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Title: THE DETERMINATION AND CHARACTERIZATION OF
PREFERRED ORIENTATION AND THE ELASTIC ANISOTROPY
OF ROLLED COPPER SHEET

Abstract approved Redacted for Privacy
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Relatively simple methods are developed for obtaining discreet
orientation characterization of preferred orientation from x-ray
diffraction-determined pole figure data. It is demonstrated how this
information, along with the single crystal elastic constants, can be
used to calculate Young's modulus as a function of sheet direction.
Computer programs are developed for the application of these methods
and also for pole figure plotting.

The textural characteristics of heavily cold rolled copper are
evaluated and compared with the results of other investigators.
Curves relating the variation of Young's modulus with direction in the
sheet are calculated with the Voigt average, the Reuss average, and
the Hill approximation. These are essentially parallel to curves
obtained experimentally from tensile test data with the experimental
values falling slightly below the values calculated with the Hill approximation. The possible effect of grain shape and dislocation arrangement on the anisotropy is also explored.

A method utilizing textural symmetry is developed for establishing the defocusing correction necessary for the x-ray analysis used in the determination of the pole figures. Sample material identical to that used for the pole figures is employed, and this avoids certain difficulties associated with textureless specimen procedures. The new method is compared with earlier methods, and factors affecting defocusing are evaluated.
The Determination and Characterization of Preferred Orientation and the Elastic Anisotropy of Rolled Copper Sheet

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A polycrystalline aggregate consists of crystals with orientations distributed in a continuous three-dimensional space. The crystal orientations may be distributed through the medium randomly (all possible orientations have equal probability for each crystal) or preferentially (crystal orientations tend to cluster about certain orientations). In the latter case, the material is said to exhibit preferred orientation or texture. Metals very seldom have random or even nearly random orientation of crystals since almost any process such as casting, mechanical working, or annealing impart texture. This texture is for the most part distinctive for a given process, material, and temperature of processing; however, a slight deviation in one of these can significantly change the resulting texture.

With preferred orientation, elastic properties are no longer isotropic, since the single-crystal elastic properties are generally not isotropic. It thus becomes important to be able to determine the preferred orientation of a metal and evaluate its effect upon the elastic properties. It is the purpose of this study to perfect methods for accomplishing this in a practical manner.

Preferred orientation information can be represented with "pole
figures" [17, 26, 27, 50]; "inverse pole figures" [3, 23, 24, 28, 35, 51, 52]; "biaxial pole figures" [53]; "three-dimensional orientation distribution functions" [2, 9, 10, 11, 41, 42, 48]; or by a combination of discreet crystallographic orientations, often called "ideal orientations" [4, 22, 23, 26, 27, 37].

A pole figure is an intensity plot on a stereographic projection representing the distribution of a particular set of crystallographic plane normals (poles) in respect to the axes of the material object. They are conveniently obtained directly by x-ray diffraction techniques and a set of them may be used for determining any one of the other representations. They are therefore used extensively for preferred orientation representation and analysis. An inverse pole figure is an intensity plot on a unit stereographic triangle representing the distribution of a particular object direction in the material in respect to the crystallographic coordinates. Neither a pole figure nor an inverse pole figure gives a full description of the preferred orientation; however, the information presented by an inverse pole figure is in a more usable form for physical property analysis for a given material direction.

A biaxial pole figure represents the preferred orientation with a density distribution in a unique three-dimensional volume using conventional crystallographic notations. A three-dimensional orientation distribution function represents the preferred orientation with a
function, \( W(\xi, \Phi, \phi_2) \) which is the distribution function for crystals with the orientations \( (\xi, \Phi, \phi_2) \). \( \xi \) is equal to \( \cos \phi_1 \) and \( \phi_1, \Phi \) and \( \phi_2 \) are the Eulerian angles. See Figure 10. With the more common discreet orientation characterization, the preferred orientation of rolled sheet is represented by a list of one or more discreet crystallographic orientations, \( \{hkl\} <uvw> \), where \( \{hkl\} \) signifies the Miller indices of the plane which is parallel to the normal direction of the sheet and \( <uvw> \) gives the direction in that plane which points in the rolling direction. A biaxial pole figure, a three-dimensional orientation distribution function, or a discreet orientation characterization give the full description of the preferred orientation, and are therefore appropriate for elastic property anisotropy determinations.

It would be more rigorous to employ either the three-dimensional distribution function or the biaxial pole figure for this work since they represent continuous functions; however, the three-dimensional distribution function is conceptually difficult for those not familiar with generalized spherical harmonics, and manipulations involve excessive computer expense. The biaxial pole figure is also costly of computer time. It would, therefore, be worthwhile if a method were devised which is inexpensive and yet will produce results that are accurate enough for physical property anisotropy determinations. The discreet orientation characterization seems a logical
choice for this, and rolled OFHC copper sheet should be a good material for evaluation of this choice, since copper has been extensively studied in the literature and is easy to work with.

Once the preferred orientation is characterized in a mathematically workable form, elastic properties such as Young's modulus can be evaluated for any direction in the sheet if certain assumptions are made in regard to the distribution of stress and strain through the material. For this, one may use the Voigt average [49], the Reuss average [39], or the Hill approximation [25].

The pole figures from which the full preferred orientation characterization is determined may be experimentally obtained from x-ray diffraction measurements using the Schulz reflection technique [44] with an offset specimen [29, 30, 33, 36]. Only two basic problems have existed in using this method and producing plotted pole figures. These are in the areas of defocusing correction evaluation and plotting.

The next subsection will cover the basic Schulz reflection and offset specimen techniques.
II. TECHNICAL BASIS

Pole Figure Theory and Technique

This subsection covers the basic theory and techniques necessary for obtaining pole figure data by the offset specimen, x-ray reflection method. Offset specimen preparation and use are covered. Also the equations for plotting in the stereographic net are developed.

The Schulz reflection method for the measurement of preferred orientation [44] utilizes special specimen rotational axes and a special slit geometry that minimizes intensity errors. Figure 1 shows the essential elements for this method.

Axis $q-q$ is the tilt axis for the specimen and $s-s$ the azimuthal axis. Angular measurements about the tilt axis are designated as $\phi$ with the tilt position shown in Figure 1 corresponding to $\phi = 0^\circ$, and angular measurements about the azimuthal axis are designated as $\alpha$ with $\alpha = 0^\circ$ dependent upon the specimen type and the plotting net zero. Utilizing rotations about these axes, the surface of the specimen can be brought to all positions necessary to obtain complete pole figure information while maintaining the line $c-f$ parafocused at the Bragg angle. Since the parafocusing conditions are maintained only along line $c-f$ as the specimen is tilted, the parafocusing geometry becomes less ideal as the beam width is...
Figure 1. Schulz reflection method for pole figure determination [15].
increased. A trade-off must then be made between ideal geometry and intensity level.

When a practical beam width is used, the beam will start to leave the surface of the specimen before $\phi = 80^\circ$ is reached, placing this very definite restriction on the tilt angle. Therefore, this method can only be used from $\phi = 0^\circ$ out to at most $\phi = 80^\circ$. The remaining 10° must be covered by some other method such as the transmission technique of Decker, Asp and Harker [16].

Since the transmission method is an entirely different procedure with different intensity levels, a region of overlap with the Schulz method must be balanced to obtain a correlation between the intensity levels from the two methods. Also, the thin transmission specimen is difficult to prepare. This has led Mueller and Knott [36] and then later Lopata and Kula [30] and Meieran [33] to develop a technique for obtaining a complete pole figure of a material with orthotropic symmetry from one reflection specimen. With this technique, laminated sheets are cut in such a way that the normal to the surface of the specimen is equidistant (54.7°) from the rolling plane normal, rolling direction and cross direction. Thus, one quadrant of the pole figure is obtained, but this is all that is required for orthotropic sheet symmetry such as is obtained with cold rolling when the direction of rolling is reversed for each pass. Specific laminating and cutting procedures for this method are presented by Leber [29] and Elias and
Heckler [18].

Since Leber's rotational geometry has some specimen alignment simplifications and since pole figure nets were available from him for hand plotting, his geometry was used in this study. The following is thus directed toward his geometry.

To get the sheet into the proper orientation for cutting, the sheet coordinates are rotated 45° clockwise about the rolling plane normal followed by a 54.7° clockwise rotation about the specimen edge direction. See Figure 2. These rotations can be accomplished for the sheet by cutting the sheet into square coupons at 45° to the rolling direction, thus establishing the specimen edge; laminating the coupons together; and finally rotating the laminate clockwise 54.7° about the specimen edge for the final cut. See Figure 3.

Then to use Leber's plotting net (similar to Figure 16), the specimen edge direction is taken to be \( \alpha = 0° \). This simplifies specimen alignment in the pole figure apparatus. It is noted also that Leber's net requires a clockwise \( \alpha \) rotation for increasing \( \phi \) angles.

All of this can be accomplished with the Norelco type No. 52495 pole figure device used in this study by beginning the sweep with the specimen rotated clockwise as viewed from the counter to the highest \( \phi \) angle to be measured. The specimen edge direction should point downward with the \( \alpha \) angle set at 0°.
Figure 2. Rotations necessary to obtain Leber's offset orientation.

a. Original sheet orientation
b. A 45° rotation about ND
c. A 54.7° rotation about SED.

RD = Rolling direction
ND = Normal direction
SED = Specimen edge direction
CD = Cross direction
SND = Specimen normal direction
The data is collected with the sheet coordinates in Leber's offset position (Figure 2c) and then rotated back to the original position (Figure 2a) for plotting. This is done automatically by using Leber's net for hand plotting. For computer plotting, the XY coordinates must be transformed within the computer program.

The coordinate transformations are made first in the unit sphere and then transformed to the stereographic net for plotting. Letting CD be the $X'_1$ axis, RD the $X'_2$ axis, and ND the $X'_3$ axis, the transformation equation becomes

$$X_i = \sum_{ij} f_{ij} X_j$$

where the summation convention is used, the prime indicates the new coordinate system and the $f_{ij}$'s are the directional cosines between the new and the old axes which in matrix form can be represented by

$$
\begin{pmatrix}
\cos \theta \cos \beta & -\sin \theta & \sin \theta \sin \beta \\
\sin \theta \cos \beta & \cos \theta & \sin \theta \sin \beta \\
-\sin \beta & 0 & \cos \beta
\end{pmatrix}
$$

For this case $\theta = 45^\circ$ and $\beta = 54.7^\circ$.

In order to compute the $X_j$ values in the old coordinates in terms of the specimen orientation angles $\phi$ and $\alpha$, two
orthographic views of the reference sphere will be observed, one looking down the $a$ axis and the other looking down the $\phi$ axis. See Figure 4. From this figure the following relationships are immediately seen:

\[
\begin{align*}
    h &= \sin \phi \\
    X_3 &= \cos \phi \\
    X_1 &= (h)(\sin a) = \sin \phi \sin a \\
    X_2 &= (h)(\cos a) = \sin \phi \cos a.
\end{align*}
\]

Substituting these values along with the $i_j$'s from the transformation matrix (with the functions of the $\beta$ and $\theta$ angles evaluated) into the transformation equation, the following is obtained:

\[
\begin{align*}
    X'_1 &= .4082704 \sin \phi \sin a - .707107 \sin \phi \cos a + .5772877 \cos \phi \quad (1) \\
    X'_2 &= .4082704 \sin \phi \sin a + .707107 \sin \phi \cos a + .5772877 \cos \phi \quad (2) \\
    X'_3 &= -.8165408 \sin \phi \sin a + .5772877 \cos \phi. \quad (3)
\end{align*}
\]

The final $X$ and $Y$ coordinates for plotting in the stereographic net are indicated in Figure 5. From this figure the following relationships are obtained by proportional triangles

\[
\begin{align*}
    \frac{Z}{1+X'_3} &= \frac{h_2}{h_1} \\
    \frac{X}{X'_1} &= \frac{h_2}{h_1}
\end{align*}
\]
a. Top view of block  

b. Side view of block

Figure 3. Offset specimen preparation.

a. α axis view  
b. φ axis view or section A-A

Figure 4. Relationships between spherical angles and rectangular coordinates.
Figure 5. The relationship between the rectangular coordinates of a point on a unit sphere and the coordinates of that point projected on a stereographic net.
From these the following two equations are derived:

\[ X = \frac{2X_1}{1 + X_3} \]

\[ Y = \frac{2X_2}{1 + X_3} \]

Substituting the values for \( X_1 \), \( X_2 \), and \( X_3 \) from Equations 1, 2, and 3, into these equations, the following is obtained:

\[ X = \frac{.8165404 \sin \phi \sin \alpha - 1.414214 \sin \phi \cos \alpha + 1.1545754 \cos \phi}{1 - .8165408 \sin \phi \sin \alpha + .5772877 \cos \phi} \quad (4) \]

and

\[ Y = \frac{.8165408 \sin \phi \sin \alpha + 1.414214 \sin \phi \sin \alpha + 1.1545754 \cos \phi}{1 - .8165408 \sin \phi \sin \alpha + .5772877 \cos \phi} \quad (5) \]

These \( X \) and \( Y \) values are based on a sphere of unit radius. Therefore, they must each be multiplied by a scale factor equal to one-half the radius of the stereographic net.

The basic geometric factors that are involved in obtaining pole figure information and plotting pole figures using the offset specimen reflection technique have been presented. The data obtained from these techniques must now be corrected for defocusing and background and then normalized to a "times random" scale. The next subsection
will be concerned with this.

Data Correction and Normalization

Defocusing Correction

Chernock and Beck [13] pointed out that a correction due to the defocusing of an x-ray beam of practical width is frequently necessary with the Schulz method at tilt angles \( \phi \) much less than the 75° noted in Schulz's original paper [44].

The major geometric considerations that give rise to this defocusing can be ascertained from Figure 6. The upper portion of the figure shows a view of the focusing circle, while the lower portion shows the section A-A taken from the upper view. With a beam width \( h_s \) and a tilt angle \( \phi \), the area "b d e g" on the specimen is irradiated by the source "a," but only rays in the central layer "ac-af" are parafocused at "k" in conventional diffractometer manner. Rays in other layers get progressively more out of focus with increasing deviation from the central layer. This gives rise to a spreading in approximate proportion to \( h_s \tan \phi / \sin \theta \) with other small terms to account for three-dimensional effects. Also, the beam penetrates the specimen an amount indicated by the dotted lines, causing further spreading "x." It is assumed in this analysis that the specimen is large enough to contain the area "b d e g" at all \( \phi \)
Figure 6. Geometry of x-ray optics showing factors that cause defocusing. The x-ray beam enters at "a", irradiates area "bdeg" of the specimen tilted at angle $\phi$, and is Bragg diffracted toward the receiving slit at "k".
angles of interest and thick enough so that an insignificant amount of intensity passes through the specimen.

If only the above considerations are taken and the spreading "x" is ignored, the standard deviation of the intensity distribution across the receiver slit $w_r$ can be approximated by $h_s \tan \phi / \sin \theta$ so that the intensity distribution is flattened for higher $\phi$, larger $h_s$ or smaller $\theta$. Figure 7 shows this effect for a change in $\phi$. From this figure, it is seen that a significant portion of the total intensity distribution is not counted for high $\phi$ angles, resulting in the necessity for a defocusing correction.

If the receiver slit width $w_r$ is increased, the defocusing correction can, of course, be reduced, but increasing the receiver slit width introduces the undesirable side effects of much higher background intensities and poorer resolution between adjacent diffraction peaks. The higher background intensity is a particularly detrimental effect when numerical integration methods are used to normalize the intensity. For these reasons, it is frequently necessary to choose a receiver slit of such width that the defocusing correction is significant.

In the past, the defocusing correction evaluation has been accomplished by observing the change in intensity as a function of the $\phi$ angle for a specimen of randomly oriented crystals or alternately by observing the change in intensity as a function of the $\phi$ angle for the background radiation, as suggested by Bragg and Packer [6].
Figure 7. Flattening of the x-ray intensity distribution across the receiving slit due to defocusing. As the tilt angle is increased from $\phi_1$ to $\phi_2$, an increasing percentage of the total diffracted beam is rejected by the receiving slit.
The first method is fundamentally accurate; however, it has been found difficult to prepare a high density specimen of randomly oriented crystals for some materials, and the density has been found to be important due to the spreading "x" in Figure 6. The second method does not appear to be fundamentally accurate, since the background radiation is not affected by the defocusing geometry in the same way as the Bragg diffraction radiation.

For these reasons an alternate method for defocusing evaluation for materials with textured mirror symmetry has been developed in this study. The method is fundamentally correct and has the additional advantage that the evaluation specimen may have the same density and defect structure as the specimen under study; in fact, in most cases the specimen under study can be used for defocusing evaluation.

If the specimen normal is located on the great circle trace of a plane that is normal to a plane of mirror symmetry but displaced from the mirror plane, the intensities measured at equal angular distances from the mirror plane along the trace of the normal plane will be equal except for defocusing. The defocusing correction can then be evaluated.

Using a conventionally offset specimen for a material with orthotropic texture symmetry, three such traces exist with the specimen normal displaced from the mirror plane approximately 35.27° in each
case. These traces are shown by the dotted lines in Figure 8. By rotating the specimen in $\alpha$ about SND the $\phi$ rotational axis can be brought into coincidence with point "a" to produce the "A" trace, with point "b" to produce the "B" trace, or point "c" to produce the "C" trace.

Letting $\Delta$ equal the angle from the mirror plane to the $\phi$ angle of interest measured along the trace and $\xi$ equal the angular separation of the specimen normal and the mirror plane again measured along the trace, the defocusing function

$$D(\phi) = \left[ \frac{I(\phi)}{I(0)} \right]_{\text{textureless}}$$  \hspace{1cm} (6)$$

can be determined after background corrections are made from the relationship

$$\left[ \frac{I(\xi+\Delta)}{I(\xi-\Delta)} \right]_{\text{textureless}} = \left[ \frac{I(\xi+\Delta)}{I(\xi-\Delta)} \right]_{\text{textured}}$$  \hspace{1cm} (7)$$

It was found in this study that the defocusing error was not appreciable for $\phi$ angles less than $\sim 35^\circ$. In this case, simple arithmetic ratios may be used to determine the defocusing functions for all $\phi$ angles as per Equation 7.

The particular trace used for defocusing evaluation should yield intensity considerably above background at all points of interest for best results. If none of the three available traces for the
Figure 8. The three available traces "A", "B", and "C" for defocusing evaluation when an orthotropic sheet specimen with a conventional $45^\circ \times 54.73^\circ$ offset is used.
conventionally offset specimens fulfill this requirement, a specially offset specimen may be required. Figure 9 shows the geometry of such a specimen with the SND offset 10° from the horizontal mirror plane and the trace of interest being that of the plane normal to RD. It is noted that trace "A" is not suitable for defocusing evaluation, since the SND is not offset from the mirror plane. It is generally desirable for orthotropic material symmetry that the SND be located at an angular distance greater than 45° from ND so that many of the individual sheets will be simultaneously irradiated by the x-ray beam for better statistical averaging. SND should also be chosen at an angular distance $\xi$ from the mirror plane which is large, compared to the angular error associated with the specimen preparation.

**Background Correction**

The background intensity level is normally obtained by measuring and then averaging the intensities as a function of $\phi$ for 20 angles adjacent to and on each side of the Bragg angle.

**Data Normalization**

In order to use the corrected pole figure data in a quantitative way, it must be normalized to a times random scale. This is done by dividing the corrected intensities of each data point by the random
Figure 9. The one available trace "B" for defocusing evaluation with the "SND" offset as shown. Trace "A" is not available for defocusing evaluation.
intensity factor. The random intensity factor can be found by measuring the intensity from a specimen of randomly oriented crystals, using the same experimental condition as for the test or by numerical integration of the pole figure data.

For the numerical integration method using Simpson's rule over an octant of a sphere, either of the following two equations may be used:

\[
\text{XIR} = \frac{\Delta \phi \Delta \alpha}{\pi} \sum_{1}^{N} \text{XIC}(\phi, \alpha) N \sin N \phi
\]

or

\[
\text{XIR} = \frac{1}{\sum_{1}^{N} \sin N \phi} \sum_{1}^{N} \text{XIC}(\phi, \alpha) N \sin N \phi
\]

where \( \text{XIR} \) is the random intensity factor in the same notation used in the computer programs and \( \text{XIC}(\phi, \alpha) \) is the corrected intensity level for the data point located with the coordinates \( \phi \) and \( \alpha \).

Now that the techniques for obtaining corrected and normalized pole figure data have been covered, the next subsection will deal with the conversion of this data to other, more usable forms.
Representation and Characterization of Texture

Since pole figures can be accurately and conveniently obtained by x-ray analysis, it would be fortunate if physical property information could be deduced directly from them. This is not easily accomplished; however, some attempts have been made along these lines, for instance, by Elias, Heyer and Smith [19]. Preferred orientation information must be extracted from the pole figures and presented in some other form if physical property information is desired.

Only two position parameters are required for pole figures, since the distribution of poles can be represented on the surface of a sphere. Nonetheless, three-dimensional information and inverse pole figure information can be obtained from a set of pole figures due to the multiplicity of poles coming from each crystal orientation. The task for doing this is one of separating the overlapping orientation in a systematic manner.

Inverse Pole Figures

In converting the information to inverse pole figures, the direction through the specimen for which crystallographic plane normal population is desired is chosen. Then the intensity data is averaged over the rotation about this direction \((a)\) for all \(\phi\) angles. For the continuous case, this can be represented with the equation
This integrated intensity is equal to the integrated intensity in the inverse pole figure at a similar \( \phi \) angle from the corresponding pole.

The process of obtaining this equality for all \( \phi \) angles from a number of poles has been done graphically by Jetter, McHargue and Williams [28]; (later improved by Williams [52]); and with spherical harmonic analysis by Roe and Krigbaum [40]; Bunge and Haessner [10]; and Morris [34]. The graphic method of Williams consists essentially of adjusting the intensity variation along various \( \phi \) sweeps in an iterative manner to obtain equal intensities at all intersections while still maintaining the total intensity for each \( \phi \) sweep equal to the experimental values obtained from the pole figures. Quite a number of pole figures are required for this technique.

The spherical harmonic method makes use of the Eulerian angles for fixing the position of the crystals in respect to the sheet coordinate system rather than Miller indices. Three independent parameters are needed to fix a rigid object in space. The Eulerian angles are a convenient choice of parameters for use with spherical harmonics. The Eulerian angles \((\phi_1, \Phi, \phi_2)\) are described by a rotation about the original \( Z \) axis followed by a rotation about the
X axis in its new position and then finally another rotation about the Z axis in its new position. See Figure 10.

Inverse pole figures are independent of the first rotation $\phi_1$, so that only $\Phi$ and $\phi_2$ are needed. Using these angles, an orientation distribution function $W(\xi, \phi)$ where $\xi = \cos \Phi$, giving the probability of finding a crystal with orientation $(\xi, \phi_2)$ will exist over the plane of the unit triangle. This distribution function can be expanded into a series of spherical harmonics

$$W(\xi, \phi_2) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} W_{\ell m} P_{\ell}^{m}(\xi) e^{-im\phi_2} \tag{10}$$

where $P_{\ell}^{m}(\xi)$ is the normalized associated Legendre polynomial. Tables for these functions can be found in the literature [7].

Similarly, the function $I_i(\phi')$ where $i$ stands for the $i$th pole figure can be expanded into a series of normalized Legendre polynomials

$$I_i(\phi') = \sum_{\ell=0}^{\infty} Q_i^{\ell} P_{\ell}(\cos \phi'). \tag{11}$$

The coefficients $W_{\ell m}$ of Equation 10 can be related to the coefficients $Q_i^{\ell}$ of Equation 11 by
Figure 10. Illustration of the definition of the Eulerian angles [10].
\[(Q_{\ell})_i = \sum_{m=-\ell}^{\ell} (2\ell+1)^{-1/2} W_{\ell m} P_{\ell}^m(X_1, X_2)_i. \tag{12}\]

\[X_1 \text{ and } X_2 \text{ are the directional cosines that the plane normal makes with the } [100] \text{ and } [010] \text{ axes respectively. It is seen that } 2\ell + 1 \text{ terms will appear in Equation 12 for a given order } \ell. \]

Therefore for no symmetry, \(2\ell + 1\) pole figures are required for a given order \(\ell\). Fortunately, symmetry reduces this significantly.

For instance, with cubic crystal symmetry and orthotropic sheet symmetry, only two pole figures are required up through order 22.

**Three-Dimensional Orientation Distribution Function**

The spherical harmonic analysis can be extended to obtain an orientational distribution function, called a three-dimensional orientation distribution function by Bunge and Haessner [10], which gives a complete description of the crystal orientation. With Roe's method [41], the function \(W(\xi, \phi, \phi_2)\) where \(\xi = \cos \phi_1\) describing the probability of finding a grain of orientation \((\xi, \phi, \phi_2)\) in angular space is expanded into a series of generalized spherical harmonics

\[W(\xi, \phi, \phi_2) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \sum_{n=-\ell}^{\ell} W_{\ell m n} Z_{\ell m n}(\xi) e^{\text{im} \phi} e^{\text{-in} \phi_2} \tag{13}\]
where the $Z_{\ell mn}'s$ are normalized generalizations of the associated Legendre polynomials.

Similarly, the function $I(\cos \phi', \alpha)$ describing the density distribution of poles on a pole figure is expanded into a series of spherical harmonics,

$$I_i(\cos \phi', \alpha) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} Q^i_{\ell m} P^m_\ell(\cos \phi') e^{-ima}. \quad (14)$$

The $i$ stands for the $i$th pole figure.

The $W_{\ell mn}'s$ from Equation 13 are related to the $Q_{\ell m}'s$ from Equation 14 by the following relationship:

$$Q^i_{\ell m} = (2\pi)(\frac{2}{2\ell+1})^{1/2} \sum_{n=-\ell}^{\ell} W_{\ell mn} P^n_\ell(\Sigma_i) e^{in\phi_i}. \quad (15)$$

where $\Sigma_i$ is the directional cosine between the $C$ axis of the crystal and the pole of pole figure $i$.

Here again, $2\ell + 1$ pole figures are required for no symmetry, but specific crystal and material symmetries reduce this appreciably. For example, with cubic crystal symmetry and orthotropic material symmetry, only two pole figures are required for $\ell$ up to 22.

Bunge and Haessner's method [10] is essentially the same,
except that they use special functions which take into account the crystal and material symmetries.

This method of characterization is lengthy and mathematically complicated indeed, requiring not only the solution to many simultaneous equations, but also the evaluation of many associated Legendre polynomials and generalized associated Legendre polynomials. In fact, no three-dimensional orientation function had been evaluated until Bunge and his co-worker's endeavors in 1968 and 1969 [8, 9, 10, 11].

**Biaxial Pole Figure**

Williams [53], has developed a different method for obtaining and presenting the orientation distribution in three dimensions. His distribution function exists in a volume which he calls a biaxial pole figure. Each level in the volume is an inverse pole figure of plane normals pointing in the sheet normal direction which have rolling directions corresponding to the vertical height of that level. Figure 11, which was taken from one of Williams' articles, explains the method more fully.

The combination of points A and B in the figure fix a position in the three-dimensional space. Point A defines a crystallographic plane whose normal points in the sheet normal direction, while point B defines the direction in that plane which points in the
Figure 11. Illustration of the definition of the biaxial volume [53].
rolling direction. These points are located in the biaxial volume with
the angles $\alpha$, $\beta$, and $\gamma$. $\beta$ is used in preference to $\omega$ since $\omega$
becomes undefined for small $\beta$ angles.

His technique for obtaining the distribution function is based on
an iterative program to bring a series of equations from two or more
pole figures, relating the intensities in the pole figures to those in the
density volume, to equality. This procedure required eight minutes
for him on a CDC 1604 computer for each biaxial pole figure.

**Discreet Orientation Characterization**

Discreet orientation-type characterization of rolling textures has
been used for years. This method consists of characterizing the tex-
ture with a list of one or more discreet crystallographic orientations
$\{hkl\} <uvw>$ where $\{hkl\}$ signifies the plane which is parallel to
the normal direction and $<uvw>$ gives the direction in that plane
which points in the rolling direction.

In the past, discreet orientations have been deduced qualitatively
by plotting the poles of various orientations on the pole figures and
observing which combination fits the pole figure contour the best.
More recently, they have been quantitatively taken from three-
dimensional orientation distribution information by Perlwitz, Lücke
and Pitsch [37]; Haessner, Jakubowski and Wilkens [23]; and Bunge
and Haessner [10].
The method used in this study is related to the discreet orientation technique, but refined so that quantitative distributions can be found. First, the poles of various ideal orientations are plotted on two or more pole figures (preferably more), until a combination is found which best fits the contour and accounts for all of the intensity maxima. Then using a partly assumed and partly calculated variance for a circular Gaussian distribution of poles on the pole figure, a quantitative measure of the contribution from each orientation is found. In order to minimize the effect of the variance, plotted poles which are the most isolated from the others are selected for the simultaneous equations, thereby relating the relative intensities. If no isolated pole is found for one of the orientations, either that orientation is not needed or an ambiguity exists, in which case more pole figures may be required.

For a first approximation of the variance of the assumed circular Gaussian distribution, the intensity variation between maximum points on a ridge and minimum points in a valley is observed. This variance is then used in the simultaneous equations to arrive at the individual pole intensities. Since orientations of the form \( \{hk\ell\}uvw \) represent four different orientations for orthotropic sheet symmetry, there will be \( 4 \times 1/8 = 1/2 \) times the multiplicity factor of the pole, corresponding to the pole figure, individual poles from each orientation appearing within an octant. The individual pole intensities must
therefore be multiplied by one-half the multiplicity factor to obtain the effective over-all maximum intensity for the orientation. The percentage of the orientation present can then be found by the relationship

\[
\text{% of orientation } A = \left( \frac{2 \pi \sigma^2}{\pi} \right) (I_A) = 4\sigma^2 (I_A)
\]

where \( \sigma^2 \) is the variance in radians and \((I_A)\) is the effective maximum intensity of orientation \( A \) in pole figure \( i \).

A typical equation relating the orientations designated as \( B \), \( C \) and \( A \) for the determination of the amount of the \( B \) orientation might be

\[
\frac{B}{3} + C[F(9.5^\circ)] + 2A[F(13^\circ)] = I(4, 61)_{200}.
\]

The 3 under the \( B \) is the multiplicity factor for the \{200\} pole figure; the 9.5° and the 13° are the angular separations between the pole from the \( B \) orientation and the \( C \) and \( A \) orientations respectively. The 2 before the \( A \) indicates that this pole is repeated twice in the orthotropic symmetry and the \( I(4, 61)_{200} \) is the pole figure intensity at the location of the \( B \) pole. \( F(9.5^\circ) \) can be calculated with the equation

\[
F(9.5^\circ) = e^{-\left(\frac{9.5}{\sigma}\right)^2/2}
\]
where \( \sigma \) is the standard deviation in degrees or alternately by the equation

\[
F(9.5) = (\sqrt{2\pi}) n\left(\frac{9.5}{\sigma}\right)
\]

where \( n\left(\frac{9.5}{\sigma}\right) \) is the ordinate of the normal density function obtained from statistical tables.

The mathematics necessary for characterizing the texture in a usable form have now been developed. The next task will be to use this information to calculate the anisotropy of Young's modulus. The mathematics necessary for this will be developed in the next subsection.

**Elastic Theory Development**

The purpose of this section is to develop the mathematics necessary for the calculations of Young's modulus as a function of the direction in the sheet from the preferred orientation of the crystals and the single crystal elastic properties. Many of the equations developed here are found in the literature; however, the development of them is at best difficult to find anywhere. These notions are presented systematically so that the subject may be more clearly understood and slight deviations from conventional analysis can be readily made. The convention of summing on repeated indices will be used throughout.
When brackets are placed around a subscripted variable, this is intended to mean the entire matrix of variables indicated by the subscripted variable notation.

**Elasticity**

The generalized relationship between stress and strain (Hooke's Law) for a homogeneous material can be written in either of the forms

\[
e_{ij} = S_{ijkl} \tau_{kl} \quad (16)
\]

or

\[
\tau_{ij} = C_{ijkl} e_{kl} \quad (17)
\]

where

\[
[S_{ijkl}] = [C_{ijkl}]^{-1}
\]

the (-1) means to invert the matrix.

The \( S_{ijkl} \)'s are called the elastic compliance constants and the \( C_{ijkl} \)'s are the elastic stiffness constants. From considerations of symmetries in the stress and in the strain tensors \( e_{ij} \) and \( \tau_{ij} \), the number of independent constants can be reduced to 36 from the original 34 or 81. This has led to the following conversions in order to obtain matrices more adaptable to engineering calculations. See Sokolnikoff [46]:
\[ \begin{align*}
\tau_{11} &= \tau_1', \quad \tau_{22} = \tau_2', \quad \tau_{33} = \tau_3', \\
\tau_{23} &= \tau_4', \quad \tau_{31} = \tau_5', \quad \tau_{12} = \tau_6' \\
e_{11} &= e_1', \quad e_{22} = e_2', \quad e_{33} = e_3', \quad 2e_{23} = e_4', \quad 2e_{31} = e_5', \quad 2e_{12} = e_6'.
\end{align*} \] (18)

Hooke's Law then becomes

\[ e_i = S_{ij} \tau_j \] (19)

or

\[ \tau_i = C_{ij} e_j. \] (20)

If symmetry in the material is now considered, the number of independent constants can be reduced even further. The symmetries of interest for this study are: (1) orthotropic (elastic symmetry about three mutually orthogonal planes); (2) cubic (elastic symmetry in three orthogonal directions); and (3) isotropic (elasticity independent of direction).

For orthotropic media, the number of independent elastic constants is reduced to 9 so that the compliance tensor becomes

\[
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\
S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\
S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & S_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & S_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & S_{66}
\end{bmatrix}
\]
Cubic symmetry reduces the number of independent constants to 3 and the compliance tensor becomes

\[
\begin{pmatrix}
S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\
S_{12} & S_{11} & S_{12} & 0 & 0 & 0 \\
S_{12} & S_{12} & S_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & S_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & S_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & S_{44}
\end{pmatrix}
\]

For isotropic materials, the number of independent constants is reduced to 2 from that of the cubic case by the following relationship:

\[
S_{11} - S_{12} - \frac{1}{2} S_{44} = 0. 
\tag{21}
\]

The development for the stiffness matrix is the same as for the compliance matrix with the exception that the factors of 2 in Equations 18 cause Equation 21 to become

\[
C_{11} - C_{12} - 2C_{44} = 0. 
\tag{22}
\]

In dealing with anisotropic material properties, it is necessary to transform the compliance and stiffness tensors from one coordinate system to another. Unfortunately, the 6 x 6 matrix is not a tensor and thus will not submit to transformation. It is, therefore, necessary to
return to the fourth order tensor for transformation.

The transformation of a tensor requires the product of as many directional cosines as the order of the tensor. For the case of the compliance tensor, the transformation equation is

\[ S'_{ijk\ell} = f_{im} f_{jn} f_{ko} f_{lp} S_{mnop} \] (23)

where the prime indicates the tensor in the new coordinate system and the \( f \)'s are the directional cosines between the old and the new coordinate axes. Equation 23 represents 81 equations with 81 terms each. Fortunately, symmetry reduces these numbers considerably.

For the case of orthotropic symmetry, the expansion of \( S'_{1111} \) will become

\[
S'_{1111} = f_{11}^4 S_{1111} + f_{12}^4 S_{2222} + f_{13}^4 S_{3333} + 2f_{11}^2 f_{12}^2 S_{1122} \\
+ 2f_{11}^2 f_{13}^2 S_{1133} + 2f_{12}^2 f_{13}^2 S_{2233} + 2f_{12}^2 f_{13}^2 S_{2323} \\
+ 2f_{13}^2 f_{11}^2 S_{3131} + 2f_{12}^2 f_{11}^2 S_{1212}. \] (24)

From Equations 18 the following conversions from the fourth order matrix to the second order matrix are obtained:
\[ S_{1111} = S_{11} \quad S_{1123} = S_{12} \quad S_{3131} = \frac{1}{2} S_{55} \]
\[ S_{222} = S_{22} \quad S_{1133} = S_{13} \quad S_{1212} = \frac{1}{2} S_{66} \]
\[ S_{3333} = S_{33} \quad S_{2323} = \frac{1}{2} S_{44} \quad S_{2233} = S_{23} \]

Equation 24 will then reduce to

\[ S_\prime_{11} = \ell_{11}^4 S_{11} + \ell_{12}^4 S_{22} + \ell_{13}^4 S_{33} + 2\ell_{11}^2 \ell_{12}^2 S_{12} + 2\ell_{11}^2 \ell_{13}^2 S_{13} \]
\[ + 2\ell_{12}^2 \ell_{13}^2 S_{23} + \ell_{12}^2 \ell_{13}^2 S_{44} + \ell_{13}^2 S_{55} + \ell_{12}^2 S_{66}. \]  

(25)

For cubic symmetry this can be further reduced to

\[ S_\prime_{11} = S_{11} (\gamma_1^4 + \gamma_2^4 + \gamma_3^4) + 2S_{12} (\gamma_1^2 \gamma_2^2 + \gamma_1^2 \gamma_3^2 + \gamma_2^2 \gamma_3^2) + S_{44} (\gamma_2 \gamma_3^2 + \gamma_3 \gamma_1^2 + \gamma_1 \gamma_2^2) \]

where

\[ \gamma_1 = \ell_{11} \quad \gamma_2 = \ell_{12} \quad \text{and} \quad \gamma_3 = \ell_{13}. \]  

Taking advantage of the fact that

\[ \gamma_1^2 + \gamma_2^2 + \gamma_3^2 = 1 \]

and

\[ (\gamma_1^2 + \gamma_2^2 + \gamma_3^2)^2 = 1, \]

Equation 27 can be changed to the following familiar form:
\[ S'_{11} = S_{11} - 2[S_{11} - S_{12} - \frac{1}{2} S_{44}] [\gamma_1 y_2^2 + \gamma_2 y_3^2 + \gamma_3 y_1^2]. \] (28)

The development to this point for the \( C'_{ij} \) is the same as that for the \( S_{ij} \) except for the \( 2' \) in Equations 18. Equation 28 then becomes

\[ C'_{11} = C_{11} - 2[C_{11} - C_{12} - 2C_{44}] [\gamma_1 y_2^2 + \gamma_2 y_3^2 + \gamma_3 y_1^2]. \] (29)

The transformations for the remaining constants can be developed in a similar fashion. The results are completely given in the literature by Stadelmaker, Pritchard and Grund [47].

**Young's Modulus for Single Crystals**

For the calculations of the Young's modulus, the following two equations involving uniaxial tension are fundamental:

\[ E = \frac{\tau_1}{\varepsilon_1} \] (30)

and

\[ \varepsilon_1 = S_{11} \tau_{11}. \] (31)

If Equation 31 is substituted into Equation 30 the following is obtained:

\[ E = \frac{\tau_1}{S_{11} \tau_1} = \frac{1}{S_{11}}. \] (32)
Thus, the Young's modulus is simply the reciprocal of $S'_{11}$ for the
direction of interest.

This is the basis for the equation found in the literature for cal-
culating the Young's modulus in an arbitrary direction for a cubic
single crystal. This equation is:

$$E = \frac{1}{S_{11} - 2(S_{11} - S_{12} - \frac{1}{2} S_{44}) (\gamma_1^2 \gamma_2^2 + \gamma_2^2 \gamma_3^2 + \gamma_3^2 \gamma_1^2)}.$$

(33)

Young's Modulus for Polycrystalline Aggregates

For an aggregate of differently oriented crystals, neither the
stress nor the strain can be uniform throughout since the stress sys-
tem on each individual grain is no longer uniaxial. Therefore, before
calculations involving elasticity can begin, some assumptions must be
made concerning the variation of stress and of strain through the
media. If uniform stress is assumed, the components of the com-
pliance tensor must be averaged in order to arrive at a composite
elastic tensor. This corresponds to series loading of the crystals.
If uniform strain is assumed, the components of the stiffness tensor
must be averaged and this corresponds to parallel loading of the
crystals.

The uniform stress average is the Reuss average [39] given by
where \( A_N \) is the fraction of grains having orientation \( N \).

The uniform strain average is the Voigt [49] average given by

\[
S_{ij}^{R} = \sum_{1}^{m} (S_{ij})_N A_N
\]  

(34)

It has been mathematically proven by Hill [25] that these two averages represent upper and lower bounds of values bracketing the actual values. Hill suggested that the matrices from the two averages be averaged in order to obtain a more useful approximation. This is known as the Hill average and is given by

\[
S_{ij}^{H} = \frac{1}{2} [S_{ij}^{V} + S_{ij}^{R}]
\]  

(36)

where

\[
S_{ij}^{V} = [C_{ij}^{V}]^{-1}
\]

The Young's modulus for each type of average is the reciprocal of \( S_{11} \). For the Voigt average, the \([C_{ij}]\) matrix must be inverted in order to obtain the corresponding \( S_{11} \).

At this point, the fundamental mathematics for Young's modulus calculations have been developed. These mathematics will now be
used to obtain equations which are useful for the ideal orientation-type characterisation. All equations won't be given, however, since they can be taken from the computer program listing in the Appendix.

Using the coordinate system of Figure 12, the directional cosines for a specific crystallographic orientation \((hkl) [uvw]\) can be obtained from the following formula:

\[
N_1 = \frac{h}{\sqrt{h^2 + k^2 + l^2}} \quad N_2 = \frac{k}{\sqrt{h^2 + k^2 + l^2}} \quad N_3 = \frac{l}{\sqrt{h^2 + k^2 + l^2}}
\]

\[
R_1 = \frac{u}{\sqrt{u^2 + v^2 + w^2}} \quad R_2 = \frac{v}{\sqrt{u^2 + v^2 + w^2}} \quad R_3 = \frac{w}{\sqrt{u^2 + v^2 + w^2}}
\]

\[
T_1 = R_2 N_3 - R_3 N_2 \quad T_2 = R_3 N_1 - R_1 N_3 \quad T_3 = R_1 N_2 - R_2 N_1.
\]

In order to study the elastic properties as a function of the direction in the sheet, these directional cosines must now be transformed to new axes by a rotation \(\theta\). See Figure 12. The transformation equations listed in a matrix form for such a \(\theta\) rotation are:

\[
\begin{bmatrix}
R'_1 & R'_2 & R'_3 \\
N'_1 & N'_2 & N'_3 \\
T'_1 & T'_2 & T'_3 \\
\end{bmatrix}
= \begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta \\
\end{bmatrix}
\begin{bmatrix}
R_1 & R_2 & R_3 \\
N_1 & N_2 & N_3 \\
T_1 & T_2 & T_3 \\
\end{bmatrix}
\]

(37)

where \(\theta\) is the clockwise rotation from the rolling direction. The
Figure 12. The definition of the crystal coordinates and the sheet coordinates for elastic property calculations. Coordinates 1, 2, 3 are the crystal coordinates; R, N, T are the original sheet coordinates; and R', N', T' are the rotated sheet coordinates.
nine individual equations may be easily deduced from this matrix representation, and they are also given in the computer program listing in the Appendix.

For a counterclockwise $\theta$ rotation, the equations are exactly the same except for an interchange of signs between the $\sin \theta$ terms. With orthotropic elastic symmetry, the elastic properties will be identical for either a clockwise or a counterclockwise $\theta$ rotation. The elastic constants obtained from the directional cosines resulting from these two rotations can then be averaged.

It is now noted that only squared directional cosines appear in the single crystal transformation equations for materials with orthotropic symmetry. Therefore, by performing the above averages the elastic constants of all of the grains of the type \{hkl\} <uvw> which are related by the orthotropic symmetry are, in effect, averaged.

The calculation of the Reuss average Young's modulus is quite simple at this point, since only one compliance matrix element is required, \( S'_{11} \). \( S'_{11} \) is calculated by substituting the values of the directional cosines along with the single crystal elastic compliances into Equation 28. Then the Young's modulus is found by taking the reciprocal of \( S'_{11} \) by Equation 32.

The calculation of the Voigt average isn't quite this simple, since the terms of the $6 \times 6$ stiffness matrix must be evaluated and then the tensor inverted to obtain \( S'_{11} \). The equation for obtaining
one of the terms in the matrix is Equation 29. The remaining terms are given by Stadelmaker, Pritchard and Grund [47] and are in the computer program listing in the Appendix. The procedure for inverting matrices is given in the literature, for example, by Rubinstein [43].

For an aggregate of randomly oriented crystals, the medium is isotropic and the Reuss [39] and Voigt [49] averages for the constants of interest become

\[
\begin{align*}
\overline{V} & = \frac{1}{5} [3S_{11} + 2S_{12} + S_{44}] \\
\overline{R} & = \frac{1}{5} (3S_{11} + 2S_{12} + 4S_{44}) \\
\overline{R} & = \frac{1}{5} (S_{11} + 4S_{12} - 2S_{44}) \\
\overline{R} & = \frac{1}{5} (S_{11} - S_{12} + 3S_{44}).
\end{align*}
\]

This completes the basic theory necessary for the determination and characterization of orthotropic textures and the calculation of the anisotropy of Young's modulus. This theory will now be applied to rolled copper.
III. EXPERIMENTAL

Specimen Preparation

The starting material for the anisotropic study of copper was .750 thick OFHC plate obtained as a gift from Tektronix, Inc. The \{111\} offset pole figure for this material was established and is shown in Figure 17. The specimen for the pole figure was obtained by milling a surface 54.7° from each of three surfaces of a cube cut from the material.

All rolling was done at the Bureau of Mines Department of Metallurgical Research at Albany, Oregon, using their 8-inch diameter, 2-high rolling mill, moving at 10 lineal feet per minute, which was the slowest calibrated position available for the machine. The 8" by approximately 3" by 3/4" starting pieces were rolled in the 3-inch direction, which in each case was the direction of original rolling for the material as indicated by the \{111\} pole figure. The direction of rolling was reversed between passes.

Two different rolling schedules were used. These were designated as 'R' and as 'N.' The R rolling schedule consisted of rolling at room temperature with 5% reduction per pass, to a total reduction of 95%. The sheets were immersed in water between passes to remove the heat due to rolling.

With the N rolling schedule, the sheets were immersed in liquid nitrogen between passes and left there until all bubbling ceased. The rolling table was insulated with a thin sheet of fiberglass, and
liquid nitrogen was allowed to drop on the sheets as they entered the rolls. At the early stages of reduction, the surface temperature of the sheet as it left the rolls was measured to be -147°C., while at the later stages as the sheets became thin and long, the leaving surface temperature rose to -114°C. The temperature of the material going through the rolls was thought to be considerably colder than this, since the material was heated by the ambient air before the measurement could be made. The resulting texture corresponds to that obtained by Alers and Liu [1] at -130°C. Two and one-half percent reduction per pass was used with a total reduction of 94.5%. There was considerable ice formation on the rolls, which slightly roughened the surface of the sheet. This roughness was later removed by polishing. It is conceivable that the surface roughness could have had some small effect upon the texture.

Portions of material from the R and from the N rolling schedules were annealed at 1200°F. for one hour in a flowing natural gas atmosphere. The as rolled sheets were designated 'R1' and 'N1' respectively and the annealed sheets were designated 'R2' and 'N2' respectively.

For the preferred orientation study, coupons measuring 1" x 1" were sheared at a 45° angle to the rolling direction of the sheet. These coupons were cleaned in a 20% H₂SO₄ solution and stacked alternately in the four available positions that share a common rolling direction. They were then bonded together with Buehler No. 20-8130 AB epoxy and cured at room temperature. For the annealed specimens, the
shearing was done prior to annealing to prevent bending.

The 1-inch cubic block was then bonded to a 1" x 1.4" x .12" bar with holes, using Eastman brand 910 cement. Care was taken at this point to insure that the block was attached to the bar in such a way that the proper rotation could be made to satisfy Leber's [29] geometry. See Figures 2 and 3. This block was next attached to a fixture which had been machined accurately to 54.7°. Then the fixture was attached to the moving table of an American Instrument Co., Inc. cutting-off machine No. 5-2418. The block was subsequently cut at the 54.7° angle, using an American Instrument Co., Inc. No. 5-2212 wheel, which is the softest one available for the machine. It was found necessary to bond a thin sheet of copper on the backside of the specimen after the initial cut to prevent the specimen from coming apart during the second cut.

After being cut to the proper shape, the specimen was metallurgically polished through .05 micron gamma alumina. It was then alternately heavily etched with dichromate etchant and repolished on the alumina wheel until all visible traces of worked metal disappeared. The same procedure was followed for all four preferred orientation specimens.

Tensile specimens were cut from these same sheets, two at each 10° angle from the rolling direction through the cross direction, including two for the rolling direction, thus producing a total of 20 specimens for each condition studied. These specimens were machined in accordance with ASTM designation E8-66, subsize
specimen, on a milling machine using a generous supply of coolant.

The same procedure for specimen preparation was used for a special defocusing correction specimen 'A4' as that used for specimen 'R1'; except that the sheet was sheared at 0° and 90° to the rolling direction and then after lamination rotated 10° about the cross direction for the final cut.

A specimen of randomly oriented crystals designated 'R12' was prepared with the assistance of John E. Kelley at the Bureau of Mines, Department of Metallurgical Research in Albany, Oregon, by sintering in flowing hydrogen a compact of 200 mesh spherical particles. The resulting density was 77% of normal.

**X-ray Diffraction Procedure and Pole Figure Determination**

This subsection covers the experimental procedures used in obtaining, correcting, and normalizing the pole figure data; and the presentation of the resulting pole figures. An evaluation of the symmetric texture method for determining the defocusing correction is also included.

In the preliminary studies the sensitivity of the scintillation counter was found non-linear across its face, and the strip chart recording circuitry was found to be non-linear above $2 \times 10^4$ counts per second. The first problem was solved by tilting the scintillation counter until the most linear region was found and then using the
narrower of the two available slits to further reduce this non-linear effect. The narrower slit also had the advantage of lowering the background level, which as has been previously pointed out, is desirable whenever integrated intensity is used. The narrow slit used measured .203 cm., which corresponds to .0120 radians as measured about the 2θ axis.

The second problem was overcome by slightly reducing the intensity of the incoming beam so that a lower scale could be used for recording the output.

The radiation used was copper Kα with a nickel filter to reduce the Kβ radiation. A pulse height analyser was employed to reject all pulses whose voltages were not within a 95% distribution limit of the mean CuKα pulse voltage. The use of pulse height discrimination reduced the background level considerably.

With the combination of chart speed and specimen rotation speed used, each .10 inch division on the chart paper corresponded to 4½° rotation in θ. Intensity readings were taken at each division or every 4½° of a rotation so that no intensity maxima would be missed. The path followed by this spiral scan in respect to the sheet coordinates is shown in Figure 16.

Common instrument settings for all pole figure determinations are listed below:
Detector Voltage 900
Baseline Setting 2.00
Window Setting 5.28
Attenuation 5
Full Scale Reading $2 \times 10^4$ cts/sec
Time Constant Setting .5 seconds
Chart Speed .50 inches per minute
Specimen Translation Setting 9

$\phi$ Angular Velocity 5° in 16 min.

$\alpha$ Angular Velocity 360° in 16 min.

Settings for the individual pole figure determinations are tabulated in Table I:

Table I. Some parameters for the pole figure determinations.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Diffracting Plane</th>
<th>X-ray Tube Levels</th>
<th>Diffracting Angle</th>
<th>BKG. Level</th>
<th>BKG. Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>{111}</td>
<td>50 3.5</td>
<td>43.34 41°</td>
<td>$1.6 \times 10^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>{200}</td>
<td>50 16</td>
<td>50.48 47.5° &amp; 53.3°</td>
<td>$7.6 \times 10^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>{220}</td>
<td>50 16</td>
<td>74.18 71° &amp; 77°</td>
<td>$7.4 \times 10^2$</td>
<td></td>
</tr>
<tr>
<td>N1</td>
<td>{111}</td>
<td>45 7.8</td>
<td>43.34 41°</td>
<td>$3.2 \times 10^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>{200}</td>
<td>50 16</td>
<td>50.48 47.5° &amp; 53.3°</td>
<td>$7.6 \times 10^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>{220}</td>
<td>50 16</td>
<td>74.18 71° &amp; 77°</td>
<td>$7.4 \times 10^2$</td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>{111}</td>
<td>45 2</td>
<td>43.34 41°</td>
<td>.8 $\times 10^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>{200}</td>
<td>50 3</td>
<td>50.48 47.5° &amp; 53.3°</td>
<td>1.4 $\times 10^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>{220}</td>
<td>50 9</td>
<td>74.18 71° &amp; 77°</td>
<td>4.0 $\times 10^2$</td>
<td></td>
</tr>
<tr>
<td>N2</td>
<td>{111}</td>
<td>45 7.8</td>
<td>43.34 41°</td>
<td>$3.2 \times 10^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>{200}</td>
<td>50 10</td>
<td>50.48 47.5° &amp; 53.3°</td>
<td>$4.6 \times 10^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>{220}</td>
<td>50 16</td>
<td>74.18 71° &amp; 77°</td>
<td>$7.2 \times 10^2$</td>
<td></td>
</tr>
</tbody>
</table>

Starting Material {111} 45 7.8 43.34 41° 3.2 $\times 10^2$
The background correction was obtained by adjusting the $2\theta$ angle away from the diffraction peak and scanning in $\phi$ with $\alpha$ held constant at $0^\circ$. A special rapid scanning mode with a rotational speed of $11.25^\circ$/min. was used for this purpose. There was found to be no significant deviation in the background intensity from $\phi = 0^\circ$ to $\phi = 60^\circ$; therefore, a constant background intensity was used.

The defocusing correction was obtained with the symmetrical texture method. The same alignment procedure was used for the special defocusing specimen as for the others, except for the starting position.

Before using the symmetric texture method for determining the defocusing correction, a few preliminary tests were run to establish the effectiveness of the method. The results of these tests are given in Figures 13 through 15.

Figure 13 shows the results of the tests used to verify the basic importance of the receiving slit width. The curves were both obtained with CuK$_\alpha$ {111} diffraction from specimen A4 (described earlier). Receiving slit widths of .012 and .028 radians measured about the $2\theta$ axis of the instrument were used for the lower and upper curves respectively. With a table of error functions, it can be shown that these results agree quite well with the assumption of a Gaussian distribution of x-ray intensity that is cut off by the receiving slit. The symmetric texture method has thus proven effective in demonstrating
Figure 13. Results of the defocusing evaluation test by the symmetric texture method that show the effect of changing the receiver slit width.
Figure 14. Comparison of the results from three different methods for defocusing evaluation.
Figure 15. Results of defocusing evaluation tests by the symmetric texture method which show the effect of changing the Bragg angle.
that the major defocusing error is due to the "cut-off" effect.

Figure 14 shows the results of three independent methods for obtaining the defocusing correction for CuKα \{111\} diffraction from copper. The previously described pressed and sintered specimen "R12" was used for the random orientation technique, and the previously described specimen "A4" was used for the symmetric texture and the background methods. The background measurements were made at 2θ = 41°, and the narrow receiving slit (0.012 radians) was used for all three curves.

The similarity between the curves obtained with the random orientation technique and the symmetric texture method is noted. The greater defocusing for the random orientation method can be explained partly at least on the basis of the lower density resulting in a greater spreading of the type "x" in Figure 6. The background method on the other hand fails to show the slit "cut-off" effect starting at approximately φ = 35° and is therefore deemed unsuitable for defocusing evaluation of this type.

Figure 15 shows the effect of a change in the Bragg angle on the defocusing correction. The lower curve was established with the specially offset specimen "A4" and the upper curve with the conventionally offset specimen "N1." The symmetric texture method was used in each case with the narrow receiver slit (0.012 radians) and CuKα radiation. The difference between the two curves is in
accord with the analysis of Figure 6.

It is now felt that the symmetric texture method has been successfully demonstrated. The defocusing functions for the \{111\} and the \{220\} pole figures were obtained directly from the curves in Figure 15. The defocusing function for the \{200\} pole figure was found by interpolation between the \{111\} and \{220\} curves.

The collected data, along with the determined correction functions, were next computer processed to produce plotted pole figures. Computer data cards were punched from the strip charts obtained from the pole figure determination tests using an Auto-trol model 3700 digitizer. The data cards were then fed to the Control Data Corp. 3300 computer using the computer program given in the Appendix, which was developed in conjunction with this study for use at Oregon State University, where the data was corrected for background and defocusing; the data was normalized to a times random basis; and pole figures were plotted. In this way \{111\}, \{200\} and \{220\} pole figures were prepared for each of the four specimens. They are presented in Figures 18 through 29. These pole figures agree very well with corresponding figures from the literature and this confirms the effectiveness of the methods used in their determinations. This will be covered more fully in the discussion and conclusions.

Now that the pole figures have been obtained, the next task will be to extract useful preferred orientation information from them.
Figure 16. Spiral scan used for pole figure determination. Data points indicated with (+). The data points for $I = 12$, $J = 1$, and for $I = 12$, $J = 80$ are shown for reference purposes.
Figure 17. One quadrant of the \{111\} pole figure for the starting material. Normalized.
Figure 18. (111) pole figure for specimen "R1" OFHC copper reduced 95% by rolling at room temperature. Normalized.
Figure 19. \{200\} pole figure for specimen "R1," OFHC copper reduced 95\% by rolling at room temperature. Normalized.
Figure 20. (220) pole figure for specimen "R1," OFHC copper reduced 95% by rolling at room temperature.
Figure 21. (111) pole figure for specimen "Ni, " OFHC copper reduced 94.5% by rolling at -130° C. Normalized.
Figure 22. (200) pole figure for specimen "Ni," OFHC copper reduced 94.5% by rolling at -130° C. Normalized.
Figure 23. (220) pole figure for specimen "Ni, OFHC copper reduced 94.5% by rolling at -130° C. Normalized.
Figure 24. (111) pole figure for specimen "R2," OFHC copper reduced 95% by rolling at room temperature and then annealed for one hour at 1200° F. Normalized.
Figure 25. (200) pole figure for specimen "R2," OFHC copper reduced 95% by rolling at room temperature and then annealed for one hour at 1200° F. Normalized.
Figure 26. (220) pole figure for specimen "R2," OFHC copper reduced 95% by rolling at room temperature and then annealed for one hour at 1200° F. Normalized.
Figure 27. \((111)\) pole figure for specimen "N2," OFHC copper reduced 94.5% by rolling \(-130^\circ C\) and then annealed for one hour at 1200°F. Normalized.
Figure 28. (200) pole figure for specimen "N2," OFHC copper reduced 94.5% by rolling at -130° C and then annealed for one hour at 1200° F. Normalized.
Figure 29. (220) pole figure for specimen "N2, "OFHC copper reduced 94.5% by rolling at -130° C and then annealed for one hour at 1200° F. Normalized.
Texture Determination

The poles of various discreet orientations were plotted and mirrored for orthotropic symmetry and then matched to the pole figures to obtain the set that best described the texture and accounted for all the pole maxima. The discreet orientations tried were selected from the works of Haessner, Jakubowski and Wilkens [23]; Bunge and Haessner [10]; Perlwitz, Lücke and Pitsch [37]; and Hu and Goodman [27]. The orientations found most appropriate for the R1 and N1 textures were:

<table>
<thead>
<tr>
<th>Letter Designation</th>
<th>Orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>{110} &lt;112&gt;</td>
</tr>
<tr>
<td>B</td>
<td>{146} &lt;211&gt;</td>
</tr>
<tr>
<td>C</td>
<td>{112} &lt;465&gt;</td>
</tr>
<tr>
<td>D</td>
<td>{123} &lt;745&gt;</td>
</tr>
<tr>
<td>E</td>
<td>{112} &lt;111&gt;</td>
</tr>
<tr>
<td>F</td>
<td>{110} &lt;001&gt;</td>
</tr>
</tbody>
</table>

The plotted {111}, {200}, and {220} poles for the orientations are presented in Figure 30.

The Gaussian distribution method was used to determine the relative amounts of the orientations. The computer program for this may be found in the Appendix. The constants \( X(1) \) through \( X(20) \)
Figure 30. Illustration showing the poles from all orthotropically related discrete orientations of the types shown at lower left.

a. \{111\} poles.  b. \{200\} poles.  c. \{220\} poles.
were evaluated by interpolation in the processed pole figure data
computer readout. The interpolating equations used are:

\[ X(1) = 0.25\left( \frac{XIB(11,34)+XIB(12,34)}{3} + \frac{XIB(11,35)+XIB(12,35)}{6} \right) \{111\} \]

\[ X(2) = 0.50\left( \frac{XIB(8,25)}{1.33} + \frac{XIB(8,24)}{4} \right) \{220\} \]

\[ X(3) = 0.25[XIB(5,39)+XIB(5,40)] \{220\} \]

\[ X(4) = 0.376[XIB(6,24)+XIB(6,25)] + 0.125[XIB(5,24)+XIB(5,25)] \{220\} \]

\[ X(5) = 0.333[XIB(6,16)+XIB(6,17)] + 0.167[XIB(5,16)+XIB(5,17)] \{200\} \]

\[ X(6) = 0.333[XIB(4,61)+XIB(4,62)] + 0.167[XIB(5,61)+XIB(5,62)] \{200\} \]

\[ X(7) = 0.167[XIB(10,37)+XIB(10,38)] \]

\[ \quad + 0.083[XIB(9,37)+XIB(9,38)] \{111\} \]

\[ X(8) = 0.188[XIB(10,32)+XIB(10,33)] \]

\[ \quad + 0.063[XIB(11,32)+XIB(11,33)] \{220\} \]

\[ X(9) = 0.25[XIB(5,19)+XIB(5,20)+XIB(4,19)+XIB(4,20)] \{220\} \]

\[ X(10) = XIB(6,40) \{111\} \]

\[ X(11) = 0.75[XIB(3,35)] + 0.25[XIB(4,35)] \{111\} \]

\[ X(12) = XIB(8,34) \{220\} \]

\[ X(13) = 0.50[XIB(5,20)+XIB(6,20)] \{220\} \]

\[ X(14) = XIB(6,50) \{220\} \]

\[ X(15) = XIB(5,18) \{200\} \]

\[ X(16) = 0.4[XIB(5,13)] + 0.1[XIB(6,13)] \{220\} \]
Intensities From Pole Figure

\[ X(17) = 0.083[X_{1B}(11, 34)+X_{1B}(12, 34)] \]
+ \[0.042[X_{1B}(11, 35)+X_{1B}(12, 35)] \] \{220\}
\[ X(18) = 0.167[X_{1B}(12, 8)] + 0.083[X_{1B}(12, 7)] \] \{111\}
\[ X(19) = 0.167[X_{1B}(12, 8)] + 0.083[X_{1B}(12, 7)] \] \{200\}
\[ X(20) = 0.125[X_{1B}(7, 48)+X_{1B}(8, 48)] \] \{200\}

The constants for \( R_1 \) as calculated from the interpolation equations are:

\[
\begin{align*}
X(1) &= 0.69 & X(2) &= 1.02 & X(3) &= 0.57 & X(4) &= 2.28 \\
X(5) &= 4.33 & X(6) &= 3.00 & X(7) &= 2.11 & X(8) &= 1.17 \\
X(9) &= 1.79 & X(10) &= 1.70 & X(11) &= 1.55 & X(12) &= 1.70 \\
X(13) &= 1.80 & X(14) &= 1.91 & X(15) &= 4.04 & X(16) &= 0.75 \\
X(17) &= 0.84 & X(18) &= 1.10 & X(19) &= Nil & X(20) &= Nil
\end{align*}
\]

The constants for \( N_1 \) as calculated from the interpolation equations are:

\[
\begin{align*}
X(1) &= 0.81 & X(2) &= 1.00 & X(3) &= 0.63 & X(4) &= 1.86 \\
X(5) &= 3.08 & X(6) &= 2.69 & X(7) &= 1.48 & X(8) &= 0.73 \\
X(9) &= 1.38 & X(10) &= 1.09 & X(11) &= 0.72 & X(12) &= 1.57 \\
X(13) &= 1.36 & X(14) &= 1.63 & X(15) &= 3.36 & X(16) &= 0.39 \\
X(17) &= 0.52 & X(18) &= 0.57 & X(19) &= 1.19 & X(20) &= 1.19
\end{align*}
\]
A standard deviation of $5^\circ$ was deduced from the observed shape of the ridge on the $\{111\}$ pole figure, and the calculations were performed on this basis. In using this standard deviation the total amount of the orientations $A$ through $F$ equaled approximately 80% for both the $R1$ and the $N1$ textures. This agrees roughly with the 15% remainder of Perlwitz, Lücke and Pitsch [37]. The results are tabulated below:

| Table II. The discreet orientation characterization for textures R1 and N1. |
|-------------------------|---------------------|-----------------|
| Orientation | Effective Max. Pole Figure Intensity | $4\sigma^2$ | % of this Orientation in the Material |
| R1 Texture | | | |
| A = 4.11 x 0.0304 x 100 = 12.5 | | |
| B = 8.52 x 0.0304 = 25.9 | | |
| C = 5.56 x 0.0304 = 16.9 | | |
| D = 5.62 x 0.0304 = 17.1 | | |
| E = 3.11 x 0.0304 = 9.5 | | |
| F = 0 x 0.0304 = 0 | | |
| Total = 81.9% | | |
| N1 Texture | | | |
| A = 4.00 x 0.0304 x 100 = 12.1 | | |
| B = 6.63 x 0.0304 = 20.1 | | |
| C = 3.13 x 0.0304 = 9.4 | | |
| D = 6.73 x 0.0304 = 20.4 | | |
| E = 1.94 x 0.0304 = 5.9 | | |
| F = 3.57 x 0.0304 = 10.7 | | |
| Total = 77.6% | | |

At this point the preferred orientation has been determined and characterized. The remaining item is the calculation of the anisotropy
of Young's modulus.

Young's Modulus Calculations

The equations developed in the elastic theory subsection and presented in the computer program in the Appendix were used for the Young's modulus calculations, but first the preferred orientation characterization was slightly modified to take into account the distribution of intensities around the discreet orientation peaks. This was done simply by assuming all intensity lying beyond \( \sigma = 1.6 \) to have the same effect on Young's modulus as a random component of intensity would have.

From Borington and May's Handbook [5], 72.2% of the circular Gaussian distribution lies within 1.6 \( \sigma \), or in this case, 8° of the mean. Using the above assumption the effective percentage of random orientations for the two textures becomes

\[
R_1 \text{ random} = (1.00 - 81.9) + (81.9)(.278) \\
= 18.1 + 22.8 = 40.9
\]

rounded to the nearest 5% \( R_1 \text{ random} = 40\% \)

\[
N_1 \text{ random} = (1.00 - 77.6) + (77.6)(.278) \\
= 22.4 + 21.6 = 44.0
\]

rounded to the nearest 5% \( N_1 \text{ random} = 45\% . \)

The orientations of the remaining crystals were then assumed to all
lie discreetly on the ideal orientations A through F in their respective percentages.

The percentage of the orientation for the purpose of Young's modulus calculations then becomes

<table>
<thead>
<tr>
<th>Texture</th>
<th>R1 Texture</th>
<th>N1 Texture</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>9.20</td>
<td>8.47</td>
</tr>
<tr>
<td>B</td>
<td>18.90</td>
<td>14.03</td>
</tr>
<tr>
<td>C</td>
<td>12.40</td>
<td>6.60</td>
</tr>
<tr>
<td>D</td>
<td>12.60</td>
<td>14.23</td>
</tr>
<tr>
<td>E</td>
<td>6.90</td>
<td>4.10</td>
</tr>
<tr>
<td>random</td>
<td>40.00</td>
<td>45.00</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>7.57</td>
</tr>
</tbody>
</table>

These values were used in the computer program listed in the Appendix to obtain the Reuss, Voigt and Hill average Young's modulus. These results are given in Tables III and IV.

**Tensile Testing Procedure**

In order to obtain a comparison for the calculated values of Young's modulus related to sheet directions, stress-strain curves were obtained experimentally from the tensile specimens which were cut at various angles to the rolling direction.

The stress was applied and measured with an Instron Engineering
Table III. Young's modulus related to sheet directions for the R1 material (copper reduced 95% by rolling at approximately -130°C.) as calculated with the Reuss average, the Voigt average, and the Hill approximation, using the discreet orientation characterization and the single crystal orientation elastic constants.

<table>
<thead>
<tr>
<th>Angle From RD</th>
<th>Young's modulus ($10^{11}$ dynes/cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reuss Average</td>
</tr>
<tr>
<td>0°</td>
<td>13.140</td>
</tr>
<tr>
<td>5°</td>
<td>13.020</td>
</tr>
<tr>
<td>10°</td>
<td>12.687</td>
</tr>
<tr>
<td>15°</td>
<td>12.221</td>
</tr>
<tr>
<td>20°</td>
<td>11.669</td>
</tr>
<tr>
<td>30°</td>
<td>10.691</td>
</tr>
<tr>
<td>35°</td>
<td>10.348</td>
</tr>
<tr>
<td>40°</td>
<td>10.135</td>
</tr>
<tr>
<td>45°</td>
<td>10.061</td>
</tr>
<tr>
<td>50°</td>
<td>10.128</td>
</tr>
<tr>
<td>55°</td>
<td>10.336</td>
</tr>
<tr>
<td>60°</td>
<td>10.672</td>
</tr>
<tr>
<td>65°</td>
<td>11.116</td>
</tr>
<tr>
<td>70°</td>
<td>11.633</td>
</tr>
<tr>
<td>75°</td>
<td>12.164</td>
</tr>
<tr>
<td>80°</td>
<td>12.635</td>
</tr>
<tr>
<td>90°</td>
<td>13.080</td>
</tr>
</tbody>
</table>
Table IV. Young's modulus related to sheet directions for the Ni material (copper reduced 94.5% by rolling at approximately -130°C.) as calculated with the Reuss average, the Voigt average, and the Hill approximation, using the discreet orientation characterization and the single crystal elastic constants.

<table>
<thead>
<tr>
<th>Angle From RD</th>
<th>Young's modulus (10^{11} dynes/cm^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reuss Average</td>
</tr>
<tr>
<td>0°</td>
<td>11.924</td>
</tr>
<tr>
<td>5°</td>
<td>11.869</td>
</tr>
<tr>
<td>10°</td>
<td>11.715</td>
</tr>
<tr>
<td>15°</td>
<td>11.488</td>
</tr>
<tr>
<td>20°</td>
<td>11.224</td>
</tr>
<tr>
<td>25°</td>
<td>10.964</td>
</tr>
<tr>
<td>30°</td>
<td>10.739</td>
</tr>
<tr>
<td>35°</td>
<td>10.577</td>
</tr>
<tr>
<td>40°</td>
<td>10.496</td>
</tr>
<tr>
<td>45°</td>
<td>10.507</td>
</tr>
<tr>
<td>50°</td>
<td>10.616</td>
</tr>
<tr>
<td>55°</td>
<td>10.819</td>
</tr>
<tr>
<td>60°</td>
<td>11.108</td>
</tr>
<tr>
<td>65°</td>
<td>11.464</td>
</tr>
<tr>
<td>70°</td>
<td>11.855</td>
</tr>
<tr>
<td>75°</td>
<td>12.241</td>
</tr>
<tr>
<td>80°</td>
<td>12.571</td>
</tr>
<tr>
<td>85°</td>
<td>12.795</td>
</tr>
<tr>
<td>90°</td>
<td>12.874</td>
</tr>
</tbody>
</table>
Corporation Universal testing instrument Model TT-C, and the strain was measured with a Baldwin Model P-3M extensometer in conjunction with the strain gage preamplifier and XY chart drive system which were a part of the Instron instrument. A crosshead speed of .020 inches per minute was used for all tests.

The calibration of the extensometer (1.000 inches of chart motion per .001 inch extensometer movement) was checked by measuring the chart motion resulting from controlled extensometer movements which were measured with a .0001 inch division dial indicator. The load weighing system for the Instron machine was calibrated by rotating the calibration potentiometer so that the chart pen would indicate the weight of a set of calibration weights hung from the load cell in the manner described in the operating instruction manual for the machine.

Typical load-displacement curves are given in Figure 31, and the resulting values of Young's modulus are given in Table V.
Figure 31. Typical load-extension curves obtained from tensile tests. Each horizontal division corresponds to a 1.000 inch gage length movement of .001 inches.
Table V. Young's modulus related to sheet direction for the R1 material (copper reduced 95% by rolling at room temperature) and the N1 material (copper reduced 94.5% by rolling at approximately -130°C.) as measured directly in tensile tests.

<table>
<thead>
<tr>
<th>Angle From RD</th>
<th>R1 Material</th>
<th>N1 Material</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Specimen 1</td>
<td>Specimen 2</td>
</tr>
<tr>
<td>0°</td>
<td>14.01</td>
<td>13.70</td>
</tr>
<tr>
<td>10°</td>
<td>13.65</td>
<td>13.81</td>
</tr>
<tr>
<td>20°</td>
<td>12.69</td>
<td>12.48</td>
</tr>
<tr>
<td>30°</td>
<td>11.78</td>
<td>11.63</td>
</tr>
<tr>
<td>40°</td>
<td>10.73</td>
<td>10.73</td>
</tr>
<tr>
<td>50°</td>
<td>10.82</td>
<td>10.95</td>
</tr>
<tr>
<td>60°</td>
<td>11.40</td>
<td>11.49</td>
</tr>
<tr>
<td>70°</td>
<td>12.37</td>
<td>12.92</td>
</tr>
<tr>
<td>80°</td>
<td>13.66</td>
<td>13.31</td>
</tr>
<tr>
<td>90°</td>
<td>13.85</td>
<td>14.00</td>
</tr>
</tbody>
</table>
IV. DISCUSSION AND CONCLUSIONS

Pole Figures

To establish the effectiveness of the methods used in determining the pole figures, two maxima, designated A and B in Figure 18, from the \{111\} pole figure for copper reduced 95% by rolling at room temperature are compared with similar regions of pole figures found in the literature in the table below:

Table VI. Comparison of \{111\} pole figure of hard rolled copper from various investigations. The A and B intensities are in 'times random' units.

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Maxima</th>
<th>% Reduction</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bunge and Haessner [10]</td>
<td>9.6</td>
<td>8.3</td>
<td>95</td>
</tr>
<tr>
<td>Goodman and Hu [21]</td>
<td>14</td>
<td>15</td>
<td>95</td>
</tr>
<tr>
<td>Haessner, Jakubowski and Wilkens [23]</td>
<td>10</td>
<td>7</td>
<td>95</td>
</tr>
<tr>
<td>Haessner, Jakubowski and Wilkens [23]</td>
<td>7</td>
<td>9</td>
<td>95</td>
</tr>
<tr>
<td>Hu and Goodman [27]</td>
<td>15</td>
<td>12</td>
<td>96.6</td>
</tr>
<tr>
<td>Perlwitz, Lücke and Pitsch [37]</td>
<td>10</td>
<td>10</td>
<td>95</td>
</tr>
<tr>
<td>Perlwitz, Pitsch and Lücke [38]</td>
<td>9.5</td>
<td>10</td>
<td>95</td>
</tr>
<tr>
<td>Williams [38]</td>
<td>5</td>
<td>6</td>
<td>92</td>
</tr>
<tr>
<td>This Study</td>
<td>9.0</td>
<td>9.8</td>
<td>95</td>
</tr>
</tbody>
</table>

*Notes:*
1. Determined with selected area electron diffraction.
2. Determined with x-ray diffraction in reflection from a spherical specimen.
3. Determined with x-ray diffraction in reflection from a 45° x 54.7° offset specimen.
4. Determined with a combination of the reflection and transmission techniques from conventional specimens.
It is seen that the pole figure from this study compares favorably with the majority of those found in the literature; however, there is quite a spread in the reported maximum intensities. Bunge and Haessner [10] calculated pole figure errors with spherical harmonic analysis in their studies with rolled copper. For a combination of the transmission and the reflection techniques with a plane specimen, they calculated a mean statistical error of about 4.5% and a mean systematic error of 7.5% with maximum errors as high as 39%. They attributed the systematic errors to "absorption and defocusing effects, inaccurate specimen adjustment, inaccurate adaption of the transmission and back-reflection region, and others" [10, p. 5512]. Other systematic errors might be inaccurate determination of the random intensity factor and the background intensity level.

The systematic errors should be greatly reduced with the techniques used in this study such as: The new texture symmetry method for defocusing function determination; the offset specimen reflection technique; and the numerical integration method for determining the random intensity factor.

It has been shown that significant defocusing errors frequently exist with the use of the Schulz reflection technique for pole figure determination and that a defocusing function should be evaluated in order to properly correct the data. This function has been evaluated in the past by measuring the intensity as a function of the tilt angle.
from a specimen of randomly oriented crystals or by measuring the background intensity as a function of the tilt angle. The texture symmetry method has an advantage over the random orientation method because material identical to that under study can be used. This is important, since material properties such as density and defect structure very likely affect defocusing. The background method was found unsuitable for defocusing function evaluation since it failed to show the slit cut-off effect.

The offset specimen reflection technique avoids the need for intensity adjustments between two regions, such as the transmission and reflection regions of conventional procedures.

The numerical integration method for determining the random intensity factor avoids the possible errors due to adjusting a textureless specimen to the identical diffraction conditions of the study and possible errors due to any slight texture in the textureless specimen.

The techniques for obtaining processed pole figure data and plotting pole figures have been successful. They are also convenient and of low cost. For instance, the computer cost for correcting and normalizing the pole figure data and plotting the pole figures averaged approximately $6 per pole figure with the CDC 3300 computer.

These techniques could be further improved by the incorporation of a system to transfer the pole figure data directly from the x-ray diffraction electronics to computer data cards or to storage tape in the
computer itself. This would eliminate the human errors that are possible with the use of the digitizer.

**Anisotropy**

The values of Young's modulus from Tables III, IV and V are plotted in Figures 32 and 33. It is noted that the calculated anisotropy of Young's modulus using the Hill approximation lies above the curve obtained by direct measurements for both the $R_1$ and the $N_1$ textures, and that the two curves have approximately the same form for the $R_1$ texture while the curve obtained by direct measurements for the $N_1$ texture is tipped down in the rolling direction in respect to the Hill curve. It is not expected that the curves be superimposed; however, any difference in form between the Hill curve and the curve obtained by direct measurements is indicative of anisotropic factors other than preferred orientation coming into play.

One possible factor for this might be grain shape. If the grains are preferentially elongated in one direction, tensile testing in that direction should more nearly approach the constant strain case than the constant stress case, while tensile tests in directions orthogonal to this should more nearly approach the constant stress case than the constant strain case. Then since the values for the Young's modulus are higher for the constant strain average, Young's modulus should be higher in the direction of grain elongation. Using selected area
Figure 32. Young's modulus versus angle in sheet measured from RD for copper reduced 95% by rolling at room temperature. The curves designated V, R, and H are calculated with the Voigt, Reuss and Hill averages respectively from the preferred orientation characterization and the single crystal elastic constants. Circles refer to direct tensile test evaluation.
Figure 33. Young's modulus versus angle in sheet measured from RD for copper reduced 94.5% by rolling at approximately -130°C. The curves designated V, R, and H are calculated with the Voigt, Reuss and Hill averages respectively from the preferred orientation characterization and the single crystal elastic constants. Circles refer to direct tensile test evaluation.
electron diffraction, Haessner, Jakubowski and Wilkens [23] found cells approximately twice as long in the rolling direction as the cross direction for copper reduced 95% by rolling at room temperature. Therefore for this case, the grain shape factor should cause the experimental curve to be tipped up near the RD and tipped down near the CD in respect to the Hill curve.

Another factor which could significantly affect the Young's modulus anisotropy is dislocation distribution. The explanation for this follows.

Tensile elongations of between .1% and 23% of annealed copper have been found to lower the Young's modulus measured at room temperature by as much as 11%. See references by Smith [45], Fantozzi, Boulanger and Gobin [20], and Chelnokov [14]. This reduction in modulus has been found to diminish as the elongation increases beyond approximately 7%, but as pointed out by Smith [45], this latter effect could be the result of simultaneous changes in preferred orientation.

The most widely accepted theory for the reduction of Young's modulus with plastic deformation is that unrestrained lengths of dislocations can move in a reversible manner under the action of an applied stress thus reducing the localized stress level and hence Young's modulus.

The reduction in the elastic modulus due to dislocation motion
will be most effective for slip systems of high resolved shear stress and for slip systems of certain (undetermined) dislocation densities. Therefore, this effect will be anisotropic in respect to the sheet coordinates, the anisotropy being influenced by the preferred orientation of the crystals and the dislocation densities in the slip systems. The mathematics for determining the influence of this factor upon the results of this study were not developed. It is suggested that this be done in future work.

The apparent modulus as measured in a tensile test might be further reduced by local yielding (irreversible movement of dislocations) at points of high residual tensile stress. However, for this study there was no significant change in the apparent Young's modulus after repeated loading and unloading cycles and the hysteresis loop tended to be closed for each cycle. See Figure 31. It is therefore concluded that irreversible effects are not significant in this study.

Figure 32 reveals that for copper rolled at room temperature the calculated Young's modulus anisotropy curve is essentially parallel to the one obtained by direct measurement. The grain shape factor requires the direct-measured curve to be tipped up near the rolling direction. Therefore, there must be a factor operating which has an effect upon the Young's modulus opposite to the effect of the grain shape factor, and this factor could be the dislocation distribution factor.
Figure 33 reveals that the direct-measured anisotropy curve for copper rolled at reduced temperature is actually tipped downward near the rolling direction in respect to the calculated curves. This indicates that the other factor, perhaps the dislocation distribution factor, is much more significant than the grain shape factor at the lower rolling temperature.

Characterization of Texture

The success with the Young's modulus anisotropy calculations demonstrates the effectiveness of the circular Gaussian distribution method for the characterization of texture in the form of discreet orientations. The circular Gaussian distribution assumption is definitely an oversimplification; however, by choosing points on the pole figures that contain an isolated pole from the orientation being evaluated, the errors in the assumption have been minimized.

The main advantages in using this method are those of ease of comprehension, convenience and low processing cost.
BIBLIOGRAPHY


APPENDIX

COMPUTER PROGRAMS

General Note for Pole Figure Plotting Programs

The data is processed in a matrix form \( XIB(I, J), X(I, J), Y(I, J), \) etc. Each \( I \) signifies a complete rotation in \( \alpha \) with \( \phi \) increasing 5°. The \( I \)'s go from 1 through 12 with \( I = 1 \) corresponding to \( \phi \) increasing from \( \phi = 0 \) to \( \phi = 5^\circ \). Each \( J \) denotes an \( \alpha \) angle within the \( \phi \) scan. The \( J \)'s go from 1 through 80 with \( J = 1 \) corresponding to \( \alpha = 0^\circ \) and \( J = 80 \) corresponding to \( \alpha = 355\frac{1}{2}^\circ \). The \( I \)'s and \( J \)'s form a matrix, which is convenient for plotting, since the adjacency of data points is preserved.

Program Coord

Purpose

This program calculates the \( \phi \) angle and the \( X \) and \( Y \) coordinates of each data point and stores the result in a file for future use.

LUN Equip

10 = file
Input

None

Output into Storage

\( B(I, J) \) - the \( \phi \) angle in radians of each data point.

\( X(I, J) \) - the \( X \) dimension to each data point in terms of a unit reference sphere.

\( Y(I, J) \) - the \( Y \) dimension to each data point in terms of a unit reference sphere.

Save

10 = BAT

Program Description

Equations 4 and 5 from the pole figure theory section are used to calculate the \( X \) and \( Y \) coordinates. Note that the argument for the trigonometric functions must be in radians. The float function converts its argument from an integer number to a floating point number.

Program BAT

Purpose

This program calculates the \( \phi \) angles to which the background and defocusing correction apply and stores this in a file for future use.
LUN Equip

10 = file

Input

None

Output into Storage

BA(K) - The $\phi$ angle to which the background and defocusing corrections 'K' apply.

Save

10 = BAT

Program Description

This program simply converts the $\phi$ angle from degrees to radians at 2° intervals.
PROGRAM COORD
DIMENSION B(12,80),X(12,80),Y(12,80)
DC 5 I=1,12
DC 5 J=1,80
9(I,J)=FLOAT((I-1)*80+J-1)*1.0908307825E-3
A=FLOAT(J-1)*7.853981634E-2
FA=.8165408*SIN(B(I,J))*SIN(A)
F3=.5772877*COS(B(I,J))
FC=1.414214*SIN(B(I,J))*COS(A)
X(I,J)=(FA-FC+2.0)/(-FA+F3)
Y(I,J)=(FA+FC+2.0)/(-FA+F3)
WRITE(10,100)B,X,Y
WRITE(61,100)B,X,Y
100 FORMAT(12F10.5)
END

PROGRAM BAT
DIMENSION BA(34)
DC 10 K=1,34
10 BA(K)=FLOAT(K-1)*3.4906504E-2
WRITE(61,100)BA
WRITE(10,100)BA
100 FORMAT(12F10.5)
END
Program Polfig

Purpose

This program

1. Reads raw data, corrections and other variables.
2. Corrects data for background and defocusing.
3. Normalizes data to times random scale.
4. Writes corrected and normalized data on output paper.
5. Calls the "Axis Subroutine" and the "Sketch Subroutine."

LUN Equip

10 = Corda
01 = Plot
20 = BAT

Label

01/name

Input

XI(I, J) - Raw intensity data
BKG(K) - Background intensity
ABT(K) - Defocusing correction
NC - Number of contours
C(L) - Contour levels
SF - Scale factor = 1/2 radius of stereographic net.
Output

XIB(I, J) - Corrected and normalized intensity
XIR - Random intensity factor.

Program Description

The raw data XI(I, J) are corrected for background and defocusing, using the correction factor at the \( \phi \) angle nearest to the data point \( \phi \) angle.

The product of each of the corrected intensities XIC(I, J) that lie within the octant and the sin of the corresponding \( \phi \) angle is summed and divided by the sum of the sins (229.88). This gives the random intensity factor XIR. Each of the corrected intensities XIC(I, J) is next divided by XIR to give the corrected and normalized intensities, XIB(I, J).

Subroutine Axis

Purpose

This subroutine defines the coordinates for the plot and draws the border for the projection of the octant.

Description

The Saxes subroutine defines the plotting axis through the use of the long list of variables in the argument.

The first Plotxy subroutine calls the plotter pin to coordinates (0, 0) with the pin up. The second Plotxy subroutine draws the
vertical portion of the border. The third and fourth Plotxy subroutines draw the curved portion of the border, and the fifth Plotxy subroutine draws the horizontal portion of the border.

Subroutine Sketch

Purpose

This subroutine draws the contour lines for the pole figure. It is an adaptation of Love's plotting program [31, 32].

Description

Each group of four adjacent data points is taken in turn and a new data point is generated in the center by interpolation. The four triangles thus generated are now used one at a time for the actual plotting. See Figure 34. The block diagram, Figure 35, gives the plotting routine for triangle U(1)-U(2)-U(5). All other triangles are plotted similarly.

Function Brace

This function determines if the contour line passes between the two data points and returns and answers yes or no.

Function Guess

This function uses linear interpolation to find the X and Y coordinates of the intersection of the contour line and the line between
(a) A rectangle taken from data array

(b) The assignment of new variable names and the interpolation of a new data point

Figure 34. Illustration of the generation of the triangles to be used in the plotting routine.
Figure 35. Block diagram of plotting routine for triangle U(1)-U(2)-U(5).
the two data points.

Subroutine Plotxy

This is a library subroutine with the Oregon State University OS3 system. It controls the plotter.
PROGRAM POLFI
DIMENSION XI(12*80),XIC(12*80),XIR(12*80),R(12*80),X(12*80)
COMMON SF,SF,X,Y,XIR,C,NC
READ (60,2)((XI(I,J),J=1,80),I=1,12)
READ (60,3)(HK(I,K)*ART(K),K=1,14)
READ (60,4)NC,CL(I=1,NC)
READ (60,5)SF,NC
5 FORMAT(F5.3,15)
4 FORMAT(12/(1AF4.1))
2 FORMAT(20F4.1)
3 FORMAT(6(F5.1,F6.3))
READ (60,6) SF,NC,PR
FORMAT(60,100)8,X,Y
PRAN(100,100)8A
100 FORMAT(12F10.5)
N=1
DC 70 I=1,12
DC 70 J=1,80
DC 70 K=N,34
IF (RA(K)-R(I,J)*.0175)20,25
20 CONTINUE
25 N=K
RKGX=RKG(K)
ARSX=ART(K)
XIC(I,J)=(XI(I,J)-RKGX)*ABSX
30 CONTINUE
SUM=0
DC 40 I=1,7
DC 40 J=1,80
40 SUM=SUM+XIC(I,J)*SIN(B(I,J))
DC 41 J=1,17
41 SUM=SUM+XIC(A,J)*SIN(B(A,J))
DC 42 J=25,42
42 SUM=SUM+XIC(B,J)*SIN(B(B,J))
DC 43 J=54,67
43 SUM=SUM+XIC(C,J)*SIN(B(C,J))
DC 44 J=2,13
44 SUM=SUM+XIC(D,J)*SIN(B(D,J))
DC 45 J=30,38
45 SUM=SUM+XIC(E,J)*SIN(B(E,J))
DC 46 J=57,65
46 SUM=SUM+XIC(F,J)*SIN(B(F,J))
DC 47 J=5,10
47 SUM=SUM+XIC(G,J)*SIN(B(G,J))
DC 48 J=32,36
48 SUM=SUM+XIC(H,J)*SIN(B(H,J))
DC 49 J=59,62
49 SUM=SUM+XIC(I,J)*SIN(B(I,J))
DC 50 J=72,80
50 XIR(I,J)=XIC(I,J)/XIR
WRITF(61,100)XIR,XIR
CALL AXIS
CALL SKETCH
END
SUBROUTINE AXIS
COMMON SF,NR
Ix=4*SF+1.0
RT=Ix
CALL SAXES(0..20..20.,10.,10.,10.,10.,10..0)
CALL PLOTXY(0..0.,0..0)
CALL PLOTXY(0..2.*SF,1.0)
X=0
DC 20 I=1,399
X=X+.005*SF
FX=4.*SF**2-X**2
Y=SORT(FX)
20 CALL PLOTXY(X,Y,1.0)
CALL PLOTXY(20.*SF+0.1,0)
CALL PLOTXY(0,0,1,0)
RETURN
END

SUBROUTINE SKETCH
DIMENSION XIB(12,80),X(12,80),Y(12,80),C(14),U(5),O(5),Z(5)
COMMON SFH,X,Y,XIP,CONC
E=2.1
DC 120 I=1,11
DC 120 J=1,80
IF(T1.TG.7)5+1
1 IF(J.E0.80)2,3
2 U(2)=XIB(I+1,1) $R(2)=X(I+1,1) $Z(2)=Y(I+1,1)
U(3)=XIB(I+2,1)
R(3)=X(I+2,1)
Z(3)=Y(I+2,1)
GO TO 4
3 U(2)=XIB(I,J+1) $R(2)=X(I,J+1) $Z(2)=Y(I,J+1)
U(3)=XIB(I+1,J+1)
R(3)=X(I+1,J+1)
Z(3)=Y(I+1,J+1)
4 U(1)=XIB(I,J) $R(1)=X(I,J) $Z(1)=Y(I,J)
U(4)=XIB(I+1,J)
R(4)=X(I+1,J)
Z(4)=Y(I+1,J)
U(5)=.25*(U(1)+U(2)+U(3)+U(4))
R(5)=.25*(R(1)+R(2)+R(3)+R(4))
Z(5)=.25*(Z(1)+Z(2)+Z(3)+Z(4))
DC 120 K=1,12
DC 120 MS=1+4
I1=MS $I2=MS+1 $ IF(I2.GT.4)2A,27
28 IF(RPACF(U(11), U(12), C(K)) GT 0) GO TO 21
27 IF(RPACF(U(11), U(12), C(K)) GT 0) GO TO 21
21 X1 = GUESS(C(K) + U(11) + U(12) + R(11) + R(12)) * SF
Y1 = GUESS(C(K) + U(11) + U(12) + Z(11) + Z(12)) * SF
CALL PLCTXY(X1, Y1, 0, 0)
IF(RPACF(U(11) + U(5) + C(K)) GT 0) GO TO 22
22 X1 = GUESS(C(K) + U(11) + U(5) + R(11) + R(5)) * SF
Y1 = GUESS(C(K) + U(11) + U(5) + Z(11) + Z(5)) * SF
GO TO 24
23 X1 = GUESS(C(K) + U(12) + U(5) + R(12) + R(5)) * SF
Y1 = GUESS(C(K) + U(12) + U(5) + Z(12) + Z(5)) * SF
CALL PLCTXY(X1, Y1, 0, 0) GO TO 120
24 IF(RPACF(U(11) + U(5) + C(K)) GT 0) GO TO 25
25 X1 = GUESS(C(K) + U(11) + U(5) + R(11) + R(5)) * SF
Y1 = GUESS(C(K) + U(11) + U(5) + Z(11) + Z(5)) * SF
CALL PLCTXY(X1, Y1, 0, 0) GO TO 23
120 CONTINUE
CALL AXISXY(0, 0, 0, 0, 0, 0, 0, 0, 0)
RETURN
END

FUNCTION RHACE(A, B, Y)
IF(A - X) GT Y GO TO 2
2 IF(X - A) GT 4
1 IF(X - A) GT 4
3 RPACF = 1.
RETURN
4 RPACF = 0
RETURN
END

FUNCTION GUESS(A, B, C, D, F, SF)
GUESS = (D + (F - D) * (A - R) / (C - B)) * SF
RETURN
END
Program Text

Purpose

This program calculates the effective maximum pole figure intensities of each of a group of previously determined discreet orientations, using the assumption of a circular Gaussian distribution with $\sigma = 5^\circ$.

Input

$X(J)$ - The constants determined from the corrected and normalized pole figure data.

Output

A - The effective maximum pole figure intensity of the $\{110\} <112>$ orientations.

B - The effective maximum pole figure intensity of the $\{146\} <211>$ orientations.

C - The effective maximum pole figure intensity of the $\{112\} <465>$ orientations.

D - The effective maximum pole figure intensity of the $\{123\} <745>$ orientations.

E - The effective maximum pole figure intensity of the $\{112\} <111>$ orientations.

F - The effective maximum pole figure intensity of the $\{110\} <001>$ orientations.
Description

A total of ten iterations are used to solve a set of simultaneous equations relating the individual pole intensities from the group of previously determined orientations.
PROGRAM TEXT
DIMENSION X(21)
READ (60*1) (X(J), J=1, 20)

A1=1.50
A2=2.00
A3=1.00
B1=1.50
B2=2.00
B3=1.00
C1=1.50
C2=2.00
C3=1.00
D1=1.50
D2=2.00
D3=1.00
F1=1.50
F2=2.00
F3=1.00
F4=1.50
F5=2.00
F6=1.00

DC 10 I=1+10
AA=(X(1)-.0198*X1)*4
RA=(X(2)-.071*R3-.027*D3)*6
CA=(X(3)-.034*R3)*6
A=(AA+RA+CA)/3
A1=A/4
A2=A/3
A3=A/6
AR=(X(4)-.375*F3-.178*A3+.056*C3)*6
RR=(X(5)-.556*A2-.165*D2)*3
CR=(X(6)-.165*C2-.068*A2)*3
DR=(X(7)-.245*D1-.051*A3-.178*E3)*6
R=(AR+RR+CR+DR)/4
R1=R/4
R2=R/3
R3=R/6
AC=(X(8)-.328*F3-.071*D3)*6
RC=(X(9)-.375*E3-.056*H3-.178*E3)*6
CC=(X(10)-.490*F1-.071*D1)*4
DC=(X(11)-.556*F1-.071*D1)*4
C=(AC+HC+CC+DC)/4
C1=C/4
C2=C/3
C3=C/6
AD=(X(12)-.325*R3-.142*C3)*6
BD=(X(13)-.375*R3-.375*C3)*6
CD=(X(14)-.178*C3-.328*B3)*6
DD=(X(15)-.198*C2-.135*B2)*3
D=(AD+BD+CD+DD)/4
D1 = D/4
D2 = D/3
D3 = D/6
AE = (X(16) - 0.184*C3)*6
BE = (X(17) - 0.168*C3)*6
CE = (X(18) - 0.198*C1 - 0.026*D1)*4
E = (AE + BE + CE)/3
E1 = E/4
E2 = E/3
E3 = E/6
AF = X(19)*3
RF = X(20)*3
F = (AF + HF)/2
F1 = F/4
F2 = F/3
F3 = F/6
WRITE (61, ?) A, R, C, D, E, F
10 CONTINUE
END
Program Elast

This program calculates the Voigt, Reuss and Hill average values of the Young's modulus from the previously determined components of discrete orientations present in the materials at 5° intervals from the rolling direction to the cross direction.

Input

R1(K), R2(K), R3(K) - The directional cosines between the rolling direction and the axes of the discrete orientation 'K'.

XN1(K), XN2(K), XN3(K) - The directional cosines between the normal direction and the axes of the discrete orientation 'K'.

X1 - The percentage of the 'A' orientation present.

X2 - The percentage of the 'B' orientation present.

X3 - The percentage of the 'C' orientation present.

X4 - The percentage of the 'D' orientation present.

X5 - The percentage of the 'E' orientation present.

X6 - The percentage of the random component of orientations present.

X7 - The percentage of the 'F' orientation present.
Output

ER - The Reuss average for the Young's modulus.

EV - The Voigt average for the Young's modulus.

EH - The Hill average for the Young's modulus.

Description

The Reuss average is obtained by averaging $S'_{11}$ obtained from Equation 28 in the Elastic Theory Section and inverting this per Equation 32.

The Voigt average is obtained by averaging the components of the elastic stiffness matrix and inverting the matrix to obtain $S'_{11}$. Then Equation 32 is used.

The Hill average is obtained by averaging the $S'_{11}$ obtained from the two above averages and then applying Equation 32.

The single crystal elastic stiffness constants for copper

$$C_{11} = 1.700$$
$$C_{12} = 1.225$$
$$C_{44} = .758,$$

were determined by Chang and Himmel [12] while the single crystal elastic compliance constants

$$S_{11} = 1.53$$
$$S_{12} = -.64$$
$$S_{44} = 1.32,$$

were obtained by inverting the elastic stiffness matrix.
PROGRAM ELAST
DIMENSION R1(7), R2(7), R3(7), XN1(7), XN2(7), XN3(7), XN4(7), T1(2), T2(2), T3(2), X(7)

DO 30 K=1,6
   T1(K) = R2(K)*XN3(K) + R3(K)*XN1(K)
   T2(K) = R3(K)*XN2(K) - R1(K)*XN3(K)
   T3(K) = R1(K)*XN2(K) - R2(K)*XN1(K)

CONTINUE

RANC(1) = 2.116
RANC(2) = 2.116
RANC(3) = 2.116
RANC(4) = 1.0164
RANC(5) = 1.0164
RANC(6) = 1.0164

DO 20 L=1,19
   YAE = (L-1)*5
   YE = YAE*0.175329
   S = COS(YE)
   T = SIN(YE)
   R11(1) = R1(K)*S*T1(K)*T
   R11(2) = R1(K)*S*T1(K)*T
   R22(1) = R2(K)*S*T2(K)*T
   R22(2) = R2(K)*S*T2(K)*T
   R33(1) = R3(K)*S*T3(K)*T
   R33(2) = R3(K)*S*T3(K)*T
   XN11(1) = XN1(K)
   XN11(2) = XN1(K)
   XN22(1) = XN2(K)
   XN22(2) = XN2(K)
   XN33(1) = XN3(K)
   XN33(2) = XN3(K)
   T11(1) = R1(K)*X*T1(K)*S
   T11(2) = T1(K)*S*R1(K)*T
   T22(1) = T2(K)*S*R2(K)*T
   T22(2) = T2(K)*S*R2(K)*T
   T33(1) = T3(K)*S*R3(K)*T
   T33(2) = T3(K)*S*R3(K)*T

DO 30 M=1,2
   A1 = R11(M)**2
   A2 = R22(M)**2
   A3 = R33(M)**2
   X11(M) = XN11(M)**2
   X22(M) = XN22(M)**2
   X33(M) = XN33(M)**2
   C1 = T11(M)**2
   C2 = T22(M)**2
   C3 = T33(M)**2

DC 30 M=1,2
DC 20 L=1,19
DC 10 K=1,6
GA = A1 * A2 + A3 + A1
GR = H1 * H2 + H3 + H1
GC = C1 * C2 + C3 * C4 + C1
GARC = GA - GH - GC
GRCA = GB - GA - GC
GCAH = GC - GA - GH

CX(1, M, K) = 1.700 + 2.08^2 * GA
CX(2, M, K) = 1.700 + 2.08^2 * GB
CX(3, M, K) = 1.700 + 2.08^2 * GC
CX(4, M, K) = 1.225 + 1.041 * GARC
CX(5, M, K) = 1.225 + 1.041 * GRC
CX(6, M, K) = 1.225 + 1.041 * GCA

CY(1, M, K) = 1.700 + 2.08^2 * GA
CX(2, M, K) = 1.700 + 2.08^2 * GB
CX(3, M, K) = 1.700 + 2.08^2 * GC
CX(4, M, K) = 1.225 + 1.041 * GARC
CX(5, M, K) = 1.225 + 1.041 * GRC
CX(6, M, K) = 1.225 + 1.041 * GCA

CONTINUE

SUMC(N) = X1 * ((SX(1, 1) + SX(2, 1)) / 2) + X2 * ((SX(1, 2) + SX(2, 2)) / 2)
1 * X3 * ((SX(1, 3) + SX(2, 3)) / 2) + X4 * ((SX(1, 4) + SX(2, 4)) / 2)
2 * X5 * ((SX(1, 5) + SX(2, 5)) / 2) + X6 * 0.0000000000000001
3 * X7 * ((SX(1, 6) + SX(2, 6)) / 2)

CONTINUE

DIV = SUMC(1) + SUMC(2) + SUMC(3) + SUMC(6) + SUMC(5) + SUMC(4) + SUMC(2)
1 - SUMC(5) + SUMC(5) + SUMC(2) - SUMC(4) + SUMC(4) + SUMC(1)
2 - SUMC(6) + SUMC(6) + SUMC(3)
CS11 = SUMC(2) + SUMC(3) + SUMC(4) * CS11 / DIV
HSUM = (SUMC + CS11) / 2
FR = 1 / HSUM
EV = 1 / CS11
EH = 1 / HSUM
WRITE (61, 3) FR, EV, FH

CONTINUE

END