Faustmann in the Sea
Optimal Rotation in Aquaculture

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Abstract: In this paper an extended version of the well-known Faustmann model is developed for solving the rotation problem in fish farming. Two particularly important aspects of the problem are emphasized: namely first, the possibilities for cycles in relative price relationships and second, the problem with limitations in release time for certain species. An illustration of the model based on assumptions from salmon farming shows that the inclusion of these two features has major influences on rotation time, and hence on harvesting weight.

Keywords: Fish farming, optimal rotation, dynamic programming, relative prices

1. INTRODUCTION

As fish farms become larger and the industry becomes more competitive, optimal production planning and efficient management practices become key factors for profitability. Among the most important managerial activities in production planning is that of determining the optimal rotation, i.e. finding the best sequence of release and harvesting that maximizes overall profit. This plan impacts on the cash flow from the farm as well as on the allocation of limited resources in production, such as feed, fish, space and environmental resources (Cacho 1997).

The *rotation* problem in fish farming has, together with other fish farming management problems, a lot in common with problems already solved in forestry and animal husbandry. Bjørndal (1988) states it this way: "Conceptually, aquaculture is more similar to forestry and animal husbandry than to traditional ocean fisheries", (Bjørndal 1988, p.139) whereas Karp, Sadeh and Griffin (1986) established the link between the rotation problem in fish farming and that in forestry.[1] During the last decade several models for the *optimal harvesting* of farmed fish have been developed. However, most of these studies consider only a one-shot decision, instead of treating the problem in a dynamic contest focusing on decisions for optimal rotation. With space (volume) as a constraint, this is a serious shortcoming of the traditional models. As the marginal value decreases over time, harvesting makes room for new releases of younger and faster growing fish. I will argue that considering a one-time investment gives at best only a rough estimate of the optimal harvesting time.

This paper presents a dynamic programming model that solves the rotation problem in aquaculture. The model can be used for different species of fish and different farming technologies. Two particularly important aspects of the optimal rotation problem will be emphasized. First, the possibility of releasing juvenile fish at any time of the year is limited for many species. Second, due to seasonalities in supply and demand, relative price relationships between different sizes of fish vary throughout the year. Hence, large fish would reach relatively higher prices than small fish at some times of the year, while the opposite might be the case at other times of the year. When solving for the optimal harvesting time, the model should include all appropriate relative price relationships.

The model can be used for different species of fish and different farming technologies. However, the phrasing and illustration of the model will be in terms of salmon farming. There are two major reasons for this. Salmon is one of the most successfully farmed species, and salmon farming is a complex production process with features that illustrate important aspects of the models.

In what follows, the rotation problem in fish farming is outlined, the main characteristics of fish farming are stressed, and links to similar problems in other industries are described. Then, previous models from the literature are briefly reviewed before the new model is presented. Finally, the usefulness of the model is illustrated before the findings are summarized.

[1] A correct solution to the forestry rotation problem is attributed to the German forester, Faustmann, who wrote a treatise on the subject as early as 1849.

2. OPTIMAL ROTATION IN AQUACULTURE

Farming techniques and practices vary across and within species, some species being cultivated in ponds while others are cultivated in pens immersed in seawater. However, the principles are the same. Very simplified, the process of fish farming can be described as follows: the farmer releases certain amounts of recruits (juvenile fish) into pens or ponds, feeds them for some time and harvests them when they have reached an appropriate marketing weight. When the fish are harvested, space becomes available for new juvenile fish. The farmer can then decide if he or she wants to market small fish by short rotations or larger fish with longer rotations. For some species it is possible to start a new generation at any time of the year, while starting new rotations are limited to certain times of the year for other species. The farmer's two most important decisions in the production process are then: 1) when to transfer the juvenile fish to the pen and 2) when to harvest the fish, i.e. when to start and when to end a rotation.

The Faustmann solution has long been established as the correct approach to solving rotation problems.[23] The "tree"-solution can be explained as follows. The tree should be cut at age T when the marginal increment to the value of the trees equals the sum of the opportunity cost of investment tied up in the standing trees and in the site (independent of whether this site should continue as forest or be converted, for example, into parking lots). The Faustmann model in it simplest form requires that a new rotation is started at the same time as the previous one ends. This is not realistic for several farmed species. Salmon smolts, for instance, can only be released at certain periods in the year.[4] Hence, if the Faustmann model prescribes that a salmon should be harvested after 21 months in the sea, and the rotation starts in March, harvesting will be in November. The farmer will then have empty pens until March of the next year. An optimal harvesting model for aquaculture should consequently manage to take this aspect into account. Note that including limitations on starting time means that we will not have any universal optimal harvest weight. Instead, the optimal rotation will be different for different groups of fish based on when the rotation started.

A problem related to prices that is apparent in fish farming, but not so relevant for forestry, is relative price relationships among sizes of fish. While a tree in the forest will be of only a marginally larger size as time goes by, a growing salmon will "jump" from one quality class to another with certain distinct characteristics every time it develops into a new weight-class. Several studies indicate that the farmer receives different prices for different sizes. If the relationships between prices for different weight classes are constant this can easily be incorporated into the Faustmann model. However Asche and Guttormsen (2001) examine relative prices (i.e. relationships between prices for different weight classes) for salmon and find that relative prices vary throughout the year, i.e., there exist patterns in the relative price relationships. For some part of the year large fish receive a higher price per kilo than small fish, and at other parts of the year the situation is reversed. A harvesting model should manage to take these different deterministic price relationships into account.

A third aspect that also might create problems in the traditional Faustmann model is the growth functions. The Faustmann model requires that the fish grow according to well-defined growth functions that are independent of starting time. Fish growth, however, is (among other factors) a function of water temperature, and water temperature varies throughout the year. For species living in tropical areas where changes in water temperature are small, it might be reasonable to operate with just one general growth function. However, for species farmed in areas with high variability in water temperature such as salmon, temperature varies in such a way that there are different growth functions based on what time (in the year) the fish are released. Fish will consequently grow at different speeds during the year. A fish that starts life in January, say, will have a different growth curve than a fish that starts in August.

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[2] The Faustmann article originally appeared in German, but have later been translated to English. See for instance the translation in *Journal of Forest Economics*, Faustmann (1995)

[3] In addition to the similarities between fish farming and forestry, rotation problems in aquaculture also share similarities with replacement problems in traditional livestock production. See for instance Kennedy (1986).

[4] Due to biological and economic reasons, smolts can only be transferred to sea during a certain period of the year (March-August). In nature, salmon spawn during late spring and hatch normally in January. Therefore, most salmon produced "are born" in January. The supply of smolts is consequently limited in other periods. Smolts are not very fond of cold weather, so release during the winter months is connected with great risk of loss in other periods.

2.1 Previous Research on Optimal Harvesting in Aquaculture

While several studies exist on optimal harvesting problems for farmed aquatic species, I will argue that most of these studies do not address all the important aspects of the problem. In a chronological review of the period 1974 to 1996, Cacho (1997) finds that the most popular species for modeling are shrimp, prawn and salmon. While some of the papers focus on specific species and technologies, others claim to be more general and applicable for different technologies and species. In the following a selection of important studies will be briefly reviewed.[5]

Karp, Sadeh and Griffin (1986) consider the problem of determining optimal harvest and restocking time and level for farmed shrimp. They first consider the case where production occurs continuously, modeled as a deterministic, continuous time autonomous control problem. Harvest and subsequent restocking are modeled as "jumps" in the biomass. Their contribution to the traditional Faustmann solution is that the optimality conditions determine the restocking level as well as the harvest level. Second, they consider the situation where the environment is uncontrolled, modeled as a stochastic control problem. They then proceed to solve it with dynamic programming. However, their model is not flexible enough to include different relative price relationships, and because shrimps are a tropical species, rotation can start at any time of the year. Hence they do not look at limitations on starting times.

One of Bjørndal's (1988; 1990) main points is that fish in a pen are nothing else than a particular form of growing capital. Hence the objective of finding the optimal harvesting times is similar to maximizing the present value of an investment. Bjørndal presents a model in which he illustrates the changes in biomass value over time as a function of growth, natural mortality and fish prices. He then adds costs to the model and presents a comparative statics analysis of the effects of changes in the parameters on optimal harvest date. However, the model is in terms of a one-time investment and what happens after the harvest is not considered. Bjørndal admits that: "it is not sufficient to merely consider a single harvesting time. The problem in question represents an infinite series of investments rather than a one time investment." (Bjørndal 1988, p.153). While he briefly presents a Faustmann-like solution to the problem, the model can neither treat the problem containing limitations on release time nor treat dynamics in relative price relationships.

Several authors have extended Bjørndal's model to emphasize specific aspects of the problem. Arnason (1992) introduces dynamic behavior and presents a general comparative dynamic analysis. He also introduces feeding as a decision variable. Heaps (1993) deals with density-independent growth, whereas Heaps (1995) allows for density-dependent growth and also looks at the culling of farmed fish. Mistiaen and Strand (1998) demonstrate general solutions for optimal feeding schedules and harvesting time under conditions of piecewise-continuous, weight-dependent prices. None of these studies considers the rotation problem.

As this brief review illustrates, only the Karp, Sadeh and Griffin (1986) and Bjørndal (1988; 1990) papers discuss the rotation problem. However both assume that when one year-class is harvested, the next one is released immediately. This again implies that recruits are available throughout the year, which is not the case for a number of important species (salmon among others). None of the papers discusses the problems of dynamics in relative price relationships. This is a serious weakness of the model because changes in relative prices are significant for some aquaculture species (Asche and Guttormsen 2001).

2.2 The Faustmann Solution[6]

We have claimed that the Faustmann model in its standard format is inadequate for finding the optimal rotation in fish farming. However, the model provides important insights into the nature of rotation problems, and will therefore be presented. Among the several methodological approaches to solving the Faustmann problem, the presentation here will be based on dynamic programming. [7]

[5] Other papers related to optimal harvesting in aquaculture include: Lillestøl (1986), Leung (1986), Leung and Shang (1989), Leung, Hochman, Wanitprapa, Shang and Wang (1989), Hean (1994), Rizzo and Spagnolo (1996) and Forsberg (1999

[6] For a thorough presentation and discussion of the problem see Clarke (1990).

[7] This presentation (and the notation) of the dynamic programming Faustmann model relies heavily on the presentation in Kennedy (1986).

The Faustmann problem for a fish farm can be explained as follows. Suppose that a pen has been committed indefinitely to fish farming. If there are fish already swimming in the pen, the decision to be made at every stage is whether to let the fish grow for another period, or to harvest the fish and release juvenile fish. We denote the juvenile fish and a release cost c_r, and assumes that these costs are incurred at the beginning of the period when the pen is empty. The return from harvesting and selling the fish (net of all harvesting, transport and selling costs) is a function of the age of the fish,[8] and can be referred to as net fish value $b\{t\}$. We simplify by assuming that below a certain age (read size) t^0 the fish have no commercial value so that $b\{t\}=0$ for $t<t^0$. For $t \ge t^0$, $b\{t\}$ is positive, initially increasing with t but decreasing with t for $t>T$ (we can think of T as the time of sexual maturity or natural mortality). For this model we further assume that the release of new juvenile fish immediately follows harvesting. The fish must be harvested if it reaches T years of age.

The objective is to maximize the present value of net income streams to infinity, i.e. to maximize the present value of an investment (the living biomass) by determining the optimal rotation time. The release costs and discount factor a are assumed constant through time. Since we assume immediately release, the prospects for a pen with fish aged t years in all rotations are identical, the optimal decision and present value of net income $V\{t\}$ are the same in all rotations. A dynamic programming specification of $V\{t\}$ and the decision alternatives for each age of the pen, t, are then:

$$V\{t\}=\begin{cases} -c_r+aV\{1\} & \text{release} \quad \text{for } t=0 \\ \max\left[aV\{t+1\}, b\{t\}+V\{0\}\right] & \text{harvest or wait} \quad \text{for } 0<t<T \\ b\{t\}+V\{0\} & \text{harvest} \quad \text{for } t=T \end{cases} \tag{1}$$

The typical optimal policy for this problem can be defined in terms of the optimal rotation period $t^* \le T$ as follows:

$$\begin{array}{ll} \text{release} & \text{if } t=0 \\ \text{wait} & \text{if } t<t^* \\ \text{harvest} & \text{if } t \ge t^* \end{array} \tag{2}$$

We then have the present value function for this policy, videlicit:

$$V\{t\}=\begin{cases} -c_r+aV\{1\} & (\text{release}) \quad \text{for } t=0 \\ aV\{t+1\} & (\text{wait}) \quad \text{for } 0<t<t^* \\ b\{t\}+V\{0\} & (\text{harvest}) \quad \text{for } t \ge t^* \end{cases} \tag{3}$$

Now t^* is the rotation period for which $V\{t\}$ is maximized. Kennedy (1986) illustrates how t^* can be found. Let $\hat{V}\{t\}$ denote the present value of net income to infinity derived from a pen of fish aged t if the rotation period is, instead, $\hat{t}$. It then follows that for sufficiently small interval stages and $t=t^*+1$, $\hat{V}\{t\}-V\{t\}$ would be inconsequentially small. It is of particular interest that the fish farmer would be indifferent between $\hat{V}\{0\}$ and $V\{0\}$, and between $\hat{V}\{t^*\}$ and $V\{t^*\}$. Another way of expressing this is that when the age of the biomass is t^* years the farmer is indifferent between harvesting the entire cohort and receiving $b\{t^*\}+V\{0\}$, and postponing the harvest one period, the present value of which is $aV\{t^*+1\}=a\left(b\{t^*+1\}+V\{0\}\right)$, i.e.

$$V\{t^*\}=b\{t^*\}+V\{0\}=a\left(b\{t^*+1\}+V\{0\}\right) \tag{4}$$

Substituting $1/(1+r)$ for a and rearranging, this becomes

$$b\{t^*+1\}-b\{t^*\}=r\left(b\{t^*\}+V\{0\}\right)=rV\{t^*\} \tag{5}$$

Equation (5) is a discrete version of the historical Faustmann formula,[9] which specifies the condition for the optimal rotation period t^*. The term $V\{0\}$ is the capitalized value of the pen immediately prior to releasing new juvenile fish. In the forestry literature this is referred to as the site value or soil expectation, and can be thought

[8] For simplicity we assume here that fish weight is only a function of age. This is of course a simplification since temperature, day length, feeding etc. are also influential.
[9] To be rigorous the stage interval must be approaching zero. For historical background and a full exposition of the analysis of optimal rotation, see Samuelson (1976).

of as the opportunity cost of the pen. Using (5) and expressing $V\{0\}$ in terms of $c_r, b\{t^*\}$ and a , provides an alternative version of the Faustmann formula:

$$b\{t^*+1\}-b\{t^*\}=r\left(b\{t^*\}-c_r\right)\big/1-a^{t^*} \qquad (6)$$

A diagrammatic interpretation of the optimal rotation is shown in Figure 1. The net biomass value is added to the pen value and gives the curve $b\{t\}+V\{0\}$. The vertical intercept of the curve marked $a^{t^*-t}\left(b\{t^*\}+V\{0\}\right)$ shows the present value at $t=0$ of the net income from harvesting the entire pen. From (3) $V\{t\}=a^{t^*-t}\left(b\{t^*\}+V\{0\}\right)$ for $t \le t^*$ t and $b\{t\}+V\{0\}$ for $t \ge t^*$. At time $t=0$, before planting, the fish farmer is indifferent between selling the pen for its maximum capitalized opportunity cost, $V\{0\}$, and releasing new smolts at a cost of c_r , obtaining net income from harvesting the entire cohort at the optimal rotation time, and then again having a free pen worth $V\{0\}$. Note that the two curves are tangential at $t=t^*$, which is the Faustmann formula.

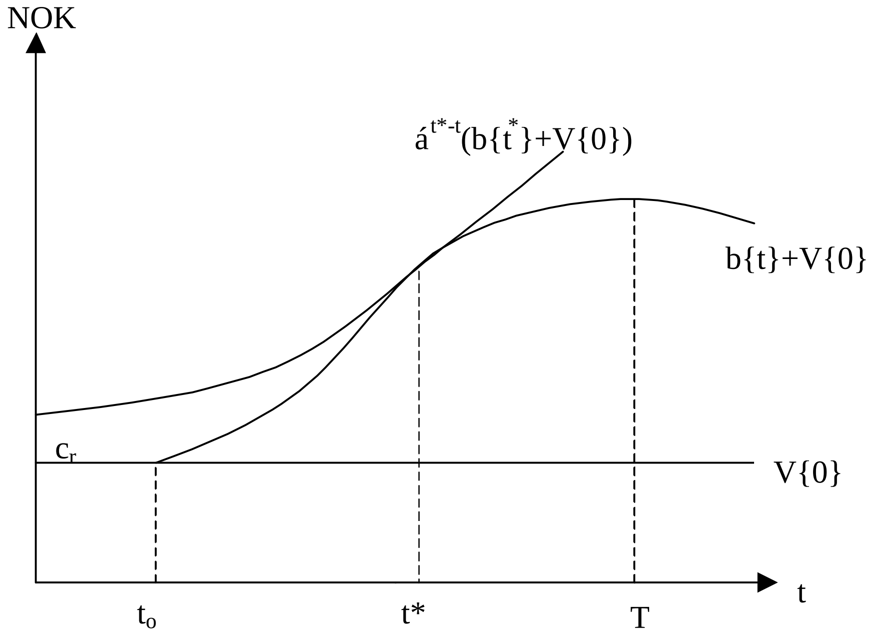

Figure 1: Conditions for the optimal rotation period, t^*

2.3 The Extended Model

The above analytical solution of the Faustmann model is in line with those presentations that do not take into account the problems discussed in previous sections. To include these problems we will therefore extend the Faustmann model so that it becomes more flexible and hence usable for fish farming.

For pedagogical reasons the model is divided into two parts, one part for the case when there are fish in the pens, and one part for the case when the pen is empty. When there are fish in the pen the farmer can, at every decision stage, either harvest the fish or wait. If he harvests he will have an empty pen in which he can either start a new rotation or not. If he harvests he will obtain the biomass value $b\{t\}$ (to simplify, harvesting cost is ignored) plus the value of the empty pen (we will return to the value of the empty pen later). The value of waiting is then the discounted value for the next period minus the incurred feeding cost, i.e. $-c^f + aV\{t+1\}$. The object is then to maximize the present value of the decision by deciding to harvest or not. This decision can then be written as $\underset{d}{\mathrm{Max}}\left[b\{t\}d-c^f+aV\{t+1\}\right]$, where d is the decision variable, taking 0 for wait and 1 for harvest. Biomass is dependent upon the number of fish and the weight of each fish. We assume homogeneity among the fish (i.e. all fish grow at the same speed, so that we can speak of a representative fish). The biomass of a year class $b(t)$ can then be calculated as:[10]

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[10]This follows the biological model developed by Bjørndal (1990).

$$b_t = n_t w_t \tag{7}$$

where n_t is the number of fish and w_t is the weight of the fish. As time increases, two processes will influence the growth of biomass. Some fish will die and the others will gain weight. The number of fish is consequently a function of the number of fish at the beginning, n_0, and mortality. We will have $n_{t+1} = (1-m)n_t$. Mortality, m, can be treated as constant or varying throughout the year as a function of the size of the fish and the time of year. Presenting the formula a little more formally, we can write this as:

$$\begin{aligned}
&V_t(w_t, n_t, k_t) = \max_{d_t}\left\{P_t(w,k)\,w_t n_t d_t - c\left(f_t\left(n_{t+1}w_{t+1} - n_t w_t\right)\right) + aV_{t+1}(w_{t+1}, n_{t+1}, k_{t+1})\right\}\\
&\text{where}\\
&d_t = \begin{cases} 0 - \text{wait} \\ 1 - \text{harvest} \end{cases}\\
&w_{t+1} = g(w_t, k)\,w_t(1-d_t)\\
&n_{t+1} = (1-m)\,n_t
\end{aligned} \tag{8}$$

Here $P_t(w,k)$ is the per kilo price for fish of size w at week k, w_t is the weight of the representative fish and n_t is number of fish at time t. Since this model can be solved numerically the modeler can choose whatever price scheme he wants. The term $c\left(f_t\left(n_{t+1}w_{t+1} - n_t w_t\right)\right)$ is the cost of feeding n fish from w_t to w_{t+1} kilos, and finally, g is the growth rate as a function of initial weight and the time of year.[11]

Equation (8) shows the value of a pen containing fish. Immediately after harvesting the pen is empty and the farmer must decide whether to release new fish or to wait. Hence, he must at every stage decide whether releasing juvenile fish immediately, or waiting, maximizes net present value of the pen. The value of the empty pen can then be written as $\max_s\{-c_r s + aV_{t+1}\}$ where s is a decision variable taking 1 for releasing and 0 for waiting. V_{t+1} is the value the next period. More formally, this maximization problem can be written as:

$$\begin{aligned}
&V_t(0, k_t) = \max_{s_t}\left\{-c_r s_t + aV_{t+1}(w_{t+1}, n_{t+1}, k_{t+1})\right\}\\
&\text{where}\\
&s_t = \begin{cases} 0 \text{ not release} \\ 1 \text{ release} \end{cases}\\
&w_{t+1} = (1-d_t)\,s_t w_t
\end{aligned} \tag{9}$$

The value when releasing fish will be the discounted future value of the pen minus the cost of releasing, $-c_r$. With a decision to release, the weight of the representative fish for the next period will be $(1-d_k)w_t$, where d_k is then a first day death rate. This parameter will handle the problems of limitations in release time. In periods where it is impossible to release juvenile fish, we set d_k equal to one.[12]

The extended model puts neither restrictions on growth, release time nor relative price relationships. The model can also easily incorporate all different costs and then examine what happens with rotation time when changes occur in one or several of the parameters. However, if the growth function is independent of release time, d_k equals zero (which means that it is possible to release fish during the whole year) and there are no seasonalities or fluctuations in prices or costs, the extended version of the problem will collapse to the traditional Faustmann problem developed in the previous section.

3. A NUMERICAL ILLUSTRATION

To illustrate the usefulness of the extended model some simple illustrations will be presented in which the results from the Faustmann model are compared with results from the extended one. Hence, the two models described

[11] Growth functions in practical fish farming are usually tabulated, i.e. the table says how much a fish of size w will grow in one day with a water temperature of t degrees.
[12] We have for simplicity assumed weight gain to be zero in the first period. This assumption is reasonable because the fish need some time in the pen to adapt to the new environment.

above are programmed and solved with fairly general assumptions.[13] To emphasize the difference between the two models, unnecessary details are ignored. The model is, as mentioned in the introduction, applied on salmon farming. This is based on the fact that Salmon is one of the most successfully farmed species, and that salmon farming exhibit features that illustrate important aspects of the models.

Biomass is defined as the number of fish times fish weight, and for weight gains, a slightly updated version of the growth function provided in Bjørndal (1990) is used.[14] The growth function is as follows:

$$w(t) = 2.8t^2 - 0.7t^3 \tag{10}$$

where t is the year. The function is tabulated, divided into months, with monthly mortality rate set to 0.8% and the interest rate set to 7%. To simplify, constant cost is assumed. The models are then solved with different assumptions about release time and relative price relationship.
The standard Faustmann model was solved first. With a price per kilogram of salmon of NOK 26, fish should be harvested at 21 months age when the fish have reached 4.8 kilos marketing weight. However, this solution implies that recruits are available throughout the year, and that it is possible to release throughout the year. As this is not the case, release was limited to March, April, May, August, September and October. The model solution then gives completely different rotations. The results for the different release times are presented in Table 1. It can be seen that only the fish released in August will be harvested at 4.8 kilos, and fish released at other times of the year will be harvested between month 19 and month 23 (4.2–5.3 kilos). This is a quite substantial difference, with a major impact on production planning and profit.

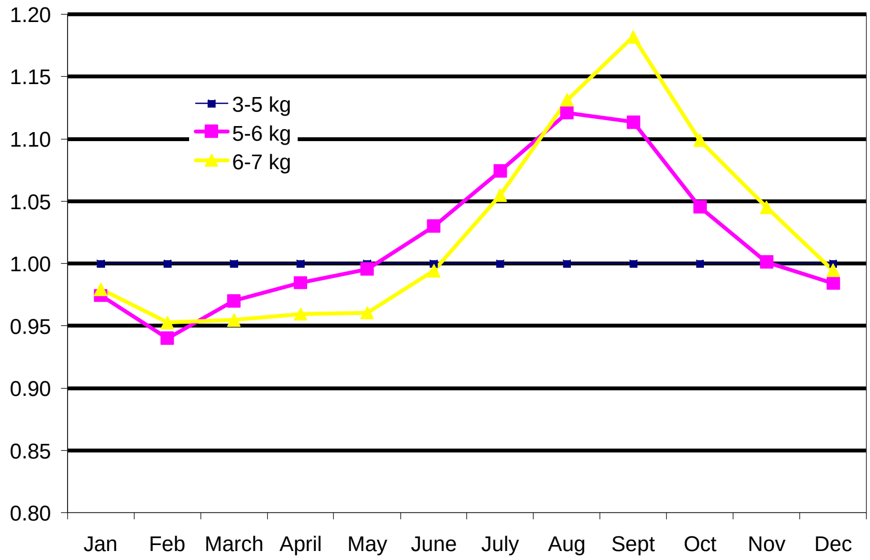

Figure 2: Price Index, relative price relationship. Based on historical observation 1992-1998.

To illustrate another of the advantages with the extended model the same problem as above is formulated, but in addition a non-constant relative price relationship is included. This relative price relationship is based on the results in Asche and Guttormsen (2001) and included in the following way: based upon price observations for salmon of different sizes from 1993 to 1998, a monthly relative price index is constructed for each weight class. The amount 3–5 kilos is used as a base weight i.e. the price equals one. The price index for the other weight classes, 5–6 kilos for instance in January, is then calculated as

$$\sum_{i=1992}^{n=1998} \left(p_{\text{i,january}}^{5-6\text{kilos}} \Big/ p_{\text{i,january}}^{3-5\text{kilos}} \right) \tag{11}$$

A graph of the price indexes is provided in Figure 2. As can be seen, larger fish will usually be more valuable than smaller fish. However, this changes during the year. Looking then at the optimal harvesting results in Table 1, we see that the numbers change significantly. Fish released in April should now be harvested at 3.32 kilos

[13] The models are both programmed in MATLAB from MathWorks Inc. and solved with a toolbox developed by Mario Miranda and Paul Fackler (Miranda and Fackler 2001).
[14] Bjørndal's (1990) growth functions are based on data from salmon farmed in 1988. Since then selective breeding, feed and feeding technology have improved the farming such that the fish grow much faster.

after 16 months in the sea instead of at 5.36 kilos after 23 months in the sea. Also, for all the other release times, the inclusion of relative price relationship substantially changes the harvesting time.

Table 1: Optimal harvesting weight and time with and without relative price relationship.

	Harvest weight in kilo (age in months)		
Release time	Faustmann	Constant prices	Relative price Relationship
March	4.82 (21)	4.24 (19)	3.63 (17)
April	4.82 (21)	5.36 (23)	3.32 (16)
May	4.82 (21)	5.10 (22)	6.47 (29)
August	4.82 (21)	4.82 (21)	6.03 (26)
September	4.82 (21)	4.54 (20)	5.60 (24)
October	4.82 (21)	5.10 (22)	5.36 (23)

4. CONCLUDING REMARKS

The rotation problem in fish farming shares many features with rotation problems in forestry and traditional terrestrial livestock production. However, fish farming also exhibits specific features that demand a more flexible model than the ones constructed for other industries. In this paper such a model is presented. This model is general and flexible enough to treat different species and technologies. Two specific features with aquaculture are stressed: limitations on release times and dynamic relative price relationships. To illustrate the strength of the model and the importance of extending traditional models, a simple example of the use of the model is presented. The empirical illustration well illustrates the importance of a model that can treat different relative price relationships as well as limitations on when it is possible to start a new rotation.

Asche and Guttormsen (2001) claim that “we in general cannot say anything about the direction of the changes in the harvesting time due to the cycles in relative prices”. This claim is confirmed with the extended model since fish released at distinct times of the year will have higher harvesting weights when relative price relationships are included, while fish released at other times of the year will have lower harvesting weights.

As fish farm enterprises become larger and the industry becomes more competitive, the timing of harvesting and marketing become key factors for success. The production plan has an impact on the cash flow from the farm as well as on the allocation of limited resources in production, such as feed, fish, space and environmental resources. Consequently, a well-developed production plan can mean the difference between loss and profit for a fish farm.

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REFERENCES

Arnason, R., Optimal feeding schedules and harvesting time in aquaculture. *Marine Resource Economics* 70(1): 15-35, 1992.

Asche, F. and A.G. Guttormsen., Patterns in relative price for different sizes of farmed fish. *Marine Resource Economics*. 16( ) , 2001

Bjørndal, T., Optimal harvesting of farmed fish. *Marine Resource Economics* 5(2): 139-59, 1988.

Bjørndal, T., *The Economics of Aquaculture*. Oxford: Blackwell Scientific Publications, 1990.

Cacho, O. J, Systems modeling and bioeconomic modeling in aquaculture. *Aquaculture Economics and Management* 1(1): 45-64, 1997

Clarke. C. W, *Mathematical Bioeconomics. The Optimal Management of Renewable Resources.* New York: John Wiley & Sons, 1990

Faustmann, M., Calculation of the value of which forest land and immature stands possess for forestry. Republished with permission from Commonwealth Forestry Association in *Journal of Forest Economics* 1(1): 7-44, 1995

Forsberg, O. I, Optimal harvesting of farmed Atlantic salmon at two cohort management strategies and different harvest operation restrictions. *Aquaculture Economics and Management*. 3(2):143-158, 1999.

Hean, R. L., An optimal management model for intensive aquaculture - an application in Atlantic salmon. *Australian Journal of Agricultural Economics* 38(1): 31-47, 1994.

Heaps, T., The optimal feeding of farmed fish. *Marine Resource Economics* 8(2): 89-99,

Heaps, T. Density dependent growth and the culling of farmed fish. *Marine Resource Economics* 10(3): 285-298, 1995

Karp, L., A. Sadeh and W. L. Griffin. Cycles in agricultural production. The case of aquaculture. *American Journal of Agricultural Economics* 68: 553-561, 1986

Kennedy, J.O.S., *Dynamic programming: applications to agriculture and natural resources*. London: Elsevier Applied Science Publications, 1986

Leung, P.S., Applications of systems modeling in aquaculture. *Aquacultural Engineering*, 5, 171-182, 1986

Leung, P. S. and Y. C. Shang. Modeling prawn production management system. A dynamic markov decision approach. *Agricultural Systems*, 29, 5-20, 1989

Leung, P.S., E. Hochman, K. Wanitprapa, Y. C. Shang and J.K. Wang. Optimal Harvest schedule for marincultured shrimp: A stochastic sequential decision model. *HITAR College of Tropical Agriculture and Human Resources, University of Hawaii Research 060.* 1989.

Lillestøl, J., On the problem of optimal timing of slaughtering in fish farming. *Modeling, Identification and Control* 7(7): 199-207, 1986.

Mirando, M.J and P. L. Fackler, *Applied Computational Economics and Finance*. US: MIT Press. 2001

Mistiaen, J.A. and I. Strand, Optimal feeding and harvest time for fish with weight-dependent prices. *Marine Resource Economics* 13(4): 231-246, 1998.

Rizzo, G. and Spagnolo,.M. A Model for the optimal Management of Sea Bass Dicentrachus Labrax Aquaculture. *Marine Resource Economic*s 11(4) 267-286, 1996.

Samuelson, P. A., Economics of forestry in an evolving society. *Economic Inquiry* 14:466-492, 1976.