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A theoretical derivation of the dependence of the remotely sensed reflectance of the ocean on the inherent optical properties

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Abstract. An expression for the ratio of the upwelling nadir radiance $L(\pi, z)$ and the downwelling scalar irradiance $E_{od}(Z)$ is derived from the following equation of radiative transfer. This expression is given by $\mathrm{RSR}(z)=[L(\pi, z)]/E_{od}(z) = [f_b(z)b_b(z)]/2\pi[k(\pi, z) + c(z) - f_L(z)b_f(z)]$, where $b_b(z)$ is the backscattering coefficient, $k(\pi, z)$ is the vertical attenuation coefficient of the nadir radiance, $c(z)$ is the beam attenuation coefficient, and $f_b(z)$ and $f_L(z)$ are shape parameters that depend on the shape of the volume scattering function and the radiance distribution. Successive approximations are subsequently applied to the above exact equation. These are $f_b(z) = [2\pi\beta(\pi - \theta_m, z)]/[b_b(z)]$, where $\beta(\pi - \theta_m, z)$ is the volume scattering function at 180° minus the zenith angle of the maximum radiance, and $k(\pi, z) = am = c[1 - 0.52\ b/c - 0.44\ (b/c)^2]$, where m is a parameter that is numerically equal to the inverse of the average cosine of the asymptotic light field for a medium with the same inherent optical properties, a is the absorption coefficient, and b/c is the single scattering albedo. Together with $f_L(z) = 1.05$ and application of Gershun's equation, it is shown that for nearly all oceanic cases $\mathrm{RSR}(z) \equiv L(\pi, z)/E_{od}(z) = [\beta(\pi - \theta_m, z)]/\{a(z)[1 + m(z)]\}$.

Introduction

The diffuse or irradiance reflectance R at a depth z is defined as the ratio of the upwelling irradiance E_u and the downwelling irradiance E_d. Hence $R(z) = E_u(z)/E_d(z)$. This parameter has been extensively measured and modeled (see, for example, *Gordon et al.* [1988], *Morel* [1988], *Gordon* [1989], and *Morel and Gentili* [1990]), primarily because of its ease of measurement since the irradiance sensor does not require absolute calibration. Remote sensing satellites sense radiance rather than irradiance, so that the models were subsequently modified to look at the ratio of the upwelling radiance L_u and the downwelling irradiance [*Gordon et al.*, 1988; *Morel and Gentili*, 1993]. The ratio $R_{rs}(z) = L_u(z)/E_d(z)$ as used in the later papers is often called the remote sensing reflectance. Instrumentation was also developed to measure the upwelling radiance spectrum.

All the references cited above contain models that are based on measurements or Monte Carlo modeling and so are semianalytic. In these models, new calculations need to be made if one wants to apply the results to a specific volume scattering function. A purely analytical model might reveal simplifying relationships between the inherent optical properties (IOP) (the scattering and absorption properties of natural waters) and the upwelling radiance. This approach was taken by *Zaneveld* [1982], who derived an exact expression for the remotely sensed reflectance ratio $\mathrm{RSR}(z) = L(\pi, z)/E_{od}(z)$, where $L(\pi, z)$ is the upwelling radiance as seen by a nadir-viewing radiance sensor (termed "nadir radiance" in the remainder of this paper) and $E_{od}(z)$ is the downwelling scalar irradiance. This expression uses the scalar downwelling irradiance rather than the plane downwelling irradiance in the denominator. As will be shown, the scalar irradiance appears naturally in the derivation. In addition, the scalar irradiance is far less dependent on the angular structure of the incident light field than the plane irradiance, so that some complexity is bound to be removed by the use of this parameter.

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Practical use of the *Zaneveld* [1982] expression was limited due to the presence of a number of difficult to measure shape factors. The present paper will give a general expression for the remotely sensed reflectance for upward radiance in any direction. It clarifies the original paper in that it contains explicit expressions for the shape factors and derives the final expression without use of the mean value theorem. It then applies a number of approximations to the remotely sensed reflectance for the nadir radiance, resulting in expressions that contain measurable inherent optical properties only.

Theory

The equation of radiative transfer in a plane parallel medium without internal sources and inelastic scattering effects is given by

$$\cos\theta \frac{dL_u(\theta, \phi, z)}{dz} = -c(z)L_u(\theta, \phi, z) + L_u^*(\theta, \phi, z), \quad (1)$$

where

$$L_u^*(\theta, \phi, z) = \int_0^{2\pi}\int_0^{\pi} \beta(\theta, \phi; \theta', \phi', z)L(\theta', \phi', z) \sin\theta'\, d\theta'\, d\phi' \quad (2)$$

and $\beta(\theta, \phi; \theta', \phi', z)$ is the volume scattering function of the angle between the unscattered ray traveling in direction (θ', ϕ') and the scattered ray traveling in direction (θ, ϕ). We are interested in deriving expressions for upwelling light. It should be borne in mind that the radiances on the left-hand side of the equations refer to upwelling light as indicated by the subscript u. The path function $L^*(\theta, \phi, z)$ describes the light that is scattered into direction (θ, ϕ) from all other directions.

We divide the path function $L^*(\theta, \phi, z)$ into components due to downwelling and upwelling radiance.

$$L_u^*(\theta, \phi, z) = \int_0^{2\pi}\int_0^{\pi/2} \beta(\theta, \phi; \theta', \phi', z)L(\theta', \phi', z) \sin\theta'\, d\theta'\, d\phi' + \int_0^{2\pi}\int_{\pi/2}^{\pi} \beta(\theta, \phi; \theta', \phi', z)L(\theta', \phi', z) \sin\theta'\, d\theta'\, d\phi'.$$

The first component is due to scattered downwelling light, so that $0 \leq \theta' \leq \pi/2$. The scattering angle is not necessarily less than $\pi/2$, however. We introduce a shape factor $f_b(\theta, \phi, z)$ that indicates the ratio of the first component to the product of the average value of the volume scattering function in the backward direction and the scalar downwelling irradiance:

$$f_b(\theta, \phi, z) \equiv \frac{\int_0^{2\pi}\int_0^{\pi/2} \beta(\theta, \phi; \theta', \phi', z)L(\theta', \phi', z) \sin\theta'\, d\theta'\, d\phi'}{\frac{b_b(z)}{2\pi} E_{od}(z)}. \tag{3}$$

It should be noted that the 2π in the denominator of the above equation carries units of steradian, so that the shape factor f_b is dimensionless. The shape factor is seen to be the ratio of the light actually scattered into the direction (θ, ϕ) by scattering of downward traveling radiance to the light that would be received if the scattering function was constant and equal to $[b_b(z)]/2\pi$.

The second component of the path function is due to upwelling light being scattered. We introduce another shape parameter, $f_L(\theta, \phi, z)$ that indicates the relative magnitude of the second component of the path function and the product of the forward scattering function and the radiance:

$$f_L(\theta, \phi, z) \equiv \frac{\int_0^{2\pi}\int_{\pi/2}^{\pi} \beta(\theta, \phi; \theta', \phi', z)L(\theta', \phi', z) \sin\theta'\, d\theta'\, d\phi'}{b_f(z)L_u(\theta, \phi, z)}. \tag{4}$$

This second shape parameter is the ratio of light scattered into the direction (θ, ϕ) by scattering of upwelling radiance and the amount of light that would be scattered into the direction (θ, ϕ) if the upwelling radiance were uniform and equal to $L_u(\theta, \phi, z)$ and if the scattering function were uniform and equal to $b_f/2\pi$.

Substitution of the above two expressions allows us to rewrite the path function:

$$L_u^*(\theta, \phi, z) = f_b(\theta, \phi, z) \frac{b_b(z)}{2\pi} E_{od}(z) + f_L(\theta, \phi, z)L_u(\theta, \phi, z)b_f(z). \tag{5}$$

Substitution of (3), (4), and (5) into the equation of radiative transfer gives

$$\cos\theta \frac{dL_u(\theta, \phi, z)}{dz} = -c(z)L_u(\theta, \phi, z) + f_b(\theta, \phi, z)\frac{b_b(z)}{2\pi} E_{od}(z) + f_L(\theta, \phi, z)L_u(\theta, \phi, z)b_f(z). \tag{6}$$

We can define the attenuation coefficient for radiance as follows:

$$k(\theta, \phi, z) \equiv -\frac{dL(\theta, \phi, z)}{L(\theta, \phi, z)\, dz}.$$

Substitution of the above equation into (6) and factoring gives the desired remotely sensed reflectance

$$\mathrm{RSR}(\theta, \phi, z) \equiv \frac{L_u(\theta, \phi, z)}{E_{od}(z)} = \frac{f_b(\theta, \phi, z)\frac{b_b(z)}{2\pi}}{-\cos\theta\, k(\theta, \phi, z) + c(z) - f_L(\theta, \phi, z)b_f(z)}. \tag{7}$$

Equation (7) is exact as it is a restatement of the equation of radiative transfer for $\theta \geq \pi/2$.

We will now look at the equations for the vertically upwelling radiance (or nadir radiance with $\theta = \pi$; θ is measured from the zenith direction). In that case the angle between the unscattered downwelling ray and the upwelling nadir radiance is 180° minus the zenith angle of the incoming ray. $\beta(\theta, \phi; \theta', \phi', z)$ may then be replaced by $\beta(\pi - \theta', z)$, resulting in the following expressions:

$$f_b(\pi, z) = \frac{\int_0^{2\pi}\int_0^{\pi/2} \beta(\pi - \theta', z)L(\theta', \phi', z) \sin\theta'\, d\theta'\, d\phi'}{\frac{b_b(z)}{2\pi} E_{od}(z)}. \tag{8}$$

The numerator in (8) is entirely due to backscattered light of downwelling radiance as $\pi - \theta'$ ranges from $\pi/2$ to π.

$$f_L(\pi, z) = \frac{\int_0^{2\pi}\int_{\pi/2}^{\pi} \beta(\pi - \theta', z)L(\theta', \phi', z) \sin\theta'\, d\theta'\, d\phi'}{b_f(z)L(\pi, z)}. \tag{9}$$

Here the numerator is due entirely to forward scattered light of upwelling radiance. If we define the attenuation coefficient for the nadir radiance as

$$k(\pi, z) \equiv -\frac{dL(\pi, z)}{L(\pi, z)\, dz}, \tag{10}$$

and substitute (8), (9), and (10) into (7), we obtain

$$\mathrm{RSR}(\pi, z) \equiv \frac{L(\pi, z)}{E_{od}(z)} = \frac{f_b(\pi, z)\, \dfrac{b_b(z)}{2\pi}}{k(\pi, z) + c(z) - f_L(\pi, z) b_f(z)}. \tag{11}$$

Equation (10) is also exact, as it simply is a restatement of the equation of radiative transfer for nadir radiance. This expression was obtained previously by *Zaneveld* [1982] using a different derivation. The expression contains three parameters that depend on the submarine light field, $k(\pi, z)$, $f_b(\pi, z)$, and $f_L(\pi, z)$. If the expression is to be useful for experimental work, these parameters need to approximated in terms of measurable inherent optical properties.

Approximations

Of the various parameters that occur in (11), $f_b(\pi, z)$ is the most critical, as the RSR is directly proportional to it. *Zaneveld* [1982] showed that $f_b(\pi, z)$ ranged from approximately 0.8 to 1.3. The parameter obviously depends on the shape of the volume-scattering function (VSF) in the backward direction and the shape of the downward radiance distribution. Radiance distributions such as shown by *Jerlov* [1976] nearly always show a well-defined maximum. Near the surface this normally occurs at the angle of the refracted solar disk. The cosine of this angle is also very close to the average cosine of the light field, as it dominates the radiance. The angle at which this maximum occurs changes only slowly with depth. At any depth that contributes significantly to remotely detectable radiance we can then argue that most of the light that eventually travels in the nadir direction is a result of light that is backscattered from the radiance maximum or its immediate neighborhood. A more or less linear slope of the scattering function results in light to one side of the maximum being scattered more than light on the other side, but the amounts nearly cancel. We thus assume that for most remote sensing viewing situations the incident light is dominated by light with a well-defined zenith angle θ_m. (If the volume scattering function is known, the shape parameters for a diffuse sky can be calculated from (3) and (4)). Near the surface in relatively clear waters this angle will be the solar zenith angle. We thus hypothesize that most of the nadir radiance can be reasonably modeled as being derived from single scattering of light near the maximum radiance. This light scatters through an angle of $\pi - \theta_m$, as the incoming ray has a zenith angle of θ_m and the scattered ray has a zenith angle of π. We can thus approximate the expression for $f_b(\pi, z)$ as follows:

$$f_b(\pi, z) \approx \beta(\pi - \theta_m, z) \cdot \frac{\displaystyle\int_0^{2\pi}\int_0^{\pi/2} L(\theta', \phi', z) \sin\theta'\, d\theta'\, d\phi'}{\dfrac{b_b(z)}{2\pi} E_{od}(z)}. \tag{12}$$

Note that this does not imply that the volume scattering function in the backward direction has to be flat but that a nearly linear scattering function with angle largely cancels errors on either side of the radiance maximum. Since the integral term is precisely equal to $E_{od}(z)$, we can simplify the above to

$$f_b(\pi, z) \approx \frac{2\pi\beta(\pi - \theta_m, z)}{b_b(z)}, \qquad 0 \le \theta_m \le \theta_c. \tag{13}$$

where θ_m is the zenith angle of the maximum radiance and θ_c is the critical angle. Equation (13) was tested extensively using Monte Carlo models [*Stavn and Zaneveld*, 1994; *Weidemann et al.*, this issue] and was found to have typical errors of 5%, with maximum errors of 12%.

Substitution of (13) into (11) gives

$$\mathrm{RSR}(\pi, z) \equiv \frac{L(\pi, z)}{E_{od}(z)} \approx \frac{\beta(\pi - \theta_m, z)}{k(\pi, z) + c(z) - f_L(\pi, z) b_f(z)}. \tag{14}$$

Equation (14) entirely removes the unmeasurable $f_b(z)$ factor but requires us to know the shape of the volume scattering function in the backward direction and the zenith angle of the maximum radiance.

The attenuation coefficient for the upwelling radiance $k(\pi, z)$ can be measured directly using the vertical structure of the upwelling radiance, which is now measured routinely. For inversion it is necessary to describe this parameter in terms of the IOP, so that eventually it can be related to particulate properties. We do not know of a study that directly relates $k(\pi, z)$ to the IOP. *Aas* [1987] describes relationships between the absorption coefficient, the ratio of the upwelling and downwelling diffuse attenuation coefficients K_u/K_d, and the ratio of the average cosines of the upwelling and downwelling radiance fields $\bar{\mu}_u/\bar{\mu}_d$; Aas' model does not provide an explicit dependence of $k(\pi, z)$ on the IOP only. Earlier studies such as that of *Lundgren and Højerslev* [1971] provide valuable insight into the relationship of K_u with other apparent and inherent optical properties, but they also do not provide the relationship needed here. We will therefore present a plausible relationship here, but this issue should be further investigated. The modeled value of $k(\pi, z)$ obtained below will be indicated by $k_m(\pi, z)$ in order to distinguish it from the actual value. *Tyler* [1960] and *Jerlov* [1976] show that the shape of the upward radiance distribution is nearly constant with depth. Depth profiles of $\ln L(\pi, z)$ [*Tyler*, 1960; *Jerlov and Fukuda*, 1960; *Timofeeva*, 1974] (as shown by *Jerlov* [1976]) generally have a constant slope as a function of depth, while the slope of the radiances at smaller angles vary a great deal near the surface. At greater depth all radiances have the same slope. This slope is the asymptotic diffuse attenuation coefficient. This implies that $k(\pi, z)$ is nearly constant with depth over the entire water column. If that is the case, it must equal the asymptotic diffuse attenuation coefficient K_∞, even near the surface. What this really indicates is that, to a large extent, the upwelling radiance attenuation coefficient is decoupled from the downwelling radiance distribution. In addition, $k(\pi, z)$ is independent of the magnitude of $L(\pi, z)$. We thus see that there are physical and experimental reasons why the attenuation coefficient for nadir radiance can reasonably be modeled as the asymptotic diffuse attenuation coefficient. Future research will undoubtedly generate more complete models for $k(\pi, z)$.

A number of expressions for K_∞ as a function of the shape of the volume scattering function and the input radiance distribution have been derived [*Gordon et al.*, 1993]. This reference shows that the dependence of K_∞ on the shape of the scattering function is weak. For realistic applications the dependence of K_∞/c on b/c as obtained by *Prieur and Morel* [1971] would appear to be sufficiently accurate. Alternatively, one could use the dependence of K_∞ on the IOP obtained by *Timofeeva* [1974]. *Zaneveld* [1989] obtained a simple fit to the *Prieur and Morel* [1971] data. We will use this relationship as the modeled value of the attenuation coefficient of the nadir radiance $k_m(\pi, z)$. We then get

$$k_m(\pi, z) = c[1 - 0.52b/c - 0.44(b/c)^2]. \tag{15}$$

Substitution of (15) into (14) and using $c = a + b$ then yields

$$\mathrm{RSR}(\pi, z) \equiv \frac{L(\pi, z)}{E_{od}(z)} \approx \frac{\beta(\pi - \theta_m, z)}{a(z)[1 + m(z)] + b(z) - f_L(\pi, z)b_f(z)}, \tag{16}$$

where

$$m(z) \equiv \frac{k_m(\pi, z)}{a(z)} = \frac{1 - 0.52b(z)/c(z) - 0.44[b(z)/c(z)]^2}{1 - b(z)/c(z)}. \tag{17}$$

While in the context of this paper, $m(z)$ has no physical meaning, it is equal in magnitude to the inverse of the average cosine of the asymptotic light field for a medium, with a K_∞ as given in (15). This can be seen by application of Gershun's equation

$$K = a/\bar{\mu}, \tag{18}$$

where $\bar{\mu}$ is the average cosine of the light field. Note that m is a function of the single scattering albedo b/c only. Since b/c can vary with depth, m is a depth dependent function also.

We now need to replace the term containing $f_L(\pi, z)$, as it is not a readily measurable parameter. The dependence of $f_L(\pi, z)$ on the shape of the upwelling radiance distribution can be seen by rewriting (9) as follows:

$$f_L(\pi, z) \approx \frac{1}{b_f(z)} \int_0^{2\pi} \int_{\pi/2}^{\pi} \beta(\pi - \theta', z) \cdot \frac{L_u(\theta', \phi', z)}{L(\pi, z)} \sin\theta' \, d\theta' \, d\phi'. \tag{19}$$

Typically, $L(\pi, z)$ is the minimum value in the upwelling radiance distribution, so that $[L_u(\theta', \phi', z)]/L(\pi, z) \geq 1$. We thus see that usually, $f_L(\pi, z) \geq 1$. We now rewrite (18) as follows using $b(z) = b_f(z) + b_b(z)$:

$$\mathrm{RSR}(\pi, z) \equiv \frac{L(\pi, z)}{E_{od}(z)} \approx \frac{\beta(\pi - \theta_m, z)}{a(z)[1 + m(z)] + b_b(z) + [1 - f_L(\pi, z)]b_f(z)}. \tag{20}$$

Calculations for extreme cases showed that $f_L(\pi, z)$ varies between 1.0 and 1.1 [*Zaneveld*, 1982]. We make, at most, a 5% error in $f_L(z)$ by setting it equal to 1.05. This results in

$$\mathrm{RSR}(\pi, z) \equiv \frac{L(\pi, z)}{E_{od}(z)} \approx \frac{\beta(\pi - \theta_m, z)}{a(z)[1 + m(z)] + b_b(z) - 0.05b_f(z)}. \tag{21}$$

We can use this formula for case 2 waters when the scattering is very large compared to the absorption. In very clear waters the molecular backscattering is important and b_b cannot be ignored in the denominator. In nearly all oceanic cases, however, we can further simplify this expression with negligible loss in accuracy if we can meet the condition that

$$a(z)[1 + m(z)] \gg b_b(z) - 0.05b_f(z). \tag{22}$$

The left-hand side of the inequality is greater than $2a$, whereas the right-hand side is a few percent of b. (For estimation purposes it is useful to bear in mind that m has the same value as the inverse of the asymptotic average cosine for homogeneous waters with the same IOP; m must thus always be greater than unity.) In most oceanic and coastal situations the particulate scattering coefficient is approximately two times the particulate absorption coefficient [*Jerlov*, 1974]. In addition, the scattering coefficient for pure water is much less than the absorption coefficient in the visible region of the spectrum. The presence of yellow matter further strengthens the inequality. Inequality (22) is thus not a severe condition and is only likely to be not satisfied in regions of very heavy inorganic particulate loads. If inequality (22) is satisfied, the remote sensing reflectance can be further approximated by

$$\mathrm{RSR}(\pi, z) \equiv \frac{L(\pi, z)}{E_{od}(z)} \approx \frac{\beta(\pi - \theta_m, z)}{a(z)[1 + m(z)]}. \tag{23}$$

We thus have a family of expressions for the remotely sensed reflectance that can be used, depending on the accuracy desired. In descending order of accuracy, with the first two having no approximations at all, these are equation (7)

$$\mathrm{RSR}(\theta, \phi, z) \equiv \frac{L_u(\theta, \phi, z)}{E_{od}(z)} = \frac{f_b(\theta, \phi, z)\dfrac{b_b(z)}{2\pi}}{-\cos\theta k(\theta, \phi, z) + c(z) - f_L(\theta, \phi, z)b_f(z)},$$

equation (11)

$$\mathrm{RSR}(\pi, z) \equiv \frac{L(\pi, z)}{E_{od}(z)} = \frac{f_b(\pi, z)\dfrac{b_b(z)}{2\pi}}{k(\pi, z) + c(z) - f_L(\pi, z)b_f(z)},$$

equation (14)

$$\mathrm{RSR}(\pi, z) \equiv \frac{L(\pi, z)}{E_{od}(z)} \approx \frac{\beta(\pi - \theta_m, z)}{k(\pi, z) + c(z) - f_L(\pi, z)b_f(z)},$$

equation (20)

$$\mathrm{RSR}(\pi, z) \equiv \frac{L(\pi, z)}{E_{od}(z)}$$

$$\approx \frac{\beta(\pi - \theta_m, z)}{a(z)[1 + m(z)] + b_b(z) + [1 - f_L(\pi, z)]b_f(z)},$$

where

$$m(z) \equiv \frac{k_m(\pi, z)}{a(z)} \approx \frac{1 - 0.52b(z)/c(z) - 0.44[b(z)/c(z)]^2}{1 - b(z)/c(z)},$$

equation (21)

$$\mathrm{RSR}(\pi, z) \equiv \frac{L(\pi, z)}{E_{od}(z)}$$

$$\approx \frac{\beta(\pi - \theta_m, z)}{a(z)[1 + m(z)] + b_b(z) - 0.05b_f(z)},$$

equation (23)

$$\mathrm{RSR}(\pi, z) \equiv \frac{L(\pi, z)}{E_{od}(z)} \approx \frac{\beta(\pi - \theta_m, z)}{a(z)[1 + m(z)]}.$$

Discussion and Conclusions

The equations derived above show the dependence of the remotely sensed reflectance on the inherent and apparent optical properties. Equation (7) retains the full richness of the equation of radiative transfer (ERT). The radiance distribution is present in $f_b(\theta, \phi, z)$, $k(\theta, \phi, z)$, and $f_L(\theta, \phi, z)$. This is therefore not a true solution but is a restatement of the ERT. It provides a starting point for the approximations, however. The path function of the radiance is contained in $f_b(\theta, \phi, z)$ and $f_L(\theta, \phi, z)$, as can be seen from (3) and (4). The essence of solutions to the ERT is the calculation of the path function. Owing to the unique geometry of this problem, f_b and f_L vary in a relatively narrow range, making the approximations possible. It would be possible to do modeling studies of the dependence of $f_b(\theta, \phi, z)$, $k(\theta, \phi, z)$, and $f_L(\theta, \phi, z)$ on the input radiance and the IOP. It is preferred, however, to develop models that retain as much as possible of the IOP in functional form. The approximations used are based on that concept.

Since $f_b(\pi, z)$ is the integral of the product of the downwelling radiance and the backward part of the scattering function $\beta_b(\theta)$, it is seen immediately that $f_b(\pi, z)$ is strongly dependent on $\beta_b(\theta)$. The shape of $\beta_b(\theta)$ is highly variable, depending on the size, shape, and index of refraction distribution of the particles. Evidence that the shape depends on internal structure as well has recently been demonstrated [*Zaneveld and Kitchen*, this issue]. It is thus desirable to develop models that contain the shape of $\beta_b(\theta)$ explicitly and that can be inverted to give that shape. Equation (13) proposes that the upwelled radiance used in remote sensing (i.e., upwelling light that is within the critical angle of the vertical) can be modeled as being due to single scattering from light at the maximum of the radiance distribution. *Morel and Gentili* [1993] showed that the number of scattering events undergone by upwelling photons is approximately equal to c/a. The above assumption is thus a priori correct if this c/a ratio remains sufficiently low. For higher c/a ratios it should be noted that while one of the scattering events has to be a large angle one, the other scattering events are likely to be small angle ones due to the sharp peak in the volume scattering function in the near-forward direction and hence the high probability of forward scattering events compared to backscattering ones. These small angle scattering events will only slowly diffuse the $\beta_b(\theta)$ shape of the upwelling radiance. This was pointed out already by *Morel and Gentili* [1993]. The radiance distribution within the critical angle will stay closer in shape to $\beta_b(\theta)$ than those outside, as surface reflections do not affect it. As c/a increases, we no longer expect the peak of the radiance distribution to remain at the refracted image of the sun. By using the maximum radiance rather than the refracted solar zenith angle, the assumption of (13) retains a small error, even for media with high c/a ratios.

The advantage of the approximation embodied in (13) is that the RSR becomes directly proportional to the shape of the volume scattering function in functional form. The RSR can then be modeled readily with any shape of $\beta_b(\theta)$. This overcomes a limitation of the Monte Carlo models in which shapes are not readily changed due to computational time. Also, when we wish to use various observed $\beta_b(\theta)$ shapes, results can more readily be obtained using the theoretical approach.

Gordon [1989] showed that the shape of $\beta_b(\theta)$ can be obtained by inversion from the irradiance ratio as a function of the solar zenith angle. For the formulation obtained here a similar result is obtained immediately. In (14), only the numerator is dependent on the input light field. Taking the ratio of two RSRs for different θ_m then yields

$$\frac{\mathrm{RSR}(\theta_{m1}, \pi)}{\mathrm{RSR}(\theta_{m2}, \pi)} = \frac{\beta(\pi - \theta_{m1})}{\beta(\pi - \theta_{m2})}.$$

The shape of $\beta_b(\theta)$ within the critical angle is thus readily obtained from the remotely sensed reflectance as a function of maximum radiance angle. The shape of $\beta_b(\theta)$, in turn, should provide some insight into the various particulate properties by means of inversion.

Error Analysis

The choice of which approximation to use depends on the application and the optical properties of the water. The most sensitive potential error comes from (13), as f_b is directly proportional to the RSR. As stated above, (13) was tested extensively [*Stavn and Zaneveld*, 1994; *Weidemann et al.*, this issue]. It was found that for many different scattering functions and lighting conditions the average error was around 5% and the maximum error was 12%.

The approximation suggested here for $k(\pi, z)$ (equation (15)) is difficult to assess, as we know of no Monte Carlo model studies in which this parameter has been studied directly. Results from a study in Lake Pend Oreille, in which an extensive suite of IOP was measured in conjunction with the vertical structure of the nadir radiance, suggest that the approximation shown here is at least as accurate as our ability to measure $k(\pi, z)$ in the field. Field measurements of $k(\pi, z)$ have all the problems usually associated with the measurement of other apparent optical properties near the surface. Naturally, the idea that $k(\pi, z)$ should always be asymptotic is only an approximation that allows us to model it in terms of the IOP. The somewhat limited data that exist support the notion, however. Undoubtedly, Monte Carlo calculations, when carried out, will show that under extreme conditions such as large solar zenith angles and black sky this will not be the case. These extreme Monte Carlo situations tend not to be realistic, however. We estimate the typical error in our formulation of $k_m(\pi, z)$ to be about 10%, based on the Lake Pend Oreille data. Future improved models for the dependence of $k(\pi, z)$ on the IOP and solar zenith angle can be used when they become available.

It is of some interest to look at the relative magnitude of terms in the denominator of (20) since use of subsequent approximations should be based on these magnitudes. Since $a/k \leq 1$, $[1 + m(z)] \geq 2$. For oceanic particle ensembles, typically, $b_p \approx 2a_p$ [*Jerlov*, 1974], so that for those particles, $a_p(1 + m) \geq b_p$. The term $b_b(z) + [1 - f_L(z)]\, b_f(z)$ for particles can be estimated as follows: $b_{bp}(z) \approx 0.02b_p$ (NUC station 2040 [*Petzold*, 1972]); $-0.1 \leq [1 - f_L(z)] \leq 0$ [*Zaneveld*, 1982], so that for typical oceanic particles, $-0.08b_p \leq b_b(z) + [1 - f_L(z)]b_f(z) \leq 0.02\, b_p$. If we now use (21) by setting $f_L = 0.05$, we see that the error in the last two terms of the denominator is $\pm 0.05b_p$. If we use (23), ignoring the last two terms of the denominator in (20), the maximum error is $-0.08\, b_p$. Together with $b_p \approx 2a_p$, we see that in (21) the maximum error in the denominator is 4% and in the case of (23), 8%. In the case of pure water, b_{bw} can be as high as $0.12a_w$ in the blue region of the spectrum. For pure water $[1 - f_L(z)]b_{fw}(z)$ can be ignored relative to b_{bw}, as $b_{fw} = b_{bw}$. For pure water near 450 nm an error of, at most, 12% is made in the denominator if the last two terms in the denominator in (20) are ignored. This error does not exist, however, if (21) is used. For larger wavelengths this error becomes rapidly smaller.

Using some of the above arguments, it can be seen when one can or cannot ignore certain terms. For high ratios of b/a as well as for high ratios of b_b/a it is necessary to use (21), but for almost all oceanic cases the simpler (23) can be used with an estimated overall average error of less than 10%. This exceeds the accuracy with which the reflectance can be measured experimentally at present, so that there is little justification in using more complex formulas.

The Q Question

The ratio of the upwelling irradiance to the upwelling nadir radiance just below the sea surface is defined as Q, thus

$$Q \equiv \frac{E_u}{L_u}. \tag{24}$$

Neither Q nor E_u appears in the derivations presented above. The derivation from the equation of radiative transfer lent itself to the use of the scalar, not the plane, irradiance. What is commonly studied, however, is the ratio of the nadir radiance to the downwelling plane irradiance (see, for example, *Gordon et al.* [1988] and *Morel and Gentili* [1993]). If we use the definition of the downwelling average cosine,

$$\frac{E_d}{E_{od}} \equiv \bar{\mu}_d, \tag{25}$$

the formulations can be readily converted into ratios using the downwelling plane irradiance. For example, (23) then becomes, ignoring depth dependence,

$$R_{rs} \equiv \frac{L_u}{E_d} = \frac{1}{\bar{\mu}_d}\frac{L_u}{E_{od}} \approx \frac{1}{\bar{\mu}_d}\frac{\beta(\pi - \theta_m)}{a(1 + m)}. \tag{26}$$

We have now introduced the downwelling irradiance, but at the loss of simplicity. We must now know the average cosine of the light field just beneath the surface. It is thus recommended that experimental work related to remote sensing use the scalar rather than the plane irradiance ratio. It stands to reason that the scalar irradiance is far less sensitive to solar zenith angle, tilt of the instrument, etc. It is thus a more benign measurement to deal with from a theoretical point of view. There are no real instrumental barriers to using scalar irradiance instead of plane irradiance [*Maffione*, 1994].

Much of the early work was carried out using the irradiance ratio E_u/E_d. We can convert all the equations for L_u/E_{od} to those for E_u/E_d by multiplying by $Q/\bar{\mu}_d$. Equation (23) then becomes

$$R \equiv \frac{E_u}{E_d} = \frac{Q}{\bar{\mu}_d}\frac{L_u}{E_{od}} \approx \frac{Q}{\bar{\mu}_d}\frac{\beta(\pi - \theta_m)}{a(1 + m)}. \tag{27}$$

Once again, we have increased the complexity of the ratio. In addition to the average cosine of the light field, we must now also know Q, an indicator of the shape of the upwelling light field. It is interesting to note that, historically, the progression has been in the opposite direction. First, the irradiance ratio was used, primarily for experimental reasons. Then, the radiance-irradiance ratio was used as satellite remote sensing matured. It is hoped that the next experimental phase will see the use of radiance-scalar irradiance ratios.

The most commonly used formulation for the irradiance reflectance is [*Gordon et al.*, 1975; *Morel and Prieur*, 1977]

$$R \equiv \frac{E_u}{E_d} \approx 0.33\, \frac{b_b}{a}\,(1 + \Delta) = f\,\frac{b_b}{a}, \tag{28}$$

where f and Δ are parameters that depend on the shape of the light field and the volume scattering function (f should not be confused with the shape parameters used in this paper). It has been found that the irradiance reflectance is inversely proportional to $\bar{\mu}_d$ [*Kirk*, 1984; *Jerome et al.*, 1988; *Gordon*, 1989; *Morel and Gentili*, 1990]. This is entirely in agreement with (27). *Gordon* [1989] also found that the slope of the dependence on $\bar{\mu}_d$ depended on the shape of the scattering function. This is also clearly seen in (27). It would predict the steepest slopes for scattering functions with steep slopes in the region of 180 to $180 - \theta_{c^\circ}$ (where θ_c is the critical angle), although the Q factor possibly intervenes.

Q appears in the irradiance ratio E_u/E_d in our derivation, whereas it appears in the reflectance L_u/E_d in the formulation of *Morel and Gentili* [1993]. Morel and Gentili noted that the ratio of f/Q is quite well behaved and that much of the fluctuations in f are canceled by Q. This implies that f is quite "Q-like" in its functional dependence. We can demonstrate this by deriving f from (27) and (28).

$$f \approx \frac{Q}{\bar{\mu}_d}\frac{\beta(\pi - \theta_m)}{b_b(1 + m)} = \frac{Q}{\bar{\mu}_d}\frac{f_b}{2\pi(1 + m)}. \tag{29}$$

Equation (29) is the approximate expression based on (23). It is also possible to write an exact expression based on the equation of radiative transfer:

$$f \equiv \frac{aE_u}{b_b E_d} = \frac{a}{b_b}\frac{Q}{\bar{\mu}_d}\frac{L(\pi)}{E_{od}} = \frac{Q}{\bar{\mu}_d}\frac{f_b \dfrac{a}{2\pi}}{k(\pi) + c - f_L b_f}, \tag{30}$$

where all the parameters are defined in the theory section. The Q-like behavior of f is shown by (29) and (30). The dependence on the shape of the VSF and the radiance distribution is also present via the shape factors f_b and f_L. The upwelling radiance distribution is present in f_L. Here f_L has a relatively weak influence on f, however, whereas f is directly proportional to Q.

Morel and Gentili [1993] have carried out Monte Carlo calculations of the dependence of f and f/Q on the direction of the upwelling radiance. Using (3) through (7), it is possible to write an exact expression for this bidirectional dependence:

$$f(\theta, \phi, z) = \frac{a}{b_b}\frac{Q(\theta, \phi, z)}{\bar{\mu}_d}\frac{L_u(\theta, \phi, z)}{E_{od}}$$

$$= \frac{Q(\theta, \phi, z)}{\bar{\mu}_d}\frac{f_b(\theta, \phi, z)\dfrac{a}{2\pi}}{\cos\theta k(\theta, \phi, z) + c - f_L(\theta, \phi, z) b_f}, \tag{31}$$

where $Q(\theta, \phi, z) \equiv E_u/L_u(\theta, \phi, z)$. It is again clear that $f(\theta, \phi, z)$ is directly proportional to $Q(\theta, \phi, z)$ and the remaining dependence on the shape of the upwelling light field is in $f_L(\theta, \phi, z)$, where it does not exert a strong influence due to its narrow range of possible values. Q depends very strongly on the radiance distribution in the region of 90 to 180 $- \theta_{c°}$ zenith angle. This range of angles has little to do with the upwelling radiance in a direction that can be sensed by a satellite. Only if light is scattered at a relatively large angle can light in the angular range of 90 to 180 $- \theta_{c°}$ affect light at an angle detectable by satellite. Unless there is significant multiple scattering, E_u and $L_u(\theta, \phi, z)$ at remote sensing angles are relatively decoupled. Thus the dependence of $L_u(\theta, \phi, z)/E_d$ on Q is weak at remote sensing angles, whereas the dependence of E_u/E_d on Q is strong.

Equation (29) shows that f is a function of f_b, Q, $\bar{\mu}_d$, and b/c (via m). Using (29), it is possible to evaluate the dependence of f on these parameters. The usual value cited for f is 0.33, with a total range of 0.25 to 0.55 [*Morel and Gentili*, 1993]. We will only show here that reasonable choices of parameters lead to results in that range. If, in (29), we choose $f_b = 1$, $Q = 5$, $\bar{\mu}_d = 0.9$, and $1/m = 0.6$, we find that $f = 0.33$. The above values correspond well to a zenith Sun. For an overcast sky we might choose $\bar{\mu}_d = 0.8$, giving $f = 0.37$. The $1/m = 0.6$ implies that $b/c = 0.86$, as shown in (17). For $b/c = 0.69$, $1/m = 0.7$, so that $f = 0.36$.

In conclusion, it has been demonstrated that an expression for the remotely sensed reflectance, L_u/E_{od} can be derived directly from the equation of radiative transfer. This expression lends itself well to various approximations that lead to models of the remotely sensed reflectance that depend on the IOP only.

Notation

- a absorption coefficient, m^{-1}.
- b volume scattering coefficient, m^{-1}.
- b_b backscattering coefficient, m^{-1}.
- b_f forward scattering coefficient, m^{-1}.
- c beam attenuation coefficient, m^{-1}.
- E_d downwelling irradiance, W m^{-2}.
- E_{od} downwelling scalar irradiance, W m^{-2}.
- f parameter relating reflectance to the ratio of backscattering and absorption, nondimensional.
- f_b shape parameter, nondimensional.
- f_L shape parameter, nondimensional.
- k vertical attenuation coefficient for radiance, m^{-1}.
- k_m modeled vertical attenuation coefficient for radiance, m^{-1}.
- K_∞ asymptotic diffuse attenuation coefficient, m^{-1}.
- L radiance, W m^{-2} sr^{-1}.
- L_d downwelling radiance, W m^{-2} sr^{-1}.
- L_u upwelling radiance, W m^{-2} sr^{-1}.
- L^* path function, W m^{-3} sr^{-1}.
- m the ratio of k_m and a, nondimensional.
- Q the ratio of the upwelling irradiance and the nadir radiance, sr.
- R irradiance reflectance, nondimensional.
- R_{rs} remote sensing reflectance; the ratio of the nadir radiance and the downwelling irradiance, sr^{-1}.
- RSR remotely sensed reflectance; the ratio of the nadir radiance and the downwelling scalar irradiance, sr^{-1}.
- z depth, m.
- β volume scattering function, m^{-1} sr^{-1}.
- β_b backscattering part of the volume-scattering function, m^{-1} sr^{-1}.
- Δ parameter used in relating R to b_b/a, nondimensional.
- ϕ azimuth angle, radians.
- $\bar{\mu}$ average cosine of the light field, nondimensional.
- $\bar{\mu}_d$ average cosine of the downwelling light field, nondimensional.
- θ zenith angle, radians.
- θ_c critical angle, radians.
- θ_m zenith angle of the maximum radiance, radians.

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