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The social and economic objectives of society in the United States are many and varied. All objectives, in turn, cannot be achieved simultaneously because of a finite resource base. The problem of finding an optimum, in some sense, level of achievement for the various objectives is complicated by lack of a common measure of value. As a result, trade-offs must be calculated and presented to a decision body for choice. The purpose of this study was to define and indicate the elements affecting the trade-off ratio. A guiding premise of the study was that any planning group, private or public, is involved in isolating a multiple product production function. Multiple output production theory provided the basis for development of a conceptual model useful in the calculation of trade-offs.

Water resource production processes were described in the

context of multiple output production theory. Several possible production relations among water products were outlined. It was argued that production relations descriptive of water resource development have to be understood to calculate trade-off ratios. Several empirical cases were examined to illustrate problems and procedures in the trade-off calculation process. The absolute level of investment was shown to be very important in the trade-off calculation process. Other recommended approaches for calculating trade-offs were compared with the recommended approach in this study. Most other approaches were found to be deficient in many respects. It was shown that net dollar benefits can be traded-off for increases in non-money valued products only if the investment levels for the various alternatives are the same and the money valued products are kept in equilibrium proportions. The theoretical concepts of joint products, joint costs, interdependence, independence, complementarity, and competitiveness were also defined concisely in the study. The Federal document used in planning water and related land resource development projects was evaluated in light of the results of the study. The approach to delineation of the number of plans should be modified. Alternative plans should reflect the same investment cost. Also, the notions of complementary and conflicting products should be defined more concisely. Underlying production relations need to be identified to facilitate plan formulation and trade-off calculations.

Multiple Objective Planning Procedures in Water Resource Development: The Trade-Off Ratio

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I. INTRODUCTION

The social and economic objectives of society in the United States are many and varied. A great deal of concern has been expressed by individuals and groups of this society, especially in recent years, over the direction the Nation should take relative to the achievement of the various objectives. The various social and economic objectives relating to income and poverty, health and illness, learning, science, and art, participation-alienation, public order and safety, social mobility, and physical environment cannot all be achieved simultaneously.[1] The resources of the Nation, while abundant by most standards, are indeed limited. As a result, choices must be made.

Public investment can be used to encourage the attainment of various objectives at the cost of lower levels of achievement for other objectives. Public expenditure affects the direction society takes over the Nation's transformation surface (or several surfaces) in the

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[1] These social objectives were outlined in U.S. Dept. of HEW ("Toward a Social Report", 1969). Any attempt at describing the relations among these objectives raises several significant conceptual and empirical issues. See Castle ("Economics of ...", 1973).

attempt to reach a social and economic optimum. The correct allocation of resources by the public sector, in turn, requires the use of multiple objective planning procedures. Alternative trade-offs among objectives can then be calculated and presented to society for choice. Explicit recognition of a need to consider a multiple objective function in the planning of public investment has had a profound effect on procedures used by many government agencies. The planning procedures used by government agencies involved in natural resource development, especially water resource development, are no exception.

Evolution of Water Resource Planning Procedures

Natural resource development has always been of concern to citizens of the United States. In early periods of the history of this country, natural resources were readily available and essentially free for use by anyone having access to the man-made capital items necessary to transform natural resources into products useable by society. This state of affairs lasted for over a hundred years. Private developers, with very little public interference through government controls, converted vast amounts of the natural resource base of this country into a great number of products desired by the populace here and abroad. Natural resources were abundant relative to the demands placed on the use of natural deposits and natures flows. This was true for several of the natural resources including water. As society progressed and the population increased, however, the public became aware of a need for conservation and orderly development of the remaining natural resource base of the Nation. This was especially evident in water and related land development. By the beginning of the twentieth century, most of the easily obtained water resources were under the control of private developers. Also, much of the remaining public land was lacking the necessary water to produce agricultural crops and support an expanded population base. The arid west brought forth calls for "making the deserts bloom". The water resource had entered the public eye. Legislation was enacted to involve the public, through the Federal treasury, in water and related land resource development and management. The reclamation, irrigation, and flood control acts had a tremendous impact on water resource development in this Nation. Public investment in development, conservation, and management expanded rapidly in an attempt to improve social welfare. As water resource budgets expanded, the birth of water resource planning occurred. Expenditures on development and management of the water resource could not be made until a plan had been developed and accepted.

Planning legislation pertaining to water and related land resource development has taken a long and arduous path to arrive at the procedures in use today. Prior to 1936, there was, essentially, no requirement for detailed planning. The famous phrase that "...benefits to whomsoever they should accrue shall exceed the costs... ", which was imbedded in the Flood Control Act of 1936 (U.S. Congress, "Flood Control... ", 1936, p. 1570), set the stage for the development (and many revisions) of water resource and related land development planning procedures. Emphasis was placed on calculating dollar benefits and costs in the earlier planning legislation.[2] Dollar benefit-cost ratios were calculated for every potential project in the attempt to gain support from the Federal treasury. In more recent years, objectives other than maximization of net dollar benefits have been made explicit in planning procedures. This is a reflection of social concern over the direction of this Nation in striving to discover the optimum point on the transformation surface. Water and related land resource development, it is sensed, contributes in some manner to the achievement of various social objectives. The most recent planning document, which reflects this mood, outlines the broad conceptual base and some general techniques for the application of multiple objective planning procedures in water resource planning. The title of this document is "Principles and Standards for Planning

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[2] The earlier planning legislation and guidelines included the 1950 'Green Book', the revised version in 1958 (U.S. Federal... , 1958), and Senate Document 97 (U.S. Water Resources Council, "Policies, Standards, ... ", 1962).

Water and Related Land Resources Development" (Water Resources Council, "Establishment of. . . ", 1973). This document is to guide all water and related land resource planning after October, 1973.

Need for Study

Multiple objective planning procedures have emerged from a felt need for dealing explicitly with incommensurables in planning. Many products provided through water and related land resource development do not have known market prices. In turn, it is felt these products contribute to the achievement of several of the social objectives. The purpose of multiple objective planning procedures, then, is to provide a means for assembling the information needed by a decision body to make rational decisions regarding the proposed public investment. The various features of a project must be displayed in such a manner that a decision body can decide if the project would contribute the correct combination of products as dictated by social preferences. The implicit assumption behind multiple objective planning procedures, of course, is that a decision body does exist that can accurately reflect the preferences of society toward various products from a water resource project.

The final product from the application of multiple objective planning procedures is, in most simple terms, a set of trade-off ratios. The concept of a trade-off ratio has not, however, been defined rigorously. The conceptual base and procedures for calculating trade-offs have not been developed to a point where the actual ratios can be calculated by resource planners at the practical, real world project level. The concept of a trade-off needs to be defined concisely if it is to become a useful tool in multiple objective planning such as to facilitate the eventual selection of the optimum product mix. Some means must be found for relating objectives. Multiple objective planning procedures could lead to distorted resource allocation if the resource planner, unknowingly, calculates the trade-off ratios incorrectly. There is a great deal at stake as public investment in water and related land resource development is large. Yet, the trade-off ratio, which is the needed end product of planning, has not been defined.

Objectives of the Study

The overall purpose of this study was to define and indicate the elements affecting the trade-off ratio. More specific objectives were:

1. to develop the conceptual base for the trade-off ratio calculation process,
2. to outline some actual trade-off ratio calculation procedures for water resource planning and development based on the conceptual base provided.

A base of reference or line of reasoning is provided in the study by the contention that any planning group, private or public, is, essentially, charged with isolating a multiple product production function. It can be argued that planning (at least economic planning) is concerned with finding the optimum combination of factors--land, labor, capital, and management--such as to provide the optimum (in some sense) product mix. The need for calculating trade-offs, in turn, arises when some of the products and/or factors do not have known, market determined prices.

Organization of Thesis

The remainder of the thesis is divided into five chapters and four appendices. A review of the literature on trade-off ratio calculation and use is presented in Chapter II. The conceptual model is developed in Chapter III and used to describe the production processes of water resource development in Chapter IV. Several procedural issues are raised in Chapter IV. Some actual examples of trade-off calculations, developed with the conceptual models of Chapters III and IV, are presented in Chapter V. The material of Chapter V is used to illustrate and reconcile the procedural issues raised in Chapter IV. The summary, conclusions, and recommendations for further research are presented in Chapter VI. The appendices provide supplementary information relating to, but not having a direct bearing on, the main plan and conclusions of the study.

II. REVIEW OF LITERATURE

Several studies dealing with the problem of incommensurable benefits (and costs) have been accomplished in recent years. Recent revision of the planning procedures for water resource development as well as the earlier work on the planning, programming, and budgeting system of the Federal government (U.S. Congress, "The Analysis and...", 1969) has led to concern for discovering the theoretical base for handling multiple objective functions. Government, at all levels, has become increasingly involved in providing many products to satisfy the multiple objectives of society. By the very nature of many of these products, the problem of incommensurable outputs becomes reality very early in the optimization process. Discussions of the optimization process under such conditions eventually reduce down to one central element--the concept of a "trade-off ratio" and/or the procedure for calculating that ratio. All of the literature on multiple objective functions and planning procedures has this central element.

An overview and discussion of past work on the nature of the optimization process with incommensurable outputs is provided in Freeman (1969). Expenditures in the public sector are made to improve human welfare through changes in such outputs as education, transportation, cleaner air, and flood control benefits (Freeman, 1969, p. 565). Choices must be made among all of the outputs as more of everything is not a realistic alternative. The choice process, of course, requires some criterion for choice. This is the central problem as not all outputs can be expressed in the same unit of measure. A criterion of efficiency, for example, may not be sufficient if one or more of the outputs lack money prices. The multiple objective problem emerges as a problem in valuation (Freeman, 1969, p. 566); i.e., the multiple objective problem would not exist if all outputs had market values or if all outputs could be valued in some other measure of value. Several approaches have been suggested for dealing with the multiple objective problem where noncommensurate benefits and costs are involved (Freeman, 1969, p. 569). The outputs of the particular project must be measurable, in some sense, in order to implement any of the techniques.

An approach using a schedule of money and non-money valued outputs has been recommended by McKean (Freeman, 1969, p. 569). The schedule is presented to the decision making entity. The net money benefits and the unvalued (in a money sense) benefits are determined for several alternative plans of development. The particular plan selected by the decision making entity gives an implicit value of the non-money valued output. The value so obtained for the non-money valued benefit reflects the subjective evaluation of society's preferences as viewed by the decision making entity (Freeman, 1969, p. 570).

Another approach was attributed to Marglin (Freeman, 1969, p. 571). In this approach, some minimum level of one output serves as a constraint in determining the amounts of other outputs provided. This approach is basically the same as one recommended by McKean (Freeman, 1969, p. 571). The choice of the minimum value of one or more outputs implies an implicit valuation of those outputs in terms of the money valued outputs produced. In both cases choice of a particular product mix implies a certain money value for the non-money valued outputs. Freeman argues that value should be allowed to determine choice rather than choice determine value as in the McKean and Marglin approaches. The use of explicit weights on non-money valued benefits would allow value to determine choice (Freeman, 1969, p. 571).

The alternative of assigning explicit weights has been suggested by Eckstein and Marglin (Freeman, 1969, p. 571). It is argued that weights on non-money valued outputs should be assigned independently of any particular project. The same set of weights can then be used to evaluate designs of projects and, eventually, to select projects. This approach would entail assigning "money units" to all measurable outputs before a choice of project is made. The criteria then becomes one of maximizing the sum of valued benefits (Freeman, 1969, p. 572). Freeman argues that value should be made explicit as "...decisions are more likely to reflect the general preferences of society, and less likely to be influenced by the pressures from special interest groups--

(and) likely to be more consistent over a wider range of choices and a longer period of time" (Freeman, 1969, p. 572).

Several approaches to assigning explicit weights have been suggested. Freeman sees these approaches as finding the weights in the political and administrative spheres. Eckstein has suggested that weights be derived from past decisions on resource allocation (expenditure) and taxation (Freeman, 1969, p. 573). Whether accomplished from use of expenditure data or taxation (marginal effective tax rates), the difficulty of defining the transformation conditions still remains; i.e., the manner in which one benefit can be exchanged for another benefit must be known (Freeman, 1969, p. 574). Rather than extracting the weights from previous resource allocations and/or taxation decisions, Freeman suggests that several alternative weighting functions be presented the decision making entity. The choice of functional form will then reflect the preferences of that entity. The particular function chosen can then be used to evaluate proposed project plans (Freeman, 1969, pp. 575-577).

Maass has also proposed that trade-offs be calculated when a multiple objective function prevails. The arguments of the multiple objective function in that paper were economic efficiency and income distribution. With these two objectives, "...all that is needed to solve the maximization equation is to specify the trade-off ratio between efficiency and income redistribution" (Maass, 1966, p. 210). Maass speaks of maximizing one of the objectives subject to a constraint on the other objective or the assignment of explicit weights to each objective. Both of these approaches are attributed to Marglin (Maass, 1966, pp. 209-210).

Maass views economic efficiency as meaning the maximization of net national income from a particular project. In essence, then, the trade-offs to be calculated are between net national income and net income distribution benefits (Maass, 1966, p. 220). Further, it was recommended that trade-off calculations should be accomplished between "...economic efficiency and the most important non-efficiency objective... if income distribution is not of concern" (Maass, 1966, p. 225). In any case, net national income is to be "traded-off" for other objectives in the multiple objective function.

The concern in another paper was also with trade-offs between net national income benefits and net regional income benefits (Major, 1969, pp. 1174-1178). The discussion in the article is framed in terms of water resource projects. Accounts for various objectives should be determined. The figure presented in that paper is reproduced, in part, in Figure 1. The vertical axis represents net national income benefits and the horizontal axis represents net regional income benefits only to a specific region (Major, 1969, p. 1175). The "transformation curve", TT, is assumed to represent income effects only in one region; i.e., the effects of the project on the income to

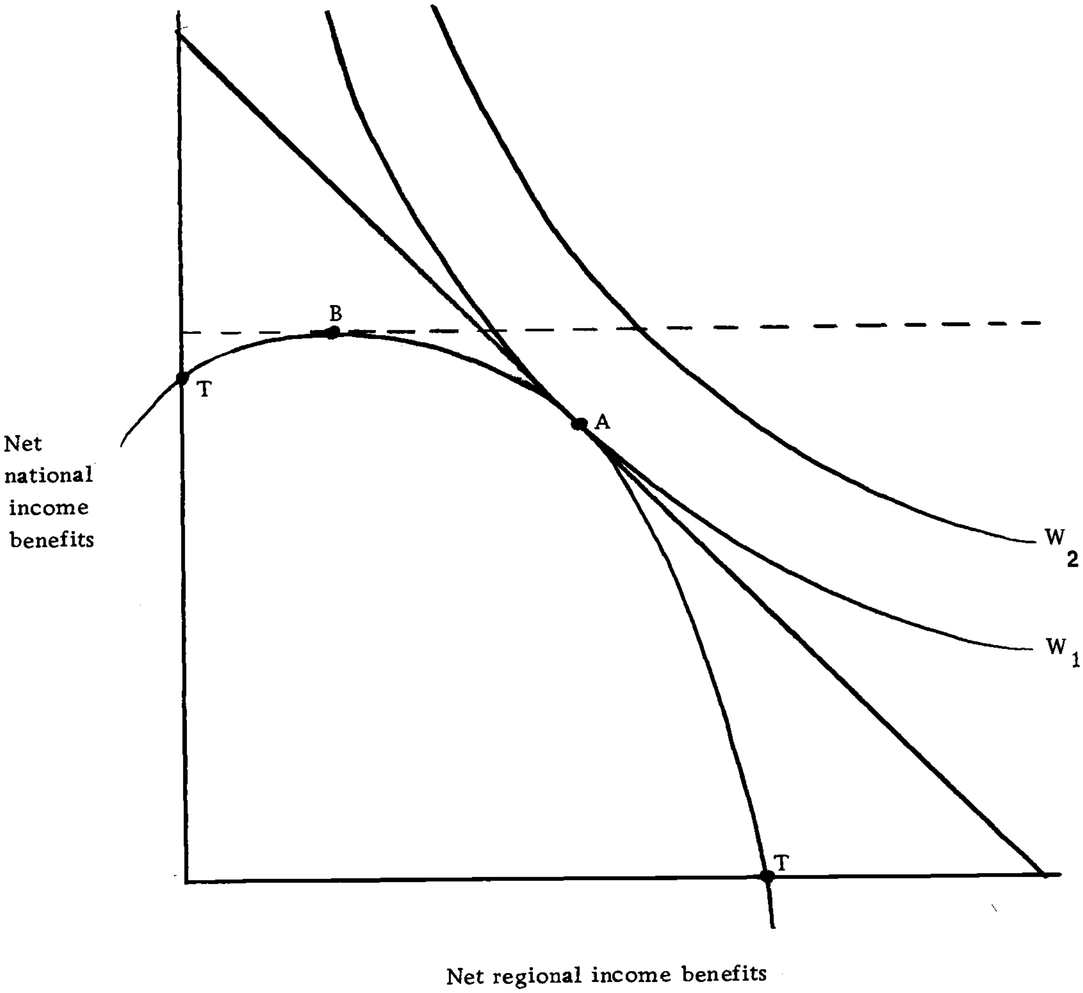

Figure 1. Transformation and social indifference curves.

other regions are assumed away. The net benefit "transformation curve" is purported to represent "...net benefits available from projects that can be designed in the water resources program...a wider analysis would deal with a curve for all government investment sectors" (Major, 1969, p. 1176). The social indifference curves can be found by "...(can use) decisions reached in an informed legislative process" (Major, 1969, p. 1176). The optimum combination of net national and regional income is then given by point A in Figure 1. The tangency point at A gives the "...relative marginal social values placed on net national income benefits and net regional income benefits to the specified region" (Major, 1969, p. 1176). In Major's view, point A is obtained from the maximization of the expression:

$$1(B_n - C_n) + 0.4(B_r - C_r) ,$$

where,

B = (gross) benefits,

C = (gross) costs,

n and r = national and regional, respectively.

The values of 1.0 and 0.4 were used for illustrative purposes. The implication is that $1.00 of net national income is equivalent to $2.50 of net regional income.

The benefit-cost ratio is also presented and is said to be:

$$\frac{1.0(B_n) + 0.4(B_r)}{1.0(C_n) + 0.4(C_r)} .$$

The weights, again, are given by the tangency of the social indifference curve to the net benefit transformation curve.[3] In addition, the weights will change "... as preferences and the transformation curve shift" (Major, 1969, p. 1176). The factors that may shift these functions are not delineated. The balance of the paper is devoted to arguing that evaluation procedures in water resource development should be modified to use the benefit-cost ratio which includes weights. Other use of benefit-cost ratios will give points such as B in Figure 1 where zero weights are attached to B_r and C_r (Major, 1969, p. 1177).

The approach to determining the optimum levels of achievement of more than one objective, as supported by Maass and Major, is also recommended by Marglin (1967, pp. 24-39). All three authors propose to determine net effect transformation curves. The relation is usually presented graphically with net national income on the vertical axis and some other "objective" on the horizontal axis. Implicit weights result from finding the tangency of a welfare function or social indifference curve to the determined net effect transformation

[3] "Net national income" and "net dollar benefits" are used interchangeably in the article.

curve. The effect of various shapes of the transformation curve is also discussed (Marglin, 1967, pp. 34-39).

Further discussion of concepts related to optimization of a multiple objective function was encouraged by the development of multiple objective planning procedures in 1969 to be used in water resource planning and development. The "Blue Book" (U.S. Water Resources Council, "Procedures for...", 1969) was an attempt at providing a better set of planning procedures than available at the time for water resource planning.[4] Several recommendations for revision in the planning procedures were made in that document. The national objectives of water resource development were determined to include national economic development, regional development, environmental enhancement, and improvements in the well-being of people (WRC, "Procedures for...", 1969, p. 3). The WRC delineated many areas for further discussion and study.

Evaluation standards were to be developed that would take full account of all tangible and intangible benefits and costs of a particular project (WRC, "Procedures for...", 1969, p. 11). Water resource projects were viewed as contributing to multi-objectives and, as a result, "...projects (should be) designed in accordance with some balance among these objectives..." (WRC, "Procedures for...",

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[4]The U.S. Water Resources Council is herein after referred to by the acronym "WRC".

1969, p. 17). More than one objective was to be considered in planning. Indeed, projects and plans were

> ...to be designed on the basis of several different relative emphasis on various objectives...to provide information to the Executive Branch and the Congress on what exactly can be achieved through water resource development (WRC, "Procedures for...", 1969, p. 18).

Different mixes of objectives were to be included in plans and presented to the decision making entity. A system of accounts was to be used to relate each project benefit to one or more of the multiple objectives. For those benefits that do not have a market price, the public planning process can establish the value (WRC, "Procedures for...", 1969, p. 37). The respective categories of benefits were viewed as market valued and non-market valued benefits. Some unit of measure was to be attached to non-market valued benefits when possible. Non-market valued benefits could be evaluated in a monetary context, however, by determining the market valued benefits foregone by providing non-market valued benefits or by the alternative cost method (WRC, "Procedures for...", 1969, p. 38).

Plans were to be formulated in economic terms; i.e., national income should be estimated. The effect of considering non-money valued benefits can then be determined by using the national income levels foregone to achieve non-money valued benefits (WRC, "Procedures for...", 1969, p. 54). In essence, "...the trade-off between objectives can be described and measured to some extent"

(WRC, "Procedures for...", 1969, p. 55). The four account system was to "...reveal the cost in national income terms of achieving the other objectives" (WRC, "Procedures for...", 1969, p. 57). The opportunities foregone were defined as the "...differences between net benefits of the plan under analysis as compared with the relevant alternative for each objective" (WRC, "Procedures for...", 1969, p. 58).

The "Blue Book" also includes references to complementary and supplementary projects and programs. Outputs such as open space, park land acquisitions, natural beauty enhancement, model cities, and urban renewal are viewed as being complementary or supplementary to water resource development (WRC, "Procedures for...", 1969, p. 68). All projects and programs of the government must be taken into consideration in water resource development (WRC, "Procedures for...", 1969, p. 69).

The Water Resources Council requested that tests be made of the proposed water resource planning guidelines as presented in the "Blue Book". Most of these tests were accomplished during the period September, 1969 to July, 1970. The test teams were to use the guidelines to measure all benefits and costs and to formulate alternative plans. A total of 19 tests were conducted on 10 projects (WRC, "A Summary...", 1970, pp. 1-2).

One of the most significant test reports to this study was the test accomplished by Major (1970). A net benefit transformation curve was utilized in the analysis. The concept of a net benefit transformation curve was justified by the reasoning that "...(since) a unit of benefit cancels a unit of cost for a given objective, we are interested only in the net benefits toward each objective of each alternative project or program design" (Major, 1970, p. 5). Each combination of net benefits toward the objectives of concern are plotted in an "objective-objective" diagram. The boundary of the resulting set of points is the transformation curve. This boundary then represents the "...technically feasible choices..." (Major, 1970, pp. 5-6).

A net benefit transformation curve was then derived for net dollar benefits vs ecological acres saved. The net benefit transformation curve, so derived, is reproduced in Figure 2. Net benefits were found to increase at the same time as ecological acres saved increased to approximately 200 acres. Net benefits were then determined to decrease for further increases in ecological acres. The optimum combination of net national income and ecological acres was found by imposing a social indifference curve on the diagram and finding the tangency point.[5] The "trade-off", then, was viewed as the

[5] Net dollar benefits from the project were used as a measure of net national product (or income). Also, the various combinations were generated with alternative plans for development.

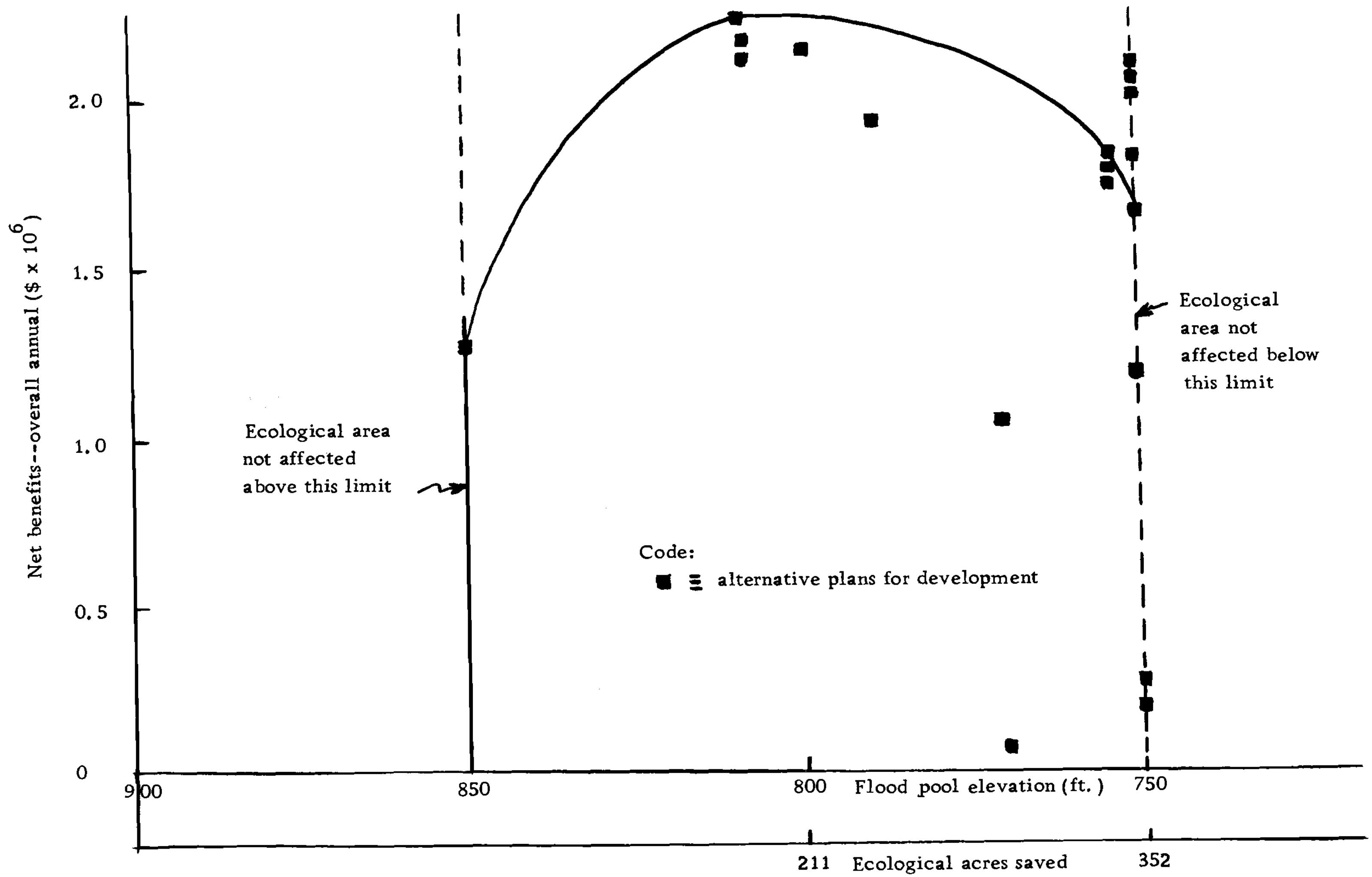

Figure 2. Net dollar benefits vs ecological acres saved. Reproduced from (Major, 1970, p. 1175).

exchange of ecological acres saved for net dollar benefits. In essence, the "slope" of the net benefit transformation curve reproduced in Figure 2 was viewed as the trade-off ratio.

Several other test teams also dealt explicitly with the question of non-commensurate benefits and costs and the trade-off ratio concept. The procedure for calculating the trade-offs were found lacking by most teams (WRC, "A Summary...", 1970, p. 21). One test team specified that agencies should only define the nature of trade-offs involved and not try to define the goals of society (WRC, "A Summary ...", p. II-C-6). Another team discovered that "...the Task Force report (Blue Book) had not presented a means whereby...trade-offs between each component (can be) highlighted..." (WRC, "A Summary ...", 1970, p. II-D-8). Still another test team determined that the trade-offs cannot be calculated if project costs are arbitrarily allocated between various objectives (WRC, "A Summary...", 1970, p. III-A-15). Another team felt one trade-off was represented by the difference in return from a 160 acre farm as compared to the most efficient size farm (implicitly assumed larger). The 160 acre limitation on irrigable acres was assumed to represent a return to the well-being account. The value of the addition to well-being was assumed measurable by the difference in returns from the "efficient" vs. the 160 acre farm (WRC, "A Summary...", 1970, p. III-B-1).

A trade-off ratio between net dollar benefits and acres of greenbelt was calculated by one team. Several alternative plans for development, all with different costs, were used in the calculation process (Tennessee Valley Authority, 1970, pp. 36-38). General directions of change were found in another study (U.S. Corp of Engineers, 1970). Irrigation benefits were thought to decline for increases in environmental quality. An interesting statement was that "...the trade-offs (for that particular project) might carry an increasingly high price tag" (U.S. Corp of Engineers, 1970, p. 62). A hypothetical goal function showing the relationship between net national income benefits and environmental quality benefits was discussed by another team (Schramm, 1970, pp. 28-33). The functional relation was referred to as a "trade-off function". In general, nearly all teams recommended further research into means for calculating trade-offs.

The recommendations of the test teams, the public, federal agencies, and further revision by the Special Task Force of the Water Resources Council resulted in a revised planning document in July, 1970 (WRC, "Principles...", and "Standards...", 1970). A few more recommendations were made to the Water Resources Council in August, 1970. The Water Resources Council made some more revisions, which gave rise to the Principles and Standards as published in December, 1971 (WRC, "Proposed Principles...", 1971). The Principles and Standards changed considerably from the "Blue Book" in 1969. Much more detail regarding non-commensurate benefits and trade-off calculations was presented in the 1971 publication.

Several alternative plans for development were to be formulated for each potential project site. The number of plans formulated, in addition to other factors, "...will depend upon complementarities or conflicts among specified components of the objectives..." (WRC, "Proposed Principles...", 1971, p. 24148). If trade-offs are expected to be large, an alternative plan for each objective is to be formulated. One alternative plan will represent maximum national income and one plan should achieve a very high level of environmental quality. Other combinations may be formed so no best plan is missed. The trade-offs are to be displayed in order to facilitate the determination of the most desirable mix of beneficial effects. Trade-offs are to be presented such as "...to facilitate administrative and legislative review and decision" (WRC, "Proposed Principles...", 1971, p. 24148). It was also noted that several of the components of the multi-objectives may affect one another in adverse (or beneficial) ways. This phenomenon, common in plan formulation, was referred to as a "joint effect relationship" (WRC, "Proposed Principles...", 1971, p. 24151). At one point, a specific type of trade-off was mentioned, as specified in the following quotation (WRC, "Proposed Principles ...", 1971, p. 24151):

> Explicit recognition should be given to the desirability of diverting a portion of the Nation's resources from production of more conventional market oriented goods and services in order to accomplish environmental objectives. As incomes and living levels increase, society appears less willing to accept environmental deterioration in exchange for additional goods and services in the market place (underlining added).

The calculation of trade-offs is seen as a very integral part of the planning and development process. The planning process, as described by the Water Resources Council, was represented as (WRC, "Proposed Principles...", p. 24168):

The Planning Process

1. Specify components of the multi-objectives relevant to planning setting.
2. Evaluate resource capabilities and expected conditions without any plan.
3. Formulate alternative plans to achieve varying levels of contributions to the specified components of the multiobjectives.
4. Analyze the differences among alternative plans to show trade-offs among the specified components of the multi-objectives.
5. Review and reconsider the specified components and formulate additional alternative plans.
6. Select a recommended plan from among the alternatives based upon an evaluation of the trade-offs among the various objectives.

The process for formulation of alternative plans was also outlined in the WRC guidelines. It was noted that complementary components are related in such a manner that "...the satisfaction of one component need does not preclude the satisfaction of the other component needs..." (WRC, "Proposed Principles...", 1971, p. 24171). Given components (products, benefits) are complementary, only one plan for development is needed. The implication was that trade-offs were not needed. The need for alternative plans is seen as arising, in part, from components that are "conflicting". Components (products, benefits) that are in conflict are related in such a manner that "...the satisfaction of one will reduce the satisfaction of others". The complementary components are seen as "...the building blocks for the formulation of alternative plans" (WRC, "Proposed Principles...", 1971, p. 24172). The number of alternative plans presented is a function of the number of component needs; i.e., there should be "...at least one plan that generally satisfies each specified component need of the multiobjectives". The trade-off calculation process is seen to be facilitated by forming one plan with emphasis on the national economic development objective. In addition, another plan should emphasize environmental quality. Plans with varying degrees of contribution to each of these objectives are also formulated. Given all these plans, trade-offs can be calculated. Movement away from the maximum environmental quality level obtained in one plan toward higher levels of national income achievement in another plan is seen as a "trade-off" (WRC, "Proposed Principles...", 1971, p. 24172).

The multiple objective planning procedures were then submitted to more testing and review. The WRC made some more revisions and published the document in 1973. This version was approved by the President and placed in effect on October 25, 1973. The most significant change in the document was a reduction in the number of objectives from four to two, as given by (WRC, "Establishment of...", 1973, p. 24781):

1. to enhance national economic development by increasing the value of the Nation's output of goods and services and improving national economic efficiency,

2. to enhance the quality of the environment by the management, conservation, preservation, creation, restoration, or improvement of the quality of certain natural and cultural resources and ecological systems.

The effects of proposed projects on regional development and social well-being were still to be displayed. These two elements were no longer to be considered, however, as objectives. Several other changes were also made, but none that changed the basic recommendations regarding trade-off ratio calculations.

Trade-offs were still to be calculated among the components of the objectives. Also, trade-offs were to be used to select a recommended plan from all those developed by the planning agency (WRC, "Establishment of...", 1973, p. 24824). At least one plan emphasizing national economic development and one with emphasis on environmental quality were to be developed. Also, "...other alternative plans reflecting significant trade-offs between the national economic development and environmental quality objectives may be formulated so as not to overlook a best overall plan" (WRC, "Establishment of...", 1973, p. 24830). The relative value of components measured in non-monetary terms can be estimated by calculating what is given up or traded-off among plans. It was also noted that the purpose of calculating trade-offs was _not_ "...to convert such effects to monetary equivalents but to gain an insight with respect to the relative value of such effects by understanding their impact upon monetary values..." (WRC, "Establishment of...", 1973, p. 24831). Trade-offs, as in the earlier documents, were not defined explicitly. Trade-offs were, however, most frequently referred to as signifying the monetary loss from providing some non-monetary (non-money valued) component of an objective. In fact, in one hypothetical (numerical) example, a trade-off between _net_ dollar benefits and contributions to the environmental quality objective was discussed. It appears the designers of the new planning procedures felt that no further definition of a "trade-off ratio" was necessary.

A few papers and articles have been published regarding very specific approaches to calculating the trade-offs among incommensurables. One approach to calculating trade-offs involves use of total and marginal benefit functions (DeVine, 1966). Some (hypothetical) total benefit functions were used to illustrate that approach. Two of the tables used to calculate the "trade-offs" in that approach are reproduced here in Tables 1 and 2. The total costs of producing either water for reclamation or recreation hours is presented in Table 1. A total of $1,000,000, for example, can be used to provide $1,375,000 in total dollar benefits or 3,700,000 user-hours (benefits) of recreation. Combinations are also possible (DeVine, 1966, p.385). A total of $700,000 could be allocated to water reclamation leaving $300,000 for provision of recreation hours, for example. This allocation would give $1,225,000 of total benefits and 1,950,000 user-hours of recreation (Table 1). Trade-offs can be determined by looking at the marginal changes (DeVine, 1966, p. 385). The trade-offs calculated by this process are presented in Table 2. Starting with an initial allocation of $800,000 to water reclamation (WR) and $200,000 to recreation (R), a reduction of $100,000 to WR results in a $75,000 reduction in total dollar benefits. The increase in recreation benefits, however, is 700,000 user-hours. The implicit value of user-hours of R is $0.11 per hour (Table 2). A reduction in the allocation of cost to WR from $100,000 to 0 results in a decrease in total dollar benefits of $250,000 and an increase in total recreation benefits of 25,000 user-hours. The implicit value of recreation for that case is $10.00 per hour.

Table 1. Total costs and total benefits.

Total Cost	Incremental Cost	Water Reclamation Total Benefit	Water Reclamation Incremental Benefit	Recreation Total Benefit	Recreation Incremental Benefit
		--------(dollars)--------		---------(hours)---------	
100, 000	100, 000	250, 000	250, 000	600, 000	600, 000
200, 000	100, 000	475, 000	225, 000	1, 250, 000	650, 000
300, 000	100, 000	675, 000	200, 000	1, 950, 000	700, 000
400, 000	100, 000	850, 000	175, 000	2, 650, 000	700, 000
500, 000	100, 000	1, 000, 000	150, 000	3, 150, 000	500, 000
600, 000	100, 000	1, 125, 000	125, 000	3, 450, 000	300, 000
700, 000	100, 000	1, 225, 000	100, 000	3, 550, 000	100, 000
800, 000	100, 000	1, 300, 000	75, 000	3, 625, 000	75, 000
900, 000	100, 000	1, 350, 000	50, 000	3, 675, 000	50, 000
1, 000, 000	100, 000	1, 375, 000	25, 000	3, 700, 000	25, 000

Reproduced from (DeVine, 1966, p. 385).

Table 2. Marginal changes and trade-offs.

Reduce Investment in Water Reclamation in $100, 000 Increments to:	Trade-offs: Give Up Benefits Valued at:	Trade-offs: To Get Additional User-Hours of Recreation of:	Implied Minimum Value of Recreation (per hour)
(dollars)			(dollars)
700, 000	75, 000	700, 000	0. 11
600, 000	100, 000	700, 000	0. 14
500, 000	125, 000	500, 000	0. 25
400, 000	150, 000	300, 000	0. 50
300, 000	175, 000	100, 000	1. 75
200, 000	200, 000	75, 000	2. 67
100, 000	225, 000	50, 000	4. 50
0	250, 000	25, 000	10. 00

Reproduced from (DeVine, 1966, p. 386).

This is a different approach than used by many of the teams that tested the proposed Principles and Standards as total benefits are "traded-off" for a non-money valued output and costs were held constant. Net benefits were considered as the relevant "loss" figure in most other recommendations and costs were allowed to vary.

Another recommended approach for calculating trade-offs appeared in the literature in 1973 (Marshall, 1973). The recommendation was "...based on that (approach) followed by the Water Resources Council and a technique described in the literature for treating incommensurable values in benefit cost analysis" (Marshall, 1973, p. 2). The "technique described in the literature" was the article by DeVine (1966). It is not apparent to this author that either of the recommendations by the Water Resources Council or DeVine were followed exactly.

The approach recommended by Marshall also makes use of marginal values. The table of hypothetical total benefit functions used by Marshall is reproduced in Table 3. Expenditures were incremented by $100,000 to give various levels of national economic development (NED), environmental quality (EQ), and regional development (RD) benefits (Table 3). The measures of benefits are total; i.e., net benefits can only be obtained by subtracting the total expenditure figures (Marshall, 1973, p. 2). As an example, $500,000 can be used to provide $660,000 of NED and 2 "units" of EQ and $850,000 of RD

Table 3. Alternative project designs with multiobjectives.

Design	Expenditures $\$10^3$		Benefits to NED $\$10^3$		Benefits EQ Unit Index		Benefits to RD $\$10^3$	
	Marginal	Total	Marginal	Total	Marginal	Total	Marginal	Total
1	100	100	200	200	0	0	250	250
2	100	200	160	360	0	0	200	450
3	100	300	130	490	0	0	150	600
4	100	400	100	590	-2	-2	150	750
5	100	500	70	660	4	2	100	850
6	100	600	40	700	2	4	80	930
7	100	700	10	710	6	10	50	980

Reproduced from (Marshall, 1973, p. 3).

benefits. These are different forms of total benefit functions than recommended by DeVine (1973). In order for the two approaches to be identical, the \$500,000 expenditure in Table 3 would have to allow provision of \$660,000 of NED <u>or</u> 2 units of EQ <u>or</u> \$850,000 of RD benefits. The benefit figures in Table 3 are, however, total figures which is consistent with DeVine (1973) but apparently inconsistent with recommendations made by the Water Resources Council (WRC, "Establishment of...", 1973).[6]

The trade-off ratio calculation process recommended by Marshall, as a result, is different. Marginal benefits are said to be equal to marginal expenditures for Design 4.[7] Designs 5 and 6 represent emphasis on "...marginal investments for environmental enhancement..." (Marshall, 1973, p. 3). Controversy develops at Design 4 as EQ was negative; thus, trade-off calculations are needed for Designs 4-7 (Marshall, 1973, p. 3). The trade-off, under this approach, is said to be determined by the choice of design. Choice of Design 4 over Design 3 implies that RD benefits are worth at least an EQ decline of -2 (Marshall, 1973, p. 4). If Design 5 is chosen, the

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[6] The exact recommendation made by the Water Resources Council is not clear to this author. Many of the teams that did test studies, however, used net benefits in trade-off calculations (see pages 16-27, this chapter).

[7] It should be noted, however, that marginal expenditures also contribute to EQ and RD benefits.

implication is that the 4 unit increase in EQ combined with the $100,000 increase in RD is worth the "...$30,000 by which marginal expenditures exceed marginal NED benefits" (Marshall, 1973, p. 4). Further, selecting Design 6 rather than 5 implies that "...$60,000 of NED equivalent benefits are willingly foregone for 2 units of EQ benefits and $80,000 of RD benefits" (Marshall, 1973, p. 4). This is a completely different approach than recommended by DeVine (1966) for calculating trade-offs. In fact, the actual values of the trade-offs are not calculated in Marshall's study (1973).

Another approach to calculating trade-offs is presented in Miller (1973). The model used is attributed to the study by Major (1969) referenced earlier in this chapter. A "net effect transformation function" is developed (Miller, 1973, p. 12). The optimum combination of two objectives, say economic efficiency and environmental quality, is determined in the following manner (Miller, 1973, p. 12):

> The combination of social objectives is optimal at the point where the net effect transformation function is tangent to the highest attainable social indifference function. At that point the slope of a line drawn tangent to both functions is equal to the negative of the marginal weight on environmental quality relative to economic efficiency and to the negative of the marginal trade off between these two objectives (underlining added).

The objective function used in the programming model was the present value of net benefits; i.e., the difference between the present value of total benefits and total costs was maximized. This quantity was then changed by constraining the solution with various environmental constraints. The environmental constraints were then changed systematically through the use of parametric programming (Miller, 1973, p. 13). One of the trade-off functions so developed is reproduced in Table 4. In order to reduce sediment phosphorous from 7,521 to 6,201 pounds (a change of 1320 pounds), net dollar benefits had to be reduced from $63,409 to $57,790 (a change of $5,619). The "trade-off" ratio is $5,619/1320 lbs = $4.26/pound. Similarly, the trade-off ratio was determined to be $5.95/pound for the next step (Table 4). Net dollar benefits are "traded-off" for improvements in the environment. Each of the points along the "trade-off" function is a different plan for development (which more than likely has a different cost).

Table 4. Trade-off between net dollar benefits and sediment phosphorous for West Boggs Creek.

Net National Benefit	Sediment Phosphorous Level, lbs	Trade-Off Ratio, $/lbs
63,409	11,190	
63,409	7,521	0.0
57,790	6,201	4.26
50,289	4,940	5.95

Reproduced from (Miller, 1973, p. 16).

Several alternative ways of optimizing a multiple objective function, where some benefits (outputs, objectives) are incommensurable, have been proposed in the literature. Some writers on the subject evaluate "trade-offs" by sacrificing net dollar benefits and others recommend using total dollar benefits. Others used net national income. In some cases, no mention is made of the type of "benefits" (total or net) used in the trade-off calculation. Some authors keep total expenditures or costs constant when evaluating trade-offs while most make absolutely no reference to cost. At least one writer felt that trade-off ratios were impossible to calculate given that joint costs had been allocated among various project outputs, while others see no relation between joint costs (and the allocation problem) and trade-offs (at least no mention is made of any possible relation). These authors are not speaking of the same "trade-off" ratio. There seems to be considerable disagreement, at best, and confusion, at worst, regarding the process for arriving at trade-offs. The conceptual base for a trade-off ratio (or ratios) needs to be developed to serve as a guideline for the development of procedures useful in determining trade-offs.

III. THEORY OF MULTIPLE OUTPUT PRODUCTION

The purposes of this chapter are: 1) to discuss the physical characteristics of multiple output production, 2) to discuss the economic optimization process, and 3) to indicate some problems associated with the economic optimization process which give rise to a need for trade-off calculations.

Physical Production Relationships

Multiple output production processes result whenever inputs can (or must) be used jointly or can be used alternatively in the production of more than one product; i.e., the products are technically connected.[8] This description of multiple output production includes only those situations where products are related in some technical way via the resources used in production. As a result, multiple output production processes result completely from physical, technical phenomenon. Products produced by a single firm, for example, would not be considered multiple outputs unless the products were related in some manner to the same resources.

The most general case of multiple output production can be described by the following system of production relations:

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[8] This definition of multiple output production follows closely that of Frisch (1965, p. 269). The remainder of this chapter was developed mainly from the basic framework provided in Frisch (1965, pp. 269-289).

$$\begin{aligned}
F^1(q_1, q_2, \ldots, q_m; x_1, x_2, \ldots, x_n) &= 0 \\
F^2(q_1, q_2, \ldots, q_m; x_1, x_2, \ldots, x_n) &= 0 \\
\vdots \qquad\qquad \vdots \qquad\qquad &\vdots \\
F^\mu(q_1, q_2, \ldots, q_m; x_1, x_2, \ldots, x_n) &= 0
\end{aligned} \tag{3.1}$$

where,

q_i = products, $(i = 1, 2, \ldots, m)$,

x_j = inputs, $(j = 1, 2, \ldots, n)$,

F^k = production relations, $(k = 1, 2, \ldots, \mu)$.

Each production relation, F^k, is assumed independent; i.e., no one production relation can be deduced from any other production relation or combination of relations.[9] A production relation is taken to mean any relation among products and/or factors. To clarify this definition, a convention in this study will be to label various forms of production relations in the following manner:

1. $F^k(q_1, q_2, \ldots, q_m) = 0$ shall be referred to as "factor free" relations,
2. $F^k(x_1, x_2, \ldots x_n) = 0$ shall be referred to as "product free" relations,
3. $F^k(q_1, q_2, \ldots, q_m; x_1, x_2, \ldots, x_n) = 0$ shall be referred to as "production function" relations.

[9]This type of independence says nothing of the relations among products.

As a result of these specifications, any of the functions in (3.1) may be factor free, product free, or production function relations.[10] The production relation label, then, is a more general description of the functions F^k, $(k = 1, 2, \ldots, \mu)$.

The system of equations represented in (3.1) are perfectly general and, as a result, can be used to represent all production processes involving one or more products. Consider, for example, the relation between wool (W), mutton (M), feed (E), labor (L), other operating expenditure (O), and capital stock (C) represented by the system of equations:

$$W = f(E, L, O, C)$$
$$M = g(E, L, O, C)$$

or,

$$F^1(W; E, L, O, C) = 0 \qquad (3.2)$$
$$F^2(M; E, L, O, C) = 0 \ .$$

In (3.2), there are two production function relations, F^1 and F^2 $(\mu = 2)$. The wool and mutton production process is characterized by both products (simultaneously) using the same set of resources. A slightly different set of equations can be used to describe grass seed production (G) where smoke (S) from field burning is one of the products. Defining the resources to be land (N), labor (L), capital

[10] "Free" in the sense that the factor or product does not appear in the function.

(C), and management (M), the system of equations could be of the form:

$$
\begin{aligned}
S &= f(G) \\
G &= g(N, L, C, M), \\
\text{or,} \qquad F^1(S, G) &= 0 \\
F^2(G; N, L, C, M) &= 0 \;.
\end{aligned}
\tag{3.3}
$$

In (3.3), there is one factor free relation and one production function relation. Grass seed (G) is characterized as using the resources while the other product (S) is always produced concurrently.

Another type of production process that can be represented in the general system (3. 1) is durum wheat (D) and malting barley (B) production. Again, using the same inputs N, L, C, and M, the system of equations can be represented by:

$$
\begin{aligned}
f(D, B) &= g(N, L, C, M) \\
\text{or,} \qquad F^1(D, B; N, L, C, M) &= 0.
\end{aligned}
\tag{3.4}
$$

In (3.4), there is only one production relation and the specific form is the "production function" relation. This production process is best characterized by one where the resources can be used alternatively in the production of the two products. Other examples of production processes could be given.[11]

Several specific classes of production processes need to be identified in order to facilitate analysis. As noted earlier, equation set (3. 1) can be used to represent and analyze all production processes; i. e., it is perfectly general. It becomes useful, therefore, to delineate several major classes of production. The values of importance to defining the classes include the degree of assortment (α) and the degree of coupling (κ) (Frisch, 1965, pp. 269-270).

The degree of assortment (α) is defined as the difference between the number of products (m) and the number of production relations (μ) as represented in equation set (3. 1); i. e., $\alpha = m - \mu$. The degree of assortment gives an indication of the amount of flexibility inherent in the physical production process with respect to the possibility for choice in determination of the product mix. A higher degree of assortment (α large) is generally associated with greater flexibility in determining the relative amounts of each q_i $(i = 1,2,\ldots,m)$ produced. The least amount of flexibility in choice of the product mix

[11] The equation set (3. 1) encompasses single as well as multiple output production. The system of equations for a single output q_1 from factors $x_1, x_2, \ldots, x_n$ may be represented by $q_1 = f(x_1, x_2, \ldots, x_n)$ or $F^1(q_1; x_1, x_2, \ldots, x_n)$. In this case, there is one production relation, namely a "production function" relation.

is associated with $\alpha = 0$.[12] These are, however, only general tendencies. The flexibility in choice of the product mix is also related to the degree of coupling (κ).

The degree of coupling (κ) is defined as the number of factor free relations that can be deduced from the general system (3. 1).[13] This concept, as with the degree of assortment, affects the flexibility inherent in the production process with respect to choice in the product mix. In general, a production process with a level of $\kappa \geq 1$ reflects a system with no flexibility for the products of concern, while $\kappa = 0$ implies more flexibility.

The amount of flexibility inherent in production processes, then, is dependent upon the values of both α and κ. The possible combinations that can occur and that must be reconciled are:[14]

---

[12] The possibility also exists for $\alpha < 0$. When this occurs, a "product free" relation exists among the relations in equation set (3.1). This relation was not considered in this study other than to note the existence of such phenomenon. See Frisch (1965, p. 270 and pp. 279-280).

[13] A "factor free" relation results whenever two or more products are related in some definite manner irregardless of the level or allocation of factors. Stated in slightly more mathematical terms, the function describing the relation between two products (or among several) does not have any factors as arguments.

[14] Again, the cases where $\alpha < 0$ have been excluded from the presentation. See Footnote 12. Frisch does not discuss "flexibility" in production processes in the manner of this section. The only reference to flexibility is some mention of choice under different types of production (Frisch, 1965, p. 273).

1. $\alpha = 0, \quad \kappa > 0$

2. $\alpha = 0, \quad \kappa = 0$

3. $\alpha > 0, \quad \kappa = 0$

4. $\alpha > 0, \quad \kappa > 0$.

The following equations serve to illustrate the effect of various values for α and κ on flexibility in production processes involving <u>two</u> products:

$$\begin{aligned} &F^1(q_1, q_2) = 0 \\ &F^2(q_2; x_1, x_2, \ldots, x_n) = 0 \\ &\alpha = (m - \mu) = (2 - 2) = 0, \quad \kappa = 1 \end{aligned} \tag{3.5}$$

$$\begin{aligned} &F^1(q_1; x_1, x_2, \ldots, x_n) = 0 \\ &F^2(q_2; x_1, x_2, \ldots, x_n) = 0 \\ &\alpha = (m - \mu) = (2 - 2) = 0, \quad \kappa = 0 \end{aligned} \tag{3.6}$$

$$\begin{aligned} &F^1(q_1, q_2; x_1, x_2, \ldots, x_n) = 0 \\ &\alpha = (m - \mu) = (2 - 1) = 1, \quad \kappa = 0. \end{aligned} \tag{3.7}$$

The case where $\alpha > 0$ and $\kappa > 0$ does not exist for two products. The most flexible set of product relations among (3.5), (3.6), and (3.7) is represented by the production function relation in (3.7) where $\alpha = 1$ and $\kappa = 0$. Given the total amounts of each resource available, the manager can still decide what product mix to provide. The least amount of flexibility available in choosing the product mix for two products is represented in (3.5), where $\alpha = 0$ and $\kappa = 1$. The manager can have no further influence on the product mix once the total amount of each resource has been specified. Some degree of flexibility between these extremes is represented in (3.6) for $\alpha = 0$ and $\kappa = 0$. For two products, then, the greatest flexibility in choice of the product mix will result with $\alpha > 0$ and $\kappa = 0$, while the least flexibility will result when $\alpha = 0$ and $\kappa > 0$.

Consider now an example set of equations for $m > 2$ products, in the general form of equations (3.5)-(3.7), as given by:

$$\begin{aligned}
&F^1(q_1, q_m) &&= 0 \\
&F^2(q_2, q_m) &&= 0 \\
&\vdots \qquad \vdots && \ \vdots \\
&F^{m-1}(q_{m-1}, q_m) &&= 0 \\
&F^{\mu}(q_m; x_1, x_2, \ldots, x_n) &&= 0 \\
&\alpha = (m-\mu) = 0, \quad \kappa = (m-1) > 0
\end{aligned} \tag{3.8}$$

$$\begin{aligned}
&F^1(q_1; x_1, x_2, \ldots, x_n) &&= 0 \\
&F^2(q_2; x_1, x_2, \ldots, x_n) &&= 0 \\
&\vdots \qquad \vdots && \ \vdots \\
&F^{\mu}(q_m; x_1, x_2, \ldots, x_n) &&= 0 \\
&\alpha = (m-\mu) = 0, \quad \kappa = 0
\end{aligned} \tag{3.9}$$

$$F^1(q_1, q_2, \ldots, q_m; x_1, x_2, \ldots, x_n) = 0 \qquad (3.10)$$
$$\alpha = (m-\mu) = (m-1) > 0, \quad \kappa = 0$$

$$F^1(q_1, q_2, \ldots, q_{m-1}) = 0$$
$$F^2(q_{m-1}, q_m; x_1, x_2, \ldots, x_n) = 0 \qquad (3.11)$$
$$\alpha = (m-\mu) = (m-2) > 0, \quad \kappa > 0.$$

Given m products $(m > 2)$ the most flexible system is, again, characterized by $\alpha > 0$ and $\kappa = 0$ as represented in equation set (3.10). Any other possible equation set representing a production process will always have less flexibility in choice of the ultimate product mix. Also, the least flexible system is, as before, characterized by $\alpha = 0$ and $\kappa > 0$ as represented in equation set (3.8). The classification process is complicated slightly, however, with the introduction of $m > 2$ because of the possibility of $\alpha > 0$ and $\kappa > 0$, represented in (3.11). The relative amounts of flexibility in (3.9) and (3.11), although within the extremes identified in (3.8) and (3.10), has yet to be established.

Many different sets of production relations can be illustrated for the case where both α and κ are greater than zero. One such case is given in equation set (3.11). Other illustrations of $\alpha > 0$ and $\kappa > 0$ are represented in the following sets of equations:

$$F^1(q_1, q_2) = 0$$
$$F^2(q_1, q_3; x_1, x_2, \ldots, x_n) = 0 \qquad (3.12)$$
$$\alpha = (m-\mu) = (3-2) = 1, \quad \kappa = 1$$

$$F^1(q_1, q_2) = 0$$
$$F^2(q_2, q_4, q_5) = 0$$
$$F^3(q_2, q_3; x_1, x_2, \ldots, x_n) = 0 \qquad (3.13)$$
$$\alpha = (m-\mu) = (5-3) = 2, \quad \kappa = 2$$

$$F^1(q_1, q_2, q_3) = 0$$
$$F^2(q_2, q_4; x_1, x_2, \ldots, x_n) = 0 \qquad (3.14)$$
$$\alpha = (m-\mu) = (4-2) = 2, \quad \kappa = 1 .$$

These are but three of many examples that could be shown. The only common element among equations sets (3.11)-(3.14) is that $\alpha > 0$ and $\kappa > 0$. The important pattern to note is the greater flexibility in the production processes represented by (3.11)-(3.14) than the process in (3.9). Consider, for example, equation set (3.12). There is more flexibility in (3.12) than in (3.9) as $\alpha = 1$ implies that for given levels of factor quantities only a <u>relationship</u> exists between products q_1 and q_3 and the exact product mix has yet to be determined. This may be called a one-dimensional assortment (Frisch, 1965, p. 277). This would not be the case given a production process characterized by $\alpha = 0$ and $\kappa = 0$, as in (3.9). Similar arguments can be made for equation sets (3.13) and (3.14); i.e., there is more flexibility in (3.13) and (3.14) than in (3.9).[15]

The inherent flexibilities in production processes can now be categorized for the general situation of m products, μ production relations, and n factors. The following possibilities are listed in ascending order representing the least to the most flexible type of production process:

1. $\alpha = 0, \quad \kappa > 0$
2. $\alpha = 0, \quad \kappa = 0$
3. $\alpha > 0, \quad \kappa > 0$
4. $\alpha > 0, \quad \kappa = 0$.

These four combinations of parameters also define the major classes of production as given by:[16]

1. factorially determined production:
   a. with coupling $\alpha = 0, \quad \kappa > 0$,
   b. without coupling $\alpha = 0, \quad \kappa = 0$,

[15] The flexible relations in (3.13) are the q_2, q_3 and q_4, q_5 relations. The flexibility in (3.14) exists for the q_2, q_4 and the q_1, q_3 relations.

[16] Factorially determined production with and without coupling and assorted production without coupling are production laws delineated by Frisch (1965, pp. 269-281). The assorted production with coupling, although not specifically excluded, is not discussed in Frisch.

2. assorted production:

   a. with coupling $\alpha > 0$, $\kappa > 0$,

   b. without coupling $\alpha > 0$, $\kappa = 0$.

A verbal description of each of these classes can now be given.

Factorially Determined Production

Factorially determined production is characterized by the degree of assortment equal to zero $(\alpha = 0)$. This type of production process results whenever the amounts of each product produced are known as soon as the factor quantities are given (Frisch, 1965, p. 270). Stated in another manner, there is no flexibility in choice of the product mix once the factor quantities are specified.[17] The major subclasses of this type of production evolve from the value of κ.

Factorially Determined With Coupling. Factorially determined production with coupling $(\kappa > 0)$ is a type of production process where the relationships among the products are invariant with respect to the factor quantities used. A somewhat general formulation of this type of production is given in the equations:

[17] Dillon characterized this type of production as "multiple response without input control" (Dillon, 1968, pp. 42-43) instead of "factorially determined production". Both labels, however, describe the same type of production.

$$\begin{aligned} &F^1(q_1, q_m) \\ &F^2(q_2, q_m) \\ &\quad\vdots \\ &F^{\mu-1}(q_{m-1}, q_m) \\ &F^{\mu}(q_m; x_1, x_2, \ldots, x_n) = 0. \end{aligned} \tag{3.15}$$

The products $q_1, q_2, \ldots q_m$ in (3.15) are considered joint in production; i.e., there are several product relations that are factor free as represented by $F^1 - F^{\mu-1}$ in (3.15). The forms of $F^1 - F^{\mu-1}$, of course, may vary. Linear functional forms would imply fixed proportions among the products, while a non-linear form would imply the relative amounts of products produced could vary for changes in factor quantities. In any case, $(\mu-1)$ of the functional relations among the products are invariant with respect to the factor quantities; i.e., $F^1 - F^{\mu-1}$ are factor free relations.

The exact nature of the relationship among the products, of course, has to be determined for each and every production process under consideration. Consider, for example, the case of factorially determined production with coupling in the case of two products and two factors as represented in Figures 3 and 4. The isoquants or iso-product curves for products q_1 and q_2 are represented by the solid and dotted lines, respectively, in Figure 3. Resource levels x_1^o of x_1 and x_2^o of x_2 can be used to produce q_1^o and q_2^o

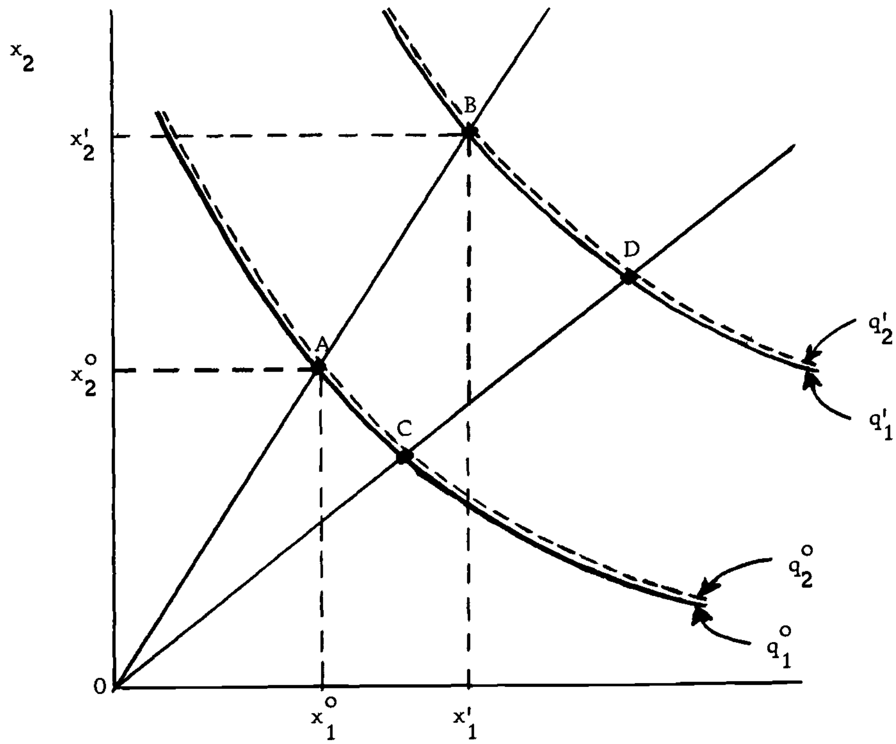

Figure 3. Isoquants for the factorially determined with coupling production law.

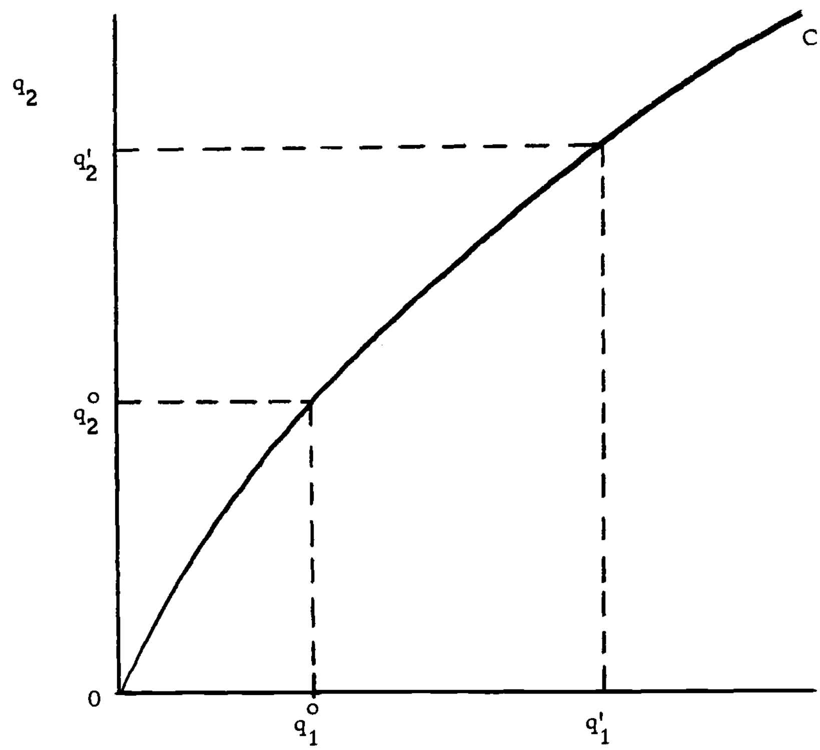

Figure 4. Relationship between two joint products.

of q_1 and q_2, respectively. The isoquants representing q_1^o and q_2^o overlay one another; i.e., the products are joint. Similarly, q_1' and q_2' of q_1 and q_2, respectively, can be produced with x_1' and x_2' of the respective resources. Again, the products are joint as the isoquants for the two products coincide completely (Figure 3).

Several alternative forms of relationships between the two products are now possible. Curve 0C in Figure 4 represents only one of the many possible relationships. The manner in which q_2 changes with a change in q_1 can easily be discerned given knowledge of one of these curves. Assume an increase in the amount of resources represented by a movement from A to B in Figure 3. The amount of q_1 increased from q_1^o to q_1' and q_2 increased from q_2^o to q_2'. Assuming the relevant relationship between q_1 and q_2 to be 0C, the change in the resource base caused an increase in both q_1 and q_2 as represented by the movement from (q_1^o, q_2^o) to (q_1', q_2') along curve 0C (Figure 4). For this particular case, q_2 increased at a diminishing rate for an increase in q_1. The same change in q_1 and q_2 could result from a different initial and subsequent allocation of resources as is shown by the movement from C to D in Figure 3. The relationship between q_1 and q_2 is invariant with respect to the factor quantities; i.e., the relationship is factor free. Also, it must be emphasized that curve 0C is <u>not</u> a transformation (iso-factor, iso-resource) curve. The transformation "curve" for joint products is only a point in product space. The product-product relation normally discussed in the economic literature does not exist for this case.

Factorially Determined Without Coupling. Another case of factorially determined production is characterized by $\kappa = 0$ or no coupling. The relationship among the products now depends upon the factor quantities used. The general formulation of this type of production is given in the equations:

$$\begin{aligned} F^1(q_1; x_1, x_2, \ldots, x_n) &= 0 \\ F^2(q_2; x_1, x_2, \ldots, x_n) &= 0 \\ \vdots \qquad\qquad & \vdots \\ F^\mu(q_m; x_1, x_2, \ldots, x_n) &= 0. \end{aligned} \tag{3.16}$$

The products $q_1, q_2, \ldots, q_m$ in (3.16) are considered separable in production. The relative amounts of each product produced can be varied (within limits) by changing the factor quantities. Separability simply means that $\kappa = 0$ (Frisch, 1965, p. 271).

The case of factorially determined production without coupling is illustrated in Figures 5 and 6. The isoquant maps for products q_1 and q_2 overlap but do not coincide completely (Figure 5). The resource level represented at point A can be used to produce q_1^o of q_1 and q_2^o of q_2. Increases in x_1 and x_2 along 0F give

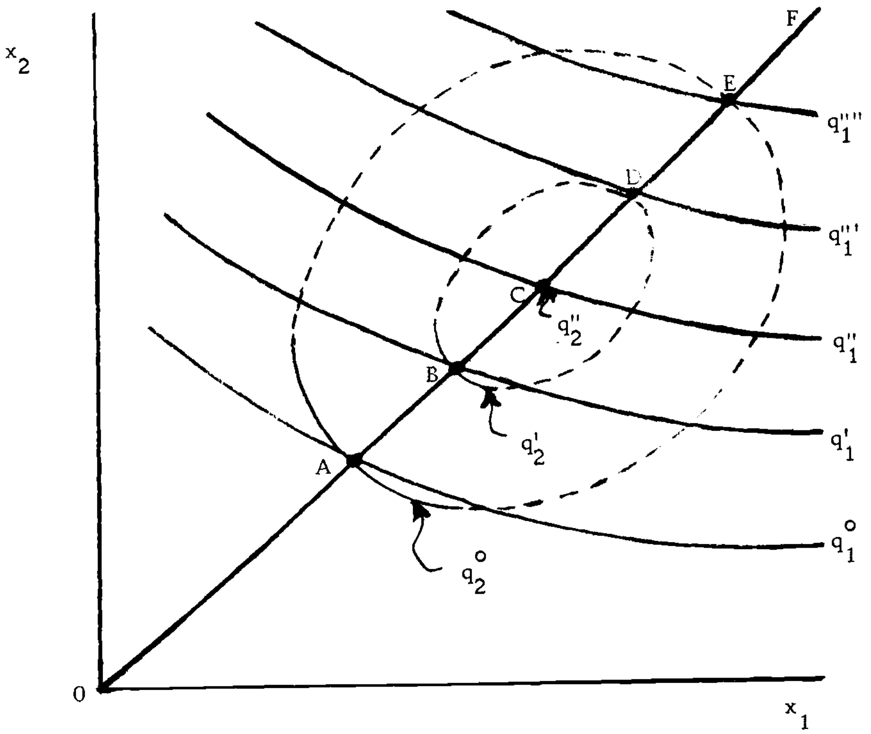

Figure 5. Isoquants for the factorially determined without coupling production law.

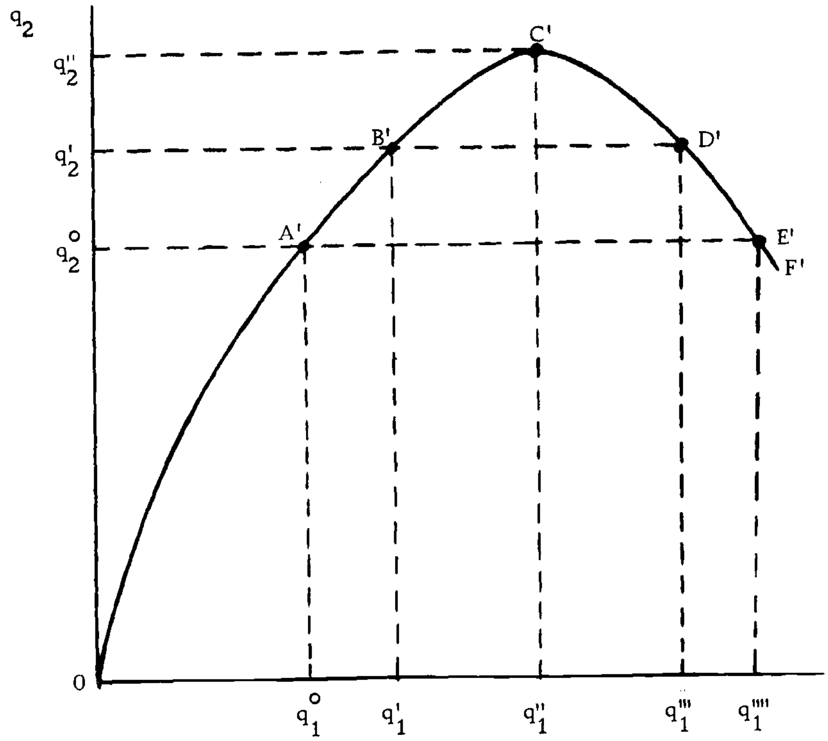

Figure 6. Relationship between two separable products.

increases in q_1 and q_2 up to q_1'' and q_2'' at C. Further increases in the resource level beyond point C results in further increases in q_1 but a decline in q_2 (Figure 5). The nature of the relationship between q_1 and q_2 for movements along 0F in Figure 5 is illustrated by 0F' in Figure 6. The amount of q_2 and q_1 both increase up to C', while q_2 decreases for further increases in q_1 beyond C' (Figure 6). Movement along 0F in Figure 5, of course, requires changes in the amounts of resource used.

Many different relationships between q_1 and q_2 will result from different factor combinations. A slightly different path than 0F in Figure 5 will give a different relationship between q_1 and q_2 in Figure 6. The relationship among products is <u>not</u> invariant with respect to the factor quantities; i.e., the relationship is <u>not</u> factor free. Also, as in the previous case, there is no transformation (iso-factor, iso-resource) relation. The physical amounts of the various resources vary along all curves such as 0F' in Figure 6. Also, then, the product-product relation as normally presented in the economic literature does not exist for this case.

Assorted Production

Assorted production is characterized by the degree of assortment being greater than zero ($\alpha > 0$). This type of production process describes a situation where a choice (regarding product mix) arises even though the factor quantities are given (Frisch, 1965, p. 273). Stated in another manner, there is flexibility in choice of the product mix once the factor quantities are specified. The major subclasses of this type of production, again, depend on the value of κ.

Assorted With Coupling. Assorted production with coupling $(\kappa > 0)$ is a type of production process where there is flexibility in choice (regarding product mix) for some of the products and essentially no choice for other products once the factor quantities are specified. A somewhat general formulation of this type of production is given by the equations:

$$\begin{aligned}
&F^{1}(q_1, q_2) &&= 0 \\
&F^{2}(q_2, q_m) &&= 0 \\
&\quad\vdots \\
&F^{\mu-1}(q_{m-2}, q_m) &&= 0 \\
&F^{\mu}(q_{m-1}, q_m; x_1, x_2, \ldots, x_n) &&= 0.
\end{aligned} \tag{3.17}$$

The products $q_1, q_2, \ldots, q_{m-2}, q_m$ in (3.17) are considered joint in production while the relationship between q_{m-1} and q_m is considered assorted. The assorted production with coupling case is essentially a mixture of the factorially determined with coupling and the assorted without coupling cases.[18]

[18] This terminology is somewhat confusing. A production process classified as "assorted with coupling" implies only that there is at

Assorted Without Coupling. Assorted production without coupling ($\kappa = 0$) is a type of production process exhibiting greater flexibility in choice of the product mix. This case is discussed most frequently in the economic literature, but under different names.[19] A relationship among the products results from specification of the factor quantities. The specific amounts of each product to produce must still be determined. Stated in another manner, the product mix is no longer factorially determined. Some further criteria beyond the technical relations must be used to determine the relative amounts of each product produced, even though factor quantities are given. The general formulation of this type of production is given in the equation:

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least one factor free relation (a coupling) and at least one production function relation (such as F^{μ} in 3.17). All factor free relations, in turn, arise from factorially determined production. This subclass could also, then, be considered as representing "assorted and factorially determined with coupling" production. It is, indeed, a mixture. The important consideration is whether or not the allocations of each factor are known. If every allocation is known, the assorted classification must be applied. If some are not known, there is some mixture.

[19] Dillon characterizes this type of production as "multiple response with input control" (Dillon, 1968, pp. 44-45). Carlson, on the other hand, calls this form of production process "joint production" (Carlson, 1939, pp. 76-95). This case has also been described as representing "joint products", but technically independent products were (supposedly) excluded from the discussion (Henderson, 1971, pp. 89-98). The numerical example used in Henderson, however, is a case of technically independent products by the Carlson definition (see Appendix B for a discussion of independence and interdependence).

$$F^1(q_1, q_2, \ldots, q_m; x_1, x_2, \ldots, x_n) = 0 . \qquad (3.18)$$

The products $q_1, q_2, \ldots, q_m$ in (3.18) are considered assorted in production. The underlying production functions are assumed to be of the form:

$$\begin{aligned} q_1 &= q_1(x_{11}, x_{21}, \ldots, x_{n1}) \\ q_2 &= q_2(x_{12}, x_{22}, \ldots, x_{n2}) \\ &\vdots \\ q_m &= q_m(x_{1m}, x_{2m}, \ldots, x_{nm}) \end{aligned} \qquad (3.18a)$$

where x_{ji} $(j = 1, 2, \ldots, n;\ i = 1, 2, \ldots, m;\ i \neq j)$ represents the amount of the jth resource allocated to the ith product.[20]

The case of assorted production for two products and two factors is illustrated in Figures 7 and 8. Given x_1^o of x_1 and x_2^o of x_2, many relationships between q_1 and q_2 can be generated. The production functions of (3.18a) are represented in Figure 7 for two factors and two products. One of the relationships between q_1 and q_2 can be determined by a mapping of 0R from factor space (Figure 7) into product space (Figure 8) to give the production

[20] Another possible form of the underlying production function is given by: $q_i = q_i(x_{1i}, x_{2i}, \ldots, x_{ni}; q_k)$, $(i, k = 1, 2, \ldots, m;\ i \neq k)$. This form of production function will also give rise to the summary relation shown in (3.18) (Mundlak, 1971, p. 493; Samuelson, "The Fundamental...", 1966, pp. 34-35).

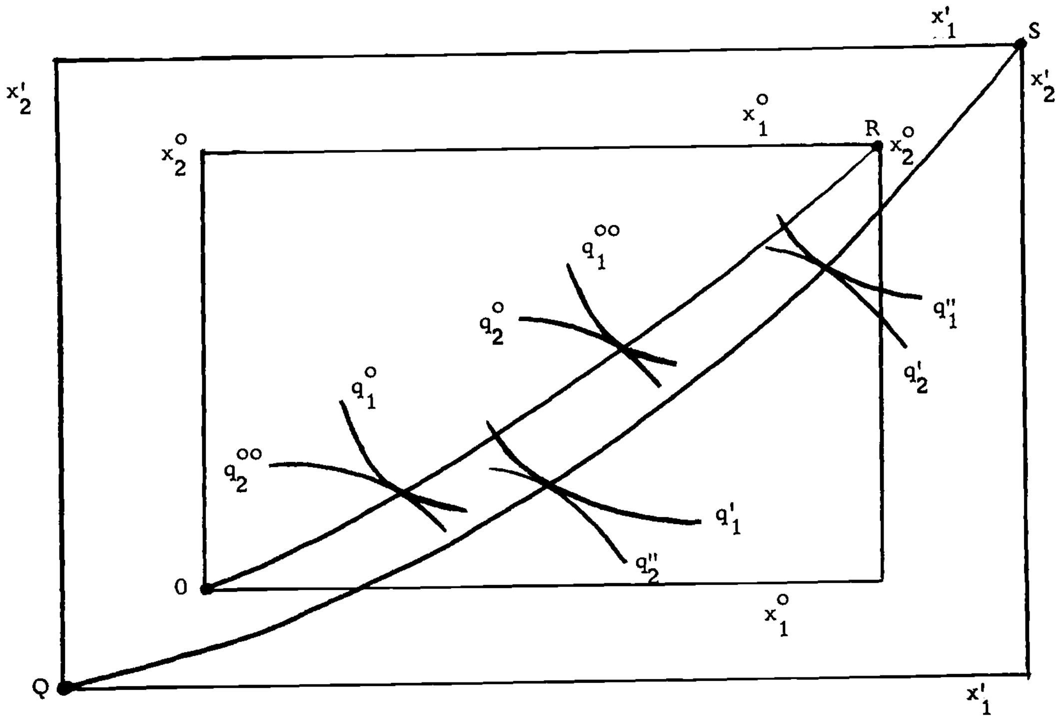

Figure 7. Edgeworth-Bowley box diagram, two factors and two products.

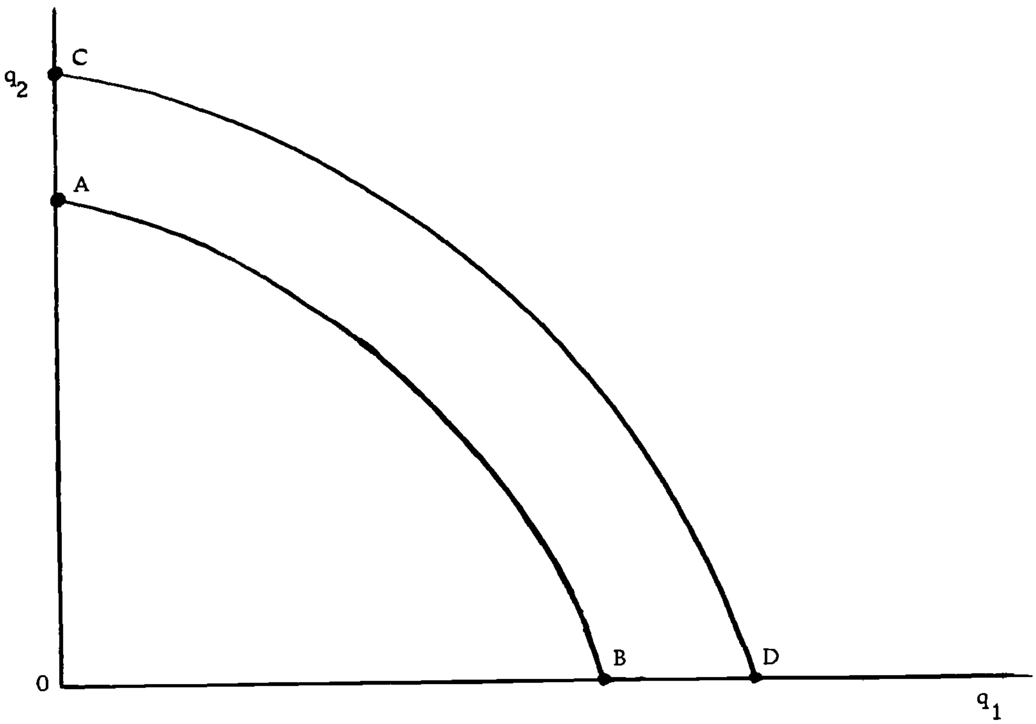

Figure 8. Transformation (iso-resource) curves.

possibility or transformation curve AB.[21] A larger amount of resources, x'_1 and x'_2, will give another relationship between q_1 and q_2 as determined by the mapping of QS (Figure 7) into product space. The resulting transformation curve in product space is CD (Figure 8). A change in the factor quantities gave rise to an entirely new relationship between q_1 and q_2. A decrease in q_2 is always associated with an increase in q_1 along transformation curves AB and CD (Figure 7); the total resource levels along any one curve, however, are constant. This is the product-product relation normally discussed in the economic literature.

Economic Optimization

The economic optimization process must now be identified for the different types of production. The economic optimization process will vary dependent upon the type of underlying physical production relations.

Factorially Determined Production

Economic optimization under factorially determined production can be viewed from either the input or the output side. Viewed from

[21] A transformation curve can also be called a factor isoquant (Frisch, 1965, p. 277) or iso-factor curve. In any case, this form of relationship is defined for constant amounts of the factors. The derivation of a transformation curve is presented in Ferguson ("Transformation...", 1962, pp. 96-102).

the input side, the process is essentially the same as for single output production (Frisch, 1965, p. 282). Consider the case for two factors and two products as represented in the equations:

$$q_1 = q_1(x_1, x_2) \tag{3.19}$$

$$q_2 = q_2(x_1, x_2) \tag{3.19a}$$

$$R_x = p_1 q_1(x_1, x_2) + p_2 q_2(x_1, x_2) \tag{3.19b}$$

$$C_x = r_1 x_1 + r_2 x_2 \tag{3.19c}$$

where,

q_1, q_2 = products,

x_1, x_2 = factors,

R_x = total revenue,

C_x = total cost,

p_1, p_2 = prices of the products,

r_1, r_2 = prices of the factors.

The prices of the products and the factors in the revenue (R) and cost (C) functions, respectively, are assumed constant. Using (3.19b) and (3.19c), the (unconstrained) profit function is given by:

$$\pi = p_1 q_1(x_1, x_2) + p_2 q_2(x_1, x_2) - r_1 x_1 - r_2 x_2. \tag{3.19d}$$

Profits will be maximized when the first partial derivatives of π with respect to each of the factors is equal to zero and the differential

$d^2\pi$ is negative (Dillon, 1968, p. 43). Stated symbolically, these conditions are represented by:

$$\frac{\partial \pi}{\partial x_1} = p_1 \frac{\partial q_1}{\partial x_1} + p_2 \frac{\partial q_2}{\partial x_1} - r_1 = 0 \tag{3.19e}$$

$$\frac{\partial \pi}{\partial x_2} = p_1 \frac{\partial q_1}{\partial x_2} + p_2 \frac{\partial q_2}{\partial x_2} - r_2 = 0 . \tag{3.19f}$$

The requirement of $d^2\pi < 0$ guarantees a concave (from above) profit function and, as a consequence, maximum rather than minimum profits where equations (3.19e) and (3.19f) are satisfied (Dillon, 1968, pp. 43-44). Using equations (3.19e) and (3.19f), an expansion path is given by:

$$\frac{p_1 \frac{\partial q_1}{\partial x_1} + p_2 \frac{\partial q_2}{\partial x_1}}{r_1} = \frac{p_1 \frac{\partial q_1}{\partial x_2} + p_2 \frac{\partial q_2}{\partial x_2}}{r_2} . \tag{3.19g}$$

The maximum profit level of production is conditional on the relative product prices. The product prices influence the location of the "path" and the maximum profit combination of outputs. Reference to Figure 9 will help clarify this point.

A hypothetical case for two products and two factors of the general form of (3.19)-(3.19a) is represented in Figure 9. The isoquants for product q_1 are represented by q_1^o, q_1', and q_1'', in

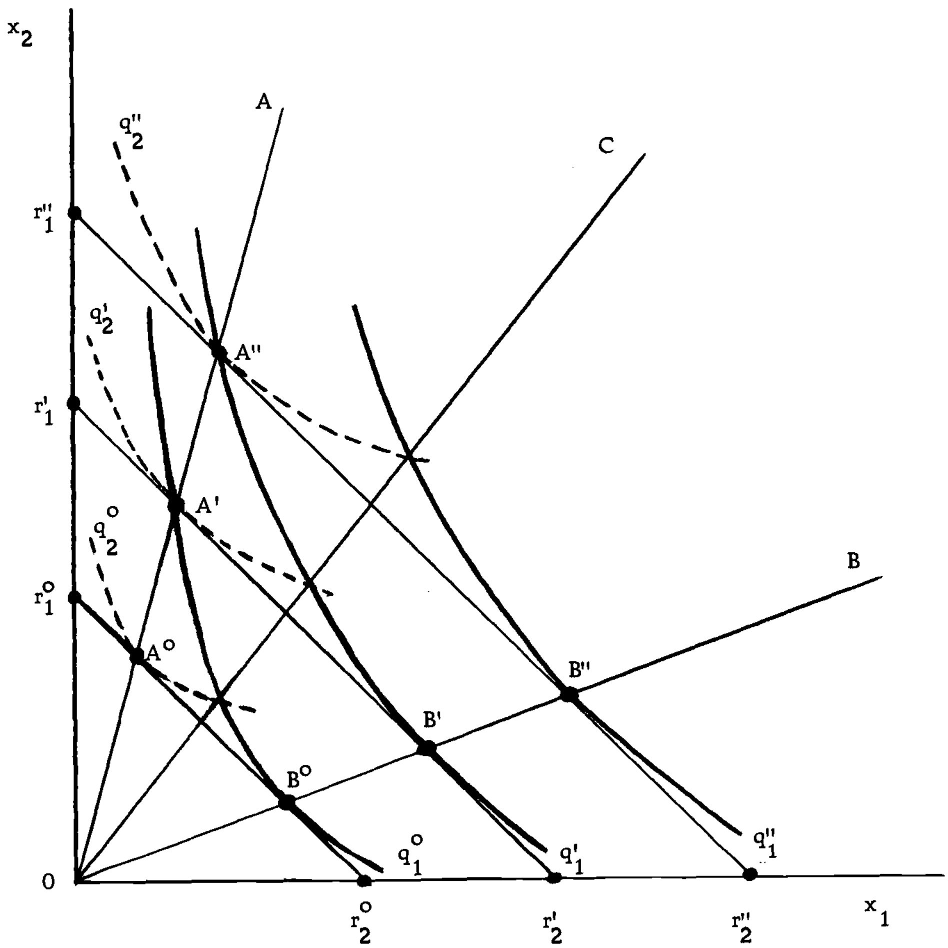

Figure 9. Isoquants and expansion paths, factorially determined without coupling production.

ascending order of magnitude. Assume, for example, the price of q_1 is zero ($p_1 = 0$). In this case, equation (3.19g) reduces to:

$$\frac{\frac{\partial q_2}{\partial x_1}}{\frac{\partial q_2}{\partial x_2}} = \frac{r_1}{r_2}. \tag{3.19h}$$

This path is represented in Figure 9 by 0A. The amount of q_1 produced along 0A does not affect the maximum profit level of q_2. Similarly, a zero price for q_2 ($p_2 = 0$) is represented by 0B. Again, output will be expanded along the path regardless of the level of the other output (in this case q_2) until profits are maximized from the production of q_1. Another possible case is represented by 0C for some non-zero prices for both q_1 and q_2.

Identification of the expansion path, of course, does not give the point at which profits are maximized. Given the actual forms of the production functions and the actual prices for factors and products, however, this point can be easily found. The maximum profit point is given by the simultaneous solution of the equations (3.19e) and (3.19f). In essence, the resources x_1 and x_2 are increased in the proportions specified on the relevant expansion path as long as the marginal revenue from the products concerned exceeds the marginal cost. This becomes clearer when the process is viewed from the "output side".

The total revenue and cost function can also be expressed as functions of the product quantities. Equations (3.19b) and (3.19c) then become of the form:[22]

$$R_q = p_1 q_1 + p_2 q_2 = R_q(q_1, q_2) \tag{3.19i}$$

$$C_q = C_q(q_1, q_2) \tag{3.19j}$$

where,

R_q = total revenue,

C_q = total cost.

All other variables are defined as before. The effect of changes in relative product prices on shifts in the production emphasis can now be isolated (Frisch, 1965, p. 283). The goal of maxiumum profits is reached when the iso-cost curves are tangent to the respective iso-revenue curves. The profit function is now represented by the form:

$$\pi = p_1 q_1 + p_2 q_2 - C_q(q_1, q_2). \tag{3.20}$$

Profits are maximized when the first partial derivatives of the profit function in (3.20) are set equal to zero and the differential $d^2\pi$ is negative. Production of both products is expanded as long as the partial marginal revenue exceeds the partial marginal cost for each

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[22] See Appendix A for the derivation of the iso-cost curve in product space.

product (Frisch, 1965, p. 285).

Assorted Production

The economic optimization process under assorted production can also be viewed either from the input or the output side. The actual process of optimizing according to economic criteria is, in fact, very similar to factorially determined production. Consider the case for two factors and two products as represented in the equations:

$$F^1(q_1, q_2; x_1, x_2) = 0 \tag{3.21}$$

$$R_q = p_1 q_1 + p_2 q_2 \tag{3.21a}$$

$$C_x = r_1 x_1 + r_2 x_2 \, . \tag{3.21b}$$

The variables are defined as before. Several approaches to economic optimization can now be followed. One approach involves the assumption that each q_i is independent of the amount of q_k, $i \neq k$, produced (Dillon, 1968, pp. 44-45). The underlying production functions reflecting this assumption are given by:

$$q_1 = q_1(x_{11}, x_{21}) \tag{3.21c}$$

$$q_2 = q_2(x_{12}, x_{22}) \, . \tag{3.21d}$$

Given production functions (3.21c) and (3.21d) the profit function becomes of the form:

$$\pi = \sum_{i=1}^{m} \pi_i \, .$$

The maximum profit (maximum π) will be attained when each of the π_i are at a maximum (Dillon, 1968, p. 44). For the present case of two factors and two products, the individual (unconstrained) profit functions are of the form:

$$\pi_1 = p_1 q_1(x_{11}, x_{21}) - r_1 x_{11} - r_2 x_{21} \tag{3.21e}$$

$$\pi_2 = p_2 q_2(x_{12}, x_{22}) - r_1 x_{12} - r_2 x_{22} \, . \tag{3.21f}$$

The resulting mn equalities (m products and n factors) must be satisfied, along with second order conditions, for maximum profits (Dillon, 1968, p. 45). The optimization process as viewed from the output side is well documented elsewhere.[23]

Non-Market Goods and Production Trade-Off Ratios

The economic decision process is usually described for a case where markets exist for all the products and factors of concern. The choice process is necessarily different given the existence of

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[23] See, for example, Henderson (1971, pp. 89-93). An approach for deriving the iso-cost curve in product space is outlined in Appendix A of this study.

non-market goods. One approach, given the lack of some prices, is to identify the production conditions. The alternative product and factor combinations can then be presented to some decision making entity for consideration. Assuming the decision body can reflect societies' preferences, choice of a product mix gives an implicit value for the non-money valued product and/or factor.

The important production conditions for this process are the iso-cost and production function relations. Given the transformation ratios along an iso-cost surface, choice of a point on the surface gives the implicit price ratios among the products. This transformation ratio is referred to as a product-product trade-off ratio in this study. Similarly, given the transformation ratios along a production function surface, choice of a point gives the implicit factor-product price ratio. This transformation ratio is referred to as a factor-product trade-off ratio in this study. The elements affecting these two ratios must now be identified.

Nature of the Product-Product Trade-Off Ratio

Given the absence of a market price for one or more products, the entity providing the products must be especially concerned with the effect of various actions on the eventual product-product trade-off ratio. Trade-off ratio calculations may be misleading unless the underlying technical relations are known. Also, product-product trade-offs must be calculated along the correct curve; i.e., trade-offs cannot be calculated along interior iso-cost curves or between curves. In addition, the effects of intermediate product relationships must be isolated.

Effects of Technical Interdependence and Independence. The type of technical relationships among the products can have a very significant effect on the product-product trade-off ratio. The various types of possible technical production relations are presented in Carlson (1939, pp. 84-102). The classification system for products is based on the sign of the mixed derivative, $(\partial^2 C / \partial q_1 \partial q_2)$, where C represents the cost of production.[24] Products are classified according to the following scheme (Carlson, 1939, pp. 82-83):[25]

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[24]The costs of production are defined along the least cost factor expansion paths for each of the products in assorted production. The relationships are maintained such that, for $C = f(q_1, q_2, \ldots, q_m)$:

$$\frac{\partial C}{\partial q_i} = \frac{r_1}{\partial q_i / \partial x_1} = \frac{r_2}{\partial q_i / \partial x_2} = \ldots = \frac{r_n}{\partial q_i / \partial x_n},$$

for all the products q_i $(i = 1, 2, \ldots, m)$ (Carlson, 1939, pp. 87-89). In the case of factorially determined production, the cost function is also defined for least cost combinations of factors, but not along any factor expansion path as defined in the usual sense.

[25]A geometric approach to product classification is presented in Appendix B. Also, there is further discussion of the notions of technical interdependence and independence.

$$\frac{\partial^2 C}{\partial q_1 \partial q_2} < 0 \qquad \text{technically complementary,}$$

$$\frac{\partial^2 C}{\partial q_1 \partial q_2} > 0 \qquad \text{technically competing,} \tag{3.22}$$

$$\frac{\partial^2 C}{\partial q_1 \partial q_2} = 0 \qquad \text{technically independent.}$$

The characteristics of the various types of technical relations in (3.22) can be described with reference to a particular example. Consider three specific production relations, given by:

$$C = a_1 + q_1^2 + q_2^2 - q_1 q_2 \, ; \qquad \frac{\partial^2 C}{\partial q_1 \partial q_2} = -1 < 0 \tag{3.22a}$$

$$C = a_2 + q_1^2 + q_2^2 + q_1 q_2 \, ; \qquad \frac{\partial^2 C}{\partial q_1 \partial q_2} = 1 > 0 \tag{3.22b}$$

$$C = a_3 + q_1^2 + q_2^2 \, ; \qquad \frac{\partial^2 C}{\partial q_1 \partial q_2} = 0 \tag{3.22c}$$

where,

C = cost of production,

q_1, q_2 = products,

a_1, a_2, a_3 = constants.

The products are classified as technically complementary, technically competitive, and technically independent in (3.22a), (3.22b), and

(3.22c), respectively. The relations are illustrated in Figure 10 for $a_1 = a_2 = a_3 = 0$ and $C = C^o$. The upper, middle, and lower curves represent technically complementary, technically independent, and technically competitive products, respectively, for the same level of resource use (constant costs).

The technical relations among the products determine the manner in which q_1 and q_2 respond to a given level of resource application. Given that q_1^o of q_1 (Figure 10) is produced, for example, a greater amount of q_2 can be obtained if the production relations are technically independent and/or complementary than if the relation is technically competitive. This is represented by $q_2'' > q_2' > q_2^o$ in Figure 10. The same amount of resource, C^o, gave rise to greater amounts of q_2 when the products were not technically competitive. The same relation holds for any given amount of q_2, such as q_2^o. The greatest amount of q_1 can be produced for the given $q_2 = q_2^o$ when the products are technically complementary (Figure 10).

The product-product trade-off ratio is affected by the technical relation among the products. The trade-off ratio at $q_2 = q_2^o$ and the given $C = C^o$, for example, is the greatest at point C; i.e., the trade-off ratio is the highest for the case exemplified when the products are technically complementary. The trade-off ratios at points A and B are less than at point C.

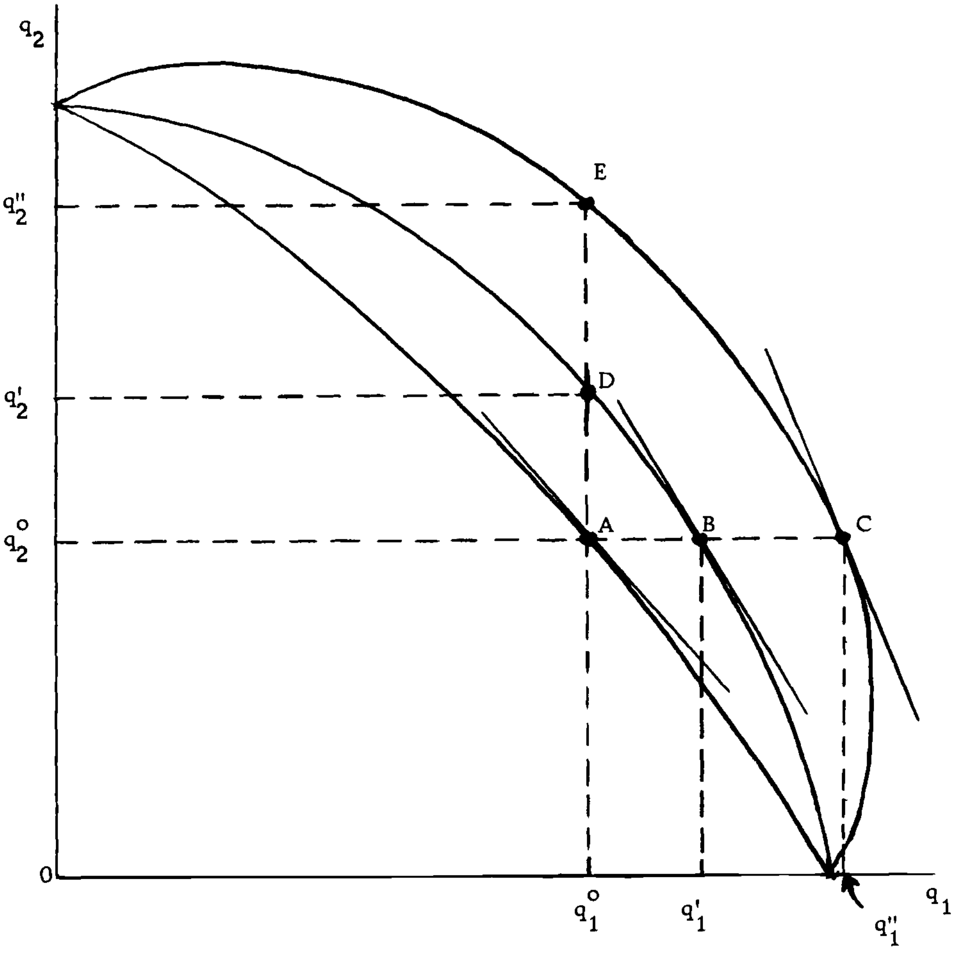

Figure 10. Iso-cost curves in product space illustrating technical interdependence and independence.

The effects of not using the correct iso-cost curve can be outlined by imposing community indifference curves on the product diagram. Assume the entity involved in producing products q_1 and q_2 has knowledge of the implicit price ratio between q_1 and q_2, as represented by the slopes of the curves at points A, B, and C in Figure 11. Assume the true relation between q_1 and q_2 is one of technical independence. Also, assume the entity producing the two products determines the "correct" transformation curve to be QAR; i.e., the products are incorrectly classified as technically competitive. Also, assume the entity is providing q_1 and q_2 at the ratio represented at S, but would like to provide the level q_2' of q_2 as represented at A (Figure 11). An increase in cost and change in allocation of resources from that at S to the allocation of resources and cost at A, such as to generate q_2' of q_2, would appear to result in a decrease in q_1. This, indeed, would be the case if the true relationship between q_1 and q_2 was represented along QAR; i.e., if q_1 and q_2 were actually technically competitive. Given, however, the products were technically independent, attempts to move from S to A would result in reaching point D, where q_1 <u>increased</u> from q_1^o to q_1''. The more preferable point from the communities point of view, however, was to reach point B. The entities decision to provide q_2' of q_2 resulted in point D (assuming the true relation was technical independence) when point B was preferred.

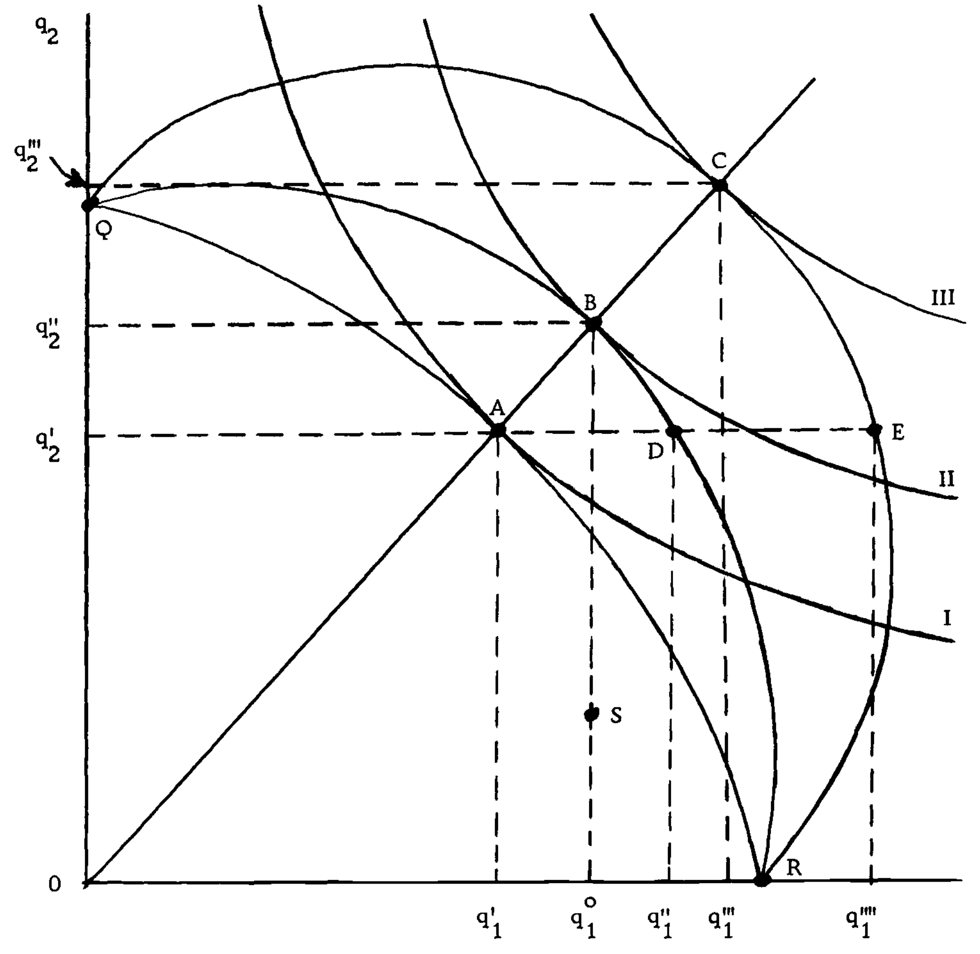

Figure 11. Iso-cost curves in product space, community indifference curves, and expansion path.

Another type of interdependence and independence that is also "technical" in nature arises when transformation curves are related in some manner. If there are common factors for various groups of products, the transformation curves for each group of products are not independent (Samuelson, 1947, pp. 235-236). Consider, for example, two transformation functions representing the relations among four products and two factors as given by:

$$F^1(q_1, q_2; x_1, x_2) = 0 \tag{3.22d}$$

$$F^2(q_3, q_4; x_1, x_2) = 0 \ . \tag{3.22e}$$

The two transformation functions in (3.22d) and (3.22e) are not independent; i.e., x_1 and x_2 must be allocated among all four products. Consideration of these two transformation relations in isolation could also lead to distortion in trade-off estimates. In fact, the trade-offs (dq_1/dq_3), (dq_1/dq_4), (dq_2/dq_3), and (dq_2/dq_4) would not even be evaluated. The two transformation functions represented in (3.22d) and (3.22e) should be considered as one function in any analysis.

Effects of Calculating Trade-Offs Along Internal Curves and Among Curves. Product-product trade-off ratios may also be misleading if calculated along production possibility curves rather than transformation curves. A transformation curve is defined for a given amount of resources while a production possibility curve is defined for particular allocations of the resources among the products (Ferguson, 1962, p. 99). Every point on a transformation curve is on a different production possibility curve. Consider, for example, the case represented in Figure 12. The transformation curve for a given amount of resources is represented by curve AE. Curves FH and KM represent particular and fixed allocations of the resources among the products; i.e., the allocation of at least one resource between products q_1 and q_2 is fixed along curves FH and KM. Curves FH and KM are production possibility curves. Points B and D are common points generated along the efficiency locus.[26]

The production trade-off ratio will be considerably different for any given q_i depending upon the curve used. The trade-off ratio between q_1 and q_2 for q_2^o produced, for example, is different at each of the points L, G, and C. The trade-off ratio is higher at point G and lower at point L, as compared to the ratio at point C. The use of production possibility curve FH instead of the transformation curve, for example, would result in underestimation of the trade-off from F to B and overestimation from B to H. The same argument can be made for using the "frontier" iso-cost curve versus internal iso-cost curves. The relevant iso-cost curve is the iso-minimum cost

[26] The efficiency locus is defined as the connecting line where the slopes of the respective isoquants are equal (Bator, 1968, p. 387).

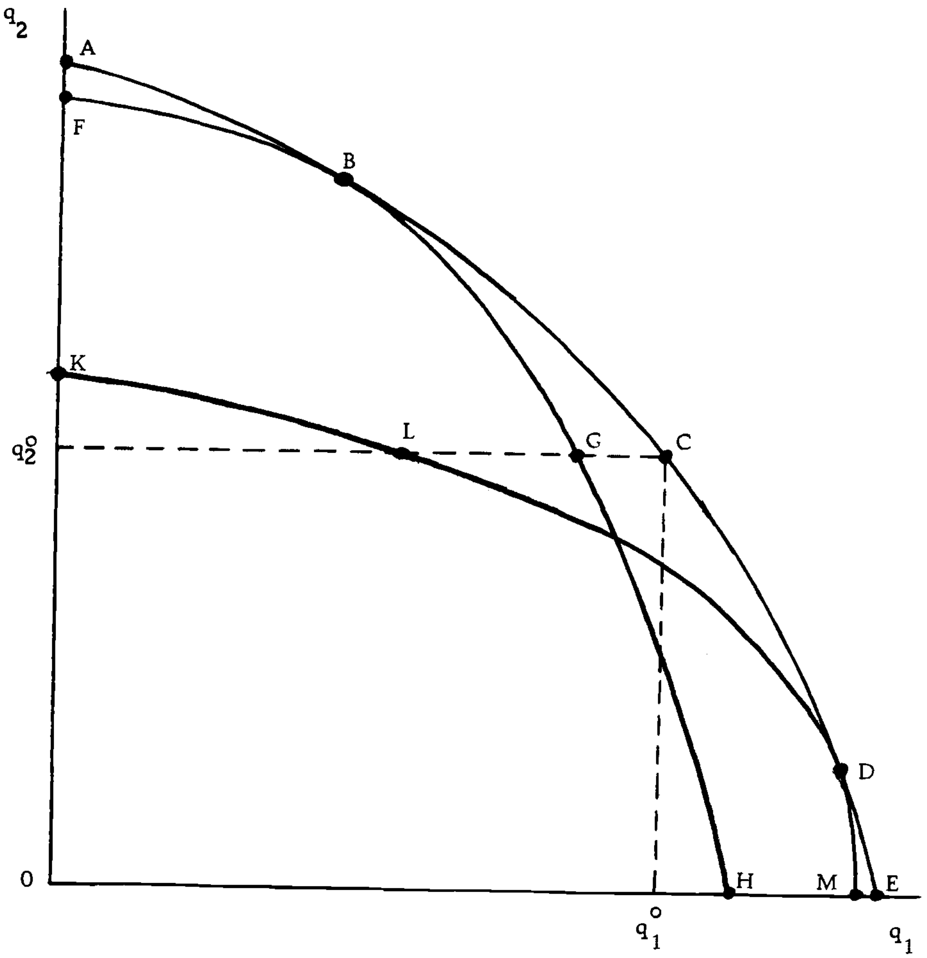

Figure 12. Transformation and production possibility curves in product space.

curve or the curve furthest out in the product-product plane for any given dollar expenditure. Trade-offs calculated along any internal iso-cost curve would be analogous to calculating trade-offs along production possibility curves. Again, the trade-offs will be misleading unless the correct curve is used in the analysis.[27]

Product-product trade-off ratios may also be misleading if calculated between curves; i.e., erroneous ratios may be obtained unless costs are held constant. Consider, for example, the case illustrated in Figure 13. The "trade-off" in moving from B to E might possibly be thought to be the relevant trade-off; i.e., BG of q_2 was "sacrificed" to gain GE of q_1. Using this approach, the trade-off ratio is given by BG/GE. The actual trade-off ratios, however, are given by the slopes at B, G, and E. By moving from B to E, the product-product trade-off ratio has changed (as the slopes are different at B and E). The actual movement, however, did not give rise to the trade-off ratio. The costs must be held constant before the trade-off ratio becomes useful; i.e., trade-off ratios must be calculated along transformation or iso-cost curves.

[27]See Appendix A for a discussion of the similarities and differences between iso-cost and iso-factor relations. The concepts are identical under certain conditions. This is always true if there is only one resource; for that case, it becomes meaningless to speak of any differences.

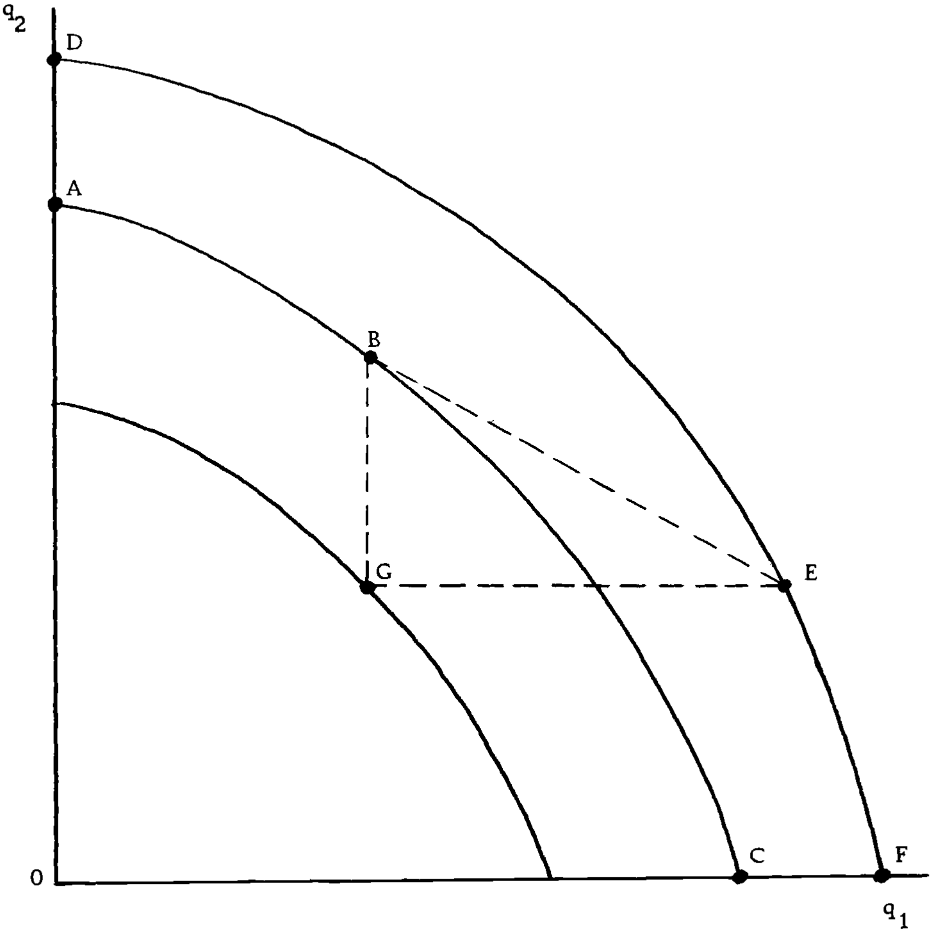

Figure 13. Iso-cost curves in product space with trade-off calculations among curves.

Effects of Intermediate Product Relations. The various cases of intermediate products can also complicate the trade-off calculation process. The trade-off ratio could, again, be misleading if the intermediate product relations are not included in the analysis. Several possible types of relationships can exist. Only four different forms are presented here for discussion purposes, namely:

$$\begin{aligned} q_1 &= q_1(x_{11}, x_{21}, q_2) \\ q_2 &= q_2(x_{12}, x_{22}) \end{aligned} \tag{3.23}$$

$$\begin{aligned} q_1 &= q_1(x_{11}, x_{21}, q_2) \\ q_2 &= q_2(x_{12}, x_{22}, q_1) \end{aligned} \tag{3.24}$$

$$\begin{aligned} q_1 &= q_1(x_{11}, x_{21}, q_3) \\ q_2 &= q_2(x_{12}, x_{22}) \\ q_3 &= q_3(x_{13}, x_{23}) \end{aligned} \tag{3.25}$$

$$\begin{aligned} q_1 &= q_1(x_{11}, x_{21}, q_3) \\ q_2 &= q_2(x_{12}, x_{22}, q_3) \\ q_3 &= q_3(x_{13}, x_{23}) \end{aligned} \tag{3.26}$$

where,

x_{ji} = amount of jth resource allocated to the production of the ith product $(i, j = 1, 2)$,

q_i = products and/or intermediate products $(i = 1, 2)$.

Product q_2 serves as an intermediate product and a product in (3.23). Both q_1 and q_2 require x_1 and x_2, but q_1 also requires q_2 (for $(\partial q_1/\partial q_2) > 0$). Both q_1 and q_2 serve as intermediate products in (3.24). Again, both q_1 and q_2 use x_1 and x_2, but each product also requires some of the other product. Product q_3 serves as an intermediate product in the production of q_1 in (3.25). All products $(q_1, q_2$, and $q_3)$ use the resources x_1 and x_2. In (3.26), q_3 is an intermediate product in the production of both q_1 and q_2. Again, all three products use some of the resources x_1 and x_2.[28]

The relationships expressed in (3.23)-(3.26) are, however, very general in nature.[29] Any one of the relations in (3.23)-(3.26) can summarize a number of specific cases. It is useful, therefore, to outline some less general cases to isolate some of the effects of intermediate products on the transformation relationship. Such a set of less general forms are represented in the following set of equations:

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[28]All examples represented in (3.23)-(3.26) represent assorted production; i.e., the allocation of each factor among the products can be determined in all cases.

[29]It should be noted that all the subscripts used in equations (3.23)-(3.26) are very crucial to this discussion. It would appear on the surface, for example, that the following relation is identical to equation (3.23): $q_1 = q_1(x_1, x_2, q_2)$, $q_2 = q_2(x_1, x_2)$. This, however, is an entirely different production process than described in (3.23). The additional subscripts in (3.23) represent the fact that x_1 and x_2 are to be allocated among the products. The allocation, in turn, is under the control of the manager.

$$\begin{aligned} q_1 &= q_1' \\ &= f(x_{11}, x_{21}) \\ q_2 &= q_2' - a_1 q_1' \\ &= g(x_{12}, x_{22}) - a_1 f(x_{11}, x_{21}) \end{aligned} \tag{3.23a}$$

$$\begin{aligned} q_1 &= q_1' - a_2 q_2' \\ &= f(x_{11}, x_{21}) - a_2 g(x_{12}, x_{22}) \\ q_2 &= q_2' - a_1 q_1' \\ &= g(x_{12}, x_{22}) - a_1 f(x_{11}, x_{21}) \end{aligned} \tag{3.24a}$$

$$\begin{aligned} q_1' &= f(x_{11}, x_{21}) \\ q_2' &= g(x_{12}, x_{22}) \\ q_3' &= h(x_{13}, x_{23}) \\ q_3 &= q_{31} \\ &= a_1 q_1 \end{aligned} \tag{3.25a}$$

$$\begin{aligned} q_1' &= f(x_{11}, x_{21}) \\ q_2' &= g(x_{12}, x_{22}) \\ q_3' &= h(x_{13}, x_{23}) \\ q_3' &= q_{31} + q_{32} \\ &= a_1 q_1 + a_2 q_2 . \end{aligned} \tag{3.26a}$$

The equation sets (3.23a)-(3.26a) represent more specific examples of (3.23)-(3.26), respectively. The "prime" on the q_j products, such as q_1' in (3.23a), represents the production function relation for that product if the intermediate products are ignored. In (3.23a), for example, the q_1' and q_2' functions could be used to generate the transformation curve ignoring the fact that q_2 is also an intermediate product required for q_1 production. The q_j products not having a "prime", then, represent the transformation curve after the effects of the intermediate products have been included. The equations represented in (3.23)-(3.26) all represent the production functions necessary to find the transformation curve when the intermediate product relations are included. In essence, the q_j values represent the product levels after the q_j' values have been adjusted to include the effects of intermediate products.

The case represented in (3.23) and (3.23a) is illustrated in Figure 14. The transformation curve is given by AB before the effect of the intermediate product is included; i.e., curve AB is derived from the relations:

$$q_1' = f(x_{11}, x_{21})$$
$$q_2' = g(x_{12}, x_{22}) \cdot$$

The transformation curve becomes AC after the intermediate product q_2 is also considered as an input into q_1 production. This is represented by a reduction in the amount of q_2 available for "sale" by $a_1 q_1'$ in (3.23a). The amount $a_1 q_1'$ of q_2 is used in the

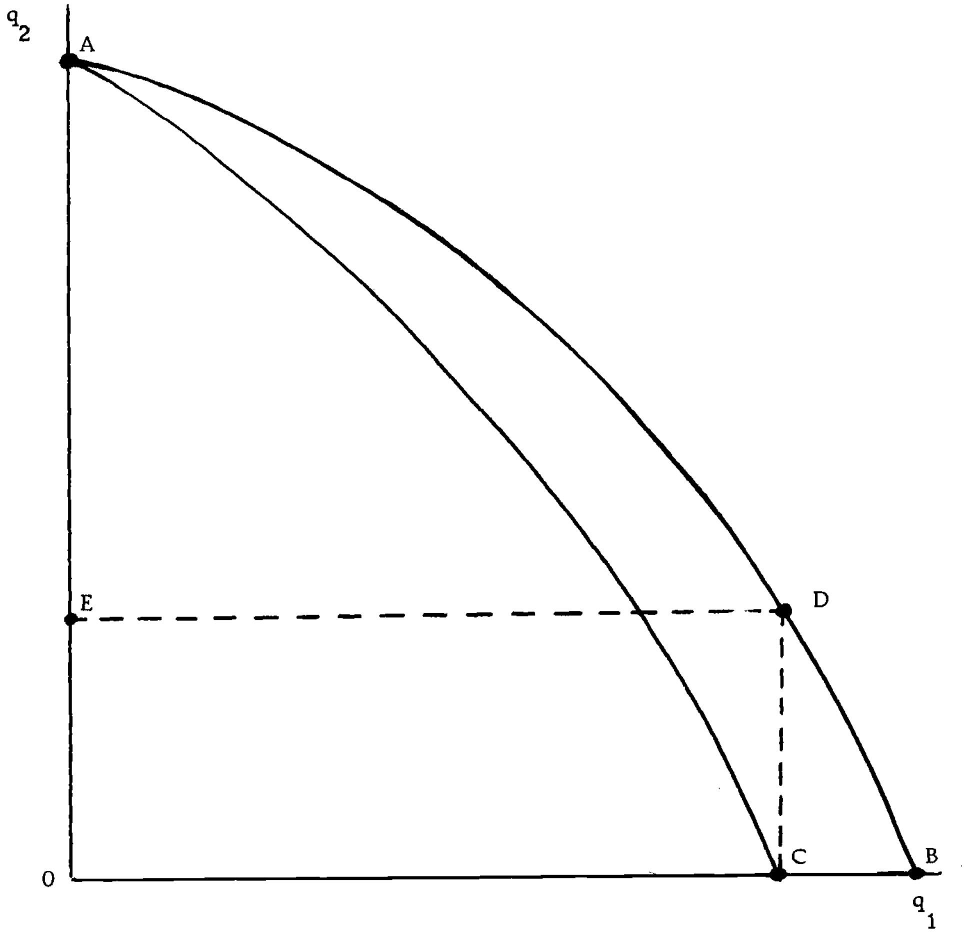

Figure 14. Transformation curves given q_2 as an intermediate product in q_1 production.

production of q_1. Reference to Figure 14 clarifies this point. Prior to consideration of q_2 as an intermediate product (as well as a final product), q_1 could be produced at the level indicated at point B. At that level of production, all the resources were assumed allocated to the production of q_1. This is not the case, however, if q_1 also requires some q_2. The intermediate product, in turn, requires some of the resource base. As a result, the level of q_1 production must be reduced to that indicated at C such as to release some resources for the required production of q_2. All of the q_2 produced at C, however, went into the production of q_1 so no q_2 was offered for "sale"; i.e., q_2 is shown to be at the zero level when q_1 is at the C level. The amount of q_2 produced is given by the relationship (from 3.23a):

$$q_2 = g(x_{12}, x_{22}) - a_1 f(x_{11}, x_{21}) = 0$$

$$g(x_{12}, x_{22}) = a_1 f(x_{11}, x_{21})$$

$$q_2' = a_1 q_1' \ .$$

In terms of Figure 14, $q_2' = 0E$ of q_2 was used in the production of q_1 at point C. A similar argument can be made for all points along AC. When q_1 is zero, of course, no q_2 is used in q_1

production. As a result, AC and AB have a common point at A (Figure 14).[30]

The case represented in (3.24a) is illustrated in Figure 15.[31] Again, (3.24a) is a more specific case of (3.24). Manipulation of (3.24a) shows how CD must necessarily be under AB at all points. The correct relationship between q_1 and q_2 (as given along CD) is derived in the following manner:

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[30] It must be understood that equation sets (3.23)-(3.26) can follow from each of those in (3.23a)-(3.26a). Caution must be exercised, however, in making the transition, especially when using general equation forms such as these. Consider, for example, trying to manipulate (3.23a) to arrive at (3.23), as follows:

$$q_1 = q_1'$$
$$q_2 = q_2' - a_1 q_1'$$

or,

$$q_1 = f(x_{11}, x_{21})$$
$$q_2 = g(x_{12}, x_{22}) - a_1 f(x_{11}, x_{21})$$

or,

$$q_1 = f(x_{11}, x_{21})$$
$$q_2 = g(x_{12}, x_{22}, q_1) \ .$$

The derived result would seem to imply that q_1 was an intermediate product (input) into the production of q_2. This is <u>not</u> the case. Similar (invalid) conclusions could be drawn from casual observance of the functional relations represented in (3.24a)-(3.26a). These relations are essentially <u>ex ante</u> (before the correct curve is found) while (3.23)-(3.26) are essentially <u>ex post</u> (already account for the effects of intermediate products).

[31] This illustration of intermediate products was developed from the basic framework presented in Vanek (1963, p. 132-133).

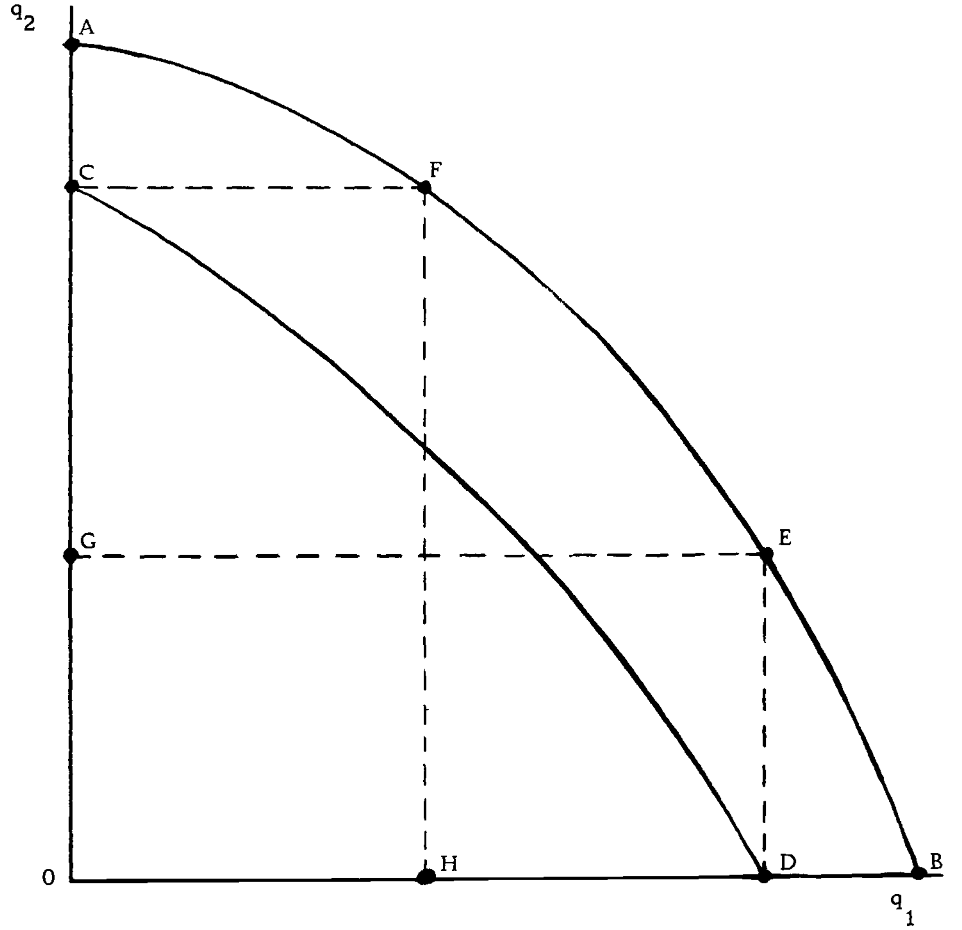

Figure 15. Transformation curves given $q_2(q_1)$ as an intermediate product in the production of $q_1(q_2)$.

$$q_1 = q_1' - a_2 q_2'$$
$$q_2 = q_2' - a_1 q_1'$$

or,

$$q_2 = q_2' - a_1(q_1 + a_2 q_2')$$
$$= (1 - a_1 a_2) q_2' - a_1 q_1 \; .$$

Point D on CD is found in the following manner:

$$q_2 = (1 - a_1 a_2) q_2' - a_1 q_1 = 0$$

$$(1 - a_1 a_2) q_2' = a_1 q_1$$

$$q_2' = (\frac{a_1}{1 - a_1 a_2}) q_1 .$$

In terms of Figure 15, $q_2' = 0G$ of q_2 is used in q_1 production at point D; i.e., $q_2' = DE = 0G$. Point C can be found in a similar manner such as to show that 0H = CF of q_1 was used to provide the level of q_2 represented at C. The transformation curve CD is necessarily internal to AB as $q_2(q_1)$ production requires that some resources be used in $q_1(q_2)$ production.

A slightly different type of intermediate product relationship is represented in (3.25a) and illustrated in Figure 16. Product q_3 uses a portion of the same resource base used by q_1 and q_2, but q_3 serves only as an intermediate product (input). It is assumed that q_3

is used only in the production of q_1.[32] A transformation surface is represented by ABD in Figure 16 when the product q_3 is not considered an input into the production of q_1. Surface ABD is derived from q_1', q_2', and q_3' of (3.25a). The transformation curve AB represents the relation between q_1 and q_2 when q_3 is ignored (q_3 is zero). Product q_3, however, cannot be at the zero level if q_1 is to be produced as is represented by $q_3 = q_{31} = a_1 q_1$. In addition, if q_3 is produced at some level, some of the resources must be diverted from q_1 and/or q_2 production. As a result, AB is not the correct transformation curve when q_3 is an input to q_1 production. The correct transformation curve is again necessarily below AB, such as given by AE in Figure 16. Line 0C in the $q_1 q_3$ plane represents the relation between q_1 (a product) and q_3 (an intermediate product or input). The slope of 0C is given by $(dq_1/dq_3) = 1/a_1$. The correct transformation curve between q_1 and q_2 is dictated by conditions along line 0C. At point C, for example, the entire resource base is used in the production of q_1 and q_3. As a result, no resources are available for producing q_2 at point C. Also, of course, q_1 cannot be produced at the level represented at B because some resources are devoted to q_3 production. The resulting point on the correct transformation curve is

[32] This illustration of intermediate products was developed from the basic framework presented in Batra (1973, pp. 297-300).

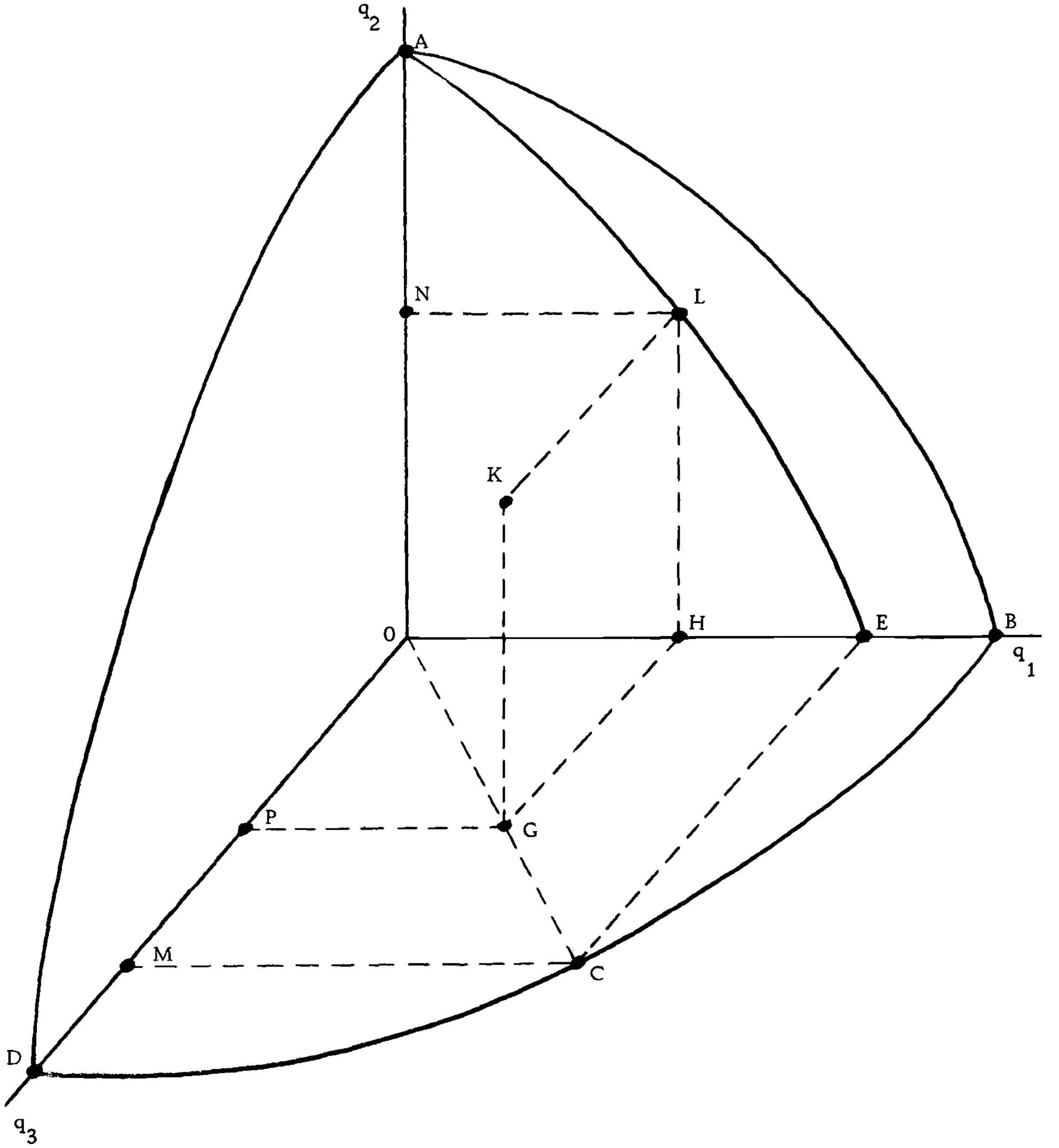

Figure 16. Transformation surface with q_3 an intermediate product in the production of q_1.

represented at E where CE = 0M of q_3 is used in q_1 production and $q_2 = 0$. By moving back on 0C to point G, resources are released for the production of q_2 as represented by the height of the transformation surface ABD at point G, or KG. This level of KG is represented by LH in the q_1, q_2 plane, or 0N of q_2 is produced in conjunction with 0H of q_1. Again, some resources were used in q_3 production to provide GH = 0P of q_3. The amount 0P of q_3 was, in turn, used in q_1 production. Similar reasoning can be used to derive the rest of curve AE. When $q_1 = q_3 = 0$, of course, all the resources are allocated to q_2 production which gives the common point A between AB and AE.

The product q_3 could also serve as an intermediate product (input) into the production of both q_1 and q_2. This case in represented in (3.26) and (3.26a) and illustrated in Figure 17.[33] Curve AB and AE from Figure 16 are reproduced in Figure 17 to show the additional effects on the curve by requiring q_3 in q_2 production as well as q_1 production. The correct transformation curve is below AE as increases in q_2 production from the zero level at point E now requires further increases in q_3 beyond the q_3 required to produce q_1. Curves AE and GE share a common point at E because q_2 production is zero and all q_3 produced is used in q_1

[33]See Footnote 32.

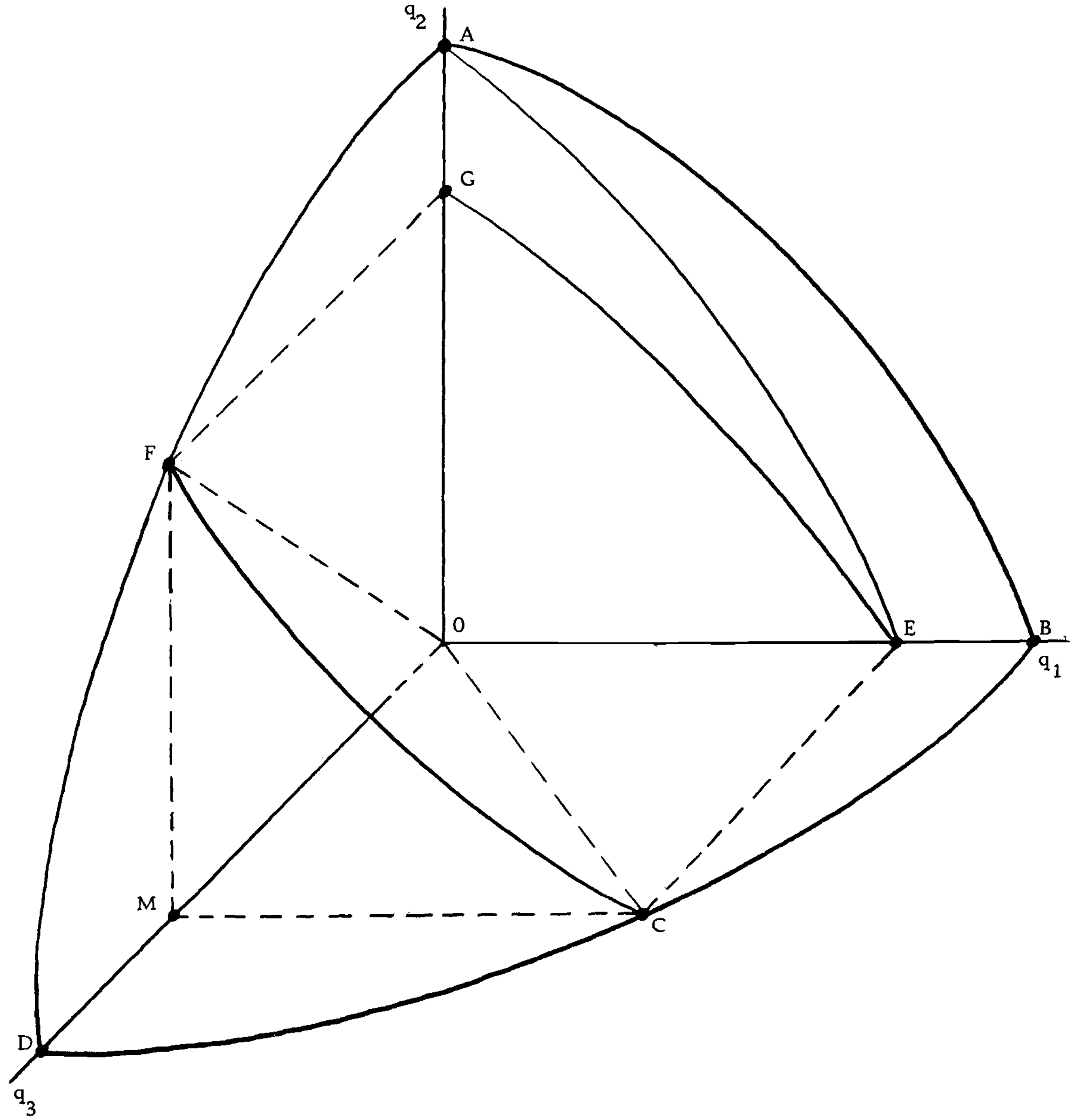

Figure 17. Transformation surface with q_3 an intermediate product in both q_1 and q_2 production.

production. Product q_2 is at level $G < A$ when q_1 is zero because some of the resources must be devoted to q_3 production. At all points on GE between G and E, an even larger amount of q_3 must be provided in order to produce both q_1 and q_2. Product q_3 is allocated between q_1 and q_2 production according to the relation $q_{32} + q_{31} = q_3$ in (3.26a).

Nature of the Factor-Product Trade-Off Ratio

The notion of factor-product relations is well established in economic theory. This is especially so with regard to the single factor-single product case. The marginal physical product of a factor x_i, given by $(\partial q_i/\partial x_j)$, $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$ for m products is also discussed frequently in economic theory text books and in application. Further theoretical discussion of this concept is easily found if desired by the reader. The importance of this theoretical notion in this study, however, relates to calling the well-established concept of marginal productivity a "trade-off ratio".

The marginal product of a factor is a trade-off ratio in the sense that reducing the amount of the factor applied results in reducing the quantity of product produced. This change, in turn, results in making more of the factor available for use in other production processes or for final demand. A certain amount of product must be "traded-off" or sacrificed to make the factor available for other use. Given there is no known, market price for the factor of concern, selection of a product combination by the decision body reflects an implicit price for the factor of concern. Consider, for example, the selection of the product combination at point C in Figure 12. Given assorted production without coupling, selection of point C implies that (for two products and two factors):

$$1.\quad \frac{dx_2}{dx_1} = -\frac{r_1}{r_2}$$

$$2.\quad \frac{\partial q_1}{\partial x_1} = \frac{r_1}{p_1}; \quad \frac{\partial q_1}{\partial x_2} = \frac{r_2}{p_1} \tag{3.27}$$

$$3.\quad \frac{\partial q_2}{\partial x_1} = \frac{r_1}{p_2}; \quad \frac{\partial q_2}{\partial x_2} = \frac{r_2}{p_2}$$

where,

x_j = factors,

r_j = factor prices ($j = 1, 2$),

q_i = products,

p_i = product prices ($i = 1, 2$).

Given the prices r_2, p_1, p_2, and the "trade-off" ratios $(\partial q_i/\partial x_j)$ $(i, j = 1, 2)$ the implicit price of x_1 is given. On the other hand, given factorially determined production, it is the case at point C that:

$$\frac{p_1 \frac{\partial q_1}{\partial x_1} + p_2 \frac{\partial q_2}{\partial x_1}}{p_1 \frac{\partial q_1}{\partial x_2} + p_2 \frac{\partial q_2}{\partial x_2}} = \frac{r_1}{r_2} . \tag{3.28}$$

Again, given the prices r_2, p_1, p_2 and the trade-off ratios the implicit price of x_1 is given. It should be noted that the common characteristic in (3.27) and (3.28) is that $(dq_i/dq_k) = -(p_k/p_i)$. This condition must hold to use the factor-product trade-off ratios to discover the price (given the choice of the optimum product combination) of the factor.

IV. PRODUCTION AND CHOICE PROCESSES IN WATER RESOURCE DEVELOPMENT

The nature of the production and choice processes in water resource development must now be identified. The resource planner is faced with planning resource allocation and the determination of alternative product mixes that can either be accepted or rejected by the body politic. The framework within which the resource planner must work is generally a governmental agency that has been formed to accomplish improvements in human welfare. The procedure used by the planner within these planning agencies must be conditioned by the proper conceptual base. The purposes of this chapter are: 1) to identify the types of information the planner should provide, 2) to indicate the production processes conditioning the types of information the planner should provide to the decision making body, and 3) to indicate the choice process in water resource development as dictated by the production processes and the realities of production faced by a planning agency.

Planning Agencies and the Planning Problem

Government agencies involved in resource planning and development are, in essence, "firms". These firms are formed by public interest in providing some products not normally provided or not provided at all by private business firms. This is not to imply that private business firms do not or cannot provide some of the same products. Witness, for example, the provision of water for irrigation by many individual business (farm) firms as well as by large federal agencies such as the Bureau of Reclamation. Many local, state, and/or federal government bodies, however, are typically requested to provide many water related products. Part of the reason for this trend has been the sheer magnitude of some proposed projects. Many water development projects encompass various municipalities, counties, and even states. The felt need for public intervention in the provision of water related products became apparent at the turn of the century. The Irrigation and Reclamation Act of 1902, for example, was felt justified in part by President Theodore Roosevelt because the "...construction (of great storage works) has been conclusively shown to be an undertaking too vast for private effort" (Richardson, 1905, p. 433). This act set the stage for Federal government participation with local and state governments in land reclamation through water resource development. Similar acts of Congress resulted in government involvement in the production of most "water outputs".[34]

Several government agencies, then, are involved in planning and

[34] An excellant discussion of early legislation leading to present government involvement in water resource development is presented in U.S. Dept. of Ag. (1972). For a discussion of more recent legislation, especially planning legislation, see Lynne, Castle, and Gibbs (1973) or Howe (1971).

actually implementing the transformation of inputs into water outputs. Some theory of the firm probably could be derived to explain the actions of these public agencies. While most assuredly interesting, the discovery of the intricacies of such a theory of the firm would not be especially useful to the purposes of this study. The economic theory of the firm, however, can provide some guidelines to the analyst involved in exploring the problems faced by the water resource planner in the process of planning for the optimum product mix. The government agency, like the economic firm, does have a production function which must be utilized in the planning and optimization process. This concept is useful in the analysis of planning procedures. Government agencies as noted earlier, produce several products. The production process is best described by the multiple output production function.

The water resource planner must discover the production relations for the multiple products such as to discover the resource allocation pattern needed to best satisfy the goals of the firm. Supposedly, the goals are set by society through elected representatives in Congress and the Executive branches of government. These branches of government, in turn, set the goals to be attained by the government agencies. Whether this occurs in reality, of course, may be subject to debate.[35] Overall, then, the resource planner must plan the development of the water resource such as to improve conditions relating to human welfare. Plans for alternative developments are submitted to the body politic for review and acceptance or rejection. The nature of this process, in a rough sense, is akin to finding the equilibrium conditions as specified in the economic theory of the firm.

A very important problem faced by resource planners is one of valuation. Many of the outputs of a water resource development project do not have accurate, observable market prices (indeed, many products do not have markets). Water for recreation use, for example, is a product having no easily observable market price. A wild and scenic river or a unique wilderness area, while not necessarily a "water output", also will not normally have prices that are easily discovered. The planning problems are made even more complex as some of these products actually serve as inputs into the production of products such as water for recreation. The optimum allocation of "acres of wilderness area" as an input into the production of "water for recreation" is not easily determined. At best, a common

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[35] It may also be the case, of course, that agencies have goals independent of social goals. An agency created to provide a particular product may continue to provide the product long after society has changed objectives.

measure of value should be developed for all factors and products. Given that all quantities (and qualities) of factors and products cannot be assigned a dollar value, the possibility of assigning some "welfare" measure should not be overlooked.[36]

The water resource planner, as a result, is faced with a most difficult problem. Society, through elected representatives, requests the production of various water products. Each water product may have several attributes which gives rise to various commodities that individuals in society can "consume". Many of these products do not have dollar values which are most easily used in any economic optimization process. At the same time, the planner is asked to produce all products efficiently; i.e., to produce the desired (useful) products in a least-cost manner. This is to be accomplished even in cases where many of the factors used in production do not have observable dollar values. An alternative criteria for guiding the resource allocation and product mix determination process must be developed as long as there are incommensurable outputs and/or factors.

Several alternative ways of viewing this problem have already been discussed in this study.[37] The work of the Special Task Force of

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[36]Freeman speaks of a "welfare" value but gives no insights into how such a value could be determined (Freeman, 1969, p. 566).

[37]See Chapter II.

the Water Resources Council as presented in the "Principles and Standards for Planning Water and Related Land Resources" (WRC, 1973) addresses this problem. The central element of any new criteria for organizing production such as to achieve the desired product mix, however, must include a discussion of the basis for such criteria. This is nearly always lacking in the many proposals and test studies. The basis for any new criteria, in turn, must be found in the production conditions faced by the agency.

The problem faced by the resource planner is roughly summarized in Figure 18. Given some relation between the products of concern, such as that depicted in Figure 18, the decision must be made as to the proper mix of the two water outputs. Given market prices for W_1 and W_2, the optimum product mix is given by the tangency of the price line p_1/p_2 with the curve TT, assuming TT is a product-product relation. In this case the resource planner can determine the desired product mix. If prices are not given, the resource planner is faced with providing the curve TT to the decision making entity such that the desired point can be chosen.[38] Faced with curve TT, the decision making entity must choose some point reflecting the

[38] As noted by Castle, "Decision makers, within the institutional framework, are (then) relied upon to choose that combination of welfare components which, in their judgment, and in the light of political realities, is most appropriate" (Castle, "Economics and. . . ", 1973, p. 727).

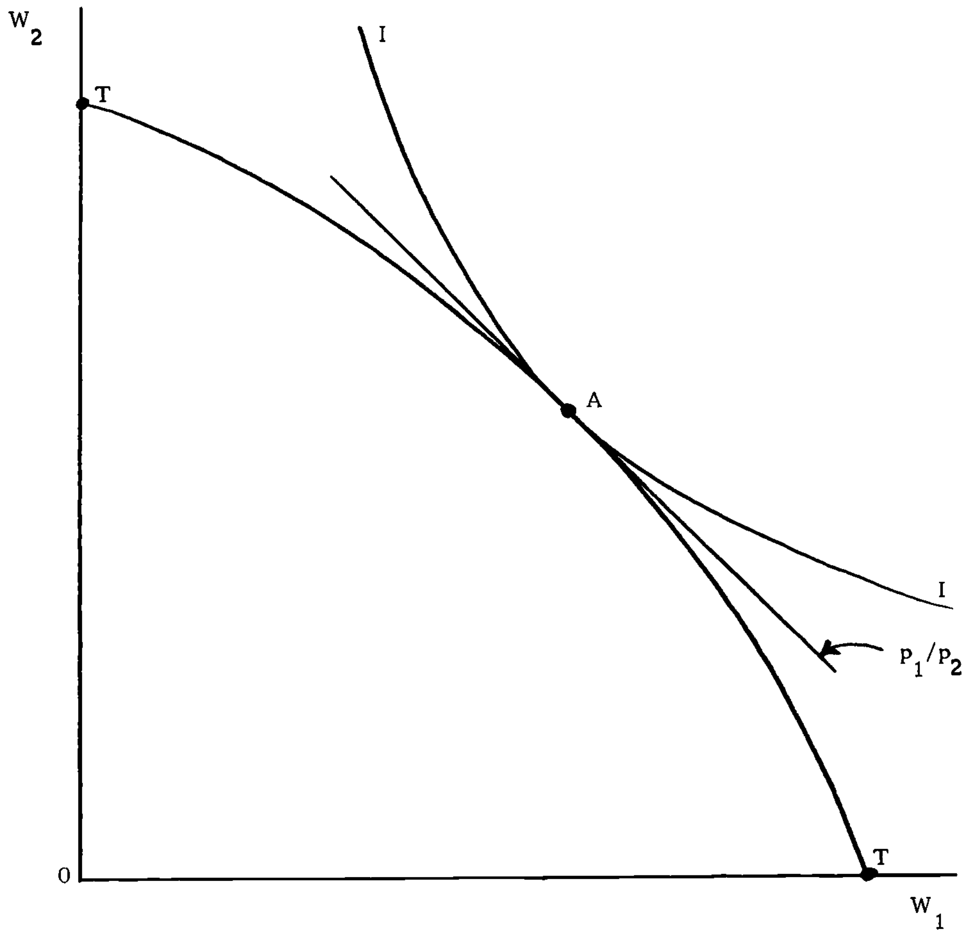

Figure 18. Iso-cost and community indifference curve, illustration of product-product trade-off ratio.

correct price ratio. The decision body must provide the "consumption" trade-off ratio or the slope of curve II in Figure 18. In the case of water resource development, Congress and the Executive branch are faced with choosing the correct point on TT given limited knowledge of social preferences regarding water products W_1 and W_2. In essence, the body politic is faced with determining the product price ratio which will maximize benefits to society. Assuming one of the prices is known at the point selected, the other price can be made explicit. Given p_1, for example, p_2 is made explicit by the selection of point A in Figure 18. A first step toward finding the desired solution, then, involves discovering the production processes.

Production Processes in Water Resource Development

The production processes in water resource development must be known by the water resource planner in order to achieve the optimum resource allocation and the optimum product mix. Production processes must be known no matter what criteria is used for solving the decision problem. A more fundamental problem, however, is defining the factors and products associated with water resource development.[39]

[39] Further references to water resource development in this study will pertain mainly to dam and reservoir configurations.

Factors and Products

The factors and products forming the basis for the determination of production relations are not easily identified. The problem of separating the various levels of production adds to the difficulty. Some factors in water resource development projects may be considered consumption items by some elements of society. Consider, for example, "five miles of scenic river". This "product" has various attributes that result in various commodities for final demand (consumption). This same "product" is essentially an input into the production of water for irrigation purposes. A viewpoint must be established. The viewpoint of the resource planner provides the guiding framework for discovering and classifying factors and products in this study.

Some insight into identifying products can be gained by looking at the purposes of a particular project development. A multiple purpose project designed to provide for flood water protection, land reclamation through irrigation, and electrical power generation has at least three discernible products. These products are water for irrigation (W_I), water for power generation (W_P), and reservoir storage and stream flow control of flood waters (F). The relation of purposes to products is not sufficient, however. The multiple purpose structure designed to provide W_I, W_P, and F may also supply water for recreation, water for fish production, water for municipal and industrial use, and improved drainage for surrounding lands. In fact, a project designed for only <u>one</u> "purpose" may provide several "products". A dam built primarily to generate electrical power (a single purpose project) may also provide flow augmentation for downstream fishery development and irrigation. In addition, the reservoir behind the dam may provide for water based recreation activity and waterfowl production. Regulated flows may also affect water quality. The purposes of a project, however, do serve as a starting point for identifying the products of water resource development.

The factors of water resource development are also somewhat "nebulus" and hard to identify. The general classification system inherent in the economic factors of production--land, labor, capital, and management--provides, however, a convenient starting point for identifying the factors in water resource development. The economic factor land encompasses all of the natural resources in the area of a proposed project that could be used (including the zero level) in the production process. The water available in the area, for example, is part of the land factor. A wilderness area identified by some measure such as acres of forest land still in a primitive condition would be a factor under the general category of land. Geological resources inundated by a reservoir could be considered part of the land factor. Similarly, the economic factor labor encompasses a large number of factors. All of the technical and non-technical, mental and physical, human input made into the planning and production processes would be encompassed in the labor factor. This factor includes all the human input from planners, construction workers, engineers, economists, and "ditch-diggers" involved in all phases of the project. The capital input encompasses all those man-made items used in planning and production. Items accountable under this factor include the heavy equipment used in construction, the engineer's hard hat, and the economist's pencil. The iron and steel and other construction materials would also fit the capital factor because of the shape and form in which they are used. A brick is a capital item while the clay used to make it is a natural resource.

The entrepreneurial factor is extremely complex in water resource development. This factor must reflect the policies of Congress and the Executive branch regarding resource allocation. This factor, essentially, organizes the production and makes the decisions. In a way, then, the bureacracy represented in the agency is part of this factor. Several "minor" decisions are certainly made at that level. In a larger sense, the elected representatives of society serve as the entreprenurial input. These elected representatives make decisions based on perception of public desires (and individual desires). In a still larger sense, the people of the Nation provide the entreprenurial input. This statement, of course, cannot be defended very easily.[40] Suffice it to say, decisions are made regarding the organization of production; these decisions are part of the entrepreneurial factor.[41]

Many specific products and factors can be identified. A most comprehensive list of elements, both products and factors, to be considered in water resource development was developed by the U.S. Water Resources Council (WRC, "Establishment of...", 1973). These elements are described as "beneficial and adverse effects". Several beneficial and adverse effects were identified for each of three objectives--national economic development, environmental quality, and regional economic development--in that document. There were also some elements identified relative to social well-being. The beneficial and adverse effects in that document correspond, in a general sense, to the products and factors of this study. Beneficial effects on the national economic development objective, for example, result from several outputs including water for irrigation and power generation

[40] There is considerable discussion regarding the sources of the public interest and how well the views of society are articulated in government planning and implemented through government action. See, for example, Steiner (1969, pp. 13-43).

[41] The entrepreneurial factor, of course, is very crucial. In fact, one of the main purposes of this study is to present a way of planning and analysis useful in the decision making process when incommensurable outputs are produced. In a sense, the determination of choice indicators is a part of the entrepreneurial factor; this is true to the extent that the public is involved in defining the planning procedures and making final decisions regarding project developments.

(products). Adverse effects are seen to include "...actual expenditures for construction; transfers from other projects, such as costs for reservoir storage; development costs; and interest during construction" (WRC, "Establishment of...", 1973, p. 24807). These are, essentially, factors. The list derived from the WRC (1973, pp. 24797-24870) is as follows:

1. elements affecting national economic development:
   a. water supply to agricultural (N_1), municipal and domestic (N_2), and industrial users (N_3),
   b. flood control (N_4), land stabilization (N_5), and drainage (N_6),
   c. water for power production (N_7)
   d. water for (navigation) transportation (N_8),
   e. water for recreation (N_9),
   f. water for commercial fish production (N_{10}),
   g. water and related land for commercial game production (N_{11}).
2. elements affecting environmental quality:
   a. open and green space (E_1),
   b. wild and scenic rivers (E_2),
   c. lakes (E_3),
   d. beaches and shores (E_4),
   e. mountains and wilderness areas (E_5),

f. estuaries (E_6),

g. other areas of natural beauty (E_7),

h. archeological resources (E_8),

i. historical resources (E_9),

j. biological resources (E_{10}),

k. geological resources (E_{11}),

l. ecological systems (E_{12}),

m. water quality (E_{13}),

n. air quality (E_{14}),

o. land quality (E_{15}),

p. freedom of choice (E_{16}), for future generations regarding resource use.

3. elements affecting regional economic development:

   a. regional income and employment (R_1),

   b. population distribution (R_2),

   c. regional economic base and stability (R_3),

   d. environmental conditions of regional concern (R_4).

4. elements affecting social well-being:

   a. income distribution to particular groups of people (S_1),

   b. security of life, health, and safety (S_2),

   c. educational, cultural, and recreational opportunities (S_3),

   d. emergency preparedness (S_4).

All of the elements listed under each of the objectives are to be considered in project evaluation under the proposed guidelines. All "beneficial and adverse effects" are to be noted for all plans (WRC, "Establishment of...", 1973, pp. 24796-24797). While useful in the accounting scheme proposed by the Water Resources Council a more useful classification system for analysis is to classify in terms of products and factors. The issues involved in finding the optimum product mix will then become more apparent.

The "translation" of beneficial and adverse effects into products and factors is not an easy process. This approach may, by necessity, have to be ad hoc as products of one project may be factors in another project. The components of the national economic development objective are most easily translated. The beneficial effects of an improved water supply, for example, can be included in the evaluation process as various water supply outputs. Water supplied for agricultural use, primarily irrigation (W_I), is essentially an intermediate product that has a price. The price of this product dictates how much is used by the agricultural sector.[42] The increase in income (after deducting associated costs) gives the "beneficial effect". The price of the product W_I, however, should influence the determination of the optimum product mix. Other products produced by the government

---

[42] This implicitly assumes, of course, that institutional constraints are not affecting the allocation of the water, which may not be the case in most situations.

agencies and affecting national economic development can be similarly derived. The water for municipal, domestic (N_2) and industrial uses (N_3) are very similar in nature to (product) N_1. The beneficial effect, however, must be determined in a different manner for N_2 as the water goes to final demand. The products related to the environmental quality objective are not as easily identified. Open and green space (E_1), for example, could be a product of the government water development agency. A situation can be envisioned where a dam and reservoir configuration would insure the preservation of some associated land resource in a particular use which keeps the land "open and green". In this case, E_1 is a product of the agency. In other cases, E_1 may be a factor. An open and green space may have to be inundated by the reservoir of water used to provide water products. A certain level of water quality (E_{13}) may be a product of the agency through flow augmentation and/or an intermediate product to the production of water for some use such as irrigation. Similar arguments can be found for the other elements, E_2-E_{12} and E_{14}-E_{16}. In general, however, the elements of concern with regard to effects on environmental quality are factors in the production of other "water outputs".

Many of the elements of the regional economic development objective also may be viewed as products or factors depending upon the particular situation. This objective is very closely related to the national economic development and the environmental quality objective. While some of the "beneficial and adverse effects" will be different, the classification scheme for products and factors remains essentially the same. This objective is concerned with the question of the distribution of effects.

The elements of the social well being objective may also be considered as products of a government agency. Provision of navigable waters (N_8) may contribute to the production of emergency preparedness (S_4). Improvements in water quality through changes in the water for quality (E_{13}) output may enable changes in health levels (S_2). In a sense, then, the government agency produces a particular health level. Most of the products related to the social well being objective use other products of the government agency as factors.

Delineation of the Planning Unit

A crucial element of the planning process involves the delineation of the area for study. It was argued earlier that production relations provide the basis for the planning process. The area delineated for study, in turn, determines the form of these production relations. The production relations for a country (region, state, river basin) are most probably different than the production relations in a particular segment of the country (sub-region, county, dam site). The relation describing the production of water for irrigation (W_I) in

Oregon may be considerably different than the production relation describing W_I production in the Willamette River Basin. Similarly, the production relations representative of the production of W_I for that Basin may be of an entirely different form from the relation describing the production of W_I by one farmer-irrigator in the Basin.

A criteria useful in the determination of the relevant boundaries of a study area results directly from the nature of iso-cost and/or transformation curves. All products that use the same factors, or group of factors, must be considered simultaneously in the optimization process. As a result, the optimum product mix in one group of products is dependent upon the product levels in other groups of products.[43] This concept can be illustrated by a simple example. Assume the relationships among water used for irrigation (W_I), power production (W_P), domestic (W_D), and municipal (W_M) purposes are represented by:

$$G(W_I, W_P; L_{IP}, N, C, M) = 0 \tag{4.1}$$

$$H(W_D, W_M; L_{DM}, N, C, M) = 0 \tag{4.2}$$

where,

N = labor,

C = capital,

M = management,

---

[43] A "group" of products defines the products of concern in the analysis that can be represented in a transformation surface.

L_{IP} = economic land (possibly water in a river) allocated to the production of W_I (L_I) plus the allocation to W_P (L_P); i.e., $L_{IP} = (L_I+L_P)$,

L_{DM} = economic land (again, possibly water) allocated to the production of W_D (L_D) plus the allocation to W_M (L_M); i.e., $L_{DM} = (L_D+L_M)$.

The factor L would be composed of $[L_{IP}+L_{DM}]$ or $[(L_I+L_P)+(L_D+L_M)] = L$. The transformation curves represented by the implicit functions G and H in equations (4.1) and (4.2) should be combined to form the one transformation function:[44]

$$F(W_I, W_P, W_D, W_M; L, N, C, M) = 0 \tag{4.3}$$

The transformation curve represented in equation (4.3) would require consideration during the optimization process of all the following relations:

$$\frac{dW_I}{dW_P}, \frac{dW_D}{dW_P}, \frac{dW_M}{dW_P}, \frac{dW_I}{dW_D}, \frac{dW_I}{dW_M}, \frac{dW_D}{dW_M}. \tag{4.4}$$

[44]The function F does not really have to be "formed". Given the L factor is used in both G and H, however, the product mix cannot be optimized in G (H) without consideration of the levels of production in H (G).

Use of equations (4.1) and (4.2) (treating the two groups of products as independent) would require only consideration of the relations:

$$\frac{dW_I}{dW_P}, \frac{dW_D}{dW_M}. \tag{4.5}$$

As a result, the true optimum would never be discovered (except by chance) using (4.1) and (4.2) when, in fact, the groups of products are non-independent.[45]

The relevance of non-independent transformation curves to the delineation of an area for study now becomes apparent. An incorrect product mix will most probably result if transformation curves which indeed are non-independent are treated as if they are independent. Non-independent transformation curves must be considered as one relation in order to achieve the optimum, in some sense, product mix. A river basin, for example, will be the relevant planning area if the transformation curve(s) representing production in that basin are independent of transformation curves in other basins or regions of the country. Further, a particular segment of a river basin can be isolated for planning purposes if it can be shown that the transformation

[45] This argument is consistent with a point made by Castle ("Criteria and. . .", 1961, p. 297), where it was stated ". . . (a) transformation curve for a particular natural resource is not an iso-resource curve for society". The correct unit for planning must be outlined.

curve(s) representing the products in that segment is independent of the transformation curves in other segments of the basin.[46] In fact, the determination of the optimum product mix is simplified greatly if transformation curves are independent as the number of relations (dq_i/dq_j) (among products) is reduced significantly. In the above described case, as an example, there were only two relations of concern when (4.1) and (4.2) were used (as shown in 4.5) while there were six relations when (4.3) was used (as shown in 4.4). The non-independent transformation curves must be used in the optimization process, however, if one or more of the factors are used by the products.

Transformation curves also may require slightly different interpretations dependent upon the type of underlying production relations. In fact, there is no transformation (iso-factor, iso-resource) curve in the case of factorially determined production.[47] There is an iso-cost curve, however, for all classes of production processes except when joint products exist. The characteristics along that curve

---

[46] The possibility exists, of course, that water products cannot be separated from the analysis of other investments in production. For example, it is highly possible that government investment in health care should be examined under the same iso-cost relationship as investment in water resources.

[47] See Chapter III, p. 51 and p. 53.

are somewhat different dependent upon the type of production process.[48]

Iso-Cost and Transformation Curves

A more useful curve for the analysis of optimization problems involving independent transformation curves is the very closely related iso-cost curve. The iso-cost curve is a parallel construction to the transformation (iso-factor) curve and either curve can be used in the analysis.[49] Also, the iso-cost curve can be used when two or more factors are involved in the production process. In most "real world" operations there are, of course, usually two or more factors. This is most certainly the case in water resource development.

The iso-cost curve is essentially the opportunity cost curve. This relation shows costs in terms of the amount of a product sacrificed to gain greater amounts of the desired product. In essence, one product serves as an input to the production of the other product. Consider, for example, the iso-cost curve illustrated in Figure 19. The relationship between water for recreation (W_R) and water for irrigation (W_I) is such that initial increases in W_R also results in an increase in W_I up to the point labeled B. From B to D, W_I drops

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[48] See Appendix A.

[49] See Appendix A.

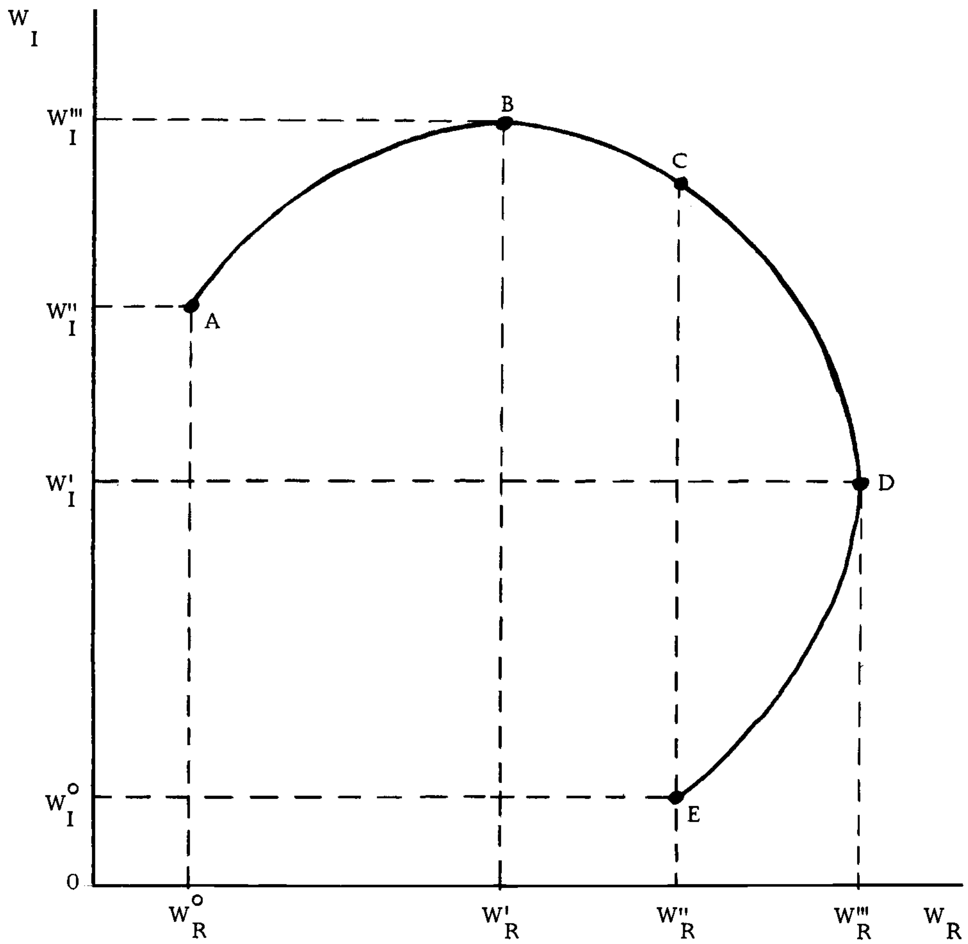

Figure 19. Iso-cost relation in product space between water for irrigation (W_I) and water for recreation (W_R).

for further increases in W_R. Beyond D, both products decline. The change in W_I for a change in W_R represents the cost (opportunity cost) of producing more W_R, measured in terms of W_I foregone. Knowledge of the direct cost of providing the quantities of W_R and W_I along the iso-cost curve can then be used in conjunction with the opportunity costs resulting from movements along the curve to arrive at a decision regarding the optimum product mix. An understanding of what the iso-cost curve represents, however, requires knowledge of the underlying production function relations.

The general form of the iso-cost curve represented in Figure 19 can result from either factorially determined or assorted production. The underlying production relations can, of course, only be determined by empirical estimation. The characteristics of the various types of production can be identified, however. Given the general shape of the relation in Figure 19 the underlying production relations are best classified as:

1. factorially determined production, if the allocation of each factor among the water products under consideration cannot be determined,
2. assorted production, if the allocation among the water products can be determined.

It is expected that both types of production (and, possibly, various combinations) occur in water resources development dependent upon the products and factors under consideration. Some simplified production situations can be used to illustrate some of the possible relations.

Assume a potential dam and reservoir site can be isolated in a river basin based on the criteria for independent iso-cost curves. Assume further that all factors of production can be represented in the economic factors capital (K) and land (L). To simplify the problem even further, assume the capital factor is composed only of "height of dam" and the land factor represents only the land under the dam, reservoir, and associated structures. Assume further that resource prices are constant. Further, assume only two products can be produced, namely W_R and W_I. Also, each product can be produced only during a particular time period. This hypothetical iso-cost relationship between W_R and W_I is represented in Figure 19. If the relationship in Figure 19 represents an iso-cost curve from assorted production, it will be possible to determine the allocation of each factor between the two products; i.e., it will be possible to find the equations:

$$
\begin{aligned}
L &= L_{WR} + L_{WI} \\
K &= K_{WR} + K_{WI}
\end{aligned}
\qquad (4.6)
$$

where,

L_{Wi} = land (acres of land) allocation to water product W_i,

(i = R, I)

K_{Wi} = capital (height of dam) allocated to water product W_i,

(i = R, I).

Further, it must be possible to find the relations:[50]

$$\begin{aligned} W_I &= f(L_{WI}, K_{WI}) \\ W_R &= g(L_{WR}, K_{WR}). \end{aligned} \tag{4.7}$$

Assorted production for W_R and W_I production exists, then, only if the equations (or some related form) in (4.7) exist given the present level of knowledge concerning the production relations. It must be possible to find the technological, physical phenomenon that allocates "height of dam" between W_R and W_I. Also, it must be possible to separate the land under the structures between the products based on technological, physical phenomenon.

Given the process represented in (4.7) where W_R and W_I are assorted, certain characteristics of the iso-cost curve can be isolated.

---

[50]They may also be of the form:

$$\begin{aligned} W_I &= f(L_{WI}, K_{WR}, W_R) \\ W_R &= g(L_{WR}, K_{WR}, W_I). \end{aligned}$$

This form, however, merely complicates the estimation procedure. This, again, is an empirical question.

The region of the iso-cost curve in Figure 19 from point A to B results from one of the marginal products of one of the factors becoming negative. The same is true in the part of the iso-cost curve from D to E. The implication is that a large amount of one resource was allocated to one of the products in those areas of the curve.[51] In region AB (DE), for example, one factor was allocated to the production of W_I (W_R) at such a level as to cause reductions in the total amount of W_I(W_R) produced. The marginal products of both factors for both products are positive in region BD of the iso-cost curve (Figure 19).

Another characteristic of the iso-cost curve for this simplified example relates to the changes in the total amounts of the resources used. Movements along the iso-cost curve derived from the relations in (4.7) necessitates changes in the total amounts of each factor used when prices are constant. The exact changes are given by the relation, for fixed (iso)-cost C^o:

$$C^o = r_1 L + r_2 K$$
$$K = \frac{C^o - r_1 L}{r_2} \quad . \tag{4.8}$$

---

[51] The amount allocated was great enough such as to cause a reduction in the amount of the product produced. Diminishing returns to each factor is implicitly assumed.

Capital (K) changes at the rate $-r_1/r_2$ as labor (L) changes. Also, the allocations are changing along the iso-cost curve such that K_{WR}, K_{WI}, L_{WR} and L_{WI} are different at each point along the curve. All of these allocations will be known if assorted production describes the underlying production relations.

It can be argued that it is impossible to determine K_{WR}, K_{WI}, L_{WR}, and L_{WI} for the simplified example defined. There is no way of determining the allocation of "height of dam" between W_R and W_I. Similarly, it is not possible to determine the allocation of "acres of land" between the products. If the empirical test validated this argument, the description of the production process is not given by the concept of assorted production.

An iso-cost curve derived from factorially determined production has quite different characteristics than an iso-cost curve derived from assorted production. It is impossible to determine the allocation of each factor among the products in factorially determined production. The level of knowledge about the production process, of course, determines whether it is impossible or not.[52] The above simplified case

[52] There is a difference between the lack of <u>knowledge</u> and the lack of <u>information</u> regarding a production process. Advances in knowledge may change the production process from factorially determined to assorted production. A lack of information regarding the allocation of each factor among products does not justify classification of a process as factorially determined. The factorially determined classification arises from a lack of <u>knowledge</u> regarding the allocation of each factor among the products; i.e., it is a physical phenomenon fixed by the current, prevailing technology.

can now be described on the basis of the factorially determined classification process. If this is the correct classification, the following production function relations would exist:

$$\begin{aligned} W_I &= f(L, K) \\ W_R &= g(L, K)\,. \end{aligned} \qquad (4.9)$$

Factors L and K are defined as before. Assume the relationship between W_I and W_R in (4.9) is given by the iso-cost curve in Figure 19. Movement along the iso-cost curve still represents a change in the total quantities of each factor used. The allocation at each point, however, cannot be determined based on any physical (or economic) phenomenon.

The segments AD and EF in Figure 19 now have completely different meanings. The marginal products of all factors for all products can be positive in these regions in contrast to assorted production. The positive sloped regions AD and EF merely represent different proportions of the factorially determined products.[53] The correct determination of the relation between W_R and W_I at this example site must, of course, be based on empirical determination of the factor allocations. It is entirely possible the production relation

[53] It should be noted the products W_R and W_I are assumed separable (not joint, $\kappa = 0$). The iso-cost "curve" for joint products is one point in the product diagram. See Appendix A.

describing W_R and W_I production processes may vary from site to site and project to project.

The general approach outlined above for determining the form of production processes can also be applied to a larger system of structures. The delineation of groups of water resource products may result in an independent transformation curve being defined over several possible dam sites and reservoir configurations. Again, the ability to determine the allocation of each factor among products enables the type of production to be identified. Some simplified examples can be presented to illustrate the procedure.

Consider a case where three dam sites can be used to produce W_R, W_I, and water for power production (W_P). The three sites are assumed to be located on the river system as represented in Figure 20. Assume that one water product is produced at each site. The factors of production are again assumed to be acres of land (L) covered by the reservoir and some measure of the structures, such as height of dam, represented by capital (K). The allocation of each factor among the products must be determined. In this highly simplified example, it would be possible to find the relations:

$$\begin{aligned} L_R + L_I + L_P &= L, \\ K_R + K_I + K_P &= K \end{aligned} \tag{4.10}$$

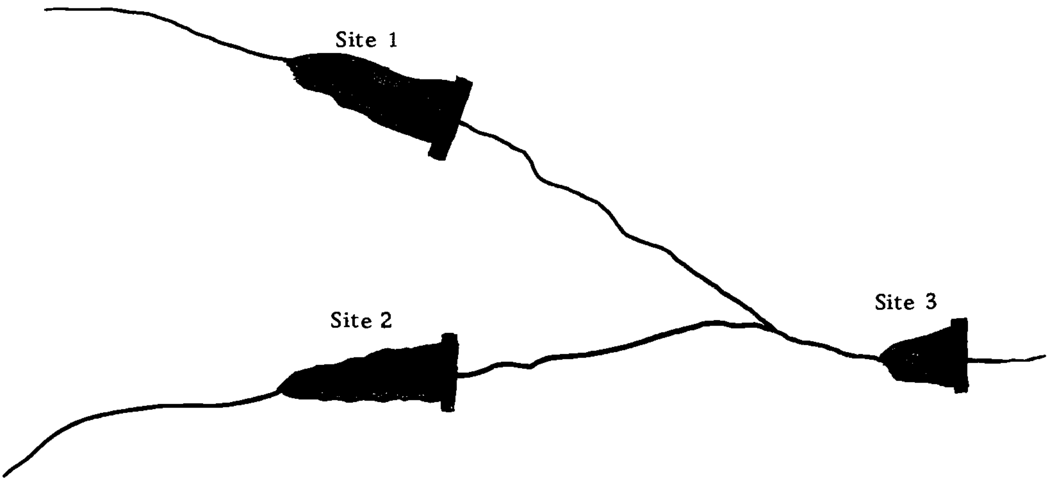

Figure 20. Diagrammatic representation of reservoir site locations, hypothetical river basin.

where,

L_i = amount of land allocated to product W_i $(i = R, I, P)$,

K_i = amount of capital allocated to product W_i $(i = R, I, P)$.

These relations, in turn, imply that some functions exist of the form:[54]

$$
\begin{aligned}
W_I &= f(L_I, K_I) \\
W_R &= g(L_R, K_R) \\
W_P &= h(L_P, K_P) \ .
\end{aligned}
\tag{4.11}
$$

Given the production relations in (4.11), the iso-factor and iso-cost relations can be derived to give:

$$F(W_I, W_R, W_P, L, K) = 0$$

or,

$$G(W_I, W_R, W_P, C) = 0 \ . \tag{4.12}$$

The resulting classification would be assorted production with all the characteristics described earlier regarding the iso-cost curve. Again, of course, the classification must be based on an empirical test. For this simplified example, however, the assorted production

---

[54]The forms may also be represented by:

$$
\begin{aligned}
W_I &= f(L_I, K_I, W_P) \ , \\
W_R &= g(L_R, K_R, W_P) \ , \\
W_P &= h(L_P, K_P) \ ,
\end{aligned}
$$

or some other variation.

classification can be assumed correct.

A slightly more complicated case is given by a slight modification of the previous case. Assume all three uses are allowed at each of the sites in Figure 20. Again, only two factors, L and K, are assumed. The classification of the production relation for this project is again ascertainable only after determining if the allocations of each factor can be isolated. If it is possible to determine such allocations, the relations would be represented by:

$$
\begin{aligned}
L &= L_1 + L_2 + L_3 \\
L_1 &= L_{1R} + L_{1I} + L_{1P} \\
L_2 &= L_{2R} + L_{2I} + L_{2P} \\
L_3 &= L_{3R} + L_{3I} + L_{3P} \\
K &= K_1 + K_2 + K_3 \\
K_1 &= K_{1R} + K_{1I} + K_{1P} \\
K_2 &= K_{2R} + K_{2I} + K_{2P} \\
K_3 &= K_{3R} + K_{3I} + K_{3P}
\end{aligned}
\qquad (4.13)
$$

where,

L_i = land used at site i (i = 1, 2, 3),

K_i = capital used at site i (i = 1, 2, 3),

L_{ij} = land used at site i for production of product W_j (j = R, I, P),

K_{ij} = capital used at site i for production of product W_j (j = R, I, P).

The production relation would be classified as assorted if all the allocations in (4.13) can be determined. The possibility that L_i and K_i could be determined while L_{ij} and K_{ij} could not is a very distinct possibility, however. If that did occur, the production process would have to be classified as factorially determined. Again, the exact description of the production process is necessarily based on empirical estimation of the actual production relations.

The exact changes in resource allocation for movements along an iso-cost (opportunity cost, transformation) curve are impossible to determine without knowledge of the underlying production processes. Movement along the iso-cost curve does, however, represent the opportunity cost of increasing the level of one water resource product (in terms of losses in the other product) no matter what the underlying production relations.[55] The types of technical and intermediate product relations in turn, affect the location and shape of iso-cost curves descriptive of production processes in water resource development.

Technical Relations Among Water Products.[56] The existence of alternative types of relations among water resource products leads to

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[55] This is a very crucial point. The iso-cost curve is the most important relation no matter how it arises. The conditions along the curve are different, of course, dependent upon the underlying relations.

[56] See Appendix B for a more thorough discussion of technical relations.

concern for defining the various forms of interdependent and independent relations. Classifying water resource products as "complementary" or "competitive" may be misleading unless these terms are identified concisely. Several intuitive definitions of complementary and competitive products can be found in the water resource literature. Some more exact definitions can be given.

Consider the iso-cost relationship between W_I and W_R illustrated in Figure 21. In sections AB and EF of the curve, the products are complementary in an intuitive sense; i.e., both products can be increased simultaneously. In sections BC and DE, one product can be increased without affecting the level of the other product. In CD, one of the water resource products can be increased only if the other product is reduced. Section CD of the curve in Figure 21 is consistent with the notion of "conflicting" products in the WRC planning document (WRC, "Establishment of...", 1973, p. 24829). All other sections of the curve, in turn, would be consistent with the notion of complementary products in the same document. As is shown in Appendix B, however, areas AB and EF of the curve are better described as irrational areas of production. The water products are complementary in that region of the curve only in the sense that movement out of an uneconomic region of production gives rise to more of both water products. Production should take place in area CD of the curve, however, if both water products do have positive prices.

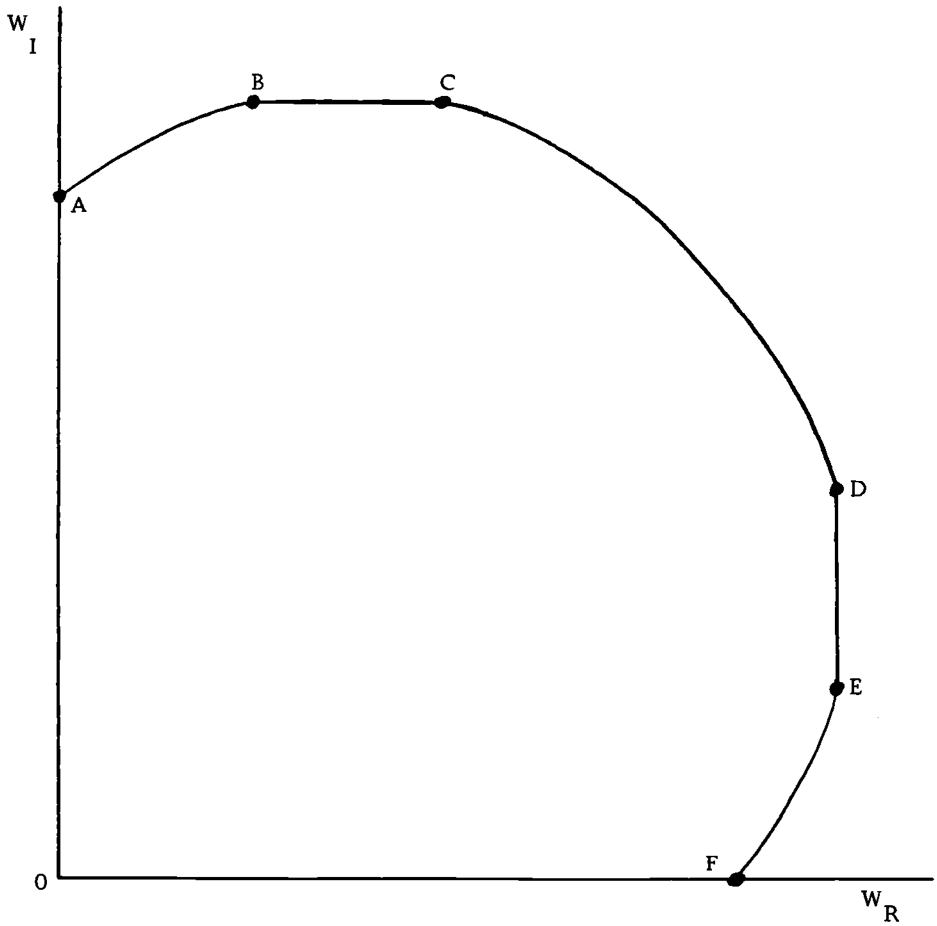

Figure 21. Iso-cost relation illustrating positive, zero, and negative slopes.

This notion of complementary-competitive products is completely different than the concepts of technically complementary, competitive, or independent products. Water products that are technically interdependent (complementary or competitive) or independent may also be represented in a figure such as Figure 21. The slope of the iso-cost curve does not have any relationship to the concepts of technical interdependence or independence. Technical interdependence or independence is related to the response of the marginal cost for one product due to a change in level of another product.[57] Really, then, this concept of complementary, competitive, and supplementary water resource products deals with movements among iso-cost curves. Products W_I and W_R would be considered complementary if for given positive changes in W_I and C the positive change in W_R became larger as more total cost was added. Viewed in another manner, technical interdependence or independence affects the response of the water products to investment. A greater amount of both W_I and W_R can be produced from a given investment if these water products are technically complementary. The least amount of both products would result from the same investment if these water products were technically competitive.

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[57] Mathematically this is represented by $(\partial^2 C/\partial q_1 \partial q_2)$. See Chapter III, pp. 67-68 and Appendix B.

Intermediate Product Relations Among Water Products. Some water resource products may also serve as intermediate products in the production of other water resource products, which has an effect on the location of the iso-cost curve in the product diagram. Water quality improvements (W_Q), for example, may be considered an intermediate product in the production of water for irrigation (W_I) in some instances. Ignoring that relation could lead to viewing the incorrect iso-cost curve. Consider, for example, the equation set:

$$\begin{aligned} W_I &= f(x_{1I}, x_{2I}, W_Q) \\ W_Q &= g(x_{1Q}, x_{2Q}) \end{aligned} \tag{4.14}$$

where,

W_I = water for irrigation,

W_Q = water for quality control,

x_{iI} = amount of resource i allocated to W_I $(i = 1, 2)$,

x_{iQ} = amount of resource i allocated to W_Q .

It is assumed that both W_I and W_Q are in demand. Product W_Q, then, can go to some other use (or user) as well as be used for the production of W_I. The iso-cost curve derived without consideration of the effect of intermediate products is represented by ADMHB in Figure 22. The relevant curve for analysis, however, is represented by curve CGB as derived from (4.14). Only 0C of W_I can be

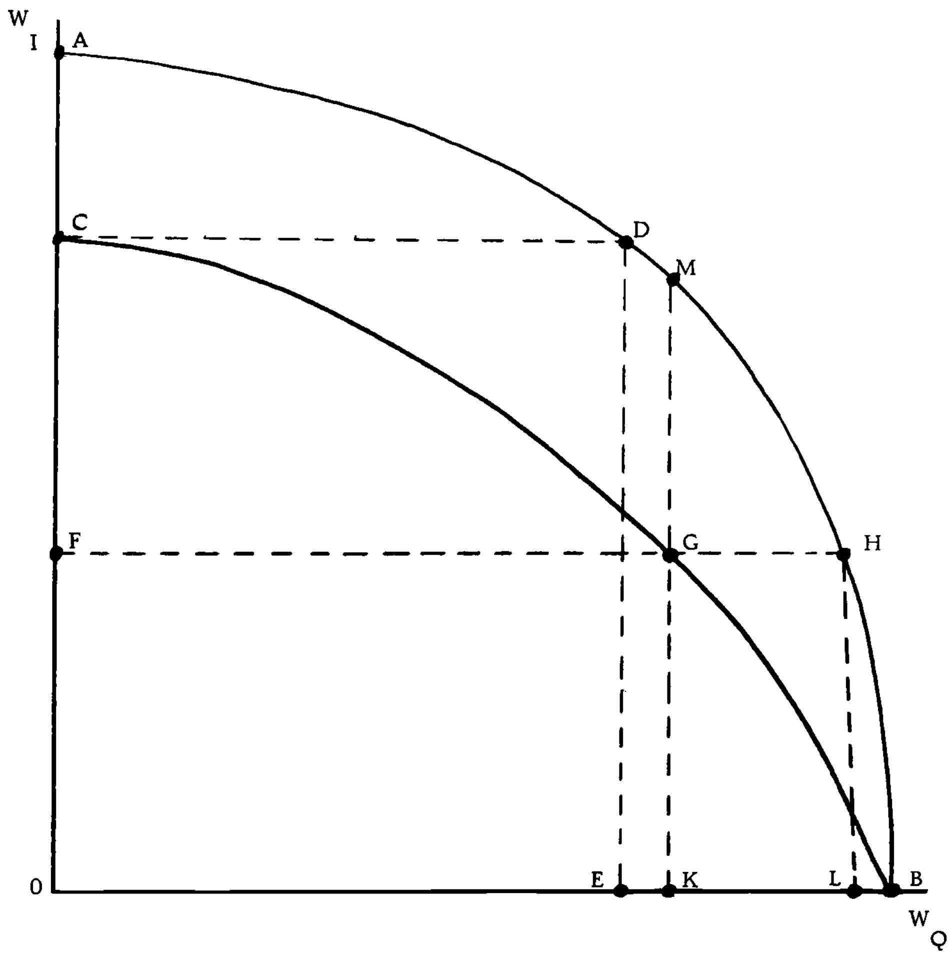

Figure 22. Transformation curves between water for irrigation (W_I) and water for quality control (W_Q), W_Q as an intermediate product and final product.

provided as some of the resources are now used in the production of 0E = CD of W_Q (in order to produce W_I at the level 0C). Viewed in another manner, W_Q for <u>other</u> uses (or users) is now at the zero level when 0C of W_I is produced. Similarly, at level F of W_I, (L-K) = (H-G) of W_Q is used in W_I production. In addition, W_I has been reduced by MG to GK (for the same W_Q level) because some resources are now devoted to W_Q production.

The possibility that W_Q serves only as an intermediate product must also be considered. Consider, for example, a case where W_Q is used in the production of W_I and W_D. The incorrect curve representing the relationship between W_I and W_D ignoring W_Q is represented by AB in Figure 23. Consideration of W_Q as an input, however, gives iso-cost curve CD. Curve CD is internal to AB as some resources must be devoted to W_Q production in order to produce W_D and/or W_I. The relationship given in CB is summarized by the set of equations:

$$\begin{aligned} W_I &= f(x_{1I}, x_{2I}, W_Q) \\ W_D &= g(x_{1D}, x_{2D}, W_Q) \qquad (4.15) \\ W_Q &= h(x_{1Q}, x_{2Q}) \end{aligned}$$

where,

x_{iI} = resource i allocated to W_I,

x_{iD} = resource i allocated to W_D,

x_{iQ} = resource i allocated to W_Q (i = 1, 2).

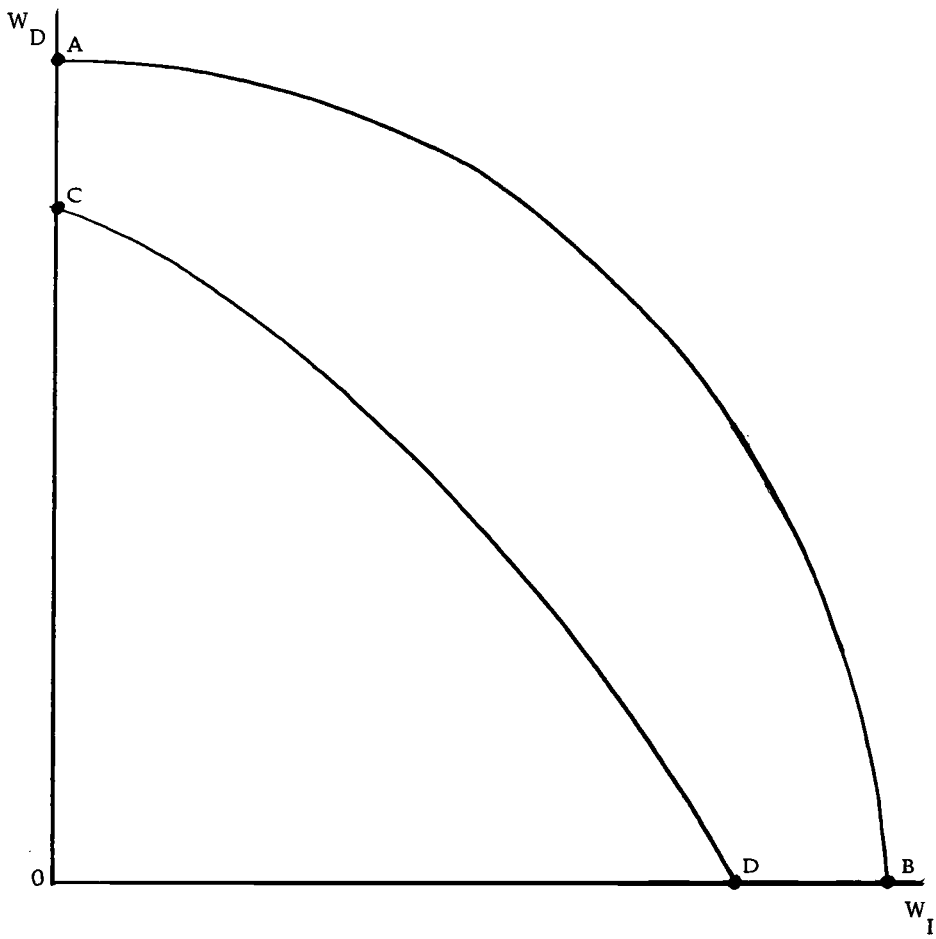

Figure 23. Transformation relations between water for domestic use (W_D) and water for irrigation (W_I), both products serve as intermediate and final products.

The products are defined as before. Curve AB is represented by the relations in (4.15) ignoring W_Q. Iso-cost curve AB, of course, is irrelevant given that W_Q is an intermediate product in the production of W_I and W_D as represented in (4.15). The determination of the correct relationship among W_Q, W_I, and W_D, of course, is necessarily an empirical determination. It is expected, however, that W_Q is an intermediate product in most water resource systems.

The identification of several water resource products which may in fact serve as intermediate products (inputs) into the production of other water resource products necessitates an understanding by the resource planner of the various levels of production. Concrete, water, and engineer hard hats, without question, serve as factors into the production of all water resource outputs. The relationships may not, however, be as clearly defined at other levels. Water for irrigation (W_I) becomes an intermediate product in the production of agricultural crops. Industrial water serves as an intermediate product in the production of various manufactured goods. Water also serves as an intermediate product in the production of electricity. All of these and many others, however, are at a different level of production. It would make little sense, for example, to examine an iso-cost relation in the product diagram between water for irrigation and agricultural crops produced. These two products are on different levels in the production process. The relevant model for analysis in this case would be a factor-product production model rather than the iso-cost (product-product) model. Only products at the same level can be compared correctly along an iso-cost (opportunity cost, transformation) curve.

Choice Processes in Water Resources Development

The choice process is necessarily complex because of the many levels of production, the myriad of products and factors, the difficulty of identifying products and factors, and the lack of market forces to allocate resources toward providing an "optimum" product mix. The very well-known and often quoted phrase "...the benefits to whomsoever they may accrue are in excess of the estimated costs..." (U.S. Congress, "Flood Control...", 1936, p. 1570) provides an all encompassing view of the choice criteria in water resource development. This concept, however, says little about the conceptual base or the procedures needed to discover the desired product mix given a lack of some product and/or factor prices. The basic problem is the lack of market forces that give prices for many of the products and factors.[58]

[58] The previous list of products and factors (pp. 106-107, this chapter) gives some indication of the magnitude of the problem. Items E_1-E_{16}, $R_2, R_3, R_4, S_1, S_2, S_3$, and S_4 do not have easily discernible prices. All of these elements, however, must be considered by the water resource planner in the consideration of development.

Optimization Among Factors

The optimum combination of factors in water resource development is assumed to be the least-cost combination. Given assorted production, this is accomplished when the following equation is satisfied for every pair of factors:

$$\frac{dx_i}{dx_j} = -\frac{r_j}{r_i}\,, \quad (i, j = 1, 2, \ldots, n;\ i \neq j)\,. \tag{4.16}$$

In the case of factorially determined production, the least-cost combinations are defined by:

$$\frac{\sum_i p_i \frac{\partial q_i}{\partial x_j}}{\sum_i p_i \frac{\partial q_i}{\partial x_k}} = \frac{r_j}{r_k}\,, \quad \begin{matrix}(i = 1, 2, \ldots, m; \\ j, k = 1, 2, \ldots, n)\end{matrix} \tag{4.17}$$

for every pair of factors. Resource planners, of course, most likely do <u>not</u> know the exact value of the ratios in (4.16) and/or (4.17) such as to know if the least-cost combination of resources has been achieved. Given all the factor prices, however, it is expected that resource planners will at least approximate the least-cost solution.

Problems in finding the least-cost solution arise when some of the factor prices are not known. Miles of wild and scenic river, for example, may be a factor in the production of water for irrigation (W_I)

and power production (W_P). Given there is no known price for the river per mile, it becomes nearly impossible to find the optimum combination of resources. Assume, for example, the factors required to provide W_I and W_P can be identified by capital (K), labor (N), and miles of scenic river (L). Given this simplified situation, the following conditions must hold in order to achieve the least-cost combination of factors, namely:

$$\frac{\partial K}{\partial L} = -\frac{r_L}{r_K}, \quad \frac{\partial N}{\partial L} = -\frac{r_L}{r_N}, \quad \frac{\partial K}{\partial N} = -\frac{r_N}{r_K}. \tag{4.18}$$

Given the prices for K (r_K) and N (r_N), the least cost position still cannot be found because of the lack of knowledge regarding r_L. It can be said, of course, that the following relation must hold:

$$-r_K \frac{\partial K}{\partial L} = -r_N \frac{\partial N}{\partial L} = r_L . \tag{4.19}$$

This may help in achieving a near optimum position. However, the level of L still cannot be determined. Some further choice criteria must still be specified. One approach to resource evaluation is to allow the decision making body to make a choice of the product mix. As a result of that choice, there is an implicit price for the factors used in the process.

Optimization Among Products

The resource planner must define the product-product relations in order that the optimum product mix can be selected. Given all the product prices, the optimum product mix can be determined by the resource planner (assuming a goal of maximum profits or net benefits). The optimum mix is then given by:

$$\frac{dq_i}{dq_j} = -\frac{p_j}{p_i} \quad (i, j = 1, 2, \ldots, m;\ i \neq j). \qquad (4.20)$$

The optimum level of each product is determined by solving for every pairwise product combination in (4.20). The resource planner may not know all these ratios. If prices are known and there is a goal of maximizing net benefits, however, these optimum conditions will at least be roughly met by the resource planners. Given a lack of some product prices, the resource planner is forced to define the iso-cost or transformation curves from which the (dq_i/dq_j) ratios can be calculated. These are the trade-off ratios that must be presented to the decision making body to facilitate determination of the desired product mix.

Implicit Prices and the Product-Product Trade-Off Ratio: Two Product Case. Given the planning unit has been formulated properly, the resource planner must then discover the iso-cost surface in order to calculate the necessary product-product trade-off ratios. These trade-off ratios, in turn, are affected by the technical and intermediate product relations encountered in the analysis.

Effects of Technical Interdependence and Independence. Distortion in resource allocation among water resource products can occur given the correct technical interdependence or independence relations are not known. Consider, for example, the technical relations as represented in Figure 24 for the water products W_I and W_R. Assume the correct price ratio, as derived from the decision making body, is given by the (equal) slopes of each curve at points B, D, and E. The water products W_I and W_R are considered technically complementary, technically independent, and technically competitive along curves ABC, ADC, and AEC, respectively. Assume the resource planner (incorrectly) provides curve AEC to the decision making body. Given that relation, point E will be selected with the expectation that 0N of W_R and 0Q of W_I will actually be produced. This will not occur, however, if the true relationship is either technically independent (curve ADC) or technically complementary (curve ABC). If product W_I is provided at level Q, the resulting level of W_R may be as high as that represented at points K or L. On the other hand, if 0N of W_R is provided, W_I may be produced at the levels represented at G or F. The greatest distortion will occur, of course, if the assumed (incorrect) relation is curve AEC when in fact the water

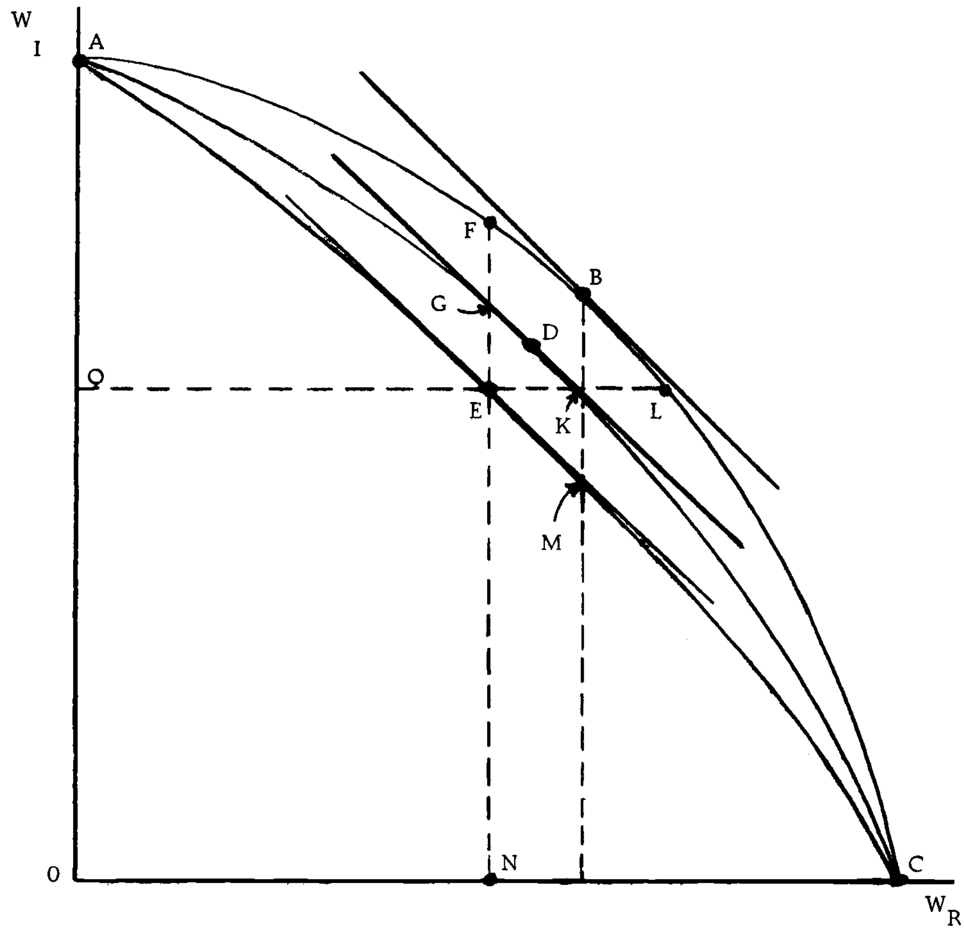

Figure 24. Transformation curves between water for irrigation (W_I) and water for recreation (W_R) under alternative assumptions regarding technical relations, all three cases.

products are technically complementary as represented in curve ABC. The same is true in reverse; i.e., if the assumed (incorrect) relationship is ABC, selection of point B may result in only the level of W_I represented at point M actually being produced rather than the level of W_I at point B.

Effects of Intermediate Product Relations. Ignoring intermediate product relations can also lead to the calculation of incorrect product-product trade-offs. The opportunity costs of producing some product may be distorted unless the correct iso-cost curve is discovered. Consider, for example, the case presented earlier in Figure 22 and reproduced in Figure 25. Product W_Q is assumed both an intermediate and a final product. If water for quality improvements (W_Q) was considered (incorrectly) only as a final product in the analysis, curve ABC would be presented to the decision making body. Choice of point B by the decision making body would give the implicit price of W_Q at $P_{WQ} = P^o_{WQ}$ (assuming P^o_{WI} is known and constant). Assume the goal is to produce 0G of W_Q for other uses. As a result only 0K of W_I actually results. The resulting implicit price for W_Q is $P'_{WQ} > P^o_{WQ}$ at point F (Figure 25). In fact, if the correct iso-cost curve relationship DEFC had been presented to the decision making body, only 0G' of W_Q for other uses would have been required. The price of W_Q is assumed equal at points E and B for constant P_{WI}. The W_I level would have been 0K' instead of the

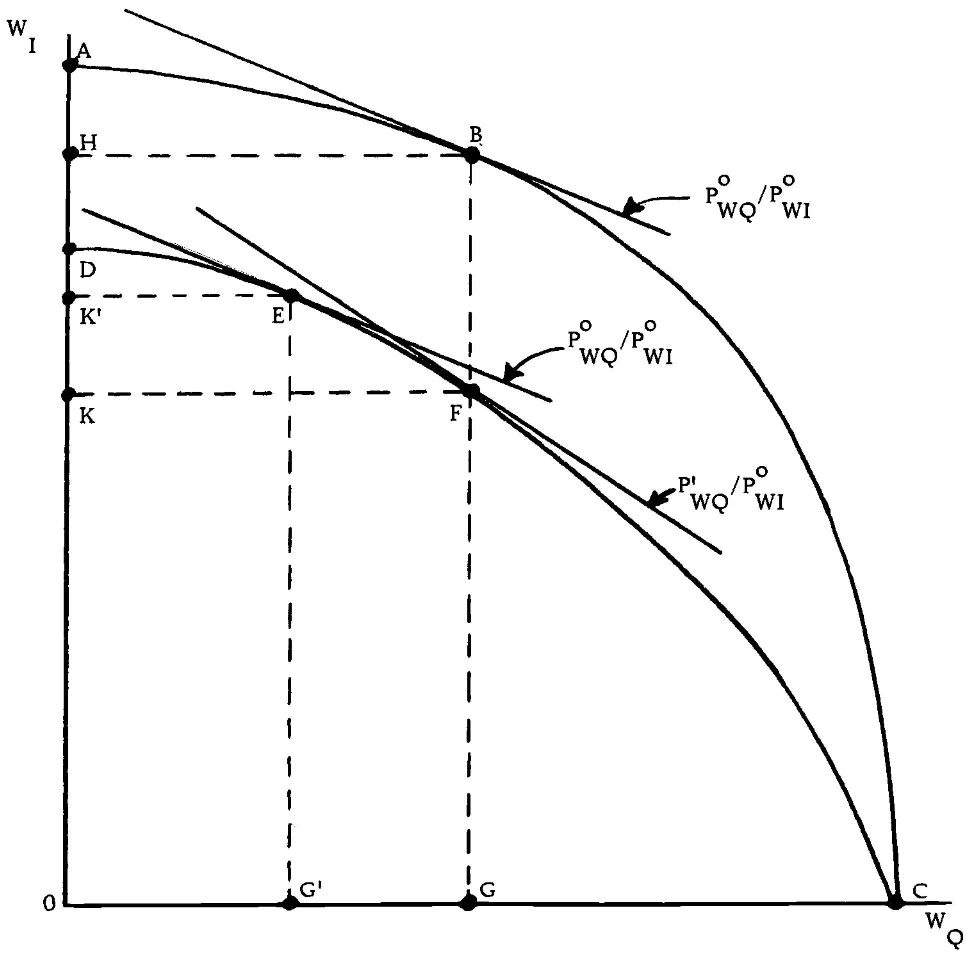

Figure 25. Transformation curves for water quality (W_Q) as an intermediate product in the production of water for irrigation (W_I).

resulting 0K level caused by the presentation of the incorrect curve. The optimum product mix would not have been chosen given curve ABC.

Another possible source of distortion in resource allocation could be caused by ignoring the intermediate product relationship depicted in Figure 23 and reproduced in Figure 26. The incorrect curve is represented by ABC as water for quality improvements (W_Q), the intermediate product, is ignored. Assume point B is chosen by the decision making body, which implies a price of $P_{WI} = P^o_{WI}$ (assuming $P_{WD} = P^o_{WD}$ is known and constant). Requiring 0N of W_I to be produced results in only 0L of W_D provided. The resulting implicit price for W_I is $P_{WI} = P'_{WI} > P^o_{WI}$ at point F. Given the correct curve (DEFG) initially, the decision making body would have selected point E (rather than B) as $P_{WI} = P^o_{WI}$. The resulting product mix would have then been represented by 0M of W_I and 0K of W_D.

<u>Implicit Prices and the Net Benefit Curve: Two Product Case.</u> The analyst must also be extremely careful in developing net benefit curves from iso-cost relations in the product diagram for presentation to the body politic for choice. One approach is to calculate the net benefits from the production of one of the products. Consider, for example, a net benefit curve generated from an iso-cost curve such as AE in Figure 19. Product W_I is assumed to have a known, constant price, P_{WI}, with P_{WR} not known. The net benefit function is given by the relation:

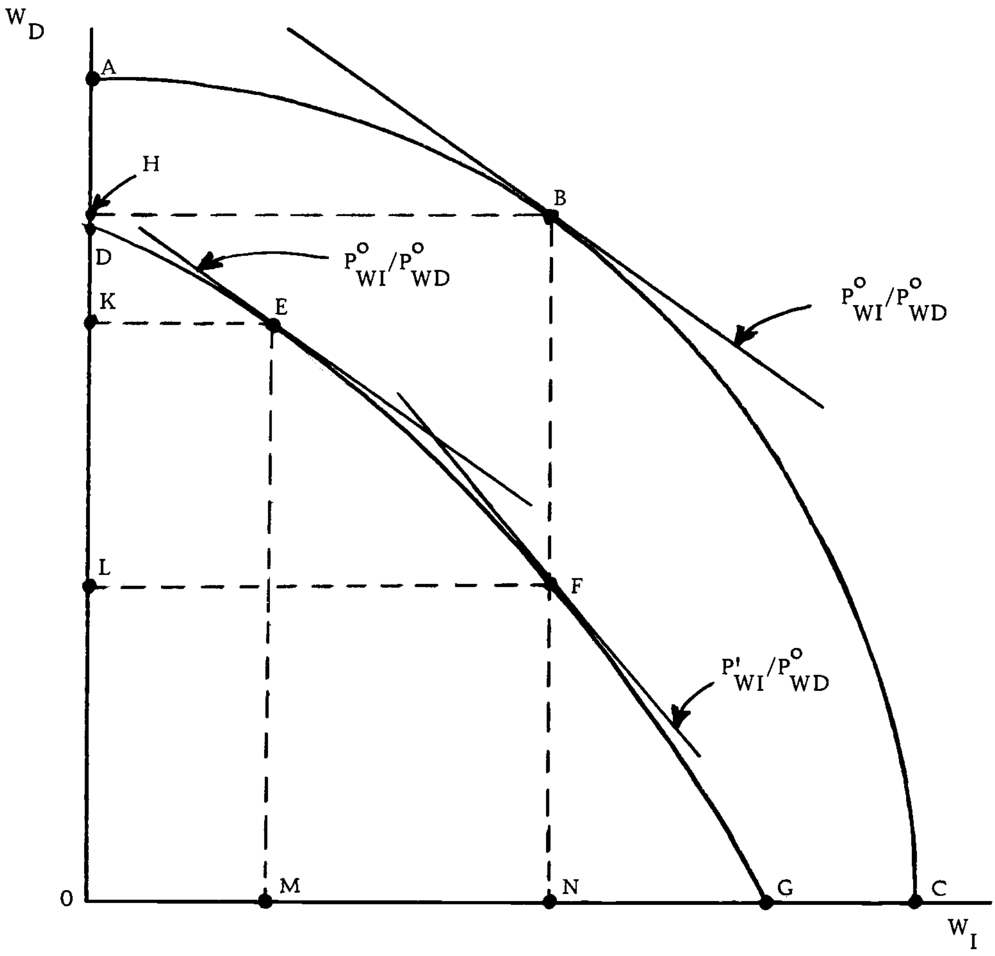

Figure 26. Transformation curves with water for domestic use (W_D) and water for irrigation (W_I) as intermediate products.

$$NB = P_{WI}W_I - C^o \tag{4.21}$$

where C^o represents the (minimum) cost of producing W_I and W_R along the iso-cost curve. Product W_I can be expressed as a function of W_R along the iso-cost curve, however, so the NB function in (4.21) can be represented as:

$$\begin{aligned} W_I &= f(W_R) \\ NB &= P_{WI}f(W_R) - C^o . \end{aligned} \tag{4.22}$$

Maximum net benefits are achieved at the zero level of W_R. Increases in W_R require reductions in W_I; as a consequence, NB declines as W_R is increased. One measure of the price of W_R can now be found by calculating the sacrifices in net benefits for increases in W_R. The slope of the net benefit curve is given by:

$$\frac{d(NB)}{dW_R} = P^o_{WI} \frac{dW_I}{dW_R} . \tag{4.23}$$

This is a negative value as $(dW_I/dW_R) < 0$. Now, in order to have an optimum product mix, it must be the case that:

$$\frac{dW_I}{dW_R} = - \frac{P_{WR}}{P_{WI}} . \tag{4.24}$$

Substituting (4.24) into (4.23), the change in net benefits for an increase in W_R is given by:

$$-\frac{d(NB)}{dW_R} = P_{WR} . \qquad (4.25)$$

The price of W_R is given along the net benefit curve. Viewed in another manner, the absolute value of the slope is the implicit price of W_R. It must be emphasized that this net benefit relation was derived from a constant (and minimum) cost curve. Also, the assumed price of W_R was $P_{WR} = 0$.

<u>Implicit Prices and the Product-Product Trade-Off Ratio: Three Product Case.</u> The resource planner is seldom faced with finding the iso-cost relations for only two products. As was noted earlier in this study, there are many water resource products in most planning units. The optimization process is complicated if there are more than two products. In general, however, the same optimization principles will apply no matter how many products (and factors) must be considered. Consider a slightly more complex case where there are three water products. Assume the general form of the relationship among water for recreation (W_R), water for irrigation (W_I), and water for power (W_P) is represented by:

$$F(W_R, W_I, W_P; C) = 0 . \qquad (4.26)$$

Also, assume the price of $W_R (P_{WR})$ is not known. There is now a three dimensional surface, illustrated in Figure 27, representing information needed by the decision making body. Assume point L is chosen. In order for point L to represent the optimum product mix, the following conditions must be met:

$$\frac{dW_I}{dW_R} = - \frac{P_{WR}}{P_{WI}} = \text{slope of FG and F'G'},$$

$$\frac{dW_I}{dW_P} = - \frac{P_{WP}}{P_{WI}} = \text{slope of HK and H'K'}, \qquad (4.27)$$

$$\frac{dW_P}{dW_R} = - \frac{P_{WR}}{P_{WP}} = \text{slope of DE and D'E'} .$$

The projections of the iso-cost curves at the optimum point into the respective planes are represented by iso-cost curves A'B', A'C', and B'C'. In essence, each of these "adjusted" iso-cost curves represent the transformation conditions given the level of some other product. Curve A'B', for example, represents the iso-cost relation between W_I and W_R given that 0N = QM of W_P is provided. By the same reasoning, the iso-cost relation between W_I and W_P is represented by A'C' given that 0Q = NM of W_R is provided.[59] At the optimum

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[59] The outer boundaries of the iso-cost surface represent production of one of the water products at the zero level. Curve AB, for example, represents the iso-cost relation between W_I and W_R when W_P is at the zero level. It should also be noted that A'B' would even be lower in the $W_I W_R$ plane if W_P was also an intermediate product

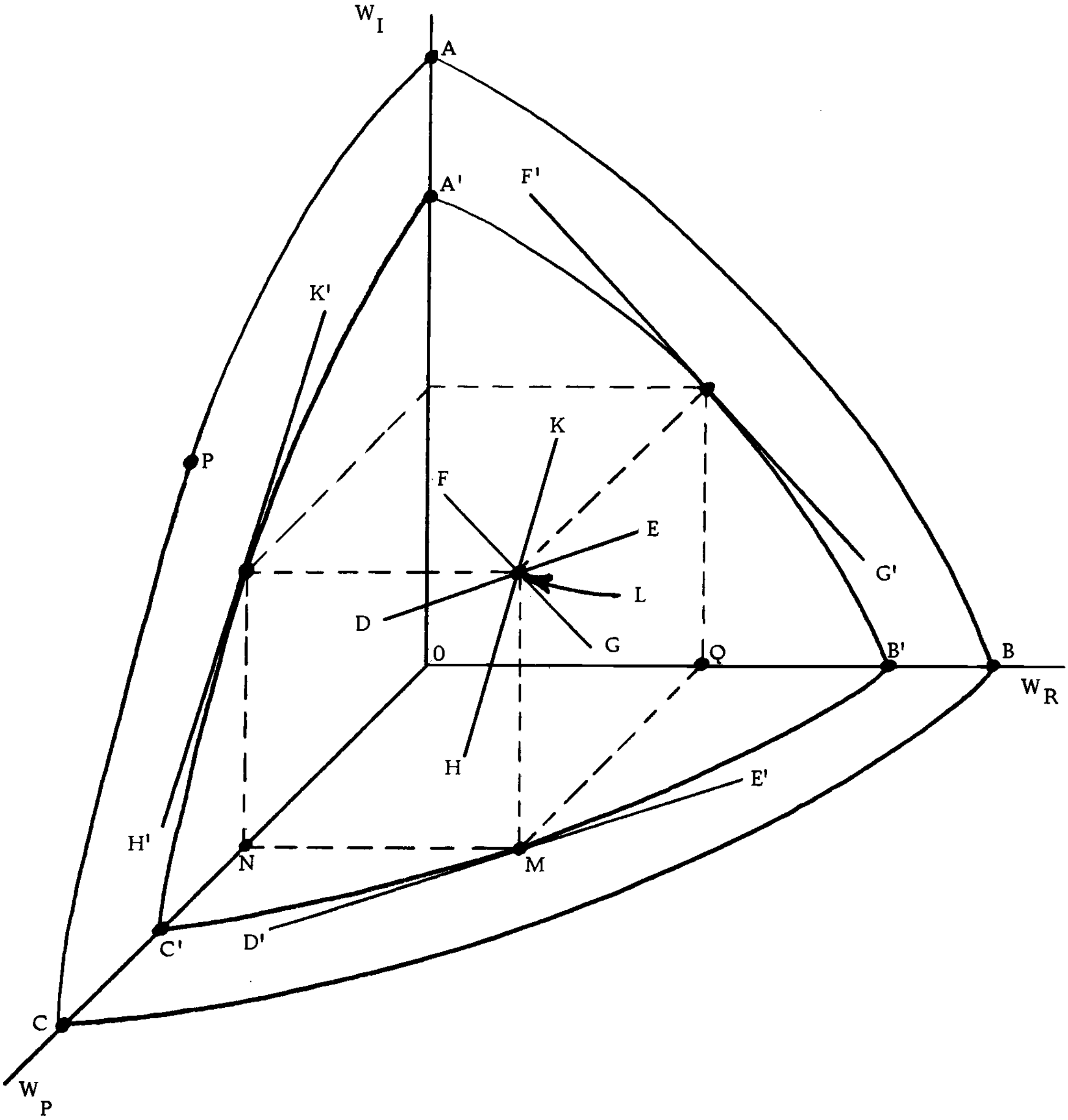

Figure 27. The transformation surface and the optimization process.

point, the quantities 0Q, 0N, and ML of the products W_R, W_P, and W_I, respectively, are produced.

The implicit price of W_R is now given at point L from the conditions in (4.27). These conditions can be used to derive the relations:

$$\frac{dW_I}{dW_R} = -\frac{P_{WR}}{P_{WI}} \quad \text{or} \quad -P_{WI}\frac{dW_I}{dW_R} = P_{WR} \tag{4.28}$$

$$\frac{dW_I}{d_{WP}} = -\frac{P_{WP}}{P_{WI}} \tag{4.29}$$

$$\frac{dW_P}{dW_R} = -\frac{P_{WR}}{P_{WP}} \quad \text{or} \quad -P_{WP}\frac{dW_P}{dW_R} = P_{WR}\,. \tag{4.30}$$

Given the optimum levels of W_I and W_P from (4.29), the level of W_R must be chosen such as to make P_{WR} the same value in both (4.28) and (4.30).

Once a particular point (such as L) is chosen by the decision making body, the resource planner can then determine if net dollar benefits are positive or negative. It may be that net dollar benefits are indeed negative at point L. All that is known for sure at point L is that the optimum product mix is given there as perceived by the

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in the production of W_I and W_R. Also, points A', B', and C' do not necessarily have to be points in common. Point B' on A'B' represents the situation where 0N of W_P and zero W_I are provided. Point B' on B'C' represents a situation where ML of W_I is provided and W_P is at the zero level. Point B' is the same on both curves only if the amount of resources required to produce 0N of W_P is identical with the amount of resources required to provide ML of W_I.

decision making body. The net dollar benefits at point L may indeed be negative especially if P_{WR} cannot be extracted from any users to reimburse the costs of the project. This does not necessarily mean, of course, that point L should not be chosen. Human welfare may be improved by selection of point L far in excess of the loss in dollar benefits. This is necessarily a political decision and is appropriately decided in the political arena.

Implicit Prices and the Net Benefit Curve: Three Product Case. A very complex net benefit surface results when three products are considered. In fact, the surface is in the fourth dimension so it is impossible to visualize. The net benefit curve for various levels of the product W_R can be examined, however, given some specific assumptions about the levels of W_I and W_P. Consider, for example, keeping W_I and W_P in such a ratio as to satisfy the relation:

$$\frac{dW_I}{dW_P} = - \frac{P_{WP}}{P_{WI}} . \tag{4.31}$$

Assume this condition is met at points P and L in Figure 27. Also, consider moving across the iso-cost surface from point P to L. Product W_R is increased from 0 to 0Q in moving from P to L. The net benefit function is then given by:

$$NB = P_{WI}W_I + P_{WP}W_P + P_{WR}W_R - C^o \tag{4.32}$$

where C^o represents the iso-(minimum)cost way of providing W_I, W_P, and W_R along the surface. Net benefits are necessarily at a maximum at point P as movement toward L represent reductions in W_I and W_P. These two products, in turn, contribute all the dollar benefits as $P_{WR} = 0$.

Movement along the net benefit curve now represents the sacrifices in net benefits from providing W_R. In a sense, net benefits are an "input" into the production of W_R. The change along the net benefit curve is now given by:

$$d(NB) = P_{WI}dW_I + P_{WP}dW_P + P_{WR}dW_R - dC . \qquad (4.33)$$

It was specified, however, that:

$$\frac{dW_I}{dW_P} = -\frac{P_{WP}}{P_{WI}}$$

or, (4.34)

$$P_{WP} = -P_{WI}\frac{dW_I}{dW_P} .$$

Substituting (4.34) into (4.33) results in:

$$d(NB) = P_{WI}dW_I - P_{WI}\frac{dW_I}{dW_P}dW_P + P_{WR}dW_R - dC. \quad (4.35)$$

The rate of change in NB for a change in W_R is then given by (for $dC = 0$):

$$\frac{d(NB)}{dW_R} = P_{WR} \ . \tag{4.36}$$

As long as the other two products are varied in the correct proportions such as to insure the equilibrium condition is met at every point, $(dW_I/dW_P) = -(P_{WP}/P_{WI})$, the net benefit (revenue) curve can be used to estimate the price of W_R. This is equivalent to saying that:

$$\begin{aligned} \frac{dW_I}{dW_P} &= -\frac{P_{WP}}{P_{WI}} \\ \frac{dW_I}{dW_R} &= -\frac{P_{WR}}{P_{WI}} \\ \frac{dW_P}{dW_R} &= -\frac{P_{WR}}{P_{WP}} \ . \end{aligned} \tag{4.37}$$

It must be assumed, of course, the decision body can select the correct point on this net benefit curve.

Use of this net benefit curve may, however, lead to distorted estimates of P_{WR} if costs are not held constant. Given costs are allowed to vary (movement is <u>across several surfaces</u>), the change in NB is given by (from 4.35):

$$\frac{d(NB)}{dW_R} = P_{WR} - \frac{dC}{dW_R} \ . \tag{4.38}$$

The estimate of P_{WR}, given the decision makers choice, will now be distorted. This result highlights the need to estimate prices only on iso-cost surfaces and not for movements between or among surfaces.

Optimization Among Factors and Products

Factor prices may also be missing in water resource production processes. The resource planner is then faced with discovering the factor-product trade-off ratios for presentation to the decision body. The optimum level of application of all water resource factors is given by the relationships:

$$\frac{\partial q_i}{\partial x_j} = \frac{r_j}{p_i}, \quad (i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n). \qquad (4.39)$$

Given prices for all the water resource factors and products in (4.39), these equilibrium conditions will at least be approximated in the planning process given a goal of maximizing net dollar benefits. If some price is not known, the implicit price of the factor is made explicit when the product mix is chosen (assuming knowledge of the marginal products).

Implicit Prices and the Factor-Product Trade-Off Ratio. Consider, for example, a case where varying amounts of scenic river must be inundated to produce water for irrigation (W_I). The scenic river, in this case, serves as a factor in the production of W_I. Some measure of the scenic river, such as L = "miles of scenic river", must be combined with the other factors of production to provide W_I. Assume a market price for L is not known. A factor-product trade-off ratio must then be calculated. Stated symbolically, the problem is one of finding the proper level of L to use from:

$$\frac{\partial W_I}{\partial L} = \frac{r_L}{P_{WI}} \tag{4.40}$$

where the price of L, r_L, is not known. The solution to the problem is to present the total product curve or the trade-offs from that curve to the decision body for consideration. Assuming the decision body can perceive society preferences toward L, the selection of a point on the total product curve (selection of a trade-off value) will give the implicit price. This process can be illustrated with the hypothetical total (TP_L) and marginal (MP_L) product relations as represented in Figure 28. Assume there is only one other factor, capital (K), which is held constant at $K = K^o$. The optimum level of use for factor L is given by the intersection of MP_L with the factor-product price ratio at point D to produce W_I' of W_I. If r_L was known, the resource planner would provide W_I' of W_I using L' of L. If r_L is not known, choice of the W_I' level of W_I would yield the implicit value of r_L at point B given P_{WI} and MP_L at B are known. In fact, the implicit price of L, r_L, is given by the slope of the TP_L curve at point B multiplied by P_{WI}; i.e.,

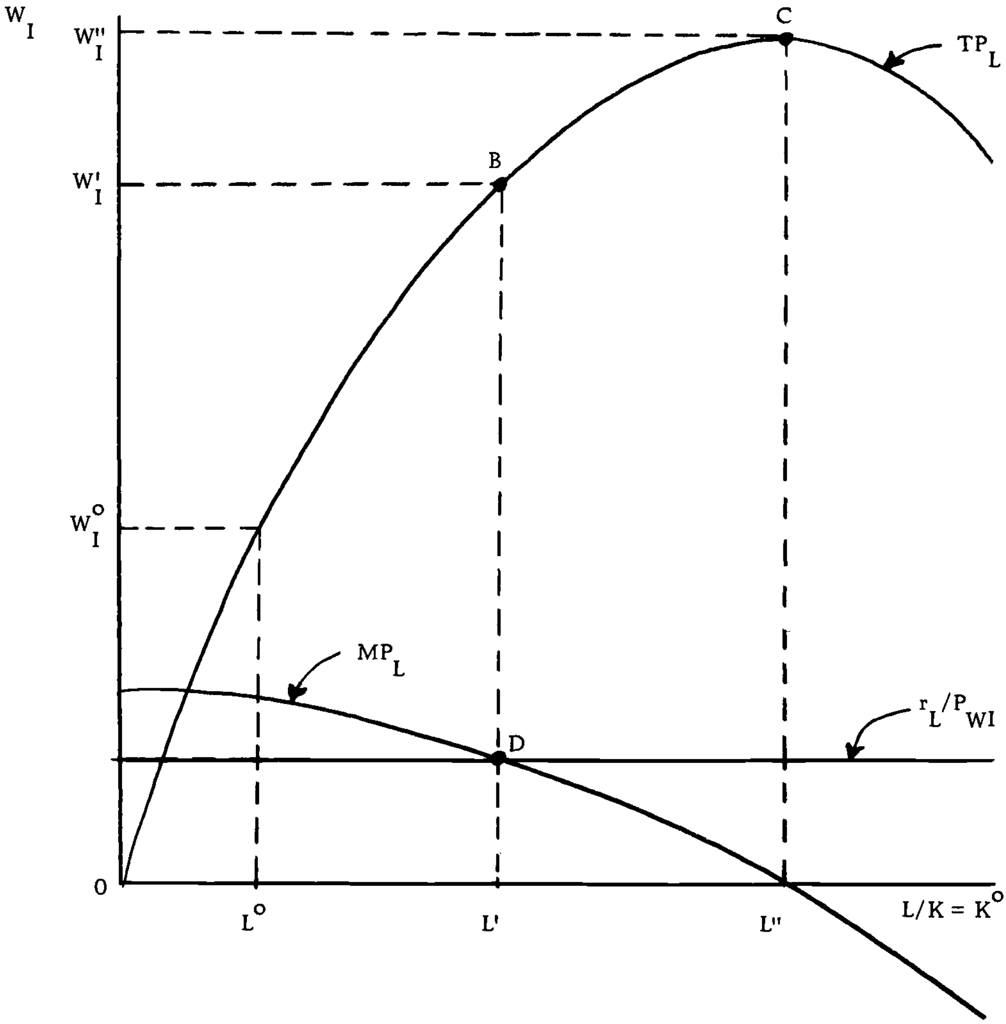

Figure 28. Factor-product relation with water for irrigation (W_I) produced from various levels of economic land.

$$P_{WI} \cdot MP_L = r_L \, . \qquad (4.41)$$

This is the simplest case for the application of the factor-product trade-off ratio as one factor (K) is held constant and there is only one product.

The slope of the TP curve at point B can, then, be viewed as a "trade-off". The TP curve is a trade-off curve in the sense that moving toward the origin from levels of L such as L' results in reductions (sacrifices) in W_I for increases (gains) in L available for other purposes. Curve TP is not, however, an iso-cost curve. An iso-cost curve of the nature discussed in earlier sections of this chapter is derived on the assumption that all the costs underlying the curve can be used in production of either product or some combination. If curve TP was an iso-cost curve, K^o of K would be required to not use any L in W_I production, which is not the case. The miles of scenic river factor is available for some other uses without any expenditure for K and L.[60] The reason for the difference between this trade-off curve (curve TP) and the previous iso-cost (product-product trade-off) curve lies in the distinction between the levels of production. Miles of scenic river is a factor which enters the production function relation to provide W_I as depicted in Figure 28. The

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[60] At least for some lesser expenditure on K and L, like possibly providing access roads.

scenic river is not a product from the viewpoint of water resource development and the resource planner.

Another approach to finding the factor price r_L would be to utilize the total benefit curve (TB_L) in Figure 29. This relationship is entirely parallel to the TP_L relationship in Figure 28 assuming a constant price for W_I; only the scale on the vertical axis has changed. The slope of the TB_L curve in Figure 29 gives the implicit price of the factor L directly. If the decision making body chose point B for total benefits of $P_{WI} \cdot W_I'$, the implicit price for the factor L is given by the slope of TB_L at point B. This slope is represented by the level of the marginal benefit (MB_L) curve at point D or $r_L = 0F = r_L'$. Selection of point A for total benefits of $P_{WI} \cdot W_I^o$ would given an implicit price of $r_L = 0G = r_L^o$ at point E.

Choosing a level of production such as represented at point B ($TB_L = P_{WI} \cdot W_I'$) does not guarantee, of course, that net benefits are positive at that point. Curve TB_L represents total benefits from combining various levels of L with K^o of K. Net benefits could, in fact, be negative for level $L = L'$. Net benefits are, however, at a maximum level at L' of L (assuming $r_L = r_L'$). If net benefits are negative, the point represents a minimum loss level from using L in combination with K^o of K.[61] A net benefit curve can be constructed

[61] Assuming that L is applied at all. A better alternative may be to not provide any W_I by using all of the L (and K^o) for other production.

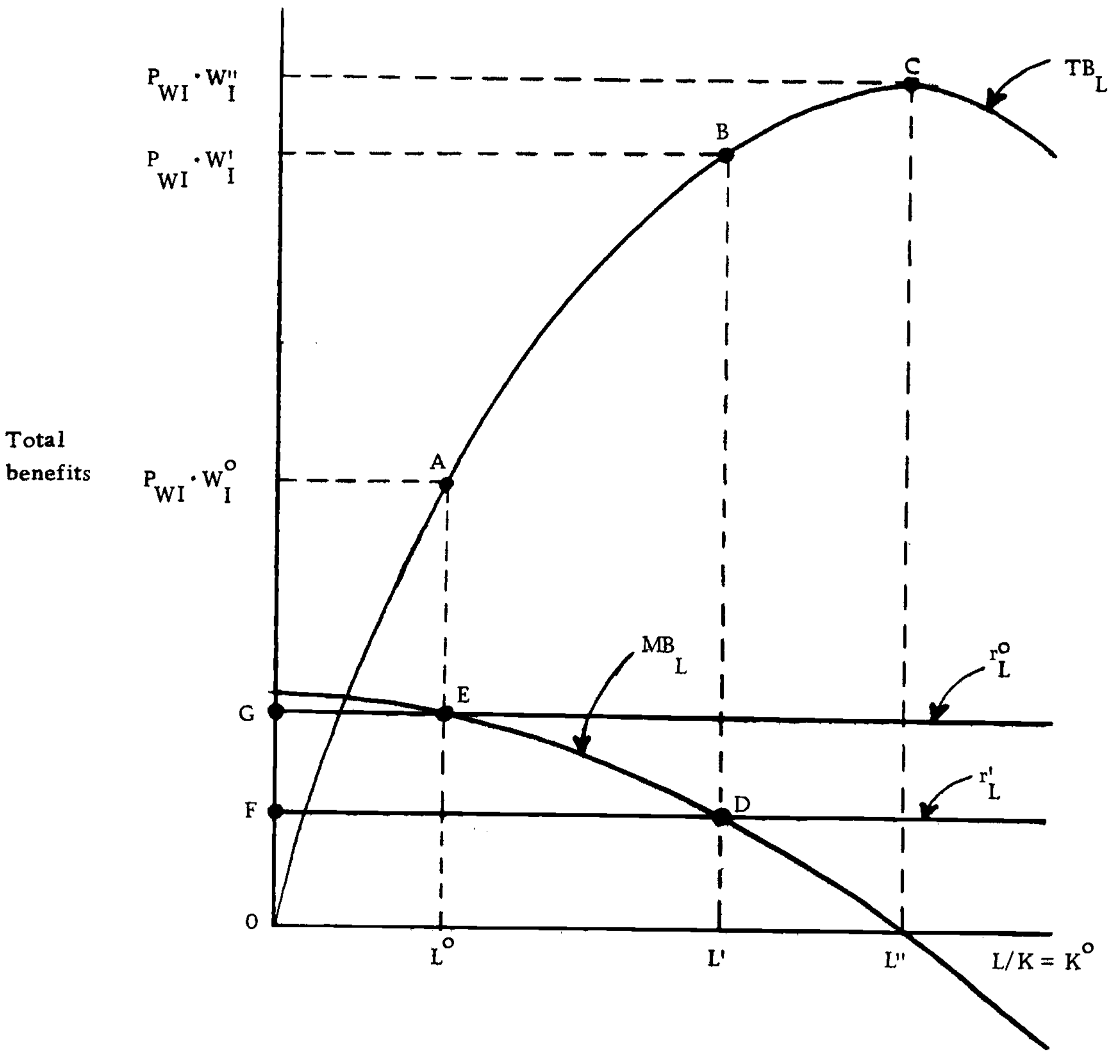

Figure 29. Total and marginal benefit relationships, water for irrigation (W_I) produced from various levels of economic land.

from relationships such as those represented in Figures 28 and 29 to illustrate the concept. One such curve is represented by 0ABC in Figure 30.

Generation of a net benefit curve requires specification of all the product and resource prices. Given the price of L, r_L, net benefits can be generated for each level of W_I by finding the value:

$$NB = TB_L - C$$

or,

$$NB = P_{WI}W_I - (r_L L + r_K K^o) . \tag{4.42}$$

Net benefits will be a maximum where the MB_L is equal to the marginal factor cost in Figure 29. Given a price of $r_L = r_L'$, maximum net benefits will occur at W_I' of W_I. This is illustrated by NB' at W_I' in Figure 30. If net benefits were negative at W_I', the peak of the NB curve would be below the horizontal axis. The peak, however, would be the nearest point on the NB curve to the horizontal axis; as a consequence, utilization of L' of L would represent the minimum loss level.

Implicit Prices and the Net Benefit Curve. A means of finding the implicit price of a non-money valued resource from a net benefit curve can now be outlined. Movement along curve 0ABC in Figure 30 measures the net benefits that must be sacrificed, in terms of lower quantities of W_I valued at P_{WI}, to use some level of L other than

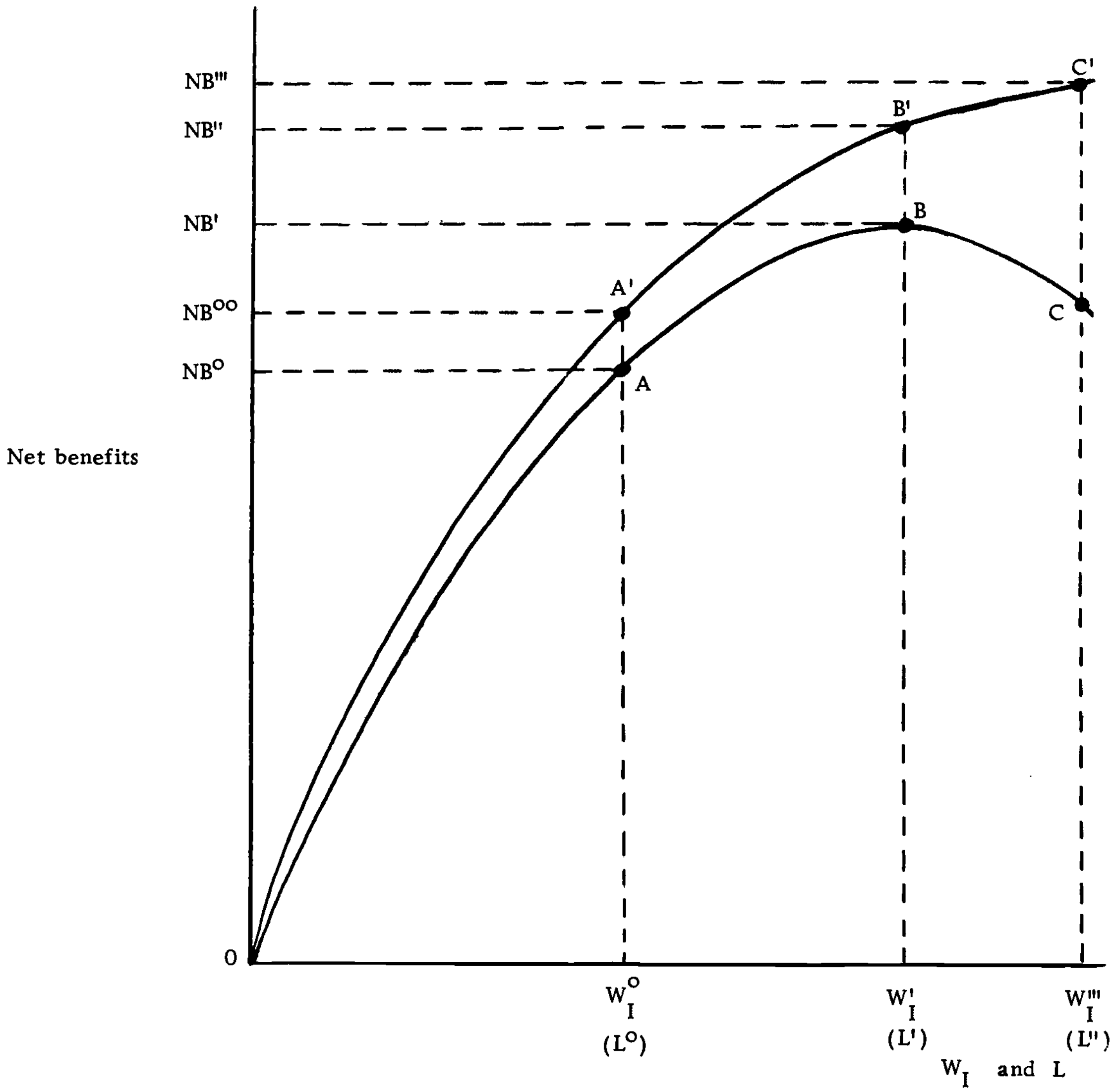

Figure 30. Net benefits function for varying levels of water for irrigation (W_I).

L'. A reduction in L from L' to L^o constitutes a loss in net benefits of $(NB'-NB^o)$ (Figure 30). It may appear that the relevant price for L is given by:

$$\frac{(NB'-NB^o)}{(L'-L^o)} = \frac{(\Delta NB)}{\Delta L} = r''_L .$$

This is not the case, of course, as $r_L = r'_L$ along 0ABC and $r'_L \neq r''_L$.

One approach to finding the price of a resource through the use of the net benefit curve is to set $r_L = 0$. The necessity for using the net benefit curve for the zero price of r_L must be understood.[62] If a non-zero price for r_L is used in the derivation of the curve, the net benefits will be given by:

$$NB = P_{WI}W_I - r^s_L L - r_K K = P_{WI}W_I - r^s_L L - C^o \quad (4.43)$$

where C^o is essentially a fixed cost. Upon differentiation of (4.43), the condition along the NB curve is given by:

$$\frac{d(NB)}{dL} = P_{WI}\frac{dW_I}{dL} - r^s_L . \quad (4.44)$$

[62]There are, of course, as many net benefit curves as there are prices for the resource. As a result, the price calculated by this method will be different for every net benefit curve used. The only way to be sure the method will give the correct price is to use the net benefit curve derived for $r_L = 0$.

Net benefit changes for reductions in L will now be affected not only by the marginal value product but also by the "starting" price for $L(r_L^s)$. The actual price of L will never be discovered unless the starting price, r_L^s, is accounted for in the analysis. The real price (r_L^R) is now given by:

$$P_{WI} \frac{dW_I}{dL} - r_L^s = r_L^w$$

or, (4.45)

$$P_{WI} \frac{dW_I}{dL} = r_L^s + r_L^w = r_L^R .$$

The incorrect price of L, r_L^w, will be discovered by using the sacrifice along a net benefit curve derived from a non-zero price for L. It is expected this could easily occur in water resource development. Acres of wilderness area (L) may, for example, have a price the owners of the area will accept for the property which would affect the location and shape of the net benefit curve. Including that price in the analysis would lead to distortion in the price derived from the choice made by the decision maker. The correct approach can be given by:

$$NB = P_{WI}W_I - r_K K - r_L L . \tag{4.46}$$

Given (4.46), the rate of change in NB is given by:

$$\frac{d(NB)}{dL} = P_{WI} \frac{dW_I}{dL} - r_L$$

or, (4.47)

$$r_L = P_{WI} \frac{dW_I}{dL} - \frac{d(NB)}{dL} .$$

The "starting" price of r_L, however, was $r_L = 0$. This implies that:

$$\frac{d(NB)}{dL} = P_{WI} \frac{dW_I}{dL} . \qquad (4.48)$$

This was shown earlier to be the value of r_L when in equilibrium. As a result, the price of L is given directly from the net benefit curve.[63]

A net benefit curve with a starting price of $r_L = 0$ is represented by curve 0A'B'C' in Figure 30. Movement along the curve toward the zero level of L used in W_I production represents sacrifices in net benefits. The price of L can be estimated at any point by finding the value:

$$P_{WI} \frac{dW_I}{dL} = r_L . \qquad (4.49)$$

[63] It must be remembered the marginal product schedule for the hypothetical production relation was downward sloping. The second order conditions are satisfied only when this condition holds (see Chapter III, p. 60). The use of net benefit curves for estimating value is also valid only when this condition holds.

Deriving this value at point B' on 0A'B'C', for example, results in the value $r_L = r'_L$ from Figures 28 and 29. The resource planner may, of course, have to work with discrete data or, at best, with continuous curves and not mathematical functions. In that case, the price of the resource is approximated by:

$$P_{WI} \frac{\Delta W_I}{\Delta L} \doteq r_L \, . \tag{4.50}$$

The resulting price is an average over the curvature of the zero price net benefit curve.

Another way to view the resource evaluation process using net benefit curves is to change the axis on Figure 30 such as to represent non-use of the resource in W_I production. Movement along the horizontal axis from left to right then would represent reductions in the amount of L devoted to W_I production. The curve could be used, directly, to evaluate the price of the resource. The slope of that curve gives the (negative) price of L.

The net dollar benefit curve can also be used for the case of two or more products. Again, the zero price net benefit curve must be utilized. Also, all the water products must be kept in the combinations $(dq_i/dq_j) = -(p_j/p_i)$, $(i, j = 1, 2, \ldots, m, \; i \neq j)$ for all m products. This is necessarily more complex, but the same principles apply.

V. CALCULATION AND INTERPRETATION OF TRADE-OFFS IN NATURAL RESOURCE PLANNING

The conceptual models must now be quantified. Only in this manner can the theoretical models be useful in guiding the trade-off calculation process for the resource planner. Also, the many recommendations regarding the definition and calculation of trade-offs must be compared with the approach recommended in this study. The purposes of this chapter are: 1) to indicate, based on the conceptual models developed in Chapters III and IV, guidelines for calculating trade-offs as necessitated by multiple objective planning, 2) to indicate the differences and similarities among the proposed definitions of trade-offs and the approaches to calculating trade-offs, and 3) to provide an evaluation of presently used Federal planning procedures for water and related land resources planning.

Problems in the Calculation of Trade-Off Ratios--Some Empirical Cases

Multiple objective planning is necessary, as noted previously in this study, because of a lack of some common unit of measure for all products and resources. As a result, the type of information needed by the decision body is of a different nature. Trade-offs must be calculated and provided to the decision body. The iso-cost and production function frameworks provide the necessary conceptual base for the calculation of trade-offs.

Several major problem areas were outlined in Chapter IV as areas that should be of concern to water resource planners. The water resource planner should:

1) define the products, factors, and the planning unit such as to find all the independent and non-independent transformation curves,

2) define the products and factors not having prices which, in turn, defines the set of elements for which the trade-off calculation process must be used,

3) define the production relations and the trade-offs.

These three steps, very briefly, define the planning process. Actual implementation of these steps for some "real world" data is illustrated in this chapter.

Delineation of the Planning Unit--The Knife River Basin

The actual data needed to illustrate application of the conceptual models was derived from an earlier study by this author and others (North Dakota State Water Commission, "The Plan...", 1970). That particular study concentrated on the determination of the usefulness of mathematical programming models in water resource planning. A proposed plan for development of the Knife River Basin in North Dakota was derived from the results. The alternative forms of development possible in that basin were large in number because of the possibility for 18 alternative locations of dams and the multiple purposes of irrigation, power production, recreation, and municipal use possible at nearly every site (Figure 31). The "optimum" plan for development was selected by finding the maximum net dollar benefits obtainable.[64] All products and factors were assigned prices (where prices did not exist) using various methods, some of which were nothing more than rough estimates of market value based on "best guesses". In other words, trade-off calculations were not needed as all factors and products were assumed to have "known" market prices.

The conceptual models of this study would allow a different approach to finding the optimum plan for development of the Knife River Basin. The North Dakota State Water Commission would be viewed as a "firm" in the sense that factors could be combined in varying proportions by the agency to produce (or, at least plan for production) alternative product mixes. The Water Commission, in essence, would be considered as having a production function, or at least of having the capability of defining the production relations in the relevant planning unit. Also, the rough guesses of various prices

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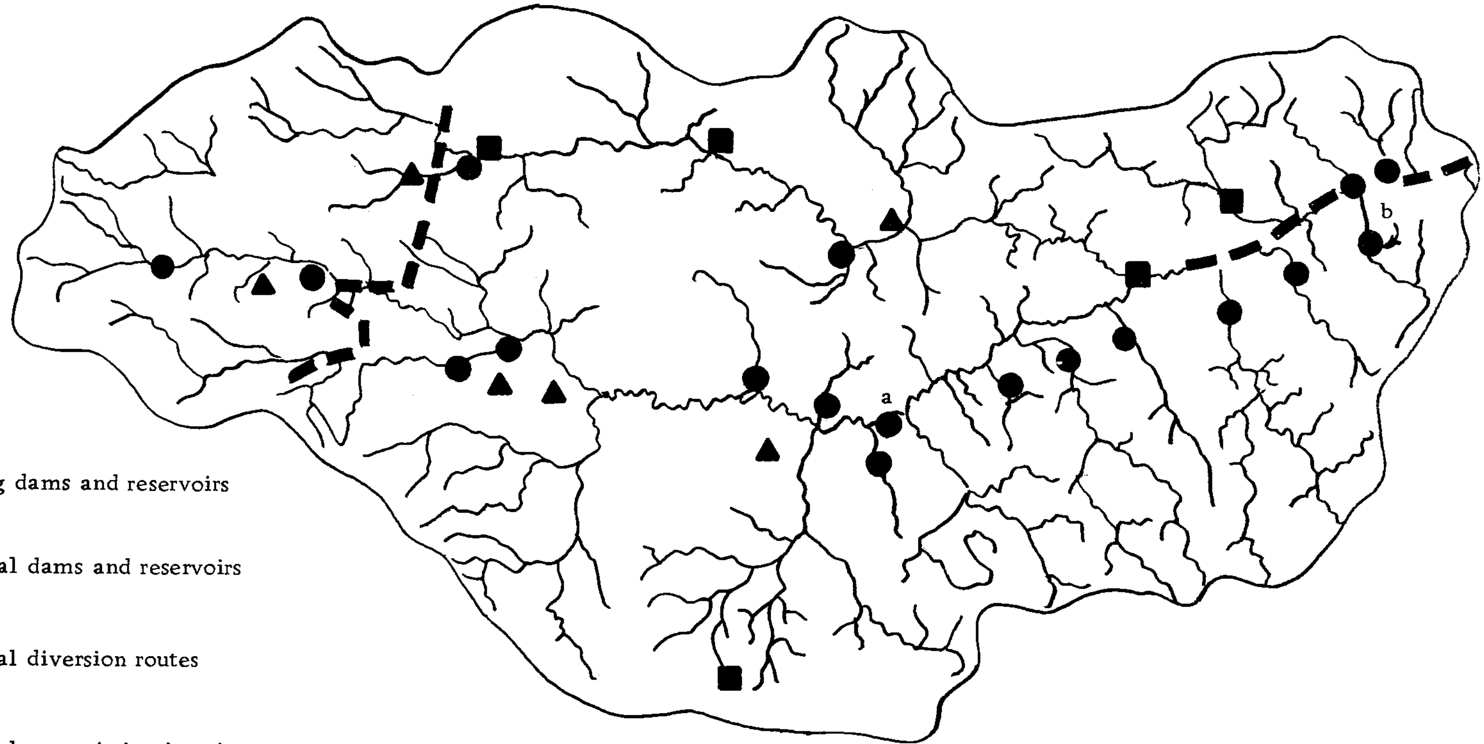
[64] A linear programming model was used, in combination with separable programming (to account for non-linearities), to isolate the plan yielding the maximum dollar benefits.



Key:

■ Existing dams and reservoirs

● Potential dams and reservoirs

▬ Potential diversion routes

▲ Potential pump irrigation sites

a Bronco site

b Kineman site

Figure 31. Present and proposed water resource development in the Knife River Basin, North Dakota.

would not be needed. Trade-off ratios would be calculated and submitted to the decision body.

The production processes in the Knife River Basin, one of several basins in the state of North Dakota, are most assuredly not independent of processes in other basins of the state or region (or even Nation). As a result, choice of the Knife River Basin as the relevant planning unit implies the choice of a non-independent transformation surface. This very important characteristic of the production processes in the Basin must be emphasized; i.e., any plan for development that is recommended within a non-independent planning unit may be sub-optimal when the viewpoint is from some larger planning unit (the independent unit). The resulting transformation surface for the Knife River Basin is necessarily a non-independent surface. Many of the factors (such as investment capital by the State) are useable in other production processes (such as the school system, highway building programs, and health care programs). The transformation surface from any water resource based planning unit, it is expected, is just one of the non-independent transformation surfaces needed by the body politic for the rational allocation of resources.

Several segments of the Knife River Basin were separated out for illustrative purposes. Any of the 18 alternative sites (or any combination) could have been used. The Bronco and Kineman sites (Figure 31) were chosen, however, because of the close proximity of the sites and the possibility for several uses of water at each site. Each segment separated out for further study in this work represents part of a transformation surface that could be developed for the entire Basin. The various segments used here for illustrative purposes, then, represent other non-independent surfaces (not independent of the other present and potential sites in the Basin).

Products, Factors, and Prices in the Knife River Basin

Several water related products can be produced in the Knife River Basin. The area is primarily agricultural, but encompasses large areas of lignite coal reserves. Water can be used in the production of a great variety of agricultural crops as well as for the production of electricity (as cooling water and for steam generation) from the coal. Also, there are frequent floods in the area. Control of river flows could provide some benefits. The potential for recreation development also exists because of the close proximity of the Basin to a city-urban area. The type of recreation activity would be mainly water based, for such surface uses as water skiing, boating, and aesthetic viewing. All of these water products were considered in the earlier study (North Dakota State Water Commission, "The Plan...", 1970). Water for irrigation (W_I), water for municipal and industrial use (W_M), and water for recreation (surface area of reservoir, W_R) were chosen for further consideration in this study.

Various assumptions were made regarding prices in the study. The prices used for W_M were set arbitrarily. This was done only to facilitate the illustration of models developed in earlier chapters. It is expected that the price of water used for municipal and industrial purposes will be known in most project situations.[65] Further, it was assumed the price of water for recreation (P_{WR}) was not known.[66] As a result, trade-off ratios must be calculated for varying levels of W_R. A value of the water used for irrigation was developed by finding the residual return to irrigation water from farm budget data provided in the earlier study (North Dakota State Water Commission, "The Plan...", 1970, pp. 97-102). This value was estimated at $32.91 per acre foot; the price of W_I was set at P_{WI} = $30.00 in this study.[67]

The factors of concern at the selected sites in the Knife River Basin are at two completely different levels in the production processes of the Basin. The first level of factors are those elements necessary in the planning, construction, operation, and maintenance of the necessary structures. These factors were assumed to be

[65] The price used in the earlier study was P_{WM} = $500 per acre foot (North Dakota State Water Commission, "The Plan...", p. 101).

[66] The alternative cost approach to establishing value was used in the earlier study (North Dakota State Water Commission, "The Plan...", 1971, p. 104).

[67] See Appendix D for the procedures used in the derivation of the residual return to water. Also, the implicit assumptions involved in using the residual return are examined.

combined in a least cost manner. The dollar value of all these factors is referred to as "cost" in the balance of this study. The other factors of concern in the analysis were the water resource factors such as wilderness areas, scenic views, and wild rivers. These elements are essentially factors in the production of water products. The specific water resource factor considered in an empirical case was acres of wilderness area at the Bronco site. There was no known, market value for an acre of wilderness area. The concept of a factor-product trade-off ratio is utilized to provide insights into the allocation problem for that case.

The natural water flows at the various sites must also be considered a factor in the production of W_I, W_R, and/or W_M. The capital, operation, and maintenance costs are applied to a particular site in a river basin to transform the natural resource, water, into water products. The amount of water flow at a particular site, then, must be known. These flows were estimated at the Bronco and Kineman sites in the earlier study (North Dakota State Water Commission, "The Plan...", 1971, pp. 80-81 and "Appendix... The Plan...", 1971, pp. 27-58). The total available (annual) flows were estimated at about 30,000 acre feet at the Bronco site and 700 acre feet at the Kineman site. The largest proportion of these amounts becomes available in late spring. No attempt was made in this study to deal with the dynamics of water availability. In fact, no attempt was made to separate out the planned developments that could actually be sustained based on flows at Bronco and Kineman. The iso-cost surfaces presented here reflect the range of alternative levels of W_I, W_R, and W_M that could be produced given enough water.

Product-Product Trade-Off Ratios Between Two Water Products at Two Alternative Sites

The simplest procedures for calculating product-product trade-offs result from the case where only two water products are provided. The two product case is illustrated here for the Bronco and Kineman sites in isolation and for the two sites in combination. The analyses for the individual sites is presented first.

Product-Product Trade-Offs at the Bronco Site. Water for irrigation (W_I) and water for recreation (W_R) can be produced at the Bronco site of the Knife River Basin. The investment requirements, operation costs, repair costs, and maintenance costs summarized in Table 5 can be "applied" to produce various combinations of W_I and W_R. An initial investment of $3,329,726 and annual operation, maintenance, and repair costs (OMR) of $16,649 can be applied, for example, to provide 61,000 acre feet of reservoir capacity and 3260 surface acres (Table 5). These physical measures of production can, in turn, be used for W_I and W_R production. A more useful measure of the cost requirement, annual cost, is also

Table 5. Total investment requirements, annual costs, reservoir capacity, and reservoir surface area, Bronco dam site, Knife River Basin, North Dakota.

Dam Height	Total Investment Requirement	Annual Amortization[a]	Annual OMR	Total Annual Costs[b]	Reservoir Capacity[c]	Reservoir Surface Area[d]
(feet)	(- - - - - -	- - - - - dollars	- - - - - -	- - - - - -)	(acre-feet)	(acres)
30	1,042,994	58,239	5,215	63,454	7,000	700
40	1,523,326	85,060	7,617	92,677	16,000	1,350
50	2,341,294	130,833	11,706	142,439	33,500	2,250
60	3,329,726	185,926	16,649	202,575	61,000	3,260
70	4,402,063	245,803	22,010	267,813	100,000	4,450
80	5,543,504	309,539	27,718	337,257	151,000	5,860
90	7,228,102	403,604	36,141	439,745	215,000	7,130
100	8,874,718	495,547	44,374	539,921	297,000	9,140
110	10,750,898	600,308	53,754	654,062	400,000	11,650

[a] Based on an interest rate of 5 1/8 percent and a 50 year repayment period.

[b] Sum of annual amortization and annual OMR.

[c] Capacity measured at spillway level.

[d] Surface area of reservoir behind dam when reservoir full to spillway level.

Basic data used to derive table from (North Dakota State Water Commission, "The Plan...", 1971, pp. 87-89).

given in Table 5. The annual (minimum) costs are composed of the annual amortization payment (principal plus interest at 5 1/8 percent) for each of 50 years plus the annual OMR costs. At the 60 foot dam height, for example, total annual costs were determined to be \$202,575 (Table 5). The other estimates of capacity, surface area, and annual cost for dams varying from 30 to 110 feet are interpreted similarly.[68]

The basic cost-physical relations data of Table 5 provided the necessary data base for the development and use of the iso-cost framework. The important relations, in general form, are given by:

$$TC = f(AC) \tag{5.1}$$

$$SA = g(TC) \tag{5.2}$$

where,

AC = total annual (minimum) cost,

TC = total reservoir capacity at spillway level, measured in acre feet,

and

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[68] The 30 foot minimum size was chosen somewhat arbitrarily. The engineers involved in the planning of development in the Knife River Basin were asked to provide the data of Table 5 for 10 foot increments in dam height up to the maximum possible size at each proposed site in the Basin. The 30 foot minimum, selected at nearly every site, limited the number of sites that had to be considered in the Basin and the amount of basic cost data needed at every site chosen for evaluation. In order to consider all alternatives, of course, even smaller dams may have to be considered in some cases.

SA = surface area of reservoir when full to capacity, measured in acres.

The discrete forms of equations (5.1) and (5.2) are given in Table 5 and illustrated in Figures 32 and 33, respectively. These discrete forms can be used directly in the iso-cost framework. It is useful, however, to convert the data points into smooth, continuous functions. The entire range of alternatives can then be considered. This was accomplished by using ordinary least squares regression (OLS) to fit quadratic functions to the data points of Table 5.[69] The continuous relations were given by:

$$TC = 17893.00 + 0.31070(AC) + 0.00000050357(AC)^2 \qquad (5.3)$$
$$R^2 = .99939$$

$$SA = 749.33 + 0.03653(TC) - 0.000000025712(TC)^2 \qquad (5.4)$$
$$R^2 = .99267$$

The continuous functions represented in (5.3) and (5.4) are also represented in Figures 32 and 33, respectively. Slightly greater

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[69] The constant term was used to account for the fact that no data was provided for a dam less than 30 feet in size. It should also be noted that the usual statistical measures are not applicable here. The discrete functions were deterministic; the least squares technique was used only to provide a continuous function. The R^2 values serve only to give an indication of the fit achieved.

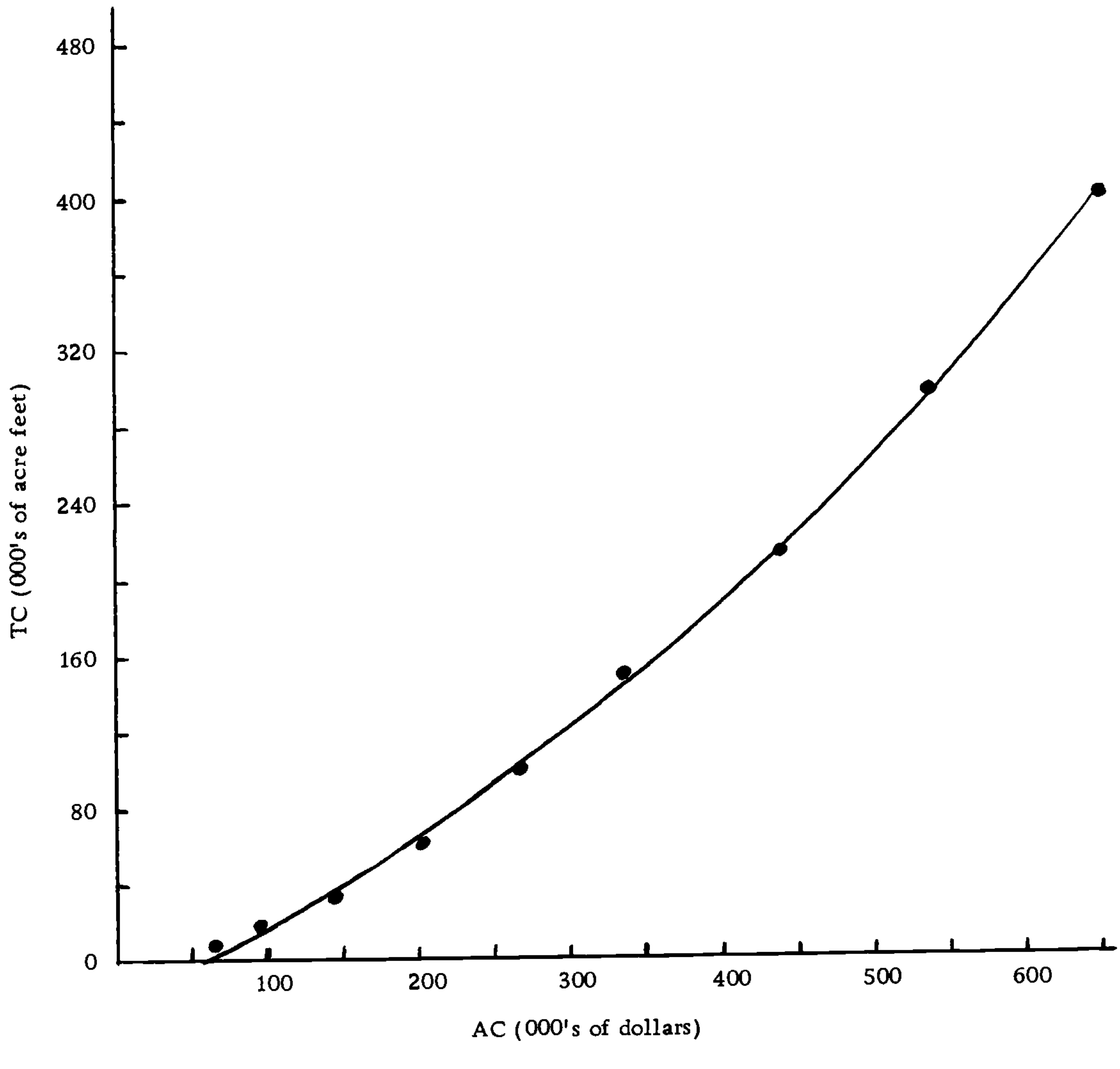

Figure 32. Reservoir capacity (TC) as a function of annual cost (AC, amortization and OMR), Bronco site, Knife River Basin, North Dakota.

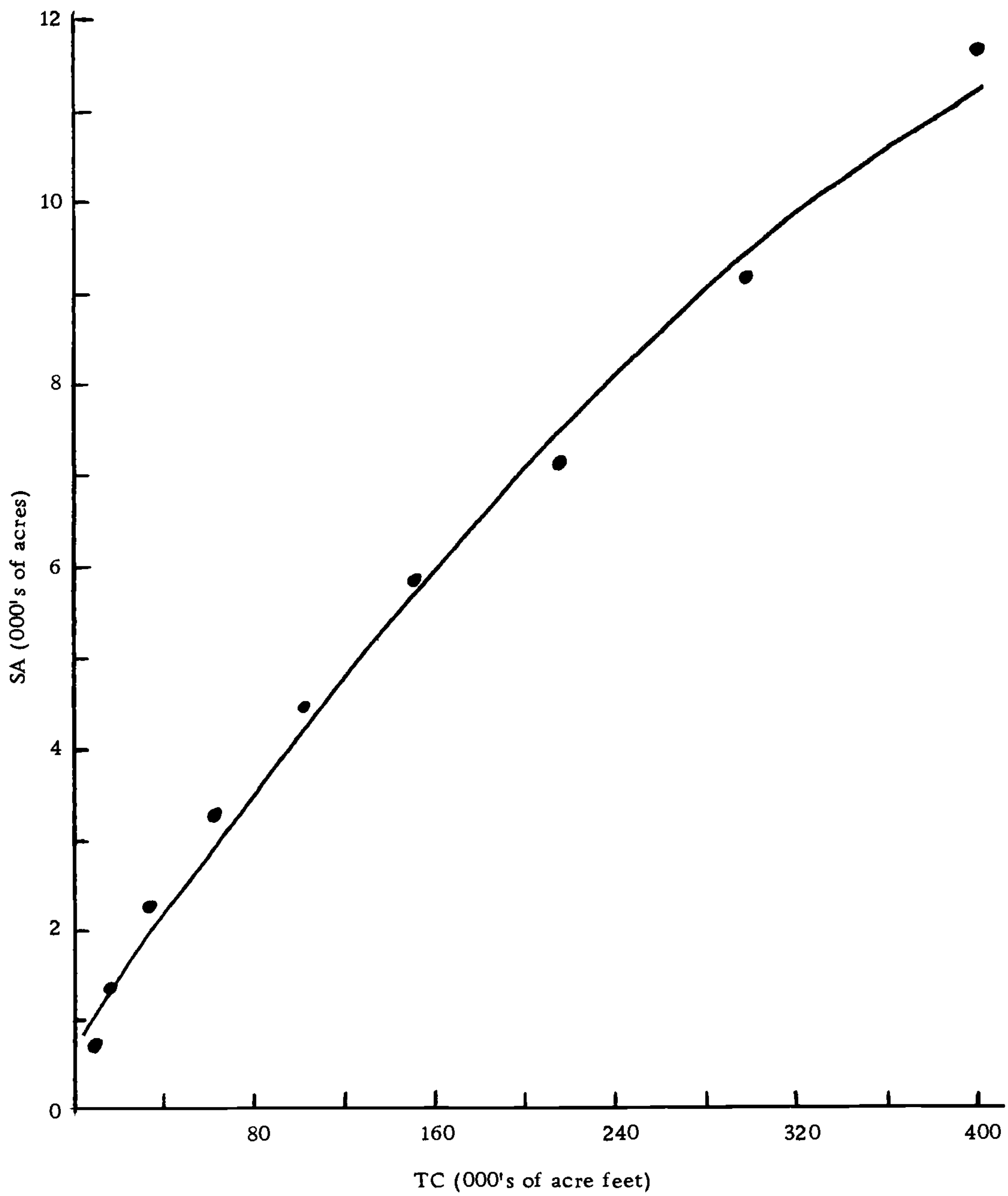

Figure 33. Surface area (SA) as a function of water in the reservoir (TC), Bronco site, Knife River Basin, North Dakota.

accuracy could have been obtained by using two functions in the case of surface area vs. capacity (Figure 33). In this author's judgement, however, an R^2 of .99267 was of sufficient magnitude to justify the use of one function. Two functions could easily be utilized if desired by the planner.

The iso-cost surface can now be derived. The problem can be defined in the following manner. Information is now available on:

1. the annual cost versus capacity relation,
2. the capacity vs surface area relation,
3. the water products for which there is a demand,
4. the price of one of those products,
5. the availability of water in the Basin at the site of concern.

The resource planner must organize this information in such a way as to make it possible for the decision making body to select the optimum size of the structure and the optimum product mix. The necessary information can be represented in the iso-cost surface.

The iso-cost surface for W_I and W_R was given at the Bronco site by:

$$W_R = 749.33 + 0.03653(TC-W_I) - 0.000000025712(TC-W_I)^2 . \tag{5.5}$$

A computer program was written (in Fortran) to "simulate" the alternative product mixes possible as dictated by the coefficients in equation (5.5).[70] Various product mixes were generated for several annual cost levels. The capacity-annual cost function in (5.3) was used to estimate the total capacity available for various (annual) cost levels. The capacity was then allocated, arbitrarily, between water for irrigation or surface area for recreation.[71] It was assumed that 10 percent of the capacity, for all capacity levels, would constitute "dead" storage and 30 percent of total capacity must be left for low stream flow years. As a result, only 60 percent of the total capacity was assumed available for irrigation in any irrigation season. Further, it was assumed that any water allocated for irrigation use could not be used for recreation. This assumption may have to be modified in every particular situation. It is a most reasonable assumption in the Knife River Basin, however, as any water used for irrigation cannot be used for recreation (as surface area) as recreation and irrigation demands occur during the same season. Also, no attempt was made to account for the fact that water in the reservoir in say, July, could be used for recreation that month and for irrigation in August. The allocation of water to irrigation was made for the entire irrigation-recreation season. No attempt was made to handle the dynamics of water flow and changes over time in the use pattern.

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[70] The algebraic manipulations used to derive (5.5) and the computer program are presented in Appendix C.

[71] All estimates are on an annual basis.

Water is allocated to irrigation (by drawing the reservoir down) or to recreation (by not drawing the reservoir down). The same water could not be allocated to both recreation and irrigation. This assumption does not, however, affect the validity of the iso-cost framework. The iso-cost surface is essentially an ex post description of product mixes anyway; i.e., the operating rule chosen affects the product combination. A different assumption regarding the relation between W_I and W_R will merely give a different iso-cost surface.

The iso-cost relations between water for irrigation, W_I, and water for recreation, W_R, (surface area) are presented in Figure 34.[72] Each relation (curve) represents a different (constant and minimum) annual cost. Costs were incremented by $75,000 from $200,000 to $650,000 (Figure 34). Movement along any one of the curves represents changes in W_I for changes in W_R. Movement from point A to point B along the $575,000 iso-cost relation, for example, represents a decrease in W_I of approximately 79,000 acre feet of water allocated to irrigation and an increase of 2200 acres of surface area available for recreation. This is the product-product "trade-off". A sacrifice of 79,000 acre feet of W_I gave rise to 2200 acres for the recreation activity. If the price of an acre of surface area was known, the optimum product combination on the $575,000

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[72] The data used to develop this figure is presented in Appendix D.

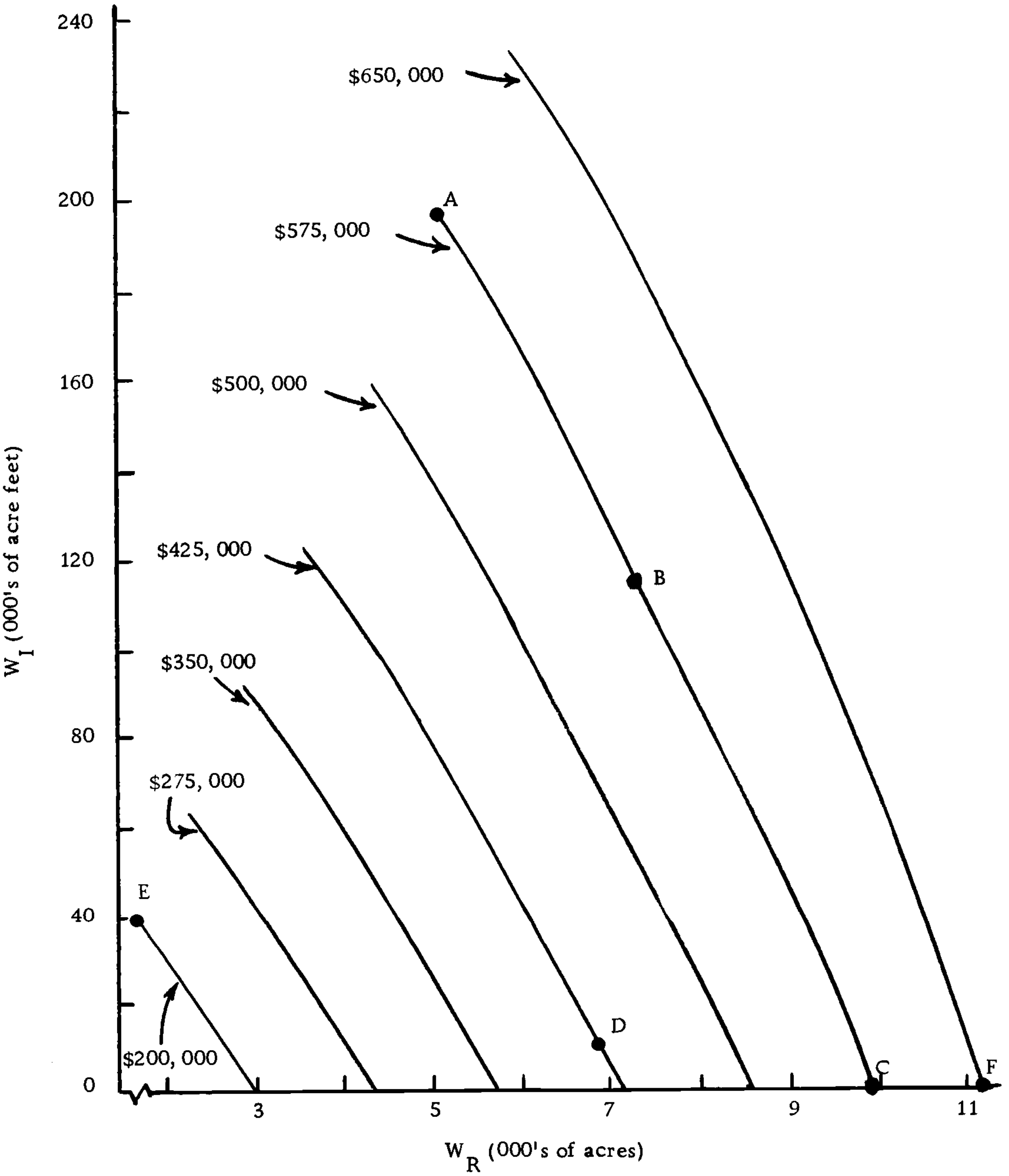

Figure 34. Iso-cost curves (annual cost: amortization and OMR), water for irrigation (W_I) vs water for recreation (W_R), Bronco dam site, Knife River Basin, North Dakota.

iso-cost curve could be chosen. Using the estimated residual value of W_I developed in the farm budget analysis as price, where P_{WI} = \$30.00, a "price" of W_R can be obtained. Assume the decision making entity chooses point B over point A; i.e., the entity was willing to sacrifice 79,000 acre feet at a price of \$30.00 to obtain 2200 acres of surface area. Using the conceptual model from previous chapters, it must be the case that:

$$\frac{\Delta W_I}{\Delta W_R} = - \frac{P_{WR}}{\$30.00}$$

or,

$$(-\$30.00)(- \frac{79,000}{2,200}) = P_{WR}$$

or,

$$P_{WR} = \$1077.00 \; .$$

The implicit price of one acre of surface area is approximately \$1077.00.

The production trade-offs for much smaller increments in W_R are provided in Appendix D for the \$200,000-\$650,000 iso-cost curves. The approximate trade-offs are shown for 100 acre increases in surface area while the exact trade-offs are calculated at a point from the derivative of the $W_I = f(W_R, TC)$ function derived in the analysis.[73] The point estimates of the trade-off are, of course,

[73]See Appendix C for derivation of the trade-off (slope) equation.

the most accurate. The point estimate at point B (Appendix D, Table D-7, Figure 34), for example, is given to be:

$$-P_{WI}\frac{dW_I}{dW_R} = P_{WR}$$

$$= (\$30.00)(-38.88) = \$1166.40 \ .$$

The estimate, using the 100 acre increase is given by:

$$(-\$30.00)(-38.79) = \$1163.70.$$

The latter estimate of $1163.70 acre of surface area is, of course, the average price over the curve up to point B from a point to the left of B by 100 acres. The two estimates are very close as the change in the slope of the curve is slight. The difference between point and interval estimates would be greater for relations having more curvature.

The trade-off ratios become greater as W_I is reduced for increases in W_R. Using the point estimates of the trade-offs, the ratio changes from -33.56 at point A to about -50.50 at point C (Appendix D, Table D-7, Figure 34). If the decision making entity selected point A, the implicit price of W_R is approximately (-$30.00)(-33.56) = $1006.80 as compared to (-$30.00)(-50.50) = $1515.00 at point C. The same type of relations exist along all the iso-cost relations illustrated in Figure 34.

Consider, for example, the trade-offs along the $200,000 iso-cost curve as shown in Figure 34. The curve represented in Figure 34 appears to be almost a straight line. The change in the slope is very slight, given as -28.41 for all the useable storage allocated to W_I (38,634 acre feet) and approximately -30.00 when the entire reservoir is used for recreation (Appendix D, Table D-1). The change is in the same direction, but the changes are larger, for the $650,000 iso-cost relation. The trade-off is only -35.25 when all the useable capacity is allocated to W_I (238,092 acre feet) and approximately -61.00 when the entire reservoir is used for recreation (Appendix D, Table D-8). For comparison purposes among the curves, the lowest trade-off is represented by -28.41 at point E on the $200,000 curve and the highest trade-off is represented by -61.00 at point F on the $650,000 iso-cost curve. In all cases, the trade-offs increase (become more negative) for movements "down" the curve of concern.

The relations in Figure 34 are the type of information needed by the decision making group for one dam site and two products, where the prices of one of the products is missing.[74] Other types of recommendations have been made elsewhere. Some of the various

[74] In addition, of course, the planning agency must provide information regarding water availability at the dam site. It may be impossible to reach any point on some of the curves because of available flows.

types of misleading trade-off ratios isolated with the conceptual model can now be illustrated using the iso-cost relations of Figure 34.

The calculation of trade-offs using net benefit changes between iso-cost curves can lead to very misleading estimates of the actual trade-offs. Assume, for example, the decision making entity is presented with only points A and D in Figure 34. Point A can be achieved with an annual cost of \$575,000 while point D can be obtained with \$150,000 less annual cost or only \$425,000. The "trade-off" would, intuitively, appear to be (data from Appendix D, Tables D-5 and D-7):

Point	W_I	W_R	NB
A	196,351	5090	\$5,315,542
D	10,919	6873	- 97,430
	-185,432	+1783	-\$5,412,972

which gives the trade-off at:

$$\frac{\Delta W_I}{\Delta W_R} = -\frac{185,432}{1783} = -104.00$$

or,

$$\frac{\Delta(NB)}{\Delta W_R} = -\frac{\$5,412,972}{1783} = -\$3036.00 .$$

A total of 104 acre feet of water evaluated at \$30.00 (gross) or \$3120 per acre are "sacrificed" to obtain one more acre of surface area (on the average, from point A to D). Stated in net benefit terms, \$3036.00 are given up to obtain one more acre of surface area (on the average) for recreation (as the reduction in cost is \$84.00 per acre foot). This is not the correct estimate of the value of W_R. Using the correct approach, the value of W_R at each of the points is given by:

Point	dW_I/dW_R	$(-P_{WI})(dW_I/dW_R) = P_{WR}$
A	-33.56	\$1006.80
D	-37.68	\$1130.40

The distortion is nearly three-fold using the "net-benefits-between-curves" approach. Prices or value must be estimated at points on particular curves. It is impossible to choose between points A and D without knowledge of the iso-cost curves for the \$425,000 and \$575,000 levels. Neither physical or net dollar benefit measures of trade-offs between curves can be used without some knowledge of the shape of the curves through those points.

The type of underlying production relation cannot be identified exactly given only the iso-cost relation at the Bronco site. It can be argued, however, the major class of production law (using the technical jargon of Chapter III) is factorially determined without coupling ($\alpha = 0$, $\kappa = 0$). It is technically <u>impossible</u> to find the allocation of the annual cost figure (or of any of the factors that constitute the annual cost figure) between W_I and W_R. The changes along the iso-cost curves of Figure 34 can be interpreted as changes in product level at one point in the factor space. This is illustrated in Figure 35.

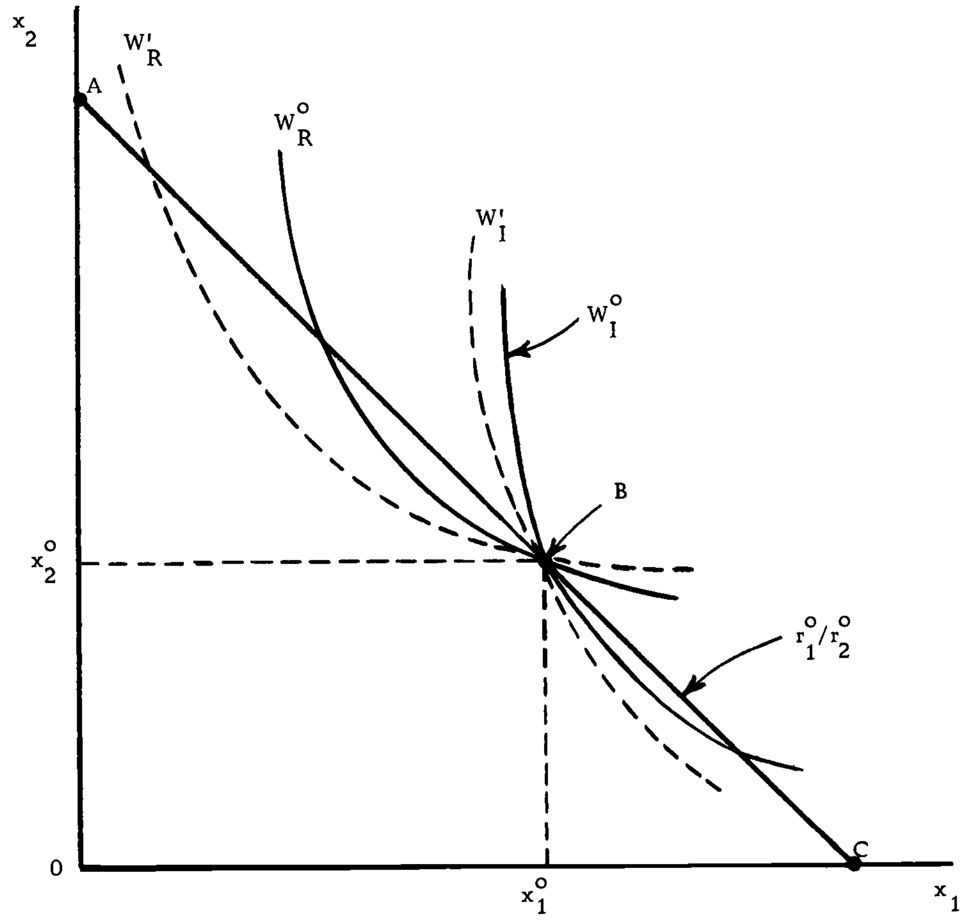

Figure 35. Hypothetical isoquants for factorially determined production without coupling.

Assume that line ABC represents a cost (minimum) of \$575,000. The factors x_2 and x_1 (say concrete and labor, among other factors) can be combined in varying proportions to produce the various levels of W_R and W_I for the same cost. Assume $x_1 = x_1^o$ and $x_2 = x_2^o$. Given one operating rule (one point on the \$575,000 iso-cost curve), $W_I = W_I^o = 196,351$ and $W_R = W_R^o = 5090$ as at point B in Figure 35. Given another operating rule, $W_I = W_I' = 117,097$ and $W_R = W_R' = 7290$, such as point B in Figure 34. The change in the operating rule curve, in essence, gives the different sets of isoquants in Figure 35. With the operating rule curve used at point A in Figure 34, the isoquant set W_R^o and W_I^o exists. With the operating rule at point B in Figure 34, the isoquants W_R' and W_I' exist. The various operating rule curves, it can be argued, change the coefficients on the factors in the factorially determined functions:

$$\begin{aligned} W_I &= f(x_1, x_2, \ldots, x_n) \\ W_R &= g(x_1, x_2, \ldots, x_n) \end{aligned} \qquad (5.6)$$

where,

x_j = factors, $(j = 1, 2, \ldots, n)$.

The functional forms remain the same in (5.6) while the coefficients change. The actual type of production relations, of course, can be determined only with an empirical test.[75]

The implications of classifying the underlying relations as factorially determined without coupling are great. As noted in Chapter IV, the actual trade-offs are not affected. The possibility for cost allocation, however, is not good. In fact, it is impossible to separate out the costs that should be allocated to each product. It is also interesting to note that the operating rule curve chosen determines the trade-off ratio. This gives a great deal of flexibility to the resource planner. Any trade-off (within the physical constraints defined by the iso-cost curve) can be calculated that is desired. The decision maker will not have enough information unless he has the whole iso-cost surface.[76]

The effect of technical interdependence and non-independence on the trade-off ratio can also be illustrated using the curves of Figure 34. The procedure for determination of technical relations was outlined in Chapter III and is applied here.[77] Figure 34 is reproduced in

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[75] Data on the actual factor levels would be needed. The data used here gave only the cost levels of all the factors used to build the structure.

[76] The relevant iso-cost surface can be delineated somewhat by the amount of water available at the dam site. There would be little need to present the iso-cost curve for the largest dam that could be built at a site if there was only sufficient flow to fill a reservoir half that size, for example. At the same time, all dam sizes up to the maximum dictated by the water availability in the basin may have to be considered.

[77] See Appendix B for a more detailed explanation of this approach to discovering technical relations.

Figure 36. Construction of box ABCH results in point C not being on the next iso-cost curve. As a result, products W_I and W_R are classified as technically complementary. The same result occurs for the construction of "boxes" among the other iso-cost curves. Points D, E, F, and G would lie on the iso-cost curve if the products were technically independent. In all cases, however, the products must be classified as technically complementary as points D', E', F', and G' are all above the "corner" of the respective boxes. This classification says, in essence, that the marginal cost of providing $W_I(W_R)$ goes down as more $W_R(W_I)$ is produced. This would be expected in a dam-reservoir configuration and probably explains the many multipurpose structures built in this country. Note, however, that W_I does in fact decrease for increases in W_R even though the two products are technically complementary.

Product-Product Trade-Offs at the Kineman Site. Similar types of results were also obtained when the same approach was used at the Kineman site (Figure 31). Another non-independent iso-cost curve was derived using the basic data representative of this site as shown in Table 6. Again, annual cost was calculated by adding the amortization and O, M, and R costs. The capacity versus annual cost and surface area versus capacity relations of Table 6 are represented by the discrete points of Figures 37 and 38, respectively. Ordinary least squares regression was used to fit quadratic equations to the

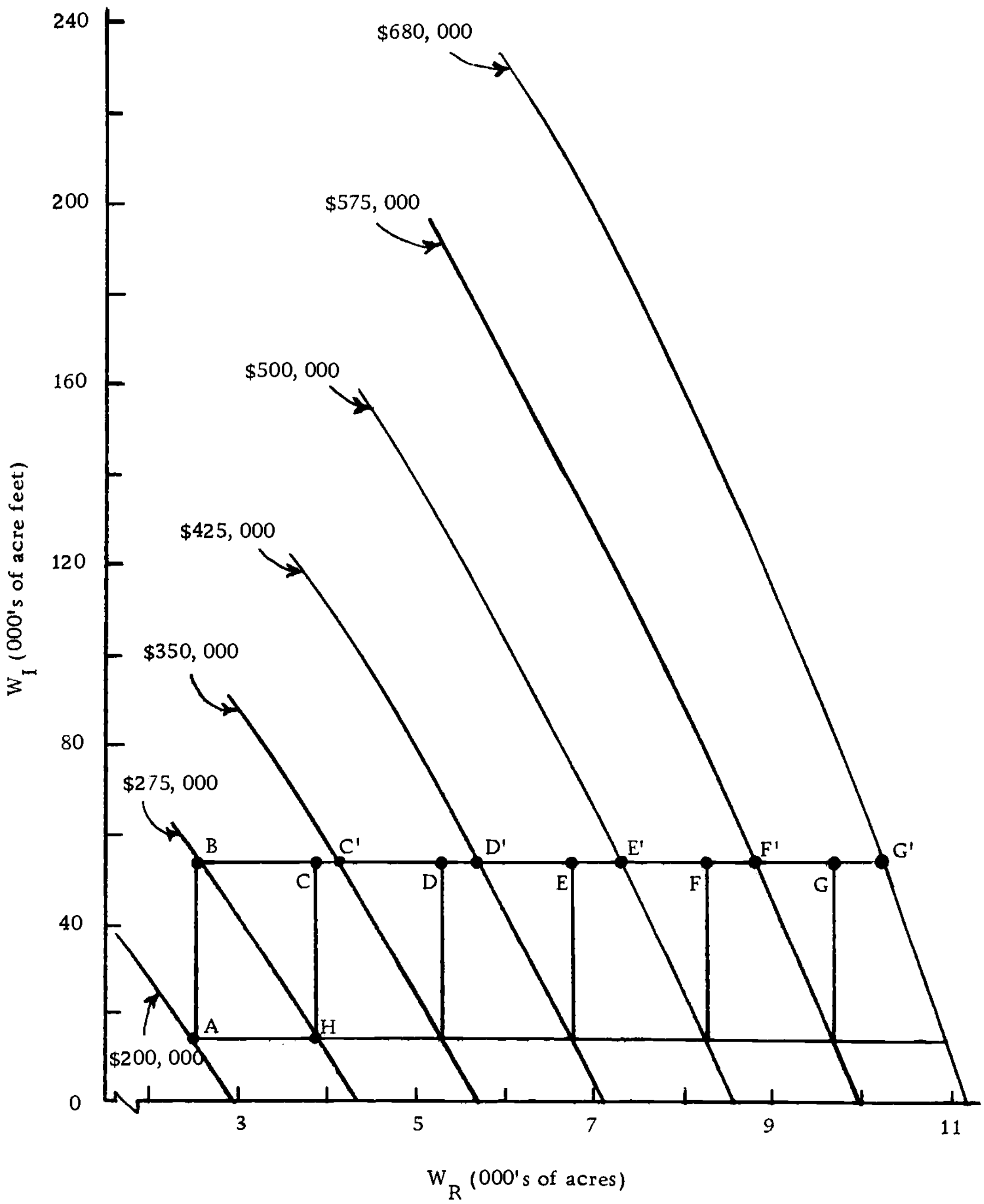

Figure 36. Iso-cost curves (annual costs: amortization and OMR), water for irrigation (W_I) vs water for recreation (W_R), Bronco dam site, Knife River Basin, with geometric illustration of technical complementarity.

Table 6. Total investment requirements, annual costs, reservoir capacity, and reservoir surface area, Kineman dam site, Knife River Basin, North Dakota.

Dam Height	Total Investment Requirement	Annual Amortization[a]	Annual OMR	Total Annual Costs[b]	Reservoir Capacity[c]	Reservoir Surface Area[d]
(feet)	(- - - - - - - - - - - -	dollars- - - - - - - - - - - - -		- -)	(acre-feet)	(acres)
30	210,980	11,781	1,055	12,836	166	15
40	312,556	17,453	1,563	19,016	363	25
50	336,265	18,776	1,681	20,457	683	39
60	385,116	21,504	1,926	23,430	1,228	70
70	608,552	33,980	3,043	37,023	2,408	166
80	903,500	50,450	4,518	54,968	4,757	304
90	1,315,930	73,479	6,580	80,059	8,507	447
95	1,513,112	84,489	7,566	92,055	11,161	531

[a] Based on an interest rate of 5 1/8 percent and a 50 year repayment period.

[b] Sum of annual amortization and annual OMR.

[c] Capacity measured at spillway level.

[d] Surface area of reserovir behind dam when reservoir full to spillway level.

Basic data used to derive table from (North Dakota State Water Commission, "The Plan. . .", 1971, pp. 87-89).

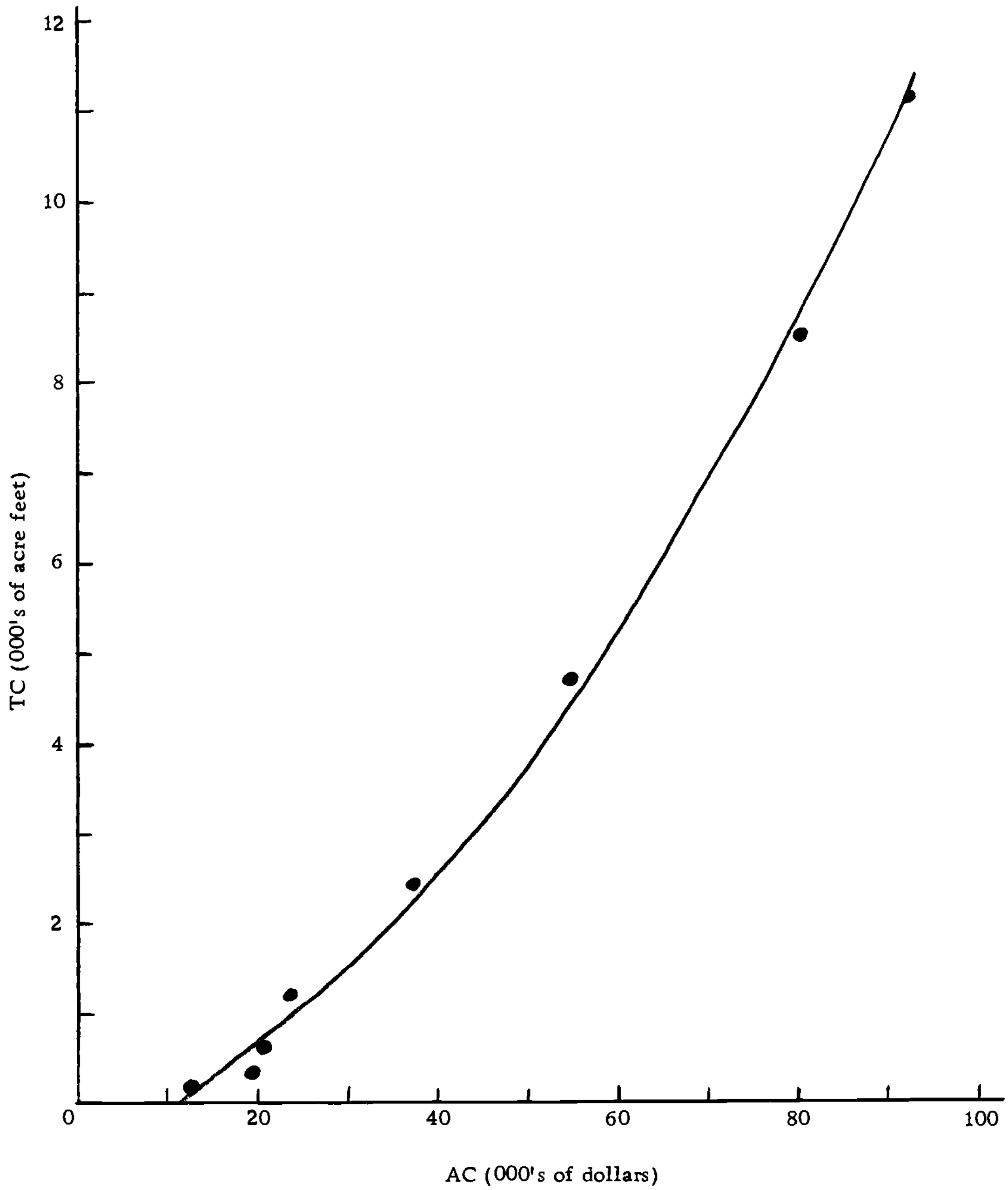

Figure 37. Reservoir capacity (TC) as a function of annual cost (AC, amortization and OMR), Kineman site, Knife River Basin, North Dakota.

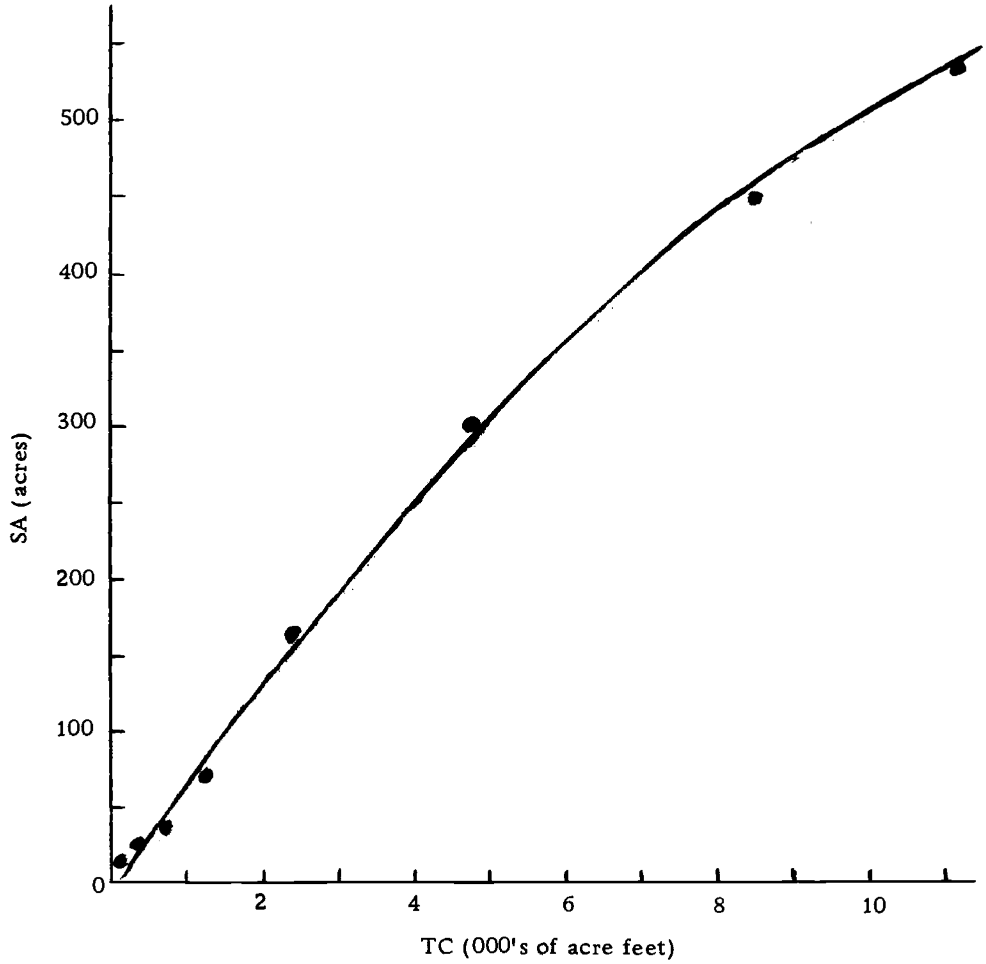

Figure 38. Surface area (SA) as a function of water in the reservoir (TC), Kineman site, Knife River Basin, North Dakota.

discrete data points to give the continuous curves shown in each figure. The continuous relations were given by:

$$TC = -715.63 + 0.054476(AC) + 0.00000079369(AC)^2 \qquad (5.7)$$

$$R^2 = .9984$$

$$SA = -4.8355 + 0.074593(TC) - 0.0000024083(TC)^2 \qquad (5.8)$$

$$R^2 = .9984$$

These equations provide the necessary information to start the iso-cost calculation process. The same approach as used for the Bronco site resulted in the iso-cost curves at the Kineman site as represented in Figure 39. Again, the same general relations exist. Trade-offs increase as W_R is increased for sacrifices in the level of W_I for any given annual cost. The water in storage, water for irrigation, and surface area levels at the Kineman site for various annual cost levels are presented in Appendix D, Tables D-9 to D-16, for increases in annual cost of \$10,000. Again, trade-offs have a considerable range. The lowest trade-off estimate is given at point B on the \$20,000 iso-cost curve (Appendix D, Table D-9). At that point, the slope is -13.65 (Appendix D, Table D-9). The highest trade-off is given at point C, where the slope of the \$90,000 iso-cost curve is approximately -40.20 (Appendix D, Table D-16). For P_{WI} = \$30.00, this is a range in value (assuming there is a point in the space that

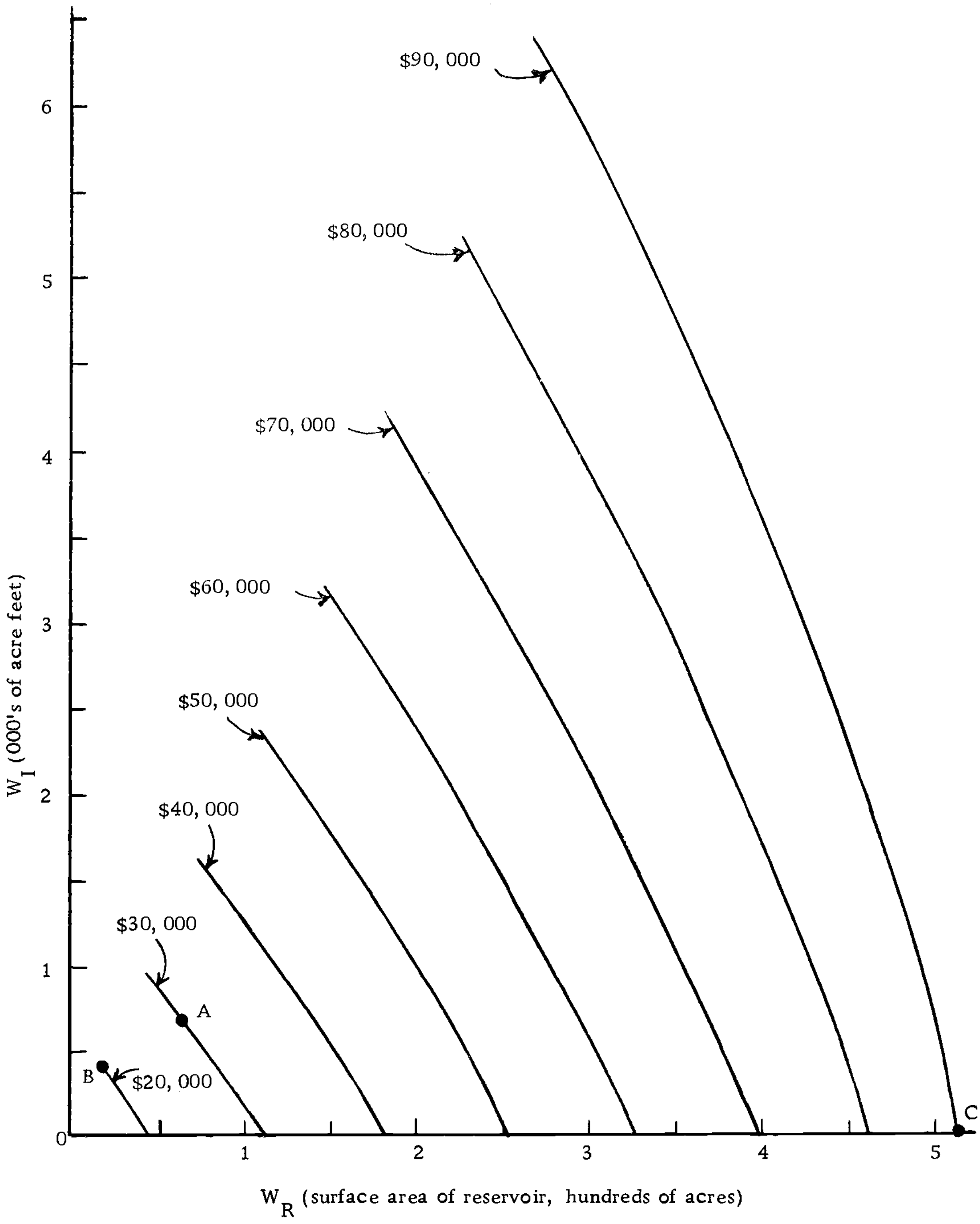

Figure 39. Iso-cost curves (annual costs: amortization and OMR), water for irrigation (W_I) vs water for recreation (W_R), Kineman site, Knife River Basin, North Dakota.

would be selected) of (-\$30.00)(-13.65) = \$409.50 to (-40.20)(-\$30.00) = \$1206.00 for an additional acre of surface area.

Product-Product Trade-Offs for Both the Bronco and Kineman Sites. The resource planner will, in many cases, be faced with planning development where more than one structure could be built. The available investment budget must now be allocated between the alternative sites as well as among the uses at each site. The physical setting is illustrated in Figure 31. Water can be diverted from the Bronco site to irrigate all the land down stream. The Kineman site could, however, be used to irrigate part of the acreage below the Kineman site (Figure 31). Water for recreation can be provided at the Bronco and/or the Kineman site. The surface area-capacity and capacity-annual cost relations at each site were combined to give the iso-cost curves.[78] The goal, of course, is to find the iso-cost curve the furthest from the origin for any given cost. This curve will be the true iso-(minimum)cost curve which is needed by the decision making body. The assumptions used in the derivation of the iso-cost relations are, however, very important.

One approach is to assume a certain constant proportion of the capacity of each reservoir must be kept in "dead" and/or emergency storage. For illustrative purposes, assume a minimum of 40 percent

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[78]See Appendix C for a detailed description of the mathematical manipulations.

of capacity must be maintained in each reservoir. Also, assume that all the useable capacity (60 percent) at the Kineman site is used for irrigation. Given some annual cost, the iso-cost curves between W_I and W_R can be derived by varying W_R. The curves resulting from these assumptions are presented in Figure 40 for the \$300,000 and \$600,000 cost levels.

The assumption regarding useable capacity is reflected in the location of the curves. The amount of land irrigated with water from the Kineman site is constant along each iso-cost curve. Iso-cost curve AA', for example, represents all the possible (W_I, W_R) combinations for the particular cost allocation given 60 percent of the water capacity at the Kineman site is used for irrigation. This results in the production of approximately 5233 acre feet of W_I at the Kineman site along the entire AA' curve. This is the reason AA' does not reach the horizontal axis; i.e., point A' reflects the fact that approximately 5233 acre feet of W_I is always provided at the Kineman site. Also, there is always a minimum of about 226 acres of surface area (W_R) produced at the Kineman site. The amount of W_I and W_R provided at the Kineman site decreases, of course, as the allocation of expenditure on the Kineman site decreases. This is reflected by the iso-cost curve EE' touching the horizontal axis when the cost allocation to Kineman is zero. The W_I and W_R levels produced at the Kineman site are zero along the entire curve EE'. The same type

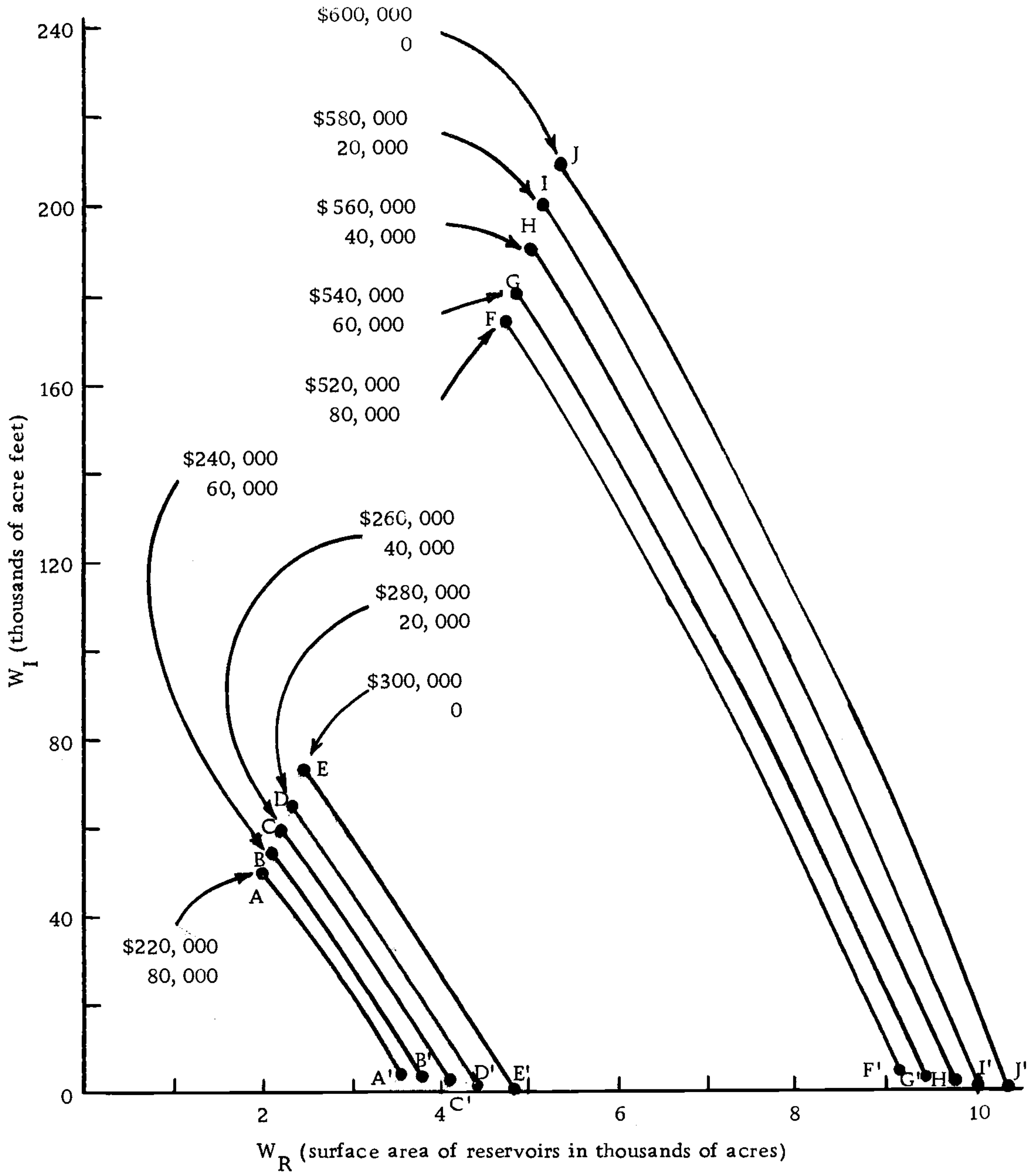

Figure 40. Iso-cost curves (annual costs: amortization and OMR), water for irrigation (W_I) vs water for recreation (W_R), Bronco and Kineman sites, Knife River Basin, North Dakota.

of relationships are obtained for an expenditure of \$600,000 as represented in curves FF'-JJ' (Figure 40). Curve JJ', the frontier curve, represents the zero expenditure allocation to the Kineman site. In either case then, the maximum amount of W_I and W_R can be obtained by allocating all available capital to the Bronco site (curves EE' and JJ'). Given that W_I and W_R have the same prices at the different sites, only the frontier curves, such as EE' and JJ' need to be presented to the decision making body. It should be noted that curves EE' and JJ' (and others like them) are the only true iso-cost curves depicted in Figure 40.

The cost curves of Figure 40 do not, however, exhaust the possibilities for the two sites. As was noted, the level of W_I and W_R at one of the sites was required to be constant along any one curve (for any one total cost and expenditure allocation) in the previous formulation. Other cost curves can be generated in the space given a different assumption. The assumption regarding constant W_I and W_R at Kineman was relaxed to generate the cost curves of Figure 41. Curves AA', CC' and EE' are reproduced (from Figure 40) to provide a reference point. Curve KK' was generated by reducing the amount of land irrigated at the Kineman site to zero. In other words, curves CC' and KK' represent the same total cost and same allocation of expenditure, but represent different amounts of land irrigated with water from the Kineman site. Curve KK', then, represents the cost

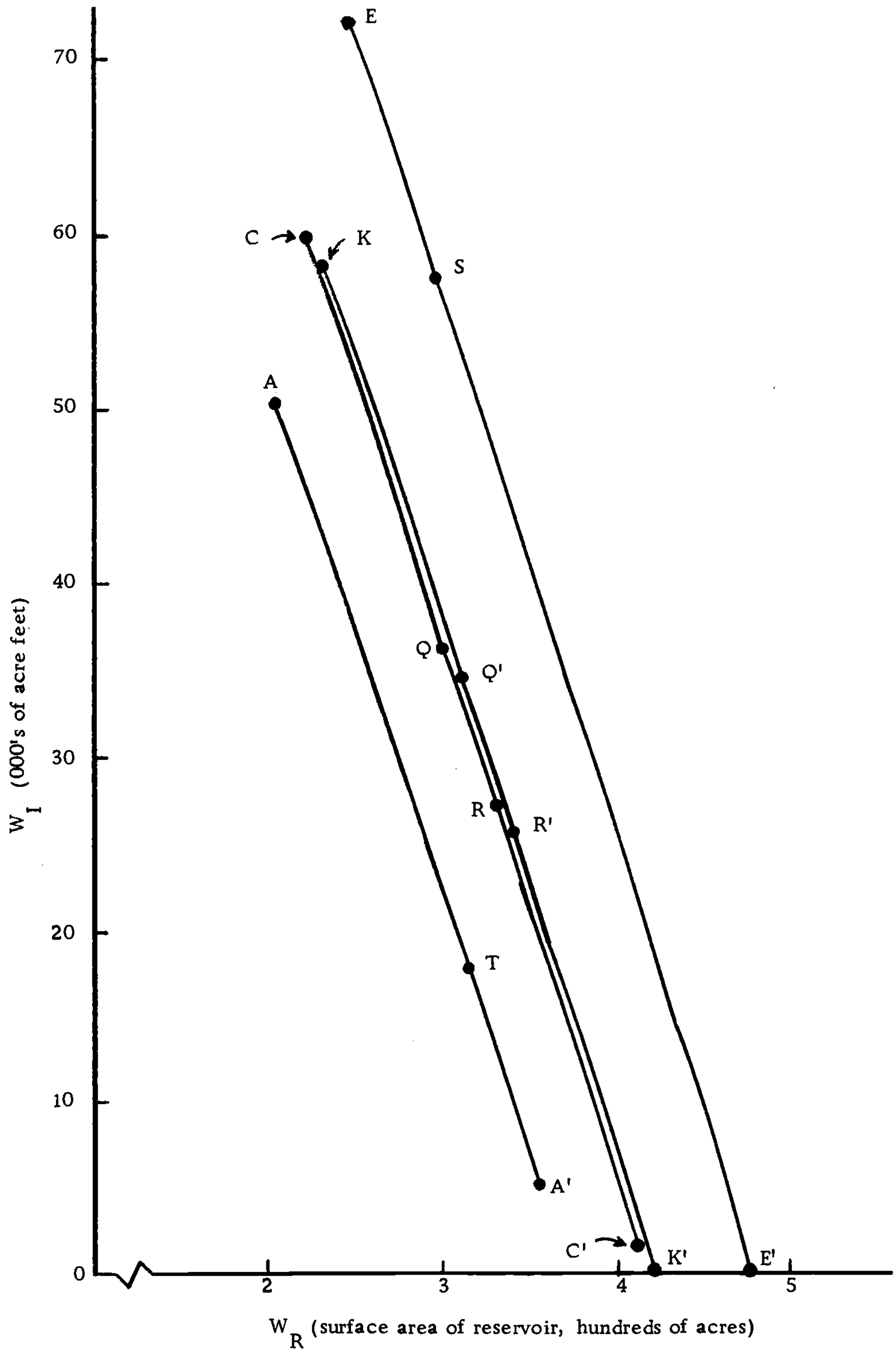

Figure 41. Iso-cost curves (annual costs: amortization and OMR), water for irrigation (W_I) vs water for recreation (W_R), Bronco and Kineman sites, variable levels of irrigation at Kineman site, Knife River Basin, North Dakota.

relation for the particular expenditure allocation when the Kineman site is used entirely for recreation. Other possibilities also exist. If, for example, 50 percent of the capacity at Kineman was used for irrigation (as compared to 60 percent on CC' and 0 percent on KK') the curve would fall between CC' and KK'. The area between CC' and KK' could be filled with an entire family of curves each representing a different amount of water for irrigation (and W_R) produced at the Kineman site. Curve CC' and KK', then, represent the boundaries of concern for the allocation of \$260,000 and \$40,000 to the Bronco and Kineman sites, respectively.

Another interesting aspect of the relaxation of this assumption arises with regard to the slope changes of the curves. The slope on CC' at point Q is the same as the slope on KK' at point Q' (the slopes are equal at a value of approximately -30.00). Similarly, the slopes at R and R' are the same (at approximately -31.00). As a result, if curve CC' was provided to the decision making body when KK' was further out in the production space, distortions would result.[79] Assume, for example, the consumption trade-off was -30.00. Given curve CC', point Q would be chosen. On the other hand, if KK' was presented to the decision body, point Q' would be chosen. An incorrect allocation of reservoir capacity would have occurred as too much

[79] Distortions in addition to that provided by presenting CC' rather than EE'.

water would be allocated to irrigation at point Q.

Even greater distortion will occur, of course, if the "frontier" curve is not found and presented to the decision body. Even with the relaxed assumption (to allow varying amounts of W_I and W_R at Kineman) the "frontier" iso-cost curves are given by the zero allocation of the expenditure levels to the Kineman site. At the $300,000 total annual expenditure level, for example, the frontier is represented by EE' in Figures 40 and 41. This is the only true iso-cost (iso-<u>minimum</u> cost) curve for the $300,000 expenditure level. Curve EE' should be presented to the decision making body. Assume, however, some internal curve such as AA', CC', or KK' is presented to the decision body (Figure 41). Also, assume the consumption tradeoff ratio (as perceived by the decision body) is -30.00. The optimum point on the frontier curve EE' is given by point S. At point S, approximately 57,531 acre feet of W_I and 2952 acres of W_R (surface area) are produced. Given any of the internal curves, however, that combination will not be chosen (indeed, it cannot even be produced). Given AA', for example, T is the optimum point as the slope is approximately -30.00 at point T. The resulting product combination is 17,933 acre feet of W_I and 3156 acres of W_R (surface area). Similar, but less extreme, distortion in resource allocation would occur at point Q on CC' and Q' on KK' (where the slopes are also -30.00).

One can also envision a situation where (incorrect) trade-offs would be calculated between curves such as those depicted in Figure 41. This mistake could be made especially when trade-offs are calculated as changes in net benefits for alternative project designs. Curves AA' and EE' can be viewed as iso-cost curves for two different project designs. Consider, for example, the change in net benefits in moving from point S on EE' to point T on AA'. The "trade-offs" are given by, for $P_{WI} = \$30.00$:

Point	W_I	W_R	NB
S	57,531	2952	$1,425,930
T	17,933	3146	237,990
	-39,598	+194	-$1,187,940

which gives the trade-off at:

$$\frac{\Delta W_I}{\Delta W_R} = -\frac{39,598}{194} = -204.00$$

or,

$$\frac{\Delta(NB)}{\Delta W_R} = -\frac{\$1,187,940}{194} = -\$6123.00.$$

The loss in net benefits for an additional acre of surface area in moving from point S to point T is approximately $6123.00. This is (grossly) incorrect. The true product-product trade-off is -30.00 at point T and at point S (or in dollar values, \$900).[80] The "trade-off" as calculated by that approach is not a trade-off in the sense of the iso-cost curve. In fact, movement from point S to point T represents moving away from the iso-(minimum) cost curve to a less efficient, internal curve; i.e., the "net-benefit-between-curves" trade-off in reality reflects inefficient use of resources and not the sacrifice in W_I necessary to produce more W_R. Similar results could be shown for the \$600,000 (or any other) expenditure level. This type of "trade-off" appears to be the type of trade-off recommended by Maass (1966, see p. 11, Chapter II). Maass speaks of trade-offs between efficiency and some product not having a known market price. An efficiency trade-off seems akin to moving to some internal curve (less efficient curve) as just discussed. It would appear a better approach would be to move to an interior curve by reducing expenditure. This could be accomplished by moving to, for example, a frontier (most efficient) iso-cost curve for only a \$200,000 expenditure rather than to a curve internal to the most efficient (frontier) curve at the \$300,000 level.

Product-Product Trade-Off Ratios Among Three or More Water Products at One Site

The resource planner is generally faced with planning a

[80] The estimated value of W_R at points S and T is given by (-\$30.00)(-30.00) or \$900.00.

development where more than two products are produced. Consider, for example, the possibility of providing water for municipal and industrial uses (W_M) as well as W_I and W_R. This possibility existed at the Bronco site and is used for illustrative purposes here.[81] The basic equations needed are, again, the surface area-capacity and capacity-annual cost relations as determined in equations (5.3) and (5.4). Assume there is a known price for $W_I(P_{WI})$ and for $W_M(P_{WM})$. Also, assume P_{WR} is unknown. The resource planner is now faced with providing an iso-cost surface as the consideration of three products adds another dimension to the problem. Trade-offs can still be calculated between W_I and W_R; the location of the iso-cost curve in the W_I, W_R plane is now, however, affected by the level of W_M. Two alternative assumptions were made regarding the affect of W_M on the shape and location of the iso-cost surface.

Product-Product Trade-Offs at the Bronco Site, Constant Ratios Between Two of Three Products. The case of a constant trade-off ratio between two of three water products was generated by assuming that water could be released from the reservoir at the Bronco site either for W_I or W_M. Given this case, an increase in $W_M(W_I)$ by

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[81]The earlier study (North Dakota State Water Commission, "The Plan. . .", 1971) did not result in recommending multiple use of the Bronco Reservoir. The possibility of such use, however, did exist in earlier versions of the original plan. The three uses were chosen here for purposes of illustrating the use of the iso-cost framework in trade-off calculations.

one acre foot causes a reduction in $W_I(W_M)$ by one acre foot. The iso-cost relation is given by:

$$W_R = 749.33 + 0.03653(TC-W_I-W_M) - 0.000000025712(TC-W_I-W_M)^2 \quad (5.9)$$

The coefficients of equation (5.9) were derived from equation (5.4), where surface area is used as a measure of W_R. The product-product trade-off ratio between W_M and W_I in this formulation is constant and equal to -1.00.[82] The iso-cost relations between W_I and W_R generated from equation (5.9) are presented in Table 7 and Figure 42. The expenditure level is constant at \$200,000 for all four iso-cost curves. Each of the four curves, in turn, represent different levels of W_M. Curve EE', for example, represents the iso-cost (trade-off) curve for W_I and W_R when $W_M = 0$. The iso-cost curve, in essence, "shifts" down for increases in W_M. Curve DD' represents the iso-cost relation between W_I and W_R when $W_M = 10,000$ acre feet. An increase in W_M gives rise to curves interior to the iso-cost relation for the zero level of W_M. Point A represents the iso-cost relation between W_I and W_R when all the useable capacity is allocated to the production of $W_M (W_I = 0)$.

---

[82] See Appendix C for derivation of the slope equations.

Table 7. Combinations of water for irrigation, municipal (and industrial) uses and recreation, for $200,000 annual cost, Bronco dam site, Knife River Basin, North Dakota.

Water for Municipal and Industrial (W_M)	Water for Irrigation (W_I)	Water for Recreation (W_R)	$\frac{dW_I}{dW_R} = \frac{dW_M}{dW_R}$ [a]
(acre feet)	(acre feet)	(acres)	
0	38,634	1673	-28.41
0	32,929	1873	-28.64
0	27,176	2073	-28.89
0	21,372	2273	-29.14
0	15,519	2473	-29.40
0	9,612	2673	-29.66
0	3,653	2873	-29.94
0	0	2995	-30.10
10,000	28,634	1673	-28.41
10,000	22,929	1873	-28.64
10,000	17,176	2073	-28.89
10,000	11,372	2273	-29.14
10,000	5,519	2473	-29.40
10,000	0	2660	-29.60
20,000	18,634	1673	-28.41
20,000	12,929	1873	-28.64
20,000	7,176	2073	-28.89
20,000	1,373	2273	-29.14
20,000	0	2320	-29.20
30,000	8,634	1673	-28.41
30,000	2,929	1873	-28.64
30,000	0	1975	-28.80

[a] The rates of change (trade-offs) in W_I and W_M for changes in W_R. Both are equal at every point. Also, $(dW_I/dW_M) = -1.0$ at every point on the surface.

A better picture of the iso-cost relation can be gained by reference to Figure 43. Figure 43 was also developed from data in Table 7. Point A and curves BB', CC', DD', and EE' of Figure 42 are represented in three-space in Figure 43. Curve BB', for example,

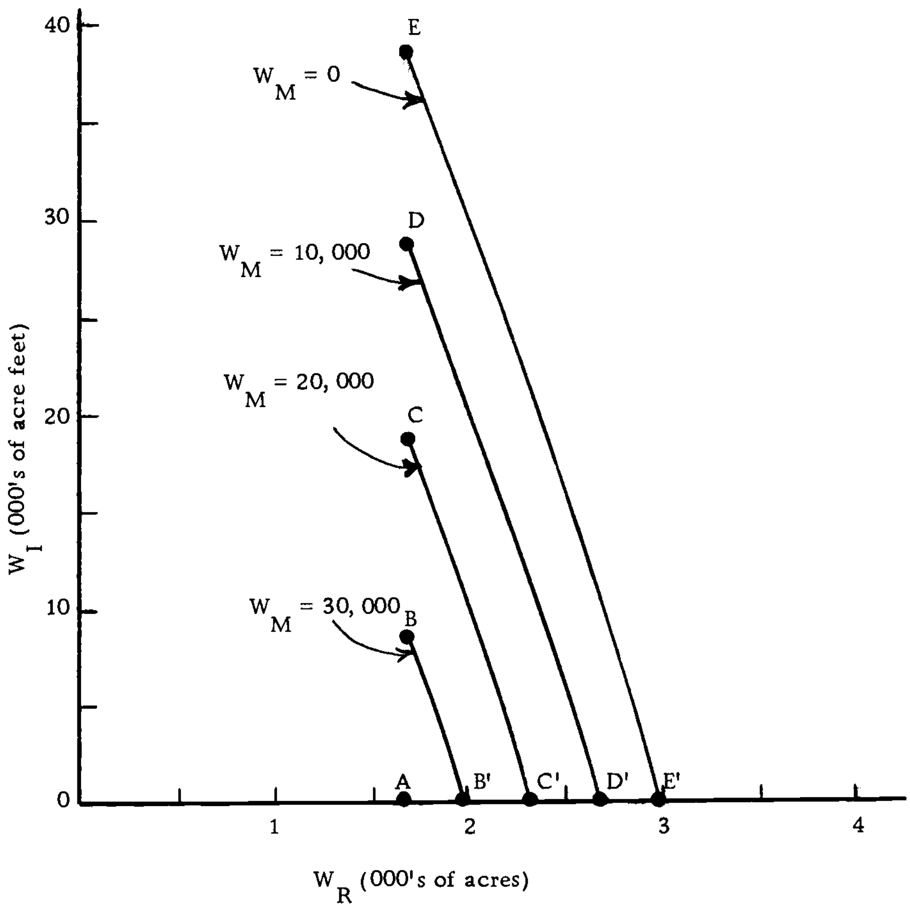

Figure 42. Iso-cost curves (annual costs: amortization and OMR), three products: water for irrigation (W_I), recreation (W_R), and municipal use (W_M), constant trade-offs at Bronco site, Knife River Basin, North Dakota.

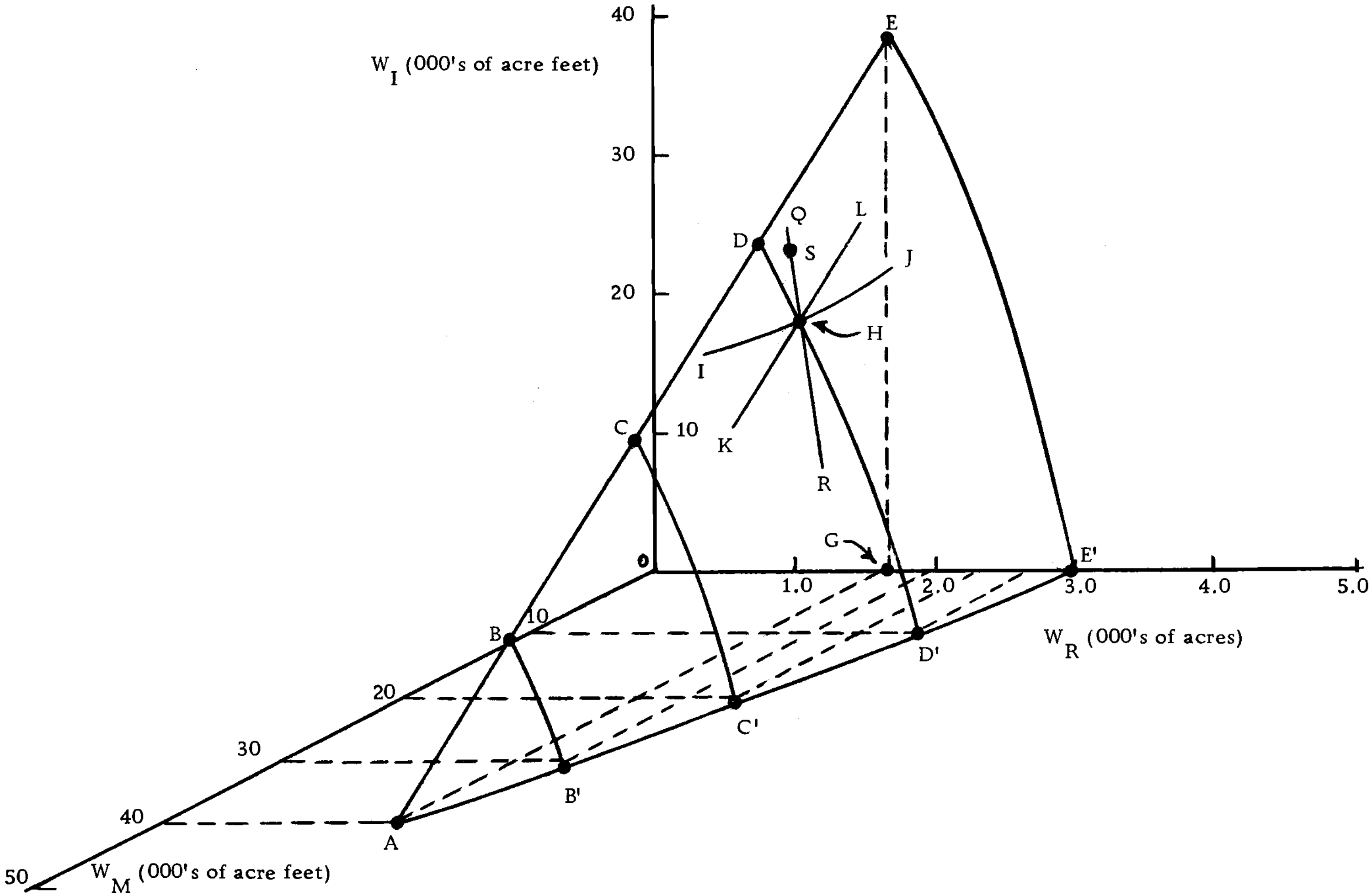

Figure 43. Iso-cost surface (annual costs: amortization and OMR), three products: water for irrigation (W_I), recreation (W_R), and municipalities (W_M), constant trade-off between two products, Bronco site, Knife River Basin, North Dakota.

represents the iso-cost relation between W_I and W_R when W_M = 30,000 acre feet. Point A represents the situation where all useable capacity (approximately 38,000 acre feet) is used in the production of W_M. Point A lies in the "floor" (the W_R, W_M plane) of the three-space diagram. Any point above the W_R, W_M plane represents some level of W_I production greater than zero. The shape and the location of the surface in three-space is influenced, of course, by the physical relations at the site of concern. Also, the assumptions utilized in the development of the surface can influence the location and shape. Point G, for example, represents the assumption regarding minimum reservoir storage levels. As noted earlier, it was assumed that 40 percent of the capacity was to be maintained; i.e., useable capacity was set at 60 percent. The surface area of the reservoir, as a result, is always at least 1674 acres as represented at point G. The total useable capacity at point G can then be allocated entirely to W_I (point E), entirely to W_M (point A) or some combination of W_I and W_M (line ABCDE). Product W_I can be "sacrificed" for increases in W_M along ABCDE or any other line originating on EE' and running down the surface parallel to ABCDE. The other defining outline of the surface is curve AB'C'D'E'. This curve represents the iso-cost relation between W_R and W_M for $W_I = 0$.

There are now three trade-offs of concern that can be calculated. Movement along curves such as BB', CC', DD', and EE' represent varying trade-offs between W_I and W_R at different levels of W_M. Movement along lines such as ABCDE represent trade-offs between W_I and W_M for different levels of W_R. Similarly, movement along curves such as AB'C'D'E' represent trade-offs between W_R and W_M. All curves across the surface represent iso-(minimum)cost curves. At point H, for example, the three trade-off ratios are given by:

$$\begin{aligned} &dW_I/dW_R, && \text{along DD}', \\ &dW_R/dW_M, && \text{along IJ}, \\ &dW_I/dW_M, && \text{along KL}. \end{aligned}$$

As shown in previous chapters, in order for point H to be an optimum point, it must be the case that:

$$\begin{aligned} \frac{\Delta W_I}{\Delta W_R} &= -\frac{P_{WR}}{P_{WI}}, \\ \frac{\Delta W_R}{\Delta W_M} &= -\frac{P_{WM}}{P_{WR}}, \\ \frac{\Delta W_I}{\Delta W_M} &= -\frac{P_{WM}}{P_{WI}}. \end{aligned} \tag{5.10}$$

Given P_{WI}, P_{WM}, and the production trade-off ratios, the price of $W_R (P_{WR})$ can be found from (5.10).

The particular slopes along the surface as derived at the Bronco site can now be isolated. The trade-off between W_I and W_M was constant at -1.0 at all points, such as H, on the surface. This trade-off is reflected in Figure 43 by the slope of line ABCDE which is -1.0 over the entire range. The slopes of curves such as DD' and IJ, were found to vary over the surface, but are equal at any one point on the surface; i.e., $(dW_I/dW_R) = (dW_M/dW_R)$ at every point on the surface. The slope of DD' (dW_I/dW_R) at point H, for example, is -28.64 (Figure 43 and Table 7). The slope of IJ (dW_M/dW_R), at the same point, is also -28.64. This was also due to the nature of the problem and the assumptions made. The rate of change in W_I for changes in W_R, given by (dW_I/dW_R), is not affected by the level of W_M. Similarly, (dW_M/dW_R) is not affected by the level of W_I. The relations among the trade-offs can be summarized in the following manner:

$$\frac{dW_I}{dW_R} = \frac{dW_M}{dW_R} = \text{variable},$$

$$\frac{dW_I}{dW_M} = -1.0 = \text{constant}, \tag{5.11}$$

$$\frac{\partial}{\partial W_M}\left(\frac{\partial W_I}{\partial W_R}\right) = \frac{\partial}{\partial W_I}\left(\frac{\partial W_M}{\partial W_R}\right) = \frac{\partial}{\partial W_R}\left(\frac{\partial W_I}{\partial W_M}\right) = 0.$$

The production trade-off between $W_I(W_M, W_I)$ and $W_R(W_R, W_M)$ is not affected by the level of $W_M(W_I, W_R)$. The equations in (5.11) hold over the entire iso-cost surface illustrated in Figure 43.

The implicit price of W_R can now be determined given prices of the other products and the point selected by the decision body. Assume, for example, that $P_{WI} = P_{WM} = \$30.00$. Also, assume point H ($W_R = 1873$, $W_I = 22{,}929$, $W_M = 10{,}000$) is selected by the decision body. The implicit price of W_R is then given by:

$$
\begin{aligned}
\frac{dW_I}{dW_R} &= -28.64 = -\frac{P_{WR}}{P_{WI}} = -\frac{P_{WR}}{\$30.00} \\
\frac{dW_M}{dW_R} &= -28.64 = -\frac{P_{WR}}{P_{WM}} = -\frac{P_{WR}}{\$30.00} \qquad (5.12) \\
\frac{dW_I}{dW_M} &= -1.00 = -\frac{P_{WM}}{P_{WI}} = -\frac{\$30.00}{\$30.00} .
\end{aligned}
$$

The equations in (5.12) are all satisfied if and only if the price of W_R is:[83]

$$
-\frac{P_{WR}}{\$30.00} = -28.64
$$

$$
P_{WR} = \$859.35
$$

This implies the last acre of surface area added was worth \$859.35 to the decision body. Value has been made explicit. Assuming the decision body can accurately reflect society's demand curve for W_R, the estimated price of W_R will range from (-28.41)(-\$30.00) = \$852.19

[83] The estimated trade-off was -28.6450 giving an estimated price of (-28.6450)(-\$30.00) = \$859.35. The trade-offs were rounded to two significant digits for presentation in the text.

all along curve ABCDE to (-30.10)(-\$30.00) = \$903.00 at point E' (Table 7 and Figure 43). The range would be much greater for iso-cost surfaces having more variable slopes.

The same estimates of price can also be derived by using the changes in net dollar benefits (NB) along the surface.[84] The trade-off ratio between W_I and W_M, however, must be equal to the relevant inverse product price ratio at all points at which net benefits are calculated. Stated in mathematical notation, this means the condition $(dW_I/dW_M) = -(P_{WM}/P_{WI})$ must be satisfied at the points being compared.[85] This requirement is satisfied at every point on the surface in Figure 43 for $P_{WI} = P_{WM}$. The price of W_R, then, can be calculated for any movement (in any direction) across the surface. Consider, for example, movement along DD' in the vicinity of point H as given by:

W_M	W_I	W_R	NB
10,000	25,787	1773	\$873,610
10,000	22,929	1873	787,870
	-2,858	+100	-\$ 85,740

which gives the trade-off at:

[84] Gross dollar benefits can also be used as the change in gross benefits will equal the change in net dollar benefits when costs are constant.

[85] See Chapter IV, pp. 151-153, for the derivation of this condition.

$$\frac{\Delta W_I}{\Delta W_R} = -\frac{2858}{100} = -28.58 \quad \text{or} \quad \frac{\Delta(NB)}{\Delta W_R} = -\frac{\$85{,}740}{100} = -\$857.40.$$

This would be the identical value to that calculated in (5.12) except for the interval estimate in this case.

The same trade-off would also result in this case if all three products were allowed to vary such as by some movement across the surface in Figure 43 as depicted by QR. Consider, for example, movement from point S toward point H in Figure 43. Using net benefits, the resulting trade-off estimate of P_{WR} is given by:

Point	W_M	W_I	W_R	NB
S	9,000	26,787	1773	$873,610
H	10,000	22,929	1873	787,870
		-2,858	+100	-$ 85,740

which gives the trade-off at:

$$\frac{\Delta W_I}{\Delta W_R} = -\frac{2858}{100} = -28.58 \quad \text{or} \quad \frac{\Delta(NB)}{\Delta W_R} = -\frac{\$85{,}740}{100} = -\$857.40$$

This is the identical estimate derived for movement toward H along DD'. Net (or gross) dollar benefits <u>cannot</u> be used, however, for calculating trade-offs along a surface when $(dW_I/dW_M) \neq -(P_{WM}/P_{WI})$ at the points being compared.[86]

[86] This is illustrated in the next section where a variable trade-off ratio exists among all three products.

The other possible price situation that can be examined is where $P_{WI} \neq P_{WM}$. Consider, for example, a case where $P_{WI} = \$15.00$ and $P_{WM} = \$30.00$. The decision making body would then select the zero level of W_I, as given by:

$$\frac{dW_I}{dW_M} = -1 > -\frac{P_{WM}}{P_{WI}} = -2.00$$

or,

$$-\frac{dW_I}{dW_M} < \frac{P_{WM}}{P_{WI}} \quad .$$

Product W_I would not be produced at all. The relevant points on the surface in Figure 43 are then given by curve AB'C'D'E'. The point on that curve, however, must still be selected. Assume the decision body decides to provide 10,000 acre feet of municipal water and 2660 surface acres at D' in Figure 43. The trade-off ratio is about -29.60 at that point (Table 7). The relevant equation is then given by:

$$\frac{dW_M}{dW_R} = -29.60 = -\frac{P_{WR}}{\$30.00}$$

$$P_{WR} = \$888.00.$$

The estimated value is \$888.00 based on the trade-off ratio selected and the given prices.

The surface presented in Figure 43 is the ideal type of information needed to make a choice regarding the product mix when the trade-off ratio between two of the three products is constant. The product-product trade-off ratio between W_I and W_M may also, however, be variable in some cases.

Product-Product Trade-Offs at the Bronco Site, Variable Ratios Among All Three Products. The case of variable product-product trade-off ratios among all three water products was generated by assuming that water releases for W_M would reduce the amount of water available for irrigation by the relation:

$$R_{WI} = d(W_M) - e(W_M)^{1/2} \qquad (5.13)$$

where,

R_{WI} = reductions in water available for irrigation,

d, e = constants.

Equation (5.13) reflects a situation where increases in W_M lead to reductions in water available for other uses at an ever increasing rate. The releases for W_M, for example, could be accomplished in the off-(irrigation) season. As W_M is increased, higher percentages of the water are diverted from irrigation use. Some examples can be given to illustrate the concept more clearly.

The estimated withdrawals for given levels of W_M given various values for d and e are presented in Table 8. With d = 1 and e = 50, for example, only 5000 acre feet of water are unavailable for other uses (W_I) when W_M is 10,000 acre feet. When

W_M = 40,000 acre feet, the amount available for W_I is reduced by 30,000. The case of d = 3 and e = 100 leads to even greater reductions in water available for W_I use. Product W_I must be reduced by 45,858 acre feet when W_M = 20,000. This could represent a situation where a reserve of 25,858 acre feet must be diverted from other uses to insure that enough water is available to produce 20,000 acre feet of W_M. The case of d = 1 and e = 50 was chosen to illustrate the effect of a varying trade-off ratio between W_I and W_M on the iso-cost surface.

Table 8. Estimated releases of water for the relation $R_{WI} = d(W_M) - e(W_M)^{1/2}$ for varying values of d and e.

W_M	d = 1 e = 50	d = 1 e = 100	d = 2 e = 50	d = 3 e = 50	d = 3 e = 100
0	0	0	0	0	0
10,000	5,000	0	15,000	25,000	20,000
20,000	12,928	5,857	32,929	52,929	45,858
30,000	21,340	12,679	51,339	81,340	72,679
40,000	30,000	20,000	70,000	110,000	100,000

Given d = 1.00 and e = 50.00, the iso-cost relation is given by:[87]

$$W_R = 749.33 + 0.03653[TC - W_I - W_M + (50.00)(W_M)^{1/2}] - 0.000000025712[TC - W_I - W_M + (50.00)(W_M)^{1/2}]^2. \quad (5.14)$$

[87] See Appendix C for the algebraic manipulations and the slope equations.

Again, various levels of TC were generated from varying levels of annual cost (AC) using equation (5.7). The iso-cost surfaces were then generated by varying the levels of W_M, W_I, and W_R. The \$200,000 iso-cost surface is presented in Table 9 and Figure 44. Each of the curves represent different levels of W_M. Curve EE', for example, represents the iso-cost curve between W_I and W_R when $W_M = 0$. This curve is located at the same location in the plane as curve EE' of Figure 43. Again, increases in W_M lead to "shifts" in the iso-cost relations. The iso-cost relations do not, however, remain equal spaced as in the previous case. An increase in W_M by 10,000 acre feet from 0 to 10,000, for example, causes a reduction in W_I of 5000 acre feet. The same increase in W_M from 20,000 to 30,000 acre feet gives a decrease in W_I of 8412 acre feet (Tables 8 and 9 and Figure 44).

The iso-cost surface representing the relation between W_I, W_M, and W_R is illustrated in Figure 45. This surface is very similar in nature to the surface generated in the previous case of constant trade-offs between W_I and W_M. In this case, however, the boundary of the surface represented by ABCDE is a curve rather than a straight line. Product W_I is reduced at an increasing rate as W_M is increased; i.e., the production trade-off ratio between W_I and W_M (dW_I/dW_M) becomes larger (more negative) as W_M is increased. In fact, both W_I and W_M can be increased for values of W_M from 0

Table 9. Combinations of water for irrigation, municipal (and industrial) uses, and recreation, for \$200,000 annual cost, reduction in available water determined by $R_{WI} = (1.0)W_M - (50.0)W_M$, Bronco dam site, Knife River Basin, North Dakota.

Water for and Municipal Industrial (W_M)	Water for Irrigation (W_I)	Water for Recreation (W_R)	$\frac{dW_I}{dW_R}$	$\frac{dW_M}{dW_R}$	$\frac{dW_I}{dW_M}$
(acre feet)	(acre feet)	(acres)			
0	38,634	1673	-28.41	0	∞[a]
0	32,929	1873	-28.64	0	∞
0	27,176	2073	-28.89	0	∞
0	21,373	2273	-29.14	0	∞
0	15,519	2473	-29.40	0	∞
0	9,613	2673	-29.66	0	∞
0	3,653	2873	-29.94	0	∞
0	0	2995	-30.10	0	∞
10,000	33,634	1673	-28.41	-37.88	-0.75
10,000	27,929	1873	-28.64	-38.19	-0.75
10,000	22,176	2073	-28.89	-38.52	-0.75
10,000	16,372	2273	-29.14	-38.85	-0.75
10,000	10,519	2473	-29.40	-39.20	-0.75
10,000	4,612	2673	-29.66	-39.55	-0.75
10,000	0	2828	-29.87	-39.83	-0.75
20,000	25,705	1673	-28.41	-34.51	-0.82
20,000	20,000	1873	-28.64	-34.80	-0.82
20,000	14,247	2073	-28.89	-35.09	-0.82
20,000	8,444	2273	-29.14	-35.40	-0.82
20,000	2,590	2473	-29.40	-35.71	-0.82
20,000	0	2561	-29.51	-35.85	-0.82
30,000	17,293	1673	-28.41	-33.20	-0.86
30,000	11,589	1873	-28.64	-33.48	-0.86
30,000	5,836	2073	-28.89	-33.76	-0.86
30,000	33	2273	-29.14	-34.06[b]	-0.86
30,000	0	2274	-29.14	-34.06[b]	-0.86
40,000	8,634	1673	-28.41	-32.46	-0.88
40,000	2,929	1873	-28.64	-32.74	-0.88
40,000	0	1975	-28.77	-32.88	-0.88

[a]The derivative dW_I/dW_M is a very large positive number when W_M is very near zero. The algebraic function for (dW_I/dW_M) is not defined at $W_M = 0$.

[b]Values the same due to rounding.

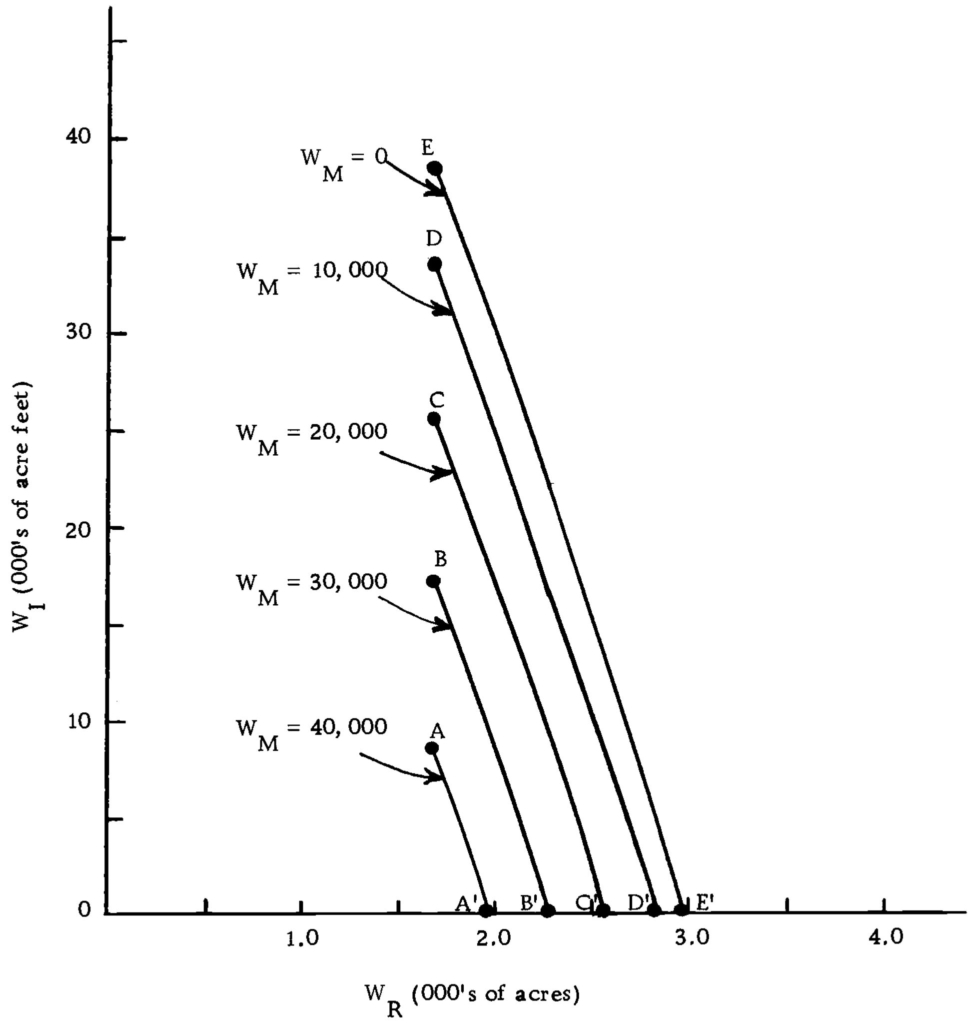

Figure 44. Iso-cost curves (annual costs: amortization and OMR), three products: water for irrigation (W_I), recreation (W_R), and municipal use (W_M), variable trade-offs at Bronco site, Knife River Basin, North Dakota.

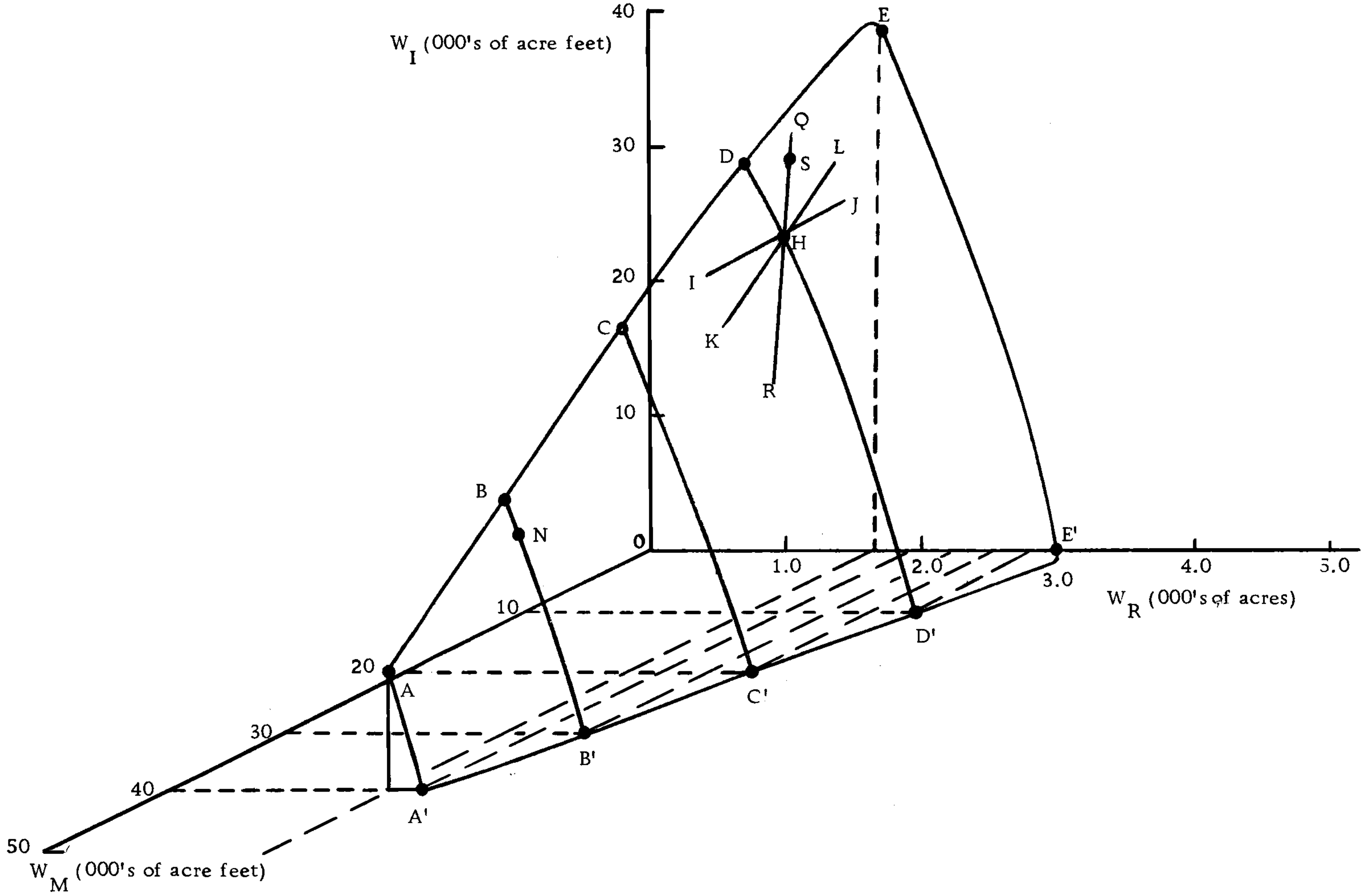

Figure 45. Iso-cost surface (annual costs: amortization and OMR), three products: water for irrigation (W_I), recreation (W_R), and municipal use (W_M), variable trade-offs at Bronco site, Knife River Basin, North Dakota.

to 625 acre feet.[88] At the 625 acre foot level, the production trade-off ratio is zero. The production trade-off ratio between W_I and W_M is negative for values of $W_M > 625$. As a result, the iso-cost surface has a negative slope (with respect to the W_M axis) for all values of $W_M > 625$ acre feet. The boundary curve A'B'C'D'E' also has a different slope in Figure 45 as compared to the earlier case. The product-product trade-off ratio between W_M and W_R is still negative and increasing. The ratios (dW_I/dW_R) and (dW_M/dW_R) are not, however, equal in Figure 45. It should also be noted that $(dW_I/dW_R)[(dW_I/dW_M), (dW_M/dW_R)]$ is not affected by the level of W_M (W_R, W_I). This relationship is the same as before. The various relations among the production trade-off ratios can be summarized in the following manner:

$$\frac{dW_I}{dW_R} \neq \frac{dW_M}{dW_R}, \quad \text{both vary,}$$

$$\frac{dW_I}{dW_M} \quad \text{is variable,} \tag{5.15}$$

$$\frac{\partial}{\partial W_M}\left(\frac{\partial W_I}{\partial W_R}\right) = \frac{\partial}{\partial W_I}\left(\frac{\partial W_M}{\partial W_R}\right) = \frac{\partial}{\partial W_R}\left(\frac{\partial W_I}{\partial W_M}\right) = 0 .$$

---

[88] Products W_I and W_M are, as a consequence, complementary in the sense that both can be increased simultaneously. This is not the notion of technical complementarity described in Chapter III, however (see Chapter III, pp. 67-69 and Appendix B).

The relations in (5.15) hold over the entire surface in Figure 45.

The implicit price of W_R can now be determined given prices of the other products and the point selected by the decision body. Assume, for example, that $P_{WI} = \$30.00$ and $P_{WM} = \$22.50$ per acre foot. Also, assume point H ($W_R = 1873$, $W_I = 27{,}929$, $W_M = 10{,}000$) is selected by the decision body. The implicit price of W_R is then given by (Table 9):

$$\frac{dW_I}{dW_R} = -28.64 = -\frac{P_{WR}}{P_{WI}} = -\frac{P_{WR}}{\$30.00}$$

$$\frac{dW_M}{dW_R} = -38.19 = -\frac{P_{WR}}{P_{WM}} = -\frac{P_{WR}}{\$22.50} \qquad (5.16)$$

$$\frac{dW_I}{dW_M} = -0.75 = -\frac{P_{WM}}{P_{WI}} = -\frac{\$22.50}{\$30.00}.$$

The equations in (5.16) are all satisfied if and only if the price of W_R is:[89]

$$P_{WR} = (-\$30.00)(-28.64) = (-\$22.50)(-38.19) = \$859.35.$$

The implicit price, given by the choice of point H, is now explicit. The last acre of W_R added has a value of \$859.35. The range in

[89]The actual production trade-off values used were rounded to two significant digits for presentation here and in Table 9.

price for all points along the DD' iso-cost curve is

$[(-\$30,00)(-28.41) = (\$22.50)(-37.88)] = \$852.19$ at point D to

$[(-\$30.00)(-29.87) = (-\$22.50)(-39.83)] = \$896.22$ at point D' (Table 9 and Figure 45).

The range in possible prices for W_R across the entire surface must now, in contrast to the previous case, be given for varying P_{WM}/P_{WI} price ratios. In the previous case, (dW_I/dW_M) was constant and equal to -1.00. As a result, an optimum point was on the surface (away from all the axes) only when $-P_{WM}/P_{WI}) = -1.00$. In this case, the optimum point may be on the surface (away from the axes) for several P_{WM}/P_{WI} ratios. Assuming $P_{WI} = \$30.00$, the optimum point will be on the surface for $0 \leq P_{WM} \leq \$26.25$.[90] If $P_{WM} > \$26.25$ per acre foot, W_I should not be produced. The relevant curve would then be A'B'C'D'E'. The rest of the surface would not provide any useful information given $P_{WM} > \$26.25$. Assuming, however, that $P_{WM} = \$26.25$, the value for P_{WR} is $[(-\$30.00)(-28.77) = (-\$26.25)(-32.88)] = \$863.07$ at point A'. The implicit price of W_R varies from

$[(-\$30.00)(-28.41) = (-\$26.25)(-32.46)] = \$852.19$ at point A to

---

[90] Calculated from $(dW_I/dW_M) = -0.875$ (-0.88 in Table 9) at the 40,000 acre foot level for W_M; i.e.,

$$(dW_I/dW_M) = -0.875 = -(P_{WM}/\$30.00)$$

or

$$P_{WM} = (-\$30.00)(-0.875) = \$26.25.$$

\$863.07 at point A' on the AA' iso-cost curve (Table 9, Figure 45). The highest implicit price for W_R (given P_{WI} = \$30.00) would result when $P_{WM} = 0$ and $W_M = 625$. This price was estimated at \$904.00 per acre.[90a] At least 625 acre feet of W_M should be produced at all times given $P_{WM} \geq 0$.

Net (or total) dollar benefits can also be used to find the price of W_R for movements along this iso-cost surface. Again, it must be the case that $(dW_I/dW_M) = -(P_{WM}/P_{WI})$ at the points compared. The necessity to meet this requirement can be illustrated for this case as (dW_I/dW_M) is not constant at every point on the surface. Consider, first, a movement between two points where the condition is satisfied. This is given for movement toward H by:

W_M	W_I	W_R	NB
10,000	30,787	1773	\$948,610
10,000	27,929	1873	862,870
	-2,858	+100	-\$ 85,740

which gives the trade-off at:

$$\frac{\Delta W_I}{\Delta W_R} = -\frac{2858}{100} = -28.58 \quad \text{or} \quad \frac{\Delta(NB)}{\Delta W_R} = -\frac{\$85,740}{100} = -\$857.40.$$

[90a] This implicit price would occur at $W_I = 0$, $W_M = 625$, and W_R at approximately 3015 acres. The following trade-offs exist at that point:

$(dW_I/dW_M) = 0$, $(dW_I/dW_R) = -30.1345$, $(dW_M/dW_R) = \infty$.

Therefore, $P_{WR} = (-\$30.00)(-30.1345) = \$904.00$ per acre.

A different value results for a movement across the surface in a direction such that $(dW_I/dW_M) \neq -(P_{WM}/P_{WI})$. Consider, for example, the movement from S to H along QR as given by:

Point	W_M	W_I	W_R	NB
S	9,000	31,531	1773	$948,430
H	10,000	27,929	1873	862,870
			+100	-$ 85,560

which gives the trade-off at:

$$\frac{\Delta(NB)}{\Delta W_R} = -\$855.60.$$

As expected, the estimated value of W_R is incorrect. The derivative (dW_I/dW_M) has a different value at each of the points S and H.[91]

The estimates of value will be even more distorted if larger changes are considered. Consider, for example, the movement across the surface in Figure 45 from point N to point H. The trade-off is then given by:

Point	W_M	W_I	W_R	NB
N	30,000	14,448	1773	$908,440
H	10,000	27,929	1873	862,870
			+100	-$ 45,570

which gives the trade-off at:

[91] The estimated value of (dW_I/dW_M) was -0.7365 at point S and -0.7500 at point H.

$$\frac{\Delta(NB)}{\Delta W_R} = -\frac{\$45,570}{100} = -\$455.70 \ .$$

The 100 acre increase in W_R for the movement from N to H resulted in a loss in net dollar benefits of \$45,570. The average loss per acre is then \$455.70. This is an underestimate of value, however, as the actual price of W_R at point H was shown in (5.16) to be \$859.35. Again, the failure to use a path across the surface where $(dW_I/dW_M) = -(P_{WM}/P_{WI})$ resulted in the incorrect estimate of P_{WR}.

The use of net benefit trade-offs between iso-cost surfaces can also lead to very misleading estimates of the value of a non-money valued product. Consider the three points from three different surfaces as given by:[92]

Iso-cost Level	W_M	W_I	W_R	NB	$\Delta(NB)/\Delta W_R$
\$200,000	10,000	27,929	1873	\$862,870	
275,000	10,000	28,608	3247	808,180	-\$39.80
350,000	40,000	5,170	4682	705,100	-\$71.83

The three points were selected such as to illustrate the (incorrect) procedure for calculating trade-offs as advocated by McKean (Freeman, 1969, p. 570); i.e., net dollar benefits decrease for

[92]The computer model could be used to generate the iso-cost surface for any given iso-cost level. The data supplied here represents one point on each of the \$200,000, \$275,000, and \$350,000 surfaces.

increases in W_R. Each of these points can be viewed, essentially, as alternative designs or plans for development at a particular project site. Net dollar benefits are shown to decrease for increases in W_R, the non-money valued product. The first increase in W_R of about 1400 acres, for example, results in a reduction in net benefits of \$54,690, or a "trade-off" of -\$39.80. The second increase in W_R of about 1400 acres gave a reduction in net benefits of \$103,080 or a "trade-off" of -\$71.83. Assume, now, the decision body selects the point on the \$275,000 surface. The implication of this selection, according to the McKean approach (Freeman, 1969, p. 570), is that W_R has a value between \$39.80 and \$71.83 per acre. Selection of the point where $W_R = 3247$, the argument goes, implies the decision body was willing to sacrifice at least \$39.80 per acre but not \$71.83 per acre. It is concluded that W_R is worth at least \$39.80 per acre but less than \$71.83 to the decision body. This is not correct. The true trade-off can only be determined by examining the iso-cost surfaces in the vicinity of each point. The true value of product W_R, as perceived by the decision body for $P_{WM} = \$22.50$ and $P_{WI} = \$30.00$, is given by satisfying the optimum conditions at the point on the \$275,000 surface, which are:

$$\frac{dW_M}{dW_R} = -40.62 = -\frac{P_{WR}}{\$22.50}$$

$$\frac{dW_I}{dW_R} = -30.46 = -\frac{P_{WR}}{\$30.00} \tag{5.17}$$

$$\frac{dW_I}{dW_M} = -0.75 \quad = -\frac{\$22.50}{\$30.00} .$$

Using (5.17), the price of W_R is given by:

$$P_{WR} = (-\$22.50)(-40.62) = (-\$30.00)(-30.46) = \$983.85 .$$

Use of the McKean approach would lead to an underestimation of the value of W_R in this case.

The most significant point to be demonstrated in the analysis of the three-product case, then, is that net or gross benefit trade-offs will give an estimate of the value of the non-money valued output (given the choice of the decision maker) only if the optimum combination of the other two products is used. Also, the calculations must be made along an iso-cost surface. This point is very significant to the case where there are more than three products.

Product-Product Trade-Offs for the Case of Four or More Products. The resource planner may also be faced with calculating product-product trade-offs for the case of four or more products. Trade-offs must still be calculated in the vicinity of a point on the multi-dimensional iso-cost surface. Given four water products, for example, there is a three-dimensional surface such as illustrated in Figures 43 and 45 for every level of the fourth product. A whole family of three dimensional surfaces would then exist for any given iso-(minimum) cost level. One approach, in this case, would by necessity be iterative. The trade-offs between a money and a non-money valued output would have to be presented for many alternative levels of the other products.

There is, however, a solution to the problem when more than three products are involved. The iso-cost relations for the products must first be identified. Given m products and the multi-dimensional surface, all the (dW_i/dW_j), $(i, j = 1, 2, \ldots, m;\ i \neq j)$, derivatives can be determined (or at least approximated). The levels of each money valued product, in turn, will be given at the various points on the surface where all these derivatives are equal to the relevant price ratios. These various levels of each money valued product can then be combined into a common measure of value using the respective prices as weights. The result will be a measure of total dollar benefits at various points along the path on the iso-cost surface where $(dW_i/dW_j) = -(P_{Wj}/P_{Wi})$. Net dollar benefits could also be used at this point, if desired, as costs are constant at every point on the path. The value of the non-money valued output can then be determined (given the choice of the decision body) from the selection of the point on the dollar vs. non-money valued output curve. This process can be illustrated for the previous three product case.

Assume, for example, that $P_{WM} = \$22.50$ and $P_{WI} = \$30.00$. The relevant path across the surface in Figure 45 is then given by all product combinations where $(dW_I/dW_M) = -(\$22.50/\$30.00) = -0.75$. The path, for this case, is the curve DD' in Figure 45. The various money valued product combinations along DD' can then be converted to gross or net dollars of benefits using the product prices as weights. The resulting estimates are given in Table 10. An increase in W_R from 1773 to 1873 acres, for example, gives an average reduction in total and net benefits of \$857.40. A further increase of 100 acres to 1973 acres gives a reduction in total and net benefits of \$861.30 per acre. The value of W_R at the 1873 level is, as a result, given by $\$857.40 < P_{WR} < \$861.30$. As was shown in (5.16), the value is exactly \$859.35. Such total or net benefit curves could be developed for as many products as there are of concern in the project. The most complex empirical problem, then, is the estimation of the iso-cost relations.

Factor-Product Trade-Off Ratios at One Site

Another problem faced by the resource planner arises when one of the factors does not have an observable market price. The relevant trade-off is now given by the factor-product trade-off ratio. Assume,

Table 10. Estimates of total and net dollar benefits and value of W_R along the $200,000 iso-cost surface where dW_I/dW_M = -0.75, Bronco dam site, Knife River Basin, North Dakota.

Total Benefits (TB)	Net Benefits (NB)	Surface Area (W_R)	$\frac{\Delta TB}{\Delta W_R} = \frac{\Delta NB}{\Delta W_R}$
(dollars)	(dollars)	(acres)	(dollars)
1,234,020	1,034,020	1673	
1,148,610	948,610	1773	-854.10
1,062,870	862,870	1873	-857.40
976,740	776,740	1973	-861.30
890,280	690,280	2073	-864.60
803,400	603,400	2173	-868.80
716,160	516,160	2273	-872.40
628,560	428,560	2373	-876.00
540,570	340,570	2473	-879.90
452,160	252,160	2573	-884.10
363,360	163,360	2673	-888.00
274,170	74,170	2773	-891.90
225,000	25,000	2828	-894.00[a]

[a]Last increment in W_R is only 55 due to the constraint on capacity; i.e., the maximum surface area for the $200,000 expenditure level is 2828 acres.

for illustrative purposes, the proposed dam and reservoir at the Bronco site would inundate a wilderness area.[93] Investment capital, then, can be combined with the wilderness area to produce water for irrigation (W_I). As a result, wilderness area is an input into the production of W_I. The basic relations in (5.3) and (5.4) can also be used to solve this problem.

The surface area of the reservoir was used as a measure of the number of acres of wilderness area used to produce water products. This reflects the assumption that a wilderness area is lost to all other uses once it has been covered with water (even for a short period of time). The production function relation is, as a result, given directly from (5.4) by:[94]

$$W_S = \frac{0.03653 - \sqrt{(0.03653)^2 - (4)(0.000000025712)(W_A - 749.33)}}{(2)(0.000000025712)} \tag{5.18}$$

where,

W_S = water in storage,

W_A = wilderness area inundated.

[93] This was not considered a problem in the previous study (North Dakota State Water Commission, "The Plan...", 1971). There were groups in the State at the time, however, that did not want to inundate the river valley. Rather, those groups wanted to maintain present use which was primarily as range land for livestock. Only in this sense is the present example descriptive of the real situation at the Bronco site.

[94] See Appendix C for the algebraic manipulations.

The level of W_A was then changed arbitrarily to give various levels of W_S. The W_S was then allocated to the production of W_I. The resulting relations from (5.18) are represented in Figure 46 for annual cost levels of \$200,000-\$650,000 in \$75,000 increments.

The basic data used in the development of the \$200,000 curve is presented in Table 11. At the \$200,000 expenditure level, for example, the "application" of 1673 acres of wilderness area (W_A) gives the zero level of W_I (point A, Figure 46 and Table 11). An increase in the use of W_A to 2273 acres results in about 17,261 acre feet of water available for W_I at point Q. If W_A is increased to approximately 3000 acres, W_I will be about 38,614 acre feet as represented at point A' in Figure 46. At point A', then, nearly 3000 acres of wilderness area must be used (as an input) to produce about 38,000 acre feet of W_I, the water product. Increases in the annual cost resulted in shifts, to the right, in the production function relation. This occurs due to the assumption made regarding useable reservoir capacity. As was noted earlier, it was assumed that the reservoir would always be at 40 percent of capacity. As a result, larger structures (as shown by increases in annual cost in Figure 46) result in larger areas of W_A inundated. For the \$200,000 expenditure, for example, 1673 acres of W_A were covered with water when the reservoir was at 40 percent of capacity. This level of use for W_A is represented at point A in Figure 46. At point H, 5900 acres of W_A

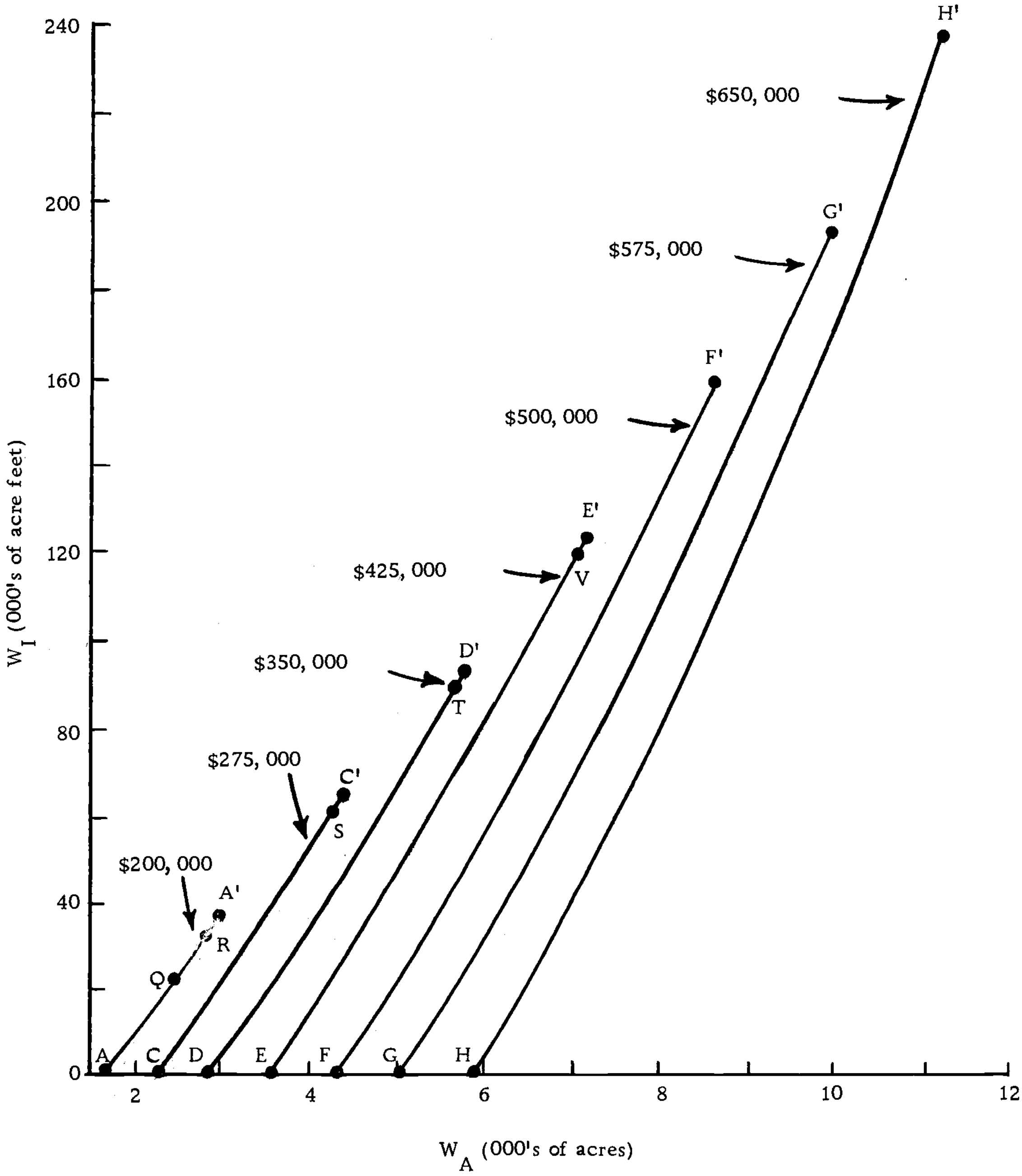

Figure 46. Production function relations, water for irrigation (W_I) as a function of acres of wilderness area inundated (W_A), Bronco site, Knife River Basin, North Dakota.

Table 11. Water for irrigation as produced from various levels of wilderness area inundated, Bronco dam site, Knife River Basin, North Dakota.

Water for Irrigation (W_I)	Wilderness Area Inundated (W_A)	Remaining Wilderness Area (W_{AN})	Net Benefits (NB)	dW_I/dW_A [a]	$d(NB)/dW_A$ [b]
(acre feet)	- - - - - - - - - (acres)	- - - - - - - - - -	(dollars)		
0	1673	1321	-200,000	28.41	
2,846	1773	1221	-114,620	28.52	853.96
5,705	1873	1121	- 28,850	28.64	857.54
8,576	1973	1021	57,280	28.77	861.17
11,458	2073	921	143,740	28.89	864.84
14,354	2173	821	230,620	29.01	868.56
17,261	2273	721	317,830	29.14	872.33
20,182	2372	621	405,460	29.27	876.15
23,115	2473	521	493,450	29.40	880.02
26,062	2573	421	581,860	29.53	883.94
29,021	2673	321	670,630	29.66	887.91
31,994	2773	221	759,820	29.80	891.94
34,981	2873	121	849,430	29.94	896.02
37,982	2973	21	939,460	30.07	900.16
38,614	2994	0	958,420	30.10	903.00

[a] Rate of change in W_I for a change in W_A; i.e., the factor-product trade-off ratio.

[b] Rate of change in net benefits (NB) for a change in W_A.

are covered when the reservoir level is at 40 percent of capacity. Point H is on the production function for the $650,000 expenditure level.

The factor-product trade-off ratio was found to increase for increases in W_A at any given expenditure level. Moving from point A toward point A' on the $200,000 production function, for example, results in the changes:[95]

Point	W_A	ΔW_A	W_I	ΔW_I	$\Delta W_I / \Delta W_A$
A	1673		0		
Q	2273	600	17,261	17,261	28.77
A'	2873	600	34,981	17,720	29.53

The first increase in W_A of 600 acres resulted in an increase in W_I of 17,261 acre feet for a factor-product trade-off ratio of 28.77. The second increase in W_A of 600 acres gave a trade-off ratio of 29.53. The point estimates of the slopes (dW_I/dW_A) are presented in Table 11. The same type of relations prevail throughout the entire family of curves in Figure 46, with the transformation ratio (point estimate) varying from 28.41 at point A to 63.30 at point H'.

The point estimates of the factor-product trade-off ratios are illustrated in Figure 47. Curve AM is a continuous curve generated over the entire range of expenditures from $200,000 to $650,000.

[95] Point A' is actually at $W_A = 2995$. The 2873 level was chosen such as to keep ΔW_A constant for illustrative purposes.

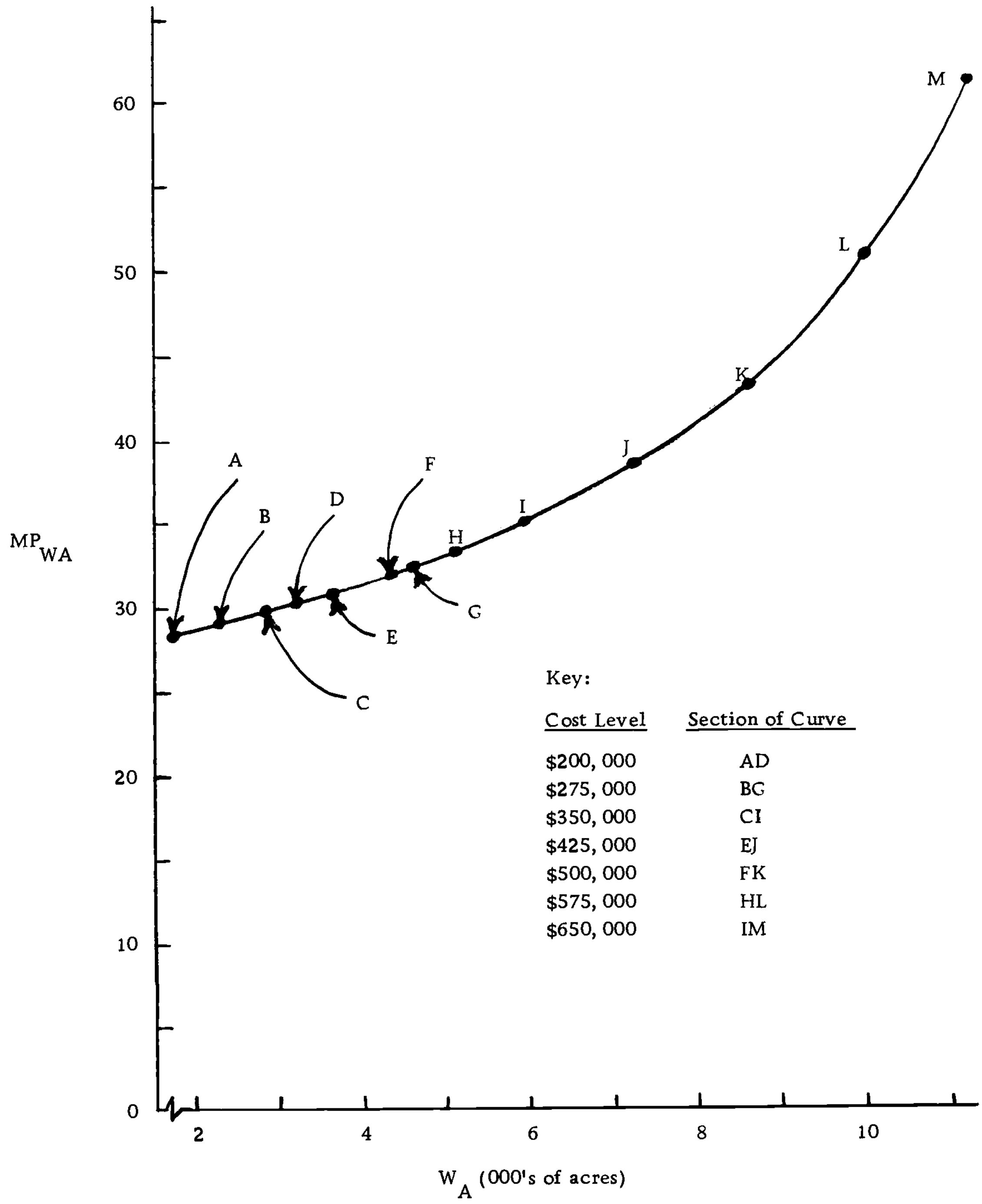

Figure 47. Marginal productivity (MP_{WA}) of wilderness area (W_A) as used in production of water for irrigation, Bronco site, Knife River Basin, North Dakota.

Only segments of the curve, however, represent particular expenditure levels. The trade-off ratios for the \$200,000 expenditure level (from Table 11), for example, are represented on segment AD of curve AM in Figure 47. Similarly, the trade-off ratios for the \$650,000 expenditure level are represented in the IM segment of the curve. The various segments were also found to be overlapping, which indicates the areas on the respective production functions having equal slopes. The trade-off ratios in the BD segment of the AM curve, for example, occur on both the \$200,000 and \$275,000 production relations (Figure 47). The relation AM in Figure 47 also reflects the increasing returns situation. The curve, essentially a summary of several marginal product relations, is increasing rather than decreasing in every segment.

The factor-product trade-off ratios for this case, then, increase as W_A is increased. This necessitates a different interpretation of a decision to produce at a particular point. The value of W_A, P_{WA}, is not revealed directly from the choice of a point on the total product schedules of Figure 46 or of a particular trade-off ratio in Figure 47. This point can be clarified with reference to curve CEB in Figure 48. Curve CEB represents the factor-product trade-off ratios for the \$200,000 cost level and is identical with section AD of curve AM in Figure 47 except for scale of the illustration. Assume the decision body selects point A' on the \$200,000 production function in Figure 46.

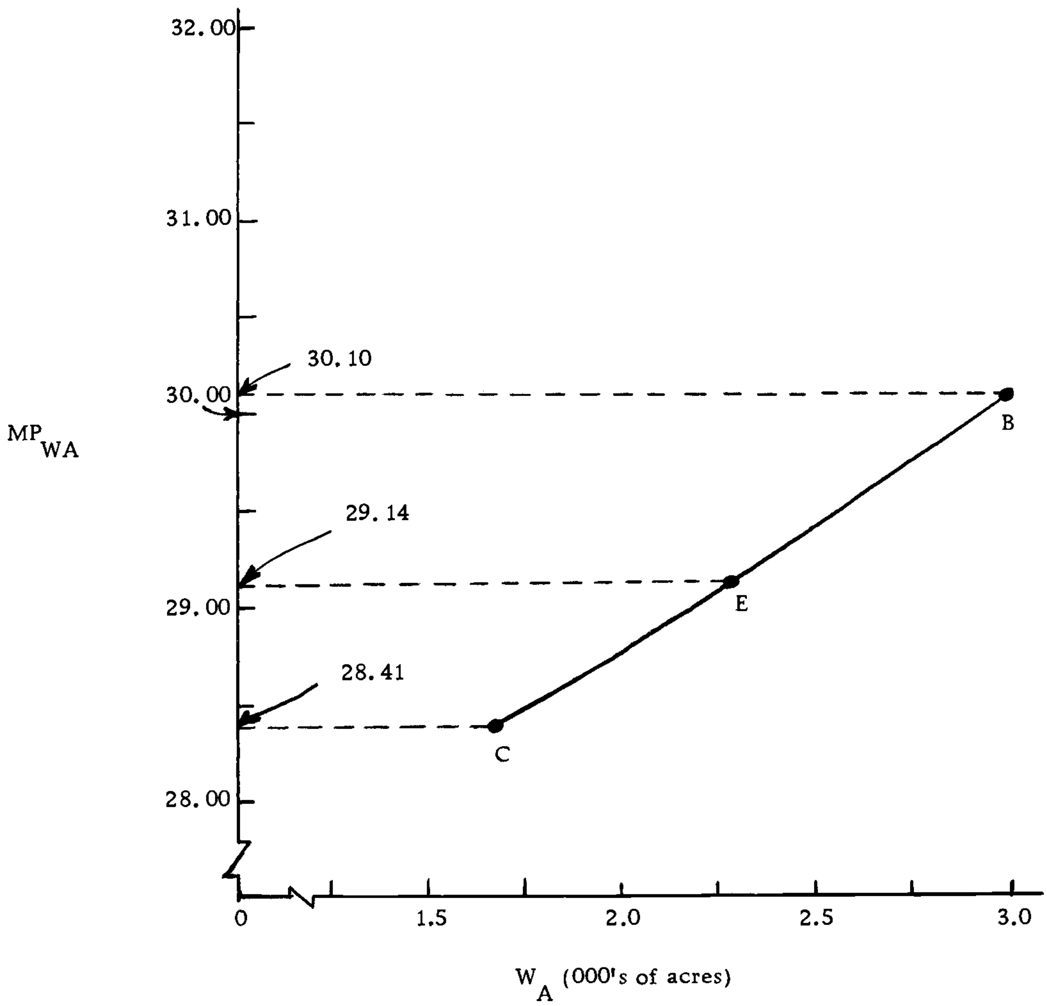

Figure 48. Factor-product trade-off ratios (MP_{WA}), wilderness area (W_A) as a factor in the production of water for irrigation, $200,000 cost level, Bronco site, Knife River Basin, North Dakota.

The trade-off at that point is 30.10 as represented at point B in Figure 48. Using the equilibrium condition developed in previous chapters, it would appear that:

$$\frac{dW_I}{dW_A} = 30.10 = \frac{P_{WA}}{P_{WI}} = \frac{P_{WA}}{\$30.00}$$

or,

$$P_{WA} = (\$30.00)(30.10) = \$903.00.$$

The price of W_A is not, however, \$903.00 per acre. If the price of W_A were \$903.00, the net benefits at point A' in Figure 46 would be:

$$\begin{aligned} NB &= P_{WI}W_I - P_{WA}W_A - C \\ &= (\$30.00)(38{,}614) - (\$903.00)(2994) - \$200{,}000 \\ &= -\$1{,}945{,}162\,. \end{aligned}$$

There would be a loss of nearly \$2,000,000 at point A'. In fact, using the price of W_A at A' implies the decision body had the goal of maximizing losses. All the prices taken directly from choice of a particular trade-off ratio on CEB in Figure 48 are the maximum loss prices of W_A. The second order conditions are not met at any point on AA' in Figure 46 (or CEB in Figure 48). The important point to note is that the price of any factor cannot be taken directly from a factor-product trade-off curve that has a positive slope.

Further information regarding the choice process used by the decision body must be available before the implicit price can be made explicit. Assume, for example, the decision body requires that net dollar benefits are non-negative. Given this additional information, choice of point A' over any other point on AA' in Figure 46 implies that marginal dollar benefits of using W_A to reach A' exceed the marginal dollar costs by a sufficient magnitude to make net dollar benefits at least zero. An approximation of price can be found, then, by finding the price of W_A that makes net benefits equal to zero at point A' (point B in Figure 48). This price is given by (data from Table 11):

$$\begin{aligned} NB &= P_{WI}W_I - P_{WA}W_A - C = 0 \\ &= (\$30.00)(38,614) - (P_{WA})(2994) - \$200,000 \end{aligned}$$

or,

$$P_{WA} = \frac{\$958,420}{2994} = \$320.11.$$

This is the price of W_A that exactly exhausts all the net dollar benefits from W_I production up to W_R = 2994 acres.

The implicit price of W_A is even lower than \$320.11 per acre when some point between A and A' is selected. Assume, for example, the decision body chooses point Q in Figure 47. The factor-product trade-off ratio at point Q is 29.14 as represented at point E in Figure 48. Using the (incorrect) direct approach, P_{WA} is given by:

$$\frac{dW_I}{dW_A} = 29.14 = \frac{P_{WA}}{P_{WI}} = \frac{P_{WA}}{\$30.00}$$

which implies that:

$$P_{WA} = (\$30.00)(29.14) = \$874.20.$$

If the trade-off schedule were downward sloping, this would be the implicit price. Assuming, again, the decision body requires net benefits to be at least zero, the implicit price of W_A is given by:

$$\begin{aligned} NB &= P_{WI}W_I - P_{WA}W_A - C \\ &= (\$30.00)(17{,}261) - P_{WA}(2273) - \$200{,}000 = 0 \end{aligned}$$

or,

$$P_{WA} = \$139.83.$$

A total of 2273 acres of wilderness area were, in essence, "purchased" at \$139.83 per acre to provide 17,261 acre feet of water. The implicit price is considerably below the level of \$874.00 per acre estimated with the direct approach.

The importance of the foregoing illustrative examples becomes apparent only after trying to apply the techniques advocated by some authors for evaluating elements such as wilderness areas, wild and scenic rivers, and other resources. The most favored technique is to represent net dollar benefits as some function of the non-money valued element. Such a "net effect" curve is represented in Figure 49 for the

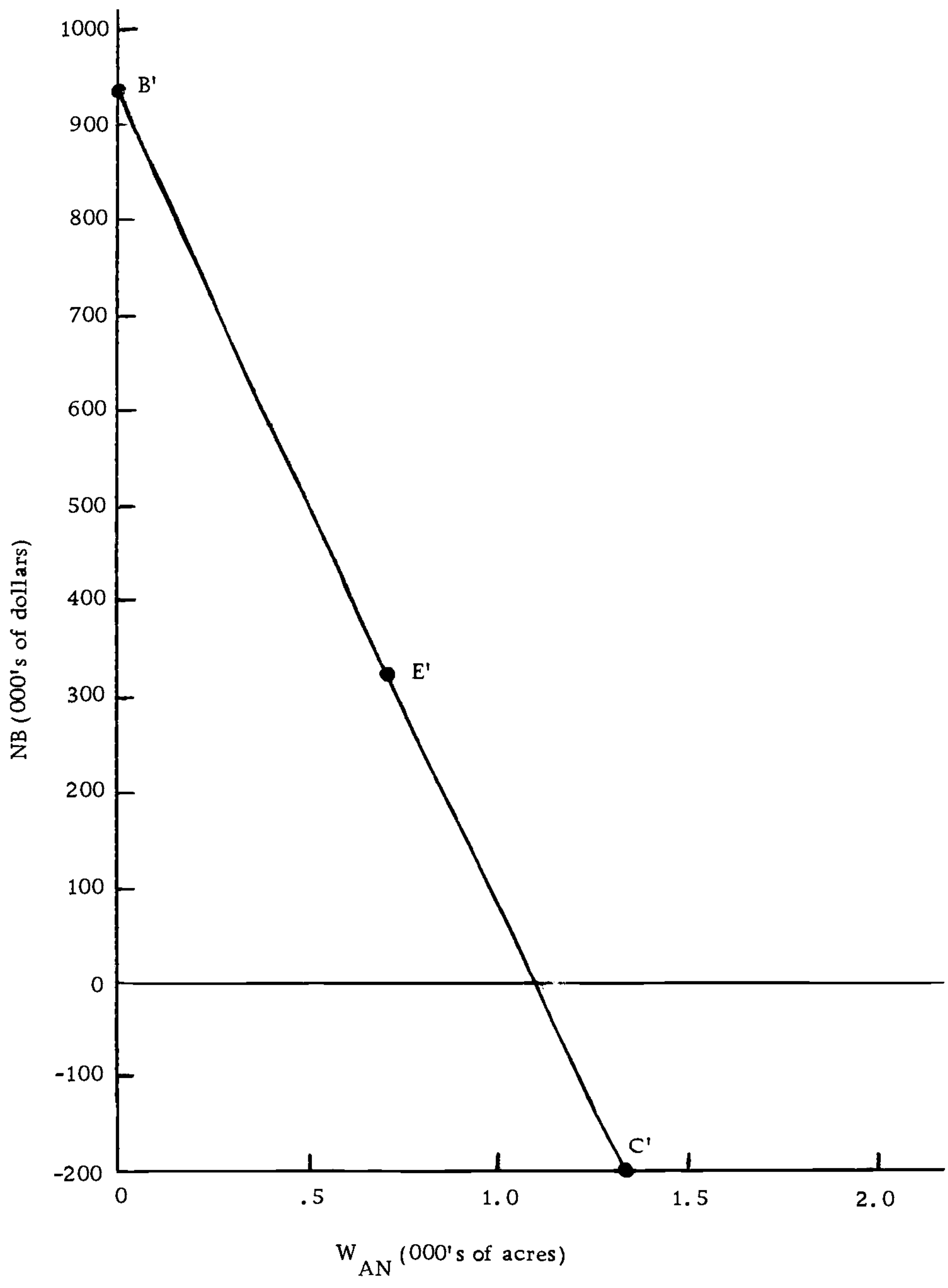

Figure 49. Net benefits (NB) from "non-use" of wilderness area (W_{AN}) in production of water for irrigation, price of water for irrigation at $30.00 per acre foot, $200, 000 expenditure level, Bronco site, Knife River Basin, North Dakota.

present case. Movement from C' toward B' in Figure 49 represents increases in the amount of W_A used in W_I production. Points B', E', and C' correspond directly to points B, E, and C in Figure 48. The horizontal axis, then, represents "non-use" of $W_A (W_{AN})$ in W_I production. The remaining wilderness area (W_{AN}) and the net benefit levels are also shown in Table 11. The "trade-off", it is argued by some, results as "non-use" is increased from 0 toward point C' in Figure 49. The slope of B'C' is said to be the trade-off, and as a result, gives an indication of the value of W_A (as soon as the decision maker chooses a point on C'B'). Assume the decision body (as in the previous example) selects point E' where $W_{AN} = 721$. An estimate of the slope at point E' is given by:

W_{AN}	NB	$\Delta(NB)/\Delta W_{AN}$
621	\$405,460	\$874.20
821	230,620	

The implicit price of W_A, it is argued, is given by \$874.20 per acre. This value was shown to be incorrect in the previous example. Assuming the decision body wants to insure that net benefits are at least zero, the value of W_A was shown to be at most, \$139.83 per acre. The reason for this distortion, of course, is the positive slope of the trade-off function.

The use of net benefit curves may also lead to misleading estimates of the value of W_A when some initial price was used to develop the net benefit curve. Assume, for example, the wilderness area was owned by private individuals willing to sell W_A for $50.00 per acre. The "starting" price of W_A would then affect the estimated value of W_A. The net benefit values for this situation are presented in Table 12. Assume the decision body, again, selects point Q on the production function relation AA'. The estimated price at point E (and E') is now given by (from Table 12):

W_{AN}	NB	$\Delta(NB)/\Delta W_{AN}$
621	$286,810	$824.20
821	121,970	

This is distorted by the amount $874.20 - $824.20 = $50.00, which was the starting price. Even this estimate, of course, is incorrect because of the increasing factor-product trade-off values. If the trade-offs were declining, this adjustment would have to be made; i.e., the "starting" price of W_A would have to be added to the price estimated with the net benefit curve.[96]

[96] The problems associated with calculating trade-off ratios with net benefit curves when there are two or more products were not illustrated here. The conceptual models of Chapter IV indicated, however, that net benefit trade-offs must also be used with caution for that case.

Table 12. Water for irrigation as produced from various levels of wilderness area inundated, purchase price of W_A equal to $50.00 per acre, Bronco dam site, Knife River Basin, North Dakota.

Water for Irrigation (W_I)	Wilderness Area Inundated (W_A)	Remaining Wilderness Area (W_{AN})	Net Benefits (NB)
(acre feet)	(acres)	(acres)	(dollars)
0	1673	1321	-283,650
2,846	1773	1221	-203,270
5,705	1873	1121	-122,500
8,576	1973	1021	- 41,370
11,458	2073	921	40,090
14,354	2173	821	121,970
17,261	2273	721	204,180
20,182	2373	621	286,810
23,115	2473	521	369,800
26,062	2573	421	453,210
29,021	2673	321	536,980
31,994	2773	221	621,170
34,981	2873	121	705,780
37,982	2973	21	790,810
38,614	2995	0	808,720

Comparison of Recommendations Regarding Trade-Off Ratio Concepts and Procedures

Several recommended approaches to trade-off ratio calculations have already been outlined in this study. The most frequently recommended approach to trade-off ratio calculations has been to "trade-off" net dollar benefits for some resource or product not having a known, market determined price. This approach is recommended by several authors.[97] There is never any discussion, however, regarding the role of costs in calculating the net benefit functions. Also, none of these authors appear concerned over the necessity for keeping resources and products separated in the trade-off calculation process.

The net benefit trade-off calculation models must be viewed with great caution. The test study accomplished for the Water Resources Council by Major (1970) is a case in point. A net benefit curve (net effect curve) was obtained by subtracting a cost curve from an overall benefit curve (Major, 1970, p. 37). This net benefit curve was then plotted against flood pool elevation. The flood pool elevation axis was then transformed into acres of ecological area preserved. The resulting curve was labeled a "net benefit transformation curve". The curve derived in that study is reproduced in Figure 50.

The trade-offs calculated along a curve such as the one depicted in Figure 50 are completely different than any conceivable "trade-off" well based in economic theory. Every point on the curve represents a different project with different costs. As a result, the curve is not an iso-cost curve. Also, the relation is not a production function relation as production functions are representative of physical phenomenon. The nature and meaning of the relation in Figure 50 is at best, confusing and, at worst, misleading. The relation illustrated in Figure 50

[97] See Chapter II.

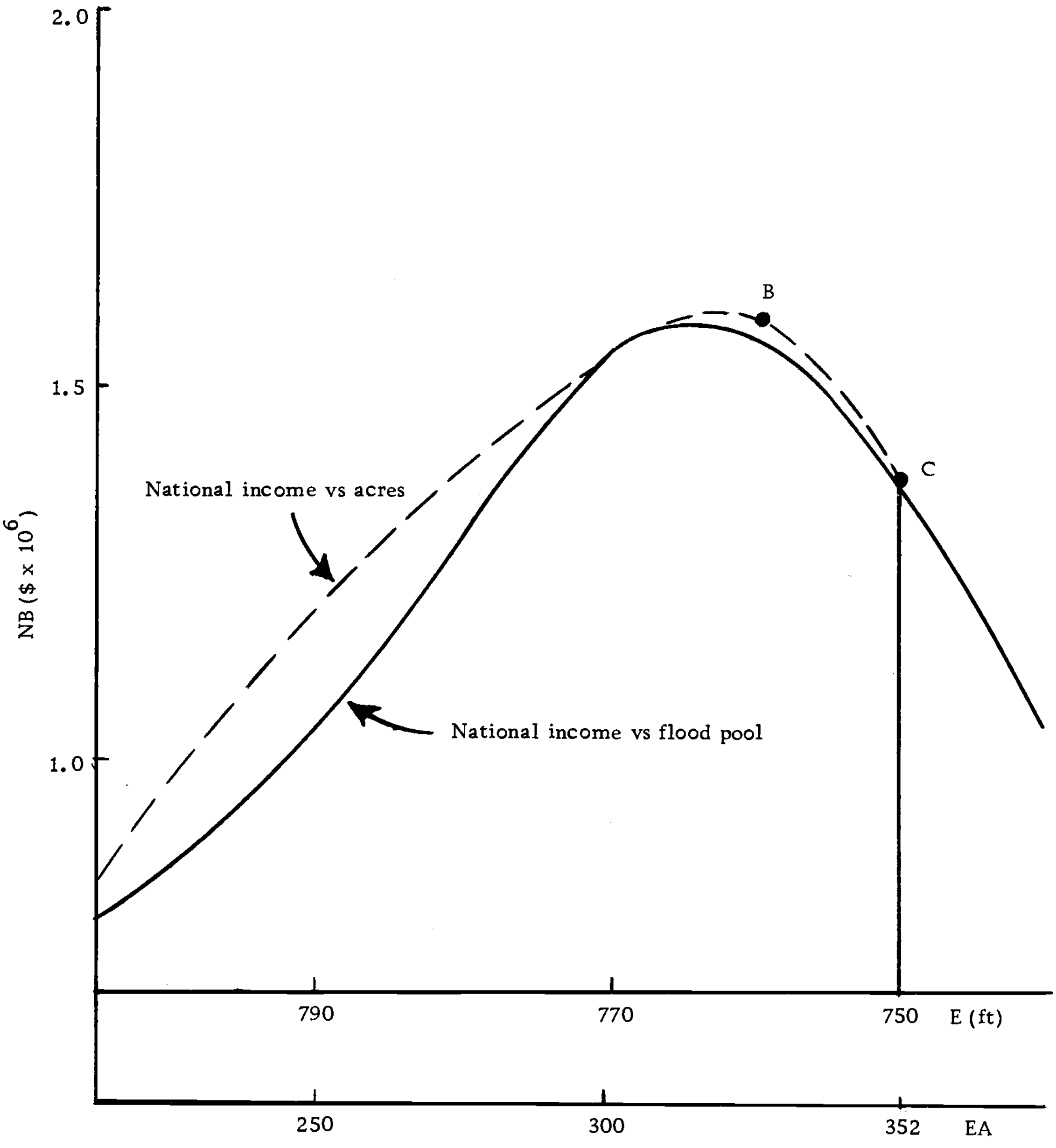

Figure 50. Net dollar benefits (NB) vs flood pool elevation (E) and ecological acres saved(EA).

can best be described as a net benefit curve from a long run production function (all factors varying) where ecological acres used (moving from right to left along horizontal axis) is a factor in the production of other products (which give rise to net dollar benefits). Ecological acres cannot be considered a product of water resource development, but rather a factor in the production of water products. As a result, ecological acres are <u>transformed</u> into water products. It follows that factor-product trade-off ratios should have been calculated. Also, <u>net</u> dollar benefits could only be calculated with some implicit price on ecological acres. This value is not mentioned in the study report.

Preference functions were also developed in the Major study. The preference functions were then placed on the net effect curve, yielding points B and C in Figure 50. Point B was selected from the preference function representative of the Corps of Engineers. Point C was selected based on the preference function of a group of conservationists (Major, 1970, pp. 41-44). On the basis of those selections, it was suggested that the Corps placed a value of $1700 on the last acre of ecological area received up to point B. The conservationists, on the other hand, valued the last acre at $15,000. It is beyond the perception of this author, however, how any group or individual can choose a point on a net benefit curve such as that illustrated in Figure 50 without knowledge of the attendant costs and total benefits used in the calculation process. It seems possible that point B (or any other point) could result from an investment in the high millions (maybe billions) or for a much lower investment level. There is one entire dimension (cost) missing in such a diagram. Individuals or groups may prefer a plan that generates slightly over $1M (such as point C) to one generating slightly over $1.5 M (such as point B) merely because less resources were used at point C (not because of a strong preference for ecological acres). If, indeed, less resources were used in the plan represented at point C, other products (possibly non-water related) could have been generated. As noted previously, however, it was impossible to determine from the study report what the costs were at each of the points on the "net effect" curve. Costs evidently were not considered relevant to the choice process.

Another interesting aspect of using net benefit curves relates to the characteristic of any economic optimization process. The calculus of economic optimization does not require net benefits to be positive. Economic optimization requires only that net benefits be maximized. The net dollar benefits may, indeed, be negative. The actual level of net returns (net benefits) are of concern only after the optimum product mix and resource use levels are determined. At that point, the decision to produce or not produce is made. The absolute level of net benefits does not play any role in economic optimization until the optimum point in the multi-dimensioned product space is discovered. Based on that argument alone, it makes little sense to find tangency points of preference functions to net benefit curves.

A slightly different approach, discussed in Chapter II of this study, was developed by DeVine (1966). In that approach total cost is held constant while various allocations of cost are made among the various water products. Total dollar benefits from water reclamation are "traded-off" for increases in total benefits to recreation (measured in user-hours) for a constant level of cost. The data provided in Tables 1 and 2 (Chapter II, p. 29), as reproduced from DeVine, is illustrated by curve ABCD in Figure 51. Point A, for example, represents a \$1 M allocation to water reclamation while point D represents a \$1 M allocation to recreation. Other points along the curve represent different allocations of \$1 M. At point B, for example, \$0.5 M was allocated to each of the water related outputs. Trade-offs are interpreted to be the slope of curve ABCD. Movement from B to C, for example, results in a loss of \$150,000 in water reclamation benefits and gives a gain of 300,000 hours of recreation. The average slope of the curve is given by:

$$\frac{\Delta(TB)}{\Delta \text{ hrs}} = -\frac{\$150,000}{300,000} = -\$0.50$$

The implied minimum value of recreation is \$0.50 per hour. This concept of a "trade-off" is consistent with the iso-cost framework of this study if it is assumed that all the water products included in

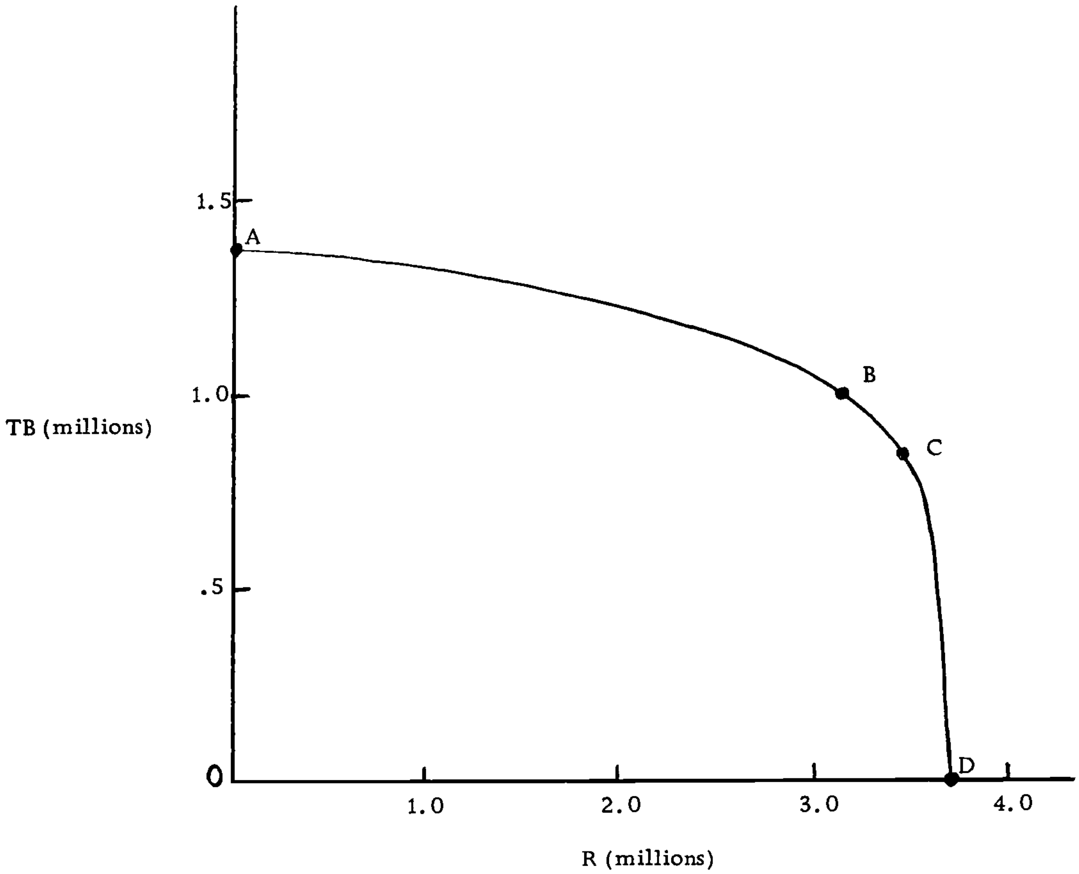

Figure 51. Iso-cost relation between total dollar benefits for reclamation (TB) and user-hours of recreation (R).

"total reclamation benefits" are in the proper ratios. Stated in another manner, the approach is identical to the approach recommended here if the equilibrium conditions $(dW_i/dW_j) = -(P_{Wj}/P_{Wi})$ $(i, j = 1, 2, \ldots, m; \; i \neq j)$ are satisfied for all the m water products considered in the water reclamation classification.

Another approach was taken by Marshall (1970). An understanding of the underlying production relation is crucial to a correct interpretation of the trade-off calculation process as envisioned in that paper. It is impossible to determine exactly, from the information provided, whether the production process is best described as factorially determined, assorted, or some variation of one of these major classes. Some possible relations can be postulated, however.

Assume the functions used by Marshall were factorially determined without coupling $(\alpha = 0, \; \kappa = 0)$. Given that determination, the production function relations can be represented by:

$$\begin{aligned} NED &= f(x_1, x_2, \ldots, x_n) \\ EQ &= g(x_1, x_2, \ldots, x_n) \\ RD &= h(x_1, x_2, \ldots, x_n) \end{aligned} \tag{5.19}$$

where,

NED = national economic development effects (presumably gross or net national product) measured in dollars,

EQ = index of environmental quality,

RD = regional economic development effects, measured in dollars,

x_i = factors of production ($i = 1, 2, \ldots, n$).

Given (5.19), the values represented in Table 3 (Chapter II, p. 31) imply a path across several iso-cost surfaces has already been chosen prior to the determination of trade-offs. Each of the points (designs) in Table 3 (Chapter II, p. 31) represents a point on a different iso-cost surface. It follows that product-product trade-offs cannot be determined from the data in Table 3. There is, however, an implicit (but unreported by Marshall) trade-off among the products at every point (design). Stated in another manner, there is an implicit price for each product already in the data provided. The value of EQ cannot be discovered because it has already been assigned. This point can be more clearly understood by reference to Figure 52. Points A, B, C, D, E, F, and G correspond to Designs 1-7, respectively, in Table 3 (Chapter II, p. 31). The cost is different at every point; i.e., each point is on a different iso-cost surface. The trade-offs at any one point, then, are given by the slope of the iso-cost surface at that point for the products of concern. The trade-off between NED and EQ at point G, for example, would be given by the slope of the iso-cost surface (for a cost of $700,000) at point G with respect to the NED, EQ plane. The trade-off between NED and EQ at point G is, as a

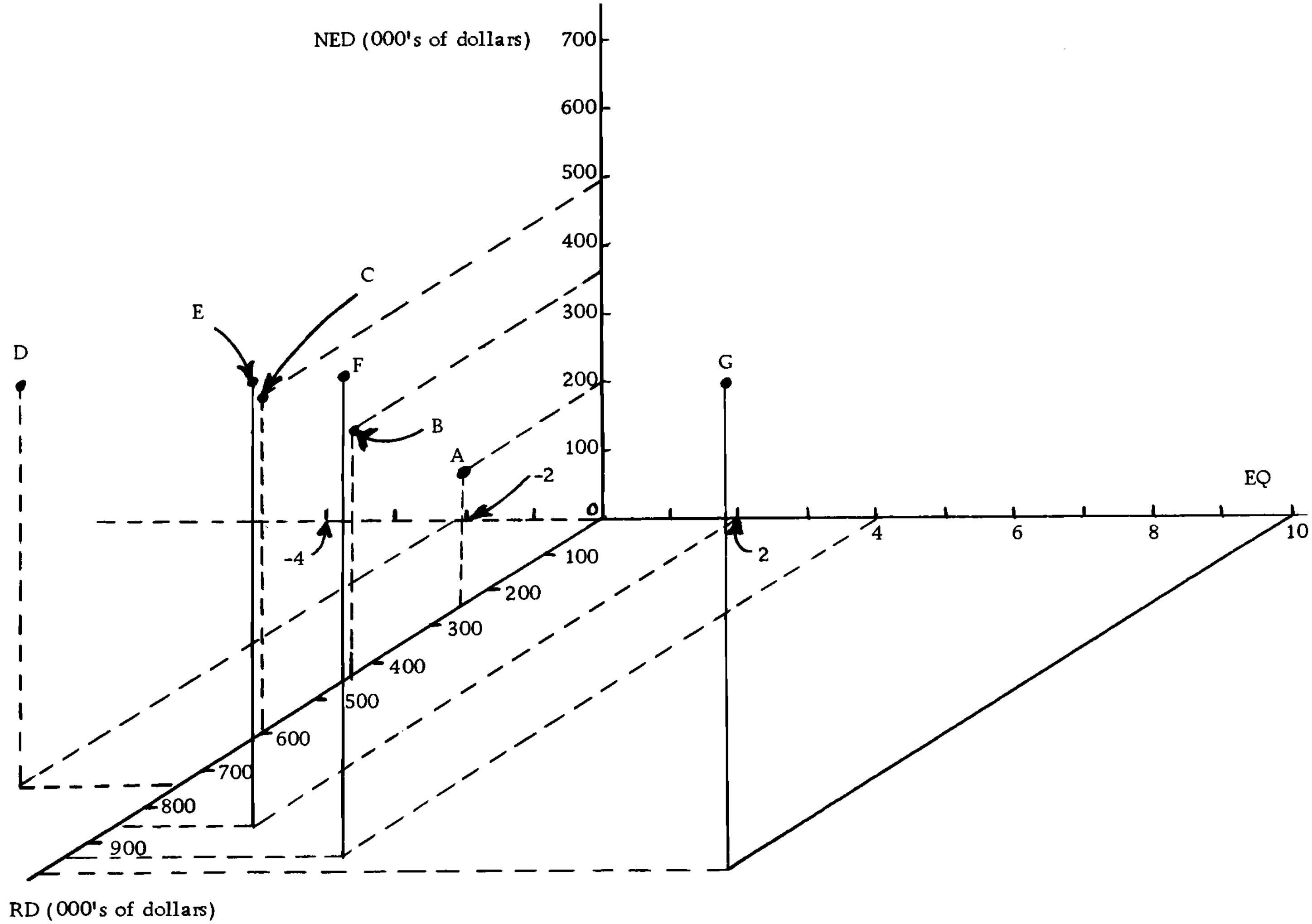

Figure 52. Iso-cost surfaces for national economic development (NED), regional economic development (RD), and environmental quality (EQ).

result, also affected by the level of RD. Marshall does not, however, refer to trade-offs in that manner.

The real result of using the approach recommended by Marshall (based on the previously noted assumption of factorially determined production without coupling) is finding the optimum scale of the project without making the trade-offs explicit. Choice of Design 6 (point F) over Design 5 (point E) was said to indicate that "...$60,000 of NED equivalent benefits are willingly foregone for 2 units of EQ benefits and $80,000 of RD benefits" (Marshall, 1973, p. 4). The choice of Design 6 does not, however, give any indication of the trade-off (or the slope) along the iso-cost surface for Design 6. If, indeed, Design 6 is chosen over Design 5, all that can be said is that marginal benefits (from all products, <u>including</u> EQ and RD) exceeded the marginal costs (in this case $100,000) for the movement. There are no trade-offs involved. In fact, if the production relations illustrated in Figure 52 are factorially determined without coupling, the resource planner has already used a set of implicit prices to derive the path across an iso-cost surface (in this case, across several iso-cost surfaces).

A slightly different interpretation of the data supplied by Marshall must be made if the true underlying production relations are still factorially determined, but the products are joint ($\alpha = 0$, $\kappa > 0$). The production relations would then be given by some relations like:

$$RD = f(NED)$$
$$EQ = g(NED) \qquad (5.20)$$
$$NED = h(x_1, x_2, \ldots, x_n) \; .$$

Given a particular level of each resource, the product mix is automatically known without any further product substitution possible. Using this approach, the products are joint if for the particular levels of each resource used in Design 5 (for a total cost of $500,000), $660,000 of NED and 2 units of EQ and $850,000 of RD were produced without any choice regarding the product mix. The particular resource levels used in Design 5 must necessarily result in the product mix shown for that design with absolutely no freedom of choice by the analyst regarding the product mix. The only influence the analyst-planner has is in the choice of the various levels of resources used. Movement from Design 5 to Design 6, again, says nothing of trade-offs. The relation among the products is fixed once the resource levels are chosen. It follows there is no way to calculate trade-offs along an iso-cost surface, because an iso-(minimum) cost surface for this case is only a point. Again, the only thing that can be determined from a choice of Design 6 over Design 5 was that the marginal benefits exceeded the marginal costs for the movement. Product-product trade-offs are not relevant for the case of joint products.

Another possible description of the data used in Marshall (1970) is assorted production without coupling ($\alpha > 0$, $\kappa = 0$). The implications for trade-off calculations are even different for this case. The relations could then be represented by:

$$\begin{aligned} RD &= f(x_{1R}, x_{2R}, \ldots, x_{nR}) \\ EQ &= g(x_{1E}, x_{2E}, \ldots, x_{nE}) \\ NED &= h(x_{1N}, x_{2N}, \ldots, x_{nN}) \end{aligned} \qquad (5.21)$$

$$x_i = \sum_j x_{ij}$$

where,

x_{ij} = amount of the ith factor allocated to production of the jth product, ($i = 1, 2, \ldots, n$; j = RD, EQ, NED).

These relations can also be presented as:

$$F(NED, EQ, RD; x_1, x_2, \ldots, x_n) = 0,$$

or, (5.22)

$$G(NED, EQ, RD; C) = 0.$$

Given the relations in (5.22), changing from Design 1 toward Design 7 again implies movement across several iso-cost surfaces (much the same as for factorially determined production without coupling). Again, trade-offs cannot be calculated from the information given in Table 3 (Chapter II, p. 31). The trade-off ratios would be the slopes of each iso-cost surface in the neighborhood of the data points. Movement along some path across the several surfaces will not give the trade-offs. All that can be said is that marginal benefits exceed (or at least equal) marginal costs in the movement from Design 5 to Design 6 (if, indeed, 6 was chosen over 5). Also, Marshall implicitly assumed that all the money valued products were, indeed, in the correct ratios as defined in the equilibrium conditions $(dW_i/dW_j) = -(P_{Wi}/P_{Wj})$ $(i, j = 1, 2, \ldots, m;\ i \neq j)$ for m water products. In fact, all of these conditions would have to hold at every point on the path Marshall chose to follow among and across the various iso-cost surfaces.

Other literature on trade-off calculations as discussed in Chapter II of this study include mainly variations on the approach utilized by Major. The paper by Miller and Byers (1973), for example, is a direct application of the approach advocated by Major. Nearly all authors that have written on the subject of trade-offs in water resource development in relatively recent years have followed a similar approach to that used by Major. The only exceptions, as discovered by this author, include the DeVine and Marshall reports just reviewed.[98]

[98] The article by Castle ("Economics of...", 1973) raised many potential problem areas in trade-off calculations, some of which were dealt with here.

Evaluation of Federal Planning Procedures

The latest document guiding the plan formulation processes carried out directly by the Federal government and by State or other entities receiving Federal support was approved by the President in October of 1973. This document, entitled "Principles and Standards for Planning Water and Related Land Resources" (WRC, "Establishment of...", 1973, pp. 24778-24869), evolved over several years of effort by the Water Resources Council.[99] Basically, this document outlines in some detail the theoretical and technical concepts (principals and standards, respectively) that are to guide water and related land resource planning in the United States. Future planning efforts are to include explicit and direct consideration of possibilities for the enhancement of national economic development and environmental quality. The designers of the new principals and standards, then, clearly recognized a need for multiple objective planning. A problem in valuation is explicitly recognized. Trade-offs are to be calculated among incommensurables.

A curious feature of the document, however, is the lack of a concise definition of a trade-off. Only some vague notions of the concept can be extracted from the document. In fact, there seems to be

[99] See Chapter II for a discussion of the development process and major features of the document.

some confusion as regards what trade-off is to be calculated. The product-product and factor-product trade-off ratios are of concern in some sections, while the consumption trade-off ratio is discussed in other sections, with no delineation of the different concepts. Consider, for example, the paragraph (WRC, "Establishment of...", 1973, p. 24796):

> The priorities and preferences of the various individuals affected will vary and, accordingly, there will likely not be full agreement among all affected on whether certain effects are beneficial or adverse or on the relative trade-offs between objectives. However, when any plan is recommended from among alternative plans, there is an implicit expression of what is considered to be the affected group's priorities and preferences (underlining added).

This notion of a trade-off is the consumption trade-off as defined in this study. At another point in the new planning document, it is noted that (WRC, "Establishment of...", 1973, p. 24830);

> In this (plan) formulation step, an analysis and comparison of alternative plans is outlined to make the following determinations:
>
> 1. the effectiveness of given alternative plans in meeting the component needs of the objectives;
> 2. the differences among alternative plans in terms of their contributions to the objectives and where appropriate their effects on regional development and social well-being; and
> 3. the relative value of those beneficial and adverse effects that are essentially presented in nonmonetary terms, in terms of what is given up or traded off among plans with varying degrees of contributions to the objectives (underlining added).

This notion of a trade-off ratio is somewhat akin to the product-product and factor-product trade-off ratios of this study. Very intuitive notions of the trade-off concept appear to have been used because of the inconsistency in the use of the term.

The most consistent use of the term trade-off was with reference to reductions in net dollar benefits for some increase in a non-market valued good. In one of the hypothetical numerical examples included in the planning document, trade-offs were viewed as reductions in net national economic development benefits for an increase in the contribution to the environmental quality objective. Trade-offs were calculated between a plan selected by the resource planners (Plan C) and one not selected. The example data used for illustrative purposes in the planning document is reproduced in Table 13. The trade-off value for this example was said to be a reduction of ($30-20=) $10 in net benefits to achieve the improvement in environmental quality represented by improvements in water quality for 100 miles of river in Plan C. This was accomplished, in this hypothetical case, by adding reservoir capacity for downstream low flow augmentation. It should be noted, however, that costs were reduced by $10 at the same time that net benefits were reduced by $10 (Table 13).

The net benefit trade-off ratio, as calculated from the data in Table 13, must be used with caution. Several important relations were not considered in that analysis. The conceptual models of this study and the empirical examples of this Chapter provide some insight into the manner in which the actual trade-offs for the hypothetical data of

Table 13 should have been calculated. Assume, for example, the prices used in the development of the total benefit estimates of Table 13 are given by (chosen arbitrarily for discussion purposes):

$$P_{FC} = \$1.50$$

$$P_R = \$1.00$$

$$P_P = \$2.00 .$$

Using these prices, the physical units produced at the site for each alternative plan are given in Table 14. The data of Table 14 is identical with the data derived from the planning document (Table 13) except beneficial effects are represented in physical units. Note that "miles of quality water (M)" has been added as one of the beneficial effects. The data of Table 14 can now be illustrated in a three-dimensional figure to better demonstrate the notion of a trade-off. The flood control level is the same in both plans so this element can be ignored in the diagrammatic representation.

The plan with EQ deleted (miles of quality water = 0) is shown at point A (Figure 53). Point B represents plan C where recreation was reduced to 20 units, power was reduced to 15 units, and "miles of quality water" (M) was increased to 100 miles. The cost at points A and B was given as $90 and $80, respectively. Assume, now that other combinations of FC, P, R, and M were possible at <u>each</u> of the given cost levels. Assume, also, that all these other combinations

Table 13. Project data (original).

	Recommended Plan with Service to EQ Objective Deleted	Recommended Plan C
NED objective:		
Beneficial effects:		
FC	$ 50	$ 50
Recreation	30	20
Power	40	30
Total	$120	$100
Adverse effects:		
Project construction and OM & R	$90	$80
Net beneficial effects	$30	$20
EQ objective:		
Beneficial and adverse effects	1.	1. Meets State water quality standards over 100 miles stream.
	2. 3, 500 acres flat water.	2. 3, 000 acres flat water.
	3. Inundate 11 miles free flowing stream.	3. Inundate 10 miles free flowing stream.

Reproduced from (WRC, "Establishment of ...", 1973, p. 24855).

Table 14. Project data (derived).

	Plan with EQ Deleted	Plan C
NED objective:		
Beneficial effects:		
Flood control (FC)	33.33	33.33
Recreation (R)	30.00	20.00
Power (P)	20.00	15.00
Miles of quality water (M)	0	100.00
Total dollar benefits	$120.00	$100.00
Adverse effects:		
Project construction and OM & R	$90.00	$80.00
Net beneficial effects	$30.00	$20.00

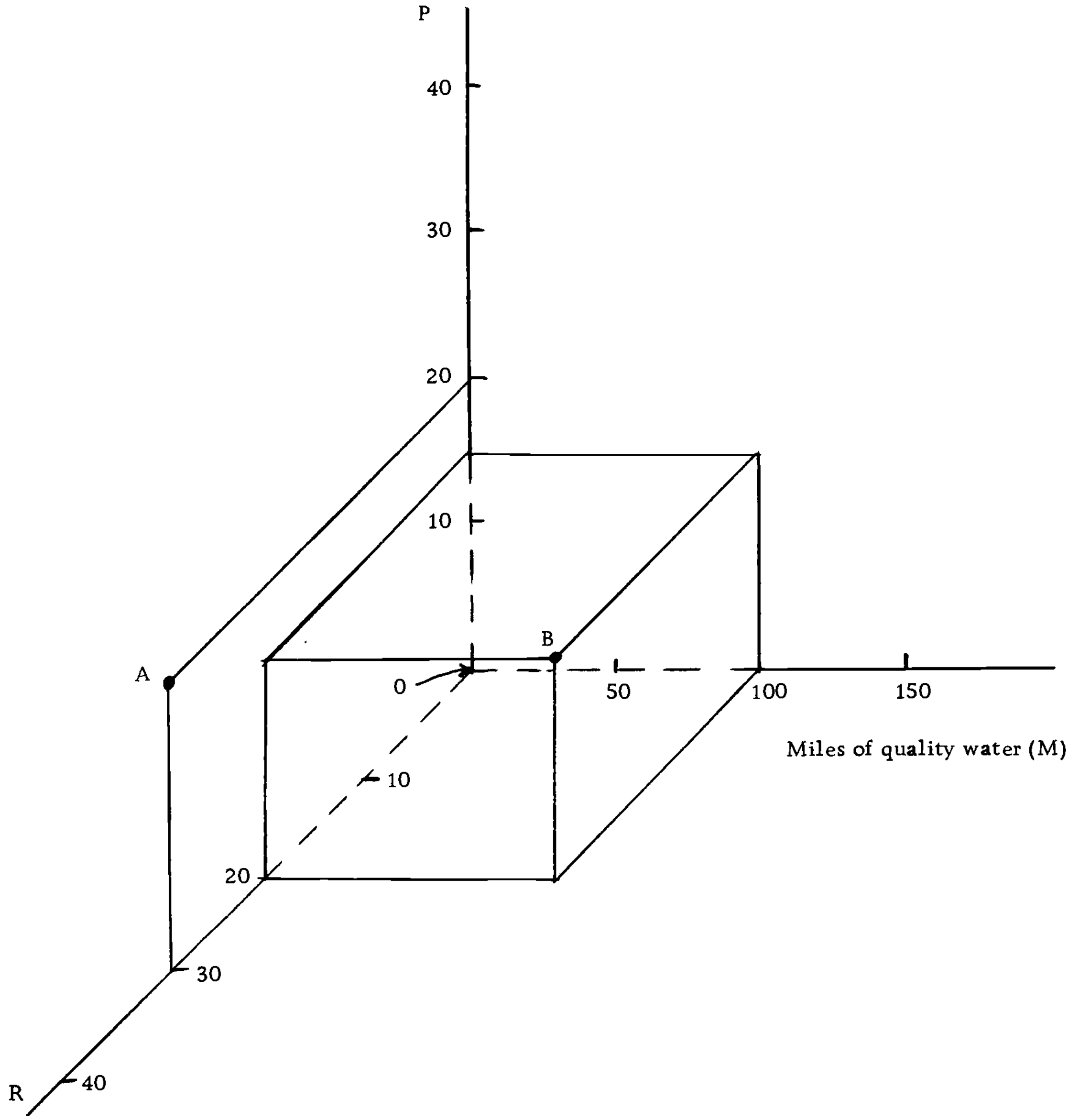

Figure 53. Alternative plans for provision of power (P), recreation (R), and environmental quality improvements (M).

(as well as the combinations at A and B) were produced at least cost. As a result of these specifications, points A and B now are on iso-cost surfaces of the general nature shown in previous sections of this Chapter. Point A lies on the \$90 surface and point B is on the \$80 surface. Trade-offs are now given <u>at</u> points A and B and <u>not</u> for movements between A and B as accomplished in the planning document. Points A and B are not directly comparable as the cost level is different at each point. The trade-offs, for this case, are the values of the derivatives (dP/dR) and (dP/dM) at each point. These derivatives cannot be obtained from the data of Table 13 but could be approximated once the two different surfaces were identified. The trade-off of -\$10 for an increase in M by 100 miles, as discussed in the planning document, is not a trade-off as defined in this study.

Given that points A and B do in fact represent points on iso-cost surfaces, an implicit value of the M product was already assigned by the resource planners at each point. Point A, one of the alternative plans obtainable with an expenditure of \$90, could not have been selected from all the other designs on the \$90 surface unless a value had been assigned to M by the resource planners. Similarly, point B on the \$80 surface could not have been selected from all the designs on that surface without assigning a value to product M. As a result, the relative value of M cannot be determined by the change in net benefits occurring for the movement from point A to point B (from one surface to another) as recommended in the planning document. The value of M had to be known to define the path between A and B.

The real meaning of the choice of point B (plan C) over point A can only be ascertained after explicit recognition of the implicit expansion path used for the movement between the points. The expansion path, in turn, is defined by the relative price ratios at each of the points A and B. The resource planner, as a result, has tremendous flexibility in influencing the final outcome as the location of the path between the various surfaces is determined by the planners perception of the value of M. Assume some path has been determined. Given that information, selection of point B over point A would then imply that the marginal benefits, both monetary and non-monetary, were <u>less</u> than the marginal costs for the movement from B toward A; i.e., net benefits (both monetary and non-monetary) were greater at point B than at point A. As a result, the decision body decided to "stay" at point B. Migration up the path from B toward A (the movement between surfaces) was not "profitable"; marginal social costs evidently exceeded marginal social benefits in the movement from B toward A. This is all that can be said given only points A and B. No trade-offs are involved.

Another interpretation of the data in Table 13, as reproduced from the planning document, is also possible. In the previous interpretation, it was assumed that iso-cost surfaces did in fact exist.

Based on the empirical estimation at the Bronco site, this assumption is defensible for most dam-reservoir configurations. The possibility exists, however, that FC, P, R, and M are joint products. In that case, there is no iso-cost surface, per se, but rather a series of points (such as A and B) in the multi-dimensional space. Each point, in turn, has a particular (minimum) cost. Even with this interpretation, however, trade-offs are not defined for movements from one point to another over varying levels of cost. Choice of point B over point A would again imply only that marginal benefits were less than marginal costs in moving from B toward A; i.e., the resource planning group decided to "stay" at point B where (evidently) net benefits were at a maximum. Given FC, P, R, and M are joint products, there is some set of factor-free relations such as:[100]

$$\begin{aligned} FC &= f(P) \\ P &= g(R) \\ R &= h(M) \\ M &= e(C) \end{aligned} \tag{5.23}$$

or,

[100] Many other possibilities also exist. One of these is given by:

$$FC = f(C) \qquad R = h(C)$$
$$P = g(C) \qquad M = e(C).$$

$$
\begin{aligned}
F^1(FC, P) &= 0 \\
F^2(P, R) &= 0 \\
F^3(R, M) &= 0 \\
F^4(M, C) &= 0
\end{aligned}
\qquad (5\text{-}23a)
$$

where C = cost.

The degree of assortment, α, is zero and the degree of coupling, κ, is three. Given some level of cost, the levels of FC, P, R, and M are determined by the physical relations. The "expansion path", as a result, is given as soon as C is known. The "expansion path", in this case, is not influenced by the resource planners' perception of relative prices but rather by the physical production relations.

Another important problem area in the new planning document also relates to the use of net dollar benefits in trade-off calculations and, as a result, for estimates of value. The principles and standards actually describe a process for discovering trade-offs among various social objectives at the macro, aggregate level. The recommendation to use net income in trade-off calculations is valid at the macro level. The "net income" to be used at this level, however, is net national income.[101] One can envision (vaguely) the transformation surface

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[101] Net national income is an aggregate concept, more commonly referred to as net national product. Net national income, then, is the difference between gross national product and the capital consumption allowance. Gross national product, in turn, represents the dollar

for the Nation having net national income on one axis. All the other social objectives, represented by some other measures of achievement, would form the other axes. Trade-offs and resulting estimates of value would then arise for movements along this surface. Net national income, for example, may have to decline (or increase) to achieve a higher level of environmental quality. The implicit value of environmental quality achievements is then given, for movements over the Nation's transformation surface, at the point selected by society.

The problem with the recommendations in the planning document regarding trade-off calculations is the use of net dollar benefits generated at the project level as a measure of net national income. This is confusing a very important, but subtle, set of issues. Net dollar benefits are a micro concept and must not be confused with the macro concept of net national income. In fact, a measure of net national income at the project (micro) level is given by total (gross) dollar benefits, less depreciation on capital items, generated from the government investment. All the factor payments (for project construction, maintenance, operation, and repair) must also be included to provide a measure of net national income. All these factor payments have been subtracted in a measure of net dollar benefits generated at the project level. The failure to recognize this very important

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value of all goods and services produced during some known period of time (usually a year). Net national income is also sometimes referred to only as "national income". See Ackley (1961, pp. 25-37).

difference has led to some confusion over the measure of money valued benefits to be used in trade-off calculations.

Net national income, as measured by total dollar benefits, should be used in trade-off calculations. This would be consistent with calculating trade-offs along the Nation's transformation surface. It is implicitly assumed, then, that resources or costs are constant. As a result, net dollar benefits can also be used in trade-off calculations as long as trade-offs are calculated along an iso-cost surface. The reason for this, of course, is that <u>changes</u> in net dollar benefits are identical to <u>changes</u> in total dollar benefits (the measure of net national income) when costs are constant. The recommendation to use net dollar benefits in trade-off calculations at the project level is valid only when the trade-offs are accomplished along an iso-cost surface.

The definition of a component should also be made explicit in the planning document. Given that trade-offs are to be calculated among components, it becomes important to discover the various levels. There is little difficulty with the "components" of the national economic development objective, some examples of which are (WRC, "Establishment of...", 1973, p. 24826):

1. water for irrigation,
2. water for recreation,
3. water for hydroelectric power production,
4. water for transportation,
5. provision of flood control facilities,
6. water for municipal and domestic use,
7. water for industrial use.

All of these components or products can be considered as being on the same level. The product-product trade-off models for these pairs of components can be applied directly. There is some question, however, as to the meaning of several of the components of the environmental quality objective, such as (WRC, "Establishment of...", 1973, p. 24826):

1. miles of scenic river having specified characteristics,
2. acres of ecological areas of specified type preserved or enhanced,
3. reach of river meeting specified water quality standards,
4. number of open space areas of specified types.

Some of these components are <u>not</u> on the same level as the components (products) delineated above for the economic development objective. Miles of scenic river having a particular characteristic, for example, is better classified as a factor in most project situations. This factor can be combined with other factors to produce water for irrigation or some other product. Similarly, acres of ecological area and an open area may indeed be factors that could potentially be used in the production of water products. Even further complexity is added by noting that "reach of river meeting specified water quality standards" could be an intermediate product necessary to the production of some water product such as water for irrigation. A case can also be envisioned where open space may also be a product. Consider, for example, the possibility that open space is essentially "produced" by building a water project to irrigate agricultural land. The act of producing water for irrigation, as a result, also provided the product, open space. Some different classification system is needed. The single classification of "component" is not sufficient as the type of trade-off to be calculated is difficult to identify.

The confusion can be eliminated by using the already well established concepts and jargon of multiple output production economic theory. Using that jargon, there are three types of "components", namely, factors, intermediate products, and products. The procedure followed in developing the necessary planning information is then conditioned by the type of components prevailing in the project area under investigation. The classification of a particular component is necessarily ad hoc. This classification system, however, is extremely useful in calculating the necessary product-product and factor-product trade-off ratios. A product-product trade-off ratio should be calculated if the non-money valued component is a product. If the non-money valued component is a factor, a factor-product trade-off ratio has to be provided.

The concepts of complementary and conflicting components of the objectives as used in the planning document also need clarification. Alternative plans are to be formulated based on a nucleus of complementary components. The following approach is to be followed (WRC, "Establishment of...", 1973, p. 24829):

> Within a given set of assumptions concerning future change and the component needs associated there to, the number and types of alternative plans to be developed will be determined by applying the following:
>
> 1. on a first approximation basis array component needs that are essentially complementary--that is, the satisfaction of one of these component needs does not preclude satisfaction of the other component needs or does not result in materially adding to the cost of satisfying the other component needs in the array; and
> 2. from the above approximation, it should be possible to group component needs and the elements of a plan to satisfy those needs that are essentially in harmony, each set representing the nucleus for an alternative plan (underlining added).

This is a very curious approach to defining the "nucleus for alternative plans". The notion of complementarity among components (products) described in the document describes a production situation where the production process is being carried on in an uneconomic area of production; i.e., the iso-cost curve is positively sloped.[102] More of both products can be obtained without increasing costs. As a result, more of both should be produced until the economic region is reached. Stopping production prior to reaching the economic region is equivalent to saying that one of the products has a negative price. It is questionable procedure, therefore, to let such product combinations (in uneconomic regions) form the nucleus for alternative plans. Also, it must be cautioned that this notion of complementarity is not consistent with the concept of technical complementarity in production

[102] See Appendix B for a discussion of complementarity, competitiveness, and independence.

economic theory.

The definition of conflicting components used in the document, on the other hand, is consistent with the conceptual framework developed here. The need for alternative plans is said to be attributable, at least in part, to the existence of conflicting components (WRC, "Establishment of...", 1973, p. 24829). The conceptual models of this study would suggest that alternative plans should be developed such as to define the economic region of the iso-cost curve. "Conflicting" products, in turn, are best described by negatively sloped iso-cost curves.

The possibility does exist, of course, where two or more joint products could be provided such that all products could be increased simultaneously. This does not describe the case of technically complementary products, but it does describe a situation where the products are, essentially, complementary.[103] This type of joint relationship may exist in water resource production. One possible example of this is the relation between water for recreation and an aesthetically pleasing view of the reservoir (man-made lake). The relationship may be as follows:

[103] In this case, the derivatives (dq_i/dq_j), $i \neq j$, would be all positive. Costs, however, would not be constant along the joint product curves. See Appendix B.

$$AP = f(W_R)$$
$$W_R = g(C) \tag{5.24}$$

where,

AP = aesthetically pleasing views,

W_R = water for recreation, measured in surface area of a reservoir,

C = cost.

If $d(AP)/dW_R > 0$ in (5.24), AP and W_R are complementary products in the sense that AP will increase with increases in W_R. This is not, however, an iso-cost curve. Costs will vary along the entire length of the curve plotted from the $AP = f(W_R)$ relation. Given these types of joint relations, no harm would be incurred by using such products as the "nucleus" for alternative plans. Caution must be exercised in finding these complementary groups of products, however, as errors could easily be made. The uneconomic regions of production may be discovered instead of the joint product case.

The procedure recommended in the planning document for formulating alternative plans could be improved with use of the conceptual models in this study. The need for alternative plans arises because of negatively sloped portions of iso-cost surfaces. This is where product-product trade-offs exist. As a result, the number of alternative plans that must be provided to the decision body is conditioned by the nature of the iso-cost surface. It is expected that large areas of the surface could be eliminated based on agency perception of social preferences. If this is impossible, the entire surface must be presented (or possibly several surfaces). Any delimitation of the surface (or surfaces), of course, must be documented. If this is not done, the resource planner could be accused of making decisions on relative values which possibly should be made by some higher level decision body. All joint product relationships should also be identified. Caution must be exercised in evaluating these relationships. The previous example is a case in point. It was assumed that $(d(AP)/dW_R) > 0$ in the hypothesized joint relation between AP and W_R. The possibility also exists that $(d(AP)/dW_R) < 0$ when W_R gets large enough. Also, positively sloped regions of iso-cost surfaces are really uneconomic regions of the iso-cost surface. These regions of the surface can be eliminated as alternatives early in the planning process. It should also be noted that the search for "...alternative means of satisfying a component need..." (WRC, "Establishment of...", 1973, p. 24830) is essentially a search for the most efficient level of production. The frontier iso-cost surface, for any given investment level, must be discovered.

The need to calculate trade-offs along an iso-cost surface also has some implications for setting a criteria regarding the type and number of plans to be formulated. It was recommended in the planning document that at least three plans should be developed for every project area. One plan was to emphasize the national economic development objective and one was to emphasize the environmental quality objective. A third, or several more plans, was to represent alternative achievement levels of both objectives (WRC, "Establishment of...", 1973, p. 24830). The changes in net dollar benefits among the alternative plans were then to be utilized in trade-off calculations. Based on the findings of this study, this approach is valid if and only if all such plans are on the same iso-(minimum) cost surface in product space.

VI. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

The social and economic objectives of society in the United States are many and varied. A great deal of concern has been expressed by individuals and groups of this society, especially in recent years, over the direction the Nation should take relative to the achievement of various objectives. All objectives cannot be achieved simultaneously because of a finite resource base. Trade-offs must be calculated along the Nation's transformation surface. Public investment, in turn, can be used to influence the direction taken across the surface. Investment in water resource development, for example, will affect the point attained on the surface, and as a result, the trade-off ratios.

Planning principles and procedures used in the public sector reflect the felt need to deal explicitly with trade-off calculations. The final product from the application of multiple objective procedures is a set of trade-off ratios. The problem to which this study was addressed was to discover the nature of these trade-off ratios that must be calculated in water and related land resource planning. The overall purpose of the study, then, was to define and indicate the elements affecting the trade-off ratio. More specific objectives were:

1. to develop the conceptual base for the trade-off ratio calculation process,
2. to outline some actual trade-off ratio calculation procedures for water resource planning and development based on the conceptual base provided.

The approach taken in the study was conditioned by the premise that any planning group, private or public, is involved in isolating a multiple product production function. The need for calculating trade-offs arises when some of the products and/or factors do not have known, market determined prices.

Review of Literature

Several alternative ways of optimizing a multiple objective function have been proposed in the literature. Several inconsistencies and contradictions were discovered. The most significant inconsistency related to what money measure of value was to be "traded-off" for non-money valued goods. Some authors used net dollar benefits. Other examples include total dollar benefits, economic efficiency, net national income, and national income. Very little was said about the role of dollar costs in trade-off calculations. In some cases, dollar costs were subtracted before trade-offs were calculated. Most authors appeared unconcerned with the level of investment required to achieve various money and non-money valued benefits.

Theory of Multiple Output Production

The theory of multiple output production provided the base for developing a model useful in isolating the conceptual and procedural issues relating to trade-off calculations. The major production laws were delineated by the values of α (degree of assortment) and κ (degree of coupling). The production laws were given by:

1. factorially determined production:
   a. with coupling, $\alpha = 0$, $\kappa > 0$
   b. without coupling, $\alpha = 0$, $\kappa = 0$
2. assorted production:
   a. with coupling, $\alpha > 0$, $\kappa > 0$
   b. without coupling, $\alpha > 0$, $\kappa = 0$.

The concept of flexibility was introduced. The least flexibility in choice available to the decision maker regarding the eventual product mix exists in case 1a and the most flexibility in case 2b. Case 2b is the production law most frequently (almost exclusively) discussed in the economic literature on multiple output production. The case of joint products exists, it was argued, only when $\kappa > 0$. The number of factor free relations defines the value of κ.

The economic optimization calculus was outlined for the main production classes. The factorially determined case is very similar in nature to the optimization process for single output production. The case of assorted production is well documented in the literature. The iso-cost and transformation surfaces were derived and discussed in the context of economic optimization. It was shown that iso-cost surfaces can be used to summarize both factorially determined and assorted production. The product substitution along an iso-cost curve or surface was then shown to represent different phenomenon dependent upon the underlying production function relations. It was also shown that transformation (iso-factor, iso-resource) curves and surfaces do not exist in the factorially determined case. Also, given assorted production, a transformation surface and an iso-cost surface will plot identically in product space given the correct changes in relative prices.

The product-product trade-off ratio was defined as the slope of the iso-cost curve in product space. This definition, it was argued, applies given either major class of production law. The nature of the product-product trade-off ratio was then outlined. The effects of technical relations on trade-off values were discussed. Incorrect trade-offs may result unless the frontier iso-cost curve is discovered. Intermediate products may affect the frontier curve and have to be accounted for in order to insure the correct curve is found. The nature of production possibility (internal, but tangent to the frontier) curves was discussed. The "frontier" iso-cost curve, it was argued, must be found. Several other incorrect iso-cost curves may be discovered unless technical relations and intermediate product effects are properly included in any analysis. Also, it was shown that the use of production possibility curves may lead to incorrect estimates of product-product trade-offs.

The factor-product trade-off ratio was defined as the slope of a total product function. The factor-product trade-off ratio is more commonly referred to, in the economic literature, as the marginal physical product. This ratio represents the amount of product that must be sacrificed to remove some of the factor from planned or present use. In this sense, it is a trade-off ratio.

Nature of the Production and Optimization Processes in Water Resource Development

The theoretical models were then used to isolate the conceptual and procedural issues regarding trade-off calculations. It was argued that planning agencies must identify the underlying production relations to insure the proper trade-off ratios are calculated. A very important phase of this process, it was argued, was the identification of the products and factors. A viewpoint must be established to accomplish that process. Factors from the water resource planners viewpoint, for example, may be products from the viewpoint of society. These various levels of production must be separated and delineated to calculate trade-offs. A wilderness area, for example, may have to be treated as a factor in the planning process. In that case, the factor-product trade-off ratio should be utilized. If wilderness area is classified as a product, the product-product trade-off ratio should be utilized.

The importance of delineating an independent planning unit, if possible, was also noted. An independent unit is given for every independent group of products. An independent group of products, in turn, can be represented in one transformation or iso-cost surface. In fact, all interdependent groups should be summarized in one transformation surface. The products of water resource development, it was argued, usually cannot be considered as forming an independent group. There is, probably, substantial interdependence among water products and the products of other public and private investment. The optimum water product mix may be sub-optimal, then, unless the other (non-water) products are also considered in the analysis.

Some possible production relations among water products were then outlined. Several empirical questions were raised. The production law describing the production of water for irrigation (W_I) and water for recreation (W_R) would be described as assorted if the allocations of each factor to both products can be determined. It was argued that W_I and W_R would be factorially determined if such allocations could not be determined. The actual relationship has to be determined by empirical estimation. Several other possible physical relations among water products were also discussed--such as technical and intermediate product relations.

A means of identifying the value of non-money valued water products and factors used in the production of water products was then outlined. The product-product and factor-product trade-off ratios were introduced and discussed in the context of water product production processes. It was shown that technical relations, unless identified by water planning agencies, could lead to distorted estimates of trade-offs. Also, the need to identify intermediate water product relations to insure the correct trade-offs are calculated was discussed. If water quality (as a water product) is an intermediate product to the provision of W_I (as well as a final product for aesthetic purposes, for example) the trade-off ratio may be affected. Again, several empirical questions were raised. The nature of net benefit curves derived from iso-cost surfaces was also discussed. It was argued, for example, that net benefit trade-offs may be misleading unless calculated along an iso-(minimum) cost surface. Also, the path taken across that surface must be such that $(dW_i/dW_j) = -(P_{Wj}/P_{Wi})$, $(i, j = 1, 2, \ldots, m;\ i \neq j)$, for the m water products.

The factor-product trade-off was also discussed in the context of water resource planning. It was argued that many of the components affected by water resource development should be viewed as factors in the planning process. A wild and scenic river, for example, may be a factor in the production of some water product such as water for power production. The validity of using net dollar benefits for these trade-off calculations was also outlined. It was argued that net benefit trade-offs may be misleading unless the price of the non-money valued factor is set at zero during the estimation of the net benefit curve. Also, great caution must be used in developing net benefit curves for the factor-product case when two or more products are concerned. It was argued that the product mix should be taken from the product levels when $(dW_i/dW_j) = -(P_{Wj}/P_{Wi})$, $(i, j = 1, 2, \ldots, m;\ i \neq j)$ for all m water products.

Problems in the Calculation of Trade-Off Ratios--Some Empirical Cases

The conceptual models were quantified using actual data from the proposed Bronco and Kineman dam sites in the Knife River Basin in North Dakota. Each site formed a non-independent planning unit. In fact, even when both sites were combined the planning unit still did not represent an independent group of products. Three products--water for irrigation (W_I), water for municipal-industrial use (W_M), and water for recreation (W_R)--were identified for analysis. Several product-product trade-offs were estimated for these water products. Wilderness area was viewed as a factor to illustrate some of the problems associated with calculating and interpreting factor-product trade-off ratios.

The important relations underlying the iso-cost surfaces are given by the general forms:

$$\begin{aligned} TC &= f(AC) \\ SA &= g(TC) \end{aligned} \tag{6.1}$$

where,

AC = total annual (minimum) cost,

TC = total reservoir capacity at spillway level, measured in acre feet,

SA = surface area of reservoir when full to capacity, measured in acres.

The actual functions representative of the general forms in (6.1) were then estimated using ordinary least squares regression. The general approach followed was to set alternative levels of AC to give particular levels of TC. This estimate of capacity was then allocated among the various water uses. Surface area of the reservoir was used as a proxy variable for the recreation product. The trade-offs among W_I, W_M, and W_R were then estimated along several iso-cost surfaces representative of production relations in the Knife River Basin.

The first models developed involved only W_I and W_R. Trade-offs were estimated to range at the Bronco site from -28.41 to -61.00. This implied a range in the implicit value of W_R of approximately

\$852.00 to \$1830.00 per acre. The product-product trade-off ratio was also found to decline (become more negative) for increases in W_R at all expenditure levels. Similar types of relations were also estimated at the Kineman site and when the sites were combined. The simultaneous analysis of both sites highlighted the need to calculate trade-offs along the frontier or iso-(minimum) cost surface. Trade-off estimates would be distorted if the frontier curve was not discovered. Also, use of net dollar benefits as a measure of the sacrifice for increases in W_R was shown to give distorted estimates of the value of W_R for movements among iso-cost curves or for movements among internal curves.

Iso-cost surfaces were then estimated for the three product case at the Bronco site. Two cases were considered. It was first assumed that the trade-off ratio between two of the three products was constant and equal to -1.00. The range in estimated values of P_{WR} was approximately \$852.00 to \$903.00 per acre for the \$200,000 iso-cost surface. Net dollar benefits were also used to calculate P_{WR}. The trade-offs, as expected, were distorted when costs were allowed to vary; i.e., the sacrifices in money valued outputs were distorted in absolute value for movements among iso-cost surfaces. It was also shown that net dollar benefits and/or total dollar benefits could be used to estimate P_{WR} for movements, in any direction, across the iso-cost surface for this case.

The second assumption regarding the relation among the three products gave rise to variable trade-off ratios among all three products. Again, it was shown that net dollar benefits cannot be used to estimate value for movements among iso-cost surfaces. One such example movement gave an estimated price of W_R in the range $\$39.80 < P_{WR} < \$71.83$. The actual price of W_R at the optimum point, estimated by the correct approach, was shown to be \$983.25.

The calculation and use of the factor-product trade-off ratio was also illustrated at the Bronco site. Wilderness area (W_A) measured in acres, was viewed as a factor in the production of W_I. Viewed in another manner, W_I had to be reduced to make W_A available for uses other than the production of water products. The underlying production relations were found to be extremely crucial to the correct interpretation of the factor-product trade-off ratio. The marginal product of W_A must be declining at the point chosen by the decision body before the estimated price of W_A could be taken directly from the curve. One point selected for illustrative purposes gave an estimated price of \$874.20 per acre. The actual price was found to be no more than \$139.83 per acre at the same point. The distortion was nearly seven-fold. It was shown that net benefit curves derived from marginal product schedules must be interpreted very carefully. The slope of the marginal product schedule must be known.

Comparison of Recommendations Regarding Trade-Off Ratio Concepts and Procedures

The recommendations in the literature on trade-off concepts and procedures was compared with the results of this study. A common error in the other literature was to ignore the existence of product-product and factor-product trade-off ratios. The various levels of production should be separated to calculate trade-offs. Net benefit curves are frequently used to calculate trade-offs with no concern expressed for keeping costs constant. This approach is valid, it is argued, as long as costs are held constant.

Evaluation of Federal Planning Procedures

A document to be used in the plan formulation processes carried out directly by the Federal government and by State or other entities receiving Federal support was approved by the President in October of 1973. This document, entitled "Principles and Standards for Planning Water and Related Land Resources" (WRC, "Establishment of. . . ", 1973), covers many problem areas related to planning. Several segments of the document were evaluated based on the results of this study. It was argued that the concept of a trade-off was not defined concisely, which could lead to misleading estimates. The concepts of consumption trade-offs and production (product-product and factor-product) trade-offs are used interchangeably in the report.

The most significant problem discovered in the report was the recommendation that net dollar benefits be used to calculate trade-offs at the project level. The recommendations are valid at the aggregate level where net national income is to be traded-off for the achievement of other objectives. It was argued that net dollar benefits at the project level are not the proper measure of net national income levels unless costs are held constant. Net dollar benefits can be used for estimating value of a non-money valued output if costs are held constant and all money valued outputs are kept in the ratios dictated by the equilibrium conditions $(dW_i/dW_j) = -(P_{Wj}/P_{Wi})$ $(i, j = 1, 2, \ldots, m,\ i \neq j)$ for all m water products. It was further argued that net benefit trade-offs among alternative plans as recommended in the planning document are valid only if these conditions were met.

Conclusions

The major findings and conclusions of this study revolve mainly around the nature of the net dollar benefit measure of the value of non-money valued products and factors. Based on the results of this investigation, it can be concluded that:

1. if the non-money valued component is a product, the product-product trade-off ratio should be used. Ideally, the iso-cost surface should be developed. This is a comparatively simple

task when only three (or fewer) products are of concern. The net benefit curve can also be used for this case if the following conditions are met:

a. dollar costs are constant at every point on the curve,

b. all the money valued products are in the combination dictated by the equilibrium conditions $(dW_i/dW_j) = -(P_{Wj}/P_{Wi})$ $(i, j = 1, 2, \ldots, m, \; i \neq j)$ for the m water products,

2. if the non-money valued component is a factor, the factor-product trade-off ratio must be used. Ideally, the production function relations should be determined. The factor-product trade-off ratios are the marginal product schedules. The net benefit curves can also be used for this case if the following conditions are met:

a. the value of the resource of concern is initialized at zero when the net benefit curve is calculated,

b. the marginal product schedule is declining,

c. all the products are in the equilibrium combinations given by $(dW_i/dW_j) = -(P_{Wj}/P_{Wi})$ $(i, j = 1, 2, \ldots, m; \; i \neq j)$ for the m water products.

These two major conclusions reflect an ideal to be strived for by the resource planner. It is doubtful the resource planner will ever have sufficient data, realistically speaking, to calculate the exact slopes of an iso-cost surface or a production function. Only approximations can be achieved. It is expected, however, at least rough approximations could be made to these conditions.

Other findings of the study are better stated as recommendations for changes in the present Federal planning document, given by:

1. underlying production relations should be identified whenever possible to insure that mistakes are not made in the estimation of prices (given the choice of the decision body). At a minimum, resource planners should be encouraged to use the framework of multiple output production theory as the conceptual base for planning. Planners, eseentially, are charged with identifying the supply alternatives. Multiple output production theory, in turn, helps describe those alternatives;
2. three classes of components should be developed in the water resource planning procedures to insure the correct trade-off process is utilized. It is recommended that the various components of each major objective be classified as factors, intermediate products, or products. As soon as this is accomplished in each project area, the type of trade-off process that must be utilized is automatically known;
3. the concept of independent and interdependent transformation surfaces should be utilized by the resource planner to insure

the correct planning unit is delineated. If it is impossible to define the independent unit, the extent of the interdependence should at least be noted;

4. the number of alternative plans that should be developed and presented to the decision body is not a function of the number of groups of complementary components, but rather is determined by the negatively sloped regions of the iso-cost surface. As a result, the number of alternative plans that should be developed should be based on the extent of the region where trade-offs are negative. Enough plans should be developed, all for the same cost, to insure the range in trade-offs is covered;
5. the difference between net dollar benefits at the project level and net national income should be highlighted and clarified to avoid confusion;
6. the notions of complementary, competitive, and independent products should be defined consistent with the definitions provided in Appendix B of this study to avoid unnecessary confusion.

Other findings are all of a theoretical nature. As a result, the following are not "conclusions", but rather can be labeled as interesting results from multiple output production theory, as given by:

1. given assorted production without coupling, a transformation curve and an iso-cost curve will plot identically in product space given the correct change in relative factor prices. As a result, either curve will give the same product combination when the maximum profit condition is satisfied (see Appendix A);
2. given factorially determined production, there is no transformation curve, but there is an iso-cost curve (in product space). The iso-cost curve, in turn, is only a point when there is coupling;
3. there is considerable confusion in the economic literature regarding the notions of technical interdependence and independence. The Heady and Carlson approaches to classifying products, while different, have been used interchangeably in economic analysis (see Appendix B);
4. there is considerable mis-use of the concept of "joint products" in the literature. Most of the confusion is because of semantic difficulties. All confusion can be eliminated if all multiple output production processes are described for what they are--namely, multiple output production processes. The labels of "joint products" and "joint production" should be reserved for the only true case of joint products. This case results only when there is a factor free relation between the

two products of concern; i.e., there must be a coupling;

5. joint products can exist in either the factorially determined or the assorted case as long as there is a coupling;
6. the existence of joint products implies the existence of joint costs; i.e., products are joint only when it is impossible to determine the allocation of each factor among the products. Given the factor allocations cannot be determined, the costs cannot be allocated;
7. the existence of joint costs does <u>not</u> imply the existence of joint products. Joint costs can also exist when there are no joint products. This situation is described by the factorially determined without coupling case; i.e., the factor allocations cannot be determined and the products are separable (not joint) in production.

The ability of the decision body to reflect society's preferences is, of course, very crucial to the use of any trade-off ratio. It must be emphasized that prices <u>cannot</u> be determined from physical production relations or the net benefit curves derived from such relations or any other means without the imposition of the demand conditions dictated in a social welfare function. The price as discovered along an iso-cost surface, a production function curve, or along the appropriate net benefit curve, is an accurate reflection of value only to the extent the demand information is available and correct.

Recommendations for Further Research

Further research should be initiated to test the conceptual models of this study. This is especially true with regard to the use of the factor-product trade-off ratio. Further knowledge of the factor-product relations in water resource development must be obtained because of the great number of factors not having known, market prices. It is expected, for example, that most of the components of the environmental quality objective will be classified as factors in most project situations. Most environmental quality components must be used as inputs to the production of water products. Given this argument, it becomes especially important to identify all the conceptual issues and an approach for using the factor-product trade-off ratio in water resource planning and development. The empirical example of the use of the factor-product trade-off ratio was presented for a very simple case in this study.

An effort should also be made to better understand the underlying production relations descriptive of water resource production processes. Knowledge would be improved in this area, of course, through further study of the factor-product and product-product trade-off ratio calculation process. There may, however, be an even greater pay-off from this type of research. The possibility for cost allocation can only be discovered with knowledge of underlying production relations. It was shown in this study, for example, that joint costs can arise only when the products are factorially determined; i.e., a factor (or the cost of that factor) cannot be allocated on any economic basis among the products when the products are factorially determined. There are, on the other hand, no joint costs in assorted production (without coupling). It is probable that water resource production processes are a mixture of factorially determined and assorted production. As a result, some of the costs can be allocated among the products on an economic basis and some cannot. Identification of the correct production relations, then, would identify a process for cost allocation.

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APPENDICES

APPENDIX A

DERIVATION OF ISO-COST CURVES

The purpose of this appendix is to illustrate the derivation of iso-cost curves for the factorially determined without coupling and assorted without coupling production laws. The general form of the iso-cost function is given by $C_q = C_q(q_1, q_2)$ in both cases. This iso-cost curve is, essentially, the opportunity cost curve (Heady, 1952, p. 214). The process for arriving at this relation, however, is very different dependent upon the type of production law.

Factorially Determined Without Coupling

The nature of the iso-cost curve derived under the conditions of factorially determined without coupling production can be illustrated with reference to Figures A-1 and A-2. Consider, for example, the isoquant maps for q_1 and q_2 in Figure A-1. The maps are overlapping, but not by enough to make the products joint.[104] The isoquants intersect at an infinite number of points such as

[104] The location of the isoquant maps in factor space is dictated by the response of the products to various ratios of factor quantities. Viewed in the x_1, x_2 plane, isoquant maps located in the north-west direction represent products highly responsive to high x_2/x_1 ratios. Maps located in the north-east directions require lower x_2/x_1 ratios.

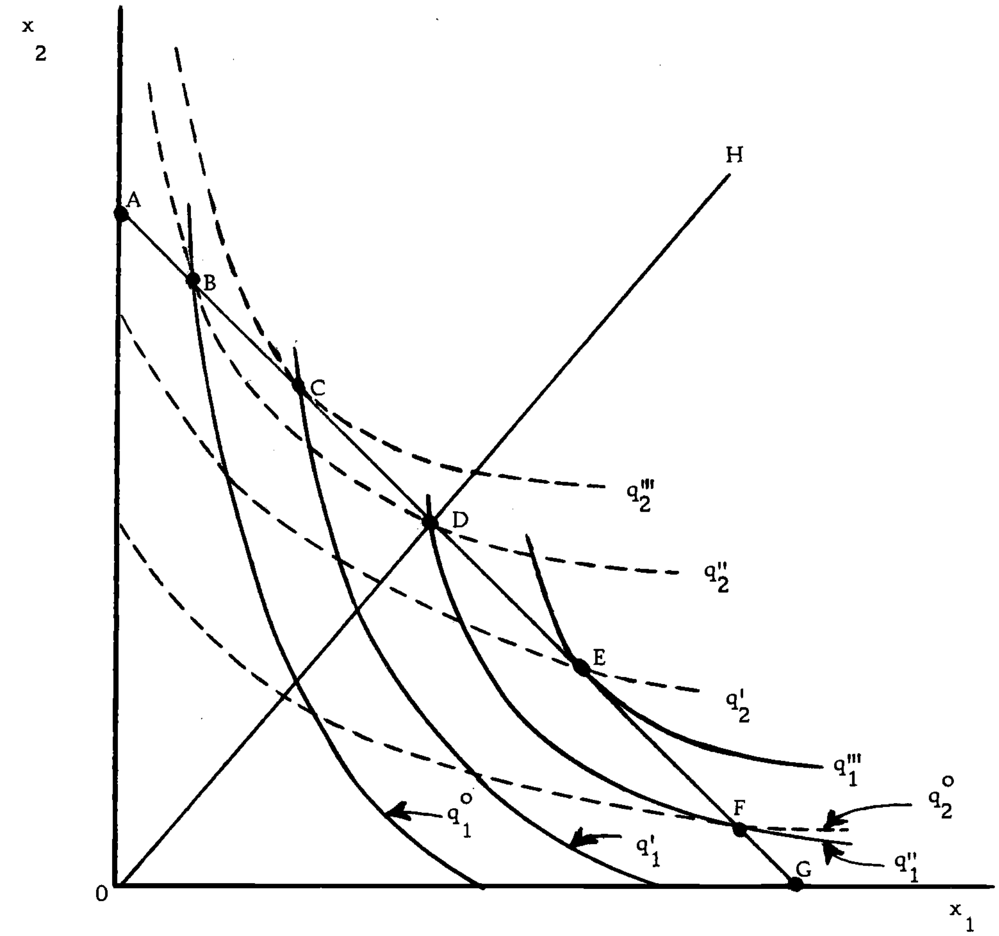

Figure A-1. Isoquants for factorially determined without coupling production, derivation of iso-cost relation.

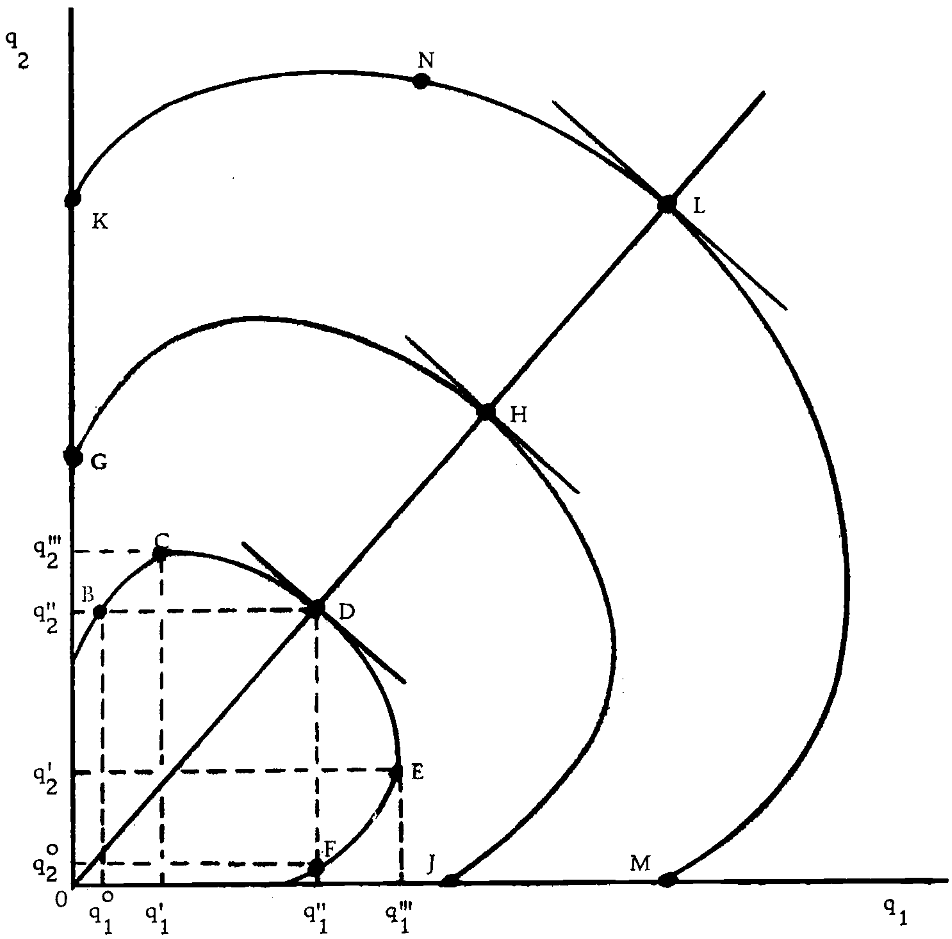

Figure A-2. Iso-cost curves.

represented at points B, C, D, E, and F. The iso-cost curve in product space can now be derived by finding those quantities of q_1 and q_2 that require the same level of expenditure. This procedure results in finding the least cost way of providing q_1 and q_2 as there is only one point in Figure A-1 for each q_1, q_2 combination.[105] All of the points having the same cost level (iso-cost in the product diagram) can be found by finding all q_1, q_2 combinations along a constant cost curve in the factor diagram, such as AG. The product combinations along AG derived for constant resource prices can then be plotted into the product diagram to give an iso-cost relation of the nature expressed in Figure A-2 along BF. The isoquants are shown to intersect along AG at points B, C, D, E, and F. Consequently, the costs at each of these points are equal. At point B, q_1^o of q_1 and q_2'' of q_2 can be produced (Figure A-1). This level of productive activity is represented at point B in Figure A-2. Movement along AG in Figure A-1 from B toward C results in an increase in q_1 (from q_1^o to q_1') and an increase in q_2 (from q_2'' to q_2'''). Both outputs were increased for a slightly different factor combination. Movements from C toward E (Figure A-1), however, requires reductions in q_2

[105]There are at least two points where the output combination (q_1, q_2) is the same only if the isoquants are drawn such as to represent negative marginal products. Even in that case, however, one combination will represent the least-cost combination.

but result in increases in q_1. This results in the CE portion of the iso-cost curve in Figure A-2. Further changes in the factor combination below point E in Figure A-1 results in reductions in both products q_1 and q_2 as represented by EF in Figure A-2.[106] Higher levels of cost, which would be represented by constant cost curves further out in the x_1, x_2 plane would give rise to iso-cost curves such as GJ and KM in product space (Figure A-2).

The substitution that takes place between the products in the product diagram is now, in actuality, a substitution between the factors. Movements from N toward L (Figure A-2), for example, represents changes in the total amounts of x_1 and x_2 utilized when cost is held constant. For any given amount of cost (C) and constant resource prices, movements along the iso-cost curve represent <u>factor</u> substitution, and <u>not product</u> substitution. The product substitution is made possible only through factor substitution. The product substitution is accomplished only to achieve factor substitution in the factor diagram.[107]

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[106] It must be emphasized that the marginal physical products of both products are positive in the areas BC and EF. This is true because of the assumed shape of the isoquants in Figure A-1; i.e., negative marginal products were simply not considered.

[107] This is true even if factor prices are specified to be of the form: $r_1 = r_1(x_1, x_2)$, $r_2 = r_2(x_1, x_2)$; i.e., prices are assumed variable. Product substitution is still accomplished only to find the least cost expansion path which involves factor substitution.

Assorted Without Coupling

The derivation of the iso-cost curve in the case of assorted production with constant resource prices can be illustrated in the following manner. Using the cost function $C^o = r_1^o x_1 + r_2^o x_2$, the relation between x_2 and x_1 can be shown to be:

$$x_2 = \frac{C^o}{r_2^o} - \frac{r_1^o}{r_2^o} x_1. \qquad \text{(A. 1)}$$

The relation in (A. 1) is illustrated in Figure A-3. In addition to this relation, the two isoquant maps (for two products) must be known. These are illustrated in Figures A-4 and A-5 for q_1 and q_2, respectively, and constant resource prices. The correct allocation of resources to each product can now be determined by the imposition of Figure A-3 on Figures A-4 and A-5; i.e., the least cost expansion paths for each of the products can be found from the tangency points of the isoquants to the inverse (negative) resource prices. The resulting expansion paths are represented as 0LA in Figure A-4 and 0FD in Figure A-5. These expansion paths are also represented in Figure A-3 by 0LA and 0FD. A starting point for the generation of the iso-cost curve is to assume that all of the dollar expenditure (cost) is allocated to the production of q_2. This decision results in the production of q_2' of q_2 at point D in Figure A-5. The cost function is

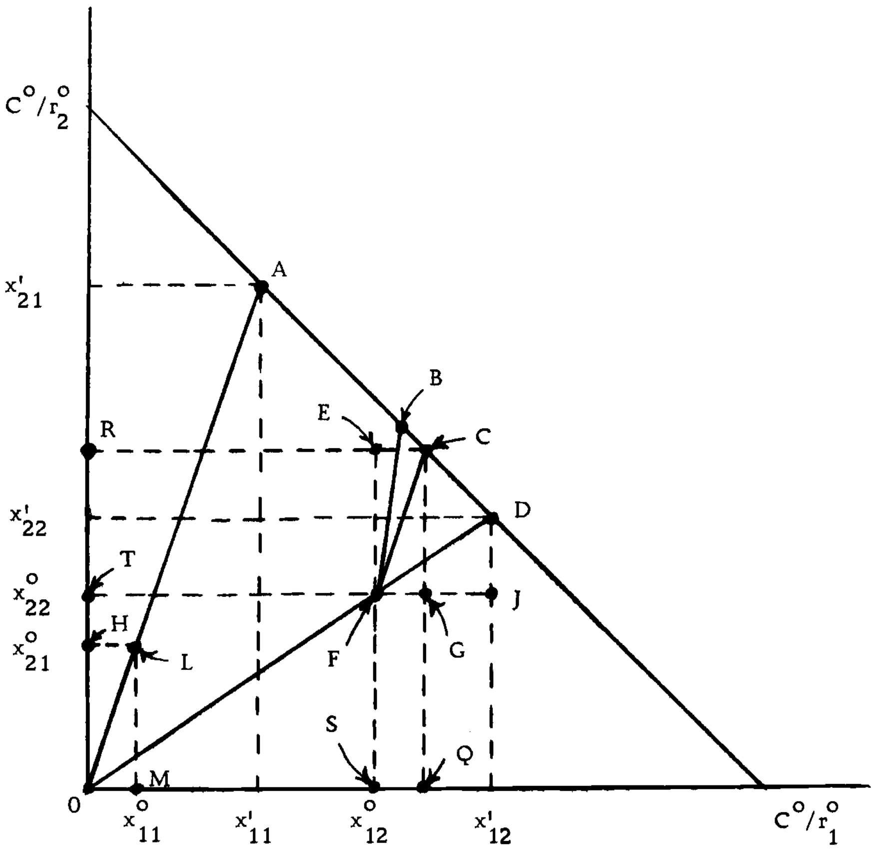

Figure A-3. Iso-cost curve in factor space.

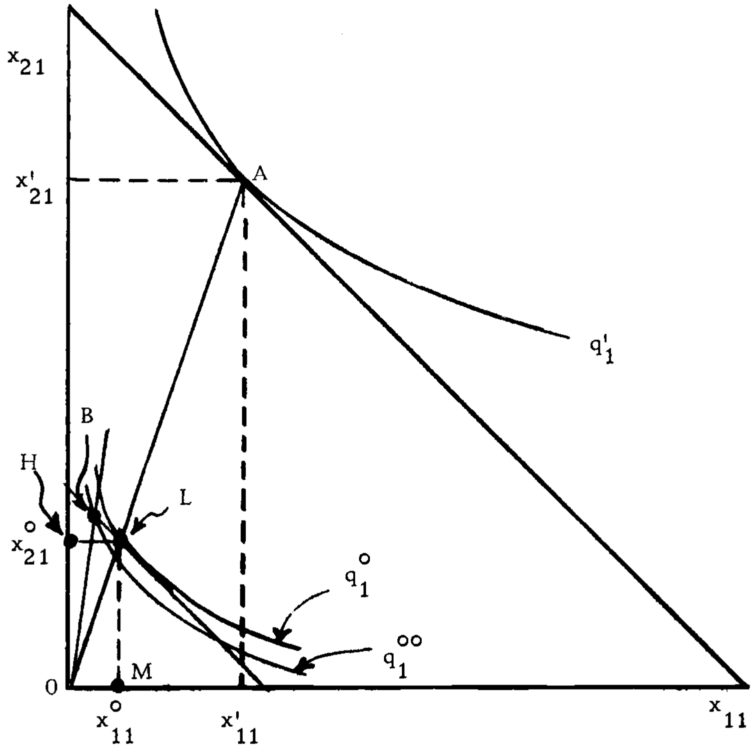

Figure A-4. Isoquants and iso-cost curves in factor space, product q_1.

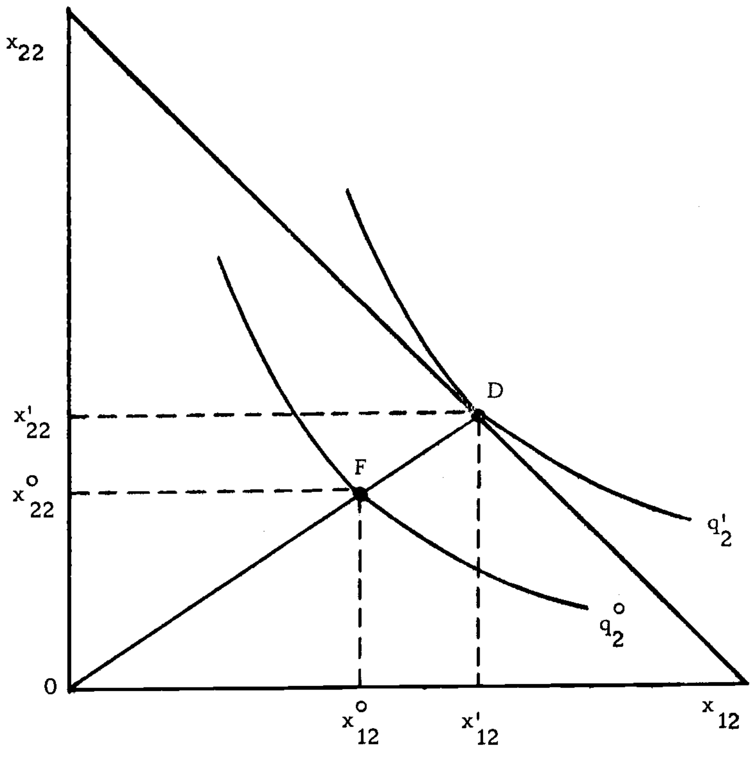

Figure A-5. Isoquants and iso-cost curves in factor space, product q_2.

now of the form:

$$C = r_1(x_{11}+x'_{12}) + r_2(x_{21}+x'_{22}) = r_1x'_{12} + r_2x'_{22} \ . \qquad (A.2)$$

The amount of each resource allocated to q_1 production is zero ($x_{11} = x_{21} = 0$). The q_2 level was produced at least cost and is defined where the iso-cost curve ABFC intersects the vertical axis in Figure A-6. All of the available capital could also be allocated to the production of q_1. In this case, q'_1 of q_1 could be produced (again along the least cost expansion path) using x'_{21} of x_2 and x'_{11} of x_1 (Figure A-4). The cost at each of the points A and C in Figure A-6 so far derived is identical; i.e., the costs are:

$$C_t = r_1x'_{12} + r_2x'_{22} = r_1x'_{11} + r_2x'_{21} \ . \qquad (A.3)$$

The q'_1 level is represented by the intersection of the iso-cost curve ABFC with the horizontal axis in Figure A-6.

Other points along the iso-cost curve ABFC in Figure A-6 can now be generated by changing the allocation of resources within the constraint of constant cost (and constant resource prices) from one of the extremes. Consider, for example, moving away from q'_2 and producing more q_1. This movement involves moving away from point D in Figure A-3, as at point D the entire cost was allocated to the production of q_2. Assume a new allocation of the resources to

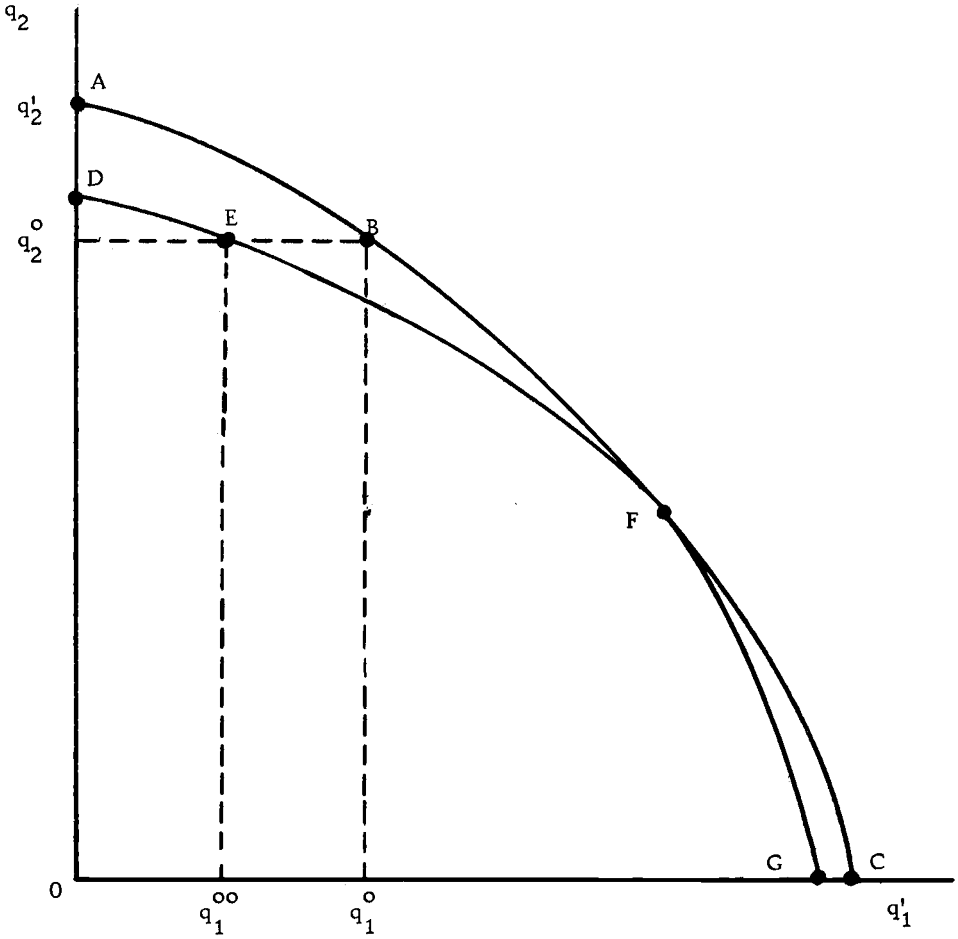

Figure A-6. Iso-cost curves in product space.

q_2 as represented at point F in Figure A-3. At point F, x_{12}^{o} and x_{22}^{o} of x_1 and x_2 are used in the production of q_2, giving q_2^{o} (Figure A-5) from a total cost of:

$$C_2 = r_1 x_{12}^{o} + r_2 x_{22}^{o} < C_t \ . \tag{A.4}$$

The amount of capital available to allocate to the production of q_1 has now increased from 0 to $(C_t - C_2) = C_1 > 0$. In order to produce q_1 at least cost, expansion of the production of q_1 should proceed along the 0LA expansion path in Figure A-4. This is the same as moving along FC in Figure A-3, where FC is parallel to 0LA, until the total budget C is expended. As a result of this movement, the capital required to purchase (J-F) of x_1 and (D-J) of x_2 can be re-allocated to the production of q_1 to give $q_1 = q_1^{o}$.[108] In essence, the "box" ECFG in Figure A-3 was purchased and used to produce some q_1. This same "box" is represented in both Figures A-3 and A-4 by 0HLM. The resulting (q_1, q_2) point in Figure A-6 is represented by (q_1^{o}, q_2^{o}) at B. The rest of the iso-cost curve could be generated in a similar way. The important point to note is that every point on the iso-cost curve ABFC in Figure A-6 results

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[108] This point requires some clarification. In moving from D to F, cost was reduced by lowering the levels of resource use by $x'_{12} - x_{12}^{o}$ for x_1 and by $x'_{22} - x_{22}^{o}$ for x_2. These physical amounts were not, however, allocated to q_1. The reduction in cost was allocated to q_1 production.

from a least-cost factor combination. Stated in another manner, the factor substitution has been accomplished irrespective of product prices.

The ramifactions of producing one of the products with factors in a ratio other than specified along an expansion path can also be determined. Assume an initial allocation of resources such as to produce $q_2 = q_2^o$ given at point F in Figure A-3. Again, assume constant resource prices. Movement up FB in Figure A-3 is the same as movement up 0B in Figure A-4. As soon as the total budget is expended (at point B in Figures A-3 and A-4) the firm would be producing q_1^{oo} of q_1 as represented in Figures A-4 and A-6. Production of one of the products off the relevant expansion path leads to a curve interior to the iso-cost curve ABFC which is represented by curve DEFG. The two curves are tangent, however, as both represent the same total cost. At point F, the resources are being used in expansion path proportions for both products. In fact, the marginal rate of factor substitution for each product is equal to the negative inverse resource price ratio at every point on the iso-cost curve.[109]

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[109] Symbolically, this statement is represented by:

$$-\frac{(\partial q_1/\partial x_{11})}{(\partial q_1/\partial x_{21})} = -\frac{(\partial q_2/\partial x_{12})}{(\partial q_2/\partial x_{22})} = -\frac{r_1^o}{r_2^o}\ .$$

This condition holds along an iso-cost curve with constant resource prices.

A very similar approach to the determination of the iso-cost curve is followed if the resource base is considered constant, cost is considered constant, and prices are allowed to vary. The cost function is now of the form:

$$C^o = r_1 x_1^o + r_2 x_2^o . \tag{A.5}$$

Under these assumptions, the iso-cost curve generated in the product diagram is of the same nature as the transformation curve derived earlier.[110] The relation between prices is of the form:

$$r_2 = \frac{C^o}{x_2^o} - \frac{x_1^o}{x_2^o} r_1 . \tag{A.6}$$

As r_1 increases, r_2 must necessarily decrease. The significant characteristic of this iso-cost curve is that the resource base, x_1^o and x_2^o, is held constant. This curve can be considered an "iso-resource" curve.

The iso-cost and iso-resource curves will always be of the same shape and located in the same position in the product diagram as long as certain conditions are met. The cost functions and the respective differentials are given by:

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[110] Pages 56-58, Chapter III.

$$C_x = r_1^o x_1 + r_2^o x_2; \quad dC_x = r_1^o dx_1 + r_2^o dx_2 \qquad (A.7)$$

$$C_r = r_1 x_1^o + r_2 x_2^o; \quad dC_r = x_1^o dr_1 + x_2^o dr_2 \qquad (A.8)$$

where,

C_x = iso-cost curve (constant prices) with variable resources,

C_r = iso-resource curve (constant resources) with variable prices.

Given the cost $C_x^o = C_r^o = C^o$ and setting $dC_x^o = dC_r^o = 0$, the requirement for $C^o = C_x^o(x_1, x_2) = C_r^o(r_1, r_2)$ is:

$$r_1^o dx_1 + r_2^o dx_2 = x_1^o dr_1 + x_2^o dr_2$$

or, (A.9)

$$\frac{dx_2}{dx_1} = \frac{x_1^o}{r_2^o}\frac{dr_1}{dx_1} + \frac{x_2^o}{r_2^o}\frac{dr_2}{dx_1} - \frac{r_1^o}{r_2^o} .$$

By introducing the required variables, (A.9) can be written in the more familiar terms:

$$\frac{dx_2}{dx_1} = \frac{r_1^o}{r_2^o}\left(\frac{x_1^o}{r_1^o}\frac{dr_1}{dx_1}\right) + \frac{x_2^o}{x_1^o}\left(\frac{x_1^o}{r_2^o}\frac{dr_2}{dx_1}\right) - \frac{r_1^o}{r_2^o}$$

$$= \frac{r_1^o}{r_2^o}(\lambda_o) + \frac{x_2^o}{x_1^o}(\lambda_c) - \frac{r_1^o}{r_2^o} \qquad (A.10)$$

$$= \frac{x_2^o}{x_1^o}(\lambda_c) + \frac{r_1^o}{r_2^o}(\lambda_o - 1)$$

where,[111]

λ_c = cross-price flexibility

λ_o = own-price flexibility.

The nature of the relationship between iso-resource and iso-cost curves can now be specified. If prices are constant, the derivative (dx_2/dx_1) is constant. This condition prevails at all points on the iso-cost curve derived with constant resource prices as $\lambda_c = \lambda_o = 0$. If resources are held constant and prices are allowed to vary, the derivative (dx_2/dx_1) varies along the entire iso-resource curve.[112] In either case, however, the product-product relation in the product diagram will have the same shape and location as long as the same level of cost is used in the derivation. This is very significant in that either curve can be used in the optimization process. Either curve will given the identical optimum product mix.

The substitution that takes place along the iso-cost curve in the product diagram is now (in contrast to the factorially determined case) a substitution <u>only between the products</u>. The substitution between the factors has already been accomplished by finding the least cost

[111] These terms are defined in Ferguson ("The Neoclassical...", 1971, p. 237).

[112] The case of variable resource prices could also have been described in the manner illustrated in Figures A-3, A-4, and A-5. In that case, the cost lines of Figures A-4 and A-5 would have been curved rather than straight. Again, the derivative (dx_2/dx_1) would vary in value along the iso-cost curve.

expansion paths in the respective factor diagrams or the locus of tangencies between the isoquants in the Edgeworth-Bowley Box. Movements from B toward F (Figure A-6), for example, represents changes in the allocation of each factor. In addition, the total amounts of each resource and/or relative resource prices may be changing. In any case, the factor substitution has already been accomplished along an iso-cost curve as derived in assorted production. Movements along the curve are accomplished only to achieve the desired product mix through <u>product</u> substitution and <u>not factor</u> substitution.

APPENDIX B

TECHNICAL INTERDEPENDENCE AND INDEPENDENCE IN PRODUCTION

The type of technical relations prevailing between products can be found from the sign on the second partial derivative of the cost function, $C = f(q_1, q_2, \ldots, q_m)$, for m products. The resulting classification of the relation between each pair of products is given by (Carlson, 1938, pp. 82-83):

$$\frac{\partial^2 C}{\partial q_1 \partial q_2} > 0, \qquad \text{technically competitive,}$$

$$\frac{\partial^2 C}{\partial q_1 \partial q_2} = 0, \qquad \text{technically independent,} \qquad \text{(B. 1)}$$

$$\frac{\partial^2 C}{\partial q_1 \partial q_2} < 0, \qquad \text{technically complementary.}$$

The mathematical definitions in (B. 1) are not particularly enlightening, however, as regards a verbal definition. A geometric approach to classification provides more insight into the nature of each relationship.

The geometric approach to classification of products requires that at least three iso-cost curves be located in product-product space.[113] The change in cost, ΔC, must be constant among the three curves. The product relation can then be determined by construction of a "box" in the product space having one corner on the lowest iso-cost curve. The relationship between the products is then determined by the location of the box in the space. Consider, for example, the iso-cost curves represented in Figure B-1. Any point can be chosen as a starting point on the lower curve. Point A on p_0p_0 was chosen in Figure B-1. There is some value of the ratio $\Delta C/\Delta q_1$, where $\Delta q_1 = (E-A)$, at point A on p_0p_0. The technical classification is then determined by changing the level of q_2 by $\Delta q_2 = (F-A)$ and monitoring the change in q_1. Note that points F and E, two defining corners of the box, are on the second, or p_1p_1, iso-cost curve. The amount of change in q_1 now determines the technical relation. The discrete partial is given by the relation:

$$T = \frac{\Delta}{\Delta q_2} \left(\frac{\Delta C}{\Delta q_1} \right)$$

where ΔC is constant. The technical relation (T) will be positive, zero, or negative for a change in q_2 dependent upon how Δq_1 changes. If the new Δq_1 is smaller than (the same as, larger than) the Δq_1 at point A, the products are technically competitive

[113]This approach was developed by Dr. Albert N. Halter, Professor, Department of Agricultural Economics, Oregon State University.

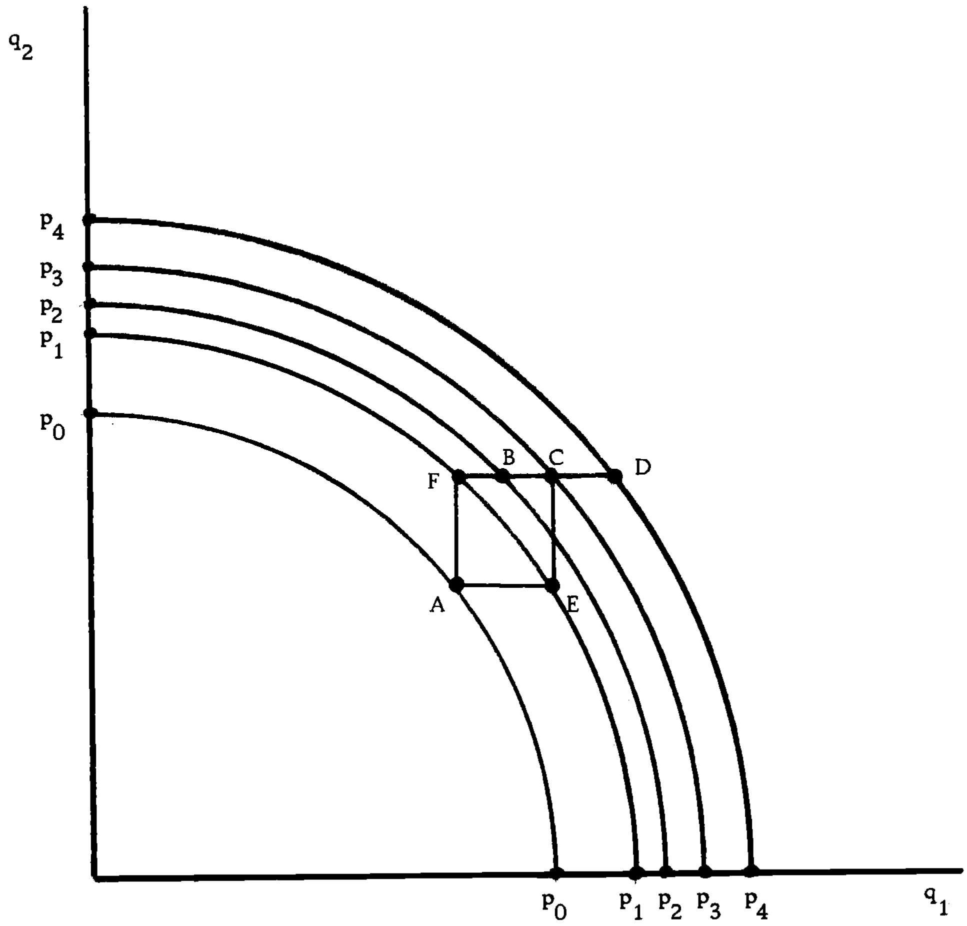

Figure B-1. Classification of technical relations between q_1 and q_2, diagrammatic representation of the cross partial $\partial^2 C/\partial q_1 \partial q_2$.

(independent, complementary). The technical classification, then, depends on the location of the (upper most) third iso-cost curve. The three possible locations of the third curve are represented by curves p_2p_2, p_3p_3, and p_4p_4 in Figure B-1. Given curve p_2p_2, the products are classified as technically competitive as the "new" Δq_1 = (B-F) is smaller than Δq_1 at point A. An increase in q_2 by Δq_2 = (F-A) led to a decline in Δq_1 (from E-A to B-F). If the relevant curve was p_3p_3, an increase in q_2 by Δq_2 = (F-A) led to no change in Δq_1 and (C-F) = (E-A); the products are technically independent. Similarly, the products are classified as technically complementary if Δq_1 increased as shown by (D-F) > (E-A) for curve p_4p_4. The location of the upper right hand corner of the box relative to the third (upper-most) curve, then, determines the classification. If that corner is above (on, below) the highest curve, the products are technically competitive (independent, complementary).[114]

The nature of different technical relations can also be illustrated with an actual algebraic equation. Consider the quadratic form of the cost equation given by:

$$C = aq_1^2 + bq_2^2 + cq_1q_2 , \qquad (B.2)$$

[114]The "highest" curve is the "third" curve as represented by p_2p_2, p_3p_3, or p_4p_4.

where a, b, c = constants.

The cross partial is given by (from B. 2):

$$\frac{\partial^2 C}{\partial q_2 \partial q_1} = c . \qquad \text{(B. 3)}$$

The constant multiplying the interaction term determines the type of technical relation. The products are technically competitive (independent, complementary) if the constant on the interaction term is positive (zero, negative).

The technically competitive case is presented in Table B-1 and Figure B-2 for varying levels of cost from the equation:

$$C = (1.0)q_1^2 + (1.0)q_2^2 + (1.0)q_1 q_2 . \qquad \text{(B. 4)}$$

The value of the cross partial is:

$$\frac{\partial^2 C}{\partial q_1 \partial q_2} = +1.0. \qquad \text{(B. 5)}$$

Products q_1 and q_2 are technically competitive in the entire q_1, q_2 plane of Figure B-2. Using the geometric approach, the upper right hand corner of the box is shown to be above the third curve (point D). The upper right hand corner of the box will be above the third curve for all such boxes that could be constructed in Figure B-2;

Table B-1. Product combinations at varying cost levels, technically competitive products for cost function $C = q_1^2 + q_2^2 + q_1q_2$.

	Cost at:					
	$2.00		$4.00		$6.00	
q_1	q_2	dq_2/dq_1	q_2	dq_2/dq_1	q_2	dq_2/dq_1
0	1.414	-0.500	2.000	-0.500	2.450	-0.500
.1	1.362	-0.553	1.948	-0.538	2.398	-0.531
.2	1.304	-0.607	1.892	-0.575	2.343	-0.561
.3	1.240	-0.662	1.833	-0.614	2.856	-0.592
.4	1.171	-0.719	1.770	-0.652	2.225	-0.624
.5	1.096	-0.778	1.703	-0.692	2.161	-0.656
.6	1.015	-0.842	1.631	-0.733	2.094	-0.688
.7	0.928	-0.911	1.556	-0.776	2.023	-0.721
.8	0.833	-0.987	1.476	-0.820	1.950	-0.755
.9	0.730	-1.072	1.392	-0.866	1.872	-0.791
1.0	0.618	-1.171	1.303	-0.916	1.791	-0.827
1.1	0.495	-1.289	1.209	-0.969	1.707	-0.866
1.2	0.359	-1.438	1.109	-1.027	1.618	-0.906
1.3	0.206	-1.639	1.003	-1.090	1.525	-0.948
1.4	0.028	-1.942	0.891	-1.160	1.428	-0.993
1.4142	0	-2.000	*	*	*	*
1.5			0.771	-1.240	1.327	-1.042
1.6			0.642	-1.332	1.220	-1.094
1.7			0.504	-1.442	1.108	-1.151
1.8			0.353	-1.577	0.989	-1.214
1.9			0.187	-1.753	0.864	-1.285
2.0			0	-2.000	0.732	-1.366
2.1					0.591	-1.460
2.2					0.440	-1.572
2.3					0.276	-1.710
2.4					0.096	-1.889
2.450					0	-2.000

*Estimate was not calculated.

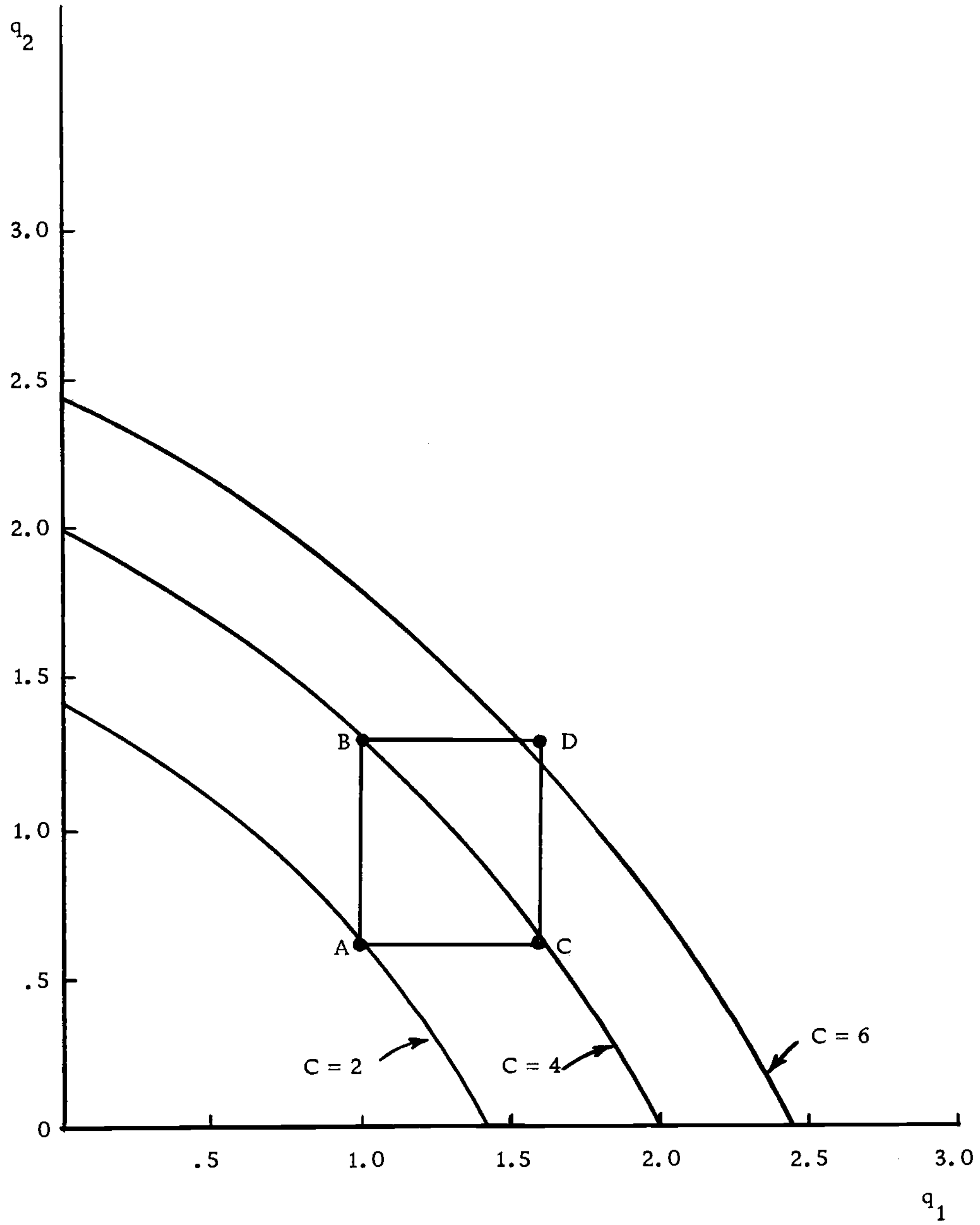

Figure B-2. Transformation curves illustrating technically competing products from the relation $C = q_1^2 + q_2^2 + q_1 q_2$.

i.e., the cross-partial is positive in the entire plane. It should also be noted that the slope of each respective curve is not related to the technical classification. Product q_2 decreases at an increasing rate for increases in q_1 along each of the curves. The slope of each iso-cost curve at various points is shown in Table B-1. The slope at point A, for example, was determined to be -1.171 for $q_1 = 1.0$. Increases in q_1 beyond point A (for the same cost) gave an increase in the slope (becomes more negative). The slopes of these curves are, of course, the product-product trade-off ratios between q_1 and q_2 as defined in Chapter III of this study.

Products q_1 and q_2 are technically independent in the cost equation:

$$C = (1.0)q_1^2 + (1.0)q_2^2 + (0.0)q_1q_2 \, . \tag{B.6}$$

The value of the cross partial derivative is given by (from B.6):

$$\frac{\partial^2 C}{\partial q_2 \partial q_1} = 0. \tag{B.7}$$

The estimated values of q_2 for alternative levels of q_1 and cost are presented in Table B-2 and Figure B-3. Again, the slopes (trade-offs) become greater as q_2 is reduced for increases in q_1 at any given cost level. Also, the calculated values of the slopes are not

Table B-2. Product combinations at varying cost levels, technically independent products for cost function $C = q_1^2 + q_2^2$.

	Cost at:					
	$2.00		$4.00		$6.00	
q_1	q_2	dq_2/dq_1	q_2	dq_2/dq_1	q_2	dq_2/dq_1
0	1.414	0	2.000	0	2.450	0
.1	1.411	-0.071	1.998	-0.050	2.447	-0.041
.2	1.400	-0.143	1.990	-0.100	2.441	-0.082
.3	1.382	-0.217	1.977	-0.152	2.431	-0.123
.4	1.356	-0.295	1.960	-0.204	2.417	-0.166
.5	1.323	-0.378	1.936	-0.258	2.400	-0.208
.6	1.281	-0.468	1.908	-0.314	2.375	-0.253
.7	1.229	-0.570	1.874	-0.374	2.347	-0.298
.8	1.166	-0.686	1.833	-0.436	2.315	-0.346
.9	1.091	-0.825	1.786	-0.504	2.278	-0.395
1.0	1.000	-1.000	1.732	-0.577	2.236	-0.447
1.1	0.889	-1.238	1.670	-0.659	2.189	-0.503
1.2	0.748	-1.604	1.600	-0.750	2.135	-0.562
1.3	0.557	-2.335	1.520	-0.855	2.076	-0.626
1.4	0.200	-7.000	1.428	-0.980	2.010	-0.696
1.414	0	$-\infty$	*	*	*	*
1.5			1.323	-1.134	1.936	-0.775
1.6			1.200	-1.333	1.855	-0.863
1.7			1.054	-1.614	1.764	-0.964
1.8			0.872	-2.065	1.661	-1.084
1.9			0.624	-3.042	1.546	-1.229
2.0			0	$-\infty$	1.414	-1.414
2.1					1.261	-1.665
2.2					1.077	-2.043
2.3					0.843	-2.730
2.4					0.490	-4.899
2.450					0	$-\infty$

*Estimate was not calculated for this cost level.

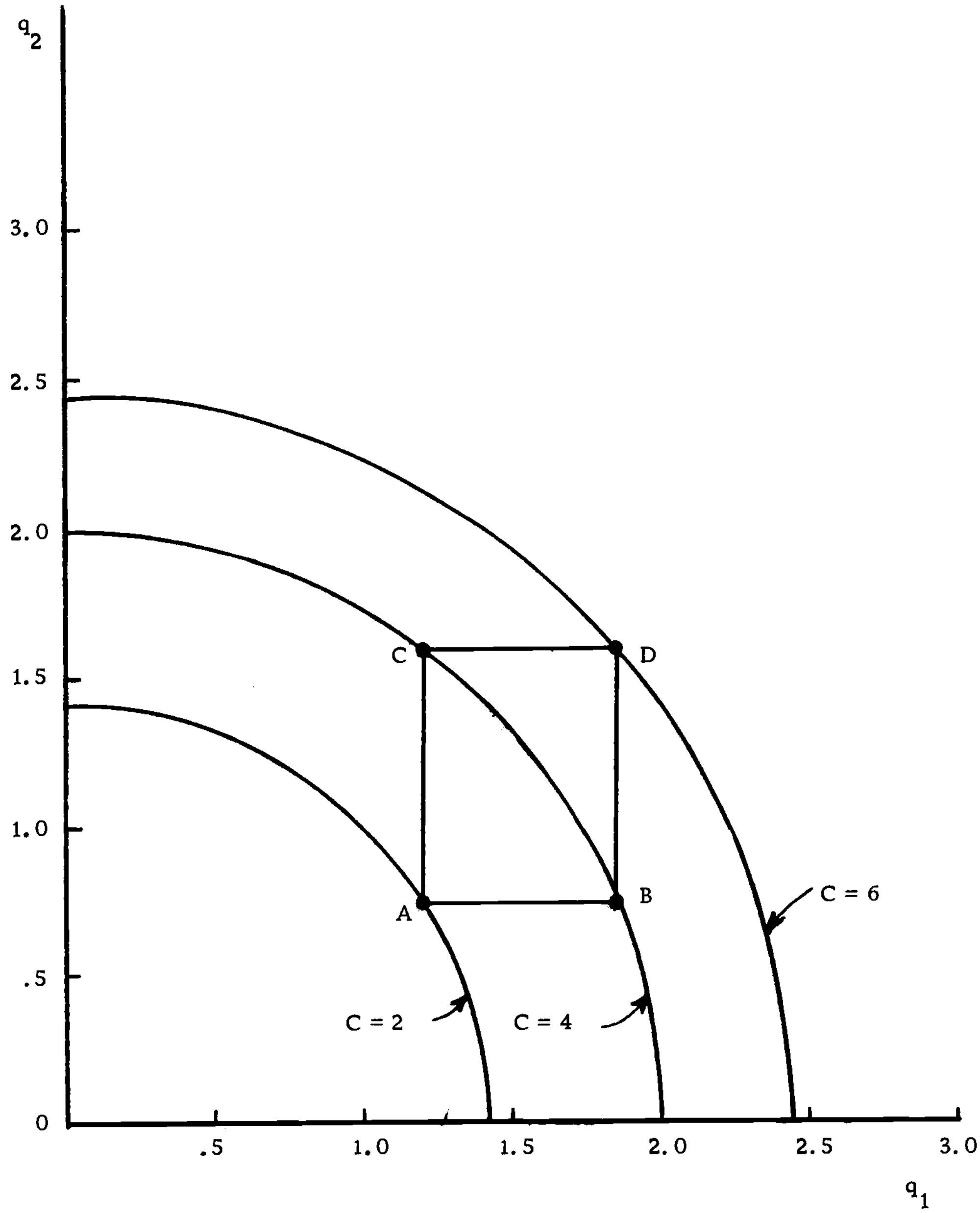

Figure B-3. Transformation curves illustrating technically independent products from the relation $C = q_1^2 + q_2^2$.

related to the technical classification scheme. As expected, the upper right corner of the "box" was found to be located on the third curve (point D). That corner would be on the third curve for all such boxes that could be constructed in Figure B-3 as the cross-partial is zero at every point in the q_1, q_2 plane.

The technically complementary case exists in the cost function:

$$C = (1.0)q_1^2 + (1.0)q_2^2 - (1.0)q_1q_2 . \tag{B.8}$$

The cross-partial derivative is given by (from B.8):

$$\frac{\partial^2 C}{\partial q_2 \partial q_1} = -1.0 . \tag{B.9}$$

The estimated values of q_1 and q_2 are presented in Tables B-3 and B-4 and Figure B-4. Products q_1 and q_2 are technically complementary in the entire q_1, q_2 plane. Using the geometric approach, the upper right hand corner of the box is below the third curve (point D). This same relationship will occur for all such boxes constructed in Figure B-4; i.e., the cross-partial is negative in the entire plane.

The slopes of the curves in this case, while not affecting the technical classification, do not remain negative over the entire range. The slope of the iso-cost curve for the $4.00 expenditure level, for example, is positive for $0 < q_1 < 1.1$ and $2.000 < q_2 < 2.309$ (Table B-3). The slope (trade-off) is negative for $1.1 < q_1 < 2.309$

Table B-3. Product combinations at varying cost levels, technically complementary products from cost function $C = q_1^2 + q_2^2 - q_1q_2$, positive branch of quadratic equation.

	Cost at:					
	$2.00		$4.00		$6.00	
q_1	q_2	dq_2/dq_1	q_2	dq_2/dq_1	q_2	dq_2/dq_1
0	1.414	0.500	2.000	0.500	2.450	0.500
.1	1.462	0.446	2.048	0.462	2.498	0.469
.2	1.504	0.393	2.092	0.425	2.543	0.439
.3	1.540	0.338	2.133	0.386	2.586	0.408
.4	1.571	0.281	2.170	0.348	2.625	0.376
.5	1.596	0.222	2.203	0.308	2.661	0.344
.6	1.615	0.158	2.231	0.267	2.694	0.312
.7	1.628	0.089	2.556	0.224	2.723	0.279
.8	1.633	0.013	2.276	0.180	2.750	0.245
.9	1.630	-0.072	2.292	0.134	2.772	0.209
1.0	1.618	-0.171	2.303	0.084	2.791	0.173
1.1	1.595	-0.289	2.309	0.031	2.807	0.134
1.2	1.559	-0.438	2.309	-0.027	2.818	0.094
1.3	1.506	-0.639	2.303	-0.090	2.825	0.052
1.4	1.428	-0.942	2.291	-0.160	2.828	0.007
1.5	1.309	-1.512	2.271	-0.240	2.827	-0.042
1.6	1.083	-3.743	2.242	-0.332	2.820	-0.094
1.633	.816	$-\infty$	*	*	*	*
1.7			2.204	-0.442	2.808	-0.151
1.8			2.153	-0.577	2.789	-0.214
1.9			2.087	-0.753	2.764	-0.285
2.0			2.000	-1.000	2.732	-0.366
2.1			1.882	-1.393	2.691	-0.460
2.2			1.708	-2.213	2.640	-0.572
2.3			1.330	-9.069	2.576	-0.710
2.3094			1.155	$-\infty$	*	*
2.4					2.496	-0.889
2.450					2.449	-1.000
2.5					2.396	-1.137
2.6					2.264	-1.522
2.7					2.080	-2.275
2.8					1.746	-5.562
2.828					1.414	$-\infty$

*Estimates were not calculated.

Table B-4. Product combinations at varying cost levels, technically complementary products from cost function $C = q_1^2 + q_2^2 - q_1 q_2$, negative branch of quadratic equation.

	Cost at:					
	\$2.00		\$4.00		\$6.00	
q_1	q_2	dq_2/dq_1	q_2	dq_2/dq_1	q_2	dq_2/dq_1
1.414	0					
1.5	0.191	2.512				
1.6	0.517	4.743				
1.633	0.816	$-\infty$				
2.0			0	2.000		
2.1			0.218	2.393		
2.2			0.492	3.213		
2.3			0.970	10.069		
2.3094			1.155	$-\infty$		
2.450					0	
2.500					0.104	2.137
2.60					0.336	2.522
2.7					0.620	3.275
2.8					1.054	6.562
2.828					1.414	$-\infty$

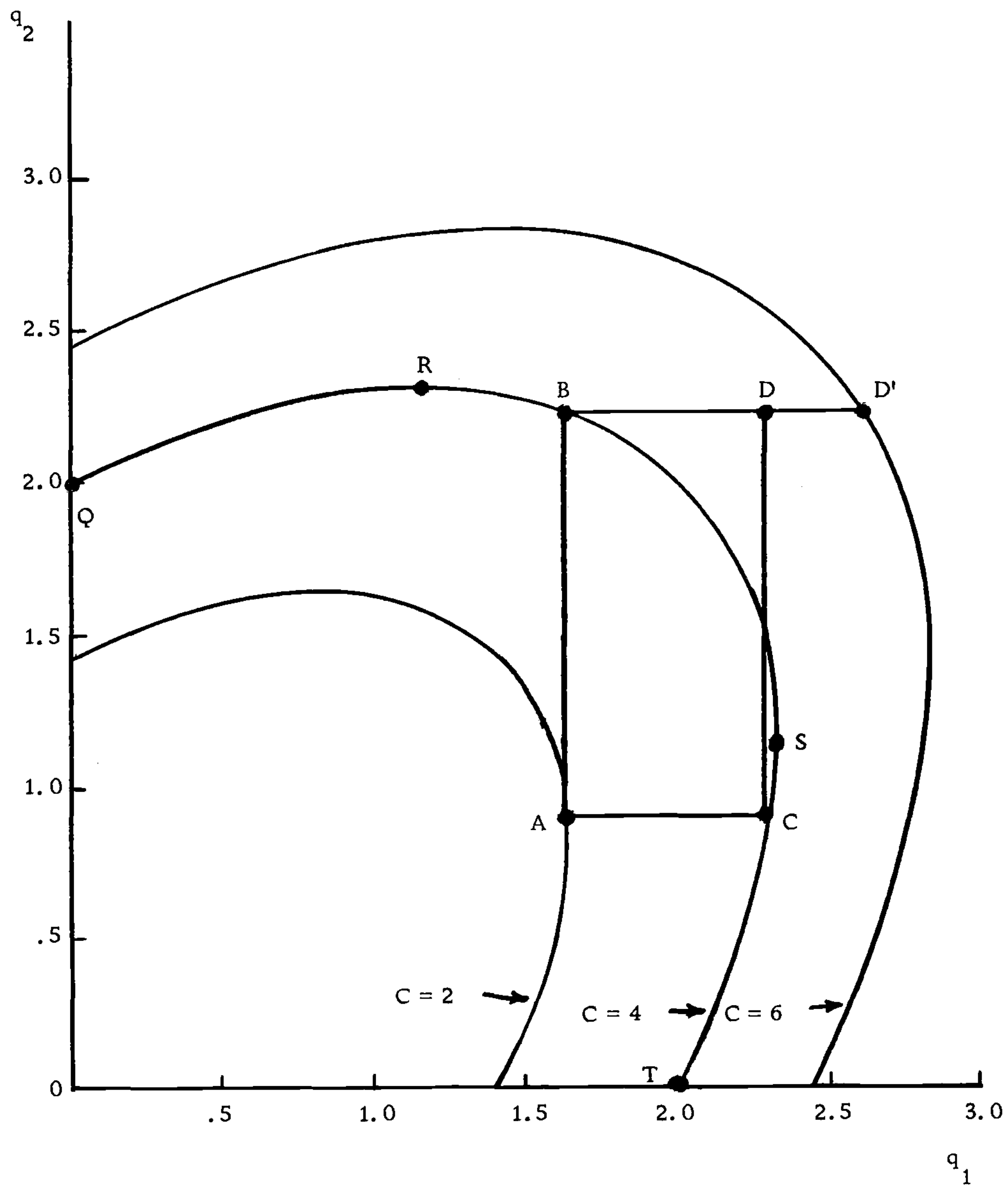

Figure B-4. Transformation curves illustrating technically complementary products from the relation $C = q_1^2 + q_2^2 - q_1q_2$.

and $1.155 < q_2 < 2.309$ (Table B-3). The slope becomes positive again for $2.000 < q_1 < 2.309$ and $0 < q_2 < 1.155$ (Table B-4). As a result, q_2 increases for increases in q_1 in region QR and ST of the iso-cost curve for the \$4.00 expenditure level (Figure B-4). Increases in q_1 result in reductions in q_2 in region RS.

There is a great deal of confusion in the economic literature regarding the classification of technical relations. The classification system based on the value of the cross partial derivative (as just discussed) can be attributed to Carlson (1938, pp. 78-83). An alternative system of classification, based on the slope of an iso-cost curve is presented in Heady (1952, pp. 221-234). Both systems of classification use the same identifying names to describe completely unrelated phenomenon. Heady, for example, would classify the relationship between q_1 and q_2 along curve QT of Figure B-4 as technically complementary in regions QR and ST, technically competitive in region RS, and technically independent (supplementary) at the points R and S. This classification system is based on the <u>slopes</u> (trade-offs) along iso-cost curves, as given by Heady (1952, p. 234):

$$\begin{aligned} dq_2/dq_1 &< 0, &&\text{technically competitive,} \\ dq_2/dq_1 &= 0, &&\text{technically independent,} \\ dq_2/dq_1 &> 0, &&\text{technically complementary.} \end{aligned} \tag{B.10}$$

This is a completely different notion of interdependence and independence as the cross partial $(\partial^2 C/\partial q_2 \partial q_1)$ in (B. 1) is <u>not</u> related to the derivative (dq_2/dq_1) in (B. 10). The confusion has arisen, then, due to a failure to recognize the difference between the slope of an iso-cost curve at constant cost as compared to a change over the surface for variable cost.

It should also be noted that the reason for a varying sloped (positive, zero, and negative) iso-cost curve can only be determined by knowledge of the underlying production functions. The iso-cost curves of Figure B-4, for example, could result from either the factorially determined without coupling $(\alpha = 0, \ \kappa = 0)$ or the assorted without coupling $(\alpha > 0, \ \kappa = 0)$ cases discussed in Chapter III of this study. Given that the underlying production relations were factorially determined (without coupling), for example, regions QR and ST are really "uneconomic" regions of production in the sense that more of both products could be obtained by changing the ratios of the <u>total available</u> factors. The regions QR and ST can also be considered "uneconomic" given assorted production. In that case, however, both q_1 and q_2 can be increased by changing the <u>allocations</u> and/or the <u>total available</u> amounts of factors between the products.[115] Also,

[115] Both the allocation and the total amount must change along the iso-cost curve from assorted production. Only the allocation changes along the iso-resource curve from assorted production.

the marginal products of each factor can all be positive in regions QR and ST given factorially determined production. This is not the case in assorted production; i.e., at least one of the marginal products is negative in each of the QR and ST regions for that case.

Both of the approaches to product classification have usefulness in economic analysis. The labels attached to the different concepts, however, leave a great deal to be desired. The real, significant difference between the two concepts should be reflected in the labels to avoid confusion and eliminate misuse. The real difference between the concepts, in turn, relates to the manner in which costs are changing. The concepts defined by Heady are relevant for a constant cost (or constant resource) situation while the concepts described by Carlson refer to changes in cost (or resource levels).[116] A more suitable identification scheme then is given by:

$dq_j/dq_i < 0$, iso-cost competitiveness,

$dq_j/dq_i = 0$, iso-cost independence,

$dq_j/dq_i > 0$, iso-cost complementarity,

---

[116] The Carlson approach refers to movements across an iso-resource or iso-cost surface in such a manner to change costs or the resource levels. The use of derivatives, of course, also requires consideration at a point; i.e., in the limit, costs are also constant in the Carlson approach.

$$\frac{\partial^2 C}{\partial q_i \partial q_j} > 0, \qquad \text{variable-cost competitiveness,}$$

$$\frac{\partial^2 C}{\partial q_i \partial q_j} = 0, \qquad \text{variable-cost independence,}$$

$$\frac{\partial^2 C}{\partial q_i \partial q_j} < 0, \qquad \text{variable-cost complementarity.}$$

Once these concepts were defined in any particular analysis, the researcher could resort to abbreviations such as "ic-competitiveness", "vc-independence", etc. The confusion in the literature regarding these concepts would, eventually, disappear.

APPENDIX C

EMPIRICAL MODELS UTILIZED FOR DERIVATION OF TRADE-OFF RELATIONS

The basic relations required for the derivation of all the trade-off curves and surfaces of Chapter V are given by the general forms:

$$\begin{aligned} TC &= f(AC) \\ SA &= g(TC) \end{aligned} \qquad (C.1)$$

where,

TC = total capacity of reservoir,

AC = annual cost,

SA = surface area of reservoir.

These physical relations were estimated at each of two sites in the Knife River Basin. The actual algebraic manipulations utilized to develop the empirical sections of Chapter V are discussed below.

Product-Product Trade-Offs at the Bronco Site

The specific forms of the physical relations discovered at the Bronco site, and used in all subsequent analysis at that site, are given by:

$$\begin{aligned} TC &= a + b(AC) + c(AC)^2 \\ &= -17893. + 0.31070(AC) + 0.00000050357(AC)^2 \end{aligned} \qquad (C.2)$$

$$SA = d + e(TC) - f(TC)^2$$

$$= 749.33 + 0.036528(TC) - 0.000000025712(TC)^2. \quad (C.3)$$

Equations (C.2) and (C.3) provided the basis for the iso-cost relation. In order to derive the iso-cost relation, however, several assumptions had to be made. First of all, it was assumed that any water allocated from the reservoir for irrigation was totally unavailable for recreation (surface area). This precludes, for example, the possibility that water available in the reservoir in the early part of the season could be used for recreation and then released later in the season for irrigation use. Water had to be used for irrigation or left in the reservoir for recreation. This assumption is reflected in the equation:

$$SA = d + e(TC-W_I) - f(TC-W_I)^2. \quad (C.4)$$

Equation (C.4) was then solved for W_I, with the quadratic formula, to give:

$$W_I = \frac{-[e-2f(TC)] \pm \sqrt{[e-af(TC)]^2 - 4f[-d-e(TC)+f(TC)^2+SA]}}{2f} \quad (C.5)$$

The plus sign gives the correct branch of the iso-cost curve for this derivation. Equation (C.5) was then used to give estimates of W_I for alternative levels of recreation (surface area, $SA = W_R$). The second assumption made was that only 60 percent of the total capacity could be used for W_I production; i.e., 40 percent must remain in the reservoir for "dead" storage and contingency needs. This figure can be changed easily and most likely will vary by project area. This restriction enters this model by mere subtraction.

The rate of change in W_I for a change in the recreation level, W_R (surface area) can now be formed from the first differential of (C.5), where $d(TC) = 0$ and $SA = W_R$, to give:

$$\frac{dW_I}{dW_R} = - \frac{1}{\sqrt{[e-af(TC)]^2 - 4f[-d-e(TC)+f(TC)^2 + W_R]}} . \quad (C.6)$$

This is the product-product trade-off ratio between W_R and W_I for constant iso-(minimum) cost. The second derivative was found to be (from C.6):

$$\frac{d^2W_I}{dW_R^2} = -2f\{[e-2f(TC)] - 4f[-d-e(TC)+f(TC)^2 + W_R]\}^{-3/2} . \quad (C.7)$$

Equation (C.7) is negative over all values of TC and W_R within the positive quadrant. The iso-cost curves are, as a result, concave downward (or convex from below) as expected.

A simple Fortran program was then utilized to generate iso-cost curves for varying levels of cost. The program used for this process is reproduced in Table C-1. The annual cost value is provided as input to the program (line 14). The total capacity of the

Table C-1. Computer program utilized in generation of iso-(minimum) cost curves at Bronco site, Knife River Basin, North Dakota.

```
      PROGRAM BISOCOS
C THIS PROGRAM USED TO CALCULATE ISOCOST AT THE BRONCO SITE
      WR1=749.33
      WR2=.036528
      WR3=.000000025712
      WI3=2.0*WR3
      CAP1=17893.
      CAP2=.31070
      CAP3=.00000050357
      WIO=0.0
      WRO=0.0
      PWI=30.
      XNBOLD=0.0
 1000 READ(30,2000)ANC
 2000 FORMAT(F6.0)
      IF(ANC.EQ.999999.0)GO TO 4500
      ANC2=ANC**2
      TC=-CAP1+CAP2*ANC+CAP3*ANC2
      UC=TC*.60
C MINIMUM SURFACE AREA FOR ALL UC TO OTHER USES
      TC2=TC**2
      REMR=TC-UC
      REMR2=REMR**2
C THIS SURFACE AREA IS THE MINIMUM AMT, WITH ALL UC TO OTHER
C USES
      WR=WR1+WR2*REMR-WR3*REMR2
C THIS IS THE MAX AMT TO REC
      WRMAX=WR1+WR2*TC-WR3*TC2
      WRITE(31,3000)TC,UC,ANC,WRMAX
 3000 FORMAT(1H1,'TC=',F9.1,2X,'TOT.UC=',F9.1,2X,'AN COST=', F12.1,
     11X,'WRMAX=', F9.1)
      TC2=TC**2
      WI1=WR2-2.0*WR3*TC
      WI12=WI1**2
      WI2=WR1-WR2*TC+WR3*TC2
      WRMAXA=(WRMAX-WR)/100.
      JK=IFIX(WRMAXA+.5)+3
      WRITE(31,3100)
 3100 FORMAT(1H0,'WS',8X,'WI',8X,'WR',8X,'EST. DIDR', 1X,
     1'EXT. DIDR', 1X,'NET BEN.',4X,'EST. DBDR')
      DO 3500 I=1,JK
      SR=SQRT(WI12-4.0*WR3*(WI2+WR))
      WI=(-WI1+SR)/WI3
      WS=TC-WI
      WIN=WI
      DWI=WIN-WIO
      WIO=WIN
```

Table C-1. Continued.

```
      WRN=WR
      DWR=WR-WRO
      WRO=WR
      DIDR=DWI/DWR
      EDIDR=-1.0/SR
      XNEB=PWI*WI-ANC
      XNBNEW=XNEB
      DB=XNBNEW-XNBOLD
      XNBOLD=XNBNEW
      DBDR=DB/DWR
      WRITE(31,4000)WS, WI, WR, DIDR, EDIDR, XNEB, DBDR
 4000 FORMAT(3F10.1,2F10.4,F12.1,F10.4)
      WR=WR+100.
 3500 CONTINUE
      GO TO 1000
 4500 CALL EXIT
      END
```

reservoir is then calculated for that given cost (line 18). The assumption regarding useable capacity is implemented in line 19; i.e., only 60 percent of reservoir capacity was assumed useable in any given year. The various levels of W_I are then calculated by incrementing W_R (surface area) from the minimum surface area (line 26) when the reservoir is at 40 percent of capacity to the maximum surface area (line 28) when the reservoir is full. The increment chosen was 100 acres (line 60). The production trade-offs are calculated over intervals of the curve (line 51) and at points on the curve (line 52). Net dollar benefits from W_I production are also monitored (line 53) and the rate of change in net dollar benefits for changes in W_R is calculated (line 57).

The program in Table C-1 is, essentially, a small "simulation" model. The entire iso-cost surface can be generated for the two product case with this program. Varying assumptions regarding useable capacity and prices are easily changed. Trade-offs are automatically given at every point on the surface. Very similar types of Fortran programs were utilized to generate the other surfaces used in the empirical analysis of Chapter V.

Product-Product Trade-Offs at the Kineman Site

The specific forms of the physical relations at the Kineman site are given by:

$$TC = -715.63 + 0.054476(AC) + 0.00000079369(AC)^2 \quad (C.8)$$

$$SA = -4.8355 + 0.074593(TC) - 0.0000024083(TC)^2. \quad (C.9)$$

The same general approach was used to find the slope equations (product-product trade-offs) as for the Bronco site.

Product-Product Trade-Offs for Both the Bronco and Kineman Sites

The iso-cost surfaces for both sites were developed by combining equations sets (C.2), (C.3), (C.8), and (C.9) such as to permit the generation of composite surfaces. It was first assumed that total capacity and surface area for recreation were (each) additive as given by:

$$\begin{aligned} TC_T &= TC_B + TC_K \\ SA_T &= W_{RT} = W_{RB} + W_{RK} \end{aligned} \quad (C.10)$$

where,

TC_T = total capacity (TC_T) is composed of the TC at Bronco site (TC_B) plus the TC at Kineman site (TC_K),

$SA_T = W_{RT}$ = total surface area of reservoir available for recreation (W_{RT}) is composed of the W_R at the Bronco site (W_{RB}) plus the W_R at the Kineman site (W_{RK}).

The two relations in (C.10) were then used to form the following relations:

$$W_{RB} = a + b(TC_B - W_{IB}) + c(TC_B - W_{IB})^2$$

$$= 749.33 + 0.036528(TC_B - W_{IB}) - 0.000000025712(TC_B - W_{IB})^2$$

$$W_{RK} = d + e(TC_K - W_{IK}) + f(TC_K - W_{IK})^2 \qquad (C.11)$$

$$= -4.8355 + 0.074593(TC_K - W_{IK}) - 0.0000024083(TC_K - W_{IK})^2$$

to give:

$$W_{RT} = W_{RB} + W_{RK}$$

$$= (a + bTC_B - bW_{IT} + bW_{IK} + cTC_B^2 - 2cTC_BW_{IT}$$

$$+ 2cTC_BW_{IK} + cW_{IT}^2 - 2cW_{IT}W_{IK} + cW_{IK}^2)$$

$$+ (d + e(TC_K - W_{IK}) + f(TC_K - W_{IK})^2) . \qquad (C.12)$$

The quadratic formula was then used to solve equation (C.12) for W_{IT} from the relations,

$$L = c$$

$$M = -b - 2cW_{IK} - 2cTC_B$$

$$N = a + bTC_B + bW_{IK} + cTC_B^2 + 2cTC_BW_{IK} + cW_{IK}^2 + d$$

$$+ e(TC_K - W_{IK}) + f(TC_K - W_{IK})^2 - W_{RT}$$

to give:

$$W_{IT} = \frac{-M \pm \sqrt{M^2 - 4LN}}{2L} . \qquad (C.13)$$

The resulting estimates of W_{IT} were generated by changing the levels of annual cost (which gives different levels of TC), the levels of W_{IK}, and W_{RT}.

Product-Product Trade-Offs at the Bronco Site, Constant Ratios Between Two of Three Products

The three product case, with the constant trade-off ratio among the three products was generated by modifying equations (C.2) and (C.3) to give:

$$\begin{aligned} W_R &= a + b(TC-W_I-W_M) - c(TC-W_I-W_M)^2 \\ &= 749.33 + 0.036528(TC-W_I-W_M) \qquad \text{(C.14)} \\ &\quad - 0.000000025712(TC-W_I-W_M)^2 . \end{aligned}$$

The quadratic equation was then used on the relations,

$$L = c$$

$$M = b - 2cTC + 2cW_M$$

$$N = -a - bTC + bW_M + cTC^2 - 2cW_M TC + cW_M^2 + W_R$$

to give:

$$W_I = \frac{-M \pm \sqrt{M^2 - 4LN}}{2L} . \qquad \text{(C.15)}$$

The plus sign gave the relevant branch. The trade-off ratio between W_I and W_M is given by (from C.15):

$$\frac{dW_I}{dW_M} = -1.0 + \frac{1}{\sqrt{M^2-4LN}}\left[(b-2cTC+2cW_M)-(b-2cTC+2cW_M)\right]. \tag{C.16}$$

The trade-off ratio between W_I and W_M is constant over the entire surface. The trade-off ratio between W_I and W_R is given by (from C.15):

$$\frac{dW_I}{dW_R} = -\frac{1}{\sqrt{M^2-4LN}}. \tag{C.17}$$

After the cancellations are made in the denominator of (C.17), dW_I/dW_R becomes a function of only W_R. The trade-off ratio dW_M/dW_R is also given by equation (C.17).

Product-Product Trade-Offs at the Bronco Site, Variable Ratios Among All Three Products

The three product case with variable trade-offs was generated by modifying equations (C.2) and (C.3) to give (where a, b, c have the same values as in C.16):

$$W_R = a + b(TC-W_I-R_{WI}) - c(TC-W_I-R_{WI})^2 \tag{C.18}$$

where,

$$\begin{aligned} R_{WI} &= dW_M - e(W_M)^{1/2} \\ &= (1.0)W_M - (50.00)(W_M)^{1/2}. \end{aligned} \tag{C.19}$$

The quadratic formula was used to solve the equation for W_I from the relations,

$$L = c$$

$$M = b - 2cTC + 2cdW_M - 2ce(W_M)^{1/2}$$

$$N = -a - bTC + bdW_M - be(W_M)^{1/2} + cTC^2 - 2cdW_M TC$$
$$+ 2ce(W_M)^{1/2} TC + cd^2 W_M^{\ 2} - 2cdeW_M^{\ 3/2} + ce^2 W_M + W_R$$

to give:

$$W_I = \frac{-M \pm \sqrt{M^2 - 4LN}}{2L} . \tag{C.20}$$

The positive sign was used on the square root in (C. 20). The trade-off ratio between W_I and W_M is then found from the derivative of (C. 20) to give:

$$\frac{dW_I}{dW_M} = -d + \frac{e}{2} (W_M)^{-1/2} . \tag{C.21}$$

The trade-off between W_R and W_M was derived from equation (C. 18) to give:

$$\begin{aligned} \frac{dW_R}{dW_M} = -b[d - \frac{e}{2} \frac{1}{W_M^{\ 1/2}}] \\ -2c(TC - W_I - R_{WI})(-d + \frac{e}{2} \frac{1}{W_M^{\ 1/2}}) \end{aligned} \tag{C.22}$$

where R_{WI} is given in equation (C. 19). The trade-off (dW_I/dW_R) is the same as in equation (C. 17).

Factor-Product Trade-Off Ratios at One Site

The factor-product ratio model was also developed from the basic relations in (C.2) and (C.3), but with a different approach. It can be argued that:

$$W_S = f(W_A)$$
$$W_I = g(W_S) \qquad \text{(C.23)}$$

where,

W_S = total capacity,

W_A = wilderness area inundated,

W_I = water for irrigation.

Also, from (C.3), using $SA = W_A$ and $TC = W_S$,

$$SA = W_A = d + e(W_S) - f(W_S)^2$$
$$= 749.33 + 0.036528(W_S) - 0.000000025712(W_S)^2$$

or,

$$W_S = \frac{e \pm \sqrt{e^2 - 4f(W_A - d)}}{2f}\ . \qquad \text{(C.24)}$$

Using this formulation, W_S was allocated to W_I. The negative sign was used on the square root in (C.24). Surface area, in essence, is a factor in W_I production. The factor product trade-off ratio is then given by (from C.24):

$$\frac{dW_S}{dW_I} = \frac{dW_I}{dW_A} = \frac{1}{\sqrt{e^2 - 4f(W_A - d)}}\ . \qquad \text{(C.25)}$$

The total product curve, as a result, is convex.

APPENDIX D

DATA USED FOR ESTIMATING VALUE OF IRRIGATION WATER AND FOR ILLUSTRATION OF ISO-COST SURFACES

Value of Irrigation Water

The farm budget data used to develop the value of irrigation water used as a proxy for price (P_{WI}) is summarized in Table D-1. Both an irrigation and a dryland budget were developed using an assumed rotation of 2 years of row crop, 1 year of small grain, and 3 years alfalfa. The rotation was then "combined" to get a composite acre, the associated composite return, and the composite irrigation water requirement. The prices used reflect the factor and product market conditions existing in the Basin in 1969. Productivity conditions reflect better than average management strategies.

The return to management for the dryland composite acre was determined to be \$1.34 (Table D-1). This value results from using the following weighting scheme, based on the assumed rotation, to give:

$$\frac{(1)(\$4.18)+(2)(-\$7.05)+(3)(\$5.99)}{6} = \$1.34$$

where the return to management for dryland barley, corn silage, and alfalfa production were determined to be \$4.18, -\$7.05 and \$5.99 per acre, respectively. The return to management <u>and irrigation water</u>

Table D-1. Costs, returns, and value of an acre foot of water for irrigation, dryland and irrigated crops, Bronco site, Knife River Basin, North Dakota.

	Unit of Measure-ment	Irrigated			Dryland		
		Barley	Potatoes	Alfalfa	Barley	Corn Silage	Alfalfa
Variable costs, w/o labor and capital charges	$/acre	44.72	129.86	40.52	14.95	18.93	10.02
Labor costs	$/acre	2.62	30.83	10.21	2.50	7.84	5.22
Land capital @ 7%	$/acre	5.04	5.04	5.04	5.04	5.04	5.04
Operating capital @ 9%	$/acre	2.01	5.84	1.82	.59	.68	.36
Equipment capital @ 9%	$/acre	7.67	9.97	7.11	2.74	4.56	2.17
Yield/acre	b	62.80	256.90	4.35	30.00	5.00	1.60
Price of crop	$/b	1.00	1.40	18.00	1.00	6.00	18.00
Total returns	$/acre	62.80	359.66	78.30	30.00	30.00	28.80
Total cost	$/acre	62.06	171.54	64.70	25.82	37.05	22.81
Water requirements (irrigation)	acre feet	1.66	1.74	1.97	---	---	---
Return to management[c]	$/acre	.74	188.12	13.60	4.18	-7.05	5.99
Rotation	acres	1	2	3	1	2	3
Return to composite acre[c]	$/acre		69.63			1.34	
Return to irrigation water	$/acre foot		32.91				

[a]Figures derived from (North Dakota State Water Commission, "The Plan...", 1971).

[b]Yield measured in bushels/acre for barley, hundredweight/acre for potatoes, and tons/acre for alfalfa and corn silage.

[c]Return to management and irrigation water for the irrigated crops.

was determined to be \$69.63 for the composite acre. An adjustment was then made in the return to an irrigated composite acre to account for additional management requirements. It was assumed that management requirements could be related directly to gross dollar returns. Using this approach, the manager of the dryland (composite) acre realized a return of 4.56 percent on the (composite) gross return, or:

$$\frac{(1)(\$30.00)+(2)(\$30.00)+(3)(\$28.80)}{6} = \$29.40$$

$$\frac{\$1.34}{\$29.40} = 0.0456.$$

This factor was applied to the gross returns to the irrigated (composite) acre to give a management return of \$7.73, by:

$$\frac{(1)(\$62.80)+(2)(\$359.66)+(3)(\$78.30)}{6} = \$169.50$$

$$(\$169.50)(0.0456) = \$7.73\ .$$

The residual return to irrigation water was then determined by subtracting the management return from dryland <u>and</u> from irrigation to arrive at an estimate of the additional return, less associated costs, attributable to irrigation. Numerically, this is represented by:

$$\$69.63 - 7.73 - 1.34 = \$60.56.$$

This value was then converted to the (residual) value of irrigation water by using the (composite) acre foot requirement of 1.84 acre feet. The resulting value of irrigation water per acre foot is given by (Table D-1):

$$\frac{\$60.56}{1.84} = \$32.91.$$

This is the "adjusted for management and dryland returns" estimated value of an acre foot of water applied to a composite acre in the Knife River Basin near the Bronco and Kineman dam sites.

The approach followed here for estimating value is consistent with procedures followed in benefit-cost analysis as previously used by some of the Federal planning agencies. Associated costs for irrigation, such as additional fertilizer and labor requirements, are usually subtracted (as accomplished here) to arrive at the additional net dollar income generated by the application of irrigation water. This approach, however, raises a set of very significant issues.

A measure of the net national income generated from the public investment in irrigation facilities would be the difference in the <u>gross</u> dollar returns in changing from dryland to irrigation agriculture (after adjustments for capital consumption); i.e., associated costs would not be subtracted. Additional fertilizer expenditures, for example, are still a part of net national income.[117] Even if it is

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[117] Net national income is defined as the dollar value of all goods and services produced (less capital consumption) during some period of time (Ackley, 1961, p. 35).

argued that additional purchases of factors, such as fertilizer, merely represent transfers from other segments of agriculture, the purchase is still a part of the net national income. The procedure of subtracting associated costs seems inconsistent with attempts to measure the net national income generated from a project.

The return to irrigation water generated in this study represents a measure of the amount the farmer--irrigator will pay for the irrigation water. The estimate of $32.91 per acre foot would exhaust all the additional net income received by the user. This value, then, is a measure of the net benefits generated (<u>not</u> net national income) from using public investment to produce one acre foot of water for irrigation. This value was used to keep the results of this study consistent with other studies; the validity of this value may be subject to question, however.

Data for Iso-Cost Surfaces

The following set of tables represents the basic data used to illustrate the iso-cost surfaces at the Bronco and Kineman sites. Data on the amount of water stored in the reservoir (W_S), the amount allocated to irrigation (W_I) and recreation (surface area of reservoir, W_R), and the trade-offs are represented in each table. Further, the data in each table represents one iso-(minimum) cost curve. Several of the points on the iso-cost surface estimated for the

Bronco site are represented in Tables D-2 to D-8. The estimates for the Kineman site are represented in Tables D-9 to D-16.

Table D-2. W_S, W_I, and W_R for annual cost of $200,000 and total capacity of 64,300 acre feet, Bronco site.

W_S	W_I	W_R	$\Delta W_I/\Delta W_R$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
25,756	38,634	1673		-28.41
28,602	35,787	1773	-28.46	-28.52
31,461	32,929	1873	-28.59	-28.64
34,332	30,058	1973	-28.70	-28.77
37,214	27,176	2073	-28.83	-28.89
40,110	24,280	2173	-28.95	-29.01
43,017	21,373	2273	-29.08	-29.14
45,938	18,452	2373	-29.20	-29.27
48,871	15,519	2473	-29.33	-29.40
51,818	12,572	2573	-29.46	-29.53
54,777	9,612	2673	-29.60	-29.66
57,750	6,639	2773	-29.73	-29.80
60,737	3,653	2873	-29.87	-29.94
63,738	652	2973	-30.00	-30.07

Table D-3. W_S, W_I, and W_R for annual cost of $275,000 and total capacity of 105,632 acre feet, Bronco site.

W_S	W_I	W_R	$\Delta W_I/\Delta W_R$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
42,253	63,379	2247		-29.11
45,170	60,462	2347	-29.17	-29.24
48,100	57,532	2447	-29.30	-29.36
51,043	54,589	2547	-29.43	-29.50
53,999	51,633	2647	-29.56	-29.63
56,969	48,663	2747	-29.70	-29.76
59,952	45,680	2847	-29.83	-29.90
63,949	42,683	2947	-29.97	-30.04
65,960	39,672	3047	-30.11	-30.18
68,984	36,648	3147	-30.25	-30.32
72,024	33,608	3247	-30.39	-30.46
75,078	30,554	3347	-30.54	-30.61
78,146	27,486	3447	-30.68	-30.76
81,230	24,402	3547	-30.84	-30.91
84,328	21,304	3647	-30.99	-31.06
87,443	18,189	3747	-31.14	-31.22
90,572	15,060	3847	-31.30	-31.38
93,718	11,914	3947	-31.46	-31.54
96,880	8,752	4047	-31.62	-31.70
100,058	5,574	4147	-31.78	-31.86
103,253	2,379	4247	-31.95	-32.03

Table D-4. W_S, W_I, and W_R for annual cost of $350,000 and total capacity of 152,539 acre feet, Bronco site.

W_S	W_I	W_R	$\Delta W_I/\Delta W_R$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
61,016	91,524	2882		-29.95
64,018	88,522	2982	-30.02	-30.09
67,033	85,506	3082	-30.16	-30.23
70,063	82,476	3182	-30.30	-30.37
73,108	79,432	3282	-30.44	-30.52
76,167	76,372	3382	-30.59	-30.66
79,241	73,298	3482	-30.74	-30.81
82,330	70,210	3582	-30.89	-30.96
85,434	67,105	3682	-31.04	-31.12
88,554	63,986	3782	-31.17	-31.28
91,689	60,850	3882	-31.35	-31.43
94,840	57,699	3982	-31.51	-31.59
98,008	54,531	4082	-31.68	-31.76
101,192	51,347	4182	-31.84	-31.92
104,393	48,146	4282	-32.01	-32.09
107,611	44,929	4382	-32.18	-32.26
110,846	41,694	4482	-32.35	-32.44
114,098	38,441	4582	-32.53	-32.62
117,369	35,170	4682	-32.70	-32.80
120,658	31,882	4782	-32.89	-32.98
123,965	28,575	4882	-33.07	-33.16
127,290	25,249	4982	-33.26	-33.35
130,635	21,904	5082	-33.45	-33.54
134,000	18,540	5182	-33.64	-33.74
137,384	15,156	5282	-33.84	-33.94
140,788	11,751	5382	-34.04	-34.14
144,213	8,327	5482	-34.25	-34.35
147,658	4,881	5582	-34.45	-34.56
151,125	1,414	5682	-34.67	-34.77

Table D-5. W_S, W_I, and W_R for annual cost of $425,000 and total capacity of 205,112 acre feet, Bronco site.

W_S	W_I	W_R	$\Delta W_I/\Delta W_R$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
82,045	123,067	3573		-30.95
85,148	119,964	3673	-31.03	-31.10
88,266	116,846	3773	-31.18	-31.26
91,400	113,712	3872	-31.34	-31.42
94,550	110,562	3973	-31.50	-31.58
97,716	107,396	4073	-31.66	-31.74
100,898	104,214	4173	-31.82	-31.91
104,098	101,014	4273	-31.99	-32.08
107,314	97,798	4373	-32.16	-32.25
110,547	94,564	4473	-32.33	-32.42
113,798	91,314	4573	-32.51	-32.60
117,067	88,045	4673	-32.69	-32.78
120,354	84,758	4773	-32.87	-32.96
123,660	81,452	4873	-33.05	-33.15
126,984	78,128	4973	-33.24	-33.34
130,327	74,785	5073	-33.43	-33.53
133,689	71,423	5173	-33.62	-33.72
137,071	68,040	5273	-33.82	-33.92
140,474	64,638	5373	-34.02	-34.12
143,896	61,215	5473	-34.23	-34.33
147,340	57,772	5573	-34.44	-34.54
150,805	54,307	5673	-34.65	-34.75
154,291	50,821	5773	-34.86	-34.97
157,800	47,312	5873	-35.08	-35.19
161,330	43,782	5973	-35.31	-35.42
164,884	40,228	6073	-35.54	-35.65
168,461	36,651	6173	-35.77	-35.89
172,062	33,050	6273	-36.01	-36.13
175,686	29,425	6372	-36.25	-36.37
179,336	25,776	6473	-36.50	-36.62
183,011	22,101	6573	-36.75	-36.88
186,712	18,400	6673	-37.01	-37.14
190,439	14,673	6773	-37.27	-37.40
194,193	10,919	6873	-37.54	-37.68
197,974	7,138	6973	-37.81	-37.95
201,784	3,328	7073	-38.10	-38.24

Table D-6. W_S, W_I and W_R for annual cost of $500,000 and total capacity of 263,350 acre feet, Bronco site.

W_S	W_I	W_R	$\Delta W_I/\Delta W_R$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
105,340	158,010	4312		-32.14
108,563	154,787	4412	-32.23	-32.32
111,803	151,547	4512	-32.40	-32.49
115,061	148,289	4612	-32.58	-32.67
118,337	145,013	4712	-32.76	-32.85
121,631	141,714	4812	-32.94	-33.03
124,943	138,406	4912	-33.12	-33.22
128,275	135,075	5012	-33.31	-33.41
131,625	131,724	5112	-33.50	-33.60
134,995	128,354	5212	-33.70	-33.80
138,385	124,964	5312	-33.90	-34.00
141,795	121,554	5412	-34.10	-34.20
145,226	118,123	5512	-34.31	-34.41
148,678	114,672	5612	-34.52	-34.62
152,151	111,198	5712	-34.73	-34.84
155,646	107,704	5812	-34.95	-35.06
159,163	104,187	5912	-35.17	-35.28
162,702	100,647	6012	-35.40	-35.51
166,265	97,085	6112	-35.63	-35.74
169,851	93,499	6212	-35.86	-35.98
173,461	89,889	6312	-36.10	-36.22
177,095	86,254	6412	-36.34	-36.47
180,755	82,595	6512	-36.59	-36.72
184,440	78,910	6612	-36.85	-36.98
188,150	75,199	6712	-37.11	-37.24
191,888	71,462	6812	-37.37	-37.51
195,652	67,697	6912	-37.64	-37.78
199,445	63,905	7012	-37.92	-38.06
203,265	60,084	7112	-38.21	-38.35
207,115	56,234	7212	-38.50	-38.64
210,904	52,355	7312	-38.79	-38.94
214,904	48,445	7412	-39.10	-39.25
218,845	44,505	7512	-39.41	-39.57
222,818	40,532	7612	-39.73	-39.89
226,823	36,527	7712	-40.05	-40.22
230,862	32,488	7812	-40.34	-40.56

Table D-6. Continued.

W_S	W_I	W_R	$\Delta W_I/\Delta W_R$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
234,935	28,415	7912	-40.73	-40.90
239,043	24,306	8012	-41.08	-41.26
243,187	20,162	8112	-41.44	-41.63
247,369	15,981	8212	-41.82	-42.00
251,589	11,761	8312	-42.20	-42.39
255,847	7,502	8412	-42.59	-42.79
260,146	3,203	8512	-42.99	-43.20

Table D-7. W_S, W_I, and W_R for annual cost of \$575,000 and total capacity of 327,252 acre feet, Bronco site.

W_S	W_I	W_R	$\Delta W_I/\Delta W_R$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
130,901	196,351	5090		-33.56
134,267	192,986	5190	-33.66	-33.76
137,652	189,600	5290	-33.86	-33.96
141,058	186,194	5390	-34.06	-34.16
144,484	182,768	5490	-34.26	-34.37
147,932	179,321	5590	-34.47	-34.58
151,400	175,852	5690	-34.68	-34.79
154,890	172,362	5790	-34.90	-35.01
158,402	168,850	5890	-35.12	-35.23
161,937	165,315	5990	-35.35	-35.46
165,494	161,758	6090	-35.58	-35.69
169,076	158,177	6190	-35.81	-35.93
172,680	154,572	6290	-36.05	-36.17
176,309	150,943	6390	-36.29	-36.41
179,963	147,289	6490	-36.54	-36.66
183,643	143,610	6590	-36.79	-36.92
187,348	139,904	6690	-37.05	-37.18
191,080	136,173	6790	-37.32	-37.45
194,838	132,414	6890	-37.59	-37.72
198,624	128,628	6990	-37.86	-38.00
202,439	124,813	7090	-38.14	-38.29
206,282	120,970	7190	-38.43	-38.58
210,155	117,097	7290	-38.79	-38.88
214,058	113,194	7390	-39.03	-39.18
217,992	109,260	7490	-39.34	-39.10
221,958	105,294	7590	-39.66	-39.82
225,956	101,296	7690	-39.98	-40.15
229,988	97,265	7790	-40.31	-40.48
234,053	93,199	7890	-40.66	-40.83
238,154	89,098	7990	-41.01	-41.18
242,290	84,962	8090	-41.36	-41.55
246,464	80,788	8190	-41.73	-41.92
250,675	76,577	8290	-42.11	-42.31
254,926	72,327	8390	-42.50	-42.70
259,216	68,036	8490	-42.90	-43.11
263,547	63,705	8590	-43.32	-43.52

Table D-7. Continued.

W_S	W_I	W_R	$\Delta W_I/\Delta W_R$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
267,921	59,331	8690	-43.74	-43.96
272,339	54,914	8790	-44.18	-44.40
276,801	50,451	8890	-44.62	-44.86
281,310	45,942	8990	-45.09	-45.33
285,867	41,385	9090	-45.57	-45.81
290,474	36,779	9190	-46.06	-46.32
295,131	32,121	9290	-46.57	-46.84
299,841	27,411	9390	-47.10	-47.37
304,606	22,646	9490	-47.65	-47.93
309,428	17,824	9590	-48.22	-48.51
314,308	12,944	9690	-48.80	-49.10
319,250	8,003	9790	-49.41	-49.72
324,254	2,998	9890	-50.04	-50.37

Table D-8. W_S, W_I and W_R for annual cost of $650,000 and total capacity of 396,820 acre feet, Bronco site.

W_S (acre feet)	W_I (acre feet)	W_R (surface acres)	$\Delta W_I/\Delta W_R$	dW_I/dW_R
158,728	238,092	5900		-35.25
162,265	234,556	6000	-35.37	-35.48
165,825	230,996	6100	-35.60	-35.71
169,408	227,413	6200	-35.83	-35.95
173,015	223,806	6300	-36.07	-36.19
176,646	220,174	6400	-36.31	-36.44
180,302	216,518	6500	-36.56	-36.69
183,984	212,836	6600	-36.82	-36.94
187,692	209,129	6700	-37.08	-37.21
191,426	205,394	6800	-37.34	-37.48
195,187	201,633	6900	-37.61	-37.75
198,976	197,844	7000	-37.89	-38.03
202,793	194,027	7100	-38.17	-38.31
206,639	190,181	7200	-38.46	-38.61
210,515	186,306	7300	-38.76	-38.91
214,421	182,400	7400	-39.06	-39.21
218,358	178,463	7500	-39.37	-39.53
222,326	174,494	7600	-39.69	-39.85
226,328	170,493	7700	-40.01	-40.18
230,362	166,458	7800	-40.35	-40.52
234,431	162,389	7900	-40.69	-40.86
238,535	158,286	8000	-41.04	-41.22
242,675	154,146	8100	-41.40	-41.58
246,852	149,969	8200	-41.77	-41.96
251,066	145,754	8300	-42.15	-42.34
255,320	141,500	8400	-42.54	-42.74
259,614	137,206	8500	-42.94	-43.14
263,950	132,870	8600	-43.35	-43.56
268,328	128,493	8700	-43.78	-44.00
272,749	124,071	8800	-44.22	-44.44
277,216	119,604	8900	-44.67	-44.90
281,730	115,091	9000	-45.13	-45.37
286,291	110,529	9000	-45.61	-45.86
290,902	105,918	9200	-46.11	-46.36
295,564	101,256	9300	-46.62	-46.88
300,280	96,541	9400	-47.15	-47.42

Table D-8. Continued.

W_S	W_I	W_R	$\Delta W_I/\Delta W_R$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
305,050	91,771	9500	-47.70	-47.98
309,877	86,944	9600	-48.27	-48.56
314,762	82,058	9700	-48.86	-49.16
319,710	77,111	9800	-49.47	-49.78
324,720	72,100	9900	-50.10	-50.43
329,796	67,024	10000	-50.76	-51.10
334,941	61,879	10100	-51.45	-51.80
340,158	56,662	10200	-52.16	-52.53
345,449	51,371	10300	-52.91	-53.29
350,818	46,002	10400	-53.69	-54.09
356,268	40,552	10500	-54.50	-54.92
361,804	35,016	10600	-55.36	-55.80
367,429	29,392	10700	-56.25	-56.71
373,148	23,673	10800	-57.19	-57.67
378,965	17,855	10900	-58.17	-58.68
384,886	11,934	11000	-59.21	-59.75
390,918	5,903	11100	-60.31	-60.88

Table D-9. W_S, W_I, and W_R for annual cost of $20,000 and total capacity of 691 acre feet, Kineman site.

W_S	W_I	W_R	$\Delta W_I/\Delta W_R$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
276	415	16		-13.65
414	278	26	-13.71	-13.77
552	139	36	-13.84	-13.90
692	0	46	-13.97	-14.03

Table D-10. W_S, W_I, and W_R for annual cost of $30,000 and total capacity of 1633 acre feet, Kineman site.

W_S	W_I	W_R	$\Delta W_I/\Delta W_T$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
653	980	43		-14.00
794	839	53	-14.06	-14.13
936	697	63	-14.20	-14.27
1079	554	73	-14.34	-14.41
1224	409	83	-14.48	-14.56
1370	263	93	-14.63	-14.78
1518	115	103	-14.78	-14.86

Table D-11. W_S, W_I, and W_R for annual cost of \$40,000 and total capacity of 2733 acre feet, Kineman site.

W_S	W_I	W_R	$\Delta W_I/\Delta W_R$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
1093	1640	74		-14.42
1238	1495	84	-14.50	-14.57
1385	1348	94	-14.65	-14.72
1533	1200	104	-14.80	-14.88
1682	1051	114	-14.96	-15.04
1834	900	124	-15.12	-15.21
1986	747	134	-15.29	-15.38
2141	592	144	-15.47	-15.56
2298	436	154	-15.65	-15.74
2456	277	164	-15.84	-15.93
2616	117	174	-16.03	-16.13

Table D-12. W_S, W_I, and W_R for annual cost of \$50,000 and total capacity of 3992 acre feet, Kineman site.

W_S	W_I	W_R	$\Delta W_I/\Delta W_R$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
1597	2395	108		-14.95
1747	2245	118	-15.03	-15.11
1899	2093	128	-15.20	-15.28
2053	1940	138	-15.37	-15.45
2208	1784	148	-15.54	-15.64
2366	1627	158	-15.73	-15.82
2525	1468	168	-15.92	-16.02
2686	1306	178	-16.12	-16.22
2849	1143	188	-16.32	-16.43
3015	978	198	-16.54	-16.65
3182	810	208	-16.76	-16.87
3352	640	218	-16.99	-17.11
3524	468	228	-17.23	-17.36
3699	293	238	-17.48	-17.61
3877	116	248	-17.75	-17.88

Table D-13. W_S, W_I, and W_R for annual cost of \$60,000 and total capacity of 5410 acre feet, Kineman site.

W_S	W_I	W_R	$\Delta W_I/\Delta W_R$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
2164	3246	145		-15.58
2321	3089	155	-15.68	-15.77
2480	2931	165	-15.86	-15.96
2640	2770	175	-16.06	-16.16
2803	2608	185	-16.26	-16.37
2968	2443	195	-16.48	-16.58
3134	2276	205	-16.70	-16.81
3304	2106	215	-16.92	-17.04
3475	1935	225	-17.16	-17.28
3649	1761	235	-17.41	-17.54
3826	1584	245	-17.67	-17.80
4006	1405	255	-17.94	-18.08
4188	1222	265	-18.23	-18.38
4373	1037	275	-18.53	-18.68
4562	849	285	-18.84	-19.00
4753	657	295	-19.17	-19.34
4948	462	305	-19.52	-19.70
5147	263	315	-19.89	-20.08
5350	60	325	-20.28	-20.48

Table D-14. W_S, W_I, and W_R for annual cost of $70,000 and total capacity of 6987 acre feet, Kineman site.

W_S (acre feet)	W_I (acre feet)	W_R (surface acres)	$\Delta W_I/\Delta W_R$	dW_I/dW_R
2795	4192	185		-16.36
2959	4027	195	-16.46	-16.57
3126	3861	205	-16.68	-16.80
3295	3691	215	-16.91	-17.03
3467	3520	225	-17.15	-17.27
3641	3346	235	-17.40	-17.53
3817	3169	245	-17.66	-17.79
3997	2990	255	-17.93	-18.07
4179	2808	265	018.21	-18.36
4364	2623	275	-18.51	-18.66
4552	2435	285	-18.82	-18.99
4744	2243	295	-19.16	-19.32
4939	2048	305	-19.50	-19.68
5138	1849	315	-19.87	-20.06
5340	1647	325	-20.26	-20.46
5547	1440	335	-20.67	-20.89
5758	1229	345	-21.11	-21.34
5974	1013	355	-21.58	-21.82
6194	792	365	-22.08	-22.34
6421	566	375	-22.62	-22.90
6653	334	385	-23.20	-23.50
6891	96	395	-23.82	-24.15

Table D-15. W_S, W_I and W_R for annual cost of $80,000 and total capacity of 8722 acre feet, Kineman site.

W_S	W_I	W_R	$\Delta W_I/\Delta W_R$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
3489	5233	226		-17.30
3663	5059	236	-17.43	-17.56
3840	4882	246	-17.69	-17.83
4020	4702	256	-17.96	-18.10
4202	4520	266	-18.25	-18.40
4388	4334	276	-18.55	-18.70
4576	4146	286	-18.87	-19.03
4768	3954	296	-19.20	-19.37
4964	3758	306	-19.55	-19.73
5163	3559	316	-19.92	-20.11
5366	3356	326	-20.31	-20.51
5573	3149	336	-20.73	-20.94
5785	2937	346	-21.17	-21.40
6002	2721	356	-21.64	-21.89
6223	2499	366	-22.15	-22.41
6450	2272	376	-22.69	-22.97
6683	2039	386	-23.27	-23.58
6922	1800	396	-23.91	-24.24
7168	1554	406	-24.59	-24.96
7421	1301	416	-25.34	-25.74
7683	1039	426	-26.16	-26.60
7953	769	436	-27.07	-27.56
8234	488	446	-28.08	-28.63
8526	195	456	-29.22	-29.83

Table D-16. W_S, W_I, and W_R for annual cost of $90,000 and total capacity of 10,616 acre feet, Kineman site.

W_S	W_I	W_R	$\Delta W_I/\Delta W_R$	dW_I/dW_R
(acre feet)	(acre feet)	(surface acres)		
4246	6370	268		-18.47
4433	6183	278	-18.62	-18.78
4622	5994	288	-18.94	-19.11
4815	5801	298	-19.28	-19.45
5011	5605	308	-19.64	-19.82
5211	5405	318	-20.01	-20.20
5416	5201	328	-20.41	-20.61
5624	4992	338	-20.83	-21.05
5837	4780	348	-21.28	-21.51
6054	4562	358	-21.76	-22.01
6277	4339	368	-22.27	-22.54
6506	4111	378	-22.82	-23.12
6739	3877	388	-23.42	-23.73
6980	3636	398	-24.06	-24.41
7228	3388	408	-24.77	-25.14
7483	3133	418	-25.53	-25.94
7747	2869	428	-26.37	-26.82
8070	2596	438	-27.30	-27.80
8303	2313	448	-28.34	-28.90
8598	2018	458	-29.51	-30.14
8907	1709	468	-30.83	-31.55
9230	1386	478	-32.35	-33.18
9571	1045	488	-34.11	-35.10
9933	683	498	-36.20	-37.39
10321	295	508	-38.74	-40.19