}$ of $G$ constrains singular values as the trace of the product $GG^H = \sum_{i=1}^{\min(n_t, n_r)} v_i^2$. Then, from that we notice that we have two cases. One, the worst case is when all singular values are zero except one which can be a result of aligned line of sight transmission where antenna arrays cannot resolve individual paths i.e., $G$ has rank 1. the other case is the best case; where

$^{7}$Using SVD i.e., $G = \Phi D \Psi^H$
all singular values are equal which occurs under favorable propagation conditions that makes entries of $G$ as i.i.d random variables. With respect to theses cases, the channel capacity can be seen bounded by them as follows:

$$\log_2(1 + \rho n_r) \leq C \leq \min(n_t, n_r) \log_2(1 + \frac{\rho \max(n_t, n_r)}{n_t})$$ (3.28)

which is obtained after normalizing the entries of $G$ such that $tr(GG^H) \approx n_t n_r$. It implies that under preferable propagation conditions and with high SNR the capacity is proportional to $\min(n_t, n_r)$. However, in the case of low SNR, the capacity does not benefit from the multiplexity i.e, no multiplexing gain. That further can be seen as

$$C_{\rho \to 0} \approx \frac{\rho Tr(GG^H)}{n_t \ln 2} \approx \frac{\rho n_r}{\ln 2}$$ (3.29)

To restore the multiplexing gain in the low SNR regime, we could increase the number of antennas in one end and fix them in the other end with the assumption of asymptotic orthogonality of the propagation vectors. That is, increasing $n_t$ and keeping $n_r$ fixed yields $(\frac{GG^H}{n_t})_{n_t \gg n_r} \approx I_{n_r}$ as long as the row vectors of $G$ are asymptotically orthogonal. That results in an achievable rate

$$C_{n_t \gg n_r} \approx \log_2 \det(I_{n_r} + \rho I_{n_r}) = n_r \log_2(1 + \rho)$$ (3.30)

Or by increasing $n_r$ and fixing $n_t$, we can see that $(\frac{GG^H}{n_r}) \approx I_{n_t}$ under the assumption of orthogonal propagation vectors. It yields that the achievable rate
is
\[
C_{n_r,n_t} \approx \log_2 \det(I_{n_t} + \frac{\rho}{n_t} G G^H) = n_t \log_2 (1 + \frac{\rho n_r}{n_t})
\] (3.31)

Both ways satisfies the upper bound in (3.28) \cite{23}.

### 3.2.5 Multiuser MIMO

One drawback of increasing the number of antennas at the receiver side \((n_r)\) is that it increases the complexity of terminals for the forward link. The shortcomings of point-to-point MIMO can be resolved by deploying massive MIMO technique. That can be achieved by constructing \(M\) antennas array at the BS that serve simultaneously \(K\) independent terminals where \(M \gg K\). This MU-MIMO configuration makes possible that the \(K\) terminals (users) are separated of the order of multiple wavelength as well as being independent from each other. Assuming time division duplex access for this multiuser MIMO communication link, the reverse link propagation matrix will be the transpose of the forward link matrix. In fact, this reverse link propagation matrix \(G\) can be represented as:

\[
G = HD_{\beta}^{\frac{1}{2}}
\] (3.32)

where \(H\) is \(M \times K\) that represents the channel’s small-scale fading and \(D_{\beta}\) is \(K \times K\) matrix that represents the large scale fading. Typically, the large-scale fading coefficients are normalized such that the small-scale fading coefficients would have unit magnitude. Thus implementing massive MIMO with very large num-
ber of antennas at BS and under desirable propagation conditions will make the propagation vectors of $G$ asymptotically orthogonal. That is

$$\left( \frac{G^H G}{M} \right)_{M \gg K} = D_\beta^\frac{1}{2} \left( \frac{H^H H}{M} \right) D_\beta^\frac{1}{2} \approx D_\beta \quad (3.33)$$

For each channel use, the reverse link yields that the $K$ terminals transmit $(K \times 1)$ vector $q_r$ to be received by the array of $M$ antennas as $(M \times 1)$ $x_r$ received vector i.e.,

$$x_r = \sqrt{\rho_r} G q_r + w_r \quad (3.34)$$

where $\rho_r$ is proportional to the SNR and $w_r$ is the noise vector in which the components are i.i.d $\mathcal{CN}(0,1)$. With a power constraint on each terminal such that $E[|q_r|^2] = 1$ and assuming the BS knowledge of the channel state, the total throughput of this reverse MU-MIMO link is

$$C_{\text{sum}_r} = \log_2 \det(I_K + \rho_r G^H G) \quad (3.35)$$

which becomes under similar conditions of (3.33) as

$$C_{\text{sum}_r \ M \gg K} = \log_2 \det(I_K + \rho_r D_\beta) = \sum_{k=1}^{K} \log_2(1 + M \rho_r \beta_k) \quad (3.36)$$

It follows that with the assumption of asymptotic orthogonal propagation vectors of $G$, the BS can use a matched-filter (MF) to separate transmitted signals according
to the corresponding terminal that sent it i.e.,

\[ G^H x_r = \sqrt{\rho_r} G^H G q_r + w_r = M \sqrt{\rho_r} D \beta q_r + G^H w_r \]  

(3.37)

Hence, retrieving information from \( k \)th terminal would be as simple as obtaining the \( k \)th element in this MF.

The forward link of the MU-MIMO embodies the opposite of the reverse link such that an \( M \times 1 \) transmit vector \( s_f \) is transmitted by the \( M \) antennas to be received by \( K \) terminals as a \( K \times 1 \) received vector:

\[ x_f = \sqrt{\rho_f} G^T s_f + w_f \]  

(3.38)

where \( w_f \) is the noise vector which has an i.i.d \( \sim \mathcal{CN}(0,1) \) components and \( \rho_f \) is proportional to SNR of the channel use. With power constraint on the total transmit power such that \( E[\|s_f\|^2] = 1 \) and assuming knowledge of the channel by both ends as well as assuming a diagonal matrix \( D, \) that contains \( K \times 1 \) vectors, the sum- capacity is the following optimization problem [23]:

\[
C_{\text{sum-}f} = \max_{\gamma_k} \log_2 \det(I_M + \rho_f G D \gamma G^H) \\
\text{subject to } \sum_{k=1}^{K} \gamma_k = 1, \gamma_k \geq 0, \forall k
\]  

(3.39)
This optimization of the sum-capacity under conditions similar to \((3.33)\) becomes:

\[
C_{\text{sum}_{-fM \gg K}} = \max_{\{\gamma_k\}} \log_2 \det(I_K + \rho_f D^{1/2} G^H G D^{1/2})
\]

\[
= \max_{\{\gamma_k\}} \log_2 \det(I_K + M \rho_f D, D_{\beta})
\]

\[
= \max_{\{\gamma_k\}} \sum_{k=1}^{K} \log_2 (1 + M \rho_f \gamma_k \beta_k)
\]

(3.40)

Then, a MF precoder can be used in the transmitter such as;

\[
s_f = \frac{1}{\sqrt{M}} G^* D^{1/2}_{\beta} D^{1/2}_{\beta} q_f
\]

(3.41)

where \(q_f\) is the symbol vector to the terminals under unit expected power per terminal i.e., \(E[|q_{fk}|^2] = 1\) and \(p\) is the power vector such that \(\sum_{k=1}^{K} p_k = 1\). Thus, (3.41) can be rewritten as

\[
s_f \approx \sqrt{M \rho_f} D^{1/2}_{\beta} D^{1/2}_{\beta} q_f + w_f
\]

(3.42)

which have similar sum-rate as in (3.40) where \(\gamma = p\) i.e., the sum rate is \(\sum_{k=1}^{K} \log_2 (1 + M \rho_f p_k \beta_k)\).

3.2.6 Antenna and propagation aspects of massive MIMO

It is known that the performance of any type of MIMO system is directly dependent on the propagation environment and the characteristics of its antenna arrays.
The addition of ideal directional antennas to MIMO system as well as sufficient complexity of the propagation environment yield a degree of freedom per antenna element. However, this enhancement is merely theoretical since there is no (practically speaking) ideal antennas plus the environment is not always complex enough for the antenna arrays to exploit it for gaining better resolution. To overcome the lack of ideal directional antennas, the transmit antennas array needs a way to focus their transmission into one receiver or more. One way to do so is by the means of precoding. [23] provides an illustrative model showing how the addition or the increased number of antennas in the transmit array enhanced the spatial focusing toward an intended receiver (MISO case). Furthermore, the model used MF precoded to focus the transmitted signal at receiver’s point. It turns out that the larger the number of antennas in transmit array, the more focused the field strength at a certain point rather than a certain direction and the less interference experienced between terminals. Another technique for the spatial focusing is by time-reversal beamforming (TRBF) [23]. In TR-based communications, the transmitted signal is just the time-reversed copy of the channel impulse response. That is, after the communication is initiated by the terminal, each antenna element in the transmit array convolve transmitted signal by the each terminal with the time-reversed estimate of the channel response to the terminal. Then, the sum of the convolutions would be finally propagated by the antenna element. This TRBF technique provides both spatial focusing of the field strength and mitigation (or cancellation) of the inter-symbol interference. Thus it exploits spatial and temporal domains giving it the advantage over MF precoding.
The fact that the performance of MIMO system is influenced by the ability to focus the spatial signatures at the point of the receiver of interest urges the discussion about the physical aspects of the antennas. These antennas are in practice non isotropic and not unipolar which in turns affect the spatial resolution of the MIMO channel. That is, the more spatial focusing in the channel, the less spatial correlation between antennas in the transmit array and thus the higher the channel capacity. Yet, this spatial uncorrelation in terms of massive MIMO introduces challenges such ads the mutual coupling between adjacent antenna elements within transmit array especially for fixed array aperture. This mutual coupling cannot be ignored unless the antenna elements are well-spaced in that fixed array aperture which is impossible in the case of very large MIMO deployment due to the large number of antenna elements in a compact area. Therefore, the mutual coupling effect is inevitable and that would result in a performance degradation. To overcome the mutual coupling between antennas, compensating techniques are implemented such as impedance matching RF circuits. The problem with these coupling mitigation/cancellation methods is that they cause bandwidth reduction which in turn makes the whole antenna array acts like a single antenna. Additionally, it causes the so called ohmic loss resulting in gain loss. Another challenge considering the topological aspects of the antennas structure is that the antenna array in massive MIMO gets to be installed in 2-D or 3-D setup rather than a linear setup due to the physical limitations that linear arrays impose. Again this choice of antenna array structure brings about the problem of coupling especially when the antennas are densely implemented. A dense 2-D or 3-D array gives a
narrow range of elevation angles for the system to exploit the spatial resolution offered by them. The SU-MIMO model in [6] shows how mutual coupling (when dealt with by impedance matching circuits) reduces the performance of the system in terms of the instantaneous capacity. That is [23],

\[
C_{mc} = \log_2 \det(I_n + \frac{\rho}{n_t} \hat{G}_{mc} \hat{G}_{mc}^H)
\]  

(3.43)

where \( \hat{G}_{mc} = 2r_{11} R_t^{\frac{1}{2}} (Z_t + Z_r)^{-1} G R_t^{\frac{1}{2}} \) is the overall SU-MIMO channel, \( G \) is the propagation channel, \( R_t = Z_t, R_t = Z_t \) and \( \hat{G}_{mc} \) is the normalization of \( G_{mc} \). That can be further noticed when examining the effect of mutual coupling in small and the moderate antenna separation where in the earlier the off-diagonal elements of the impedance matrices are closer to the values of diagonal elements but far in the latter. In the case of MU-MIMO system, because terminals are considered autonomous, the transmit array is assumed to be uncorrelated and uncoupled.

The model provided in [23] analyzes the ergodic capacity per use of the reverse link versus antenna spacing under the uniform linear array and uniform square array setup. Relative to the capacity of an i.i.d channel which is the highest, the capacity under uniform square array (USA) configuration scored the worst capacity with both the coupling and correlation are being considered [23]. It also shows that the effect of coupling and correlation reduces as the spacing increases in both ULA and USA structures. The performance of the channel with consideration to coupling and correlation referred in \( \hat{G}_{mc} \) drastically degrades as the antenna separation gets smaller since the elements within USA are compact tightly and so
they lack the angular separation \[ 23 \, [11]. \] Furthermore, for much larger antenna elements in USA than terminals, the assumption of asymptotic orthogonality in (3.33) means that the propagation matrix \( \hat{G}_{mc} \) has a condition number approximately 1 i.e., well conditioned. However, that is not true with coupling presence since the large number of antennas in USA brings high mutual coupling which will result in performance degradation. If we ignore the coupling and consider spatial correlation only, we find that the performance is roughly close to that under the assumption of asymptotic orthogonal \( \hat{G}_{mc} \) because the size of USA is large enough to provide that spatial resolution, meaning that the correlation in the USA of the reverse link will not have significant impact on the overall capacity of the channel.

3.2.7 Measured channels

From the previous discussion, it is clear that the ability to obtain orthogonal propagation matrix \( G \) and separate users/data streams is confined to the correlation properties of the channel and the number of antennas at both ends of the MIMO link. One can look at the ratio of number of antennas at BS to those in the terminals as a measure of the possible spatial diversity gain. The bigger the ratio\(^8\), the larger the channel matrix and the larger the values of its singular valuers and the more stable they become. It is known in the conventional MIMO system, where \( M = K \), that the scatterers are clustered to have the same angle of arrival (AOA), angle of departure (AOD) and delay where the individual multipath components

---

\(^8\)The ratio in the conventional MIMO is usually 1 but in the massive MIMO it is much bigger than 1.
within a cluster are correlated. These clusters contribute in correlation between the terminals at the other end of the communication. However, this joint correlation (stemmed from the joint clusters) can be addressed using massive MIMO making these joint clusters to appear differently from the side of the transmit array and thus reducing the internal correlation between the individual multipath components. That is shown by the given results in [23, 17], where the increase of the number of antennas provided higher spatial resolution. The model used the commutative distribution functions CDF’s of the eigenvalues of $G^H G$. That is, in the massive MIMO the eigenvalues appeared to be less spread and had low variances i.e., more stable. On the other hand, for conventional MIMO the eigenvalues were less stable i.e., they had higher variances and tended to have larger spread between them compared to what is in massive MIMO case. Thus, these statistical measures prove the ability to resolve the correlation of joint clusters as well as the individual multipath components. That can be difficult in the case of the reverse link since the antenna arrays receive different power levels from different terminals which causes higher eigenvalues spread and lack of spatial gain.

3.2.8 Acquiring the CSI

In MU-MIMO system, the knowledge of the the CSI is crucial to achieve high spatial degrees of freedom. Thus, to reduce the complexity in the terminal devices the BS needs to have the sufficient knowledge of CSI in order to perform the precoding f the downlink channel. To do so, the terminals simultaneously transmit
symbols via TDD assuming constant frequency response for \( N_{coh} \) subcarriers used by \( N_{con} \) terminals. Then, the BS performs an estimation of the channel i.e., \( \hat{G}_{kk}^T \) accordingly to be used in the precoding of the downlink.

### 3.2.9 Precoding in the forward link

For downlink, the \( k^{th} \) terminal receives the corresponding component of the vector \( x_f \) as [23]

\[
x_f = G^T s_f + w_f
\]

where \( G^T \) is the downlink channel matrix and the \( s_f \) is the precoded version of \( q_f \) and \( w_f \) is the additive Gaussian noise. There are different techniques for precoding forward channel. For reference and comparison of the performance of precoders, the system with no interference between the terminals i.e., IF system is referred to be the best performing system since it has an SNR per terminal of \( \rho_f \) as \( M \rightarrow \infty \). With that being set as the benchmark of precoding performance, an actual precoded such as zero forcing (ZF) is examined under the assumption of the growth of the number of antennas \( M,K \) with constant ratio \( \alpha \). The vector \( s_f \) in the above expression in (3.44) is expressed by ZF precoded as

\[
s_f = \frac{1}{\sqrt{\gamma}} (G^T)^+ q_f = \frac{1}{\sqrt{\gamma}} G^* (G^T G^*)^{-1} q_f
\]

\(^9\alpha = \frac{M}{K}\)
where $\gamma$ normalizes the power in $s_f$ to $\rho_f$. So, the received symbol becomes

$$x_{fk} = \frac{q_{fk}}{\sqrt{\gamma}} + w_{fk}$$

and the received SNR per terminal is $SNR = \frac{\rho_f}{K\gamma}$. Choosing $\gamma = \frac{Tr(G^TG^*)^{-1}}{K}$ yields

$$SNR = \frac{\rho_f}{Tr(G^TG^*)^{-1}}$$  \hspace{1cm} (3.46)

Furthermore, if we allow the growth of $M,K$, the denominator $Tr(G^TG^*)^{-1}$ will converge to $\frac{1}{\alpha-1}$. Then, the ZF achieves $SNR = \rho_f(\alpha - 1)$. In real world systems, obtaining the term $(G^TG^*)^{-1}$ i.e., the inverse of $(K \times K)$ matrix is computationally expensive and would brings more complexity to the BS. However, this inverse can be approximated since the term $(G^TG^*)^{-1}$ would converge to identity matrix as $M \to \infty$. That implies that the ZF precoder can be performed much easier when we increase the number of antennas $M$ at BS. Thus, the precoded vector $s_f$ can be rewritten as

$$s_f = \frac{1}{\sqrt{\gamma}} G^* q_f$$  \hspace{1cm} (3.47)

which turns to be the MF precoder where $\gamma = \frac{Tr(G^TG^*)}{K}$.

In the case of faulty CSI, we estimate the forward link channel by means of MMSE. That is,

$$\hat{G}^T = \zeta G^T + \sqrt{1-\zeta^2}E$$  \hspace{1cm} (3.48)

where $\zeta$ is reliability measure of the estate which takes values $0 \leq \zeta \leq 1$ and $E$ is a $(K \times M)$ matrix with i.i.d $\mathcal{C}\mathcal{N}(0,1)$ entries. Here, the performance of the precoding methods is given via their signal to interference plus noise ratios (SINR)s. So the
SINR expressions for ZF and MF respectively are [23]:

\[
SINR_{ZF} = \frac{\zeta^2 \rho_f (\alpha - 1)}{(1 - \zeta^2) \rho_f + 1} \\
SINR_{MF} = \frac{\zeta^2 \rho_f \alpha}{\rho_f + 1}
\]  

From the SINR expressions above, one can conclude that SINR can be increased (with any value of \(\zeta\)) by growing up the antenna arrays. However, once we have a MIMO system with \(M \approx K\), the performance of linear precoders such as ZF and MF reduces significantly relative to the IF benchmark. Hence, the nonlinear precoding techniques such as Dirty paper coding (DPC) and vector perturbation (VP) are better performers since they have smaller performance gap relative to IF benchmark. The example given in [23] shows that when \(K = M = 15\) the performance of DPC precoder is much better than ZF but when \(M \gg K\) this performance gap turns to be tight between them and both perform close to IF level. This tight gap between the performance of DPC and ZF implies that performance gain does not require increasing the complexity of the system. Another interesting result tells that although MF was not having high performance in the different cases, it shows better sum-rate over ZF in low SNR regime.

Letting the size of arrays grows infinitely i.e., \(M \to \infty\) carries advantages as well as disadvantages. The advantages are dissipation of thermal noise and the small scale Rayleigh fading [14]. A disadvantage is that the unlimited increase of \(M\) with multicell MIMO system increases the so called pilot contamination. Pilot contamination is the intercell interference occurs during pilot training phase at the
beginning of communication link establishment in which the channel estimation in one BS gets contaminated by pilot signals from terminal neighboring cell which in turns results in partial beamforming to those terminals and hence that interfere with the beamforming coming from their BS. For a multi cell MIMO system, there are K terminals in each cell that has M antennas per BS. The terminals in the \( k^{th} \) cell receive the vector \( x_{fk} \) transmitted by the BS in that same cell. That is expressed as

\[
x_{fk} = \rho_f \sum_j G_{fj}^T s_{fj} + w_{fk}
\]  

(3.50)

where \( G_{fj}^T \) is the \((K \times M)\) forward link channel between BS in cell j and terminals in cell k and \( s_{fj} \) is the precoded copy of data symbols \( q_{fj} \) and \( \rho_f \) is the normalized average power. Using its pilot observations, the BS in the \( n^{th} \) cell computes channel estimation \( \hat{G}_{Tnn}^T \) as:

\[
\hat{G}_{Tnn}^T = \sqrt{\rho_p} G_{nn}^T + \sqrt{\rho_p} \sum_{i \neq n} G_{mi}^T + V_n^T
\]  

(3.51)

where \( G_{nn}^T \) is the forward channel matrix between the BS and terminals in the cell n, \( G_{mi}^T \) is the forward channel matrix between the BS at cell n and terminals at cell i, \( V_n^T \) is the receiver noise matrix during pilot training which consists of i.i.d \( \mathcal{CN}(0, 1) \) entries and it is uncorrelated with other matrices and \( \sqrt{\rho_p} \) is a measure of SNR during pilot transmission which indicates that the channel estimate \( \hat{G}_{Tnn}^T \) initializes identical pilot signals from all terminals. The channel estimate \( \hat{G}_{Tnn}^T \) will be highly contaminated if pilot signals of all cells including cell n are synchronized.
The idea of using different pilots for different cells is limited due to the finite signal space. The $l^{th}$ terminal in cell $j$ would receive the $l^{th}$ corresponding component of the vector $\mathbf{x}_{fj} = [x_{fj1}, x_{fj2}, \ldots, x_{fjk}]^T$. Thus, the MF precoder\footnote{Precoding matrix $(\hat{\mathbf{G}}^T_{nn})^H \triangleq \mathbf{G}^{*}_{nn}$} yields \cite{23}: 

$$
\mathbf{x}_{fj} = \sqrt{\rho_f} \sum_n \hat{\mathbf{G}}^T_{jn} \hat{\mathbf{G}}^*_{nn} \mathbf{q}_f + \mathbf{w}_{fj} = \sqrt{\rho_f} \sum_n \hat{\mathbf{G}}^T_{jn} \left[ \sqrt{\rho_p} \sum_i \mathbf{G}^T_{in} + \mathbf{V}^T_n \right] \mathbf{q}_f + \mathbf{w}_{fj}
$$ 

As $M$ increases, the precoding above would have the only significant terms when $j=i$. That is the term $\sum_n \frac{\mathbf{G}^T_{jn} \mathbf{G}^*_{jn}}{M} \mathbf{q}_f$ and by letting $M \to \infty$ we get $\frac{\mathbf{G}^T_{jn} \mathbf{G}^*_{jn}}{M} \to \mathbf{D}_{\beta_{jn}}$ which means the small-scale fading is cancelled. So, the received component by the $l^{th}$ terminal in cell $j$ is 

$$
\frac{\mathbf{x}_{fil}}{M \sqrt{\rho_f \rho_p}} \to \beta_{jl} q_{fil} + \sum_{n \neq j} \beta_{jnl} q_{fil}
$$ 

(3.53)

Based on that, the SIR of the $l^{th}$ terminal is 

$$
SIR = \frac{\beta_{jl}^2}{\sum_{n \neq j} \beta_{jnl}^2}
$$ 

(3.54)

This SIR shows that in the case of synchronized pilot signal transmissions, investing more power in the pilot training phase does not change the contamination level whereas if they are not synchronized, the power increasing helps in mitigating the interference at least between the neighboring cells.

The ZF precoded takes the pseudo inverse of the channel estimate \((\mathbf{G}^T_{nn})^+ = \mathbf{G}^{*}_{nn}\)
\( \hat{G}_{nn}^*(G_{nn}^T \hat{G}_{nn}^*)^{-1} \) yielding:

\[
(G_{nn}^T)^+ = \left[ \sqrt{\rho_p} \sum_i G_{in}^* + V_n^* \right] \left( \left[ \sqrt{\rho_p} \sum_i G_{in}^T + V_n^T \right] \left[ \sqrt{\rho_p} \sum_i G_{in}^* + V_n^* \right] \right)^{-1}
\]

Similarly to MF, as \( M \) grows the only correlated terms remain significant i.e.,

\[
(G_{nn}^T)^+ \to \frac{1}{M \rho_p} \left[ \sqrt{\rho_p} \sum_i G_{in}^* + V_n^* \right] \left( \sum_i D_{\beta in} + \frac{1}{\rho_p} I_k \right)
\]

Then, the received vector \( x_{fj} \) by terminals in the \( j^{th} \) cell becomes

\[
\sqrt{\frac{\rho_p}{\rho_f}} x_{fj} \to \sum_n D_{\betajn} \left( \sum_i D_{\beta in} + \frac{1}{\rho_p} I_k \right)^{-1} q_{fn}
\]

and so, the \( l^{th} \) terminal in the \( j^{th} \) cell receives the component

\[
\sqrt{\frac{\rho_p}{\rho_f}} x_{fj} \to \sum_i \beta_{ijl} + \frac{1}{\rho_p} q_{fil} + \sum_{n \neq j} \beta_{jnl} \sum_i \beta_{inl} + \frac{1}{\rho_p} q_{fil}
\]

where \( \beta_{jnl} \) is the large-scale fading factor between the \( l^{th} \) terminal in cell \( j \) and BS in the same cell. The SIR will be:

\[
SIR = \frac{\beta_{jil}^2 / (\sum_i \beta_{ijl} + \frac{1}{\rho_p})^2}{\sum_{n \neq j} \beta_{jnl}^2 / (\sum_i \beta_{inl} + \frac{1}{\rho_p})^2}
\]

which depends on \( \rho_p \) making its performance approaches MF’s performance as \( \rho_p \to 0 \). Another precoding regularizes the ZF pseudo inverse in a way such that
\(G_{nn}^{*}(G_{nn}^{T}G_{nn}^{*} + \delta I_{K})^{-1}\), where the parameter \(\delta\) is to be optimized according to the desired performance. In this regularized ZF precoder, we can eliminate the dependence of ZF precoder on \(\rho_{p}\) by setting \(\delta = \frac{-M}{\rho_{p}}\).

3.2.10 Detection in the reverse link

Detection in the uplink of MU-MIMO can be performed by different techniques. The simplest is the linear detectors that can achieve nearly optimal detection when \(M \gg K\). Other techniques such as iterative linear filtering, random step methods and tree-based methods are used when \(M \approx K\). In the iterative linear methods, the detection of the signaling vector \(q\) is resolved iteratively via linear filters which use the propagated information from previous estimate of \(q\). These methods are matrix-inversion dependent where the inversions are needed for every iteration and for every realization. The propagated information for each terminal/user can be either hard which contain decisions on \(q\) to soft which has probabilistic measures about the transmitted symbols. One of the iterative filter algorithm that is soft-information based is the conditional MMSE with soft interference cancellation (MMSE-SIC). Its working steps are: 1) a LMMSE (\(\tilde{q}\)) is obtained for the signaling vector \(q\). 2) An interference cancelled signal \(x_{ik}\) is constructed for each user to remove interuser interference. 3) Because the interference will not be fully removed due to errors in symbols estimation, an estimate of the remaining interference plus noise power is computed. 4) Based on that estimation, an MMSE

\[i.k\] respectively denote iteration number and user number.
filter conditioned on previous filtered output is computed for each user \((k)\). 5) Remove the bias to get a soft MMSE estimate of each symbol and then to be propagated to the next iteration. 6) The steps are repeated \(N_{iter}\) times. Since this algorithm needs an inverse of the channel matrix for every iteration and for every user, there will be \(KN_{iter}\) matrix inversions which brings high computation complexity. However, this complexity can be reduced by taking the inversion for user \(k\) as a rank-one update of a general matrix inversion at each iteration. An example of hard-information based algorithm is the Block-iterative generalized decision feedback equalizer (BI-GDFE). Unlike MMSE-SIC, it does not depend on the received vector \(x\) since its filters (input decision correlation IDC) vary with different users and iterations while the channel matrix \(G\) is fixed which in turns makes it possible to precompute the IDCs.

The random step methods do not relay on inversion of the channel matrix. \(^\text{12}\) The general working principle of random step algorithms states that the algorithm begins with initial vector and then it evaluates the MSE of neighboring vectors \((V_{\text{neigh}})\). Then, the neighboring vector with smallest MSE is picked for the next iteration and the algorithm is repeated \(N_{iter}\) times. Examples on random step algorithms are Likelihood ascent search (LAS) and Tabu search (TS). They differ in the searching process i.e., LAS constraints transitions to states with lower MSE values whereas TS allows the transition of search to the states with larger MSE values. Thus, that makes TS surpasses LAS plus the advantage of keeping track of searched point to be avoided in the next iteration.

\(^\text{12}\)Except those needed for obtaining MMSE.
In tree-decision class, the sphere decoding algorithm (SD) represents the optimal ML decoder in a sense that signaling points are considered in side a sphere with certain radius. SD complexity increases exponentially with K which makes it unsuitable for massive MIMO systems. Therefore, light weight algorithms of this class are proposed such as stack decoder and fixed complexity sphere decoder (FCSD). In the earlier, the search is reduced by means of expanding only search tree-nodes that have the least euclidian distance to the received signal. In the latter, it takes combinations of first 'r' symbols of q and enumerate them for each combination it detects the remaining K-r symbols by ZF-decision feedback (ZF-DF). This strategy of searching provides suboptimal performance and in the same time keeps the complexity constant. It turns out that FCSD algorithm works best for MIMO systems where M \( \approx \) K since it enhances channel matrix conditioning number for such systems. The soft detection methods can be driven from the above hard-based detection schemes. The analytical results for CDMA under a large \( \alpha \) shows similar performance (by means of spectral efficiency ) among MF, ZF and linear MMSE as well as optimum joint detectors. On the contrary, under small \( \alpha \), ZF has worse performance than others. Furthermore, the analysis of density evolution of both conditional and unconditional MMSE-SIC under CDMA shows suboptimal coded BER waterfall levels. In fact, MMSE-SIC has similar specral efficiency as MAP detector. The random step and tree-based schemes search mainly for best fitting candidate q vectors to approximate log-likelihood ratio evaluation. That is done in TS and FCSD detectors such that they begin with hard detection and

\footnotesize{\textsuperscript{13}with a priori probability.}
then search candidate vectors to have an approximate max-log LLR. For large scale soft-output MIMO, statistical techniques such as Markov chain Monte-Carlo (MCMC) can be used as soft detectors.
Chapter 4: Simulation Results and Analysis

In this chapter, we investigate the massive MIMO channels using correlation based stochastic models (CBSM), specifically we use the Rayleigh fading model for the small-scale fading channels. By using the Rayleigh fading channel model, we gain the simplicity of the model and the ability to model richly scattering environments as well as clustered. Additionally, the simulations at first account for the single cell situation where the large-scale fading effect is not included. The goal here is to characterize the massive MIMO channel in different scenarios. Our approach here is that we model the channel first as a narrowband or flat fading channel, and second as a wideband or frequency selective fading. In both types we examine and analyze the channel in slow or fast fading scenarios. Later in the chapter, we include the multicell system of massive MIMO and the large-scale fading alongside the analysis of the obtained results for all different scenarios. The analysis here is on the uplink of the massive MIMO channel under the assumption of time division duplexing.

4.1 Flat Fading Massive MIMO Channels

In this part, we model the massive MIMO channel as flat and either slow or fast fading. Thus, the main object here is to study the small-scale fading channel
i.e., $\mathbf{H}$. Each coefficient of the small-scale fading matrix i.e., $[\mathbf{H}]_{mk}$, is merely the aggregation or the sum of all non-resolvable paths between antenna $m$ at the basestation and mobile terminal $k$. As mentioned in chapter 2, for a dense or rich scattering environments CLT can be invoked and the result is Rayleigh distributed taps magnitudes and uniformly distributed phases of all non-resolvable paths that compose every tap in the narrowband channel i.e., $[\mathbf{H}]_{mk} \sim \mathcal{CN}(0,1)$ . This aggregation of paths implies that they do not have very distinct delays and thus they are lumped at the antenna array as one single resolvable path for each transmitted signal. It also implies that the delay spread $T_d$ of the channel is very small compared to the inverse of signal bandwidth $1/Bw$. As a result of this very short delay spread, the flat fading channel can be represented in a single tap. It also has a negligible intersymbol interference (ISI) since the delay between the paths of each tap is much smaller than the resolvability period i.e., the symbol period $T_s$. Generally, a channel is said to be narrowband or flat fading channel if it fulfills the condition,

$$T_s >> T_d \quad \text{or} \quad Bw << B_c$$ \hspace{1cm} (4.1)

where $B_c$ is the channel coherence bandwidth and $Bw$ is the symbol bandwidth.
4.1.1 Simulation parameters

The simulations here consider two main configurations of massive MIMO cellular systems.

First, a system that consists of $M = 64$ or $128$ as the number of transmit antennas at the BS and $K = 8$ or $16$ as the number of terminals or mobile users. The initial carrier frequency $f_c$ for the simulation of both configurations is $29 \text{ GHz}$ and can be increased to $60 \text{ GHz}$. We assume a BPSK modulation and thus the data rate or bit rates of the signal transmission is equal to the symbol modulation rate. Then the choice of the bandwidth is based on the potential of massive MIMO of having very high data rates as well as to abide by those of realistic values such as in LTE where the bandwidth is of $20 \text{ Mhz}$. Then, this choice of bandwidth dictates the symbol period to be $T_s = 5 \times 10^{-8} \text{ seconds} (50 \text{ ns})$. The initial value of Doppler spread of the channel which is calculated as $D_s = \frac{f_c v}{c}$ is equal to $1.0741 \times 10^3 \text{ Hz}$ for initial speed of mobile $v$ of $40 \text{ km/h} (11.1111 \text{ m/s})$. Hence, the channel coherence time $T_c = \frac{1}{4D_s}$ is $2.3276 \times 10^{-4} \text{ second} (\approx 232.8 \mu\text{sec})$. Other important parameters are the channel coherence bandwidth $B_c$ and its reciprocal the delay spread $T_d$.

As defined in chapter 2, the coherence bandwidth is $B_c = 1/2T_d$ and according to condition in 4.1 the channel has a very short delay spread $T_d$ and much wider coherence bandwidth $B_c$. Thus, in the simulation of flat fading case the coherence bandwidth is 10 times bigger than the symbol bandwidth which yields a $B_c$ of $200 \text{ Mhz}$ for the initial values of bandwidth and a $2.5 \text{ ns}$ as a delay spread parameter.

In the simulations, we extend the aforementioned parameters to examine situations
<table>
<thead>
<tr>
<th>Number of BS antennas</th>
<th>$M = 64$ or $128$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of terminal</td>
<td>$K = 8$ or $16$</td>
</tr>
<tr>
<td>carrier frequency $f_c$</td>
<td>$29$Ghz or $60$ Ghz</td>
</tr>
<tr>
<td>mobile speed $v$</td>
<td>$40$km/h or $60$ km/h</td>
</tr>
<tr>
<td>Doppler spread $D_s$</td>
<td>$D_s = \frac{f_c v}{c}$</td>
</tr>
<tr>
<td>Symbol bandwidth $Bw$</td>
<td>$20$Mhz up to $50$Mhz</td>
</tr>
<tr>
<td>Symbol time $T_s$</td>
<td>$T_s = 1/Bw$</td>
</tr>
<tr>
<td>Coherence time $T_c$</td>
<td>$T_c = \frac{1}{4D_s} = 232.8$ or $75\mu sec$</td>
</tr>
<tr>
<td>Coherence bandwidth $B_c$</td>
<td>$B_c = 1/2T_d = 10 \times Bw$</td>
</tr>
<tr>
<td>Delay spread $T_d$</td>
<td>$2.5$ ns</td>
</tr>
</tbody>
</table>

Table 4.1: simulation parameters for flat fading channel model

similar to those in a real life propagation environment. For instance, the carrier frequency can be changed to $60$ GHz, the speed of mobiles can be increased to $60$ km/h and the symbol bandwidth can be increased up to $50$ MHz for a very high data rate demanding application such as ultra high definition multimedia (UHD). Simulation parameters are summarizer in table 4.1.

4.1.2 Slow fading scenario

The channel would have a slow fading when the coherence time is much greater than the symbol time $T_s$ or equivalently the doppler spread $D_s$ is much smaller than the signal’s $Bw$. The channel is said to undergo both flat and slow fading if the following condition is met

$$T_s \gg T_d \text{ or } Bw \ll B_c$$  \hspace{1cm} (4.2)
for being flat fading and,

$$T_s << T_c \quad \text{or} \quad Bw >> D_s$$ (4.3)

for being slow faded channel. The channel here, stays constant for much longer time (compared to $T_s$) making many symbols to have the same channel response or we can say that the channel’s frequency response remains flat for $T_c/T_s$ symbols. It is crucial for the channel to act flat in terms of its frequency response to have a larger coherence bandwidth i.e., $B_c$ has to be much larger than the symbol bandwidth $Bw$. Thus, in this model the $B_c$ is 10 times bigger than symbol bandwidth of the signal being transmitted through this channel i.e., $B_c = 200$ Mhz. Thus, there will be $T_c/T_s \lfloor = 4655$ transmitted symbols in a single coherence period $T_c$.

The following results shows the channel variations over the time which is shown over 7 $T_c$ periods for both system configurations. The small scale coefficients of the channel’s matrix $H$ that we are interested in change significantly in each single $T_c$.

This change is depicted throughly in figures 4.1 and 4.2 representing the flat and slow fading case of the massive MIMO channel. As we can see from both figures 4.1 and 4.2 the channel models a rich scattering environment i.e., Rayleigh channel model in which the channel matrix coefficients are nonzero for all terminal vectors in the channel. Taking this representation into more details would be best if a snapshot of the channel is taken for a single coherence period as shown in figure 4.3. Further, this channel remains constant for each link allowing 4655 symbols
The flat and slow fading channel over time

Figure 4.1: Channel variations over time as $7 \, T_c$ for a massive MIMO channel of $M = 64$ antenna, $K = 8$ users
Figure 4.2: Channel variations over time as $8 \ T_c$ for a massive MIMO channel of $M = 128$ antenna, $K = 16$ users
Figure 4.3: Channel variations over single $T_c$
Figure 4.4: Example of flat and slow fading over selected links
to be transmitted in the given amount of $T_c$ which is as stated above 232.8$\mu$sec. For example, in figure 4.4 the links between first serving antenna and first user, fifth antenna and fourth user as well as forty fourth antenna at BS and eighth user, respectively, show the slow fading of the channel over the course of time as multiple coherence times of the channel. The time in figure 4.4 as 6 consecutive coherence intervals would represent a total of 27930 symbol transmissions per terminal. That is the equivalent of roughly 27 kbits per 6 channel uses or about 4 kbits per a channel use considering our choice of BPSK modulation.

Another way to look deeper into this flat and slow faded massive MIMO channel is by looking at the channel with respect of each individual user/terminal over the course of the whole channel evolution i.e., seven coherence periods.

Each of the figures 4.5, 4.6 and 4.7 contains 8 subplots of which each represents the channel evolution for the corresponding terminal with all $M$ BS antennas. It produces clearly the channel magnitudes between all the basestation antennas and every single mobile user in that cell. Every coherence period is shown in a different color than the others to further distinguish the significant changes every $T_c$ and for the purpose of comparison with different configuration for the same and different configurations that come next. It can been seen from those figures that the terminal channels within each single $T_c$ have different magnitude with different antenna elements of the BS array. Such disparity in the magnitudes of those antenna elements comes from the fact that the environment is richly scattered as well as the fact that each tap of this channel i.e., the magnitude of the channel between one antenna and an individual user is the aggregation of many
Channel magnitudes between all BS antennas and each single terminal over time

Figure 4.5: The magnitude of the flat and slow fading coefficients between each terminal and all $M$ BS antennas over a span of time as multiple of coherence period for the a massive MIMO channel of $M = 64$ antennas and $K = 8$ terminals.
Figure 4.6: The magnitude of the flat and slow fading coefficients between each terminal and all $M$ BS antennas over a span of time as multiple of coherence period for the a massive MIMO channel of $M = 128$ antennas and $K = 16$ terminals. This figure is for the channels of the first 8 users.
Channel magnitudes between all BS antennas and each single terminal over time

Figure 4.7: The magnitude of the flat and slow fading coefficients between each terminal and all $M$ BS antennas over a span of time as multiple of coherence period for the a massive MIMO channel of $M = 128$ antennas and $K = 16$ terminals. This figure is for the channel of the other 8 users.
nonresolvable paths that might add constructively or destructively depending on how the total phase shift of those nonresolvable paths might add up. That is, if they add up to be an even multiple of $\pi$ then they will constructively add up the magnitude of that tap otherwise, they will destructively add and result in less magnitude of the tap or the link between the terminal and the antenna element of the BS antenna.

We know change the carrier frequency $f_c$ and the mobile user speed $v$ to 60 Ghz and 60 km/h, respectively. We assume that all $K$ mobile terminals move at the same speed. This parameter change results in a larger Doppler spread of the channel i.e., $D_s = 3333.3Hz$ which is roughly 3 folds of what was initially modeled. As a result of this increased doppler spread, the coherence time of the channel is smaller than the previous value i.e., $T_c = 7.5 \times 10^{-5}sec$ or 75 $\mu$sec. Thus, the amount of possible transmitted symbol per channel coherence time or say per channel use decreases to about 1499 symbols for a fixed bandwidth of 20 Mhz. This decrease of possible transmitted symbols is equivalent to 1 kbit per channel use, and it is remarkably less than the previous case when the coherence period of the channel was higher. However, the channel remains flat and slow faded since the coherence bandwidth $B_c = 200MHz$ is much much greater than the signal’s bandwidth $Bw = 20MHz$. In fact, the channel for this simulation will be always flat even if we opt for a higher bandwidth, as we see next, because we set the coherence bandwidth to be ten times bigger than the signal bandwidth.

Furthermore, if we change the signal bandwidth, say to accommodate for a very high data rate demanding application like UHD multimedia, to $Bw =$50
Mhz, then within the range of the given mobile speeds and carrier frequencies the effect of doppler spread on the coherence time will not change. But, it will effect the symbol rate per each coherence period of the channel. That is, the possible transmitted symbols per $T_c$ in this massive MIMO narrowband channel is 11637 bits or symbols i.e., roughly 11 kbit per channel coherence period for a mobile speed of 40km/h and carrier frequency of 29 Ghz. Moreover, if the speed of user increases to 60 km/h while the carrier frequency fixed on 29 Ghz, the coherence time decreases to 155 microseconed. Thus the symbol rate per channel use is 7758 bits or approximately 7 kbits per channel use. This rate further decreases when operating at higher carrier frequency $f_c$ i.e., 60 Ghz to 3749 bits or roughly 3 kbits per channel use.

Therefore, the conclusion to be drawn here, is that the coherence time of the flat and slow fading channel decreases by either the increase of carrier frequency or mobile users speed. This decrease is more dramatic at higher carrier frequencies. As a result of the shorter coherence time of the channel, which is most likely parameter to change more frequently than the signal bandwidth, the less the transmission symbol rate per channel use which can substantially impact the performance of the channel when considering any sort of performance evaluation. It also suggests that for higher carrier frequencies a higher bandwidth would lessen the effect of doppler spread on the channel’s coherence time and thus allowing for higher symbol transmission rate.
4.1.3 Simulation parameters

The simulation parameters for this case are similar to those already introduced in the slow case which is also summarized at table\[4.1\]. It is important to note to the fact that this case requires choosing different values of bandwidth as we will see in the analysis.

4.1.4 Fast fading scenario

We now investigate the characteristics of the massive MIMO channel that undergoes flat and fast fading. It is well known that in terms of how fast a wireless channel fluctuates depends on the coherence time \( T_c \). Hence, if this interval is very small compared to a symbol period, then, the channel is called fast faded i.e., \( T_c < T_s \). Or equivalently, the symbol bandwidth is smaller than the channel’s Doppler spread i.e, \( Bw < D_s \). That can be summarized as follows:

\[
T_s \gg T_d \quad \text{or} \quad Bw << B_c \quad (4.4)
\]

for flat fading and,

\[
T_s > T_c \quad \text{or} \quad Bw < D_s \quad (4.5)
\]

for fast faded channel. In terms of how fast the channel frequency response changes, it depends on the channel’s coherence bandwidth \( B_c \) which is the spectrum of bandwidth in which the channel response does not change. This parameter is compared to the symbols bandwidth as shown in the\[4.4\].
Since the coherence bandwidth $B_c$ is set to be 10 times bigger than the signal’s, the channel will always act flat at any given value of $Bw$. However, the choice of the bandwidth must follow the famous rule of thumb indicated at 4.3 in which the bandwidth of the transmitted symbol must be less than the doppler spread of the channel. This is achieved in this model for the given initial values where it is clear that $D_s$ is a bit more than $Bw$ thus meeting the condition for the channel to act as fast faded. The nature of the channel during fast fading dictates rapid changes in the magnitude of the small scale coefficients i.e., elements of $H$ matrix since it has a very short $T_c$ when compared to the symbol time $T_s$. Thus, a single symbol needs to be sent via multiple taps. Meaning, there will be $\lfloor T_s/T_c \rfloor$ taps for a single symbol to be transmitted per each link in the channel. For example, this model with initial parameter values indicated at table 4.1 would have roughly 4 taps or say 4 fluctuations in the channel during the transmission of a single symbol.

The fluctuations per single symbol transmission from all terminals are shown in figures 4.8 and 4.9 for the channel of $M = 64$ or 128 and $K = 8$ or 16, respectively. We can take a deeper look and more details of that in figures 4.10, 4.11 and 4.12. They explicitly depict the channel variations with respect of each individual terminal for every $T_c$ in a single symbol period $T_s$. In addition to the state of the channel being fast faded, the disparity in the magnitudes of each link in the channel i.e., the link between single mobile terminal to single receive antenna element, is due to the same reasons introduced in the slow fading case. In fact, this disparity does not depend on the channel being fast or slow faded, rather it depends on the channel being flat faded. Then, we can recall that a flat channel has a very
Figure 4.8: The channel variations over time as $4 \ T_c$ needed for single symbol transmission. The massive MIMO channel in this figure contains 64 antennas at BS serving simultaneously 8 users.
Figure 4.9: The channel variations over time as $4\ T_c$ needed for single symbol transmission. The massive MIMO channel in this figure contains 128 antennas at BS serving simultaneously 16 users.
Channel magnitudes between all BS antennas and each single terminal over time

Figure 4.10: Flat and fast fading channel representation with respect to each individual terminal for a massive MIMO channel of $M = 64$ and $K = 8$ terminals
Channel magnitudes between all BS antennas and each single terminal over time

Figure 4.11: Flat and fast fading channel representation with respect to each individual terminal for a massive MIMO channel of $M = 128$ and $K = 16$ terminals. This figure depicts the channel magnitudes of the first 8 terminals.
Channel magnitudes between all BS antennas and each single terminal over time

Figure 4.12: Flat and fast fading channel representation with respect to each individual terminal for a massive MIMO channel of $M = 128$ and $K = 16$ terminals. This figure depicts the channel magnitudes of the other terminals.
small amount of delay spread and thus, the nonresolvable paths that make up the channel taps either aggregate constructively or destructively. Eventually all the presented terminal channel magnitude disparity with the antenna array elements are due to the constructive/destructive interference of the nonresolvable paths that make up the channel taps.

Taking a single element of the fast fading channel $H$ can show explicitly details how the channel behaves in this type of fading. For instance, if we take the channel link between the first serving antenna at the base station with the first user, the 8th with 5th user and 44th with 8th user, then we plot them to have a result as in figure 4.13. As noted in the figure, it is obvious that during the transmission of a single symbol the channel over each of those links changes substantially after each $232.76\mu sec$ i.e. a single $T_c$.

Changing the carrier frequency $f_c$ and the mobile velocities to 60 GHz and 60 km/h, respectively, while fixing the symbol bandwidth yields a 13 channel fluctuations per single symbol period $T_s$ as shown in figure 4.14. This increased number of changes in the channel status or magnitude is due to the fact that the increase of both carrier frequency and the mobile user speed causes a substantial decrease in the amount of the coherence time of the channel e.g., $T_c = 75\mu sec$. The decrease in coherence time can be obtained even for a fixed carrier frequency $f_c$ when the increasing the terminal velocity. However, it is less severe that having both parameters increased. The opposite is true where the slow velocity and or low carrier frequency cause the changes in the channels magnitude to remain for longer period i.e., $T_c$ increases. As long as condition 4.5 is fulfilled, increasing the bandwidth of
Figure 4.13: Examples of flat and fast fading of single channel links during single $T_s$
Figure 4.14: Total channel variations over time as 13 $T_c$ i.e., in single $T_s$ for $f_c = 60\text{Ghz}$, $v = 60\text{km/h}$
the signal is upper bounded by the amount of the doppler spectrum for any fast
fading channel. Thus, one can conclude that the doppler spectrum parameter of
fast fading channel plays major role in determining the channel coherence time
and so the channel fluctuations per symbol period. This role can be extended to
the carrier frequency as well as the mobile velocity parameters due to their pro-
portional relationship. Another conclusion can be drawn based on these results
and notes, that is, the channels that undergo fast fading will suffer very low data
rates due to the fact that for a single symbol transmission the symbol pulse adds
constructively or destructively many times before detected at the receiver.

We have seen how the flat massive MIMO channels act in temporal domain
and we can use the angular domain to check if they provides full or partial spatial
degrees of freedom gain and what can be infer based on that. Therefore, we
transform the flat and fast/slow fading channel $H$ from the spatial domain into
angular domain to obtain $H^a$ as;

$$H^a = U_M^* H U_K$$  (4.6)

where $U_M$ and $U_K$ are $M \times M$ and $K \times K$ unitary matrices, respectively each
of which provide the orthonormal basis of the received and transmitted symbols
respectively. The angular channel matrix $H^a$ is depicted in figure 4.15. It shows
that the flat fading massive MIMO channel provide full angular spread and thus,
provides spatial degrees of freedom gain of $\min(M,K)$ i.e., 8 for $64 \times 8$ channel
and 16 for $128 \times 16$ channel. This spatial degree of freedom gain is true whether
Figure 4.15: The angular spread of flat fading massive MIMO channel
the flat fading channel undergoes slow or fast fading.

4.2 Frequency-Selective Fading Massive MIMO Channel

It is well known that a wideband channel has large delay spread $T_d$ due to the multipath effect on the propagated signal. That yields multiple replicas of the transmitted signal at the receiving end of the communication. For a linear modulated signal, it is considered to be a pulse with a duration of $T_s$ or equivalently $1/Bw$ that is much smaller than the channel delay spread $T_d$. Thus, the transmission of the pulse will be received at the receiver as $L$ different copies where each copy has its own delay $\tau_l$ and phase $\phi_l$. Unlike the narrowband channels, the wideband channels have much higher symbol resolution. This high resolution enables resolving the major multipath components of the original transmitted pulse. That is, when $T_s \ll T_d$ there will be $\lfloor T_d/T_s \rfloor$ resolvable paths. However, such channels suffer from the fact that the consecutive symbols or pulses during the period of $T_d$ will interfere with each other resulting the so called intersymbol interference (ISI) that degrades the performance of the communication channel. This ISI can be mitigated (or in some cases cancelled) using techniques such as spectrum spreading and multicarrier modulation.

The received signal is in fact the sum of all major resolvable delayed taps. That is\textsuperscript{1}

\textsuperscript{1}Assuming noise free environment.
\[
y(t) = \sum_{l=1}^{L} c_l(t) x(t - \tau_l(t)),
\]  
(4.7)

where \(c_l(t)\) is the baseband equivalent of the time varying channel impulse response and \(x(t)\) is the transmitted pulse. Equation (4.7) merely represents the case of a single link of communication i.e., single transmit antenna and a single receive antenna. That is not the case in a massive MIMO channel where there are \(M\) serving antennas at the base station that serve simultaneously \(K\) different single antenna mobile terminals. We can take the uplink channel where the \(K\) mobile users transmit symbols to the \(M\) antennas at the same time-frequency resource, and obtain the received signal at the \(M\) antenna array as

\[
y = C x
\]  
(4.8)

where \(y\) is \(M \times 1\) and \(C\) is \(\in C^{M \times KL}\) and \(x\) is \(KL \times 1\) vector and \(l = 1, 2, \ldots, L\). That is represented in a matrix form as;

\[
\begin{bmatrix}
y_1(t) \\
y_2(t) \\
\vdots \\
y_M(t)
\end{bmatrix} =
\begin{bmatrix}
c_{11}(t) & c_{12}(t) & \cdots & c_{1K}(t) \\
c_{21}(t) & c_{22}(t) & \cdots & c_{2K}(t) \\
\vdots & \vdots & \ddots & \vdots \\
c_{M1}(t) & c_{M2}(t) & \cdots & c_{MK}(t)
\end{bmatrix}
\begin{bmatrix}
x_1(t - \tau_{1l}) \\
x_2(t - \tau_{2l}) \\
\vdots \\
x_K(t - \tau_{KL})
\end{bmatrix}
\]  
(4.9)

This channel model is called the tapped delay frequency selective fading channel where each received signal is the result of the aggregation of taps of the major
multipath clusters contribution.

4.2.1 Simulation parameters

This model is for a single cell massive MIMO cellular system that consists of one of two configurations. The first consists of $M = 64$ as the number of serving antenna at the BS and $K = 8$ as the number of terminals or mobile users. The second configuration consists of $M = 128$ and $K = 16$. The carrier frequency is in both configurations is either $f_c = 29$ GHz or 60 GHz. We assume BPSK modulation thus, the bit rate will be same as the symbol bandwidth. Then, the symbol bandwidth is chosen to be LTE-like bandwidth of initially 20 MHz for simulating the slow fading scenario of this wideband channel. The symbol period $T_s$ will be $5 \times 10^{-8}$ seconds (50 ns). Further, the initial value of Doppler spread parameter of the channel is $D_s = 1.0741 \times 10^3$ Hz for initial speed of mobile $v$ of 40 km/h (11.1111 m/s). Hence, the channel coherence time $T_c$ is $2.3276e - 04$ second ($\approx 232.8\mu$sec). Now, it is crucial to set the channel delay spread parameter $T_d$ to be 10 fold larger than the symbol period $T_s$ to satisfy the condition of frequency selectivity of the channel i.e., $T_d >> T_s$ [8]. That results in a $T_d$ equal to 500 ns based on the initial value of the symbol period chosen above. Based on this delay spread, the channel coherence bandwidth $B_c$ is the reciprocal of $T_d$ and using the relationship in [25] i.e., $B_c = 1/2T_d$, it is 1 MHz. In the simulations, we extend the aforementioned parameters to examine various situations similar to those in real life propagation environment. For instance, the carrier frequency can be changed
to 60 GHz, the speed of mobiles can be increased to 60 km/h and the symbol bandwidth can be increased up to 50 MHz for a very high data rate demanding application such as ultra high definition multimedia (UHD). Parameters are summarized in table 4.1. Furthermore, we assume that $L = 4$ i.e., 4 aggregated taps that represent four major clusters in the propagation environment. We model these taps as Rayleigh fading magnitudes. With these assumptions, the small-scale fading channel denoted above as $C$ is a matrix of circular complex gaussian random variables elements that has a dimensions $\in \mathbb{C}^{M \times 4K}$. It follows that such propagation environment gives the so-called preferable propagation condition where the transmission from any terminal is seen independent from the transmission of other terminals in the same cell covered by the same $M$ array of antennas. In other words, the columns in $C$ corresponding to the transmission of one terminal is different enough or asymptotically orthogonal to those corresponding to others. This is illustrated at figure 4.16.

4.2.2 Slow fading scenario

In what follows, we will show the result of simulating a massive MIMO channel that undergoes both frequency selectivity and slow fading effect. We represent their effect on the channel figuratively and numerically using the fading coefficients’ magnitudes for further understanding of those effects and what we expect when trying to further process the signal in the receiver or the transmitter ends of the
Figure 4.16: The preferable propagation condition occurs when the Gramian matrix $C^* C$ yields a diagonal matrix
communication channel. In this scenario the channel fulfills the conditions of

\[ T_s << T_d \quad \text{or} \quad Bw >> B_c \] (4.10)

for frequency selectivity (wideband) and

\[ T_s << T_c \quad \text{or} \quad Bw >> D_s \] (4.11)

for slow fading. These conditions imply that the channel is underspread i.e., \( T_d < T_c \) or equivalently it has a spread factor \( d_C = T_d \times D_s \) that is smaller than one (here, it is \( 5.3704 \times 10^{-4} \)). The channel magnitudes due to the transmission of each individual user is represented in figures 4.17, 4.18, and 4.19 for both configurations of \( M = 64 \) antennas, \( K = 8 \) terminals and \( M = 128, K = 16 \) respectively.

We notice that the variations in the channel coefficients magnitude is distinct in every link and for every resolvable multipath component. It is understood that the delayed replicas have to be different from each other simply because they impinge from different and physically separated clusters of scatterers. In addition, one can reason that from the statistical point of view those paths or taps are due to the transmission of physically separated users which means that their phases are mutually independent. That is, the phase of the \( l^{th} \) tap at time \( t \) is given by

\[ \phi_l(t) = 2\pi f_c \tau_l(t) \] (4.12)

and, a substantial change in \( \phi_l(t) \) occurs when the delay of the path \( l \) (that repre-
Figure 4.17: Channel magnitudes with respect to each user including the multipath taps
Figure 4.18: Channel magnitudes with respect to each user including the multipath taps
channel magnitudes including the resolvable delayed taps due to terminal transmission over time

Figure 4.19: Channel magnitudes with respect to each user including the multipath taps
sents the lth cluster) changes by \( \frac{c}{4f_c v} \) or equivalently when the path length changes by \( \frac{c}{4f_c} = \frac{1}{4\lambda} \) i.e., a quarter of the carrier wavelength. In the context of massive MIMO being implemented over mm waves as in our model, such changes happen in an extremely short period of time of the order of microseconds. That implies that the envelope of that delayed pulse is going to change from peak to valley i.e., \( \pi/2 \) very rapidly. Furthermore, since we assume that each of the delayed taps comes from different cluster of scatterers, we can refer to the fact that each of these taps is the result of the aggregation of large number of locally scattered paths having roughly the same delay within each cluster. All of that yields that those phases are uniformly distributed for such a dense propagation environment.

For the channel between the \( M \) serving antennas and each individual terminal, the magnitude variations do not contradict the fact that the overall channel matrix \( \mathbf{C} \) has mutually asymptotically orthogonal columns each of which corresponds to a transmitting terminal (or each antenna element in the BS for downlink channel) as shown in [4.16] even for very small separation between the serving \( M \) antennas. This is true as long as the separation between antennas is larger or equal to the half of the wavelength \( \lambda \) i.e., sparsely or critically spaced [25]. One can show that by the means of the angular domain representation of the channel. That is, the angular domain of the channel matrix \( \mathbf{C} \) is given by\(^2\):

\[
\mathbf{C}^a = \mathbf{U}_M^* \mathbf{C} \mathbf{U}_K
\]

\(^2\) The time index is dropped since this \( \mathbf{C} \) matrix is for each single period of coherence time.
where $\mathbf{U}_M$ and $\mathbf{U}_K$ are $M \times M$ and $KL \times KL$ unitary matrices, respectively.

The elements of $\mathbf{C}^a$ are nonzero for all rows which means that this configuration fulfills the angular separation condition for the antenna arrays to be spatially separated and leverage higher degrees of freedom and eventually full spatial multiplexing capabilities. This angular spread is illustrated in figure 4.20.

Another way to analyze the channel under this scenario is by taking the collective magnitude of the these delayed taps. Mathematically, the collective magnitude of the resolvable taps is given by

$$c_{\text{collective}}(t, \tau) = \sum_{l=1}^{L} \alpha_l(t) \delta(t - \tau(t)) = \sum_{l=1}^{L} c_l(t)$$  \hspace{1cm} (4.14)

which is merely the sum of all time-variant impulse responses of the subpath clusters due to the transmission from each individual terminal. Thus the collective channel magnitudes are shown in figures 4.21, 4.22, and 4.23 for both configurations of $M = 64$ antennas, $K = 8$ terminals and $M = 128$, $K = 16$ respectively. In those figures and in the ones before, the plotting is of the channel magnitudes for the communication link between each user and all the $M$ receiving antennas at the base station during the course of several coherence periods for the purpose of showing the slowly fading magnitudes of the channel. As mentioned above, the fading slowly changes (compared to the symbol period $T_s$) when the delay of each aggregated tap changes by the amount of $T_c$. One can infer from the collective gains shown in those figures that the constructive/destructive interference effect due to the multipath delay spread on the elements of the antenna array at the BS.
Figure 4.20: The angular domain representation of the channel matrix showing a complete angular spread of the antennas
Figure 4.21: The collective channel magnitudes due to user transmissions over time as $4 \, T_c$
Figure 4.22: The collective channel magnitudes due to first 8 users transmissions for channel with M=128 and K=16

is what makes some elements to have higher magnitude and lower for others during the coherence time of the channel $T_c$. Moreover in such multipath fading propagation, the aggregated taps from each symbol transmission add constructively when the change in their phases add to be an even multiple of $2\pi$ rad resulting in a higher gain (strong signal). Otherwise, the phase shifts add to be an odd multiple of $\pi$ rad which leads to a destructive pattern resulting of lower magnitude of that received symbol (weak signal).

Further explicit examples on the collective magnitudes for both configurations due to user 1 transmission are given in figures 4.24 and 4.25.

Now we turn our attention to the effect of the intersymbol interference in this scenario. As mentioned previously, the symbol resolution in the frequency selective
The collective magnitudes due to terminals transmission over time

With respect to terminal 9

With respect to terminal 10

With respect to terminal 11

With respect to terminal 12

With respect to terminal 13

With respect to terminal 14

With respect to terminal 15

With respect to terminal 16

Figure 4.23: The collective channel magnitudes due to other 8 users transmissions
Figure 4.24: Example for the collective magnitude due to terminal 1 with M=64 elements
Figure 4.25: Example for the collective magnitude due to terminal 1 with M=128 elements
fading channels is much higher than in the flat fading ones. That translates into a compulsory difference between the delay of consecutive multipath components larger than the pulse period $T_s$ i.e., $|\tau_i - \tau_j| >> T_s$ for paths $i \neq j$ coming from different clusters.

Figures 4.26 and 4.27 provides a visual representation for such effect. In both figures, due to the delay spread of the channel i.e., $T_d$ the consecutive transmitted symbols from terminal 1 to the antennas at the base station and during the time window of $T_d$ will have a significant interference with the delayed replicas of each preceding symbol. According to our set up of the model, there will be a maximum of 10 resolvable paths for the symbols that have been transmitted in the time less than the maximum delay spread $T_d$ of the channel. However, since we consider fewer clusters to represent such multipath environment there are only 4 multipath components due to each symbol transmission that are shown in the figures. Furthermore, for clarity of the representation we show the intersymbol interference of two consecutive symbols being transmitted by terminal 1 to be received by the first antennal element in the base station. We notice that the second symbol will be delayed alongside its replicas by the time of the previous symbol pulse the subpath cluster delay that each replica propagates through. Additionally in figures 4.26 and 4.27, we show the ISI effect in a single $T_d$ period and two $T_c$ intervals where the blue lines indicate ISI on symbols at the first $T_c$ and the black lines for the second $T_c$. The two symbols are shown in the figures as: the first symbol is represented with a dotted line and the second symbol with a solid line at both coherence intervals. Clearly, both figures show the slow fading where the magnitude of the
Figure 4.26: Intersymbol interference between two consecutive symbols during the transmission between user 1 and antenna 1
Figure 4.27: Intersymbol interference between two consecutive symbols during the transmission between user 1 and antenna 1
two interfering symbols are roughly the same during every single $T_c$ interval. The massive MIMO system does not suffer from the interuser interference within a cell for the reason of the spatially separated user channels that are in dense scattering environment as described in this model sufficiently different i.e., asymptotically orthogonal.

If we want to test the scenario where the users move faster, say 60 km/h or equivalently 16.6667 m/s , then for sure the characteristics of the channel will change accordingly. The change in the speed of the mobile as we assume all terminals move at the same speed dictates changes in the doppler spread of the channel $D_s$ which leads to a change in the coherence time of the channel $T_c$. The increase of the speed alone would make a doppler spread to increase about 600 Hz more for the same carrier frequency i.e., 29GHz. As a result the coherence time $T_c$ decreases to roughly 155 microseconds. Furthermore, the spread factor $d_C$ changes to $8.0555 \times 10^{-4}$ which is still smaller than one i.e., underspread channel. Also, if we consider the system to work with higher carrier frequency such as 60 GHz then there will be even more changes in the channel characteristics due to doppler spread change. With a speed $v$ of mobiles 60 km/h and $f_c$ of 60 GHz, the channel will have a maximum doppler spread $D_s$ of roughly 3333 Hz. As a result and just like the previous case, the amount of $T_c$ is shorter i.e., roughly 75 microsecond. That is understood since both parameters $T_c$ and $D_s$ are reciprocal and thus their relationship is inversely proportional. We notice the these changes in the carrier frequency and the speed of the terminals are proportional to the change in doppler
spread of the channel. The higher the carrier frequency and the terminal speed, the less coherence time the channel will have thus fewer transmitted pulses per $T_c$. One can infer that the short channel coherence interval will eventually effect the ability to estimate the channel accurately since that causes the required orthogonal uplink pilot sequences to be limited and thus resulting on their reuse in other cells causing the so-called pilot contamination impact [17, 11].

The impact on the delay spread of the channel here is not present on the symbol period since the signal bandwidth is the same. But if the signal bandwidth is to be raised up to 50 MHz, say for some ultra high definition multimedia transmission then, the channel will have different characteristics as well. For this model, we assumed that the delay spread of the channel is 10 fold of the symbol period $T_s$ which for this choice of bandwidth and modulation is equal to $2 \times 10^{-8}$ seconds i.e., 20 nanoseconds hence the $T_d$ is equal to 200 nanosecond. The channel’s coherence bandwidth $B_c$ is 2.5 MHz. Similar to the relationship between the coherence time and doppler spread, the coherence bandwidth is inversely proportional to the channel’s delay spread as noted here. The increase in the symbol bandwidth allows for more symbols to be transmitted during the channel coherence time $T_c$ even for small $T_c$ since the channel undergoes a slow fading. At least for the slow fading channels, one can argue that the more bandwidth the signal has the longer the delay spread effect on the multipath components of the transmitted signal. This statement might seem unclear but considering the relationship between the signal bandwidth $B_w$ and the coherence bandwidth $B_c$ of the channel for the wideband channels implies the previous interpretation. Precisely, it is always true
that $B_c << Bw$ in a wideband channel, and from the reciprocal relationship between the time domain and the frequency domain we can infer that the time domain parameters will have inverse relationship i.e., $T_d >> T_s$.

The conclusion that can be drawn from the above discussion is this: for a fixed signal bandwidth the changes in the channel’s doppler spread parameter for any reason e.g., change in the terminal speed or carrier frequency will lead to shorter coherence intervals of the wireless channel resulting in either faster or slower rate of changes in the channel fading coefficients depending on the change in doppler spectrum spread factor. However, the behavior of the wideband channels is dependent on the signal bandwidth since that is what dictates the symbol period at least for linear modulation techniques. That is, the larger the bandwidth the higher the impact of the delay spread on the transmitted symbols thus, the more severe the intersymbol interference between them.

4.2.3 Fast fading scenario

A wireless channel that undergoes a frequency selective and fast fading must meet the conditions:

\[ T_s << T_d \text{ or } Bw >> B_c \]  \hspace{1cm} (4.15)

for frequency selectivity (wideband) and

\[ T_s > T_c \text{ or } Bw < D_s \]  \hspace{1cm} (4.16)
for fast fading.
These conditions in (4.15) and (4.16) imply an overspread channel where the channels spread factor $d_C$ is larger than 1 i.e., here it is 10.7407. That is for a doppler spread $D_s$ of 1.074 Khz when the mobile speed is $v = 40km/h$ and $f_c = 29Ghz$. The delay spread of such spread factor is 0.01 second i.e., 10 ms. In the case of fast fading, the channel changes several times within a single symbol interval. Precisely, there will be $\lfloor T_s/T_c \rfloor$ fluctuations of the channel during the course of a symbol interval.

The channel variations during fast fading are very rapid and that can be seen in figure 4.28. It shows that the channel magnitudes changed 4 times as each terminal transmits one single pulse of width $T_s$ of 1 ms. Alternatively, the channel magnitudes due the transmission of every user is shown explicitly in figures 4.29, 4.30 and 4.31 for both configurations of $M = 64$ antennas, $K = 8$ terminals and $M = 128$, $K = 16$ respectively. From the physical point of view, the variations in this scenario are due to the same facts that were presented in the slow fading scenario. The difference here is in the values of the physical parameters that comply with the conditions above. Although the coherence time of the channel does not differ from the aforementioned slow fading scenario, it is the symbol period that got increased making it much larger than the coherence interval of the channel hence, causing the fast variations in the magnitude of the channel within the transmission of one symbol.

The collective magnitudes of the channel links are given in figures 4.32, 4.33 and 4.34 for the the two configurations i.e., $M=64$, $K=8$ and $M=128$, $K=16$, re-
Channel variations of frequency selective and fast fading during single $T_s$

Figure 4.28: Channel variations including the multipath taps during the transmission of one symbol
Figure 4.29: Channel magnitudes with respect to each user including the multipath taps
channel magnitudes including the resolvable delayed taps due to terminal transmission over time

Due to terminal 1

Due to terminal 2

Due to terminal 3

Due to terminal 4

Due to terminal 5

Due to terminal 6

Due to terminal 7

Due to terminal 8

Figure 4.30: Channel magnitudes with respect to each user including the multipath taps for system with M=128 and K=16 (1)
Figure 4.31: Channel magnitudes with respect to each user including the multipath taps for system with $M=128$ and $K=16$ (2)
The collective magnitudes due to terminals transmission over time

With respect to terminal 1

With respect to terminal 2

With respect to terminal 3

With respect to terminal 4

With respect to terminal 5

With respect to terminal 6

With respect to terminal 7

With respect to terminal 8

Figure 4.32: The collective channel magnitudes due to user transmissions of one symbol
Figure 4.33: The collective channel magnitudes due to first 8 users transmissions for channel with $M=128$ and $K=16$
The collective magnitudes due to terminals transmission over time

With respect to terminal 9

With respect to terminal 10

With respect to terminal 11

With respect to terminal 12

With respect to terminal 13

With respect to terminal 14

With respect to terminal 15

With respect to terminal 16

Figure 4.34: The collective channel magnitudes due to other 8 users transmissions
respectively. Similar to the previous scenario, these figures display the collective amplitude of each transmission link with respect to every uplink from the terminals who are being served by \( M \) antennas in the base station simultaneously. The collective magnitudes may add constructively for some antenna elements at the base station and may add destructively for others. An example of this is shown in figures [4.35] and [4.36] for the transmission link between user 1 and the antenna array at the base station in both configurations, respectively. They illustrate how a transmitted symbol undergoes a fast fading for the duration of its width i.e., \( T_s \). The fact that the channel is frequency selective and fast faded implies that if we were taking the random fading process, we will see that the fading process decorrelates several times i.e., after \( 4 T_c \) here, during the transmission of a pulse. This decorrelation can help in detecting and distinguishing between successive received symbols for those receiver decisions based on the observed symbols. That means there will be no burst errors in successive symbols but only isolated errors.

The intersymbol interference of this case is severe due to the longer delay spread \( T_d \) of the channel which is as mentioned above of 10 ms for this setup. Figure [4.37] shows the ISI effect on two consecutive symbols transmitted over the channel between the the first mobile user and first serving antenna in the BS. We notice that each symbol alongside its replicas fluctuates very rapidly due to the short amount of coherence time the channel has. The magnitude of the first symbol in solid blue lines changes four \( T_c \) periods and we see that its replicas are interfering with the next transmitted symbol which is represented in black dotted lines.

Similar to the case of slow fading, the increase of doppler spread \( D_s \) of the
Figure 4.35: Example for the collective magnitude due to terminal 1 with $M=64$ elements
Figure 4.36: Example for the collective magnitude due to terminal 1 with $M=128$ elements
Figure 4.37: Intersymbol interference between two consecutive symbols during the transmission between user 1 and antenna 1, where symbol 1 is represented by the blue solid lines and the symbol 2 is the black dotted lines.
channel yields shorter coherence time and hence much rapid variations. For instance, the doppler spread parameter is about 3.3 KHz when the operating carrier frequency is 60GHz with mobiles moving at speed of 60 km/h. With these values the symbol with pulse width of 1ms will have roughly $13 T_c$ intervals during its propagation which tells how much fading this channel is going through. It also indicates that if we consider the frequency domain analysis of the channel then the power spectral density of the samples of the frequency response will be at maximum at those band of the doppler spread. By looking at the coherence bandwidth of the channel, we can infer that there is high frequency selectivity since $B_c$ is much smaller than the symbol bandwidth i.e., 1KHz, 50Hz, respectively. On the other hand, when the signal bandwidth increases the signal or the symbol period $T_s$ decreased thus, the delay spread $T_d$ becomes about 3.3 ms for a 3KHz signal bandwidth. That means that the transmitted symbols will have lesser delay spread intervals but the interference among them will be there always since it is a wide-band channel. The difference that this decrease in the symbol time makes is in how much channel variations the symbol undergoes i.e., 4 with a coherence time of 75 microsecond.

It is of importance to note that almost all wireless communications are considered to be underspread [15]. The only exception for that is underwater channels where they might have a spread factor $d_C$ that is more than one (overspread). Therefore, the analysis of such channel above is for extreme case and to showcase all the possibilities. Additionally, the overspread channel will be unpractical for the
massive MIMO systems since the maximum number of orthogonal pilot sequences is upper bounded by the ratio \( \frac{T_c}{T_d} \) which is less than one for an overspread channel [11]. Overall, a channel that undergoes frequency selectivity and fast fading will be highly faded especially for low data rates as shown in the analysis.

4.3 Multicell Multiuser Massive MIMO Cellular System Scenario

After we discussed the different scenarios that consider both flat and frequency selective fading channels combined with either a slow or fast fading in a single cell, we turn our attention to the bigger picture of the cellular system that is composed of seven cells. Such a system is referred in the literature by multicell MU-Massive MIMO. In multicell system, the issue of pilot contamination, described in chapter 3, emerges since the basestations in each cell do not only learn or estimate their own terminals, but other cells’ too due to the frequency reusing factor among the cells. That is, when a basestation in the cell estimates the channel of the terminals within that cell, it ends up estimating the channel of the terminals from other cells which have the same pilot sequence or have partially correlated pilots with its terminals’ pilot sequence. As a result of this contamination, the uplink and downlink channels to/from any basestation array will be highly affected by the inter-cell interference. Furthermore, this inter-cell interference is considered to be the most limiting constraint to the full potential of massive MU-MIMO technology [23, 19, 13].

For a multicell system of massive MIMO having \( L \) cells, all the \( K \) terminals at
all $L$ cells simultaneously transmit signals to their serving basestations. Thus, the
basestation at the $j^{th}$ cell receives a $M \times 1$ vector

$$y_j(t) = \sum_{l=1}^{L} G_{jl}(t) x_l(t) + n_j(t)$$

(4.17)

which can be seen explicitly as:

$$y_j(t) = G_{jj}(t) x_j(t) + \sum_{l \neq j}^{L-1} G_{jl}(t) x_l(t) + n_j(t)$$

(4.18)

where the first summation is for the cell of interest i.e., the propagation coefficients
between basestation antenna array at cell $j$ and terminals at that cell, and the
second summation represents the intercell interference due to pilot contamination
from other cells.\footnote{Commonly, the channel matrix $G_{jl}$ is decomposed\footnote{We ignore the noise term for this analysis for the fact that the noise is additive at the receiver
and we are only concerned about the propagation channel analysis} as we
mentioned in chapter (3). That is:

$$G_{jl} = H_{jl} D_{jl}^{1/2}$$

(4.19)

where $H_{jl}$ is the $M \times K$ small-scale fading matrix that accounts for fast fading
coefficients between terminals at cell $l$ and antenna array of the basestation of
cell $j$. Those coefficients i.e., $h_{jlmk}$ are i.i.d complex circular Gaussian random

\footnote{Time index is dropped since this channel model is per a single coherent time instance}
variables with a Rayleigh distributed magnitudes and uniform phases. The matrix $D_{\beta jl}^{1/2}$ is a $K \times K$ diagonal matrix of the large-scale coefficients that account for the combined effect of path loss and shadowing of which each of its diagonal entries i.e., $[\beta_{jl}]_{kk}$ represents the shadowing between terminals in cell $l$ and the basestation antennas at the $j^{th}$ cell. There is a consensus in the literature about the modeling of those elements as log-normally distributed that can be described explicitly as:

$$[\beta_{jl}]_{kk} = \frac{Z_{jlk}}{d_{jlk}^n}$$  \hspace{0.5cm} (4.20)

where $d_{jlk}$ is the distance between the $k^{th}$ terminal in cell $l$ and basestation in cell $j$, and $n$ is the path loss exponent. $Z_{jlk}$ is the log-normal shadowing that have the distribution:

$$p(z) = \frac{\zeta}{z \sqrt{2\pi \sigma_{sh(dB)}}} \exp \left[ - \frac{(10 \log_{10} z - \mu_{sh(dB)})^2}{2\sigma_{sh(dB)}^2} \right], z > 0; \quad \zeta = 10 \ln 10$$  \hspace{0.5cm} (4.21)

In fact if $Z$ is in dB units, then it is normally distributed with zero mean and $\sigma_{sh}$ standard deviation in dB units.

In massive MIMO systems, the coefficients of the large-scale fading i.e., $[\beta_{jl}]_{kk}$ are assumed to be static over all terminal channels since the distance between terminals and basestations is far greater than the size of the basestation antenna array [23, [17], 9] and the fact that they change very slowly comparing to the changes of those of $h_{jlmk}$ coefficients. By substituting (4.19) into (4.18) we have,
\[ y_j = H_{jj} D_{\beta jj}^{\frac{1}{2}} x_j + \sum_{l \neq j}^{L-1} H_{jl} D_{\beta jl}^{\frac{1}{2}} x_l + n_j \quad (4.22) \]

Notably, the antenna array of the basestation \( j \) i.e., at the cell of interest, will be blocked by large objects such as large buildings and hills which in turn causes the shadow fading onto their received signal due to users transmissions (uplink). The effect of shadow fading on the multicell massive MIMO channel is depicted in figures 4.38, 4.39, 4.40 and 4.41 for all the scenarios of flat and slow fading, flat and fast fading, frequency selective and slow fading, frequency selective and fast fading channels, respectively. To obtain these results, we make a couple of assumptions for simplicity. First, as the case for a single cell, we assume that all users move with the same velocity \( v \) as well as the distance between the users and the serving BS at each cell is fixed. Furthermore, it assumes that each cell has a radius of 500 meters and mobile users are excluded from the proximity of their local BS by 100 meters. This assumption of 100 meters as a cell-hole is chosen based on the empirical measurements [26, 8] which found that the effect of shadowing is obvious at such distance. Second assumption is that this model does not consider for the fact that different antenna elements in the basestation array observe different blocking objects. In other words, it assumes that the channel is spatially invariant at least for the coherent period \( T_c \) at which the channel is modeled.

In all of these figures, we notice that some terminal channels have magnitudes that are much smaller than others. This severe difference is due to the fact that the smaller terminal channels are blocked and thus, their attenuation is greater than
Figure 4.38: The flat and slow fading massive MIMO channel for a multicell system undergoes shadow fading
Figure 4.39: The flat and fast fading massive MIMO channel for a multicell system undergoes shadow fading
Figure 4.40: The frequency selective and slow fading massive MIMO channel for a multicell system undergoes shadow fading
Figure 4.41: The frequency selective and fast fading massive MIMO channel for a multicell system undergoes shadow fading
those are not shadowed by large blockages. It can be inferred that the larger the blocking object the more severe the effect of the shadowing on the massive MIMO channel. In fact, this slow fading becomes the dominant fading effect when the system is extended to multiple cells as in this section. That is, the known preferable propagation feature of massive MIMO channels indicates that the average power of the small-scale fading coefficients in \(H_{jl}\) i.e., \([H]_{jl}\) would be very small for mutual terminal channel vectors but asymptotically close to those of the large-scale fading coefficients. Therefore, the shadow fading becomes dominant for multicell massive MIMO system. The preferable propagation condition for channels in cell \(j\) is:

\[
\frac{G^H_{jj}G_{jj}}{M} = D_{\beta jj}^{\frac{1}{2}} \left( \frac{H^H_{jj}H_{jj}}{M} \right) D_{\beta jj}^{\frac{1}{2}} \approx D_{\beta jj} \tag{4.23}
\]

For example, figure 4.42 shows the preferable propagation condition for the channel presented in figure 4.40. This convergence to \(D_{\beta}\) becomes much more closer when increasing the number of antenna at the BS array i.e., \(M\) is increased as shown in figure 4.43 as compared to the magnitude of \(D_{\beta}\) in figure 4.44.

As far as the resolvability of the multipath channels, one can use the channel angular domain to determine the resolvability of multipath channels. The flat fading channels will always have full angular spread whether they are fast or slow faded. The angular spread of flat or narrowband channel is shown in figure 4.45.

The flat massive MIMO channel in a multicell as in this case has an angular domain matrix \(G^a\) that under any array configuration has a full rank of \(\min(M, K)\). Despite the effect of shadowing, one can deduce that the angular bins of such richly
Figure 4.42: Preferable propagation condition for a frequency selective and slow fading massive MIMO channel results in a diagonal matrix asymptotically approaches the large-scale fading matrix $D_\beta$
Figure 4.43: Preferable propagation condition for a frequency selective and slow fading massive MIMO channel with BS having $M = 128$ antennas
Figure 4.44: Magnitude of large-scale fading $D_\beta$. The preferable propagation conditions converges more closely to the $D_\beta$ as the number of antennas $M$ grows.
Figure 4.45: Full angular spread of flat fading massive MIMO channel
scattered propagation environment aggregate strongly, thus forming nonzero and independent resolvable paths as the entries of \( G^a \). With a full rank and full angular spread, the channel is well conditioned and thus, it surely provides high spatial degree of freedom gains and so higher multiplexing capability.

On the other hand, a frequency selective massive MIMO channel in a multicell configuration does not have a complete angular spread among its terminal channels as it is depicted in figure 4.46. This is true for any array configuration and whether the channel undergoes a fast or slow fading, as long as the channel is assumed to be spatially invariant at the coherence period. This partial angular spread is because of three reasons. One is the assumption that all terminal channels undergoes the same amount of shadowing yields that all the multiple taps of each transmitted signal by the terminals undergoes the equal shadowing coefficient. Two, the equal shadowing coefficients across all the cluster paths as well as the dominance of shadowing effect in this multicell configuration makes the received replicas from every terminal transmission to be seen as one lumped resolvable path. Three, since the BS antenna array already packs large number of antennas and even with critical spacing between its elements, the shadowed cluster-paths would not have enough angular resolution when they are received by some antennas at the BS array of that cell. Moreover, a closer look at figure 4.46 suggests that at least two or three columns of the angular domain matrix \( G^a \) have all zero elements which is another way to point out the lack of angular separation at one side of the channel. That is, the angular spread of the channel indicates that the cluster paths of uplinking terminals, which have a zero column, do not have the required amount of angular
Figure 4.46: Angular spread is limited in frequency selective fading massive MIMO channel for multicell system where the cluster paths undergoes the same shadowing when assuming spatially-invariant channel.
separation between them to be resolved in the BS array as individual resolvable paths. The rank of such angular matrix will be $\min(M, K)$ which for this example of $M = 64$ and $K = 8$ is 8, indicating that the diversity of the wideband channel replicas is not achievable mainly due to the lack of proper angular separation of the terminals. One possible approach to overcome the shadowing effect on the wideband channels is to increase the size of the antenna array of BS so the antennas become sparsely spaced thus allowing more angular separation for the upcoming cluster paths.

From the above results, one can conclude that in a multicell massive MIMO communication system the shadow fading is dominant and its effect on narrowband channels depends on the size of the shadowing object. However, in the wideband type of channels shadowing effect can cause the channel to lose their diversity when the cluster paths are not separated enough due to the closeness of the blockage to their corresponding terminals.
Chapter 5: Conclusion

Throughout this thesis we have studied the general wireless channel characterizations and applied that knowledge toward further understanding of the behavior of massive MIMO channels in very different scenarios. In first chapter, we introduced the concept of fading and its two main categories. Alongside a brief literature review, the massive MIMO technology was introduced as well as the goal of this thesis.

Chapter two provides the basic background for this research. It started with a primitive ray tracing technique with gradual complexity of different scenarios. Then, due to the complexity of ray tracing technique another modeling techniques were introduced. Mainly, the input/output relationship based physical model gives unique and simple way to characterize wireless channels based on the relationship on input and output between the transmitter and receiver. Using the input/output physical model, the wireless channel is characterized by its impulse response and how fast this response changes in time domain as well as in frequency domain. As a result of the physical channel modeling, we learned the main characterization factors that determines the main features of the channel. Toward the end of chapter two, the second main modeling technique was introduced, namely, statistical models such as Rayleigh and Racian channel models.

The third chapter introduced a the current technology that is being used in
wireless channel communication, namely, multiple input multiple output (MIMO). The first part discussed in details about MIMO channels in a resemblance with chapter two. Within this part, we have seen how and what setup of MIMO channels guarantees a full spatial degrees of freedom as well as power gain. That can be easily monitored by transforming the channel matrix into its angular domain. From the angular domain of the channel matrix, one can see whether or not the MIMO channel has full angular separation between antennas at both channel end that guarantees having spatial degrees of freedom and thus gaining higher multiplexing capability. With enough insight on MIMO channels, the scaled up version is presented alongside concise details and literature review of massive MIMO technology. Details included, the major differences between MIMO and its scaled up version massive MIMO, the potentials of the latter, channel radio propagation challenges such as mutual coupling between adjacent antennas in the BS array, techniques for acquiring the channel state information (CSI) where different beamforming and detection methods were discussed such as matching filter, zero forcing, random step methods etc.

In chapter four, we presented our results as well as the analysis on the results. We divided the simulation into two main cases that include a narrowband massive MIMO channel and a wideband massive MIMO channel characterization based on Rayleigh fading model as the small-scale fading matrix of the channel. In each case the slow and the fast fading effect are included for a single cell scenario. Then a multicell system is included at the end of the chapter. The main findings in our simulation can be summarized as follow:
• In a single cell scenario, massive MIMO channels harden i.e., the effect of small-scale fading asymptotically vanishes as the number of antennas at the BS increases. This is true whether the massive MIMO channels are frequency selective or flat faded.

• As a result of that the channel hardening feature, the interuser interference is eliminated leaving only the intersymbol interference per single link transmission in wideband channels.

• Massive MIMO channels in richly scattered environments such as in our simulation provides full spatial degree of freedom gain for both flat and frequency selective cases with more diversity for the latter.

• In multicell scenario, due to the hardening feature the shadow fading becomes dominant.

• When assuming spatial invariant shadowing i.e., all terminals have same shadowing with all BSs, the dominance of shadowing affect frequency selective fading massive MIMO channels greatly causing them to lose the diversity of their resolvable paths due the lack of required angular separation for the incoming paths from the shadowed terminals.

• The flat fading massive MIMO channels, however, do not suffer degradation in their spatial degrees of freedom since they have a single tap per link between an antenna element in BS and a terminal which is subject to a shadow fading independent from other taps.
• In both flat and frequency selective fading massive MIMO channel the fast fading case results of low rates of symbol transmission due to the fact that in fast fading the symbol bandwidth is upper bounded by the maximum doppler spread of the channel.

• Overspread channels are impractical for any massive MIMO configuration that use TDD since there is very short coherence time of the channel to acquire orthogonal pilots and correctly estimate the channel.
Bibliography


