Mathematics of Alignment Chart Construction Without the Use of Determinants

JAMES R. GRIFFITH

Bulletin Series, No. 12
November 1939

Engineering Experiment Station
Oregon State System of Higher Education
Oregon State College

THE Oregon State Engineering Experiment Station was established by act of the Board of Regents of the College on May 4, 1927. It is the purpose of the Station to serve the state in a manner broadly outlined by the following policy:

(1) To stimulate and elevate engineering education by developing the research spirit in faculty and students.

(2) To serve the industries, utilities, professional engineers, public departments, and engineering teachers by making investigations of interest to them.

(3) To publish and distribute by bulletins, circulars, and technical articles in periodicals the results of such studies, surveys, tests, investigations, and researches as will be of greatest benefit to the people of Oregon, and particularly to the state's industries, utilities, and professional engineers.

To make available the results of the investigations conducted by the Station three types of publications are issued. These are:

(1) *Bulletins* covering original investigations.

(2) *Circulars* giving compilations of useful data.

(3) *Reprints* giving more general distribution to scientific papers or reports previously published elsewhere, as for example, in the proceedings of professional societies.

Single copies of publications are sent free on request to residents of Oregon, to libraries, and to other experiment stations exchanging publications. As long as available, additional copies, or copies to others are sent at prices covering cost of printing. The price of this bulletin is 25 cents.

For copies of publications or for other information address

Oregon State Engineering Experiment Station,

Corvallis, Oregon

Mathematics of Alignment Chart Construction Without the Use of Determinants

By
James R. Griffith
Professor of Structural Engineering

Bulletin Series, No. 12
November 1939

Engineering Experiment Station
Oregon State System of Higher Education
Oregon State College

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

DEFINITIONS AND EQUATIONS

Mathematics of
Alignment Chart Construction
Without the Use of Determinants

By

JAMES R. GRIFFITH
Professor of Structural Engineering

I. FOREWORD

1. **Introduction.** The term "Nomograph" comes from the Greek *nomos* meaning law; and *graphein* meaning to write or to draw. Thus a nomograph might be said to be a chart drawn according to certain laws. Professor F. T. Mavis, of the University of Iowa, defines a nomograph as "not merely a graphical representation of a function, but it is rather a grapho-mechanical computing device."

The nomograph consists of a number of scales and is solved by a straight line known as an "isopleth." This term, first used by Vogler in 1880, is from the Greek *isos* meaning equal, or the same; and *pleth* meaning quantity. Thus an isopleth can be said to give values of equal quantity, or weight, in the equation plotted.

Those nomographs having parallel straight-line scales are conventionally termed "Alignment Charts." They represent but one special type of nomograph, and as such are likewise solved by the use of an isopleth.

The characteristics of any alignment chart consist of the relative spacing of the scales and the relative magnitude of the co-ordinates used thereon. The mathematics of alignment chart construction involves the mathematical determination of the necessary characteristics to permit construction.

2. **Objectives.** The conventional approach to the mathematics of nomograph construction is by means of determinants. Without fear of dissent, it can be said that relatively few practicing engineers are able to handle determinants with any facility, so the reader of the conventional text on the subject is in most cases discouraged at the start.

The method presented herein for the determination of alignment chart characteristics is by basic equations and the solution of two equations simultaneously. The method is original and has been considered by many to require less effort for mastery than that utilizing determinants. It is objectionable in that it is necessarily confined to those equations that are adjustable to the type of expression resulting in a parallel-scale nomograph.

Maximum facility in nomographic construction can be gained by familiarity with different methods. No one method is always the easiest, and some, like that herein presented, are limited in application. Each method is but a tool. The reader is referred to the bibliography for the graphical method, and that requiring determinants.

Some will argue as to the merits of various type charts. The increasing popularity of the nomograph, however, would lead one to assume that it has its full share of merits. In the many charts published by the author, the problem has always been one of production, seldom that of finding an interested publisher.

In preparing this bulletin, the attempt has been made to afford an additional simple tool to those who have problems they wish to solve by alignment charts. While many readers may never have occasion to construct an alignment chart, a knowledge of the mechanics of construction should make them more intelligent users of such charts. To many, the construction of an alignment chart has a fascination comparable to that which the assembling of a jig-saw puzzle or the completion of a cross-word puzzle has to addicts of that type. Many organizations utilize slack periods for the construction of time-saving devices, such as charts, for use in rush periods.

3. **Acknowledments.** The enthusiasm and inspiration of Otto A. Steller, former editor of *Concrete*, was largely responsible for the author's continued application and for such proficiency as he has developed. Likewise, J. I. Ballard, editor of *Western Construction News*, has contributed in the publication of a monthly chart by the author during the past four years. It has been largely due to the instigation and counsel of Professor S. H. Graf, Director of Engineering Research, that the author has prepared the present publication.

II. BASIC THEORY AND GENERAL EQUATIONS

1. **Characteristics Defined.** Each alignment chart is based primarily on a three-variable equation although the issue may be confused as will be demonstrated later. In Figure 1, the relative positions of scales for the three funda-

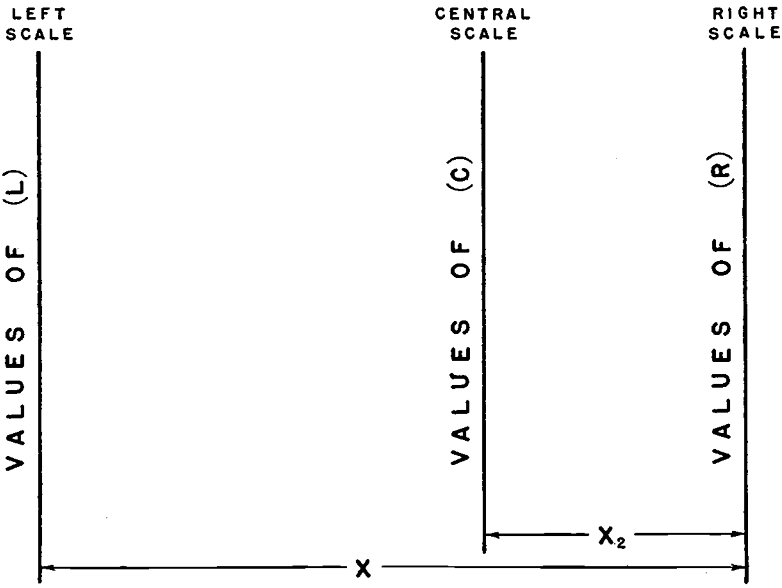

Figure 1. Basic Positions of Scales.

mental variables are indicated. By agreement, the left scale will contain values of (L), the central scale values of (C), and the right scale values of (R).

The relative spacing of the three lines containing the scales will be defined by the expression

$$n = \frac{X_2}{X} \qquad \text{Definition 1.}$$

The relative sizes of co-ordinates on the separate scales will be defined by the following expressions

$$r = \frac{\text{Size of } (R) \text{ co-ordinates}}{\text{Size of } (L) \text{ co-ordinates}} \qquad \text{Definition 2.}$$

$$m = \frac{\text{Size of } (R) \text{ co-ordinates}}{\text{Size of } (C) \text{ co-ordinates}} \qquad \text{Definition 3.}$$

If rectangular co-ordinates are to be used, "size" might be considered as the linear distance between successive co-ordinates. Thus, if on the right scale 1-inch = 10 units were used, and on the central scale values of 1-inch = 20 units were used, each unit on the (R) scale would be twice the size of those on the (C) scale. By Definition 3, then

$$m = \frac{\text{Size of } (R) \text{ co-ordinates}}{\text{Size of } (C) \text{ co-ordinates}} = 2$$

Likewise, if on the same chart, values were laid out to a scale of 1-inch = 30 units on the left scale, each unit on the (R) scale would be three times the size of those on the (L) scale, and by Definition 2, then

$$r = \frac{\text{Size of } (R) \text{ co-ordinates}}{\text{Size of } (L) \text{ co-ordinates}} = 3$$

In comparing the size of logarithmic co-ordinates, relative length must involve a cycle, or the linear measurement of the same number of units. Referring to the ordinary slide rule, if the (D) scale is used for laying out values on the right scale, and the (A) scale used for values on the central scale, by Definition 3, then

$$m = \frac{\text{Size of } (R) \text{ co-ordinates}}{\text{Size of } (C) \text{ co-ordinates}} = 2$$

In other words, one cycle of the (D) scale of a slide rule is twice as long as a cycle of the (A) scale, for it is thus that squares and square roots are possible of solution. Likewise, the units of the (D) scale are three times the size of those on the (K) or cube scale.

The three definitions given will be spoken of as the "Characteristics." If the numerical values of these three characteristics are known, the alignment chart may then be drawn. The mathematics of alignment chart construction involve the determination of these three essential characteristics.

2. **Spacing of Scales.** Temporarily now let us ignore the fact that units will eventually be used, and consider only linear values on the three parallel lines. In Figure 2, three intersecting lines, or isopleths, hi, ij, and jk,

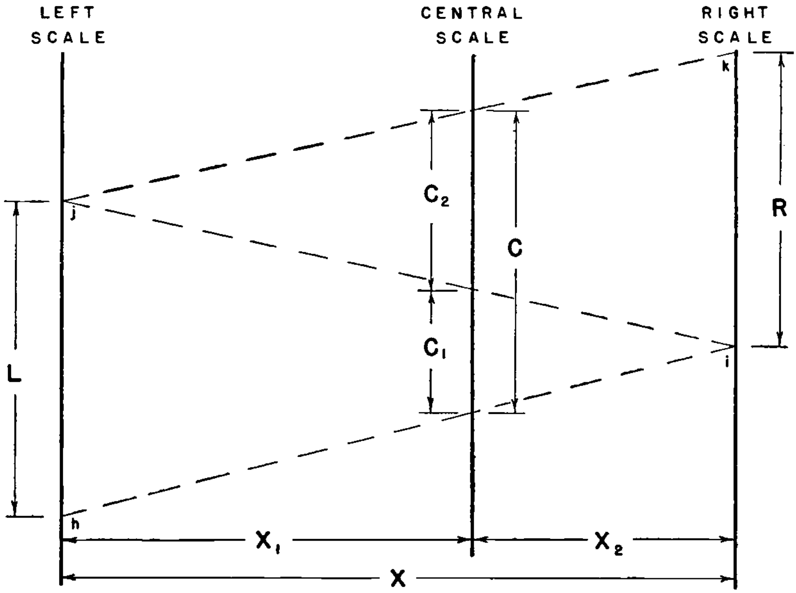

Figure 2. Geometric Relations.

have been drawn on the basic parallel lines of Figure 1, so as to form two sets of similar triangles. The linear intercepts, L, C, and R, are as indicated. By similar triangles then, the following proportions exist:

$$\frac{C_1}{L} = \frac{X_2}{X} \qquad \text{or} \qquad C_1 = \frac{X_2}{X} L$$

and

$$\frac{C_2}{R} = \frac{X_1}{X} \qquad \text{or} \qquad C_2 = \frac{X_1}{X} R$$

However,

$$C_1 + C_2 = C$$

so

$$C = \frac{X_2}{X} L + \frac{X_1}{X} R$$

or

$$C = \frac{X_2}{X} L + \frac{(X - X_2)}{X} R$$

By substitution of Definition 1, $n = \dfrac{X_2}{X}$

the above expression simplifies to the form

$$C = n\,L + (1 - n)\,R \qquad \text{Equation (a)}$$

Equation (a) can then be said to give the relation between linear intercepts on the three scales in terms of the spacing of scales.

3. **Rectangular co-ordinates.** If rectangular co-ordinates are used, with a different size on each line, some adjustment must be made in order to reduce the values intercepted to terms of linear intercepts. Or, we might say it is necessary to reduce all intercepts to terms of the same scale.

Reducing all intercepts to terms of the units on the (R) scale, by use of Definitions 2 and 3, Equation (a) then becomes

$$\frac{C}{m} = \frac{n}{r}\,L + (1 - n)\,R$$

or

$$C = \frac{mn}{r}\,L + m\,(1 - n)\,R \qquad \text{Equation 1}$$

Equation 1 is the general equation for alignment charts involving rectangular co-ordinates. While its significance and application may not now be apparent, it will be used later to determine the necessary characteristics for the construction of an alignment chart.

4. **Logarithmic co-ordinates.** Logarithmic co-ordinates are but logarithms of numbers plotted to rectangular co-ordinates and labeled the corresponding anti-logarithm. Addition of logarithms results in the multiplication of the numbers. Likewise, multiplication of logarithms results in raising the number to the power of the multiplier. So when logarithmic co-ordinates are used, the sum of the variables in Equation 1 becomes a product, and the coefficients become exponential values, as in the following expression:

$$C = L^{(mn/r)} \cdot R^{\,m(1-n)} \qquad \text{Equation 2.}$$

Equation 2 is the general expression involving logarithmic co-ordinates, and will be used to determine the characteristics of later charts.

5. **Recapitulation.** In order to separate the definitions and general equations for more ready reference, the following summarization is given:

$$n = \frac{X_2}{X} \qquad \text{Definition 1.}$$

$$r = \frac{\text{Size of } (R) \text{ co-ordinates}}{\text{Size of } (L) \text{ co-ordinates}} \qquad \text{Definition 2.}$$

$$m = \frac{\text{Size of } (R) \text{ co-ordinates}}{\text{Size of } (C) \text{ co-ordinates}} \qquad \text{Definition 3.}$$

General expression for rectangular co-ordinates

$$C = \frac{mn}{r} L + m(1 - n) R \qquad \text{Equation 1.}$$

General expression for logarithmic co-ordinates

$$C = L^{(mn/r)} \cdot R^{m(1-n)} \qquad \text{Equation 2.}$$

III. CO-ORDINATES AND DEVICES FOR OBTAINING

1. **Rectangular co-ordinates.** An engineer's scale, a metric scale, and geometric projection have always supplied sufficient variations to meet the author's needs for rectangular co-ordinates. Standard co-ordinate paper can be used to an advantage by drawing diagonal lines thereon to obtain the desired size of co-ordinates. Others prepare a reduction diagram for rectangular co-ordinates similar to that which will be illustrated for logarithmic co-ordinates.

2. **Logarithmic co-ordinates.** The engineer's first reaction is to use his slide rule to obtain logarithmic co-ordinates. Thereon will usually be found three sizes. If he has slide rules of different lengths available, he may have a sufficient range to satisfy his ordinary demands. The use of a slide rule for logarithmic co-ordinates is objectionable for one major reason; namely, that of transferring the co-ordinates to paper. Maximum facility and accuracy are obtained by preparing the chart on tracing cloth, or vellum, tracing the co-ordinates from some master sheet that can be moved under the drawing.

A photostat of a slide rule does not have sufficient accuracy due to the distortion of the lens and the uneven shrinkage of the paper. The author has had made up a high-grade photographic negative of a slide rule, which does give an accuracy within the necessary limits.

The author also has a triangular scale containing six different logarithmic scales from the K-scale of a 5-inch slide rule, to the D-scale of a 20-inch slide rule. This was one of a special order, however, and a recent check with

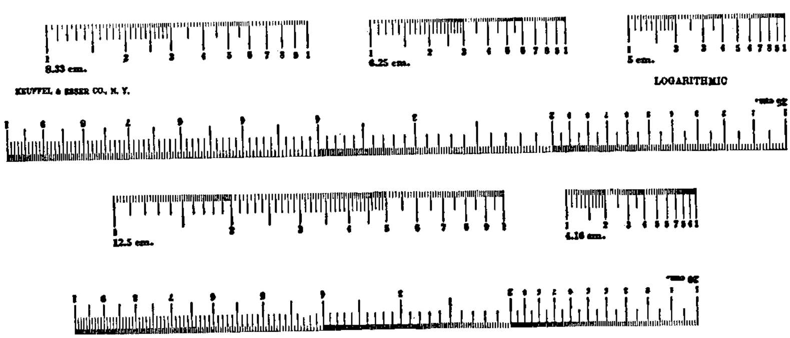

Figure 3. Statistician's Scale.
(Reproduced by permission of Keuffel & Esser Co.)

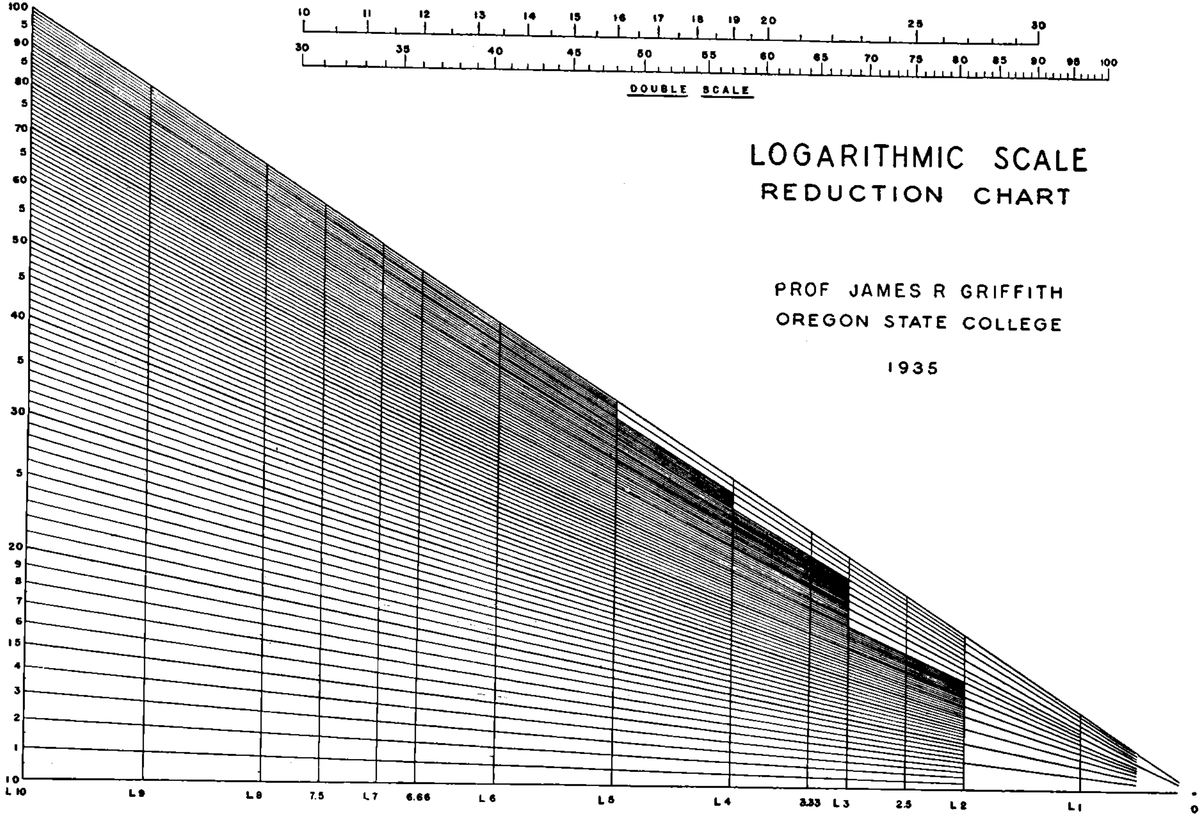

Figure 4. Logarithmic Reduction Diagram.

the manufacturer resulted in a report that no more of them were available. Its one objectionable feature, like the slide rule, is the difficulty of transferring co-ordinates to the drawing paper.

There are available several types of so-called "Statistician's Scales," with a cycle based on the metric system. One such, illustrated in Figure 3, is printed on heavy paper and sells for 30 cents.

3. **Reduction Diagram.** Maximum facility can be obtained by the use of a reduction diagram similar to that shown in Figure 4. Many chart enthusiasts have prepared their own reduction diagrams. It is not practicable to include a full-scale reduction diagram in this bulletin; however, original reproductions of Figure 4 printed on 8½ x 11-inch paper, may be obtained direct from the author at 25 cents a copy or $1.00 per dozen copies. Figure 4 has been reduced to the same scale as example charts later shown, for convenient comparison by the reader.

In using the reduction diagram, any relative size of co-ordinates may be obtained by drawing a line at the proper relative position. Thus at the line marked "6.66," the co-ordinates are $\frac{6.66}{10}$ the size of the full standard ones on the *L*10 line. In the examples to be solved later, logarithmic co-ordinates used will be referred to this diagram.

IV. EXAMPLES OF APPLICATION

1. **Two variables—comparison scale.** While seldom used by itself, the comparison scale is a device which is frequently useful in conjunction with an alignment chart. Its possibilities should therefore be recognized for inclusion when desirable.

Briefly, the comparison scale is but a line on which graduations of two variables appear. Equations of the first degree (linear) may be plotted to either rectangular or to logarithmic co-ordinates. For example, let us consider the relation between feet and inches.

$$\frac{\text{Inches}}{12} = \text{Feet}$$

In Figure 5 is shown a comparison scale of this expression, plotted to rectangular co-ordinates. It will be noted that the co-ordinates for inches are one-twelfth the size of those used for feet. If a comparison scale of this same expression is plotted to logarithmic co-ordinates, as in Figure 6, a scale of the same length cycle is used for each variable. Thus the linear distance

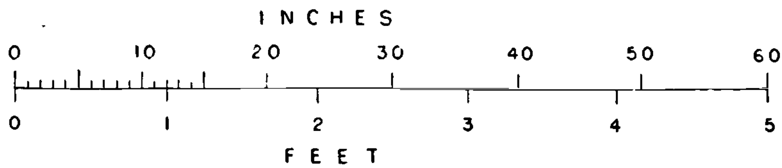

Figure 5. Comparison Scale, Inches to Feet, Rectangular Co-ordinates.

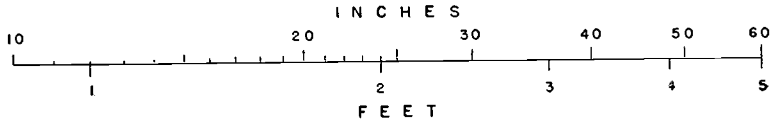

Figure 6. Comparison Scale, Inches to Feet, Logarithmic Co-ordinates.

between 10 and 20 on the inch scale is identical to that between 1 and 2 on the foot scale. When the exponent of both variables is unity, the same size co-ordinates are used for each.

If now an equation other than linear is taken, such as the expression showing the relation between the area and sides of a square, a marked difference will be noted. Figure 7 is a comparison scale of the equation $Area = (Side)^2$, in which rectangular co-ordinates are used to plot "*Area*." The co-

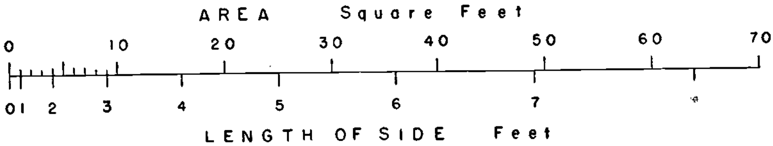

Figure 7. Comparison Scale, $Area = (Side)^2$, Area in Rectangular Co-ordinates.

ordinates for the "*Side*" are of no regular system, which necessitates computing each value before plotting.

In Figure 8, this second expression is plotted to logarithmic co-ordinates. It will be noted that two cycles of the "*Area*" scale are equivalent in length

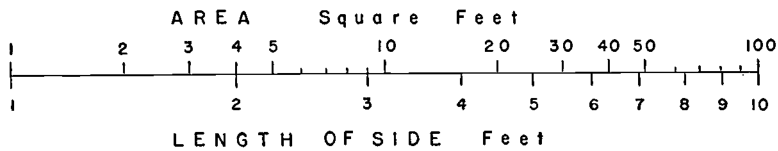

Figure 8. Comparison Scale, $Area = (Side)^2$, Logarithmic Co-ordinates.

to one cycle of the "*Side*" scale. If the relation were cubic, as in the case of the slide rule *K*-scale, it would require three cycles of one to a single cycle of the other, to show the proper relation.

2. **Three variables—rectangular co-ordinates.** The equation for the perimeter of a rectangle may be written

$$P = 2\,B + 2\,H; \qquad \text{Equation 3.}$$

where P = Perimeter,

B = Base,

and H = Altitude.

Equation 3 will be seen to be of the type represented by the general expression, Equation 1, involving the sum of two variables, requiring rectangular

$$C = \frac{mn}{r} L + m(1 - n) R \qquad \text{Equation 1.}$$

co-ordinates for nomographic construction. This general equation contains three variables (C), (L), and (R), in which one (C) is equated to the sum of the other two. The coefficient of (L) is $\left(\frac{mn}{r}\right)$, and that of (R) is $m(1 - n)$. The exponent of each variable is unity.

In Equation 3, there are likewise three variables (P), (B), and (H), in which the variable (P) is equated to the sum of the other two. Thus in a nomograph of this equation, values of (P) will appear on the central (C) scale; values of (B) will appear on the left (L) scale; and values of (H) on the right (R) scale.

In order to determine the nomographic characteristics (m), (n), and (r), it is necessary to equate the general coefficients of Equation 1 to the specific ones of the given expression, Equation 3. Thus

$$\frac{mn}{r} = 2$$

and $m(1 - n) = 2$.

Thus, two equations containing the three unknown characteristics are obtained. It then becomes necessary to assume the value of one unknown in order to solve for the other two.

If the two outside scales, containing values of (B) and (H), are assumed to have the same desired range of values, the same size co-ordinates may be used for each. Then, by Definition 2,

$$r = 1.$$

The two expressions containing the characteristics now reduce to

$$mn = 2$$

and $m(1 - n) = 2$;

which, when solved simultaneously, give the values

$$n = 1/2$$

and $m = 4$.

From Definitions 1 and 3, it is now seen that the central scale will be located midway between the outside scales, and will be calibrated with co-ordinates 1/4 the size of those used on the right (R) scale.

The nomograph in Figure 9 has been drawn according to the characteristics thus found. The values on the central scale have been located by means of a construction isopleth through the values $B = 2$ and $H = 2$. By substitution in Equation 3

$$P = 2\,B + 2\,H = 2\,(2) + 2\,(2) = 8$$

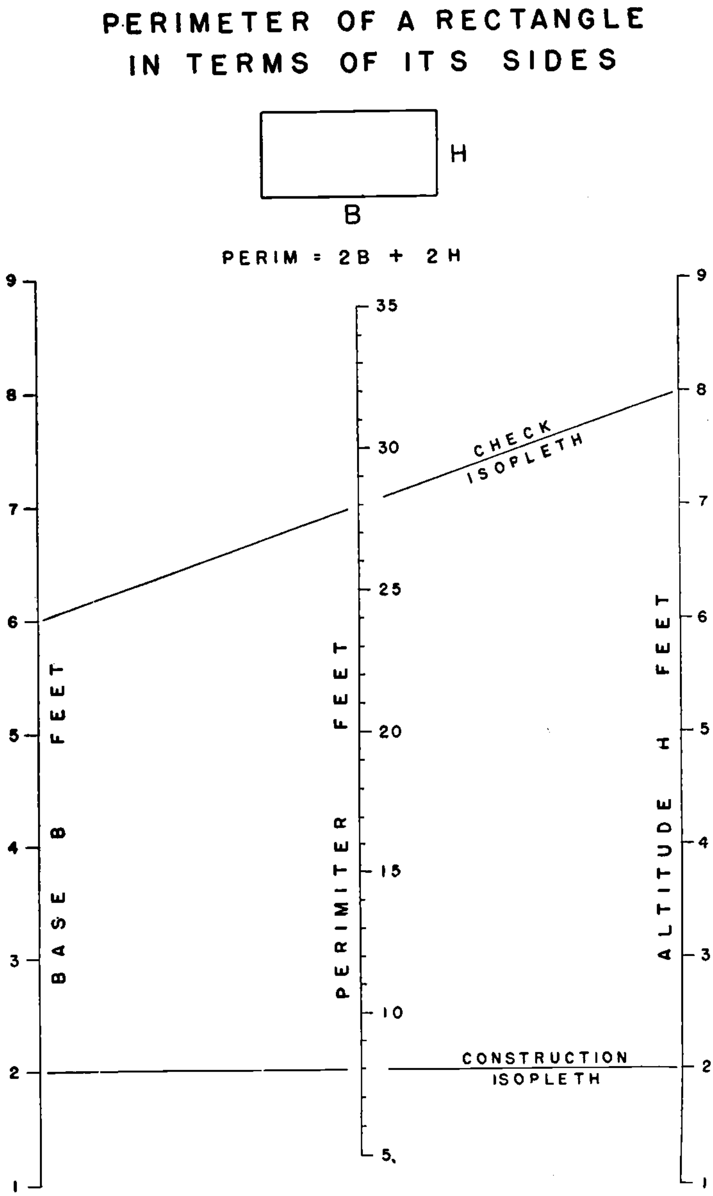

Figure 9. Alignment Chart, Perimeter of Rectangle in Terms of its Sides. Rectangular Co-ordinates.

which is the value on the central scale at the intersection of the construction isopleth.

In order to check the accuracy of the chart, a check isopleth has been drawn through the values $B = 6$ and $H = 8$. Then by substitution in Equation 3

$$P = 2\,B + 2\,H = 2\ (6) + 2\ (8) = 28.$$

This value will be seen to agree with that given by the check isopleth on the central scale.

3. **Three variables—logarithmic co-ordinates.** The general expression for determining the characteristics of a 3-variable alignment chart involving logarithmic co-ordinates, was given by the expression

$$C = L^{(mn/r)} \cdot R^{m(1-n)} \qquad \text{Equation 2.}$$

in which the characteristics appear as exponential values in a two-variable product. If an equation can be arranged in a form similar to this basic expression, the characteristics may easily be determined.

One equation which is of the type under consideration, is that for the maximum bending moment in simple beams under uniform loading.

$$M = \frac{w\,L^2}{8}, \qquad \text{Equation 4.}$$

where w = Unit load, uniformly distributed, lb per ft,
L = Span of simple beam, ft,
and M = Maximum bending moment, ft lb.

The effect of a coefficient in an equation involving a product of two variables, such as the value 1/8 in Equation 4, is to produce a vertical shifting of the scales. It has no effect on the nomographic characteristics and may be ignored in finding such.

Equation 4 is seen to be similar in form to Equation 2, as it stands. The central (C) scale will contain values of (M); the left (L) scale will contain values of (w); and the right scale will contain values of the span (L). If the general exponents of Equation 2 are equated to the specific ones of Equation 4, we have

$$\frac{mn}{r} = 1$$

and $m(1 - n) = 2.$

Since three unknowns exist in but two equations, one must be assumed in order to determine the other two. In most cases, the co-ordinates used on the outside scales will be controlled by the space available and the range of values desired. In defining logarithmic co-ordinates, reference will be made to Figure 4, on which the $L10$ scale is the unit scale to be found at the left side of the reduction diagram. The $L5$ scale is then one-half the length of the $L10$ scale.

The following assumptions will be made for the alignment chart of Equation 4:

Right scale: values of span (L), $L10$ co-ordinates.
Left scale: values of load (w), $L5$ co-ordinates.
Thus by Definition 2

$$r = \frac{L10}{L5} = 2.$$

Substituting this value of $r = 2$ in the first expression involving the characteristics to be determined, we have

$$mn = 2$$

and $$m(1 - n) = 2.$$

When these two expressions are solved simultaneously, the following values are obtained:

$$n = 1/2$$

and $$m = 4.$$

Thus by Definition 1, since $n = 1/2$, the central scale containing values of (M) will be midway between the outside scales. By Definition 3, since $m = 4$, the co-ordinates for the central scale containing values of (M) will be one-fourth the size of those used on the right scale. Since the $L10$ units were assumed for the right scale, $\frac{L10}{4} = L2.5$ units will be necessary on the central scale.

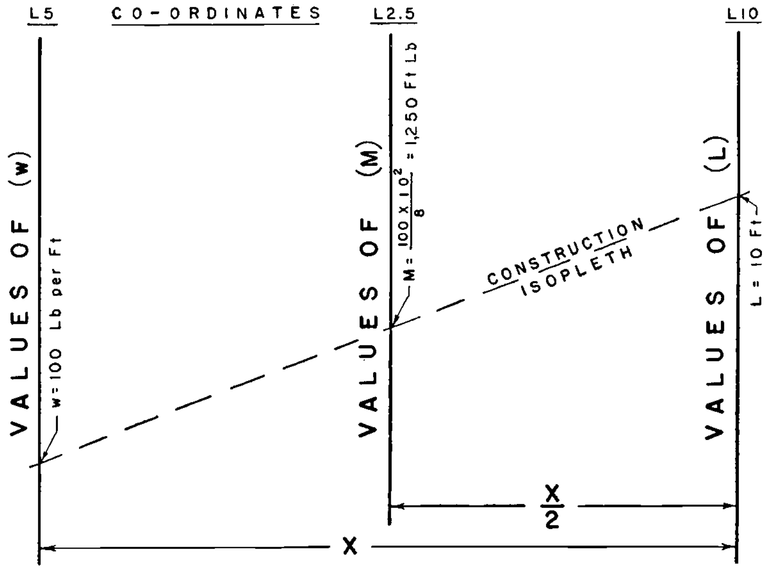

Figure 10. Data for chart of $M = \frac{w L^2}{8}$.

The characteristics thus determined have been summarized in Figure 10. In order to locate the central scale vertically, a construction isopleth was drawn, as shown, through the values

$$w = 100 \text{ lb per ft}$$

and $$L = 10 \text{ ft},$$

which when substituted in Equation 4 result in a value on the central scale of

$$M = \frac{w\,L^2}{8} = \frac{100 \times 10^2}{8} = 1{,}250 \text{ ft lb.}$$

Let us now consider the relation between moments in foot-pounds and in inch-kips, wherein

$$\frac{12\ M_{FL}}{1{,}000} = M_{IK}$$

From the examples discussed under comparison scales in Chapter IV it will be seen that a comparison scale of these two units will require the same co-ordinates, since both are of the first power. At the point on the central scale where the construction isopleth crosses, we would have a value of

$$M = \frac{12 \times 1{,}250}{1{,}000} = 15 \text{ inch-kips.}$$

Figure 11 has been thus constructed. In order to check for possible errors in computation or construction, a solution isopleth has been drawn on Figure 11 through the values

$$w = 4{,}000 \text{ lb per ft}$$

and $$L = 30 \text{ ft.}$$

Substituting these values in Equation 4, we obtain

$$M = \frac{w\,L^2}{8} = \frac{4{,}000 \times 30^2}{8} = 450{,}000 \text{ ft-lb}$$

and $$M = \frac{12 \times 450{,}000}{1{,}000} = 5{,}400 \text{ inch-kips.}$$

Both values will be seen to check reasonably well at the intersection of the solution isopleth and the central scale.

4. **Three variables—inverted scale.** While limits of space in a bulletin such as this prohibit the inclusion of all possible variations, an attempt has been made to cover the major ones. As one common variation, let us take the expression previously discussed,

$$M = \frac{w\,L^2}{8}. \qquad \text{Equation 4.}$$

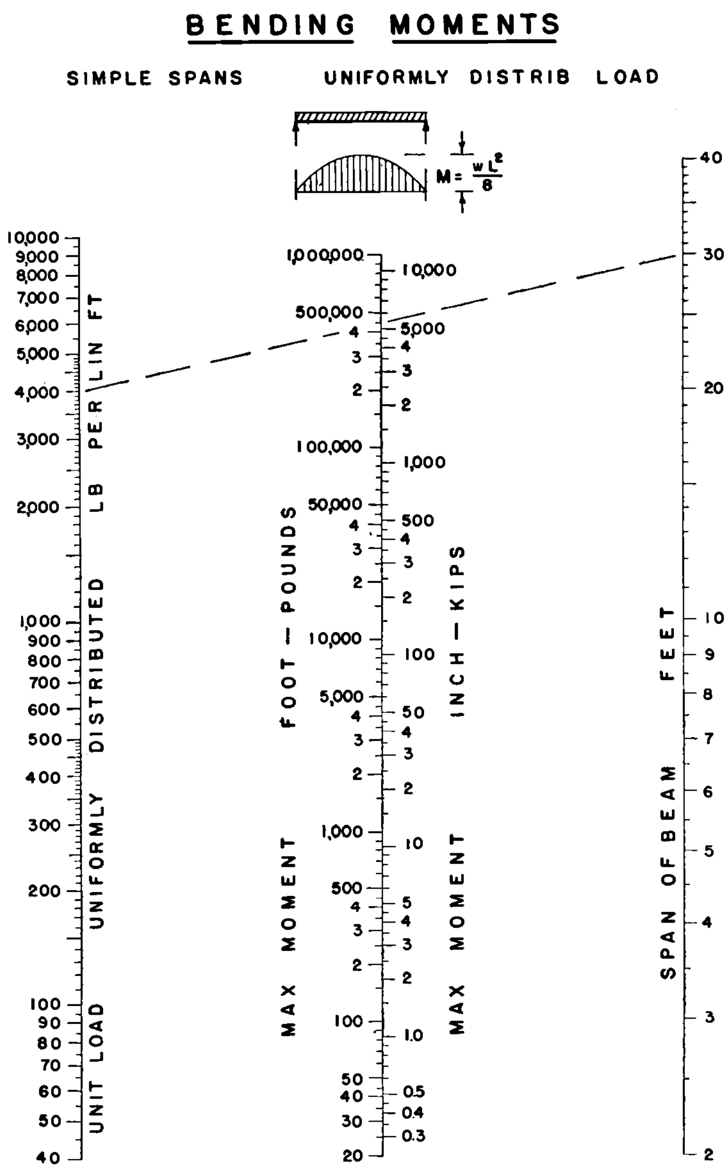

Figure 11. Alignment Chart of Equation $M = \frac{wL^2}{8}$.

Now let us assume that for some reason the chart designer wishes to rearrange this expression so that values of (w) will appear on the central scale, values of (L) on the left scale, and values of (M) on the right scale. The above equation may be written

$$w = \frac{8\,M}{L^2} = 8\left(\frac{1}{L^2}\right)M \qquad \text{Equation 5.}$$

which will accomplish the desired scale distribution. The value $\left(\frac{1}{L^2}\right)$ will result in a reciprocal or "inverted" scale. In equating the general to the specific exponents to determine the nomographic characteristics, the fact that the variable (L) is in the denominator may be ignored. Thus we have

$$\frac{mn}{r} = 2$$

and $m(1 - n) = 1.$

For the outside scales, the following assumptions will be made:

> Right scale, values of moment (M), $L3.33$ co-ordinates;
> Left scale, values of span (L), $L10$ co-ordinates.

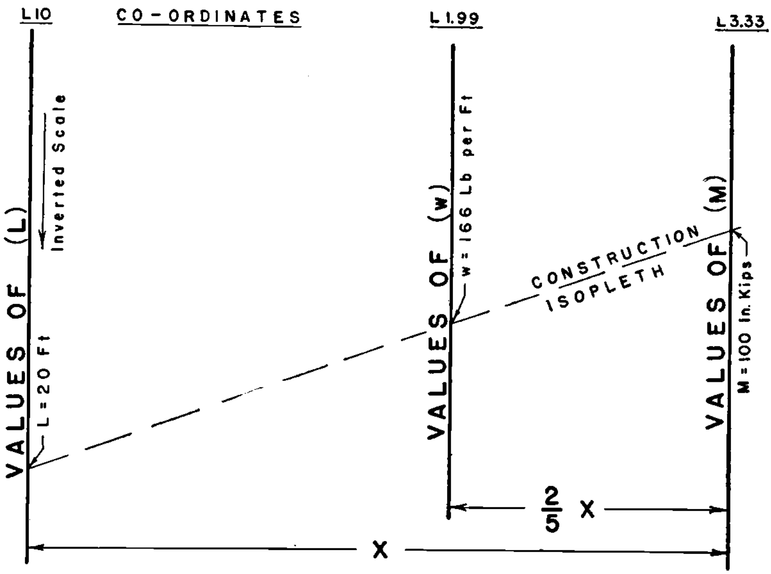

Figure 12. Data for Chart of Equation $w = \frac{8\,M}{L^2}$.

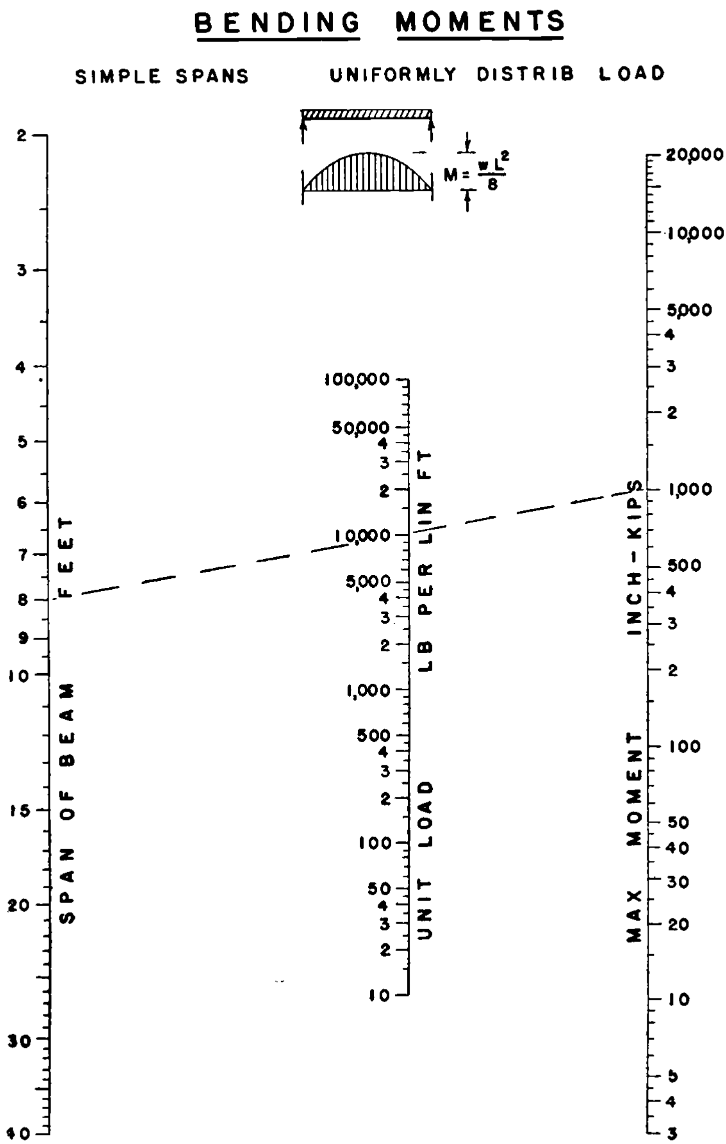

Figure 13. Alignment Chart of Equation $w = \frac{8M}{L^2}$, Inverted Scale.

By Definition 2, we then have the value

$$r = \frac{L\ 3.33}{L\ 10} = \frac{1}{3}.$$

Substituting this value of $r = 1/3$ in the expressions for the determination of the characteristics, we then have

$$mn = 2/3$$

and $\quad m(1 - n) = 1.$

Solving these two expressions simultaneously, the following values are obtained:

$$n = 2/5$$

and $\quad m = 5/3.$

From Definition 1, since $n = 2/5$, it is seen that the central scale will be two-fifths the distance from the right to the left scale. From Definition 3, since $m = 5/3$, the co-ordinates on the central scale will be

$$\tfrac{3}{5}L3.33 = L1.99 \text{ co-ordinates.}$$

The necessary data for construction of the final chart has been summarized in Figure 12. It was demonstrated in Figure 11 that units of the bending moment (M) in either foot-pounds or inch-kips, would require the same size co-ordinates. In the chart for Equation 5, only inch-kips will be included. A construction isopleth has been assumed through the values

$$L = 20 \text{ ft}$$

and $\quad M = 100 \text{ inch-kips} = 8{,}330 \text{ ft-lb}.$

By substituting these values in Equation 5, in order to determine the value on the central scale at the construction isopleth, we find that

$$w = \frac{8\ M}{L^2} = \frac{8 \times 8{,}330}{20 \times 20} = 166.6 \text{ lb per ft.}$$

Using the data shown on Figure 12, the final chart, Figure 13, has been drawn. A check isopleth has been drawn through the values

$$L = 8 \text{ ft}$$

and $\quad M = 1{,}000$ inch-kips gives the result 83,300 foot-pounds.

The value on the central scale will be seen to check the following value obtained by substitution in Equation 5:

$$w = \frac{8\ M}{L^2} = \frac{8 \times 83{,}300}{8 \times 8} = 10{,}400 \text{ lb per ft.}$$

5. **Parenthetic variables.** Some equations seemingly are not adjustable to the forms of either Equation 1 or Equation 2. One such equation from civil engineering literature is Duchemin's formula for computing the wind load component normal to a sloping roof surface.

$$P_n = P \left(\frac{2 \text{ Sin } A}{1 + \text{Sin}^2 A} \right),$$

where P_n = Pressure normal to sloping roof surface, lb per sq ft,
P = Wind pressure against vertical surface, lb per sq ft,
and A = Angle of inclination of surface with horizontal.

It will be noted that the entire quantity within the parentheses contains but one variable. If now we let

$$f = \frac{2 \text{ Sin } A}{1 + \text{Sin}^2 A},$$

Duchemin's formula reduces to the form

$$P_n = P f, \qquad \text{Equation 6.}$$

which is similar to the general expression, Equation 2, requiring logarithmic co-ordinates for nomographic construction. The central (C) scale will contain values of (P_n), the left (L) scale will contain values of (P), and values of (f) will appear on the right (R) scale. Equating the general exponents of Equation 2 to the specific ones of Equation 6, we have

$$\frac{mn}{r} = 1$$

and $m(1 - n) = 1.$

If the $L10$ co-ordinates are used for the (R) scale, and the $L5$ co-ordinates for the (L) scale, by Definition 2

$$r = \frac{L10}{L5} = 2.$$

Then by substitution

$$mn = 2$$

and $m(1 - n) = 1,$

which, when solved simultaneously, results in the values

$$n = 2/3$$

and $m = 3.$

From Definition 1, since $n = 2/3$, it is seen that the (C) scale containing values of (P_n) will be two-thirds the distance from the (R) scale to the (L) scale. By Definition 3, since $m = 3$, the co-ordinates for the central scale will be one-third the size of those used on the (R) scale, or the

$$\frac{L10}{3} = L3.33$$

co-ordinates will be used for values of (P_n).

Thus far the co-ordinates determined for the (R) scale are for values of (f) where

$$f = \frac{2 \text{ Sin } A}{1 + \text{Sin}^2 A}.$$

If values of (A) are substituted, however, their corresponding values of (f) can be laid off on the (R) scale, but labeled as values of (A). Thus a direct solution is possible. This method is practical only when there are relatively few values of the parenthetic variable. Tabular values are given below for the major angles and more common values of roof pitch.

Pitch $\frac{\text{Rise}}{\text{Span}}$	(A) Degrees	Sin A	$f = \frac{2 \text{ Sin } A}{1 + \text{Sin}^2 A}$
	10	0.1736	0.336
	20	0.3420	0.584
	30	0.5000	0.800
	40	0.6428	0.908
	50	0.7660	0.968
1/6	18°-26′-06″	0.3162	0.573
1/5	21°-48′-05″	0.3714	0.653
1/4	26°-33′-54″	0.4472	0.745
1/3	33°-41′-24″	0.5547	0.848
1/2	45°-00′-00″	0.7071	0.942

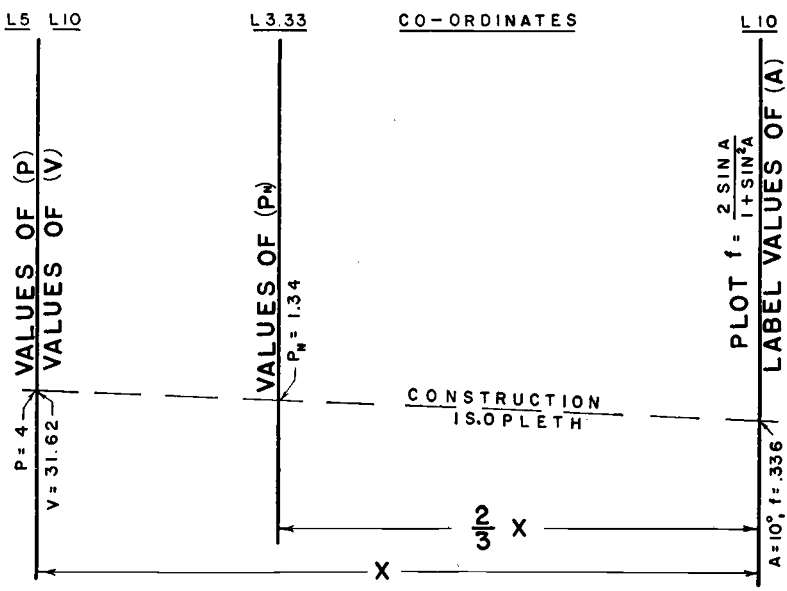

Figure 14. Data for Chart of Equation $P_n = P \dfrac{2 \text{ Sin } A}{1 + \text{Sin}^2 A}$.

These values of (f) will be laid off on the (R) scale and labeled their corresponding values of (A) or "Pitch."

Since the conventional formula for finding the wind pressure (P) is

$$P = 0.004\ V^2,$$

where P = Unit pressure against a vertical flat surface, lb per sq ft.
and V = Wind velocity in miles per hour,

a comparison scale may be constructed showing values of both (V) and (P). Since the $L5$ co-ordinates were assumed for values of (P) on the left side, it will be necessary to use the $L10$ co-ordinates for values of (V).

All the data thus obtained for the construction of the alignment chart have been summarized in Figure 14. As noted, a construction isopleth has been assumed through the values

$$P = 4 \text{ lb per sq ft}$$

and

$$A = 10°.$$

Then by substitution in the necessary equations, we have

$$V = \sqrt{\frac{P}{0.004}} = \sqrt{\frac{4}{0.004}} = 31.62 \text{ miles per hour}$$

and

$$P_n = P\,\frac{2 \text{ Sin } A}{1 + \text{Sin}^2 A} = 4 \times \frac{2 \times 0.1736}{1 + 0.1736^2} = 1.34 \text{ lb per sq ft.}$$

The final chart is shown in Figure 15, with a check isopleth drawn through the values

$$P = 25 \text{ lb per sq ft}$$

and

$$\text{Roof pitch} = 1/4.$$

By substitution, this will be found to give a value of

$$P_n = 18.6 \text{ lb per sq ft},$$

which will be seen to check that found on the chart.

The reader who becomes sufficiently interested in the subject will find it possible to construct nomographs of such equations by means of curved scales. But this involves the use of determinants, or the graphical method, both of which have been avoided in this publication. The solution herein given has been such as to bring parenthetic equations into a type resulting in parallel-scale nomographs or alignment charts. The method is applicable to all cases where a single variable is contained inside the parentheses. As previously noted, the method is limited to those equations where the parenthetic variable has relatively few values that need to be shown on the chart.

6. **Multiple variables involving two common ones.** There are numerous examples in engineering literature where several three-variable equations are to be found containing two common variables. One example is to

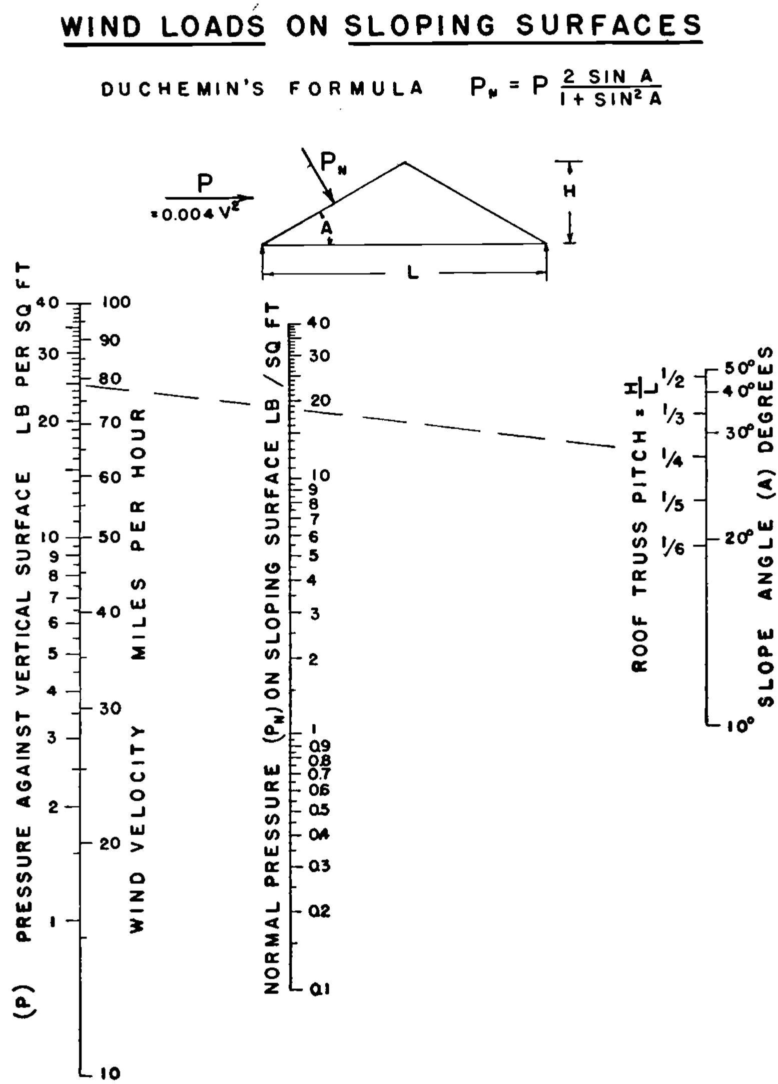

Figure 15. Chart Illustrating Parenthetic Equation $P_n = P \dfrac{2 \sin A}{1 + \sin^2 A}$.

be found in the solution of a rectangular concrete beam with balanced tensile reinforcing:

$$M = K\ b\ d^2$$

and $$A_s = p\ b\ d.$$

$$\text{Approx. dead-load} = \frac{b\ d}{144}\ 150$$

where M = Bending moment, in-lb,
A_s = Area of tensile reinforcing, sq in.,
b = Breadth of beam, in.,
d = Effective depth of beam, in.,
and K and p are constants depending upon the ultimate strength of concrete to be used.

In each of the above three equations, (b) and (d) appear as common variables. An alignment chart may be constructed for the simultaneous solution of all three equations by superposing individual charts of each equation with common (b) and (d) scales. Examples, by the author, of this particular chart may be found in the July 1937 issue of *Western Construction News*, a complete explanation of the construction of which was given in the May 1939 issue of the same publication. The April 1928 issue of *Concrete* contains not only the chart, but also an explanation of its construction.

Another well-known example of such group-equations is to be found in the so-called "exponential" equation for the flow of water in pipes. If a coefficient ($C = 100$) is used in the Chezy formula for the flow of water in pipes, we have

$$V = 100\ \sqrt{R\ S},$$

where V = Velocity, ft per sec,
S = Slope of hydraulic gradient,
and R = Mean hydraulic radius, ft.

However, $$R = \frac{\text{Area}}{\text{Perimeter}} = \frac{\pi d^2}{4(\pi d)} = \frac{d}{4},$$

where d = Inside pipe diameter, ft.

Also $$S = \frac{H_f}{100}, \text{ ft per hundred feet of pipe,}$$

where H_f = Friction head, ft.

The original equation then reduces to the form

$$V = 100 \sqrt{\frac{d}{4}\ \frac{H_f}{100}} = 5\ d^{0.5}\ H^{0.5} \qquad \text{Equation 7.}$$

The discharge is given by the expression

$$Q = A\ V = \frac{\pi}{4}\ d^2\ V, \qquad \text{Equation 8.}$$

where Q = Discharge, cu ft per sec,

and A = Sectional area of pipe, sq ft.

In both Equations 7 and 8, the common variable (d) and (v) will be seen to appear.

The choice of scales involves a consideration of size of drawing, range of values to be shown, and significant figures to be read. These are questions that must be decided by the individual who is to use the chart. When a choice of scales is made herein, it will be done so without explanation.

It is usually advisable to rough out a chart before spending too much time on the final drawing. An approximate check can then be obtained so as to catch any error in nomographic characteristics. The truly useful chart is frequently the result of evolution, additional factors becoming more evident after completion and use.

Equation 7, involving a product of two variables, corresponds to the general expression, Equation 2, requiring logarithmic co-ordinates. As arranged, values of (d) will appear on the left scale, values of (V) on the central scale, and values of (H) will be on the right scale. The $L10$ co-ordinates of Figure 4 will be used for the two outside scales, resulting in a value of $r = 1$ by Definition 2. Equating the general exponents of Equation 2 to the specific ones of Equation 7, we have

$$\frac{m\,n}{r} = 0.5$$

and $m(1 - n) = 0.5$.

When the above assumed value of $r = 1$ is substituted, the resulting two equations contain but two unknowns:

$$m\,n = 0.5$$

and $m(1 - n) = 0.5$.

Solving the two expressions simultaneously, the following values are obtained:

$$n = 1/2$$

and $m = 1$.

Thus by Definition 1, the central scale, containing values of (V), will be halfway between the two outside scales. By Definition 3, the co-ordinates for the central scale will be the same $(L10)$ as used for the outside scales.

Equation 8 is likewise similar in form to the general expression, Equation 2, requiring logarithmic co-ordinates. An alignment chart of Equation 8 will then have values of (d) on the left scale, values of (Q) on the central scale, and values of (V) on the right scale. In the preceding solution for the characteristics of Equation 7, the scales for values of (d) and (V) were found to require the same size $(L10)$ co-ordinates. So for Equation 8, the two outside scales are already determined, resulting by Definition 2 in a value of $r = 1$. Equating the general exponents of Equation 2 to the specific ones of Equation 8, we have

$$\frac{m\,n}{r} = 2$$

and $m(1 - n) = 1.$

Substituting the above value of $r = 1$, we obtain the two expressions

$$m\,n = 2$$

and $m(1 - n) = 1.$

Solving these two equations simultaneously for values of (m) and (n), we obtain

$$n = 2/3$$

and $m = 3.$

From Definition 1, it will be seen that the scale containing values of (Q) will lie 2/3 the distance from the (V) scale to the (d) scale. By Definition 3, the co-ordinates for the (Q) scale will be 1/3 the size of those used for values of (V), resulting in the use of the

$$\frac{L10}{3} = L3.33 \text{ co-ordinates.}$$

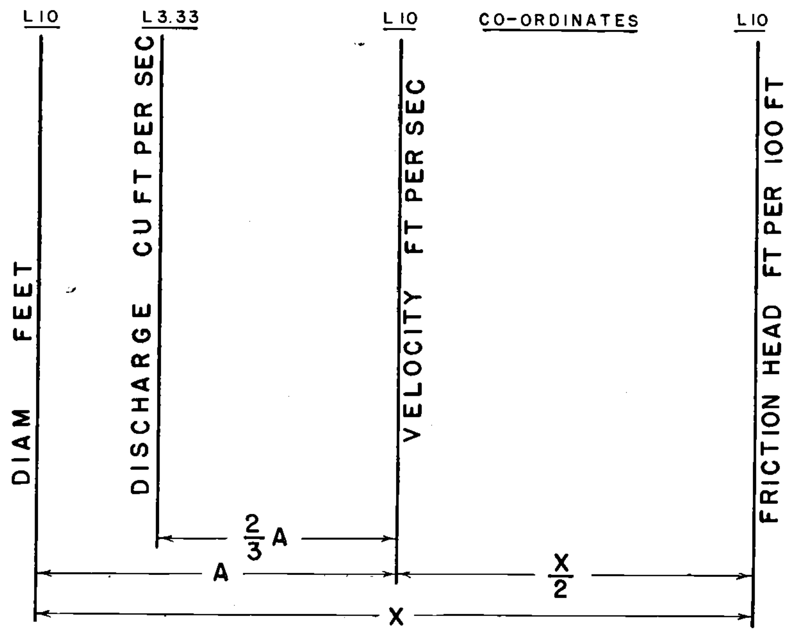

Figure 16. Summary of Characteristics for Equations $V = 5\ d^{0.5}\ H_f^{0.5}$ and $Q = \frac{\pi}{4}\ d^2\ V.$

From our knowledge of comparison scales (Figure 6) it is known that if values of (d) were calibrated in both feet and inches, the same size logarithmic co-ordinates would be used. Since pipe sizes are conventionally given in terms of inches, only those values commercially available will be included. The reader will note, from the discussion of comparison scales, that such a shift from feet to inches does not involve any change of co-ordinates.

Figure 16 is a summary of the characteristics thus far obtained.

A comparison scale could be constructed for discharge. Thus

$$G = 448.8\ Q,$$

where G = Discharge, gal per min,
and Q = Discharge, cu ft per sec.

Since no exponential values, other than unity, are involved in this relation, the same co-ordinates ($L3.33$) will be used for the discharge in gallons per minute.

In writing Bernoulli's theorem between two points in a hydraulic system, it becomes advantageous to have the velocity (V) in terms of velocity head (H_v). This is obtained from the expression

$$H_v = \frac{V^2}{2g} = \frac{V^2}{64.4}.$$

So a comparison scale may be constructed with velocity head on one side and

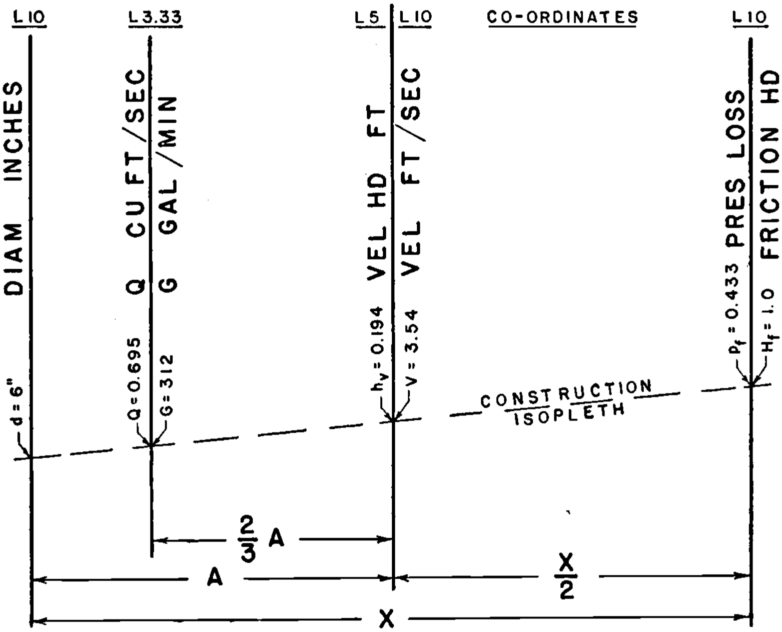

Figure 17. Summary of Information for Construction of Alignment Chart for Flow of Water in Pipes.

velocity on the other. Since the $L10$ co-ordinates are to be used for values of (V), by reference to the comparison scale in Figure 8, it will be seen necessary to use the $\dfrac{L10}{2} = L_5$ co-ordinates for values of (H_v).

The friction loss has been given in terms of lost head (H_f) in feet per 100 feet of pipe. Many times it is convenient to know what the loss may be in terms of pressure. So we have the expression

$$P_f = \frac{H_f}{2.31}$$

where P_f = Pressure loss, lb per sq in. per 100 ft,

and H_f = Friction head, ft per 100 ft of pipe.

A comparison scale involving these two variables will require the same size co-ordinates $(L10)$ for each.

In Figure 17 a summary is given of all the information necessary for the construction of the chart. In order to locate all scales, a construction isopleth was assumed through the values $d = 0.5$ ft $= 6$ in. and $H_f = 1$ ft per 100 ft of pipe. Substituting these values in the expressions previously given, we have by slide rule

$$V = 5\sqrt{d\ H} = 5\sqrt{0.5 \times 1} = 3.54 \text{ ft per sec,}$$

$$H_v = \frac{V^2}{2g} = \frac{(3.54)^2}{64.4} = 0.194 \text{ ft,}$$

$$Q = A\ V = 0.1963 \times 3.54 = 0.695 \text{ cu ft per sec,}$$

$$G = 448.8\ Q = 448.8 \times 0.695 = 312 \text{ gal per min,}$$

and

$$P_f = \frac{H_f}{2.31} = \frac{1}{2.31} = 0.433 \text{ lb per sq in. per 100 ft.}$$

These values have been included in Figure 17 as indicated on the construction isopleth.

The chart in Figure 18 has been drawn from the information shown in Figure 17. In order to check the accuracy of Figure 18, a solution line has been drawn through the values $d = 24$ in. $= 2$ ft and $H_f = 10$ ft per 100 ft. A substitution of these values in the proper equations gives the following results:

$$V = 5\sqrt{d\ H} = 5\sqrt{1 \times 4} = 10 \text{ ft per sec,}$$

$$H_v = \frac{V^2}{2g} = \frac{(10)^2}{64.4} = 1.522 \text{ ft,}$$

$$Q = A\ V = 0.7854 \times 10 = 7.854 \text{ cu ft per sec,}$$

$$G = 448.8\ Q = 448.8 \times 7.854 = 3525 \text{ gal per min,}$$

and

$$P_f = \frac{H_f}{2.31} = \frac{4}{2.31} = 1.732 \text{ lb per sq in. per 100 ft.}$$

These values will be seen to check reasonably well those obtained from the solution isopleth in Figure 18.

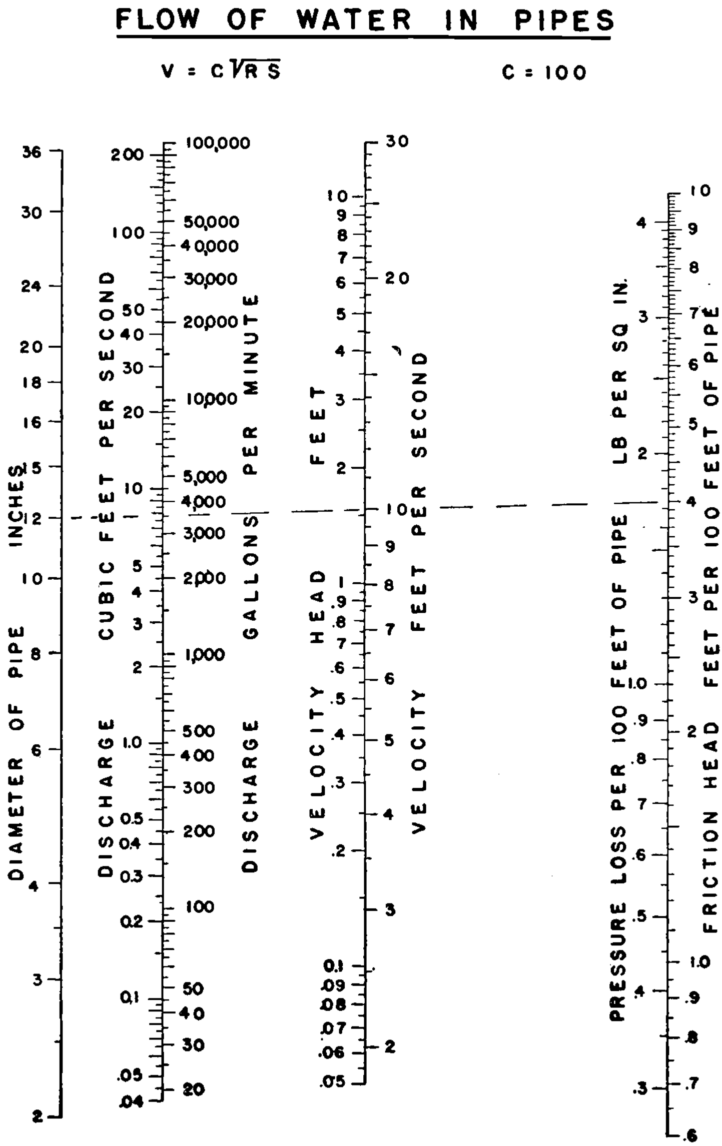

Figure 18. Alignment Chart for Flow of Water in Pipes.

Somewhat similar charts by the author will be found in the following publications:

Civil Engineering; November 1934,

Weymann-Aeryns formula for flow of water in clean cast iron pipe.

$$V = 182.5\ R^{0.723}\ S^{0.539}$$

Western Construction News; April 1937,

Manning's formula, $n = 0.014$, for flow of water in pipes.

Western Construction News; April 1938.

Flow of water in small copper and brass pipes, from Bridgeport Brass Co. tabular data.

7. **Four variables—logarithmic co-ordinates.** The alignment chart, as has been demonstrated, is based on an equation of three variables. Equations containing more than three variables must be separated into groups of three variable equations before they can be put into alignment chart form. Thus a four-variable expression such as

$$A = \frac{x\,y}{B}$$

may be written $A\ B = x\ y$.

Equating both sides of this expression to a common function (f), we have

$$f = A\ B$$

and $$f = x\,y,$$

resulting in two three-variable equations with a common variable (f).

An alignment chart could now be constructed for each of these two equations. The central scale of each will contain values of the common variable (f). If the same size co-ordinates were used for values of (f) on each chart, the two charts could be superposed with a common (f) scale. Since values of (f) are only a means to an end, values of it are of no interest. Therefore, the co-ordinates of the (f) scale are usually omitted. This results in a line without co-ordinates, which some authorities term a "turning scale," but which the author prefers to call a "support."

As an example of this type of chart, let us take the general equation for finding the maximum bending moment in a continuous beam subjected to a uniformly distributed load.

$$M = 12\ C\ w\ L^2 \qquad \text{Equation 9.}$$

where M = Maximum bending moment, in-lb,
w = Uniformly distributed load, lb per lin ft,
L = Span of beam, ft,
and C = Coefficient, value of which depends upon location in beam and degree of restraint. The more common values of (C) are: 1/8; 1/10; 1/12; 1/16, the application of which will be evident to anyone familiar with building design. Such coefficients are conventionally specified in building codes.

In the design of any continuous beam, there may be a number of moments to be computed with the span and the load remaining fixed. Therefore Equation 9 will be written with these two variables on the same side of the equality sign.

$$\frac{M}{C} = 12\ w\ L^2$$

Equating each side to a common function (f), we obtain

$$f = 12\ w\ L^2 \qquad \text{Equation 9a.}$$

$$f = \frac{1}{C} M \qquad \text{Equation 9b.}$$

Since each of these two expressions is similar to the general expression, Equation 2, alignment charts may be constructed for each by the use of logarithmic co-ordinates.

If for Equation 9a, the $L9$ co-ordinates of Figure 4 are assumed for the two outside scales containing values of (w) and (L), by Definition 2 we have the value $r = 1$. Equating the general exponents of Equation 2 to the specific ones of Equation 9a, the following two expressions are obtained:

$$\frac{m\,n}{r} = 1$$

and $\quad m(1 - n) = 2.$

Substituting the assumed value of $r = 1$, we obtain two equations

$$m\,n = 1$$

and $\quad m(1 - n) = 2,$

which may be solved simultaneously, resulting in the following values:

$$n = 1/3$$

and $\quad m = 3.$

From Definition 1, it will be seen that the central scale containing values of (f) will be 1/3 the distance from the right scale containing the values of (L), to the left scale containing values of (w). From Definition 3, the co-ordinates for the central scale must be 1/3 the size of those used on the right scale. Thus for values of (f) the

$$\frac{L9}{3} = L3 \text{ co-ordinates}$$

of Figure 4 must be used. In Figure 19 these data are summarized. A construction isopleth has been assumed through the values $w = 1{,}000$ lb per ft and $L = 10$ ft. By substitution, these values result in a central scale value of

$$f = 12\ w\ L^2 = 12 \times 10^2 = 1{,}200{,}000.$$

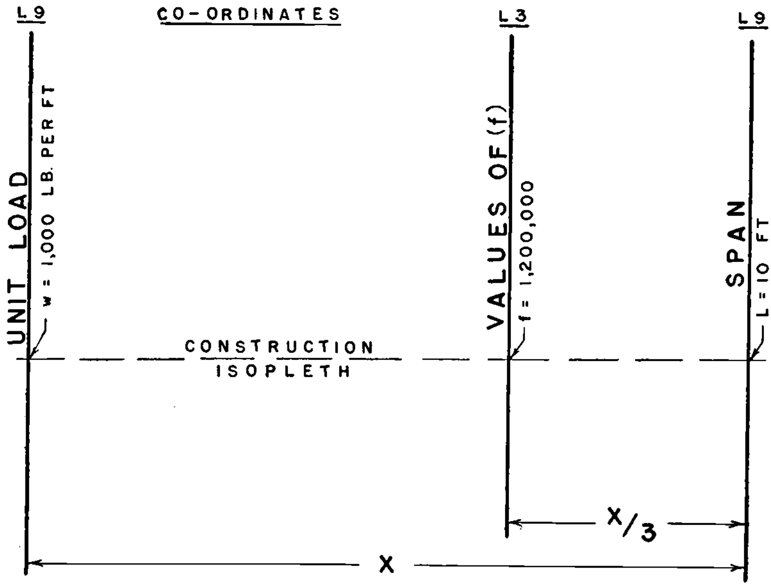

Figure 19. Data for Chart of Equation $f = 12\, w\, L^2$.

Since the two charts are to be superposed utilizing a common (f) scale, for Equation 9b we must start with the knowledge that the co-ordinates for the (f) scale are the $(L3)$ ones of Figure 4. If now the $(L6)$ co-ordinates are chosen for values of (M) of the right scale, by Definition 3 we know that

$$m = \frac{L6}{L3} = 2.$$

Equating the general exponents of Equation 2 to the specific ones of Equation 9b, the following expressions are obtained:

$$\frac{m\,n}{r} = 1$$

and $\quad m(1 - n) = 1.$

Substituting the value $m = 2$, found above, in the second expression, we find that

$$n = 1/2.$$

Substituting the known values of (m) and (n) in the first expression, we obtain a value of

$$r = 1.$$

Therefore, by Definition 1 the central scale containing values of (f) will lie midway between the right scale containing values of (M) and the left scale containing values of (C). Also, from the discussion of inverted scales, it will be seen that the reciprocal $(1/C)$ results in a system of co-ordinates which progress in reverse to those for (M). From Definition 2, it is seen that the left scale utilizes the same co-ordinates $(L6)$ as the right scale. In Figure 20 these data thus found have been summarized. A construction isopleth as shown has been assumed through the value $f = 1{,}200{,}000$ taken from Figure 19 and $M = 240{,}000$ in-lb. By substitution in Equation 9b, the value

$$C = \frac{M}{f} = \frac{240{,}000}{1{,}200{,}000} = 0.2$$

is obtained for locating the (C) co-ordinates.

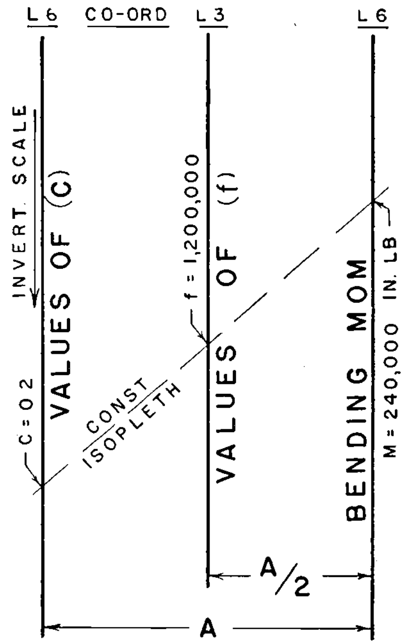

Figure 20. Data for Chart of Equation $f = \frac{M}{C}$.

In Figure 21, the two charts have been superposed, omitting the co-ordinates for values of (f) on the "support." Only the four values of (C) have been included although a complete scale of these coefficients could have been calibrated.

Instead of labeling the bending moments as "inch pounds," they have been indicated at their corresponding values of "inch kips." This involves no change in co-ordinates. At the construction isopleth, we then have a value of

$$M = 240{,}000 \text{ in-lb} = 240 \text{ in-kips}.$$

In order to avoid confusion as to which scales are to be used together, the author has found that such scales can best be indicated with a common letter at the top, as in Figure 21. Thus a solution isopleth must extend between the

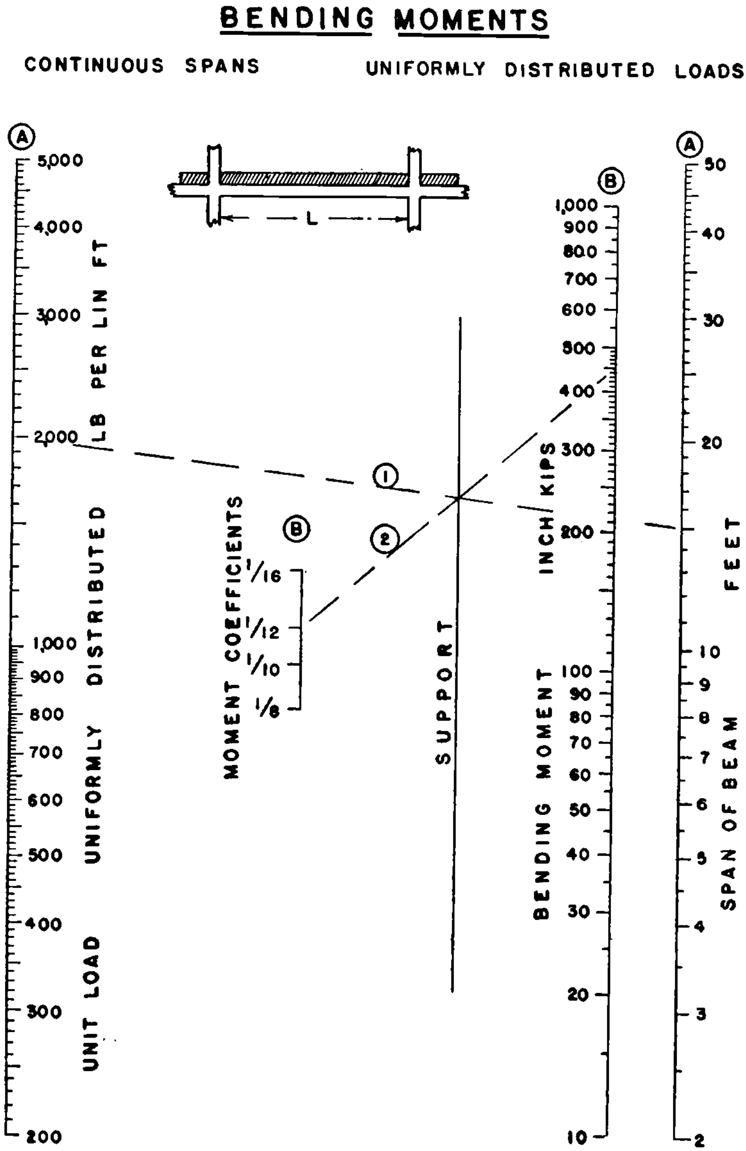

Figure 21. Alignment Chart for Determination of Bending Moments in Continuous Beams.

"A" scales, and another one between the "B" scales. For any one solution, the isopleths thus drawn must intersect on the "support."

For the solution shown on Figure 21, the following values were assumed:

Load, $w = 2{,}000$ lb per lin ft,
Span, $L = 15$ ft,
Mom. Coef., $C = 1/12$.

Solution isopleth (1) has been drawn between the values of (w) and (L) on the "A" scales. Isopleth 2 has been drawn from the value $C = 1/12$ through the intersection of line 1 on the support. On the moment scale, line 2 indicates a value of $M = 450$ inch-kips. If the assumed values are substituted in Equation 9, we find that

$$M = 12\ C\ w\ L^2 = 12 \times \frac{1}{12} \times 2{,}000 \times 15^2 = 450{,}000 \text{ in.-lb}$$
$$= 450 \text{ inch-kips.}$$

V. CONCLUSIONS

In conclusion, it might be said that the method herein presented for the determination of alignment chart characteristics, consists of the following procedure:

(1) If the equation to be plotted is not already in such form, transpose it until it is in a form similar to one of the general expressions, Equation 1 or Equation 2.

(2) By comparison of the variables in the specific equation to those in the similar general expression, the order of the scales becomes obvious.

(3) Equate the general coefficients, or exponents, to the specific ones of the equation to be plotted.

(4) Assume one of the three unknown characteristics, and solve for the remaining two from the expressions developed in step (3).

While the method is limited to equations that result in nomographic charts with parallel scales, it is likewise the first mathematical solution that does not require the use of determinants. Because of the lack of facility in the use of determinants, many elementary students of nomography have been unable to follow the conventional text utilizing that presentation. Therefore, it is hoped that by presenting a method involving only simple algebraic manipulations, many individuals will learn that the construction of such charts is as simple as their application. Modern engineering literature indicates a marked increase in the use of nomographic charts, and a knowledge of the general principles involved should result in a more intelligent use of such charts.

VI. GENERAL RECOMMENDATIONS

In order to justify the time spent on the design and construction of any chart, an increased efficiency should be realized in the application of the problem solved. Routine problems of many applications are frequently worthy of such effort. In the trigonometric reduction of many thousands of test observations, one organization had stenographic help making the substitutions by means of a large nomographic chart. Another organization reported time saved by using the author's *Concrete* charts to check design computations.

Since increased efficiency is the principal objective, simplicity should be the keynote. The author has one chart, prepared by a commercial design organization, which is so complicated that a book of instructions is necessary for its use. Such a chart may save time for the constant user who is thoroughly familiar with its application. For the occasional user, however, a complicated chart is always of questionable value. The simplicity of application of the nomographic chart is probably responsible for its popularity.

Above all, a chart should indicate the problem to be solved, the equation used when any uncertainty might exist, and the method of solution. Since convenient charts are frequently copied and find wide distribution, they should preferably carry the name of the individual or organization originating them in order that proper credit may be received.

The accuracy of a solution by chart is dependent on both the accuracy of construction and the method of reproduction. Vertical lines and graduations should be no wider than for proper visibility and legibility. Graduations should be so labeled and varied in length that readings can be made at a glance without tedious counting. Reproductions made by any of the so-called "wet" processes, may result in uneven shrinkage and resulting errors. Lens distortion in photostating may result in an inaccurate reproduction. Contact reproductions made by either the ammonia gas or continuous damp-process are probably among the most accurate of those available to the usual engineering office.

Above all, patience and persistence are necessary attributes for those who desire reasonable success in the field of nomography.

VII. BIBLIOGRAPHY

1. Traité de Nomographie, by M. d'Ocagne (Gauthier-Villars, Paris).
2. Design of Diagrams for Engineering Formulas, by Hewes and Seward (McGraw-Hill Book Company, Inc.).
3. The Nomogram, by Allcock and Jones (Sir Isaac Pitman & Sons, Ltd.).
4. The Construction of Graphical Charts, by Peddle (McGraw-Hill Book Company, Inc.).
5. How to Make Alignment Charts, by Van Voorhis (McGraw-Hill Book Company, Inc.).
6. Construction of Alignment Charts, by Swett (John Wiley & Sons, Inc.).
7. Graphical and Mechanical Computations, by Lipka, Vol. 1, Alignment Charts (John Wiley & Sons, Inc.).
8. Graphical Solutions, by Mackey (John Wiley & Sons, Inc.).
9. Charts and Graphs, by Karsten (Prentice-Hall).
10. The Construction of Nomographic Charts, by Mavis (International Text Book Co.).
11. Nomographic Solutions for Formulas of Various Types, by Strachan (Trans. A. S. C. E., Vol. 78, 1915).
12. Alignment Charts, their Construction and Use, by LeHoczky, Circular No. 34, Eng. Exp. Sta., Ohio State Univ.
13. Series of monthly articles and charts in *Concrete* 1928-1931, by James R. Griffith. Explanation of construction of each chart is included.
14. The Construction of Nomographs, by James R. Griffith, *Western Construction News* (San Francisco), May 1939.
15. Series of monthly charts by James R. Griffith, in *Western Construction News,* September 1935 ——.

OREGON STATE COLLEGE
ENGINEERING EXPERIMENT STATION
CORVALLIS, OREGON

LIST OF PUBLICATIONS

Bulletins—

No. 1. Preliminary Report on the Control of Stream Pollution in Oregon, by C. V. Langton and H. S. Rogers. 1929.
None available.

No. 2. A Sanitary Survey of the Willamette Valley, by H. S. Rogers, C. A. Mockmore, and C. D. Adams. 1930.
Forty cents.

No. 3. The Properties of Cement-Sawdust Mortars, Plain, and with Various Admixtures, by S. H. Graf and R. H. Johnson. 1930.
Twenty cents.

No. 4. Interpretation of Exhaust Gas Analyses, by S. H. Graf, G. W. Gleeson, and W. H. Paul. 1934.
Twenty-five cents.

No. 5. Boiler-Water Troubles and Treatments with Special Reference to Problems in Western Oregon, by R. E. Summers. 1935.
Twenty-five cents.

No. 6. A Sanitary Survey of the Willamette River from Sellwood Bridge to the Columbia, by G. W. Gleeson. 1936.
Twenty-five cents.

No. 7. Industrial and Domestic Wastes of the Willamette Valley, by G. W. Gleeson and F. Merryfield. 1936.
Fifty cents.

No. 8. An Investigation of Some Oregon Sands with a Statistical Study of the Predictive Values of Tests, by C. E. Thomas and S. H. Graf. 1937.
Fifty cents.

No. 9. Preservative Treatments of Fence Posts.
1938 Progress Report on the Post Farm, by T. J. Starker, 1938.
Twenty-five cents.

No. 10. Precipitation-Static Radio Interference Phenomena Originating on Aircraft, by E. C. Starr, 1939.
Seventy-five cents.

No. 11. Electric Fence Controllers with Special Reference to Equipment Developed for Measuring Their Characteristics, by F. Alton Everest. 1939.
Forty cents.

No. 12. Mathematics of Alignment Chart Construction without the Use of Determinants, by James R. Griffith. 1940.
Twenty-five cents.

Circulars—

No. 1. A Discussion of the Properties and Economics of Fuels Used in Oregon, by C. E. Thomas and G. D. Keerins. 1929.
Twenty-five cents.

No. 2. Adjustment of Automotive Carburetors for Economy, by S. H. Graf and G. W. Gleeson. 1930.
None available.

No. 3. Elements of Refrigeration for Small Commercial Plants, by W. H. Martin. 1935.
None available.

No. 4. Some Engineering Aspects of Locker and Home Cold-Storage Plants, by W. H. Martin. 1938.
Twenty cents.

Reprints—

No. 1. Methods of Live Line Insulator Testing and Results of Tests with Different Instruments, by F. O. McMillan. Reprinted from 1927 Proc. N. W. Elec. Lt. and Power Assoc.
Twenty cents.

No. 2. Some Anomalies of Siliceous Matter in Boiler Water Chemistry, by R. E. Summers. Reprinted from Jan. 1935, Combustion.
Ten cents.

No. 3. Asphalt Emulsion Treatment Prevents Radio Interference, by F. O. McMillan. Reprinted from Jan. 1935, Electrical West.
None available.

No. 4. Some Characteristics of A-C Conductor Corona, by F. O. McMillan. Reprinted from Mar. 1935, Electrical Engineering.
None available.

No. 5. A Radio Interference Measuring Instrument, by F. O. McMillan and H. G. Barnett. Reprinted from Aug. 1935, Electrical Engineering.
Ten cents.

No. 6. Water-Gas Reaction Apparently Controls Engine Exhaust Gas Composition, by G. W. Gleeson and W. H. Paul. Reprinted from Feb. 1936, National Petroleum News.
Ten cents.

No. 7. Steam Generation by Burning Wood, by R. E. Summers. Reprinted from April 1936, Heating and Ventilating.
Ten cents.

No. 8. The Piezo Electric Engine Indicator, by W. H. Paul and K. R. Eldredge. Reprinted from Nov. 1935, Oregon State Technical Record.
Ten cents.

No. 9. Humidity and Low Temperatures, by W. H. Martin and E. C. Willey. Reprinted from Feb. 1937, Power Plant Engineering.
None available.

No. 10. Heat Transfer Efficiency of Range Units, by W. J. Walsh. Reprinted from Aug. 1937, Electrical Engineering.
None available.

No. 11. Design of Concrete Mixtures, by I. F. Waterman. Reprinted from Nov. 1937, Concrete.
None available.

No. 12. Water-wise Refrigeration, by W. H. Martin and R. E. Summers. Reprinted from July 1938, Power.
None available.

No. 13. Polarity Limits of the Sphere Gap, by F. O. McMillan. Reprinted from Vol. 58, A.I.E.E. Transactions, Mar. 1939.
Ten cents.

No. 14. Influence of Utensils on Heat Transfer, by W. G. Short. Reprinted from Nov. 1938, Electrical Engineering.
Ten cents.

No. 15. Corrosion and Self-Protection of Metals, by R. E. Summers. Reprinted from Sept. and Oct. 1938, Industrial Power.
Ten cents.

No. 16. Monocoque Fuselage Circular Ring Analysis, by B. F. Ruffner. Reprinted from Jan. 1939, Journal of the Aeronautical Sciences.
Ten cents.

No. 17. The Photoelastic Method as an Aid in Stress Analysis and Structural Design, by B. F. Ruffner. Reprinted from Apr. 1939, Aero Digest.
Ten cents.

No. 18. Fuel Value of Old-Growth vs. Second-Growth Douglas Fir, by Lee Gabie. Reprinted from June 1939, the Timberman.
Ten cents.

No. 19. Stoichiometric Calculations of Exhaust Gas, by G. W. Gleeson and F. W. Woodfield, Jr. Reprinted from November 1, 1939, National Petroleum News.
Ten cents.

Research Papers—

(Published as indicated. Not available from the Station.)

No. 1. Electric Fish Screens, by F. O. McMillan. Bulletin of the U. S. Bureau of Fisheries, vol. 44, 1928. Also in pamphlet form, U. S. Bureau of Fisheries, Document No. 1042.

No. 2. Water Control of Dry Mixed Concrete, by G. W. Gleeson. Concrete Products, December 1929.

No. 3. High-voltage Gaseous Conductor Lamps, by F. O. McMillan and E. C. Starr. Trans. American Institute of Electrical Engineers, vol. 48, no. 1, pp. 11-18, 1929.

No. 4. The Influence of Polarity in High-voltage Discharges, by F. O. McMillan and E. C. Starr. Trans. American Institute of Electrical Engineers, vol. 50, no. 1, pp. 23-35, 1931.

No. 5. Progress Report on Radio Interference from High-voltage Transmission Lines —Pin and Pedestal Type Insulators, by F. O. McMillan. Trans. 8th annual general meeting, Engineering Section, Northwest Electric Light and Power Assoc., 1931.

No. 6. Aggregate Grading for Tamped Concrete Pipe, by G. W. Gleeson. Concrete, June 1932. Rock Products, 1932. Concrete Products, June 1932 and May-June 1934.

No. 7. Water Control of Dry Mixed Concrete, by G. W. Gleeson. Concrete Products, September 1932, and Rock Products, November 1932.

No. 8. Litharge and Glycerine Mortars, by G. W. Gleeson. Paper Trade Journal, October 13, 1932.

No. 9. Radio Interference from Insulator Corona, by F. O. McMillan. Trans. American Institute of Electrical Engineers, vol. 51, no. 2, pp. 385-391, 1932.

No. 10. The Coordination of High-voltage Transmission Lines with Radio, by F. O. McMillan. Trans. 9th annual general meeting, Engineering Section, Northwest Electric Light and Power Assoc., 1932.

No. 11. Asphalt Emulsion Reduces Insulator Radio Troubles, by F. O. McMillan. Electrical World, vol. 102, no. 6, August 5, 1933.

No. 12. Silicon, a Major Constituent of Boiler Scales in Western Oregon, by R. E. Summers and C. S. Keevil. Paper presented at annual meeting, American Society of Mechanical Engineers, 1933. Abstracts published in Mechanical Engineering, vol. 55, p. 720, November 1933; Power, vol. 77, p. 687, mid-Dec. 1933; and Power Plant Engineering, vol. 37, p. 519, December 1933, and vol. 38, p. 219, May 1934.

No. 13. Study of the Frequency of Fuel Knock Noises, by W. H. Paul and A. L. Albert. National Petroleum News, August 9, 1933.

No. 14. The Pollutional Character of Flax Retting Wastes, by G. W. Gleeson, F. Merryfield, and E. F. Howard. Sewage Works Journal, May 1934.

No. 15. Siliceous Scales in Boilers of Western Oregon and Washington, by R. E. Summers and C. S. Keevil. The Timberman, vol. 35, p. 30, May 1934.

No. 16. How Much Phosphate? by R. E. Summers. Power, vol. 78, p. 452, August 1934.

No. 17. The Carbon Dioxide-Hydrogen Ratio in the Products of Combustion from Automotive Engines, by G. W. Gleeson and W. H. Paul. National Petroleum News, September 15, 1934.

No. 18. Exhaust Gas Analysis, by G. W. Gleeson and W. H. Paul. Parts I, II, III. National Petroleum News, September 26, October 3 and 10, 1934.

No. 19. Simplified Measurements of Sound Absorption, by A. L. Albert and T. B. Wagner. Electrical Engineering, vol. 53, no. 8, p. 1,160, August 1934.

No. 20. Treatment and Recovery of Sulfite Waste, by F. Merryfield. Civil Engineering, June 1936.

No. 21. Industrial Wastes in the Willamette Valley, by F. Merryfield. Civil Engineering, October 1936.

No. 22. Flow Characteristics in Elbow Draft-Tubes, by C. A. Mockmore. Proc. American Society of Civil Engineers, vol. 63, no. 2, pp. 251-286, Feb. 1937.

No. 23. Some Simple Experiments Dealing with Rates of Solution, by G. W. Gleeson. Journal of Chemical Education, vol. 15, no. 4, April 1938.

No. 24. Heat Transfer Coefficient in Boiling Refrigerant, by W. H. Martin. Refrigerating Engineering, vol. 36, no. 3, September 1938.

No. 25. Kiln Drying Guaranteed Moisture Content Spruce Lumber, by Glenn Voorhies. West Coast Lumberman, vol. 66, no. 1. January 1939.

No. 26. Steam Demand in Drying Douglas Fir Lumber, by Glenn Voorhies. The Timberman, vol. 40, no. 4, February 1939.

THE ENGINEERING EXPERIMENT STATION STAFF

R. H. DEARBORN, Dean and Director of Engineering.
S. H. GRAF, Director of Engineering Research.
A. L. ALBERT, Communication Engineering.
F. A. EVEREST, Radio Engineering.
G. W. GLEESON, Chemical Engineering.
BURDETTE GLENN, Highway Engineering.
J. R. GRIFFITH, Structural Engineering.
F. O. MCMILLAN, Electrical Engineering.
W. H. MARTIN, Mechanical Engineering.
E. G. MASON, Forestry.
FRED MERRYFIELD, Sanitary Engineering.
C. A. MOCKMORE, Civil and Hydraulic Engineering.
W. H. PAUL, Automotive Engineering.
B. F. RUFFNER, Aeronautical Engineering.
E. C. STARR, Electrical Engineering.
C. E. THOMAS, Engineering Materials.
GLENN VOORHIES, Wood Products.

Technical Counselors

R. H. BALDOCK, State Highway Engineer, Salem.
R. G. DIECK, Consulting Civil Engineer, Portland.
C. V. LANGTON, Professor of Hygiene, Oregon State College.
J. C. H. LEE, Colonel, Corps of Engineers, Division Engineer, North Pacific Division, Portland.
PAUL B. MCKEE, President, Portland Gas and Coke Company, Portland.
J. H. POLHEMUS, President, Portland Electric Power Company.
J. C. STEVENS, Consulting Civil and Hydraulic Engineer, Portland.
C. E. STRICKLIN, State Engineer, Salem.

Oregon State College

Corvallis

RESIDENT INSTRUCTION

Liberal Arts and Sciences
- The Lower Division (Junior Certificate)
- School of Science (B.A., B.S., M.A., M.S., Ph.D. degrees)

The Professional and Technical Curricula
- School of Agriculture (B.S., M.S., Ph.D. degrees)
- School of Education (B.A., B.S., B.S. in Ed., M.A., M.S., Ed.M., Ed.D. degrees)
- School of Engineering and Industrial Arts (B.S., M.S., Ch.E., C.E., E.E., M.E. degrees)
- School of Forestry (B.S., M.S., M.F., F.E. degrees)
- School of Home Economics (B.A., B.S., M.A., M.S. degrees)
- School of Pharmacy (B.S., M.S. degrees)
- Secretarial Science (B.S.S. degree)

The Graduate Division (M.A., M.S., Ed.M., M.F., Ch.E., C.E., E.E., F.E., M.E., Ed.D., Ph.D. degrees)

The Summer Sessions

The Short Courses

RESEARCH AND EXPERIMENTATION

The General Research Council

The Agricultural Experiment Station—
- The Central Station, Corvallis
- The Union, Moro, Hermiston, Talent, Burns, Astoria, Hood River, Pendleton, Medford, and Squaw Butte Branch Stations

The Engineering Experiment Station, Corvallis

EXTENSION

Federal Cooperative Extension (Agriculture and Home Economics)

General Extension Division