AN ABSTRACT OF THE THESIS OF

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Two types of boundary element models are developed for the interaction of waves with trenches. The first type is for a two-dimensional domain in the horizontal plane and employs the linear long wave approximations. It is shown that appropriate selection of pit geometries leads to a significant reduction in wave height behind the pits. Wave heights in the lee can be reduced to 10% of the incident wave height by two similar pits that have approximate geometries of one or more wave lengths in the shore parallel direction, one-half a wave length in the cross-shore, a depth three times that of the adjacent water, and spaced approximately one-half wave length apart. Examples are presented which show how to select pit geometries to provide shoreline protection or a harbor area. An example is also presented showing how a properly placed pit can significantly reduce wave heights in a navigation channel.
The second type of boundary element models is developed for a two-dimensional domain in the vertical plane and are valid for all water depths. These models are for an infinitely long trench, which can contain rubble. Energy dissipation due to rubble in the trench is estimated using a linearized friction coefficient. Results indicate that highly permeable material in the pit does not significantly degrade (and some specific cases actually improves) the performance of the trench as a breakwater. Therefore, it is not necessary to remove all of the coarse aggregate from the trench. This would significantly reduce the construction cost.
Wave Diffraction Due to Trenches and Rubble

by

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LIST OF SYMBOLS

a  pit (trench) width
b  pit (trench) length
C_t  dimensionless turbulent resistance coefficient
C_M  virtual mass coefficient of the stones
d  pit (trench) depth
E_{DD}  total energy in the diffracted wave
E_i  total energy in the incident wave
E_{RR}  total energy in the reflected wave
E_{TT}  total energy in the transmitted wave
f  linearized damping coefficient in the rubble
G  free space Green's function
g  gravitational acceleration
H  wave height
h  water depth
K  intrinsic permeability
K_D  diffraction coefficient
\bar{K_D}  average diffraction coefficient
K_L  loss coefficient
K_R  reflection coefficient
\bar{K_R}  average reflection coefficient
LIST OF SYMBOLS (continued)

\( K_T \) \hspace{1em} \text{transmission coefficient}
\( \overline{K_T} \) \hspace{1em} \text{average transmission coefficient}
\( k \) \hspace{1em} \text{wave number in wave ray direction} \quad (= 2\pi/L)
\( L \) \hspace{1em} \text{wave length}
\( L_b \) \hspace{1em} \text{distance from the front of the pit to the back of the shadow zone}
\( L_f \) \hspace{1em} \text{distance from the front of the pit to the front of the shadow zone}
\( L_s \) \hspace{1em} \text{length of the shadow zone}
\( l \) \hspace{1em} \text{projection of the wave number onto the x-axis}
\( m \) \hspace{1em} \text{projection of the wave number onto the y-axis}
\( P \) \hspace{1em} \text{singular (base) point in the domain or on the boundary}
\( Q \) \hspace{1em} \text{nodal point on the boundary}
\( \vec{v} \) \hspace{1em} \text{fluid velocity vector}
\( S \) \hspace{1em} \text{inertia coefficient of the rubble}
\( S_{\Pi} \) \hspace{1em} \text{incident wave spectrum}
\( S_{RR} \) \hspace{1em} \text{reflected wave spectrum}
\( S_{TT} \) \hspace{1em} \text{transmitted wave spectrum}
\( T \) \hspace{1em} \text{wave period}
\( t \) \hspace{1em} \text{time}
\( x,y,z \) \hspace{1em} \text{cartesian coordinate system with z positive upwards from SWL}
\( \alpha \) \hspace{1em} \text{relative pit width (a/L)}
LIST OF SYMBOLS (continued)

\( \beta \) relative pit (trench) length \((b/L)\)

\( \gamma \) relative pit (trench) depth \((d/h)\)

\( \delta \) relative pit spacing to the wave length

\( \varepsilon \) porosity of the rubble

\( \eta \) free surface elevation

\( \Phi \) complex total velocity potential

\( \Phi^I \) incident wave velocity potential

\( \phi \) spacially dependent component of \( \Phi \)

\( \omega \) wave angular frequency \((2\pi/T)\)

\( \theta \) angle of the incident wave

\( \rho \) density of water
WAVE DIFFRACTION DUE TO TRENCHES AND RUBBLE

1.0 INTRODUCTION

1.1 Purpose of the Study

Coastal engineers often try to reduce waves in the near shore to control erosion, provide safe navigation or develop harbors. Rubble mound breakwaters have been the most common structural means of accomplishing this goal. Floating breakwaters, concrete caissons, submerged breakwaters and even flexible structures have also been employed. However, all of these structures occupy some portion of water.

A pit or trench in the sea bottom is a means of providing wave protection without occupying any portion of water column. Intuitively, pits or trenches are most effective for shallow water or long waves. These types of waves typically correspond to higher energy conditions and therefore, these structures selectively provide more protection under more energetic conditions. This type of shoreline protection is most applicable in areas with shallow nearshores. Coast lines with wide, shallow coral reefs are particularly well suited for this kind of protection. These structures provide no view degradation and have minimum impact on water circulation and quality.
A pit or trench can be constructed by excavating or blasting into sea bed. The trench will be partially filled by sand, gravel or coral during construction and later due to sediment transport. Maintaining an open trench is very difficult in areas with high sediment transport. Therefore, the trench breakwater is most suitable on coastlines with no or low sediment transport. This would include wide wave cut terraces on rocky shorelines and coral reefs. For these shorelines, backfilling due to sediment transport is not a significant problem. Highly permeable materials left in the trench do not degrade the performance. Therefore, it is not necessary to remove all of the coarse aggregate from the trench. This would significantly reduce construction costs.

1.2 Background

The interaction of normally and obliquely incident surface waves with a rectangular trench has been investigated by several authors. Lee and Ayer (1981) developed an analytical solution for two-dimensional linear waves over a rectangular trench. Experimental results were presented and compared favorably with the theory.

Kirby and Dalrymple (1983) developed an analytical solution for a rectangular trench for the cases of large angles of wave incidence and asymmetric geometry. They noted that a large reduction in wave transmission due to by a local refracting barrier could be caused by small differences in trench depth. The large reduction in wave transmission occurred when the trench-parallel component of the wave number in the incident wave region exceeded the wave number for
freely propagating waves in the trench. A numerical solution and an approximate solution based on plane-wave modes were presented.

These previous studies were for cases with no material in the trench. Trenches will tend to be partially filled, unless protective measures against scour and sedimentation are taken. Liu, et al. (1986) studied wave interactions with a sediment trench both theoretically and experimentally. They showed that potential theory adequately described the velocity and pressure fields when the trench is not filled with sand. They also showed, if the trench is filled with sand, Biot consolidation theory gives a better description of the pore water pressure field under wave loading than other theoretical models.

In all of these studies, the trench was assumed to be infinitely long. The influence of a finite lateral dimension was not examined. In the present study, the wave field around a finite length pit is examined. It is found that the effects of the finite length are quite significant. A finite length pit with waves approaching at arbitrary angles results in a 3-D boundary value problem. A solution to this problem is computationally intensive. Therefore, three simplified geometries are considered in this study. The domains for these are 2-D horizontal, 2-D vertical and quasi 3-D. Since several geometries are examined, the following terminology is used to distinguish the various cases. Pit is used to describe a finite length depression in the bottom. Trench is used when referring to an infinitely long depression in the bottom.

The three different domains considered are shown in Figure 1.1. The 2-D horizontal domain examines wave height variations in the horizontal plane around
a finite length pit. In this formulation, long wave theory is used. The boundary conditions around the pit are vertically integrated, so there is no z dependency in this formulation. The wave field is characterized through a diffraction coefficient. This is defined as the ratio of the modulus of the local wave height to the incident wave height. In this domain, wave height changes result from reflection, refraction and diffraction. Again, the 2-D horizontal formulation is only valid for shallow water.

The 2-D vertical domain is for an infinitely long trench in the wave crest direction. In this case, there is no variation in the y direction and a 2-D solution in the x-z plane is obtained. Solutions developed in this domain are depth dependent and are valid for arbitrary water depths. Changes in wave height are due to reflection and are characterized by reflection and transmission coefficients. These are defined as the ratio of the modulus of the reflected and transmitted wave heights to the incident wave height, respectively.

The quasi 3-D domain is for an infinitely long trench but with waves obliquely incident. The term quasi 3-D is used to describe this domain because the trench geometry is 2-D but the wave are 3-D. For this case, Snell’s law allows the quasi 3-D problem to be reduced to a 2-D solution technique. Wave changes are due to reflection and refraction. The wave changes on each side of the trench are again characterized by the reflection and transmission coefficients.

In addition to showing sketches of each of these three domains, Figure 1.1 also gives the field equation and Green’s function for each domain. These
equations are derived in this study. However, they are presented here to help orient the reader to the different classes of solutions which are developed.
[2-D horizontal domain]

\[
\frac{\partial^2 \phi_j}{\partial x^2} + \frac{\partial^2 \phi_j}{\partial y^2} + k_j^2 \phi_j = 0 \quad ; \quad j = 1, 2
\]

\[
G_j = \frac{i \pi}{2} H_0^{(i)}(k_j r) \quad ; \quad j = 1, 2
\]

\[
r^2 = (x - x')^2 + (y - y')^2
\]

[2-D vertical domain]

\[
\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0
\]

\[
G = \ln r
\]

\[
r^2 = (x - x')^2 + (z - z')^2
\]

[Quasi 3-D domain]

\[
\frac{\partial^2 \phi_j}{\partial x^2} + \frac{\partial^2 \phi_j}{\partial z^2} - m^2 \phi_j = 0
\]

\[
(j = 1, 2, 3)
\]

\[
G = -K_0(mr)
\]

\[
r^2 = (x - x')^2 + (z - z')^2
\]

Figure 1.1 Domain definitions: a) 2-D horizontal, b) 2-D vertical, c) quasi 3-D
2.0 WAVE DIFFRACTION DUE TO A PIT

2.1 Theoretical Formulation

2.1.1 Governing Equation

A definition sketch of the boundary value problem under consideration is shown in Figure 2.1 for a single pit. The pit has dimensions of length \( a \) and width \( b \). The water depth within the pit is denoted by \( d \) while that of the surrounding fluid region is denoted by \( h \). Cartesian coordinates are employed with the \( x \) and \( y \) axes in the horizontal plane and the \( z \)-axis taken positive upward from the still water level. The fluid is assumed to be incompressible and irrotational so that a velocity potential exists.

The incident wave propagates from left to right at an angle \( \theta \) to the positive \( x \)-axis. The velocity potential can be expressed by the real part of a complex potential

\[
\Phi(x,y,z,t) = \text{Re} \left[ \Phi(x,y,z) e^{-i\omega t} \right] \tag{2.1}
\]

where, \( \omega \) is the angular frequency of the simple harmonic wave motion, \( \Phi \) is the velocity potential. From conservation of mass, the velocity potential must satisfy Laplace's equation

\[
\nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \tag{2.2}
\]

Taking the bottom to be impermeable and horizontal, the bottom boundary condition is
\[ \frac{\partial \Phi}{\partial z} = 0 \quad ; \quad z = -h, \, z = -d \quad (2.3) \]

The fluid domain is divided into two regions, an interior region 1 (pit) and an exterior region 2 consisting of the surrounding fluid domain. Therefore, the depth dependency of the velocity potentials in region 1 and region 2 are given by

\[ \Phi_1(x,y,z) = \phi_1(x,y) \cosh k_1(z+d) \quad (2.4) \]

\[ \Phi_2(x,y,z) = \phi_2(x,y) \cosh k_2(z+h) \quad (2.5) \]

in which \( k \) is the wave number determined by the linear wave theory dispersion equation

\[ \omega^2 = g k_1 \tanh k_1 d \quad (2.6) \]

\[ \omega^2 = g k_2 \tanh k_2 h \quad (2.7) \]

Substituting (2.4) and (2.5) into Laplace's equation yields,

\[ \frac{\partial^2 \phi_j}{\partial x^2} + \frac{\partial^2 \phi_j}{\partial y^2} + k_j^2 \phi_j = 0 \quad ; \quad j = 1, 2 \quad (2.8) \]

It is assumed that wave heights are small and that the water depth to wave length ratio is small so that linear shallow water wave theory is applicable. Considering the above assumptions, the fluid motion may be described in terms of a long wave velocity potential. Therefore, the wave numbers for each region are defined by
\[
k_1 = \frac{\omega}{\sqrt{gd}} ; \quad k_2 = \frac{\omega}{\sqrt{gh}} \quad (2.9a,b)
\]

2.1.2 Boundary Conditions

Matching conditions require continuity of mass flux and pressure across the interface between the two regions,

\[
d \frac{\partial \phi_1}{\partial x} = h \frac{\partial \phi_2}{\partial x} \quad \text{on} \ x=0,b, \ 0 \leq y \leq a \quad (2.10)
\]

\[
d \frac{\partial \phi_1}{\partial y} = h \frac{\partial \phi_2}{\partial y} \quad \text{on} \ y=0,a, \ 0 \leq x \leq b \quad (2.11)
\]

and

\[
\phi_1 = \phi_2 \quad \text{on} \ x=0,b, \ 0 \leq y \leq a \quad y=0,a, \ 0 \leq x \leq b \quad (2.12)
\]

The scattered component of the velocity potential in region 2 must satisfy an appropriate radiation condition at large distances from the pit,

\[
\lim_{r \to \infty} \left[ \frac{\partial}{\partial r} - i k_2 \right] (\phi_2 - \phi_2') = 0 \quad (2.13)
\]

where, \( \phi_2' \) is the velocity potential of the incident wave and is given by

\[
\phi_2'(x,y) = \frac{-igH}{2\omega} e^{ik_2(x \cos \theta + y \sin \theta)} \quad (2.14)
\]

in which \( H \) is the incident wave height and \( \theta \) is the incident wave angle.
2.1.3 Integral Equations

The boundary value problem described in previous section can be solved numerically by utilizing the boundary element method (BEM). The BEM converts the boundary value problem in the domain to an integral equation for the unknown velocity potential and normal derivative of the velocity potential along the boundary.

Figure 2.2 shows the fluid domain under consideration for a single pit. Region 1 is the domain inside the pit, and region 2 is the surrounding fluid domain.

Applying Green's Second Identity to the velocity potential \( \phi_j \) and Green's function \( G_j \) over regions 1 and 2 yields

\[
\iint_A [\phi_j (\nabla^2 G_j + k_j^2 G_j) - G_j (\nabla^2 \phi_j + k_j^2 \phi_j)] dA = \int_s (\phi_j \nabla G_j - G_j \nabla \phi_j) \mathbf{n} ds
\]

(2.15)

\( \phi_j \) and \( G_j \) each satisfy the governing equation.

\[
\nabla^2 \phi_j + k_j^2 \phi_j = 0
\]

(2.16)

\[
\nabla^2 G_j + k_j^2 G_j = 0
\]

(2.17)

An appropriate Green's function must be selected that satisfies (2.17). It follows that

\[
\int_s (\phi_j \nabla G_j - G_j \nabla \phi_j) \mathbf{n} ds = 0
\]

(2.18)

Using the vector relationships,
\[ \nabla G_j \cdot \mathbf{n} = \frac{\partial G_j}{\partial n} \quad \nabla \phi_j \cdot \mathbf{n} = \frac{\partial \phi_j}{\partial n} \quad (2.19) \]

(2.18) is rewritten as,

\[ \int_s (\phi_j \frac{\partial G_j}{\partial n} - G_j \frac{\partial \phi_j}{\partial n}) ds = 0 \quad (2.20) \]

The free space Green's function for the Helmholtz equation is (Chester, 1971)

\[ G_1 = \frac{i\pi}{2} H_0^{(1)}(k_1 r) \quad G_2 = \frac{i\pi}{2} H_0^{(1)}(k_2 r) \quad (2.21a,b) \]

where, \( H_0^{(1)} \) is the Hankel function of the first kind of order zero,

\[ r^2 = (x-x')^2 + (y-y')^2 \quad (2.22) \]

and

\[ P = (x,y) \quad Q = (x',y') \quad (2.23a,b) \]

where \( P \) is a base point, \( Q \) is a moving point; \( k_1 \) and \( k_2 \) are the wave numbers given by (2.9). The above Green's functions satisfy the respective governing equations in the fluid region except at \( P=Q \), where they exhibit a logarithmic singularity.

In region 1, to perform the integral of (2.20), a small circle is taken around the singular point in the region as shown in Figure 2.3. Then, (2.20) is divided into two parts.
\[
\int_{S_0} \left( \Phi_1 \frac{\partial G_1}{\partial n} - G_1 \frac{\partial \Phi_1}{\partial n} \right) \, ds
\]
\[= \lim_{\epsilon \to 0} \int_{\sigma} \left( \Phi_1 \frac{\partial G_1}{\partial n} - G_1 \frac{\partial \Phi_1}{\partial n} \right) \, d\sigma \quad (2.24)
\]

where, \( S_0 \) is the boundary of the pit and \( \sigma \) is a small circle around the singularity. The integral around the circle \( \sigma \) becomes

\[
\lim_{\epsilon \to 0} \int_0^{2\pi} \left( \Phi_1 \frac{\partial G_1}{\partial n} - G_1 \frac{\partial \Phi_1}{\partial n} \right) \, d\theta
\]
\[= \lim_{\epsilon \to 0} \int_0^{2\pi} \left( -\phi_1 \frac{i \pi}{2} \frac{2i}{\pi \epsilon} + \frac{i \pi}{2} \frac{2i}{\pi} \ln \epsilon \frac{\partial \Phi_1}{\partial r} \right) \, d\theta
\]
\[= 2\pi \phi(P) \quad (2.25)
\]

Since, when \( r \) is small

\[
H_0^{(1)}(k_1 r) = \frac{2i}{\pi} \ln k_1 r
\]
\[= -\frac{\partial}{\partial r} H_0^{(1)}(k_1 r) = H_1^{(0)}(k_1 r) = -\frac{2i}{\pi r} \quad (2.26)
\]

(2.24) can be rewritten as

\[
2\pi \phi_1(P)
\]
\[+ \int_{S_0} \left[ \Phi_1(Q) \frac{\partial G_1}{\partial n}(P,Q) - G_1(P,Q) \frac{\partial \Phi_1}{\partial n}(Q) \right] \, ds = 0 \quad (2.28)
\]

If the point \( P \) is moved to the boundary (Figure 2.3), (2.28) becomes,
\[ \pi \phi_1(P) \]
\[ + \int_{s_0} [\phi_1(Q) \frac{\partial G_1(P,Q)}{\partial n} - G_1(P,Q) \frac{\partial \phi_1(Q)}{\partial n}] ds = 0 \tag{2.29} \]

Similarly, applying Green's Second Identity to \( \phi_2 \) and \( G_2 \) over region 2 results in the following integral equation. In region 2, small circle is taken as Figure 2.4.

\[ \int_s \left( \phi_2 \frac{\partial G_2}{\partial n} - G_2 \frac{\partial \phi_2}{\partial n} \right) ds \]
\[ = \int_s \left( \phi_2 \frac{\partial G_2}{\partial n} - G_2 \frac{\partial \phi_2}{\partial n} \right) ds \]
\[ + \lim_{\epsilon \to 0} \int_{\sigma} \left( \phi_2 \frac{\partial G_2}{\partial n} - G_2 \frac{\partial \phi_2}{\partial n} \right) d\sigma \]
\[ + \int_{s_2} \left( \phi_2 \frac{\partial G_2}{\partial n} - G_2 \frac{\partial \phi_2}{\partial n} \right) ds = 0 \tag{2.30} \]

The second term in (2.30) becomes,

\[ \lim_{\epsilon \to 0} \int_{\sigma} \left( \phi_2 \frac{\partial G_2}{\partial n} - G_2 \frac{\partial \phi_2}{\partial n} \right) d\sigma \]
\[ = \lim_{\epsilon \to 0} \int_0^{2\pi} \left( \phi_2 \left( \frac{\partial G_2}{\partial \theta} - \frac{\partial \phi_2}{\partial \theta} \right) \epsilon d\theta \right) \]
\[ = \lim_{\epsilon \to 0} \int_0^{2\pi} \left( -\phi_2 \frac{1}{\epsilon} - \ln(\epsilon) \frac{\partial \phi_2}{\partial \theta} \right) \epsilon d\theta \]
\[ = -2\pi \phi_2(P) \tag{2.31} \]

The third term in (2.30) becomes,
\[ \int_{s_{1}} (\phi_{2} \frac{\partial G_{2}}{\partial n} - G_{2} \frac{\partial \phi_{2}}{\partial n})ds \]
\[ = \int_{s_{1}} [(\phi_{2}' + \phi_{2}) \frac{\partial G_{2}}{\partial n}ds - G_{2} \frac{\partial (\phi_{2}' + \phi_{2})}{\partial n}ds \]
\[ = \int_{s_{1}} (\phi_{2}' \frac{\partial G_{2}}{\partial n} - G_{2} \frac{\partial \phi_{2}'}{\partial n})ds \]
\[ + \int_{s_{1}} (\phi_{2} \frac{\partial G_{2}}{\partial n} - G_{2} \frac{\partial \phi_{2}'}{\partial n})ds \]  
\[ = 2 \pi \phi_{2}(P) \]  
\[ + \int_{s_{1}} (\frac{\partial G_{2}}{\partial n} - ik_{2} G_{2}) \phi_{2}' ds \]
\[ - \int_{s_{1}} (\frac{\partial \phi_{2}'}{\partial n} - ik_{2} \phi_{2}') G_{2} ds \]
\[ = 2 \pi \phi_{2}(P) \]

Since second and third terms of (2.32) tend to zero because of the radiation condition, (2.30) becomes

\[
\int_{s_{0}} [\phi_{2}(Q) \frac{\partial G_{2}}{\partial n}(P,Q) - G_{2}(P,Q) \frac{\partial \phi_{2}}{\partial n}(Q)] ds
\]
\[ + 2 \pi \phi_{2}(P) = \alpha \phi_{2}(P) \]  

where \( \alpha = 2 \pi \) if \( P \) is inside region 2 and \( \alpha = \pi \), if \( P \) is on a smooth portion of the boundary. Using the matching conditions between region 1 and 2 given by (2.10)-(2.12), (2.29) and (2.33) become,

\[
\int_{s_{0}} [\phi_{2}(Q) \frac{\partial G_{1}}{\partial n}(P,Q) - \frac{h}{d} G_{1}(P,Q) \frac{\partial \phi_{2}}{\partial n}(Q)] ds
\]
\[ + \pi \phi_{2}(P) = 0 \]
\[
\int_{S_0} \left[ \phi_2(Q) \frac{\partial G_2(P,Q)}{\partial n} - G_2(P,Q) \frac{\partial \phi_2(Q)}{\partial n} \right] ds \\
+ 2\pi \phi'(P) = \pi \phi_2(P)
\] (2.35)

The matching boundary conditions have allowed \( \phi_1 \) to be eliminated. Since \( G_1 \) and \( G_2 \) are known, the only unknown is \( \phi_2 \) along the boundary of the trench, \( S_0 \). The system is forced by the incident potential, \( \phi^i \).

2.2 Numerical Analysis Procedure

The integral equations may be solved by discretizing the integration contour \( S_0 \) into a finite number of small segments. The potentials and normal derivatives of the potentials are assumed to be approximated by an interpolation function, i.e. constant, linear, quadratic, etc. In this study, constant elements are used with each value being represented by the segment mid-point value.

Equations (2.34) and (2.35) can be rewritten in matrix form.

\[
[A_1] [\phi_2] + [B_1] [\partial \phi_2/\partial n] = 0
\] (2.36)

\[
[A_2] [\phi_2] + [B_2] [\partial \phi_2/\partial n] = [J]
\] (2.37)

where

\[
a^1_y = \delta_y + \frac{1}{\pi} \int_{S_y} \frac{\partial G_1(P,Q)}{\partial n} ds
\] (2.38)
\[ b^1_{ij} = -\frac{h}{d} \frac{1}{\pi} \int_{s_j} g_1(P,Q) \, ds \]  
(2.39) 

\[ a^2_{ij} = \delta_{ij} - \frac{1}{\pi} \int_{s_j} \frac{\partial G_2(P,Q)}{\partial n} \, ds \]  
(2.40) 

\[ b^2_{ij} = \frac{1}{\pi} \int_{s_j} G_2(P,Q) \, ds \]  
(2.41) 

\[ [I_j] = 2\phi'(p) \]  
(2.42) 

The coefficient matrices \( a_{ij} \) and \( b_{ij} \) can be integrated numerically by the Gaussian quadrature method or trapezoidal formula.

Solving for \( (\partial \phi / \partial n) \) yields

\[ ([B_2] - [A_2] [A_1^{-1}][B_1]) [\partial \phi / \partial n] = [I] \]  
(2.43) 

This procedure results in a system of simultaneous matrix equations for the normal derivative of the velocity potential. They can be solved by standard matrix techniques. The potential can then be recovered from (2.36). Once the potentials and normal derivatives are obtained, the potential at any point in the fluid can be calculated using (2.28) and (2.33).

Finally, the instantaneous free-surface elevations may be determined from the dynamic free surface boundary condition.

\[ \eta_j = \frac{1}{g} \frac{\partial \phi_j}{\partial t} \]  
(2.44) 

where, \( j \) is a nodal point on the free surface.
Figure 2.1 Definition sketch for a 2-D horizontal domain
Figure 2.2 Definition for the fluid domains
Figure 2.3 Region 1 surrounded by the boundaries $S_0$ and $\sigma$
Figure 2.4 Region 2 surrounded by the boundaries $S_0$, $\sigma$ and $S_\infty$. 
2.3 Results and Discussion

The notation used to define the geometry of the pits is given in Figure 2.5. The width, length and distance between pits are scaled by the wave length in region 2. The depths of the pits are scaled by the adjacent still water depth. This results in the dimensionless variables $\alpha$, $\beta$, $\delta$ and $\gamma$, respectively.

2.3.1 Single Pit Results

Because of the complexity of the multiple pit response, first the single pit is examined in some detail. The reference case is $h=1m$, $T=12sec$, $\alpha=1.0$, $\beta=0.5$ and $\gamma=3.0$. Figure 2.6 shows the diffraction pattern around a single pit. The diffraction coefficient is the modulus of the local wave height divided by the incident wave height. The diffraction pattern in the vicinity of the pit is quite complex. The location of the pit is shown on the figure. The contour lines in Figure 2.6a correspond to the values of the diffraction coefficient. In front of the pit, a pattern of partial standing waves is developed. However, in the lee of the pit, there is a shadow zone in which wave heights are reduced down to values of $K_D=0.4-0.6$. The shadow zone can be clearly seen in the surface projection shown in Figure 2.6b. This single pit provides a level of wave protection for long waves which exceeds the performance of many types of floating breakwaters. This performance also exceeds that of many common breakwaters, such as impermeable surface piercing caissons, which have the same horizontal size as the pit. This is shown in later section.

Variations in the parameters around the reference case are examined.
2.3.1a Sensitivity to the pit width

First consider the sensitivity to $\alpha$, the width of the pit. Figure 2.7 shows that for $\alpha$ values ranging from 1 to 5, the minimum diffraction coefficient in the lee of the pit is on the order of 0.25. It is not significantly improved as $\alpha$ increases.

The location and size of the shadow zone are strongly related to $\alpha$ as shown in Figure 2.8. $L_f$ indicates the distance from the front edge of the pit to the front edge of the shadow zone. $L_b$ is the distance from the front of the pit to the back edge of the shadow as shown in Figure 2.9. In Figure 2.6a, the shadow zone ($K_D < 0.5$) begins approximately 1 to 2 wave lengths behind the pit and has a length of about 4 wave lengths. For a pit width of 3 wave lengths, the shadow zone ($K_D < 0.5$) begins 7 wave lengths behind the structure and is more than 40 wave lengths long. The best fit to the data of $L_f$ and $L_b$ can be obtained by a least square analysis on the logarithm of the values. The best fit curves are expressed as,

$$\frac{L_f}{L} = 0.5931 e^{0.7746\alpha} \quad (2.45a)$$

$$\frac{L_b}{L} = 3.0987 e^{0.8652\alpha} \quad (2.45b)$$

These best fit curves are shown in Figure 2.8. From Figure 2.8, it is concluded that to provide reasonable protection, the minimum pit width is approximately one wave length. Otherwise, a very small shadow zone is developed. On the other hand, if the pit width exceed 3 wave lengths, the
shadow is very far behind the pit. This would require an extremely wide shelf. Therefore, practical pit widths are between 1 and 3 wave lengths.

The variation of the width of the shadow zone ($K_D < 0.5$) is shown in Figure 2.10 relative to the pit width ($\alpha L$). The width of the shadow zone relative to the pit width tends to increase slightly as $\alpha$ increases. For reasonable pit widths ($1 < \alpha < 3$), the shadow width is approximately the same as the pit width.

### 2.3.1b Sensitivity to the pit length

The second geometrical consideration is the length of the pit ($\beta L$). The dependency of the minimum diffraction coefficient on $\beta$ is shown in Figure 2.11. The reference values are employed for other parameters, ($T=12$sec, $h=1$m, $\alpha=1.0$, $\gamma=3$). It is seen that the diffraction coefficients decrease as $\beta$ increases except for $\beta=0.6$. This corresponds to a natural frequency of the system. It is also seen that for $\beta>0.5$, the minimum diffraction coefficient is generally 0.4 or less. Doubling the pit length from $\beta=0.5$ to $\beta=1.0$ results in $K_{D\min} \approx 0.1$. Figure 2.12 shows dependency of the shadow length on $\beta$. The shadow length increases rapidly as $\beta$ increases up to $\beta \approx 0.8$. However, after $\beta=0.8$, the increase is quite slow. The front of the shadow zone ($L_1$) is not sensitive to $\beta$. Since little protection is provided for $\beta < 0.5$, $\beta=0.5$ represents an minimum length. While larger values of $\beta$ provide somewhat better protection, the required excavation is significantly increased.
2.3.1c Sensitivity to the pit depth

Figure 2.13 shows the dependency of the minimum diffraction coefficient on the pit depth. As the pit depth increases, the protection provided also increases. Figure 2.14 shows the dependency of the shadow length on the pit depth. The shadow length increases as the pit depth increases. The rate of improvement in protection decreases as for \( \gamma > 3 \). The distance to the shadow is quite large for \( \gamma > 3 \). From these figures, it appears that an optimum value of \( \gamma \) is approximately 3.

In all of the above cases, the wave period was 12 seconds (\( h/L = 0.027 \)). Wave periods of 10-20 seconds (\( h/L = 0.016-0.032 \)) were also examined. Results are shown in Figure 2.15. It is seen that there is a very small dependency of the minimum diffraction coefficient and shadow length on the wave period. As shown in Figure 2.16, the same lack of sensitivity is also true for the water depth, provided the period is large enough to have long waves.

2.3.1d Sensitivity to the wave angle

Up to this point, all waves have been normally incident to the structure. Typically, waves can come from a variety of directions. However, because the pit is located in shallow water, significant refraction would be expected. For an offshore wave angle of 70°, the angle at the pit is less than 20°. Figure 2.17 shows the local wave angle as a function of the offshore angle, computed by Snell's law. In general, wave angles at the pit will be 20° or less.

Figure 2.18 shows the influence of wave direction on the minimum diffraction
coefficient. Wave direction has little influence on the minimum diffraction coefficient. The nomenclature used to define the location and the size of the shadow zone are shown in Figure 2.19. As shown in Figure 2.20, the location of the shadow zone is nearly constant and is same as that for normal incidence. Wave direction does not affect the width of the shadow zone as shown Figure 2.21. However, the shore parallel width of shadow zone increases slightly. It is clear that the width of the shadow zone can be well estimated given the wave angle and pit width.

If waves approach from some range of angles, $\pm \theta$, the overlap width can be considered as follows. Figure 2.22 shows the idealized geometry of the shadow zones. The shadow zone ($K_D < 0.5$) is taken to be a rectangular shape and of width $\alpha L$. This follows from Figure 2.21. The distance between front edge of the pit and front edge of the shadow zone, $L_f$ is independent on the wave angle (Figure 2.20) and is approximately given by (2.45).

From the geometry, the various length are given by

\[ L_1 = L_f \tan \theta \]
\[ L_2 = \frac{\alpha L}{2} - L_1 = \frac{\alpha L}{2} - L_f \tan \theta \]
\[ L_3 = L_2 \cos \theta \]
\[ = \left( \frac{\alpha L}{2} - L_f \tan \theta \right) \cos \theta \]

The overlap width $2L_3$ relative to the pit width $\alpha L$ is expressed as,
\[
\frac{2L_3}{\alpha L} = \left( 1 - \frac{2L}{\alpha L} \tan \theta \right) \cos \theta
\]  
(2.47)

Substituting (2.45) into above equation yields

\[
\frac{2L_3}{\alpha L} = \left( 1 - \frac{2}{\alpha} 0.5931 e^{0.7746 \alpha \tan \theta} \right) \cos \theta
\]  
(2.48)

Figure 2.23 shows the overlap length related to the pit width for different wave angles and pit widths. For the reference case data, \(T=12\text{sec}, h/L_0 \approx 0.0044\) an offshore wave angle of \(\pm 45^\circ\) would have a local angle of approximately \(\pm 10^\circ\) (Figure 2.17). The width of the shadow would be approximately 0.54 wave lengths (Figure 2.23).
Figure 2.5 Definition sketch for the pit geometries
Figure 2.6a  Diffraction coefficient contours (single pit: $T=12$ sec, $h=1$ m, $\alpha=1$, $\beta=0.5$, $\gamma=3$)
Figure 2.6b  Surface projection of diffraction coefficient (single pit: $T=12\text{sec}$, $h=1m$, $\alpha=1$, $\beta=0.5$, $\gamma=3$)
Figure 2.7  Dependency of the minimum diffraction coefficient on the pit width $\alpha L$ (single pit: $T=12$ sec, $h=1$ m, $\beta=0.5$, $\gamma=3$)
Figure 2.8  Dependency of the location of the shadow zone on the pit width $\alpha L$ (single pit: $T=12$ sec, $h=1$ m, $\beta=0.5$, $\gamma=3$)
Figure 2.9  Definition of the size of the shadow zone (normal incidence case)
Figure 2.10 Dependency of the shadow width on the pit width $\alpha L$ (single pit: $T=12$ sec, $h=1$ m, $\beta=0.5, \gamma=3$)
Figure 2.11  Dependency of the minimum diffraction coefficient on the pit length $\beta L$ (single pit: $T=12$ sec, $h=1$ m, $\alpha=1$, $\gamma=3$)
Figure 2.12 Dependency of the location of the shadow zone on the pit length $\beta L$ (single pit: $T=12\text{sec}$, $h=1\text{m}$, $\alpha=1$, $\gamma=3$)
Figure 2.13  Dependency of the minimum diffraction coefficient on the pit depth $\gamma h$ (single pit: $T=12$ sec, $h=1$ m, $\alpha=1$, $\beta=0.5$)
Figure 2.14  Dependency of the location of the shadow zone on the pit depth $\gamma h$ (single pit: $T=12$ sec, $h=1$ m, $\alpha=1$, $\beta=0.5$)
Figure 2.15 Variation of the minimum diffraction coefficient and the shadow length on the relative water depth (single pit: $h=1\text{m}$, $\alpha=1$, $\beta=0.5$, $\gamma=3$)
Figure 2.16  Dependency of the minimum diffraction coefficient on the water depth \( h \) (single pit: \( T=20 \text{ sec}, \alpha=1, \beta=0.5 \))
Figure 2.17  Local wave angle at pit location (h=1m)
Figure 2.18 Dependency of the minimum diffraction coefficient on the wave angle $\theta$ (single pit: $T=12$ sec, $h=1$ m, $\alpha=1$, $\beta=0.5$, $\gamma=3$)
Figure 2.19 Definition sketch of the size of the shadow zone (oblique incidence case)
Figure 2.20  Dependency of the location of the shadow zone on the wave angle $\theta$ (single pit: $T=12$ sec, $h=1$ m, $\alpha=1$, $\beta=0.5$, $\gamma=3$)
Figure 2.21  Dependency of the width of the shadow zone on the wave angle $\theta$
(single pit: $T=12$ sec, $h=1$ m, $\alpha=1$, $\beta=0.5$, $\gamma=3$)
Figure 2.22  Overlap of the shadow zones
Figure 2.23  Overlap length of the shadow zones ($K_D < 0.5$) on the wave angle (single pit: $T=12\text{sec}, h=1\text{m}, \beta=0.5, \gamma=3$)
2.3.2 Multiple Pit Results

2.3.2a Sensitivity to the pit width

We now proceed to examine the diffraction field due to two pits. Although a variety of pit locations were examined, placing the second pit in the lee of the first pit yields optimum results. A typical result for the diffraction pattern is shown in Figure 2.24 for two identical pits with $\alpha=1.0$, $\beta=0.5$, $\gamma=3.0$, and $\delta=0.5$; that is distance between the two pits is half a wave length. Figure 2.24a shows diffraction coefficient contours and Figure 2.24b, a surface projection. The minimum diffraction coefficient is much less than that of single pit. Diffraction coefficients as low as $K_d=0.1$ are observed behind the pit.

As with the single pit model, the size of the shadow zone can be expressed as a function of the width of pits ($\alpha_1L$, $\alpha_2L$). However, since the diffraction coefficient is much less than that for the single pit, the shadow zone is taken as the area in which the diffraction coefficient is less than 0.3. These results are shown in Figure 2.25 and are quite similar to a single pit. From a least square analysis on the logarithms of the values, $L_f$ and $L_b$ are approximated as,

\[
\frac{L_f}{L} = 1.1035 e^{0.7772\alpha} \quad (2.49a)
\]

\[
\frac{L_b}{L} = 2.0938 e^{0.9678\alpha} \quad (2.49b)
\]
2.3.2b Sensitivity to the pit spacing

The sensitivity of the minimum diffraction coefficient to the pit spacing $\delta$ is shown in Figure 2.26. There is a surprising lack of sensitivity of the minimum diffraction coefficient to spacing between the pits. Somewhat lower diffraction coefficients are obtained for specific values. However, this effect is rather minor. In general, low diffraction coefficients are obtained over a wide range of pit spacings. Figure 2.27 shows the length of shadow zone behind the two pits with a diffraction coefficient, $K_D < 0.3$. The length of this shadow zone is also relatively insensitive to $\delta$.

2.3.2c Three pit results

The model was then run with three pits in a variety of locations. Figure 2.28 shows typical results for three identical pits with $\alpha=1.0$, $\beta=0.5$, $\gamma=3.0$ and $\delta=0.5$. A rather surprising result is that the performance as a breakwater is not significantly improved by adding a third pit. A number of placement schemes were considered. Pits were placed in so that shadow zones all superimposed at the same location. They were also placed such that the one pit was in the shadow zone of the preceding pit. It was found that little improvement was obtained over the two pit scenario. Therefore, only two pits are required in this application as the third pit provides little additional benefit for the costs associated construction.
2.3.2d Sensitivity to the wave angle

Figure 2.29 shows the width of the shadow zone \((K_D < 0.3)\) as a function of the incident wave angle \(\theta\). As in the single pit case, wave direction has little affect the shadow zone width for reasonable angles.

The shadow overlap width for a variable wave direction of \(\pm \theta\) can be estimated in the same manner as for the single pit. \(L_I\) is approximately given by (2.48) and, substituting into (2.46) yields,

\[
\frac{2L_i}{L} = \left(1 - \frac{2}{\alpha} \cdot 1.1035 e^{0.7772 \alpha \tan \theta} \cos \theta \right)
\]

These results are shown in Figure 2.30 for several pit widths, \(\alpha\). The dependency on wave direction is similar to the single pit; the greater the wave angle, the narrower the shadow zone. However, in the two pit case, the overlap width increases as \(\alpha\) increases which is opposite to the single pit case. For the reference data and \(\delta = 0.5\), the shadow width for two pits with and offshore wave angle of \(\pm 45^\circ\) is approximately 0.45 wave length.
Figure 2.24a  Diffraction coefficient contours (two pits: $T=12$ sec, $h=1$ m, $\alpha_1=\alpha_2=1$, $\beta_1=\beta_2=0.5$, $\gamma_1=\gamma_2=3$)
Figure 2.24b  Surface projection of diffraction coefficient (two pits: $T=12\text{sec}$, $h=1\text{m}$, $\alpha_1=\alpha_2=1$, $\beta_1=\beta_2=0.5$, $\gamma_1=\gamma_2=3$)
Figure 2.25  Dependency of the length of the shadow zone on the pit width \( \alpha L \) (two pits: \( T=12 \text{sec}, h=1 \text{m}, \beta_1=\beta_2=0.5, \gamma_1=\gamma_2=3 \))
Figure 2.26  Dependency of the minimum diffraction coefficient on the pit spacing $\delta L$ (two pits: $T=12$ sec, $\alpha=1$, $\beta_1=\beta_2=0.5$, $\gamma_1=\gamma_2=3$)
Figure 2.27  Dependency of the shadow length on the pit spacing \( \delta L \) (two pits: \( T=12 \text{sec} \), \( h=1 \text{m} \), \( \alpha_1=\alpha_2=1 \), \( \beta_1=\beta_2=0.5 \), \( \gamma_1=\gamma_2=3 \))
Figure 2.28a  Diffraction coefficient contours (three pits: $T=12\text{sec}$, $h=1\text{m}$, $\alpha_1=\alpha_2=\alpha_3=1$, $\beta_1=\beta_2=\beta_3=0.5$, $\gamma_1=\gamma_2=\gamma_3=3$)
Figure 2.28b  
Surface projection of diffraction coefficient (three pits: \( T = 12\, \text{sec} \), \( h = 1\, \text{m} \), 
\( \alpha_1 = \alpha_2 = \alpha_3 = 1 \), \( \beta_1 = \beta_2 = \beta_3 = 0.5 \), \( \gamma_1 = \gamma_2 = \gamma_3 = 3 \))
Figure 2.29  Dependency of the shadow width ($K_D < 0.3$) on the wave angle $\theta$
(two pits: $T = 12$ sec, $h = 1$ m, $\alpha_1 = \alpha_2 = 1$, $\beta_1 = \beta_2 = 0.5$, $\gamma_1 = \gamma_2 = 3$)
Figure 2.30  Overlap length of the shadow zones \((K_D < 0.3)\) (two pits: \(T = 12\) sec, \(h = 1\) m, \(\beta_1 = \beta_2 = 0.5\), \(\gamma_1 = \gamma_2 = 3\))
2.3.3 Frequency Response

Real ocean waves are usually composed of several different frequency components. Since this pit is a linear system, the response to random waves can be estimated using superposition. The JONSWAP spectrum (Appendix A) is used as the incident wave model. The responses of the diffraction coefficients at three fixed locations in the lee of the pit are examined. Figure 2.31 shows the responses of the diffraction coefficient at these three fixed points. However, since this numerical model is based on long wave approximation, the results for \( f < 0.12 \) Hz are less accurate.

Diffraction coefficients approach unity as \( f \to 0 \). In this limit the waves are much longer than the pit. As \( \alpha \to 0 \), the influence of the pit diminishes. As the wave frequency increases, diffraction coefficients tend to decrease except at \( f \approx 0.095 \). This value corresponds to \( \beta \approx 0.6 \) in Figure 2.11.

Figures 2.32-2.34 show wave spectra, for incident and diffracted waves at the three locations. Integrating the spectrum yields an average or bulk diffraction coefficient

\[
\overline{K_D} = \sqrt{\frac{E_{DD}}{E_{II}}} \quad (2.51)
\]

where \( E \) is the total energy in the incident or diffracted wave. The bulk diffraction coefficient at \( x = 100 \) m is \( \overline{K_D} = 0.48 \), \( \overline{K_D} = 0.495 \) at \( x = 200 \) m, and \( \overline{K_D} = 0.552 \) at \( x = 300 \).
Figure 2.31 Frequency response of the diffraction coefficient at three locations in the lee of the pit (single pit: \( h = 1 \text{m} \), \( d = 3 \text{m} \), \( a = 37.4 \text{m} \), \( b = 18.7 \text{m} \))
Figure 2.32  Spectrum at 100m behind the pit (single pit: h=1m, d=3m, a=37.4m, b=18.7m)
Figure 2.33  Spectrum at 200m behind the pit (single pit: $h=1m$, $d=3m$, $a=37.4m$, $b=18.7m$)
Figure 2.34  Spectrum at 300m behind the pit (single pit: h=1m, d=3m, 
a=37.4m, b=18.7m)
2.3.4 Wave Reduction in a Navigation Channel

The pit concept can be applied to a variety of applications. Considering the previous discussion, an offshore pit is selected to protect a dredged navigation channel from long swell waves having a period of 18 sec. The channel is 100 m wide, 1000 m long and 12 m deep. The adjacent, undredged depth is 6 m. Figure 2.35 shows a contour map and a surface projection of the diffraction coefficients for the original navigation channel. At the seaward end of the channel, the incident wave height is amplified by more than 40% due to interactions with the channel. There are also several locations within the channel where wave heights are amplified.

To reduce the wave height, an offshore pit is dredged seaward of the navigation channel. It is noted in Figure 2.35 that the seaward 50% of the channel experiences the worst wave conditions. Therefore, it is necessary to select a pit that has a shadow in this region approximately 500 m long. From the single pit results, it is estimated that the pit width should be more than one wave length and it should be placed less than two wave length seaward of the end of the channel. The length of the pit corresponds to $\beta = 0.5$ and the depth to $\gamma = 2$.

The resulting diffraction coefficients are shown in Figure 2.36. Again, both a contour and a surface projection are given. In Figure 2.36a, it is seen that the wave heights in the channel have been significantly reduced. Maximum diffraction coefficients are on the order of $K_D = 0.5$. Figure 2.36b
shows how the shadow falls on the channel. The result is a valley of low waves which correspond to the location of the navigation channel. The diffraction coefficients along the center line of the channel are shown in Figure 2.37. There is clearly a significant reduction in wave heights in the navigation channel.

2.3.5 Comparison with Other Structures

The degree of wave reduction provided by a pit can be compared with other common structures. These include a submerged breakwater and a surface piercing impermeable breakwater, as shown in Figure 2.38. The same numerical method is utilized for the submerged breakwater as for the pit. For surface piercing breakwater, a BEM solution was developed using the long wave approximations. To make a fair comparison, the horizontal sizes for all structures are taken as same. That is, they all have same base dimensions, width a and length b. Figures 2.39 - 2.41 show contour maps and surface projections for each case respectively.

Intuitively, the diffraction fields in the lee of the structures are different for each case. However, the shadow zone behind the pit is the widest and has the lowest diffraction coefficients. The diffraction field behind the submerged breakwater actually increases the wave height. This is because refraction/diffraction over a bump (submerged breakwater) focuses wave energy. The diffraction field in the lee of the surface piercing breakwater is
similar to the pit. However, the values are lower for the pit.

This comparison is a simple case with relatively small sized structures. However, these results indicate the pit is, in fact, a more effective means of providing protection in long waves than conventional structures.
Figure 2.35a  Diffraction coefficient contours for a navigation channel (T=18sec, h=6m, d=12m)
Figure 2.35b  Surface projection for a navigation channel ($T=18\text{sec}$, $h=6\text{m}$, $d=12\text{m}$)
Figure 2.36a  Diffraction coefficient contours for a navigation channel with an offshore pit
(T=18sec, h=6m, d=12m)
Figure 2.36b  Surface projection for a navigation channel with an offshore pit (T=18sec, h=6m, d=12m)
Figure 2.37  Diffraction coefficients along the center line of the navigation channel (T=18sec, h=6m, d=12m)
Figure 2.38 Structures compared with a pit (h=1m, d=3m, d₁=0.5m, b=18.7m)
Figure 2.39a  Diffraction coefficient contours for a single pit (T=12sec, h=1m, d=3m, α=1, β=0.5)
Figure 2.39b  Surface projection for a single pit (T=12sec, h=1m, d=3m, α=1, β=0.5)
Figure 2.40a Diffraction coefficient contours for a submerged breakwater (T=12sec, h=1m, $d_t=0.5m$, $\alpha=1$, $\beta=0.5$)
Figure 2.40b  Surface projection for a submerged breakwater (T=12sec, h=1m, d₁=0.5m, α=1, β=0.5)
Figure 2.41a  Diffraction coefficient contours for a surface piercing breakwater ($T=12\text{sec}$, $h=1\text{m}$, $\alpha=1$, $\beta=0.5$)
Figure 2.41b  Surface projection for a surface piercing breakwater (T=12sec, h=1m, α=1, β=0.5)
3.0 WAVE DIFFRACTION DUE TO AN OPEN TRENCH

3.1 Theoretical Formulation

3.1.1 Boundary Value Problem

3.1.1a 2-D vertical domain (normal incidence)

The fluid domain under consideration is shown in Figure 3.1. The problem is idealized as two-dimensional in Cartesian coordinates. A coordinate system is defined with the z-axis positive upwards, and the x-axis directed to onshore from the center of the trench along the SWL.

The fluid is assumed to be incompressible and irrotational. The velocity potential can be expressed by

\[ \Phi(x,z,t) = \text{Re}[\phi(x,z)e^{-j\omega t}] \]  

where, \( \phi(x,z) \) is the spatially dependent component of the velocity potential, \( i = \sqrt{-1} \), and \( \omega \) is the angular frequency of the simple harmonic wave motion. The fluid velocity vector is given by

\[ \vec{q} = -\nabla \Phi \]  

Continuity of an incompressible fluid requires that the velocity potential \( \phi(x,z) \) satisfy the 2-D Laplace's equation.

\[ \nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \]
At the still water level, the linear combined free surface boundary condition is applied, and at the bottom a no flow condition.

\[ \phi_x - \frac{\omega^2}{g} \phi = 0 \quad \text{on } S_f \]  
\[ \phi_n = 0 \quad \text{on } S_{B},S_{P} \]

where the subscripts on the velocity potentials denote partial differentiation, \( n \) is the normal direction to the boundary outward from the fluid domain and \( g \) is the acceleration due to gravity. Far from the trench, the velocity potential must satisfy the radiation condition

\[ \lim_{x \to \pm \infty} \left( \frac{\partial \phi}{\partial x} + ik \phi \right) = 0 \quad \text{on } S_{\infty} \]  

From linear wave theory, the incident velocity potential \( \phi^i \) can be expressed as

\[ \phi^i = \frac{gH}{2\omega} \frac{\cosh k(z+h)}{\cosh kh} e^{ikx} e^{-i\omega t} \]  

and the wave number \( k \) is determined from the dispersion relation.

\[ \frac{\omega^2}{g} = k \tanh kh \]  

3.1.1b Quasi 3-D domain (oblique incidence)

In this section, an infinitely long trench is considered. The fluid domain is divided into 3 regions as shown in Figure 3.2. The trench has dimensions \((d-h) \times b\) in \( x-z \) plane and is infinitely long in \( y \)-direction. The water depth
outside the trench is uniform, \( h \). The incident wave has an angle \( \theta_i \) to the positive x-axis and propagates from left to right. The coordinate system is defined the same as the previous section for x and z-axes, while the y-axis is extending along the trench.

The velocity potential \( \phi(x,y,z) \) must satisfy the 3-D Laplace’s equation.

\[
\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{3.9}
\]

The wave number of the incident wave in region 1 is denoted by \( k_1 \). The y-component of the wave number \( m \), is

\[
m = k_1 \sin \theta_1 \tag{3.10}
\]

Snell’s law for refraction due to changes in the water depth yields,

\[
m = k_j \sin \theta_j = \text{constant} \quad (j=1,2,3) \tag{3.11}
\]

Therefore, the velocity potential \( \phi_j(x,y,z) \) in each region may be written as

\[
\phi_j(x,y,z) = \phi_j(x,z)e^{imy} \quad (j=1,2,3) \tag{3.12}
\]

Substituting \( \phi_j(x,y,z) \) into Laplace’s equation (3.9) yields a modified Helmholtz equation.

\[
\frac{\partial^2 \phi_j}{\partial x^2} + \frac{\partial^2 \phi_j}{\partial z^2} - m^2 \phi_j = 0 \quad (j=1,2,3) \tag{3.13}
\]

The boundary conditions are the same as the previous section, except the radiation condition. In the quasi 3-D case, the radiation condition is expressed as,
\[
\lim_{x \to \pm \infty} (\frac{\partial \phi_j}{\partial x} \pm il_j \phi_j) = 0
\]  
(3.14)

where,

\[
k_j^2 = l_j^2 + m^2
\]  
(3.15)

The incident wave in x-z plane can be expressed as

\[
\phi^I(x,z) = \frac{gH}{2\omega} \frac{\cosh k_1(z+h)}{\cosh k_1 h} e^{il_1 x}
\]  
(3.16)

where,

\[
\frac{\omega^2}{g} = k_1 \tanh k_1 h_1
\]  
(3.17)

### 3.1.2 Method of Analysis by the Boundary Element Method

The boundary value problem described in the previous section can be solved numerically utilizing the Boundary Element Method. Although the methodology is similar to the 2-D horizontal problem described in Chapter 2, this model can be applied in an arbitrary water depth.

Applying Green's Second Identity to the potential \(\phi\) and the free space Green's function for (3.3) yields,

\[
\iint_A (\phi \nabla^2 G_1 - G_1 \nabla^2 \phi) \, dA = \int_S (\phi \nabla G_1 - G_1 \nabla \phi) \cdot \hat{n} \, ds
\]  
(3.18)
Also, using (3.13) yields

\[ \int \int_A [\Phi(\nabla^2 G_2 - m^2 G_2) - G_2(\nabla^2 \phi - m^2 \phi)] dA \]

\[ = \int \int_A (\phi \nabla^2 G_2 - G_2 \nabla^2 \phi) dA \]

\[ = \int_S (\phi \nabla G_2 - G_2 \nabla \phi) \, \hat{n} ds \]  \hspace{1cm} (3.19)

where \( G_1 \) and \( G_2 \) are the free space Green's functions for each governing equation. \( \phi \) and \( G \) are chosen so that they satisfy the governing equations; namely

\[ \nabla^2 \phi = \nabla^2 G_1 = 0 \]  \hspace{1cm} (3.20)

\[ \nabla^2 \phi - m^2 \phi = \nabla^2 G_2 - m^2 G_2 = 0 \]  \hspace{1cm} (3.21)

Then, both (3.18) and (3.19) become

\[ \int_S (\phi \frac{\partial G}{\partial n} - G_2 \frac{\partial \phi}{\partial n}) ds = 0 \]  \hspace{1cm} (3.22)

The free space Green's functions can be chosen as

\[ G_1 = \ln r \]  \hspace{1cm} (3.23)

\[ G_2 = -K_0(mr) \]  \hspace{1cm} (3.24)

where, \( K_0 \) is the modified Bessel function of the second kind of order zero, and \( r \) is the distance between a base point \( P(x,z) \) and a moving point \( Q(x',z') \),

\[ r^2 = (x-x')^2 + (z-z')^2 \]  \hspace{1cm} (3.25)

\( G_1 \) and \( G_2 \) have logarithmic singularities at \( P=Q \).
As in the 2-D horizontal problem in Chapter 2, (3.22) can be integrated by taking small semi-circle \( \sigma \) on the boundary shown in Figure 3.3.

In the 2-D vertical problem,

\[
\int_s \left( \phi \frac{\partial G_1}{\partial n} - G_1 \frac{\partial \phi}{\partial n} \right) ds + \lim_{\epsilon \to 0} \int_\sigma \left( \phi \frac{\partial G_1}{\partial n} - G_1 \frac{\partial \phi}{\partial n} \right) d\sigma = 0 \tag{3.26}
\]

Since,

\[
G_1 = \ln r, \quad \frac{\partial G_1}{\partial n} = -\frac{\partial G_1}{\partial r} = -\frac{1}{r}
\]

the second term becomes

\[
\lim_{\epsilon \to 0} \int_0^{\pi} \left( -\frac{1}{\epsilon} + \ln \epsilon \frac{\partial \phi}{\partial r} \right) \epsilon d\theta = -\pi \phi(p) \tag{3.27}
\]

so (3.26) can be rewritten as,

\[
\pi \phi(p) = \int_s \left( \phi \frac{\partial G_1}{\partial n} - G_1 \frac{\partial \phi}{\partial n} \right) ds \tag{3.28}
\]

In the quasi 3-D problem, (3.22) can be rewritten as,

\[
\int_s \left( \phi \frac{\partial G_2}{\partial n} - G_2 \frac{\partial \phi}{\partial n} \right) ds + \lim_{\epsilon \to 0} \int_\sigma \left( \phi \frac{\partial G_2}{\partial n} - G_2 \frac{\partial \phi}{\partial n} \right) d\sigma = 0 \tag{3.29}
\]

where \( G_2 = \ln mr, \quad \frac{\partial G_2}{\partial n} = -\frac{\partial G_2}{\partial r} = -\frac{1}{r} \quad \text{when} \quad r \to 0 \)

and the second term becomes
\[
\lim_{e \to 0} \int_0^\pi \left( -\phi \frac{1}{e} + \ln m \epsilon \frac{\partial \phi}{\partial r} \right) e d\theta = -\pi \phi(p) \tag{3.30}
\]

Therefore, (3.22) can be written as same as (3.28).

\[
\pi \phi(p) = \int_s \left( \phi \frac{\partial G_2}{\partial n} - G_2 \frac{\partial \phi}{\partial n} \right) ds \tag{3.31}
\]
Figure 3.1  Definition sketch for a 2-D vertical domain
Figure 3.2 Definition sketch for a quasi 3-D domain
Figure 3.3 Definition of the semi-circle $\sigma$
3.2 Numerical Analysis Procedure

The integral equations can be solved by discretizing the boundary. It is assumed the potentials and their normal derivatives are constant in each segment. Equation (3.28) can be expressed in matrix form

\[ [A][\phi] + [B]\left[\frac{\partial \phi}{\partial n}\right] = 0 \]  

(3.31)

where,

\[ [a_{ij}] = -\delta_{ij} + \frac{1}{\pi} \int_{s_j} \frac{\partial \ln r_{ij}}{\partial n} ds \]  

(3.32)

\[ [b_{ij}] = -\frac{1}{\pi} \int_{s_j} \ln r_{ij} ds \]  

(3.33)

\[ \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \]  

(3.34)

The integral of the Green's function and its normal derivative can be evaluated for the geometry given in Figure 3.4.

\[ E_{ij} = \int_{s_j} \ln r_{ij} ds_p \]  

(3.35)

\[ F_{ij} = \int_{s_j} \frac{\partial}{\partial n} (\ln r_{ij}) ds_p \]  

(3.36)

In Figure 3.4, P is a nodal point on the element C_i, Q is a moving point on the element C_j, r is a distance from P to Q, n is the normal vector of the element
C, and \( ds_p \) is a small line segment of \( C \). In the general case, that is, \( C \neq C \), the \( \xi \)-axis and \( \eta \)-axis are taken such that the \( \xi \)-axis is parallel to \( C \). This yields

\[
r = \frac{D}{\cos \theta}
\]

(3.37)

\[
ds_p = \frac{D}{\cos^2 \theta} d\theta
\]

(3.38)

\[
E_{ij} = \int_{\theta_1}^{\theta_2} \ln \left( \frac{D}{\cos \theta} \right) \frac{D}{\cos^2 \theta} d\theta
\]

\[
= D[\tan \theta \ln(D \sec \theta) - 1]_{\theta_1}^{\theta_2}
\]

(3.39)

\[
= [r \sin \theta \ln(r - 1) + \theta \cos \theta]_{\theta_1}^{\theta_2}
\]

\[
= r_2[\sin \theta_2 \ln(r_2 - 1) + \theta_2 \cos \theta_2] - r_1[\sin \theta_1 \ln (r_1 - 1) + \theta_1 \cos \theta_1]
\]

\[
F_{ij} = \int_{\theta_1}^{\theta_2} \frac{\partial}{\partial r} (\ln r) \left( \frac{\partial r}{\partial \xi} \frac{\partial \xi}{\partial \eta} + \frac{\partial r}{\partial \eta} \frac{\partial \eta}{\partial \eta} \right) ds_p
\]

\[
= \int_{\theta_1}^{\theta_2} \frac{1}{r} \cos \theta D \sec^2 \theta d\theta
\]

(3.40)

\[
= \int_{\theta_1}^{\theta_2} \frac{\cos \theta D \sec^2 \theta}{D \sec \theta} d\theta
\]

\[
= \int_{\theta_1}^{\theta_2} d\theta
\]

\[
= (\theta_2 - \theta_1)
\]
For the case when $C_i = C_j$, a special integration is required.

\[ E_{ii} = 2 \int_0^{l_2} \ln r \, dr \]
\[ = 2[r \ln r - r]_0^{l/2} \]
\[ = 2\left[\frac{l_i}{2} \ln \frac{l_i}{2} - \frac{l_i}{2}\right] \]
\[ = l_i \left[\ln \frac{l_i}{2} - 1\right] \quad (3.41) \]

\[ F_{ii} = \int \frac{\partial}{\partial r} (\ln r) \frac{\partial r}{\partial n} ds_p \]
\[ = \int \frac{1}{c_i r} \left( \frac{\partial r}{\partial x} \frac{\partial x}{\partial n} + \frac{\partial r}{\partial z} \frac{\partial z}{\partial n} \right) ds_p \]
\[ = \int \frac{1}{c_i r} \left[ \cos \alpha \sin \alpha + \sin \alpha (-\cos \alpha) \right] ds_p \]
\[ = \int \frac{1}{c_i r} \left( 0 \right) ds_p = 0 \quad (3.42) \]

In the quasi 3-D domain, the coefficient matrices are generally computed numerically using Gaussian quadrature or trapezoidal methods. At the singular element $(i=j)$ and at elements at close distances, special integrals are used.
\[ [a_{ij}] = -\delta_{ij} + \frac{1}{\pi} \int s_j \frac{\partial}{\partial n} [-K_o(mr)] ds \quad (3.43) \]

\[ [b_{ij}] = -\frac{1}{\pi} \int s_j [-K_o(mr)] ds \quad (3.44) \]

In a singular element,

\[ [a_{ii}] = -1 + \frac{1}{\pi} \int s_i \frac{\partial}{\partial n} \{-\ln(mr)\} ds \]

\[ = -1 - \frac{1}{\pi} \int s_i \frac{1}{r \partial n} ds \quad (3.45) \]

\[ = -1 \]

\[ [b_{ii}] = -\frac{1}{\pi} \int s_i \ln(mr) ds \]

\[ = -\frac{2}{\pi} \int_0^{\pi/2} \ln(mr) dr \]

\[ = -\frac{2}{\pi} m \left[ r \ln(mr) - r \right]_0^{\pi/2} \]

\[ = -\frac{m}{\pi} \ln \left[ \frac{ml_i}{2} - 1 \right] \]
When points P and Q are very close, that is, r is small, a special integration is required because numerical integration tends to give large errors. When r is small, the modified Bessel function is approximately

\[ G = -K_0(mr) = \ln(mr) \] (3.47)

\[ \frac{\partial G}{\partial r} = m \frac{1}{mr} - \frac{1}{r} \]

In the local coordinate system (ξ-η) described in Figure 3.5, it is possible to integrate these functions analytically. Analytical integration corresponding to the ξ-η local coordinate system can be done as follows.

\[ E_y = \frac{1}{\pi} \int G_{ij} ds_j \]

\[ = \frac{1}{\pi} \int \ln(mr) ds_j \]

\[ = \frac{1}{\pi} \int \ln(m) + \frac{1}{2} \ln(\xi^2 + \eta_i^2)] ds_j \]

\[ = \frac{1}{\pi} \int_{\xi_1}^{\xi_2} \ln(m) + \frac{1}{2} \ln(\xi^2 + \eta_i^2) d\xi \]

\[ = \frac{1}{\pi} \ln(m) \Delta \xi \]

\[ + \frac{1}{2\pi} [\xi \ln(\xi^2 + \eta_i^2) - 2\xi + 2\eta_i \tan^{-1} \frac{\xi}{\eta_i}]_{\xi_1}^{\xi_2} \]

If \( \eta_i = 0 \), \( E_{ij} \) becomes
\[ E_{ij} = \frac{1}{\pi} \int_{\xi_1}^{\xi_2} (\ln m + \frac{1}{2} \ln \xi^2) d\xi \]

\[ = \frac{1}{\pi} \ln m \Delta l_j + \frac{1}{2\pi} [\xi \ln \xi^2 - 2\xi^2]_{\xi_1}^{\xi_2} \]

where, \( \Delta l_j \) is the length of element \( j \).

\[ F_{y_j} = \frac{1}{\pi} \int_j \frac{\partial G_{yj}}{\partial n} ds_j \]

\[ = \frac{1}{\pi} \int_j \frac{\partial G_{yj}}{\partial r} \frac{\partial r}{\partial n} ds_j \]

\[ = \frac{1}{\pi} \int_{\xi_1}^{\xi_2} \frac{1}{\sqrt{\xi^2 + \eta_i^2}} \frac{\eta_i}{\sqrt{\xi^2 + \eta_i^2}} d\xi \]

\[ = \frac{\eta_i}{\pi} \int_{\xi_1}^{\xi_2} \frac{1}{\xi^2 + \eta_i^2} d\xi \]

\[ = \frac{1}{\pi} \left( \tan^{-1} \frac{\xi}{\eta_i} \right)_{\xi_1}^{\xi_2} \]

If \( \eta_i = 0 \), then

\[ F_{y_j} = 0 \quad \text{since,} \quad \frac{\partial r}{\partial n} = 0 \]
Figure 3.4 Line integral for the Green's function
Figure 3.5 The $\xi-\eta$ local coordinate system
3.3 Results and Discussion

In wave reduction studies, the effect of the structure is typically characterized by the transmission and reflection coefficients. For waves approaching at an angle, it is the x-component of the wave which is used to describe reflection and transmission. The transmission coefficient is denoted by $K_T$ and reflection coefficient is $K_R$. Following the 2-D horizontal results, the base geometry for the trench is taken as, $h=1\text{m}$, $d=3\text{m}$ ($\gamma=3$), $b=18.7\text{m}$ ($\beta=0.5$ for a 12sec wave).

3.3.1 2-D Vertical Domain

A comparison of the numerical results for $K_T$ computed by the present study with the results of Lee and Ayer (1981) is shown in Figure 3.6. It is seen that the numerical results of the present study generally agree with the theoretical results of Lee and Ayer. $K_T$ tends to unity as $h/L\rightarrow 0$. This is due to the vanishing width of the trench relative to the incident wave length. In addition, $K_T$ becomes larger as $h/L$ increases since the shorter the wave, the less the influence of a trench in the bottom. It is interesting to note that the minimum transmission coefficient occurs near $h/L=1/20$. This depth corresponds to the shallow water limit in linear wave theory.

3.3.1a Sensitivity to the trench width

As in Chapter 2, the effects of the trench geometry are examined. The
first geometrical consideration is the length of the trench (b). Figure 3.7 shows that $K_T$ and $K_R$ for the relative trench length ($b/L$) ranging 0.1 to 1.0 for three different wave periods (6, 8 and 12 sec). $K_T$ and $K_R$ are strongly related to $b/L$ when the trench depth is fixed. At $b/L=0.9$, the wave is almost completely transmitted. However, the dependency on the wave period is minor.

3.3.1b Sensitivity to the trench depth

The second geometrical consideration is the trench depth. Figures 3.8 and 3.9 show the dependency of $K_T$ and $K_R$ on the relative trench depth $\gamma$. Transmission and reflection characteristics are similar for the waves of periods 12 and 14 sec because these are both long waves. The responses of 8 sec wave are smaller and the 6 sec wave even smaller yet. This is because the relative water depth increases as the wave period decreases.

3.3.1c Frequency response

Since this is a linear solution, superposition may be used to examine the responses to random waves. A JONSWAP spectrum is used with a peak period $T_p=12$ sec and significant wave height $H_{1/3}=30$ cm. Figure 3.10 shows the frequency response of $K_T$ and $K_R$ for the base geometry. These results are basically same as Figure 3.7. For example, $f=0.15$ Hz corresponds to $b/L=0.9$ and the next node, $f=0.27$ Hz corresponds to $b/L=1.7$. The amplitude of oscillation of $K_T$ and $K_R$ become smaller as frequency increases.
Figure 3.11 shows wave spectra for the incident, transmitted and reflected waves. The transmitted and reflected spectra are determined as

\[ S_{TT}(f) = \frac{1}{\Delta f} A(f)^2 |K_T(f)|^2 \]   (3.52a)

\[ S_{RR}(f) = \frac{1}{\Delta f} A(f)^2 |K_R(f)|^2 \]   (3.52b)

where, \( \Delta f \) is the frequency interval, and \( A(f) \) is the amplitude of each component of the random wave. Integrating each spectrum yields, average or bulk transmission and reflection coefficients

\[ \overline{K_T} = \sqrt{\frac{E_{TT}}{E_{HT}}} \quad , \quad \overline{K_R} = \sqrt{\frac{E_{RR}}{E_{II}}} \]   (3.53)

where, \( E \) indicates the total energy of each wave. In this case,

\( \overline{K_T} = 0.896 \) and \( \overline{K_R} = 0.445 \). This \( K_T \) value is a little larger than for a regular wave with characters corresponding to the peak frequency, and \( \overline{K_R} \) is a little smaller than the case of regular wave.

3.3.1d Examination of the trench side slope

In practice, the side walls of the trench are not likely to be vertical. The shape may will probably be more trapezoidal. Therefore, the effect of trench side slope is examined briefly for a trapezoidal trench which has a side wall slope 1V:3H as shown in Figure 3.12. Results for a rectangular trench
and a trapezoidal trench is shown in Figure 3.13. The response for the trapezoidal trench tends to vary uniformly. This is because the vertical walled trench have very sharply defined frequency responses. In the trapezoidal trench, these are less sharply defined. For long period waves, the rectangular trench gives slightly better wave reduction than the trapezoidal trench. However, the magnitude of the transmission coefficients are quite similar for both cases.

3.3.2 Quasi 3-D Domain

The numerical results of the present study are compared in Figure 3.14 with theoretical results from Kirby and Dalrymple(1983). Overall, the agreement is very good.

3.3.2a Sensitivity to the wave angle

Figure 3.15 shows the dependency of the transmission coefficient on the incident wave angle $\theta_1$ for three different wave periods. All periods show similar results. An interesting result is that waves transmit perfectly at $\theta_1=30^\circ$. For $\theta_1>30^\circ$, a large reduction occurs in transmission. The reason waves pass through at $\theta_1=30^\circ$ can be explained by Snell’s Law. To demonstrate this, take the wave be a long wave. Then from Snell’s Law,
If the x-components of the wave lengths between region 1 and region 2 match, the wave transmits perfectly. This condition gives

\[
\frac{\sin \theta_1}{\sqrt{gh}} = \frac{\sin \theta_2}{\sqrt{gd}} \tag{3.54}
\]

From (3.54) and (3.55),

\[
L_{s1} = L_{s2}
\]

\[
\cos \theta_1 = \sqrt{\gamma} \cos \theta_2 \tag{3.55}
\]

\[
\gamma^2 \sin^2 \theta_1 + \cos^2 \theta_1 - \gamma = 0 \tag{3.56}
\]

For uniform water depths before and after the trench, the \( \theta_1 \) which satisfies (3.56) gives perfect transmission over the trench. Figure 3.16 shows incident wave angles \( \theta_1 \) which transmit perfectly for given \( \gamma \). This \( \theta_1 \) is independent of the wave period. \( \gamma = 1 \) is a singularity since any wave transmits perfectly.

Kirby and Dalrymple (1983) suggest that reductions in transmission occur for large angles because the trench-parallel wave number component \( m = k_j \sin \theta_j \) exceeds the wave number for propagating waves in the trench \( k_2 \). This is shown in Figure 3.15, where \( \theta_1 > 37^\circ \) corresponds to \( m > k_2 \). For these large angles, there is significant wave trapping in the trench.
3.3.2b Sensitivity to the trench length

Figure 3.17 shows the transmission coefficient for different trench length and wave angles. At \( \theta_1 = 30^\circ \), waves transmit perfectly independent of \( b/L \) because of the reason discussed above. When \( \theta_1 = 15^\circ \), transmission coefficients are similar to results with \( \theta_1 = 0^\circ \). \( K_r \) decreases uniformly as \( b/L \) increases for \( \theta_1 = 45^\circ \). This is in agreement with Figure 3.15.

3.3.2c Sensitivity to the trench depth

The dependency on the trench depth is shown in Figure 3.18. As the trench depth increases, the wave length in region 2 increases. This results in a smaller \( k_2 \), while the trench-parallel wave number component \( m \) remains same according to Snell’s Law.
Figure 3.6  Comparison of the transmission coefficient with results from Lee and Ayer (1981)
Figure 3.7  Dependency of the transmission and reflection coefficients on the relative trench length $b/L$ ($h=1m$, $d=3m$, $\theta=0^\circ$)
Figure 3.8  Dependency of the transmission coefficient on the relative trench depth $\gamma h$ ($h=1\text{m}, b=18.7\text{m}, \theta=0^\circ$)
Figure 3.9  Dependency of the reflection coefficient on the relative trench depth $\gamma h$ ($h=1m$, $b=18.7m$, $\theta=0^\circ$)
Figure 3.10 Frequency response of the transmission and reflection coefficients (open trench: \( h=1 \) m, \( d=3 \) m, \( b=18.7 \) m, \( T_p=12 \) sec, \( H_{1/3}=30 \) cm)
Figure 3.11 Spectra of incident, transmitted and reflected waves (open trench: \( h=1 \text{m} \), \( d=3 \text{m} \), \( b=18.7 \text{m} \), \( T_p=12 \text{sec} \), \( H_{1/3}=30 \text{cm} \))
Figure 3.12  Trapezoidal trench (h=1m, d=3m, $b_T=18.7m$, $b_B=6.7m$, slope=$1V:3H$)
Figure 3.13  Comparison of the transmission and reflection coefficients for the rectangular and trapezoidal trenches
Figure 3.14  Comparison for the transmission coefficient with the results from Kirby and Dalrymple (1983) (b/h=10, d/h=3, θ₁=45°)
Figure 3.15  Dependency of the transmission coefficient on the incident wave angle \( \theta \) (\( h=1\text{m}, d=3\text{m}, b=18.7\text{m} \))
Figure 3.16  Dependency of the incident wave angles perfectly transmitted on the relative trench depth $\gamma h$
Figure 3.17  Dependency of the transmission coefficient on the relative trench length $b/L$ ($T=12$ sec, $h=1$ m, $d=3$ m, $b=18.7$ m)
Figure 3.18  Dependency of the transmission coefficient on the relative trench depth $\gamma h$ ($T=12$ sec, $h=1$m, $b=18.7$m)
4.0 WAVE DIFFRACTION DUE TO A RUBBLE TRENCH

4.1 Introduction

Rubble structures, such as rubble mound breakwaters, are highly dissipative. Therefore, rubble in the trench may also result in high energy dissipation. As mentioned in Chapter 1, if highly permeable materials in the pit do not degrade the performance of the pit, it may not be necessary to remove coarse aggregate from the pit. In addition, this will reduce maintenance costs associated with dredging the pit. Therefore, in this Chapter, the effects of rubble in the trench on wave transmission are examined.

4.2 Theoretical Formulation

4.2.1 Governing Equations

A definition sketch of the boundary value problem under consideration is shown in Figure 4.1 for a 2-D vertical domain and Figure 4.2 for a Quasi 3-D domain. The analysis of wave propagation over a rubble trench requires a determination of the energy dissipated due to the porous media flow in the trench. Energy dissipation is estimated using a linearized drag term described in this section.

4.2.1a Equation of fluid motion in the porous medium

The motion of the fluid in the porous media is described in terms of the seepage velocity and local pressure. The equation of motion may be written in the following form
\[
\frac{d\vec{v}}{dt} = -\frac{1}{\rho} \nabla(p + \rho gz) + \text{(resistance forces)}
\] (4.1)

where \(\vec{v}\) is the local seepage velocity vector, \(p\) is the corresponding pressure and \(\rho\) is the fluid mass density. The resistance forces are primarily due to drag and inertia. Previous studies have shown the drag forces may be estimated by the Frochheimer equation.

\[
[\text{drag force}] = \alpha \vec{v} + \beta \vec{v} \cdot \vec{v}
\] (4.2)

The inertia term is given by

\[
[\text{inertia term}] = \frac{1 - \epsilon}{\epsilon} C_M \frac{\partial \vec{v}}{\partial t}
\] (4.3)

where \(\epsilon\) is the porosity of the medium and \(C_M\) is the virtual mass coefficient of the medium grains (Sulisz and McDougal, 1989). Therefore, the equation of motion can be rewritten in the following form.

\[
[1 + \frac{1 - \epsilon}{\epsilon} C_M] \frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho} \nabla(p + \rho gz) - \alpha \vec{v} - \beta \vec{v} \cdot \vec{v}
\] (4.4)

The drag forces were modeled by Sollitt and Cross (1972) as

\[
\alpha \vec{v} + \beta \vec{v} \cdot \vec{v} = \frac{\nu \epsilon}{K} \vec{v} + \frac{C_f \epsilon^2}{\sqrt{K}} \vec{v} \cdot \vec{v}
\] (4.5)

where \(\nu\) is the kinematic viscosity, \(K\) is the intrinsic permeability and \(C_f\) is a dimensionless turbulent resistance coefficient.

The terms representing the drag forces in the equation of motion are
linearized based on Lorentz’s hypothesis of equivalent work. The terms responsible for drag forces in (4.4) are replaced by a term linear in the velocity.

\[ S \frac{\partial \bar{v}}{\partial t} = -\frac{1}{\rho} \nabla (p + \rho g z) - f \omega \bar{v} \]  

(4.6)

where \( \omega \) is the angular frequency of periodic motion, \( f \) is the linear damping coefficient, and \( S \) is an inertia coefficient defined by

\[ S = 1 + \frac{1 - \epsilon}{\epsilon} C_m \]  

(4.7)

According to the hypothesis of equivalent work, the energy dissipation in the porous domain over one wave period due to the nonlinear terms and linear term are equal. Therefore, the linear damping coefficient, \( f \) is equal to

\[ f = \frac{1}{\omega} \left\{ \int_T^t \left\{ \frac{\nu \bar{v} - \frac{C_f^2}{\sqrt{K}} \bar{v}}{K} + \frac{C_f^2}{\sqrt{K}} \bar{v} \right\} \cdot \bar{v} dt dR \right\} \]  

(4.8)

where \( T \) is the wave period and \( R \) is the volume over which the average damping coefficient is determined.

4.2.1b Linear damping coefficient

In (4.8) \( f \) depends on the form of \( \bar{v} \). In this study, we take \( \bar{v} \) as simple harmonic, but dependent on position in the porous domain. However, the spacial dependency is approximated by using the computed average velocity in the
domain, $\vec{q}$

$$\vec{v} = \vec{q} \cos \omega t \quad (4.9)$$

Substituting into (4.8), finally $f$ is expressed by

$$f = \frac{1}{\omega} \left( \frac{\nu e}{K} + \frac{8}{3} \frac{C_f e^2}{\sqrt{K}} \frac{\sqrt{q}}{\pi} \right) \quad (4.10)$$

However, on the right hand side of (4.10), $|\vec{q}|$ is unknown. Therefore, to determine $f$, iteration is required.

In region 2 in Figure 4.1 and region 4 in Figure 4.2, the governing equation can be derived as follows. The linearized equation of motion in the porous domain is

$$(f-iS) \omega \vec{v} = -\frac{1}{\rho} \nabla(p + \gamma z) \quad (4.11)$$

Taking the gradient of both sides yields,

$$(f-iS) \omega \nabla \cdot \vec{v} = -\frac{1}{\rho} \nabla^2 p \quad (4.12)$$

Conservation of mass for an incompressible fluid gives

$$\nabla \cdot \vec{v} = 0 \quad (4.13)$$

Therefore,

$$\nabla^2 p = 0 \quad (4.14)$$
Since,

\[ p = \frac{1}{g} \Phi_t = -\frac{i \omega}{g} \Phi \quad (4.15) \]

Equation (4.14) can be rewritten as

\[ \nabla^2 \Phi = 0 \quad (4.16) \]

The governing equations for the entire fluid domain, including the rubble, are the Laplace equation.

Similarly, for a Quasi 3-D domain, the governing equation is expressed by the modified Helmholtz equation (3.13) in all regions in Figure 4.2.

4.2.2 Boundary Conditions

Boundary conditions are same as Chapter 3 for solid boundaries, the free surface and radiation boundaries. The appropriate matching conditions between the porous trench and upper fluid domain are continuity of pressure and vertical mass flux at \( z = -h \). The matching conditions may be summarized as follows.

[2-D vertical domain]

\[ \frac{\partial \phi_1}{\partial z} = \epsilon \frac{\partial \phi_2}{\partial z} \quad (4.17) \]

\[ \phi_1 = (S + if) \phi_2 \quad (4.18) \]
[Quasi 3-D domain]

\[ \frac{\partial \phi_2}{\partial z} = \epsilon \frac{\partial \phi_4}{\partial z} \]  

(4.19)

\[ \phi_2 = (S + i f) \phi_4 \]  

(4.20)
Figure 4.1  Definition sketch for a 2-D vertical rubble trench
Figure 4.2  Definition sketch for a quasi 3-D rubble trench
4.3 Permeability

The dissipation of energy in the rubble is strongly dependent on the permeability. The permeability itself depends on the grain size and porosity. In this section, simple relationships among these are obtained. The permeability $k$ is usually related to the discharge velocity through unit area due to a hydraulic gradient as

$$v = k \frac{\partial h}{\partial x}$$

(4.21)

where, $v$ is the seepage velocity [L/T], and $\partial h / \partial x$ is the hydraulic gradient [0]. Therefore, $k$ has units of [L/T]. This $k$ is called the engineering permeability. Muskat (1937) relates the engineering permeability $k$ and the intrinsic permeability $k$ as,

$$v = K \frac{\rho g \partial h}{\mu} = k \frac{\partial h}{\partial x}$$

(4.22)

where, $v$ is the seepage velocity, $\mu$ is the molecular viscosity, $K$ is the intrinsic permeability [L$^2$] and $\rho$ is a fluid mass density [M/L$^3$]. It follows that,

$$K = \frac{k \mu}{\rho g}$$

(4.23)

4.3.1 Variation in Permeability with Size of Materials

Data to develop a relationship between $k$ and stone size $D$ were taken from Cedergren (1967). These data are summarized in Table 4.1. It is assumed that
\[ \mu = 1.14 \times 10^3 \text{ N/s/m}^2, \rho = 1026 \text{ kg/m}^3 \text{ for } t=15^\circ \text{C}, \text{ and } g = 9.81 \text{ m/s}^2 \text{ to convert } k \text{ to } K. \]

Table 4.1  Relationship between permeability and stone size  
(Cedergren, 1967)

<table>
<thead>
<tr>
<th>D (mm)</th>
<th>k (m/sec)</th>
<th>K(m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>1E-6</td>
<td>1.133E-13</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0001</td>
<td>1.133E-11</td>
</tr>
<tr>
<td>0.635</td>
<td>0.01</td>
<td>1.133E-9</td>
</tr>
<tr>
<td>1.0</td>
<td>0.01</td>
<td>1.133E-9</td>
</tr>
<tr>
<td>6.35</td>
<td>0.6</td>
<td>6.796E-8</td>
</tr>
<tr>
<td>5.0</td>
<td>0.5</td>
<td>5.663E-8</td>
</tr>
<tr>
<td>12.0</td>
<td>0.79</td>
<td>8.948E-8</td>
</tr>
<tr>
<td>25.0</td>
<td>1.65</td>
<td>1.869E-7</td>
</tr>
<tr>
<td>63.5</td>
<td>7.0</td>
<td>7.928E-7</td>
</tr>
<tr>
<td>635</td>
<td>35.0</td>
<td>3.964E-6</td>
</tr>
<tr>
<td>1000</td>
<td>60.0</td>
<td>6.796E-6</td>
</tr>
</tbody>
</table>

Although these data are rather approximate and somewhat scattered, a quadratic relationship exists between D and K. This is given by

\[ K = \frac{\mu}{\rho g} C_1 D^{C_2} \]  

A best fit can be obtained by a least square analysis on the logarithm of the values. It was determined that \( C_1 = 0.006724 \) and \( C_2 = 1.571729 \). These data and a best fit relation are shown in Figure 4.3. Therefore, the intrinsic permeability is approximately,
\[ K = \frac{\mu}{\rho g} (0.006724) D^{1.571} - 0.789 \times 10^{-9} (D)^{1.57} \]  \hspace{1cm} (4.25)

It is convenient to use a dimensionless form of (4.25) using known values of \( K \) and \( D \) as a reference values. Taking the reference diameter as \( D_0 = 10 \text{ mm} \) and from (4.25), \( K(D_0=10 \text{ mm})=2.943 \times 10^{-6} \text{ m}^2 \), yields

\[ K(D) = K(D_0) \left( \frac{D}{D_0} \right)^{1.57} \]  \hspace{1cm} (4.26)

where, \( K(D_0)=0.2943 \times 10^{-7} \text{ m}^2 \) was evaluated at a porosity \( \varepsilon = 0.442 \)

4.3.2 Variation in Permeability with the Porosity

From Muskat (1937), the permeability \( K \) can be expressed as a function of the porosity \( \varepsilon \)

\[ K(\varepsilon) = C_3 \frac{\varepsilon^3}{(1-\varepsilon)^2} \]  \hspace{1cm} (4.27)

where, \( C_3 \) is a constant. To determine \( C_3 \), a known value of the permeability was employed. That is, \( K=1.923\times10^{-7} \text{ m}^2 \) for \( \varepsilon=0.442 \) and \( D=25\text{mm} \) (Sulisz, 1985). Substituting these values into (4.27) yields,

\[ C_3 = 6.934 \times 10^{-7} \]  \hspace{1cm} (4.28)

Therefore, for \( D=25\text{mm} \), the dependency of permeability on porosity is,
\[ K(\varepsilon) = 6.934 \times 10^{-7} \frac{\varepsilon^3}{(1-\varepsilon)^2} \]  

(4.29)

Figure 4.4 shows the intrinsic permeability over a range of porosities as predicted by (4.29).

4.3.3 Variation in Permeability with the Porosity and Stone Size

Combining (4.26) and (4.27), the permeability can be expressed as,

\[ K(\varepsilon, D) = K(\varepsilon_0, D_0)(\frac{D}{D_0})^\beta \frac{\varepsilon^3}{(1-\varepsilon)^2} \]  

(4.30)

where, \( \varepsilon_0 = 0.442 \), \( D_0 = 10\text{mm} \), \( \beta = 1.571729 \)

Substituting known values (\( K = 1.923 \times 10^{-7} \text{ m}^2 \), \( \varepsilon = 0.442 \), \( D = 25\text{mm} \)) into (4.30), then

\[ K(\varepsilon_0, D_0) = 1.643 \times 10^{-7} \text{ m}^2 \]  

(4.31)

The permeability can be expressed as a function of both porosity and stone size,

\[ K(\varepsilon, D) = 1.643 \times 10^{-7} \left(\frac{D}{D_0}\right)^{1.57} \frac{\varepsilon^3}{(1-\varepsilon)^2} \]  

(4.32)

where, \( D \) is in mm. Results from equation (4.32) are shown in Figure 4.5.
Figure 4.3  Relationship between the intrinsic permeability and the stone size (Cedergren, 1967)

\[ K = C_1 \, (D)^{C_2} \]

\[ C_1 = 0.762E-9, \quad C_2 = 1.571729 \]
Figure 4.4 Intrinsic permeability as a function of the porosity
Figure 4.5a  Intrinsic permeability as a function of porosity and stone size (large stone size variation)
Figure 4.5b     Intrinsic permeability as a function of porosity and stone size (typical stone sizes)
4.4 Method of Analysis

As discussed in section 4.2, iteration is required to determine the value of $f$. At first $|\bar{q}|$ is assumed equal to the bottom velocity of upper fluid domain. Then $f$ is computed based on equation (4.10). $|\bar{q}|$ can be computed again after determining the velocity potential in the rubble trench. It is necessary to compute $|\bar{q}|$ repeatedly until it converges to a specified convergence criterion. In this study, 3% error is allowed. Figure 4.6 shows this computation process.

![Diagram of computation process for the porous trench](image)

Figure 4.6 Computation process for the porous trench
The velocity inside the rubble trench is computed at 9 points as shown in Figure 4.7. At each point, velocity potentials are computed at 4 additional mesh points, and the velocity at the center point is approximated by finite differences.

[2-D vertical]

$$\tilde{q}_j = u_j \tilde{i} + w_j \tilde{k}$$  \hspace{1cm} (4.33)

where,

$$u_j = -\frac{\partial \phi_j}{\partial x} \, , \, \, \, w = -\frac{\partial \phi_j}{\partial z}$$  \hspace{1cm} (4.34)

[Quasi 3-D]

$$\tilde{q}_j = u_j \tilde{i} + v_j \tilde{j} + w_j \tilde{k}$$  \hspace{1cm} (4.35)

where,

$$u_j = -\frac{\partial \phi_j}{\partial x}$$

$$v_j = -im \phi_j$$  \hspace{1cm} (4.36)

$$w_j = -\frac{\partial \phi_j}{\partial z}$$
Figure 4.7  Locations in the rubble trench used to compute average velocity
4.5 Results and Discussion

The porosity $\epsilon$ and the stone size $D$ characterize the rubble in the trench. The geometry of the trench is characterized by $h$, $b$ and $d$ and waves by $T$ and $H$. Variations in the parameters around a base case are examined numerically. The base values of parameters are taken as $T=12$ sec, $H=30$ cm, $h=1$ m, $\epsilon=0.5$, $D=10$ cm, $b=18.7$ m ($=0.5L_{12}$), $d=3$ m.

4.5.1 2-D Vertical Domain

4.5.1a Sensitivity to the porosity

Physically, the reasonable range of porosity is approximately 0.3 to 0.7. However, the response for a wider range of porosity values is examined to reveal the response at the limits of no rubble ($\epsilon=1.0$) and no trench ($\epsilon=0.0$). In Figure 4.8, the linearized damping coefficients are plotted against the porosity. For $\epsilon<0.2$, the linearized damping coefficient $f$ tends to be large because the intrinsic permeability is very small according to (4.10). This occurs even though the average velocity inside the rubble becomes small in this range of porosity as shown in Figure 4.9. In the range of $0.2<\epsilon<0.7$, both $f$ and velocity inside the rubble vary slightly. The velocity goes to the no rubble condition as the porosity approaches 1.0. Figures 4.10 - 4.12 show transmission, reflection and the loss coefficient dependencies on the porosity. The loss coefficient is given as

$$K_L^2 = 1 - K_T^2 - K_R^2$$  \hspace{1cm} (4.37)
From Figure 4.10, the rubble in the trench causes a significant reduction of wave height behind the trench. The reflection coefficient is also smaller than that of the no rubble trench. From Figure 4.12, high values of the loss coefficient ($K_L > 0.7$) exist for $0.6 < \epsilon < 0.9$ for the three wave periods examined. A surprising result is that the longer waves experience less dissipation. This is because the trench length was the same in all cases. Since the length scale of wave decay is the wave length, more decay is seen for the shorter waves. All of these waves are intermediate water depth waves. For very short waves, the trench, of course has no influence.

4.5.1b Sensitivity to the stone size

As mentioned above, the intrinsic permeability depends on the stone size. Figure 4.13 shows variations of $K_T$, $K_R$ and $K_L$ with the stone size. As expected, when the stone size is very small, the trench effect is quite small. That is, near perfect transmission occurs. However, as the stone size increases, energy dissipation effects occur. Finally, at $D = 50$cm, energy dissipation tends to a maximum for these wave conditions.

4.5.1c Sensitivity to the wave height

The linearized damping coefficient depends on the amplitude of velocity from equation (4.10) for given a $K$ and $T$. In linear wave theory, velocity is proportional to the wave height. However, the velocity inside the rubble is not proportional to the wave height. As shown in Figure 4.14, the relative velocity
decreases as the wave height increases. Therefore, \( f \) shows the opposite behavior. Figure 4.15 shows \( K_T, K_R \) and \( K_L \) against the wave height. Again, it is shown that the results are not linear in the wave height.

**4.5.1d Frequency response**

Since the response for the rubble trench is not linear in the wave height, the response to the random waves is more difficult to determine. Superposition of the energy loss using each frequency component in the wave spectrum is not valid. One approach is to solve the problem in the time domain. While this is a technically valid approach, it provides little insight into the frequency response of the rubble trench. As an alternative, we employ the following approximation in the friction term of (4.2).

\[
drag = \alpha v + \beta |v| v
= \alpha \sum V_i + \beta |(\sum v_i)| \sum v_i
= \alpha \sum v_i + \beta v_{rms} \sum v_i
= (\alpha + \beta v_{rms}) \sum v_i
\]

(4.38)

The same wave spectrum is employed as in the previous section to examine the response with random waves and a rubble trench. Figure 4.16 shows that transmission coefficient is nearly constant with wave frequency. The reflection coefficient oscillates slightly. In Figure 4.17, the bulk transmission and reflection coefficients are computed as
This transmission value is a little smaller than open trench case. The reflection value is much smaller than that of open trench case. The difference is due to dissipation.

4.5.1e Examination of the trench side slope

As in Chapter 3, the comparison of a rectangular rubble trench and a trapezoidal rubble trench is presented. The trapezoidal trench shape is the same in Figure 3.12. Both trenches are assumed to be filled by rubble with $D=10\text{cm}$ and $\epsilon=0.5$. Figure 4.18 shows transmission and reflection coefficients for both cases. The difference in transmission characteristics is not significantly different for the trapezoidal and rectangular cross-sections.

4.5.2 Quasi 3-D Domain

4.5.2a Sensitivity to the wave angle

The response to the rubble trench to obliquely incident waves is examined. Figure 4.19 shows the transmission coefficient as a function of the incident wave angle for the open trench and the rubble trench. As can be seen, the transmission coefficients for the rubble trench are nearly constant up to wave angles of about $40^\circ$. There is no special angle which wave transmits perfectly as the case of no rubble. For angles less than $40^\circ$, the rubble trench transmission coefficients are a
little smaller than that of no stone.
Figure 4.8 Dependency of the linear damping coefficient on the porosity of the rubble ($\theta=0^\circ$, $h=1\text{m}$, $b=18.7\text{m}$, $d=3\text{m}$, $D=10\text{cm}$)
Figure 4.9  Dependency of the average velocity in side the rubble on the porosity ($\theta=0^\circ$, $h=1m$, $b=18.7m$, $d=3m$, $D=10cm$)
Figure 4.10  Dependency of the transmission coefficient on the porosity 
\((\theta=0^\circ, h=1\text{m}, b=18.7\text{m}, d=3\text{m}, D=10\text{cm})\)
Figure 4.11  Dependency of the reflection coefficient on the porosity ($\theta=0^\circ$, $h=1m$, $b=18.7m$, $d=3m$, $D=10cm$)
Figure 4.12  Dependency of the loss coefficient on the porosity ($\theta=0^\circ$, $h=1m$, $b=18.7m$, $d=3m$, $D=10cm$)
Figure 4.13  Dependency of the transmission, reflection and loss coefficients on the stone size ($\theta=0^\circ$, $T=12$sec, $h=1$m, $d=3$m, $b=18.7$m, $\epsilon=0.5$)
Figure 4.14  Dependency of the velocity in the rubble and the linear damping coefficient on the wave height (T=12sec, h=1m, b=18.7m, d=3m, \(\epsilon=0.5\), D=10cm)
Figure 4.15  Dependency of the transmission, reflection and loss coefficients on the wave height ($T=12\text{sec}$, $h=1\text{m}$, $b=18.7\text{m}$, $d=3\text{m}$, $\epsilon=0.5$, $D=10\text{cm}$)
Figure 4.16  Frequency response of the transmission and reflection coefficients for a rubble trench (h=1m, b=18.7m, d=3m, \( \varepsilon = 0.5 \), D=10cm)
Figure 4.17 Spectra for $S_{II}$, $S_{TT}$ and $S_{RR}$ ($T_p = 12$ sec, $H_{1/3} = 30$ cm)
Figure 4.18  Comparison of reflection and transmission coefficients for rectangular and trapezoidal rubble trenches ($h=1 \text{m}$, $b_T=18.7 \text{m}$, $b_B=6.7 \text{m}$, $d=3 \text{m}$, $\varepsilon=0.5$, $D=10 \text{cm}$)
Figure 4.19 Dependency of the transmission coefficients for an open and rubble trench on the incident wave angle ($T=12\text{sec}, h=1\text{m}, b=18.7\text{m}, d=3\text{m}, D=10\text{cm}$)
5.0 CONCLUSIONS

The wave fields around pits with and without rubble are examined numerically. The linearized boundary value problems are solved by boundary element method. The following conclusions can be drawn.

1. A long wave model has been developed for the interaction of waves with multiple pits of finite length. It is shown that a high level of wave protection can be provided by appropriate placement of pits. Although the diffraction field in the vicinity of the pits is very complex, rather simple design cases can be identified. These are:
   1) The pit width should be 1 to 3 wave lengths.
   2) The pit length should be approximately 1/2 wave length.
   3) The pit depth should be 3 times local water depth.
   4) The pit spacing should be approximately 1/2 the wave length.

2. Finite length pits can provide wave reduction which is superior to submerged or surface piercing impermeable breakwaters for long waves. This protection is provided without degrading nearshore circulation or water quality.

3. The pit concept can be applied in a variety of applications. An example using an offshore pit is presented to reduce the wave heights in a dredged navigation channel.

4. Construction costs for pit breakwaters can significantly less than those for conventional breakwaters. This is because the coarse stones need not be
removed from the pit.

5. Energy dissipation due to rubble in the trench is evaluated by considering drag in the rubble. A linearized friction coefficient is formulated as a function of the porosity, intrinsic permeability and the seepage velocity in the rubble. The intrinsic permeability is derived as a function of porosity and the stone size. The model can evaluate the energy dissipation for a wide range of porosity and stone sizes.

6. Transmission coefficients for a rubble trench are much smaller than those of open trench if the porosity and stone size are chosen properly.

7. The response of obliquely incident waves over the open trench is quite sensitive to the angle of incident wave. Depending on the angle, the waves may be transmitted or trapped in the trench. The response of the rubble trench is much less sensitive to the wave angle.

8. Before the pit breakwater concept is employed in a prototype application, detailed hydraulic model studies should be conducted.
REFERENCES


Hilaly, N. 1969, "Water Waves over a Rectangular Channel through a Reef," Journal of Waterway and Harbors Division, ASCE, 95:77-94


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APPENDIX A: JONSWAP SPECTRUM

This JONSWAP spectrum is given by (Goda, 1985)

\[ S_{\eta \eta}(f) = \frac{\alpha H_{1/3}^2}{T_p^4 f^3} \exp \left[ - \frac{1.25}{(T_p f)^4} \right] \exp \left[ - \frac{(f - f_p)^2}{2 \sigma_f^2} \right] \]  \hspace{1cm} (A1)

in which

\[ \alpha = \frac{0.0624}{0.230 + 0.0336 \gamma - 0.185 (1.9 + \gamma)^{-1}} \]  \hspace{1cm} (A2)

\[ \sigma = \begin{bmatrix} \sigma_a \\ \sigma_b \end{bmatrix} = \begin{bmatrix} \sigma_a \\ \sigma_b \end{bmatrix} = \begin{bmatrix} f \leq f_p \\ f \geq f_p \end{bmatrix} \]  \hspace{1cm} (A3)

\[ \gamma = 1 - 7 \text{(average3.3)}, \quad \sigma_a = 0.07, \quad \sigma_b = 0.09 \]  \hspace{1cm} (A4)

This spectrum is shown in Figure A1 for \( T_p = 12 \text{sec}, \ (f_p = 0.083 \text{ Hz}), \ H_{1/3} = 30 \text{cm} \).

The corresponding wave amplitudes are shown in Figure A2. The phase spectrum is assumed to be uniform random between ±\( \pi \). A typical time series is shown in Figure A3.
Figure A1  Wave spectrum based on JONSWAP spectrum ($T_p = 12\text{ sec}$, $H_{1/3} = 30\text{ cm}$, $N = 1024$)
Figure A2  Amplitude of the each component of wave based on JONSWAP spectrum \( (T_p=12\text{sec}, \, H_{1/3}=30\text{cm}, \, N=1024) \)
Figure A3  Wave profile based on JONSWAP spectrum ($T_p=12\text{sec}$, $H_{1/3}=30\text{cm}$, $N=1024$)