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EFFECT OF DIRECTION OF GROWTH RINGS ON THE RELATIVE AMOUNT OF SHRINKAGE IN WIDTH AND THICKNESS OF LUMBER AND EFFECT OF RADIAL AND TANGENTIAL SHRINKAGE ON DIMENSIONS OF ROUND TIMBERS

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EFFECT OF DIRECTION OF GROWTH RINGS ON THE RELATIVE AMOUNT OF SHRINKAGE IN WIDTH AND THICKNESS OF LUMBER AND EFFECT OF RADIAL AND TANGENTIAL SHRINKAGE ON DIMENSIONS OF ROUND TIMBERS

By

J. D. MacLEAN, Engineer

Introduction

The principal factors affecting the volumetric shrinkage of wood, disregarding the effect of collapse and of "set" under stress, are the amount of wood substance per unit volume, which is shown by the density or specific gravity, and the amount of water that is removed below the fiber saturation point. Other factors that may affect the results to a variable extent are drying stresses and chemical substances in the wood. The relative amount of shrinkage that occurs in the width and thickness of a rectangular section depends upon the radial and tangential shrinkage and the angle the rings make with the sawed surfaces.

There are occasions when it is of interest to know how the angle that the rings make with the surfaces of a sawed timber affect the dimensions of width and thickness after shrinkage or swelling occurs. As an illustration, it is not always possible to avoid a certain amount of variation in the slope of the annual rings in the cross section of matched test specimens cut from a particular stick. Variability of this kind is dependent on such factors as size of the log from which the stick is cut, length and cross sectional dimensions of the piece, and size of test specimens. If, for the purpose of test, it is necessary to prepare the specimens in finished form while green and they are subsequently seasoned for strength tests, it may be important to know whether the variations in ring slope that may occur in different specimens will have a significant effect on the proportion of shrinkage in width and thickness. Differences in shrinkage in these two dimensions will of course have more or less effect on the strength factors computed from bending strength tests. For example, work to proportional limit and work to maximum load vary inversely as the product of width and thickness; stress at proportional limit and modulus of rupture vary inversely as the product of width and square of the thickness, and the modulus of elasticity varies inversely as the product of width and cube of the thickness.

Various other examples could be given where information on the effect of slope of rings or the relative amount of shrinkage in width and thickness is of assistance.

The purpose of this paper is to discuss the theoretical considerations that are involved in determining the relative amount of change in width and thickness that may occur when moisture changes take place and the annual rings make various angles with the surfaces of sawed material. The formulas developed will be found convenient to use in determining the approximate change in width and thickness when lumber or pieces having moderately small cross-sectional dimensions are seasoned and the average slope of the rings is known. A discussion of the effect of radial and tangential shrinkage on the dimensions of round timbers is also included.

Effect of Radial, Tangential, and Longitudinal Shrinkage on Timber Volume and Dimensions

Since longitudinal shrinkage is usually slight in normal wood, its effect on volumetric shrinkage can be disregarded, and only shrinkages in the radial and tangential directions need to be considered as having an important effect on the timber dimensions. The ratio of the radial shrinkage, or dimension change that takes place at right angles to the annual rings, to the tangential shrinkage, or shrinkage parallel to the rings, varies considerably for different species; but a rough average value would be about 1 to 2.

Figure 1-A represents the cross section of a timber W inches wide and V inches thick in which the rings are approximately parallel with the horizontal faces while figure 1-B represents the cross section of a timber of the same dimensions with the rings at right angles to the horizontal faces. The shrinkage in the width W and thickness V will therefore be different for the two sections, since the unit radial shrinkage P_R is less than the unit tangential shrinkage P_T.

Assuming the length constant, because of negligible shrinkage in the longitudinal direction, the volume after shrinkage for the transverse section shown in figure 1-A will be, $[W(1 - P_T)]\ [V(1 - P_R)] = (WV)(1 - P_T)(1 - P_R)$ per unit length. Likewise the volume after shrinkage for the section shown in figure 1-B will be $[W(1 - P_R)]\ [V(1 - P_T)] = (WV)(1 - P_R)\ (1 - P_T)$. In each section the volumetric shrinkage per unit length $= WV - [(WV)\ (1 - P_T)\ (1 - P_R)] = WV\ [P_T + P_R - P_TP_R]$. The volumetric shrinkage is therefore the same although the amount of shrinkage in the directions of width and thickness is different for the two sections.

When the rings are approximately parallel to two of the faces, as in figure 1, it is a simple matter to compute the change in the width W and the thickness V that may be expected when the average unit shrinkages P_R and P_T in the radial and tangential directions are known or assumed for the species under consideration.

More often, however, the rings make some angle θ with the horizontal faces, which is between 0° and 90° as indicated in figures 2 and 3. For the purpose of this discussion the rings will be assumed as straight lines. This can be done without serious error if the dimensions of width and thickness are not too large. Assuming other conditions the same, the volumetric shrinkage would not be affected by the slope of the rings with respect to the faces, but the relative amount of shrinkage in width and thickness would vary depending on the angle θ. This necessarily would result from the fact that the rings vary in length over the surface of the cross section. In addition both the width and thickness are affected by components of radial and tangential shrinkage.

If the rings are assumed at right angles to two opposite faces (as in fig. 1) the section remains rectangular after shrinking even when the radial and tangential shrinkage are different, but the ratio of the two dimensions changes from that existing before shrinkage took place. On the other hand, if the rings make an angle θ with two opposite faces, and θ is some angle between 0° and 90°, the shape of the section could not remain rectangular when the radial shrinkage is different from the tangential. Figure 2 illustrates the effect of radial and tangential shrinkage separately and also the combined effect of shrinkage in the two directions when the rings are assumed as straight lines. In this figure the angle $\theta = 45°$, and the radial shrinkage is assumed as one-half the tangential shrinkage. Figure 3 shows the shape after shrinking when the angle θ is 30° assuming the same values of P_R and P_T as in figure 2. In these two figures the original width W and thickness V have been assumed the same so that before shrinkage the cross section is square. It may be noted that when $\theta = 45°$, the unit shrinkage, measured on the edges of the cross section is the same in both directions. At this angle the change per unit width and thickness is the same regardless of the ratio of radial to tangential shrinkage because the shrinkage components are the same. In order to emphasize the effect of shrinkage in figures 2 and 3 the unit radial shrinkage P_R was taken as 0.125 and the unit tangential shrinkage P_T was taken as 0.25. These values are approximately 2-1/2 times greater than the radial and tangential shrinkage factors for white oak in seasoning from the green to the oven-dry condition.

Theoretically if P_R were equal to P_T (assuming that the short lengths of the annual rings are straight lines) the shape of any cross section would remain the same after shrinking, and the change in the two dimensions, width and thickness, would be in the same proportion regardless of the direction of the rings with respect to the surfaces. For example, in figures 4 and 5 the shrinkage in each direction has been assumed as 50 percent, and the volume after shrinking will therefore be one-fourth the original volume. The angle θ has been taken as 15° for figure 4 and 30° for figure 5; but it may be noted that both the shape and the ratio of width to thickness remain the same regardless of the angle assumed.

Formula for Computing Shrinkage in Width and Thickness for Any Angle θ

The following discussion outlines the method used in developing a formula for computing the shrinkage in width and thickness for any angle θ that the rings make with the horizontal faces. This assumes that the annual rings can be considered as approximately straight lines over the cross section under consideration.

The direction of the rings with respect to the horizontal faces is shown in figure 6, which also shows the shrinkage components affecting the final width and thickness. In the figure the axis XX has been rotated through the angle θ so that it is in the position X_1X_1 parallel to the direction of the rings. Similarly the axis YY is rotated into the position Y_1Y_1, which is perpendicular to the annual rings and thus parallel to the direction of radial shrinkage. All tangential shrinkage is therefore in a direction parallel to the axis X_1X_1, and all radial shrinkage is in a direction parallel to the axis Y_1Y_1. In this figure the various distances to the axis Y_1Y_1 (measured parallel to the axis X_1X_1 from the edges of the section W inches wide and V inches thick) are designated as L. Similarly all distances from the edges to the axis X_1X_1 and measured parallel to the axis Y_1Y_1 are designated as L_1.

It may be noted from figure 6 that the distance L from any point (x, y) to the axis Y_1Y_1 is equal to $(x + y \tan \theta)(\cos \theta)$ or,

$L = x \cos \theta + y \sin \theta$ -(A)

Likewise the distance L_1 from any point (x, y) to the axis X_1X_1 is equal to $(y - x \tan \theta)(\cos \theta)$ or $L_1 = y \cos \theta - x \sin \theta$ - - - - - - - - - - - -(B)

The shrinkage parallel to the horizontal axis XX =

$LP_T \cos \theta - L_1 P_R \sin \theta = P_T [x \cos^2 \theta + y \sin \theta \cos \theta] -$
$P_R [y \sin \theta \cos \theta - x \sin^2 \theta]$ -(C)

$= (x_1 + x_2)$ where $x_1 = L P_T \cos \theta = P_T [x \cos^2 \theta + y \sin \theta \cos \theta]$ and $x_2 = -L_1 P_R \sin \theta = -P_R [y \sin \theta \cos \theta - x \sin^2 \theta]$.

Then $x - (x_1 + x_2)$ is the distance of the point xy from the vertical axis YY after shrinking. Similarly the shrinkage parallel to the vertical axis YY = $L P_T \sin \theta + L_1 P_R \cos \theta =$

$P_T [x \sin \theta \cos \theta + y \sin^2 \theta] + P_R [y \cos^2 \theta - x \sin \theta \cos \theta]$ - - - - -(D)

$= (y_1 + y_2)$ where $y_1 = L P_T \sin \theta$ and $y_2 = L_1 P_R \cos \theta$. After shrinking, the distance of the point xy from the horizontal axis XX is then, $y - (y_1 + y_2)$. The new coordinates of a point (x,y) in a given cross section are therefore $[x - (x_1 + x_2)]$ and $[y - (y_1 + y_2)]$ after shrinkage has occurred.

If W represents the width and V the thickness, as indicated in figures 2 and 3, the original width W will shrink to the length of side ab where the length ab = length cd = length A_1A_1. Similarly the thickness, V will shrink to the length ac, which = length bd = length B_1B_1.

The length A_1A_1 (figs. 3 and 4) which is equal to the length of the side ab or cd, can be readily found by determining the x and y coordinates of the points A on the two vertical faces after shrinkage. For points on this axis, y in formula (C) becomes zero. Hence $x_1 = \frac{W}{2} (P_T \cos^2 \theta)$ and $x_2 = \frac{W}{2} (P_R \sin^2 \theta)$ since at points A, $x = \frac{W}{2}$. In this case, $\frac{W}{2}$ will have a plus or minus sign depending on whether it is measured to the right or left of the vertical axis. The values of x at the ends of line A_1A_1 (figs. 3 and 4) are then $\frac{W}{2} [1 - (P_T \cos^2 \theta + P_R \sin^2 \theta)] = x_m$. Similarly, since $y = 0$ at points A the values of y for the points $A_1 = 0 - (y_1 + y_2)$ $= - [\frac{W}{2} (P_T - P_R) (\cos \theta \sin \theta)] = y_m$. Likewise the length B_1B_1 can be found by determining the x and y coordinates of the points B on the two horizontal faces after shrinkage. For these points x in formula D becomes zero and $y_1 = \frac{V}{2} (P_T \sin^2 \theta)$ and $y_2 = \frac{V}{2} (P_R \cos^2 \theta)$ where $\frac{V}{2}$ may have a plus or minus sign depending on whether it is measured above or below the horizontal axis. The values of x at the ends of line B_1B_1 are then $- [\frac{V}{2} (P_T - P_R) \cos \theta \sin \theta] = x_n$ and the values of y at the ends of line $B_1B_1 = \frac{V}{2} [1 - (P_T \sin^2 \theta + P_R \cos^2 \theta)] = y_n$. The length A_1A_1 (figs. 2 and 3) is then $2\sqrt{x_m^2 + y_m^2}$

$$= W\sqrt{[1 - (P_T \cos^2 \theta + P_R \sin^2 \theta)]^2 + [(P_T - P_R) \cos \theta \sin \theta]^2} \text{ - - - - - -(E)}$$

and the length B_1B_1 (figs. 2 and 3) $= 2\sqrt{x_n^2 + y_n^2}$

$$= V\sqrt{[1 - (P_T \sin^2 \theta + P_R \cos^2 \theta)]^2 + [(P_T - P_R) \cos \theta \sin \theta]^2} \text{ - - - - -(E}_1\text{)}$$

Computation of Dimensions After Shrinkage

Since the term $[(P_T - P_R) \cos \theta \sin \theta]^2$ in equations (E) and (E_1) is extremely small, the difference between the true length A_1A_1 and the length $[W - 2 (x_1 + x_2)]$, which equals W_s in figures 2 and 3, is negligible even for relatively large values of thickness or width. It is then sufficiently accurate to compute the width after shrinkage (W_s) as,

$$W_s = [W - 2(x_1 + x_2)] = W [1 - (P_T \cos^2 \theta + P_R \sin^2 \theta)] \text{ - - - - - - - - - -(F)}$$

The thickness after shrinkage (V_s) would, for the same reason, be computed as, $V_s = [V - 2(y_1 + y_2)] = V[1 - (P_T \sin^2\theta + P_R \cos^2\theta)]$ - - - - - (F_1) These values, W_s and V_s, the length of the sides after shrinking measured on the horizontal and vertical axes AA and BB respectively, will be approximately the same as the true lengths A_1A_1 and B_1B_1 determined from equations (E) and (E_1).

Computation of Unit Shrinkage

When W and V become unity, equations (F) and (F_1) show the corresponding dimensions after shrinkage, per unit width and unit thickness. If, therefore, H represents the average unit horizontal shrinkage (shrinkage in width) and T represents the average unit vertical shrinkage (shrinkage in thickness) $H = 2(x_1 + x_2) = (P_T \cos^2\theta + P_R \sin^2\theta)$ - - - - - - - - (G) and $T = 2(y_1 + y_2) = (P_T \sin^2\theta + P_R \cos^2\theta)$ - - - - - - - - - - - - - - - (G_1) The dimensions after shrinkage, for any width W and thickness V, are therefore computed as $W[1 - H]$ and $V[1 - T]$ respectively.

Values of P_R and P_T are given in table 1 for various species. These are average values for the species listed and will serve as a guide in comparing the shrinkage of different woods. Data for other woods can be found in U.S.D.A. Technical Bulletin No. 479, "Strength and Related Properties of Woods Grown in the United States."

The following example will illustrate the procedure for computing the shrinkage when the rings make an angle θ with the horizontal faces. The wood will be assumed as sugar maple that has an average shrinkage from the green to the oven-dry condition of 4.9 percent radially = 100 (P_R) and 9.5 percent tangentially = 100 (P_T).

Assume $\theta = 50°$, the angle the rings make with the horizontal surfaces. Compute the total shrinkage in width and thickness in seasoning from the green to the oven-dry condition if the width W (measured in the horizontal direction) is 1.5 inches and the thickness V is 3/4 inch.

The sine of 50° = 0.766 and the cosine = 0.6428 $\sin^2 50$ is then 0.587 and $\cos^2 50$ is 0.413 approximately. From equation (G) the shrinkage per unit width $H = [(0.095)(0.413) + (0.049)(0.587)] = 0.068$. From equation ($G_1$) the shrinkage per unit thickness T $= [(0.095)(0.587) + (0.049)(0.413)] = 0.076$.

The total horizontal shrinkage for the width of 1.5 inches is then (1.5) (0.068) = 0.102 inch while the shrinkage for the thickness of 3/4-inch is (0.75)(0.076) = 0.057 inch. The width after shrinking (W_s) is then, (1.5 - 0.102) = 1.398 or approximately 1.4 inches, and the thickness after shrinking (V_s) is (0.75 - 0.057) = 0.693 or about 0.69 inch.

Neglecting the slight change from a right angle where the horizontal and vertical faces meet, the volume after shrinkage = $WV - [W(1 - H)][V(1-T)]$, per unit length along the fibers, = $WV (H + T - HT)$. The volumetric shrinkage for the section would then =

(1.5)(0.75) [0.068 + 0.076 - (0.068)(0.076)] = (1.125)(0.1388) = 0.1560 cu. in.; and the volume after shrinking would be, (1.125 - 0.1560) = 0.969 or approximately 0.97 cubic inch.

Table 2 shows values of $\sin^2 \theta$ and $\cos^2 \theta$ between 0° and 90° in 5° intervals that will be convenient to use in making calculations. Figure 7 shows curves for $\sin^2 \theta$ and $\cos^2 \theta$, and figure 8 shows a curve for $\sin \theta \cos \theta$. Figure 8 can be used if it is desired to compute the length A_1A_1 or B_1B_1 from equation (E) or (E_1) or to compute values of x_1 and x_2 or y_1 and y_2 for any point by means of formulas (C) and (D). Values of $\sin^2 \theta$, $\cos^2 \theta$, and $\sin \theta \cos \theta$, can be found from these charts with sufficient accuracy for computing shrinkage by means of the equations given.

If it is desired to find the shrinkage to any moisture content M (percent) between the fiber saturation point and the oven-dry condition the shrinkage values H and T, can be multiplied by $(1 - \frac{M}{30})$ where 30 is taken as the percent moisture content at the fiber saturation point. For example, assume it is desired to find the horizontal shrinkage H and the vertical shrinkage T for coast Douglas-fir when the wood is seasoned from the green condition to 12 percent moisture and the rings make an angle of approximately 15° with the horizontal faces. From table 1, P_R is found to be 0.05 and P_T is 0.078. For θ = 15° figure 7 shows $\sin^2 15°$ = 0.07 and $\cos^2 15°$ = 0.93 approximately. From equation (G), the unit shrinkage $H = [(0.078)(0.93) + (0.05)(0.07)] = 0.076$ or 7.6 percent. Likewise the unit vertical shrinkage $T = [(0.078)\ 0.07 + (0.05)(0.93)] = 0.052$. At 12 percent moisture H would be computed as $7.6\ (1 - \frac{12}{30}) = 4.6$ percent while the unit vertical shrinkage T would be computed as $5.2\ (1 - \frac{12}{30}) = 3.1$ percent approximately.

The foregoing discussion is based on the assumption that, for the cross-sectional area under consideration, the change in ring curvature is relatively small, and it is sufficiently close to determine θ from the average slope of the rings. Even when the section is wide enough or thick enough to allow a considerable change in ring curvature, as in figure 9, it is often possible to compute the approximate shrinkage in the horizontal and vertical directions by assuming the cross-sectional area divided into smaller sections as indicated by the heavy vertical lines in figure 9.

The average slope of the rings for each area 1, 2, 3, and 4 could then be determined as shown by the tangent lines AB, CD, EF, and GH making angles θ_1, θ_2, θ_3, and θ_4 with the horizontal face. The total horizontal shrinkage would then be taken as the sum of the computed horizontal shrinkages for the divisions shown. Similarly the vertical shrinkage could be computed for each section to determine the variation in thickness at different points. For figure 9 the thickness would evidently be decreased more on the left side after shrinking, than on the right side because of the steeper angle of the rings (slope AB) and the greater effect of the component of tangential shrinkage. The shrinkage of the section on the right side, however, would have a greater effect on the horizontal shrinkage and therefore contribute more to reducing the total width because of the larger component of tangential shrinkage in the horizontal direction.

Round timbers

If radial shrinkage only is considered and the effect of checking on the resulting dimensions is disregarded, the change in diameter because of radial shrinkage is $D\ (1 - P_R)$, where D is the diameter before seasoning and P_R is the unit radial shrinkage.

If tangential shrinkage only is considered, the change in diameter because of circumferential shrinkage would be $\frac{\pi D(1 - P_T)}{\pi} = D(1 - P_T)$.

Since P_T, the unit tangential shrinkage, is greater than P_R, the diameter $D(1 - P_T)$ is less than $D\ (1 - P_R)$. In this case $D(1 - P_R) - D(1 - P_T) = D\ [(1 - P_R) - (1 - P_T)] = D(P_T - P_R)$. The difference of P_T and P_R sets up a stress that tends to cause checking.

Neglecting the effect of checking, the final diameter after seasoning would probably be between that resulting from radial shrinkage, or $D(1 - P_R)$, and that resulting from tangential shrinkage, $D(1 - P_T)$.

If it is desired to compute the maximum shrinkage that could occur under any conditions, it is sufficiently accurate to assume that the final diameter is not less than that resulting from tangential shrinkage only, or $D\ (1 - P_T)$. For example, assume a loblolly pine pole is green at the time it is set and it seasons to a moisture content of about 15 percent as a minimum. Table 1 shows that the average tangential shrinkage P_T is 0.074 in seasoning from the green to the oven-dry condition. In seasoning to 15 percent moisture, the shrinkage is about one-half this amount, or about 0.037. If the diameter where shrinkage is to be determined is 8 inches, the final diameter after seasoning is computed as $8(1 - 0.037) = 7.7$ inches.

Summary

Formulas for width and thickness after shrinkage

The following formulas were derived for making shrinkage calculations for width and thickness when the rings make an angle θ with the horizontal surfaces and θ is greater than 0° and less than 90°.

$$W_s = W\sqrt{[1 - (P_T \cos^2\theta + P_R \sin^2\theta)]^2 + [(P_T - P_R)\cos\theta \sin\theta]^2} \quad \text{-- -- -- --(1)}$$

$$V_s = V\sqrt{[1 - (P_T \sin^2\theta + P_R \cos^2\theta)]^2 + [(P_T - P_R)\cos\theta \sin\theta]^2} \quad \text{-- -- -- --(2)}$$

In these equations W_s = the width after shrinkage, W = the width before shrinkage, V_s = the thickness after shrinkage, V = the thickness before shrinkage, P_T = the unit tangential shrinkage and P_R = the unit radial shrinkage from the green to the oven-dry condition.

Since the term $[(P_T - P_R)\cos\theta \sin\theta]^2$ in equations (1) and (2) is extremely small, it is sufficiently accurate to compute the width after shrinkage as,

$$W_s = W[1 - (P_T \cos^2\theta + P_R \sin^2\theta)] \quad \text{-- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- (1a)}$$

Similarly the thickness after shrinkage may be computed as,

$$V_s = V[1 - (P_T \sin^2\theta + P_R \cos^2\theta)] \quad \text{-- -- -- -- -- -- -- -- -- -- -- -- -- -- -- -- (2a)}$$

Unit shrinkage in width and thickness

If H represents the average unit horizontal shrinkage and T represents the average unit vertical shrinkage (shrinkage in thickness) from the relation shown by equations (1a) and (2a)

$$H = (P_T \cos^2\theta + P_R \sin^2\theta) \quad \text{-- (3)}$$

$$T = (P_T \sin^2\theta + P_R \cos^2\theta) \quad \text{-- (4)}$$

The total shrinkage in width is therefore WH, and the total shrinkage in thickness is VT. (This neglects the very slight change in angle at the corners, which can be neglected.) The width after shrinking may then be computed as $W(1 - H)$, and the thickness after shrinking as $V(1 - T)$.

Similarly, after shrinking the volume (per unit length along the fibers) may be computed as $WV(1 - H)(1 - T)$ and the volumetric shrinkage as $WV - WV(1 - H)(1 - T) = WV(H + T - HT)$ where the very slight variation from 90° at the corners, is neglected.

Shrinkage of round timbers

Since tangential shrinkage is greater than radial shrinkage for practical purposes it should be sufficiently accurate to assume that the maximum shrinkage would not exceed that computed on the basis of tangential shrinkage only. The computed shrinkage would then be $D(P_T)$ or the diameter after shrinking would be $D(1 - P_T)$, where D is the diameter before seasoning and P_T is the unit tangential shrinkage over the moisture range under consideration.

Table 1.--Average radial, tangential, and volumetric shrinkage of various commercial species grown in the United States[1]

Species	Average shrinkage from green to oven-dry condition (percent of dimensions when green)[2] Radial $100(P_R)$	Tangential $100(P_T)$	Volumetric	Species	Average shrinkage from green to oven-dry condition (percent of dimensions when green)[2] Radial $100(P_R)$	Tangential $100(P_T)$	Volumetric
	Percent	Percent	Percent		Percent	Percent	Percent
Ash, white	4.9	7.9	13.3	Oak, red[5]	4.3	9.0	14.8
Basswood, American	6.6	9.3	15.8	Oak, white[6]	5.4	9.3	16.0
Baldcypress	3.8	6.2	10.5	Pecan[7]	4.9	8.9	13.6
Birch, yellow	7.2	9.2	16.7	Pine, Eastern white	2.3	6.0	8.2
Cherry, black	3.7	7.1	11.5	Pine, loblolly	4.8	7.4	12.3
Chestnut, American	3.4	6.7	11.6	Pine, lodgepole	4.5	6.7	11.5
Cottonwood, Eastern	3.9	9.2	14.1	Pine, longleaf	5.1	7.5	12.2
Cottonwood, black	3.6	8.6	12.4	Pine, ponderosa	3.9	6.3	9.6
Douglas-fir, Coast Region	5.0	7.8	11.8	Pine, red	4.6	7.2	11.5
Douglas-fir, Inland Empire Region	4.1	7.6	10.9	Pine, shortleaf	4.4	7.7	12.3
Douglas-fir, Rocky Mountain Region	3.6	6.2	10.6	Pine, slash	5.8	8.2	12.7
Elm, American	4.2	9.5	14.6	Pine, sugar	2.9	5.6	7.9
Elm, rock	4.8	8.1	14.1	Pine, Western white	2.6	5.3	11.8
Elm, slippery	4.9	8.9	13.8	Redcedar, Eastern	3.1	4.7	7.8
Fir, balsam	2.8	6.6	10.8	Redcedar, Western	2.4	5.0	7.7
Fir, commercial white[3]	3.2	7.1	9.8	Redwood	2.6	4.4	~~8.6~~ 6.8
Hackberry	4.8	8.9	13.8	Spruce, Eastern[8]	4.3	7.7	12.6
Hemlock, Eastern	3.0	6.8	9.7	Spruce, Engelmann	3.4	6.6	10.4
Hemlock, Western	4.3	7.9	11.9	Spruce, Sitka	4.3	7.5	11.5
Hickory[4]	7.3	11.4	17.9	Sugarberry	5.0	7.3	12.7
Honeylocust	4.2	6.6	10.8	Sweetgum	5.2	9.9	15.0
Larch, Western	4.2	8.1	13.2	Sycamore, American	5.1	7.6	14.2
Locust, black	4.4	6.9	9.8	Tamarack	3.7	7.4	13.6
Maple, bigleaf	3.7	7.1	11.6	Tupelo, black	4.4	7.7	13.9
Maple, black	4.8	9.3	14.0	Tupelo, water	4.2	7.6	12.5
Maple, red	4.0	8.2	13.1	Walnut, black	5.2	7.1	11.3
Maple, silver	3.0	7.2	12.0	White-cedar, Northern	2.1	4.7	7.0
Maple, sugar	4.9	9.5	14.9	Yellow-poplar	4.0	7.1	12.3

[1] Data from U. S. Dept. Agr. Tech. Bul. 479, "Strength and Related Properties of Woods Grown in the United States."

[2] To compute the shrinkage in seasoning to any moisture content multiply figure from the table by $(1 - \frac{M}{30})$, when M is the moisture content (percent) to which wood is seasoned.

[3] Average of lowland white fir and white fir.

[4] Average of shellbark hickory, mockernut hickory, pignut hickory, and shagbark hickory.

[5] Average of black oak, laurel oak, pin oak, Northern red oak, scarlet oak, Southern red oak, swamp red oak, water oak, and willow oak.

[6] Average of bur oak, chestnut oak, post oak, swamp chestnut oak, swamp white oak, and white oak.

[7] Average of bitternut hickory, nutmeg hickory, water hickory, and pecan.

[8] Average of black spruce, red spruce, and white spruce.

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Table 2.--Values of $\sin^2\theta$ and $\cos^2\theta$ between 0° and 90°

Angle θ	$\sin^2\theta$	$\cos^2\theta$	Angle θ	$\sin^2\theta$	$\cos^2\theta$
Degrees			Degrees		
0	0.0000	1.0000	50	0.5868	0.4132
5	.0076	.9924	55	.6710	.3290
10	.0302	.9698	60	.7500	.2500
15	.0670	.9330	65	.8214	.1786
20	.1170	.8830	70	.8830	.1170
25	.1786	.8214	75	.9330	.0670
30	.2500	.7500	80	.9698	.0302
35	.3290	.6710	85	.9924	.0076
40	.4132	.5868	90	1.0000	.0000
45	.5000	.5000			

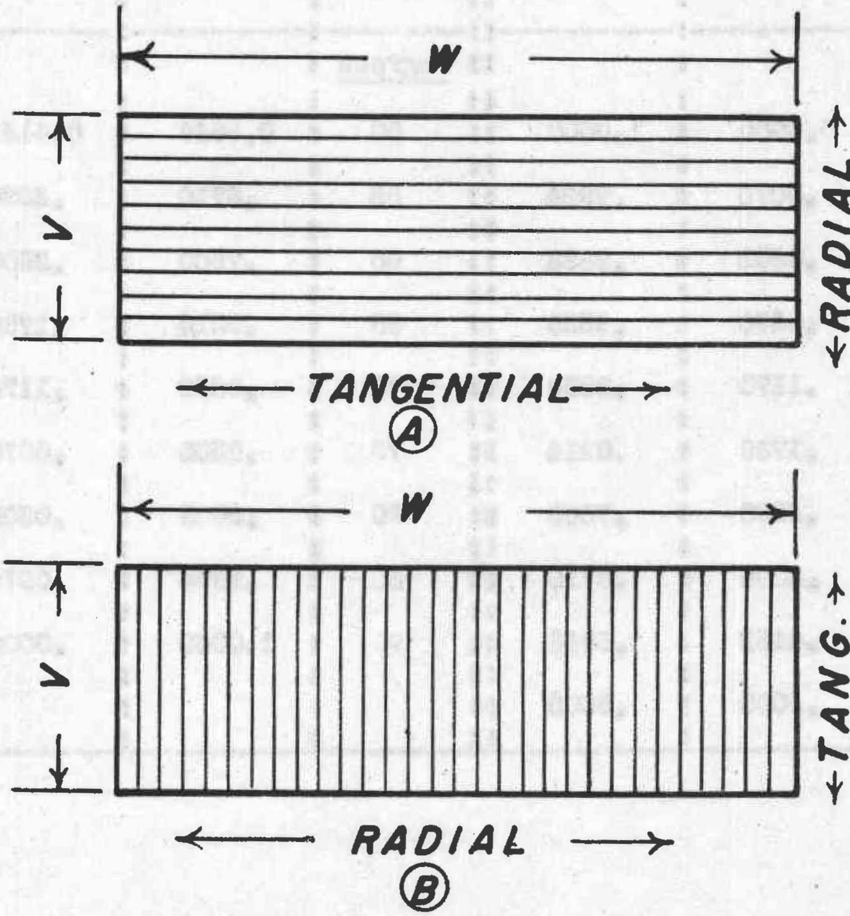

Figure 1.--Cross sections of a timber with (A) rings approximately parallel with the horizontal faces, and (B) rings at right angles to the horizontal faces.

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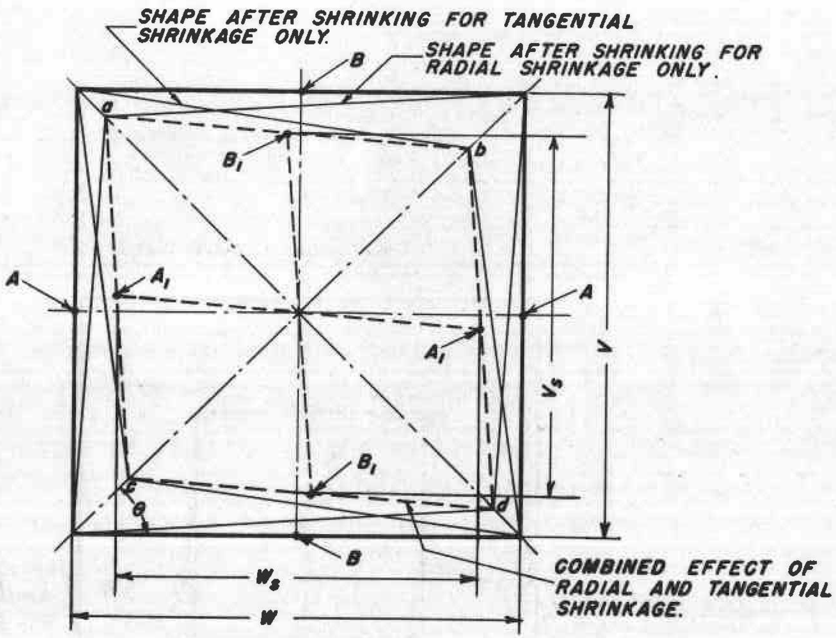

Figure 2.--Radial and tangential shrinkage in a timber and the combined effect of shrinkage in these two directions when the annual rings make an angle of $\theta = 45°$ to the horizontal faces. Radial shrinkage P_R was taken as 0.125, and the tangential shrinkage P_T as 0.250.

W = width before shrinkage
W_s = width after shrinkage measured on axis AA
A_1A_1 = position of axis AA after shrinking
B_1B_1 = position of axis BB after shrinking
V = thickness before shrinkage
V_s = thickness after shrinkage measured on axis BB
abcd = shape after shrinking
$ab = A_1A_1 = cd$
$ac = B_1B_1 = bd$

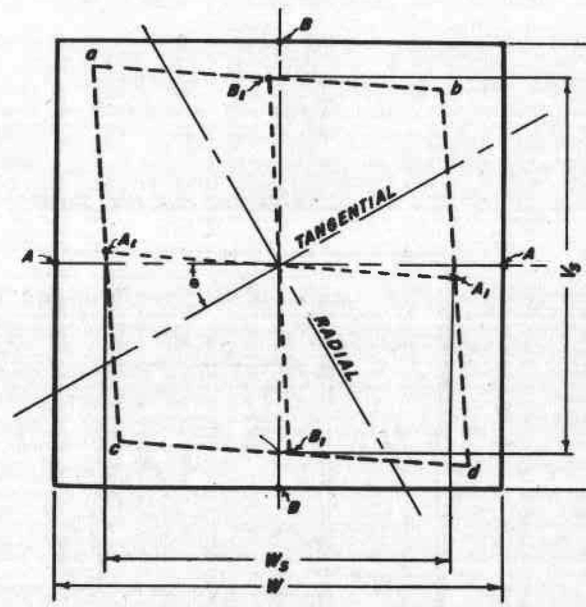

Figure 3.--Combined effect of radial and tangential shrinkage when annual rings make an angle of $\theta = 30°$ to the horizontal faces. Radial shrinkage P_R was taken as 0.125 and tangential shrinkage P_T as 0.250.

W = width before shrinkage
W_s = width after shrinkage measured on axis AA
A_1A_1 = position of axis AA after shrinking
B_1B_1 = position of axis BB after shrinking
V = thickness before shrinkage
V_s = thickness after shrinkage measured on axis BB
abcd = shape after shrinking
$ab = A_1A_1 = cd$
$ac = B_1B_1 = bd$

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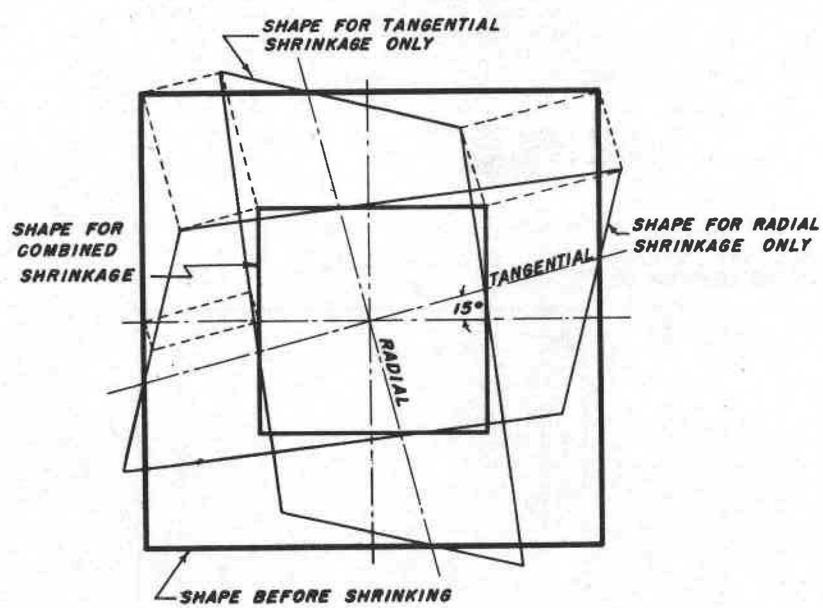

Figure 4.--Effect of shrinkage when radial and tangential shrinkage are assumed equal (0.5) and the slope of the rings θ = 15°.

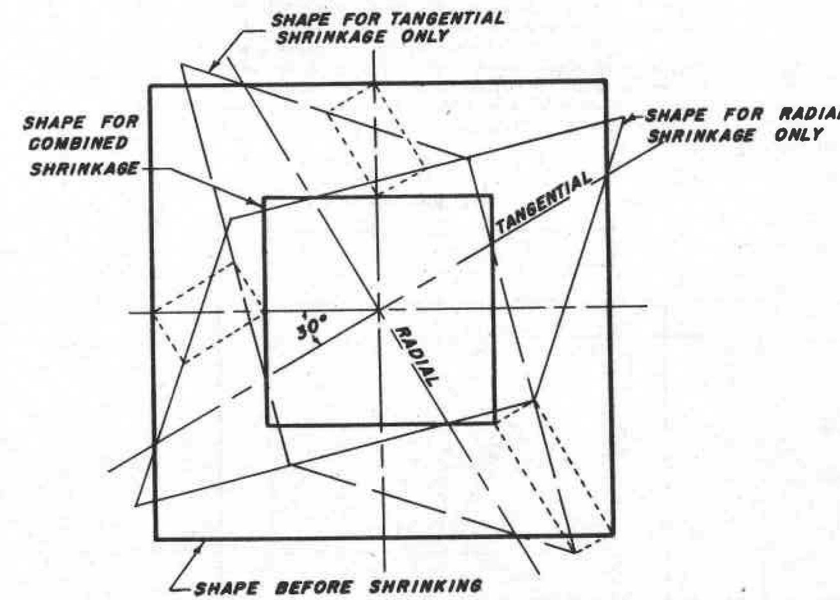

Figure 5.--Effect of shrinkage when radial and tangential shrinkage are equal (0.5) and the slope of the rings θ = 30°.

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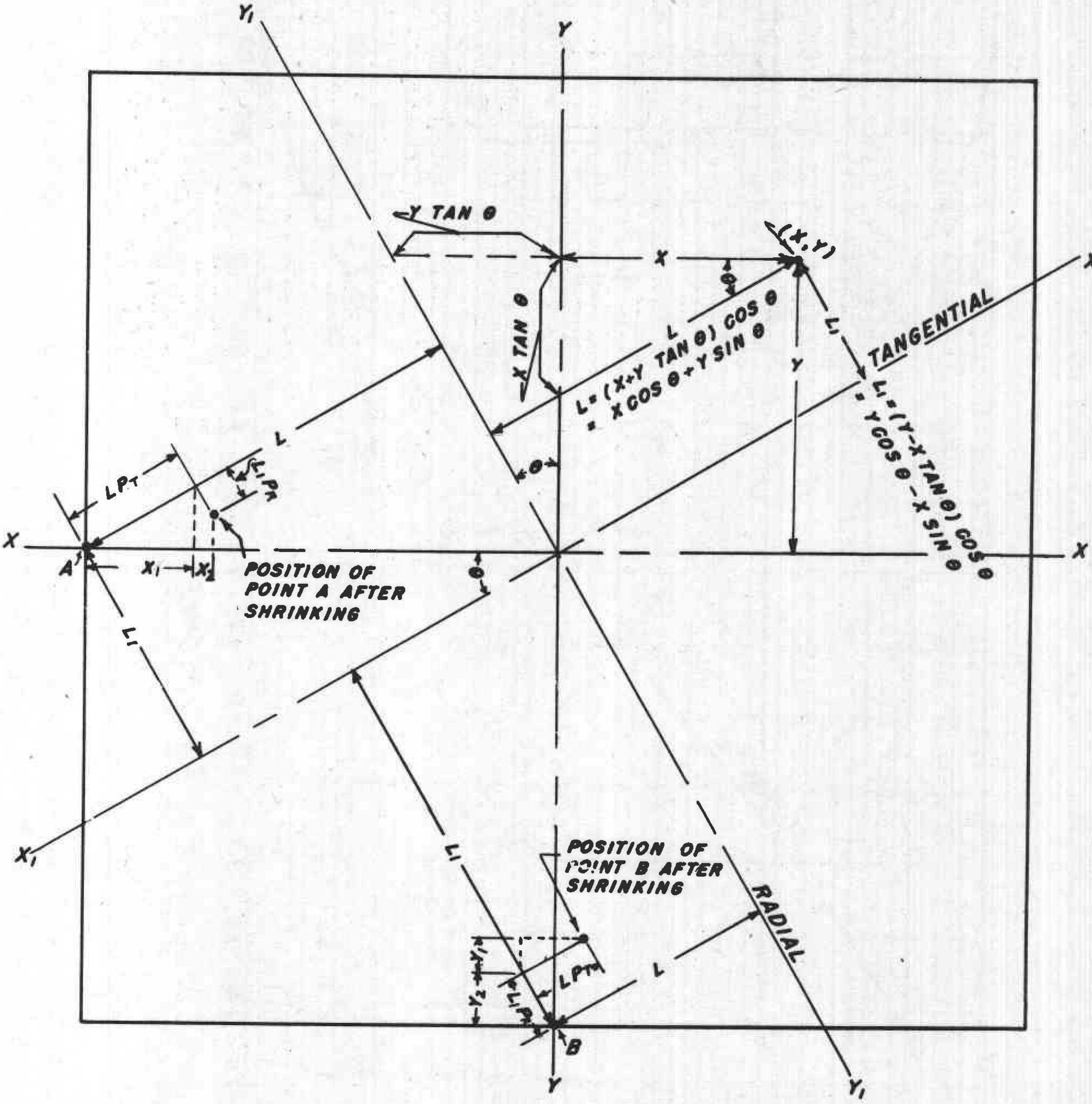

Figure 6.--Shrinkage components that affect the final width and thickness of a timber with the direction of the rings at any angle θ to the horizontal faces.

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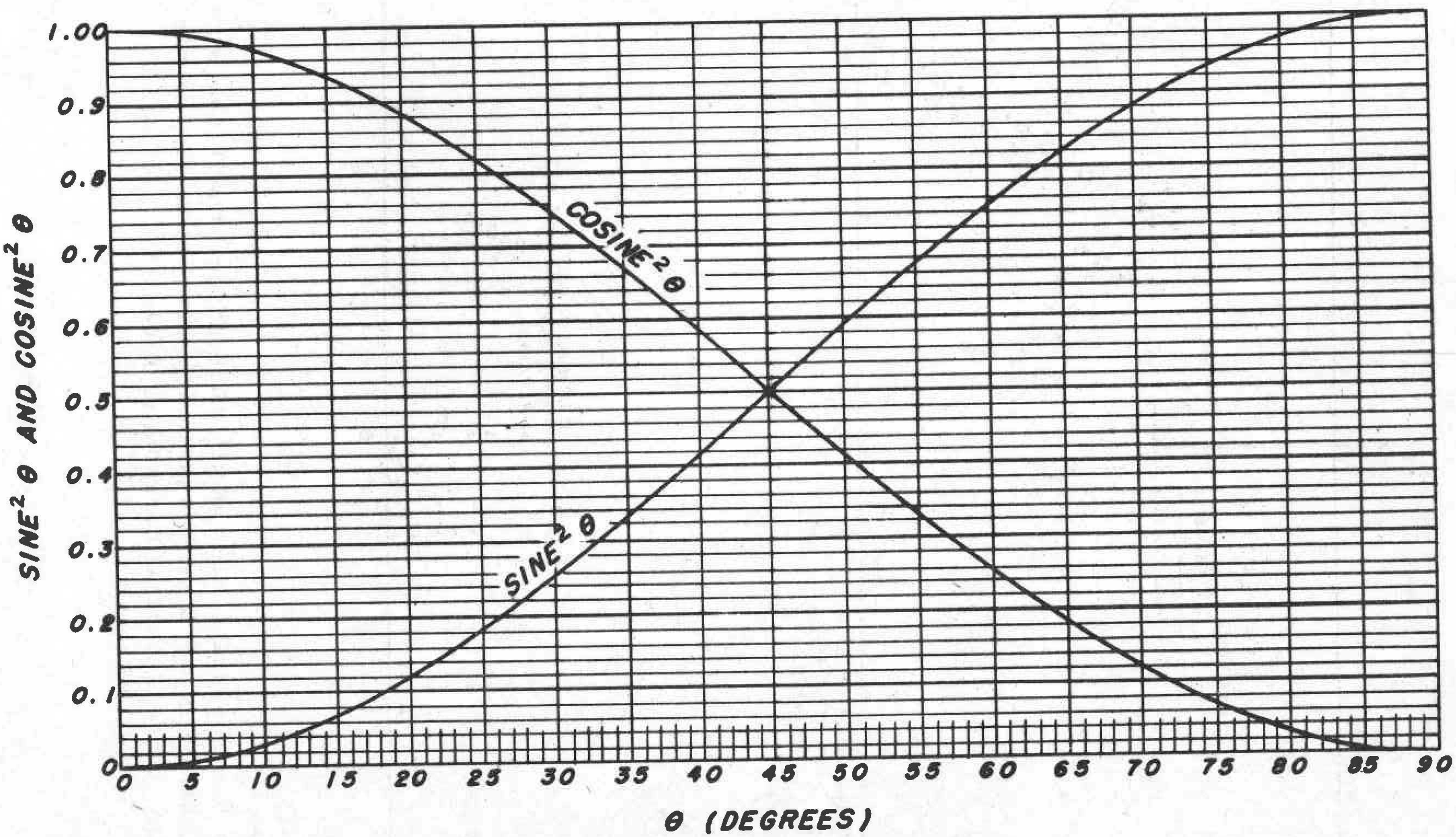

Figure 7.--Curves of $\sin^2 \theta$ and $\cos^2 \theta$.

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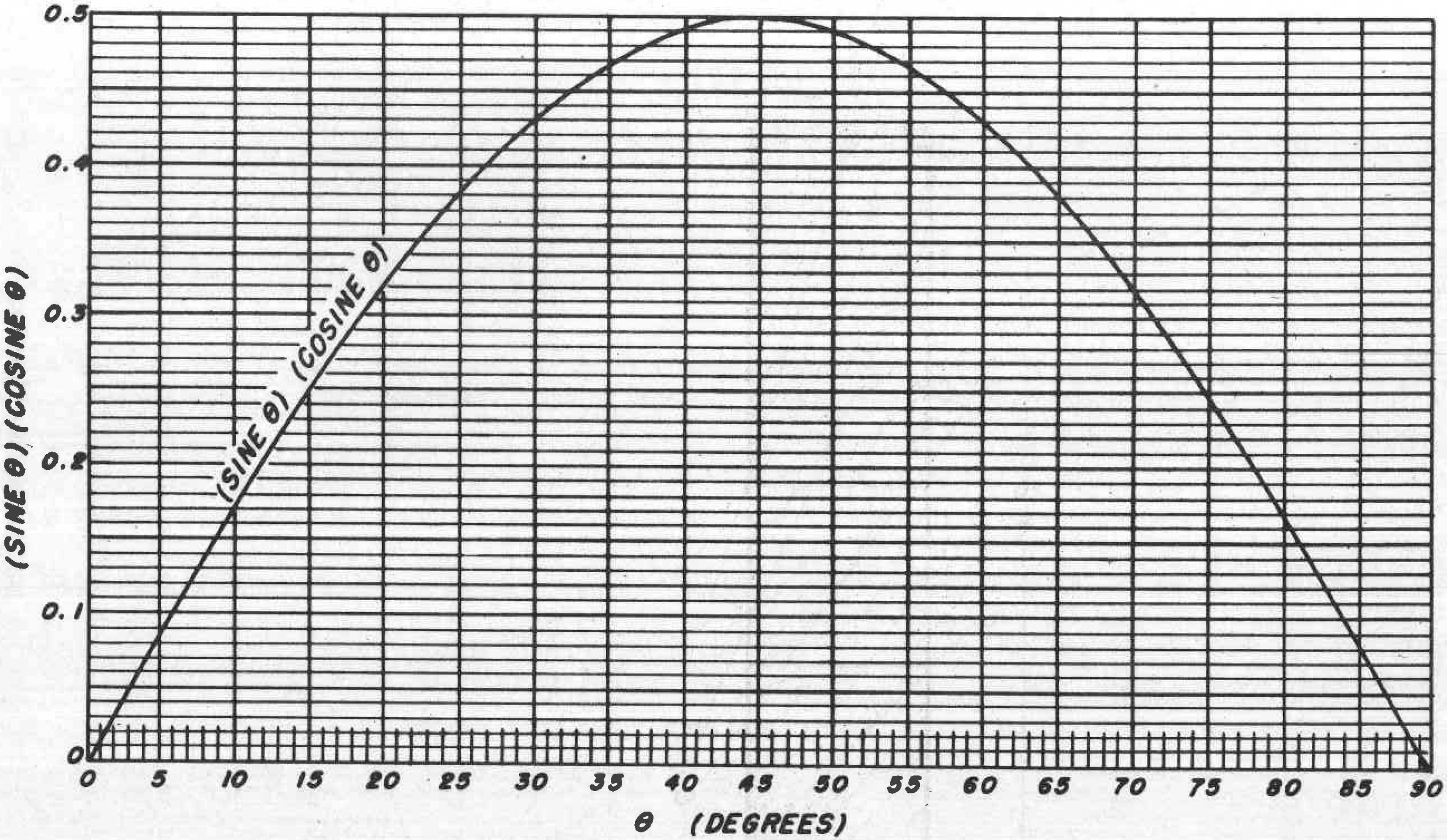

Figure 8.--Curve of sin θ cos θ.

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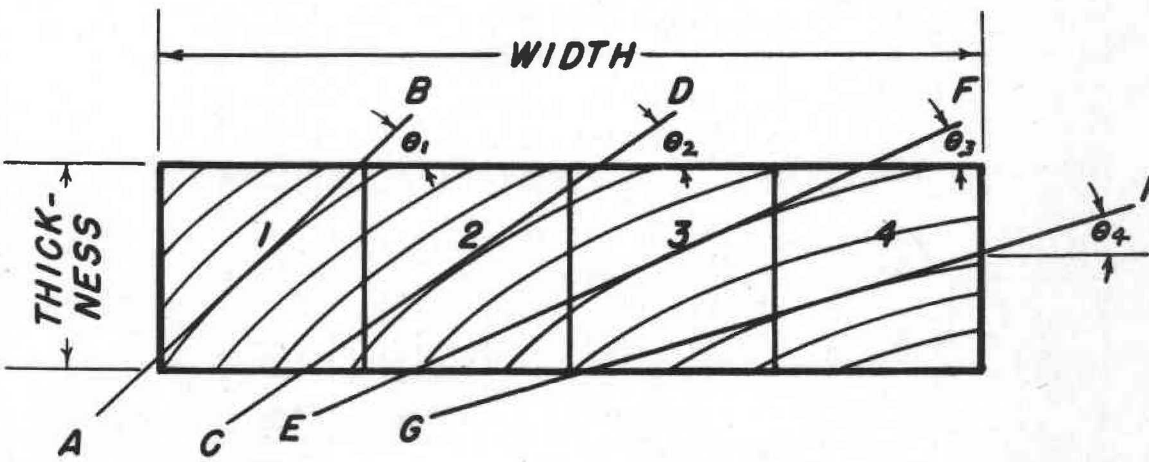

Figure 9.--Method of finding approximate slope of rings for different portions of timber.

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