AN ABSTRACT OF THE THESIS OF

KIMBERLY ILES for the MASTER OF SCIENCE
(Name of student) (Degree)

in Forest Management presented on 1 May 74
(Major) (Date)

Title: GEOMETRICAL CONSIDERATIONS AFFECTING THE DETERMINATION OF BASAL AREA GROWTH BY INCREMENT BORING METHODS

Abstract approved: Signature redacted for privacy.
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The purpose of this study was to examine the geometry of enclosed circles in order to predict the probable effects of increment boring methods used in determining basal area growth. Of specific concern was the comparison of areas derived from sets of borings taken from opposite sides of the tree contrasted with sets taken at right angles to each other. A simple function for bias using opposite borings is given. Four borings in each of 107 sample trees were combined in several ways, and the differences between areas produced by these combinations were statistically tested. The differences between areas computed using opposite sets of borings were significantly smaller than those using sets at right angles at the .01 level, and the variance of those differences was also significantly smaller at the .01 level.

GEOMETRICAL CONSIDERATIONS AFFECTING THE DETERMINATION OF BASAL AREA GROWTH BY INCREMENT BORING METHODS

by

Kimberly Iles

A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Master of Science

June 1974

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Date thesis is presented 1 May 74

Typed by Karen Bland for Kimberly Iles

ACKNOWLEDGMENTS

The author wishes to thank several people for their assistance during the course of this study. He is lastingly indebted to Dr. John F. Bell for the benefit of two years of excellent educational opportunity in the field of biometrics. He would like to thank Dr. Roger G. Peterson for his assistance with the statistical approaches. Mr. Lester J. Wilson aided in appraising the mathematics in a long search for more elegant and simple means of expressing relationships, and it is much appreciated. Lastly he wishes to acknowledge Dr. H. D. Brunk whose example as a teacher and master of his field was an inspiration.

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INTRODUCTION

For many years the use of basal area as a measure of growth or relative growth has been widespread in the field of forestry. The determination of the area of a ring of wood on a stem cross section, composed of one or several years' growth, has been a problem whenever the tree cannot be felled and the cross section directly examined. The common approach has been through the use of increment borings, normally two in number, and often at right angles to each other. Regardless of objections that basal area at a single height is not a total indicator of volume growth the method continues to be used extensively in the field, for management and for research. Recent work indicates that borings at opposite sides of the tree may have advantages. Although it is commonly agreed and often repeated that trees do not have a circular cross section, the circular model is almost always used in practice, but has been only slightly examined, particularly with regard to eccentricity in the enclosed circles. It is the purpose of this study to examine the geometry of the circular model, in hopes that it might yield additional information on the preferred method of using increment borings, and perhaps on the reasons for the experimental results noted in past studies.

LITERATURE REVIEW

Because of the obvious effect of the basal area of geometrical forms on the resultant volumes, interest in accurate determination of basal area was evident some years ago. The geometry of the stem cross section was of concern to McArdle (1928) and Robertson (1928) who commented on the relative accuracy of calipers vs. diameter tape. As was often the case in later years their work was based on empirical study with little reference to geometrical theory. The main concern was with the repeatability of the two methods, and the estimation of the amount of the usual area underestimation using calipers compared to the diameter tape. The very excellent work by Chaturvedi (1926) and the addition to it by the Imperial Forestry Institute (1944) cover very well the bias incurred in elliptical forms when diameter tape or caliper are used. Chaturvedi also points out that the common formulas used to obtain area from diameter measurements are not correct for an elliptical form even though they are often invoked under the argument that tree stems are elliptical. Chacko (1961) concluded from the four species he studied that tree cross sections more closely resembled a circle than an ellipse.

Probably the most complete work in the field was done by Matern (1956) who examined the geometry of closed convex curves. Matern defined a diameter (D) as the distance between parallel lines which are tangent to the convex figure. This is the same as a caliper measurement when the caliper arms are straight and parallel. He used a theorem first published by Cauchy to prove that the perimeter of the figure, divided by pi, gave the average of all possible diameters (D_o). This was quite an important observation, since it showed that the expectation of a random caliper measurement was exactly the same as the value given by a diameter tape. Brickell (1970) later referred to this work by Matern after the publication of an article by Ross (1968). Ross had been examining the use of optical calipers to get upper stem measurements, and had questioned the comparability of such measurements with diameter tape estimates.

Matern further points out that the quadratic mean of randomly measured radii offers an unbiased method of determining basal area, even if the figure is not convex. This is of particular interest to foresters who might wish to consider the pith as a point from which to measure such radii. Six examples of convex regions were tested by Matern, using several equations for area, and a slight overestimate was obtained in almost all cases. This overestimate was never greater than 2 percent for the elliptical forms when more than one measurement was used.

The work discussed up to this point has been concerned with the calculation of total basal area rather than with the basal area of rings, but it is important, particularly in respect to the validity of the circular model, since the ring area is commonly calculated as the difference between two circles of given diameter.

A later paper by Matern (1961) examined the precision of diameter growth estimates using increment borings. He found that the standard error of the calculated area was smaller when using borings from opposite directions than those at right angles, and further notes that:

> It is seen that the average for two cores from opposite directions is as precise as an average for four cores taken independently and at random.

No geometric causes are stated in relation to this observation. Siostrzonek (1958) did work with ring areas on an empirical basis, particularly as regards log disks, and suggested that the quadratic mean should be used when several radii are measured. Reukema (1971) makes several observations which bear upon the work presented in this thesis. He states the following:

> The geometric center of each ring generally shifts from the pith in an approximately consistent direction.
>
> The average of measurements along two radii at right angles to each other, as recommended by some, is very likely to either grossly underestimate or grossly overestimate growth.

USE OF THE INCREMENT BORE

The high cost of felling and sectioning trees, as well as the loss of material and destructive nature of the operation makes the use of log disks unreasonable in most cases, even though the results are excellent and sometimes warranted for research work. In the vast majority of instances a nondestructive indication of growth on many trees is needed, and no better measurement scheme seems to have been found than the use of the increment borer. It provides a fast estimate of current growth in terms of ring width, which can then be reasonably extended to basal area growth. Some aspects of the care and use of the increment borer are covered by Hermann (1971) and the rachet assembly described in that publication was used in the field test described in this paper. The increment borer is essentially a hollow steel shaft with a cutting edge. This shaft is screwed into the tree stem and a core of wood is extracted from the hollow center. The distance between growth rings, along the wood core, is then used as an indication of the ring width.

Several aspects of the field use of increment borings became apparent during the study. It was difficult to accurately locate opposite points on the tree stem. Lacking some sort of complicated mechanical aid the only practical method of assistance in this task seemed to be circumference division using the diameter tape. The diameter tape also aided in insuring that the boring was done at a uniform height.

No satisfactory method was discovered for insuring that borings were directed towards a common point in the tree or parallel to another boring. Bark thickness was measured from the edge of the diameter tape to the edge of the wood as suggested by Mesavage (1969).

There is a simple procedure that eliminates many of the problems of multiple increment borings when the trees are small enough to use it, and this is simply to bore entirely through the stem. This procedure has several advantages.

1. It eliminates most of the problems of bark thickness, which is usually noted only to get inside bark diameter.

2. There is no question about the measurements from opposite sides of rings being parallel and directed at the same point.

3. The chord distances can be directly measured, without errors from the subtraction of other measurements, which may be cumulative in nature.

4. Any regular error from the boring, in either the horizontal or vertical direction is more easily detectable and measurable, is identical on both sides of the tree, and therefore can be compensated for more easily and more accurately.

5. There are several theoretical advantages to the measurement of this chord distance which will be covered later in this paper.

It is recognized by the author that this type of measurement is often impossible, principally because of the size of the trees sampled, but where possible it is the preferred method. The pith of the tree can often be found by utilizing branching angles, since the branch often points directly toward the pith. Care should be taken to avoid boring through areas which are close to wounds, branch stubs or other obvious disturbances. It is wise to check the desirability of all boring points before the first one is taken, so that if adjustment is necessary it can be done in such a way as to maintain the same angular relationship between the borings. The placement of these borings will be discussed separately.

THEORETICAL CONSIDERATIONS

The Cross Section as a Circle

However vigorous the protests that stems do not have circular cross sections, the common practice is to use equations that consider them circular. If the investigator insists upon using an elliptical model for the stem cross section, then the proper formula for an ellipse would be Area $= \frac{\pi a b}{4}$ where a and b are the major and minor diameters of the ellipse. Instead of this formula the most common formulas used for cross sectional area are:

$$(1) \quad A = \frac{\pi}{4}\left(\frac{a^2 + b^2}{2}\right) \quad \text{or} \quad (2) \quad A = \frac{\pi}{4}\left(\frac{a + b}{2}\right)^2$$

Where a and b are two diameters measured across the cross section. These more closely resemble situations where a circle was being measured, but the measurement scheme was not too precise, and therefore some sort of average was taken of several measurements. The work done by Matern (1956) seems to indicate that the circular model is very adequate, and this would seem even more warranted for stem sections well above the root swell and for younger trees. The application of the circular model is obviously in order in many cases, and lacking any simple figure for a substitute is often applied to all trees in a particular inventory or research study.

Opposite Boring System

Concentric Circles

This is the most simple situation, and the calculations are quite simple as well. Let us consider the two situations in Figure 1. In the first situation (a) the diameters are measured through the centers of both circles and the ring area computed by the formula:

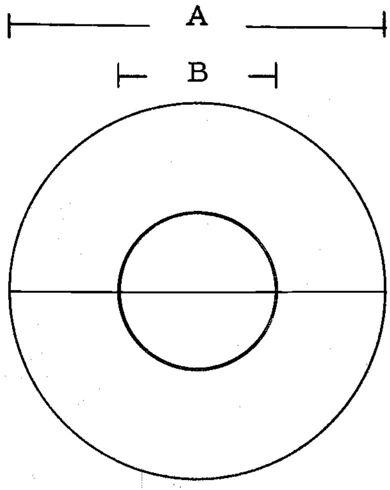
$$AR = \frac{A^2\pi}{4} - \frac{B^2\pi}{4}$$

In the second case (b) the distances shown are from the edges of the circles, but do not pass through the centers of them. The area of the ring is again computed by

$$AR = \frac{A^2\pi}{4} - \frac{B^2\pi}{4}$$

exactly as before, and furnishes exactly the same answer. This equation is valid for calculating the ring area using <u>any</u> line which passes through, or is simply tangent to, the inner circle. It does not, of course, yield the proper basal area for either of the two circles individually, but the area of the ring is correct since the bias is identical for both of the individual circles and cancels out during the subtraction of their areas.

a. distances measured through centers of concentric circles



b. distances not measured through centers of concentric circles

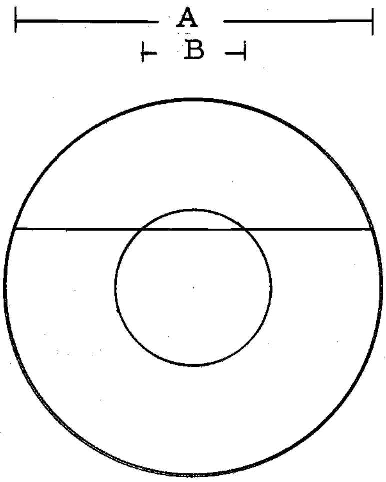


Figure 1. Illustration of opposite boring system through concentric rings.

The properties of this ring calculating chord will be further examined, since it affords an elegant way of viewing future problems of this type.

This method of calculating the ring area is given by Gardner (1959). The third dimensional analogue is sometimes used as an interesting exercise in calculus. As it concerns the problem at hand, the statement of the theorem is this:

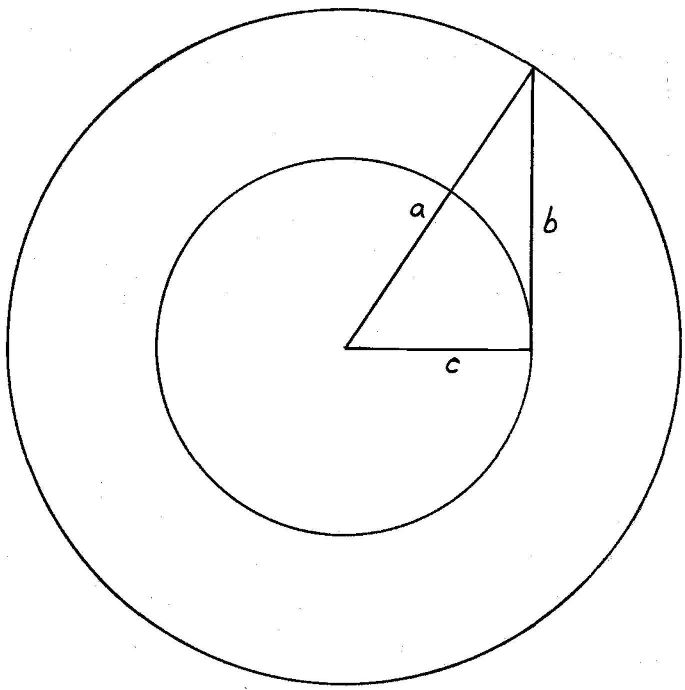
> The half chord length b (see Figure 2) is the radius of a circle, the area of which is the same as the area which lies between two concentric circles of radii a and c, one of which is tangent to the chord 2b, the other which has 2b as a chord.

The proof is an adaptation of the pythagorean theorem and is included with Figure 2 for those who are interested.

The areas of any sets of rings through which the borer passes may thus be calculated even though the borer does not pass through the center of the circles. The distances across the rings are simply used as though they were the diameters themselves. Note that this is in no way a sampling procedure, but a measurement in the same sense that diameters would normally be measured. This mathematical concept does not seem to be widely known in the field of forestry, or perhaps has simply not been published.

Eccentric Circles

The term eccentric is used here to indicate that the circles do



Proof

$a^2 = b^2 + c^2$ by the pythagorean theorem multiplying by π on both sides yields

$a^2\pi = b^2\pi + c^2\pi$

solving for $b^2\pi$ yields

$b^2\pi = a^2\pi - c^2\pi$

at which point we recognize that

$b^2\pi$	=	$a^2\pi$	-	$c^2\pi$
(area of ring)	=	(area of large circle)	-	(area of small circle)

Figure 2. Illustration of the application of the pythagorean theorem to the calculation of ring area.

not have a common center, not that they are other than perfectly circular in shape. Two other terms will be used throughout the paper and should be explained at this point.

Direction of offset - is used to refer to the direction in which the center of an interior circle has moved with respect to the center of the outer circle.

Axis of offset - refers to a line drawn through the center of two circles which are eccentric with respect to each other, and extending in both directions (see Figure 3a).

There are two instances of measurement with eccentric circles illustrated in Figure 3. The first case (b) is the situation where the measurement is made through the centers of both circles, and therefore necessarily along the axis of offset. In this case the ring area is computed exactly with the formula RA = $\frac{A^2\pi}{4} - \frac{B^2\pi}{4}$. In the second case (c) the measurement is also parallel to the axis of offset, but goes through the center of neither of the circles. In this case the area of the ring is also given exactly by the formula RA = $\frac{A^2\pi}{4} - \frac{B^2\pi}{4}$. The proof is slightly more tedious but quite simple to see intuitively. As the circle slides along the axis of offset neither distance A nor B are changed in the slightest, therefore any results from an equation with the inner circle at the center of the outer circle (the concentric case) will be duplicated at any point along the axis of offset. This system of course requires one continuous increment core or two at

Terminology used for eccentric circles

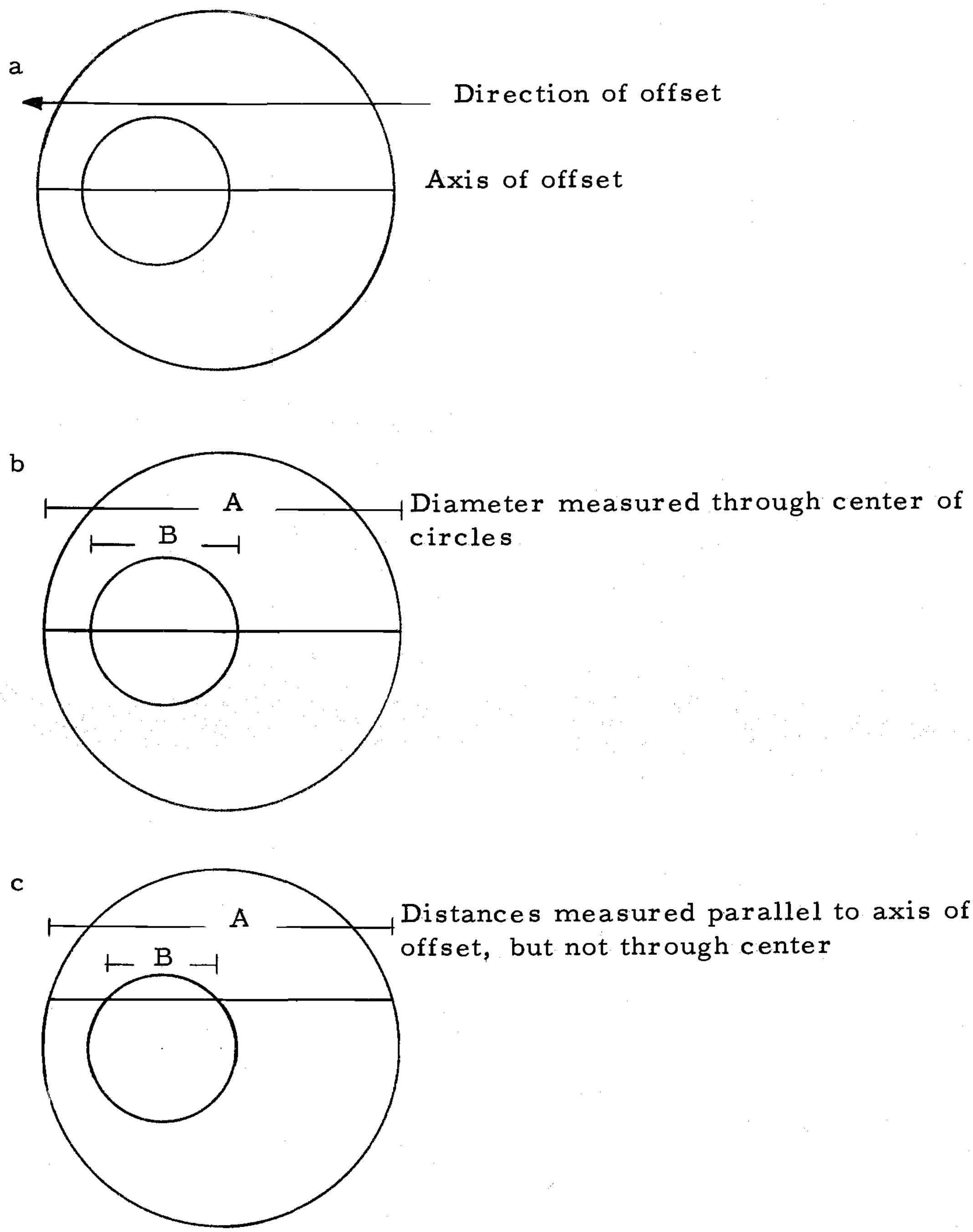


Figure 3. Illustration of opposite boring system through eccentric rings, and terminology.

opposite sides with a caliper measurement between their entrances. There is no error in determining the area as long as the measurement is along the axis of offset. Approximately this same result has been noted experimentally, but the underlying reason for it was not noted.

Reukema (1971) states the following:

> Examinations of several cross sections indicated that patterns of growth tended to be more consistent on the long and short radii than on other radii. Therefore, measurements along radii whose orientation closely approximates these two should generally give as good an estimate as can be obtained by any method other than with a planimeter.

When the radius is measured from the center of an inner circle to the edge of an outer circle the long and short radii necessarily lie along the axis of offset.

It should be stated at this point that the formula used for all four cases is exact and correct. The use of a quadratic or geometric mean diameter is incorrect. It may certainly be said that the use of some other method may compensate for errors in measurement, non-circularity of the stem, errors of alignment, etc., but these are considerations quite apart from our discussion here.

Bias in Ring Area

Even though we may have good reason to suspect that the axis of offset is known, either because of the slope, prevailing wind or branching indicators, it will not always be anticipated correctly and

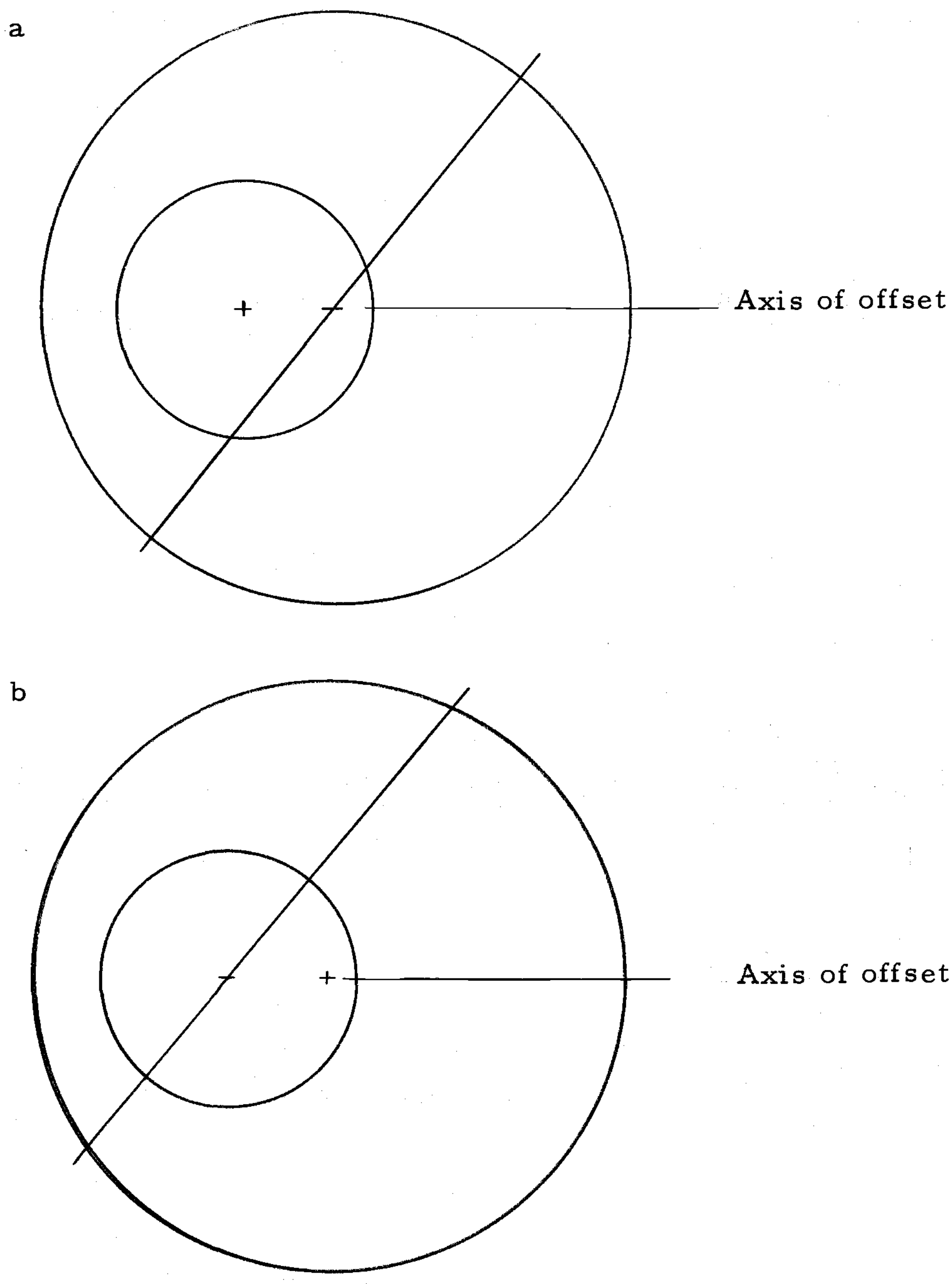


Figure 4. Illustration of the case where the bore passes through the center of one circle, but misses the center of the other.

some attention must be given the bias incurred by boring at an angle to the axis of offset. Consider Figure 4. There are two possible situations shown.

The first situation (a) is where the borer passes through the center of the outer circle, but misses the center of the inner circle. In the case where the formula $RA = \frac{\pi}{4}A^2 - \frac{\pi}{4}B^2$ is used, $\frac{\pi}{4}A^2$ would correctly estimate the outer circle but $\frac{\pi}{4}B^2$ would underestimate the inner circle. The difference, or calculated ring area, would then be too large. In the second case (b) the smaller circle would be correctly estimated, but the larger one underestimated, leading to an underestimate of the ring area.

In practice one technique is to bore through the pith, assuming that it is the center of the inner circle, thus eliminating bias for the inner circle; and then to use a diameter tape to obtain the diameter of the outer circle without bias. This system is not feasible with the many layers of interior rings which may be of interest, and which cannot be measured with a diameter tape. The problem is to find the bias in individual areas of pairs of circles. If both have equal or zero bias, the ring area is correct. If the inner circle has a larger negative bias (the bias will never be positive) the calculated ring area will be too large. Conversely, if the outer ring has a larger negative bias the ring area will be calculated to be too small.

The amount of area bias for an individual circle is not difficult to compute, nor is the chord length across the circle. Let the distance along the axis of offset from the center of the circle be designated d, and the angle of the boring from the axis of offset be designated θ as in Figure 5. The chord length C is calculated using θ, d, and r which is the radius of the circle, by the following formula:

$$C = 2\sqrt{r^2 - (d * \sin\theta)^2}$$

This is derived from an application of the pythagorean theorem, and is mentioned in case the reader is interested. The bias can be calculated without it. The bias is found by making use of the theorem earlier proven in relation to the ring calculating chord. The area calculated from the chord is really the area of a circle, excluding a smaller circle tangent to the chord and having the same center.

Figure 5 indicates such a circle and its radius f. The bias is simply the difference between the true area of the outer circle and the area calculated by the chord as a diameter. The bias must then be the area of this tangent circle, since the area of a circle with the chord as diameter has been shown to be the area of the ring. The radius of the tangent circle is equal to d*sin θ, and the area is $(d * \sin\theta)^2 \pi$.

The bias for any circle through which a chord passes is therefore the same as the area of a smaller circle with the same center and which is tangent to the chord. The bias in the area of a ring

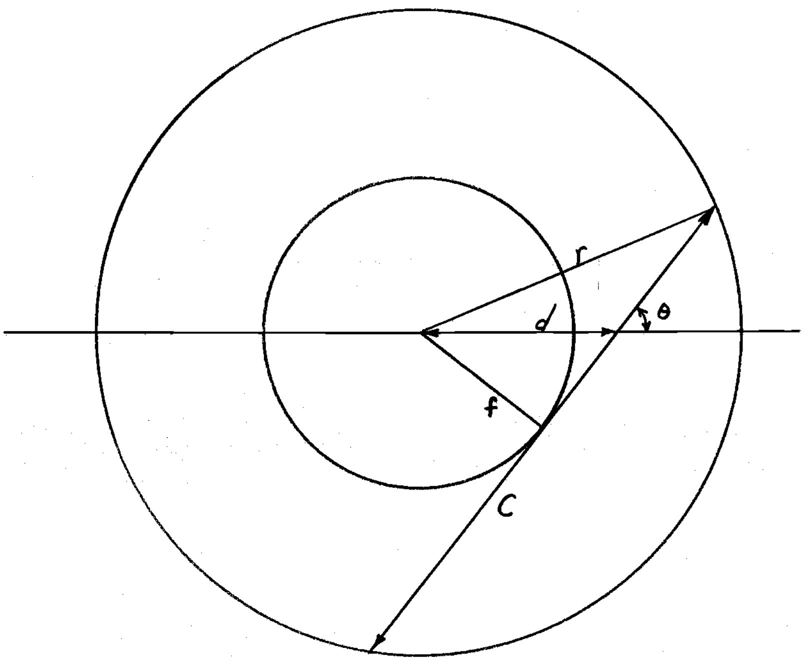
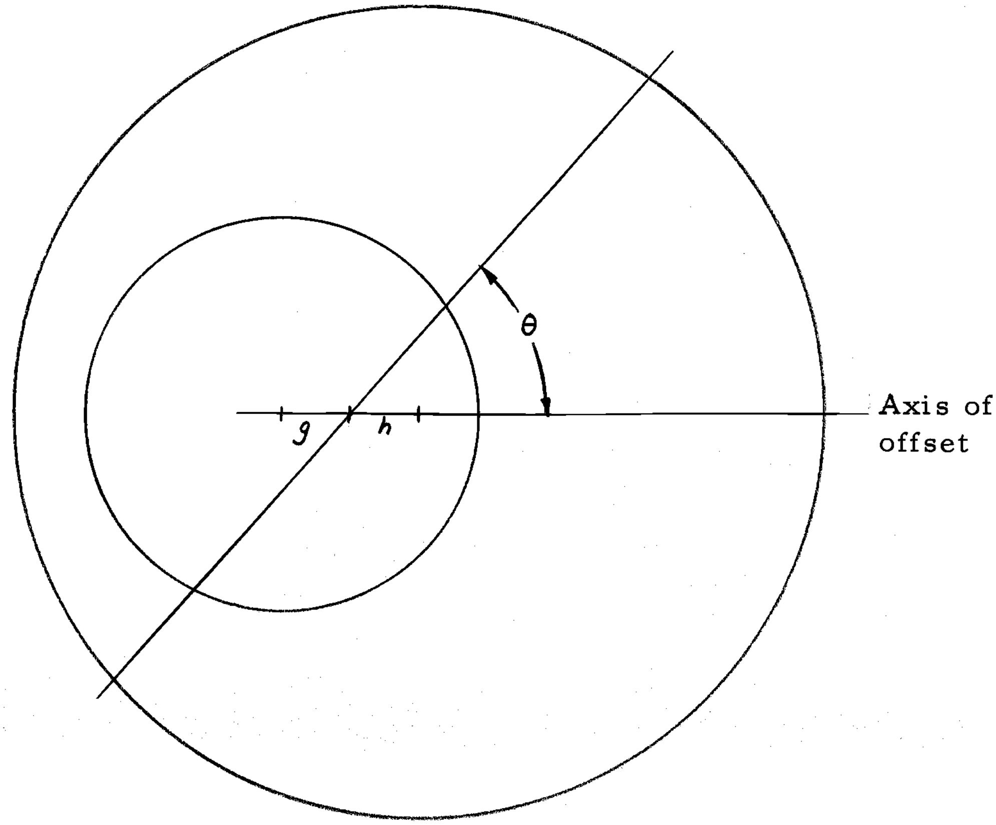


Figure 5. Indication of the bias incurred in circle area by using a chord as diameter rather than the true diameter.

between circles is dependent on their individual biases. If the circles have the same center there is no bias in the calculated ring area. The biases also decrease to zero when the angle of the increment boring approaches the axis of offset, thus decreasing the radius of the tangent circle to zero. Note that the bias does not depend upon the size of the circle through which the chord passes.

The final question to be examined was whether there existed a system for measuring through eccentric circles, such that the correct ring area between them could be determined regardless of the relationship between the boring angle and the axis of offset. The key to the problem was simply to insure that the bias in both circles was identical. Since the bias is dependent only on the distance from the chord to the center of the circle, and not the size of the circle, it was necessary only to insure that the chord (increment bore) passes the same distance from the center of either circle. This situation is shown in Figure 6.

There remains the problem of locating such a point. There would not appear to be any simple way of doing this in the field, but that such a target point exists at all, for any two circles, is of considerable interest. Since the offset of tree rings generally progresses in a constant direction, it would be reasonable to expect this point to be between the geometric center of the tree and the pith. It would also be expected that less bias would be incurred by boring through any



Where g + h is the distance between circle centers measured along the axis of offset, and g = h.

Figure 6. A system which gives the correct value for ring area for any angle θ, and any offset.

point between the pith and geometric center, than by boring through the pith itself. When the areas among three or more circles are desired, all of which have different centers, there appears to be no single target point which will give an unbiased result. The advantages of a single increment core, extending completely through the tree, should be obvious from the foregoing discussions.

Right Angle Boring System

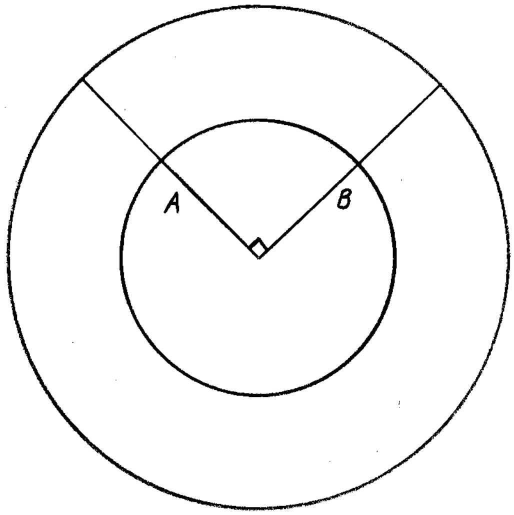
General Discussion

It is common practice for foresters to take two increment cores at right angles to each other, and to average the ring radii found, using either a simple arithmetic mean or the quadratic mean. This discussion will be limited almost exclusively to the simple arithmetic mean. This system was probably an outgrowth of the practice followed for determining the basal area of a tree using calipers, in which two measurements at right angles were commonly used. Chaturvedi (1926) credits the popular adoption of this procedure to Klauprect. Recent work indicating the superiority of opposing borings will probably be slow to change this practice, however conclusive the studies may be.

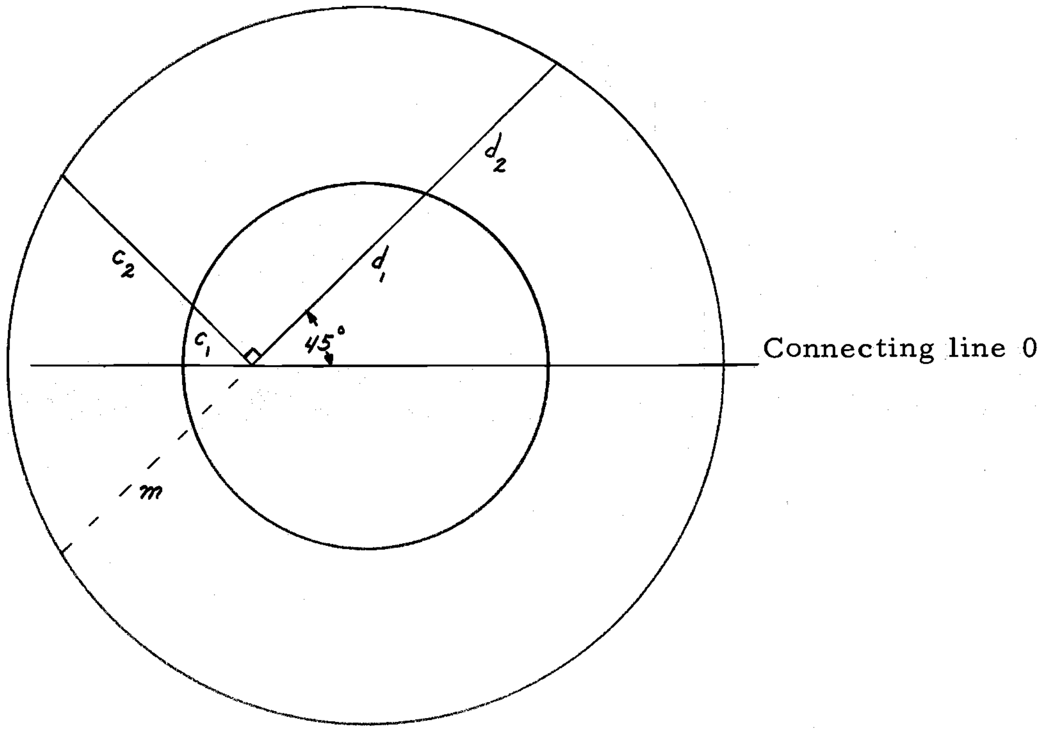
Concentric Circles

Two situations regarding the measurement of concentric rings are shown in Figure 7. In the first case (a) the mathematics are quite obvious when the vertex of the two measurements is at the center of the circles. Since both distances are equal to the true radius, the area of the ring as calculated by the formula RA $= \frac{\pi}{4}A^2 - \frac{\pi}{4}B^2$ will yield the correct answer. There exists a situation in the use of perpendicular borings which is almost exactly the same as previously discussed for opposing borings. The second illustration (b) in Figure 7 shows this rather unusual situation. In this case the formula $\frac{\pi}{4}A^2 - \frac{\pi}{4}B^2 =$ RA will give precisely the correct answer for the area of the ring. A good deal of laborious calculation can be avoided by simply recognizing that the distance along the shorter core (C_1+C_2) is exactly the same length as indicated by the dotted line m. This really is a situation of a reflection of the incident angle of the first core with respect to the line passing through the vertex of the cores and the center of the circles. Since this is true the same proof as provided earlier with Figure 2 will apply. It can then be concluded that no bias in calculated ring area will exist, provided that each of the perpendicular cores forms an angle of 45° to a line passing from the centers of the circles through the vertex of the cores. This proof actually holds whenever the angles of the cores to the connecting line (0) are equal, but with the stipulation that the cores also be at right angles to each other, the restriction to 45° is automatic.

a



b



Where

$$A = C_2 + C_1 + d_1 + d_2$$

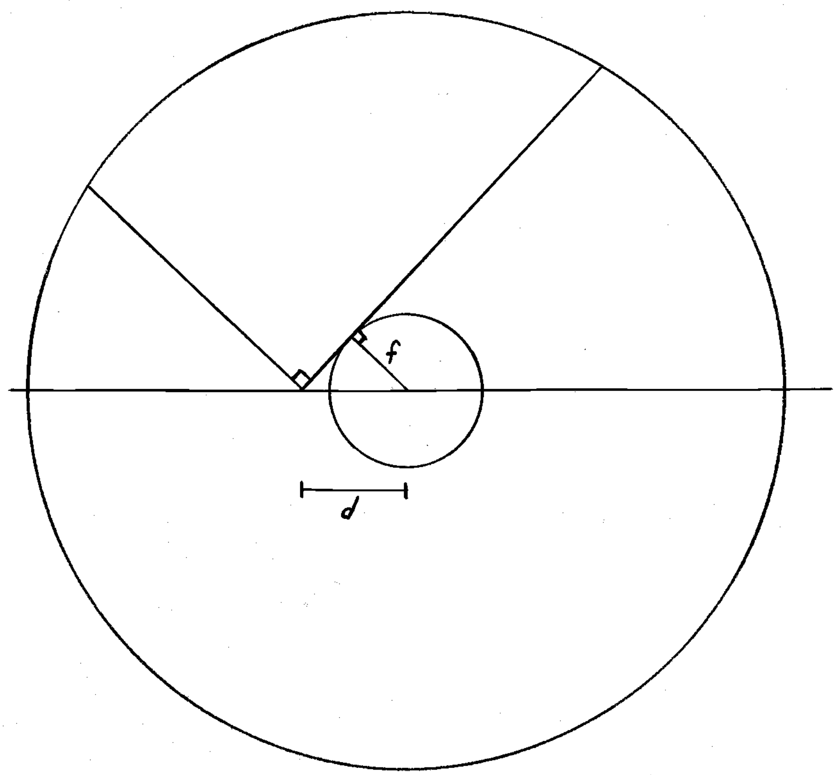
$$B = C_1 + d_1$$

Figure 7. Adjacent borings through concentric circles.

Eccentric Circles

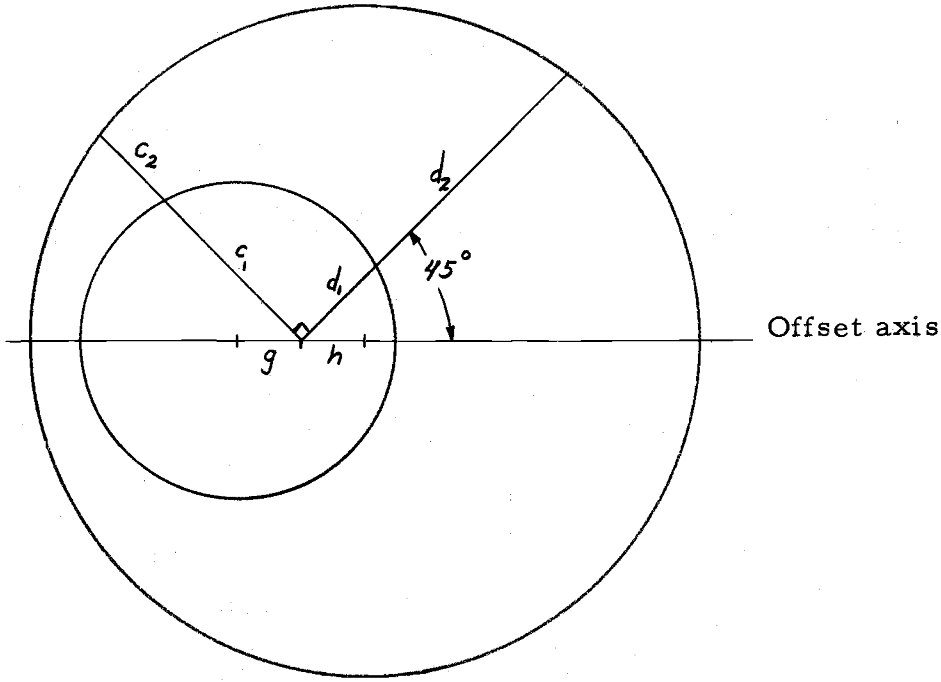
The situation with respect to eccentric circles is complicated when increment borings are at right angles to each other. No simple relationships were found for expressing bias, except in the case where the vertex of the boring lay along the axis of offset, and both borings were at a 45° angle to that axis. In such situations the bias is exactly the same as with opposing borings. The pith, commonly used as a meeting point for the two borings is generally along the axis of offset, so there is some practical advantage in establishing this relationship. The bias in an individual circle area is dependent on the distance (f) from the center of the circle to the partial chord. The bias is the same as that of a circle with radius f. Figure 8 shows a typical example, and the circle of radius f which expresses the bias. When calculating the ring area between two circles the bias for both is identical when the vertex of the bores is equidistant from the centers of both circles, along the axis of offset, and with both bores at an angle of 45° to the offset axis. In this case there is no resulting bias in the calculated ring area. Figure 9 shows a scheme for determining ring area without bias, which closely resembles the method used with opposite borings, but with the restriction that the angle θ can only be 45°.

The function for the length of the partial chord from the vertex



$f = d * \sin 45^{\circ}$

Figure 8. Illustration of the bias incurred by missing the center of a circle with adjacent borings.



ring area $= \frac{A^2}{4}\pi - \frac{B^2}{4}\pi$

where

$A = C_2 + C_1 + d_1 + d_2$

$B = C_1 + d_1$

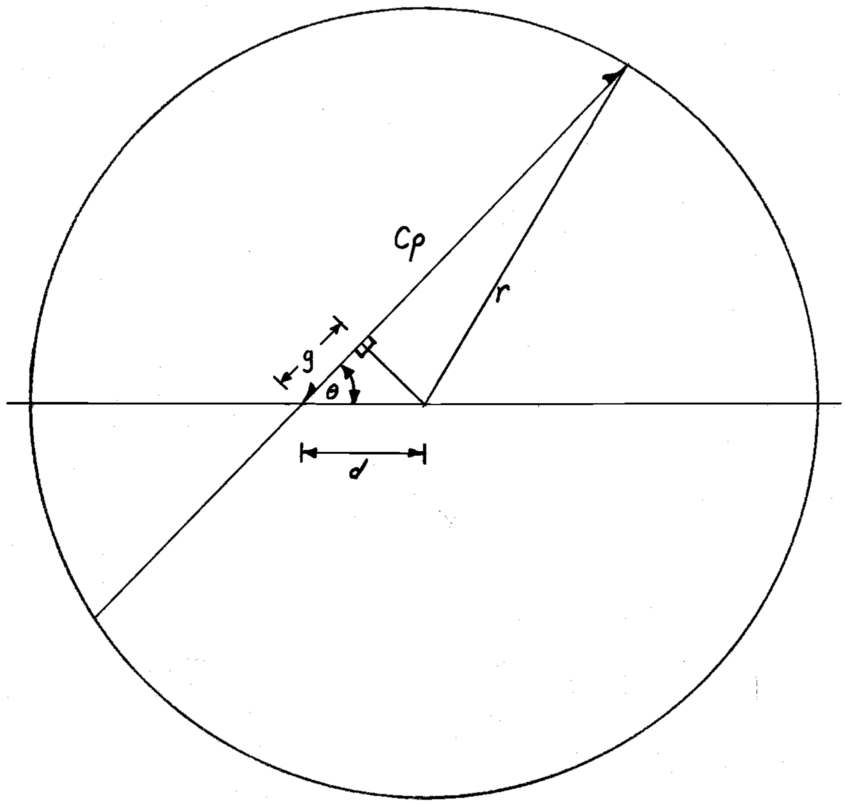
$g = h$

Figure 9. An unbiased system for measuring ring area between eccentric circles with adjacent borings.

of the cores to the edge of the circle is not difficult to calculate. Let the distance along the partial chord be designated C_p (see Figure 10) with the distance from the center of the circle d; r is the radius of the circle; and the angle of the partial chord to the axis of offset is θ. The distance C_p is defined by the equation

$$C_p = \sqrt{r^2 - (d * \sin \theta)^2} + (d * \cos \theta)$$

This can of course be solved for any angle θ, and for $\theta + 90$ to get the length for both legs. These summed figures can then be squared and multiplied by $\pi/4$ to get estimated circle area and then compared to $r^2\pi$ to obtain the bias. This, however, is a tedious procedure and offers no intuitive comparison to the very simple bias calculation discussed earlier in relation to opposite borings. When the borings are made at angles to the offset axis of other than 45° the area bias may be positive for one circle, while being negative for the other. This is to be contrasted to the opposite system where area bias is always negative in both circles. This would lead us to expect both larger and smaller area estimates with right angle borings, and very probably a larger variation among those estimates, whenever the angle of boring is random with respect to the axis of offset.



$$C_p = 1/2 \text{ chord length} \pm \text{length } g$$

$$C_p = \sqrt{r^2 - (d*\sin\theta)^2} + (\cos\theta * d)$$

Figure 10. Principles used to determine equation of partial chord length.

FIELD TEST

Purpose of the Study

The study was undertaken to determine whether the area differences between the two basic boring systems would yield a result which differed significantly in either mean or variance under actual field conditions. Even though the theoretical considerations indicated that the system of boring from opposite sides would be less sensitive to changes in the orientation of the increment borer, it was not known if this would result in any substantial difference under actual field conditions. The general procedure was to calculate the area using a pair of borings, and to compare it to the area derived from a pair initiated at right angles to the first pair. The assumption was that the system which was least sensitive to change of insertion angle relative to the unknown axis of offset would yield the most uniform calculated areas.

Field Procedures

During the summer of 1973, 107 trees were sampled. They ranged in diameter inside bark from 11 to 23 inches and averaged 18.6 inches. Four increment borings at 90° angles were taken from each tree at breast height, penetrating about 30 years' growth in each case. In every case an attempt was made to bore towards the geometric center of the tree. The first core was taken from the uphill side on each tree, and the downslope azimuth was recorded. Diameter was measured to the nearest 1/10 inch with a diameter tape. Bark thickness was measured from the diameter tape to the wood in increments of 1/100 inch. The cores extracted were stored in labeled plastic straws. The cores were later dipped in a solution of copper sulphate to inhibit mold, and frozen when not being examined. The necessity of these precautions became apparent when mold obscured the sapwood growth rings from samples taken early in the experiment. The trees were not selected at random, but on the basis of having a crown which was essentially free from competition and a bole free from mechanical damage. The selection was based on the requirement of the primary study being made on soil compaction response, but should not have any effect on the analysis of the data for this study. Additional data on crown width and length, and tree age were gathered for the primary study, but do not affect this analysis. The four areas from which the trees were gathered are noted on Table 1. The number of rings counted on each tree was not always identical, but the author attaches no real importance to this fact, since the ring width differences are well within the range which would normally occur simply because of differences in individual tree growth. The study was concerned with the problems of establishing ring widths, and there was no reason to restrict those ring widths to a particular number of

Table 1. Description of Trees Used in Study.

Group	Number of Trees Used	Area of Location	Number of Rings Counted
A	11	T. 11S., R. 7W., Sec. 21	13
B	38	T. 3S., R. 6W., Sec. 27	12
C	6	T. 3S., R. 6W., Sec. 23, W1/2	15
D	18	T. 3S., R. 6W., Sec. 23, E1/2	11
E	34	T. 3S., R. 6W., Sec. 9	11

Total number of trees = 107

years of growth. Abnormal growth or knots were avoided when taking cores. The direction in which the first core was taken for each sample tree is shown in Figure 11. The length of the lines shown are proportional to the number of trees first bored from that direction. The first core, which was on the uphill side, shows a good distribution through the range of directions. Slopes on which the trees were located ranged from 0 to 50 percent. When the ground was level the first core was taken from the north side.

Ring Width and Ring Area

The width of the ring along each increment core was measured to the nearest .005 inch by a single interpreter. Measurements were made using a graduated rule and Luxo magnifier, providing approximately 2X magnification through the center of a fluorescent light. The combined ring width of six combinations of two cores was calculated for each tree. Using this combined ring width, the average bark thickness, and the diameter of the tree, the ring area was calculated. A diameter tape was used when obtaining outside bark diameters since this represents common field practice and the size of the trees prevented boring the entire chord distance. The calculations required were performed using the SIPS subsystem for the Oregon State University CDC 3300 computer.

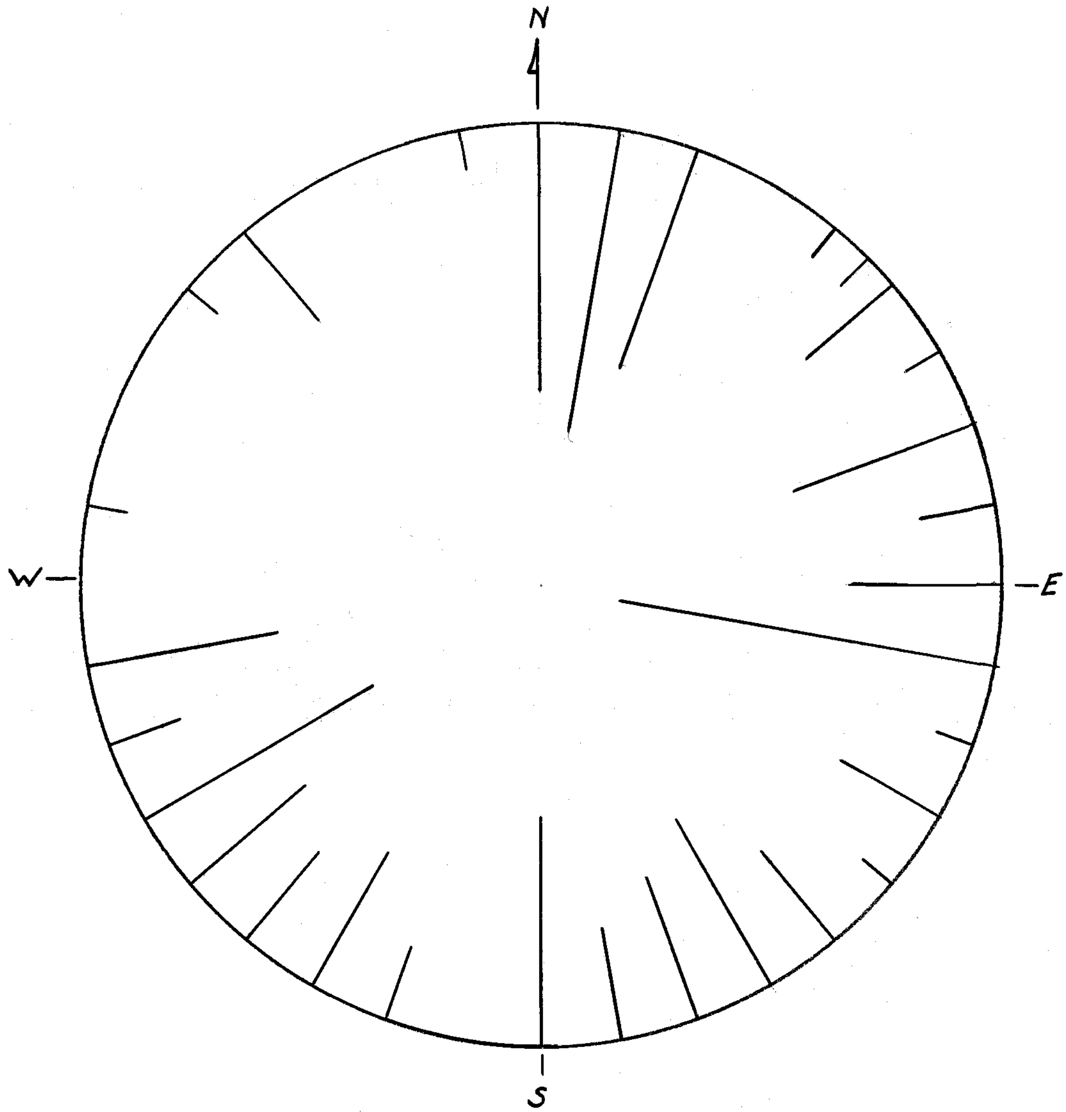


Figure 11. Direction and relative frequency of the first bore for 107 trees.

Statistical Techniques

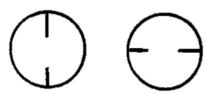
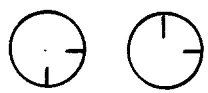
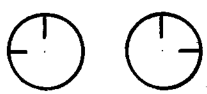
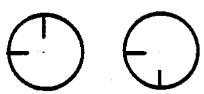
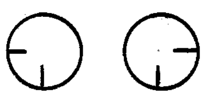
Individual ring widths, as well as the paired ring widths were tested for differences in mean and variance through the use of Students t and F tests, respectively. The same tests were also applied to the areas generated from these ring widths. Finally, the differences bebween the areas generated for each ring using a set of two borings was compared to the area generated by a set initiated at right angles to the first set. The calculated differences between the system of opposite borings were compared to those using adjacent borings.

PRESENTATION OF RESULTS

No significant differences were found between single ring widths at the .10 probability level. The maximum difference in average ring width was only 5.1 percent. There was no significant difference in variance between the width of these four rings. These results seemed to indicate a good distribution of the direction of offset between the four borings taken. The same results were noted with the areas generated from pairs of these ring widths. The areas calculated from opposing borings were in the middle of the range, with the areas formed from adjacent sets occupying the extremes. No difference in values of either mean or variance was significant at the .10 probability level. The maximum difference in mean ring area between sets of combined borings was less than 4 percent of their combined mean.

The absolute value of the area differences as calculated from two different sets of borings, initiated at right angles to each other, showed results which were very highly significant. Table 2 and Figure 12 illustrate these results. The mean difference between areas using sets of opposing borings was significantly smaller than the mean difference between areas using sets of adjacent borings at the .01 level in every instance. The differences in variance between areas calculated from opposing versus adjacent borings were also significant at the .01 level. No significant difference in mean or variance

Table 2. Mean Absolute Difference Between Ring Areas Calculated by Sets of Borings in Square Inches.

Diagram of Ring Combinations	Combinations	Mean	Variance	Standard Error
	1, 2 - 3, 4	5.239	24.717	.4806
	2, 4 - 1, 4	7.215	33.309	.558
	1, 3 - 1, 4	7.769	38.578	.600
	1, 3 - 2, 3	7.212	32.875	.554
	2, 3 - 2, 4	7.759	38.520	.600

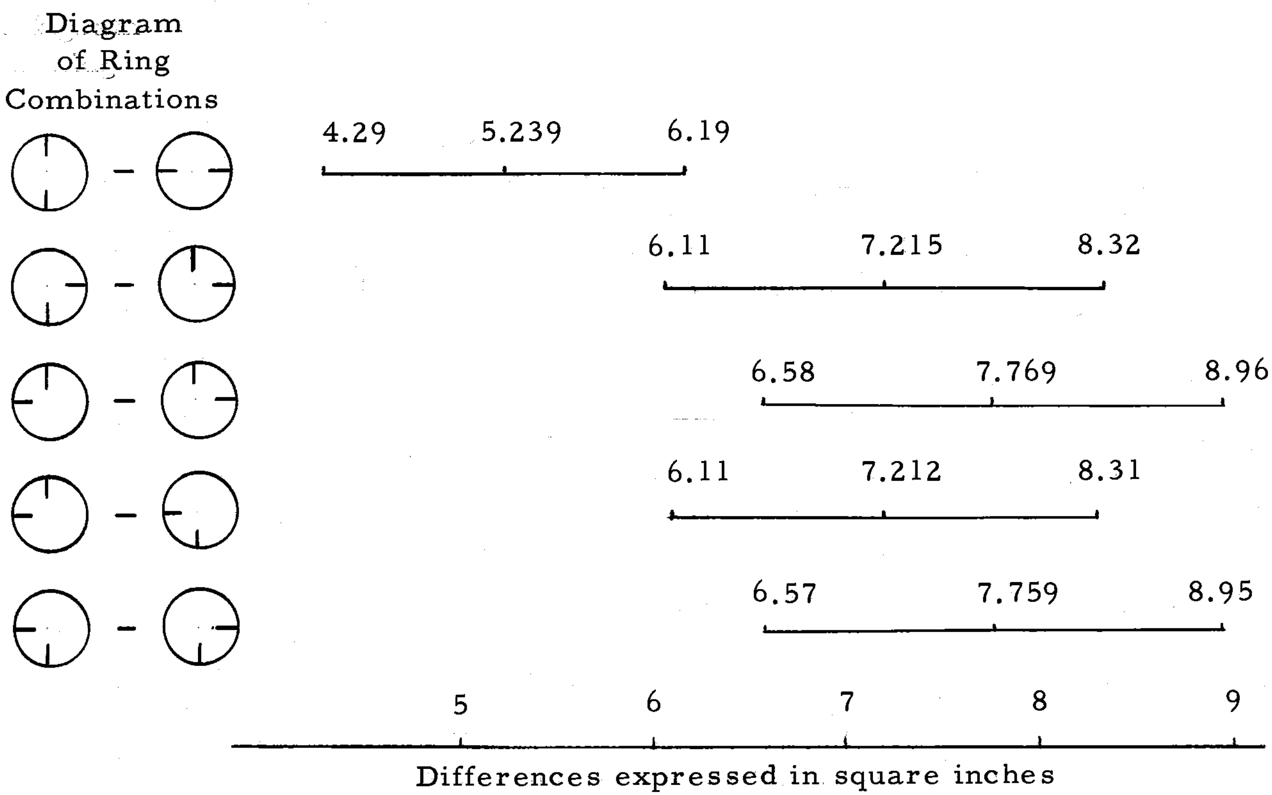


Figure 12. Mean absolute difference between ring areas calculated by different pairs of borings, showing means and 95 percent confidence intervals.

was noted among the four comparisons of sets of adjacent borings. The average difference in calculated area between sets of opposite borings was 10.5 percent of their combined mean, while the average difference between pooled sets of adjacent borings was 15.5 percent of their combined mean.

When the signed differences between calculated areas of sets of borings were averaged, the positive and negative values largely canceled out. None of these averages was significantly different than any other, nor were any significantly different than zero at the .10 level. The variance of the area difference using opposite borings is still significantly smaller than the variance of the area difference using adjacent borings at the .01 level. The average difference between areas when the sign is taken into account drops to .5 percent in the case of opposite borings, and an average of 1.7 percent in the case of adjacent borings, when compared to the mean ring area.

DISCUSSION

The results showed clearly that the area calculated by a pair of opposing borings varied less, when compared to an area calculated from another set at right angles to the first, than the areas of two sets of adjacent borings which were at right angles to each other. The increased variation caused by using a system with cores at right angles to each other does not seem to strongly affect the mean difference, which was not significantly different from zero for 107 trees. Even if the difference had been significant it would have been in the range of one to two percent, which would be tolerably small from a practical viewpoint. Where the sample size was small or where accuracy on individual trees is important, the system of opposite borings would be desirable, due to the smaller variance. The field test indicated that opposite borings gave areas that tended towards the middle of the range of calculated values, though this was not significant. The higher variance for ring area on an individual tree which was indicated by the field test was fully in agreement with theory developed earlier.

CONCLUSIONS

The use of a single chord length to estimate ring area, rather than two partial chord lengths at right angles is preferred due to smaller variance. When using opposite borings, or a single boring passing through the cross section, bias is reduced by boring parallel to a line passing through the centers of the circles of interest, or by passing as evenly as possible between the centers of the two circles of interest. When using two borings at right angles both borings should be at a 45° angle to the axis of offset, and their vertex should be located midway between the centers of the circles of interest. When any substantial number of trees are sampled the variation between tree ring areas can be expected to mask differences between the two systems studied, but repeated borings of the same tree should show smaller variance with the opposite boring technique when the axis of offset is unknown.

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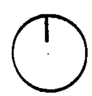
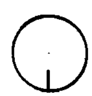
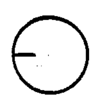
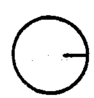
APPENDICES

APPENDIX A. RESULTS OF STATISTICAL ANALYSIS

Table 3. Mean Single Ring Width for 107 Trees.

Ring Number	Diagram	Relative Size Code	Mean	Variance	Standard Error
1		- -	.88079	.19474	.04266
2		+ +	.93621	.15525	.03809
3		-	.90383	.19838	.04306
4		+	.93009	.20378	.04364

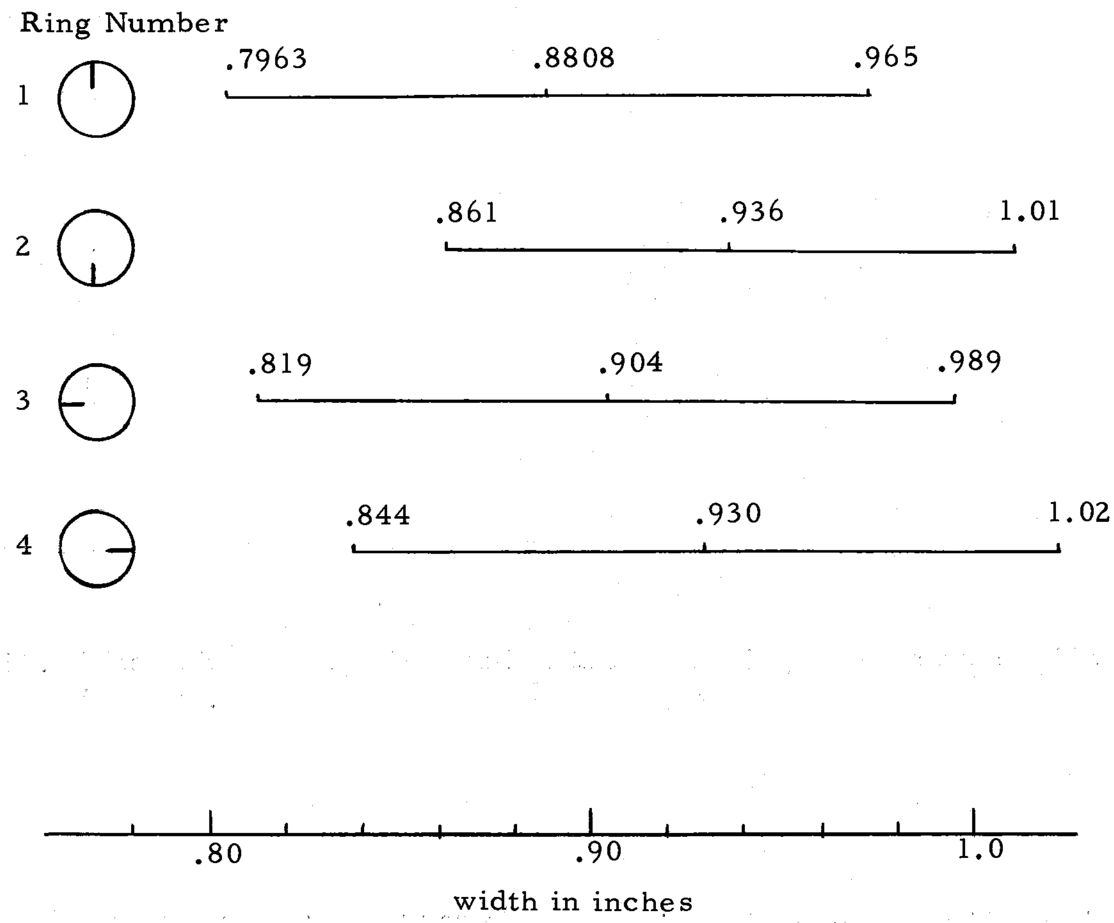
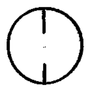
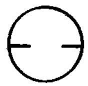
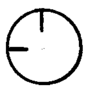
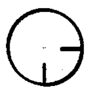
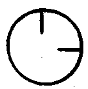
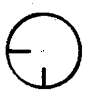


Figure 13. Mean single ring width of 107 trees showing 95 percent confidence interval.

Table 4. Mean Combined Ring Width for 107 Trees.

Diagram	Ring Combination	Mean	Variance	Standard Error
	1 + 2	1.817	.5601	.07235
	3 + 4	1.834	.6560	.0783
	1 + 3	1.785	.7057	.08121
	2 + 4	1.866	.6150	.07581
	1 + 4	1.811	.7083	.08136
	2 + 3	1.840	.6109	.07556
	Pooled opposites	1.825	.6053	.05318
	Pooled adjacents	1.825	.6563	.03916

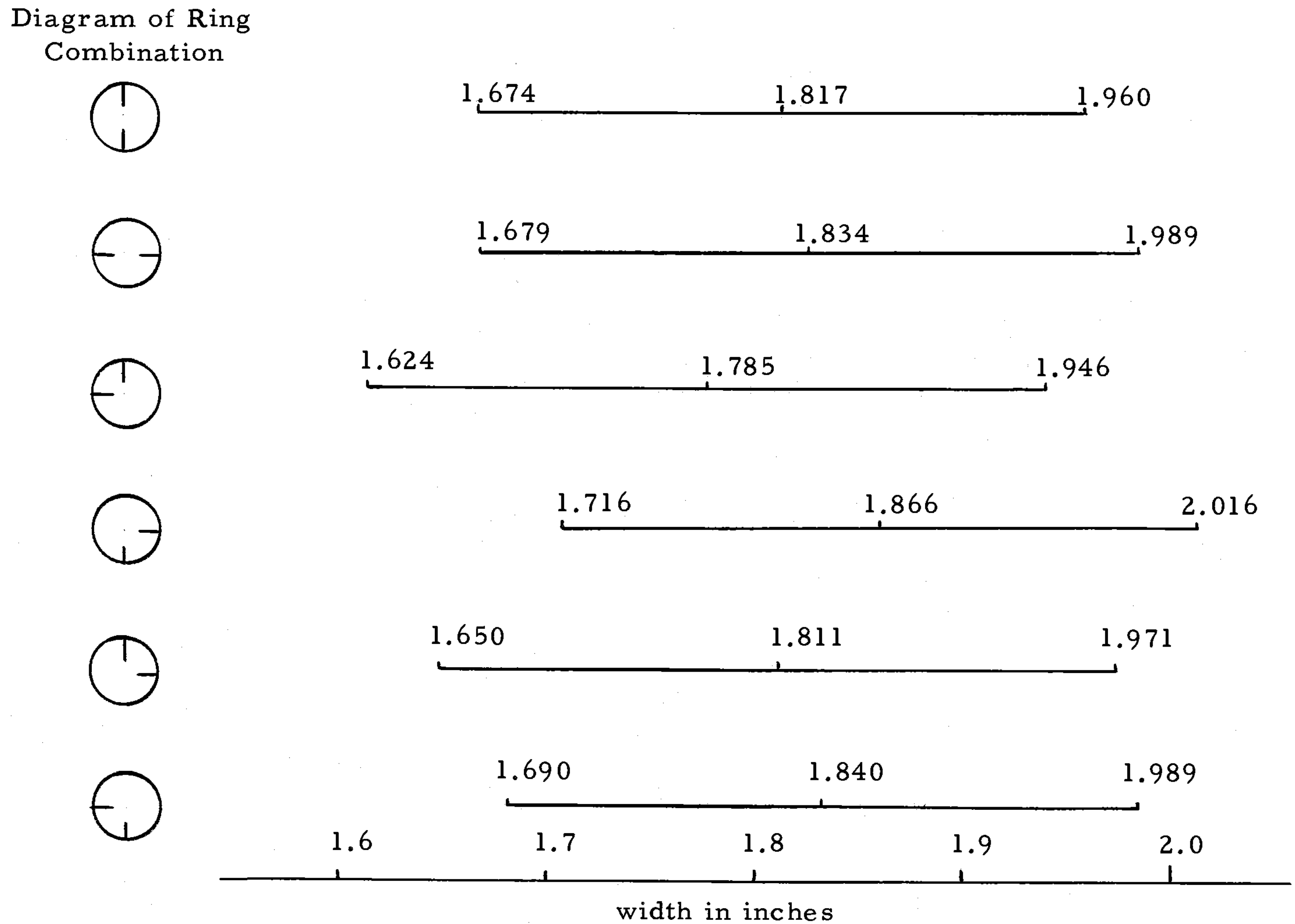
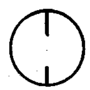
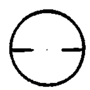
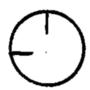
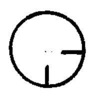
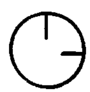
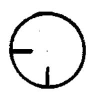


Figure 14. Mean combined ring width for 107 trees showing 95 percent confidence intervals.

Table 5. Mean Calculated Ring Area for 107 Trees.

Diagram of Ring Combination	Ring Combination	Mean	Variance	Standard Error
	1, 2	48.731	344.95	1.796
	3, 4	48.988	396.41	1.925
	1, 3	47.849	427.00	1.998
	2, 4	49.787	374.53	1.871
	1, 4	48.474	437.45	2.022
	2, 3	49.166	369.67	1.859
	Pooled opposites	48.86	368.96	1.313
	Pooled adjacents	48.82	399.87	.967

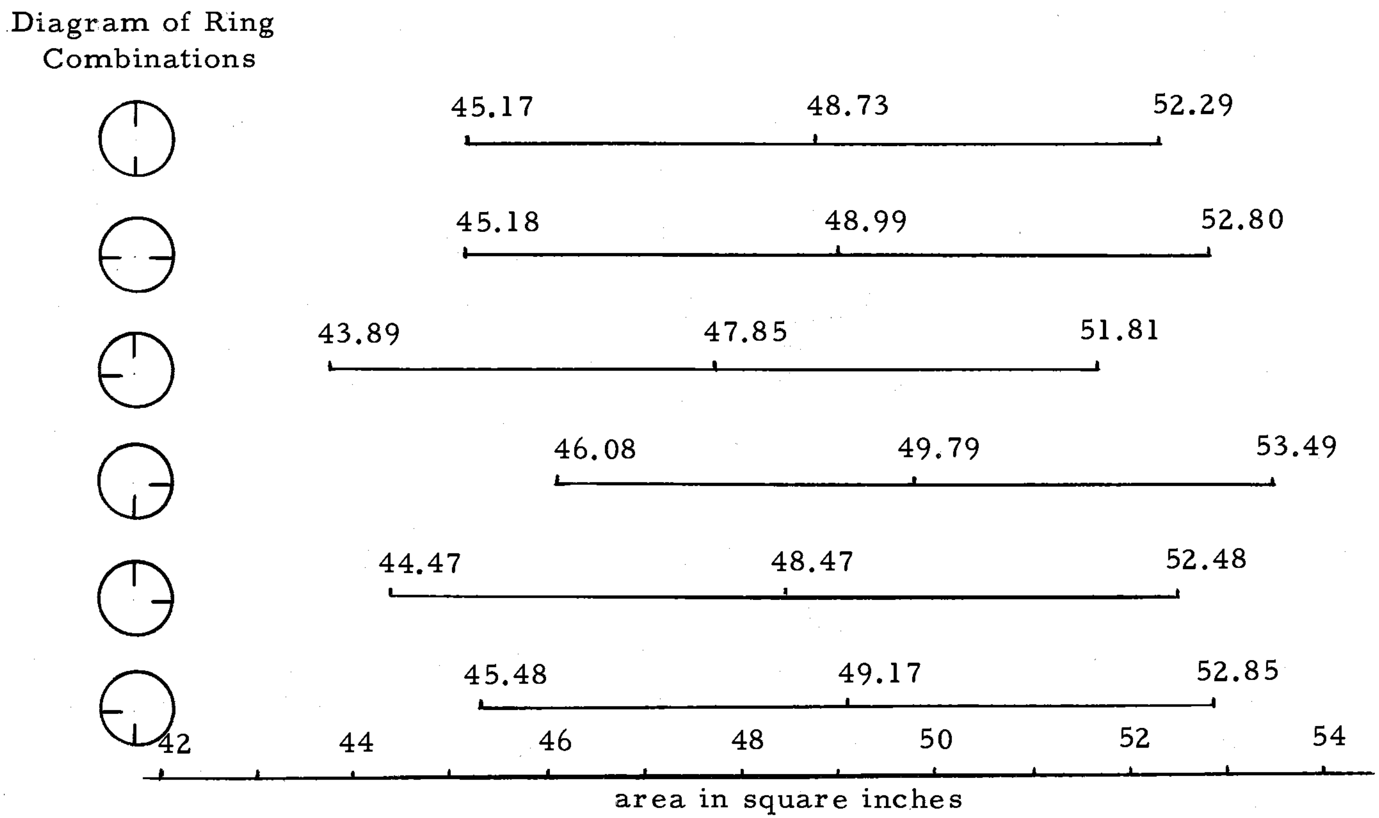
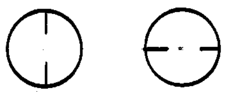
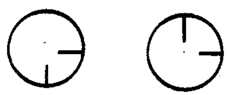
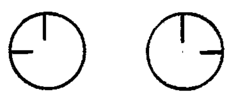
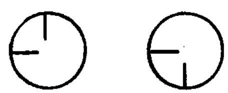
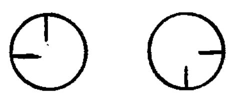


Figure 15. Mean calculated ring area of 107 trees showing 95 percent confidence interval.

Table 6. Mean Differences Between Ring Areas Calculated by Sets of Borings.

Diagram of Ring Combination	Ring Combinations	Mean	Variance	Standard Error
	1, 2 - 3, 4	-.2546	52.365	.6996
	2, 4 - 1, 4	1.3134	84.108	.8866
	1, 3 - 1, 4	.6241	99.111	.9624
	1, 3 - 2, 3	-1.3168	83.630	.8841
	2, 3 - 2, 4	-.6207	98.894	.9614

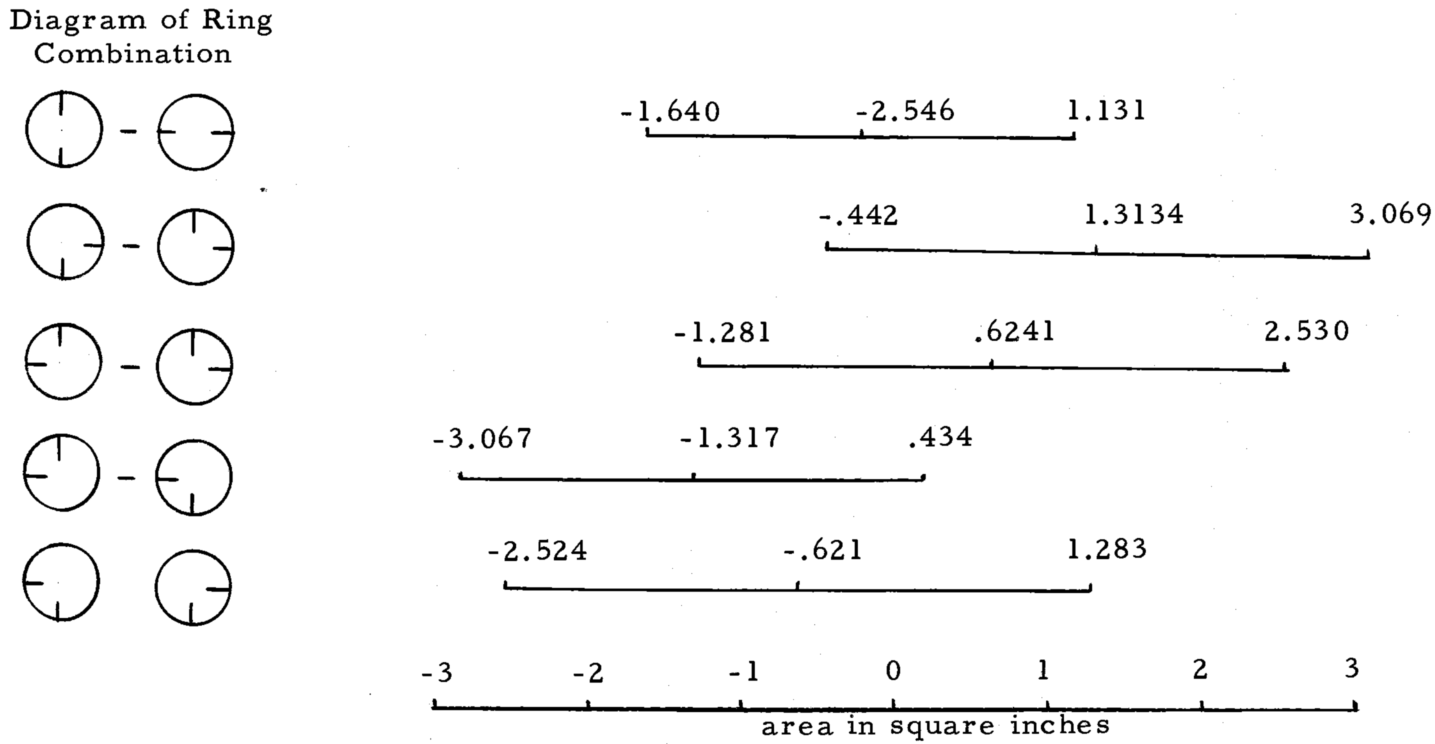


Figure 16. Mean differences between ring areas calculated by sets of borings, showing 95 percent confidence intervals.